Computer Algebra
Independent Integration Tests

Summer 2023 edition

7-Inverse-hyperbolic-functions/7.1-Inverse-hyperbolic-sine/187-7.1.4-f-x-^m-d+e-x^2-^p-a+b-arcsinh-c-x-^n

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [663]. This is test number [187].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (663)	0.00 (0)
Mathematica	98.94 (656)	1.06 (7)
Maple	76.32 (506)	23.68 (157)
Maxima	38.46 (255)	61.54 (408)
Fricas	36.35 (241)	63.65 (422)
Sympy	30.02 (199)	69.98 (464)
Mupad	20.06 (133)	79.94 (530)
Giac	14.33 (95)	85.67 (568)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

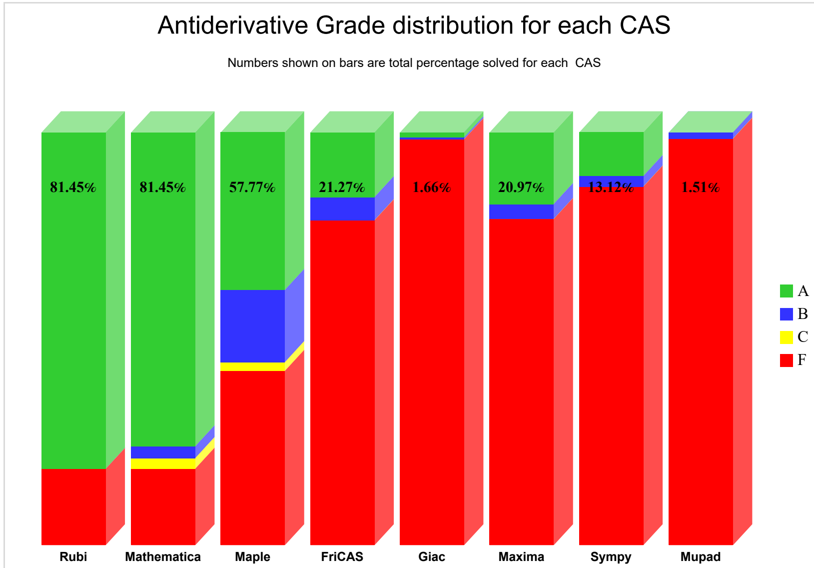
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

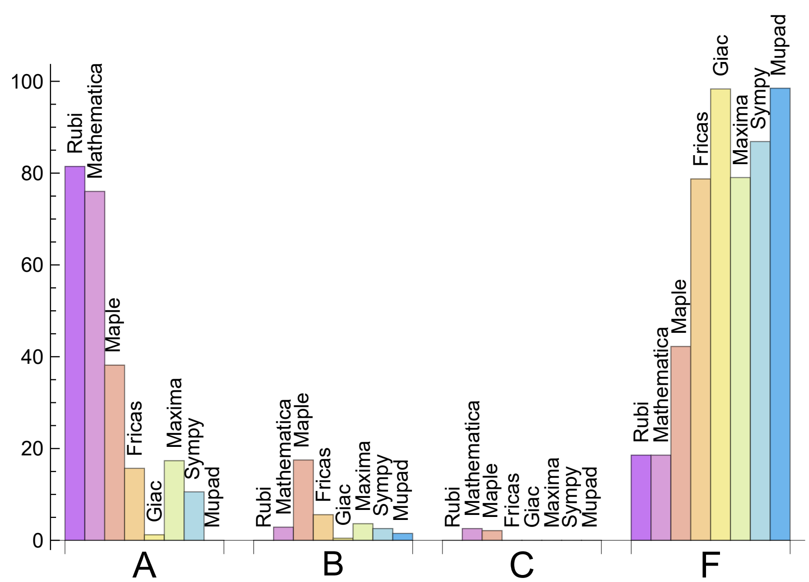
System	% A grade	% B grade	% C grade	% F grade
Rubi	81.448	0.000	0.000	18.552
Mathematica	76.018	2.866	2.564	18.552
Maple	38.160	17.496	2.112	42.232
Maxima	17.345	3.620	0.000	79.035
Fricas	15.686	5.581	0.000	78.733
Sympy	10.558	2.564	0.000	86.878
Giac	1.207	0.452	0.000	98.341
Mupad	0.000	1.508	0.000	98.492

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	7	0.00	100.00	0.00
Maple	157	100.00	0.00	0.00
Fricas	422	84.83	0.00	15.17
Maxima	408	78.19	1.96	19.85
Sympy	464	86.85	12.93	0.22
Giac	568	48.06	0.35	51.58
Mupad	530	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.24
Fricas	0.27
Maple	0.30
Maxima	0.32
Giac	0.40
Mathematica	2.15
Mupad	2.69
Sympy	9.80

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	24.11	1.00	26.00	1.00
Giac	29.62	1.07	23.00	1.00
Sympy	115.20	1.24	26.00	0.96
Fricas	144.33	1.86	99.00	1.46
Maxima	199.68	4.27	101.00	1.00
Rubi	201.28	1.00	156.00	1.00
Mathematica	231.62	1.11	145.50	1.07
Maple	301.52	1.67	163.00	1.22

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

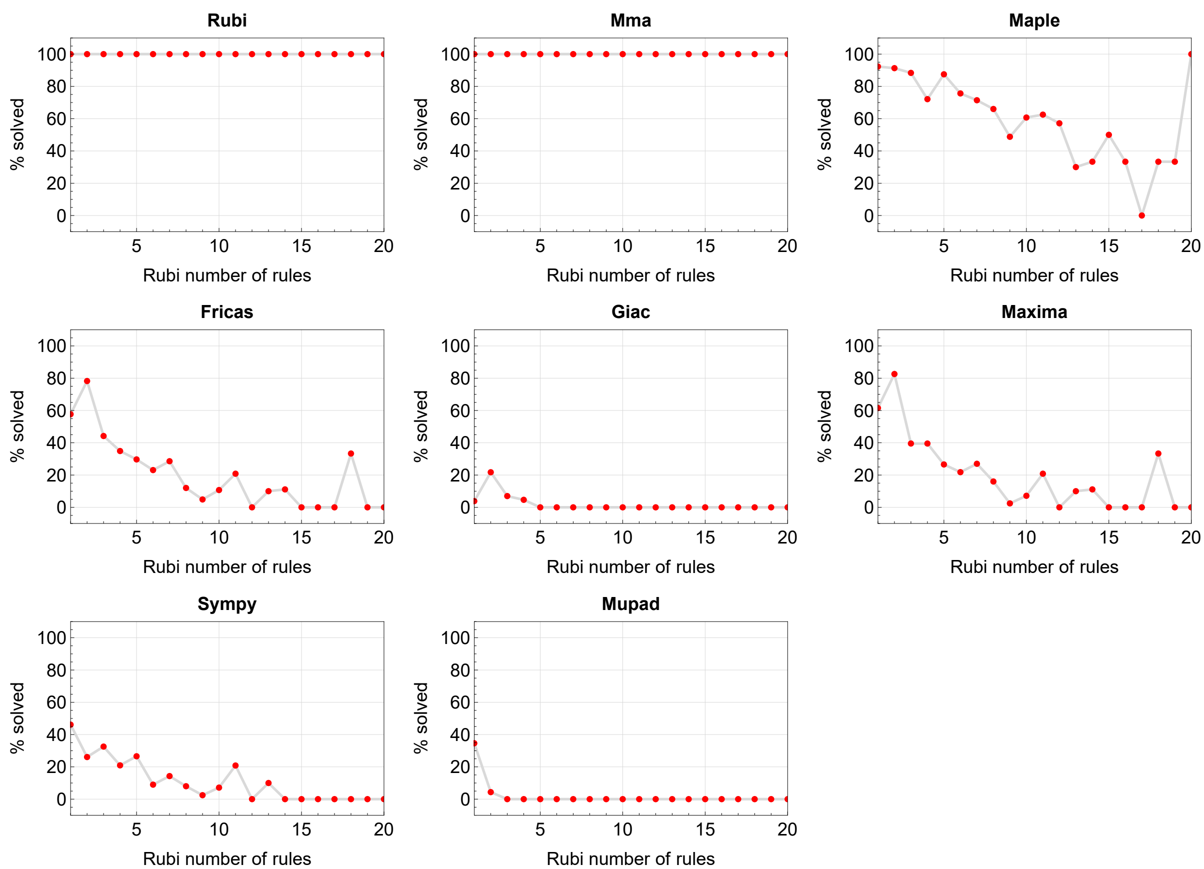


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

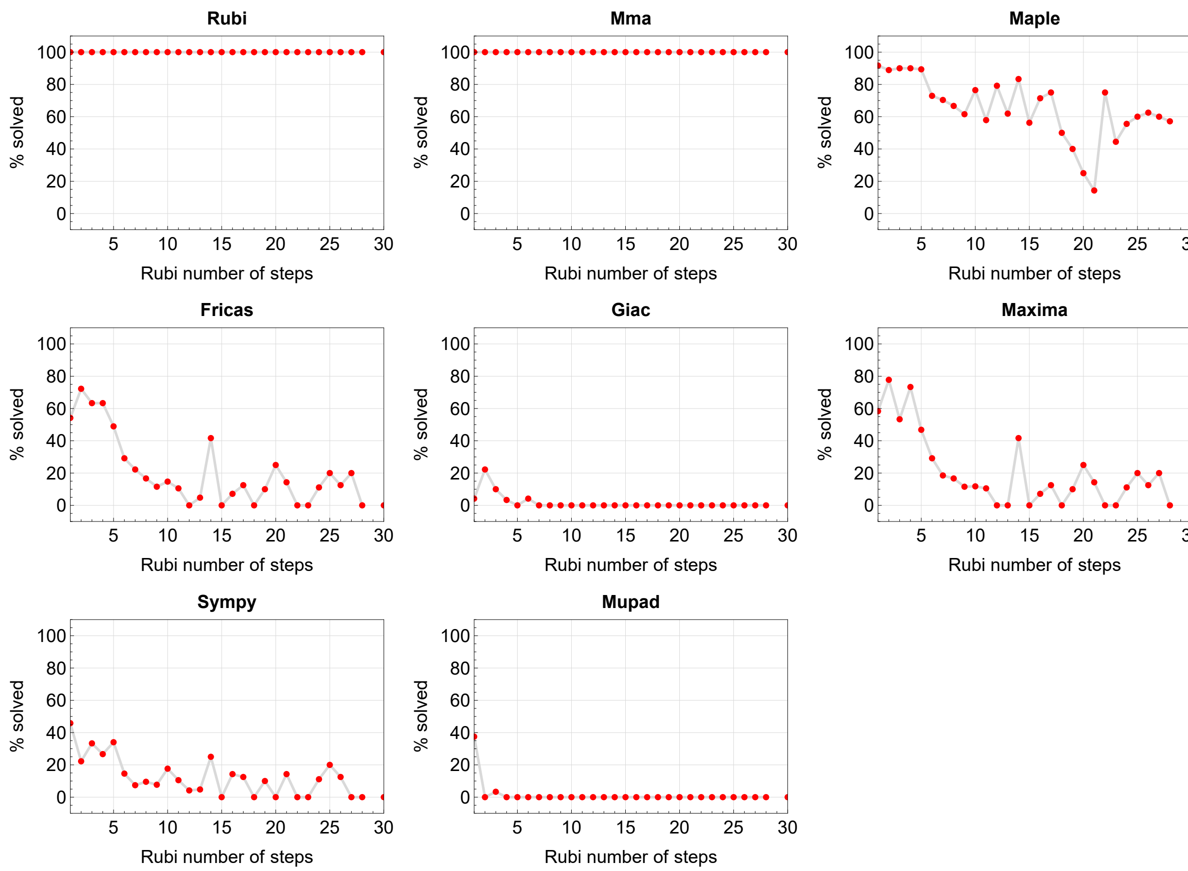


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

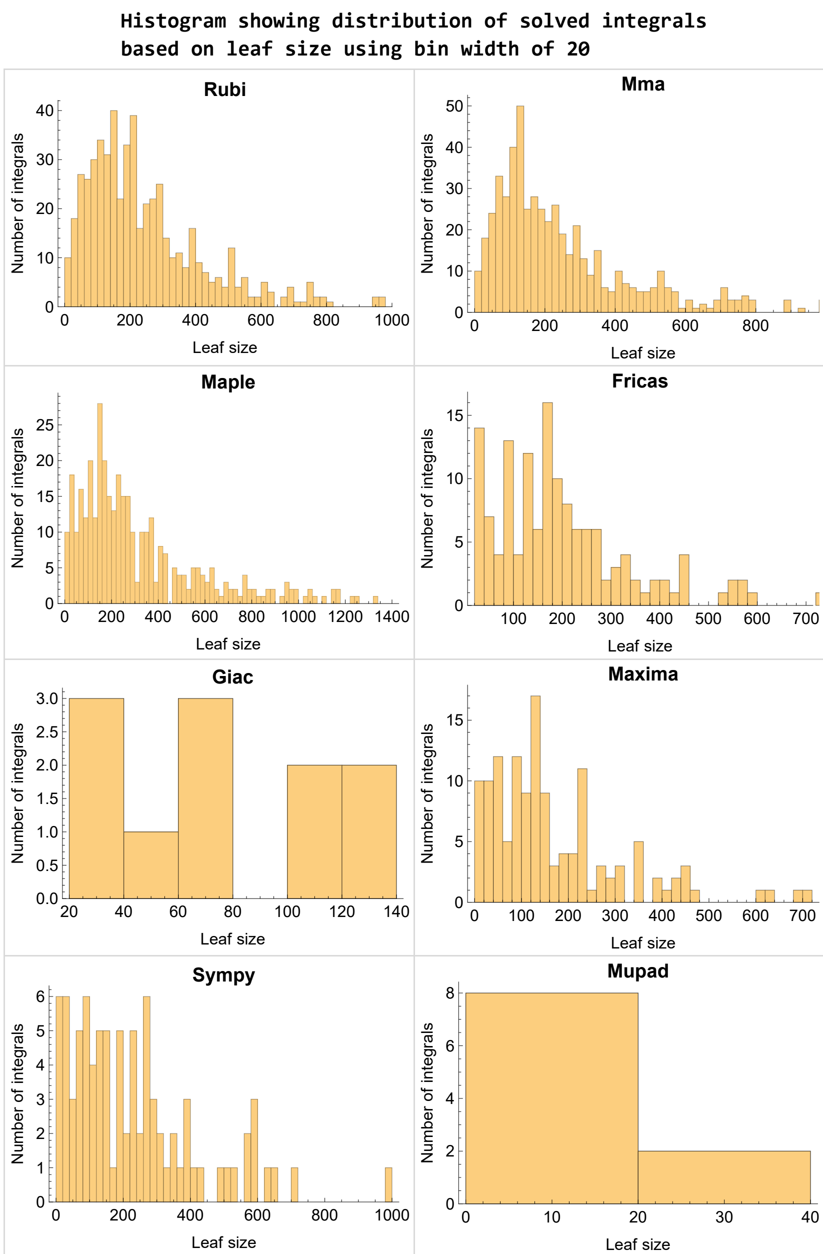


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

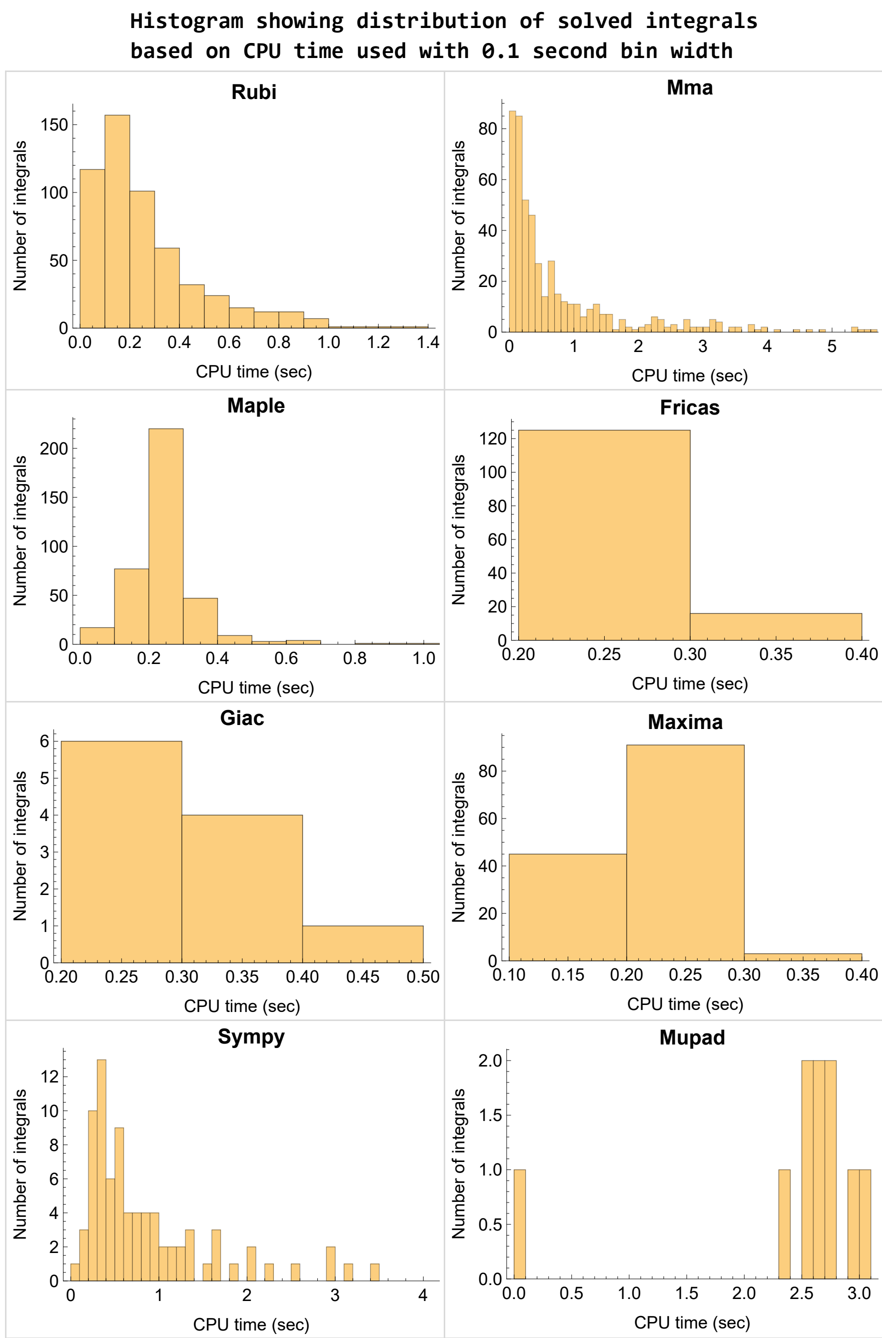


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

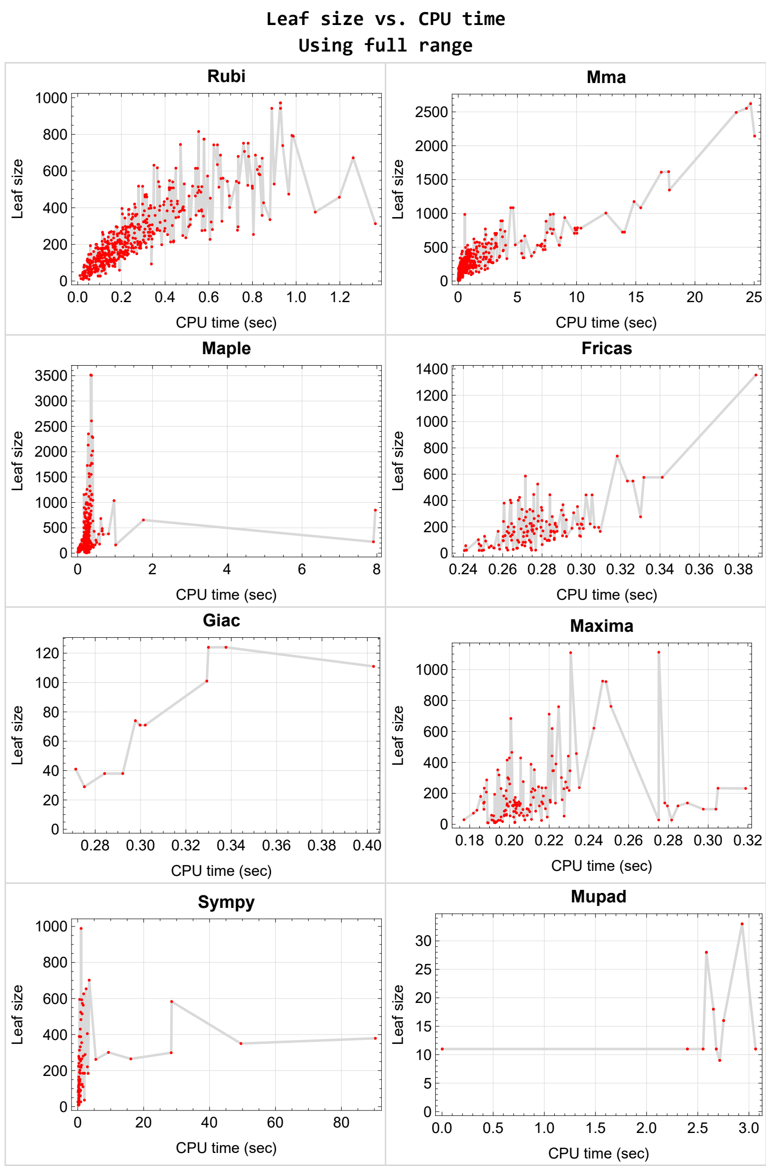


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{188, 189, 190, 321, 322, 323, 324, 325, 326, 327, 341, 353, 354, 360, 361, 362, 363, 368, 369, 370, 371, 376, 377, 378, 379, 386, 387, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 409, 410, 415, 416, 417, 418, 423, 424, 425, 426, 431, 432, 433, 434, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 466, 471, 475, 476, 480, 484, 488, 489, 493, 499, 500, 505, 506, 510, 511, 515, 516, 520, 521, 525, 526, 527, 532, 533, 622, 623, 627, 628, 632, 633, 636, 637, 641, 642, 645, 646, 647, 648, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {564, 575, 581, 587, 593, 594, 595, 596, 599, 600, 601, 602, 603, 604}

Maple {612}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
```

```
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

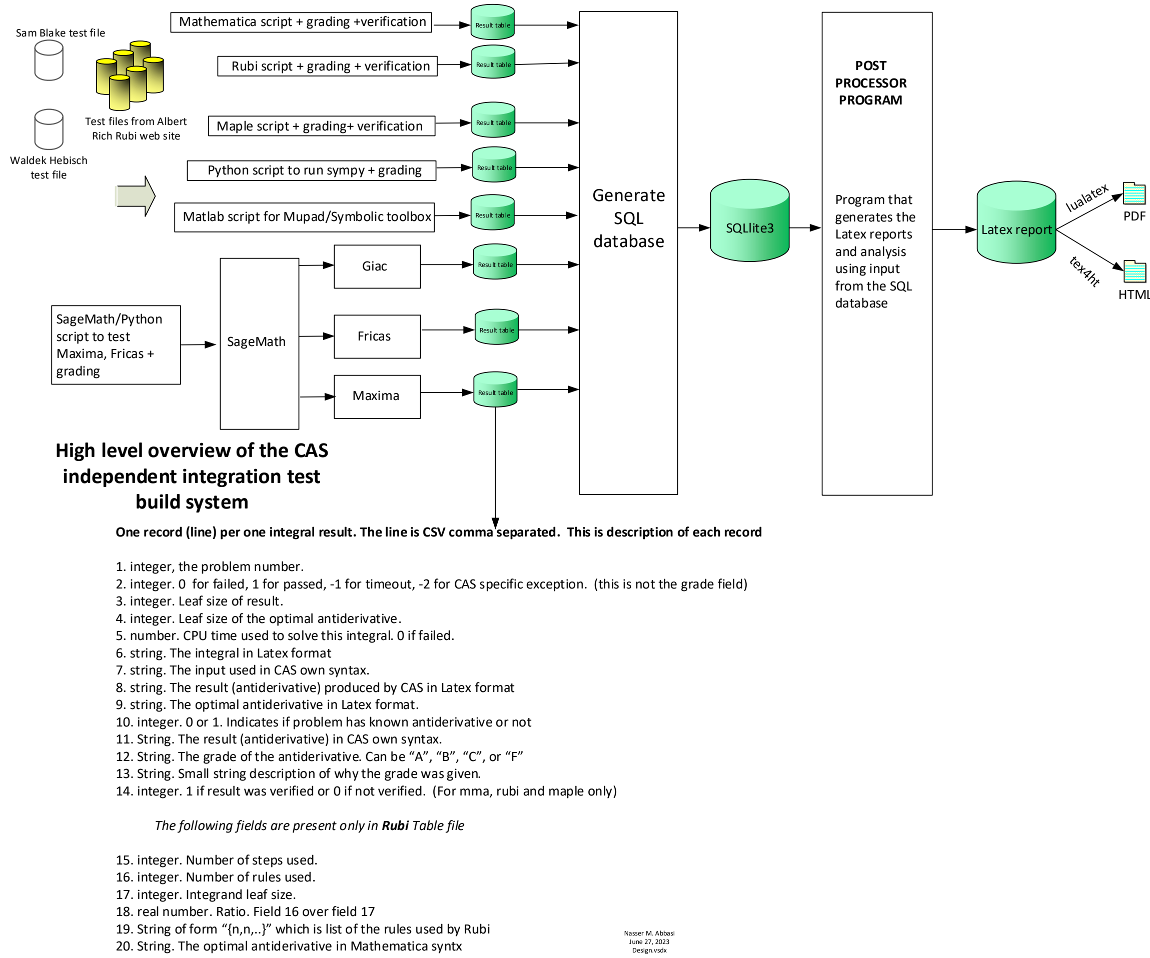
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 364, 365, 366, 367, 372, 373, 374, 375, 380, 381, 382, 383, 384, 385, 388, 389, 390, 391, 392, 393, 406, 407, 408, 411, 412, 413, 414, 419, 420, 421, 422, 427, 428, 429, 430, 435, 436, 437, 438, 439, 440, 461, 462, 463, 464, 465, 467, 468, 469, 470, 472, 473, 474, 477, 478, 479, 481, 482, 483, 485, 486, 487, 490, 491, 492, 494, 495, 496, 497, 498, 501, 502, 503, 504, 507, 508, 509, 512, 513, 514, 517, 518, 519, 522, 523, 524, 528, 529, 530, 531, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 624, 625, 626, 629, 630, 631, 634, 635, 638, 639, 640, 643, 644, 649, 650, 651 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 36, 37, 39, 40, 41, 46, 47, 48, 49, 50, 51, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 229, 231, 234, 236, 237, 238, 240, 242, 243, 244, 245, 246, 247, 249, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 349, 350, 351, 352, 355, 356, 357, 358, 359, 364, 365, 366, 367, 372, 373, 374, 375, 380, 381, 382, 383, 384, 385, 388, 389, 390, 391, 392, 393, 406, 407, 408, 411, 412, 413, 414, 419, 420, 421, 422, 427, 428, 429, 430, 435, 436, 437, 438, 439, 440, 461, 462, 463, 464, 465, 467, 468, 469, 470, 472, 473, 474, 477, 478, 479, 481, 482, 483, 485, 486, 487, 490, 491, 492, 494, 495, 496, 497, 498, 501, 502, 503, 504, 507, 508, 509, 512, 513, 514, 517, 518, 519, 522, 523, 524, 528, 529, 530, 531, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 582, 583, 584, 585, 588, 589, 590, 592, 593, 595, 596, 597, 599, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 613, 614, 615, 616, 617, 618, 619, 620, 621, 624, 625, 626, 629, 630, 631, 634, 635, 638, 639, 640, 643, 644 }

B grade { 31, 33, 35, 42, 44, 228, 230, 233, 295, 551, 564, 581, 586, 587, 591, 594, 598, 600, 601 }

C grade { 38, 43, 45, 52, 54, 226, 232, 235, 239, 241, 248, 250, 348, 612, 649, 650, 651 }

F normal fail { }

F(-1) timedout fail { 449, 451, 453, 632, 633, 636, 637 }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 61, 63, 64, 66, 67, 69, 71, 72, 74, 75, 76, 77, 79, 80, 81, 82, 83, 84, 86, 88, 96, 98, 107, 109, 112, 113, 114, 115, 116, 117, 119, 124, 126, 127, 132, 133, 134, 135, 140, 141, 142, 143, 144, 150, 151, 153, 154, 156, 158, 161, 162, 163, 165, 167, 172, 174, 175, 177, 178, 179, 180, 181, 182, 184, 198, 199, 200, 201, 202, 204, 206, 207, 208, 209, 210, 211, 213, 215, 216, 217, 218, 219, 220, 222, 224, 226, 228, 235, 237, 244, 246, 252, 253, 254, 271, 272, 279, 280, 282, 283, 284, 285, 286, 287, 288, 289, 300, 301, 302, 304, 305, 320, 328, 329, 330, 334, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 364, 365, 366, 367, 372, 373, 374, 375, 380, 381, 382, 383, 384, 385, 388, 389, 390, 391, 392, 393, 406, 407, 408, 412, 414, 420, 422, 428, 430, 436, 438, 440, 461, 474, 479, 483, 487, 492, 498, 504, 509, 531, 606, 607, 608, 609, 610, 613, 614, 615, 616, 618, 619, 620, 621, 625, 626 }

B grade { 57, 60, 62, 65, 68, 70, 73, 78, 85, 87, 89, 91, 93, 95, 97, 99, 100, 102, 104, 106, 108, 110, 111, 118, 120, 121, 122, 123, 125, 128, 129, 130, 131, 136, 137, 138, 139, 145, 146, 147, 148, 149, 152, 160, 164, 169, 171, 173, 176, 183, 203, 205, 212, 214, 221, 223, 230, 232, 239, 241, 248, 250, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 273, 274, 275, 276, 277, 278, 281, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 303, 307, 309, 310, 311, 312, 313, 314, 315, 317, 319, 411, 413, 419, 421, 427, 429, 435, 437, 439, 624 }

C grade { 90, 92, 94, 101, 103, 105, 155, 157, 159, 166, 168, 170, 611, 612 }

F normal fail { 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 225, 227, 229, 231, 233, 234, 236, 238, 240, 242, 243, 245, 247, 249, 251, 306, 308, 316, 318, 331, 332, 333, 462, 463, 464, 465, 467, 468, 469, 470, 472, 473, 477, 478, 481, 482, 485, 486, 490, 491, 494, 495, 496, 497, 501, 502, 503, 507, 508, 512, 513, 514, 517, 518, 519, 522, 523, 524, 528, 529, 530, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 617, 629, 630, 631, 634, 635, 638, 639, 640, 643, 644, 649, 650, 651 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 19, 20, 21, 22, 23, 25, 27, 40, 47, 49, 55, 63, 79, 80, 82, 90, 101, 112, 113, 114, 115, 118, 120, 122, 128, 130, 136, 138, 144, 145, 147, 149, 154, 155, 157, 159, 166, 168, 170, 177, 178, 179, 180, 183, 198, 199, 200, 201, 202, 207, 208, 209, 210, 211, 216, 217, 218, 219, 220, 244, 258, 260, 266, 268, 274, 276, 282, 283, 284, 285, 290, 292, 294, 328, 329, 330, 342, 343, 344, 345, 393, 440, 487, 492, 504, 509, 606, 607, 608, 609, 610, 613, 614, 615 }

B grade { 7, 9, 16, 18, 57, 62, 65, 71, 73, 84, 87, 89, 92, 94, 103, 105, 116, 127, 152, 181, 237, 246, 286, 346, 385, 461, 531, 539, 556, 557, 561, 566, 567, 616, 649, 650, 651 }

C grade { }

F normal fail { 6, 8, 15, 17, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 48, 50, 51, 52, 53, 54, 56, 58, 59, 60, 61, 64, 66, 67, 68, 69, 70, 72, 74, 75, 76, 77, 78, 81, 83, 85, 86, 88, 91, 93, 95, 96, 97, 98, 99, 100, 102, 104, 106, 107, 108, 109, 110, 111, 117, 119, 121, 123, 124, 125, 126, 129, 131, 132, 133, 134, 135, 137, 139, 140, 141, 142, 143, 146, 148, 150, 151, 153, 156, 158, 160, 161, 162, 163, 164, 165, 167, 169, 171, 172, 173, 174, 175, 176, 182, 184, 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 203, 204, 205, 206, 212, 213, 214, 215, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 245, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 259, 261, 262, 263, 264, 265, 267, 269, 270, 271, 272, 273, 275, 277, 278, 279, 280, 281, 287, 288, 289, 291, 293, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 347, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 364, 365, 366, 367, 372, 373, 374, 375, 380, 381, 382, 383, 384, 388, 389, 390, 391, 392, 406, 407, 408, 411, 412, 413, 414, 419, 420, 421, 422, 427, 428, 429, 430, 435, 436, 437, 438, 439, 512, 513, 514, 517, 518, 519, 522, 523, 524, 528, 529, 530, 534, 535, 536, 537, 538, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 558, 559, 560, 562, 563, 564, 565, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 611, 612, 617, 618, 619, 620, 621, 624, 625, 626 }

F(-1) timedout fail { }

F(-2) exception fail { 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 488, 489, 490, 491, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 505, 506, 507, 508, 510, 511, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646 }

Maxima

A grade { 1, 2, 3, 4, 5, 7, 9, 11, 12, 14, 16, 18, 25, 27, 55, 57, 63, 65, 71, 73, 79, 80, 82, 84, 85, 89, 92, 95, 97, 103, 106, 110, 111, 112, 113, 114, 115, 116, 118, 120, 122, 127, 128, 130, 136, 138, 144, 145, 147, 149, 150, 152, 154, 157, 160, 162, 168, 169, 171, 175, 176, 177, 178, 179, 180, 181, 183, 198, 200, 258, 260, 266, 268, 274, 276, 283, 285, 286, 290, 292, 294, 295, 328, 329, 330, 337, 343, 345, 346, 385, 393, 440, 461, 531, 539, 555, 556, 557, 561, 562, 563, 566, 567, 568, 569, 591, 606, 607, 608, 609, 610, 613, 614, 615, 616 }

B grade { 10, 13, 19, 20, 21, 22, 23, 62, 87, 104, 199, 201, 202, 207, 208, 209, 210, 211, 216, 217, 218, 219, 220, 255 }

C grade { }

F normal fail { 6, 8, 15, 17, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 59, 60, 61, 67, 69, 75, 77, 86, 88, 90, 91, 93, 94, 96, 98, 99, 100, 101, 102, 105, 107, 108, 109, 117, 119, 124, 125, 126, 132, 134, 140, 142, 151, 153, 155, 156, 158, 159, 161, 163, 164, 165, 166, 167, 170, 172, 173, 174, 182, 184, 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 203, 204, 205, 206, 212, 213, 214, 215, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 256, 257, 262, 263, 264, 265, 270, 272, 278, 280, 282, 284, 287, 288, 289, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 331, 332, 333, 338, 339, 340, 342, 344, 347, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 364, 365, 366, 367, 372, 373, 374, 375, 380, 381, 382, 383, 384, 388, 389, 390, 391, 392, 406, 407, 408, 411, 412, 413, 414, 419, 420, 421, 422, 427, 428, 429, 430, 435, 436, 437, 438, 439, 462, 463, 464, 465, 467, 468, 469, 470, 472, 473, 474, 477, 478, 479, 481, 482, 483, 485, 486, 487, 490, 491, 492, 494, 495, 496, 497, 498, 501, 502, 503, 504, 507, 508, 509, 512, 513, 514, 517, 518, 519, 522, 523, 524, 528, 529, 530, 537, 538, 544, 545, 550, 551, 554, 558, 559, 560, 564, 565, 573, 574, 580, 586, 590, 592, 594, 595, 596, 597, 598, 605, 618, 619, 620, 621, 624, 625, 626, 629, 630, 631, 634, 635, 638, 639, 640, 643, 644, 650, 651 }

F(-1) timedout fail { 575, 581, 587, 593, 600, 601, 602, 603 }

F(-2) exception fail { 56, 58, 64, 66, 68, 70, 72, 74, 76, 78, 81, 83, 121, 123, 129, 131, 133, 135, 137, 139, 141, 143, 146, 148, 252, 253, 254, 259, 261, 267, 269, 271, 273, 275, 277, 279, 281, 291, 293, 334, 335, 336, 534, 535, 536, 540, 541, 542, 543, 546, 547, 548, 549, 552, 553, 570, 571, 572, 576, 577, 578, 579, 582, 583, 584, 585, 588, 589, 599, 604, 611, 612, 617, 632, 636, 647, 648, 649, 652, 653, 654 }

Giac

A grade { 111, 115, 176, 180, 285, 345, 531, 610 }

B grade { 118, 183, 616 }

C grade { }

F normal fail { 30, 31, 32, 33, 34, 35, 36, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 56, 64, 72, 79, 81, 83, 84, 85, 86, 87, 88, 89, 93, 94, 95, 96, 97, 98, 99, 102, 104, 105, 106, 107, 108, 109, 110, 112, 114, 116, 117, 119, 121, 129, 137, 144, 146, 148, 149, 150, 151, 152, 153, 154, 158, 159, 160, 161, 162, 163, 164, 167, 169, 170, 171, 172, 173, 174, 175, 177, 179, 181, 182, 184, 194, 195, 196, 197, 227, 228, 229, 230, 231, 232, 233, 236, 237, 238, 239, 240, 241, 242, 243, 245, 246, 247, 248, 249, 250, 251, 255, 256, 257, 259, 267, 275, 282, 284, 286, 287, 289, 291, 293, 294, 295, 296, 297, 298, 299, 303, 304, 305, 306, 307, 308, 309, 311, 313, 314, 315, 316, 317, 318, 319, 331, 332, 333, 337, 338, 342, 344, 346, 347, 349, 350, 351, 352, 355, 357, 358, 359, 365, 367, 373, 375, 380, 382, 383, 384, 385, 389, 391, 392, 393, 406, 407, 408, 412, 413, 414, 420, 422, 428, 430, 436, 438, 439, 440, 461, 463, 465, 468, 470, 474, 479, 483, 485, 486, 487, 490, 491, 492, 494, 495, 496, 497, 498, 501, 502, 503, 504, 507, 508, 509, 512, 517, 522, 529, 530, 537, 538, 539, 543, 545, 553, 554, 555, 556, 557, 560, 561, 562, 565, 566, 567, 573, 574, 575, 579, 589, 590, 591, 592, 593, 596, 597, 598, 602, 603, 611, 612, 617, 618, 619, 620, 621, 624, 625, 626, 629, 630, 631, 635, 638, 639, 640, 643, 644, 649, 650, 651 }

F(-1) timedout fail { 544, 580 }

F(-2) exception fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 37, 38, 47, 55, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 80, 82, 90, 91, 92, 100, 101, 103, 113, 120, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 145, 147, 155, 156, 157, 165, 166, 168, 178, 185, 186, 187, 191, 192, 193, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 234, 235, 244, 252, 253, 254, 258, 260, 261, 262, 263, 264, 265, 266, 268, 269, 270, 271, 272, 273, 274, 276, 277, 278, 279, 280, 281, 283, 288, 290, 292, 300, 301, 302, 310, 312, 320, 321, 322, 323, 328, 329, 330, 334, 335, 336, 339, 340, 343, 348, 356, 360, 362, 364, 366, 368, 370, 372, 374, 376, 378, 381, 388, 390, 394, 397, 399, 401, 402, 403, 411, 415, 417, 419, 421, 423, 425, 427, 429, 431, 433, 435, 437, 441, 443, 445, 447, 449, 451, 453, 455, 456, 457, 462, 464, 466, 467, 469, 471, 472, 473, 477, 478, 481, 482, 513, 514, 515, 516, 518, 519, 520, 521, 523, 524, 525, 526, 528, 534, 535, 536, 540, 541, 542, 546, 547, 548, 549, 550, 551, 552, 558, 559, 563, 564, 568, 569, 570, 571, 572, 576, 577, 578, 581, 582, 583, 584, 585, 586, 587, 588, 594, 595, 599, 600, 601, 604, 605, 606, 607, 608, 609, 613, 614, 615, 634 }

Mupad

A grade { }

B grade { 116, 181, 286, 346, 385, 393, 440, 461, 531, 610 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 347, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 364, 365, 366, 367, 372, 373, 374, 375, 380, 381, 382, 383, 384, 388, 389, 390, 391, 392, 406, 407, 408, 411, 412, 413, 414, 419, 420, 421, 422, 427, 428, 429, 430, 435, 436, 437, 438, 439, 462, 463, 464, 465, 467, 468, 469, 470, 472, 473, 474, 477, 478, 479, 481, 482, 483, 485, 486, 487, 490, 491, 492, 494, 495, 496, 497, 498, 501, 502, 503, 504, 507, 508, 509, 512, 513, 514, 517, 518, 519, 522, 523, 524, 528, 529, 530, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 624, 625, 626, 629, 630, 631, 634, 635, 638, 639, 640, 643, 644, 649, 650, 651 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 19, 20, 21, 22, 23, 64, 66, 72, 74, 80, 81, 82, 83, 84, 112, 113, 114, 115, 116, 177, 178, 179, 180, 181, 198, 199, 200, 202, 207, 208, 209, 211, 216, 217, 218, 220, 282, 283, 284, 285, 286, 328, 329, 330, 342, 343, 344, 345, 346, 385, 461, 606, 607, 608, 609, 610, 613, 614, 615, 616 }

B grade { 55, 57, 60, 63, 65, 71, 73, 85, 201, 210, 219, 252, 253, 255, 393, 440, 531 }

C grade { }

F normal fail { 6, 7, 8, 9, 15, 16, 17, 18, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 58, 59, 61, 62, 67, 68, 69, 70, 75, 76, 77, 78, 79, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 182, 183, 184, 185, 186, 187, 193, 194, 195, 196, 197, 203, 204, 205, 206, 212, 213, 214, 215, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 254, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 276, 277, 278, 279, 280, 281, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 331, 332, 333, 335, 336, 337, 338, 339, 340, 347, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 364, 365, 366, 367, 372, 373, 374, 375, 380, 381, 382, 383, 384, 388, 389, 390, 391, 392, 406, 407, 408, 411, 412, 413, 414, 419, 420, 421, 422, 427, 428, 429, 430, 435, 436, 437, 438, 439, 462, 463, 464, 465, 467, 468, 469, 470, 472, 473, 474, 478, 479, 485, 486, 487, 491, 492, 494, 496, 497, 498, 502, 503, 504, 507, 508, 509, 512, 513, 514, 528, 529, 530, 535, 536, 537, 538, 539, 541, 542, 543, 544, 545, 553, 554, 555, 556, 557, 559, 560, 561, 562, 565, 566, 567, 571, 572, 573, 574, 575, 578, 579, 580, 581, 589, 590, 591, 592, 593, 595, 596, 597, 598, 601, 602, 603, 611, 612, 617, 618, 619, 620, 621, 624, 625, 626, 629, 630, 631, 634, 635, 638, 639, 640, 643, 644, 649, 650 }

F(-1) timedout fail { 136, 137, 191, 192, 274, 275, 321, 322, 334, 455, 477, 481, 482, 483, 484, 490, 495, 501, 506, 510, 511, 518, 519, 521, 522, 523, 524, 525, 526, 534, 540, 546, 547, 548, 549, 550, 551, 552, 558, 563, 564, 568, 569, 570, 576, 577, 582, 583, 584, 585, 586, 587, 588, 594, 599, 600, 604, 605, 628, 651 }

F(-2) exception fail { 517 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	87	120	184	113	151	0	0
N.S.	1	1.00	0.70	0.97	1.48	0.91	1.22	0.00	0.00
time (sec)	N/A	0.076	0.073	0.064	0.197	0.286	0.691	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	88	109	166	109	138	0	0
N.S.	1	1.00	0.73	0.91	1.38	0.91	1.15	0.00	0.00
time (sec)	N/A	0.074	0.041	0.033	0.198	0.281	0.599	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	78	101	145	103	126	0	0
N.S.	1	1.00	0.76	0.99	1.42	1.01	1.24	0.00	0.00
time (sec)	N/A	0.069	0.063	0.045	0.200	0.286	0.369	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	77	85	127	98	117	0	0
N.S.	1	1.00	0.89	0.98	1.46	1.13	1.34	0.00	0.00
time (sec)	N/A	0.030	0.040	0.077	0.205	0.268	0.297	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	86	73	97	83	90	0	0
N.S.	1	1.00	1.15	0.97	1.29	1.11	1.20	0.00	0.00
time (sec)	N/A	0.045	0.032	0.015	0.187	0.274	0.166	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	113	159	0	0	0	0	0
N.S.	1	1.00	1.02	1.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.130	0.041	0.154	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	74	67	64	156	0	0	0
N.S.	1	1.00	1.12	1.02	0.97	2.36	0.00	0.00	0.00
time (sec)	N/A	0.061	0.023	0.018	0.199	0.292	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	111	151	0	0	0	0	0
N.S.	1	1.00	0.87	1.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.123	0.038	0.119	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	93	83	91	169	0	0	0
N.S.	1	1.00	1.16	1.04	1.14	2.11	0.00	0.00	0.00
time (sec)	N/A	0.059	0.026	0.023	0.198	0.285	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	119	163	319	165	230	0	0
N.S.	1	1.00	0.66	0.90	1.76	0.91	1.27	0.00	0.00
time (sec)	N/A	0.139	0.074	0.234	0.195	0.262	1.151	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	115	152	292	161	218	0	0
N.S.	1	1.00	0.64	0.84	1.62	0.89	1.21	0.00	0.00
time (sec)	N/A	0.113	0.063	0.236	0.200	0.266	0.996	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	111	144	261	153	202	0	0
N.S.	1	1.00	0.71	0.92	1.66	0.97	1.29	0.00	0.00
time (sec)	N/A	0.115	0.062	0.184	0.200	0.263	0.649	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	104	116	234	149	190	0	0
N.S.	1	1.00	0.87	0.97	1.95	1.24	1.58	0.00	0.00
time (sec)	N/A	0.045	0.094	0.217	0.187	0.276	0.530	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	95	116	194	133	165	0	0
N.S.	1	1.00	0.74	0.91	1.52	1.04	1.29	0.00	0.00
time (sec)	N/A	0.075	0.069	0.214	0.193	0.261	0.321	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	173	222	0	0	0	0	0
N.S.	1	1.00	1.01	1.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.170	0.132	0.216	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	124	112	143	228	0	0	0
N.S.	1	1.00	1.03	0.93	1.19	1.90	0.00	0.00	0.00
time (sec)	N/A	0.100	0.087	0.197	0.187	0.287	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	151	240	0	0	0	0	0
N.S.	1	1.00	0.81	1.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.173	0.164	0.242	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	133	112	137	243	0	0	0
N.S.	1	1.00	1.06	0.89	1.09	1.93	0.00	0.00	0.00
time (sec)	N/A	0.103	0.086	0.228	0.187	0.278	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	143	202	465	201	289	0	0
N.S.	1	1.00	0.63	0.89	2.06	0.89	1.28	0.00	0.00
time (sec)	N/A	0.192	0.099	0.207	0.201	0.287	2.235	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	139	191	429	197	280	0	0
N.S.	1	1.00	0.70	0.96	2.16	0.99	1.41	0.00	0.00
time (sec)	N/A	0.114	0.086	0.219	0.200	0.279	1.657	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	135	183	388	189	265	0	0
N.S.	1	1.00	0.67	0.91	1.92	0.94	1.31	0.00	0.00
time (sec)	N/A	0.173	0.098	0.228	0.211	0.295	1.234	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	128	143	352	185	253	0	0
N.S.	1	1.00	0.88	0.99	2.43	1.28	1.74	0.00	0.00
time (sec)	N/A	0.050	0.118	0.168	0.194	0.277	0.877	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	119	153	301	169	221	0	0
N.S.	1	1.00	0.70	0.90	1.77	0.99	1.30	0.00	0.00
time (sec)	N/A	0.111	0.082	0.209	0.199	0.280	0.580	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	216	271	0	0	0	0	0
N.S.	1	1.00	0.98	1.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.232	0.166	0.283	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	163	149	231	276	0	0	0
N.S.	1	1.00	1.02	0.93	1.44	1.72	0.00	0.00	0.00
time (sec)	N/A	0.144	0.112	0.162	0.196	0.330	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	244	289	0	0	0	0	0
N.S.	1	1.00	0.98	1.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.235	0.154	0.293	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	171	153	208	289	0	0	0
N.S.	1	1.00	0.98	0.88	1.20	1.66	0.00	0.00	0.00
time (sec)	N/A	0.170	0.106	0.159	0.188	0.291	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	170	208	0	0	0	0	0
N.S.	1	1.00	1.09	1.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.167	0.175	0.351	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	181	146	0	0	0	0	0
N.S.	1	1.00	1.34	1.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.145	0.155	0.280	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	121	170	0	0	0	0	0
N.S.	1	1.00	1.12	1.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.102	0.168	0.168	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	167	84	0	0	0	0	0
N.S.	1	1.00	2.29	1.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.083	0.051	0.230	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	135	143	0	0	0	0	0
N.S.	1	1.00	1.93	2.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.048	0.073	0.219	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	207	161	0	0	0	0	0
N.S.	1	1.00	3.39	2.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.087	0.068	0.212	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	182	176	0	0	0	0	0
N.S.	1	1.00	1.80	1.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.108	0.113	0.195	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	240	217	0	0	0	0	0
N.S.	1	1.00	2.12	1.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.142	0.182	0.241	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	247	215	0	0	0	0	0
N.S.	1	1.00	1.58	1.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.184	0.143	0.171	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	268	224	0	0	0	0	0
N.S.	1	1.00	1.57	1.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.177	0.220	0.205	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	241	140	0	0	0	0	0
N.S.	1	1.00	1.66	0.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.140	0.158	0.261	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	221	195	0	0	0	0	0
N.S.	1	1.00	1.74	1.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.098	0.170	0.161	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	74	61	0	65	0	0	0
N.S.	1	1.00	1.35	1.11	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.035	0.099	0.213	0.000	0.280	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	216	195	0	0	0	0	0
N.S.	1	1.00	1.74	1.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.075	0.114	0.219	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	234	217	0	0	0	0	0
N.S.	1	1.00	2.13	1.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.127	0.309	0.210	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	253	227	0	0	0	0	0
N.S.	1	1.00	1.51	1.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.142	0.383	0.230	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	326	242	0	0	0	0	0
N.S.	1	1.00	2.23	1.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.185	0.315	0.229	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	311	268	0	0	0	0	0
N.S.	1	1.00	1.30	1.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.219	0.425	0.234	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	341	247	0	0	0	0	0
N.S.	1	1.00	1.83	1.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.170	0.489	0.168	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	79	108	0	99	0	0	0
N.S.	1	1.00	0.81	1.11	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.060	0.139	0.229	0.000	0.283	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	340	247	0	0	0	0	0
N.S.	1	1.00	1.85	1.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.130	0.193	0.176	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	56	76	0	98	0	0	0
N.S.	1	1.00	0.70	0.95	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	0.040	0.116	0.237	0.000	0.274	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	341	248	0	0	0	0	0
N.S.	1	1.00	1.92	1.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.105	0.149	0.168	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	289	280	0	0	0	0	0
N.S.	1	1.00	1.82	1.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.203	0.448	0.257	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	298	291	0	0	0	0	0
N.S.	1	1.00	1.34	1.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.184	0.775	0.168	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	353	344	0	0	0	0	0
N.S.	1	1.00	1.52	1.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.264	0.605	0.312	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	374	333	0	0	0	0	0
N.S.	1	1.00	1.27	1.13	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.263	1.056	0.237	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	106	164	134	158	221	0	0
N.S.	1	1.00	0.97	1.50	1.23	1.45	2.03	0.00	0.00
time (sec)	N/A	0.094	0.189	0.224	0.204	0.286	0.900	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	79	156	0	0	0	0	0
N.S.	1	1.00	0.66	1.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.132	0.189	0.211	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	63	108	73	127	141	0	0
N.S.	1	1.00	1.03	1.77	1.20	2.08	2.31	0.00	0.00
time (sec)	N/A	0.046	0.125	0.214	0.206	0.260	0.344	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	69	100	0	0	0	0	0
N.S.	1	1.00	1.03	1.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.048	0.157	0.158	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	131	171	0	0	0	0	0
N.S.	1	1.00	1.47	1.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.144	0.192	0.227	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	75	155	0	0	110	0	0
N.S.	1	1.00	1.23	2.54	0.00	0.00	1.80	0.00	0.00
time (sec)	N/A	0.082	0.217	0.169	0.000	0.000	1.610	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	185	231	0	0	0	0	0
N.S.	1	1.00	1.64	2.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.144	2.484	0.184	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	96	501	133	217	0	0	0
N.S.	1	1.00	1.55	8.08	2.15	3.50	0.00	0.00	0.00
time (sec)	N/A	0.063	0.168	0.164	0.217	0.300	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	100	195	145	199	301	0	0
N.S.	1	1.00	0.80	1.56	1.16	1.59	2.41	0.00	0.00
time (sec)	N/A	0.112	0.198	0.277	0.216	0.263	9.329	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	154	214	0	0	262	0	0
N.S.	1	1.00	0.93	1.30	0.00	0.00	1.59	0.00	0.00
time (sec)	N/A	0.221	0.378	0.211	0.000	0.000	5.488	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	72	139	85	167	221	0	0
N.S.	1	1.00	0.94	1.81	1.10	2.17	2.87	0.00	0.00
time (sec)	N/A	0.053	0.146	0.171	0.213	0.273	2.929	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	111	152	0	0	185	0	0
N.S.	1	1.00	1.00	1.37	0.00	0.00	1.67	0.00	0.00
time (sec)	N/A	0.084	0.246	0.194	0.000	0.000	1.578	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	180	228	0	0	0	0	0
N.S.	1	1.00	1.34	1.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.203	0.366	0.222	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	122	208	0	0	0	0	0
N.S.	1	1.00	1.13	1.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.117	0.370	0.168	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	292	291	0	0	0	0	0
N.S.	1	1.00	1.88	1.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.215	1.275	0.221	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	125	622	0	0	0	0	0
N.S.	1	1.00	1.09	5.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.162	0.300	0.164	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	108	226	156	263	379	0	0
N.S.	1	1.00	0.77	1.60	1.11	1.87	2.69	0.00	0.00
time (sec)	N/A	0.121	0.260	0.249	0.202	0.301	90.306	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	196	267	0	0	350	0	0
N.S.	1	1.00	0.92	1.25	0.00	0.00	1.64	0.00	0.00
time (sec)	N/A	0.303	0.648	0.234	0.000	0.000	49.520	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	80	170	96	225	299	0	0
N.S.	1	1.00	0.86	1.83	1.03	2.42	3.22	0.00	0.00
time (sec)	N/A	0.057	0.154	0.195	0.210	0.265	28.365	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	153	202	0	0	265	0	0
N.S.	1	1.00	0.93	1.22	0.00	0.00	1.61	0.00	0.00
time (sec)	N/A	0.112	0.408	0.216	0.000	0.000	16.114	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	257	284	0	0	0	0	0
N.S.	1	1.00	1.44	1.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.289	0.447	0.154	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	168	261	0	0	0	0	0
N.S.	1	1.00	1.07	1.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.158	0.515	0.201	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	349	348	0	0	0	0	0
N.S.	1	1.00	1.70	1.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.290	1.489	0.167	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	179	692	0	0	0	0	0
N.S.	1	1.00	1.08	4.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.197	0.480	0.218	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	28	26	28	40	0	0	0
N.S.	1	1.00	0.88	0.81	0.88	1.25	0.00	0.00	0.00
time (sec)	N/A	0.024	0.011	0.181	0.275	0.262	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	108	193	174	161	184	0	0
N.S.	1	1.00	0.72	1.30	1.17	1.08	1.23	0.00	0.00
time (sec)	N/A	0.190	0.241	0.204	0.203	0.285	3.148	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	111	165	0	0	185	0	0
N.S.	1	1.00	0.88	1.31	0.00	0.00	1.47	0.00	0.00
time (sec)	N/A	0.166	0.337	0.161	0.000	0.000	2.078	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	82	133	117	132	124	0	0
N.S.	1	1.00	0.84	1.36	1.19	1.35	1.27	0.00	0.00
time (sec)	N/A	0.111	0.196	0.205	0.199	0.299	1.323	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	69	107	0	0	122	0	0
N.S.	1	1.00	0.92	1.43	0.00	0.00	1.63	0.00	0.00
time (sec)	N/A	0.088	0.244	0.165	0.000	0.000	1.027	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	49	72	55	96	60	0	0
N.S.	1	1.00	1.17	1.71	1.31	2.29	1.43	0.00	0.00
time (sec)	N/A	0.049	0.149	0.201	0.197	0.284	0.797	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	53	28	0	87	0	0
N.S.	1	1.00	1.00	2.12	1.12	0.00	3.48	0.00	0.00
time (sec)	N/A	0.030	0.014	0.209	0.191	0.000	0.617	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	96	118	0	0	0	0	0
N.S.	1	1.00	1.71	2.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.088	0.201	0.214	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	52	84	101	132	0	0	0
N.S.	1	1.00	1.27	2.05	2.46	3.22	0.00	0.00	0.00
time (sec)	N/A	0.064	0.139	0.158	0.203	0.290	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	185	216	0	0	0	0	0
N.S.	1	1.00	1.61	1.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.154	1.949	0.196	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	106	373	121	222	0	0	0
N.S.	1	1.00	1.09	3.85	1.25	2.29	0.00	0.00	0.00
time (sec)	N/A	0.133	0.175	0.194	0.202	0.304	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	131	279	0	196	0	0	0
N.S.	1	1.00	0.96	2.04	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.143	0.287	0.196	0.000	0.309	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	147	261	0	0	0	0	0
N.S.	1	1.00	1.12	1.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.190	0.453	0.293	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	87	163	119	165	0	0	0
N.S.	1	1.00	1.01	1.90	1.38	1.92	0.00	0.00	0.00
time (sec)	N/A	0.111	0.242	0.171	0.285	0.292	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	78	168	0	0	0	0	0
N.S.	1	1.00	0.98	2.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.114	0.390	0.210	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	52	103	0	127	0	0	0
N.S.	1	1.00	1.16	2.29	0.00	2.82	0.00	0.00	0.00
time (sec)	N/A	0.053	0.181	0.157	0.000	0.292	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	66	110	58	0	0	0	0
N.S.	1	1.00	1.29	2.16	1.14	0.00	0.00	0.00	0.00
time (sec)	N/A	0.029	0.120	0.234	0.205	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	143	155	0	0	0	0	0
N.S.	1	1.00	1.52	1.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.170	0.393	0.184	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	102	244	119	0	0	0	0
N.S.	1	1.00	1.10	2.62	1.28	0.00	0.00	0.00	0.00
time (sec)	N/A	0.116	0.250	0.226	0.215	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	269	224	0	0	0	0	0
N.S.	1	1.00	1.66	1.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.247	2.927	0.170	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	168	604	0	0	0	0	0
N.S.	1	1.00	1.10	3.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.126	0.307	0.171	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	202	970	0	0	0	0	0
N.S.	1	1.00	1.05	5.05	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.287	0.596	0.331	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	132	237	0	218	0	0	0
N.S.	1	1.00	0.90	1.62	0.00	1.49	0.00	0.00	0.00
time (sec)	N/A	0.150	0.318	0.178	0.000	0.287	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	166	897	0	0	0	0	0
N.S.	1	1.00	1.19	6.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.193	0.491	0.171	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	93	157	138	187	0	0	0
N.S.	1	1.00	0.89	1.50	1.31	1.78	0.00	0.00	0.00
time (sec)	N/A	0.111	0.281	0.170	0.290	0.301	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	88	729	137	0	0	0	0
N.S.	1	1.00	1.10	9.11	1.71	0.00	0.00	0.00	0.00
time (sec)	N/A	0.085	0.256	0.193	0.223	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	72	121	0	165	0	0	0
N.S.	1	1.00	0.96	1.61	0.00	2.20	0.00	0.00	0.00
time (sec)	N/A	0.056	0.215	0.168	0.000	0.310	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	100	619	126	0	0	0	0
N.S.	1	1.00	0.93	5.73	1.17	0.00	0.00	0.00	0.00
time (sec)	N/A	0.063	0.150	0.196	0.211	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	209	208	0	0	0	0	0
N.S.	1	1.00	1.41	1.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.232	0.703	0.229	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	125	778	0	0	0	0	0
N.S.	1	1.00	0.83	5.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.139	0.409	0.182	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	331	314	0	0	0	0	0
N.S.	1	1.00	1.34	1.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.326	4.107	0.178	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	239	1155	236	0	0	0	0
N.S.	1	1.00	1.15	5.55	1.13	0.00	0.00	0.00	0.00
time (sec)	N/A	0.177	0.398	0.169	0.218	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	121	363	143	0	0	124	0
N.S.	1	1.00	0.60	1.82	0.72	0.00	0.00	0.62	0.00
time (sec)	N/A	0.087	0.145	0.277	0.221	0.000	0.000	0.338	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	63	74	83	83	82	0	0
N.S.	1	1.00	0.73	0.86	0.97	0.97	0.95	0.00	0.00
time (sec)	N/A	0.108	0.038	0.158	0.202	0.270	0.500	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	48	82	59	55	65	0	0
N.S.	1	1.00	0.69	1.17	0.84	0.79	0.93	0.00	0.00
time (sec)	N/A	0.076	0.032	0.209	0.199	0.254	0.357	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	42	40	55	62	42	0	0
N.S.	1	1.00	0.86	0.82	1.12	1.27	0.86	0.00	0.00
time (sec)	N/A	0.061	0.048	0.204	0.200	0.270	0.337	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	47	26	38	24	38	0
N.S.	1	1.00	1.00	1.68	0.93	1.36	0.86	1.36	0.00
time (sec)	N/A	0.029	0.038	0.222	0.194	0.268	0.240	0.292	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	23	10	0	11
N.S.	1	1.00	1.00	0.92	0.85	1.77	0.77	0.00	0.85
time (sec)	N/A	0.015	0.008	0.234	0.193	0.263	0.198	0.000	2.398

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	57	89	0	0	0	0	0
N.S.	1	1.00	1.68	2.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.058	0.109	0.231	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	29	56	25	39	0	71	0
N.S.	1	1.00	1.07	2.07	0.93	1.44	0.00	2.63	0.00
time (sec)	N/A	0.040	0.049	0.244	0.192	0.256	0.000	0.302	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	126	150	0	0	0	0	0
N.S.	1	1.00	1.58	1.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.101	0.555	0.200	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	120	578	134	158	0	0	0
N.S.	1	1.00	0.69	3.30	0.77	0.90	0.00	0.00	0.00
time (sec)	N/A	0.094	0.128	0.271	0.206	0.271	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	129	338	0	0	0	0	0
N.S.	1	1.00	0.71	1.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.135	0.617	0.175	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	92	321	73	127	0	0	0
N.S.	1	1.00	0.88	3.06	0.70	1.21	0.00	0.00	0.00
time (sec)	N/A	0.047	0.109	0.232	0.204	0.271	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	120	256	0	0	0	0	0
N.S.	1	1.00	1.08	2.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.043	0.400	0.166	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	168	331	0	0	0	0	0
N.S.	1	1.00	0.95	1.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.136	0.401	0.203	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	129	251	0	0	0	0	0
N.S.	1	1.00	1.23	2.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.082	0.342	0.209	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	223	334	0	0	0	0	0
N.S.	1	1.00	1.11	1.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.139	2.252	0.184	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	117	137	133	217	0	0	0
N.S.	1	1.00	1.10	1.29	1.25	2.05	0.00	0.00	0.00
time (sec)	N/A	0.069	0.330	0.199	0.199	0.284	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	130	872	145	199	0	0	0
N.S.	1	1.00	0.60	4.02	0.67	0.92	0.00	0.00	0.00
time (sec)	N/A	0.112	0.194	0.216	0.205	0.266	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	251	799	0	0	0	0	0
N.S.	1	1.00	0.99	3.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.217	0.640	0.219	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	102	559	85	167	0	0	0
N.S.	1	1.00	0.70	3.83	0.58	1.14	0.00	0.00	0.00
time (sec)	N/A	0.059	0.134	0.204	0.197	0.275	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	200	496	0	0	0	0	0
N.S.	1	1.00	1.11	2.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.082	0.696	0.182	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	248	428	0	0	0	0	0
N.S.	1	1.00	1.00	1.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.231	0.631	0.270	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	200	230	0	0	0	0	0
N.S.	1	1.00	1.13	1.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.124	0.834	0.249	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	352	289	0	0	0	0	0
N.S.	1	1.00	1.30	1.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.224	3.471	0.237	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	217	257	0	0	0	0	0
N.S.	1	1.00	1.18	1.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.162	0.634	0.249	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	140	996	156	263	0	0	0
N.S.	1	1.00	0.53	3.74	0.59	0.99	0.00	0.00	0.00
time (sec)	N/A	0.114	0.229	0.230	0.220	0.276	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	388	1165	0	0	0	0	0
N.S.	1	1.00	1.15	3.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.317	0.896	0.224	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	112	863	96	225	0	0	0
N.S.	1	1.00	0.58	4.47	0.50	1.17	0.00	0.00	0.00
time (sec)	N/A	0.058	0.152	0.216	0.207	0.274	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	317	801	0	0	0	0	0
N.S.	1	1.00	1.25	3.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.116	0.664	0.185	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	339	540	0	0	0	0	0
N.S.	1	1.00	1.03	1.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.308	0.712	0.244	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	270	285	0	0	0	0	0
N.S.	1	1.00	1.05	1.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.181	1.332	0.201	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	355	424	339	0	0	0	0	0
N.S.	1	1.00	1.19	0.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.326	6.407	0.243	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	286	322	0	0	0	0	0
N.S.	1	1.00	1.08	1.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.231	0.992	0.224	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	28	26	28	40	0	0	0
N.S.	1	1.00	0.88	0.81	0.88	1.25	0.00	0.00	0.00
time (sec)	N/A	0.023	0.007	0.232	0.282	0.255	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	119	625	174	161	0	0	0
N.S.	1	1.00	0.55	2.91	0.81	0.75	0.00	0.00	0.00
time (sec)	N/A	0.190	0.173	0.217	0.212	0.273	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	151	519	0	0	0	0	0
N.S.	1	1.00	0.79	2.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.165	0.691	0.205	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	93	358	117	132	0	0	0
N.S.	1	1.00	0.65	2.52	0.82	0.93	0.00	0.00	0.00
time (sec)	N/A	0.114	0.140	0.196	0.204	0.272	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	121	273	0	0	0	0	0
N.S.	1	1.00	1.02	2.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.091	0.656	0.211	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	74	148	55	96	0	0	0
N.S.	1	1.00	1.16	2.31	0.86	1.50	0.00	0.00	0.00
time (sec)	N/A	0.051	0.152	0.236	0.207	0.283	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	76	77	28	0	0	0	0
N.S.	1	1.00	1.62	1.64	0.60	0.00	0.00	0.00	0.00
time (sec)	N/A	0.024	0.078	0.206	0.208	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	129	233	0	0	0	0	0
N.S.	1	1.00	1.06	1.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.091	0.292	0.249	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	77	169	101	132	0	0	0
N.S.	1	1.00	1.22	2.68	1.60	2.10	0.00	0.00	0.00
time (sec)	N/A	0.064	0.192	0.235	0.209	0.284	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	229	333	0	0	0	0	0
N.S.	1	1.00	1.13	1.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.159	2.285	0.203	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	131	166	121	222	0	0	0
N.S.	1	1.00	0.93	1.18	0.86	1.57	0.00	0.00	0.00
time (sec)	N/A	0.129	0.235	0.221	0.205	0.300	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	174	238	0	197	0	0	0
N.S.	1	1.00	0.82	1.12	0.00	0.93	0.00	0.00	0.00
time (sec)	N/A	0.136	0.398	0.230	0.000	0.307	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	161	274	0	0	0	0	0
N.S.	1	1.00	0.78	1.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.178	0.550	0.244	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	130	176	119	166	0	0	0
N.S.	1	1.00	0.96	1.29	0.88	1.22	0.00	0.00	0.00
time (sec)	N/A	0.108	0.355	0.231	0.280	0.288	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	146	232	0	0	0	0	0
N.S.	1	1.00	1.12	1.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.101	0.331	0.195	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	86	125	0	128	0	0	0
N.S.	1	1.00	1.23	1.79	0.00	1.83	0.00	0.00	0.00
time (sec)	N/A	0.057	0.264	0.240	0.000	0.300	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	100	143	58	0	0	0	0
N.S.	1	1.00	1.32	1.88	0.76	0.00	0.00	0.00	0.00
time (sec)	N/A	0.028	0.174	0.177	0.209	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	231	271	0	0	0	0	0
N.S.	1	1.00	1.19	1.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.161	0.683	0.237	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	163	239	119	0	0	0	0
N.S.	1	1.00	1.14	1.67	0.83	0.00	0.00	0.00	0.00
time (sec)	N/A	0.103	0.605	0.194	0.202	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	369	342	0	0	0	0	0
N.S.	1	1.00	1.29	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.260	6.182	0.224	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	216	968	0	0	0	0	0
N.S.	1	1.00	0.95	4.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.123	0.324	0.224	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	222	410	0	0	0	0	0
N.S.	1	1.00	0.79	1.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.283	0.936	0.253	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	177	400	0	219	0	0	0
N.S.	1	1.00	0.84	1.90	0.00	1.04	0.00	0.00	0.00
time (sec)	N/A	0.133	0.417	0.262	0.000	0.298	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	191	345	0	0	0	0	0
N.S.	1	1.00	0.94	1.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.194	0.576	0.217	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	136	263	138	188	0	0	0
N.S.	1	1.00	0.94	1.83	0.96	1.31	0.00	0.00	0.00
time (sec)	N/A	0.118	0.363	0.190	0.278	0.299	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	118	413	137	0	0	0	0
N.S.	1	1.00	0.99	3.47	1.15	0.00	0.00	0.00	0.00
time (sec)	N/A	0.092	0.281	0.255	0.204	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	103	198	0	166	0	0	0
N.S.	1	1.00	0.90	1.74	0.00	1.46	0.00	0.00	0.00
time (sec)	N/A	0.055	0.286	0.192	0.000	0.286	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	143	425	126	0	0	0	0
N.S.	1	1.00	0.97	2.89	0.86	0.00	0.00	0.00	0.00
time (sec)	N/A	0.061	0.188	0.276	0.203	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	247	364	0	0	0	0	0
N.S.	1	1.00	0.94	1.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.260	1.001	0.245	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	227	1257	0	0	0	0	0
N.S.	1	1.00	1.06	5.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.126	0.329	0.248	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	400	400	409	406	0	0	0	0	0
N.S.	1	1.00	1.02	1.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.352	5.360	0.231	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	267	372	236	0	0	0	0
N.S.	1	1.00	0.90	1.25	0.79	0.00	0.00	0.00	0.00
time (sec)	N/A	0.176	0.360	0.216	0.216	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	121	363	143	0	0	124	0
N.S.	1	1.00	0.60	1.82	0.72	0.00	0.00	0.62	0.00
time (sec)	N/A	0.084	0.083	0.184	0.203	0.000	0.000	0.330	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	63	74	83	83	82	0	0
N.S.	1	1.00	0.73	0.86	0.97	0.97	0.95	0.00	0.00
time (sec)	N/A	0.105	0.023	0.209	0.203	0.272	0.417	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	48	82	59	55	65	0	0
N.S.	1	1.00	0.69	1.17	0.84	0.79	0.93	0.00	0.00
time (sec)	N/A	0.074	0.020	0.232	0.191	0.271	0.351	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	42	40	55	62	42	0	0
N.S.	1	1.00	0.86	0.82	1.12	1.27	0.86	0.00	0.00
time (sec)	N/A	0.065	0.013	0.199	0.192	0.258	0.299	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	47	26	38	24	38	0
N.S.	1	1.00	1.00	1.68	0.93	1.36	0.86	1.36	0.00
time (sec)	N/A	0.031	0.010	0.243	0.196	0.267	0.232	0.284	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	23	10	0	11
N.S.	1	1.00	1.00	0.92	0.85	1.77	0.77	0.00	0.85
time (sec)	N/A	0.023	0.001	0.228	0.193	0.277	0.204	0.000	0.002

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	57	89	0	0	0	0	0
N.S.	1	1.00	1.68	2.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.060	0.092	0.201	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	29	56	25	39	0	71	0
N.S.	1	1.00	1.07	2.07	0.93	1.44	0.00	2.63	0.00
time (sec)	N/A	0.041	0.011	0.211	0.216	0.274	0.000	0.300	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	126	150	0	0	0	0	0
N.S.	1	1.00	1.58	1.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.102	0.114	0.250	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	313	313	257	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.363	0.352	0.000	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	217	217	188	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.200	0.013	0.000	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	128	118	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.091	0.033	0.000	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	36	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	1.50	1.08	1.08
time (sec)	N/A	0.045	3.334	0.260	0.298	0.280	2.258	0.287	2.577

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	41	54	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.71	2.25	1.08	1.08
time (sec)	N/A	0.110	4.911	0.283	0.283	0.269	16.142	0.278	2.735

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	52	71	26	26
N.S.	1	1.00	1.08	1.00	1.08	2.17	2.96	1.08	1.08
time (sec)	N/A	0.190	5.239	0.286	0.283	0.284	117.915	0.291	2.706

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	618	618	332	0	0	0	0	0	0
N.S.	1	1.00	0.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.364	0.904	0.000	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	390	390	233	0	0	0	0	0	0
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.232	0.370	0.000	0.000	0.000	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	240	240	179	0	0	0	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.140	0.049	0.000	0.000	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	161	161	129	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.073	0.044	0.000	0.000	0.000	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	268	268	206	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.165	0.161	0.000	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	402	402	286	0	0	0	0	0	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.264	0.273	0.000	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	102	97	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.050	0.026	0.000	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	201	329	441	260	388	0	0
N.S.	1	1.00	0.71	1.16	1.56	0.92	1.37	0.00	0.00
time (sec)	N/A	0.308	0.197	0.166	0.230	0.268	0.921	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	186	266	442	240	332	0	0
N.S.	1	1.00	0.94	1.34	2.23	1.21	1.68	0.00	0.00
time (sec)	N/A	0.384	0.192	0.109	0.221	0.282	0.786	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	177	247	346	225	313	0	0
N.S.	1	1.00	0.86	1.20	1.68	1.09	1.52	0.00	0.00
time (sec)	N/A	0.228	0.179	0.114	0.222	0.275	0.548	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	155	182	347	204	269	0	0
N.S.	1	1.00	1.15	1.35	2.57	1.51	1.99	0.00	0.00
time (sec)	N/A	0.101	0.206	0.085	0.222	0.275	0.432	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	135	166	230	178	224	0	0
N.S.	1	1.00	1.08	1.33	1.84	1.42	1.79	0.00	0.00
time (sec)	N/A	0.098	0.108	0.067	0.212	0.275	0.235	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	226	415	0	0	0	0	0
N.S.	1	1.00	1.36	2.50	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.219	0.292	0.171	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	192	239	0	0	0	0	0
N.S.	1	1.00	1.47	1.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.207	0.316	0.120	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	241	404	0	0	0	0	0
N.S.	1	1.00	1.34	2.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.246	0.226	0.195	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	245	240	0	0	0	0	0
N.S.	1	1.00	1.55	1.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.257	0.609	0.171	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	386	251	427	760	368	563	0	0
N.S.	1	1.00	0.65	1.11	1.97	0.95	1.46	0.00	0.00
time (sec)	N/A	0.488	0.269	0.299	0.225	0.291	1.676	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	237	343	762	348	515	0	0
N.S.	1	1.00	0.80	1.16	2.57	1.18	1.74	0.00	0.00
time (sec)	N/A	0.734	0.240	0.217	0.251	0.280	1.320	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	227	345	619	327	483	0	0
N.S.	1	1.00	0.75	1.14	2.04	1.08	1.59	0.00	0.00
time (sec)	N/A	0.433	1.131	0.237	0.221	0.290	0.982	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	208	247	621	307	430	0	0
N.S.	1	1.00	1.02	1.21	3.04	1.50	2.11	0.00	0.00
time (sec)	N/A	0.145	1.155	0.250	0.242	0.296	0.788	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	191	264	457	278	389	0	0
N.S.	1	1.00	0.89	1.23	2.14	1.30	1.82	0.00	0.00
time (sec)	N/A	0.185	0.926	0.202	0.234	0.285	0.461	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	326	573	0	0	0	0	0
N.S.	1	1.00	1.27	2.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.349	0.411	0.310	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	306	364	0	0	0	0	0
N.S.	1	1.00	1.34	1.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.340	0.878	0.243	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	329	584	0	0	0	0	0
N.S.	1	1.00	1.21	2.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.397	0.754	0.277	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	357	363	0	0	0	0	0
N.S.	1	1.00	1.44	1.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.481	0.662	0.257	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	465	465	299	519	1109	444	702	0	0
N.S.	1	1.00	0.64	1.12	2.38	0.95	1.51	0.00	0.00
time (sec)	N/A	0.694	0.284	0.306	0.231	0.284	3.485	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	376	285	414	1112	424	654	0	0
N.S.	1	1.00	0.76	1.10	2.96	1.13	1.74	0.00	0.00
time (sec)	N/A	1.088	0.280	0.295	0.275	0.269	2.552	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	275	437	922	403	626	0	0
N.S.	1	1.00	0.72	1.14	2.41	1.05	1.64	0.00	0.00
time (sec)	N/A	0.555	1.295	0.234	0.249	0.264	1.809	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	256	306	925	383	573	0	0
N.S.	1	1.00	0.98	1.17	3.54	1.47	2.20	0.00	0.00
time (sec)	N/A	0.191	1.376	0.220	0.247	0.264	1.392	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	239	354	712	354	524	0	0
N.S.	1	1.00	0.82	1.22	2.45	1.22	1.80	0.00	0.00
time (sec)	N/A	0.276	1.227	0.234	0.220	0.298	0.897	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	419	687	0	0	0	0	0
N.S.	1	1.00	1.24	2.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.516	0.693	0.330	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	466	463	0	0	0	0	0
N.S.	1	1.00	1.52	1.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.515	1.074	0.281	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	496	772	0	0	0	0	0
N.S.	1	1.00	1.40	2.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.575	0.621	0.335	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	461	468	0	0	0	0	0
N.S.	1	1.00	1.41	1.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.660	0.792	0.257	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	277	277	365	0	0	0	0	0	0
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.363	0.828	0.000	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	294	343	0	0	0	0	0
N.S.	1	1.00	1.48	1.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.275	0.346	0.283	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	198	198	317	0	0	0	0	0	0
N.S.	1	1.00	1.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.214	0.454	0.000	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	281	180	0	0	0	0	0
N.S.	1	1.00	2.68	1.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.142	0.210	0.210	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	274	0	0	0	0	0	0
N.S.	1	1.00	1.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.093	0.184	0.000	0.000	0.000	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	400	392	0	0	0	0	0
N.S.	1	1.00	3.45	3.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.155	0.276	0.244	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	204	204	363	0	0	0	0	0	0
N.S.	1	1.00	1.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.243	0.803	0.000	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	411	540	0	0	0	0	0
N.S.	1	1.00	2.12	2.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.287	1.043	0.273	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	297	602	0	0	0	0	0	0
N.S.	1	1.00	2.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.508	7.325	0.000	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	279	279	482	0	0	0	0	0	0
N.S.	1	1.00	1.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.364	1.542	0.000	0.000	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	320	320	0	0	0	0	0
N.S.	1	1.00	1.50	1.50	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.285	0.741	0.257	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	213	213	385	0	0	0	0	0	0
N.S.	1	1.00	1.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.215	1.470	0.000	0.000	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	145	147	0	185	0	0	0
N.S.	1	1.00	1.71	1.73	0.00	2.18	0.00	0.00	0.00
time (sec)	N/A	0.072	0.454	0.219	0.000	0.269	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	210	210	403	0	0	0	0	0	0
N.S.	1	1.00	1.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.178	1.307	0.000	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	428	532	0	0	0	0	0
N.S.	1	1.00	2.22	2.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.250	1.652	0.263	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	287	287	549	0	0	0	0	0	0
N.S.	1	1.00	1.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.399	7.056	0.000	0.000	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	506	586	0	0	0	0	0
N.S.	1	1.00	2.00	2.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.411	1.543	0.295	0.000	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	401	401	764	0	0	0	0	0	0
N.S.	1	1.00	1.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.695	8.041	0.000	0.000	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	320	320	552	0	0	0	0	0	0
N.S.	1	1.00	1.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.377	2.354	0.000	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	186	272	0	283	0	0	0
N.S.	1	1.00	1.11	1.63	0.00	1.69	0.00	0.00	0.00
time (sec)	N/A	0.229	0.323	0.268	0.000	0.278	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	318	318	550	0	0	0	0	0	0
N.S.	1	1.00	1.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.312	2.030	0.000	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	152	227	0	273	0	0	0
N.S.	1	1.00	1.05	1.57	0.00	1.88	0.00	0.00	0.00
time (sec)	N/A	0.117	0.466	0.232	0.000	0.272	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	309	309	546	0	0	0	0	0	0
N.S.	1	1.00	1.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.276	2.415	0.000	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	560	676	0	0	0	0	0
N.S.	1	1.00	2.04	2.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.376	3.163	0.311	0.000	0.000	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	389	389	716	0	0	0	0	0	0
N.S.	1	1.00	1.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.567	7.508	0.000	0.000	0.000	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	701	832	0	0	0	0	0
N.S.	1	1.00	1.84	2.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.567	7.930	0.358	0.000	0.000	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	529	529	937	0	0	0	0	0	0
N.S.	1	1.00	1.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.899	8.997	0.000	0.000	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	284	397	0	0	583	0	0
N.S.	1	1.00	0.95	1.32	0.00	0.00	1.94	0.00	0.00
time (sec)	N/A	0.253	1.871	0.197	0.000	0.000	28.503	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	202	289	0	0	405	0	0
N.S.	1	1.00	0.96	1.38	0.00	0.00	1.93	0.00	0.00
time (sec)	N/A	0.171	0.999	0.185	0.000	0.000	2.912	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	124	179	0	0	0	0	0
N.S.	1	1.00	1.02	1.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.085	0.505	0.198	0.000	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	72	47	0	90	0	0
N.S.	1	1.00	1.00	2.88	1.88	0.00	3.60	0.00	0.00
time (sec)	N/A	0.040	0.021	0.207	0.219	0.000	0.896	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	153	243	0	0	0	0	0
N.S.	1	1.00	1.47	2.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.140	0.604	0.259	0.000	0.000	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	293	1729	0	0	0	0	0
N.S.	1	1.00	1.44	8.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.203	0.798	0.256	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	358	358	222	1162	302	316	0	0	0
N.S.	1	1.00	0.62	3.25	0.84	0.88	0.00	0.00	0.00
time (sec)	N/A	0.354	0.259	0.377	0.226	0.272	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	207	618	0	0	0	0	0
N.S.	1	1.00	0.71	2.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.278	1.301	0.273	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	166	657	183	249	0	0	0
N.S.	1	1.00	0.92	3.65	1.02	1.38	0.00	0.00	0.00
time (sec)	N/A	0.115	0.449	0.261	0.217	0.271	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	200	480	0	0	0	0	0
N.S.	1	1.00	1.09	2.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.090	1.019	0.218	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	338	338	352	764	0	0	0	0	0
N.S.	1	1.00	1.04	2.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.246	0.977	0.323	0.000	0.000	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	232	579	0	0	0	0	0
N.S.	1	1.00	1.11	2.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.219	1.243	0.286	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	358	358	446	755	0	0	0	0	0
N.S.	1	1.00	1.25	2.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.265	3.786	0.349	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	240	1729	0	0	0	0	0
N.S.	1	1.00	0.82	5.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.227	0.681	0.335	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	482	482	251	1766	346	402	0	0	0
N.S.	1	1.00	0.52	3.66	0.72	0.83	0.00	0.00	0.00
time (sec)	N/A	0.549	0.370	0.388	0.230	0.268	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	405	405	508	1552	0	0	0	0	0
N.S.	1	1.00	1.25	3.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.483	2.111	0.361	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	198	1149	230	332	0	0	0
N.S.	1	1.00	0.74	4.30	0.86	1.24	0.00	0.00	0.00
time (sec)	N/A	0.150	1.239	0.374	0.227	0.274	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	329	959	0	0	0	0	0
N.S.	1	1.00	1.12	3.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.181	2.341	0.262	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	498	498	520	1053	0	0	0	0	0
N.S.	1	1.00	1.04	2.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.420	1.782	0.352	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	398	398	369	473	0	0	0	0	0
N.S.	1	1.00	0.93	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.336	3.497	0.309	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	541	541	771	933	0	0	0	0	0
N.S.	1	1.00	1.43	1.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.435	7.791	0.369	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	378	458	1929	0	0	0	0	0
N.S.	1	1.00	1.21	5.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.465	1.322	0.368	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	625	625	277	2014	390	525	0	0	0
N.S.	1	1.00	0.44	3.22	0.62	0.84	0.00	0.00	0.00
time (sec)	N/A	0.833	0.448	0.401	0.223	0.278	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	536	536	619	2280	0	0	0	0	0
N.S.	1	1.00	1.15	4.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.738	2.836	0.404	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	366	224	1773	274	446	0	0	0
N.S.	1	1.00	0.61	4.84	0.75	1.22	0.00	0.00	0.00
time (sec)	N/A	0.205	1.393	0.370	0.228	0.276	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	420	420	499	1568	0	0	0	0	0
N.S.	1	1.00	1.19	3.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.297	2.296	0.303	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	635	635	757	1321	0	0	0	0	0
N.S.	1	1.00	1.19	2.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.639	3.560	0.323	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	530	530	550	589	0	0	0	0	0
N.S.	1	1.00	1.04	1.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.488	2.788	0.345	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	687	687	990	720	0	0	0	0	0
N.S.	1	1.00	1.44	1.05	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.654	8.037	0.331	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	561	561	616	2297	0	0	0	0	0
N.S.	1	1.00	1.10	4.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.665	2.220	0.385	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	98	125	0	131	146	0	0
N.S.	1	1.00	0.64	0.82	0.00	0.86	0.95	0.00	0.00
time (sec)	N/A	0.193	0.055	0.216	0.000	0.279	0.592	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	79	113	101	98	121	0	0
N.S.	1	1.00	0.65	0.93	0.83	0.80	0.99	0.00	0.00
time (sec)	N/A	0.149	0.043	0.266	0.218	0.265	0.462	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	72	69	0	102	78	0	0
N.S.	1	1.00	0.83	0.79	0.00	1.17	0.90	0.00	0.00
time (sec)	N/A	0.111	0.035	0.188	0.000	0.248	0.391	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	48	64	48	70	49	74	0
N.S.	1	1.00	0.92	1.23	0.92	1.35	0.94	1.42	0.00
time (sec)	N/A	0.051	0.027	0.276	0.196	0.248	0.333	0.298	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	23	10	0	11
N.S.	1	1.00	1.00	0.92	0.85	1.77	0.77	0.00	0.85
time (sec)	N/A	0.024	0.007	0.202	0.189	0.275	0.234	0.000	3.065

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	100	144	0	0	0	0	0
N.S.	1	1.00	1.47	2.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.120	0.117	0.276	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	65	132	0	0	0	0	0
N.S.	1	1.00	0.98	2.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.122	0.264	0.190	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	188	233	0	0	0	0	0
N.S.	1	1.00	1.39	1.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.200	0.919	0.302	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	383	383	230	1227	353	319	0	0	0
N.S.	1	1.00	0.60	3.20	0.92	0.83	0.00	0.00	0.00
time (sec)	N/A	0.409	0.308	0.331	0.213	0.275	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	268	992	0	0	0	0	0
N.S.	1	1.00	0.83	3.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.301	1.047	0.256	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	176	706	243	254	0	0	0
N.S.	1	1.00	0.66	2.66	0.92	0.96	0.00	0.00	0.00
time (sec)	N/A	0.221	0.295	0.291	0.215	0.292	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	198	506	0	0	0	0	0
N.S.	1	1.00	0.97	2.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.157	1.197	0.230	0.000	0.000	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	127	296	125	179	0	0	0
N.S.	1	1.00	0.92	2.14	0.91	1.30	0.00	0.00	0.00
time (sec)	N/A	0.090	0.571	0.236	0.202	0.275	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	115	120	47	0	0	0	0
N.S.	1	1.00	2.45	2.55	1.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.035	0.146	0.211	0.202	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	266	546	0	0	0	0	0
N.S.	1	1.00	1.19	2.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.154	0.762	0.281	0.000	0.000	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	168	478	0	0	0	0	0
N.S.	1	1.00	1.01	2.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.194	0.687	0.263	0.000	0.000	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	455	780	0	0	0	0	0
N.S.	1	1.00	1.26	2.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.299	3.807	0.307	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	278	1506	0	0	0	0	0
N.S.	1	1.00	0.93	5.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.323	0.611	0.309	0.000	0.000	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	515	515	427	520	0	0	0	0	0
N.S.	1	1.00	0.83	1.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.568	0.810	0.325	0.000	0.000	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	400	400	288	579	0	0	0	0	0
N.S.	1	1.00	0.72	1.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.441	1.702	0.286	0.000	0.000	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	383	383	318	412	0	0	0	0	0
N.S.	1	1.00	0.83	1.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.305	0.703	0.301	0.000	0.000	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	215	478	0	0	0	0	0
N.S.	1	1.00	0.92	2.05	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.228	1.351	0.254	0.000	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	217	318	0	0	0	0	0
N.S.	1	1.00	1.15	1.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.132	0.906	0.184	0.000	0.000	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	152	343	0	0	0	0	0
N.S.	1	1.00	0.85	1.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.122	0.749	0.282	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	412	412	568	0	0	0	0	0	0
N.S.	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.333	1.414	0.000	0.000	0.000	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	296	1117	0	0	0	0	0
N.S.	1	1.00	0.97	3.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.329	1.332	0.298	0.000	0.000	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	573	573	884	0	0	0	0	0	0
N.S.	1	1.00	1.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.594	7.416	0.000	0.000	0.000	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	452	452	438	2608	0	0	0	0	0
N.S.	1	1.00	0.97	5.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.607	0.940	0.368	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	512	512	333	1042	0	0	0	0	0
N.S.	1	1.00	0.65	2.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.646	1.594	0.405	0.000	0.000	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	398	398	359	766	0	0	0	0	0
N.S.	1	1.00	0.90	1.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.480	1.224	0.307	0.000	0.000	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	301	704	0	0	0	0	0
N.S.	1	1.00	0.98	2.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.341	1.124	0.204	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	280	2352	0	0	0	0	0
N.S.	1	1.00	0.90	7.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.248	1.399	0.287	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	254	591	0	0	0	0	0
N.S.	1	1.00	0.94	2.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.153	1.344	0.293	0.000	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	236	2131	0	0	0	0	0
N.S.	1	1.00	0.81	7.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.206	1.449	0.279	0.000	0.000	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	518	518	547	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.526	3.211	0.000	0.000	0.000	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	421	421	408	3513	0	0	0	0	0
N.S.	1	1.00	0.97	8.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.456	2.142	0.349	0.000	0.000	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	687	687	983	0	0	0	0	0	0
N.S.	1	1.00	1.43	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.814	7.775	0.000	0.000	0.000	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	506	506	417	3506	0	0	0	0	0
N.S.	1	1.00	0.82	6.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.800	2.596	0.367	0.000	0.000	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	366	178	570	0	0	0	0	0
N.S.	1	1.00	0.49	1.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.251	0.640	0.263	0.000	0.000	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	133	0	0	28
N.S.	1	1.00	1.07	0.93	1.00	4.75	0.00	0.00	1.00
time (sec)	N/A	0.853	3.615	1.048	0.356	0.266	0.000	0.000	3.121

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	80	0	0	28
N.S.	1	1.00	1.07	0.93	1.00	2.86	0.00	0.00	1.00
time (sec)	N/A	0.419	0.914	0.852	0.312	0.266	0.000	0.000	3.066

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	40	27	0	28
N.S.	1	1.00	1.07	0.93	1.00	1.43	0.96	0.00	1.00
time (sec)	N/A	0.251	0.655	0.706	0.322	0.263	21.286	0.000	3.184

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	40	27	28	28
N.S.	1	1.00	1.07	0.93	1.00	1.43	0.96	1.00	1.00
time (sec)	N/A	0.095	3.420	0.303	0.305	0.261	7.796	0.356	3.109

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	67	27	28	28
N.S.	1	1.00	1.07	0.93	1.00	2.39	0.96	1.00	1.00
time (sec)	N/A	0.106	4.435	0.397	0.358	0.259	9.435	0.356	2.989

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	78	27	28	28
N.S.	1	1.00	1.07	0.93	1.00	2.79	0.96	1.00	1.00
time (sec)	N/A	0.111	4.604	0.455	0.359	0.275	165.867	0.377	2.998

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	23	22	23	23
N.S.	1	1.00	1.09	0.91	1.00	1.00	0.96	1.00	1.00
time (sec)	N/A	0.063	0.460	0.175	0.297	0.260	1.150	0.335	3.097

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	169	270	276	248	355	0	0
N.S.	1	1.00	0.47	0.75	0.77	0.69	0.99	0.00	0.00
time (sec)	N/A	0.542	0.191	0.215	0.207	0.280	1.189	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	137	200	210	204	262	0	0
N.S.	1	1.00	0.52	0.75	0.79	0.77	0.99	0.00	0.00
time (sec)	N/A	0.302	0.102	0.292	0.211	0.272	0.573	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	99	128	124	140	150	0	0
N.S.	1	1.00	0.65	0.84	0.81	0.92	0.98	0.00	0.00
time (sec)	N/A	0.159	0.093	0.046	0.199	0.265	0.300	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	174	174	223	0	0	0	0	0	0
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.109	0.115	0.000	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	294	294	568	0	0	0	0	0	0
N.S.	1	1.00	1.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.225	1.790	0.000	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	409	409	654	0	0	0	0	0	0
N.S.	1	1.00	1.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.377	3.736	0.000	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	509	509	177	802	0	0	0	0	0
N.S.	1	1.00	0.35	1.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.419	0.603	0.242	0.000	0.000	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	136	484	0	0	0	0	0
N.S.	1	1.00	0.39	1.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.252	0.278	0.212	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	86	231	0	0	0	0	0
N.S.	1	1.00	0.42	1.13	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.140	0.162	0.218	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	41	39	14	0	0	0	0
N.S.	1	1.00	1.02	0.98	0.35	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	0.047	0.237	0.203	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	133	262	0	0	0	0	0
N.S.	1	1.00	0.61	1.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.137	0.255	0.278	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	363	195	550	0	0	0	0	0
N.S.	1	1.00	0.54	1.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.234	0.474	0.297	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	515	515	297	888	0	0	0	0	0
N.S.	1	1.00	0.58	1.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.374	0.590	0.294	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	23	22	23	23
N.S.	1	1.00	1.09	0.91	1.00	1.00	0.96	1.00	1.00
time (sec)	N/A	0.065	0.470	0.178	0.299	0.264	2.539	0.363	2.549

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	121	156	0	166	185	0	0
N.S.	1	1.00	0.65	0.83	0.00	0.89	0.99	0.00	0.00
time (sec)	N/A	0.339	0.060	0.199	0.000	0.258	0.730	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	98	164	127	128	148	0	0
N.S.	1	1.00	0.64	1.07	0.83	0.84	0.97	0.00	0.00
time (sec)	N/A	0.226	0.049	0.260	0.204	0.264	0.518	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	83	84	0	128	100	0	0
N.S.	1	1.00	0.79	0.80	0.00	1.22	0.95	0.00	0.00
time (sec)	N/A	0.153	0.058	0.214	0.000	0.251	0.406	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	58	90	61	92	61	101	0
N.S.	1	1.00	0.91	1.41	0.95	1.44	0.95	1.58	0.00
time (sec)	N/A	0.094	0.030	0.287	0.198	0.251	0.343	0.329	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	23	10	0	11
N.S.	1	1.00	1.00	0.92	0.85	1.77	0.77	0.00	0.85
time (sec)	N/A	0.022	0.006	0.254	0.203	0.250	0.246	0.000	2.552

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	146	197	0	0	0	0	0
N.S.	1	1.00	1.43	1.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.116	0.136	0.320	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	97	187	0	0	0	0	0
N.S.	1	1.00	1.10	2.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.141	0.225	0.209	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	304	377	0	0	0	0	0
N.S.	1	1.00	1.45	1.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.261	3.265	0.244	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	43	42	0	0	0	0	0
N.S.	1	1.00	0.64	0.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.087	0.137	0.208	0.000	0.000	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	34	33	0	0	0	0	0
N.S.	1	1.00	0.68	0.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.077	0.073	0.225	0.000	0.000	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	23	22	0	0	0	0	0
N.S.	1	1.00	0.79	0.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.055	0.026	0.041	0.000	0.000	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	21	21	20	21	21
N.S.	1	1.00	1.11	1.00	1.11	1.11	1.05	1.11	1.11
time (sec)	N/A	0.021	0.491	0.200	0.247	0.245	0.526	0.294	2.587

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	21	36	36	21	21
N.S.	1	1.00	1.11	1.00	1.11	1.89	1.89	1.11	1.11
time (sec)	N/A	0.020	1.529	0.068	0.249	0.248	0.984	0.319	2.543

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	152	163	0	0	0	0	0
N.S.	1	1.00	0.74	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.311	0.297	1.015	0.000	0.000	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	135	148	0	0	0	0	0
N.S.	1	1.00	0.74	0.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.314	0.255	0.286	0.000	0.000	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	65	67	0	0	0	0	0
N.S.	1	1.00	0.79	0.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.160	0.186	0.281	0.000	0.000	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	91	100	0	0	0	0	0
N.S.	1	1.00	0.75	0.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.186	0.204	0.226	0.000	0.000	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	63	67	0	0	0	0	0
N.S.	1	1.00	0.77	0.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.103	0.231	0.308	0.000	0.000	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	27	27	22	0	27
N.S.	1	1.00	1.07	0.93	1.00	1.00	0.81	0.00	1.00
time (sec)	N/A	0.279	2.529	0.140	0.265	0.256	0.714	0.000	2.547

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	27	31	24	27	27
N.S.	1	1.00	1.07	0.93	1.00	1.15	0.89	1.00	1.00
time (sec)	N/A	0.204	2.092	0.107	0.274	0.263	0.620	0.291	2.557

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	27	31	24	0	27
N.S.	1	1.00	1.07	0.93	1.00	1.15	0.89	0.00	1.00
time (sec)	N/A	0.104	4.321	0.161	0.263	0.255	0.655	0.000	2.501

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	27	31	24	27	27
N.S.	1	1.00	1.07	0.93	1.00	1.15	0.89	1.00	1.00
time (sec)	N/A	0.085	1.529	0.174	0.272	0.240	0.809	0.276	2.635

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	179	196	0	0	0	0	0
N.S.	1	1.00	0.73	0.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.330	0.530	0.241	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	152	163	0	0	0	0	0
N.S.	1	1.00	0.74	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.276	0.428	0.404	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	136	149	0	0	0	0	0
N.S.	1	1.00	0.74	0.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.233	0.401	0.216	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	109	115	0	0	0	0	0
N.S.	1	1.00	0.76	0.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.156	0.306	0.421	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	27	27	22	0	27
N.S.	1	1.00	1.07	0.93	1.00	1.00	0.81	0.00	1.00
time (sec)	N/A	0.507	2.433	0.076	0.265	0.253	2.143	0.000	2.548

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	27	31	24	27	27
N.S.	1	1.00	1.07	0.93	1.00	1.15	0.89	1.00	1.00
time (sec)	N/A	0.382	2.628	0.078	0.263	0.249	1.630	0.297	2.529

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	27	31	24	0	27
N.S.	1	1.00	1.07	0.93	1.00	1.15	0.89	0.00	1.00
time (sec)	N/A	0.091	4.359	0.088	0.268	0.258	1.944	0.000	2.536

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	27	31	24	27	27
N.S.	1	1.00	1.07	0.93	1.00	1.15	0.89	1.00	1.00
time (sec)	N/A	0.090	1.576	0.171	0.259	0.252	2.592	0.308	2.548

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	180	197	0	0	0	0	0
N.S.	1	1.00	0.73	0.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.334	0.808	0.252	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	197	211	0	0	0	0	0
N.S.	1	1.00	0.74	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.343	0.734	0.507	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	180	197	0	0	0	0	0
N.S.	1	1.00	0.73	0.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.305	0.683	0.260	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	153	163	0	0	0	0	0
N.S.	1	1.00	0.74	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.222	0.562	0.465	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	27	44	22	0	27
N.S.	1	1.00	1.07	0.93	1.00	1.63	0.81	0.00	1.00
time (sec)	N/A	0.813	2.399	0.076	0.264	0.246	5.234	0.000	2.666

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	27	48	24	27	27
N.S.	1	1.00	1.07	0.93	1.00	1.78	0.89	1.00	1.00
time (sec)	N/A	0.635	2.408	0.079	0.273	0.252	3.881	0.319	2.682

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	27	48	24	0	27
N.S.	1	1.00	1.07	0.93	1.00	1.78	0.89	0.00	1.00
time (sec)	N/A	0.092	4.280	0.093	0.270	0.258	4.047	0.000	2.720

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	27	48	24	27	27
N.S.	1	1.00	1.07	0.93	1.00	1.78	0.89	1.00	1.00
time (sec)	N/A	0.107	1.728	0.181	0.278	0.246	4.635	0.321	2.650

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	31	30	0	0	0	0	0
N.S.	1	1.00	0.76	0.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.128	0.071	0.221	0.000	0.000	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	23	0	0	0	0	0
N.S.	1	1.00	0.81	0.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.104	0.143	0.264	0.000	0.000	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	21	0	0	0	0	0
N.S.	1	1.00	0.81	0.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.100	0.088	0.230	0.000	0.000	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	21	0	0	0	0	0
N.S.	1	1.00	0.81	0.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.093	0.008	0.000	0.000	0.000	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	0	0	0	0
N.S.	1	1.00	1.00	1.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.051	0.098	0.237	0.000	0.000	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	21	7	0	9
N.S.	1	1.00	1.00	1.11	1.00	2.33	0.78	0.00	1.00
time (sec)	N/A	0.025	0.033	0.248	0.189	0.248	0.211	0.000	2.715

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	31	20	23	23
N.S.	1	1.00	1.09	0.91	1.00	1.35	0.87	1.00	1.00
time (sec)	N/A	0.060	1.267	0.236	0.260	0.254	0.453	0.276	2.685

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	33	22	23	23
N.S.	1	1.00	1.09	0.91	1.00	1.43	0.96	1.00	1.00
time (sec)	N/A	0.063	0.116	0.067	0.268	0.248	0.451	0.289	2.794

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	136	149	0	0	0	0	0
N.S.	1	1.00	0.74	0.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.265	0.442	0.229	0.000	0.000	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	109	115	0	0	0	0	0
N.S.	1	1.00	0.76	0.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.214	0.351	0.361	0.000	0.000	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	92	101	0	0	0	0	0
N.S.	1	1.00	0.76	0.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.215	0.334	0.270	0.000	0.000	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	65	67	0	0	0	0	0
N.S.	1	1.00	0.79	0.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.163	0.289	0.232	0.000	0.000	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	46	53	0	0	0	0	0
N.S.	1	1.00	0.85	0.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.111	0.172	0.267	0.000	0.000	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	16	28	26	0	16
N.S.	1	1.00	1.00	1.06	1.00	1.75	1.62	0.00	1.00
time (sec)	N/A	0.041	0.048	0.303	0.193	0.265	0.920	0.000	2.752

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	27	45	24	0	27
N.S.	1	1.00	1.07	0.93	1.00	1.67	0.89	0.00	1.00
time (sec)	N/A	0.092	2.150	0.107	0.263	0.254	1.143	0.000	2.921

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	27	49	26	27	27
N.S.	1	1.00	1.07	0.93	1.00	1.81	0.96	1.00	1.00
time (sec)	N/A	0.098	1.221	0.065	0.272	0.256	0.934	0.286	2.803

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	27	62	24	27	27
N.S.	1	1.00	1.07	0.93	1.00	2.30	0.89	1.00	1.00
time (sec)	N/A	0.113	2.285	0.111	0.265	0.258	1.119	0.277	2.695

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	60	22	0	25
N.S.	1	1.00	1.08	0.92	1.00	2.40	0.88	0.00	1.00
time (sec)	N/A	0.065	2.226	0.115	0.258	0.258	1.126	0.000	2.672

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	59	22	24	24
N.S.	1	1.00	1.08	0.92	1.00	2.46	0.92	1.00	1.00
time (sec)	N/A	0.039	0.104	0.064	0.258	0.254	1.255	0.287	2.569

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	27	63	24	0	27
N.S.	1	1.00	1.07	0.93	1.00	2.33	0.89	0.00	1.00
time (sec)	N/A	0.102	2.323	0.159	0.276	0.255	2.753	0.000	2.618

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	27	67	26	27	27
N.S.	1	1.00	1.07	0.93	1.00	2.48	0.96	1.00	1.00
time (sec)	N/A	0.096	1.964	0.144	0.267	0.265	1.606	0.292	2.668

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	27	44	24	0	27
N.S.	1	1.00	1.07	0.93	1.00	1.63	0.89	0.00	1.00
time (sec)	N/A	0.090	1.094	0.669	0.264	0.247	178.266	0.000	2.708

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	27	27	24	0	27
N.S.	1	1.00	1.07	0.93	1.00	1.00	0.89	0.00	1.00
time (sec)	N/A	0.093	0.623	0.620	0.257	0.256	14.486	0.000	2.810

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	27	27	24	0	27
N.S.	1	1.00	1.07	0.93	1.00	1.00	0.89	0.00	1.00
time (sec)	N/A	0.088	0.164	0.507	0.257	0.258	0.735	0.000	2.911

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	27	44	24	27	27
N.S.	1	1.00	1.07	0.93	1.00	1.63	0.89	1.00	1.00
time (sec)	N/A	0.087	0.554	0.125	0.278	0.251	0.862	0.280	2.793

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	27	62	24	27	27
N.S.	1	1.00	1.07	0.93	1.00	2.30	0.89	1.00	1.00
time (sec)	N/A	0.092	0.768	0.293	0.271	0.259	2.943	0.297	2.763

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	82	106	0	0	0	0	0
N.S.	1	1.00	0.87	1.13	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.136	0.316	0.237	0.000	0.000	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	69	84	0	0	0	0	0
N.S.	1	1.00	0.90	1.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.121	0.245	0.338	0.000	0.000	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	60	0	0	0	0	0
N.S.	1	1.00	1.00	1.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.091	0.369	0.049	0.000	0.000	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	226	21	24	21	21
N.S.	1	1.00	1.11	1.00	11.89	1.11	1.26	1.11	1.11
time (sec)	N/A	0.080	1.660	0.265	0.346	0.251	0.647	0.272	2.835

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	325	36	41	21	21
N.S.	1	1.00	1.11	1.00	17.11	1.89	2.16	1.11	1.11
time (sec)	N/A	0.070	4.023	0.079	0.381	0.252	1.463	0.282	2.910

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	175	633	0	0	0	0	0
N.S.	1	1.00	0.82	2.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.438	0.518	0.405	0.000	0.000	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	82	144	0	0	0	0	0
N.S.	1	1.00	0.88	1.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.337	0.309	0.378	0.000	0.000	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	126	364	0	0	0	0	0
N.S.	1	1.00	0.85	2.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.270	0.328	0.294	0.000	0.000	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	73	138	0	0	0	0	0
N.S.	1	1.00	0.86	1.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.123	0.176	0.319	0.000	0.000	0.000	0.000	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	392	42	24	0	27
N.S.	1	1.00	1.07	0.93	14.52	1.56	0.89	0.00	1.00
time (sec)	N/A	0.156	10.986	0.268	0.471	0.263	1.083	0.000	2.815

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	378	48	26	27	27
N.S.	1	1.00	1.07	0.93	14.00	1.78	0.96	1.00	1.00
time (sec)	N/A	0.126	2.422	0.147	0.436	0.258	1.014	0.295	2.778

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	403	48	26	0	27
N.S.	1	1.00	1.07	0.93	14.93	1.78	0.96	0.00	1.00
time (sec)	N/A	0.091	16.417	0.209	0.515	0.299	1.176	0.000	2.797

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	406	48	26	27	27
N.S.	1	1.00	1.07	0.93	15.04	1.78	0.96	1.00	1.00
time (sec)	N/A	0.094	4.538	0.207	0.520	0.455	1.433	0.287	2.899

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	399	958	0	0	0	0	0
N.S.	1	1.00	1.44	3.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.583	0.673	0.308	0.000	0.000	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	306	369	0	0	0	0	0
N.S.	1	1.00	1.40	1.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.439	0.621	0.680	0.000	0.000	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	295	633	0	0	0	0	0
N.S.	1	1.00	1.38	2.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.425	0.555	0.269	0.000	0.000	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	122	255	0	0	0	0	0
N.S.	1	1.00	0.82	1.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.208	0.414	0.487	0.000	0.000	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	433	42	24	0	27
N.S.	1	1.00	1.07	0.93	16.04	1.56	0.89	0.00	1.00
time (sec)	N/A	0.281	7.858	0.115	0.583	0.270	3.027	0.000	2.721

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	443	48	26	27	27
N.S.	1	1.00	1.07	0.93	16.41	1.78	0.96	1.00	1.00
time (sec)	N/A	0.182	3.679	0.282	0.578	0.294	3.061	0.314	2.738

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	441	48	26	0	27
N.S.	1	1.00	1.07	0.93	16.33	1.78	0.96	0.00	1.00
time (sec)	N/A	0.097	12.482	0.350	0.565	0.246	3.945	0.000	2.743

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	428	48	26	27	27
N.S.	1	1.00	1.07	0.93	15.85	1.78	0.96	1.00	1.00
time (sec)	N/A	0.147	3.560	0.204	0.524	0.250	5.335	0.317	2.904

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	408	1070	0	0	0	0	0
N.S.	1	1.00	1.47	3.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.732	1.108	0.344	0.000	0.000	0.000	0.000	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	413	484	0	0	0	0	0
N.S.	1	1.00	1.47	1.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.617	1.024	0.655	0.000	0.000	0.000	0.000	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	404	958	0	0	0	0	0
N.S.	1	1.00	1.47	3.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.569	0.874	0.284	0.000	0.000	0.000	0.000	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	311	374	0	0	0	0	0
N.S.	1	1.00	1.44	1.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.274	0.601	0.560	0.000	0.000	0.000	0.000	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	480	59	24	0	27
N.S.	1	1.00	1.07	0.93	17.78	2.19	0.89	0.00	1.00
time (sec)	N/A	0.366	9.485	0.128	0.620	0.257	7.711	0.000	2.681

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	491	65	26	27	27
N.S.	1	1.00	1.07	0.93	18.19	2.41	0.96	1.00	1.00
time (sec)	N/A	0.223	3.275	0.125	0.686	0.253	8.279	0.331	2.903

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	492	65	26	0	27
N.S.	1	1.00	1.07	0.93	18.22	2.41	0.96	0.00	1.00
time (sec)	N/A	0.098	12.584	0.141	0.691	0.247	7.788	0.000	2.737

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	491	65	26	27	27
N.S.	1	1.00	1.07	0.93	18.19	2.41	0.96	1.00	1.00
time (sec)	N/A	0.090	3.826	0.523	0.666	0.257	11.445	0.372	2.722

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	158	633	0	0	0	0	0
N.S.	1	1.00	0.77	3.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.277	0.311	0.266	0.000	0.000	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	117	243	0	0	0	0	0
N.S.	1	1.00	0.83	1.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.227	0.240	0.326	0.000	0.000	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	113	364	0	0	0	0	0
N.S.	1	1.00	0.80	2.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.225	0.225	0.259	0.000	0.000	0.000	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	70	135	0	0	0	0	0
N.S.	1	1.00	0.89	1.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.171	0.167	0.283	0.000	0.000	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	60	151	0	0	0	0	0
N.S.	1	1.00	0.82	2.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.114	0.151	0.284	0.000	0.000	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	30	36	0	18
N.S.	1	1.00	1.00	1.06	1.00	1.67	2.00	0.00	1.00
time (sec)	N/A	0.032	0.014	0.254	0.197	0.258	2.024	0.000	2.653

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	416	75	26	0	27
N.S.	1	1.00	1.07	0.93	15.41	2.78	0.96	0.00	1.00
time (sec)	N/A	0.109	6.476	0.147	0.485	0.255	1.696	0.000	2.808

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	427	81	27	27	27
N.S.	1	1.00	1.07	0.93	15.81	3.00	1.00	1.00	1.00
time (sec)	N/A	0.117	2.017	0.080	0.486	0.245	1.479	0.284	2.721

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	475	105	26	0	27
N.S.	1	1.00	1.07	0.93	17.59	3.89	0.96	0.00	1.00
time (sec)	N/A	0.099	53.718	0.302	0.564	0.252	1.831	0.000	2.607

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	447	105	26	27	27
N.S.	1	1.00	1.07	0.93	16.56	3.89	0.96	1.00	1.00
time (sec)	N/A	0.147	3.146	0.123	0.464	0.252	1.798	0.314	2.600

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	461	103	24	0	25
N.S.	1	1.00	1.08	0.92	18.44	4.12	0.96	0.00	1.00
time (sec)	N/A	0.066	54.144	0.145	0.482	0.255	1.836	0.000	2.638

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	446	102	24	24	24
N.S.	1	1.00	1.08	0.92	18.58	4.25	1.00	1.00	1.00
time (sec)	N/A	0.082	2.189	0.063	0.452	0.268	2.045	0.297	2.537

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	477	107	26	0	27
N.S.	1	1.00	1.07	0.93	17.67	3.96	0.96	0.00	1.00
time (sec)	N/A	0.098	35.978	0.203	0.556	0.248	4.461	0.000	2.560

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	485	113	27	27	27
N.S.	1	1.00	1.07	0.93	17.96	4.19	1.00	1.00	1.00
time (sec)	N/A	0.094	15.906	0.186	0.543	0.259	2.952	0.297	2.587

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	0	25	584	137	26	0	27
N.S.	1	1.00	0.00	0.93	21.63	5.07	0.96	0.00	1.00
time (sec)	N/A	0.107	0.000	0.194	0.646	0.264	3.910	0.000	2.702

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	585	137	26	27	27
N.S.	1	1.00	1.07	0.93	21.67	5.07	0.96	1.00	1.00
time (sec)	N/A	0.102	10.246	0.198	0.562	0.248	3.739	0.340	2.625

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	0	23	568	135	24	0	25
N.S.	1	1.00	0.00	0.92	22.72	5.40	0.96	0.00	1.00
time (sec)	N/A	0.063	0.000	0.179	0.629	0.264	3.643	0.000	2.529

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	554	134	24	24	24
N.S.	1	1.00	1.08	0.92	23.08	5.58	1.00	1.00	1.00
time (sec)	N/A	0.093	4.409	0.178	0.497	0.253	3.826	0.289	2.582

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	0	25	584	139	26	0	27
N.S.	1	1.00	0.00	0.93	21.63	5.15	0.96	0.00	1.00
time (sec)	N/A	0.092	0.000	0.328	0.617	0.260	8.170	0.000	2.677

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	592	145	27	27	27
N.S.	1	1.00	1.07	0.93	21.93	5.37	1.00	1.00	1.00
time (sec)	N/A	0.095	11.459	0.427	0.623	0.256	6.013	0.314	2.635

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	537	58	0	0	27
N.S.	1	1.00	1.07	0.93	19.89	2.15	0.00	0.00	1.00
time (sec)	N/A	0.095	1.239	0.682	1.276	0.258	0.000	0.000	2.736

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	480	41	26	0	27
N.S.	1	1.00	1.07	0.93	17.78	1.52	0.96	0.00	1.00
time (sec)	N/A	0.095	0.752	0.562	1.092	0.247	34.669	0.000	2.824

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	424	41	26	0	27
N.S.	1	1.00	1.07	0.93	15.70	1.52	0.96	0.00	1.00
time (sec)	N/A	0.081	0.196	0.547	0.850	0.252	1.574	0.000	2.661

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	441	73	26	27	27
N.S.	1	1.00	1.07	0.93	16.33	2.70	0.96	1.00	1.00
time (sec)	N/A	0.105	0.593	0.145	0.525	0.253	2.842	0.322	2.659

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	506	105	26	27	27
N.S.	1	1.00	1.07	0.93	18.74	3.89	0.96	1.00	1.00
time (sec)	N/A	0.094	0.844	0.340	0.740	0.262	11.888	0.321	2.707

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	613	137	26	27	27
N.S.	1	1.00	1.07	0.93	22.70	5.07	0.96	1.00	1.00
time (sec)	N/A	0.094	1.115	0.361	0.834	0.256	82.637	0.340	2.767

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	23	12	0	11
N.S.	1	1.00	1.00	0.92	0.85	1.77	0.92	0.00	0.85
time (sec)	N/A	0.025	0.011	0.251	0.192	0.242	0.430	0.000	2.681

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	254	254	232	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.804	0.768	0.000	0.000	0.000	0.000	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	335	335	436	0	0	0	0	0	0
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.880	0.684	0.000	0.000	0.000	0.000	0.000	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	236	236	227	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.495	0.402	0.000	0.000	0.000	0.000	0.000	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	228	228	295	0	0	0	0	0	0
N.S.	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.324	0.643	0.000	0.000	0.000	0.000	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	0	83	0	26
N.S.	1	1.00	1.08	0.92	1.00	0.00	3.19	0.00	1.00
time (sec)	N/A	0.512	1.346	0.131	0.719	0.000	4.457	0.000	2.849

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	474	474	462	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.966	0.889	0.000	0.000	0.000	0.000	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	457	457	577	0	0	0	0	0	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.198	1.586	0.000	0.000	0.000	0.000	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	358	358	351	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.844	0.571	0.000	0.000	0.000	0.000	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	346	346	440	0	0	0	0	0	0
N.S.	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.449	1.593	0.000	0.000	0.000	0.000	0.000	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	0	131	0	28
N.S.	1	1.00	1.07	0.93	1.00	0.00	4.68	0.00	1.00
time (sec)	N/A	0.970	1.089	0.158	0.745	0.000	5.971	0.000	2.835

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	319	319	142	0	0	0	0	0	0
N.S.	1	1.00	0.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.317	0.183	0.000	0.000	0.000	0.000	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	186	186	104	0	0	0	0	0	0
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.138	0.108	0.000	0.000	0.000	0.000	0.000	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	43	36	0	0	0	0	0
N.S.	1	1.00	1.02	0.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.029	0.061	0.263	0.000	0.000	0.000	0.000	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	21	0	22	21	21
N.S.	1	1.00	1.09	0.83	0.91	0.00	0.96	0.91	0.91
time (sec)	N/A	0.068	1.729	0.283	0.385	0.000	1.428	0.456	2.617

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	21	0	22	21	21
N.S.	1	1.00	1.09	0.83	0.91	0.00	0.96	0.91	0.91
time (sec)	N/A	0.134	2.160	0.341	0.396	0.000	23.511	0.476	2.798

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	449	449	186	0	0	0	0	0	0
N.S.	1	1.00	0.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.410	0.361	0.000	0.000	0.000	0.000	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	271	271	126	0	0	0	0	0	0
N.S.	1	1.00	0.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.198	0.297	0.000	0.000	0.000	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	43	36	0	0	0	0	0
N.S.	1	1.00	1.02	0.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	0.057	0.252	0.000	0.000	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	21	0	22	21	21
N.S.	1	1.00	1.09	0.83	0.91	0.00	0.96	0.91	0.91
time (sec)	N/A	0.066	1.910	0.270	0.395	0.000	9.759	0.649	2.792

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	514	514	201	0	0	0	0	0	0
N.S.	1	1.00	0.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.552	0.373	0.000	0.000	0.000	0.000	0.000	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	298	298	135	0	0	0	0	0	0
N.S.	1	1.00	0.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.218	0.319	0.000	0.000	0.000	0.000	0.000	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	43	36	0	0	0	0	0
N.S.	1	1.00	1.02	0.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.030	0.057	0.261	0.000	0.000	0.000	0.000	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	21	0	0	21	21
N.S.	1	1.00	1.09	0.83	0.91	0.00	0.00	0.91	0.91
time (sec)	N/A	0.073	1.882	0.253	0.399	0.000	0.000	0.656	2.623

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	309	309	156	0	0	0	0	0	0
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.280	0.145	0.000	0.000	0.000	0.000	0.000	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	176	176	110	0	0	0	0	0	0
N.S.	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.116	0.068	0.000	0.000	0.000	0.000	0.000	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	0	20	0	0	0
N.S.	1	1.00	1.00	0.87	0.00	0.51	0.00	0.00	0.00
time (sec)	N/A	0.023	0.022	0.266	0.000	0.241	0.000	0.000	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	20	0	19	20	20
N.S.	1	1.00	1.09	0.82	0.91	0.00	0.86	0.91	0.91
time (sec)	N/A	0.063	1.312	0.293	0.392	0.000	1.392	0.562	2.888

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	20	0	19	20	20
N.S.	1	1.00	1.09	0.82	0.91	0.00	0.86	0.91	0.91
time (sec)	N/A	0.125	1.365	0.348	0.384	0.000	23.343	0.560	2.761

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	433	433	210	0	0	0	0	0	0
N.S.	1	1.00	0.48	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.396	0.300	0.000	0.000	0.000	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	259	259	133	0	0	0	0	0	0
N.S.	1	1.00	0.51	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.204	0.252	0.000	0.000	0.000	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	0	20	0	0	0
N.S.	1	1.00	1.00	0.87	0.00	0.51	0.00	0.00	0.00
time (sec)	N/A	0.022	0.032	0.256	0.000	0.249	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	20	0	19	20	20
N.S.	1	1.00	1.09	0.82	0.91	0.00	0.86	0.91	0.91
time (sec)	N/A	0.057	1.344	0.263	0.386	0.000	9.844	0.854	2.661

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	33	33	34	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.060	0.061	0.000	0.000	0.000	0.000	0.000	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	396	396	197	0	0	0	0	0	0
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.203	0.278	0.000	0.000	0.000	0.000	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	264	264	141	0	0	0	0	0	0
N.S.	1	1.00	0.53	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.144	0.173	0.000	0.000	0.000	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	156	156	101	0	0	0	0	0	0
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.100	0.108	0.000	0.000	0.000	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	41	36	0	0	0	0	0
N.S.	1	1.00	1.02	0.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.030	0.055	0.255	0.000	0.000	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	21	0	24	21	21
N.S.	1	1.00	1.09	0.83	0.91	0.00	1.04	0.91	0.91
time (sec)	N/A	0.031	1.832	0.265	0.410	0.000	3.699	0.498	2.743

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	21	0	24	21	21
N.S.	1	1.00	1.09	0.83	0.91	0.00	1.04	0.91	0.91
time (sec)	N/A	0.031	2.068	0.342	0.393	0.000	58.041	0.495	2.722

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	391	391	399	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.235	0.893	0.000	0.000	0.000	0.000	0.000	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	256	256	225	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.164	0.376	0.000	0.000	0.000	0.000	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	152	115	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.094	0.225	0.000	0.000	0.000	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	41	36	0	57	0	0	0
N.S.	1	1.00	1.02	0.90	0.00	1.42	0.00	0.00	0.00
time (sec)	N/A	0.031	0.057	0.263	0.000	0.241	0.000	0.000	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	21	0	24	21	21
N.S.	1	1.00	1.09	0.83	0.91	0.00	1.04	0.91	0.91
time (sec)	N/A	0.073	1.914	0.260	0.412	0.000	25.933	0.347	2.693

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	21	0	0	21	21
N.S.	1	1.00	1.09	0.83	0.91	0.00	0.00	0.91	0.91
time (sec)	N/A	0.075	1.994	0.334	0.400	0.000	0.000	0.359	2.766

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	296	296	262	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.288	0.354	0.000	0.000	0.000	0.000	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	182	182	122	0	0	0	0	0	0
N.S.	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.092	0.154	0.000	0.000	0.000	0.000	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	43	36	0	57	0	0	0
N.S.	1	1.00	1.02	0.86	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.033	0.056	0.268	0.000	0.249	0.000	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	21	0	0	21	21
N.S.	1	1.00	1.09	0.83	0.91	0.00	0.00	0.91	0.91
time (sec)	N/A	0.066	1.854	0.267	0.405	0.000	0.000	0.353	2.541

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	21	0	0	21	21
N.S.	1	1.00	1.09	0.83	0.91	0.00	0.00	0.91	0.91
time (sec)	N/A	0.066	2.024	0.307	0.407	0.000	0.000	0.384	2.655

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	235	235	171	0	0	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.253	0.750	0.000	0.000	0.000	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	355	355	229	0	0	0	0	0	0
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.264	0.810	0.000	0.000	0.000	0.000	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	235	235	160	0	0	0	0	0	0
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.164	0.497	0.000	0.000	0.000	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	28	26	0	28
N.S.	1	1.00	1.07	0.93	1.00	1.00	0.93	0.00	1.00
time (sec)	N/A	0.369	0.298	0.222	0.406	0.262	2.233	0.000	2.618

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	28	27	0	28
N.S.	1	1.00	1.07	0.93	1.00	1.00	0.96	0.00	1.00
time (sec)	N/A	0.243	0.334	0.226	0.406	0.264	4.094	0.000	2.600

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	616	616	429	0	0	0	0	0	0
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.449	2.342	0.000	0.000	0.000	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	542	542	390	0	0	0	0	0	0
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.370	1.498	0.000	0.000	0.000	0.000	0.000	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	420	420	288	0	0	0	0	0	0
N.S.	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.260	1.092	0.000	0.000	0.000	0.000	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	28	26	0	28
N.S.	1	1.00	1.07	0.93	1.00	1.00	0.93	0.00	1.00
time (sec)	N/A	0.716	0.362	0.211	0.405	0.290	125.890	0.000	2.557

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	28	0	0	28
N.S.	1	1.00	1.07	0.93	1.00	1.00	0.00	0.00	1.00
time (sec)	N/A	0.528	0.946	0.221	0.403	0.262	0.000	0.000	2.562

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	816	816	667	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.554	5.616	0.000	0.000	0.000	0.000	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	745	745	685	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.470	2.329	0.000	0.000	0.000	0.000	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	632	632	529	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.349	3.996	0.000	0.000	0.000	0.000	0.000	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	53	0	0	28
N.S.	1	1.00	1.07	0.93	1.00	1.89	0.00	0.00	1.00
time (sec)	N/A	1.171	0.398	0.189	0.426	0.260	0.000	0.000	2.440

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	53	0	0	28
N.S.	1	1.00	1.07	0.93	1.00	1.89	0.00	0.00	1.00
time (sec)	N/A	0.897	0.654	0.225	0.432	0.260	0.000	0.000	2.557

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	23	22	23	23
N.S.	1	1.00	1.09	0.91	1.00	1.00	0.96	1.00	1.00
time (sec)	N/A	0.076	0.650	0.129	0.374	0.254	3.799	0.412	2.616

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	113	113	100	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.176	0.158	0.000	0.000	0.000	0.000	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	86	0	0	0	0	0	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.143	0.212	0.000	0.000	0.000	0.000	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	49	49	43	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.080	0.101	0.000	0.000	0.000	0.000	0.000	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	17	83	34	29	33
N.S.	1	1.00	1.00	1.06	1.00	4.88	2.00	1.71	1.94
time (sec)	N/A	0.028	0.011	0.272	0.194	0.268	0.357	0.275	2.933

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	31	20	23	23
N.S.	1	1.00	1.09	0.91	1.00	1.35	0.87	1.00	1.00
time (sec)	N/A	0.070	5.709	0.108	0.365	0.265	0.678	0.428	2.683

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	33	22	23	23
N.S.	1	1.00	1.09	0.91	1.00	1.43	0.96	1.00	1.00
time (sec)	N/A	0.068	1.858	0.108	0.395	0.266	1.044	0.422	2.732

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	416	416	361	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.428	2.791	0.000	0.000	0.000	0.000	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	304	304	273	0	0	0	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.242	2.825	0.000	0.000	0.000	0.000	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	147	233	0	0	0	0	0	0
N.S.	1	1.00	1.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.132	1.261	0.000	0.000	0.000	0.000	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	158	158	227	0	0	0	0	0	0
N.S.	1	1.00	1.44	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.206	1.520	0.000	0.000	0.000	0.000	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	181	181	283	0	0	0	0	0	0
N.S.	1	1.00	1.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.282	2.550	0.000	0.000	0.000	0.000	0.000	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	187	187	141	0	219	548	0	0	0
N.S.	1	1.00	0.75	0.00	1.17	2.93	0.00	0.00	0.00
time (sec)	N/A	0.222	1.510	0.000	0.230	0.323	0.000	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	459	459	683	0	0	0	0	0	0
N.S.	1	1.00	1.49	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.308	3.170	0.000	0.000	0.000	0.000	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	247	247	352	0	0	0	0	0	0
N.S.	1	1.00	1.43	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.182	2.377	0.000	0.000	0.000	0.000	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	304	304	273	0	0	0	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.248	2.940	0.000	0.000	0.000	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	266	266	344	0	0	0	0	0	0
N.S.	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.330	5.473	0.000	0.000	0.000	0.000	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	284	284	514	0	0	0	0	0	0
N.S.	1	1.00	1.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.332	7.011	0.000	0.000	0.000	0.000	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	364	364	706	0	0	0	0	0	0
N.S.	1	1.00	1.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.271	9.809	0.000	0.000	0.000	0.000	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	344	344	481	0	0	0	0	0	0
N.S.	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.216	2.762	0.000	0.000	0.000	0.000	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	459	459	683	0	0	0	0	0	0
N.S.	1	1.00	1.49	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.316	3.162	0.000	0.000	0.000	0.000	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	416	416	565	0	0	0	0	0	0
N.S.	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.393	2.569	0.000	0.000	0.000	0.000	0.000	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	381	381	465	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.437	7.175	0.000	0.000	0.000	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	518	518	779	0	0	0	0	0	0
N.S.	1	1.00	1.50	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.279	9.864	0.000	0.000	0.000	0.000	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	472	472	1005	0	0	0	0	0	0
N.S.	1	1.00	2.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.314	12.468	0.000	0.000	0.000	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	381	381	465	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.470	7.342	0.000	0.000	0.000	0.000	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	266	266	344	0	0	0	0	0	0
N.S.	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.323	5.552	0.000	0.000	0.000	0.000	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	158	158	227	0	0	0	0	0	0
N.S.	1	1.00	1.44	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.212	1.760	0.000	0.000	0.000	0.000	0.000	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	113	0	32	0	0	0	0
N.S.	1	1.00	1.92	0.00	0.54	0.00	0.00	0.00	0.00
time (sec)	N/A	0.120	1.225	0.000	0.211	0.000	0.000	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	111	111	113	0	98	443	0	0	0
N.S.	1	1.00	1.02	0.00	0.88	3.99	0.00	0.00	0.00
time (sec)	N/A	0.164	0.874	0.000	0.304	0.305	0.000	0.000	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	295	295	143	0	233	576	0	0	0
N.S.	1	1.00	0.48	0.00	0.79	1.95	0.00	0.00	0.00
time (sec)	N/A	0.231	1.020	0.000	0.305	0.341	0.000	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	517	517	781	0	0	0	0	0	0
N.S.	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.297	10.367	0.000	0.000	0.000	0.000	0.000	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	283	283	515	0	0	0	0	0	0
N.S.	1	1.00	1.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.330	7.292	0.000	0.000	0.000	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	180	180	285	0	0	0	0	0	0
N.S.	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.283	2.711	0.000	0.000	0.000	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	112	112	94	0	98	443	0	0	0
N.S.	1	1.00	0.84	0.00	0.88	3.96	0.00	0.00	0.00
time (sec)	N/A	0.176	1.094	0.000	0.298	0.302	0.000	0.000	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	118	0	82	0	0	0	0
N.S.	1	1.00	1.15	0.00	0.80	0.00	0.00	0.00	0.00
time (sec)	N/A	0.138	1.193	0.000	0.210	0.000	0.000	0.000	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	282	282	201	0	237	0	0	0	0
N.S.	1	1.00	0.71	0.00	0.84	0.00	0.00	0.00	0.00
time (sec)	N/A	0.207	1.493	0.000	0.229	0.000	0.000	0.000	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	470	470	1083	0	0	0	0	0	0
N.S.	1	1.00	2.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.308	15.416	0.000	0.000	0.000	0.000	0.000	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	362	362	706	0	0	0	0	0	0
N.S.	1	1.00	1.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.265	10.023	0.000	0.000	0.000	0.000	0.000	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	185	185	131	0	220	548	0	0	0
N.S.	1	1.00	0.71	0.00	1.19	2.96	0.00	0.00	0.00
time (sec)	N/A	0.207	1.749	0.000	0.213	0.326	0.000	0.000	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	294	294	139	0	232	576	0	0	0
N.S.	1	1.00	0.47	0.00	0.79	1.96	0.00	0.00	0.00
time (sec)	N/A	0.239	1.059	0.000	0.319	0.332	0.000	0.000	0.000

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	282	282	202	0	237	0	0	0	0
N.S.	1	1.00	0.72	0.00	0.84	0.00	0.00	0.00	0.00
time (sec)	N/A	0.220	1.431	0.000	0.235	0.000	0.000	0.000	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	203	203	193	0	159	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.78	0.00	0.00	0.00	0.00
time (sec)	N/A	0.168	1.260	0.000	0.227	0.000	0.000	0.000	0.000

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	680	680	890	0	0	0	0	0	0
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.735	3.597	0.000	0.000	0.000	0.000	0.000	0.000

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	508	508	705	0	0	0	0	0	0
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.438	3.234	0.000	0.000	0.000	0.000	0.000	0.000

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	244	244	352	0	0	0	0	0	0
N.S.	1	1.00	1.44	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.232	1.877	0.000	0.000	0.000	0.000	0.000	0.000

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	259	259	315	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.349	2.712	0.000	0.000	0.000	0.000	0.000	0.000

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	544	544	594	0	0	0	0	0	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.726	5.325	0.000	0.000	0.000	0.000	0.000	0.000

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	518	518	783	0	0	0	0	0	0
N.S.	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.799	9.855	0.000	0.000	0.000	0.000	0.000	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	774	774	1084	0	0	0	0	0	0
N.S.	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.578	4.435	0.000	0.000	0.000	0.000	0.000	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	396	396	524	0	0	0	0	0	0
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.326	3.191	0.000	0.000	0.000	0.000	0.000	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	508	508	705	0	0	0	0	0	0
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.427	3.091	0.000	0.000	0.000	0.000	0.000	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	436	436	532	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.457	8.523	0.000	0.000	0.000	0.000	0.000	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	752	752	1174	0	0	0	0	0	0
N.S.	1	1.00	1.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.780	14.851	0.000	0.000	0.000	0.000	0.000	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F(-1)	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	580	580	1609	0	0	0	0	0	0
N.S.	1	1.00	2.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.834	17.155	0.000	0.000	0.000	0.000	0.000	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	548	548	735	0	0	0	0	0	0
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.420	3.945	0.000	0.000	0.000	0.000	0.000	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	774	774	1084	0	0	0	0	0	0
N.S.	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.578	4.634	0.000	0.000	0.000	0.000	0.000	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	680	680	890	0	0	0	0	0	0
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.783	3.725	0.000	0.000	0.000	0.000	0.000	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	615	615	723	0	0	0	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.539	14.047	0.000	0.000	0.000	0.000	0.000	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	972	972	2492	0	0	0	0	0	0
N.S.	1	1.00	2.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.928	23.484	0.000	0.000	0.000	0.000	0.000	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F(-1)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	790	790	2622	0	0	0	0	0	0
N.S.	1	1.00	3.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.987	24.699	0.000	0.000	0.000	0.000	0.000	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	615	615	723	0	0	0	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.548	13.876	0.000	0.000	0.000	0.000	0.000	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	436	436	529	0	0	0	0	0	0
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.428	8.546	0.000	0.000	0.000	0.000	0.000	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	259	259	315	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.345	2.638	0.000	0.000	0.000	0.000	0.000	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	168	0	53	0	0	0	0
N.S.	1	1.00	2.85	0.00	0.90	0.00	0.00	0.00	0.00
time (sec)	N/A	0.191	2.258	0.000	0.228	0.000	0.000	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	464	464	508	0	0	0	0	0	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.539	3.263	0.000	0.000	0.000	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	942	942	524	0	0	0	0	0	0
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.929	6.914	0.000	0.000	0.000	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	972	972	2143	0	0	0	0	0	0
N.S.	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.929	25.043	0.000	0.000	0.000	0.000	0.000	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	752	752	1346	0	0	0	0	0	0
N.S.	1	1.00	1.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.759	17.842	0.000	0.000	0.000	0.000	0.000	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	544	544	530	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.685	6.688	0.000	0.000	0.000	0.000	0.000	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	464	464	511	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.510	3.163	0.000	0.000	0.000	0.000	0.000	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	224	224	488	0	0	0	0	0	0
N.S.	1	1.00	2.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.300	3.046	0.000	0.000	0.000	0.000	0.000	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	743	743	754	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.640	9.913	0.000	0.000	0.000	0.000	0.000	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F(-1)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	794	794	2552	0	0	0	0	0	0
N.S.	1	1.00	3.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.981	24.350	0.000	0.000	0.000	0.000	0.000	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F(-1)	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	584	584	1617	0	0	0	0	0	0
N.S.	1	1.00	2.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.829	17.776	0.000	0.000	0.000	0.000	0.000	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	522	522	788	0	0	0	0	0	0
N.S.	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.779	10.072	0.000	0.000	0.000	0.000	0.000	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	942	942	528	0	0	0	0	0	0
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.888	6.957	0.000	0.000	0.000	0.000	0.000	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	743	743	757	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.623	10.009	0.000	0.000	0.000	0.000	0.000	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	386	386	642	0	0	0	0	0	0
N.S.	1	1.00	1.66	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.374	8.665	0.000	0.000	0.000	0.000	0.000	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	260	425	415	333	593	0	0
N.S.	1	1.00	0.83	1.36	1.33	1.07	1.90	0.00	0.00
time (sec)	N/A	0.247	0.213	0.295	0.199	0.270	1.202	0.000	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	187	295	287	241	389	0	0
N.S.	1	1.00	0.85	1.33	1.30	1.09	1.76	0.00	0.00
time (sec)	N/A	0.192	0.179	0.290	0.189	0.260	0.632	0.000	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	125	188	180	163	240	0	0
N.S.	1	1.00	0.85	1.28	1.22	1.11	1.63	0.00	0.00
time (sec)	N/A	0.103	0.118	0.366	0.186	0.266	0.370	0.000	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	71	97	91	94	109	0	0
N.S.	1	1.00	0.88	1.20	1.12	1.16	1.35	0.00	0.00
time (sec)	N/A	0.057	0.046	0.015	0.184	0.260	0.222	0.000	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	31	30	43	26	41	28
N.S.	1	1.00	1.00	1.03	1.00	1.43	0.87	1.37	0.93
time (sec)	N/A	0.010	0.008	0.051	0.177	0.253	0.070	0.271	2.584

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	485	485	434	224	0	0	0	0	0
N.S.	1	1.00	0.89	0.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.564	0.333	7.909	0.000	0.000	0.000	0.000	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	707	707	622	848	0	0	0	0	0
N.S.	1	1.00	0.88	1.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.763	1.317	7.960	0.000	0.000	0.000	0.000	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	559	559	443	683	684	586	989	0	0
N.S.	1	1.00	0.79	1.22	1.22	1.05	1.77	0.00	0.00
time (sec)	N/A	0.656	0.336	0.617	0.201	0.272	1.024	0.000	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	289	431	429	380	595	0	0
N.S.	1	1.00	0.88	1.31	1.30	1.16	1.81	0.00	0.00
time (sec)	N/A	0.400	0.238	0.439	0.206	0.261	0.587	0.000	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	164	215	218	209	279	0	0
N.S.	1	1.00	1.07	1.41	1.42	1.37	1.82	0.00	0.00
time (sec)	N/A	0.203	0.144	0.221	0.198	0.276	0.315	0.000	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	74	72	72	96	82	111	0
N.S.	1	1.00	1.61	1.57	1.57	2.09	1.78	2.41	0.00
time (sec)	N/A	0.048	0.087	0.112	0.182	0.260	0.108	0.403	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	739	739	985	0	0	0	0	0	0
N.S.	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.938	0.551	0.000	0.000	0.000	0.000	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	670	670	444	654	0	0	0	0	0
N.S.	1	1.00	0.66	0.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.844	0.754	1.757	0.000	0.000	0.000	0.000	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	388	388	253	380	0	0	0	0	0
N.S.	1	1.00	0.65	0.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.479	0.431	0.824	0.000	0.000	0.000	0.000	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	126	178	0	0	0	0	0
N.S.	1	1.00	0.70	0.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.256	0.185	0.584	0.000	0.000	0.000	0.000	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	45	56	0	0	0	0	0
N.S.	1	1.00	0.83	1.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.052	0.057	0.082	0.000	0.000	0.000	0.000	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	29	17	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.45	0.85	1.10	1.10
time (sec)	N/A	0.031	0.689	0.477	0.245	0.265	3.577	0.290	2.619

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	53	19	22	22
N.S.	1	1.00	1.10	1.00	1.10	2.65	0.95	1.10	1.10
time (sec)	N/A	0.035	2.876	0.499	0.257	0.242	65.608	0.293	2.553

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	495	495	356	1036	0	0	0	0	0
N.S.	1	1.00	0.72	2.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.556	2.179	0.971	0.000	0.000	0.000	0.000	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	190	438	0	0	0	0	0
N.S.	1	1.00	0.77	1.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.307	0.913	0.651	0.000	0.000	0.000	0.000	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	71	118	0	0	0	0	0
N.S.	1	1.00	0.84	1.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.118	0.190	0.095	0.000	0.000	0.000	0.000	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	766	57	19	22	22
N.S.	1	1.00	1.10	1.00	38.30	2.85	0.95	1.10	1.10
time (sec)	N/A	0.030	12.000	0.480	0.906	0.255	33.777	0.291	2.663

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	1027	98	0	22	22
N.S.	1	1.00	1.10	1.00	51.35	4.90	0.00	1.10	1.10
time (sec)	N/A	0.026	25.139	0.750	1.177	0.252	0.000	0.298	2.549

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	672	672	535	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.261	4.818	0.000	0.000	0.000	0.000	0.000	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	322	322	319	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.611	2.099	0.000	0.000	0.000	0.000	0.000	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	102	101	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.166	0.162	0.000	0.000	0.000	0.000	0.000	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	0	20	0	0	19	22	22
N.S.	1	1.00	0.00	0.91	0.00	0.00	0.86	1.00	1.00
time (sec)	N/A	0.042	0.000	0.632	0.000	0.000	0.822	1.022	2.663

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	0	20	22	0	20	22	22
N.S.	1	1.00	0.00	0.91	1.00	0.00	0.91	1.00	1.00
time (sec)	N/A	0.039	0.000	1.016	0.401	0.000	14.300	1.043	2.697

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	427	427	770	0	0	0	0	0	0
N.S.	1	1.00	1.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.851	2.287	0.000	0.000	0.000	0.000	0.000	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	251	0	0	0	0	0	0
N.S.	1	1.00	1.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.194	0.390	0.000	0.000	0.000	0.000	0.000	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	0	20	0	0	19	22	22
N.S.	1	1.00	0.00	0.91	0.00	0.00	0.86	1.00	1.00
time (sec)	N/A	0.050	0.000	0.560	0.000	0.000	9.086	1.526	2.596

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	0	20	22	0	20	22	22
N.S.	1	1.00	0.00	0.91	1.00	0.00	0.91	1.00	1.00
time (sec)	N/A	0.047	0.000	0.837	0.497	0.000	95.972	1.533	2.617

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	608	608	530	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.822	0.793	0.000	0.000	0.000	0.000	0.000	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	287	287	218	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.364	0.428	0.000	0.000	0.000	0.000	0.000	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	101	0	0	0	0	0	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.082	0.067	0.000	0.000	0.000	0.000	0.000	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	0	20	22	22
N.S.	1	1.00	1.09	0.91	1.00	0.00	0.91	1.00	1.00
time (sec)	N/A	0.042	0.165	0.559	0.379	0.000	2.101	1.172	2.643

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	0	22	22	22
N.S.	1	1.00	1.09	0.91	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.040	0.216	0.895	0.396	0.000	41.199	1.130	2.634

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	349	349	303	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.462	0.949	0.000	0.000	0.000	0.000	0.000	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	116	116	137	0	0	0	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.171	0.181	0.000	0.000	0.000	0.000	0.000	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	0	20	22	22
N.S.	1	1.00	1.09	0.91	1.00	0.00	0.91	1.00	1.00
time (sec)	N/A	0.055	0.172	0.581	0.376	0.000	7.802	0.395	2.709

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	0	22	22	22
N.S.	1	1.00	1.09	0.91	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.052	0.227	0.796	0.403	0.000	139.898	0.409	2.763

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	20	19	20	20
N.S.	1	1.00	1.10	0.90	0.00	1.00	0.95	1.00	1.00
time (sec)	N/A	0.019	3.629	0.552	0.000	0.247	1.439	0.327	2.969

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	20	19	20	20
N.S.	1	1.00	1.10	0.90	0.00	1.00	0.95	1.00	1.00
time (sec)	N/A	0.023	2.796	0.297	0.000	0.263	0.878	0.298	2.864

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	75	0	0	326	0	0	0
N.S.	1	1.00	1.07	0.00	0.00	4.66	0.00	0.00	0.00
time (sec)	N/A	0.074	0.084	0.000	0.000	0.280	0.000	0.000	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	146	146	139	0	0	738	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	5.05	0.00	0.00	0.00
time (sec)	N/A	0.129	0.244	0.000	0.000	0.318	0.000	0.000	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	227	227	191	0	0	1354	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	5.96	0.00	0.00	0.00
time (sec)	N/A	0.606	0.294	0.000	0.000	0.389	0.000	0.000	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	34	20	22	22
N.S.	1	1.00	1.09	0.91	0.00	1.55	0.91	1.00	1.00
time (sec)	N/A	0.037	13.099	0.178	0.000	0.254	0.998	0.374	2.684

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	34	20	22	22
N.S.	1	1.00	1.09	0.91	0.00	1.55	0.91	1.00	1.00
time (sec)	N/A	0.034	8.238	0.199	0.000	0.276	0.828	0.325	2.724

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	54	20	22	22
N.S.	1	1.00	1.09	0.91	0.00	2.45	0.91	1.00	1.00
time (sec)	N/A	0.032	2.760	0.683	0.000	0.267	4.451	0.330	2.641

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	99	65	20	22	22
N.S.	1	1.00	1.09	0.91	4.50	2.95	0.91	1.00	1.00
time (sec)	N/A	0.032	5.009	0.828	0.497	0.263	59.513	0.331	2.670

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	22	19	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.00	0.86	1.00	1.00
time (sec)	N/A	0.034	1.017	0.595	0.253	0.248	0.368	0.297	2.612

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	39	20	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.77	0.91	1.00	1.00
time (sec)	N/A	0.036	0.809	0.645	0.284	0.244	0.619	0.300	2.729

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	63	20	22	22
N.S.	1	1.00	1.09	0.91	1.00	2.86	0.91	1.00	1.00
time (sec)	N/A	0.038	1.396	0.662	0.258	0.243	1.698	0.313	2.530

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	87	20	22	22
N.S.	1	1.00	1.09	0.91	1.00	3.95	0.91	1.00	1.00
time (sec)	N/A	0.039	2.780	0.671	0.257	0.243	7.797	0.314	2.619

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	575	36	20	22	22
N.S.	1	1.00	1.09	0.91	26.14	1.64	0.91	1.00	1.00
time (sec)	N/A	0.033	3.566	0.654	0.660	0.274	0.649	0.353	2.653

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	591	67	22	22	22
N.S.	1	1.00	1.09	0.91	26.86	3.05	1.00	1.00	1.00
time (sec)	N/A	0.033	6.732	0.663	0.783	0.265	1.233	0.303	2.655

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	864	108	22	22	22
N.S.	1	1.00	1.09	0.91	39.27	4.91	1.00	1.00	1.00
time (sec)	N/A	0.037	13.987	0.702	1.328	0.260	4.716	0.329	2.792

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	1123	149	22	22	22
N.S.	1	1.00	1.09	0.91	51.05	6.77	1.00	1.00	1.00
time (sec)	N/A	0.036	22.713	0.668	1.908	0.272	27.889	0.340	2.627

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [250] had the largest ratio of [.7308000000000000005]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	5	1.00	22	0.227
2	A	6	6	1.00	22	0.273
3	A	5	5	1.00	22	0.227
4	A	4	3	1.00	20	0.150
5	A	5	4	1.00	19	0.210
6	A	8	8	1.00	22	0.364
7	A	6	7	1.00	22	0.318
8	A	8	8	1.00	22	0.364
9	A	6	7	1.00	22	0.318
10	A	6	6	1.00	24	0.250
11	A	7	8	1.00	24	0.333
12	A	5	5	1.00	24	0.208
13	A	5	3	1.00	22	0.136
14	A	5	5	1.00	21	0.238
15	A	12	8	1.00	24	0.333
16	A	7	7	1.00	24	0.292
17	A	12	10	1.00	24	0.417
18	A	7	8	1.00	24	0.333
19	A	5	5	1.00	24	0.208
20	A	8	7	1.00	24	0.292
21	A	5	5	1.00	24	0.208
22	A	6	3	1.00	22	0.136
23	A	5	5	1.00	21	0.238
24	A	17	8	1.00	24	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	7	7	1.00	24	0.292
26	A	17	10	1.00	24	0.417
27	A	8	8	1.00	24	0.333
28	A	12	8	1.00	24	0.333
29	A	8	8	1.00	24	0.333
30	A	8	6	1.00	24	0.250
31	A	5	5	1.00	22	0.227
32	A	6	4	1.00	21	0.190
33	A	7	5	1.00	24	0.208
34	A	10	8	1.00	24	0.333
35	A	9	7	1.00	24	0.292
36	A	15	9	1.00	24	0.375
37	A	12	9	1.00	24	0.375
38	A	8	8	1.00	24	0.333
39	A	8	6	1.00	24	0.250
40	A	2	2	1.00	22	0.091
41	A	8	6	1.00	21	0.286
42	A	9	7	1.00	24	0.292
43	A	13	11	1.00	24	0.458
44	A	12	9	1.00	24	0.375
45	A	19	12	1.00	24	0.500
46	A	12	8	1.00	24	0.333
47	A	4	3	1.00	24	0.125
48	A	10	7	1.00	24	0.292
49	A	3	3	1.00	22	0.136
50	A	10	6	1.00	21	0.286
51	A	12	8	1.00	24	0.333
52	A	16	11	1.00	24	0.458
53	A	16	10	1.00	24	0.417
54	A	23	12	1.00	24	0.500
55	A	3	4	1.00	26	0.154
56	A	5	4	1.00	26	0.154
57	A	2	1	1.00	24	0.042
58	A	3	3	1.00	23	0.130
59	A	8	6	1.00	26	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	3	3	1.00	26	0.115
61	A	8	6	1.00	26	0.231
62	A	3	2	1.00	26	0.077
63	A	4	5	1.00	26	0.192
64	A	8	6	1.00	26	0.231
65	A	3	2	1.00	24	0.083
66	A	6	5	1.00	23	0.217
67	A	10	7	1.00	26	0.269
68	A	6	5	1.00	26	0.192
69	A	11	8	1.00	26	0.308
70	A	6	5	1.00	26	0.192
71	A	4	5	1.00	26	0.192
72	A	12	8	1.00	26	0.308
73	A	3	2	1.00	24	0.083
74	A	8	6	1.00	23	0.261
75	A	13	8	1.00	26	0.308
76	A	10	8	1.00	26	0.308
77	A	13	9	1.00	26	0.346
78	A	10	7	1.00	26	0.269
79	A	3	3	1.00	12	0.250
80	A	6	4	1.00	26	0.154
81	A	5	3	1.00	26	0.115
82	A	4	4	1.00	26	0.154
83	A	3	3	1.00	26	0.115
84	A	2	2	1.00	24	0.083
85	A	1	1	1.00	23	0.043
86	A	6	4	1.00	26	0.154
87	A	2	2	1.00	26	0.077
88	A	8	6	1.00	26	0.231
89	A	4	4	1.00	26	0.154
90	A	5	6	1.00	26	0.231
91	A	7	6	1.00	26	0.231
92	A	4	6	1.00	26	0.231
93	A	3	3	1.00	26	0.115
94	A	2	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	2	2	1.00	23	0.087
96	A	8	6	1.00	26	0.231
97	A	5	6	1.00	26	0.231
98	A	11	8	1.00	26	0.308
99	A	5	6	1.00	26	0.231
100	A	11	6	1.00	26	0.231
101	A	5	7	1.00	26	0.269
102	A	7	5	1.00	26	0.192
103	A	4	6	1.00	26	0.231
104	A	4	3	1.00	26	0.115
105	A	3	3	1.00	24	0.125
106	A	4	4	1.00	23	0.174
107	A	11	7	1.00	26	0.269
108	A	5	7	1.00	26	0.269
109	A	15	10	1.00	26	0.385
110	A	5	7	1.00	26	0.269
111	A	6	4	1.00	19	0.210
112	A	5	3	1.00	21	0.143
113	A	4	4	1.00	21	0.190
114	A	3	3	1.00	21	0.143
115	A	2	2	1.00	19	0.105
116	A	1	1	1.00	18	0.056
117	A	6	4	1.00	21	0.190
118	A	2	2	1.00	21	0.095
119	A	8	6	1.00	21	0.286
120	A	3	4	1.00	26	0.154
121	A	5	4	1.00	26	0.154
122	A	2	1	1.00	24	0.042
123	A	3	3	1.00	23	0.130
124	A	8	6	1.00	26	0.231
125	A	3	3	1.00	26	0.115
126	A	8	6	1.00	26	0.231
127	A	3	2	1.00	26	0.077
128	A	4	5	1.00	26	0.192
129	A	8	6	1.00	26	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	3	2	1.00	24	0.083
131	A	6	5	1.00	23	0.217
132	A	10	7	1.00	26	0.269
133	A	6	5	1.00	26	0.192
134	A	11	8	1.00	26	0.308
135	A	6	5	1.00	26	0.192
136	A	4	5	1.00	26	0.192
137	A	12	8	1.00	26	0.308
138	A	3	2	1.00	24	0.083
139	A	8	6	1.00	23	0.261
140	A	13	8	1.00	26	0.308
141	A	10	8	1.00	26	0.308
142	A	13	9	1.00	26	0.346
143	A	10	7	1.00	26	0.269
144	A	3	3	1.00	12	0.250
145	A	6	4	1.00	26	0.154
146	A	5	3	1.00	26	0.115
147	A	4	4	1.00	26	0.154
148	A	3	3	1.00	26	0.115
149	A	2	2	1.00	24	0.083
150	A	1	1	1.00	23	0.043
151	A	6	4	1.00	26	0.154
152	A	2	2	1.00	26	0.077
153	A	8	6	1.00	26	0.231
154	A	4	4	1.00	26	0.154
155	A	5	6	1.00	26	0.231
156	A	7	6	1.00	26	0.231
157	A	4	6	1.00	26	0.231
158	A	3	3	1.00	26	0.115
159	A	2	2	1.00	24	0.083
160	A	2	2	1.00	23	0.087
161	A	8	6	1.00	26	0.231
162	A	5	6	1.00	26	0.231
163	A	11	8	1.00	26	0.308
164	A	5	6	1.00	26	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	11	6	1.00	26	0.231
166	A	5	7	1.00	26	0.269
167	A	7	5	1.00	26	0.192
168	A	4	6	1.00	26	0.231
169	A	4	3	1.00	26	0.115
170	A	3	3	1.00	24	0.125
171	A	4	4	1.00	23	0.174
172	A	11	7	1.00	26	0.269
173	A	5	7	1.00	26	0.269
174	A	15	10	1.00	26	0.385
175	A	5	7	1.00	26	0.269
176	A	6	4	1.00	19	0.210
177	A	5	3	1.00	21	0.143
178	A	4	4	1.00	21	0.190
179	A	3	3	1.00	21	0.143
180	A	2	2	1.00	19	0.105
181	A	1	1	1.00	18	0.056
182	A	6	4	1.00	21	0.190
183	A	2	2	1.00	21	0.095
184	A	8	6	1.00	21	0.286
185	A	6	7	1.00	24	0.292
186	A	5	6	1.00	24	0.250
187	A	4	5	1.00	22	0.227
188	N/A	0	0	1.00	24	0.000
189	N/A	0	0	1.00	24	0.000
190	N/A	0	0	1.00	24	0.000
191	A	9	6	1.00	26	0.231
192	A	6	5	1.00	26	0.192
193	A	3	3	1.00	26	0.115
194	A	1	1	1.00	26	0.038
195	A	3	3	1.00	26	0.115
196	A	5	3	1.00	26	0.115
197	A	1	1	1.00	21	0.048
198	A	11	10	1.00	24	0.417
199	A	14	6	1.00	24	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	9	10	1.00	24	0.417
201	A	7	6	1.00	22	0.273
202	A	6	4	1.00	21	0.190
203	A	10	10	1.00	24	0.417
204	A	12	9	1.00	24	0.375
205	A	10	10	1.00	24	0.417
206	A	16	8	1.00	24	0.333
207	A	16	11	1.00	26	0.423
208	A	25	7	1.00	26	0.269
209	A	14	11	1.00	26	0.423
210	A	9	7	1.00	24	0.292
211	A	10	5	1.00	23	0.217
212	A	17	12	1.00	26	0.462
213	A	17	11	1.00	26	0.423
214	A	17	12	1.00	26	0.462
215	A	24	10	1.00	26	0.385
216	A	21	11	1.00	26	0.423
217	A	40	9	1.00	26	0.346
218	A	19	11	1.00	26	0.423
219	A	11	7	1.00	24	0.292
220	A	14	5	1.00	23	0.217
221	A	26	13	1.00	26	0.500
222	A	24	12	1.00	26	0.462
223	A	28	15	1.00	26	0.577
224	A	31	12	1.00	26	0.462
225	A	16	9	1.00	26	0.346
226	A	10	9	1.00	26	0.346
227	A	11	8	1.00	26	0.308
228	A	6	6	1.00	24	0.250
229	A	8	5	1.00	23	0.217
230	A	9	6	1.00	26	0.231
231	A	15	10	1.00	26	0.385
232	A	12	9	1.00	26	0.346
233	A	24	11	1.00	26	0.423
234	A	15	14	1.00	26	0.538

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	10	9	1.00	26	0.346
236	A	11	8	1.00	26	0.308
237	A	3	3	1.00	24	0.125
238	A	11	8	1.00	23	0.348
239	A	12	9	1.00	26	0.346
240	A	20	14	1.00	26	0.538
241	A	17	15	1.00	26	0.577
242	A	32	15	1.00	26	0.577
243	A	16	13	1.00	26	0.500
244	A	8	6	1.00	26	0.231
245	A	15	10	1.00	26	0.385
246	A	5	5	1.00	24	0.208
247	A	15	9	1.00	23	0.391
248	A	17	11	1.00	26	0.423
249	A	27	15	1.00	26	0.577
250	A	23	19	1.00	26	0.731
251	A	43	17	1.00	26	0.654
252	A	16	8	1.00	25	0.320
253	A	10	8	1.00	25	0.320
254	A	5	5	1.00	25	0.200
255	A	1	1	1.00	25	0.040
256	A	6	6	1.00	25	0.240
257	A	9	9	1.00	25	0.360
258	A	14	8	1.00	28	0.286
259	A	10	6	1.00	28	0.214
260	A	5	4	1.00	26	0.154
261	A	5	5	1.00	25	0.200
262	A	12	8	1.00	28	0.286
263	A	7	7	1.00	28	0.250
264	A	13	10	1.00	28	0.357
265	A	9	9	1.00	28	0.321
266	A	20	14	1.00	28	0.500
267	A	17	11	1.00	28	0.393
268	A	6	6	1.00	26	0.231
269	A	10	8	1.00	25	0.320

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	17	12	1.00	28	0.429
271	A	14	13	1.00	28	0.464
272	A	18	15	1.00	28	0.536
273	A	16	11	1.00	28	0.393
274	A	27	18	1.00	28	0.643
275	A	25	14	1.00	28	0.500
276	A	6	6	1.00	26	0.231
277	A	16	8	1.00	25	0.320
278	A	23	16	1.00	28	0.571
279	A	23	15	1.00	28	0.536
280	A	25	20	1.00	28	0.714
281	A	27	15	1.00	28	0.536
282	A	10	5	1.00	23	0.217
283	A	8	7	1.00	23	0.304
284	A	5	5	1.00	23	0.217
285	A	3	3	1.00	21	0.143
286	A	1	1	1.00	20	0.050
287	A	8	5	1.00	23	0.217
288	A	6	6	1.00	23	0.261
289	A	13	10	1.00	23	0.435
290	A	14	7	1.00	28	0.250
291	A	10	5	1.00	28	0.179
292	A	9	7	1.00	28	0.250
293	A	5	5	1.00	28	0.179
294	A	4	3	1.00	26	0.115
295	A	1	1	1.00	25	0.040
296	A	8	5	1.00	28	0.179
297	A	6	6	1.00	28	0.214
298	A	13	10	1.00	28	0.357
299	A	9	9	1.00	28	0.321
300	A	22	12	1.00	28	0.429
301	A	14	11	1.00	28	0.393
302	A	13	9	1.00	28	0.321
303	A	7	7	1.00	28	0.250
304	A	7	5	1.00	26	0.192

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	A	6	6	1.00	25	0.240
306	A	15	10	1.00	28	0.357
307	A	14	10	1.00	28	0.357
308	A	26	14	1.00	28	0.500
309	A	24	11	1.00	28	0.393
310	A	26	11	1.00	28	0.393
311	A	16	9	1.00	28	0.321
312	A	16	7	1.00	28	0.250
313	A	9	9	1.00	28	0.321
314	A	9	7	1.00	26	0.269
315	A	9	9	1.00	25	0.360
316	A	24	12	1.00	28	0.429
317	A	19	14	1.00	28	0.500
318	A	38	17	1.00	28	0.607
319	A	32	15	1.00	28	0.536
320	A	13	10	1.00	21	0.476
321	N/A	0	0	1.00	28	0.000
322	N/A	0	0	1.00	28	0.000
323	N/A	0	0	1.00	28	0.000
324	N/A	0	0	1.00	28	0.000
325	N/A	0	0	1.00	28	0.000
326	N/A	0	0	1.00	28	0.000
327	N/A	0	0	1.00	23	0.000
328	A	24	13	1.00	19	0.684
329	A	17	11	1.00	19	0.579
330	A	10	7	1.00	17	0.412
331	A	10	6	1.00	19	0.316
332	A	18	10	1.00	19	0.526
333	A	28	11	1.00	19	0.579
334	A	24	9	1.00	21	0.429
335	A	14	8	1.00	21	0.381
336	A	6	5	1.00	21	0.238
337	A	1	1	1.00	21	0.048
338	A	7	7	1.00	21	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
339	A	11	10	1.00	21	0.476
340	A	17	11	1.00	21	0.524
341	N/A	0	0	1.00	23	0.000
342	A	13	4	1.00	23	0.174
343	A	10	6	1.00	23	0.261
344	A	6	4	1.00	23	0.174
345	A	4	3	1.00	21	0.143
346	A	1	1	1.00	20	0.050
347	A	10	6	1.00	23	0.261
348	A	7	7	1.00	23	0.304
349	A	18	10	1.00	23	0.435
350	A	7	3	1.00	19	0.158
351	A	6	3	1.00	19	0.158
352	A	5	3	1.00	17	0.176
353	N/A	0	0	1.00	19	0.000
354	N/A	0	0	1.00	19	0.000
355	A	12	5	1.00	27	0.185
356	A	12	5	1.00	27	0.185
357	A	6	5	1.00	27	0.185
358	A	9	5	1.00	25	0.200
359	A	6	5	1.00	24	0.208
360	N/A	0	0	1.00	27	0.000
361	N/A	0	0	1.00	27	0.000
362	N/A	0	0	1.00	27	0.000
363	N/A	0	0	1.00	27	0.000
364	A	15	5	1.00	27	0.185
365	A	12	5	1.00	27	0.185
366	A	12	5	1.00	25	0.200
367	A	9	5	1.00	24	0.208
368	N/A	0	0	1.00	27	0.000
369	N/A	0	0	1.00	27	0.000
370	N/A	0	0	1.00	27	0.000
371	N/A	0	0	1.00	27	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
372	A	15	5	1.00	27	0.185
373	A	15	5	1.00	27	0.185
374	A	15	5	1.00	25	0.200
375	A	12	5	1.00	24	0.208
376	N/A	0	0	1.00	27	0.000
377	N/A	0	0	1.00	27	0.000
378	N/A	0	0	1.00	27	0.000
379	N/A	0	0	1.00	27	0.000
380	A	5	3	1.00	23	0.130
381	A	5	3	1.00	23	0.130
382	A	4	3	1.00	23	0.130
383	A	4	3	1.00	23	0.130
384	A	2	2	1.00	21	0.095
385	A	1	1	1.00	20	0.050
386	N/A	0	0	1.00	23	0.000
387	N/A	0	0	1.00	23	0.000
388	A	12	5	1.00	27	0.185
389	A	9	5	1.00	27	0.185
390	A	9	5	1.00	27	0.185
391	A	6	5	1.00	27	0.185
392	A	4	4	1.00	25	0.160
393	A	1	1	1.00	24	0.042
394	N/A	0	0	1.00	27	0.000
395	N/A	0	0	1.00	27	0.000
396	N/A	0	0	1.00	27	0.000
397	N/A	0	0	1.00	25	0.000
398	N/A	0	0	1.00	24	0.000
399	N/A	0	0	1.00	27	0.000
400	N/A	0	0	1.00	27	0.000
401	N/A	0	0	1.00	27	0.000
402	N/A	0	0	1.00	27	0.000
403	N/A	0	0	1.00	27	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
404	N/A	0	0	1.00	27	0.000
405	N/A	0	0	1.00	27	0.000
406	A	8	4	1.00	19	0.210
407	A	7	4	1.00	19	0.210
408	A	6	4	1.00	17	0.235
409	N/A	0	0	1.00	19	0.000
410	N/A	0	0	1.00	19	0.000
411	A	22	6	1.00	27	0.222
412	A	16	7	1.00	27	0.259
413	A	14	7	1.00	25	0.280
414	A	7	7	1.00	24	0.292
415	N/A	0	0	1.00	27	0.000
416	N/A	0	0	1.00	27	0.000
417	N/A	0	0	1.00	27	0.000
418	N/A	0	0	1.00	27	0.000
419	A	28	6	1.00	27	0.222
420	A	19	6	1.00	27	0.222
421	A	22	8	1.00	25	0.320
422	A	10	6	1.00	24	0.250
423	N/A	0	0	1.00	27	0.000
424	N/A	0	0	1.00	27	0.000
425	N/A	0	0	1.00	27	0.000
426	N/A	0	0	1.00	27	0.000
427	A	34	6	1.00	27	0.222
428	A	28	6	1.00	27	0.222
429	A	28	8	1.00	25	0.320
430	A	13	6	1.00	24	0.250
431	N/A	0	0	1.00	27	0.000
432	N/A	0	0	1.00	27	0.000
433	N/A	0	0	1.00	27	0.000
434	N/A	0	0	1.00	27	0.000
435	A	13	6	1.00	27	0.222
436	A	10	6	1.00	27	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
437	A	10	6	1.00	27	0.222
438	A	7	7	1.00	27	0.259
439	A	5	5	1.00	25	0.200
440	A	1	1	1.00	24	0.042
441	N/A	0	0	1.00	27	0.000
442	N/A	0	0	1.00	27	0.000
443	N/A	0	0	1.00	27	0.000
444	N/A	0	0	1.00	27	0.000
445	N/A	0	0	1.00	25	0.000
446	N/A	0	0	1.00	24	0.000
447	N/A	0	0	1.00	27	0.000
448	N/A	0	0	1.00	27	0.000
449	N/A	0	0	1.00	27	0.000
450	N/A	0	0	1.00	27	0.000
451	N/A	0	0	1.00	25	0.000
452	N/A	0	0	1.00	24	0.000
453	N/A	0	0	1.00	27	0.000
454	N/A	0	0	1.00	27	0.000
455	N/A	0	0	1.00	27	0.000
456	N/A	0	0	1.00	27	0.000
457	N/A	0	0	1.00	27	0.000
458	N/A	0	0	1.00	27	0.000
459	N/A	0	0	1.00	27	0.000
460	N/A	0	0	1.00	27	0.000
461	A	1	1	1.00	20	0.050
462	A	27	7	1.00	26	0.269
463	A	32	7	1.00	26	0.269
464	A	17	9	1.00	24	0.375
465	A	14	7	1.00	23	0.304
466	N/A	0	0	1.00	26	0.000
467	A	32	7	1.00	28	0.250
468	A	42	7	1.00	28	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
469	A	32	9	1.00	26	0.346
470	A	19	7	1.00	25	0.280
471	N/A	0	0	1.00	28	0.000
472	A	24	11	1.00	23	0.478
473	A	10	9	1.00	23	0.391
474	A	1	1	1.00	23	0.043
475	N/A	0	0	1.00	23	0.000
476	N/A	0	0	1.00	23	0.000
477	A	26	12	1.00	23	0.522
478	A	11	9	1.00	23	0.391
479	A	1	1	1.00	23	0.043
480	N/A	0	0	1.00	23	0.000
481	A	39	14	1.00	23	0.609
482	A	13	11	1.00	23	0.478
483	A	1	1	1.00	23	0.043
484	N/A	0	0	1.00	23	0.000
485	A	24	11	1.00	22	0.500
486	A	10	9	1.00	22	0.409
487	A	1	1	1.00	22	0.045
488	N/A	0	0	1.00	22	0.000
489	N/A	0	0	1.00	22	0.000
490	A	26	12	1.00	22	0.546
491	A	11	9	1.00	22	0.409
492	A	1	1	1.00	22	0.045
493	N/A	0	0	1.00	22	0.000
494	A	6	5	1.00	17	0.294
495	A	18	6	1.00	23	0.261
496	A	13	6	1.00	23	0.261
497	A	8	6	1.00	23	0.261
498	A	1	1	1.00	23	0.043
499	N/A	0	0	1.00	23	0.000
500	N/A	0	0	1.00	23	0.000
501	A	19	7	1.00	23	0.304
502	A	14	7	1.00	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
503	A	9	8	1.00	23	0.348
504	A	1	1	1.00	23	0.043
505	N/A	0	0	1.00	23	0.000
506	N/A	0	0	1.00	23	0.000
507	A	18	10	1.00	23	0.435
508	A	7	6	1.00	23	0.261
509	A	1	1	1.00	23	0.043
510	N/A	0	0	1.00	23	0.000
511	N/A	0	0	1.00	23	0.000
512	A	6	4	1.00	28	0.143
513	A	9	4	1.00	26	0.154
514	A	6	4	1.00	25	0.160
515	N/A	0	0	1.00	28	0.000
516	N/A	0	0	1.00	28	0.000
517	A	12	4	1.00	28	0.143
518	A	12	4	1.00	26	0.154
519	A	9	4	1.00	25	0.160
520	N/A	0	0	1.00	28	0.000
521	N/A	0	0	1.00	28	0.000
522	A	15	4	1.00	28	0.143
523	A	15	4	1.00	26	0.154
524	A	12	4	1.00	25	0.160
525	N/A	0	0	1.00	28	0.000
526	N/A	0	0	1.00	28	0.000
527	N/A	0	0	1.00	23	0.000
528	A	9	4	1.00	23	0.174
529	A	6	4	1.00	23	0.174
530	A	4	3	1.00	21	0.143
531	A	1	1	1.00	20	0.050
532	N/A	0	0	1.00	23	0.000
533	N/A	0	0	1.00	23	0.000
534	A	13	8	1.00	35	0.229
535	A	8	6	1.00	35	0.171

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
536	A	4	4	1.00	35	0.114
537	A	6	5	1.00	35	0.143
538	A	8	8	1.00	35	0.229
539	A	6	6	1.00	35	0.171
540	A	12	9	1.00	35	0.257
541	A	7	6	1.00	35	0.171
542	A	8	6	1.00	35	0.171
543	A	9	7	1.00	35	0.200
544	A	10	10	1.00	35	0.286
545	A	9	9	1.00	35	0.257
546	A	9	7	1.00	35	0.200
547	A	12	9	1.00	35	0.257
548	A	13	8	1.00	35	0.229
549	A	13	7	1.00	35	0.200
550	A	7	9	1.00	35	0.257
551	A	10	8	1.00	35	0.229
552	A	13	7	1.00	35	0.200
553	A	9	7	1.00	35	0.200
554	A	6	5	1.00	35	0.143
555	A	2	2	1.00	35	0.057
556	A	5	6	1.00	35	0.171
557	A	8	8	1.00	35	0.229
558	A	7	9	1.00	35	0.257
559	A	10	10	1.00	35	0.286
560	A	8	8	1.00	35	0.229
561	A	5	6	1.00	35	0.171
562	A	3	3	1.00	35	0.086
563	A	8	8	1.00	35	0.229
564	A	10	8	1.00	35	0.229
565	A	9	9	1.00	35	0.257
566	A	6	6	1.00	35	0.171
567	A	8	8	1.00	35	0.229
568	A	8	8	1.00	35	0.229
569	A	5	5	1.00	35	0.143
570	A	23	13	1.00	37	0.351

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
571	A	13	11	1.00	37	0.297
572	A	6	6	1.00	37	0.162
573	A	8	6	1.00	37	0.162
574	A	19	13	1.00	37	0.351
575	A	20	12	1.00	37	0.324
576	A	19	15	1.00	37	0.405
577	A	11	9	1.00	37	0.243
578	A	13	11	1.00	37	0.297
579	A	11	9	1.00	37	0.243
580	A	23	15	1.00	37	0.405
581	A	21	13	1.00	37	0.351
582	A	17	9	1.00	37	0.243
583	A	19	15	1.00	37	0.405
584	A	23	13	1.00	37	0.351
585	A	17	10	1.00	37	0.270
586	A	28	19	1.00	37	0.514
587	A	25	16	1.00	37	0.432
588	A	17	10	1.00	37	0.270
589	A	11	9	1.00	37	0.243
590	A	8	6	1.00	37	0.162
591	A	2	2	1.00	37	0.054
592	A	16	11	1.00	37	0.297
593	A	30	18	1.00	37	0.486
594	A	28	19	1.00	37	0.514
595	A	23	15	1.00	37	0.405
596	A	19	13	1.00	37	0.351
597	A	16	11	1.00	37	0.297
598	A	7	7	1.00	37	0.189
599	A	21	14	1.00	37	0.378
600	A	25	16	1.00	37	0.432
601	A	21	13	1.00	37	0.351
602	A	20	12	1.00	37	0.324
603	A	30	18	1.00	37	0.486
604	A	21	14	1.00	37	0.378
605	A	10	10	1.00	37	0.270

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
606	A	5	5	1.00	18	0.278
607	A	5	5	1.00	18	0.278
608	A	5	5	1.00	18	0.278
609	A	4	3	1.00	16	0.188
610	A	3	2	1.00	8	0.250
611	A	18	6	1.00	18	0.333
612	A	26	9	1.00	18	0.500
613	A	26	7	1.00	20	0.350
614	A	17	7	1.00	20	0.350
615	A	10	7	1.00	18	0.389
616	A	3	3	1.00	10	0.300
617	A	22	7	1.00	20	0.350
618	A	42	7	1.00	20	0.350
619	A	27	7	1.00	20	0.350
620	A	15	7	1.00	18	0.389
621	A	4	4	1.00	10	0.400
622	N/A	0	0	1.00	20	0.000
623	N/A	0	0	1.00	20	0.000
624	A	26	7	1.00	20	0.350
625	A	15	7	1.00	18	0.389
626	A	5	5	1.00	10	0.500
627	N/A	0	0	1.00	20	0.000
628	N/A	0	0	1.00	20	0.000
629	A	42	9	1.00	22	0.409
630	A	23	9	1.00	20	0.450
631	A	7	6	1.00	12	0.500
632	N/A	0	0	1.00	22	0.000
633	N/A	0	0	1.00	22	0.000
634	A	32	12	1.00	20	0.600
635	A	8	7	1.00	12	0.583
636	N/A	0	0	1.00	22	0.000
637	N/A	0	0	1.00	22	0.000
638	A	39	8	1.00	22	0.364
639	A	21	8	1.00	20	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
640	A	6	5	1.00	12	0.417
641	N/A	0	0	1.00	22	0.000
642	N/A	0	0	1.00	22	0.000
643	A	21	8	1.00	20	0.400
644	A	7	6	1.00	12	0.500
645	N/A	0	0	1.00	22	0.000
646	N/A	0	0	1.00	22	0.000
647	N/A	0	0	1.00	20	0.000
648	N/A	0	0	1.00	20	0.000
649	A	6	7	1.00	20	0.350
650	A	7	9	1.00	20	0.450
651	A	8	10	1.00	20	0.500
652	N/A	0	0	1.00	22	0.000
653	N/A	0	0	1.00	22	0.000
654	N/A	0	0	1.00	22	0.000
655	N/A	0	0	1.00	22	0.000
656	N/A	0	0	1.00	22	0.000
657	N/A	0	0	1.00	22	0.000
658	N/A	0	0	1.00	22	0.000
659	N/A	0	0	1.00	22	0.000
660	N/A	0	0	1.00	22	0.000
661	N/A	0	0	1.00	22	0.000
662	N/A	0	0	1.00	22	0.000
663	N/A	0	0	1.00	22	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^4(d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 124

$$\int x^4(d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx = -\frac{2bd\sqrt{1+c^2x^2}}{35c^5} - \frac{bd(1+c^2x^2)^{3/2}}{105c^5} + \frac{8bd(1+c^2x^2)^{5/2}}{175c^5} - \frac{bd(1+c^2x^2)^{7/2}}{49c^5} + \frac{1}{5}dx^5(a+b\text{arcsinh}(cx)) + \frac{1}{7}c^2dx^7(a+b\text{arcsinh}(cx))$$

```
[Out] -1/105*b*d*(c^2*x^2+1)^(3/2)/c^5+8/175*b*d*(c^2*x^2+1)^(5/2)/c^5-1/49*b*d*(
c^2*x^2+1)^(7/2)/c^5+1/5*d*x^5*(a+b*arcsinh(c*x))+1/7*c^2*d*x^7*(a+b*arcsin
h(c*x))-2/35*b*d*(c^2*x^2+1)^(1/2)/c^5
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {14, 5803, 12, 457, 78}

$$\int x^4(d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx = \frac{1}{7}c^2dx^7(a+b\text{arcsinh}(cx)) + \frac{1}{5}dx^5(a+b\text{arcsinh}(cx)) - \frac{bd(c^2x^2+1)^{7/2}}{49c^5} + \frac{8bd(c^2x^2+1)^{5/2}}{175c^5} - \frac{bd(c^2x^2+1)^{3/2}}{105c^5} - \frac{2bd\sqrt{c^2x^2+1}}{35c^5}$$

```
[In] Int[x^4*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-2*b*d*Sqrt[1 + c^2*x^2])/(35*c^5) - (b*d*(1 + c^2*x^2)^(3/2))/(105*c^5) +
 (8*b*d*(1 + c^2*x^2)^(5/2))/(175*c^5) - (b*d*(1 + c^2*x^2)^(7/2))/(49*c^5)
 + (d*x^5*(a + b*ArcSinh[c*x]))/5 + (c^2*d*x^7*(a + b*ArcSinh[c*x]))/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{5}dx^5(a+b\operatorname{arcsinh}(cx))+\frac{1}{7}c^2dx^7(a+b\operatorname{arcsinh}(cx))-(bc)\int\frac{dx^5(7+5c^2x^2)}{35\sqrt{1+c^2x^2}}\,dx\\ &= \frac{1}{5}dx^5(a+b\operatorname{arcsinh}(cx))+\frac{1}{7}c^2dx^7(a+b\operatorname{arcsinh}(cx))-\frac{1}{35}(bcd)\int\frac{x^5(7+5c^2x^2)}{\sqrt{1+c^2x^2}}\,dx\end{aligned}$$

$$= \frac{1}{5}dx^5(a + b\text{arcsinh}(cx)) + \frac{1}{7}c^2dx^7(a + b\text{arcsinh}(cx)) - \frac{1}{70}(bcd)\text{Subst}\left(\int \frac{x^2(7+5c^2x)}{\sqrt{1+c^2x}}\,dx, x, x^2\right)$$

$$= \frac{1}{5}dx^5(a + b\text{arcsinh}(cx)) + \frac{1}{7}c^2dx^7(a + b\text{arcsinh}(cx)) - \frac{1}{70}(bcd)\text{Subst}\left(\int \left(\frac{2}{c^4\sqrt{1+c^2x}} + \frac{\sqrt{1+c^2x}}{c^4} - \frac{8(1+c^2x)^{3/2}}{c^4} + \frac{5(1+c^2x)^{5/2}}{c^4}\right) dx, x, x^2\right)$$

$$= -\frac{2bd\sqrt{1+c^2x^2}}{35c^5} - \frac{bd(1+c^2x^2)^{3/2}}{105c^5} + \frac{8bd(1+c^2x^2)^{5/2}}{175c^5} - \frac{bd(1+c^2x^2)^{7/2}}{49c^5} + \frac{1}{5}dx^5(a + b\text{arcsinh}(cx)) + \frac{1}{7}c^2dx^7(a + b\text{arcsinh}(cx))$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.70

$$\int x^4(d + c^2dx^2)(a + b\text{arcsinh}(cx))\,dx = \frac{d\left(105ax^5(7+5c^2x^2) - \frac{b\sqrt{1+c^2x^2}(152-76c^2x^2+57c^4x^4+75c^6x^6)}{c^5} + 105bx^5(7+5c^2x^2)\,\text{arcsinh}(cx)\right)}{3675}$$

```
[In] Integrate[x^4*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
[Out] (d*(105*a*x^5*(7 + 5*c^2*x^2) - (b*Sqrt[1 + c^2*x^2]*(152 - 76*c^2*x^2 + 57
*c^4*x^4 + 75*c^6*x^6))/c^5 + 105*b*x^5*(7 + 5*c^2*x^2)*ArcSinh[c*x]))/3675
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.97

method	result
parts	$da\left(\frac{1}{7}c^2x^7 + \frac{1}{5}x^5\right) + \frac{db\left(\frac{\text{arcsinh}(cx)c^7x^7}{7} + \frac{\text{arcsinh}(cx)c^5x^5}{5} - \frac{c^6x^6\sqrt{c^2x^2+1}}{49} - \frac{19c^4x^4\sqrt{c^2x^2+1}}{1225} + \frac{76c^2x^2\sqrt{c^2x^2+1}}{3675} - \cdots\right.}{c^5}$
derivativedivides	$\frac{da\left(\frac{1}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+db\left(\frac{\text{arcsinh}(cx)c^7x^7}{7} + \frac{\text{arcsinh}(cx)c^5x^5}{5} - \frac{c^6x^6\sqrt{c^2x^2+1}}{49} - \frac{19c^4x^4\sqrt{c^2x^2+1}}{1225} + \frac{76c^2x^2\sqrt{c^2x^2+1}}{3675} - \frac{152\sqrt{c^2x^2+1}}{3675}\cdots\right.}{c^5}$
default	$\frac{da\left(\frac{1}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+db\left(\frac{\text{arcsinh}(cx)c^7x^7}{7} + \frac{\text{arcsinh}(cx)c^5x^5}{5} - \frac{c^6x^6\sqrt{c^2x^2+1}}{49} - \frac{19c^4x^4\sqrt{c^2x^2+1}}{1225} + \frac{76c^2x^2\sqrt{c^2x^2+1}}{3675} - \frac{152\sqrt{c^2x^2+1}}{3675}\cdots\right.}{c^5}$

```
[In] int(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] d*a*(1/7*c^2*x^7+1/5*x^5)+d*b/c^5*(1/7*arcsinh(c*x)*c^7*x^7+1/5*arcsinh(c*x
)*c^5*x^5-1/49*c^6*x^6*(c^2*x^2+1)^(1/2)-19/1225*c^4*x^4*(c^2*x^2+1)^(1/2)+
76/3675*c^2*x^2*(c^2*x^2+1)^(1/2)-152/3675*(c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.91

$$\int x^4(d+c^2dx^2)(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\frac{525\,ac^7dx^7+735\,ac^5dx^5+105\,(5\,bc^7dx^7+7\,bc^5dx^5)\log\left(cx+\sqrt{c^2x^2+1}\right)-(75\,bc^6dx^6+57\,bc^4dx^4-76\,bc}{3675\,c^5}$$

```
[In] integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3675*(525*a*c^7*d*x^7 + 735*a*c^5*d*x^5 + 105*(5*b*c^7*d*x^7 + 7*b*c^5*d*
x^5)*log(c*x + sqrt(c^2*x^2 + 1)) - (75*b*c^6*d*x^6 + 57*b*c^4*d*x^4 - 76*b
*c^2*d*x^2 + 152*b*d)*sqrt(c^2*x^2 + 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.22

$$\int x^4(d+c^2dx^2)(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\begin{cases}\frac{ac^2dx^7}{7}+\frac{adx^5}{5}+\frac{bc^2dx^7\operatorname{asinh}(cx)}{7}-\frac{bcdx^6\sqrt{c^2x^2+1}}{49}+\frac{bdx^5\operatorname{asinh}(cx)}{5}-\frac{19bdx^4\sqrt{c^2x^2+1}}{1225c}+\frac{76bdx^2\sqrt{c^2x^2+1}}{3675c^3}-\frac{152bd\sqrt{c^2x^2+1}}{3675c^5} \\ \frac{adx^5}{5}\end{cases}$$

```
[In] integrate(x**4*(c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d*x**7/7 + a*d*x**5/5 + b*c**2*d*x**7*asinh(c*x)/7 - b*c*
d*x**6*sqrt(c**2*x**2 + 1)/49 + b*d*x**5*asinh(c*x)/5 - 19*b*d*x**4*sqrt(c*
*2*x**2 + 1)/(1225*c) + 76*b*d*x**2*sqrt(c**2*x**2 + 1)/(3675*c**3) - 152*b
*d*sqrt(c**2*x**2 + 1)/(3675*c**5), Ne(c, 0)), (a*d*x**5/5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.48

$$\int x^4\left(d+c^2dx^2\right)\left(a+b\text{arcsinh}(cx)\right)dx=\frac{1}{7}ac^2dx^7+\frac{1}{5}adx^5$$
$$+\frac{1}{245}\left(35\,x^7\,\text{arsinh}\,(cx)-\left(\frac{5\,\sqrt{c^2x^2+1}x^6}{c^2}-\frac{6\,\sqrt{c^2x^2+1}x^4}{c^4}+\frac{8\,\sqrt{c^2x^2+1}x^2}{c^6}-\frac{16\,\sqrt{c^2x^2+1}}{c^8}\right)c\right)bc^2d$$
$$+\frac{1}{75}\left(15\,x^5\,\text{arsinh}\,(cx)-\left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2}-\frac{4\,\sqrt{c^2x^2+1}x^2}{c^4}+\frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\right)bd$$

```
[In] integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*c^2*d*x^7 + 1/5*a*d*x^5 + 1/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^
2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6
- 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^2*d + 1/75*(15*x^5*arcsinh(c*x) - (3*sqr
t(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/
c^6)*c)*b*d
```

Giac [F(-2)]

Exception generated.

$$\int x^4\left(d+c^2dx^2\right)\left(a+b\text{arcsinh}(cx)\right)dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^4\left(d+c^2dx^2\right)\left(a+b\text{arcsinh}(cx)\right)dx=\int x^4\,\left(a+b\,\text{asinh}(c\,x)\right)\,\left(d\,c^2\,x^2+d\right)\,dx$$

```
[In] int(x^4*(a + b*asinh(c*x))*(d + c^2*d*x^2),x)
```

```
[Out] int(x^4*(a + b*asinh(c*x))*(d + c^2*d*x^2), x)
```

3.2 $\int x^3(d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 120

$$\int x^3\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))\,dx=\frac{bdx\sqrt{1+c^2x^2}}{24c^3}-\frac{bdx^3\sqrt{1+c^2x^2}}{36c}-\frac{1}{36}bcdx^5\sqrt{1+c^2x^2}-\frac{bd\text{arcsinh}(cx)}{24c^4}+\frac{1}{4}dx^4(a+b\text{arcsinh}(cx))+\frac{1}{6}c^2dx^6(a+b\text{arcsinh}(cx))$$

```
[Out] -1/24*b*d*arcsinh(c*x)/c^4+1/4*d*x^4*(a+b*arcsinh(c*x))+1/6*c^2*d*x^6*(a+b*
arcsinh(c*x))+1/24*b*d*x*(c^2*x^2+1)^(1/2)/c^3-1/36*b*d*x^3*(c^2*x^2+1)^(1/
2)/c-1/36*b*c*d*x^5*(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {14, 5803, 12, 470, 327, 221}

$$\int x^3\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))\,dx=\frac{1}{6}c^2dx^6(a+b\text{arcsinh}(cx))+\frac{1}{4}dx^4(a+b\text{arcsinh}(cx))-\frac{bd\text{arcsinh}(cx)}{24c^4}-\frac{1}{36}bcdx^5\sqrt{c^2x^2+1}-\frac{bdx^3\sqrt{c^2x^2+1}}{36c}+\frac{bdx\sqrt{c^2x^2+1}}{24c^3}$$

```
[In] Int[x^3*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (b*d*x*Sqrt[1 + c^2*x^2])/(24*c^3) - (b*d*x^3*Sqrt[1 + c^2*x^2])/(36*c) - (
b*c*d*x^5*Sqrt[1 + c^2*x^2])/36 - (b*d*ArcSinh[c*x])/(24*c^4) + (d*x^4*(a +
 b*ArcSinh[c*x]))/4 + (c^2*d*x^6*(a + b*ArcSinh[c*x]))/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}dx^4(a+b\text{arcsinh}(cx))+\frac{1}{6}c^2dx^6(a+b\text{arcsinh}(cx))-(bc)\int\frac{dx^4(3+2c^2x^2)}{12\sqrt{1+c^2x^2}}\,dx\\
&= \frac{1}{4}dx^4(a+b\text{arcsinh}(cx))+\frac{1}{6}c^2dx^6(a+b\text{arcsinh}(cx))-\frac{1}{12}(bcd)\int\frac{x^4(3+2c^2x^2)}{\sqrt{1+c^2x^2}}\,dx\\
&= -\frac{1}{36}bcdx^5\sqrt{1+c^2x^2}+\frac{1}{4}dx^4(a+b\text{arcsinh}(cx))\\
&\quad+\frac{1}{6}c^2dx^6(a+b\text{arcsinh}(cx))-\frac{1}{9}(bcd)\int\frac{x^4}{\sqrt{1+c^2x^2}}\,dx\\
&= -\frac{bdx^3\sqrt{1+c^2x^2}}{36c}-\frac{1}{36}bcdx^5\sqrt{1+c^2x^2}+\frac{1}{4}dx^4(a+b\text{arcsinh}(cx))\\
&\quad+\frac{1}{6}c^2dx^6(a+b\text{arcsinh}(cx))+\frac{(bd)\int\frac{x^2}{\sqrt{1+c^2x^2}}\,dx}{12c}\\
&= \frac{bdx\sqrt{1+c^2x^2}}{24c^3}-\frac{bdx^3\sqrt{1+c^2x^2}}{36c}-\frac{1}{36}bcdx^5\sqrt{1+c^2x^2}\\
&\quad+\frac{1}{4}dx^4(a+b\text{arcsinh}(cx))+\frac{1}{6}c^2dx^6(a+b\text{arcsinh}(cx))-\frac{(bd)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{24c^3}\\
&= \frac{bdx\sqrt{1+c^2x^2}}{24c^3}-\frac{bdx^3\sqrt{1+c^2x^2}}{36c}-\frac{1}{36}bcdx^5\sqrt{1+c^2x^2}-\frac{bd\text{arcsinh}(cx)}{24c^4}\\
&\quad+\frac{1}{4}dx^4(a+b\text{arcsinh}(cx))+\frac{1}{6}c^2dx^6(a+b\text{arcsinh}(cx))
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.73

$$
\begin{aligned}
&\int x^3\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))\,dx\\
&=\frac{d\left(6ac^4x^4(3+2c^2x^2)+bcx\sqrt{1+c^2x^2}(3-2c^2x^2-2c^4x^4)+3b(-1+6c^4x^4+4c^6x^6)\,\text{arcsinh}(cx)\right)}{72c^4}
\end{aligned}
$$

```
[In] Integrate[x^3*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*(6*a*c^4*x^4*(3 + 2*c^2*x^2) + b*c*x*Sqrt[1 + c^2*x^2]*(3 - 2*c^2*x^2 - 
2*c^4*x^4) + 3*b*(-1 + 6*c^4*x^4 + 4*c^6*x^6)*ArcSinh[c*x]))/(72*c^4)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.91

method	result
parts	$da\left(\frac{1}{6}c^2x^6+\frac{1}{4}x^4\right)+\frac{db\left(\frac{\operatorname{arcsinh}(cx)c^6x^6}{6}+\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}-\frac{c^5x^5\sqrt{c^2x^2+1}}{36}-\frac{c^3x^3\sqrt{c^2x^2+1}}{36}+\frac{cx\sqrt{c^2x^2+1}}{24}-\frac{\operatorname{arcsinh}(c}{24}\right.}{c^4}$
derivativedivides	$\frac{da\left(\frac{1}{6}c^6x^6+\frac{1}{4}c^4x^4\right)+db\left(\frac{\operatorname{arcsinh}(cx)c^6x^6}{6}+\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}-\frac{c^5x^5\sqrt{c^2x^2+1}}{36}-\frac{c^3x^3\sqrt{c^2x^2+1}}{36}+\frac{cx\sqrt{c^2x^2+1}}{24}-\frac{\operatorname{arcsinh}(cx)}{24}\right)}{c^4}$
default	$\frac{da\left(\frac{1}{6}c^6x^6+\frac{1}{4}c^4x^4\right)+db\left(\frac{\operatorname{arcsinh}(cx)c^6x^6}{6}+\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}-\frac{c^5x^5\sqrt{c^2x^2+1}}{36}-\frac{c^3x^3\sqrt{c^2x^2+1}}{36}+\frac{cx\sqrt{c^2x^2+1}}{24}-\frac{\operatorname{arcsinh}(cx)}{24}\right)}{c^4}$

```
[In] int(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] d*a*(1/6*c^2*x^6+1/4*x^4)+d*b/c^4*(1/6*arcsinh(c*x)*c^6*x^6+1/4*arcsinh(c*x
)*c^4*x^4-1/36*c^5*x^5*(c^2*x^2+1)^(1/2)-1/36*c^3*x^3*(c^2*x^2+1)^(1/2)+1/2
4*c*x*(c^2*x^2+1)^(1/2)-1/24*arcsinh(c*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.91

$$\int x^3\left(d+c^2dx^2\right)\left(a+b\operatorname{arcsinh}(cx)\right)dx$$
$$=\frac{12\,ac^6dx^6+18\,ac^4dx^4+3\left(4\,bc^6dx^6+6\,bc^4dx^4-bd\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(2\,bc^5dx^5+2\,bc^3dx^3-3\,bcd\right.}{72\,c^4}$$

```
[In] integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/72*(12*a*c^6*d*x^6 + 18*a*c^4*d*x^4 + 3*(4*b*c^6*d*x^6 + 6*b*c^4*d*x^4 -
b*d)*log(c*x + sqrt(c^2*x^2 + 1)) - (2*b*c^5*d*x^5 + 2*b*c^3*d*x^3 - 3*b*c*
d*x)*sqrt(c^2*x^2 + 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.15

$$\int x^3\left(d+c^2dx^2\right)\left(a+b\operatorname{arcsinh}(cx)\right)dx$$
$$=\begin{cases}\frac{ac^2dx^6}{6}+\frac{adx^4}{4}+\frac{bc^2dx^6\operatorname{asinh}(cx)}{6}-\frac{bcdx^5\sqrt{c^2x^2+1}}{36}+\frac{bdx^4\operatorname{asinh}(cx)}{4}-\frac{bdx^3\sqrt{c^2x^2+1}}{36c}+\frac{bdx\sqrt{c^2x^2+1}}{24c^3}-\frac{bd\operatorname{asinh}(cx)}{24c^4} & \text{for } c\\ \frac{adx^4}{4} & \text{othe}\end{cases}$$

```
[In] integrate(x**3*(c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d*x**6/6 + a*d*x**4/4 + b*c**2*d*x**6*asinh(c*x)/6 - b*c*
d*x**5*sqrt(c**2*x**2 + 1)/36 + b*d*x**4*asinh(c*x)/4 - b*d*x**3*sqrt(c**2*
x**2 + 1)/(36*c) + b*d*x*sqrt(c**2*x**2 + 1)/(24*c**3) - b*d*asinh(c*x)/(24
*c**4), Ne(c, 0)), (a*d*x**4/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.38

$$\int x^3(d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx = \frac{1}{6}\,ac^2dx^6 + \frac{1}{4}\,adx^4 + \frac{1}{288}\left(48\,x^6\,\text{arsinh}\,(cx) - \left(\frac{8\,\sqrt{c^2x^2+1}x^5}{c^2} - \frac{10\,\sqrt{c^2x^2+1}x^3}{c^4} + \frac{15\,\sqrt{c^2x^2+1}x}{c^6} - \frac{15\,\text{arsinh}\,(cx)}{c^7}\right)c\right)bc^2 + \frac{1}{32}\left(8\,x^4\,\text{arsinh}\,(cx) - \left(\frac{2\,\sqrt{c^2x^2+1}x^3}{c^2} - \frac{3\,\sqrt{c^2x^2+1}x}{c^4} + \frac{3\,\text{arsinh}\,(cx)}{c^5}\right)c\right)bd$$

```
[In] integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*c^2*d*x^6 + 1/4*a*d*x^4 + 1/288*(48*x^6*arcsinh(c*x) - (8*sqrt(c^2*x^
2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6
- 15*arcsinh(c*x)/c^7)*c)*b*c^2*d + 1/32*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*
x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*b*d
```

Giac [F(-2)]

Exception generated.

$$\int x^3(d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx=\int x^3\,(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)\,dx$$

```
[In] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2),x)
[Out] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2), x)
```

3.3 $\int x^2(d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 102

$$\int x^2\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))\,dx=\frac{2bd\sqrt{1+c^2x^2}}{15c^3}+\frac{bd(1+c^2x^2)^{3/2}}{45c^3}-\frac{bd(1+c^2x^2)^{5/2}}{25c^3}+\frac{1}{3}dx^3(a+b\text{arcsinh}(cx))+\frac{1}{5}c^2dx^5(a+b\text{arcsinh}(cx))$$

```
[Out] 1/45*b*d*(c^2*x^2+1)^(3/2)/c^3-1/25*b*d*(c^2*x^2+1)^(5/2)/c^3+1/3*d*x^3*(a+
b*arcsinh(c*x))+1/5*c^2*d*x^5*(a+b*arcsinh(c*x))+2/15*b*d*(c^2*x^2+1)^(1/2)
/c^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {14, 5803, 12, 457, 78}

$$\int x^2\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))\,dx=\frac{1}{5}c^2dx^5(a+b\text{arcsinh}(cx))+\frac{1}{3}dx^3(a+b\text{arcsinh}(cx))-\frac{bd(c^2x^2+1)^{5/2}}{25c^3}+\frac{bd(c^2x^2+1)^{3/2}}{45c^3}+\frac{2bd\sqrt{c^2x^2+1}}{15c^3}$$

```
[In] Int[x^2*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*b*d*Sqrt[1 + c^2*x^2])/(15*c^3) + (b*d*(1 + c^2*x^2)^(3/2))/(45*c^3) - (
b*d*(1 + c^2*x^2)^(5/2))/(25*c^3) + (d*x^3*(a + b*ArcSinh[c*x]))/3 + (c^2*d
*x^5*(a + b*ArcSinh[c*x]))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}dx^3(a+b\text{arcsinh}(cx))+\frac{1}{5}c^2dx^5(a+b\text{arcsinh}(cx))-(bc)\int\frac{dx^3(5+3c^2x^2)}{15\sqrt{1+c^2x^2}}\,dx\\
&= \frac{1}{3}dx^3(a+b\text{arcsinh}(cx))+\frac{1}{5}c^2dx^5(a+b\text{arcsinh}(cx))-\frac{1}{15}(bcd)\int\frac{x^3(5+3c^2x^2)}{\sqrt{1+c^2x^2}}\,dx\\
&= \frac{1}{3}dx^3(a+b\text{arcsinh}(cx))+\frac{1}{5}c^2dx^5(a+b\text{arcsinh}(cx))\\
&\quad -\frac{1}{30}(bcd)\text{Subst}\left(\int\frac{x(5+3c^2x)}{\sqrt{1+c^2x}}\,dx,x,x^2\right)
\end{aligned}
$$

$$= \frac{1}{3}dx^3(a + b\text{arcsinh}(cx)) + \frac{1}{5}c^2dx^5(a + b\text{arcsinh}(cx)) - \frac{1}{30}(bcd)\text{Subst}\left(\int \left(-\frac{2}{c^2\sqrt{1+c^2x}} - \frac{\sqrt{1+c^2x}}{c^2} + \frac{3(1+c^2x)^{3/2}}{c^2}\right) dx, x, x^2\right)$$

$$= \frac{2bd\sqrt{1+c^2x^2}}{15c^3} + \frac{bd(1+c^2x^2)^{3/2}}{45c^3} - \frac{bd(1+c^2x^2)^{5/2}}{25c^3} + \frac{1}{3}dx^3(a + b\text{arcsinh}(cx)) + \frac{1}{5}c^2dx^5(a + b\text{arcsinh}(cx))$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.76

$$\int x^2(d + c^2dx^2)(a + b\text{arcsinh}(cx))\,dx = \frac{1}{225}d\left(15ax^3(5 + 3c^2x^2) + \frac{b\sqrt{1+c^2x^2}(26 - 13c^2x^2 - 9c^4x^4)}{c^3} + 15bx^3(5 + 3c^2x^2)\,\text{arcsinh}(cx)\right)$$

```
[In] Integrate[x^2*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*(15*a*x^3*(5 + 3*c^2*x^2) + (b*Sqrt[1 + c^2*x^2]*(26 - 13*c^2*x^2 - 9*c^
4*x^4))/c^3 + 15*b*x^3*(5 + 3*c^2*x^2)*ArcSinh[c*x]))/225
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.99

method	result	size
parts	$da\left(\frac{1}{5}c^2x^5 + \frac{1}{3}x^3\right) + \frac{db\left(\frac{\text{arcsinh}(cx)c^5x^5}{5} + \frac{\text{arcsinh}(cx)c^3x^3}{3} - \frac{c^4x^4\sqrt{c^2x^2+1}}{25} - \frac{13c^2x^2\sqrt{c^2x^2+1}}{225} + \frac{26\sqrt{c^2x^2+1}}{225}\right)}{c^3}$	101
derivativedivides	$\frac{da\left(\frac{1}{5}c^5x^5 + \frac{1}{3}c^3x^3\right) + db\left(\frac{\text{arcsinh}(cx)c^5x^5}{5} + \frac{\text{arcsinh}(cx)c^3x^3}{3} - \frac{c^4x^4\sqrt{c^2x^2+1}}{25} - \frac{13c^2x^2\sqrt{c^2x^2+1}}{225} + \frac{26\sqrt{c^2x^2+1}}{225}\right)}{c^3}$	105
default	$\frac{da\left(\frac{1}{5}c^5x^5 + \frac{1}{3}c^3x^3\right) + db\left(\frac{\text{arcsinh}(cx)c^5x^5}{5} + \frac{\text{arcsinh}(cx)c^3x^3}{3} - \frac{c^4x^4\sqrt{c^2x^2+1}}{25} - \frac{13c^2x^2\sqrt{c^2x^2+1}}{225} + \frac{26\sqrt{c^2x^2+1}}{225}\right)}{c^3}$	105

```
[In] int(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] d*a*(1/5*c^2*x^5+1/3*x^3)+d*b/c^3*(1/5*arcsinh(c*x)*c^5*x^5+1/3*arcsinh(c*x
)*c^3*x^3-1/25*c^4*x^4*(c^2*x^2+1)^(1/2)-13/225*c^2*x^2*(c^2*x^2+1)^(1/2)+2
6/225*(c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.01

$$\int x^2\left(d+c^2dx^2\right)\left(a+b\operatorname{arcsinh}(cx)\right)dx$$
$$=\frac{45\,ac^5dx^5+75\,ac^3dx^3+15\left(3\,bc^5dx^5+5\,bc^3dx^3\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(9\,bc^4dx^4+13\,bc^2dx^2-26\,bd\right)\sqrt{}}{225\,c^3}$$

```
[In] integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/225*(45*a*c^5*d*x^5 + 75*a*c^3*d*x^3 + 15*(3*b*c^5*d*x^5 + 5*b*c^3*d*x^3)
*log(c*x + sqrt(c^2*x^2 + 1)) - (9*b*c^4*d*x^4 + 13*b*c^2*d*x^2 - 26*b*d)*s
qrt(c^2*x^2 + 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.24

$$\int x^2\left(d+c^2dx^2\right)\left(a+b\operatorname{arcsinh}(cx)\right)dx$$
$$=\begin{cases}\frac{ac^2dx^5}{5}+\frac{adx^3}{3}+\frac{bc^2dx^5\operatorname{asinh}(cx)}{5}-\frac{bcdx^4\sqrt{c^2x^2+1}}{25}+\frac{bdx^3\operatorname{asinh}(cx)}{3}-\frac{13bdx^2\sqrt{c^2x^2+1}}{225c}+\frac{26bd\sqrt{c^2x^2+1}}{225c^3} & \text{for } c\neq 0\\ \frac{adx^3}{3} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**2*(c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d*x**5/5 + a*d*x**3/3 + b*c**2*d*x**5*asinh(c*x)/5 - b*c*
d*x**4*sqrt(c**2*x**2 + 1)/25 + b*d*x**3*asinh(c*x)/3 - 13*b*d*x**2*sqrt(c*
*2*x**2 + 1)/(225*c) + 26*b*d*sqrt(c**2*x**2 + 1)/(225*c**3), Ne(c, 0)), (a
*d*x**3/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.42

$$\int x^2\left(d+c^2dx^2\right)\left(a+b\operatorname{arcsinh}(cx)\right)dx$$
$$=\frac{1}{5}\,ac^2dx^5$$
$$+\frac{1}{75}\left(15\,x^5\operatorname{arsinh}(cx)-\left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2}-\frac{4\,\sqrt{c^2x^2+1}x^2}{c^4}+\frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\right)bc^2d$$
$$+\frac{1}{3}\,adx^3+\frac{1}{9}\left(3\,x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)bd$$

```
[In] integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*c^2*d*x^5 + 1/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2
- 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*c^2*d + 1/3*a
*d*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^
2*x^2 + 1)/c^4))*b*d
```

Giac [F(-2)]

Exception generated.

$$\int x^2(d + c^2dx^2)(a + b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d + c^2dx^2)(a + b\text{arcsinh}(cx))\,dx = \int x^2\,(a + b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2 + d\right)\,dx$$

```
[In] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2),x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2), x)
```

3.4 $\int x(d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 87

$$\int x(d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx=-\frac{3bdx\sqrt{1+c^2x^2}}{32c}-\frac{bdx(1+c^2x^2)^{3/2}}{16c}-\frac{3bd\text{arcsinh}(cx)}{32c^2}+\frac{d(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{4c^2}$$

```
[Out] -1/16*b*d*x*(c^2*x^2+1)^(3/2)/c-3/32*b*d*arcsinh(c*x)/c^2+1/4*d*(c^2*x^2+1)
^2*(a+b*arcsinh(c*x))/c^2-3/32*b*d*x*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5798, 201, 221}

$$\int x(d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx=\frac{d(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{3bd\text{arcsinh}(cx)}{32c^2}-\frac{bdx(c^2x^2+1)^{3/2}}{16c}-\frac{3bdx\sqrt{c^2x^2+1}}{32c}$$

```
[In] Int[x*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-3*b*d*x*Sqrt[1 + c^2*x^2])/(32*c) - (b*d*x*(1 + c^2*x^2)^(3/2))/(16*c) -
(3*b*d*ArcSinh[c*x])/(32*c^2) + (d*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(4
*c^2)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{4c^2} - \frac{(bd)\int(1+c^2x^2)^{3/2}\,dx}{4c} \\
&= -\frac{bdx(1+c^2x^2)^{3/2}}{16c} + \frac{d(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{4c^2} - \frac{(3bd)\int\sqrt{1+c^2x^2}\,dx}{16c} \\
&= -\frac{3bdx\sqrt{1+c^2x^2}}{32c} - \frac{bdx(1+c^2x^2)^{3/2}}{16c} + \frac{d(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{4c^2} - \frac{(3bd)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{32c} \\
&= -\frac{3bdx\sqrt{1+c^2x^2}}{32c} - \frac{bdx(1+c^2x^2)^{3/2}}{16c} - \frac{3bd\text{arcsinh}(cx)}{32c^2} + \frac{d(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{4c^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.89

$$\int x\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))\,dx = \frac{d\left(cx\left(8acx(2+c^2x^2)-b\sqrt{1+c^2x^2}(5+2c^2x^2)\right)+b(5+16c^2x^2+8c^4x^4)\,\text{arcsinh}(cx)\right)}{32c^2}$$

```
[In] Integrate[x*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*(c*x*(8*a*c*x*(2 + c^2*x^2) - b*Sqrt[1 + c^2*x^2]*(5 + 2*c^2*x^2)) + b*(
5 + 16*c^2*x^2 + 8*c^4*x^4)*ArcSinh[c*x]))/(32*c^2)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{\frac{da\left(c^2x^2+1\right)^2}{4}+db\left(\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}+\frac{\operatorname{arcsinh}(cx)c^2x^2}{2}+\frac{5\operatorname{arcsinh}(cx)}{32}-\frac{cx\left(c^2x^2+1\right)^{\frac{3}{2}}}{16}-\frac{3cx\sqrt{c^2x^2+1}}{32}\right)}{c^2}$	85
default	$\frac{\frac{da\left(c^2x^2+1\right)^2}{4}+db\left(\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}+\frac{\operatorname{arcsinh}(cx)c^2x^2}{2}+\frac{5\operatorname{arcsinh}(cx)}{32}-\frac{cx\left(c^2x^2+1\right)^{\frac{3}{2}}}{16}-\frac{3cx\sqrt{c^2x^2+1}}{32}\right)}{c^2}$	85
parts	$\frac{da\left(c^2x^2+1\right)^2}{4c^2}+\frac{db\left(\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}+\frac{\operatorname{arcsinh}(cx)c^2x^2}{2}+\frac{5\operatorname{arcsinh}(cx)}{32}-\frac{cx\left(c^2x^2+1\right)^{\frac{3}{2}}}{16}-\frac{3cx\sqrt{c^2x^2+1}}{32}\right)}{c^2}$	87

```
[In] int(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/4*d*a*(c^2*x^2+1)^2+d*b*(1/4*arcsinh(c*x)*c^4*x^4+1/2*arcsinh(c*x)
*c^2*x^2+5/32*arcsinh(c*x)-1/16*c*x*(c^2*x^2+1)^(3/2)-3/32*c*x*(c^2*x^2+1)^
(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.13

$$\int x\left(d+c^2dx^2\right)(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\frac{8\,ac^4dx^4+16\,ac^2dx^2+\left(8\,bc^4dx^4+16\,bc^2dx^2+5\,bd\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(2\,bc^3dx^3+5\,bcdx\right)\sqrt{c^2x^2+}}{32\,c^2}$$

```
[In] integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/32*(8*a*c^4*d*x^4 + 16*a*c^2*d*x^2 + (8*b*c^4*d*x^4 + 16*b*c^2*d*x^2 + 5*
b*d)*log(c*x + sqrt(c^2*x^2 + 1)) - (2*b*c^3*d*x^3 + 5*b*c*d*x)*sqrt(c^2*x^
2 + 1))/c^2
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.34

$$\int x(d + c^2dx^2) (a + b\text{arcsinh}(cx)) dx$$
$$= \begin{cases} \frac{ac^2dx^4}{4} + \frac{adx^2}{2} + \frac{bc^2dx^4 \operatorname{asinh}(cx)}{4} - \frac{bcdx^3\sqrt{c^2x^2+1}}{16} + \frac{bdx^2 \operatorname{asinh}(cx)}{2} - \frac{5bdx\sqrt{c^2x^2+1}}{32c} + \frac{5bd \operatorname{asinh}(cx)}{32c^2} & \text{for } c \neq 0 \\ \frac{adx^2}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x*(c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d*x**4/4 + a*d*x**2/2 + b*c**2*d*x**4*asinh(c*x)/4 - b*c*
d*x**3*sqrt(c**2*x**2 + 1)/16 + b*d*x**2*asinh(c*x)/2 - 5*b*d*x*sqrt(c**2*x
**2 + 1)/(32*c) + 5*b*d*asinh(c*x)/(32*c**2), Ne(c, 0)), (a*d*x**2/2, True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.46

$$\int x(d + c^2dx^2) (a + b\text{arcsinh}(cx)) dx$$
$$= \frac{1}{4} ac^2dx^4 + \frac{1}{32} \left(8 x^4 \operatorname{arsinh}(cx) - \left(\frac{2 \sqrt{c^2x^2 + 1}x^3}{c^2} - \frac{3 \sqrt{c^2x^2 + 1}x}{c^4} + \frac{3 \operatorname{arsinh}(cx)}{c^5}\right)c\right)bc^2d + \frac{1}{2} adx^2 + \frac{1}{4} \left(2 x^2 \operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2 + 1}x}{c^2} - \frac{\operatorname{arsinh}(cx)}{c^3}\right)\right)bd$$

```
[In] integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*c^2*d*x^4 + 1/32*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 -
 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*b*c^2*d + 1/2*a*d*x^2 +
 1/4*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3))*
b*d
```

Giac [F(-2)]

Exception generated.

$$\int x(d+c^2dx^2)\,(a+b\operatorname{arcsinh}(cx))\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(d+c^2dx^2)\,(a+b\operatorname{arcsinh}(cx))\,dx=\int x\,(a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)\,dx$$

```
[In] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2), x)
```

3.5 $\int (d + c^2dx^2)(a + b\text{arcsinh}(cx))\, dx$

Optimal result

Integrand size = 19, antiderivative size = 75

$$\int (d + c^2dx^2)(a + b\text{arcsinh}(cx))\, dx = -\frac{2bd\sqrt{1+c^2x^2}}{3c} - \frac{bd(1+c^2x^2)^{3/2}}{9c} + dx(a + b\text{arcsinh}(cx)) + \frac{1}{3}c^2dx^3(a + b\text{arcsinh}(cx))$$

```
[Out] -1/9*b*d*(c^2*x^2+1)^(3/2)/c+d*x*(a+b*arcsinh(c*x))+1/3*c^2*d*x^3*(a+b*arcs
inh(c*x))-2/3*b*d*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {5784, 12, 455, 45}

$$\int (d + c^2dx^2)(a + b\text{arcsinh}(cx))\, dx = \frac{1}{3}c^2dx^3(a + b\text{arcsinh}(cx)) + dx(a + b\text{arcsinh}(cx)) - \frac{bd(c^2x^2+1)^{3/2}}{9c} - \frac{2bd\sqrt{c^2x^2+1}}{3c}$$

```
[In] Int[(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-2*b*d*Sqrt[1 + c^2*x^2])/(3*c) - (b*d*(1 + c^2*x^2)^(3/2))/(9*c) + d*x*(a
 + b*ArcSinh[c*x]) + (c^2*d*x^3*(a + b*ArcSinh[c*x]))/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= dx(a+b\operatorname{arcsinh}(cx))+\frac{1}{3}c^2dx^3(a+b\operatorname{arcsinh}(cx))-(bc)\int\frac{dx(3+c^2x^2)}{3\sqrt{1+c^2x^2}}\,dx\\
&= dx(a+b\operatorname{arcsinh}(cx))+\frac{1}{3}c^2dx^3(a+b\operatorname{arcsinh}(cx))-\frac{1}{3}(bcd)\int\frac{x(3+c^2x^2)}{\sqrt{1+c^2x^2}}\,dx\\
&= dx(a+b\operatorname{arcsinh}(cx))+\frac{1}{3}c^2dx^3(a+b\operatorname{arcsinh}(cx))-\frac{1}{6}(bcd)\operatorname{Subst}\left(\int\frac{3+c^2x}{\sqrt{1+c^2x}}\,dx,x,x^2\right)\\
&= dx(a+b\operatorname{arcsinh}(cx))+\frac{1}{3}c^2dx^3(a+b\operatorname{arcsinh}(cx))\\
&\quad-\frac{1}{6}(bcd)\operatorname{Subst}\left(\int\left(\frac{2}{\sqrt{1+c^2x}}+\sqrt{1+c^2x}\right)\,dx,x,x^2\right)\\
&= -\frac{2bd\sqrt{1+c^2x^2}}{3c}-\frac{bd(1+c^2x^2)^{3/2}}{9c}+dx(a+b\operatorname{arcsinh}(cx))+\frac{1}{3}c^2dx^3(a+b\operatorname{arcsinh}(cx))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.15

$$\int \left(d+c^2dx^2\right)\left(a+b\operatorname{arcsinh}(cx)\right)dx=adx+\frac{1}{3}ac^2dx^3-\frac{7bd\sqrt{1+c^2x^2}}{9c}-\frac{1}{9}bcdx^2\sqrt{1+c^2x^2}+bdx\operatorname{arcsinh}(cx)+\frac{1}{3}bc^2dx^3\operatorname{arcsinh}(cx)$$

```
[In] Integrate[(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
[Out] a*d*x + (a*c^2*d*x^3)/3 - (7*b*d*Sqrt[1 + c^2*x^2])/(9*c) - (b*c*d*x^2*Sqrt
[1 + c^2*x^2])/9 + b*d*x*ArcSinh[c*x] + (b*c^2*d*x^3*ArcSinh[c*x])/3
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

method	result	size
parts	$da\left(\frac{1}{3}x^3c^2+x\right)+\frac{db\left(\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}+\operatorname{arcsinh}(cx)cx-\frac{c^2x^2\sqrt{c^2x^2+1}}{9}-\frac{7\sqrt{c^2x^2+1}}{9}\right)}{c}$	73
derivativedivides	$\frac{da\left(\frac{1}{3}c^3x^3+cx\right)+db\left(\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}+\operatorname{arcsinh}(cx)cx-\frac{c^2x^2\sqrt{c^2x^2+1}}{9}-\frac{7\sqrt{c^2x^2+1}}{9}\right)}{c}$	76
default	$\frac{da\left(\frac{1}{3}c^3x^3+cx\right)+db\left(\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}+\operatorname{arcsinh}(cx)cx-\frac{c^2x^2\sqrt{c^2x^2+1}}{9}-\frac{7\sqrt{c^2x^2+1}}{9}\right)}{c}$	76

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
[Out] d*a*(1/3*x^3*c^2+x)+d*b/c*(1/3*arcsinh(c*x)*c^3*x^3+arcsinh(c*x)*c*x-1/9*c^
2*x^2*(c^2*x^2+1)^(1/2)-7/9*(c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.11

$$\int \left(d+c^2dx^2\right)\left(a+b\operatorname{arcsinh}(cx)\right)dx=\frac{3\,ac^3dx^3+9\,acdx+3\left(bc^3dx^3+3\,bcdx\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(bc^2dx^2+7\,bd\right)\sqrt{c^2x^2+1}}{9\,c}$$

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] 1/9*(3*a*c^3*d*x^3 + 9*a*c*d*x + 3*(b*c^3*d*x^3 + 3*b*c*d*x)*log(c*x + sqrt
(c^2*x^2 + 1)) - (b*c^2*d*x^2 + 7*b*d)*sqrt(c^2*x^2 + 1))/c
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.20

$$\int (d + c^2 dx^2) (a + b\text{arcsinh}(cx)) \, dx = \begin{cases} \frac{ac^2dx^3}{3} + adx + \frac{bc^2dx^3 \operatorname{asinh}(cx)}{3} - \frac{bcdx^2\sqrt{c^2x^2+1}}{9} + bdx \operatorname{asinh}(cx) - \frac{7bd\sqrt{c^2x^2+1}}{9c} & \text{for } c \neq 0 \\ adx & \text{otherwise} \end{cases}$$

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d*x**3/3 + a*d*x + b*c**2*d*x**3*asinh(c*x)/3 - b*c*d*x**
2*sqrt(c**2*x**2 + 1)/9 + b*d*x*asinh(c*x) - 7*b*d*sqrt(c**2*x**2 + 1)/(9*c
), Ne(c, 0)), (a*d*x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.29

$$\int (d + c^2 dx^2) (a + b\text{arcsinh}(cx)) \, dx = \frac{1}{3} ac^2dx^3 + \frac{1}{9} \left(3x^3 \operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bc^2d + adx + \frac{\left(cx \operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)bd}{c}$$

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*c^2*d*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 -
2*sqrt(c^2*x^2 + 1)/c^4))*b*c^2*d + a*d*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^
2 + 1))*b*d/c
```

Giac [F(-2)]

Exception generated.

$$\int (d + c^2 dx^2) (a + b\text{arcsinh}(cx)) \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))\,dx=\int (a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)\,dx$$

```
[In] int((a + b*asinh(c*x))*(d + c^2*d*x^2),x)
[Out] int((a + b*asinh(c*x))*(d + c^2*d*x^2), x)
```

3.6 $\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 111

$$\begin{aligned}\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x}\,dx &= -\frac{1}{4}bcdx\sqrt{1+c^2x^2}-\frac{1}{4}bd\text{arcsinh}(cx)\\ &\quad+\frac{1}{2}d(1+c^2x^2)(a+b\text{arcsinh}(cx))\\ &\quad+\frac{d(a+b\text{arcsinh}(cx))^2}{2b}\\ &\quad+d(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)\\ &\quad-\frac{1}{2}bd\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)\end{aligned}$$

```
[Out] -1/4*b*d*arcsinh(c*x)+1/2*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))+1/2*d*(a+b*arcsi
nh(c*x))^2/b+d*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b*d
*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)-1/4*b*c*d*x*(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used

$= \{5801, 5775, 3797, 2221, 2317, 2438, 201, 221\}$

$$\int \frac{(d + c^2dx^2)(a + b\text{arcsinh}(cx))}{x}\,dx = \frac{1}{2}d(c^2x^2 + 1)(a + b\text{arcsinh}(cx)) + \frac{d(a + b\text{arcsinh}(cx))^2}{2b} + d\log\left(1 - e^{-2\text{arcsinh}(cx)}\right)(a + b\text{arcsinh}(cx)) - \frac{1}{2}bd\,\text{PolyLog}\left(2, e^{-2\text{arcsinh}(cx)}\right) - \frac{1}{4}bd\text{arcsinh}(cx) - \frac{1}{4}bcdx\sqrt{c^2x^2 + 1}$$

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] -1/4*(b*c*d*x*Sqrt[1 + c^2*x^2]) - (b*d*ArcSinh[c*x])/4 + (d*(1 + c^2*x^2)*
(a + b*ArcSinh[c*x]))/2 + (d*(a + b*ArcSinh[c*x])^2)/(2*b) + d*(a + b*ArcSi
nh[c*x])*Log[1 - E^(-2*ArcSinh[c*x])] - (b*d*PolyLog[2, E^(-2*ArcSinh[c*x])
])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5801

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcSinh[c*x])/(2*p)), x] + (Dist[d
, Int[(d + e*x^2)^(p - 1)*((a + b*ArcSinh[c*x])/x), x], x] - Dist[b*c*(d^p/
(2*p)), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}d\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))+d\int\frac{a+b\text{arcsinh}(cx)}{x}\,dx-\frac{1}{2}(bcd)\int\sqrt{1+c^2x^2}\,dx\\
&= -\frac{1}{4}bcdx\sqrt{1+c^2x^2}+\frac{1}{2}d\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))\\
&\quad-\frac{d\text{Subst}\left(\int x\coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{b}-\frac{1}{4}(bcd)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx\\
&= -\frac{1}{4}bcdx\sqrt{1+c^2x^2}-\frac{1}{4}bd\text{arcsinh}(cx)+\frac{1}{2}d\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))\\
&\quad+\frac{d(a+b\text{arcsinh}(cx))^2}{2b}+\frac{(2d)\text{Subst}\left(\int\frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b}\\
&= -\frac{1}{4}bcdx\sqrt{1+c^2x^2}-\frac{1}{4}bd\text{arcsinh}(cx)+\frac{1}{2}d\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))\\
&\quad+\frac{d(a+b\text{arcsinh}(cx))^2}{2b}+d(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)\\
&\quad-d\text{Subst}\left(\int\log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{1}{4}bcdx\sqrt{1+c^2x^2} - \frac{1}{4}bd\text{arcsinh}(cx) + \frac{1}{2}d(1+c^2x^2)\,(a+b\text{arcsinh}(cx)) \\
&\quad + \frac{d(a+b\text{arcsinh}(cx))^2}{2b} + d(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right) \\
&\quad + \frac{1}{2}(bd)\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right) \\
&= -\frac{1}{4}bcdx\sqrt{1+c^2x^2} - \frac{1}{4}bd\text{arcsinh}(cx) + \frac{1}{2}d(1+c^2x^2)\,(a+b\text{arcsinh}(cx)) \\
&\quad + \frac{d(a+b\text{arcsinh}(cx))^2}{2b} + d(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right) \\
&\quad - \frac{1}{2}bd\,\text{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02

$$
\begin{aligned}
\int \frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))}{x}\,dx &= \frac{1}{2}ac^2dx^2 - \frac{1}{4}bcdx\sqrt{1+c^2x^2} + \frac{1}{4}bd\text{arcsinh}(cx) \\
&\quad + \frac{1}{2}bc^2dx^2\text{arcsinh}(cx) - \frac{1}{2}bd\text{arcsinh}(cx)^2 \\
&\quad + bd\text{arcsinh}(cx)\log\left(1-e^{2\text{arcsinh}(cx)}\right) \\
&\quad + ad\log(x) + \frac{1}{2}bd\,\text{PolyLog}\left(2, e^{2\text{arcsinh}(cx)}\right)
\end{aligned}
$$

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (a*c^2*d*x^2)/2 - (b*c*d*x*Sqrt[1 + c^2*x^2])/4 + (b*d*ArcSinh[c*x])/4 + (b
*c^2*d*x^2*ArcSinh[c*x])/2 - (b*d*ArcSinh[c*x]^2)/2 + b*d*ArcSinh[c*x]*Log[
1 - E^(2*ArcSinh[c*x])] + a*d*Log[x] + (b*d*PolyLog[2, E^(2*ArcSinh[c*x])])
/2
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.43

method	result
parts	$da\left(\frac{c^2x^2}{2}+\ln(x)\right)-\frac{db\,\text{arcsinh}(cx)^2}{2}+\frac{db\,\text{arcsinh}(cx)c^2x^2}{2}-\frac{bcdx\sqrt{c^2x^2+1}}{4}+\frac{bd\,\text{arcsinh}(cx)}{4}+db\ \text{arcsi}$
derivativedivides	$da\left(\frac{c^2x^2}{2}+\ln(cx)\right)-\frac{db\,\text{arcsinh}(cx)^2}{2}+\frac{db\,\text{arcsinh}(cx)c^2x^2}{2}-\frac{bcdx\sqrt{c^2x^2+1}}{4}+\frac{bd\,\text{arcsinh}(cx)}{4}+db\ \text{arcs}$
default	$da\left(\frac{c^2x^2}{2}+\ln(cx)\right)-\frac{db\,\text{arcsinh}(cx)^2}{2}+\frac{db\,\text{arcsinh}(cx)c^2x^2}{2}-\frac{bcdx\sqrt{c^2x^2+1}}{4}+\frac{bd\,\text{arcsinh}(cx)}{4}+db\ \text{arcs}$

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] d*a*(1/2*c^2*x^2+ln(x))-1/2*d*b*arcsinh(c*x)^2+1/2*d*b*arcsinh(c*x)*c^2*x^2
-1/4*b*c*d*x*(c^2*x^2+1)^(1/2)+1/4*b*d*arcsinh(c*x)+d*b*arcsinh(c*x)*ln(1+c
*x+(c^2*x^2+1)^(1/2))+d*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+d*b*arcsinh(c*x
)*ln(1-c*x-(c^2*x^2+1)^(1/2))+d*b*polylog(2,c*x+(c^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{(d + c^2dx^2)\,(a + b\mathrm{arcsinh}(cx))}{x}\,dx = \int \frac{(c^2dx^2 + d)(b\operatorname{arsinh}(cx) + a)}{x}\,dx$$

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d + c^2dx^2)\,(a + b\mathrm{arcsinh}(cx))}{x}\,dx = d\left(\int \frac{a}{x}\,dx + \int ac^2x\,dx + \int \frac{b\operatorname{asinh}(cx)}{x}\,dx + \int bc^2x\operatorname{asinh}(cx)\,dx\right)$$

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))/x,x)
```

```
[Out] d*(Integral(a/x, x) + Integral(a*c**2*x, x) + Integral(b*asinh(c*x)/x, x) +
 Integral(b*c**2*x*asinh(c*x), x))
```

Maxima [F]

$$\int \frac{(d + c^2dx^2)\,(a + b\mathrm{arcsinh}(cx))}{x}\,dx = \int \frac{(c^2dx^2 + d)(b\operatorname{arsinh}(cx) + a)}{x}\,dx$$

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*c^2*d*x^2 + a*d*log(x) + integrate(b*c^2*d*x*log(c*x + sqrt(c^2*x^2 +
 1)) + b*d*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,(d\,c^2\,x^2+d)}{x}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x, x)
```

3.7 $\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 66

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^2}\,dx = -bcd\sqrt{1+c^2x^2}-\frac{d(a+b\text{arcsinh}(cx))}{x}+c^2dx(a+b\text{arcsinh}(cx))-bcd\text{arctanh}\left(\sqrt{1+c^2x^2}\right)$$

```
[Out] -d*(a+b*arcsinh(c*x))/x+c^2*d*x*(a+b*arcsinh(c*x))-b*c*d*arctanh((c^2*x^2+1
)^(1/2))-b*c*d*(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {14, 5803, 12, 457, 81, 65, 214}

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^2}\,dx = c^2dx(a+b\text{arcsinh}(cx))-\frac{d(a+b\text{arcsinh}(cx))}{x}-bcd\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-bcd\sqrt{c^2x^2+1}$$

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] -(b*c*d*Sqrt[1 + c^2*x^2]) - (d*(a + b*ArcSinh[c*x]))/x + c^2*d*x*(a + b*Ar
cSinh[c*x]) - b*c*d*ArcTanh[Sqrt[1 + c^2*x^2]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\text{integral} = -\frac{d(a+b\text{arcsinh}(cx))}{x} + c^2dx(a+b\text{arcsinh}(cx)) - (bc)\int \frac{d(-1+c^2x^2)}{x\sqrt{1+c^2x^2}}\,dx$$

$$= -\frac{d(a + b\text{arcsinh}(cx))}{x} + c^2dx(a + b\text{arcsinh}(cx)) - (bcd)\int \frac{-1 + c^2x^2}{x\sqrt{1 + c^2x^2}}\,dx$$

$$= -\frac{d(a + b\text{arcsinh}(cx))}{x} + c^2dx(a + b\text{arcsinh}(cx)) - \frac{1}{2}(bcd)\text{Subst}\left(\int \frac{-1 + c^2x}{x\sqrt{1 + c^2x}}\,dx, x, x^2\right)$$

$$= -bcd\sqrt{1 + c^2x^2} - \frac{d(a + b\text{arcsinh}(cx))}{x} + c^2dx(a + b\text{arcsinh}(cx)) + \frac{1}{2}(bcd)\text{Subst}\left(\int \frac{1}{x\sqrt{1 + c^2x}}\,dx, x, x^2\right)$$

$$= -bcd\sqrt{1 + c^2x^2} - \frac{d(a + b\text{arcsinh}(cx))}{x} + c^2dx(a + b\text{arcsinh}(cx)) + \frac{(bd)\text{Subst}\left(\int \frac{1}{-\frac{1}{c^2} + \frac{x^2}{c^2}}\,dx, x, \sqrt{1 + c^2x^2}\right)}{c}$$

$$= -bcd\sqrt{1 + c^2x^2} - \frac{d(a + b\text{arcsinh}(cx))}{x} + c^2dx(a + b\text{arcsinh}(cx)) - bcd\text{arctanh}\left(\sqrt{1 + c^2x^2}\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.12

$$\int \frac{(d + c^2dx^2)(a + b\text{arcsinh}(cx))}{x^2}\,dx = -\frac{ad}{x} + ac^2dx - bcd\sqrt{1 + c^2x^2} - \frac{bd\text{arcsinh}(cx)}{x} + bc^2dx\text{arcsinh}(cx) - bcd\text{arctanh}\left(\sqrt{1 + c^2x^2}\right)$$

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] -((a*d)/x) + a*c^2*d*x - b*c*d*Sqrt[1 + c^2*x^2] - (b*d*ArcSinh[c*x])/x + b
*c^2*d*x*ArcSinh[c*x] - b*c*d*ArcTanh[Sqrt[1 + c^2*x^2]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

method	result
parts	$da\left(c^2x - \frac{1}{x}\right) + dbc\left(\text{arcsinh}\,(cx)\,cx - \frac{\text{arcsinh}(cx)}{cx} - \sqrt{c^2x^2 + 1} - \text{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)\right)$
derivativedivides	$c\left(da\left(cx - \frac{1}{cx}\right) + db\left(\text{arcsinh}\,(cx)\,cx - \frac{\text{arcsinh}(cx)}{cx} - \sqrt{c^2x^2 + 1} - \text{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)\right)\right)$
default	$c\left(da\left(cx - \frac{1}{cx}\right) + db\left(\text{arcsinh}\,(cx)\,cx - \frac{\text{arcsinh}(cx)}{cx} - \sqrt{c^2x^2 + 1} - \text{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)\right)\right)$

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] d*a*(c^2*x-1/x)+d*b*c*(arcsinh(c*x)*c*x-arcsinh(c*x)/c/x-(c^2*x^2+1)^(1/2)-
arctanh(1/(c^2*x^2+1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(62) = 124.

Time = 0.29 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.36

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^2}\,dx = \frac{ac^2dx^2 - bcdx\log\left(-cx+\sqrt{c^2x^2+1}+1\right) + bcdx\log\left(-cx+\sqrt{c^2x^2+1}-1\right) - \sqrt{c^2x^2+1}bcdx - (bc^2-b)}{x}$$

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] (a*c^2*d*x^2 - b*c*d*x*log(-c*x + sqrt(c^2*x^2 + 1) + 1) + b*c*d*x*log(-c*x
 + sqrt(c^2*x^2 + 1) - 1) - sqrt(c^2*x^2 + 1)*b*c*d*x - (b*c^2 - b)*d*x*log
(-c*x + sqrt(c^2*x^2 + 1)) - a*d + (b*c^2*d*x^2 - (b*c^2 - b)*d*x - b*d)*lo
g(c*x + sqrt(c^2*x^2 + 1)))/x
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^2}\,dx = d\left(\int ac^2\,dx + \int \frac{a}{x^2}\,dx + \int bc^2\operatorname{asinh}(cx)\,dx + \int \frac{b\operatorname{asinh}(cx)}{x^2}\,dx\right)$$

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))/x**2,x)
```

```
[Out] d*(Integral(a*c**2, x) + Integral(a/x**2, x) + Integral(b*c**2*asinh(c*x),
x) + Integral(b*asinh(c*x)/x**2, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^2}\,dx = ac^2dx + \left(cx\,\text{arsinh}(cx) - \sqrt{c^2x^2+1}\right)bcd - \left(c\,\text{arsinh}\left(\frac{1}{c|x|}\right) + \frac{\text{arsinh}(cx)}{x}\right)bd - \frac{ad}{x}$$

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] a*c^2*d*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*c*d - (c*arcsinh(1/(c*
abs(x))) + arcsinh(c*x)/x)*b*d - a*d/x
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + c^2dx^2)(a + b\text{arcsinh}(cx))}{x^2} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + c^2dx^2)(a + b\text{arcsinh}(cx))}{x^2} dx = \int \frac{(a + b\,\text{asinh}(c\,x))\,(d\,c^2\,x^2 + d)}{x^2} dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x^2, x)
```

3.8 $\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 128

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^3}\,dx = -\frac{bcd\sqrt{1+c^2x^2}}{2x}+\frac{1}{2}bc^2d\text{arcsinh}(cx) - \frac{d(1+c^2x^2)(a+b\text{arcsinh}(cx))}{2x^2} + \frac{c^2d(a+b\text{arcsinh}(cx))^2}{2b} + c^2d(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right) - \frac{1}{2}bc^2d\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)$$

```
[Out] 1/2*b*c^2*d*arcsinh(c*x)-1/2*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))/x^2+1/2*c^2*d
*(a+b*arcsinh(c*x))^2/b+c^2*d*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1
/2))^2)-1/2*b*c^2*d*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b*c*d*(c^2*x
^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used

$= \{5802, 283, 221, 5775, 3797, 2221, 2317, 2438\}$

$$\int \frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))}{x^3}\,dx = -\frac{d(c^2x^2+1)\,(a+b\text{arcsinh}(cx))}{2x^2} + \frac{c^2d(a+b\text{arcsinh}(cx))^2}{2b} + c^2d\log\left(1-e^{-2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)) - \frac{1}{2}bc^2d\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right) + \frac{1}{2}bc^2d\text{arcsinh}(cx) - \frac{bcd\sqrt{c^2x^2+1}}{2x}$$

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*d*Sqrt[1 + c^2*x^2])/x + (b*c^2*d*ArcSinh[c*x])/2 - (d*(1 + c^2*x
^2)*(a + b*ArcSinh[c*x]))/(2*x^2) + (c^2*d*(a + b*ArcSinh[c*x])^2)/(2*b) +
c^2*d*(a + b*ArcSinh[c*x])*Log[1 - E^(-2*ArcSinh[c*x])] - (b*c^2*d*PolyLog[
2, E^(-2*ArcSinh[c*x])])/2
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5802

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSinh[c
*x])/(f*(m + 1))), x] + (-Dist[b*c*(d^p/(f*(m + 1))), Int[(f*x)^(m + 1)*(1
+ c^2*x^2)^(p - 1/2), x], x] - Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m + 2
)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{2x^2} \\
&\quad + \frac{1}{2}(bcd)\int \frac{\sqrt{1+c^2x^2}}{x^2}\,dx + \left(c^2d\right)\int \frac{a+b\text{arcsinh}(cx)}{x}\,dx \\
&= -\frac{bcd\sqrt{1+c^2x^2}}{2x} - \frac{d(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{2x^2} \\
&\quad - \frac{(c^2d)\,\text{Subst}\left(\int x\coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)}{b} \\
&\quad + \frac{1}{2}\left(bc^3d\right)\int \frac{1}{\sqrt{1+c^2x^2}}\,dx \\
&= -\frac{bcd\sqrt{1+c^2x^2}}{2x} + \frac{1}{2}bc^2d\text{arcsinh}(cx) - \frac{d(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{2x^2} \\
&\quad + \frac{c^2d(a+b\text{arcsinh}(cx))^2}{2b} + \frac{(2c^2d)\,\text{Subst}\left(\int \frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bcd\sqrt{1+c^2x^2}}{2x} + \frac{1}{2}bc^2d\text{arcsinh}(cx) - \frac{d(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{2x^2} \\
&\quad + \frac{c^2d(a+b\text{arcsinh}(cx))^2}{2b} + c^2d(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right) \\
&\quad - (c^2d)\,\text{Subst}\left(\int \log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\text{arcsinh}(cx)\right) \\
&= -\frac{bcd\sqrt{1+c^2x^2}}{2x} + \frac{1}{2}bc^2d\text{arcsinh}(cx) - \frac{d(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{2x^2} \\
&\quad + \frac{c^2d(a+b\text{arcsinh}(cx))^2}{2b} + c^2d(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right) \\
&\quad + \frac{1}{2}(bc^2d)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right) \\
&= -\frac{bcd\sqrt{1+c^2x^2}}{2x} + \frac{1}{2}bc^2d\text{arcsinh}(cx) - \frac{d(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{2x^2} \\
&\quad + \frac{c^2d(a+b\text{arcsinh}(cx))^2}{2b} + c^2d(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right) \\
&\quad - \frac{1}{2}bc^2d\,\text{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.87

$$
\begin{aligned}
\int \frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))}{x^3}\,dx &= -\frac{ad}{2x^2} - \frac{bcd\sqrt{1+c^2x^2}}{2x} \\
&\quad - \frac{bd\text{arcsinh}(cx)}{2x^2} - \frac{1}{2}bc^2d\text{arcsinh}(cx)^2 \\
&\quad + bc^2d\text{arcsinh}(cx)\log\left(1-e^{2\text{arcsinh}(cx)}\right) \\
&\quad + ac^2d\log(x) + \frac{1}{2}bc^2d\,\text{PolyLog}\left(2, e^{2\text{arcsinh}(cx)}\right)
\end{aligned}
$$

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -1/2*(a*d)/x^2 - (b*c*d*Sqrt[1 + c^2*x^2])/(2*x) - (b*d*ArcSinh[c*x])/(2*x^
2) - (b*c^2*d*ArcSinh[c*x]^2)/2 + b*c^2*d*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh
[c*x])] + a*c^2*d*Log[x] + (b*c^2*d*PolyLog[2, E^(2*ArcSinh[c*x])])/2
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.18

method	result
parts	$da\left(-\frac{1}{2x^2}+c^2\ln\left(x\right)\right)+db\,c^2\left(-\frac{\operatorname{arcsinh}(cx)^2}{2}-\frac{cx\sqrt{c^2x^2+1}-c^2x^2+\operatorname{arcsinh}(cx)}{2c^2x^2}+\operatorname{arcsinh}\left(cx\right)\ln\left(1\right.\right.$
derivativedivides	$c^2\left(da\left(\ln\left(cx\right)-\frac{1}{2c^2x^2}\right)+db\left(-\frac{\operatorname{arcsinh}(cx)^2}{2}-\frac{cx\sqrt{c^2x^2+1}-c^2x^2+\operatorname{arcsinh}(cx)}{2c^2x^2}+\operatorname{arcsinh}\left(cx\right)\ln\left(1\right.\right.\right.$
default	$c^2\left(da\left(\ln\left(cx\right)-\frac{1}{2c^2x^2}\right)+db\left(-\frac{\operatorname{arcsinh}(cx)^2}{2}-\frac{cx\sqrt{c^2x^2+1}-c^2x^2+\operatorname{arcsinh}(cx)}{2c^2x^2}+\operatorname{arcsinh}\left(cx\right)\ln\left(1\right.\right.\right.$

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^3,x,method=_RETURNVERBOSE)
[Out] d*a*(-1/2/x^2+c^2*ln(x))+d*b*c^2*(-1/2*arcsinh(c*x)^2-1/2*(c*x*(c^2*x^2+1)^
(1/2)-c^2*x^2+arcsinh(c*x))/c^2/x^2+arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2)
)+polylog(2,-c*x-(c^2*x^2+1)^(1/2))+arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2)
)+polylog(2,c*x+(c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)\,(a+b\operatorname{arcsinh}(cx))}{x^3}\,dx=\int \frac{(c^2dx^2+d)(b\operatorname{arsinh}\left(cx\right)+a)}{x^3}\,dx$$

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas")
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)\,(a+b\operatorname{arcsinh}(cx))}{x^3}\,dx=d\left(\int \frac{a}{x^3}\,dx+\int \frac{ac^2}{x}\,dx+\int \frac{b\operatorname{asinh}\left(cx\right)}{x^3}\,dx+\int \frac{bc^2\operatorname{asinh}\left(cx\right)}{x}\,dx\right)$$

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))/x**3,x)
[Out] d*(Integral(a/x**3, x) + Integral(a*c**2/x, x) + Integral(b*asinh(c*x)/x**3
, x) + Integral(b*c**2*asinh(c*x)/x, x))
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^3}\,dx = \int \frac{(c^2dx^2+d)(b\,\text{arsinh}\,(cx)+a)}{x^3}\,dx$$

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] b*c^2*d*integrate(log(c*x + sqrt(c^2*x^2 + 1))/x, x) + a*c^2*d*log(x) - 1/2
*b*d*(sqrt(c^2*x^2 + 1)*c/x + arcsinh(c*x)/x^2) - 1/2*a*d/x^2
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^3}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,(d\,c^2\,x^2+d)}{x^3}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x^3, x)
```

3.9 $\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 80

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^4}\,dx = -\frac{bcd\sqrt{1+c^2x^2}}{6x^2} - \frac{d(a+b\text{arcsinh}(cx))}{3x^3} - \frac{c^2d(a+b\text{arcsinh}(cx))}{x} - \frac{5}{6}bc^3d\text{arctanh}\left(\sqrt{1+c^2x^2}\right)$$

```
[Out] -1/3*d*(a+b*arcsinh(c*x))/x^3-c^2*d*(a+b*arcsinh(c*x))/x-5/6*b*c^3*d*arctan
h((c^2*x^2+1)^(1/2))-1/6*b*c*d*(c^2*x^2+1)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {14, 5803, 12, 457, 79, 65, 214}

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^4}\,dx = -\frac{c^2d(a+b\text{arcsinh}(cx))}{x} - \frac{d(a+b\text{arcsinh}(cx))}{3x^3} - \frac{5}{6}bc^3d\text{arctanh}\left(\sqrt{c^2x^2+1}\right) - \frac{bcd\sqrt{c^2x^2+1}}{6x^2}$$

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*d*Sqrt[1 + c^2*x^2])/x^2 - (d*(a + b*ArcSinh[c*x]))/(3*x^3) - (c^
2*d*(a + b*ArcSinh[c*x]))/x - (5*b*c^3*d*ArcTanh[Sqrt[1 + c^2*x^2]])/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\text{integral} = -\frac{d(a+b\text{arcsinh}(cx))}{3x^3} - \frac{c^2d(a+b\text{arcsinh}(cx))}{x} - (bc)\int \frac{d(-1-3c^2x^2)}{3x^3\sqrt{1+c^2x^2}}\,dx$$
$$= -\frac{d(a+b\text{arcsinh}(cx))}{3x^3} - \frac{c^2d(a+b\text{arcsinh}(cx))}{x} - \frac{1}{3}(bcd)\int \frac{-1-3c^2x^2}{x^3\sqrt{1+c^2x^2}}\,dx$$
$$= -\frac{d(a+b\text{arcsinh}(cx))}{3x^3} - \frac{c^2d(a+b\text{arcsinh}(cx))}{x} - \frac{1}{6}(bcd)\text{Subst}\left(\int \frac{-1-3c^2x}{x^2\sqrt{1+c^2x}}\,dx, x, x^2\right)$$
$$= -\frac{bcd\sqrt{1+c^2x^2}}{6x^2} - \frac{d(a+b\text{arcsinh}(cx))}{3x^3} - \frac{c^2d(a+b\text{arcsinh}(cx))}{x} + \frac{1}{12}\left(5bc^3d\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+c^2x}}\,dx, x, x^2\right)$$
$$= -\frac{bcd\sqrt{1+c^2x^2}}{6x^2} - \frac{d(a+b\text{arcsinh}(cx))}{3x^3} - \frac{c^2d(a+b\text{arcsinh}(cx))}{x} + \frac{1}{6}(5bcd)\text{Subst}\left(\int \frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx, x, \sqrt{1+c^2x^2}\right)$$
$$= -\frac{bcd\sqrt{1+c^2x^2}}{6x^2} - \frac{d(a+b\text{arcsinh}(cx))}{3x^3} - \frac{c^2d(a+b\text{arcsinh}(cx))}{x} - \frac{5}{6}bc^3d\text{arctanh}\left(\sqrt{1+c^2x^2}\right)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.16

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^4}\,dx = -\frac{ad}{3x^3} - \frac{ac^2d}{x} - \frac{bcd\sqrt{1+c^2x^2}}{6x^2} - \frac{bd\text{arcsinh}(cx)}{3x^3} - \frac{bc^2d\text{arcsinh}(cx)}{x} - \frac{5}{6}bc^3d\text{arctanh}\left(\sqrt{1+c^2x^2}\right)$$

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -1/3*(a*d)/x^3 - (a*c^2*d)/x - (b*c*d*Sqrt[1 + c^2*x^2])/(6*x^2) - (b*d*Arc
Sinh[c*x])/(3*x^3) - (b*c^2*d*ArcSinh[c*x])/x - (5*b*c^3*d*ArcTanh[Sqrt[1 +
 c^2*x^2]])/6
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.04

method	result
parts	$da\left(-\frac{c^2}{x}-\frac{1}{3x^3}\right)+db\,c^3\left(-\frac{\operatorname{arcsinh}(cx)}{3c^3x^3}-\frac{\operatorname{arcsinh}(cx)}{cx}-\frac{\sqrt{c^2x^2+1}}{6c^2x^2}-\frac{5\,\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)}{6}\right)$
derivativedivides	$c^3\left(da\left(-\frac{1}{3c^3x^3}-\frac{1}{cx}\right)+db\left(-\frac{\operatorname{arcsinh}(cx)}{3c^3x^3}-\frac{\operatorname{arcsinh}(cx)}{cx}-\frac{\sqrt{c^2x^2+1}}{6c^2x^2}-\frac{5\,\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)}{6}\right)\right)$
default	$c^3\left(da\left(-\frac{1}{3c^3x^3}-\frac{1}{cx}\right)+db\left(-\frac{\operatorname{arcsinh}(cx)}{3c^3x^3}-\frac{\operatorname{arcsinh}(cx)}{cx}-\frac{\sqrt{c^2x^2+1}}{6c^2x^2}-\frac{5\,\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)}{6}\right)\right)$

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] d*a*(-c^2/x-1/3/x^3)+d*b*c^3*(-1/3*arcsinh(c*x)/c^3/x^3-arcsinh(c*x)/c/x-1/
6/c^2/x^2*(c^2*x^2+1)^(1/2)-5/6*arctanh(1/(c^2*x^2+1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(70) = 140.

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.11

$$\int \frac{(d+c^2dx^2)\,(a+b\operatorname{arcsinh}(cx))}{x^4}\,dx =$$
$$-\frac{5\,bc^3dx^3\log\left(-cx+\sqrt{c^2x^2+1}+1\right)-5\,bc^3dx^3\log\left(-cx+\sqrt{c^2x^2+1}-1\right)+6\,ac^2dx^2-2\,(3\,bc^2+b)dx}{}$$

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(5*b*c^3*d*x^3*log(-c*x + sqrt(c^2*x^2 + 1) + 1) - 5*b*c^3*d*x^3*log(-
c*x + sqrt(c^2*x^2 + 1) - 1) + 6*a*c^2*d*x^2 - 2*(3*b*c^2 + b)*d*x^3*log(-c
*x + sqrt(c^2*x^2 + 1)) + sqrt(c^2*x^2 + 1)*b*c*d*x + 2*a*d + 2*(3*b*c^2*d*
x^2 - (3*b*c^2 + b)*d*x^3 + b*d)*log(c*x + sqrt(c^2*x^2 + 1)))/x^3
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)(a+b\operatorname{arcsinh}(cx))}{x^4}\,dx = d\left(\int \frac{a}{x^4}\,dx + \int \frac{ac^2}{x^2}\,dx + \int \frac{b\operatorname{asinh}(cx)}{x^4}\,dx + \int \frac{bc^2\operatorname{asinh}(cx)}{x^2}\,dx\right)$$

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))/x**4,x)
[Out] d*(Integral(a/x**4, x) + Integral(a*c**2/x**2, x) + Integral(b*asinh(c*x)/x
**4, x) + Integral(b*c**2*asinh(c*x)/x**2, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.14

$$\begin{aligned}\int \frac{(d+c^2dx^2)(a+b\operatorname{arcsinh}(cx))}{x^4}\,dx &= -\left(c\operatorname{arsinh}\left(\frac{1}{c|x|}\right) + \frac{\operatorname{arsinh}(cx)}{x}\right)bc^2d \\ &\quad + \frac{1}{6}\left(\left(c^2\operatorname{arsinh}\left(\frac{1}{c|x|}\right) - \frac{\sqrt{c^2x^2+1}}{x^2}\right)c - \frac{2\operatorname{arsinh}(cx)}{x^3}\right)bd - \frac{ac^2d}{x} - \frac{ad}{3x^3}\end{aligned}$$

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima")
[Out] -(c*arcsinh(1/(c*abs(x))) + arcsinh(c*x)/x)*b*c^2*d + 1/6*((c^2*arcsinh(1/(
c*abs(x))) - sqrt(c^2*x^2 + 1)/x^2)*c - 2*arcsinh(c*x)/x^3)*b*d - a*c^2*d/x
 - 1/3*a*d/x^3
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)(a+b\operatorname{arcsinh}(cx))}{x^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))}{x^4}\,dx=\int \frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)\,\left(d\,c^2\,x^2+d\right)}{x^4}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x^4,x)
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x^4, x)
```

3.10 $\int x^4 \left(d+c^2 d x^2\right)^2 (a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 181

$$\begin{aligned}
&\int x^4 \left(d+c^2 d x^2\right)^2 (a+b\text{arcsinh}(cx))\,dx \\
&= -\frac{8bd^2\sqrt{1+c^2x^2}}{315c^5} - \frac{4bd^2(1+c^2x^2)^{3/2}}{945c^5} - \frac{bd^2(1+c^2x^2)^{5/2}}{525c^5} \\
&\quad + \frac{10bd^2(1+c^2x^2)^{7/2}}{441c^5} - \frac{bd^2(1+c^2x^2)^{9/2}}{81c^5} \\
&\quad + \frac{1}{5}d^2x^5(a+b\text{arcsinh}(cx)) + \frac{2}{7}c^2d^2x^7(a+b\text{arcsinh}(cx)) + \frac{1}{9}c^4d^2x^9(a+b\text{arcsinh}(cx))
\end{aligned}$$

```
[Out] -4/945*b*d^2*(c^2*x^2+1)^(3/2)/c^5-1/525*b*d^2*(c^2*x^2+1)^(5/2)/c^5+10/441
*b*d^2*(c^2*x^2+1)^(7/2)/c^5-1/81*b*d^2*(c^2*x^2+1)^(9/2)/c^5+1/5*d^2*x^5*(
a+b*arcsinh(c*x))+2/7*c^2*d^2*x^7*(a+b*arcsinh(c*x))+1/9*c^4*d^2*x^9*(a+b*a
rcsinh(c*x))-8/315*b*d^2*(c^2*x^2+1)^(1/2)/c^5
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {276, 5803, 12, 1265, 911, 1167}

$$\int x^4(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx = \frac{1}{9}c^4d^2x^9(a+b\text{arcsinh}(cx)) + \frac{2}{7}c^2d^2x^7(a+b\text{arcsinh}(cx)) + \frac{1}{5}d^2x^5(a+b\text{arcsinh}(cx)) - \frac{bd^2(c^2x^2+1)^{9/2}}{81c^5} + \frac{10bd^2(c^2x^2+1)^{7/2}}{441c^5} - \frac{bd^2(c^2x^2+1)^{5/2}}{525c^5} - \frac{4bd^2(c^2x^2+1)^{3/2}}{945c^5} - \frac{8bd^2\sqrt{c^2x^2+1}}{315c^5}$$

```
[In] Int[x^4*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-8*b*d^2*Sqrt[1 + c^2*x^2])/(315*c^5) - (4*b*d^2*(1 + c^2*x^2)^(3/2))/(945
*c^5) - (b*d^2*(1 + c^2*x^2)^(5/2))/(525*c^5) + (10*b*d^2*(1 + c^2*x^2)^(7/
2))/(441*c^5) - (b*d^2*(1 + c^2*x^2)^(9/2))/(81*c^5) + (d^2*x^5*(a + b*ArcS
inh[c*x]))/5 + (2*c^2*d^2*x^7*(a + b*ArcSinh[c*x]))/7 + (c^4*d^2*x^9*(a + b
*ArcSinh[c*x]))/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}d^2x^5(a+b\operatorname{arcsinh}(cx))+\frac{2}{7}c^2d^2x^7(a+b\operatorname{arcsinh}(cx)) \\
&\quad+\frac{1}{9}c^4d^2x^9(a+b\operatorname{arcsinh}(cx))-(bc)\int\frac{d^2x^5(63+90c^2x^2+35c^4x^4)}{315\sqrt{1+c^2x^2}}\,dx \\
&= \frac{1}{5}d^2x^5(a+b\operatorname{arcsinh}(cx))+\frac{2}{7}c^2d^2x^7(a+b\operatorname{arcsinh}(cx)) \\
&\quad+\frac{1}{9}c^4d^2x^9(a+b\operatorname{arcsinh}(cx))-\frac{1}{315}\left(bcd^2\right)\int\frac{x^5(63+90c^2x^2+35c^4x^4)}{\sqrt{1+c^2x^2}}\,dx \\
&= \frac{1}{5}d^2x^5(a+b\operatorname{arcsinh}(cx))+\frac{2}{7}c^2d^2x^7(a+b\operatorname{arcsinh}(cx))+\frac{1}{9}c^4d^2x^9(a+b\operatorname{arcsinh}(cx)) \\
&\quad-\frac{1}{630}\left(bcd^2\right)\operatorname{Subst}\left(\int\frac{x^2(63+90c^2x+35c^4x^2)}{\sqrt{1+c^2x}}\,dx,x,x^2\right) \\
&= \frac{1}{5}d^2x^5(a+b\operatorname{arcsinh}(cx))+\frac{2}{7}c^2d^2x^7(a+b\operatorname{arcsinh}(cx))+\frac{1}{9}c^4d^2x^9(a+b\operatorname{arcsinh}(cx)) \\
&\quad-\frac{\left(bd^2\right)\operatorname{Subst}\left(\int\left(-\frac{1}{c^2}+\frac{x^2}{c^2}\right)^2\left(8+20x^2+35x^4\right)\,dx,x,\sqrt{1+c^2x^2}\right)}{315c} \\
&= \frac{1}{5}d^2x^5(a+b\operatorname{arcsinh}(cx))+\frac{2}{7}c^2d^2x^7(a+b\operatorname{arcsinh}(cx))+\frac{1}{9}c^4d^2x^9(a+b\operatorname{arcsinh}(cx)) \\
&\quad-\frac{\left(bd^2\right)\operatorname{Subst}\left(\int\left(\frac{8}{c^4}+\frac{4x^2}{c^4}+\frac{3x^4}{c^4}-\frac{50x^6}{c^4}+\frac{35x^8}{c^4}\right)\,dx,x,\sqrt{1+c^2x^2}\right)}{315c}
\end{aligned}$$

$$= -\frac{8bd^2\sqrt{1+c^2x^2}}{315c^5} - \frac{4bd^2(1+c^2x^2)^{3/2}}{945c^5} - \frac{bd^2(1+c^2x^2)^{5/2}}{525c^5} + \frac{10bd^2(1+c^2x^2)^{7/2}}{441c^5} - \frac{bd^2(1+c^2x^2)^{9/2}}{81c^5} + \frac{1}{5}d^2x^5(a+b\text{arcsinh}(cx)) + \frac{2}{7}c^2d^2x^7(a+b\text{arcsinh}(cx)) + \frac{1}{9}c^4d^2x^9(a+b\text{arcsinh}(cx))$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.66

$$\int x^4(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx = \frac{d^2\left(315ac^5x^5(63+90c^2x^2+35c^4x^4) - b\sqrt{1+c^2x^2}(2104-1052c^2x^2+789c^4x^4+2650c^6x^6+1225c^8x^8) + \right.}{99225c^5}$$

```
[In] Integrate[x^4*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(315*a*c^5*x^5*(63 + 90*c^2*x^2 + 35*c^4*x^4) - b*Sqrt[1 + c^2*x^2]*(2
104 - 1052*c^2*x^2 + 789*c^4*x^4 + 2650*c^6*x^6 + 1225*c^8*x^8) + 315*b*c^5
*x^5*(63 + 90*c^2*x^2 + 35*c^4*x^4)*ArcSinh[c*x]))/(99225*c^5)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.90

method	result
parts	$d^2a\left(\frac{1}{9}c^4x^9+\frac{2}{7}c^2x^7+\frac{1}{5}x^5\right)+\frac{d^2b\left(\frac{\text{arcsinh}(cx)c^9x^9}{9}+\frac{2\,\text{arcsinh}(cx)c^7x^7}{7}+\frac{\text{arcsinh}(cx)c^5x^5}{5}-\frac{c^8x^8\sqrt{c^2x^2+1}}{81}-\frac{106c^6}{}\right.}{c^5}$
derivativedivides	$\frac{d^2a\left(\frac{1}{9}c^9x^9+\frac{2}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^2b\left(\frac{\text{arcsinh}(cx)c^9x^9}{9}+\frac{2\,\text{arcsinh}(cx)c^7x^7}{7}+\frac{\text{arcsinh}(cx)c^5x^5}{5}-\frac{c^8x^8\sqrt{c^2x^2+1}}{81}-\frac{106c^6x^6\sqrt{c^2x^2+}}{3969}\right.}{c^5}$
default	$\frac{d^2a\left(\frac{1}{9}c^9x^9+\frac{2}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^2b\left(\frac{\text{arcsinh}(cx)c^9x^9}{9}+\frac{2\,\text{arcsinh}(cx)c^7x^7}{7}+\frac{\text{arcsinh}(cx)c^5x^5}{5}-\frac{c^8x^8\sqrt{c^2x^2+1}}{81}-\frac{106c^6x^6\sqrt{c^2x^2+}}{3969}\right.}{c^5}$

```
[In] int(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] d^2*a*(1/9*c^4*x^9+2/7*c^2*x^7+1/5*x^5)+d^2*b/c^5*(1/9*arcsinh(c*x)*c^9*x^9
+2/7*arcsinh(c*x)*c^7*x^7+1/5*arcsinh(c*x)*c^5*x^5-1/81*c^8*x^8*(c^2*x^2+1)
^(1/2)-106/3969*c^6*x^6*(c^2*x^2+1)^(1/2)-263/33075*c^4*x^4*(c^2*x^2+1)^(1/
2)+1052/99225*c^2*x^2*(c^2*x^2+1)^(1/2)-2104/99225*(c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.91

$$\int x^4 \left(d+c^2dx^2\right)^2 (a+b\operatorname{arcsinh}(cx))\,dx = \frac{11025\,ac^9d^2x^9 + 28350\,ac^7d^2x^7 + 19845\,ac^5d^2x^5 + 315\,(35\,bc^9d^2x^9 + 90\,bc^7d^2x^7 + 63\,bc^5d^2x^5)\log\left(cx + \sqrt{c\right.}{99225\,c^5}$$

```
[In] integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/99225*(11025*a*c^9*d^2*x^9 + 28350*a*c^7*d^2*x^7 + 19845*a*c^5*d^2*x^5 +
315*(35*b*c^9*d^2*x^9 + 90*b*c^7*d^2*x^7 + 63*b*c^5*d^2*x^5)*log(c*x + sqrt
(c^2*x^2 + 1)) - (1225*b*c^8*d^2*x^8 + 2650*b*c^6*d^2*x^6 + 789*b*c^4*d^2*x
^4 - 1052*b*c^2*d^2*x^2 + 2104*b*d^2)*sqrt(c^2*x^2 + 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.27

$$\int x^4 \left(d+c^2dx^2\right)^2 (a+b\operatorname{arcsinh}(cx))\,dx = \begin{cases} \frac{ac^4d^2x^9}{9} + \frac{2ac^2d^2x^7}{7} + \frac{ad^2x^5}{5} + \frac{bc^4d^2x^9\operatorname{asinh}(cx)}{9} - \frac{bc^3d^2x^8\sqrt{c^2x^2+1}}{81} + \frac{2bc^2d^2x^7\operatorname{asinh}(cx)}{7} - \frac{106bcd^2x^6\sqrt{c^2x^2+1}}{3969} + \frac{bd^2x^5\operatorname{asinh}}{5} \\ \frac{ad^2x^5}{5} \end{cases}$$

```
[In] integrate(x**4*(c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**9/9 + 2*a*c**2*d**2*x**7/7 + a*d**2*x**5/5 + b*c*
*4*d**2*x**9*asinh(c*x)/9 - b*c**3*d**2*x**8*sqrt(c**2*x**2 + 1)/81 + 2*b*c
**2*d**2*x**7*asinh(c*x)/7 - 106*b*c*d**2*x**6*sqrt(c**2*x**2 + 1)/3969 + b
*d**2*x**5*asinh(c*x)/5 - 263*b*d**2*x**4*sqrt(c**2*x**2 + 1)/(33075*c) + 1
052*b*d**2*x**2*sqrt(c**2*x**2 + 1)/(99225*c**3) - 2104*b*d**2*sqrt(c**2*x*
*2 + 1)/(99225*c**5), Ne(c, 0)), (a*d**2*x**5/5, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(155) = 310.

Time = 0.19 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.76

$$\int x^4\left(d+c^2dx^2\right)^2\left(a+b\text{arcsinh}(cx)\right)dx=\frac{1}{9}ac^4d^2x^9+\frac{2}{7}ac^2d^2x^7$$
$$+\frac{1}{2835}\left(315\,x^9\,\text{arsinh}\,(cx)-\left(\frac{35\,\sqrt{c^2x^2+1}x^8}{c^2}-\frac{40\,\sqrt{c^2x^2+1}x^6}{c^4}+\frac{48\,\sqrt{c^2x^2+1}x^4}{c^6}-\frac{64\,\sqrt{c^2x^2+1}x^2}{c^8}\right.\right.$$
$$+\frac{1}{5}\,ad^2x^5$$
$$+\frac{2}{245}\left(35\,x^7\,\text{arsinh}\,(cx)-\left(\frac{5\,\sqrt{c^2x^2+1}x^6}{c^2}-\frac{6\,\sqrt{c^2x^2+1}x^4}{c^4}+\frac{8\,\sqrt{c^2x^2+1}x^2}{c^6}-\frac{16\,\sqrt{c^2x^2+1}}{c^8}\right)c\right)bc^2d$$
$$+\frac{1}{75}\left(15\,x^5\,\text{arsinh}\,(cx)-\left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2}-\frac{4\,\sqrt{c^2x^2+1}x^2}{c^4}+\frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\right)bd^2$$

```
[In] integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/9*a*c^4*d^2*x^9 + 2/7*a*c^2*d^2*x^7 + 1/2835*(315*x^9*arcsinh(c*x) - (35*
sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2
+ 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c
)*b*c^4*d^2 + 1/5*a*d^2*x^5 + 2/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^2
+ 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 -
16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^2*d^2 + 1/75*(15*x^5*arcsinh(c*x) - (3*sqr
t(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/
c^6)*c)*b*d^2
```

Giac [F(-2)]

Exception generated.

$$\int x^4\left(d+c^2dx^2\right)^2\left(a+b\text{arcsinh}(cx)\right)dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \left(d+c^2 d x^2\right)^2 (a+b \operatorname{arcsinh}(c x)) \, dx = \int x^4 \left(a+b \operatorname{asinh}(c x)\right) \left(d c^2 x^2+d\right)^2 \, dx$$

```
[In] int(x^4*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2,x)
[Out] int(x^4*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2, x)
```

3.11 $\int x^3(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 180

$$\int x^3(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))\,dx = \frac{73bd^2x\sqrt{1+c^2x^2}}{3072c^3} - \frac{73bd^2x^3\sqrt{1+c^2x^2}}{4608c} - \frac{43bcd^2x^5\sqrt{1+c^2x^2}}{1152} - \frac{1}{64}bc^3d^2x^7\sqrt{1+c^2x^2} - \frac{73bd^2\text{arcsinh}(cx)}{3072c^4} + \frac{1}{4}d^2x^4(a+b\text{arcsinh}(cx)) + \frac{1}{3}c^2d^2x^6(a+b\text{arcsinh}(cx)) + \frac{1}{8}c^4d^2x^8(a+b\text{arcsinh}(cx))$$

```
[Out] -73/3072*b*d^2*arcsinh(c*x)/c^4+1/4*d^2*x^4*(a+b*arcsinh(c*x))+1/3*c^2*d^2*
x^6*(a+b*arcsinh(c*x))+1/8*c^4*d^2*x^8*(a+b*arcsinh(c*x))+73/3072*b*d^2*x*(
c^2*x^2+1)^(1/2)/c^3-73/4608*b*d^2*x^3*(c^2*x^2+1)^(1/2)/c-43/1152*b*c*d^2*
x^5*(c^2*x^2+1)^(1/2)-1/64*b*c^3*d^2*x^7*(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

= {272, 45, 5803, 12, 1281, 470, 327, 221}

$$\int x^3\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))\,dx=\frac{1}{8}c^4d^2x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2d^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}d^2x^4(a+b\text{arcsinh}(cx))-\frac{73bd^2\text{arcsinh}(cx)}{3072c^4}-\frac{43bcd^2x^5\sqrt{c^2x^2+1}}{1152}-\frac{73bd^2x^3\sqrt{c^2x^2+1}}{4608c}+\frac{73bd^2x\sqrt{c^2x^2+1}}{3072c^3}-\frac{1}{64}bc^3d^2x^7\sqrt{c^2x^2+1}$$

```
[In] Int[x^3*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (73*b*d^2*x*Sqrt[1 + c^2*x^2])/(3072*c^3) - (73*b*d^2*x^3*Sqrt[1 + c^2*x^2]
)/(4608*c) - (43*b*c*d^2*x^5*Sqrt[1 + c^2*x^2])/1152 - (b*c^3*d^2*x^7*Sqrt[
1 + c^2*x^2])/64 - (73*b*d^2*ArcSinh[c*x])/(3072*c^4) + (d^2*x^4*(a + b*Arc
Sinh[c*x]))/4 + (c^2*d^2*x^6*(a + b*ArcSinh[c*x]))/3 + (c^4*d^2*x^8*(a + b*
ArcSinh[c*x]))/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}d^2x^4(a+b\operatorname{arcsinh}(cx))+\frac{1}{3}c^2d^2x^6(a+b\operatorname{arcsinh}(cx)) \\
&\quad+\frac{1}{8}c^4d^2x^8(a+b\operatorname{arcsinh}(cx))-(bc)\int\frac{d^2x^4(6+8c^2x^2+3c^4x^4)}{24\sqrt{1+c^2x^2}}\,dx \\
&= \frac{1}{4}d^2x^4(a+b\operatorname{arcsinh}(cx))+\frac{1}{3}c^2d^2x^6(a+b\operatorname{arcsinh}(cx)) \\
&\quad+\frac{1}{8}c^4d^2x^8(a+b\operatorname{arcsinh}(cx))-\frac{1}{24}\left(bcd^2\right)\int\frac{x^4(6+8c^2x^2+3c^4x^4)}{\sqrt{1+c^2x^2}}\,dx \\
&= -\frac{1}{64}bc^3d^2x^7\sqrt{1+c^2x^2}+\frac{1}{4}d^2x^4(a+b\operatorname{arcsinh}(cx))+\frac{1}{3}c^2d^2x^6(a+b\operatorname{arcsinh}(cx)) \\
&\quad+\frac{1}{8}c^4d^2x^8(a+b\operatorname{arcsinh}(cx))-\frac{(bd^2)\int\frac{x^4(48c^2+43c^4x^2)}{\sqrt{1+c^2x^2}}\,dx}{192c}
\end{aligned}$$

$$= -\frac{43bcd^2x^5\sqrt{1+c^2x^2}}{1152} - \frac{1}{64}bc^3d^2x^7\sqrt{1+c^2x^2} + \frac{1}{4}d^2x^4(a+b\text{arcsinh}(cx)) + \frac{1}{3}c^2d^2x^6(a+b\text{arcsinh}(cx)) + \frac{1}{8}c^4d^2x^8(a+b\text{arcsinh}(cx)) - \frac{(73bcd^2)\int \frac{x^4}{\sqrt{1+c^2x^2}}\,dx}{1152}$$

$$= -\frac{73bd^2x^3\sqrt{1+c^2x^2}}{4608c} - \frac{43bcd^2x^5\sqrt{1+c^2x^2}}{1152} - \frac{1}{64}bc^3d^2x^7\sqrt{1+c^2x^2} + \frac{1}{4}d^2x^4(a+b\text{arcsinh}(cx)) + \frac{1}{3}c^2d^2x^6(a+b\text{arcsinh}(cx)) + \frac{1}{8}c^4d^2x^8(a+b\text{arcsinh}(cx)) + \frac{(73bd^2)\int \frac{x^2}{\sqrt{1+c^2x^2}}\,dx}{1536c}$$

$$= \frac{73bd^2x\sqrt{1+c^2x^2}}{3072c^3} - \frac{73bd^2x^3\sqrt{1+c^2x^2}}{4608c} - \frac{43bcd^2x^5\sqrt{1+c^2x^2}}{1152} - \frac{1}{64}bc^3d^2x^7\sqrt{1+c^2x^2} + \frac{1}{4}d^2x^4(a+b\text{arcsinh}(cx)) + \frac{1}{3}c^2d^2x^6(a+b\text{arcsinh}(cx)) + \frac{1}{8}c^4d^2x^8(a+b\text{arcsinh}(cx)) - \frac{(73bd^2)\int \frac{1}{\sqrt{1+c^2x^2}}\,dx}{3072c^3}$$

$$= \frac{73bd^2x\sqrt{1+c^2x^2}}{3072c^3} - \frac{73bd^2x^3\sqrt{1+c^2x^2}}{4608c} - \frac{43bcd^2x^5\sqrt{1+c^2x^2}}{1152} - \frac{1}{64}bc^3d^2x^7\sqrt{1+c^2x^2} - \frac{73bd^2\text{arcsinh}(cx)}{3072c^4} + \frac{1}{4}d^2x^4(a+b\text{arcsinh}(cx)) + \frac{1}{3}c^2d^2x^6(a+b\text{arcsinh}(cx)) + \frac{1}{8}c^4d^2x^8(a+b\text{arcsinh}(cx))$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.64

$$\int x^3\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))\,dx = \frac{d^2\left(384ac^4x^4(6+8c^2x^2+3c^4x^4) - bcx\sqrt{1+c^2x^2}(-219+146c^2x^2+344c^4x^4+144c^6x^6) + 3b(-73+768c^4 \right.}{9216c^4}$$

```
[In] Integrate[x^3*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(384*a*c^4*x^4*(6 + 8*c^2*x^2 + 3*c^4*x^4) - b*c*x*Sqrt[1 + c^2*x^2]*(
-219 + 146*c^2*x^2 + 344*c^4*x^4 + 144*c^6*x^6) + 3*b*(-73 + 768*c^4*x^4 +
1024*c^6*x^6 + 384*c^8*x^8)*ArcSinh[c*x]))/(9216*c^4)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.84

method	result
parts	$d^2a\left(\frac{1}{8}c^4x^8+\frac{1}{3}c^2x^6+\frac{1}{4}x^4\right)+\frac{d^2b\left(\frac{\operatorname{arcsinh}(cx)c^8x^8}{8}+\frac{\operatorname{arcsinh}(cx)c^6x^6}{3}+\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}-\frac{c^7x^7\sqrt{c^2x^2+1}}{64}-\frac{43c^5x^5}{1}\right.}{c^4}$
derivativedivides	$\frac{d^2a\left(\frac{1}{8}c^8x^8+\frac{1}{3}c^6x^6+\frac{1}{4}c^4x^4\right)+d^2b\left(\frac{\operatorname{arcsinh}(cx)c^8x^8}{8}+\frac{\operatorname{arcsinh}(cx)c^6x^6}{3}+\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}-\frac{c^7x^7\sqrt{c^2x^2+1}}{64}-\frac{43c^5x^5\sqrt{c^2x^2+1}}{1152}-\right.}{c^4}$
default	$\frac{d^2a\left(\frac{1}{8}c^8x^8+\frac{1}{3}c^6x^6+\frac{1}{4}c^4x^4\right)+d^2b\left(\frac{\operatorname{arcsinh}(cx)c^8x^8}{8}+\frac{\operatorname{arcsinh}(cx)c^6x^6}{3}+\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}-\frac{c^7x^7\sqrt{c^2x^2+1}}{64}-\frac{43c^5x^5\sqrt{c^2x^2+1}}{1152}-\right.}{c^4}$

```
[In] int(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] d^2*a*(1/8*c^4*x^8+1/3*c^2*x^6+1/4*x^4)+d^2*b/c^4*(1/8*arcsinh(c*x)*c^8*x^8
+1/3*arcsinh(c*x)*c^6*x^6+1/4*arcsinh(c*x)*c^4*x^4-1/64*c^7*x^7*(c^2*x^2+1)
^(1/2)-43/1152*c^5*x^5*(c^2*x^2+1)^(1/2)-73/4608*c^3*x^3*(c^2*x^2+1)^(1/2)+
73/3072*c*x*(c^2*x^2+1)^(1/2)-73/3072*arcsinh(c*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.89

$$\int x^3\left(d+c^2dx^2\right)^2\left(a+b\operatorname{arcsinh}(cx)\right)dx$$
$$=\frac{1152\,ac^8d^2x^8+3072\,ac^6d^2x^6+2304\,ac^4d^2x^4+3\left(384\,bc^8d^2x^8+1024\,bc^6d^2x^6+768\,bc^4d^2x^4-73\,bd^2\right)\log\left(\right.}{9216\,c^4}$$

```
[In] integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/9216*(1152*a*c^8*d^2*x^8 + 3072*a*c^6*d^2*x^6 + 2304*a*c^4*d^2*x^4 + 3*(3
84*b*c^8*d^2*x^8 + 1024*b*c^6*d^2*x^6 + 768*b*c^4*d^2*x^4 - 73*b*d^2)*log(c
*x + sqrt(c^2*x^2 + 1)) - (144*b*c^7*d^2*x^7 + 344*b*c^5*d^2*x^5 + 146*b*c^
3*d^2*x^3 - 219*b*c*d^2*x)*sqrt(c^2*x^2 + 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.21

$$\int x^3(d+c^2dx^2)^2(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\begin{cases}\frac{ac^4d^2x^8}{8}+\frac{ac^2d^2x^6}{3}+\frac{ad^2x^4}{4}+\frac{bc^4d^2x^8\operatorname{asinh}(cx)}{8}-\frac{bc^3d^2x^7\sqrt{c^2x^2+1}}{64}+\frac{bc^2d^2x^6\operatorname{asinh}(cx)}{3}-\frac{43bcd^2x^5\sqrt{c^2x^2+1}}{1152}+\frac{bd^2x^4\operatorname{asinh}(cx)}{4}\\ \frac{ad^2x^4}{4}\end{cases}$$

```
[In] integrate(x**3*(c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**8/8 + a*c**2*d**2*x**6/3 + a*d**2*x**4/4 + b*c**4
*d**2*x**8*asinh(c*x)/8 - b*c**3*d**2*x**7*sqrt(c**2*x**2 + 1)/64 + b*c**2*
d**2*x**6*asinh(c*x)/3 - 43*b*c*d**2*x**5*sqrt(c**2*x**2 + 1)/1152 + b*d**2
*x**4*asinh(c*x)/4 - 73*b*d**2*x**3*sqrt(c**2*x**2 + 1)/(4608*c) + 73*b*d**
2*x*sqrt(c**2*x**2 + 1)/(3072*c**3) - 73*b*d**2*asinh(c*x)/(3072*c**4), Ne(
c, 0)), (a*d**2*x**4/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.62

$$\int x^3(d+c^2dx^2)^2(a+b\operatorname{arcsinh}(cx))\,dx=\frac{1}{8}ac^4d^2x^8+\frac{1}{3}ac^2d^2x^6$$
$$+\frac{1}{3072}\left(384\,x^8\operatorname{arsinh}(cx)-\left(\frac{48\sqrt{c^2x^2+1}x^7}{c^2}-\frac{56\sqrt{c^2x^2+1}x^5}{c^4}+\frac{70\sqrt{c^2x^2+1}x^3}{c^6}-\frac{105\sqrt{c^2x^2+1}x}{c^8}+\right.\right.$$
$$+\frac{1}{4}ad^2x^4$$
$$+\frac{1}{144}\left(48\,x^6\operatorname{arsinh}(cx)-\left(\frac{8\sqrt{c^2x^2+1}x^5}{c^2}-\frac{10\sqrt{c^2x^2+1}x^3}{c^4}+\frac{15\sqrt{c^2x^2+1}x}{c^6}-\frac{15\operatorname{arsinh}(cx)}{c^7}\right)c\right)bc^2$$
$$+\frac{1}{32}\left(8\,x^4\operatorname{arsinh}(cx)-\left(\frac{2\sqrt{c^2x^2+1}x^3}{c^2}-\frac{3\sqrt{c^2x^2+1}x}{c^4}+\frac{3\operatorname{arsinh}(cx)}{c^5}\right)c\right)bd^2$$

```
[In] integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*c^4*d^2*x^8 + 1/3*a*c^2*d^2*x^6 + 1/3072*(384*x^8*arcsinh(c*x) - (48*
sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^2
+ 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/c^8 + 105*arcsinh(c*x)/c^9)*c)*b*c^4
*d^2 + 1/4*a*d^2*x^4 + 1/144*(48*x^6*arcsinh(c*x) - (8*sqrt(c^2*x^2 + 1)*x^
5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*arcs
inh(c*x)/c^7)*c)*b*c^2*d^2 + 1/32*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1
)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*b*d^2
```

Giac [F(-2)]

Exception generated.

$$\int x^3 (d + c^2 d x^2)^2 (a + b \text{arcsinh}(cx)) \, dx = \text{Exception raised: TypeError}$$

[In] integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int x^3 (d + c^2 d x^2)^2 (a + b \text{arcsinh}(cx)) \, dx = \int x^3 (a + b \,\text{asinh}(c\, x)) \left(d\, c^2\, x^2 + d\right)^2 \, dx$$

[In] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2,x)

[Out] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2, x)

3.12 $\int x^2(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 157

$$\int x^2(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))\,dx = \frac{8bd^2\sqrt{1+c^2x^2}}{105c^3}+\frac{4bd^2(1+c^2x^2)^{3/2}}{315c^3}+\frac{bd^2(1+c^2x^2)^{5/2}}{175c^3}-\frac{bd^2(1+c^2x^2)^{7/2}}{49c^3}+\frac{1}{3}d^2x^3(a+b\text{arcsinh}(cx))+\frac{2}{5}c^2d^2x^5(a+b\text{arcsinh}(cx))+\frac{1}{7}c^4d^2x^7(a+b\text{arcsinh}(cx))$$

```
[Out] 4/315*b*d^2*(c^2*x^2+1)^(3/2)/c^3+1/175*b*d^2*(c^2*x^2+1)^(5/2)/c^3-1/49*b*
d^2*(c^2*x^2+1)^(7/2)/c^3+1/3*d^2*x^3*(a+b*arcsinh(c*x))+2/5*c^2*d^2*x^5*(a
+b*arcsinh(c*x))+1/7*c^4*d^2*x^7*(a+b*arcsinh(c*x))+8/105*b*d^2*(c^2*x^2+1)
^(1/2)/c^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {276, 5803, 12, 1265, 785}

$$\int x^2(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))\,dx = \frac{1}{7}c^4d^2x^7(a+b\text{arcsinh}(cx))+\frac{2}{5}c^2d^2x^5(a+b\text{arcsinh}(cx))+\frac{1}{3}d^2x^3(a+b\text{arcsinh}(cx))-\frac{bd^2(c^2x^2+1)^{7/2}}{49c^3}+\frac{bd^2(c^2x^2+1)^{5/2}}{175c^3}+\frac{4bd^2(c^2x^2+1)^{3/2}}{315c^3}+\frac{8bd^2\sqrt{c^2x^2+1}}{105c^3}$$

```
[In] Int[x^2*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (8*b*d^2*Sqrt[1 + c^2*x^2])/(105*c^3) + (4*b*d^2*(1 + c^2*x^2)^(3/2))/(315*
c^3) + (b*d^2*(1 + c^2*x^2)^(5/2))/(175*c^3) - (b*d^2*(1 + c^2*x^2)^(7/2))/
(49*c^3) + (d^2*x^3*(a + b*ArcSinh[c*x]))/3 + (2*c^2*d^2*x^5*(a + b*ArcSinh
[c*x]))/5 + (c^4*d^2*x^7*(a + b*ArcSinh[c*x]))/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 785

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}d^2x^3(a+b\text{arcsinh}(cx))+\frac{2}{5}c^2d^2x^5(a+b\text{arcsinh}(cx))\\&\quad+\frac{1}{7}c^4d^2x^7(a+b\text{arcsinh}(cx))-(bc)\int\frac{d^2x^3(35+42c^2x^2+15c^4x^4)}{105\sqrt{1+c^2x^2}}\,dx\end{aligned}$$

$$= \frac{1}{3}d^2x^3(a+b\operatorname{arcsinh}(cx)) + \frac{2}{5}c^2d^2x^5(a+b\operatorname{arcsinh}(cx)) + \frac{1}{7}c^4d^2x^7(a+b\operatorname{arcsinh}(cx)) - \frac{1}{105}(bcd^2)\int \frac{x^3(35+42c^2x^2+15c^4x^4)}{\sqrt{1+c^2x^2}}\,dx$$

$$= \frac{1}{3}d^2x^3(a+b\operatorname{arcsinh}(cx)) + \frac{2}{5}c^2d^2x^5(a+b\operatorname{arcsinh}(cx)) + \frac{1}{7}c^4d^2x^7(a+b\operatorname{arcsinh}(cx)) - \frac{1}{210}(bcd^2)\,\text{Subst}\left(\int \frac{x(35+42c^2x+15c^4x^2)}{\sqrt{1+c^2x}}\,dx, x, x^2\right)$$

$$= \frac{1}{3}d^2x^3(a+b\operatorname{arcsinh}(cx)) + \frac{2}{5}c^2d^2x^5(a+b\operatorname{arcsinh}(cx)) + \frac{1}{7}c^4d^2x^7(a+b\operatorname{arcsinh}(cx)) - \frac{1}{210}(bcd^2)\,\text{Subst}\left(\int \left(-\frac{8}{c^2\sqrt{1+c^2x}} - \frac{4\sqrt{1+c^2x}}{c^2} - \frac{3(1+c^2x)^{3/2}}{c^2} + \frac{15(1+c^2x)^{5/2}}{c^2}\right) dx, x, x^2\right)$$

$$= \frac{8bd^2\sqrt{1+c^2x^2}}{105c^3} + \frac{4bd^2(1+c^2x^2)^{3/2}}{315c^3} + \frac{bd^2(1+c^2x^2)^{5/2}}{175c^3} - \frac{bd^2(1+c^2x^2)^{7/2}}{49c^3} + \frac{1}{3}d^2x^3(a+b\operatorname{arcsinh}(cx)) + \frac{2}{5}c^2d^2x^5(a+b\operatorname{arcsinh}(cx)) + \frac{1}{7}c^4d^2x^7(a+b\operatorname{arcsinh}(cx))$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.71

$$\int x^2(d+c^2dx^2)^2(a+b\operatorname{arcsinh}(cx))\,dx = \frac{d^2(105ac^3x^3(35+42c^2x^2+15c^4x^4) - b\sqrt{1+c^2x^2}(-818+409c^2x^2+612c^4x^4+225c^6x^6) + 105bc^3x^3(35+$$

$$11025c^3$$

```
[In] Integrate[x^2*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(105*a*c^3*x^3*(35 + 42*c^2*x^2 + 15*c^4*x^4) - b*Sqrt[1 + c^2*x^2]*(-
818 + 409*c^2*x^2 + 612*c^4*x^4 + 225*c^6*x^6) + 105*b*c^3*x^3*(35 + 42*c^2
*x^2 + 15*c^4*x^4)*ArcSinh[c*x]))/(11025*c^3)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92

method	result
parts	$d^2 a\left(\frac{1}{7}c^4x^7+\frac{2}{5}c^2x^5+\frac{1}{3}x^3\right)+\frac{d^2 b\left(\frac{\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{2\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}-\frac{c^6x^6\sqrt{c^2x^2+1}}{49}-\frac{68c^4x}{}\right.}{c^3}$
derivativedivides	$\frac{d^2 a\left(\frac{1}{7}c^7x^7+\frac{2}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^2 b\left(\frac{\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{2\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}-\frac{c^6x^6\sqrt{c^2x^2+1}}{49}-\frac{68c^4x^4\sqrt{c^2x^2+1}}{1225}\right.}{c^3}$
default	$\frac{d^2 a\left(\frac{1}{7}c^7x^7+\frac{2}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^2 b\left(\frac{\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{2\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}-\frac{c^6x^6\sqrt{c^2x^2+1}}{49}-\frac{68c^4x^4\sqrt{c^2x^2+1}}{1225}\right.}{c^3}$

```
[In] int(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] d^2*a*(1/7*c^4*x^7+2/5*c^2*x^5+1/3*x^3)+d^2*b/c^3*(1/7*arcsinh(c*x)*c^7*x^7
+2/5*arcsinh(c*x)*c^5*x^5+1/3*arcsinh(c*x)*c^3*x^3-1/49*c^6*x^6*(c^2*x^2+1)
^(1/2)-68/1225*c^4*x^4*(c^2*x^2+1)^(1/2)-409/11025*c^2*x^2*(c^2*x^2+1)^(1/2
)+818/11025*(c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.97

$$\int x^2\left(d+c^2dx^2\right)^2\left(a+b\operatorname{arcsinh}(cx)\right)dx$$
$$=\frac{1575\,ac^7d^2x^7+4410\,ac^5d^2x^5+3675\,ac^3d^2x^3+105\left(15\,bc^7d^2x^7+42\,bc^5d^2x^5+35\,bc^3d^2x^3\right)\log\left(cx+\sqrt{c^2x}\right.}{11025\,c^3}$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/11025*(1575*a*c^7*d^2*x^7 + 4410*a*c^5*d^2*x^5 + 3675*a*c^3*d^2*x^3 + 105
*(15*b*c^7*d^2*x^7 + 42*b*c^5*d^2*x^5 + 35*b*c^3*d^2*x^3)*log(c*x + sqrt(c^
2*x^2 + 1)) - (225*b*c^6*d^2*x^6 + 612*b*c^4*d^2*x^4 + 409*b*c^2*d^2*x^2 -
818*b*d^2)*sqrt(c^2*x^2 + 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.29

$$\int x^2(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx$$
$$=\begin{cases} \frac{ac^4d^2x^7}{7}+\frac{2ac^2d^2x^5}{5}+\frac{ad^2x^3}{3}+\frac{bc^4d^2x^7\operatorname{asinh}(cx)}{7}-\frac{bc^3d^2x^6\sqrt{c^2x^2+1}}{49}+\frac{2bc^2d^2x^5\operatorname{asinh}(cx)}{5}-\frac{68bcd^2x^4\sqrt{c^2x^2+1}}{1225}+\frac{bd^2x^3\operatorname{asinh}(}{3} \\ \frac{ad^2x^3}{3} \end{cases}$$

```
[In] integrate(x**2*(c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**7/7 + 2*a*c**2*d**2*x**5/5 + a*d**2*x**3/3 + b*c*
*4*d**2*x**7*asinh(c*x)/7 - b*c**3*d**2*x**6*sqrt(c**2*x**2 + 1)/49 + 2*b*c
**2*d**2*x**5*asinh(c*x)/5 - 68*b*c*d**2*x**4*sqrt(c**2*x**2 + 1)/1225 + b*
d**2*x**3*asinh(c*x)/3 - 409*b*d**2*x**2*sqrt(c**2*x**2 + 1)/(11025*c) + 81
8*b*d**2*sqrt(c**2*x**2 + 1)/(11025*c**3), Ne(c, 0)), (a*d**2*x**3/3, True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.66

$$\int x^2(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx=\frac{1}{7}ac^4d^2x^7+\frac{2}{5}ac^2d^2x^5$$
$$+\frac{1}{245}\left(35x^7\operatorname{arsinh}(cx)-\left(\frac{5\sqrt{c^2x^2+1}x^6}{c^2}-\frac{6\sqrt{c^2x^2+1}x^4}{c^4}+\frac{8\sqrt{c^2x^2+1}x^2}{c^6}-\frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\right)bc^4d^2$$
$$+\frac{2}{75}\left(15x^5\operatorname{arsinh}(cx)-\left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2}-\frac{4\sqrt{c^2x^2+1}x^2}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)bc^2d^2$$
$$+\frac{1}{3}ad^2x^3+\frac{1}{9}\left(3x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bd^2$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*c^4*d^2*x^7 + 2/5*a*c^2*d^2*x^5 + 1/245*(35*x^7*arcsinh(c*x) - (5*sqr
t(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*
x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^4*d^2 + 2/75*(15*x^5*arcsinh(c*x
) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2
*x^2 + 1)/c^6)*c)*b*c^2*d^2 + 1/3*a*d^2*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(
sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*d^2
```

Giac [F(-2)]

Exception generated.

$$\int x^2 (d + c^2 d x^2)^2 (a + b \text{arcsinh}(cx)) \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (d + c^2 d x^2)^2 (a + b \text{arcsinh}(cx)) \, dx = \int x^2 (a + b \, \text{asinh}(c \, x)) \left(d \, c^2 \, x^2 + d\right)^2 dx$$

```
[In] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2,x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2, x)
```

3.13 $\int x(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 120

$$\int x(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))\,dx = -\frac{5bd^2x\sqrt{1+c^2x^2}}{96c} - \frac{5bd^2x(1+c^2x^2)^{3/2}}{144c} - \frac{bd^2x(1+c^2x^2)^{5/2}}{36c} - \frac{5bd^2\text{arcsinh}(cx)}{96c^2} + \frac{d^2(1+c^2x^2)^3 (a+b\text{arcsinh}(cx))}{6c^2}$$

```
[Out] -5/144*b*d^2*x*(c^2*x^2+1)^(3/2)/c-1/36*b*d^2*x*(c^2*x^2+1)^(5/2)/c-5/96*b*
d^2*arcsinh(c*x)/c^2+1/6*d^2*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))/c^2-5/96*b*d^
2*x*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5798, 201, 221}

$$\int x(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))\,dx = \frac{d^2(c^2x^2+1)^3 (a+b\text{arcsinh}(cx))}{6c^2} - \frac{5bd^2\text{arcsinh}(cx)}{96c^2} - \frac{bd^2x(c^2x^2+1)^{5/2}}{36c} - \frac{5bd^2x(c^2x^2+1)^{3/2}}{144c} - \frac{5bd^2x\sqrt{c^2x^2+1}}{96c}$$

```
[In] Int[x*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-5*b*d^2*x*Sqrt[1 + c^2*x^2])/(96*c) - (5*b*d^2*x*(1 + c^2*x^2)^(3/2))/(14
4*c) - (b*d^2*x*(1 + c^2*x^2)^(5/2))/(36*c) - (5*b*d^2*ArcSinh[c*x])/(96*c^
2) + (d^2*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x]))/(6*c^2)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d^2(1+c^2x^2)^3\,(a+b\text{arcsinh}(cx))}{6c^2} - \frac{(bd^2)\int (1+c^2x^2)^{5/2}\,dx}{6c} \\
&= -\frac{bd^2x(1+c^2x^2)^{5/2}}{36c} + \frac{d^2(1+c^2x^2)^3\,(a+b\text{arcsinh}(cx))}{6c^2} - \frac{(5bd^2)\int (1+c^2x^2)^{3/2}\,dx}{36c} \\
&= -\frac{5bd^2x(1+c^2x^2)^{3/2}}{144c} - \frac{bd^2x(1+c^2x^2)^{5/2}}{36c} \\
&\quad + \frac{d^2(1+c^2x^2)^3\,(a+b\text{arcsinh}(cx))}{6c^2} - \frac{(5bd^2)\int \sqrt{1+c^2x^2}\,dx}{48c} \\
&= -\frac{5bd^2x\sqrt{1+c^2x^2}}{96c} - \frac{5bd^2x(1+c^2x^2)^{3/2}}{144c} - \frac{bd^2x(1+c^2x^2)^{5/2}}{36c} \\
&\quad + \frac{d^2(1+c^2x^2)^3\,(a+b\text{arcsinh}(cx))}{6c^2} - \frac{(5bd^2)\int \frac{1}{\sqrt{1+c^2x^2}}\,dx}{96c} \\
&= -\frac{5bd^2x\sqrt{1+c^2x^2}}{96c} - \frac{5bd^2x(1+c^2x^2)^{3/2}}{144c} - \frac{bd^2x(1+c^2x^2)^{5/2}}{36c} \\
&\quad - \frac{5bd^2\text{arcsinh}(cx)}{96c^2} + \frac{d^2(1+c^2x^2)^3\,(a+b\text{arcsinh}(cx))}{6c^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.87

$$\int x(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{d^2\left(cx\left(48acx(3+3c^2x^2+c^4x^4)-b\sqrt{1+c^2x^2}(33+26c^2x^2+8c^4x^4)\right)+3b(11+48c^2x^2+48c^4x^4+16c^6x^6)\right.}{288c^2}$$

```
[In] Integrate[x*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(c*x*(48*a*c*x*(3 + 3*c^2*x^2 + c^4*x^4) - b*Sqrt[1 + c^2*x^2]*(33 + 2
6*c^2*x^2 + 8*c^4*x^4)) + 3*b*(11 + 48*c^2*x^2 + 48*c^4*x^4 + 16*c^6*x^6)*A
rcSinh[c*x]))/(288*c^2)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{\frac{d^2a(c^2x^2+1)^3}{6}+d^2b\left(\frac{\text{arcsinh}(cx)c^6x^6}{6}+\frac{\text{arcsinh}(cx)c^4x^4}{2}+\frac{\text{arcsinh}(cx)c^2x^2}{2}+\frac{11\ \text{arcsinh}(cx)}{96}-\frac{cx(c^2x^2+1)^{\frac{5}{2}}}{36}-\frac{5cx(c^2x^2+1)^{\frac{3}{2}}}{144}-\right.}{c^2}$
default	$\frac{\frac{d^2a(c^2x^2+1)^3}{6}+d^2b\left(\frac{\text{arcsinh}(cx)c^6x^6}{6}+\frac{\text{arcsinh}(cx)c^4x^4}{2}+\frac{\text{arcsinh}(cx)c^2x^2}{2}+\frac{11\ \text{arcsinh}(cx)}{96}-\frac{cx(c^2x^2+1)^{\frac{5}{2}}}{36}-\frac{5cx(c^2x^2+1)^{\frac{3}{2}}}{144}-\right.}{c^2}$
parts	$\frac{d^2a(c^2x^2+1)^3}{6c^2}+\frac{d^2b\left(\frac{\text{arcsinh}(cx)c^6x^6}{6}+\frac{\text{arcsinh}(cx)c^4x^4}{2}+\frac{\text{arcsinh}(cx)c^2x^2}{2}+\frac{11\ \text{arcsinh}(cx)}{96}-\frac{cx(c^2x^2+1)^{\frac{5}{2}}}{36}-\frac{5cx(c^2x^2+1)^{\frac{3}{2}}}{144}\right.}{c^2}$

```
[In] int(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/6*d^2*a*(c^2*x^2+1)^3+d^2*b*(1/6*arcsinh(c*x)*c^6*x^6+1/2*arcsinh(
c*x)*c^4*x^4+1/2*arcsinh(c*x)*c^2*x^2+11/96*arcsinh(c*x)-1/36*c*x*(c^2*x^2+
1)^(5/2)-5/144*c*x*(c^2*x^2+1)^(3/2)-5/96*c*x*(c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.24

$$\int x(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx = \frac{48\,ac^6d^2x^6+144\,ac^4d^2x^4+144\,ac^2d^2x^2+3\,(16\,bc^6d^2x^6+48\,bc^4d^2x^4+48\,bc^2d^2x^2+11\,bd^2)\log\left(cx+\sqrt{c^2\cdots}\right.}{288\,c^2}$$

```
[In] integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/288*(48*a*c^6*d^2*x^6 + 144*a*c^4*d^2*x^4 + 144*a*c^2*d^2*x^2 + 3*(16*b*c
^6*d^2*x^6 + 48*b*c^4*d^2*x^4 + 48*b*c^2*d^2*x^2 + 11*b*d^2)*log(c*x + sqrt
(c^2*x^2 + 1)) - (8*b*c^5*d^2*x^5 + 26*b*c^3*d^2*x^3 + 33*b*c*d^2*x)*sqrt(c
^2*x^2 + 1))/c^2
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.58

$$\int x(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx = \begin{cases} \frac{ac^4d^2x^6}{6}+\frac{ac^2d^2x^4}{2}+\frac{ad^2x^2}{2}+\frac{bc^4d^2x^6\,\text{asinh}\,(cx)}{6}-\frac{bc^3d^2x^5\sqrt{c^2x^2+1}}{36}+\frac{bc^2d^2x^4\,\text{asinh}\,(cx)}{2}-\frac{13bcd^2x^3\sqrt{c^2x^2+1}}{144}+\frac{bd^2x^2\,\text{asinh}\,(\cdots}{2} \\ \frac{ad^2x^2}{2} \end{cases}$$

```
[In] integrate(x*(c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**6/6 + a*c**2*d**2*x**4/2 + a*d**2*x**2/2 + b*c**4
*d**2*x**6*asinh(c*x)/6 - b*c**3*d**2*x**5*sqrt(c**2*x**2 + 1)/36 + b*c**2*
d**2*x**4*asinh(c*x)/2 - 13*b*c*d**2*x**3*sqrt(c**2*x**2 + 1)/144 + b*d**2*
x**2*asinh(c*x)/2 - 11*b*d**2*x*sqrt(c**2*x**2 + 1)/(96*c) + 11*b*d**2*asin
h(c*x)/(96*c**2), Ne(c, 0)), (a*d**2*x**2/2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(104) = 208$.

Time = 0.19 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.95

$$\int x(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx = \frac{1}{6}ac^4d^2x^6 + \frac{1}{2}ac^2d^2x^4 + \frac{1}{288}\left(48x^6\,\text{arsinh}(cx) - \left(\frac{8\sqrt{c^2x^2+1}x^5}{c^2} - \frac{10\sqrt{c^2x^2+1}x^3}{c^4} + \frac{15\sqrt{c^2x^2+1}x}{c^6} - \frac{15\,\text{arsinh}(cx)}{c^7}\right)c\right)bc^4 + \frac{1}{16}\left(8x^4\,\text{arsinh}(cx) - \left(\frac{2\sqrt{c^2x^2+1}x^3}{c^2} - \frac{3\sqrt{c^2x^2+1}x}{c^4} + \frac{3\,\text{arsinh}(cx)}{c^5}\right)c\right)bc^2d^2 + \frac{1}{2}ad^2x^2 + \frac{1}{4}\left(2x^2\,\text{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\text{arsinh}(cx)}{c^3}\right)\right)bd^2$$

```
[In] integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*c^4*d^2*x^6 + 1/2*a*c^2*d^2*x^4 + 1/288*(48*x^6*arcsinh(c*x) - (8*sqr
t(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1
)*x/c^6 - 15*arcsinh(c*x)/c^7)*c)*b*c^4*d^2 + 1/16*(8*x^4*arcsinh(c*x) - (2
*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5
)*c)*b*c^2*d^2 + 1/2*a*d^2*x^2 + 1/4*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2
+ 1)*x/c^2 - arcsinh(c*x)/c^3))*b*d^2
```

Giac [F(-2)]

Exception generated.

$$\int x(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx = \int x\,(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^2\,dx$$

```
[In] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2,x)
```

```
[Out] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2, x)
```

3.14 $\int (d+c^2dx^2)^2 (a+b\text{arcsinh}(cx)) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 128

$$\int (d+c^2dx^2)^2 (a+b\text{arcsinh}(cx)) \, dx = -\frac{8bd^2\sqrt{1+c^2x^2}}{15c} - \frac{4bd^2(1+c^2x^2)^{3/2}}{45c} - \frac{bd^2(1+c^2x^2)^{5/2}}{25c} + d^2x(a+b\text{arcsinh}(cx)) + \frac{2}{3}c^2d^2x^3(a+b\text{arcsinh}(cx)) + \frac{1}{5}c^4d^2x^5(a+b\text{arcsinh}(cx))$$

```
[Out] -4/45*b*d^2*(c^2*x^2+1)^(3/2)/c-1/25*b*d^2*(c^2*x^2+1)^(5/2)/c+d^2*x*(a+b*a
rcsinh(c*x))+2/3*c^2*d^2*x^3*(a+b*arcsinh(c*x))+1/5*c^4*d^2*x^5*(a+b*arcsin
h(c*x))-8/15*b*d^2*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {200, 5784, 12, 1261, 712}

$$\int (d+c^2dx^2)^2 (a+b\text{arcsinh}(cx)) \, dx = \frac{1}{5}c^4d^2x^5(a+b\text{arcsinh}(cx)) + \frac{2}{3}c^2d^2x^3(a+b\text{arcsinh}(cx)) + d^2x(a+b\text{arcsinh}(cx)) - \frac{bd^2(c^2x^2+1)^{5/2}}{25c} - \frac{4bd^2(c^2x^2+1)^{3/2}}{45c} - \frac{8bd^2\sqrt{c^2x^2+1}}{15c}$$

```
[In] Int[(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-8*b*d^2*Sqrt[1 + c^2*x^2])/(15*c) - (4*b*d^2*(1 + c^2*x^2)^(3/2))/(45*c)
- (b*d^2*(1 + c^2*x^2)^(5/2))/(25*c) + d^2*x*(a + b*ArcSinh[c*x]) + (2*c^2*
d^2*x^3*(a + b*ArcSinh[c*x]))/3 + (c^4*d^2*x^5*(a + b*ArcSinh[c*x]))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= d^2x(a+b\operatorname{arcsinh}(cx))+\frac{2}{3}c^2d^2x^3(a+b\operatorname{arcsinh}(cx)) \\ &\quad+\frac{1}{5}c^4d^2x^5(a+b\operatorname{arcsinh}(cx))-(bc)\int\frac{d^2x(15+10c^2x^2+3c^4x^4)}{15\sqrt{1+c^2x^2}}\,dx \\ &= d^2x(a+b\operatorname{arcsinh}(cx))+\frac{2}{3}c^2d^2x^3(a+b\operatorname{arcsinh}(cx)) \\ &\quad+\frac{1}{5}c^4d^2x^5(a+b\operatorname{arcsinh}(cx))-\frac{1}{15}\left(bcd^2\right)\int\frac{x(15+10c^2x^2+3c^4x^4)}{\sqrt{1+c^2x^2}}\,dx \end{aligned}$$

$$
\begin{aligned}
&= d^2x(a+b\operatorname{arcsinh}(cx)) + \frac{2}{3}c^2d^2x^3(a+b\operatorname{arcsinh}(cx)) + \frac{1}{5}c^4d^2x^5(a+b\operatorname{arcsinh}(cx)) \\
&\quad - \frac{1}{30}\left(bcd^2\right)\operatorname{Subst}\left(\int \frac{15+10c^2x+3c^4x^2}{\sqrt{1+c^2x}}\,dx, x, x^2\right) \\
&= d^2x(a+b\operatorname{arcsinh}(cx)) + \frac{2}{3}c^2d^2x^3(a+b\operatorname{arcsinh}(cx)) + \frac{1}{5}c^4d^2x^5(a+b\operatorname{arcsinh}(cx)) \\
&\quad - \frac{1}{30}\left(bcd^2\right)\operatorname{Subst}\left(\int \left(\frac{8}{\sqrt{1+c^2x}} + 4\sqrt{1+c^2x} + 3\left(1+c^2x\right)^{3/2}\right)dx, x, x^2\right) \\
&= -\frac{8bd^2\sqrt{1+c^2x^2}}{15c} - \frac{4bd^2\left(1+c^2x^2\right)^{3/2}}{45c} - \frac{bd^2\left(1+c^2x^2\right)^{5/2}}{25c} \\
&\quad + d^2x(a+b\operatorname{arcsinh}(cx)) + \frac{2}{3}c^2d^2x^3(a+b\operatorname{arcsinh}(cx)) + \frac{1}{5}c^4d^2x^5(a+b\operatorname{arcsinh}(cx))
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.74

$$
\int \left(d+c^2dx^2\right)^2(a+b\operatorname{arcsinh}(cx))\,dx = \frac{d^2\left(15acx(15+10c^2x^2+3c^4x^4) - b\sqrt{1+c^2x^2}(149+38c^2x^2+9c^4x^4) + 15bcx(15+10c^2x^2+3c^4x^4)\operatorname{arcsi}\right.}{225c}
$$

```
[In] Integrate[(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(15*a*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4) - b*Sqrt[1 + c^2*x^2]*(149 + 3
8*c^2*x^2 + 9*c^4*x^4) + 15*b*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x
]))/(225*c)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.91

method	result
parts	$d^2a\left(\frac{1}{5}c^4x^5+\frac{2}{3}x^3c^2+x\right)+\frac{d^2b\left(\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{2\operatorname{arcsinh}(cx)c^3x^3}{3}+\operatorname{arcsinh}(cx)cx-\frac{149\sqrt{c^2x^2+1}}{225}-\frac{c^4x^4\sqrt{c^2x^2}}{25}\right.}{c}$
derivativedivides	$\frac{d^2a\left(\frac{1}{5}c^5x^5+\frac{2}{3}c^3x^3+cx\right)+d^2b\left(\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{2\operatorname{arcsinh}(cx)c^3x^3}{3}+\operatorname{arcsinh}(cx)cx-\frac{149\sqrt{c^2x^2+1}}{225}-\frac{c^4x^4\sqrt{c^2x^2+1}}{25}-\frac{38c^2x^2}{}\right.}{c}$
default	$\frac{d^2a\left(\frac{1}{5}c^5x^5+\frac{2}{3}c^3x^3+cx\right)+d^2b\left(\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{2\operatorname{arcsinh}(cx)c^3x^3}{3}+\operatorname{arcsinh}(cx)cx-\frac{149\sqrt{c^2x^2+1}}{225}-\frac{c^4x^4\sqrt{c^2x^2+1}}{25}-\frac{38c^2x^2}{}\right.}{c}$

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] d^2*a*(1/5*c^4*x^5+2/3*x^3*c^2+x)+d^2*b/c*(1/5*arcsinh(c*x)*c^5*x^5+2/3*arc
sinh(c*x)*c^3*x^3+arcsinh(c*x)*c*x-149/225*(c^2*x^2+1)^(1/2)-1/25*c^4*x^4*(
c^2*x^2+1)^(1/2)-38/225*c^2*x^2*(c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.04

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{45\,ac^5d^2x^5+150\,ac^3d^2x^3+225\,acd^2x+15\,(3\,bc^5d^2x^5+10\,bc^3d^2x^3+15\,bcd^2x)\log\left(cx+\sqrt{c^2x^2+1}\right)-(9\,b}{225\,c}$$

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/225*(45*a*c^5*d^2*x^5 + 150*a*c^3*d^2*x^3 + 225*a*c*d^2*x + 15*(3*b*c^5*d
^2*x^5 + 10*b*c^3*d^2*x^3 + 15*b*c*d^2*x)*log(c*x + sqrt(c^2*x^2 + 1)) - (9
*b*c^4*d^2*x^4 + 38*b*c^2*d^2*x^2 + 149*b*d^2)*sqrt(c^2*x^2 + 1))/c
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.29

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))\,dx$$
$$=\begin{cases}\frac{ac^4d^2x^5}{5}+\frac{2ac^2d^2x^3}{3}+ad^2x+\frac{bc^4d^2x^5\operatorname{asinh}(cx)}{5}-\frac{bc^3d^2x^4\sqrt{c^2x^2+1}}{25}+\frac{2bc^2d^2x^3\operatorname{asinh}(cx)}{3}-\frac{38bcd^2x^2\sqrt{c^2x^2+1}}{225}+bd^2x\,\text{asinh}\\ ad^2x\end{cases}$$

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**5/5 + 2*a*c**2*d**2*x**3/3 + a*d**2*x + b*c**4*d*
*2*x**5*asinh(c*x)/5 - b*c**3*d**2*x**4*sqrt(c**2*x**2 + 1)/25 + 2*b*c**2*d
**2*x**3*asinh(c*x)/3 - 38*b*c*d**2*x**2*sqrt(c**2*x**2 + 1)/225 + b*d**2*x
*asinh(c*x) - 149*b*d**2*sqrt(c**2*x**2 + 1)/(225*c), Ne(c, 0)), (a*d**2*x,
 True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.52

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{1}{5}\,ac^4d^2x^5$$
$$+\frac{1}{75}\left(15\,x^5\,\text{arsinh}\,(cx)-\left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2}-\frac{4\,\sqrt{c^2x^2+1}x^2}{c^4}+\frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\right)bc^4d^2$$
$$+\frac{2}{3}\,ac^2d^2x^3+\frac{2}{9}\left(3\,x^3\,\text{arsinh}\,(cx)-c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)bc^2d^2$$
$$+ad^2x+\frac{\left(cx\,\text{arsinh}\,(cx)-\sqrt{c^2x^2+1}\right)bd^2}{c}$$

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*c^4*d^2*x^5 + 1/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^
2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*c^4*d^2 + 2
/3*a*c^2*d^2*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 -
 2*sqrt(c^2*x^2 + 1)/c^4))*b*c^2*d^2 + a*d^2*x + (c*x*arcsinh(c*x) - sqrt(c
^2*x^2 + 1))*b*d^2/c
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^2\left(a+b\text{arcsinh}(cx)\right)dx=\int \left(a+b\,\text{asinh}(c\,x)\right)\,\left(d\,c^2\,x^2+d\right)^2\,dx$$

```
[In] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^2,x)
[Out] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^2, x)
```

3.15 $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 172

$$\begin{aligned}\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}{x}\,dx &= -\frac{11}{32}bcd^2x\sqrt{1+c^2x^2} \\ &\quad -\frac{1}{16}bcd^2x(1+c^2x^2)^{3/2}-\frac{11}{32}bd^2\text{arcsinh}(cx) \\ &\quad +\frac{1}{2}d^2(1+c^2x^2)(a+b\text{arcsinh}(cx)) \\ &\quad +\frac{1}{4}d^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx)) \\ &\quad +\frac{d^2(a+b\text{arcsinh}(cx))^2}{2b} \\ &\quad +d^2(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right) \\ &\quad -\frac{1}{2}bd^2\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)\end{aligned}$$

```
[Out] -1/16*b*c*d^2*x*(c^2*x^2+1)^(3/2)-11/32*b*d^2*arcsinh(c*x)+1/2*d^2*(c^2*x^2
+1)*(a+b*arcsinh(c*x))+1/4*d^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))+1/2*d^2*(a+
b*arcsinh(c*x))^2/b+d^2*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2
)-1/2*b*d^2*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)-11/32*b*c*d^2*x*(c^2*x^2
+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5801, 5775, 3797, 2221, 2317, 2438, 201, 221}

$$\int \frac{(d+c^2dx^2)^2\,(a+b\text{arcsinh}(cx))}{x}\,dx = \frac{1}{4}d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx)) + \frac{1}{2}d^2\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx)) + \frac{d^2(a+b\text{arcsinh}(cx))^2}{2b} + d^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)) - \frac{1}{2}bd^2\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right) - \frac{11}{32}bd^2\text{arcsinh}(cx) - \frac{1}{16}bcd^2x\left(c^2x^2+1\right)^{3/2} - \frac{11}{32}bcd^2x\sqrt{c^2x^2+1}$$

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (-11*b*c*d^2*x*Sqrt[1 + c^2*x^2])/32 - (b*c*d^2*x*(1 + c^2*x^2)^(3/2))/16 -
 (11*b*d^2*ArcSinh[c*x])/32 + (d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/2 +
(d^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/4 + (d^2*(a + b*ArcSinh[c*x])^2)
/(2*b) + d^2*(a + b*ArcSinh[c*x])*Log[1 - E^(-2*ArcSinh[c*x])] - (b*d^2*Pol
yLog[2, E^(-2*ArcSinh[c*x])])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5801

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcSinh[c*x])/(2*p)), x] + (Dist[d
, Int[(d + e*x^2)^(p - 1)*((a + b*ArcSinh[c*x])/x), x], x] - Dist[b*c*(d^p/
(2*p)), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}d^2\left(1+c^2x^2\right)^2(a+b\operatorname{arcsinh}(cx)) \\
&\quad + d\int \frac{(d+c^2dx^2)\,(a+b\operatorname{arcsinh}(cx))}{x}\,dx - \frac{1}{4}(bcd^2)\int \left(1+c^2x^2\right)^{3/2}\,dx \\
&= -\frac{1}{16}bcd^2x\left(1+c^2x^2\right)^{3/2}+\frac{1}{2}d^2\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))+\frac{1}{4}d^2\left(1+c^2x^2\right)^2(a+b\operatorname{arcsinh}(cx)) \\
&\quad + d^2\int \frac{a+b\operatorname{arcsinh}(cx)}{x}\,dx - \frac{1}{16}(3bcd^2)\int \sqrt{1+c^2x^2}\,dx \\
&\quad - \frac{1}{2}(bcd^2)\int \sqrt{1+c^2x^2}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{11}{32}bcd^2x\sqrt{1+c^2x^2} - \frac{1}{16}bcd^2x\left(1+c^2x^2\right)^{3/2} \\
&\quad + \frac{1}{2}d^2\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx)) + \frac{1}{4}d^2\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx)) \\
&\quad - \frac{d^2\text{Subst}\left(\int x\coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)}{b} \\
&\quad - \frac{1}{32}\left(3bcd^2\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx - \frac{1}{4}\left(bcd^2\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx \\
&= -\frac{11}{32}bcd^2x\sqrt{1+c^2x^2} - \frac{1}{16}bcd^2x\left(1+c^2x^2\right)^{3/2} - \frac{11}{32}bd^2\text{arcsinh}(cx) \\
&\quad + \frac{1}{2}d^2\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx)) + \frac{1}{4}d^2\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx)) \\
&\quad + \frac{d^2(a+b\text{arcsinh}(cx))^2}{2b} + \frac{\left(2d^2\right)\text{Subst}\left(\int\frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b} \\
&= -\frac{11}{32}bcd^2x\sqrt{1+c^2x^2} - \frac{1}{16}bcd^2x\left(1+c^2x^2\right)^{3/2} - \frac{11}{32}bd^2\text{arcsinh}(cx) \\
&\quad + \frac{1}{2}d^2\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx)) + \frac{1}{4}d^2\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx)) \\
&\quad + \frac{d^2(a+b\text{arcsinh}(cx))^2}{2b} + d^2(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right) \\
&\quad - d^2\text{Subst}\left(\int\log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\text{arcsinh}(cx)\right) \\
&= -\frac{11}{32}bcd^2x\sqrt{1+c^2x^2} - \frac{1}{16}bcd^2x\left(1+c^2x^2\right)^{3/2} - \frac{11}{32}bd^2\text{arcsinh}(cx) \\
&\quad + \frac{1}{2}d^2\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx)) + \frac{1}{4}d^2\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx)) \\
&\quad + \frac{d^2(a+b\text{arcsinh}(cx))^2}{2b} + d^2(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right) \\
&\quad + \frac{1}{2}\left(bd^2\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right) \\
&= -\frac{11}{32}bcd^2x\sqrt{1+c^2x^2} - \frac{1}{16}bcd^2x\left(1+c^2x^2\right)^{3/2} - \frac{11}{32}bd^2\text{arcsinh}(cx) \\
&\quad + \frac{1}{2}d^2\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx)) + \frac{1}{4}d^2\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx)) \\
&\quad + \frac{d^2(a+b\text{arcsinh}(cx))^2}{2b} + d^2(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right) \\
&\quad - \frac{1}{2}bd^2\,\text{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.01

$$\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}{x}\,dx = \frac{d^2\left(-16a^2+24ab+32abc^2x^2+8abc^4x^4-13b^2cx\sqrt{1+c^2x^2}-2b^2c^3x^3\sqrt{1+c^2x^2}-16b^2\text{arcsinh}(cx)^2+32\right.}{}$$

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (d^2*(-16*a^2 + 24*a*b + 32*a*b*c^2*x^2 + 8*a*b*c^4*x^4 - 13*b^2*c*x*Sqrt[1
 + c^2*x^2] - 2*b^2*c^3*x^3*Sqrt[1 + c^2*x^2] - 16*b^2*ArcSinh[c*x]^2 + 32*
a*b*Log[1 - E^(2*ArcSinh[c*x])] + b*ArcSinh[c*x]*(-32*a + b*(13 + 32*c^2*x^
2 + 8*c^4*x^4) + 32*b*Log[1 - E^(2*ArcSinh[c*x])]) + 16*b^2*PolyLog[2, E^(2
*ArcSinh[c*x])]))/(32*b)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.29

method	result
parts	$d^2a\left(\frac{c^4x^4}{4}+c^2x^2+\ln(x)\right)-\frac{d^2bc^3x^3\sqrt{c^2x^2+1}}{16}-\frac{13bcd^2x\sqrt{c^2x^2+1}}{32}+\frac{d^2b\,\text{arcsinh}(cx)c^4x^4}{4}+d^2b\,\text{arc}$
derivativedivides	$d^2a\left(\frac{c^4x^4}{4}+c^2x^2+\ln(cx)\right)+\frac{13bd^2\,\text{arcsinh}(cx)}{32}+d^2b\,\text{arcsinh}(cx)\ln\left(1-cx-\sqrt{c^2x^2+1}\right)$
default	$d^2a\left(\frac{c^4x^4}{4}+c^2x^2+\ln(cx)\right)+\frac{13bd^2\,\text{arcsinh}(cx)}{32}+d^2b\,\text{arcsinh}(cx)\ln\left(1-cx-\sqrt{c^2x^2+1}\right)$

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*a*(1/4*c^4*x^4+c^2*x^2+ln(x))-1/16*d^2*b*c^3*x^3*(c^2*x^2+1)^(1/2)-13/3
2*b*c*d^2*x*(c^2*x^2+1)^(1/2)+1/4*d^2*b*arcsinh(c*x)*c^4*x^4+d^2*b*arcsinh(
c*x)*c^2*x^2+d^2*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+d^2*b*polylog(2,c*x+(c
^2*x^2+1)^(1/2))+13/32*b*d^2*arcsinh(c*x)+d^2*b*arcsinh(c*x)*ln(1+c*x+(c^2*
x^2+1)^(1/2))+d^2*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-1/2*d^2*b*arcs
inh(c*x)^2
```

Fricas [F]

$$\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\text{arcsinh}(cx)\right)}{x}\,dx=\int \frac{\left(c^2dx^2+d\right)^2\left(b\,\text{arsinh}\left(cx\right)+a\right)}{x}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))/x, x)
```

Sympy [F]

$$\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\text{arcsinh}(cx)\right)}{x}\,dx=d^2\left(\int \frac{a}{x}\,dx+\int 2ac^2x\,dx+\int ac^4x^3\,dx+\int \frac{b\,\text{asinh}\left(cx\right)}{x}\,dx+\int 2bc^2x\,\text{asinh}\left(cx\right)dx+\int bc^4x^3\,\text{asinh}\left(cx\right)dx\right)$$

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))/x,x)
[Out] d**2*(Integral(a/x, x) + Integral(2*a*c**2*x, x) + Integral(a*c**4*x**3, x)
 + Integral(b*asinh(c*x)/x, x) + Integral(2*b*c**2*x*asinh(c*x), x) + Integ
ral(b*c**4*x**3*asinh(c*x), x))
```

Maxima [F]

$$\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\text{arcsinh}(cx)\right)}{x}\,dx=\int \frac{\left(c^2dx^2+d\right)^2\left(b\,\text{arsinh}\left(cx\right)+a\right)}{x}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
[Out] 1/4*a*c^4*d^2*x^4 + a*c^2*d^2*x^2 + a*d^2*log(x) + integrate(b*c^4*d^2*x^3*
log(c*x + sqrt(c^2*x^2 + 1)) + 2*b*c^2*d^2*x*log(c*x + sqrt(c^2*x^2 + 1)) +
 b*d^2*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^2\,(a+b\text{arcsinh}(cx))}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^2\,(a+b\text{arcsinh}(cx))}{x}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^2}{x}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x, x)
```

3.16 $\int \frac{(d+c^2dx^2)^2(a+b\,\text{arcsinh}(cx))}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 120

$$\int \frac{(d+c^2dx^2)^2\,(a+b\text{arcsinh}(cx))}{x^2}\,dx = -\frac{5}{3}bcd^2\sqrt{1+c^2x^2}-\frac{1}{9}bcd^2\left(1+c^2x^2\right)^{3/2} - \frac{d^2(a+b\text{arcsinh}(cx))}{x}+2c^2d^2x(a+b\text{arcsinh}(cx)) + \frac{1}{3}c^4d^2x^3(a+b\text{arcsinh}(cx)) - bcd^2\text{arctanh}\left(\sqrt{1+c^2x^2}\right)$$

```
[Out] -1/9*b*c*d^2*(c^2*x^2+1)^(3/2)-d^2*(a+b*arcsinh(c*x))/x+2*c^2*d^2*x*(a+b*ar
csinh(c*x))+1/3*c^4*d^2*x^3*(a+b*arcsinh(c*x))-b*c*d^2*arctanh((c^2*x^2+1)^
(1/2))-5/3*b*c*d^2*(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {276, 5803, 12, 1265, 911, 1167, 214}

$$\int \frac{(d+c^2dx^2)^2\,(a+b\text{arcsinh}(cx))}{x^2}\,dx = \frac{1}{3}c^4d^2x^3(a+b\text{arcsinh}(cx))+2c^2d^2x(a+b\text{arcsinh}(cx)) - \frac{d^2(a+b\text{arcsinh}(cx))}{x}-bcd^2\text{arctanh}\left(\sqrt{c^2x^2+1}\right) - \frac{1}{9}bcd^2\left(c^2x^2+1\right)^{3/2}-\frac{5}{3}bcd^2\sqrt{c^2x^2+1}$$

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (-5*b*c*d^2*Sqrt[1 + c^2*x^2])/3 - (b*c*d^2*(1 + c^2*x^2)^(3/2))/9 - (d^2*(
a + b*ArcSinh[c*x]))/x + 2*c^2*d^2*x*(a + b*ArcSinh[c*x]) + (c^4*d^2*x^3*(a
 + b*ArcSinh[c*x]))/3 - b*c*d^2*ArcTanh[Sqrt[1 + c^2*x^2]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
```

```
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\operatorname{arcsinh}(cx))}{x}+2c^2d^2x(a+b\operatorname{arcsinh}(cx)) \\
&\quad+\frac{1}{3}c^4d^2x^3(a+b\operatorname{arcsinh}(cx))-(bc)\int\frac{d^2(-3+6c^2x^2+c^4x^4)}{3x\sqrt{1+c^2x^2}}\,dx \\
&= -\frac{d^2(a+b\operatorname{arcsinh}(cx))}{x}+2c^2d^2x(a+b\operatorname{arcsinh}(cx)) \\
&\quad+\frac{1}{3}c^4d^2x^3(a+b\operatorname{arcsinh}(cx))-\frac{1}{3}\left(bcd^2\right)\int\frac{-3+6c^2x^2+c^4x^4}{x\sqrt{1+c^2x^2}}\,dx \\
&= -\frac{d^2(a+b\operatorname{arcsinh}(cx))}{x}+2c^2d^2x(a+b\operatorname{arcsinh}(cx))+\frac{1}{3}c^4d^2x^3(a+b\operatorname{arcsinh}(cx)) \\
&\quad-\frac{1}{6}\left(bcd^2\right)\operatorname{Subst}\left(\int\frac{-3+6c^2x+c^4x^2}{x\sqrt{1+c^2x}}\,dx,x,x^2\right) \\
&= -\frac{d^2(a+b\operatorname{arcsinh}(cx))}{x}+2c^2d^2x(a+b\operatorname{arcsinh}(cx)) \\
&\quad+\frac{1}{3}c^4d^2x^3(a+b\operatorname{arcsinh}(cx))-\frac{\left(bd^2\right)\operatorname{Subst}\left(\int\frac{-8+4x^2+x^4}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx,x,\sqrt{1+c^2x^2}\right)}{3c} \\
&= -\frac{d^2(a+b\operatorname{arcsinh}(cx))}{x}+2c^2d^2x(a+b\operatorname{arcsinh}(cx))+\frac{1}{3}c^4d^2x^3(a+b\operatorname{arcsinh}(cx)) \\
&\quad-\frac{\left(bd^2\right)\operatorname{Subst}\left(\int\left(5c^2+c^2x^2-\frac{3}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\right)dx,x,\sqrt{1+c^2x^2}\right)}{3c} \\
&= -\frac{5}{3}bcd^2\sqrt{1+c^2x^2}-\frac{1}{9}bcd^2\left(1+c^2x^2\right)^{3/2}-\frac{d^2(a+b\operatorname{arcsinh}(cx))}{x} \\
&\quad+2c^2d^2x(a+b\operatorname{arcsinh}(cx))+\frac{1}{3}c^4d^2x^3(a+b\operatorname{arcsinh}(cx)) \\
&\quad+\frac{\left(bd^2\right)\operatorname{Subst}\left(\int\frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx,x,\sqrt{1+c^2x^2}\right)}{c} \\
&= -\frac{5}{3}bcd^2\sqrt{1+c^2x^2}-\frac{1}{9}bcd^2\left(1+c^2x^2\right)^{3/2}-\frac{d^2(a+b\operatorname{arcsinh}(cx))}{x} \\
&\quad+2c^2d^2x(a+b\operatorname{arcsinh}(cx))+\frac{1}{3}c^4d^2x^3(a+b\operatorname{arcsinh}(cx))-bcd^2\operatorname{arctanh}\left(\sqrt{1+c^2x^2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.03

$$\int \frac{(d + c^2dx^2)^2 (a + b\text{arcsinh}(cx))}{x^2} dx$$
$$= \frac{d^2(-9a + 18ac^2x^2 + 3ac^4x^4 - 16bcx\sqrt{1 + c^2x^2} - bc^3x^3\sqrt{1 + c^2x^2} + 3b(-3 + 6c^2x^2 + c^4x^4)\,\text{arcsinh}(cx) +}{9x}$$

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (d^2*(-9*a + 18*a*c^2*x^2 + 3*a*c^4*x^4 - 16*b*c*x*Sqrt[1 + c^2*x^2] - b*c^
3*x^3*Sqrt[1 + c^2*x^2] + 3*b*(-3 + 6*c^2*x^2 + c^4*x^4)*ArcSinh[c*x] + 9*b
*c*x*Log[x] - 9*b*c*x*Log[1 + Sqrt[1 + c^2*x^2]]))/(9*x)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93

method	result
parts	$d^2a\left(\frac{c^4x^3}{3} + 2c^2x - \frac{1}{x}\right) + d^2bc\left(\frac{\text{arcsinh}(cx)c^3x^3}{3} + 2\,\text{arcsinh}\,(cx)\,cx - \frac{\text{arcsinh}(cx)}{cx} - \frac{c^2x^2\sqrt{c^2x^2+}}{9}\right.$
derivativedivides	$c\left(d^2a\left(\frac{c^3x^3}{3} + 2cx - \frac{1}{cx}\right) + d^2b\left(\frac{\text{arcsinh}(cx)c^3x^3}{3} + 2\,\text{arcsinh}\,(cx)\,cx - \frac{\text{arcsinh}(cx)}{cx} - \frac{c^2x^2\sqrt{c^2x^2}}{9}\right.\right.$
default	$c\left(d^2a\left(\frac{c^3x^3}{3} + 2cx - \frac{1}{cx}\right) + d^2b\left(\frac{\text{arcsinh}(cx)c^3x^3}{3} + 2\,\text{arcsinh}\,(cx)\,cx - \frac{\text{arcsinh}(cx)}{cx} - \frac{c^2x^2\sqrt{c^2x^2}}{9}\right.\right.$

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*a*(1/3*c^4*x^3+2*c^2*x-1/x)+d^2*b*c*(1/3*arcsinh(c*x)*c^3*x^3+2*arcsinh
(c*x)*c*x-arcsinh(c*x)/c/x-1/9*c^2*x^2*(c^2*x^2+1)^(1/2)-16/9*(c^2*x^2+1)^(
1/2)-arctanh(1/(c^2*x^2+1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(108) = 216$.

Time = 0.29 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.90

$$\int \frac{(d + c^2dx^2)^2 (a + b\text{arcsinh}(cx))}{x^2} dx$$
$$= \frac{3\,ac^4d^2x^4 + 18\,ac^2d^2x^2 - 9\,bcd^2x\log\left(-cx + \sqrt{c^2x^2 + 1} + 1\right) + 9\,bcd^2x\log\left(-cx + \sqrt{c^2x^2 + 1} - 1\right) - 3\,(bc}{}$$

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/9*(3*a*c^4*d^2*x^4 + 18*a*c^2*d^2*x^2 - 9*b*c*d^2*x*log(-c*x + sqrt(c^2*x
^2 + 1) + 1) + 9*b*c*d^2*x*log(-c*x + sqrt(c^2*x^2 + 1) - 1) - 3*(b*c^4 + 6
*b*c^2 - 3*b)*d^2*x*log(-c*x + sqrt(c^2*x^2 + 1)) - 9*a*d^2 + 3*(b*c^4*d^2*
x^4 + 6*b*c^2*d^2*x^2 - (b*c^4 + 6*b*c^2 - 3*b)*d^2*x - 3*b*d^2)*log(c*x +
sqrt(c^2*x^2 + 1)) - (b*c^3*d^2*x^3 + 16*b*c*d^2*x)*sqrt(c^2*x^2 + 1))/x
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))}{x^2}\,dx = d^2\left(\int 2ac^2\,dx + \int \frac{a}{x^2}\,dx + \int ac^4x^2\,dx + \int 2bc^2\operatorname{asinh}(cx)\,dx + \int \frac{b\operatorname{asinh}(cx)}{x^2}\,dx + \int bc^4x^2\operatorname{asinh}(cx)\,dx\right)$$

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))/x**2,x)
```

```
[Out] d**2*(Integral(2*a*c**2, x) + Integral(a/x**2, x) + Integral(a*c**4*x**2, x
) + Integral(2*b*c**2*asinh(c*x), x) + Integral(b*asinh(c*x)/x**2, x) + Int
egral(b*c**4*x**2*asinh(c*x), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.19

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))}{x^2}\,dx = \frac{1}{3}\,ac^4d^2x^3 + \frac{1}{9}\left(3\,x^3\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bc^4d^2 + 2\,ac^2d^2x + 2\left(cx\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)bcd^2 - \left(c\operatorname{arsinh}\left(\frac{1}{c|x|}\right) + \frac{\operatorname{arsinh}(cx)}{x}\right)bd^2 - \frac{ad^2}{x}$$

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*c^4*d^2*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2
- 2*sqrt(c^2*x^2 + 1)/c^4))*b*c^4*d^2 + 2*a*c^2*d^2*x + 2*(c*x*arcsinh(c*x)
 - sqrt(c^2*x^2 + 1))*b*c*d^2 - (c*arcsinh(1/(c*abs(x))) + arcsinh(c*x)/x)*
b*d^2 - a*d^2/x
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\text{arcsinh}(cx)\right)}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\text{arcsinh}(cx)\right)}{x^2}\,dx = \int \frac{\left(a+b\,\text{asinh}(c\,x)\right)\,\left(d\,c^2\,x^2+d\right)^2}{x^2}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x^2, x)
```

3.17 $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 187

$$\begin{aligned}\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}{x^3}\,dx &= \frac{1}{4}bc^3d^2x\sqrt{1+c^2x^2} - \frac{bcd^2(1+c^2x^2)^{3/2}}{2x} \\ &\quad + \frac{1}{4}bc^2d^2\text{arcsinh}(cx) \\ &\quad + c^2d^2\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx)) \\ &\quad - \frac{d^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{2x^2} \\ &\quad + \frac{c^2d^2(a+b\text{arcsinh}(cx))^2}{b} \\ &\quad + 2c^2d^2(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right) \\ &\quad - bc^2d^2\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)\end{aligned}$$

```
[Out] -1/2*b*c*d^2*(c^2*x^2+1)^(3/2)/x+1/4*b*c^2*d^2*arcsinh(c*x)+c^2*d^2*(c^2*x^
2+1)*(a+b*arcsinh(c*x))-1/2*d^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/x^2+c^2*d^
2*(a+b*arcsinh(c*x))^2/b+2*c^2*d^2*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+
1)^(1/2))^2)-b*c^2*d^2*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)+1/4*b*c^3*d^2
*x*(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5802, 283, 201, 221, 5801, 5775, 3797, 2221, 2317, 2438}

$$\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}{x^3}dx = c^2d^2(c^2x^2+1)(a+b\text{arcsinh}(cx)) - \frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{2x^2} + \frac{c^2d^2(a+b\text{arcsinh}(cx))^2}{b} + 2c^2d^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)) - bc^2d^2\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right) + \frac{1}{4}bc^2d^2\text{arcsinh}(cx) - \frac{bcd^2(c^2x^2+1)^{3/2}}{2x} + \frac{1}{4}bc^3d^2x\sqrt{c^2x^2+1}$$

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (b*c^3*d^2*x*Sqrt[1 + c^2*x^2])/4 - (b*c*d^2*(1 + c^2*x^2)^(3/2))/(2*x) + (
b*c^2*d^2*ArcSinh[c*x])/4 + c^2*d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]) - (d
^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(2*x^2) + (c^2*d^2*(a + b*ArcSinh[
c*x])^2)/b + 2*c^2*d^2*(a + b*ArcSinh[c*x])*Log[1 - E^(-2*ArcSinh[c*x])] -
b*c^2*d^2*PolyLog[2, E^(-2*ArcSinh[c*x])]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5801

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcSinh[c*x])/(2*p)), x] + (Dist[d
, Int[(d + e*x^2)^(p - 1)*((a + b*ArcSinh[c*x])/x), x], x] - Dist[b*c*(d^p/
(2*p)), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5802

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSinh[c
```

```
*x])/(f*(m + 1))), x] + (-Dist[b*c*(d^p/(f*(m + 1))), Int[(f*x)^(m + 1)*(1
+ c^2*x^2)^(p - 1/2), x], x] - Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m + 2
)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))}{2x^2} + (2c^2d)\int \frac{(d+c^2dx^2)(a+b\operatorname{arcsinh}(cx))}{x}\,dx + \frac{1}{2}(bcd^2)\int \frac{(1+c^2x^2)^{3/2}}{x^2}\,dx \\
&= -\frac{bcd^2(1+c^2x^2)^{3/2}}{2x} + c^2d^2(1+c^2x^2)(a+b\operatorname{arcsinh}(cx)) - \frac{d^2(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))}{2x^2} + (2c^2d^2)\int \frac{a+b\operatorname{arcsinh}(cx)}{x}\,dx - (bc^3d^2)\int \sqrt{1+c^2x^2}\,dx + \frac{1}{2}(3bc^3d^2)\int \sqrt{1+c^2x^2}\,dx \\
&= \frac{1}{4}bc^3d^2x\sqrt{1+c^2x^2} - \frac{bcd^2(1+c^2x^2)^{3/2}}{2x} + c^2d^2(1+c^2x^2)(a+b\operatorname{arcsinh}(cx)) - \frac{d^2(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))}{2x^2} - \frac{(2c^2d^2)\operatorname{Subst}\left(\int x\coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b} - \frac{1}{2}(bc^3d^2)\int \frac{1}{\sqrt{1+c^2x^2}}\,dx + \frac{1}{4}(3bc^3d^2)\int \frac{1}{\sqrt{1+c^2x^2}}\,dx \\
&= \frac{1}{4}bc^3d^2x\sqrt{1+c^2x^2} - \frac{bcd^2(1+c^2x^2)^{3/2}}{2x} + \frac{1}{4}bc^2d^2\operatorname{arcsinh}(cx) + c^2d^2(1+c^2x^2)(a+b\operatorname{arcsinh}(cx)) - \frac{d^2(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))}{2x^2} + \frac{c^2d^2(a+b\operatorname{arcsinh}(cx))^2}{b} + \frac{(4c^2d^2)\operatorname{Subst}\left(\int \frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b} \\
&= \frac{1}{4}bc^3d^2x\sqrt{1+c^2x^2} - \frac{bcd^2(1+c^2x^2)^{3/2}}{2x} + \frac{1}{4}bc^2d^2\operatorname{arcsinh}(cx) + c^2d^2(1+c^2x^2)(a+b\operatorname{arcsinh}(cx)) - \frac{d^2(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))}{2x^2} + \frac{c^2d^2(a+b\operatorname{arcsinh}(cx))^2}{b} + 2c^2d^2(a+b\operatorname{arcsinh}(cx))\log\left(1-e^{-2\operatorname{arcsinh}(cx)}\right) - (2c^2d^2)\operatorname{Subst}\left(\int \log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\operatorname{arcsinh}(cx)\right)
\end{aligned}$$

$$= \frac{1}{4}bc^3d^2x\sqrt{1+c^2x^2} - \frac{bcd^2(1+c^2x^2)^{3/2}}{2x} + \frac{1}{4}bc^2d^2\text{arcsinh}(cx) + c^2d^2(1+c^2x^2)(a+b\text{arcsinh}(cx)) - \frac{d^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{2x^2} + \frac{c^2d^2(a+b\text{arcsinh}(cx))^2}{b} + 2c^2d^2(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right) + (bc^2d^2)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right)$$

$$= \frac{1}{4}bc^3d^2x\sqrt{1+c^2x^2} - \frac{bcd^2(1+c^2x^2)^{3/2}}{2x} + \frac{1}{4}bc^2d^2\text{arcsinh}(cx) + c^2d^2(1+c^2x^2)(a+b\text{arcsinh}(cx)) - \frac{d^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{2x^2} + \frac{c^2d^2(a+b\text{arcsinh}(cx))^2}{b} + 2c^2d^2(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right) - bc^2d^2\,\text{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right)$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.81

$$\int \frac{(d+c^2dx^2)^2\,(a+b\text{arcsinh}(cx))}{x^3}\,dx = \frac{1}{4}d^2\left(2ac^4x^2 - \frac{2bc\sqrt{1+c^2x^2}}{x} - bc^3x\sqrt{1+c^2x^2} + bc^2\text{arcsinh}(cx) + 2bc^4x^2\text{arcsinh}(cx) - \frac{2(a+b\text{arcsinh}(cx))}{x^2} - \frac{4c^2(a+b\text{arcsinh}(cx))^2}{b} + 4c^2\left(2(a+b\text{arcsinh}(cx))\log\left(1-e^{2\text{arcsinh}(cx)}\right) + b\,\text{PolyLog}\left(2, e^{2\text{arcsinh}(cx)}\right)\right)\right)$$

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (d^2*(2*a*c^4*x^2 - (2*b*c*Sqrt[1 + c^2*x^2])/x - b*c^3*x*Sqrt[1 + c^2*x^2]
 + b*c^2*ArcSinh[c*x] + 2*b*c^4*x^2*ArcSinh[c*x] - (2*(a + b*ArcSinh[c*x]))
/x^2 - (4*c^2*(a + b*ArcSinh[c*x])^2)/b + 4*c^2*(2*(a + b*ArcSinh[c*x])*Log
[1 - E^(2*ArcSinh[c*x])] + b*PolyLog[2, E^(2*ArcSinh[c*x])])))/4
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.28

method	result
parts	$d^{2} a \left(\frac{c^{4} x^{2}}{2}-\frac{1}{2 x^{2}}+2 c^{2} \ln \left(x \right)\right)+d^{2} b \,c^{2} \left(-\operatorname{arcsinh}\left(c x \right)^{2}+\frac{\left(-1+2 \,\operatorname{arcsinh}\left(c x \right)\right) \left(2 c^{2} x^{2}+1+2 c x \sqrt{c^{2} x\right.}{16}$
derivativedivides	$c^{2} \left(d^{2} a \left(\frac{c^{2} x^{2}}{2}+2 \ln \left(c x \right)-\frac{1}{2 c^{2} x^{2}}\right)+d^{2} b \left(-\operatorname{arcsinh}\left(c x \right)^{2}+\frac{\left(-1+2 \,\operatorname{arcsinh}\left(c x \right)\right) \left(2 c^{2} x^{2}+1+2 c x \sqrt{}\right.}{16}\right.\right.$
default	$c^{2} \left(d^{2} a \left(\frac{c^{2} x^{2}}{2}+2 \ln \left(c x \right)-\frac{1}{2 c^{2} x^{2}}\right)+d^{2} b \left(-\operatorname{arcsinh}\left(c x \right)^{2}+\frac{\left(-1+2 \,\operatorname{arcsinh}\left(c x \right)\right) \left(2 c^{2} x^{2}+1+2 c x \sqrt{}\right.}{16}\right.\right.$

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*a*(1/2*c^4*x^2-1/2/x^2+2*c^2*ln(x))+d^2*b*c^2*(-arcsinh(c*x)^2+1/16*(-1
+2*arcsinh(c*x))*(2*c^2*x^2+1+2*c*x*(c^2*x^2+1)^(1/2))+1/16*(-2*c*x*(c^2*x^
2+1)^(1/2)+2*c^2*x^2+1)*(1+2*arcsinh(c*x))-1/2*(c*x*(c^2*x^2+1)^(1/2)-c^2*x
^2+arcsinh(c*x))/c^2/x^2+2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*polyl
og(2,-c*x-(c^2*x^2+1)^(1/2))+2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*p
olylog(2,c*x+(c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\text{arcsinh}(cx)\right)}{x^3}\,dx=\int \frac{{\left(c^{2} d x^{2} + d\right)}^{2} {\left(b \operatorname{arsinh}\left(c x\right) + a\right)}}{x^{3}}\,{d x}$$

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\text{arcsinh}(cx)\right)}{x^3}\,dx=d^{2} \left(\int \frac{a}{x^{3}}\, dx + \int \frac{2 a c^{2}}{x}\, dx + \int a c^{4} x\, dx + \int \frac{b \operatorname{asinh}{\left(c x \right)}}{x^{3}}\, dx + \int \frac{2 b c^{2} \operatorname{asinh}{\left(c x \right)}}{x}\, dx + \int b c^{4} x \operatorname{asinh}{\left(c x \right)}\, dx\right)$$

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))/x**3,x)
```

```
[Out] d**2*(Integral(a/x**3, x) + Integral(2*a*c**2/x, x) + Integral(a*c**4*x, x)
 + Integral(b*asinh(c*x)/x**3, x) + Integral(2*b*c**2*asinh(c*x)/x, x) + In
tegral(b*c**4*x*asinh(c*x), x))
```

Maxima [F]

$$\int \frac{(d + c^2dx^2)^2 (a + b\text{arcsinh}(cx))}{x^3} dx = \int \frac{(c^2dx^2 + d)^2(b \operatorname{arsinh}(cx) + a)}{x^3} dx$$

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*c^4*d^2*x^2 + 2*a*c^2*d^2*log(x) - 1/2*b*d^2*(sqrt(c^2*x^2 + 1)*c/x +
 arcsinh(c*x)/x^2) - 1/2*a*d^2/x^2 + integrate(b*c^4*d^2*x*log(c*x + sqrt(c
^2*x^2 + 1)) + 2*b*c^2*d^2*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + c^2dx^2)^2 (a + b\text{arcsinh}(cx))}{x^3} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + c^2dx^2)^2 (a + b\text{arcsinh}(cx))}{x^3} dx = \int \frac{(a + b \operatorname{asinh}(c\,x)) \left(d\,c^2\,x^2 + d\right)^2}{x^3} dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x^3, x)
```

3.18 $\int \frac{(d+c^2dx^2)^2(a+b\,\text{arcsinh}(cx))}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 126

$$\int \frac{(d+c^2dx^2)^2(a+b\,\text{arcsinh}(cx))}{x^4}\,dx = -bc^3d^2\sqrt{1+c^2x^2}-\frac{bcd^2\sqrt{1+c^2x^2}}{6x^2}-\frac{d^2(a+b\,\text{arcsinh}(cx))}{3x^3}-\frac{2c^2d^2(a+b\,\text{arcsinh}(cx))}{x}+c^4d^2x(a+b\,\text{arcsinh}(cx))-\frac{11}{6}bc^3d^2\,\text{arctanh}\left(\sqrt{1+c^2x^2}\right)$$

```
[Out] -1/3*d^2*(a+b*arcsinh(c*x))/x^3-2*c^2*d^2*(a+b*arcsinh(c*x))/x+c^4*d^2*x*(a
+b*arcsinh(c*x))-11/6*b*c^3*d^2*arctanh((c^2*x^2+1)^(1/2))-b*c^3*d^2*(c^2*x
^2+1)^(1/2)-1/6*b*c*d^2*(c^2*x^2+1)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {276, 5803, 12, 1265, 911, 1171, 396, 214}

$$\int \frac{(d+c^2dx^2)^2(a+b\,\text{arcsinh}(cx))}{x^4}\,dx = c^4d^2x(a+b\,\text{arcsinh}(cx))-\frac{2c^2d^2(a+b\,\text{arcsinh}(cx))}{x}-\frac{d^2(a+b\,\text{arcsinh}(cx))}{3x^3}-\frac{11}{6}bc^3d^2\,\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\frac{bcd^2\sqrt{c^2x^2+1}}{6x^2}-bc^3d^2\sqrt{c^2x^2+1}$$

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -(b*c^3*d^2*Sqrt[1 + c^2*x^2]) - (b*c*d^2*Sqrt[1 + c^2*x^2])/(6*x^2) - (d^2
*(a + b*ArcSinh[c*x]))/(3*x^3) - (2*c^2*d^2*(a + b*ArcSinh[c*x]))/x + c^4*d
^2*x*(a + b*ArcSinh[c*x]) - (11*b*c^3*d^2*ArcTanh[Sqrt[1 + c^2*x^2]])/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
```

```
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\operatorname{arcsinh}(cx))}{3x^3}-\frac{2c^2d^2(a+b\operatorname{arcsinh}(cx))}{x} \\
&\quad +c^4d^2x(a+b\operatorname{arcsinh}(cx))-(bc)\int\frac{d^2(-1-6c^2x^2+3c^4x^4)}{3x^3\sqrt{1+c^2x^2}}\,dx \\
&= -\frac{d^2(a+b\operatorname{arcsinh}(cx))}{3x^3}-\frac{2c^2d^2(a+b\operatorname{arcsinh}(cx))}{x} \\
&\quad +c^4d^2x(a+b\operatorname{arcsinh}(cx))-\frac{1}{3}\left(bcd^2\right)\int\frac{-1-6c^2x^2+3c^4x^4}{x^3\sqrt{1+c^2x^2}}\,dx \\
&= -\frac{d^2(a+b\operatorname{arcsinh}(cx))}{3x^3}-\frac{2c^2d^2(a+b\operatorname{arcsinh}(cx))}{x}+c^4d^2x(a+b\operatorname{arcsinh}(cx)) \\
&\quad -\frac{1}{6}\left(bcd^2\right)\operatorname{Subst}\left(\int\frac{-1-6c^2x+3c^4x^2}{x^2\sqrt{1+c^2x}}\,dx,x,x^2\right) \\
&= -\frac{d^2(a+b\operatorname{arcsinh}(cx))}{3x^3}-\frac{2c^2d^2(a+b\operatorname{arcsinh}(cx))}{x} \\
&\quad +c^4d^2x(a+b\operatorname{arcsinh}(cx))-\frac{\left(bd^2\right)\operatorname{Subst}\left(\int\frac{8-12x^2+3x^4}{\left(-\frac{1}{c^2}+\frac{x^2}{c^2}\right)^2}\,dx,x,\sqrt{1+c^2x^2}\right)}{3c} \\
&= -\frac{bcd^2\sqrt{1+c^2x^2}}{6x^2}-\frac{d^2(a+b\operatorname{arcsinh}(cx))}{3x^3}-\frac{2c^2d^2(a+b\operatorname{arcsinh}(cx))}{x} \\
&\quad +c^4d^2x(a+b\operatorname{arcsinh}(cx))-\frac{1}{6}\left(bcd^2\right)\operatorname{Subst}\left(\int\frac{-17+6x^2}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx,x,\sqrt{1+c^2x^2}\right)
\end{aligned}
$$

$$= -bc^3d^2\sqrt{1+c^2x^2} - \frac{bcd^2\sqrt{1+c^2x^2}}{6x^2} - \frac{d^2(a+b\text{arcsinh}(cx))}{3x^3} - \frac{2c^2d^2(a+b\text{arcsinh}(cx))}{x} + c^4d^2x(a+b\text{arcsinh}(cx)) + \frac{1}{6}\left(11bcd^2\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx, x, \sqrt{1+c^2x^2}\right)$$

$$= -bc^3d^2\sqrt{1+c^2x^2} - \frac{bcd^2\sqrt{1+c^2x^2}}{6x^2} - \frac{d^2(a+b\text{arcsinh}(cx))}{3x^3} - \frac{2c^2d^2(a+b\text{arcsinh}(cx))}{x} + c^4d^2x(a+b\text{arcsinh}(cx)) - \frac{11}{6}bc^3d^2\text{arctanh}\left(\sqrt{1+c^2x^2}\right)$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06

$$\int \frac{\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))}{x^4}\,dx = \frac{d^2\left(-2a-12ac^2x^2+6ac^4x^4-bcx\sqrt{1+c^2x^2}-6bc^3x^3\sqrt{1+c^2x^2}+2b\left(-1-6c^2x^2+3c^4x^4\right)\text{arcsinh}(cx)+1\right.}{6x^3}$$

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] (d^2*(-2*a - 12*a*c^2*x^2 + 6*a*c^4*x^4 - b*c*x*Sqrt[1 + c^2*x^2] - 6*b*c^3
*x^3*Sqrt[1 + c^2*x^2] + 2*b*(-1 - 6*c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x] + 11
*b*c^3*x^3*Log[x] - 11*b*c^3*x^3*Log[1 + Sqrt[1 + c^2*x^2]]))/(6*x^3)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.89

method	result
parts	$d^2a\left(c^4x-\frac{2c^2}{x}-\frac{1}{3x^3}\right)+d^2b\,c^3\left(\text{arcsinh}\,(cx)\,cx-\frac{\text{arcsinh}(cx)}{3c^3x^3}-\frac{2\,\text{arcsinh}(cx)}{cx}-\sqrt{c^2x^2+1}-\right.$
derivativedivides	$c^3\left(d^2a\left(cx-\frac{1}{3c^3x^3}-\frac{2}{cx}\right)+d^2b\left(\text{arcsinh}\,(cx)\,cx-\frac{\text{arcsinh}(cx)}{3c^3x^3}-\frac{2\,\text{arcsinh}(cx)}{cx}-\sqrt{c^2x^2+1}-\right.\right.$
default	$c^3\left(d^2a\left(cx-\frac{1}{3c^3x^3}-\frac{2}{cx}\right)+d^2b\left(\text{arcsinh}\,(cx)\,cx-\frac{\text{arcsinh}(cx)}{3c^3x^3}-\frac{2\,\text{arcsinh}(cx)}{cx}-\sqrt{c^2x^2+1}-\right.\right.$

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*a*(c^4*x-2*c^2/x-1/3/x^3)+d^2*b*c^3*(arcsinh(c*x)*c*x-1/3*arcsinh(c*x)/
c^3/x^3-2*arcsinh(c*x)/c/x-(c^2*x^2+1)^(1/2)-1/6/c^2/x^2*(c^2*x^2+1)^(1/2)-
11/6*arctanh(1/(c^2*x^2+1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(114) = 228$.

Time = 0.28 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.93

$$\int \frac{(d+c^2dx^2)^2 (a+b\operatorname{arcsinh}(cx))}{x^4}\,dx$$
$$= \frac{6\,ac^4d^2x^4 - 11\,bc^3d^2x^3\log\left(-cx+\sqrt{c^2x^2+1}+1\right) + 11\,bc^3d^2x^3\log\left(-cx+\sqrt{c^2x^2+1}-1\right) - 12\,ac^2d^2x^2}{}$$

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*c^4*d^2*x^4 - 11*b*c^3*d^2*x^3*log(-c*x + sqrt(c^2*x^2 + 1) + 1) +
 11*b*c^3*d^2*x^3*log(-c*x + sqrt(c^2*x^2 + 1) - 1) - 12*a*c^2*d^2*x^2 - 2*
(3*b*c^4 - 6*b*c^2 - b)*d^2*x^3*log(-c*x + sqrt(c^2*x^2 + 1)) - 2*a*d^2 + 2
*(3*b*c^4*d^2*x^4 - 6*b*c^2*d^2*x^2 - (3*b*c^4 - 6*b*c^2 - b)*d^2*x^3 - b*d
^2)*log(c*x + sqrt(c^2*x^2 + 1)) - (6*b*c^3*d^2*x^3 + b*c*d^2*x)*sqrt(c^2*x
^2 + 1))/x^3
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^2 (a+b\operatorname{arcsinh}(cx))}{x^4}\,dx = d^2\left(\int ac^4\,dx + \int \frac{a}{x^4}\,dx + \int \frac{2ac^2}{x^2}\,dx + \int bc^4\operatorname{asinh}(cx)\,dx + \int \frac{b\operatorname{asinh}(cx)}{x^4}\,dx + \int \frac{2bc^2\operatorname{asinh}(cx)}{x^2}\,dx\right)$$

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))/x**4,x)
```

```
[Out] d**2*(Integral(a*c**4, x) + Integral(a/x**4, x) + Integral(2*a*c**2/x**2, x
) + Integral(b*c**4*asinh(c*x), x) + Integral(b*asinh(c*x)/x**4, x) + Integ
ral(2*b*c**2*asinh(c*x)/x**2, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.09

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))}{x^4} dx$$
$$= ac^4d^2x + \left(cx \operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)bc^3d^2 - 2\left(c \operatorname{arsinh}\left(\frac{1}{c|x|}\right) + \frac{\operatorname{arsinh}(cx)}{x}\right)bc^2d^2$$
$$+ \frac{1}{6}\left(\left(c^2 \operatorname{arsinh}\left(\frac{1}{c|x|}\right) - \frac{\sqrt{c^2x^2+1}}{x^2}\right)c - \frac{2 \operatorname{arsinh}(cx)}{x^3}\right)bd^2 - \frac{2\,ac^2d^2}{x} - \frac{ad^2}{3\,x^3}$$

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] a*c^4*d^2*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*c^3*d^2 - 2*(c*arcsi
nh(1/(c*abs(x))) + arcsinh(c*x)/x)*b*c^2*d^2 + 1/6*((c^2*arcsinh(1/(c*abs(x
))) - sqrt(c^2*x^2 + 1)/x^2)*c - 2*arcsinh(c*x)/x^3)*b*d^2 - 2*a*c^2*d^2/x
- 1/3*a*d^2/x^3
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))}{x^4} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))}{x^4} dx = \int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^2}{x^4} \,dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x^4, x)
```

3.19 $\int x^4(d+c^2dx^2)^3\,(a+b\mathbf{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 226

$$\int x^4\left(d+c^2dx^2\right)^3(a+b\operatorname{arcsinh}(cx))\,dx=-\frac{16bd^3\sqrt{1+c^2x^2}}{1155c^5}-\frac{8bd^3(1+c^2x^2)^{3/2}}{3465c^5}-\frac{2bd^3(1+c^2x^2)^{5/2}}{1925c^5}-\frac{bd^3(1+c^2x^2)^{7/2}}{1617c^5}+\frac{4bd^3(1+c^2x^2)^{9/2}}{297c^5}-\frac{bd^3(1+c^2x^2)^{11/2}}{121c^5}+\frac{1}{5}d^3x^5(a+b\operatorname{arcsinh}(cx))+\frac{3}{7}c^2d^3x^7(a+b\operatorname{arcsinh}(cx))+\frac{1}{3}c^4d^3x^9(a+b\operatorname{arcsinh}(cx))+\frac{1}{11}c^6d^3x^{11}(a+b\operatorname{arcsin}$$

```
[Out] -8/3465*b*d^3*(c^2*x^2+1)^(3/2)/c^5-2/1925*b*d^3*(c^2*x^2+1)^(5/2)/c^5-1/16
17*b*d^3*(c^2*x^2+1)^(7/2)/c^5+4/297*b*d^3*(c^2*x^2+1)^(9/2)/c^5-1/121*b*d^
3*(c^2*x^2+1)^(11/2)/c^5+1/5*d^3*x^5*(a+b*arcsinh(c*x))+3/7*c^2*d^3*x^7*(a+
b*arcsinh(c*x))+1/3*c^4*d^3*x^9*(a+b*arcsinh(c*x))+1/11*c^6*d^3*x^11*(a+b*a
rcsinh(c*x))-16/1155*b*d^3*(c^2*x^2+1)^(1/2)/c^5
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {276, 5803, 12, 1813, 1634}

$$\int x^4\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))\,dx=\frac{1}{11}c^6d^3x^{11}(a+b\text{arcsinh}(cx))+\frac{1}{3}c^4d^3x^9(a+b\text{arcsinh}(cx))+\frac{3}{7}c^2d^3x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}d^3x^5(a+b\text{arcsinh}(cx))-\frac{bd^3(c^2x^2+1)^{11/2}}{121c^5}+\frac{4bd^3(c^2x^2+1)^{9/2}}{297c^5}-\frac{bd^3(c^2x^2+1)^{7/2}}{1617c^5}-\frac{2bd^3(c^2x^2+1)^{5/2}}{1925c^5}-\frac{8bd^3(c^2x^2+1)^{3/2}}{3465c^5}-\frac{16bd^3\sqrt{c^2x^2+1}}{1155c^5}$$

```
[In] Int[x^4*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-16*b*d^3*Sqrt[1 + c^2*x^2])/(1155*c^5) - (8*b*d^3*(1 + c^2*x^2)^(3/2))/(3
465*c^5) - (2*b*d^3*(1 + c^2*x^2)^(5/2))/(1925*c^5) - (b*d^3*(1 + c^2*x^2)^
(7/2))/(1617*c^5) + (4*b*d^3*(1 + c^2*x^2)^(9/2))/(297*c^5) - (b*d^3*(1 + c
^2*x^2)^(11/2))/(121*c^5) + (d^3*x^5*(a + b*ArcSinh[c*x]))/5 + (3*c^2*d^3*x
^7*(a + b*ArcSinh[c*x]))/7 + (c^4*d^3*x^9*(a + b*ArcSinh[c*x]))/3 + (c^6*d^
3*x^11*(a + b*ArcSinh[c*x]))/11
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
```

```
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}d^3x^5(a+b\text{arcsinh}(cx)) + \frac{3}{7}c^2d^3x^7(a+b\text{arcsinh}(cx)) \\
&\quad + \frac{1}{3}c^4d^3x^9(a+b\text{arcsinh}(cx)) + \frac{1}{11}c^6d^3x^{11}(a+b\text{arcsinh}(cx)) \\
&\quad - (bc)\int \frac{d^3x^5(231+495c^2x^2+385c^4x^4+105c^6x^6)}{1155\sqrt{1+c^2x^2}}\,dx \\
&= \frac{1}{5}d^3x^5(a+b\text{arcsinh}(cx)) + \frac{3}{7}c^2d^3x^7(a+b\text{arcsinh}(cx)) + \frac{1}{3}c^4d^3x^9(a+b\text{arcsinh}(cx)) \\
&\quad + \frac{1}{11}c^6d^3x^{11}(a+b\text{arcsinh}(cx)) - \frac{(bcd^3)\int \frac{x^5(231+495c^2x^2+385c^4x^4+105c^6x^6)}{\sqrt{1+c^2x^2}}\,dx}{1155} \\
&= \frac{1}{5}d^3x^5(a+b\text{arcsinh}(cx)) + \frac{3}{7}c^2d^3x^7(a+b\text{arcsinh}(cx)) + \frac{1}{3}c^4d^3x^9(a+b\text{arcsinh}(cx)) \\
&\quad + \frac{1}{11}c^6d^3x^{11}(a+b\text{arcsinh}(cx)) - \frac{(bcd^3)\,\text{Subst}\left(\int \frac{x^2(231+495c^2x+385c^4x^2+105c^6x^3)}{\sqrt{1+c^2x}}\,dx, x, x^2\right)}{2310} \\
&= \frac{1}{5}d^3x^5(a+b\text{arcsinh}(cx)) + \frac{3}{7}c^2d^3x^7(a+b\text{arcsinh}(cx)) \\
&\quad + \frac{1}{3}c^4d^3x^9(a+b\text{arcsinh}(cx)) + \frac{1}{11}c^6d^3x^{11}(a+b\text{arcsinh}(cx)) \\
&\quad - \frac{(bcd^3)\,\text{Subst}\left(\int \left(\frac{16}{c^4\sqrt{1+c^2x}} + \frac{8\sqrt{1+c^2x}}{c^4} + \frac{6(1+c^2x)^{3/2}}{c^4} + \frac{5(1+c^2x)^{5/2}}{c^4} - \frac{140(1+c^2x)^{7/2}}{c^4} + \frac{105(1+c^2x)^{9/2}}{c^4}\right) d\right.}{2310}
\end{aligned}$$

$$= -\frac{16bd^3\sqrt{1+c^2x^2}}{1155c^5} - \frac{8bd^3(1+c^2x^2)^{3/2}}{3465c^5} - \frac{2bd^3(1+c^2x^2)^{5/2}}{1925c^5} - \frac{bd^3(1+c^2x^2)^{7/2}}{1617c^5} + \frac{4bd^3(1+c^2x^2)^{9/2}}{297c^5} - \frac{bd^3(1+c^2x^2)^{11/2}}{121c^5} + \frac{1}{5}d^3x^5(a+b\text{arcsinh}(cx)) + \frac{3}{7}c^2d^3x^7(a+b\text{arcsinh}(cx)) + \frac{1}{3}c^4d^3x^9(a+b\text{arcsinh}(cx)) + \frac{1}{11}c^6d^3x^{11}(a+b$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.63

$$\int x^4(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))\,dx = \frac{d^3(3465ac^5x^5(231+495c^2x^2+385c^4x^4+105c^6x^6) - b\sqrt{1+c^2x^2}(50488-25244c^2x^2+18933c^4x^4+11762}{4002075}$$

```
[In] Integrate[x^4*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^3*(3465*a*c^5*x^5*(231 + 495*c^2*x^2 + 385*c^4*x^4 + 105*c^6*x^6) - b*Sq
rt[1 + c^2*x^2]*(50488 - 25244*c^2*x^2 + 18933*c^4*x^4 + 117625*c^6*x^6 + 1
11475*c^8*x^8 + 33075*c^10*x^10) + 3465*b*c^5*x^5*(231 + 495*c^2*x^2 + 385*
c^4*x^4 + 105*c^6*x^6)*ArcSinh[c*x]))/(4002075*c^5)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.89

method	result
parts	$d^3a\left(\frac{1}{11}c^6x^{11}+\frac{1}{3}c^4x^9+\frac{3}{7}c^2x^7+\frac{1}{5}x^5\right)+\frac{d^3b\left(\frac{\text{arcsinh}(cx)c^{11}x^{11}}{11}+\frac{\text{arcsinh}(cx)c^9x^9}{3}+\frac{3\,\text{arcsinh}(cx)c^7x^7}{7}+\text{arcsinh}\right.}{}$
derivativedivides	$\frac{d^3a\left(\frac{1}{11}c^{11}x^{11}+\frac{1}{3}c^9x^9+\frac{3}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^3b\left(\frac{\text{arcsinh}(cx)c^{11}x^{11}}{11}+\frac{\text{arcsinh}(cx)c^9x^9}{3}+\frac{3\,\text{arcsinh}(cx)c^7x^7}{7}+\frac{\text{arcsinh}(cx)c^5x^5}{5}-91c\right.}{c^5}$
default	$\frac{d^3a\left(\frac{1}{11}c^{11}x^{11}+\frac{1}{3}c^9x^9+\frac{3}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^3b\left(\frac{\text{arcsinh}(cx)c^{11}x^{11}}{11}+\frac{\text{arcsinh}(cx)c^9x^9}{3}+\frac{3\,\text{arcsinh}(cx)c^7x^7}{7}+\frac{\text{arcsinh}(cx)c^5x^5}{5}-91c\right.}{c^5}$

```
[In] int(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] d^3*a*(1/11*c^6*x^11+1/3*c^4*x^9+3/7*c^2*x^7+1/5*x^5)+d^3*b/c^5*(1/11*arcsi
nh(c*x)*c^11*x^11+1/3*arcsinh(c*x)*c^9*x^9+3/7*arcsinh(c*x)*c^7*x^7+1/5*arc
sinh(c*x)*c^5*x^5-91/3267*c^8*x^8*(c^2*x^2+1)^(1/2)-4705/160083*c^6*x^6*(c^
2*x^2+1)^(1/2)-6311/1334025*c^4*x^4*(c^2*x^2+1)^(1/2)+25244/4002075*c^2*x^2
*(c^2*x^2+1)^(1/2)-50488/4002075*(c^2*x^2+1)^(1/2)-1/121*c^10*x^10*(c^2*x^2
+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.89

$$\int x^4(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{363825\,ac^{11}d^3x^{11}+1334025\,ac^9d^3x^9+1715175\,ac^7d^3x^7+800415\,ac^5d^3x^5+3465\,(105\,bc^{11}d^3x^{11}+385\,bc^9}{}$$

```
[In] integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] 1/4002075*(363825*a*c^11*d^3*x^11 + 1334025*a*c^9*d^3*x^9 + 1715175*a*c^7*d
^3*x^7 + 800415*a*c^5*d^3*x^5 + 3465*(105*b*c^11*d^3*x^11 + 385*b*c^9*d^3*x
^9 + 495*b*c^7*d^3*x^7 + 231*b*c^5*d^3*x^5)*log(c*x + sqrt(c^2*x^2 + 1)) -
(33075*b*c^10*d^3*x^10 + 111475*b*c^8*d^3*x^8 + 117625*b*c^6*d^3*x^6 + 1893
3*b*c^4*d^3*x^4 - 25244*b*c^2*d^3*x^2 + 50488*b*d^3)*sqrt(c^2*x^2 + 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.28

$$\int x^4(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))\,dx$$
$$=\begin{cases}\frac{ac^6d^3x^{11}}{11}+\frac{ac^4d^3x^9}{3}+\frac{3ac^2d^3x^7}{7}+\frac{ad^3x^5}{5}+\frac{bc^6d^3x^{11}\operatorname{asinh}(cx)}{11}-\frac{bc^5d^3x^{10}\sqrt{c^2x^2+1}}{121}+\frac{bc^4d^3x^9\operatorname{asinh}(cx)}{3}-\frac{91bc^3d^3x^8\sqrt{c^2x^2+1}}{3267} \\ \frac{ad^3x^5}{5}\end{cases}$$

```
[In] integrate(x**4*(c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
[Out] Piecewise((a*c**6*d**3*x**11/11 + a*c**4*d**3*x**9/3 + 3*a*c**2*d**3*x**7/7
 + a*d**3*x**5/5 + b*c**6*d**3*x**11*asinh(c*x)/11 - b*c**5*d**3*x**10*sqrt
(c**2*x**2 + 1)/121 + b*c**4*d**3*x**9*asinh(c*x)/3 - 91*b*c**3*d**3*x**8*s
qrt(c**2*x**2 + 1)/3267 + 3*b*c**2*d**3*x**7*asinh(c*x)/7 - 4705*b*c*d**3*x
**6*sqrt(c**2*x**2 + 1)/160083 + b*d**3*x**5*asinh(c*x)/5 - 6311*b*d**3*x**
4*sqrt(c**2*x**2 + 1)/(1334025*c) + 25244*b*d**3*x**2*sqrt(c**2*x**2 + 1)/(
4002075*c**3) - 50488*b*d**3*sqrt(c**2*x**2 + 1)/(4002075*c**5), Ne(c, 0)),
 (a*d**3*x**5/5, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. 2(194) = 388.

Time = 0.20 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.06

$$\int x^4\left(d+c^2dx^2\right)^3(a+b\operatorname{arcsinh}(cx))\,dx = \frac{1}{11}\,ac^6d^3x^{11}+\frac{1}{3}\,ac^4d^3x^9+\frac{3}{7}\,ac^2d^3x^7 + \frac{1}{7623}\left(693\,x^{11}\operatorname{arsinh}(cx)-\left(\frac{63\sqrt{c^2x^2+1}x^{10}}{c^2}-\frac{70\sqrt{c^2x^2+1}x^8}{c^4}+\frac{80\sqrt{c^2x^2+1}x^6}{c^6}-\frac{96\sqrt{c^2x^2+1}x^4}{c^8}\right.\right.$$
$$+\frac{1}{945}\left(315\,x^9\operatorname{arsinh}(cx)-\left(\frac{35\sqrt{c^2x^2+1}x^8}{c^2}-\frac{40\sqrt{c^2x^2+1}x^6}{c^4}+\frac{48\sqrt{c^2x^2+1}x^4}{c^6}-\frac{64\sqrt{c^2x^2+1}x^2}{c^8}+\right.\right.$$
$$+\frac{1}{5}\,ad^3x^5$$
$$+\frac{3}{245}\left(35\,x^7\operatorname{arsinh}(cx)-\left(\frac{5\sqrt{c^2x^2+1}x^6}{c^2}-\frac{6\sqrt{c^2x^2+1}x^4}{c^4}+\frac{8\sqrt{c^2x^2+1}x^2}{c^6}-\frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\right)bc^2d^3$$
$$+\frac{1}{75}\left(15\,x^5\operatorname{arsinh}(cx)-\left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2}-\frac{4\sqrt{c^2x^2+1}x^2}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)bd^3$$

```
[In] integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/11*a*c^6*d^3*x^11 + 1/3*a*c^4*d^3*x^9 + 3/7*a*c^2*d^3*x^7 + 1/7623*(693*x
^11*arcsinh(c*x) - (63*sqrt(c^2*x^2 + 1)*x^10/c^2 - 70*sqrt(c^2*x^2 + 1)*x^
8/c^4 + 80*sqrt(c^2*x^2 + 1)*x^6/c^6 - 96*sqrt(c^2*x^2 + 1)*x^4/c^8 + 128*s
qrt(c^2*x^2 + 1)*x^2/c^10 - 256*sqrt(c^2*x^2 + 1)/c^12)*c)*b*c^6*d^3 + 1/94
5*(315*x^9*arcsinh(c*x) - (35*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 +
 1)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 +
 128*sqrt(c^2*x^2 + 1)/c^10)*c)*b*c^4*d^3 + 1/5*a*d^3*x^5 + 3/245*(35*x^7*a
rcsinh(c*x) - (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 +
8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^2*d^3 + 1/75
*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*
x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*d^3
```

Giac [F(-2)]

Exception generated.

$$\int x^4\left(d+c^2dx^2\right)^3(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (d + c^2dx^2)^3 (a + b\text{arcsinh}(cx))\, dx = \int x^4 (a + b\,\text{asinh}(c\,x))\, \left(d\,c^2\,x^2 + d\right)^3 dx$$

```
[In] int(x^4*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3,x)
```

```
[Out] int(x^4*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3, x)
```

3.20 $\int x^3(d+c^2dx^2)^3\,(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 199

$$\int x^3\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))\,dx=\frac{49bd^3x\sqrt{1+c^2x^2}}{5120c^3}+\frac{49bd^3x(1+c^2x^2)^{3/2}}{7680c^3}+\frac{49bd^3x(1+c^2x^2)^{5/2}}{9600c^3}+\frac{7bd^3x(1+c^2x^2)^{7/2}}{1600c^3}-\frac{bd^3x(1+c^2x^2)^{9/2}}{100c^3}+\frac{49bd^3\text{arcsinh}(cx)}{5120c^4}-\frac{d^3(1+c^2x^2)^4\,(a+b\text{arcsinh}(cx))}{8c^4}+\frac{d^3(1+c^2x^2)^5\,(a+b\text{arcsinh}(cx))}{10c^4}$$

```
[Out] 49/7680*b*d^3*x*(c^2*x^2+1)^(3/2)/c^3+49/9600*b*d^3*x*(c^2*x^2+1)^(5/2)/c^3
+7/1600*b*d^3*x*(c^2*x^2+1)^(7/2)/c^3-1/100*b*d^3*x*(c^2*x^2+1)^(9/2)/c^3+4
9/5120*b*d^3*arcsinh(c*x)/c^4-1/8*d^3*(c^2*x^2+1)^4*(a+b*arcsinh(c*x))/c^4+
1/10*d^3*(c^2*x^2+1)^5*(a+b*arcsinh(c*x))/c^4+49/5120*b*d^3*x*(c^2*x^2+1)^(
1/2)/c^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used

= {272, 45, 5803, 12, 396, 201, 221}

$$\int x^3 (d + c^2 d x^2)^3 (a + b\text{arcsinh}(cx))\,dx = \frac{d^3 (c^2x^2+1)^5 (a + b\text{arcsinh}(cx))}{10c^4} - \frac{d^3 (c^2x^2+1)^4 (a + b\text{arcsinh}(cx))}{8c^4} + \frac{49bd^3\text{arcsinh}(cx)}{5120c^4} - \frac{bd^3x(c^2x^2+1)^{9/2}}{100c^3} + \frac{7bd^3x(c^2x^2+1)^{7/2}}{1600c^3} + \frac{49bd^3x(c^2x^2+1)^{5/2}}{9600c^3} + \frac{49bd^3x(c^2x^2+1)^{3/2}}{7680c^3} + \frac{49bd^3x\sqrt{c^2x^2+1}}{5120c^3}$$

```
[In] Int[x^3*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (49*b*d^3*x*Sqrt[1 + c^2*x^2])/(5120*c^3) + (49*b*d^3*x*(1 + c^2*x^2)^(3/2)
)/(7680*c^3) + (49*b*d^3*x*(1 + c^2*x^2)^(5/2))/(9600*c^3) + (7*b*d^3*x*(1
+ c^2*x^2)^(7/2))/(1600*c^3) - (b*d^3*x*(1 + c^2*x^2)^(9/2))/(100*c^3) + (4
9*b*d^3*ArcSinh[c*x])/(5120*c^4) - (d^3*(1 + c^2*x^2)^4*(a + b*ArcSinh[c*x]
))/(8*c^4) + (d^3*(1 + c^2*x^2)^5*(a + b*ArcSinh[c*x]))/(10*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^3(1+c^2x^2)^4\,(a+b\text{arcsinh}(cx))}{8c^4}+\frac{d^3(1+c^2x^2)^5\,(a+b\text{arcsinh}(cx))}{10c^4} \\
&\quad -(bc)\int\frac{d^3(1+c^2x^2)^{7/2}\,(-1+4c^2x^2)}{40c^4}\,dx \\
&= -\frac{d^3(1+c^2x^2)^4\,(a+b\text{arcsinh}(cx))}{8c^4}+\frac{d^3(1+c^2x^2)^5\,(a+b\text{arcsinh}(cx))}{10c^4} \\
&\quad -\frac{(bd^3)\int(1+c^2x^2)^{7/2}\,(-1+4c^2x^2)\,dx}{40c^3} \\
&= -\frac{bd^3x(1+c^2x^2)^{9/2}}{100c^3}-\frac{d^3(1+c^2x^2)^4\,(a+b\text{arcsinh}(cx))}{8c^4} \\
&\quad +\frac{d^3(1+c^2x^2)^5\,(a+b\text{arcsinh}(cx))}{10c^4}+\frac{(7bd^3)\int(1+c^2x^2)^{7/2}\,dx}{200c^3} \\
&= \frac{7bd^3x(1+c^2x^2)^{7/2}}{1600c^3}-\frac{bd^3x(1+c^2x^2)^{9/2}}{100c^3}-\frac{d^3(1+c^2x^2)^4\,(a+b\text{arcsinh}(cx))}{8c^4} \\
&\quad +\frac{d^3(1+c^2x^2)^5\,(a+b\text{arcsinh}(cx))}{10c^4}+\frac{(49bd^3)\int(1+c^2x^2)^{5/2}\,dx}{1600c^3}
\end{aligned}$$

$$= \frac{49bd^3x(1+c^2x^2)^{5/2}}{9600c^3} + \frac{7bd^3x(1+c^2x^2)^{7/2}}{1600c^3} - \frac{bd^3x(1+c^2x^2)^{9/2}}{100c^3} - \frac{d^3(1+c^2x^2)^4(a+b\operatorname{arcsinh}(cx))}{8c^4} + \frac{d^3(1+c^2x^2)^5(a+b\operatorname{arcsinh}(cx))}{10c^4} + \frac{(49bd^3)\int(1+c^2x^2)^{3/2}\,dx}{1920c^3}$$

$$= \frac{49bd^3x(1+c^2x^2)^{3/2}}{7680c^3} + \frac{49bd^3x(1+c^2x^2)^{5/2}}{9600c^3} + \frac{7bd^3x(1+c^2x^2)^{7/2}}{1600c^3} - \frac{bd^3x(1+c^2x^2)^{9/2}}{100c^3} - \frac{d^3(1+c^2x^2)^4(a+b\operatorname{arcsinh}(cx))}{8c^4} + \frac{d^3(1+c^2x^2)^5(a+b\operatorname{arcsinh}(cx))}{10c^4} + \frac{(49bd^3)\int\sqrt{1+c^2x^2}\,dx}{2560c^3}$$

$$= \frac{49bd^3x\sqrt{1+c^2x^2}}{5120c^3} + \frac{49bd^3x(1+c^2x^2)^{3/2}}{7680c^3} + \frac{49bd^3x(1+c^2x^2)^{5/2}}{9600c^3} + \frac{7bd^3x(1+c^2x^2)^{7/2}}{1600c^3} - \frac{bd^3x(1+c^2x^2)^{9/2}}{100c^3} - \frac{d^3(1+c^2x^2)^4(a+b\operatorname{arcsinh}(cx))}{8c^4} + \frac{d^3(1+c^2x^2)^5(a+b\operatorname{arcsinh}(cx))}{10c^4} + \frac{(49bd^3)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{5120c^3}$$

$$= \frac{49bd^3x\sqrt{1+c^2x^2}}{5120c^3} + \frac{49bd^3x(1+c^2x^2)^{3/2}}{7680c^3} + \frac{49bd^3x(1+c^2x^2)^{5/2}}{9600c^3} + \frac{7bd^3x(1+c^2x^2)^{7/2}}{1600c^3} - \frac{bd^3x(1+c^2x^2)^{9/2}}{100c^3} + \frac{49bd^3\operatorname{arcsinh}(cx)}{5120c^4} - \frac{d^3(1+c^2x^2)^4(a+b\operatorname{arcsinh}(cx))}{8c^4} + \frac{d^3(1+c^2x^2)^5(a+b\operatorname{arcsinh}(cx))}{10c^4}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.70

$$\int x^3(d+c^2dx^2)^3(a+b\operatorname{arcsinh}(cx))\,dx = \frac{d^3\left(1920ac^4x^4(10+20c^2x^2+15c^4x^4+4c^6x^6)-bcx\sqrt{1+c^2x^2}(-1185+790c^2x^2+3208c^4x^4+2736c^6x^6 -\right.}{76800c^4}$$

```
[In] Integrate[x^3*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^3*(1920*a*c^4*x^4*(10 + 20*c^2*x^2 + 15*c^4*x^4 + 4*c^6*x^6) - b*c*x*Sqr
t[1 + c^2*x^2]*(-1185 + 790*c^2*x^2 + 3208*c^4*x^4 + 2736*c^6*x^6 + 768*c^8
*x^8) + 15*b*(-79 + 1280*c^4*x^4 + 2560*c^6*x^6 + 1920*c^8*x^8 + 512*c^10*x
^10)*ArcSinh[c*x]))/(76800*c^4)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.96

method	result
parts	$d^3a\left(\frac{1}{10}c^6x^{10}+\frac{3}{8}c^4x^8+\frac{1}{2}c^2x^6+\frac{1}{4}x^4\right)+\frac{d^3b\left(\frac{\operatorname{arcsinh}(cx)c^{10}x^{10}}{10}+\frac{3\operatorname{arcsinh}(cx)c^8x^8}{8}+\frac{\operatorname{arcsinh}(cx)c^6x^6}{2}+\operatorname{arcsinh}\right.}{}$
derivativedivides	$\frac{d^3a\left(\frac{1}{10}c^{10}x^{10}+\frac{3}{8}c^8x^8+\frac{1}{2}c^6x^6+\frac{1}{4}c^4x^4\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^{10}x^{10}}{10}+\frac{3\operatorname{arcsinh}(cx)c^8x^8}{8}+\frac{\operatorname{arcsinh}(cx)c^6x^6}{2}+\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}-c^9\right.}{c^4}$
default	$\frac{d^3a\left(\frac{1}{10}c^{10}x^{10}+\frac{3}{8}c^8x^8+\frac{1}{2}c^6x^6+\frac{1}{4}c^4x^4\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^{10}x^{10}}{10}+\frac{3\operatorname{arcsinh}(cx)c^8x^8}{8}+\frac{\operatorname{arcsinh}(cx)c^6x^6}{2}+\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}-c^9\right.}{c^4}$

```
[In] int(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] d^3*a*(1/10*c^6*x^10+3/8*c^4*x^8+1/2*c^2*x^6+1/4*x^4)+d^3*b/c^4*(1/10*arcsi
nh(c*x)*c^10*x^10+3/8*arcsinh(c*x)*c^8*x^8+1/2*arcsinh(c*x)*c^6*x^6+1/4*arc
sinh(c*x)*c^4*x^4-1/100*c^9*x^9*(c^2*x^2+1)^(1/2)-57/1600*c^7*x^7*(c^2*x^2+
1)^(1/2)-401/9600*c^5*x^5*(c^2*x^2+1)^(1/2)-79/7680*c^3*x^3*(c^2*x^2+1)^(1/
2)+79/5120*c*x*(c^2*x^2+1)^(1/2)-79/5120*arcsinh(c*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.99

$$\int x^3\left(d+c^2dx^2\right)^3(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\frac{7680\,ac^{10}d^3x^{10}+28800\,ac^8d^3x^8+38400\,ac^6d^3x^6+19200\,ac^4d^3x^4+15\,(512\,bc^{10}d^3x^{10}+1920\,bc^8d^3x^8+256}{}$$

```
[In] integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/76800*(7680*a*c^10*d^3*x^10 + 28800*a*c^8*d^3*x^8 + 38400*a*c^6*d^3*x^6 +
 19200*a*c^4*d^3*x^4 + 15*(512*b*c^10*d^3*x^10 + 1920*b*c^8*d^3*x^8 + 2560*
b*c^6*d^3*x^6 + 1280*b*c^4*d^3*x^4 - 79*b*d^3)*log(c*x + sqrt(c^2*x^2 + 1))
 - (768*b*c^9*d^3*x^9 + 2736*b*c^7*d^3*x^7 + 3208*b*c^5*d^3*x^5 + 790*b*c^3
*d^3*x^3 - 1185*b*c*d^3*x)*sqrt(c^2*x^2 + 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.41

$$\int x^3(d+c^2dx^2)^3(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\begin{cases}\frac{ac^6d^3x^{10}}{10}+\frac{3ac^4d^3x^8}{8}+\frac{ac^2d^3x^6}{2}+\frac{ad^3x^4}{4}+\frac{bc^6d^3x^{10}\operatorname{asinh}(cx)}{10}-\frac{bc^5d^3x^9\sqrt{c^2x^2+1}}{100}+\frac{3bc^4d^3x^8\operatorname{asinh}(cx)}{8}-\frac{57bc^3d^3x^7\sqrt{c^2x^2+1}}{1600} \\ \frac{ad^3x^4}{4}\end{cases}$$

```
[In] integrate(x**3*(c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**6*d**3*x**10/10 + 3*a*c**4*d**3*x**8/8 + a*c**2*d**3*x**6/2
 + a*d**3*x**4/4 + b*c**6*d**3*x**10*asinh(c*x)/10 - b*c**5*d**3*x**9*sqrt(
c**2*x**2 + 1)/100 + 3*b*c**4*d**3*x**8*asinh(c*x)/8 - 57*b*c**3*d**3*x**7*
sqrt(c**2*x**2 + 1)/1600 + b*c**2*d**3*x**6*asinh(c*x)/2 - 401*b*c*d**3*x**
5*sqrt(c**2*x**2 + 1)/9600 + b*d**3*x**4*asinh(c*x)/4 - 79*b*d**3*x**3*sqrt
(c**2*x**2 + 1)/(7680*c) + 79*b*d**3*x*sqrt(c**2*x**2 + 1)/(5120*c**3) - 79
*b*d**3*asinh(c*x)/(5120*c**4), Ne(c, 0)), (a*d**3*x**4/4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(173) = 346$.

Time = 0.20 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.16

$$\int x^3(d+c^2dx^2)^3(a+b\operatorname{arcsinh}(cx))\,dx = \frac{1}{10}ac^6d^3x^{10}+\frac{3}{8}ac^4d^3x^8+\frac{1}{2}ac^2d^3x^6$$
$$+\frac{1}{12800}\left(1280\,x^{10}\operatorname{arsinh}(cx)-\left(\frac{128\sqrt{c^2x^2+1}x^9}{c^2}-\frac{144\sqrt{c^2x^2+1}x^7}{c^4}+\frac{168\sqrt{c^2x^2+1}x^5}{c^6}-\frac{210\sqrt{c^2x^2}}{c^8}\right.\right.$$
$$+\frac{1}{1024}\left(384\,x^8\operatorname{arsinh}(cx)-\left(\frac{48\sqrt{c^2x^2+1}x^7}{c^2}-\frac{56\sqrt{c^2x^2+1}x^5}{c^4}+\frac{70\sqrt{c^2x^2+1}x^3}{c^6}-\frac{105\sqrt{c^2x^2+1}x}{c^8}\right.\right.$$
$$+\frac{1}{4}ad^3x^4$$
$$+\frac{1}{96}\left(48\,x^6\operatorname{arsinh}(cx)-\left(\frac{8\sqrt{c^2x^2+1}x^5}{c^2}-\frac{10\sqrt{c^2x^2+1}x^3}{c^4}+\frac{15\sqrt{c^2x^2+1}x}{c^6}-\frac{15\operatorname{arsinh}(cx)}{c^7}\right)c\right)bc^2$$
$$+\frac{1}{32}\left(8\,x^4\operatorname{arsinh}(cx)-\left(\frac{2\sqrt{c^2x^2+1}x^3}{c^2}-\frac{3\sqrt{c^2x^2+1}x}{c^4}+\frac{3\operatorname{arsinh}(cx)}{c^5}\right)c\right)bd^3$$

```
[In] integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/10*a*c^6*d^3*x^10 + 3/8*a*c^4*d^3*x^8 + 1/2*a*c^2*d^3*x^6 + 1/12800*(1280
*x^10*arcsinh(c*x) - (128*sqrt(c^2*x^2 + 1)*x^9/c^2 - 144*sqrt(c^2*x^2 + 1)
```

```
*x^7/c^4 + 168*sqrt(c^2*x^2 + 1)*x^5/c^6 - 210*sqrt(c^2*x^2 + 1)*x^3/c^8 +
315*sqrt(c^2*x^2 + 1)*x/c^10 - 315*arcsinh(c*x)/c^11)*c)*b*c^6*d^3 + 1/1024
*(384*x^8*arcsinh(c*x) - (48*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2*x^2 +
1)*x^5/c^4 + 70*sqrt(c^2*x^2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/c^8 + 1
05*arcsinh(c*x)/c^9)*c)*b*c^4*d^3 + 1/4*a*d^3*x^4 + 1/96*(48*x^6*arcsinh(c*
x) - (8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(
c^2*x^2 + 1)*x/c^6 - 15*arcsinh(c*x)/c^7)*c)*b*c^2*d^3 + 1/32*(8*x^4*arcsin
h(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsi
nh(c*x)/c^5)*c)*b*d^3
```

Giac [F(-2)]

Exception generated.

$$\int x^3 (d + c^2 d x^2)^3 (a + b \operatorname{arcsinh}(cx)) \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (d + c^2 d x^2)^3 (a + b \operatorname{arcsinh}(cx)) \, dx = \int x^3 (a + b \operatorname{asinh}(c x)) \left(d c^2 x^2 + d\right)^3 dx$$

```
[In] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3,x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3, x)
```

3.21 $\int x^2(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 202

$$\int x^2(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))\,dx = \frac{16bd^3\sqrt{1+c^2x^2}}{315c^3}+\frac{8bd^3(1+c^2x^2)^{3/2}}{945c^3} + \frac{2bd^3(1+c^2x^2)^{5/2}}{525c^3}+\frac{bd^3(1+c^2x^2)^{7/2}}{441c^3}-\frac{bd^3(1+c^2x^2)^{9/2}}{81c^3} + \frac{1}{3}d^3x^3(a+b\text{arcsinh}(cx))+\frac{3}{5}c^2d^3x^5(a+b\text{arcsinh}(cx))+\frac{3}{7}c^4d^3x^7(a+b\text{arcsinh}(cx))+\frac{1}{9}c^6d^3x^9(a+b\text{arcsinh}($$

```
[Out] 8/945*b*d^3*(c^2*x^2+1)^(3/2)/c^3+2/525*b*d^3*(c^2*x^2+1)^(5/2)/c^3+1/441*b
*d^3*(c^2*x^2+1)^(7/2)/c^3-1/81*b*d^3*(c^2*x^2+1)^(9/2)/c^3+1/3*d^3*x^3*(a+
b*arcsinh(c*x))+3/5*c^2*d^3*x^5*(a+b*arcsinh(c*x))+3/7*c^4*d^3*x^7*(a+b*arc
sinh(c*x))+1/9*c^6*d^3*x^9*(a+b*arcsinh(c*x))+16/315*b*d^3*(c^2*x^2+1)^(1/2
)/c^3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {276, 5803, 12, 1813, 1634}

$$\int x^2\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))\,dx=\frac{1}{9}c^6d^3x^9(a+b\text{arcsinh}(cx))+\frac{3}{7}c^4d^3x^7(a+b\text{arcsinh}(cx))+\frac{3}{5}c^2d^3x^5(a+b\text{arcsinh}(cx))+\frac{1}{3}d^3x^3(a+b\text{arcsinh}(cx))-\frac{bd^3\left(c^2x^2+1\right)^{9/2}}{81c^3}+\frac{bd^3\left(c^2x^2+1\right)^{7/2}}{441c^3}+\frac{2bd^3\left(c^2x^2+1\right)^{5/2}}{525c^3}+\frac{8bd^3\left(c^2x^2+1\right)^{3/2}}{945c^3}+\frac{16bd^3\sqrt{c^2x^2+1}}{315c^3}$$

```
[In] Int[x^2*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (16*b*d^3*Sqrt[1 + c^2*x^2])/(315*c^3) + (8*b*d^3*(1 + c^2*x^2)^(3/2))/(945
*c^3) + (2*b*d^3*(1 + c^2*x^2)^(5/2))/(525*c^3) + (b*d^3*(1 + c^2*x^2)^(7/2
))/(441*c^3) - (b*d^3*(1 + c^2*x^2)^(9/2))/(81*c^3) + (d^3*x^3*(a + b*ArcSi
nh[c*x]))/3 + (3*c^2*d^3*x^5*(a + b*ArcSinh[c*x]))/5 + (3*c^4*d^3*x^7*(a +
b*ArcSinh[c*x]))/7 + (c^6*d^3*x^9*(a + b*ArcSinh[c*x]))/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}d^3x^3(a+b\text{arcsinh}(cx))+\frac{3}{5}c^2d^3x^5(a+b\text{arcsinh}(cx))+\frac{3}{7}c^4d^3x^7(a+b\text{arcsinh}(cx)) \\
&\quad+\frac{1}{9}c^6d^3x^9(a+b\text{arcsinh}(cx))-(bc)\int\frac{d^3x^3(105+189c^2x^2+135c^4x^4+35c^6x^6)}{315\sqrt{1+c^2x^2}}\,dx \\
&= \frac{1}{3}d^3x^3(a+b\text{arcsinh}(cx))+\frac{3}{5}c^2d^3x^5(a+b\text{arcsinh}(cx))+\frac{3}{7}c^4d^3x^7(a+b\text{arcsinh}(cx)) \\
&\quad+\frac{1}{9}c^6d^3x^9(a+b\text{arcsinh}(cx))-\frac{1}{315}\left(bcd^3\right)\int\frac{x^3(105+189c^2x^2+135c^4x^4+35c^6x^6)}{\sqrt{1+c^2x^2}}\,dx \\
&= \frac{1}{3}d^3x^3(a+b\text{arcsinh}(cx))+\frac{3}{5}c^2d^3x^5(a+b\text{arcsinh}(cx)) \\
&\quad+\frac{3}{7}c^4d^3x^7(a+b\text{arcsinh}(cx))+\frac{1}{9}c^6d^3x^9(a+b\text{arcsinh}(cx)) \\
&\quad-\frac{1}{630}\left(bcd^3\right)\text{Subst}\left(\int\frac{x(105+189c^2x+135c^4x^2+35c^6x^3)}{\sqrt{1+c^2x}}\,dx,x,x^2\right) \\
&= \frac{1}{3}d^3x^3(a+b\text{arcsinh}(cx))+\frac{3}{5}c^2d^3x^5(a+b\text{arcsinh}(cx))+\frac{3}{7}c^4d^3x^7(a+b\text{arcsinh}(cx)) \\
&\quad+\frac{1}{9}c^6d^3x^9(a+b\text{arcsinh}(cx))-\frac{1}{630}\left(bcd^3\right)\text{Subst}\left(\int\left(-\frac{16}{c^2\sqrt{1+c^2x}}-\frac{8\sqrt{1+c^2x}}{c^2}\right.\right. \\
&\qquad\left.\left.-\frac{6(1+c^2x)^{3/2}}{c^2}-\frac{5(1+c^2x)^{5/2}}{c^2}+\frac{35(1+c^2x)^{7/2}}{c^2}\right)dx,x,x^2\right) \\
&= \frac{16bd^3\sqrt{1+c^2x^2}}{315c^3}+\frac{8bd^3(1+c^2x^2)^{3/2}}{945c^3}+\frac{2bd^3(1+c^2x^2)^{5/2}}{525c^3} \\
&\quad+\frac{bd^3(1+c^2x^2)^{7/2}}{441c^3}-\frac{bd^3(1+c^2x^2)^{9/2}}{81c^3} \\
&\quad+\frac{1}{3}d^3x^3(a+b\text{arcsinh}(cx))+\frac{3}{5}c^2d^3x^5(a+b\text{arcsinh}(cx))+\frac{3}{7}c^4d^3x^7(a+b\text{arcsinh}(cx))+\frac{1}{9}c^6d^3x^9(a+b
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.67

$$\int x^2(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{d^3\left(315ac^3x^3(105+189c^2x^2+135c^4x^4+35c^6x^6)-b\sqrt{1+c^2x^2}(-5258+2629c^2x^2+6297c^4x^4+4675c^6x^6\right.}{99225c^3}$$

```
[In] Integrate[x^2*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^3*(315*a*c^3*x^3*(105 + 189*c^2*x^2 + 135*c^4*x^4 + 35*c^6*x^6) - b*Sqrt
[1 + c^2*x^2]*(-5258 + 2629*c^2*x^2 + 6297*c^4*x^4 + 4675*c^6*x^6 + 1225*c^
8*x^8) + 315*b*c^3*x^3*(105 + 189*c^2*x^2 + 135*c^4*x^4 + 35*c^6*x^6)*ArcSi
nh[c*x]))/(99225*c^3)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.91

method	result
parts	$d^3a\left(\frac{1}{9}c^6x^9+\frac{3}{7}c^4x^7+\frac{3}{5}c^2x^5+\frac{1}{3}x^3\right)+\frac{d^3b\left(\frac{\text{arcsinh}(cx)c^9x^9}{9}+\frac{3\ \text{arcsinh}(cx)c^7x^7}{7}+\frac{3\ \text{arcsinh}(cx)c^5x^5}{5}+\frac{\text{arcsinh}(cx}{3}\right.}{}$
derivativedivides	$\frac{d^3a\left(\frac{1}{9}c^9x^9+\frac{3}{7}c^7x^7+\frac{3}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^3b\left(\frac{\text{arcsinh}(cx)c^9x^9}{9}+\frac{3\ \text{arcsinh}(cx)c^7x^7}{7}+\frac{3\ \text{arcsinh}(cx)c^5x^5}{5}+\frac{\text{arcsinh}(cx)c^3x^3}{3}-\frac{c^8x^8\sqrt{}}{}\right.}{c^3}$
default	$\frac{d^3a\left(\frac{1}{9}c^9x^9+\frac{3}{7}c^7x^7+\frac{3}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^3b\left(\frac{\text{arcsinh}(cx)c^9x^9}{9}+\frac{3\ \text{arcsinh}(cx)c^7x^7}{7}+\frac{3\ \text{arcsinh}(cx)c^5x^5}{5}+\frac{\text{arcsinh}(cx)c^3x^3}{3}-\frac{c^8x^8\sqrt{}}{}\right.}{c^3}$

```
[In] int(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] d^3*a*(1/9*c^6*x^9+3/7*c^4*x^7+3/5*c^2*x^5+1/3*x^3)+d^3*b/c^3*(1/9*arcsinh(
c*x)*c^9*x^9+3/7*arcsinh(c*x)*c^7*x^7+3/5*arcsinh(c*x)*c^5*x^5+1/3*arcsinh(
c*x)*c^3*x^3-1/81*c^8*x^8*(c^2*x^2+1)^(1/2)-187/3969*c^6*x^6*(c^2*x^2+1)^(1
/2)-2099/33075*c^4*x^4*(c^2*x^2+1)^(1/2)-2629/99225*c^2*x^2*(c^2*x^2+1)^(1/
2)+5258/99225*(c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.94

$$\int x^2(d+c^2dx^2)^3(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\frac{11025\,ac^9d^3x^9+42525\,ac^7d^3x^7+59535\,ac^5d^3x^5+33075\,ac^3d^3x^3+315\,(35\,bc^9d^3x^9+135\,bc^7d^3x^7+189\,b}{}$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/99225*(11025*a*c^9*d^3*x^9 + 42525*a*c^7*d^3*x^7 + 59535*a*c^5*d^3*x^5 +
33075*a*c^3*d^3*x^3 + 315*(35*b*c^9*d^3*x^9 + 135*b*c^7*d^3*x^7 + 189*b*c^5
*d^3*x^5 + 105*b*c^3*d^3*x^3)*log(c*x + sqrt(c^2*x^2 + 1)) - (1225*b*c^8*d^
3*x^8 + 4675*b*c^6*d^3*x^6 + 6297*b*c^4*d^3*x^4 + 2629*b*c^2*d^3*x^2 - 5258
*b*d^3)*sqrt(c^2*x^2 + 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.31

$$\int x^2(d+c^2dx^2)^3(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\begin{cases}\frac{ac^6d^3x^9}{9}+\frac{3ac^4d^3x^7}{7}+\frac{3ac^2d^3x^5}{5}+\frac{ad^3x^3}{3}+\frac{bc^6d^3x^9\operatorname{asinh}(cx)}{9}-\frac{bc^5d^3x^8\sqrt{c^2x^2+1}}{81}+\frac{3bc^4d^3x^7\operatorname{asinh}(cx)}{7}-\frac{187bc^3d^3x^6\sqrt{c^2x^2+}}{3969}\\ \frac{ad^3x^3}{3}\end{cases}$$

```
[In] integrate(x**2*(c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**6*d**3*x**9/9 + 3*a*c**4*d**3*x**7/7 + 3*a*c**2*d**3*x**5/5
 + a*d**3*x**3/3 + b*c**6*d**3*x**9*asinh(c*x)/9 - b*c**5*d**3*x**8*sqrt(c*
*2*x**2 + 1)/81 + 3*b*c**4*d**3*x**7*asinh(c*x)/7 - 187*b*c**3*d**3*x**6*sq
rt(c**2*x**2 + 1)/3969 + 3*b*c**2*d**3*x**5*asinh(c*x)/5 - 2099*b*c*d**3*x*
*4*sqrt(c**2*x**2 + 1)/33075 + b*d**3*x**3*asinh(c*x)/3 - 2629*b*d**3*x**2*
sqrt(c**2*x**2 + 1)/(99225*c) + 5258*b*d**3*sqrt(c**2*x**2 + 1)/(99225*c**3
), Ne(c, 0)), (a*d**3*x**3/3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. 2(174) = 348.

Time = 0.21 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.92

$$\int x^2(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))\,dx = \frac{1}{9}\,ac^6d^3x^9+\frac{3}{7}\,ac^4d^3x^7 + \frac{1}{2835}\left(315\,x^9\,\text{arsinh}\,(cx)-\left(\frac{35\,\sqrt{c^2x^2+1}x^8}{c^2}-\frac{40\,\sqrt{c^2x^2+1}x^6}{c^4}+\frac{48\,\sqrt{c^2x^2+1}x^4}{c^6}-\frac{64\,\sqrt{c^2x^2+1}x^2}{c^8}+\right.\right. + \frac{3}{5}\,ac^2d^3x^5 + \frac{3}{245}\left(35\,x^7\,\text{arsinh}\,(cx)-\left(\frac{5\,\sqrt{c^2x^2+1}x^6}{c^2}-\frac{6\,\sqrt{c^2x^2+1}x^4}{c^4}+\frac{8\,\sqrt{c^2x^2+1}x^2}{c^6}-\frac{16\,\sqrt{c^2x^2+1}}{c^8}\right)c\right)bc^4d^3 + \frac{1}{25}\left(15\,x^5\,\text{arsinh}\,(cx)-\left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2}-\frac{4\,\sqrt{c^2x^2+1}x^2}{c^4}+\frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\right)bc^2d^3 + \frac{1}{3}\,ad^3x^3+\frac{1}{9}\left(3\,x^3\,\text{arsinh}\,(cx)-c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)bd^3$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/9*a*c^6*d^3*x^9 + 3/7*a*c^4*d^3*x^7 + 1/2835*(315*x^9*arcsinh(c*x) - (35*
sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2
+ 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c
)*b*c^6*d^3 + 3/5*a*c^2*d^3*x^5 + 3/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*
x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^
6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^4*d^3 + 1/25*(15*x^5*arcsinh(c*x) - (3
*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 +
 1)/c^6)*c)*b*c^2*d^3 + 1/3*a*d^3*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c
^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*d^3
```

Giac [F(-2)]

Exception generated.

$$\int x^2(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (d + c^2 dx^2)^3 (a + b\text{arcsinh}(cx))\, dx = \int x^2 (a + b\,\text{asinh}(c\,x))\, \left(d\,c^2\,x^2 + d\right)^3\, dx$$

```
[In] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3,x)
[Out] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3, x)
```

3.22 $\int x(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 145

$$\int x\left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))\,dx=-\frac{35bd^3x\sqrt{1+c^2x^2}}{1024c}-\frac{35bd^3x(1+c^2x^2)^{3/2}}{1536c}-\frac{7bd^3x(1+c^2x^2)^{5/2}}{384c}-\frac{bd^3x(1+c^2x^2)^{7/2}}{64c}-\frac{35bd^3\text{arcsinh}(cx)}{1024c^2}+\frac{d^3(1+c^2x^2)^4(a+b\text{arcsinh}(cx))}{8c^2}$$

```
[Out] -35/1536*b*d^3*x*(c^2*x^2+1)^(3/2)/c-7/384*b*d^3*x*(c^2*x^2+1)^(5/2)/c-1/64
*b*d^3*x*(c^2*x^2+1)^(7/2)/c-35/1024*b*d^3*arcsinh(c*x)/c^2+1/8*d^3*(c^2*x^
2+1)^4*(a+b*arcsinh(c*x))/c^2-35/1024*b*d^3*x*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5798, 201, 221}

$$\int x\left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))\,dx=\frac{d^3(c^2x^2+1)^4(a+b\text{arcsinh}(cx))}{8c^2}-\frac{35bd^3\text{arcsinh}(cx)}{1024c^2}-\frac{bd^3x(c^2x^2+1)^{7/2}}{64c}-\frac{7bd^3x(c^2x^2+1)^{5/2}}{384c}-\frac{35bd^3x(c^2x^2+1)^{3/2}}{1536c}-\frac{35bd^3x\sqrt{c^2x^2+1}}{1024c}$$

```
[In] Int[x*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-35*b*d^3*x*Sqrt[1 + c^2*x^2])/(1024*c) - (35*b*d^3*x*(1 + c^2*x^2)^(3/2))
/(1536*c) - (7*b*d^3*x*(1 + c^2*x^2)^(5/2))/(384*c) - (b*d^3*x*(1 + c^2*x^2
)^(7/2))/(64*c) - (35*b*d^3*ArcSinh[c*x])/(1024*c^2) + (d^3*(1 + c^2*x^2)^4
*(a + b*ArcSinh[c*x]))/(8*c^2)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d^3(1+c^2x^2)^4\,(a+b\operatorname{arcsinh}(cx))}{8c^2}-\frac{(bd^3)\int (1+c^2x^2)^{7/2}\,dx}{8c}\\
&= -\frac{bd^3x(1+c^2x^2)^{7/2}}{64c}+\frac{d^3(1+c^2x^2)^4\,(a+b\operatorname{arcsinh}(cx))}{8c^2}-\frac{(7bd^3)\int (1+c^2x^2)^{5/2}\,dx}{64c}\\
&= -\frac{7bd^3x(1+c^2x^2)^{5/2}}{384c}-\frac{bd^3x(1+c^2x^2)^{7/2}}{64c}\\
&\quad+\frac{d^3(1+c^2x^2)^4\,(a+b\operatorname{arcsinh}(cx))}{8c^2}-\frac{(35bd^3)\int (1+c^2x^2)^{3/2}\,dx}{384c}\\
&= -\frac{35bd^3x(1+c^2x^2)^{3/2}}{1536c}-\frac{7bd^3x(1+c^2x^2)^{5/2}}{384c}-\frac{bd^3x(1+c^2x^2)^{7/2}}{64c}\\
&\quad+\frac{d^3(1+c^2x^2)^4\,(a+b\operatorname{arcsinh}(cx))}{8c^2}-\frac{(35bd^3)\int \sqrt{1+c^2x^2}\,dx}{512c}
\end{aligned}$$

$$= -\frac{35bd^3x\sqrt{1+c^2x^2}}{1024c} - \frac{35bd^3x(1+c^2x^2)^{3/2}}{1536c} - \frac{7bd^3x(1+c^2x^2)^{5/2}}{384c} - \frac{bd^3x(1+c^2x^2)^{7/2}}{64c} + \frac{d^3(1+c^2x^2)^4(a+b\text{arcsinh}(cx))}{8c^2} - \frac{(35bd^3)\int\frac{1}{\sqrt{1+c^2x^2}}dx}{1024c}$$

$$= -\frac{35bd^3x\sqrt{1+c^2x^2}}{1024c} - \frac{35bd^3x(1+c^2x^2)^{3/2}}{1536c} - \frac{7bd^3x(1+c^2x^2)^{5/2}}{384c} - \frac{bd^3x(1+c^2x^2)^{7/2}}{64c} - \frac{35bd^3\text{arcsinh}(cx)}{1024c^2} + \frac{d^3(1+c^2x^2)^4(a+b\text{arcsinh}(cx))}{8c^2}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.88

$$\int x(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))\,dx = \frac{d^3\left(cx\left(384acx(4+6c^2x^2+4c^4x^4+c^6x^6)-b\sqrt{1+c^2x^2}(279+326c^2x^2+200c^4x^4+48c^6x^6)\right)+3b(93+51\right.}{3072c^2}$$

```
[In] Integrate[x*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^3*(c*x*(384*a*c*x*(4 + 6*c^2*x^2 + 4*c^4*x^4 + c^6*x^6) - b*Sqrt[1 + c^2
*x^2]*(279 + 326*c^2*x^2 + 200*c^4*x^4 + 48*c^6*x^6)) + 3*b*(93 + 512*c^2*x
^2 + 768*c^4*x^4 + 512*c^6*x^6 + 128*c^8*x^8)*ArcSinh[c*x]))/(3072*c^2)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{\frac{d^3a(c^2x^2+1)^4}{8}+d^3b\left(\frac{\text{arcsinh}(cx)c^8x^8}{8}+\frac{\text{arcsinh}(cx)c^6x^6}{2}+\frac{3\,\text{arcsinh}(cx)c^4x^4}{4}+\frac{\text{arcsinh}(cx)c^2x^2}{2}+\frac{93\,\text{arcsinh}(cx)}{1024}-\frac{cx(c^2x^2+1)^{\frac{7}{2}}}{64}\right.}{c^2}$
default	$\frac{\frac{d^3a(c^2x^2+1)^4}{8}+d^3b\left(\frac{\text{arcsinh}(cx)c^8x^8}{8}+\frac{\text{arcsinh}(cx)c^6x^6}{2}+\frac{3\,\text{arcsinh}(cx)c^4x^4}{4}+\frac{\text{arcsinh}(cx)c^2x^2}{2}+\frac{93\,\text{arcsinh}(cx)}{1024}-\frac{cx(c^2x^2+1)^{\frac{7}{2}}}{64}\right.}{c^2}$
parts	$\frac{d^3a(c^2x^2+1)^4}{8c^2}+\frac{d^3b\left(\frac{\text{arcsinh}(cx)c^8x^8}{8}+\frac{\text{arcsinh}(cx)c^6x^6}{2}+\frac{3\,\text{arcsinh}(cx)c^4x^4}{4}+\frac{\text{arcsinh}(cx)c^2x^2}{2}+\frac{93\,\text{arcsinh}(cx)}{1024}-\frac{cx(c^2x^2+1)}{64}\right.}{c^2}$

```
[In] int(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/8*d^3*a*(c^2*x^2+1)^4+d^3*b*(1/8*arcsinh(c*x)*c^8*x^8+1/2*arcsinh(
c*x)*c^6*x^6+3/4*arcsinh(c*x)*c^4*x^4+1/2*arcsinh(c*x)*c^2*x^2+93/1024*arcs
```

```
inh(c*x)-1/64*c*x*(c^2*x^2+1)^(7/2)-7/384*c*x*(c^2*x^2+1)^(5/2)-35/1536*c*x
*(c^2*x^2+1)^(3/2)-35/1024*c*x*(c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.28

$$\int x(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))\,dx = \frac{384\,ac^8d^3x^8+1536\,ac^6d^3x^6+2304\,ac^4d^3x^4+1536\,ac^2d^3x^2+3\,(128\,bc^8d^3x^8+512\,bc^6d^3x^6+768\,bc^4d^3x^4}{3}$$

```
[In] integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3072*(384*a*c^8*d^3*x^8 + 1536*a*c^6*d^3*x^6 + 2304*a*c^4*d^3*x^4 + 1536*
a*c^2*d^3*x^2 + 3*(128*b*c^8*d^3*x^8 + 512*b*c^6*d^3*x^6 + 768*b*c^4*d^3*x^
4 + 512*b*c^2*d^3*x^2 + 93*b*d^3)*log(c*x + sqrt(c^2*x^2 + 1)) - (48*b*c^7*
d^3*x^7 + 200*b*c^5*d^3*x^5 + 326*b*c^3*d^3*x^3 + 279*b*c*d^3*x)*sqrt(c^2*x
^2 + 1))/c^2
```

Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.74

$$\int x(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))\,dx = \begin{cases} \frac{ac^6d^3x^8}{8}+\frac{ac^4d^3x^6}{2}+\frac{3ac^2d^3x^4}{4}+\frac{ad^3x^2}{2}+\frac{bc^6d^3x^8\operatorname{asinh}(cx)}{8}-\frac{bc^5d^3x^7\sqrt{c^2x^2+1}}{64}+\frac{bc^4d^3x^6\operatorname{asinh}(cx)}{2}-\frac{25bc^3d^3x^5\sqrt{c^2x^2+1}}{384}+ \\ \frac{ad^3x^2}{2} \end{cases}$$

```
[In] integrate(x*(c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**6*d**3*x**8/8 + a*c**4*d**3*x**6/2 + 3*a*c**2*d**3*x**4/4 +
 a*d**3*x**2/2 + b*c**6*d**3*x**8*asinh(c*x)/8 - b*c**5*d**3*x**7*sqrt(c**2
*x**2 + 1)/64 + b*c**4*d**3*x**6*asinh(c*x)/2 - 25*b*c**3*d**3*x**5*sqrt(c*
*2*x**2 + 1)/384 + 3*b*c**2*d**3*x**4*asinh(c*x)/4 - 163*b*c*d**3*x**3*sqrt
(c**2*x**2 + 1)/1536 + b*d**3*x**2*asinh(c*x)/2 - 93*b*d**3*x*sqrt(c**2*x**
2 + 1)/(1024*c) + 93*b*d**3*asinh(c*x)/(1024*c**2), Ne(c, 0)), (a*d**3*x**2
/2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(125) = 250.

Time = 0.19 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.43

$$\int x(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))\,dx = \frac{1}{8}ac^6d^3x^8 + \frac{1}{2}ac^4d^3x^6 + \frac{1}{3072}\left(384\,x^8\,\text{arsinh}\,(cx) - \left(\frac{48\,\sqrt{c^2x^2+1}x^7}{c^2} - \frac{56\,\sqrt{c^2x^2+1}x^5}{c^4} + \frac{70\,\sqrt{c^2x^2+1}x^3}{c^6} - \frac{105\,\sqrt{c^2x^2+1}x}{c^8} + \right.\right.$$
$$+ \frac{3}{4}ac^2d^3x^4 + \frac{1}{96}\left(48\,x^6\,\text{arsinh}\,(cx) - \left(\frac{8\,\sqrt{c^2x^2+1}x^5}{c^2} - \frac{10\,\sqrt{c^2x^2+1}x^3}{c^4} + \frac{15\,\sqrt{c^2x^2+1}x}{c^6} - \frac{15\,\text{arsinh}\,(cx)}{c^7}\right)c\right)bc^4d$$
$$+ \frac{3}{32}\left(8\,x^4\,\text{arsinh}\,(cx) - \left(\frac{2\,\sqrt{c^2x^2+1}x^3}{c^2} - \frac{3\,\sqrt{c^2x^2+1}x}{c^4} + \frac{3\,\text{arsinh}\,(cx)}{c^5}\right)c\right)bc^2d^3$$
$$+ \frac{1}{2}ad^3x^2 + \frac{1}{4}\left(2\,x^2\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\text{arsinh}\,(cx)}{c^3}\right)\right)bd^3$$

```
[In] integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*c^6*d^3*x^8 + 1/2*a*c^4*d^3*x^6 + 1/3072*(384*x^8*arcsinh(c*x) - (48*
sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^2
+ 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/c^8 + 105*arcsinh(c*x)/c^9)*c)*b*c^6
*d^3 + 3/4*a*c^2*d^3*x^4 + 1/96*(48*x^6*arcsinh(c*x) - (8*sqrt(c^2*x^2 + 1)
*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*a
rcsinh(c*x)/c^7)*c)*b*c^4*d^3 + 3/32*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2
+ 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*b*c^2*d^3
 + 1/2*a*d^3*x^2 + 1/4*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - a
rcsinh(c*x)/c^3))*b*d^3
```

Giac [F(-2)]

Exception generated.

$$\int x(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(d+c^2dx^2)^3\,(a+b\operatorname{arcsinh}(cx))\,dx=\int x\,(a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^3\,dx$$

```
[In] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3,x)
[Out] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3, x)
```

3.23 $\int (d + c^2dx^2)^3 (a + b\,\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 170

$$\int (d + c^2dx^2)^3 (a + b\,\text{arcsinh}(cx))\,dx = -\frac{16bd^3\sqrt{1+c^2x^2}}{35c} - \frac{8bd^3(1+c^2x^2)^{3/2}}{105c} - \frac{6bd^3(1+c^2x^2)^{5/2}}{175c} - \frac{bd^3(1+c^2x^2)^{7/2}}{49c} + d^3x(a+b\,\text{arcsinh}(cx)) + c^2d^3x^3(a+b\,\text{arcsinh}(cx)) + \frac{3}{5}c^4d^3x^5(a+b\,\text{arcsinh}(cx)) + \frac{1}{7}c^6d^3x^7(a+b\,\text{arcsinh}(cx))$$

```
[Out] -8/105*b*d^3*(c^2*x^2+1)^(3/2)/c-6/175*b*d^3*(c^2*x^2+1)^(5/2)/c-1/49*b*d^3
*(c^2*x^2+1)^(7/2)/c+d^3*x*(a+b*arcsinh(c*x))+c^2*d^3*x^3*(a+b*arcsinh(c*x)
)+3/5*c^4*d^3*x^5*(a+b*arcsinh(c*x))+1/7*c^6*d^3*x^7*(a+b*arcsinh(c*x))-16/
35*b*d^3*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {200, 5784, 12, 1813, 1864}

$$\int (d+c^2dx^2)^3 (a + b\,\text{arcsinh}(cx))\,dx = \frac{1}{7}c^6d^3x^7(a+b\,\text{arcsinh}(cx)) + \frac{3}{5}c^4d^3x^5(a+b\,\text{arcsinh}(cx)) + c^2d^3x^3(a+b\,\text{arcsinh}(cx)) + d^3x(a+b\,\text{arcsinh}(cx)) - \frac{bd^3(c^2x^2+1)^{7/2}}{49c} - \frac{6bd^3(c^2x^2+1)^{5/2}}{175c} - \frac{8bd^3(c^2x^2+1)^{3/2}}{105c} - \frac{16bd^3\sqrt{c^2x^2+1}}{35c}$$

```
[In] Int[(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-16*b*d^3*Sqrt[1 + c^2*x^2])/(35*c) - (8*b*d^3*(1 + c^2*x^2)^(3/2))/(105*c
) - (6*b*d^3*(1 + c^2*x^2)^(5/2))/(175*c) - (b*d^3*(1 + c^2*x^2)^(7/2))/(49
*c) + d^3*x*(a + b*ArcSinh[c*x]) + c^2*d^3*x^3*(a + b*ArcSinh[c*x]) + (3*c^
4*d^3*x^5*(a + b*ArcSinh[c*x]))/5 + (c^6*d^3*x^7*(a + b*ArcSinh[c*x]))/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= d^3x(a+b\text{arcsinh}(cx))+c^2d^3x^3(a+b\text{arcsinh}(cx))+\frac{3}{5}c^4d^3x^5(a+b\text{arcsinh}(cx)) \\ &\quad+\frac{1}{7}c^6d^3x^7(a+b\text{arcsinh}(cx))-(bc)\int\frac{d^3x(35+35c^2x^2+21c^4x^4+5c^6x^6)}{35\sqrt{1+c^2x^2}}\,dx \\ &= d^3x(a+b\text{arcsinh}(cx))+c^2d^3x^3(a+b\text{arcsinh}(cx))+\frac{3}{5}c^4d^3x^5(a+b\text{arcsinh}(cx)) \\ &\quad+\frac{1}{7}c^6d^3x^7(a+b\text{arcsinh}(cx))-\frac{1}{35}\left(bcd^3\right)\int\frac{x(35+35c^2x^2+21c^4x^4+5c^6x^6)}{\sqrt{1+c^2x^2}}\,dx\end{aligned}$$

$$= d^3x(a + b\text{arcsinh}(cx)) + c^2d^3x^3(a + b\text{arcsinh}(cx)) + \frac{3}{5}c^4d^3x^5(a + b\text{arcsinh}(cx)) + \frac{1}{7}c^6d^3x^7(a+b\text{arcsinh}(cx)) - \frac{1}{70}(bcd^3)\text{Subst}\left(\int \frac{35 + 35c^2x + 21c^4x^2 + 5c^6x^3}{\sqrt{1+c^2x}}dx, x, x^2\right)$$

$$= d^3x(a + b\text{arcsinh}(cx)) + c^2d^3x^3(a + b\text{arcsinh}(cx)) + \frac{3}{5}c^4d^3x^5(a + b\text{arcsinh}(cx)) + \frac{1}{7}c^6d^3x^7(a + b\text{arcsinh}(cx)) - \frac{1}{70}(bcd^3)\text{Subst}\left(\int \left(\frac{16}{\sqrt{1+c^2x}} + 8\sqrt{1+c^2x} + 6(1+c^2x)^{3/2} + 5(1+c^2x)^{5/2}\right)dx, x, x^2\right)$$

$$= -\frac{16bd^3\sqrt{1+c^2x^2}}{35c} - \frac{8bd^3(1+c^2x^2)^{3/2}}{105c} - \frac{6bd^3(1+c^2x^2)^{5/2}}{175c} - \frac{bd^3(1+c^2x^2)^{7/2}}{49c} + d^3x(a+b\text{arcsinh}(cx)) + c^2d^3x^3(a+b\text{arcsinh}(cx)) + \frac{3}{5}c^4d^3x^5(a+b\text{arcsinh}(cx)) + \frac{1}{7}c^6d^3x^7(a+b\text{arcsin}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.70

$$\int (d + c^2dx^2)^3 (a + b\text{arcsinh}(cx))\,dx = \frac{d^3(105acx(35 + 35c^2x^2 + 21c^4x^4 + 5c^6x^6) - b\sqrt{1+c^2x^2}(2161 + 757c^2x^2 + 351c^4x^4 + 75c^6x^6) + 105bcx(35}{3675c}$$

```
[In] Integrate[(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^3*(105*a*c*x*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) - b*Sqrt[1 + c^2
*x^2]*(2161 + 757*c^2*x^2 + 351*c^4*x^4 + 75*c^6*x^6) + 105*b*c*x*(35 + 35*
c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6)*ArcSinh[c*x]))/(3675*c)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.90

method	result
parts	$d^3a\left(\frac{1}{7}c^6x^7 + \frac{3}{5}c^4x^5 + x^3c^2 + x\right) + \frac{d^3b\left(\frac{\text{arcsinh}(cx)c^7x^7}{7} + \frac{3\,\text{arcsinh}(cx)c^5x^5}{5} + \text{arcsinh}(cx)c^3x^3 + \text{arcsinh}(cx)cx -\right.}{c}$
derivativedivides	$\frac{d^3a\left(\frac{1}{7}c^7x^7 + \frac{3}{5}c^5x^5 + c^3x^3 + cx\right) + d^3b\left(\frac{\text{arcsinh}(cx)c^7x^7}{7} + \frac{3\,\text{arcsinh}(cx)c^5x^5}{5} + \text{arcsinh}(cx)c^3x^3 + \text{arcsinh}(cx)cx - \frac{2161\sqrt{c^2x^2+1}}{3675}\right.}{c}$
default	$\frac{d^3a\left(\frac{1}{7}c^7x^7 + \frac{3}{5}c^5x^5 + c^3x^3 + cx\right) + d^3b\left(\frac{\text{arcsinh}(cx)c^7x^7}{7} + \frac{3\,\text{arcsinh}(cx)c^5x^5}{5} + \text{arcsinh}(cx)c^3x^3 + \text{arcsinh}(cx)cx - \frac{2161\sqrt{c^2x^2+1}}{3675}\right.}{c}$

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] d^3*a*(1/7*c^6*x^7+3/5*c^4*x^5+x^3*c^2+x)+d^3*b/c*(1/7*arcsinh(c*x)*c^7*x^7
+3/5*arcsinh(c*x)*c^5*x^5+arcsinh(c*x)*c^3*x^3+arcsinh(c*x)*c*x-2161/3675*(
c^2*x^2+1)^(1/2)-1/49*c^6*x^6*(c^2*x^2+1)^(1/2)-117/1225*c^4*x^4*(c^2*x^2+1
)^(1/2)-757/3675*c^2*x^2*(c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))\,dx$$
$$= \frac{525\,ac^7d^3x^7 + 2205\,ac^5d^3x^5 + 3675\,ac^3d^3x^3 + 3675\,acd^3x + 105\,(5\,bc^7d^3x^7 + 21\,bc^5d^3x^5 + 35\,bc^3d^3x^3 + 35}{3675\,c}$$

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3675*(525*a*c^7*d^3*x^7 + 2205*a*c^5*d^3*x^5 + 3675*a*c^3*d^3*x^3 + 3675*
a*c*d^3*x + 105*(5*b*c^7*d^3*x^7 + 21*b*c^5*d^3*x^5 + 35*b*c^3*d^3*x^3 + 35
*b*c*d^3*x)*log(c*x + sqrt(c^2*x^2 + 1)) - (75*b*c^6*d^3*x^6 + 351*b*c^4*d^
3*x^4 + 757*b*c^2*d^3*x^2 + 2161*b*d^3)*sqrt(c^2*x^2 + 1))/c
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.30

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))\,dx$$
$$= \begin{cases} \frac{ac^6d^3x^7}{7} + \frac{3ac^4d^3x^5}{5} + ac^2d^3x^3 + ad^3x + \frac{bc^6d^3x^7\operatorname{asinh}(cx)}{7} - \frac{bc^5d^3x^6\sqrt{c^2x^2+1}}{49} + \frac{3bc^4d^3x^5\operatorname{asinh}(cx)}{5} - \frac{117bc^3d^3x^4\sqrt{c^2x^2+}}{1225} \\ ad^3x \end{cases}$$

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**6*d**3*x**7/7 + 3*a*c**4*d**3*x**5/5 + a*c**2*d**3*x**3 + a
*d**3*x + b*c**6*d**3*x**7*asinh(c*x)/7 - b*c**5*d**3*x**6*sqrt(c**2*x**2 +
 1)/49 + 3*b*c**4*d**3*x**5*asinh(c*x)/5 - 117*b*c**3*d**3*x**4*sqrt(c**2*x
**2 + 1)/1225 + b*c**2*d**3*x**3*asinh(c*x) - 757*b*c*d**3*x**2*sqrt(c**2*x
**2 + 1)/3675 + b*d**3*x*asinh(c*x) - 2161*b*d**3*sqrt(c**2*x**2 + 1)/(3675
*c), Ne(c, 0)), (a*d**3*x, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(150) = 300.

Time = 0.20 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.77

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))\,dx = \frac{1}{7}\,ac^6d^3x^7 + \frac{3}{5}\,ac^4d^3x^5 + \frac{1}{245}\left(35\,x^7\,\text{arsinh}\,(cx) - \left(\frac{5\,\sqrt{c^2x^2+1}x^6}{c^2} - \frac{6\,\sqrt{c^2x^2+1}x^4}{c^4} + \frac{8\,\sqrt{c^2x^2+1}x^2}{c^6} - \frac{16\,\sqrt{c^2x^2+1}}{c^8}\right)c\right)bc^6d^3 + \frac{1}{25}\left(15\,x^5\,\text{arsinh}\,(cx) - \left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\,\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\right)bc^4d^3 + ac^2d^3x^3 + \frac{1}{3}\left(3\,x^3\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)bc^2d^3 + ad^3x + \frac{\left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)bd^3}{c}$$

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*c^6*d^3*x^7 + 3/5*a*c^4*d^3*x^5 + 1/245*(35*x^7*arcsinh(c*x) - (5*sqr
t(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*
x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^6*d^3 + 1/25*(15*x^5*arcsinh(c*x
) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2
*x^2 + 1)/c^6)*c)*b*c^4*d^3 + a*c^2*d^3*x^3 + 1/3*(3*x^3*arcsinh(c*x) - c*(
sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*c^2*d^3 + a*d^3*x +
 (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*d^3/c
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))\,dx=\int (a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^3\,dx$$

```
[In] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^3,x)
[Out] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^3, x)
```

3.24 $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 221

$$\begin{aligned}\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))}{x}\,dx &= -\frac{19}{48}bcd^3x\sqrt{1+c^2x^2}-\frac{7}{72}bcd^3x(1+c^2x^2)^{3/2}\\&\quad-\frac{1}{36}bcd^3x(1+c^2x^2)^{5/2}-\frac{19}{48}bd^3\text{arcsinh}(cx)\\&\quad+\frac{1}{2}d^3(1+c^2x^2)(a+b\text{arcsinh}(cx))\\&\quad+\frac{1}{4}d^3(1+c^2x^2)^2(a+b\text{arcsinh}(cx))\\&\quad+\frac{1}{6}d^3(1+c^2x^2)^3(a+b\text{arcsinh}(cx))\\&\quad+\frac{d^3(a+b\text{arcsinh}(cx))^2}{2b}\\&\quad+d^3(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)\\&\quad-\frac{1}{2}bd^3\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)\end{aligned}$$

```
[Out] -7/72*b*c*d^3*x*(c^2*x^2+1)^(3/2)-1/36*b*c*d^3*x*(c^2*x^2+1)^(5/2)-19/48*b*
d^3*arcsinh(c*x)+1/2*d^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))+1/4*d^3*(c^2*x^2+1)
^2*(a+b*arcsinh(c*x))+1/6*d^3*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))+1/2*d^3*(a+b
*arcsinh(c*x))^2/b+d^3*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)
-1/2*b*d^3*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)-19/48*b*c*d^3*x*(c^2*x^2+
1)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5801, 5775, 3797, 2221, 2317, 2438, 201, 221}

$$\begin{aligned}\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))}{x}\,dx = & \frac{1}{6}d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx)) \\ & +\frac{1}{4}d^3(c^2x^2+1)^2(a+b\text{arcsinh}(cx)) \\ & +\frac{1}{2}d^3(c^2x^2+1)(a+b\text{arcsinh}(cx)) \\ & +\frac{d^3(a+b\text{arcsinh}(cx))^2}{2b} \\ & +d^3\log\left(1-e^{-2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)) \\ & -\frac{1}{2}bd^3\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right) \\ & -\frac{19}{48}bd^3\text{arcsinh}(cx)-\frac{1}{36}bcd^3x(c^2x^2+1)^{5/2} \\ & -\frac{7}{72}bcd^3x(c^2x^2+1)^{3/2}-\frac{19}{48}bcd^3x\sqrt{c^2x^2+1}\end{aligned}$$

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (-19*b*c*d^3*x*Sqrt[1 + c^2*x^2])/48 - (7*b*c*d^3*x*(1 + c^2*x^2)^(3/2))/72
 - (b*c*d^3*x*(1 + c^2*x^2)^(5/2))/36 - (19*b*d^3*ArcSinh[c*x])/48 + (d^3*(
1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/2 + (d^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[
c*x]))/4 + (d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x]))/6 + (d^3*(a + b*ArcSi
nh[c*x])^2)/(2*b) + d^3*(a + b*ArcSinh[c*x])*Log[1 - E^(-2*ArcSinh[c*x])] -
 (b*d^3*PolyLog[2, E^(-2*ArcSinh[c*x])])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5801

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcSinh[c*x])/(2*p)), x] + (Dist[d
, Int[(d + e*x^2)^(p - 1)*((a + b*ArcSinh[c*x])/x), x], x] - Dist[b*c*(d^p/
(2*p)), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{6}d^3\left(1+c^2x^2\right)^3(a+b\text{arcsinh}(cx))\\ &\quad+d\int\frac{\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))}{x}\,dx-\frac{1}{6}\left(bcd^3\right)\int\left(1+c^2x^2\right)^{5/2}\,dx\end{aligned}$$

$$= -\frac{1}{36}bcd^3x\left(1+c^2x^2\right)^{5/2} + \frac{1}{4}d^3\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx)) + \frac{1}{6}d^3\left(1+c^2x^2\right)^3(a+b\text{arcsinh}(cx)) + d^2\int\frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x}\,dx - \frac{1}{36}(5bcd^3)\int\left(1+c^2x^2\right)^{3/2}\,dx - \frac{1}{4}(bcd^3)\int\left(1+c^2x^2\right)^{3/2}\,dx$$

$$= -\frac{7}{72}bcd^3x\left(1+c^2x^2\right)^{3/2} - \frac{1}{36}bcd^3x\left(1+c^2x^2\right)^{5/2} + \frac{1}{2}d^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx)) + \frac{1}{4}d^3\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx)) + \frac{1}{6}d^3\left(1+c^2x^2\right)^3(a+b\text{arcsinh}(cx)) + d^3\int\frac{a+b\text{arcsinh}(cx)}{x}\,dx - \frac{1}{48}(5bcd^3)\int\sqrt{1+c^2x^2}\,dx - \frac{1}{16}(3bcd^3)\int\sqrt{1+c^2x^2}\,dx - \frac{1}{2}(bcd^3)\int\sqrt{1+c^2x^2}\,dx$$

$$= -\frac{19}{48}bcd^3x\sqrt{1+c^2x^2} - \frac{7}{72}bcd^3x\left(1+c^2x^2\right)^{3/2} - \frac{1}{36}bcd^3x\left(1+c^2x^2\right)^{5/2} + \frac{1}{2}d^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx)) + \frac{1}{4}d^3\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx)) + \frac{1}{6}d^3\left(1+c^2x^2\right)^3(a+b\text{arcsinh}(cx)) - \frac{d^3\text{Subst}\left(\int x\coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)}{b} - \frac{1}{96}(5bcd^3)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx - \frac{1}{32}(3bcd^3)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx - \frac{1}{4}(bcd^3)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx$$

$$= -\frac{19}{48}bcd^3x\sqrt{1+c^2x^2} - \frac{7}{72}bcd^3x\left(1+c^2x^2\right)^{3/2} - \frac{1}{36}bcd^3x\left(1+c^2x^2\right)^{5/2} - \frac{19}{48}bd^3\text{arcsinh}(cx) + \frac{1}{2}d^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx)) + \frac{1}{4}d^3\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx)) + \frac{1}{6}d^3\left(1+c^2x^2\right)^3(a+b\text{arcsinh}(cx)) + \frac{d^3(a+b\text{arcsinh}(cx))^2}{2b} + \frac{(2d^3)\,\text{Subst}\left(\int\frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b}$$

$$= -\frac{19}{48}bcd^3x\sqrt{1+c^2x^2} - \frac{7}{72}bcd^3x\left(1+c^2x^2\right)^{3/2} - \frac{1}{36}bcd^3x\left(1+c^2x^2\right)^{5/2} - \frac{19}{48}bd^3\text{arcsinh}(cx) + \frac{1}{2}d^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx)) + \frac{1}{4}d^3\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx)) + \frac{1}{6}d^3\left(1+c^2x^2\right)^3(a+b\text{arcsinh}(cx)) + \frac{d^3(a+b\text{arcsinh}(cx))^2}{2b} + d^3(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right) - d^3\text{Subst}\left(\int\log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)$$

$$= -\frac{19}{48}bcd^3x\sqrt{1+c^2x^2} - \frac{7}{72}bcd^3x\left(1+c^2x^2\right)^{3/2} - \frac{1}{36}bcd^3x\left(1+c^2x^2\right)^{5/2} - \frac{19}{48}bd^3\text{arcsinh}(cx) + \frac{1}{2}d^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx)) + \frac{1}{4}d^3\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx)) + \frac{1}{6}d^3\left(1+c^2x^2\right)^3(a+b\text{arcsinh}(cx)) + \frac{d^3(a+b\text{arcsinh}(cx))^2}{2b} + d^3(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right) + \frac{1}{2}\left(bd^3\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right)$$

$$= -\frac{19}{48}bcd^3x\sqrt{1+c^2x^2} - \frac{7}{72}bcd^3x\left(1+c^2x^2\right)^{3/2} - \frac{1}{36}bcd^3x\left(1+c^2x^2\right)^{5/2} - \frac{19}{48}bd^3\text{arcsinh}(cx) + \frac{1}{2}d^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx)) + \frac{1}{4}d^3\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx)) + \frac{1}{6}d^3\left(1+c^2x^2\right)^3(a+b\text{arcsinh}(cx)) + \frac{d^3(a+b\text{arcsinh}(cx))^2}{2b} + d^3(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right) - \frac{1}{2}bd^3\,\text{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right)$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.98

$$\int\frac{\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))}{x}\,dx = \frac{d^3\left(-72a^2+132ab+216abc^2x^2+108abc^4x^4+24abc^6x^6-75b^2cx\sqrt{1+c^2x^2}-22b^2c^3x^3\sqrt{1+c^2x^2}-4b^2c^5\right.}{}$$

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (d^3*(-72*a^2 + 132*a*b + 216*a*b*c^2*x^2 + 108*a*b*c^4*x^4 + 24*a*b*c^6*x^
6 - 75*b^2*c*x*Sqrt[1 + c^2*x^2] - 22*b^2*c^3*x^3*Sqrt[1 + c^2*x^2] - 4*b^2
*c^5*x^5*Sqrt[1 + c^2*x^2] - 72*b^2*ArcSinh[c*x]^2 + 144*a*b*Log[1 - E^(2*A
rcSinh[c*x])] + 3*b*ArcSinh[c*x]*(-48*a + b*(25 + 72*c^2*x^2 + 36*c^4*x^4 +
 8*c^6*x^6) + 48*b*Log[1 - E^(2*ArcSinh[c*x])]) + 72*b^2*PolyLog[2, E^(2*Ar
cSinh[c*x])]))/(144*b)
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.23

method	result
parts	$d^{3}a\left(\frac{c^{6}x^{6}}{6}+\frac{3c^{4}x^{4}}{4}+\frac{3c^{2}x^{2}}{2}+\ln\left(x\right)\right)-\frac{d^{3}b\,c^{5}x^{5}\sqrt{c^{2}x^{2}+1}}{36}-\frac{11d^{3}b\,c^{3}x^{3}\sqrt{c^{2}x^{2}+1}}{72}-\frac{25bc\,d^{3}x\sqrt{c^{2}x^{2}+1}}{48}$
derivativedivides	$d^{3}a\left(\frac{c^{6}x^{6}}{6}+\frac{3c^{4}x^{4}}{4}+\frac{3c^{2}x^{2}}{2}+\ln\left(cx\right)\right)+\frac{25b\,d^{3}\operatorname{arcsinh}(cx)}{48}+d^{3}b\,\operatorname{arcsinh}\left(cx\right)\ln\left(1+cx+\sqrt{c}\right.$
default	$d^{3}a\left(\frac{c^{6}x^{6}}{6}+\frac{3c^{4}x^{4}}{4}+\frac{3c^{2}x^{2}}{2}+\ln\left(cx\right)\right)+\frac{25b\,d^{3}\operatorname{arcsinh}(cx)}{48}+d^{3}b\,\operatorname{arcsinh}\left(cx\right)\ln\left(1+cx+\sqrt{c}\right.$

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*a*(1/6*c^6*x^6+3/4*c^4*x^4+3/2*c^2*x^2+ln(x))-1/36*d^3*b*c^5*x^5*(c^2*x
^2+1)^(1/2)-11/72*d^3*b*c^3*x^3*(c^2*x^2+1)^(1/2)-25/48*b*c*d^3*x*(c^2*x^2+
1)^(1/2)+1/6*d^3*b*arcsinh(c*x)*c^6*x^6+3/4*d^3*b*arcsinh(c*x)*c^4*x^4+3/2*
d^3*b*arcsinh(c*x)*c^2*x^2+25/48*b*d^3*arcsinh(c*x)+d^3*b*polylog(2,-c*x-(c
^2*x^2+1)^(1/2))+d^3*b*polylog(2,c*x+(c^2*x^2+1)^(1/2))-1/2*d^3*b*arcsinh(c
*x)^2+d^3*b*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+d^3*b*arcsinh(c*x)*ln(
1-c*x-(c^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int\frac{(d+c^{2}dx^{2})^{3}\,(a+b\text{arcsinh}(cx))}{x}\,dx=\int\frac{(c^{2}dx^{2}+d)^{3}(b\,\text{arsinh}\,(cx)+a)}{x}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^6*d^3*x^6 + 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 + a*d^3 + (b*c^
6*d^3*x^6 + 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 + b*d^3)*arcsinh(c*x))/x, x)
```

Sympy [F]

$$\int\frac{(d+c^{2}dx^{2})^{3}\,(a+b\text{arcsinh}(cx))}{x}\,dx=d^{3}\left(\int\frac{a}{x}\,dx+\int 3ac^{2}x\,dx+\int 3ac^{4}x^{3}\,dx+\int ac^{6}x^{5}\,dx+\int\frac{b\,\text{asinh}\,(cx)}{x}\,dx+\int 3bc^{2}x\,\text{asinh}\,(cx)\,dx+\int 3bc^{4}x^{3}\,\text{asinh}\,(cx)\,dx+\int bc^{6}x^{5}\,\text{asinh}\,(cx)\,dx\right)$$

[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))/x,x)

[Out] d**3*(Integral(a/x, x) + Integral(3*a*c**2*x, x) + Integral(3*a*c**4*x**3, x) + Integral(a*c**6*x**5, x) + Integral(b*asinh(c*x)/x, x) + Integral(3*b*c**2*x*asinh(c*x), x) + Integral(3*b*c**4*x**3*asinh(c*x), x) + Integral(b*c**6*x**5*asinh(c*x), x))

Maxima [F]

$$\int \frac{(d + c^2dx^2)^3 (a + b\operatorname{arcsinh}(cx))}{x} dx = \int \frac{(c^2dx^2 + d)^3 (b \operatorname{arsinh}(cx) + a)}{x} dx$$

[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")

[Out] 1/6*a*c^6*d^3*x^6 + 3/4*a*c^4*d^3*x^4 + 3/2*a*c^2*d^3*x^2 + a*d^3*log(x) + integrate(b*c^6*d^3*x^5*log(c*x + sqrt(c^2*x^2 + 1)) + 3*b*c^4*d^3*x^3*log(c*x + sqrt(c^2*x^2 + 1)) + 3*b*c^2*d^3*x*log(c*x + sqrt(c^2*x^2 + 1)) + b*d^3*log(c*x + sqrt(c^2*x^2 + 1))/x, x)

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + c^2dx^2)^3 (a + b\operatorname{arcsinh}(cx))}{x} dx = \text{Exception raised: TypeError}$$

[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + c^2dx^2)^3 (a + b\operatorname{arcsinh}(cx))}{x} dx = \int \frac{(a + b \operatorname{asinh}(c x)) (d c^2 x^2 + d)^3}{x} dx$$

[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x,x)

[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x, x)

3.25 $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))}{x^2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 160

$$\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))}{x^2} \, dx = -\frac{11}{5}bcd^3\sqrt{1+c^2x^2}-\frac{1}{5}bcd^3\left(1+c^2x^2\right)^{3/2} -\frac{1}{25}bcd^3\left(1+c^2x^2\right)^{5/2}-\frac{d^3(a+b\text{arcsinh}(cx))}{x} +3c^2d^3x(a+b\text{arcsinh}(cx)) +c^4d^3x^3(a+b\text{arcsinh}(cx)) +\frac{1}{5}c^6d^3x^5(a+b\text{arcsinh}(cx)) -bcd^3\text{arctanh}\left(\sqrt{1+c^2x^2}\right)$$

```
[Out] -1/5*b*c*d^3*(c^2*x^2+1)^(3/2)-1/25*b*c*d^3*(c^2*x^2+1)^(5/2)-d^3*(a+b*arcs
inh(c*x))/x+3*c^2*d^3*x*(a+b*arcsinh(c*x))+c^4*d^3*x^3*(a+b*arcsinh(c*x))+1
/5*c^6*d^3*x^5*(a+b*arcsinh(c*x))-b*c*d^3*arctanh((c^2*x^2+1)^(1/2))-11/5*b
*c*d^3*(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used

= {276, 5803, 12, 1813, 1634, 65, 214}

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))}{x^2} dx = \frac{1}{5}c^6d^3x^5(a+b\text{arcsinh}(cx))+c^4d^3x^3(a+b\text{arcsinh}(cx)) + 3c^2d^3x(a+b\text{arcsinh}(cx)) - \frac{d^3(a+b\text{arcsinh}(cx))}{x} - bcd^3\text{arctanh}\left(\sqrt{c^2x^2+1}\right) - \frac{1}{25}bcd^3\left(c^2x^2+1\right)^{5/2} - \frac{1}{5}bcd^3\left(c^2x^2+1\right)^{3/2} - \frac{11}{5}bcd^3\sqrt{c^2x^2+1}$$

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (-11*b*c*d^3*Sqrt[1 + c^2*x^2])/5 - (b*c*d^3*(1 + c^2*x^2)^(3/2))/5 - (b*c*
d^3*(1 + c^2*x^2)^(5/2))/25 - (d^3*(a + b*ArcSinh[c*x]))/x + 3*c^2*d^3*x*(a
 + b*ArcSinh[c*x]) + c^4*d^3*x^3*(a + b*ArcSinh[c*x]) + (c^6*d^3*x^5*(a + b
*ArcSinh[c*x]))/5 - b*c*d^3*ArcTanh[Sqrt[1 + c^2*x^2]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
```

```
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^3(a+b\text{arcsinh}(cx))}{x}+3c^2d^3x(a+b\text{arcsinh}(cx))+c^4d^3x^3(a+b\text{arcsinh}(cx)) \\
&\quad+\frac{1}{5}c^6d^3x^5(a+b\text{arcsinh}(cx))-(bc)\int\frac{d^3(-5+15c^2x^2+5c^4x^4+c^6x^6)}{5x\sqrt{1+c^2x^2}}\,dx \\
&= -\frac{d^3(a+b\text{arcsinh}(cx))}{x}+3c^2d^3x(a+b\text{arcsinh}(cx))+c^4d^3x^3(a+b\text{arcsinh}(cx)) \\
&\quad+\frac{1}{5}c^6d^3x^5(a+b\text{arcsinh}(cx))-\frac{1}{5}\left(bcd^3\right)\int\frac{-5+15c^2x^2+5c^4x^4+c^6x^6}{x\sqrt{1+c^2x^2}}\,dx \\
&= -\frac{d^3(a+b\text{arcsinh}(cx))}{x}+3c^2d^3x(a+b\text{arcsinh}(cx))+c^4d^3x^3(a+b\text{arcsinh}(cx)) \\
&\quad+\frac{1}{5}c^6d^3x^5(a+b\text{arcsinh}(cx))-\frac{1}{10}\left(bcd^3\right)\text{Subst}\left(\int\frac{-5+15c^2x+5c^4x^2+c^6x^3}{x\sqrt{1+c^2x}}\,dx,x,x^2\right) \\
&= -\frac{d^3(a+b\text{arcsinh}(cx))}{x}+3c^2d^3x(a+b\text{arcsinh}(cx))+c^4d^3x^3(a+b\text{arcsinh}(cx)) \\
&\quad+\frac{1}{5}c^6d^3x^5(a+b\text{arcsinh}(cx))-\frac{1}{10}\left(bcd^3\right)\text{Subst}\left(\int\left(\frac{11c^2}{\sqrt{1+c^2x}}-\frac{5}{x\sqrt{1+c^2x}}\right.\right. \\
&\quad\left.\left.+3c^2\sqrt{1+c^2x}+c^2\left(1+c^2x\right)^{3/2}\right)dx,x,x^2\right) \\
&= -\frac{11}{5}bcd^3\sqrt{1+c^2x^2}-\frac{1}{5}bcd^3\left(1+c^2x^2\right)^{3/2}-\frac{1}{25}bcd^3\left(1+c^2x^2\right)^{5/2} \\
&\quad-\frac{d^3(a+b\text{arcsinh}(cx))}{x}+3c^2d^3x(a+b\text{arcsinh}(cx))+c^4d^3x^3(a+b\text{arcsinh}(cx)) \\
&\quad+\frac{1}{5}c^6d^3x^5(a+b\text{arcsinh}(cx))+\frac{1}{2}\left(bcd^3\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+c^2x}}\,dx,x,x^2\right)
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{11}{5}bcd^3\sqrt{1+c^2x^2} - \frac{1}{5}bcd^3\left(1+c^2x^2\right)^{3/2} - \frac{1}{25}bcd^3\left(1+c^2x^2\right)^{5/2} \\
&\quad - \frac{d^3(a+b\operatorname{arcsinh}(cx))}{x} + 3c^2d^3x(a+b\operatorname{arcsinh}(cx)) + c^4d^3x^3(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{1}{5}c^6d^3x^5(a+b\operatorname{arcsinh}(cx)) + \frac{(bd^3)\operatorname{Subst}\left(\int \frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx, x, \sqrt{1+c^2x^2}\right)}{c} \\
&= -\frac{11}{5}bcd^3\sqrt{1+c^2x^2} - \frac{1}{5}bcd^3\left(1+c^2x^2\right)^{3/2} \\
&\quad - \frac{1}{25}bcd^3\left(1+c^2x^2\right)^{5/2} - \frac{d^3(a+b\operatorname{arcsinh}(cx))}{x} + 3c^2d^3x(a+b\operatorname{arcsinh}(cx)) \\
&\quad + c^4d^3x^3(a+b\operatorname{arcsinh}(cx)) + \frac{1}{5}c^6d^3x^5(a+b\operatorname{arcsinh}(cx)) - bcd^3\operatorname{arctanh}\left(\sqrt{1+c^2x^2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.02

$$
\int \frac{\left(d+c^2dx^2\right)^3(a+b\operatorname{arcsinh}(cx))}{x^2}\,dx
= \frac{d^3\left(-25a+75ac^2x^2+25ac^4x^4+5ac^6x^6-61bcx\sqrt{1+c^2x^2}-7bc^3x^3\sqrt{1+c^2x^2}-bc^5x^5\sqrt{1+c^2x^2}+5b(-5\right.}{25x}
$$

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (d^3*(-25*a + 75*a*c^2*x^2 + 25*a*c^4*x^4 + 5*a*c^6*x^6 - 61*b*c*x*Sqrt[1 +
 c^2*x^2] - 7*b*c^3*x^3*Sqrt[1 + c^2*x^2] - b*c^5*x^5*Sqrt[1 + c^2*x^2] + 5
*b*(-5 + 15*c^2*x^2 + 5*c^4*x^4 + c^6*x^6)*ArcSinh[c*x] + 25*b*c*x*Log[x] -
 25*b*c*x*Log[1 + Sqrt[1 + c^2*x^2]]))/(25*x)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.93

method	result
parts	$d^3a\left(\frac{c^6x^5}{5}+c^4x^3+3c^2x-\frac{1}{x}\right)+d^3bc\left(\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}+\operatorname{arcsinh}(cx)\,c^3x^3+3\operatorname{arcsinh}(cx)\,c\right.$
derivativedivides	$c\left(d^3a\left(\frac{c^5x^5}{5}+c^3x^3+3cx-\frac{1}{cx}\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}+\operatorname{arcsinh}(cx)\,c^3x^3+3\operatorname{arcsinh}(cx)\right.\right.$
default	$c\left(d^3a\left(\frac{c^5x^5}{5}+c^3x^3+3cx-\frac{1}{cx}\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}+\operatorname{arcsinh}(cx)\,c^3x^3+3\operatorname{arcsinh}(cx)\right.\right.$

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*a*(1/5*c^6*x^5+c^4*x^3+3*c^2*x-1/x)+d^3*b*c*(1/5*arcsinh(c*x)*c^5*x^5+a
rcsinh(c*x)*c^3*x^3+3*arcsinh(c*x)*c*x-arcsinh(c*x)/c/x-1/25*c^4*x^4*(c^2*x
^2+1)^(1/2)-7/25*c^2*x^2*(c^2*x^2+1)^(1/2)-61/25*(c^2*x^2+1)^(1/2)-arctanh(
1/(c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.72

$$\int \frac{(d+c^2dx^2)^3 (a+b\operatorname{arcsinh}(cx))}{x^2} dx = \frac{5\,ac^6d^3x^6 + 25\,ac^4d^3x^4 + 75\,ac^2d^3x^2 - 25\,bcd^3x\log\left(-cx+\sqrt{c^2x^2+1}+1\right) + 25\,bcd^3x\log\left(-cx+\sqrt{c^2x^2}\right.}{}$$

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/25*(5*a*c^6*d^3*x^6 + 25*a*c^4*d^3*x^4 + 75*a*c^2*d^3*x^2 - 25*b*c*d^3*x*
log(-c*x + sqrt(c^2*x^2 + 1) + 1) + 25*b*c*d^3*x*log(-c*x + sqrt(c^2*x^2 +
1) - 1) - 5*(b*c^6 + 5*b*c^4 + 15*b*c^2 - 5*b)*d^3*x*log(-c*x + sqrt(c^2*x^
2 + 1)) - 25*a*d^3 + 5*(b*c^6*d^3*x^6 + 5*b*c^4*d^3*x^4 + 15*b*c^2*d^3*x^2
- (b*c^6 + 5*b*c^4 + 15*b*c^2 - 5*b)*d^3*x - 5*b*d^3)*log(c*x + sqrt(c^2*x^
2 + 1)) - (b*c^5*d^3*x^5 + 7*b*c^3*d^3*x^3 + 61*b*c*d^3*x)*sqrt(c^2*x^2 + 1
))/x
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^3 (a+b\operatorname{arcsinh}(cx))}{x^2} dx = d^3\left(\int 3ac^2\,dx + \int \frac{a}{x^2}\,dx + \int 3ac^4x^2\,dx + \int ac^6x^4\,dx + \int 3bc^2\operatorname{asinh}(cx)\,dx + \int \frac{b\operatorname{asinh}(cx)}{x^2}\,dx + \int 3bc^4x^2\operatorname{asinh}(cx)\,dx + \int bc^6x^4\operatorname{asinh}(cx)\,dx\right)$$

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))/x**2,x)
```

```
[Out] d**3*(Integral(3*a*c**2, x) + Integral(a/x**2, x) + Integral(3*a*c**4*x**2,
 x) + Integral(a*c**6*x**4, x) + Integral(3*b*c**2*asinh(c*x), x) + Integra
l(b*asinh(c*x)/x**2, x) + Integral(3*b*c**4*x**2*asinh(c*x), x) + Integral(
b*c**6*x**4*asinh(c*x), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.44

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))}{x^2}\,dx$$
$$= \frac{1}{5}\,ac^6d^3x^5 + \frac{1}{75}\left(15\,x^5\,\text{arsinh}\,(cx) - \left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\,\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\right)bc^6d^3 + ac^4d^3x^3 + \frac{1}{3}\left(3\,x^3\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)bc^4d^3 + 3\,ac^2d^3x + 3\left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)bcd^3 - \left(c\,\text{arsinh}\left(\frac{1}{c|x|}\right) + \frac{\text{arsinh}\,(cx)}{x}\right)bd^3 - \frac{ad^3}{x}$$

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*a*c^6*d^3*x^5 + 1/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^
2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*c^6*d^3 + a
*c^4*d^3*x^3 + 1/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*s
qrt(c^2*x^2 + 1)/c^4))*b*c^4*d^3 + 3*a*c^2*d^3*x + 3*(c*x*arcsinh(c*x) - sq
rt(c^2*x^2 + 1))*b*c*d^3 - (c*arcsinh(1/(c*abs(x))) + arcsinh(c*x)/x)*b*d^3
 - a*d^3/x
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + c^2dx^2)^3 (a + b\text{arcsinh}(cx))}{x^2} \, dx = \int \frac{(a + b \operatorname{asinh}(c\,x)) \, (d\,c^2\,x^2 + d)^3}{x^2} \, dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x^2,x)
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x^2, x)
```

3.26 $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 249

$$\begin{aligned}\int \frac{(d+c^2dx^2)^3\,(a+b\text{arcsinh}(cx))}{x^3}\,dx =& -\frac{3}{32}bc^3d^3x\sqrt{1+c^2x^2}+\frac{7}{16}bc^3d^3x\left(1+c^2x^2\right)^{3/2}\\&-\frac{bcd^3(1+c^2x^2)^{5/2}}{2x}-\frac{3}{32}bc^2d^3\text{arcsinh}(cx)\\&+\frac{3}{2}c^2d^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))\\&+\frac{3}{4}c^2d^3\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))\\&-\frac{d^3(1+c^2x^2)^3\,(a+b\text{arcsinh}(cx))}{2x^2}\\&+\frac{3c^2d^3(a+b\text{arcsinh}(cx))^2}{2b}\\&+3c^2d^3(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)\\&-\frac{3}{2}bc^2d^3\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)\end{aligned}$$

```
[Out] 7/16*b*c^3*d^3*x*(c^2*x^2+1)^(3/2)-1/2*b*c*d^3*(c^2*x^2+1)^(5/2)/x-3/32*b*c
^2*d^3*arcsinh(c*x)+3/2*c^2*d^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))+3/4*c^2*d^3*
(c^2*x^2+1)^2*(a+b*arcsinh(c*x))-1/2*d^3*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))/x
^2+3/2*c^2*d^3*(a+b*arcsinh(c*x))^2/b+3*c^2*d^3*(a+b*arcsinh(c*x))*ln(1-1/(
c*x+(c^2*x^2+1)^(1/2))^2)-3/2*b*c^2*d^3*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))
^2)-3/32*b*c^3*d^3*x*(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5802, 283, 201, 221, 5801, 5775, 3797, 2221, 2317, 2438}

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))}{x^3} \, dx = -\frac{d^3(c^2x^2+1)^3 (a+b\text{arcsinh}(cx))}{2x^2} + \frac{3}{4}c^2d^3(c^2x^2+1)^2 (a+b\text{arcsinh}(cx)) + \frac{3}{2}c^2d^3(c^2x^2+1)(a+b\text{arcsinh}(cx)) + \frac{3c^2d^3(a+b\text{arcsinh}(cx))^2}{2b} + 3c^2d^3\log\left(1-e^{-2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)) - \frac{3}{2}bc^2d^3\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right) - \frac{3}{32}bc^2d^3\text{arcsinh}(cx) - \frac{bcd^3(c^2x^2+1)^{5/2}}{2x} + \frac{7}{16}bc^3d^3x(c^2x^2+1)^{3/2} - \frac{3}{32}bc^3d^3x\sqrt{c^2x^2+1}$$

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (-3*b*c^3*d^3*x*Sqrt[1 + c^2*x^2])/32 + (7*b*c^3*d^3*x*(1 + c^2*x^2)^(3/2))
/16 - (b*c*d^3*(1 + c^2*x^2)^(5/2))/(2*x) - (3*b*c^2*d^3*ArcSinh[c*x])/32 +
 (3*c^2*d^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/2 + (3*c^2*d^3*(1 + c^2*x^2
)^2*(a + b*ArcSinh[c*x]))/4 - (d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x]))/(2
*x^2) + (3*c^2*d^3*(a + b*ArcSinh[c*x])^2)/(2*b) + 3*c^2*d^3*(a + b*ArcSinh
[c*x])*Log[1 - E^(-2*ArcSinh[c*x])] - (3*b*c^2*d^3*PolyLog[2, E^(-2*ArcSinh
[c*x])])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5801

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcSinh[c*x])/(2*p)), x] + (Dist[d
, Int[(d + e*x^2)^(p - 1)*((a + b*ArcSinh[c*x])/x), x], x] - Dist[b*c*(d^p/
(2*p)), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5802

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSinh[c
*x])/(f*(m + 1))), x] + (-Dist[b*c*(d^p/(f*(m + 1))), Int[(f*x)^(m + 1)*(1
+ c^2*x^2)^(p - 1/2), x], x] - Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m + 2
)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^3(1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))}{2x^2} \\
&\quad +(3c^2d)\int\frac{(d+c^2dx^2)^2(a+b\operatorname{arcsinh}(cx))}{x}\,dx+\frac{1}{2}(bcd^3)\int\frac{(1+c^2x^2)^{5/2}}{x^2}\,dx \\
&= -\frac{bcd^3(1+c^2x^2)^{5/2}}{2x}+\frac{3}{4}c^2d^3(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx)) \\
&\quad -\frac{d^3(1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))}{2x^2}+(3c^2d^2)\int\frac{(d+c^2dx^2)(a+b\operatorname{arcsinh}(cx))}{x}\,dx \\
&\quad -\frac{1}{4}(3bc^3d^3)\int(1+c^2x^2)^{3/2}\,dx+\frac{1}{2}(5bc^3d^3)\int(1+c^2x^2)^{3/2}\,dx \\
&= \frac{7}{16}bc^3d^3x(1+c^2x^2)^{3/2}-\frac{bcd^3(1+c^2x^2)^{5/2}}{2x}+\frac{3}{2}c^2d^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx)) \\
&\quad +\frac{3}{4}c^2d^3(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))-\frac{d^3(1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))}{2x^2} \\
&\quad +(3c^2d^3)\int\frac{a+b\operatorname{arcsinh}(cx)}{x}\,dx-\frac{1}{16}(9bc^3d^3)\int\sqrt{1+c^2x^2}\,dx \\
&\quad -\frac{1}{2}(3bc^3d^3)\int\sqrt{1+c^2x^2}\,dx+\frac{1}{8}(15bc^3d^3)\int\sqrt{1+c^2x^2}\,dx \\
&= -\frac{3}{32}bc^3d^3x\sqrt{1+c^2x^2}+\frac{7}{16}bc^3d^3x(1+c^2x^2)^{3/2} \\
&\quad -\frac{bcd^3(1+c^2x^2)^{5/2}}{2x}+\frac{3}{2}c^2d^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx)) \\
&\quad +\frac{3}{4}c^2d^3(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))-\frac{d^3(1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))}{2x^2} \\
&\quad -\frac{(3c^2d^3)\operatorname{Subst}\left(\int x\coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{b} \\
&\quad -\frac{1}{32}(9bc^3d^3)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx-\frac{1}{4}(3bc^3d^3)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx \\
&\quad +\frac{1}{16}(15bc^3d^3)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx
\end{aligned}$$

$$= -\frac{3}{32}bc^3d^3x\sqrt{1+c^2x^2} + \frac{7}{16}bc^3d^3x\left(1+c^2x^2\right)^{3/2} - \frac{bcd^3\left(1+c^2x^2\right)^{5/2}}{2x} - \frac{3}{32}bc^2d^3\text{arcsinh}(cx) + \frac{3}{2}c^2d^3\left(1+c^2x^2\right)\left(a+b\text{arcsinh}(cx)\right) + \frac{3}{4}c^2d^3\left(1+c^2x^2\right)^2\left(a+b\text{arcsinh}(cx)\right) - \frac{d^3\left(1+c^2x^2\right)^3\left(a+b\text{arcsinh}(cx)\right)}{2x^2} + \frac{3c^2d^3(a+b\text{arcsinh}(cx))^2}{2b} + \frac{\left(6c^2d^3\right)\text{Subst}\left(\int \frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b}$$

$$= -\frac{3}{32}bc^3d^3x\sqrt{1+c^2x^2} + \frac{7}{16}bc^3d^3x\left(1+c^2x^2\right)^{3/2} - \frac{bcd^3\left(1+c^2x^2\right)^{5/2}}{2x} - \frac{3}{32}bc^2d^3\text{arcsinh}(cx) + \frac{3}{2}c^2d^3\left(1+c^2x^2\right)\left(a+b\text{arcsinh}(cx)\right) + \frac{3}{4}c^2d^3\left(1+c^2x^2\right)^2\left(a+b\text{arcsinh}(cx)\right) - \frac{d^3\left(1+c^2x^2\right)^3\left(a+b\text{arcsinh}(cx)\right)}{2x^2} + \frac{3c^2d^3(a+b\text{arcsinh}(cx))^2}{2b} + 3c^2d^3(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right) - \left(3c^2d^3\right)\text{Subst}\left(\int \log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)dx, x, a+b\text{arcsinh}(cx)\right)$$

$$= -\frac{3}{32}bc^3d^3x\sqrt{1+c^2x^2} + \frac{7}{16}bc^3d^3x\left(1+c^2x^2\right)^{3/2} - \frac{bcd^3\left(1+c^2x^2\right)^{5/2}}{2x} - \frac{3}{32}bc^2d^3\text{arcsinh}(cx) + \frac{3}{2}c^2d^3\left(1+c^2x^2\right)\left(a+b\text{arcsinh}(cx)\right) + \frac{3}{4}c^2d^3\left(1+c^2x^2\right)^2\left(a+b\text{arcsinh}(cx)\right) - \frac{d^3\left(1+c^2x^2\right)^3\left(a+b\text{arcsinh}(cx)\right)}{2x^2} + \frac{3c^2d^3(a+b\text{arcsinh}(cx))^2}{2b} + 3c^2d^3(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right) + \frac{1}{2}\left(3bc^2d^3\right)\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right)$$

$$= -\frac{3}{32}bc^3d^3x\sqrt{1+c^2x^2} + \frac{7}{16}bc^3d^3x\left(1+c^2x^2\right)^{3/2} - \frac{bcd^3\left(1+c^2x^2\right)^{5/2}}{2x} - \frac{3}{32}bc^2d^3\text{arcsinh}(cx) + \frac{3}{2}c^2d^3\left(1+c^2x^2\right)\left(a+b\text{arcsinh}(cx)\right) + \frac{3}{4}c^2d^3\left(1+c^2x^2\right)^2\left(a+b\text{arcsinh}(cx)\right) - \frac{d^3\left(1+c^2x^2\right)^3\left(a+b\text{arcsinh}(cx)\right)}{2x^2} + \frac{3c^2d^3(a+b\text{arcsinh}(cx))^2}{2b} + 3c^2d^3(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right) - \frac{3}{2}bc^2d^3\,\text{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right)$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.98

$$\int \frac{(d+c^2dx^2)^3\,(a+b\operatorname{arcsinh}(cx))}{x^3}\,dx = \frac{d^3\left(-16ab-48a^2c^2x^2+48abc^4x^4+8abc^6x^6-16b^2cx\sqrt{1+c^2x^2}-21b^2c^3x^3\sqrt{1+c^2x^2}-2b^2c^5x^5\sqrt{1+c^2x^2}\right.}{}$$

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (d^3*(-16*a*b - 48*a^2*c^2*x^2 + 48*a*b*c^4*x^4 + 8*a*b*c^6*x^6 - 16*b^2*c*
x*Sqrt[1 + c^2*x^2] - 21*b^2*c^3*x^3*Sqrt[1 + c^2*x^2] - 2*b^2*c^5*x^5*Sqrt
[1 + c^2*x^2] - 48*b^2*c^2*x^2*ArcSinh[c*x]^2 + 96*a*b*c^2*x^2*Log[1 - E^(2
*ArcSinh[c*x])] + b*ArcSinh[c*x]*(-96*a*c^2*x^2 + b*(-16 + 21*c^2*x^2 + 48*
c^4*x^4 + 8*c^6*x^6) + 96*b*c^2*x^2*Log[1 - E^(2*ArcSinh[c*x])]) + 48*b^2*c
^2*x^2*PolyLog[2, E^(2*ArcSinh[c*x])]))/(32*b*x^2)
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.16

method	result
derivativedivides	$c^2\left(d^3a\left(\frac{c^4x^4}{4}+\frac{3c^2x^2}{2}+3\ln(cx)-\frac{1}{2c^2x^2}\right)+\frac{d^3b}{2}+\frac{21b\,d^3\operatorname{arcsinh}(cx)}{32}-\frac{3d^3b\operatorname{arcsinh}(cx)^2}{2}+3d^3b\right.$
default	$c^2\left(d^3a\left(\frac{c^4x^4}{4}+\frac{3c^2x^2}{2}+3\ln(cx)-\frac{1}{2c^2x^2}\right)+\frac{d^3b}{2}+\frac{21b\,d^3\operatorname{arcsinh}(cx)}{32}-\frac{3d^3b\operatorname{arcsinh}(cx)^2}{2}+3d^3b\right.$
parts	$d^3a\left(\frac{c^6x^4}{4}+\frac{3c^4x^2}{2}-\frac{1}{2x^2}+3c^2\ln(x)\right)+\frac{d^3b\,c^2}{2}+3d^3b\,c^2\operatorname{polylog}\left(2,-cx-\sqrt{c^2x^2+1}\right)+$

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(d^3*a*(1/4*c^4*x^4+3/2*c^2*x^2+3*ln(c*x)-1/2/c^2/x^2)+1/2*d^3*b+21/32*
b*d^3*arcsinh(c*x)-3/2*d^3*b*arcsinh(c*x)^2+3*d^3*b*polylog(2,c*x+(c^2*x^2+
1)^(1/2))+3*d^3*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-1/16*d^3*b*c^3*x^3*(c^2
*x^2+1)^(1/2)-21/32*b*c*d^3*x*(c^2*x^2+1)^(1/2)+1/4*d^3*b*arcsinh(c*x)*c^4*
x^4+3/2*d^3*b*arcsinh(c*x)*c^2*x^2-1/2*d^3*b*arcsinh(c*x)/c^2/x^2-1/2*d^3*b
/c/x*(c^2*x^2+1)^(1/2)+3*d^3*b*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+3*d
^3*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))}{x^3} dx = \int \frac{(c^2dx^2+d)^3(b\,\text{arsinh}\,(cx)+a)}{x^3} dx$$

[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas")

[Out] integral((a*c^6*d^3*x^6 + 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 + a*d^3 + (b*c^6*d^3*x^6 + 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 + b*d^3)*arcsinh(c*x))/x^3, x)

Sympy [F]

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))}{x^3} dx = d^3\left(\int \frac{a}{x^3}\,dx+\int \frac{3ac^2}{x}\,dx+\int 3ac^4x\,dx+\int ac^6x^3\,dx + \int \frac{b\,\text{asinh}\,(cx)}{x^3}\,dx + \int \frac{3bc^2\,\text{asinh}\,(cx)}{x}\,dx + \int 3bc^4x\,\text{asinh}\,(cx)\,dx + \int bc^6x^3\,\text{asinh}\,(cx)\,dx\right)$$

[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))/x**3,x)

[Out] d**3*(Integral(a/x**3, x) + Integral(3*a*c**2/x, x) + Integral(3*a*c**4*x, x) + Integral(a*c**6*x**3, x) + Integral(b*asinh(c*x)/x**3, x) + Integral(3*b*c**2*asinh(c*x)/x, x) + Integral(3*b*c**4*x*asinh(c*x), x) + Integral(b*c**6*x**3*asinh(c*x), x))

Maxima [F]

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))}{x^3} dx = \int \frac{(c^2dx^2+d)^3(b\,\text{arsinh}\,(cx)+a)}{x^3} dx$$

[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima")

[Out] 1/4*a*c^6*d^3*x^4 + 3/2*a*c^4*d^3*x^2 + 3*a*c^2*d^3*log(x) - 1/2*b*d^3*(sqrt(c^2*x^2 + 1)*c/x + arcsinh(c*x)/x^2) - 1/2*a*d^3/x^2 + integrate(b*c^6*d^3*x^3*log(c*x + sqrt(c^2*x^2 + 1)) + 3*b*c^4*d^3*x*log(c*x + sqrt(c^2*x^2 + 1)) + 3*b*c^2*d^3*log(c*x + sqrt(c^2*x^2 + 1))/x, x)

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))}{x^3} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))}{x^3} dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^3}{x^3} dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x^3, x)
```

3.27 $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 174

$$\int \frac{(d+c^2dx^2)^3\,(a+b\text{arcsinh}(cx))}{x^4}\,dx = -\frac{8}{3}bc^3d^3\sqrt{1+c^2x^2}-\frac{bcd^3\sqrt{1+c^2x^2}}{6x^2} - \frac{1}{9}bc^3d^3\left(1+c^2x^2\right)^{3/2}-\frac{d^3(a+b\text{arcsinh}(cx))}{3x^3} - \frac{3c^2d^3(a+b\text{arcsinh}(cx))}{x} + 3c^4d^3x(a+b\text{arcsinh}(cx)) + \frac{1}{3}c^6d^3x^3(a+b\text{arcsinh}(cx)) - \frac{17}{6}bc^3d^3\text{arctanh}\left(\sqrt{1+c^2x^2}\right)$$

```
[Out] -1/9*b*c^3*d^3*(c^2*x^2+1)^(3/2)-1/3*d^3*(a+b*arcsinh(c*x))/x^3-3*c^2*d^3*(
a+b*arcsinh(c*x))/x+3*c^4*d^3*x*(a+b*arcsinh(c*x))+1/3*c^6*d^3*x^3*(a+b*arc
sinh(c*x))-17/6*b*c^3*d^3*arctanh((c^2*x^2+1)^(1/2))-8/3*b*c^3*d^3*(c^2*x^2
+1)^(1/2)-1/6*b*c*d^3*(c^2*x^2+1)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

= {276, 5803, 12, 1813, 1635, 911, 1167, 214}

$$\int \frac{(d+c^2dx^2)^3\,(a+b\operatorname{arcsinh}(cx))}{x^4}\,dx = \frac{1}{3}c^6d^3x^3(a+b\operatorname{arcsinh}(cx))+3c^4d^3x(a+b\operatorname{arcsinh}(cx)) - \frac{3c^2d^3(a+b\operatorname{arcsinh}(cx))}{x} - \frac{d^3(a+b\operatorname{arcsinh}(cx))}{3x^3} - \frac{17}{6}bc^3d^3\operatorname{arctanh}\left(\sqrt{c^2x^2+1}\right) - \frac{bcd^3\sqrt{c^2x^2+1}}{6x^2} - \frac{1}{9}bc^3d^3\left(c^2x^2+1\right)^{3/2} - \frac{8}{3}bc^3d^3\sqrt{c^2x^2+1}$$

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] (-8*b*c^3*d^3*Sqrt[1 + c^2*x^2])/3 - (b*c*d^3*Sqrt[1 + c^2*x^2])/(6*x^2) -
(b*c^3*d^3*(1 + c^2*x^2)^(3/2))/9 - (d^3*(a + b*ArcSinh[c*x]))/(3*x^3) - (3
*c^2*d^3*(a + b*ArcSinh[c*x]))/x + 3*c^4*d^3*x*(a + b*ArcSinh[c*x]) + (c^6*
d^3*x^3*(a + b*ArcSinh[c*x]))/3 - (17*b*c^3*d^3*ArcTanh[Sqrt[1 + c^2*x^2]])
/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1635

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^3(a+b\text{arcsinh}(cx))}{3x^3} - \frac{3c^2d^3(a+b\text{arcsinh}(cx))}{x} + 3c^4d^3x(a+b\text{arcsinh}(cx)) \\
&\quad + \frac{1}{3}c^6d^3x^3(a+b\text{arcsinh}(cx)) - (bc)\int \frac{d^3(-1-9c^2x^2+9c^4x^4+c^6x^6)}{3x^3\sqrt{1+c^2x^2}}\,dx \\
&= -\frac{d^3(a+b\text{arcsinh}(cx))}{3x^3} - \frac{3c^2d^3(a+b\text{arcsinh}(cx))}{x} + 3c^4d^3x(a+b\text{arcsinh}(cx)) \\
&\quad + \frac{1}{3}c^6d^3x^3(a+b\text{arcsinh}(cx)) - \frac{1}{3}\left(bcd^3\right)\int \frac{-1-9c^2x^2+9c^4x^4+c^6x^6}{x^3\sqrt{1+c^2x^2}}\,dx \\
&= -\frac{d^3(a+b\text{arcsinh}(cx))}{3x^3} - \frac{3c^2d^3(a+b\text{arcsinh}(cx))}{x} + 3c^4d^3x(a+b\text{arcsinh}(cx)) \\
&\quad + \frac{1}{3}c^6d^3x^3(a+b\text{arcsinh}(cx)) - \frac{1}{6}\left(bcd^3\right)\text{Subst}\left(\int \frac{-1-9c^2x+9c^4x^2+c^6x^3}{x^2\sqrt{1+c^2x}}\,dx, x, x^2\right)
\end{aligned}$$

$$= -\frac{bcd^3\sqrt{1+c^2x^2}}{6x^2} - \frac{d^3(a+b\text{arcsinh}(cx))}{3x^3} - \frac{3c^2d^3(a+b\text{arcsinh}(cx))}{x} + 3c^4d^3x(a+b\text{arcsinh}(cx)) + \frac{1}{3}c^6d^3x^3(a+b\text{arcsinh}(cx)) + \frac{1}{6}(bcd^3)\,\text{Subst}\left(\int \frac{\frac{17c^2}{2} - 9c^4x - c^6x^2}{x\sqrt{1+c^2x}}\,dx, x, x^2\right)$$

$$= -\frac{bcd^3\sqrt{1+c^2x^2}}{6x^2} - \frac{d^3(a+b\text{arcsinh}(cx))}{3x^3} - \frac{3c^2d^3(a+b\text{arcsinh}(cx))}{x} + 3c^4d^3x(a+b\text{arcsinh}(cx)) + \frac{1}{3}c^6d^3x^3(a+b\text{arcsinh}(cx)) + \frac{(bd^3)\,\text{Subst}\left(\int \frac{\frac{33c^2}{2} - 7c^2x^2 - c^2x^4}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx, x, \sqrt{1+c^2x^2}\right)}{3c}$$

$$= -\frac{bcd^3\sqrt{1+c^2x^2}}{6x^2} - \frac{d^3(a+b\text{arcsinh}(cx))}{3x^3} - \frac{3c^2d^3(a+b\text{arcsinh}(cx))}{x} + 3c^4d^3x(a+b\text{arcsinh}(cx)) + \frac{1}{3}c^6d^3x^3(a+b\text{arcsinh}(cx)) + \frac{(bd^3)\,\text{Subst}\left(\int \left(-8c^4 - c^4x^2 + \frac{17c^2}{2\left(-\frac{1}{c^2}+\frac{x^2}{c^2}\right)}\right)dx, x, \sqrt{1+c^2x^2}\right)}{3c}$$

$$= -\frac{8}{3}bc^3d^3\sqrt{1+c^2x^2} - \frac{bcd^3\sqrt{1+c^2x^2}}{6x^2} - \frac{1}{9}bc^3d^3\left(1+c^2x^2\right)^{3/2} - \frac{d^3(a+b\text{arcsinh}(cx))}{3x^3} - \frac{3c^2d^3(a+b\text{arcsinh}(cx))}{x} + 3c^4d^3x(a+b\text{arcsinh}(cx)) + \frac{1}{3}c^6d^3x^3(a+b\text{arcsinh}(cx)) + \frac{1}{6}(17bcd^3)\,\text{Subst}\left(\int \frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx, x, \sqrt{1+c^2x^2}\right)$$

$$= -\frac{8}{3}bc^3d^3\sqrt{1+c^2x^2} - \frac{bcd^3\sqrt{1+c^2x^2}}{6x^2} - \frac{1}{9}bc^3d^3\left(1+c^2x^2\right)^{3/2} - \frac{d^3(a+b\text{arcsinh}(cx))}{3x^3} - \frac{3c^2d^3(a+b\text{arcsinh}(cx))}{x} + 3c^4d^3x(a+b\text{arcsinh}(cx)) + \frac{1}{3}c^6d^3x^3(a+b\text{arcsinh}(cx)) - \frac{17}{6}bc^3d^3\text{arctanh}\left(\sqrt{1+c^2x^2}\right)$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.98

$$\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))}{x^4}\,dx$$
$$=\frac{d^3\left(-6a-54ac^2x^2+54ac^4x^4+6ac^6x^6-3bcx\sqrt{1+c^2x^2}-50bc^3x^3\sqrt{1+c^2x^2}-2bc^5x^5\sqrt{1+c^2x^2}+6b(-1\right.}{18x^3}$$

[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^4,x]

[Out] (d^3*(-6*a - 54*a*c^2*x^2 + 54*a*c^4*x^4 + 6*a*c^6*x^6 - 3*b*c*x*Sqrt[1 + c^2*x^2] - 50*b*c^3*x^3*Sqrt[1 + c^2*x^2] - 2*b*c^5*x^5*Sqrt[1 + c^2*x^2] + 6*b*(-1 - 9*c^2*x^2 + 9*c^4*x^4 + c^6*x^6)*ArcSinh[c*x] + 51*b*c^3*x^3*Log[x] - 51*b*c^3*x^3*Log[1 + Sqrt[1 + c^2*x^2]]))/(18*x^3)

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.88

method	result
parts	$d^3a\left(\frac{c^6x^3}{3}+3c^4x-\frac{3c^2}{x}-\frac{1}{3x^3}\right)+d^3b\,c^3\left(\frac{\text{arcsinh}(cx)c^3x^3}{3}+3\,\text{arcsinh}\,(cx)\,cx-\frac{\text{arcsinh}(cx)}{3c^3x^3}-\frac{3}{}\right.$
derivativedivides	$c^3\left(d^3a\left(\frac{c^3x^3}{3}+3cx-\frac{1}{3c^3x^3}-\frac{3}{cx}\right)+d^3b\left(\frac{\text{arcsinh}(cx)c^3x^3}{3}+3\,\text{arcsinh}\,(cx)\,cx-\frac{\text{arcsinh}(cx)}{3c^3x^3}-\right.\right.$
default	$c^3\left(d^3a\left(\frac{c^3x^3}{3}+3cx-\frac{1}{3c^3x^3}-\frac{3}{cx}\right)+d^3b\left(\frac{\text{arcsinh}(cx)c^3x^3}{3}+3\,\text{arcsinh}\,(cx)\,cx-\frac{\text{arcsinh}(cx)}{3c^3x^3}-\right.\right.$

[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^4,x,method=_RETURNVERBOSE)

[Out] d^3*a*(1/3*c^6*x^3+3*c^4*x-3*c^2/x-1/3/x^3)+d^3*b*c^3*(1/3*arcsinh(c*x)*c^3*x^3+3*arcsinh(c*x)*c*x-1/3*arcsinh(c*x)/c^3/x^3-3*arcsinh(c*x)/c/x-1/9*c^2*x^2*(c^2*x^2+1)^(1/2)-25/9*(c^2*x^2+1)^(1/2)-1/6/c^2/x^2*(c^2*x^2+1)^(1/2)-17/6*arctanh(1/(c^2*x^2+1)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.66

$$\int \frac{(d+c^2dx^2)^3 (a+b\operatorname{arcsinh}(cx))}{x^4} dx = \frac{6\,ac^6d^3x^6+54\,ac^4d^3x^4-51\,bc^3d^3x^3\log\left(-cx+\sqrt{c^2x^2+1}+1\right)+51\,bc^3d^3x^3\log\left(-cx+\sqrt{c^2x^2+1}-1\right)}{}$$

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/18*(6*a*c^6*d^3*x^6 + 54*a*c^4*d^3*x^4 - 51*b*c^3*d^3*x^3*log(-c*x + sqrt
(c^2*x^2 + 1) + 1) + 51*b*c^3*d^3*x^3*log(-c*x + sqrt(c^2*x^2 + 1) - 1) - 5
4*a*c^2*d^3*x^2 - 6*(b*c^6 + 9*b*c^4 - 9*b*c^2 - b)*d^3*x^3*log(-c*x + sqrt
(c^2*x^2 + 1)) - 6*a*d^3 + 6*(b*c^6*d^3*x^6 + 9*b*c^4*d^3*x^4 - 9*b*c^2*d^3
*x^2 - (b*c^6 + 9*b*c^4 - 9*b*c^2 - b)*d^3*x^3 - b*d^3)*log(c*x + sqrt(c^2*
x^2 + 1)) - (2*b*c^5*d^3*x^5 + 50*b*c^3*d^3*x^3 + 3*b*c*d^3*x)*sqrt(c^2*x^2
 + 1))/x^3
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^3 (a+b\operatorname{arcsinh}(cx))}{x^4} dx = d^3\left(\int 3ac^4\,dx+\int \frac{a}{x^4}\,dx+\int \frac{3ac^2}{x^2}\,dx+\int ac^6x^2\,dx + \int 3bc^4\operatorname{asinh}(cx)\,dx+\int \frac{b\operatorname{asinh}(cx)}{x^4}\,dx + \int \frac{3bc^2\operatorname{asinh}(cx)}{x^2}\,dx+\int bc^6x^2\operatorname{asinh}(cx)\,dx\right)$$

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))/x**4,x)
```

```
[Out] d**3*(Integral(3*a*c**4, x) + Integral(a/x**4, x) + Integral(3*a*c**2/x**2,
 x) + Integral(a*c**6*x**2, x) + Integral(3*b*c**4*asinh(c*x), x) + Integra
l(b*asinh(c*x)/x**4, x) + Integral(3*b*c**2*asinh(c*x)/x**2, x) + Integral(
b*c**6*x**2*asinh(c*x), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.20

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))}{x^4}\,dx$$
$$= \frac{1}{3}\,ac^6d^3x^3 + \frac{1}{9}\left(3\,x^3\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)bc^6d^3 + 3\,ac^4d^3x$$
$$+ 3\left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)bc^3d^3 - 3\left(c\,\text{arsinh}\left(\frac{1}{c|x|}\right) + \frac{\text{arsinh}\,(cx)}{x}\right)bc^2d^3$$
$$+ \frac{1}{6}\left(\left(c^2\,\text{arsinh}\left(\frac{1}{c|x|}\right) - \frac{\sqrt{c^2x^2+1}}{x^2}\right)c - \frac{2\,\text{arsinh}\,(cx)}{x^3}\right)bd^3 - \frac{3\,ac^2d^3}{x} - \frac{ad^3}{3\,x^3}$$

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*a*c^6*d^3*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2
- 2*sqrt(c^2*x^2 + 1)/c^4))*b*c^6*d^3 + 3*a*c^4*d^3*x + 3*(c*x*arcsinh(c*x)
 - sqrt(c^2*x^2 + 1))*b*c^3*d^3 - 3*(c*arcsinh(1/(c*abs(x))) + arcsinh(c*x)
/x)*b*c^2*d^3 + 1/6*((c^2*arcsinh(1/(c*abs(x))) - sqrt(c^2*x^2 + 1)/x^2)*c
- 2*arcsinh(c*x)/x^3)*b*d^3 - 3*a*c^2*d^3/x - 1/3*a*d^3/x^3
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))}{x^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^3\,(a+b\text{arcsinh}(cx))}{x^4}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^3}{x^4}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x^4,x)
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x^4, x)
```

3.28 $\int \frac{x^4(a+b\,\mathbf{arcsinh}(cx))}{d+c^2dx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 156

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{d+c^2dx^2}\,dx = \frac{4b\sqrt{1+c^2x^2}}{3c^5d} - \frac{b(1+c^2x^2)^{3/2}}{9c^5d} - \frac{x(a+b\operatorname{arcsinh}(cx))}{c^4d} + \frac{x^3(a+b\operatorname{arcsinh}(cx))}{3c^2d} + \frac{2(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c^5d} - \frac{ib\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{c^5d} + \frac{ib\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{c^5d}$$

```
[Out] -1/9*b*(c^2*x^2+1)^(3/2)/c^5/d-x*(a+b*arcsinh(c*x))/c^4/d+1/3*x^3*(a+b*arcs
inh(c*x))/c^2/d+2*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/c^5/d-I*
b*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d+I*b*polylog(2,I*(c*x+(c^2*x^2
+1)^(1/2)))/c^5/d+4/3*b*(c^2*x^2+1)^(1/2)/c^5/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5812, 5789, 4265, 2317, 2438, 267, 272, 45}

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{d+c^2dx^2}\,dx = \frac{2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx))}{c^5d} - \frac{x(a+b\operatorname{arcsinh}(cx))}{c^4d} + \frac{x^3(a+b\operatorname{arcsinh}(cx))}{3c^2d} - \frac{ib\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{c^5d} + \frac{ib\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{c^5d} - \frac{b(c^2x^2+1)^{3/2}}{9c^5d} + \frac{4b\sqrt{c^2x^2+1}}{3c^5d}$$

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] (4*b*Sqrt[1 + c^2*x^2])/(3*c^5*d) - (b*(1 + c^2*x^2)^(3/2))/(9*c^5*d) - (x*
(a + b*ArcSinh[c*x]))/(c^4*d) + (x^3*(a + b*ArcSinh[c*x]))/(3*c^2*d) + (2*(
a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^5*d) - (I*b*PolyLog[2, (-I)*
E^ArcSinh[c*x]])/(c^5*d) + (I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d} - \frac{\int \frac{x^2(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx}{c^2} - \frac{b\int \frac{x^3}{\sqrt{1+c^2x^2}}\,dx}{3cd} \\
&= -\frac{x(a+b\text{arcsinh}(cx))}{c^4d} + \frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d} + \frac{\int \frac{a+b\text{arcsinh}(cx)}{d+c^2dx^2}\,dx}{c^4} \\
&\quad + \frac{b\int \frac{x}{\sqrt{1+c^2x^2}}\,dx}{c^3d} - \frac{b\text{Subst}\left(\int \frac{x}{\sqrt{1+c^2x}}\,dx, x, x^2\right)}{6cd} \\
&= \frac{b\sqrt{1+c^2x^2}}{c^5d} - \frac{x(a+b\text{arcsinh}(cx))}{c^4d} + \frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d} \\
&\quad + \frac{\text{Subst}(\int (a+bx)\text{sech}(x)\,dx, x, \text{arcsinh}(cx))}{c^5d} \\
&\quad - \frac{b\text{Subst}\left(\int \left(-\frac{1}{c^2\sqrt{1+c^2x}} + \frac{\sqrt{1+c^2x}}{c^2}\right)\,dx, x, x^2\right)}{6cd} \\
&= \frac{4b\sqrt{1+c^2x^2}}{3c^5d} - \frac{b(1+c^2x^2)^{3/2}}{9c^5d} - \frac{x(a+b\text{arcsinh}(cx))}{c^4d} \\
&\quad + \frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d} + \frac{2(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^5d} \\
&\quad - \frac{(ib)\text{Subst}\left(\int \log\left(1-ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{c^5d} \\
&\quad + \frac{(ib)\text{Subst}\left(\int \log\left(1+ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{c^5d}
\end{aligned}
$$

$$= \frac{4b\sqrt{1+c^2x^2}}{3c^5d} - \frac{b(1+c^2x^2)^{3/2}}{9c^5d} - \frac{x(a+b\text{arcsinh}(cx))}{c^4d} + \frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d} + \frac{2(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^5d} - \frac{(ib)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{c^5d} + \frac{(ib)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{c^5d}$$

$$= \frac{4b\sqrt{1+c^2x^2}}{3c^5d} - \frac{b(1+c^2x^2)^{3/2}}{9c^5d} - \frac{x(a+b\text{arcsinh}(cx))}{c^4d} + \frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d} + \frac{2(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^5d} - \frac{ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^5d} + \frac{ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^5d}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.09

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx = \frac{-9acx+3ac^3x^3+11b\sqrt{1+c^2x^2}-bc^2x^2\sqrt{1+c^2x^2}-9bcx\text{arcsinh}(cx)+3bc^3x^3\text{arcsinh}(cx)+9a\arctan(c}{}$$

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
[Out] (-9*a*c*x + 3*a*c^3*x^3 + 11*b*Sqrt[1 + c^2*x^2] - b*c^2*x^2*Sqrt[1 + c^2*x
^2] - 9*b*c*x*ArcSinh[c*x] + 3*b*c^3*x^3*ArcSinh[c*x] + 9*a*ArcTan[c*x] + (
9*I)*b*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (9*I)*b*ArcSinh[c*x]*Log[1
+ I*E^ArcSinh[c*x]] - (9*I)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]] + (9*I)*b*Pol
yLog[2, I*E^ArcSinh[c*x]])/(9*c^5*d)
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{\frac{a\left(\frac{c^3x^3}{3}-cx+\arctan(cx)\right)}{d}+\frac{b\left(\frac{\text{arcsinh}(cx)c^3x^3}{3}-\text{arcsinh}(cx)cx+\text{arcsinh}(cx)\arctan(cx)-\frac{c^2x^2\sqrt{c^2x^2+1}}{9}+\frac{11\sqrt{c^2x^2+1}}{9}+\arctan(cx)\right.}{d}}{c^5}$
default	$\frac{\frac{a\left(\frac{c^3x^3}{3}-cx+\arctan(cx)\right)}{d}+\frac{b\left(\frac{\text{arcsinh}(cx)c^3x^3}{3}-\text{arcsinh}(cx)cx+\text{arcsinh}(cx)\arctan(cx)-\frac{c^2x^2\sqrt{c^2x^2+1}}{9}+\frac{11\sqrt{c^2x^2+1}}{9}+\arctan(cx)\right.}{d}}{c^5}$
parts	$\frac{a\left(\frac{\frac{1}{3}x^3c^2-x}{c^4}+\frac{\arctan(cx)}{c^5}\right)}{d}+\frac{b\left(\frac{\text{arcsinh}(cx)c^3x^3}{3}-\text{arcsinh}(cx)cx+\text{arcsinh}(cx)\arctan(cx)-\frac{c^2x^2\sqrt{c^2x^2+1}}{9}+\frac{11\sqrt{c^2x^2+1}}{9}+\right.}{}$

[In] int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)

[Out] 1/c^5*(a/d*(1/3*c^3*x^3-c*x+arctan(c*x))+b/d*(1/3*arcsinh(c*x)*c^3*x^3-arcsinh(c*x)*c*x+arcsinh(c*x)*arctan(c*x)-1/9*c^2*x^2*(c^2*x^2+1)^(1/2)+11/9*(c^2*x^2+1)^(1/2)+arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))))

Fricas [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^4}{c^2dx^2+d}\,dx$$

[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")

[Out] integral((b*x^4*arcsinh(c*x) + a*x^4)/(c^2*d*x^2 + d), x)

Sympy [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx=\frac{\int \frac{ax^4}{c^2x^2+1}\,dx+\int \frac{bx^4\,\text{asinh}\,(cx)}{c^2x^2+1}\,dx}{d}$$

[In] integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d),x)

[Out] (Integral(a*x**4/(c**2*x**2 + 1), x) + Integral(b*x**4*asinh(c*x)/(c**2*x**2 + 1), x))/d

Maxima [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^4}{c^2dx^2+d}\,dx$$

[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")

[Out] 1/3*a*((c^2*x^3 - 3*x)/(c^4*d) + 3*arctan(c*x)/(c^5*d)) + b*integrate(x^4*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d), x)

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a + b\text{arcsinh}(cx))}{d + c^2dx^2} dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b\text{arcsinh}(cx))}{d + c^2dx^2} dx = \int \frac{x^4 (a + b \,\text{asinh}(c\, x))}{d\, c^2\, x^2 + d} dx$$

```
[In] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2),x)
```

```
[Out] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2), x)
```

3.29 $\int \frac{x^3(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 135

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx = -\frac{bx\sqrt{1+c^2x^2}}{4c^3d}+\frac{b\text{arcsinh}(cx)}{4c^4d}+\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d}+\frac{(a+b\text{arcsinh}(cx))^2}{2bc^4d}-\frac{(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^4d}-\frac{b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{2c^4d}$$

```
[Out] 1/4*b*arcsinh(c*x)/c^4/d+1/2*x^2*(a+b*arcsinh(c*x))/c^2/d+1/2*(a+b*arcsinh(
c*x))^2/b/c^4/d-(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c^4/d-1/
2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c^4/d-1/4*b*x*(c^2*x^2+1)^(1/2)/c
^3/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5812, 5797, 3799, 2221, 2317, 2438, 327, 221}

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx = \frac{(a+b\text{arcsinh}(cx))^2}{2bc^4d}-\frac{\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))}{c^4d}+\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d}-\frac{b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{2c^4d}+\frac{b\text{arcsinh}(cx)}{4c^4d}-\frac{bx\sqrt{c^2x^2+1}}{4c^3d}$$

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] -1/4*(b*x*Sqrt[1 + c^2*x^2])/(c^3*d) + (b*ArcSinh[c*x])/(4*c^4*d) + (x^2*(a
 + b*ArcSinh[c*x]))/(2*c^2*d) + (a + b*ArcSinh[c*x])^2/(2*b*c^4*d) - ((a +
b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c^4*d) - (b*PolyLog[2, -E^(2*
ArcSinh[c*x])])/(2*c^4*d)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d} - \frac{\int \frac{x(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx}{c^2} - \frac{b\int \frac{x^2}{\sqrt{1+c^2x^2}}\,dx}{2cd} \\
&= -\frac{bx\sqrt{1+c^2x^2}}{4c^3d} + \frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d} \\
&\quad - \frac{\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \text{arcsinh}(cx)\right)}{c^4d} + \frac{b\int \frac{1}{\sqrt{1+c^2x^2}}\,dx}{4c^3d} \\
&= -\frac{bx\sqrt{1+c^2x^2}}{4c^3d} + \frac{b\text{arcsinh}(cx)}{4c^4d} + \frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d} \\
&\quad + \frac{(a+b\text{arcsinh}(cx))^2}{2bc^4d} - \frac{2\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \text{arcsinh}(cx)\right)}{c^4d} \\
&= -\frac{bx\sqrt{1+c^2x^2}}{4c^3d} + \frac{b\text{arcsinh}(cx)}{4c^4d} + \frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d} + \frac{(a+b\text{arcsinh}(cx))^2}{2bc^4d} \\
&\quad - \frac{(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^4d} + \frac{b\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{c^4d} \\
&= -\frac{bx\sqrt{1+c^2x^2}}{4c^3d} + \frac{b\text{arcsinh}(cx)}{4c^4d} + \frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d} + \frac{(a+b\text{arcsinh}(cx))^2}{2bc^4d} \\
&\quad - \frac{(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^4d} + \frac{b\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\text{arcsinh}(cx)}\right)}{2c^4d} \\
&= -\frac{bx\sqrt{1+c^2x^2}}{4c^3d} + \frac{b\text{arcsinh}(cx)}{4c^4d} + \frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d} + \frac{(a+b\text{arcsinh}(cx))^2}{2bc^4d} \\
&\quad - \frac{(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^4d} - \frac{b\,\text{PolyLog}\left(2, -e^{2\text{arcsinh}(cx)}\right)}{2c^4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.34

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx =$$

$$-\frac{-2ac^2x^2+bcx\sqrt{1+c^2x^2}-b\text{arcsinh}(cx)-2bc^2x^2\text{arcsinh}(cx)-2b\text{arcsinh}(cx)^2+4b\text{arcsinh}(cx)\log\Big(1}{}$$

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] -1/4*(-2*a*c^2*x^2 + b*c*x*Sqrt[1 + c^2*x^2] - b*ArcSinh[c*x] - 2*b*c^2*x^2
*ArcSinh[c*x] - 2*b*ArcSinh[c*x]^2 + 4*b*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[
c*x])/Sqrt[-c^2]] + 4*b*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c]
 + 2*a*Log[1 + c^2*x^2] + 4*b*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 4
*b*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(c^4*d)
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{\frac{a\left(\frac{c^2x^2}{2}-\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d}+\frac{b\,\text{arcsinh}(cx)^2}{2d}+\frac{b\,\text{arcsinh}(cx)c^2x^2}{2d}-\frac{bcx\sqrt{c^2x^2+1}}{4d}+\frac{b\,\text{arcsinh}(cx)}{4d}-\frac{b\,\text{arcsinh}(cx)\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right.\right.}{d}}{c^4}$
default	$\frac{\frac{a\left(\frac{c^2x^2}{2}-\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d}+\frac{b\,\text{arcsinh}(cx)^2}{2d}+\frac{b\,\text{arcsinh}(cx)c^2x^2}{2d}-\frac{bcx\sqrt{c^2x^2+1}}{4d}+\frac{b\,\text{arcsinh}(cx)}{4d}-\frac{b\,\text{arcsinh}(cx)\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right.\right.}{d}}{c^4}$
parts	$\frac{a\left(\frac{x^2}{2c^2}-\frac{\ln\left(c^2x^2+1\right)}{2c^4}\right)}{d}+\frac{b\,\text{arcsinh}(cx)^2}{2d\,c^4}+\frac{b\,\text{arcsinh}(cx)x^2}{2d\,c^2}-\frac{bx\sqrt{c^2x^2+1}}{4c^3d}+\frac{b\,\text{arcsinh}(cx)}{4c^4d}-\frac{b\,\text{arcsinh}(cx)\ln\left(1\right.}{}$

```
[In] int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(a/d*(1/2*c^2*x^2-1/2*ln(c^2*x^2+1))+1/2*b/d*arcsinh(c*x)^2+1/2*b/d*a
rcsinh(c*x)*c^2*x^2-1/4*b/d*c*x*(c^2*x^2+1)^(1/2)+1/4*b/d*arcsinh(c*x)-b/d*
arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b*polylog(2,-(c*x+(c^2*x^2
+1)^(1/2))^2)/d)
```

Fricas [F]

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{d+c^2dx^2}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x^3}{c^2dx^2+d}\,dx$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
[Out] integral((b*x^3*arcsinh(c*x) + a*x^3)/(c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{d+c^2dx^2}\,dx = \frac{\int \frac{ax^3}{c^2x^2+1}\,dx + \int \frac{bx^3\operatorname{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

```
[In] integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d),x)
[Out] (Integral(a*x**3/(c**2*x**2 + 1), x) + Integral(b*x**3*asinh(c*x)/(c**2*x**
2 + 1), x))/d
```

Maxima [F]

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{d+c^2dx^2}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x^3}{c^2dx^2+d}\,dx$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
[Out] 1/2*a*(x^2/(c^2*d) - log(c^2*x^2 + 1)/(c^4*d)) - 1/8*b*((2*c^2*x^2 - log(c^
2*x^2 + 1)^2 - 4*(c^2*x^2 - log(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))
 - 2*log(c^2*x^2 + 1))/(c^4*d) - 8*integrate(-1/2*(c^2*x^2 - log(c^2*x^2 + 1
))/(c^6*d*x^3 + c^4*d*x + (c^5*d*x^2 + c^3*d)*sqrt(c^2*x^2 + 1)), x))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{d+c^2dx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{d+c^2dx^2}\,dx=\int \frac{x^3\,\left(a+b\,\mathrm{asinh}(c\,x)\right)}{d\,c^2\,x^2+d}\,dx$$

```
[In] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2),x)
[Out] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2), x)
```

3.30 $\int \frac{x^2(a+b\,\mathbf{arcsinh}(cx))}{d+c^2dx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 108

$$\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{d+c^2dx^2}\,dx = -\frac{b\sqrt{1+c^2x^2}}{c^3d}+\frac{x(a+b\operatorname{arcsinh}(cx))}{c^2d} - \frac{2(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c^3d} + \frac{ib\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{c^3d}-\frac{ib\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{c^3d}$$

```
[Out] x*(a+b*arcsinh(c*x))/c^2/d-2*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2
))/c^3/d+I*b*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d-I*b*polylog(2,I*(c
*x+(c^2*x^2+1)^(1/2)))/c^3/d-b*(c^2*x^2+1)^(1/2)/c^3/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5812, 5789, 4265, 2317, 2438, 267}

$$\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{d+c^2dx^2}\,dx = -\frac{2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx))}{c^3d} + \frac{x(a+b\operatorname{arcsinh}(cx))}{c^2d}+\frac{ib\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{c^3d} - \frac{ib\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{c^3d}-\frac{b\sqrt{c^2x^2+1}}{c^3d}$$

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] -((b*Sqrt[1 + c^2*x^2])/(c^3*d)) + (x*(a + b*ArcSinh[c*x]))/(c^2*d) - (2*(a
 + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^3*d) + (I*b*PolyLog[2, (-I)*E
^ArcSinh[c*x]])/(c^3*d) - (I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c^3*d)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{x(a+b\text{arcsinh}(cx))}{c^2d} - \frac{\int \frac{a+b\text{arcsinh}(cx)}{d+c^2dx^2}\,dx}{c^2} - \frac{b\int \frac{x}{\sqrt{1+c^2x^2}}\,dx}{cd}$$

$$= -\frac{b\sqrt{1+c^2x^2}}{c^3d} + \frac{x(a+b\text{arcsinh}(cx))}{c^2d} - \frac{\text{Subst}\left(\int (a+bx)\text{sech}(x)\,dx, x, \text{arcsinh}(cx)\right)}{c^3d}$$

$$= -\frac{b\sqrt{1+c^2x^2}}{c^3d} + \frac{x(a+b\text{arcsinh}(cx))}{c^2d} - \frac{2(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^3d} + \frac{(ib)\text{Subst}\left(\int \log\left(1-ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{c^3d} - \frac{(ib)\text{Subst}\left(\int \log\left(1+ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{c^3d}$$

$$= -\frac{b\sqrt{1+c^2x^2}}{c^3d} + \frac{x(a+b\text{arcsinh}(cx))}{c^2d} - \frac{2(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^3d} + \frac{(ib)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{c^3d} - \frac{(ib)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{c^3d}$$

$$= -\frac{b\sqrt{1+c^2x^2}}{c^3d} + \frac{x(a+b\text{arcsinh}(cx))}{c^2d} - \frac{2(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^3d} + \frac{ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^3d} - \frac{ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^3d}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx = \frac{acx - b\sqrt{1+c^2x^2} + bcx\text{arcsinh}(cx) - a\arctan(cx) - ib\text{arcsinh}(cx)\log\left(1-ie^{\text{arcsinh}(cx)}\right) + ib\text{arcsinh}(cx)\,\text{lo}}{c^3d}$$

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] (a*c*x - b*Sqrt[1 + c^2*x^2] + b*c*x*ArcSinh[c*x] - a*ArcTan[c*x] - I*b*Arc
Sinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + I*b*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[
c*x]] + I*b*PolyLog[2, (-I)*E^ArcSinh[c*x]] - I*b*PolyLog[2, I*E^ArcSinh[c*
x]])/(c^3*d)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.57

method	result
derivativedivides	$\frac{\frac{a(cx-\arctan(cx))}{d}+\frac{b\left(-\operatorname{arcsinh}(cx)\arctan(cx)+\operatorname{arcsinh}(cx)cx-\sqrt{c^2x^2+1}-\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)+\arctan(cx)\ln\left(1-\frac{i(icx+1)}{\sqrt{c^2x^2-}}\right.}{d}}{c^3}$
default	$\frac{\frac{a(cx-\arctan(cx))}{d}+\frac{b\left(-\operatorname{arcsinh}(cx)\arctan(cx)+\operatorname{arcsinh}(cx)cx-\sqrt{c^2x^2+1}-\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)+\arctan(cx)\ln\left(1-\frac{i(icx+1)}{\sqrt{c^2x^2-}}\right.}{d}}{c^3}$
parts	$\frac{a\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{d}+\frac{b\left(-\operatorname{arcsinh}(cx)\arctan(cx)+\operatorname{arcsinh}(cx)cx-\sqrt{c^2x^2+1}-\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)+\arctan(c\right.}{d\,c^3}$

```
[In] int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(a/d*(c*x-arctan(c*x))+b/d*(-arcsinh(c*x)*arctan(c*x)+arcsinh(c*x)*c*
x-(c^2*x^2+1)^(1/2)-arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+arctan(
c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^
(1/2))-I*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{d+c^2dx^2}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)x^2}{c^2dx^2+d}\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arcsinh(c*x) + a*x^2)/(c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{d+c^2dx^2}\,dx=\frac{\int \frac{ax^2}{c^2x^2+1}\,dx+\int \frac{bx^2\operatorname{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

```
[In] integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a*x**2/(c**2*x**2 + 1), x) + Integral(b*x**2*asinh(c*x)/(c**2*x**
2 + 1), x))/d
```

Maxima [F]

$$\int \frac{x^2(a + b\text{arcsinh}(cx))}{d + c^2dx^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)x^2}{c^2dx^2 + d}\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] a*(x/(c^2*d) - arctan(c*x)/(c^3*d)) + b*integrate(x^2*log(c*x + sqrt(c^2*x^
2 + 1))/(c^2*d*x^2 + d), x)
```

Giac [F]

$$\int \frac{x^2(a + b\text{arcsinh}(cx))}{d + c^2dx^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)x^2}{c^2dx^2 + d}\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^2/(c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b\text{arcsinh}(cx))}{d + c^2dx^2}\,dx = \int \frac{x^2\,(a + b\,\text{asinh}(c\,x))}{d\,c^2\,x^2 + d}\,dx$$

```
[In] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2),x)
```

```
[Out] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2), x)
```

3.31 $\int \frac{x(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{x(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx = -\frac{(a+b\text{arcsinh}(cx))^2}{2bc^2d}+\frac{(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^2d}+\frac{b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{2c^2d}$$

```
[Out] -1/2*(a+b*arcsinh(c*x))^2/b/c^2/d+(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^
(1/2))^2)/c^2/d+1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c^2/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5797, 3799, 2221, 2317, 2438}

$$\int \frac{x(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx = -\frac{(a+b\text{arcsinh}(cx))^2}{2bc^2d}+\frac{\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))}{c^2d}+\frac{b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{2c^2d}$$

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] -1/2*(a + b*ArcSinh[c*x])^2/(b*c^2*d) + ((a + b*ArcSinh[c*x])*Log[1 + E^(2*
ArcSinh[c*x])])/(c^2*d) + (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/(2*c^2*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \text{arcsinh}(cx)\right)}{c^2d} \\
&= -\frac{(a+b\text{arcsinh}(cx))^2}{2bc^2d} + \frac{2\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \text{arcsinh}(cx)\right)}{c^2d} \\
&= -\frac{(a+b\text{arcsinh}(cx))^2}{2bc^2d} + \frac{(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^2d} \\
&\quad - \frac{b\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{c^2d} \\
&= -\frac{(a+b\text{arcsinh}(cx))^2}{2bc^2d} + \frac{(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^2d} \\
&\quad - \frac{b\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\text{arcsinh}(cx)}\right)}{2c^2d} \\
&= -\frac{(a+b\text{arcsinh}(cx))^2}{2bc^2d} + \frac{(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^2d} + \frac{b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{2c^2d}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 167 vs. 2(73) = 146.

Time = 0.05 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.29

$$\int \frac{x(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx = -\frac{b\text{arcsinh}(cx)^2}{2c^2d} + \frac{b\text{arcsinh}(cx)\log\left(1-\frac{\sqrt{-c^2}e^{\text{arcsinh}(cx)}}{c}\right)}{c^2d} + \frac{b\text{arcsinh}(cx)\log\left(1+\frac{\sqrt{-c^2}e^{\text{arcsinh}(cx)}}{c}\right)}{c^2d} + \frac{a\log(1+c^2x^2)}{2c^2d} + \frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{-c^2}e^{\text{arcsinh}(cx)}}{c}\right)}{c^2d} + \frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{-c^2}e^{\text{arcsinh}(cx)}}{c}\right)}{c^2d}$$

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] -1/2*(b*ArcSinh[c*x]^2)/(c^2*d) + (b*ArcSinh[c*x]*Log[1 - (Sqrt[-c^2]*E^Arc
Sinh[c*x])/c])/(c^2*d) + (b*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x]
)/c])/(c^2*d) + (a*Log[1 + c^2*x^2])/(2*c^2*d) + (b*PolyLog[2, -((Sqrt[-c^2
]*E^ArcSinh[c*x])/c)])/(c^2*d) + (b*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/
c])/(c^2*d)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

method	result	size
derivativedivides	$\frac{\frac{a\ln\left(c^2x^2+1\right)}{2d}+\frac{b\left(-\frac{\text{arcsinh}(cx)^2}{2}+\text{arcsinh}(cx)\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)+\frac{\text{polylog}\left(2,-\left(cx+\sqrt{c^2x^2+1}\right)^2\right)}{2}\right)}{d}}{c^2}$	84
default	$\frac{\frac{a\ln\left(c^2x^2+1\right)}{2d}+\frac{b\left(-\frac{\text{arcsinh}(cx)^2}{2}+\text{arcsinh}(cx)\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)+\frac{\text{polylog}\left(2,-\left(cx+\sqrt{c^2x^2+1}\right)^2\right)}{2}\right)}{d}}{c^2}$	84
parts	$\frac{a\ln\left(c^2x^2+1\right)}{2d\,c^2}+\frac{b\left(-\frac{\text{arcsinh}(cx)^2}{2}+\text{arcsinh}(cx)\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)+\frac{\text{polylog}\left(2,-\left(cx+\sqrt{c^2x^2+1}\right)^2\right)}{2}\right)}{d\,c^2}$	86

```
[In] int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/2*a/d*ln(c^2*x^2+1)+b/d*(-1/2*arcsinh(c*x)^2+arcsinh(c*x)*ln(1+(c*
x+(c^2*x^2+1)^(1/2))^2)+1/2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)))
```

Fricas [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x}{c^2dx^2+d}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
[Out] integral((b*x*arcsinh(c*x) + a*x)/(c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx = \frac{\int \frac{ax}{c^2x^2+1}\,dx + \int \frac{bx\,\text{asinh}\,(cx)}{c^2x^2+1}\,dx}{d}$$

```
[In] integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d),x)
[Out] (Integral(a*x/(c**2*x**2 + 1), x) + Integral(b*x*asinh(c*x)/(c**2*x**2 + 1)
, x))/d
```

Maxima [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x}{c^2dx^2+d}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
[Out] -1/8*b*((log(c^2*x^2 + 1)^2 - 4*log(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2 + 1
)))/(c^2*d) + 8*integrate(1/2*log(c^2*x^2 + 1)/(c^4*d*x^3 + c^2*d*x + (c^3*
d*x^2 + c*d)*sqrt(c^2*x^2 + 1)), x)) + 1/2*a*log(c^2*d*x^2 + d)/(c^2*d)
```

Giac [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x}{c^2dx^2+d}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)*x/(c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{d+c^2dx^2}\,dx = \int \frac{x\,(a+b\,\mathrm{asinh}(c\,x))}{d\,c^2\,x^2+d}\,dx$$

```
[In] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2),x)
[Out] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2), x)
```

3.32 $\int \frac{a+b\text{arcsinh}(cx)}{d+c^2dx^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \frac{a+b\text{arcsinh}(cx)}{d+c^2dx^2}\,dx=\frac{2(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{cd}-\frac{ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{cd}+\frac{ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{cd}$$

```
[Out] 2*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/c/d-I*b*polylog(2,-I*(c*
x+(c^2*x^2+1)^(1/2)))/c/d+I*b*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5789, 4265, 2317, 2438}

$$\int \frac{a+b\text{arcsinh}(cx)}{d+c^2dx^2}\,dx=\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{cd}-\frac{ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{cd}+\frac{ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{cd}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2),x]
```

```
[Out] (2*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*d) - (I*b*PolyLog[2, (-I
)*E^ArcSinh[c*x]])/(c*d) + (I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c*d)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (a+bx)\text{sech}(x)\,dx, x, \text{arcsinh}(cx)\right)}{cd}\\
&= \frac{2(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{cd}\\
&\quad - \frac{(ib)\text{Subst}\left(\int \log\left(1-ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{cd}\\
&\quad + \frac{(ib)\text{Subst}\left(\int \log\left(1+ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{cd}\\
&= \frac{2(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{cd} - \frac{(ib)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{cd}\\
&\quad + \frac{(ib)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{cd}\\
&= \frac{2(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{cd}\\
&\quad - \frac{ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{cd} + \frac{ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{cd}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.93

$$\int \frac{a+b\text{arcsinh}(cx)}{d+c^2dx^2}\,dx =$$
$$-\frac{c\left(a\sqrt{-c^2}\arctan(cx)-b\text{arcsinh}(cx)\log\left(1+\frac{ce^{\text{arcsinh}(cx)}}{\sqrt{-c^2}}\right)+b\text{arcsinh}(cx)\log\left(1+\frac{\sqrt{-c^2}e^{\text{arcsinh}(cx)}}{c}\right)+bc\right.}{(-c^2)^{3/2}\,d}$$

[In] Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2),x]

[Out] -((c*(a*Sqrt[-c^2]*ArcTan[c*x] - b*c*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + b*c*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + b*c*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - b*c*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c]))/((-c^2)^(3/2)*d))

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.04

method	result
derivativedivides	$\frac{\frac{a\arctan(cx)}{d}+\frac{b\left(\text{arcsinh}(cx)\arctan(cx)+\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)-\arctan(cx)\ln\left(1-\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)-i\,\text{dilog}\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)+i\,\text{dilog}(\right.}{d}}{c}$
default	$\frac{\frac{a\arctan(cx)}{d}+\frac{b\left(\text{arcsinh}(cx)\arctan(cx)+\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)-\arctan(cx)\ln\left(1-\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)-i\,\text{dilog}\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)+i\,\text{dilog}(\right.}{d}}{c}$
parts	$\frac{a\arctan(cx)}{dc}+\frac{b\left(\text{arcsinh}(cx)\arctan(cx)+\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)-\arctan(cx)\ln\left(1-\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)-i\,\text{dilog}\left(1+\frac{i(ic}{\sqrt{c^2}}\right.\right.}{dc}$

[In] int((a+b*arcsinh(c*x))/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)

[Out] 1/c*(a/d*arctan(c*x)+b/d*(arcsinh(c*x)*arctan(c*x)+arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))))

Fricas [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{d + c^2dx^2} \, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{c^2dx^2 + d} \, dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
[Out] integral((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{d + c^2dx^2} \, dx = \frac{\int \frac{a}{c^2x^2+1} \, dx + \int \frac{b \operatorname{asinh}(cx)}{c^2x^2+1} \, dx}{d}$$

```
[In] integrate((a+b*asinh(c*x))/(c**2*d*x**2+d),x)
[Out] (Integral(a/(c**2*x**2 + 1), x) + Integral(b*asinh(c*x)/(c**2*x**2 + 1), x)
)/d
```

Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{d + c^2dx^2} \, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{c^2dx^2 + d} \, dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
[Out] b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d), x) + a*arctan(c*x
)/(c*d)
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{d + c^2dx^2} \, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{c^2dx^2 + d} \, dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{d + c^2dx^2}\,dx = \int \frac{a + b\,\text{asinh}(c\,x)}{d\,c^2\,x^2 + d}\,dx$$

```
[In] int((a + b*asinh(c*x))/(d + c^2*d*x^2),x)
[Out] int((a + b*asinh(c*x))/(d + c^2*d*x^2), x)
```

3.33 $\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 61

$$\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)}\,dx = -\frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d} - \frac{b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{2d} + \frac{b\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{2d}$$

```
[Out] -2*(a+b*arcsinh(c*x))*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)/d-1/2*b*polylog(2,
-(c*x+(c^2*x^2+1)^(1/2))^2)/d+1/2*b*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5799, 5569, 4267, 2317, 2438}

$$\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)}\,dx = -\frac{2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d} - \frac{b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{2d} + \frac{b\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{2d}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)),x]
```

```
[Out] (-2*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/d - (b*PolyLog[2, -E^
(2*ArcSinh[c*x])])/(2*d) + (b*PolyLog[2, E^(2*ArcSinh[c*x])])/(2*d)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (a+bx)\text{csch}(x)\text{sech}(x)\,dx,x,\text{arcsinh}(cx)\right)}{d}\\
&= \frac{2\text{Subst}\left(\int (a+bx)\text{csch}(2x)\,dx,x,\text{arcsinh}(cx)\right)}{d}\\
&= -\frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d}\\
&\quad -\frac{b\text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx,x,\text{arcsinh}(cx)\right)}{d}\\
&\quad +\frac{b\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx,x,\text{arcsinh}(cx)\right)}{d}\\
&= -\frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d}\\
&\quad -\frac{b\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2\text{arcsinh}(cx)}\right)}{2d}+\frac{b\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{2\text{arcsinh}(cx)}\right)}{2d}
\end{aligned}$$

$$= -\frac{2(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{2\operatorname{arcsinh}(cx)}\right)}{d} - \frac{b\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{2d} + \frac{b\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(cx)}\right)}{2d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 207 vs. $2(61) = 122$.

Time = 0.07 (sec) , antiderivative size = 207, normalized size of antiderivative = 3.39

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x(d+c^2dx^2)}\,dx = -\frac{a\operatorname{arcsinh}(cx)}{d} - \frac{b\operatorname{arcsinh}(cx)\log\left(1-\frac{\sqrt{-c^2}e^{\operatorname{arcsinh}(cx)}}{c}\right)}{d} - \frac{b\operatorname{arcsinh}(cx)\log\left(1+\frac{\sqrt{-c^2}e^{\operatorname{arcsinh}(cx)}}{c}\right)}{d} + \frac{a\log\left(1-e^{2\operatorname{arcsinh}(cx)}\right)}{d} + \frac{b\operatorname{arcsinh}(cx)\log\left(1-e^{2\operatorname{arcsinh}(cx)}\right)}{d} - \frac{a\log(1+c^2x^2)}{2d} - \frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{-c^2}e^{\operatorname{arcsinh}(cx)}}{c}\right)}{d} - \frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{-c^2}e^{\operatorname{arcsinh}(cx)}}{c}\right)}{d} + \frac{b\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(cx)}\right)}{2d}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)),x]
```

```
[Out] -((a*ArcSinh[c*x])/d) - (b*ArcSinh[c*x]*Log[1 - (Sqrt[-c^2]*E^ArcSinh[c*x])
/c])/d - (b*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/d + (a*Log
[1 - E^(2*ArcSinh[c*x])])/d + (b*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])])/
d - (a*Log[1 + c^2*x^2])/(2*d) - (b*PolyLog[2, -((Sqrt[-c^2]*E^ArcSinh[c*x]
)/c)])/d - (b*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/d + (b*PolyLog[2,
E^(2*ArcSinh[c*x])])/(2*d)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.64

method	result
parts	$\frac{a\left(-\frac{\ln\left(c^2x^2+1\right)}{2}+\ln(x)\right)}{d}+\frac{b\left(\operatorname{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)+\operatorname{polylog}\left(2,-cx-\sqrt{c^2x^2+1}\right)-\operatorname{arcsinh}(cx)\ln\left(1+\left(cx\right.\right.\right.}{}$
derivativedivides	$\frac{a\left(\ln(cx)-\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d}+\frac{b\left(\operatorname{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)+\operatorname{polylog}\left(2,-cx-\sqrt{c^2x^2+1}\right)-\operatorname{arcsinh}(cx)\ln\left(1+\left(cx+\right.\right.\right.}{}$
default	$\frac{a\left(\ln(cx)-\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d}+\frac{b\left(\operatorname{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)+\operatorname{polylog}\left(2,-cx-\sqrt{c^2x^2+1}\right)-\operatorname{arcsinh}(cx)\ln\left(1+\left(cx+\right.\right.\right.}{}$

```
[In] int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] a/d*(-1/2*ln(c^2*x^2+1)+ln(x))+b/d*(arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2)
)+polylog(2,-c*x-(c^2*x^2+1)^(1/2))-arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2
))^2)-1/2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+arcsinh(c*x)*ln(1-c*x-(c^2*
x^2+1)^(1/2))+polylog(2,c*x+(c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x\left(d+c^2dx^2\right)}\,dx=\int \frac{b\operatorname{arsinh}\left(cx\right)+a}{(c^2dx^2+d)x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^2*d*x^3 + d*x), x)
```

Sympy [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x\left(d+c^2dx^2\right)}\,dx=\frac{\int \frac{a}{c^2x^3+x}\,dx+\int \frac{b\operatorname{asinh}\left(cx\right)}{c^2x^3+x}\,dx}{d}$$

```
[In] integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a/(c**2*x**3 + x), x) + Integral(b*asinh(c*x)/(c**2*x**3 + x), x)
)/d
```

Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)}\,dx=\int \frac{b\,\text{arsinh}\left(cx\right)+a}{(c^2dx^2+d)x}\,dx$$

[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d),x, algorithm="maxima")

[Out] -1/2*a*(log(c^2*x^2 + 1)/d - 2*log(x)/d) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^3 + d*x), x)

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)}\,dx=\int \frac{b\,\text{arsinh}\left(cx\right)+a}{(c^2dx^2+d)x}\,dx$$

[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d),x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)*x), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{x\,\left(d\,c^2\,x^2+d\right)}\,dx$$

[In] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)),x)

[Out] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)), x)

3.34 $\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 101

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)}\,dx = -\frac{a+b\text{arcsinh}(cx)}{dx} - \frac{2c(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d} - \frac{b\text{carctanh}\left(\sqrt{1+c^2x^2}\right)}{d} + \frac{ibc\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{d} - \frac{ibc\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{d}$$

```
[Out] (-a-b*arcsinh(c*x))/d/x-2*c*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2)
)/d-b*c*arctanh((c^2*x^2+1)^(1/2))/d+I*b*c*polylog(2,-I*(c*x+(c^2*x^2+1)^(1
/2)))/d-I*b*c*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5809, 5789, 4265, 2317, 2438, 272, 65, 214}

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)}\,dx = -\frac{2c\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d} - \frac{a+b\text{arcsinh}(cx)}{dx} + \frac{ibc\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{d} - \frac{ibc\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{d} - \frac{b\text{carctanh}\left(\sqrt{c^2x^2+1}\right)}{d}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)),x]
```

```
[Out] -((a + b*ArcSinh[c*x])/(d*x)) - (2*c*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[
c*x]])/d - (b*c*ArcTanh[Sqrt[1 + c^2*x^2]])/d + (I*b*c*PolyLog[2, (-I)*E^Ar
cSinh[c*x]])/d - (I*b*c*PolyLog[2, I*E^ArcSinh[c*x]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
```

```
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\text{arcsinh}(cx)}{dx} - c^2\int \frac{a+b\text{arcsinh}(cx)}{d+c^2dx^2}\,dx + \frac{(bc)\int \frac{1}{x\sqrt{1+c^2x^2}}\,dx}{d} \\
&= -\frac{a+b\text{arcsinh}(cx)}{dx} - \frac{c\text{Subst}\left(\int (a+bx)\text{sech}(x)\,dx, x, \text{arcsinh}(cx)\right)}{d} \\
&\quad + \frac{(bc)\text{Subst}\left(\int \frac{1}{x\sqrt{1+c^2x}}\,dx, x, x^2\right)}{2d} \\
&= -\frac{a+b\text{arcsinh}(cx)}{dx} - \frac{2c(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d} \\
&\quad + \frac{b\text{Subst}\left(\int \frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx, x, \sqrt{1+c^2x^2}\right)}{cd} \\
&\quad + \frac{(ibc)\text{Subst}\left(\int \log\left(1-ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{d} \\
&\quad - \frac{(ibc)\text{Subst}\left(\int \log\left(1+ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{d} \\
&= -\frac{a+b\text{arcsinh}(cx)}{dx} - \frac{2c(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d} - \frac{b\text{carctanh}\left(\sqrt{1+c^2x^2}\right)}{d} \\
&\quad + \frac{(ibc)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{d} - \frac{(ibc)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{d} \\
&= -\frac{a+b\text{arcsinh}(cx)}{dx} - \frac{2c(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d} \\
&\quad - \frac{b\text{carctanh}\left(\sqrt{1+c^2x^2}\right)}{d} + \frac{ibc\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{d} \\
&\quad - \frac{ibc\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.80

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)}\,dx = -\frac{a+b\text{arcsinh}(cx)+acx\arctan(cx)+bcx\text{arctanh}\left(\sqrt{1+c^2x^2}\right)+b\sqrt{-c^2}x\text{arcsinh}(cx)\log\left(1+\frac{ce^{\text{arcsinh}(cx)}}{\sqrt{-c^2}}\right.}{}$$

[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)),x]

[Out] -((a + b*ArcSinh[c*x] + a*c*x*ArcTan[c*x] + b*c*x*ArcTanh[Sqrt[1 + c^2*x^2]] + b*Sqrt[-c^2]*x*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - b*Sqrt[-c^2]*x*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - b*Sqrt[-c^2]*x*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + b*Sqrt[-c^2]*x*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(d*x))

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.74

method	result
parts	$\frac{a\left(-c\arctan(cx)-\frac{1}{x}\right)}{d}+\frac{bc\left(-\frac{\text{arcsinh}(cx)}{cx}-\text{arcsinh}(cx)\arctan(cx)-\text{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)-\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right.\right.}{d}$
derivativedivides	$c\left(\frac{a\left(-\frac{1}{cx}-\arctan(cx)\right)}{d}+\frac{b\left(-\frac{\text{arcsinh}(cx)}{cx}-\text{arcsinh}(cx)\arctan(cx)-\text{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)-\arctan(cx)\ln\left(1+\frac{i(icx+1}{\sqrt{c^2x^2+}}\right.\right.}{d}\right.$
default	$c\left(\frac{a\left(-\frac{1}{cx}-\arctan(cx)\right)}{d}+\frac{b\left(-\frac{\text{arcsinh}(cx)}{cx}-\text{arcsinh}(cx)\arctan(cx)-\text{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)-\arctan(cx)\ln\left(1+\frac{i(icx+1}{\sqrt{c^2x^2+}}\right.\right.}{d}\right.$

[In] int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)

[Out] a/d*(-c*arctan(c*x)-1/x)+b/d*c*(-arcsinh(c*x)/c/x-arcsinh(c*x)*arctan(c*x)-arctanh(1/(c^2*x^2+1)^(1/2))-arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-I*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2)))

Fricas [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2 (d + c^2dx^2)}\, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(c^2dx^2 + d)x^2}\, dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d),x, algorithm="fricas")
[Out] integral((b*arcsinh(c*x) + a)/(c^2*d*x^4 + d*x^2), x)
```

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2 (d + c^2dx^2)}\, dx = \frac{\int \frac{a}{c^2x^4+x^2}\, dx + \int \frac{b \operatorname{asinh}(cx)}{c^2x^4+x^2}\, dx}{d}$$

```
[In] integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d),x)
[Out] (Integral(a/(c**2*x**4 + x**2), x) + Integral(b*asinh(c*x)/(c**2*x**4 + x**
2), x))/d
```

Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2 (d + c^2dx^2)}\, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(c^2dx^2 + d)x^2}\, dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d),x, algorithm="maxima")
[Out] -a*(c*arctan(c*x)/d + 1/(d*x)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(
c^2*d*x^4 + d*x^2), x)
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2 (d + c^2dx^2)}\, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(c^2dx^2 + d)x^2}\, dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2 (d + c^2dx^2)} \, dx = \int \frac{a + b \,\text{asinh}(c\,x)}{x^2 \,\left(d\,c^2\,x^2 + d\right)} \,dx$$

```
[In] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)),x)
[Out] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)), x)
```

3.35 $\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 113

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)}\,dx = -\frac{bc\sqrt{1+c^2x^2}}{2dx}-\frac{a+b\text{arcsinh}(cx)}{2dx^2}+\frac{2c^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d}+\frac{bc^2\operatorname{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{2d}-\frac{bc^2\operatorname{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{2d}$$

```
[Out] 1/2*(-a-b*arcsinh(c*x))/d/x^2+2*c^2*(a+b*arcsinh(c*x))*arctanh((c*x+(c^2*x^
2+1)^(1/2))^2)/d+1/2*b*c^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d-1/2*b*c^
2*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d-1/2*b*c*(c^2*x^2+1)^(1/2)/d/x
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5809, 5799, 5569, 4267, 2317, 2438, 270}

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)}\,dx = \frac{2c^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d}-\frac{a+b\text{arcsinh}(cx)}{2dx^2}+\frac{bc^2\operatorname{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{2d}-\frac{bc^2\operatorname{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{2d}-\frac{bc\sqrt{c^2x^2+1}}{2dx}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)),x]
```

```
[Out] -1/2*(b*c*Sqrt[1 + c^2*x^2])/(d*x) - (a + b*ArcSinh[c*x])/(2*d*x^2) + (2*c^
2*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/d + (b*c^2*PolyLog[2, -
E^(2*ArcSinh[c*x])])/(2*d) - (b*c^2*PolyLog[2, E^(2*ArcSinh[c*x])])/(2*d)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
```

```
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a+b\text{arcsinh}(cx)}{2dx^2}-c^2\int\frac{a+b\text{arcsinh}(cx)}{x\,(d+c^2dx^2)}\,dx+\frac{(bc)\int\frac{1}{x^2\sqrt{1+c^2x^2}}\,dx}{2d}\\
&=-\frac{bc\sqrt{1+c^2x^2}}{2dx}-\frac{a+b\text{arcsinh}(cx)}{2dx^2}-\frac{c^2\text{Subst}\left(\int(a+bx)\text{csch}(x)\text{sech}(x)\,dx,x,\text{arcsinh}(cx)\right)}{d}\\
&=-\frac{bc\sqrt{1+c^2x^2}}{2dx}-\frac{a+b\text{arcsinh}(cx)}{2dx^2}-\frac{(2c^2)\,\text{Subst}\left(\int(a+bx)\text{csch}(2x)\,dx,x,\text{arcsinh}(cx)\right)}{d}\\
&=-\frac{bc\sqrt{1+c^2x^2}}{2dx}-\frac{a+b\text{arcsinh}(cx)}{2dx^2}+\frac{2c^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d}\\
&\quad+\frac{(bc^2)\,\text{Subst}\left(\int\log\left(1-e^{2x}\right)\,dx,x,\text{arcsinh}(cx)\right)}{d}\\
&\quad-\frac{(bc^2)\,\text{Subst}\left(\int\log\left(1+e^{2x}\right)\,dx,x,\text{arcsinh}(cx)\right)}{d}\\
&=-\frac{bc\sqrt{1+c^2x^2}}{2dx}-\frac{a+b\text{arcsinh}(cx)}{2dx^2}+\frac{2c^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d}\\
&\quad+\frac{(bc^2)\,\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2\text{arcsinh}(cx)}\right)}{2d}-\frac{(bc^2)\,\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\text{arcsinh}(cx)}\right)}{2d}\\
&=-\frac{bc\sqrt{1+c^2x^2}}{2dx}-\frac{a+b\text{arcsinh}(cx)}{2dx^2}+\frac{2c^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d}\\
&\quad+\frac{bc^2\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{2d}-\frac{bc^2\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{2d}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 240 vs. $2(113) = 226$.

Time = 0.18 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.12

$$
\int\frac{a+b\text{arcsinh}(cx)}{x^3\,(d+c^2dx^2)}\,dx
=\frac{-\frac{bc\sqrt{1+c^2x^2}}{x}-bc^2\text{arcsinh}(cx)^2-\frac{a+b\text{arcsinh}(cx)}{x^2}+\frac{c^2(a+b\text{arcsinh}(cx))^2}{b}+2bc^2\text{arcsinh}(cx)\log\left(1+\frac{ce^{\text{arcsinh}(cx)}}{\sqrt{-c^2}}\right)}{}
$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)),x]
```

```
[Out] (-((b*c*Sqrt[1 + c^2*x^2])/x) - b*c^2*ArcSinh[c*x]^2 - (a + b*ArcSinh[c*x])
/x^2 + (c^2*(a + b*ArcSinh[c*x])^2)/b + 2*b*c^2*ArcSinh[c*x]*Log[1 + (c*E^A
rcSinh[c*x])/Sqrt[-c^2]] + 2*b*c^2*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSi
nh[c*x])/c] + a*c^2*Log[1 + c^2*x^2] + 2*b*c^2*PolyLog[2, (c*E^ArcSinh[c*x]
)/Sqrt[-c^2]] + 2*b*c^2*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - c^2*(2*
(a + b*ArcSinh[c*x])*Log[1 - E^(2*ArcSinh[c*x])] + b*PolyLog[2, E^(2*ArcSin
h[c*x])]))/(2*d)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.92

method	result
derivativedivides	$c^2 \left(\frac{a\left(-\frac{1}{2c^2x^2}-\ln(cx)+\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d} + \frac{b\left(-\frac{cx\sqrt{c^2x^2+1}-c^2x^2+\operatorname{arcsinh}(cx)}{2c^2x^2}-\operatorname{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)-\text{pol}\right.}{}\right.$
default	$c^2 \left(\frac{a\left(-\frac{1}{2c^2x^2}-\ln(cx)+\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d} + \frac{b\left(-\frac{cx\sqrt{c^2x^2+1}-c^2x^2+\operatorname{arcsinh}(cx)}{2c^2x^2}-\operatorname{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)-\text{pol}\right.}{}\right.$
parts	$\frac{a\left(\frac{c^2\ln\left(c^2x^2+1\right)}{2}-\frac{1}{2x^2}-c^2\ln(x)\right)}{d} + \frac{b\,c^2\left(-\frac{cx\sqrt{c^2x^2+1}-c^2x^2+\operatorname{arcsinh}(cx)}{2c^2x^2}-\operatorname{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)-\text{polylo}\right.}{}$

```
[In] int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(a/d*(-1/2/c^2/x^2-ln(c*x)+1/2*ln(c^2*x^2+1))+b/d*(-1/2*(c*x*(c^2*x^2+1
)^(1/2)-c^2*x^2+arcsinh(c*x))/c^2/x^2-arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/
2))-polylog(2,-c*x-(c^2*x^2+1)^(1/2))+arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1
/2))^2)+1/2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)-arcsinh(c*x)*ln(1-c*x-(c^
2*x^2+1)^(1/2))-polylog(2,c*x+(c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)}\,dx=\int \frac{b\,\text{arsinh}\left(cx\right)+a}{(c^2dx^2+d)x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d*x^3), x)
```

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\,(d+c^2dx^2)}\,dx = \frac{\int \frac{a}{c^2x^5+x^3}\,dx+\int \frac{b\,\text{asinh}\,(cx)}{c^2x^5+x^3}\,dx}{d}$$

```
[In] integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d),x)
[Out] (Integral(a/(c**2*x**5 + x**3), x) + Integral(b*asinh(c*x)/(c**2*x**5 + x**
3), x))/d
```

Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\,(d+c^2dx^2)}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{(c^2dx^2+d)x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d),x, algorithm="maxima")
[Out] 1/2*(c^2*log(c^2*x^2 + 1)/d - 2*c^2*log(x)/d - 1/(d*x^2))*a + b*integrate(l
og(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^5 + d*x^3), x)
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\,(d+c^2dx^2)}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{(c^2dx^2+d)x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\,(d+c^2dx^2)}\,dx = \int \frac{a+b\,\text{asinh}(c\,x)}{x^3\,(d\,c^2\,x^2+d)}\,dx$$

```
[In] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)),x)
[Out] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)), x)
```

3.36 $\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 156

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)}\,dx = -\frac{bc\sqrt{1+c^2x^2}}{6dx^2}-\frac{a+b\text{arcsinh}(cx)}{3dx^3}+\frac{c^2(a+b\text{arcsinh}(cx))}{dx} + \frac{2c^3(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d} + \frac{7bc^3\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{6d}-\frac{ibc^3\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{d} + \frac{ibc^3\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{d}$$

```
[Out] 1/3*(-a-b*arcsinh(c*x))/d/x^3+c^2*(a+b*arcsinh(c*x))/d/x+2*c^3*(a+b*arcsinh
(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/d+7/6*b*c^3*arctanh((c^2*x^2+1)^(1/2))
/d-I*b*c^3*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d+I*b*c^3*polylog(2,I*(c*x
+(c^2*x^2+1)^(1/2)))/d-1/6*b*c*(c^2*x^2+1)^(1/2)/d/x^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used

= {5809, 5789, 4265, 2317, 2438, 272, 65, 214, 44}

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)}\,dx = \frac{2c^3\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d} + \frac{c^2(a+b\text{arcsinh}(cx))}{dx} - \frac{a+b\text{arcsinh}(cx)}{3dx^3} - \frac{ibc^3\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{d} + \frac{ibc^3\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{d} + \frac{7bc^3\text{arctanh}\left(\sqrt{c^2x^2+1}\right)}{6d} - \frac{bc\sqrt{c^2x^2+1}}{6dx^2}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)),x]
```

```
[Out] -1/6*(b*c*Sqrt[1 + c^2*x^2])/(d*x^2) - (a + b*ArcSinh[c*x])/(3*d*x^3) + (c^
2*(a + b*ArcSinh[c*x]))/(d*x) + (2*c^3*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSin
h[c*x]])/d + (7*b*c^3*ArcTanh[Sqrt[1 + c^2*x^2]])/(6*d) - (I*b*c^3*PolyLog[
2, (-I)*E^ArcSinh[c*x]])/d + (I*b*c^3*PolyLog[2, I*E^ArcSinh[c*x]])/d
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\operatorname{arcsinh}(cx)}{3dx^3}-c^2\int\frac{a+b\operatorname{arcsinh}(cx)}{x^2\left(d+c^2dx^2\right)}\,dx+\frac{(bc)\int\frac{1}{x^3\sqrt{1+c^2x^2}}\,dx}{3d}\\
&= -\frac{a+b\operatorname{arcsinh}(cx)}{3dx^3}+\frac{c^2(a+b\operatorname{arcsinh}(cx))}{dx}+c^4\int\frac{a+b\operatorname{arcsinh}(cx)}{d+c^2dx^2}\,dx\\
&\quad+\frac{(bc)\text{Subst}\left(\int\frac{1}{x^2\sqrt{1+c^2x}}\,dx,x,x^2\right)}{6d}-\frac{\left(bc^3\right)\int\frac{1}{x\sqrt{1+c^2x^2}}\,dx}{d}
\end{aligned}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6dx^2} - \frac{a+b\text{arcsinh}(cx)}{3dx^3} + \frac{c^2(a+b\text{arcsinh}(cx))}{dx} + \frac{c^3\text{Subst}(\int (a+bx)\text{sech}(x)\,dx, x, \text{arcsinh}(cx))}{d} - \frac{(bc^3)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+c^2x}}\,dx, x, x^2\right)}{12d} - \frac{(bc^3)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+c^2x}}\,dx, x, x^2\right)}{2d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6dx^2} - \frac{a+b\text{arcsinh}(cx)}{3dx^3} + \frac{c^2(a+b\text{arcsinh}(cx))}{dx} + \frac{2c^3(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d} - \frac{(bc)\text{Subst}\left(\int \frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx, x, \sqrt{1+c^2x^2}\right)}{6d} - \frac{(bc)\text{Subst}\left(\int \frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx, x, \sqrt{1+c^2x^2}\right)}{d} - \frac{(ibc^3)\,\text{Subst}(\int \log(1-ie^x)\,dx, x, \text{arcsinh}(cx))}{d} + \frac{(ibc^3)\,\text{Subst}(\int \log(1+ie^x)\,dx, x, \text{arcsinh}(cx))}{d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6dx^2} - \frac{a+b\text{arcsinh}(cx)}{3dx^3} + \frac{c^2(a+b\text{arcsinh}(cx))}{dx} + \frac{2c^3(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d} + \frac{7bc^3\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{6d} - \frac{(ibc^3)\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{d} + \frac{(ibc^3)\,\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6dx^2} - \frac{a+b\text{arcsinh}(cx)}{3dx^3} + \frac{c^2(a+b\text{arcsinh}(cx))}{dx} + \frac{2c^3(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d} + \frac{7bc^3\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{6d} - \frac{ibc^3\,\text{PolyLog}\left(2, -ie^{\text{arcsinh}(cx)}\right)}{d} + \frac{ibc^3\,\text{PolyLog}\left(2, ie^{\text{arcsinh}(cx)}\right)}{d}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.58

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\left(d+c^2dx^2\right)}\,dx$$
$$= \frac{-2a+6ac^2x^2-bcx\sqrt{1+c^2x^2}-2b\text{arcsinh}(cx)+6bc^2x^2\text{arcsinh}(cx)+6ac^3x^3\arctan(cx)+7bc^3x^3\text{arctan}}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)),x]
```

```
[Out] (-2*a + 6*a*c^2*x^2 - b*c*x*Sqrt[1 + c^2*x^2] - 2*b*ArcSinh[c*x] + 6*b*c^2*
x^2*ArcSinh[c*x] + 6*a*c^3*x^3*ArcTan[c*x] + 7*b*c^3*x^3*ArcTanh[Sqrt[1 + c
^2*x^2]] - 6*b*(-c^2)^(3/2)*x^3*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqr
t[-c^2]] + 6*b*(-c^2)^(3/2)*x^3*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[
c*x])/c] + 6*b*(-c^2)^(3/2)*x^3*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] -
 6*b*(-c^2)^(3/2)*x^3*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(6*d*x^3)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.38

method	result
derivativedivides	$c^3\left(\frac{a\left(-\frac{1}{3c^3x^3}+\frac{1}{cx}+\arctan(cx)\right)}{d}+\frac{b\left(-\frac{\text{arcsinh}(cx)}{3c^3x^3}+\frac{\text{arcsinh}(cx)}{cx}+\text{arcsinh}(cx)\arctan(cx)-\frac{\sqrt{c^2x^2+1}}{6c^2x^2}+\frac{7\,\text{arctanh}\left(\frac{}{\sqrt{c}}\right)}{6}\right.}{}\right.$
default	$c^3\left(\frac{a\left(-\frac{1}{3c^3x^3}+\frac{1}{cx}+\arctan(cx)\right)}{d}+\frac{b\left(-\frac{\text{arcsinh}(cx)}{3c^3x^3}+\frac{\text{arcsinh}(cx)}{cx}+\text{arcsinh}(cx)\arctan(cx)-\frac{\sqrt{c^2x^2+1}}{6c^2x^2}+\frac{7\,\text{arctanh}\left(\frac{}{\sqrt{c}}\right)}{6}\right.}{}\right.$
parts	$\frac{a\left(c^3\arctan(cx)-\frac{1}{3x^3}+\frac{c^2}{x}\right)}{d}+\frac{b\,c^3\left(-\frac{\text{arcsinh}(cx)}{3c^3x^3}+\frac{\text{arcsinh}(cx)}{cx}+\text{arcsinh}(cx)\arctan(cx)-\frac{\sqrt{c^2x^2+1}}{6c^2x^2}+\frac{7\,\text{arctanh}\left(\frac{1}{\sqrt{c^2x^2+}}\right)}{6}\right.}{}$

```
[In] int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(a/d*(-1/3/c^3/x^3+1/c/x+arctan(c*x))+b/d*(-1/3*arcsinh(c*x)/c^3/x^3+ar
csinh(c*x)/c/x+arcsinh(c*x)*arctan(c*x)-1/6/c^2/x^2*(c^2*x^2+1)^(1/2)+7/6*a
rctanh(1/(c^2*x^2+1)^(1/2))+arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))
-arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*c*x)/(c^2
*x^2+1)^(1/2))+I*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^4 (d + c^2dx^2)} \, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(c^2dx^2 + d)x^4} \, dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d),x, algorithm="fricas")
[Out] integral((b*arcsinh(c*x) + a)/(c^2*d*x^6 + d*x^4), x)
```

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^4 (d + c^2dx^2)} \, dx = \frac{\int \frac{a}{c^2x^6+x^4} \, dx + \int \frac{b \operatorname{asinh}(cx)}{c^2x^6+x^4} \, dx}{d}$$

```
[In] integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d),x)
[Out] (Integral(a/(c**2*x**6 + x**4), x) + Integral(b*asinh(c*x)/(c**2*x**6 + x**
4), x))/d
```

Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^4 (d + c^2dx^2)} \, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(c^2dx^2 + d)x^4} \, dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d),x, algorithm="maxima")
[Out] 1/3*(3*c^3*arctan(c*x)/d + (3*c^2*x^2 - 1)/(d*x^3))*a + b*integrate(log(c*x
 + sqrt(c^2*x^2 + 1))/(c^2*d*x^6 + d*x^4), x)
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^4 (d + c^2dx^2)} \, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(c^2dx^2 + d)x^4} \, dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{x^4 (d + c^2dx^2)}\, dx = \int \frac{a + b\,\operatorname{asinh}(c\,x)}{x^4\,\left(d\,c^2\,x^2 + d\right)}\, dx$$

```
[In] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)),x)
[Out] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)), x)
```

3.37 $\int \frac{x^4(a+b\,\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 171

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \frac{b}{2c^5d^2\sqrt{1+c^2x^2}}-\frac{b\sqrt{1+c^2x^2}}{c^5d^2}+\frac{3x(a+b\text{arcsinh}(cx))}{2c^4d^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{2c^2d^2(1+c^2x^2)}-\frac{3(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^5d^2}+\frac{3ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{2c^5d^2}-\frac{3ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c^5d^2}$$

```
[Out] 3/2*x*(a+b*arcsinh(c*x))/c^4/d^2-1/2*x^3*(a+b*arcsinh(c*x))/c^2/d^2/(c^2*x^
2+1)-3*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/c^5/d^2+3/2*I*b*pol
ylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d^2-3/2*I*b*polylog(2,I*(c*x+(c^2*x^
2+1)^(1/2)))/c^5/d^2+1/2*b/c^5/d^2/(c^2*x^2+1)^(1/2)-b*(c^2*x^2+1)^(1/2)/c^
5/d^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used

$= \{5810, 5812, 5789, 4265, 2317, 2438, 267, 272, 45\}$

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = -\frac{3\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx))}{c^5d^2} + \frac{3x(a+b\operatorname{arcsinh}(cx))}{2c^4d^2} - \frac{x^3(a+b\operatorname{arcsinh}(cx))}{2c^2d^2(c^2x^2+1)} + \frac{3ib\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{2c^5d^2} - \frac{3ib\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{2c^5d^2} - \frac{b\sqrt{c^2x^2+1}}{c^5d^2} + \frac{b}{2c^5d^2\sqrt{c^2x^2+1}}$$

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] b/(2*c^5*d^2*Sqrt[1 + c^2*x^2]) - (b*Sqrt[1 + c^2*x^2])/(c^5*d^2) + (3*x*(a
 + b*ArcSinh[c*x]))/(2*c^4*d^2) - (x^3*(a + b*ArcSinh[c*x]))/(2*c^2*d^2*(1
+ c^2*x^2)) - (3*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^5*d^2) + (
((3*I)/2)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^5*d^2) - (((3*I)/2)*b*PolyL
og[2, I*E^ArcSinh[c*x]])/(c^5*d^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{x^3(a+b\text{arcsinh}(cx))}{2c^2d^2(1+c^2x^2)} + \frac{b\int \frac{x^3}{(1+c^2x^2)^{3/2}}\,dx}{2cd^2} + \frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx}{2c^2d}$$

$$= \frac{3x(a+b\operatorname{arcsinh}(cx))}{2c^4d^2} - \frac{x^3(a+b\operatorname{arcsinh}(cx))}{2c^2d^2(1+c^2x^2)} - \frac{(3b)\int \frac{x}{\sqrt{1+c^2x^2}}\,dx}{2c^3d^2} + \frac{b\text{Subst}\left(\int \frac{x}{(1+c^2x)^{3/2}}\,dx, x, x^2\right)}{4cd^2} - \frac{3\int \frac{a+b\operatorname{arcsinh}(cx)}{d+c^2dx^2}\,dx}{2c^4d}$$

$$= -\frac{3b\sqrt{1+c^2x^2}}{2c^5d^2} + \frac{3x(a+b\operatorname{arcsinh}(cx))}{2c^4d^2} - \frac{x^3(a+b\operatorname{arcsinh}(cx))}{2c^2d^2(1+c^2x^2)} - \frac{3\text{Subst}(\int (a+bx)\operatorname{sech}(x)\,dx, x, \operatorname{arcsinh}(cx))}{2c^5d^2} + \frac{b\text{Subst}\left(\int \left(-\frac{1}{c^2(1+c^2x)^{3/2}} + \frac{1}{c^2\sqrt{1+c^2x}}\right)dx, x, x^2\right)}{4cd^2}$$

$$= \frac{b}{2c^5d^2\sqrt{1+c^2x^2}} - \frac{b\sqrt{1+c^2x^2}}{c^5d^2} + \frac{3x(a+b\operatorname{arcsinh}(cx))}{2c^4d^2} - \frac{x^3(a+b\operatorname{arcsinh}(cx))}{2c^2d^2(1+c^2x^2)} - \frac{3(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c^5d^2} + \frac{(3ib)\text{Subst}(\int \log(1-ie^x)\,dx, x, \operatorname{arcsinh}(cx))}{2c^5d^2} - \frac{(3ib)\text{Subst}(\int \log(1+ie^x)\,dx, x, \operatorname{arcsinh}(cx))}{2c^5d^2}$$

$$= \frac{b}{2c^5d^2\sqrt{1+c^2x^2}} - \frac{b\sqrt{1+c^2x^2}}{c^5d^2} + \frac{3x(a+b\operatorname{arcsinh}(cx))}{2c^4d^2} - \frac{x^3(a+b\operatorname{arcsinh}(cx))}{2c^2d^2(1+c^2x^2)} - \frac{3(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c^5d^2} + \frac{(3ib)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{2c^5d^2} - \frac{(3ib)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{2c^5d^2}$$

$$= \frac{b}{2c^5d^2\sqrt{1+c^2x^2}} - \frac{b\sqrt{1+c^2x^2}}{c^5d^2} + \frac{3x(a+b\operatorname{arcsinh}(cx))}{2c^4d^2} - \frac{x^3(a+b\operatorname{arcsinh}(cx))}{2c^2d^2(1+c^2x^2)} - \frac{3(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c^5d^2} + \frac{3ib\,\text{PolyLog}\left(2, -ie^{\operatorname{arcsinh}(cx)}\right)}{2c^5d^2} - \frac{3ib\,\text{PolyLog}\left(2, ie^{\operatorname{arcsinh}(cx)}\right)}{2c^5d^2}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.57

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx$$
$$= \frac{3acx+2ac^3x^3-b\sqrt{1+c^2x^2}-2bc^2x^2\sqrt{1+c^2x^2}+3bcx\text{arcsinh}(cx)+2bc^3x^3\text{arcsinh}(cx)-3a\arctan(cx)-}{}$$

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] (3*a*c*x + 2*a*c^3*x^3 - b*Sqrt[1 + c^2*x^2] - 2*b*c^2*x^2*Sqrt[1 + c^2*x^2
] + 3*b*c*x*ArcSinh[c*x] + 2*b*c^3*x^3*ArcSinh[c*x] - 3*a*ArcTan[c*x] - 3*a
*c^2*x^2*ArcTan[c*x] - (3*I)*b*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (3*
I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + (3*I)*b*ArcSinh[c*x]*
Log[1 + I*E^ArcSinh[c*x]] + (3*I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSin
h[c*x]] + (3*I)*b*(1 + c^2*x^2)*PolyLog[2, (-I)*E^ArcSinh[c*x]] - (3*I)*b*(
1 + c^2*x^2)*PolyLog[2, I*E^ArcSinh[c*x]])/(2*c^5*d^2*(1 + c^2*x^2))
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{\frac{a\left(cx+\frac{cx}{2c^2x^2+2}-\frac{3\arctan(cx)}{2}\right)}{d^2}+\frac{b\left(\text{arcsinh}(cx)cx+\frac{\text{arcsinh}(cx)cx}{2c^2x^2+2}-\frac{3\,\text{arcsinh}(cx)\arctan(cx)}{2}-\frac{1}{2\sqrt{c^2x^2+1}}-\frac{c^2x^2}{\sqrt{c^2x^2+1}}-\frac{3\arctan(cx)}{}\right.}{}}{c^5}$
default	$\frac{\frac{a\left(cx+\frac{cx}{2c^2x^2+2}-\frac{3\arctan(cx)}{2}\right)}{d^2}+\frac{b\left(\text{arcsinh}(cx)cx+\frac{\text{arcsinh}(cx)cx}{2c^2x^2+2}-\frac{3\,\text{arcsinh}(cx)\arctan(cx)}{2}-\frac{1}{2\sqrt{c^2x^2+1}}-\frac{c^2x^2}{\sqrt{c^2x^2+1}}-\frac{3\arctan(cx)}{}\right.}{}}{c^5}$
parts	$\frac{a\left(\frac{x}{c^4}-\frac{-\frac{x}{2(c^2x^2+1)}+\frac{3\arctan(cx)}{2c}}{c^4}\right)}{d^2}+\frac{b\left(\text{arcsinh}(cx)cx+\frac{\text{arcsinh}(cx)cx}{2c^2x^2+2}-\frac{3\,\text{arcsinh}(cx)\arctan(cx)}{2}-\frac{1}{2\sqrt{c^2x^2+1}}-\frac{c^2x^2}{\sqrt{c^2x^2+1}}-\right.}{}$

```
[In] int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(a/d^2*(c*x+1/2*c*x/(c^2*x^2+1)-3/2*arctan(c*x))+b/d^2*(arcsinh(c*x)*
c*x+1/2*c*x/(c^2*x^2+1)*arcsinh(c*x)-3/2*arcsinh(c*x)*arctan(c*x)-1/2/(c^2*
x^2+1)^(1/2)-1/(c^2*x^2+1)^(1/2)*c^2*x^2-3/2*arctan(c*x)*ln(1+I*(1+I*c*x)/(
c^2*x^2+1)^(1/2))+3/2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/2*I
*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/2*I*dilog(1-I*(1+I*c*x)/(c^2*x^2+
1)^(1/2))))
```

Fricas [F]

$$\int \frac{x^4(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)x^4}{(c^2dx^2 + d)^2}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*x^4*arcsinh(c*x) + a*x^4)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2),
x)
```

Sympy [F]

$$\int \frac{x^4(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^2}\,dx = \frac{\int \frac{ax^4}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{bx^4\,\text{asinh}\,(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

```
[In] integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
[Out] (Integral(a*x**4/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*x**4*asinh(
c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{x^4(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)x^4}{(c^2dx^2 + d)^2}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
[Out] 1/2*a*(x/(c^6*d^2*x^2 + c^4*d^2) + 2*x/(c^4*d^2) - 3*arctan(c*x)/(c^5*d^2))
 + b*integrate(x^4*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^2*d^2*x^
2 + d^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b\operatorname{arcsinh}(cx))}{(d + c^2dx^2)^2}\,dx = \int \frac{x^4\,(a + b\,\mathrm{asinh}(c\,x))}{(d\,c^2\,x^2 + d)^2}\,dx$$

```
[In] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2,x)
[Out] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2, x)
```

3.38 $\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = -\frac{bx}{2c^3d^2\sqrt{1+c^2x^2}}+\frac{b\text{arcsinh}(cx)}{2c^4d^2} -\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d^2(1+c^2x^2)}-\frac{(a+b\text{arcsinh}(cx))^2}{2bc^4d^2} +\frac{(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^4d^2} +\frac{b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{2c^4d^2}$$

```
[Out] 1/2*b*arcsinh(c*x)/c^4/d^2-1/2*x^2*(a+b*arcsinh(c*x))/c^2/d^2/(c^2*x^2+1)-1
/2*(a+b*arcsinh(c*x))^2/b/c^4/d^2+(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^
(1/2))^2)/c^4/d^2+1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c^4/d^2-1/2*b
*x/c^3/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{5810, 5797, 3799, 2221, 2317, 2438, 294, 221\}$

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = -\frac{(a+b\operatorname{arcsinh}(cx))^2}{2bc^4d^2} + \frac{\log\left(e^{2\operatorname{arcsinh}(cx)}+1\right)(a+b\operatorname{arcsinh}(cx))}{c^4d^2} - \frac{x^2(a+b\operatorname{arcsinh}(cx))}{2c^2d^2(c^2x^2+1)} + \frac{b\,\mathrm{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{2c^4d^2} + \frac{b\operatorname{arcsinh}(cx)}{2c^4d^2} - \frac{bx}{2c^3d^2\sqrt{c^2x^2+1}}$$

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] -1/2*(b*x)/(c^3*d^2*Sqrt[1 + c^2*x^2]) + (b*ArcSinh[c*x])/(2*c^4*d^2) - (x^
2*(a + b*ArcSinh[c*x]))/(2*c^2*d^2*(1 + c^2*x^2)) - (a + b*ArcSinh[c*x])^2/
(2*b*c^4*d^2) + ((a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c^4*d^2
) + (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/(2*c^4*d^2)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d^2(1+c^2x^2)}+\frac{b\int\frac{x^2}{(1+c^2x^2)^{3/2}}\,dx}{2cd^2}+\frac{\int\frac{x(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx}{c^2d}\\
&= -\frac{bx}{2c^3d^2\sqrt{1+c^2x^2}}-\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d^2(1+c^2x^2)}\\
&\quad+\frac{\text{Subst}\left(\int(a+bx)\tanh(x)\,dx,x,\text{arcsinh}(cx)\right)}{c^4d^2}+\frac{b\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{2c^3d^2}\\
&= -\frac{bx}{2c^3d^2\sqrt{1+c^2x^2}}+\frac{b\text{arcsinh}(cx)}{2c^4d^2}-\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d^2(1+c^2x^2)}\\
&\quad-\frac{(a+b\text{arcsinh}(cx))^2}{2bc^4d^2}+\frac{2\text{Subst}\left(\int\frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx,x,\text{arcsinh}(cx)\right)}{c^4d^2}\\
&= -\frac{bx}{2c^3d^2\sqrt{1+c^2x^2}}+\frac{b\text{arcsinh}(cx)}{2c^4d^2}-\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d^2(1+c^2x^2)}-\frac{(a+b\text{arcsinh}(cx))^2}{2bc^4d^2}\\
&\quad+\frac{(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^4d^2}-\frac{b\text{Subst}\left(\int\log\left(1+e^{2x}\right)\,dx,x,\text{arcsinh}(cx)\right)}{c^4d^2}
\end{aligned}
$$

$$= -\frac{bx}{2c^3d^2\sqrt{1+c^2x^2}} + \frac{b\text{arcsinh}(cx)}{2c^4d^2} - \frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d^2(1+c^2x^2)} - \frac{(a+b\text{arcsinh}(cx))^2}{2bc^4d^2} + \frac{(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^4d^2} - \frac{b\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\text{arcsinh}(cx)}\right)}{2c^4d^2}$$

$$= -\frac{bx}{2c^3d^2\sqrt{1+c^2x^2}} + \frac{b\text{arcsinh}(cx)}{2c^4d^2} - \frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d^2(1+c^2x^2)} - \frac{(a+b\text{arcsinh}(cx))^2}{2bc^4d^2} + \frac{(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^4d^2} + \frac{b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{2c^4d^2}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.66

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \frac{a - bcx\sqrt{1+c^2x^2} + b\text{arcsinh}(cx) - b\text{arcsinh}(cx)^2 - bc^2x^2\text{arcsinh}(cx)^2 + 2b\text{arcsinh}(cx)\log\left(1-ie^{\text{arcsinh}(cx)}\right.}{}$$

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] (a - b*c*x*Sqrt[1 + c^2*x^2] + b*ArcSinh[c*x] - b*ArcSinh[c*x]^2 - b*c^2*x^
2*ArcSinh[c*x]^2 + 2*b*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + 2*b*c^2*x^2
*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + 2*b*ArcSinh[c*x]*Log[1 + I*E^ArcS
inh[c*x]] + 2*b*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] + a*Log[1 +
c^2*x^2] + a*c^2*x^2*Log[1 + c^2*x^2] + 2*b*(1 + c^2*x^2)*PolyLog[2, (-I)*E
^ArcSinh[c*x]] + 2*b*(1 + c^2*x^2)*PolyLog[2, I*E^ArcSinh[c*x]])/(2*c^4*d^2
*(1 + c^2*x^2))
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{\frac{a\left(\frac{1}{2c^2x^2+2}+\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d^2}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)^2}{2}+\frac{-cx\sqrt{c^2x^2+1}+c^2x^2+\operatorname{arcsinh}(cx)+1}{2c^2x^2+2}+\operatorname{arcsinh}(cx)\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)+\cdots\right)}{d^2}}{c^4}$
default	$\frac{\frac{a\left(\frac{1}{2c^2x^2+2}+\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d^2}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)^2}{2}+\frac{-cx\sqrt{c^2x^2+1}+c^2x^2+\operatorname{arcsinh}(cx)+1}{2c^2x^2+2}+\operatorname{arcsinh}(cx)\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)+\cdots\right)}{d^2}}{c^4}$
parts	$\frac{a\left(\frac{1}{2c^4\left(c^2x^2+1\right)}+\frac{\ln\left(c^2x^2+1\right)}{2c^4}\right)}{d^2}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)^2}{2}+\frac{-cx\sqrt{c^2x^2+1}+c^2x^2+\operatorname{arcsinh}(cx)+1}{2c^2x^2+2}+\operatorname{arcsinh}(cx)\ln\left(1+\left(cx+\sqrt{c^2 \cdots}\right.\right.\right.}{d^2c^4}$

```
[In] int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(a/d^2*(1/2/(c^2*x^2+1)+1/2*ln(c^2*x^2+1))+b/d^2*(-1/2*arcsinh(c*x)^2
+1/2*(-c*x*(c^2*x^2+1)^(1/2)+c^2*x^2+arcsinh(c*x)+1)/(c^2*x^2+1)+arcsinh(c*
x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1/2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2
)))
```

Fricas [F]

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{\left(d+c^2dx^2\right)^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^3}{\left(c^2dx^2+d\right)^2}\,dx$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arcsinh(c*x) + a*x^3)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2),
x)
```

Sympy [F]

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{\left(d+c^2dx^2\right)^2}\,dx=\frac{\int \frac{ax^3}{c^4x^4+2c^2x^2+1}\,dx+\int \frac{bx^3\,\text{asinh}\,(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

```
[In] integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a*x**3/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*x**3*asinh(
c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x^3}{(c^2dx^2+d)^2}\,dx$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/8*b*(((c^2*x^2 + 1)*log(c^2*x^2 + 1)^2 - 4*((c^2*x^2 + 1)*log(c^2*x^2 +
1) + 1)*log(c*x + sqrt(c^2*x^2 + 1)) - 2)/(c^6*d^2*x^2 + c^4*d^2) + 8*integ
rate(1/2*((c^2*x^2 + 1)*log(c^2*x^2 + 1) + 1)/(c^8*d^2*x^5 + 2*c^6*d^2*x^3
+ c^4*d^2*x + (c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2)*sqrt(c^2*x^2 + 1)), x
)) + 1/2*a*(1/(c^6*d^2*x^2 + c^4*d^2) + log(c^2*x^2 + 1)/(c^4*d^2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \int \frac{x^3\,(a+b\,\mathrm{asinh}(c\,x))}{(d\,c^2\,x^2+d)^2}\,dx$$

```
[In] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2,x)
```

```
[Out] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2, x)
```

3.39 $\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 127

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2} dx = -\frac{b}{2c^3d^2\sqrt{1+c^2x^2}}-\frac{x(a+b\text{arcsinh}(cx))}{2c^2d^2(1+c^2x^2)}+\frac{(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^3d^2}-\frac{ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{2c^3d^2}+\frac{ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c^3d^2}$$

```
[Out] -1/2*x*(a+b*arcsinh(c*x))/c^2/d^2/(c^2*x^2+1)+(a+b*arcsinh(c*x))*arctan(c*x
+(c^2*x^2+1)^(1/2))/c^3/d^2-1/2*I*b*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c
^3/d^2+1/2*I*b*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^2-1/2*b/c^3/d^2/(
c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5810, 5789, 4265, 2317, 2438, 267}

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2} dx = \frac{\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c^3d^2}-\frac{x(a+b\text{arcsinh}(cx))}{2c^2d^2(c^2x^2+1)}-\frac{ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{2c^3d^2}+\frac{ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c^3d^2}-\frac{b}{2c^3d^2\sqrt{c^2x^2+1}}$$

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] -1/2*b/(c^3*d^2*Sqrt[1 + c^2*x^2]) - (x*(a + b*ArcSinh[c*x]))/(2*c^2*d^2*(1
 + c^2*x^2)) + ((a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^3*d^2) - ((
I/2)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^3*d^2) + ((I/2)*b*PolyLog[2, I*E
^ArcSinh[c*x]])/(c^3*d^2)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
```

[m, 1]

Rubi steps

$$\text{integral} = -\frac{x(a+b\text{arcsinh}(cx))}{2c^2d^2(1+c^2x^2)} + \frac{b\int \frac{x}{(1+c^2x^2)^{3/2}}\,dx}{2cd^2} + \frac{\int \frac{a+b\text{arcsinh}(cx)}{d+c^2dx^2}\,dx}{2c^2d}$$
$$= -\frac{b}{2c^3d^2\sqrt{1+c^2x^2}} - \frac{x(a+b\text{arcsinh}(cx))}{2c^2d^2(1+c^2x^2)} + \frac{\text{Subst}(\int (a+bx)\text{sech}(x)\,dx, x, \text{arcsinh}(cx))}{2c^3d^2}$$
$$= -\frac{b}{2c^3d^2\sqrt{1+c^2x^2}} - \frac{x(a+b\text{arcsinh}(cx))}{2c^2d^2(1+c^2x^2)} + \frac{(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^3d^2} - \frac{(ib)\text{Subst}\left(\int \log\left(1-ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{2c^3d^2} + \frac{(ib)\text{Subst}\left(\int \log\left(1+ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{2c^3d^2}$$
$$= -\frac{b}{2c^3d^2\sqrt{1+c^2x^2}} - \frac{x(a+b\text{arcsinh}(cx))}{2c^2d^2(1+c^2x^2)} + \frac{(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^3d^2} - \frac{(ib)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{2c^3d^2} + \frac{(ib)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{2c^3d^2}$$
$$= -\frac{b}{2c^3d^2\sqrt{1+c^2x^2}} - \frac{x(a+b\text{arcsinh}(cx))}{2c^2d^2(1+c^2x^2)} + \frac{(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^3d^2} - \frac{ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{2c^3d^2} + \frac{ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c^3d^2}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.74

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = -\frac{acx+b\sqrt{1+c^2x^2}+bcx\text{arcsinh}(cx)-a\arctan(cx)-ac^2x^2\arctan(cx)-ib\text{arcsinh}(cx)\log\left(1-ie^{\text{arcsin}}\right.}{}$$

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] -1/2*(a*c*x + b*Sqrt[1 + c^2*x^2] + b*c*x*ArcSinh[c*x] - a*ArcTan[c*x] - a*
c^2*x^2*ArcTan[c*x] - I*b*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - I*b*c^2*
x^2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + I*b*ArcSinh[c*x]*Log[1 + I*E^A
rcSinh[c*x]] + I*b*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] + I*b*(1
+ c^2*x^2)*PolyLog[2, (-I)*E^ArcSinh[c*x]] - I*b*(1 + c^2*x^2)*PolyLog[2, I
*E^ArcSinh[c*x]])/(c^3*d^2*(1 + c^2*x^2))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.54

method	result
derivativedivides	$\frac{\frac{a\left(-\frac{cx}{2\left(c^2x^2+1\right)}+\frac{\arctan(cx)}{2}\right)}{d^2}+\frac{b\left(-\frac{cx\,\mathrm{arcsinh}(cx)}{2\left(c^2x^2+1\right)}+\frac{\mathrm{arcsinh}(cx)\arctan(cx)}{2}+\frac{\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{2}-\frac{\arctan(cx)\ln\left(1-\frac{i(ic}{\sqrt{c^2}}\right.}{2}\right.}{d^2}}{c^3}$
default	$\frac{\frac{a\left(-\frac{cx}{2\left(c^2x^2+1\right)}+\frac{\arctan(cx)}{2}\right)}{d^2}+\frac{b\left(-\frac{cx\,\mathrm{arcsinh}(cx)}{2\left(c^2x^2+1\right)}+\frac{\mathrm{arcsinh}(cx)\arctan(cx)}{2}+\frac{\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{2}-\frac{\arctan(cx)\ln\left(1-\frac{i(ic}{\sqrt{c^2}}\right.}{2}\right.}{d^2}}{c^3}$
parts	$\frac{a\left(-\frac{x}{2c^2\left(c^2x^2+1\right)}+\frac{\arctan(cx)}{2c^3}\right)}{d^2}+\frac{b\left(-\frac{cx\,\mathrm{arcsinh}(cx)}{2\left(c^2x^2+1\right)}+\frac{\mathrm{arcsinh}(cx)\arctan(cx)}{2}+\frac{\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{2}-\frac{\arctan(cx)\ln\left(\right.}{}\right.}{d^2c^3}$

```
[In] int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
[Out] 1/c^3*(a/d^2*(-1/2*c*x/(c^2*x^2+1)+1/2*arctan(c*x))+b/d^2*(-1/2*c*x/(c^2*x^
2+1)*arcsinh(c*x)+1/2*arcsinh(c*x)*arctan(c*x)+1/2*arctan(c*x)*ln(1+I*(1+I*
c*x)/(c^2*x^2+1)^(1/2))-1/2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))
-1/2*I*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*dilog(1-I*(1+I*c*x)/(c^
2*x^2+1)^(1/2))-1/2/(c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{x^2(a+b\,\mathrm{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx=\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)x^2}{(c^2dx^2+d)^2}\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*x^2*arcsinh(c*x) + a*x^2)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2),
x)
```

Sympy [F]

$$\int \frac{x^2(a+b\,\mathrm{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx=\frac{\int \frac{ax^2}{c^4x^4+2c^2x^2+1}\,dx+\int \frac{bx^2\,\mathrm{asinh}\,(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

```
[In] integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
[Out] (Integral(a*x**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*x**2*asinh(
c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^2}{(c^2dx^2+d)^2}\,dx$$

[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")

[Out] -1/2*a*(x/(c^4*d^2*x^2 + c^2*d^2) - arctan(c*x)/(c^3*d^2)) + b*integrate(x^2*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)

Giac [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^2}{(c^2dx^2+d)^2}\,dx$$

[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)*x^2/(c^2*d*x^2 + d)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \int \frac{x^2\,(a+b\,\text{asinh}(c\,x))}{(d\,c^2\,x^2+d)^2}\,dx$$

[In] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2,x)

[Out] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2, x)

3.40 $\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \frac{bx}{2cd^2\sqrt{1+c^2x^2}} - \frac{a+b\text{arcsinh}(cx)}{2c^2d^2(1+c^2x^2)}$$

```
[Out] 1/2*(-a-b*arcsinh(c*x))/c^2/d^2/(c^2*x^2+1)+1/2*b*x/c/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5798, 197}

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \frac{bx}{2cd^2\sqrt{c^2x^2+1}} - \frac{a+b\text{arcsinh}(cx)}{2c^2d^2(c^2x^2+1)}$$

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] (b*x)/(2*c*d^2*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(2*c^2*d^2*(1 + c^
2*x^2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
```

```
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{a+b\operatorname{arcsinh}(cx)}{2c^2d^2(1+c^2x^2)}+\frac{b\int\frac{1}{(1+c^2x^2)^{3/2}}\,dx}{2cd^2}\\ &= \frac{bx}{2cd^2\sqrt{1+c^2x^2}}-\frac{a+b\operatorname{arcsinh}(cx)}{2c^2d^2(1+c^2x^2)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.35

$$\int\frac{x(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx=-\frac{a}{2c^2d^2(1+c^2x^2)}+\frac{bx}{2cd^2\sqrt{1+c^2x^2}}-\frac{b\operatorname{arcsinh}(cx)}{2c^2d^2(1+c^2x^2)}$$

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
[Out] -1/2*a/(c^2*d^2*(1 + c^2*x^2)) + (b*x)/(2*c*d^2*Sqrt[1 + c^2*x^2]) - (b*Arc
Sinh[c*x])/(2*c^2*d^2*(1 + c^2*x^2))
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{-\frac{a}{2d^2(c^2x^2+1)}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{2(c^2x^2+1)}+\frac{cx}{2\sqrt{c^2x^2+1}}\right)}{d^2}}{c^2}$	61
default	$\frac{-\frac{a}{2d^2(c^2x^2+1)}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{2(c^2x^2+1)}+\frac{cx}{2\sqrt{c^2x^2+1}}\right)}{d^2}}{c^2}$	61
parts	$-\frac{a}{2d^2c^2(c^2x^2+1)}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{2(c^2x^2+1)}+\frac{cx}{2\sqrt{c^2x^2+1}}\right)}{d^2c^2}$	63

```
[In] int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
[Out] 1/c^2*(-1/2*a/d^2/(c^2*x^2+1)+b/d^2*(-1/2/(c^2*x^2+1)*arcsinh(c*x)+1/2*c*x/
(c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.18

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \frac{ac^2x^2+\sqrt{c^2x^2+1}bcx-b\log\left(cx+\sqrt{c^2x^2+1}\right)}{2\left(c^4d^2x^2+c^2d^2\right)}$$

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
[Out] 1/2*(a*c^2*x^2 + sqrt(c^2*x^2 + 1)*b*c*x - b*log(c*x + sqrt(c^2*x^2 + 1)))/
(c^4*d^2*x^2 + c^2*d^2)
```

Sympy [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \frac{\int \frac{ax}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{bx\operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

```
[In] integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
[Out] (Integral(a*x/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*x*asinh(c*x)/(
c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \int \frac{(b\,\text{arsinh}(cx)+a)x}{(c^2dx^2+d)^2}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
[Out] -1/4*b*((2*log(c*x + sqrt(c^2*x^2 + 1)) + 1)/(c^4*d^2*x^2 + c^2*d^2) - 4*in
tegrate(1/2/(c^6*d^2*x^5 + 2*c^4*d^2*x^3 + c^2*d^2*x + (c^5*d^2*x^4 + 2*c^3
*d^2*x^2 + c*d^2)*sqrt(c^2*x^2 + 1)), x)) - 1/2*a/(c^4*d^2*x^2 + c^2*d^2)
```

Giac [F]

$$\int \frac{x(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^2} \, dx = \int \frac{(b \operatorname{arsinh}(cx) + a)x}{(c^2dx^2 + d)^2} \, dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)*x/(c^2*d*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^2} \, dx = \int \frac{x(a + b \operatorname{asinh}(c x))}{(d c^2 x^2 + d)^2} \, dx$$

```
[In] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2,x)
[Out] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2, x)
```

3.41 $\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 124

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^2} dx = \frac{b}{2cd^2\sqrt{1+c^2x^2}} + \frac{x(a+b\text{arcsinh}(cx))}{2d^2(1+c^2x^2)} + \frac{(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{cd^2} - \frac{ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{2cd^2} + \frac{ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2cd^2}$$

```
[Out] 1/2*x*(a+b*arcsinh(c*x))/d^2/(c^2*x^2+1)+(a+b*arcsinh(c*x))*arctan(c*x+(c^2
*x^2+1)^(1/2))/c/d^2-1/2*I*b*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2+1/
2*I*b*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2+1/2*b/c/d^2/(c^2*x^2+1)^(1
/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5788, 5789, 4265, 2317, 2438, 267}

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^2} dx = \frac{\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{cd^2} + \frac{x(a+b\text{arcsinh}(cx))}{2d^2(c^2x^2+1)} - \frac{ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{2cd^2} + \frac{ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2cd^2} + \frac{b}{2cd^2\sqrt{c^2x^2+1}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^2,x]
```

```
[Out] b/(2*c*d^2*Sqrt[1 + c^2*x^2]) + (x*(a + b*ArcSinh[c*x]))/(2*d^2*(1 + c^2*x^
2)) + ((a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*d^2) - ((I/2)*b*Poly
Log[2, (-I)*E^ArcSinh[c*x]])/(c*d^2) + ((I/2)*b*PolyLog[2, I*E^ArcSinh[c*x]
])/(c*d^2)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
```

```
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{x(a+b\text{arcsinh}(cx))}{2d^2(1+c^2x^2)} - \frac{(bc)\int \frac{x}{(1+c^2x^2)^{3/2}}\,dx}{2d^2} + \frac{\int \frac{a+b\text{arcsinh}(cx)}{d+c^2dx^2}\,dx}{2d}$$
$$= \frac{b}{2cd^2\sqrt{1+c^2x^2}} + \frac{x(a+b\text{arcsinh}(cx))}{2d^2(1+c^2x^2)} + \frac{\text{Subst}\left(\int (a+bx)\text{sech}(x)\,dx, x, \text{arcsinh}(cx)\right)}{2cd^2}$$
$$= \frac{b}{2cd^2\sqrt{1+c^2x^2}} + \frac{x(a+b\text{arcsinh}(cx))}{2d^2(1+c^2x^2)} + \frac{(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{cd^2} - \frac{(ib)\text{Subst}\left(\int \log\left(1-ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{2cd^2} + \frac{(ib)\text{Subst}\left(\int \log\left(1+ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{2cd^2}$$
$$= \frac{b}{2cd^2\sqrt{1+c^2x^2}} + \frac{x(a+b\text{arcsinh}(cx))}{2d^2(1+c^2x^2)} + \frac{(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{cd^2} - \frac{(ib)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{2cd^2} + \frac{(ib)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{2cd^2}$$
$$= \frac{b}{2cd^2\sqrt{1+c^2x^2}} + \frac{x(a+b\text{arcsinh}(cx))}{2d^2(1+c^2x^2)} + \frac{(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{cd^2} - \frac{ib\,\text{PolyLog}\left(2, -ie^{\text{arcsinh}(cx)}\right)}{2cd^2} + \frac{ib\,\text{PolyLog}\left(2, ie^{\text{arcsinh}(cx)}\right)}{2cd^2}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.74

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^2}\,dx = \frac{acx + b\sqrt{1+c^2x^2} + bcx\text{arcsinh}(cx) + a\arctan(cx) + ac^2x^2\arctan(cx) + ib\text{arcsinh}(cx)\log\left(1-ie^{\text{arcsinh}(cx)}\right)}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^2,x]
[Out] (a*c*x + b*Sqrt[1 + c^2*x^2] + b*c*x*ArcSinh[c*x] + a*ArcTan[c*x] + a*c^2*x
^2*ArcTan[c*x] + I*b*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + I*b*c^2*x^2*A
rcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - I*b*ArcSinh[c*x]*Log[1 + I*E^ArcSin
h[c*x]] - I*b*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] - I*b*(1 + c^2
*x^2)*PolyLog[2, (-I)*E^ArcSinh[c*x]] + I*b*(1 + c^2*x^2)*PolyLog[2, I*E^Ar
cSinh[c*x]])/(2*d^2*(c + c^3*x^2))
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.57

method	result
derivativedivides	$\frac{\frac{a\left(\frac{cx}{2c^2x^2+2}+\frac{\arctan(cx)}{2}\right)}{d^2}+\frac{b\left(\frac{\operatorname{arcsinh}(cx)cx}{2c^2x^2+2}+\frac{\operatorname{arcsinh}(cx)\arctan(cx)}{2}+\frac{1}{2\sqrt{c^2x^2+1}}+\frac{\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{2}-\frac{\arctan(cx)\ln(\cdots}{\cdots}\right.}{d^2}}{c}$
default	$\frac{\frac{a\left(\frac{cx}{2c^2x^2+2}+\frac{\arctan(cx)}{2}\right)}{d^2}+\frac{b\left(\frac{\operatorname{arcsinh}(cx)cx}{2c^2x^2+2}+\frac{\operatorname{arcsinh}(cx)\arctan(cx)}{2}+\frac{1}{2\sqrt{c^2x^2+1}}+\frac{\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{2}-\frac{\arctan(cx)\ln(\cdots}{\cdots}\right.}{d^2}}{c}$
parts	$\frac{a\left(\frac{x}{2c^2x^2+2}+\frac{\arctan(cx)}{2c}\right)}{d^2}+\frac{b\left(\frac{\operatorname{arcsinh}(cx)cx}{2c^2x^2+2}+\frac{\operatorname{arcsinh}(cx)\arctan(cx)}{2}+\frac{1}{2\sqrt{c^2x^2+1}}+\frac{\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{2}-\frac{\arctan(c\cdots}{\cdots}\right.}{d^2c}$

```
[In] int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a/d^2*(1/2*c*x/(c^2*x^2+1)+1/2*arctan(c*x))+b/d^2*(1/2*c*x/(c^2*x^2+1)
*arcsinh(c*x)+1/2*arcsinh(c*x)*arctan(c*x)+1/2/(c^2*x^2+1)^(1/2)+1/2*arctan
(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*arctan(c*x)*ln(1-I*(1+I*c*x)/
(c^2*x^2+1)^(1/2))-1/2*I*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*dilog
(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\left(d+c^2dx^2\right)^2}\,dx=\int \frac{b\operatorname{arsinh}(cx)+a}{\left(c^2dx^2+d\right)^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\left(d+c^2dx^2\right)^2}\,dx=\frac{\int \frac{a}{c^4x^4+2c^2x^2+1}\,dx+\int \frac{b\operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

```
[In] integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*asinh(c*x)/(c**4
*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + c^2dx^2)^2} \, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(c^2dx^2 + d)^2} \, dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(x/(c^2*d^2*x^2 + d^2) + arctan(c*x)/(c*d^2)) + b*integrate(log(c*x +
 sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + c^2dx^2)^2} \, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(c^2dx^2 + d)^2} \, dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + c^2dx^2)^2} \, dx = \int \frac{a + b \operatorname{asinh}(c\,x)}{(d\,c^2\,x^2 + d)^2} \, dx$$

```
[In] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^2,x)
```

```
[Out] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^2, x)
```

3.42 $\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 110

$$\int \frac{a+b\text{arcsinh}(cx)}{x\,(d+c^2dx^2)^2}\,dx = -\frac{bcx}{2d^2\sqrt{1+c^2x^2}}+\frac{a+b\text{arcsinh}(cx)}{2d^2\,(1+c^2x^2)} - \frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^2} - \frac{b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{2d^2}+\frac{b\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{2d^2}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))/d^2/(c^2*x^2+1)-2*(a+b*arcsinh(c*x))*arctanh((c*x+(c
^2*x^2+1)^(1/2))^2)/d^2-1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^2+1/2
*b*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d^2-1/2*b*c*x/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5811, 5799, 5569, 4267, 2317, 2438, 197}

$$\int \frac{a+b\text{arcsinh}(cx)}{x\,(d+c^2dx^2)^2}\,dx = -\frac{2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d^2} + \frac{a+b\text{arcsinh}(cx)}{2d^2\,(c^2x^2+1)} - \frac{b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{2d^2} + \frac{b\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{2d^2} - \frac{bcx}{2d^2\sqrt{c^2x^2+1}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^2),x]
```

```
[Out] -1/2*(b*c*x)/(d^2*Sqrt[1 + c^2*x^2]) + (a + b*ArcSinh[c*x])/(2*d^2*(1 + c^2
*x^2)) - (2*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/d^2 - (b*Poly
Log[2, -E^(2*ArcSinh[c*x])])/(2*d^2) + (b*PolyLog[2, E^(2*ArcSinh[c*x])])/(
2*d^2)
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
```

```
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rubi steps

$$\text{integral} = \frac{a+b\text{arcsinh}(cx)}{2d^2(1+c^2x^2)} - \frac{(bc)\int \frac{1}{(1+c^2x^2)^{3/2}}\,dx}{2d^2} + \frac{\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)}\,dx}{d}$$

$$= -\frac{bcx}{2d^2\sqrt{1+c^2x^2}} + \frac{a+b\text{arcsinh}(cx)}{2d^2(1+c^2x^2)} + \frac{\text{Subst}\left(\int (a+bx)\text{csch}(x)\text{sech}(x)\,dx, x, \text{arcsinh}(cx)\right)}{d^2}$$

$$= -\frac{bcx}{2d^2\sqrt{1+c^2x^2}} + \frac{a+b\text{arcsinh}(cx)}{2d^2(1+c^2x^2)} + \frac{2\text{Subst}\left(\int (a+bx)\text{csch}(2x)\,dx, x, \text{arcsinh}(cx)\right)}{d^2}$$

$$= -\frac{bcx}{2d^2\sqrt{1+c^2x^2}} + \frac{a+b\text{arcsinh}(cx)}{2d^2(1+c^2x^2)} - \frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^2} - \frac{b\text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{d^2} + \frac{b\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{d^2}$$

$$= -\frac{bcx}{2d^2\sqrt{1+c^2x^2}} + \frac{a+b\text{arcsinh}(cx)}{2d^2(1+c^2x^2)} - \frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^2} - \frac{b\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\text{arcsinh}(cx)}\right)}{2d^2} + \frac{b\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\text{arcsinh}(cx)}\right)}{2d^2}$$

$$= -\frac{bcx}{2d^2\sqrt{1+c^2x^2}} + \frac{a+b\text{arcsinh}(cx)}{2d^2(1+c^2x^2)} - \frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^2} - \frac{b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{2d^2} + \frac{b\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{2d^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 234 vs. $2(110) = 220$.

Time = 0.31 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.13

$$\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)^2}\,dx = -\frac{\frac{a^2}{b} - \frac{a}{1+c^2x^2} + \frac{bcx}{\sqrt{1+c^2x^2}} + 2a\text{arcsinh}(cx) - \frac{b\text{arcsinh}(cx)}{1+c^2x^2} + 2b\text{arcsinh}(cx)\log\left(1+\frac{ce^{\text{arcsinh}(cx)}}{\sqrt{-c^2}}\right) + 2b\text{arcsinh}}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^2),x]
```

```
[Out] -1/2*(a^2/b - a/(1 + c^2*x^2) + (b*c*x)/Sqrt[1 + c^2*x^2] + 2*a*ArcSinh[c*x
] - (b*ArcSinh[c*x])/(1 + c^2*x^2) + 2*b*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[
c*x])/Sqrt[-c^2]] + 2*b*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c]
 - 2*a*Log[1 - E^(2*ArcSinh[c*x])] - 2*b*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[
c*x])] + a*Log[1 + c^2*x^2] + 2*b*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]]
 + 2*b*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - b*PolyLog[2, E^(2*ArcSin
h[c*x])]])/d^2
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.97

method	result
derivativedivides	$\frac{a\left(\ln(cx)+\frac{1}{2c^2x^2+2}-\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d^2}+\frac{b\left(\frac{-cx\sqrt{c^2x^2+1}+c^2x^2+\text{arcsinh}(cx)+1}{2c^2x^2+2}+\text{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)+\text{polylog}\right.}{}$
default	$\frac{a\left(\ln(cx)+\frac{1}{2c^2x^2+2}-\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d^2}+\frac{b\left(\frac{-cx\sqrt{c^2x^2+1}+c^2x^2+\text{arcsinh}(cx)+1}{2c^2x^2+2}+\text{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)+\text{polylog}\right.}{}$
parts	$\frac{a}{2d^2(c^2x^2+1)}-\frac{a\ln(c^2x^2+1)}{2d^2}+\frac{a\ln(x)}{d^2}+\frac{b\left(\frac{-cx\sqrt{c^2x^2+1}+c^2x^2+\text{arcsinh}(cx)+1}{2c^2x^2+2}+\text{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)+\right.}{}$

```
[In] int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a/d^2*(ln(c*x)+1/2/(c^2*x^2+1)-1/2*ln(c^2*x^2+1))+b/d^2*(1/2*(-c*x*(c^2*x^2
+1)^(1/2)+c^2*x^2+arcsinh(c*x)+1)/(c^2*x^2+1)+arcsinh(c*x)*ln(1+c*x+(c^2*x^
2+1)^(1/2))+polylog(2,-c*x-(c^2*x^2+1)^(1/2))-arcsinh(c*x)*ln(1+(c*x+(c^2*x
^2+1)^(1/2))^2)-1/2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+arcsinh(c*x)*ln(1
-c*x-(c^2*x^2+1)^(1/2))+polylog(2,c*x+(c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^2}\,dx=\int \frac{b\,\text{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^2x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x)
```

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^2}\,dx = \frac{\int \frac{a}{c^4x^5+2c^2x^3+x}\,dx + \int \frac{b\,\text{asinh}\,(cx)}{c^4x^5+2c^2x^3+x}\,dx}{d^2}$$

```
[In] integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d)**2,x)
[Out] (Integral(a/(c**4*x**5 + 2*c**2*x**3 + x), x) + Integral(b*asinh(c*x)/(c**4
*x**5 + 2*c**2*x**3 + x), x))/d**2
```

Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^2}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{\left(c^2dx^2+d\right)^2x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^2,x, algorithm="maxima")
[Out] 1/2*a*(1/(c^2*d^2*x^2 + d^2) - log(c^2*x^2 + 1)/d^2 + 2*log(x)/d^2) + b*int
egrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x),
x)
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^2}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{\left(c^2dx^2+d\right)^2x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^2}\,dx = \int \frac{a+b\,\text{asinh}(c\,x)}{x\,\left(d\,c^2\,x^2+d\right)^2}\,dx$$

```
[In] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^2),x)
[Out] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^2), x)
```

3.43 $\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 168

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^2}\,dx = &-\frac{bc}{2d^2\sqrt{1+c^2x^2}}-\frac{a+b\text{arcsinh}(cx)}{d^2x(1+c^2x^2)}-\frac{3c^2x(a+b\text{arcsinh}(cx))}{2d^2(1+c^2x^2)}\\&-\frac{3c(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d^2}-\frac{b\text{carctanh}\left(\sqrt{1+c^2x^2}\right)}{d^2}\\&+\frac{3ibc\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{2d^2}-\frac{3ibc\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2d^2}\end{aligned}$$

```
[Out] (-a-b*arcsinh(c*x))/d^2/x/(c^2*x^2+1)-3/2*c^2*x*(a+b*arcsinh(c*x))/d^2/(c^2
*x^2+1)-3*c*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/d^2-b*c*arctan
h((c^2*x^2+1)^(1/2))/d^2+3/2*I*b*c*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^
2-3/2*I*b*c*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d^2-1/2*b*c/d^2/(c^2*x^2+1
)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {5809, 5788, 5789, 4265, 2317, 2438, 267, 272, 53, 65, 214}

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^2}\,dx = -\frac{3c\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d^2} - \frac{3c^2x(a+b\text{arcsinh}(cx))}{2d^2(c^2x^2+1)} - \frac{a+b\text{arcsinh}(cx)}{d^2x(c^2x^2+1)} + \frac{3ibc\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{2d^2} - \frac{3ibc\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2d^2} - \frac{b c \text{arctanh}\left(\sqrt{c^2x^2+1}\right)}{d^2} - \frac{bc}{2d^2\sqrt{c^2x^2+1}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^2),x]
```

```
[Out] -1/2*(b*c)/(d^2*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(d^2*x*(1 + c^2*x
^2)) - (3*c^2*x*(a + b*ArcSinh[c*x]))/(2*d^2*(1 + c^2*x^2)) - (3*c*(a + b*A
rcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/d^2 - (b*c*ArcTanh[Sqrt[1 + c^2*x^2]])
/d^2 + (((3*I)/2)*b*c*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^2 - (((3*I)/2)*b*c
*PolyLog[2, I*E^ArcSinh[c*x]])/d^2
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\text{arcsinh}(cx)}{d^2x\left(1+c^2x^2\right)}-\left(3c^2\right)\int\frac{a+b\text{arcsinh}(cx)}{\left(d+c^2dx^2\right)^2}\,dx+\frac{(bc)\int\frac{1}{x(1+c^2x^2)^{3/2}}\,dx}{d^2}\\
&= -\frac{a+b\text{arcsinh}(cx)}{d^2x\left(1+c^2x^2\right)}-\frac{3c^2x(a+b\text{arcsinh}(cx))}{2d^2\left(1+c^2x^2\right)}+\frac{(bc)\text{Subst}\left(\int\frac{1}{x(1+c^2x)^{3/2}}\,dx,x,x^2\right)}{2d^2}\\
&\quad+\frac{\left(3bc^3\right)\int\frac{x}{(1+c^2x^2)^{3/2}}\,dx}{2d^2}-\frac{\left(3c^2\right)\int\frac{a+b\text{arcsinh}(cx)}{d+c^2dx^2}\,dx}{2d}\\
&= -\frac{bc}{2d^2\sqrt{1+c^2x^2}}-\frac{a+b\text{arcsinh}(cx)}{d^2x\left(1+c^2x^2\right)}-\frac{3c^2x(a+b\text{arcsinh}(cx))}{2d^2\left(1+c^2x^2\right)}\\
&\quad-\frac{(3c)\text{Subst}\left(\int(a+bx)\text{sech}(x)\,dx,x,\text{arcsinh}(cx)\right)}{2d^2}\\
&\quad+\frac{(bc)\text{Subst}\left(\int\frac{1}{x\sqrt{1+c^2x}}\,dx,x,x^2\right)}{2d^2}\\
&= -\frac{bc}{2d^2\sqrt{1+c^2x^2}}-\frac{a+b\text{arcsinh}(cx)}{d^2x\left(1+c^2x^2\right)}-\frac{3c^2x(a+b\text{arcsinh}(cx))}{2d^2\left(1+c^2x^2\right)}\\
&\quad-\frac{3c(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d^2}+\frac{b\text{Subst}\left(\int\frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx,x,\sqrt{1+c^2x^2}\right)}{cd^2}\\
&\quad+\frac{(3ibc)\text{Subst}\left(\int\log\left(1-ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{2d^2}\\
&\quad-\frac{(3ibc)\text{Subst}\left(\int\log\left(1+ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{2d^2}
\end{aligned}$$

$$= -\frac{bc}{2d^2\sqrt{1+c^2x^2}} - \frac{a+b\text{arcsinh}(cx)}{d^2x(1+c^2x^2)} - \frac{3c^2x(a+b\text{arcsinh}(cx))}{2d^2(1+c^2x^2)} - \frac{3c(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d^2} - \frac{b\text{carctanh}\left(\sqrt{1+c^2x^2}\right)}{d^2} + \frac{(3ibc)\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{2d^2} - \frac{(3ibc)\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{2d^2}$$

$$= -\frac{bc}{2d^2\sqrt{1+c^2x^2}} - \frac{a+b\text{arcsinh}(cx)}{d^2x(1+c^2x^2)} - \frac{3c^2x(a+b\text{arcsinh}(cx))}{2d^2(1+c^2x^2)} - \frac{3c(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d^2} - \frac{b\text{carctanh}\left(\sqrt{1+c^2x^2}\right)}{d^2} + \frac{3ibc\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{2d^2} - \frac{3ibc\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2d^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.38 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.51

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^2}\,dx = -\frac{\frac{3a}{x} - \frac{a}{x+c^2x^3} + \frac{3b\text{arcsinh}(cx)}{x} - \frac{b\text{arcsinh}(cx)}{x+c^2x^3} + 3ac\arctan(cx) + 3b\text{carctanh}\left(\sqrt{1+c^2x^2}\right) + \frac{bc\,\text{Hypergeometric2F}}{\sqrt{1+c}}}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^2),x]
```

```
[Out] -1/2*((3*a)/x - a/(x + c^2*x^3) + (3*b*ArcSinh[c*x])/x - (b*ArcSinh[c*x])/(
x + c^2*x^3) + 3*a*c*ArcTan[c*x] + 3*b*c*ArcTanh[Sqrt[1 + c^2*x^2]] + (b*c*
Hypergeometric2F1[-1/2, 1, 1/2, 1 + c^2*x^2])/Sqrt[1 + c^2*x^2] + 3*b*Sqrt[
-c^2]*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 3*b*Sqrt[-c^2]*
ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 3*b*Sqrt[-c^2]*PolyLo
g[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 3*b*Sqrt[-c^2]*PolyLog[2, (Sqrt[-c^2]
*E^ArcSinh[c*x])/c])/d^2
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.35

method	result
derivativedivides	$c\left(\frac{a\left(-\frac{1}{cx}-\frac{cx}{2(c^2x^2+1)}-\frac{3\arctan(cx)}{2}\right)}{d^2}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{cx}-\frac{cx\operatorname{arcsinh}(cx)}{2(c^2x^2+1)}-\frac{3\operatorname{arcsinh}(cx)\arctan(cx)}{2}-\frac{3\arctan(cx)\ln\left(1+\right.}{2}\right.}{}\right.$
default	$c\left(\frac{a\left(-\frac{1}{cx}-\frac{cx}{2(c^2x^2+1)}-\frac{3\arctan(cx)}{2}\right)}{d^2}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{cx}-\frac{cx\operatorname{arcsinh}(cx)}{2(c^2x^2+1)}-\frac{3\operatorname{arcsinh}(cx)\arctan(cx)}{2}-\frac{3\arctan(cx)\ln\left(1+\right.}{2}\right.}{}\right.$
parts	$\frac{a\left(-c^2\left(\frac{x}{2c^2x^2+2}+\frac{3\arctan(cx)}{2c}\right)-\frac{1}{x}\right)}{d^2}+\frac{bc\left(-\frac{\operatorname{arcsinh}(cx)}{cx}-\frac{cx\operatorname{arcsinh}(cx)}{2(c^2x^2+1)}-\frac{3\operatorname{arcsinh}(cx)\arctan(cx)}{2}-\frac{3\arctan(cx)\ln\left(1+\frac{i(i}{\sqrt{c}}\right.}{2}\right.}{}$

```
[In] int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
[Out] c*(a/d^2*(-1/c/x-1/2*c*x/(c^2*x^2+1)-3/2*arctan(c*x))+b/d^2*(-arcsinh(c*x)/
c/x-1/2*c*x/(c^2*x^2+1)*arcsinh(c*x)-3/2*arcsinh(c*x)*arctan(c*x)-3/2*arcta
n(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/2*arctan(c*x)*ln(1-I*(1+I*c*x)
/(c^2*x^2+1)^(1/2))+3/2*I*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/2*I*dilo
g(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2/(c^2*x^2+1)^(1/2)-arctanh(1/(c^2*x^2
+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(d+c^2dx^2\right)^2}\,dx=\int \frac{b\,\text{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^2x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arcsinh(c*x) + a)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x)
```

Sympy [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^2\left(d+c^2dx^2\right)^2}\,dx=\frac{\int \frac{a}{c^4x^6+2c^2x^4+x^2}\,dx+\int \frac{b\operatorname{asinh}\left(cx\right)}{c^4x^6+2c^2x^4+x^2}\,dx}{d^2}$$

```
[In] integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a/(c**4*x**6 + 2*c**2*x**4 + x**2), x) + Integral(b*asinh(c*x)/(c
**4*x**6 + 2*c**2*x**4 + x**2), x))/d**2
```

Maxima [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^2\left(d+c^2dx^2\right)^2}\,dx=\int \frac{b\operatorname{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^2x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*((3*c^2*x^2 + 2)/(c^2*d^2*x^3 + d^2*x) + 3*c*arctan(c*x)/d^2) + b*in
tegrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2
), x)
```

Giac [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^2\left(d+c^2dx^2\right)^2}\,dx=\int \frac{b\operatorname{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^2x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^2\left(d+c^2dx^2\right)^2}\,dx=\int \frac{a+b\operatorname{asinh}(c\,x)}{x^2\,\left(d\,c^2\,x^2+d\right)^2}\,dx$$

```
[In] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^2),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^2), x)
```

3.44 $\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 146

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^2}\,dx = -\frac{bc}{2d^2x\sqrt{1+c^2x^2}} - \frac{c^2(a+b\text{arcsinh}(cx))}{d^2(1+c^2x^2)} - \frac{a+b\text{arcsinh}(cx)}{2d^2x^2(1+c^2x^2)} + \frac{4c^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^2} + \frac{bc^2\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d^2} - \frac{bc^2\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d^2}$$

```
[Out] -c^2*(a+b*arcsinh(c*x))/d^2/(c^2*x^2+1)+1/2*(-a-b*arcsinh(c*x))/d^2/x^2/(c^
2*x^2+1)+4*c^2*(a+b*arcsinh(c*x))*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)/d^2+b*
c^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^2-b*c^2*polylog(2,(c*x+(c^2*x^2
+1)^(1/2))^2)/d^2-1/2*b*c/d^2/x/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5809, 5811, 5799, 5569, 4267, 2317, 2438, 197, 277}

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^2}\,dx = \frac{4c^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d^2} - \frac{c^2(a+b\text{arcsinh}(cx))}{d^2(c^2x^2+1)} - \frac{a+b\text{arcsinh}(cx)}{2d^2x^2(c^2x^2+1)} + \frac{bc^2\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d^2} - \frac{bc^2\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d^2} - \frac{bc}{2d^2x\sqrt{c^2x^2+1}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^2),x]
```

```
[Out] -1/2*(b*c)/(d^2*x*Sqrt[1 + c^2*x^2]) - (c^2*(a + b*ArcSinh[c*x]))/(d^2*(1 +
 c^2*x^2)) - (a + b*ArcSinh[c*x])/(2*d^2*x^2*(1 + c^2*x^2)) + (4*c^2*(a + b
*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/d^2 + (b*c^2*PolyLog[2, -E^(2*A
rcSinh[c*x])])/d^2 - (b*c^2*PolyLog[2, E^(2*ArcSinh[c*x])])/d^2
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
```

```
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rubi steps

$$\text{integral} = -\frac{a+b\text{arcsinh}(cx)}{2d^2x^2\left(1+c^2x^2\right)} - \left(2c^2\right)\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^2}\,dx + \frac{(bc)\int \frac{1}{x^2(1+c^2x^2)^{3/2}}\,dx}{2d^2}$$

$$= -\frac{bc}{2d^2x\sqrt{1+c^2x^2}} - \frac{c^2(a+b\text{arcsinh}(cx))}{d^2\left(1+c^2x^2\right)} - \frac{a+b\text{arcsinh}(cx)}{2d^2x^2\left(1+c^2x^2\right)} - \frac{\left(2c^2\right)\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)}\,dx}{d}$$

$$= -\frac{bc}{2d^2x\sqrt{1+c^2x^2}} - \frac{c^2(a+b\text{arcsinh}(cx))}{d^2\left(1+c^2x^2\right)} - \frac{a+b\text{arcsinh}(cx)}{2d^2x^2\left(1+c^2x^2\right)} - \frac{\left(2c^2\right)\text{Subst}\left(\int (a+bx)\text{csch}(x)\text{sech}(x)\,dx, x, \text{arcsinh}(cx)\right)}{d^2}$$

$$= -\frac{bc}{2d^2x\sqrt{1+c^2x^2}} - \frac{c^2(a+b\text{arcsinh}(cx))}{d^2\left(1+c^2x^2\right)} - \frac{a+b\text{arcsinh}(cx)}{2d^2x^2\left(1+c^2x^2\right)} - \frac{\left(4c^2\right)\text{Subst}\left(\int (a+bx)\text{csch}(2x)\,dx, x, \text{arcsinh}(cx)\right)}{d^2}$$

$$= -\frac{bc}{2d^2x\sqrt{1+c^2x^2}} - \frac{c^2(a+b\text{arcsinh}(cx))}{d^2(1+c^2x^2)} - \frac{a+b\text{arcsinh}(cx)}{2d^2x^2(1+c^2x^2)} + \frac{4c^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^2} + \frac{(2bc^2)\,\text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{d^2} - \frac{(2bc^2)\,\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{d^2}$$

$$= -\frac{bc}{2d^2x\sqrt{1+c^2x^2}} - \frac{c^2(a+b\text{arcsinh}(cx))}{d^2(1+c^2x^2)} - \frac{a+b\text{arcsinh}(cx)}{2d^2x^2(1+c^2x^2)} + \frac{4c^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^2} + \frac{(bc^2)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\text{arcsinh}(cx)}\right)}{d^2} - \frac{(bc^2)\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\text{arcsinh}(cx)}\right)}{d^2}$$

$$= -\frac{bc}{2d^2x\sqrt{1+c^2x^2}} - \frac{c^2(a+b\text{arcsinh}(cx))}{d^2(1+c^2x^2)} - \frac{a+b\text{arcsinh}(cx)}{2d^2x^2(1+c^2x^2)} + \frac{4c^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^2} + \frac{bc^2\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d^2} - \frac{bc^2\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 326 vs. $2(146) = 292$.

Time = 0.31 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.23

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^2}\,dx$$

$$= \frac{\frac{2a^2c^2}{b} - \frac{2a}{x^2} + \frac{bc}{x\sqrt{1+c^2x^2}} + \frac{2bc^3x}{\sqrt{1+c^2x^2}} - \frac{2bc\sqrt{1+c^2x^2}}{x} + \frac{a}{x^2+c^2x^4} + 4ac^2\text{arcsinh}(cx) - \frac{2b\text{arcsinh}(cx)}{x^2} + \frac{b\text{arcsinh}(cx)}{x^2+c^2x^4} + 4}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^2),x]
```

```
[Out] ((2*a^2*c^2)/b - (2*a)/x^2 + (b*c)/(x*Sqrt[1 + c^2*x^2]) + (2*b*c^3*x)/Sqrt
[1 + c^2*x^2] - (2*b*c*Sqrt[1 + c^2*x^2])/x + a/(x^2 + c^2*x^4) + 4*a*c^2*A
rcSinh[c*x] - (2*b*ArcSinh[c*x])/x^2 + (b*ArcSinh[c*x])/(x^2 + c^2*x^4) + 4
*b*c^2*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 4*b*c^2*ArcSin
h[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 4*a*c^2*Log[1 - E^(2*ArcSin
h[c*x])] - 4*b*c^2*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])] + 2*a*c^2*Log[1
```

```
 + c^2*x^2] + 4*b*c^2*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 4*b*c^2*P
olyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 2*b*c^2*PolyLog[2, E^(2*ArcSinh[
c*x])])/(2*d^2)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.66

method	result
derivativedivides	$c^2\left(\frac{a\left(-\frac{1}{2c^2x^2}-2\ln(cx)-\frac{1}{2(c^2x^2+1)}+\ln(c^2x^2+1)\right)}{d^2}+\frac{b\left(-\frac{2\,\mathrm{arcsinh}(cx)c^2x^2+cx\sqrt{c^2x^2+1}+\mathrm{arcsinh}(cx)}{2c^2x^2(c^2x^2+1)}-2\,\mathrm{arcsinh}(cx\right.}{}\right.$
default	$c^2\left(\frac{a\left(-\frac{1}{2c^2x^2}-2\ln(cx)-\frac{1}{2(c^2x^2+1)}+\ln(c^2x^2+1)\right)}{d^2}+\frac{b\left(-\frac{2\,\mathrm{arcsinh}(cx)c^2x^2+cx\sqrt{c^2x^2+1}+\mathrm{arcsinh}(cx)}{2c^2x^2(c^2x^2+1)}-2\,\mathrm{arcsinh}(cx\right.}{}\right.$
parts	$\frac{a\left(\frac{c^4\left(-\frac{1}{c^2(c^2x^2+1)}+\frac{2\ln(c^2x^2+1)}{c^2}\right)}{2}-\frac{1}{2x^2}-2c^2\ln(x)\right)}{d^2}+\frac{b\,c^2\left(-\frac{2\,\mathrm{arcsinh}(cx)c^2x^2+cx\sqrt{c^2x^2+1}+\mathrm{arcsinh}(cx)}{2c^2x^2(c^2x^2+1)}-2\,\mathrm{arcsin}\right.}{}$

```
[In] int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(a/d^2*(-1/2/c^2/x^2-2*ln(c*x)-1/2/(c^2*x^2+1)+ln(c^2*x^2+1))+b/d^2*(-1
/2*(2*arcsinh(c*x)*c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x))/c^2/x^2/(c^2
*x^2+1)-2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-2*polylog(2,-c*x-(c^2*x^
2+1)^(1/2))+2*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+polylog(2,-(c*x+
(c^2*x^2+1)^(1/2))^2)-2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-2*polylog(
2,c*x+(c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a+b\mathrm{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^2}\,dx=\int \frac{b\,\mathrm{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^2x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^4*d^2*x^7 + 2*c^2*d^2*x^5 + d^2*x^3), x)
```

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^2}\,dx = \frac{\int \frac{a}{c^4x^7+2c^2x^5+x^3}\,dx + \int \frac{b\,\text{asinh}\,(cx)}{c^4x^7+2c^2x^5+x^3}\,dx}{d^2}$$

[In] integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d)**2,x)

[Out] (Integral(a/(c**4*x**7 + 2*c**2*x**5 + x**3), x) + Integral(b*asinh(c*x)/(c **4*x**7 + 2*c**2*x**5 + x**3), x))/d**2

Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^2}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{(c^2dx^2+d)^2x^3}\,dx$$

[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^2,x, algorithm="maxima")

[Out] 1/2*a*(2*c^2*log(c^2*x^2 + 1)/d^2 - 4*c^2*log(x)/d^2 - (2*c^2*x^2 + 1)/(c^2 *d^2*x^4 + d^2*x^2)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^ 7 + 2*c^2*d^2*x^5 + d^2*x^3), x)

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^2}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{(c^2dx^2+d)^2x^3}\,dx$$

[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^2,x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^2*x^3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^2}\,dx = \int \frac{a+b\,\text{asinh}(c\,x)}{x^3\,(d\,c^2\,x^2+d)^2}\,dx$$

[In] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^2),x)

[Out] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^2), x)

3.45 $\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 239

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^2}\,dx &= \frac{bc^3}{3d^2\sqrt{1+c^2x^2}} - \frac{bc}{6d^2x^2\sqrt{1+c^2x^2}} - \frac{a+b\text{arcsinh}(cx)}{3d^2x^3(1+c^2x^2)} \\ &+ \frac{5c^2(a+b\text{arcsinh}(cx))}{3d^2x(1+c^2x^2)} + \frac{5c^4x(a+b\text{arcsinh}(cx))}{2d^2(1+c^2x^2)} \\ &+ \frac{5c^3(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d^2} \\ &+ \frac{13bc^3\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{6d^2} - \frac{5ibc^3\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{2d^2} \\ &+ \frac{5ibc^3\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2d^2}\end{aligned}$$

```
[Out] 1/3*(-a-b*arcsinh(c*x))/d^2/x^3/(c^2*x^2+1)+5/3*c^2*(a+b*arcsinh(c*x))/d^2/
x/(c^2*x^2+1)+5/2*c^4*x*(a+b*arcsinh(c*x))/d^2/(c^2*x^2+1)+5*c^3*(a+b*arcsi
nh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/d^2+13/6*b*c^3*arctanh((c^2*x^2+1)^(
1/2))/d^2-5/2*I*b*c^3*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^2+5/2*I*b*c^3
*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d^2+1/3*b*c^3/d^2/(c^2*x^2+1)^(1/2)-1
/6*b*c/d^2/x^2/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5809, 5788, 5789, 4265, 2317, 2438, 267, 272, 53, 65, 214, 44}

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^2}\,dx = \frac{5c^3\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d^2}+\frac{5c^2(a+b\text{arcsinh}(cx))}{3d^2x(c^2x^2+1)}-\frac{a+b\text{arcsinh}(cx)}{3d^2x^3(c^2x^2+1)}+\frac{5c^4x(a+b\text{arcsinh}(cx))}{2d^2(c^2x^2+1)}-\frac{5ibc^3\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{2d^2}+\frac{5ibc^3\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2d^2}+\frac{13bc^3\text{arctanh}\left(\sqrt{c^2x^2+1}\right)}{6d^2}-\frac{bc}{6d^2x^2\sqrt{c^2x^2+1}}+\frac{bc^3}{3d^2\sqrt{c^2x^2+1}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^2),x]
```

```
[Out] (b*c^3)/(3*d^2*Sqrt[1 + c^2*x^2]) - (b*c)/(6*d^2*x^2*Sqrt[1 + c^2*x^2]) - (
a + b*ArcSinh[c*x])/(3*d^2*x^3*(1 + c^2*x^2)) + (5*c^2*(a + b*ArcSinh[c*x])
)/(3*d^2*x*(1 + c^2*x^2)) + (5*c^4*x*(a + b*ArcSinh[c*x]))/(2*d^2*(1 + c^2*
x^2)) + (5*c^3*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/d^2 + (13*b*c^3
*ArcTanh[Sqrt[1 + c^2*x^2]])/(6*d^2) - (((5*I)/2)*b*c^3*PolyLog[2, (-I)*E^A
rcSinh[c*x]])/d^2 + (((5*I)/2)*b*c^3*PolyLog[2, I*E^ArcSinh[c*x]])/d^2
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
```

```
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\text{arcsinh}(cx)}{3d^2x^3(1+c^2x^2)} - \frac{1}{3}(5c^2)\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^2}\,dx + \frac{(bc)\int \frac{1}{x^3(1+c^2x^2)^{3/2}}\,dx}{3d^2} \\
&= -\frac{a+b\text{arcsinh}(cx)}{3d^2x^3(1+c^2x^2)} + \frac{5c^2(a+b\text{arcsinh}(cx))}{3d^2x(1+c^2x^2)} + (5c^4)\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^2}\,dx \\
&\quad + \frac{(bc)\text{Subst}\left(\int \frac{1}{x^2(1+c^2x)^{3/2}}\,dx, x, x^2\right)}{6d^2} - \frac{(5bc^3)\int \frac{1}{x(1+c^2x^2)^{3/2}}\,dx}{3d^2} \\
&= -\frac{bc}{6d^2x^2\sqrt{1+c^2x^2}} - \frac{a+b\text{arcsinh}(cx)}{3d^2x^3(1+c^2x^2)} + \frac{5c^2(a+b\text{arcsinh}(cx))}{3d^2x(1+c^2x^2)} \\
&\quad + \frac{5c^4x(a+b\text{arcsinh}(cx))}{2d^2(1+c^2x^2)} - \frac{(bc^3)\,\text{Subst}\left(\int \frac{1}{x(1+c^2x)^{3/2}}\,dx, x, x^2\right)}{4d^2} \\
&\quad - \frac{(5bc^3)\,\text{Subst}\left(\int \frac{1}{x(1+c^2x)^{3/2}}\,dx, x, x^2\right)}{6d^2} \\
&\quad - \frac{(5bc^5)\int \frac{x}{(1+c^2x^2)^{3/2}}\,dx}{2d^2} + \frac{(5c^4)\int \frac{a+b\text{arcsinh}(cx)}{d+c^2dx^2}\,dx}{2d}
\end{aligned}$$

$$= \frac{bc^3}{3d^2\sqrt{1+c^2x^2}} - \frac{bc}{6d^2x^2\sqrt{1+c^2x^2}} - \frac{a+b\text{arcsinh}(cx)}{3d^2x^3(1+c^2x^2)} + \frac{5c^2(a+b\text{arcsinh}(cx))}{3d^2x(1+c^2x^2)} + \frac{5c^4x(a+b\text{arcsinh}(cx))}{2d^2(1+c^2x^2)} + \frac{(5c^3)\,\text{Subst}\left(\int (a+bx)\text{sech}(x)\,dx, x, \text{arcsinh}(cx)\right)}{2d^2} - \frac{(bc^3)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+c^2x}}\,dx, x, x^2\right)}{4d^2} - \frac{(5bc^3)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+c^2x}}\,dx, x, x^2\right)}{6d^2}$$

$$= \frac{bc^3}{3d^2\sqrt{1+c^2x^2}} - \frac{bc}{6d^2x^2\sqrt{1+c^2x^2}} - \frac{a+b\text{arcsinh}(cx)}{3d^2x^3(1+c^2x^2)} + \frac{5c^2(a+b\text{arcsinh}(cx))}{3d^2x(1+c^2x^2)} + \frac{5c^4x(a+b\text{arcsinh}(cx))}{2d^2(1+c^2x^2)} + \frac{5c^3(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d^2} - \frac{(bc)\text{Subst}\left(\int \frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx, x, \sqrt{1+c^2x^2}\right)}{2d^2} - \frac{(5bc)\text{Subst}\left(\int \frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx, x, \sqrt{1+c^2x^2}\right)}{3d^2} - \frac{(5ibc^3)\,\text{Subst}\left(\int \log\left(1-ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{2d^2} + \frac{(5ibc^3)\,\text{Subst}\left(\int \log\left(1+ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{2d^2}$$

$$= \frac{bc^3}{3d^2\sqrt{1+c^2x^2}} - \frac{bc}{6d^2x^2\sqrt{1+c^2x^2}} - \frac{a+b\text{arcsinh}(cx)}{3d^2x^3(1+c^2x^2)} + \frac{5c^2(a+b\text{arcsinh}(cx))}{3d^2x(1+c^2x^2)} + \frac{5c^4x(a+b\text{arcsinh}(cx))}{2d^2(1+c^2x^2)} + \frac{5c^3(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d^2} + \frac{13bc^3\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{6d^2} - \frac{(5ibc^3)\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{2d^2} + \frac{(5ibc^3)\,\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{2d^2}$$

$$= \frac{bc^3}{3d^2\sqrt{1+c^2x^2}} - \frac{bc}{6d^2x^2\sqrt{1+c^2x^2}} - \frac{a+b\text{arcsinh}(cx)}{3d^2x^3(1+c^2x^2)} + \frac{5c^2(a+b\text{arcsinh}(cx))}{3d^2x(1+c^2x^2)} + \frac{5c^4x(a+b\text{arcsinh}(cx))}{2d^2(1+c^2x^2)} + \frac{5c^3(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d^2} + \frac{13bc^3\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{6d^2} - \frac{5ibc^3\,\text{PolyLog}\left(2, -ie^{\text{arcsinh}(cx)}\right)}{2d^2} + \frac{5ibc^3\,\text{PolyLog}\left(2, ie^{\text{arcsinh}(cx)}\right)}{2d^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.42 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.30

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4 (d+c^2dx^2)^2}\,dx$$

$$= \frac{-\frac{5a}{3x^3}+\frac{5ac^2}{x}-\frac{5bc\sqrt{1+c^2x^2}}{6x^2}+\frac{a}{x^3+c^2x^5}-\frac{5b\text{arcsinh}(cx)}{3x^3}+\frac{5bc^2\text{arcsinh}(cx)}{x}+\frac{b\text{arcsinh}(cx)}{x^3+c^2x^5}+5ac^3\arctan(cx)+\frac{35}{6}bc$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^2),x]
```

```
[Out] ((-5*a)/(3*x^3) + (5*a*c^2)/x - (5*b*c*Sqrt[1 + c^2*x^2])/(6*x^2) + a/(x^3
+ c^2*x^5) - (5*b*ArcSinh[c*x])/(3*x^3) + (5*b*c^2*ArcSinh[c*x])/x + (b*Arc
Sinh[c*x])/(x^3 + c^2*x^5) + 5*a*c^3*ArcTan[c*x] + (35*b*c^3*ArcTanh[Sqrt[1
 + c^2*x^2]])/6 + (b*c^3*Hypergeometric2F1[-1/2, 2, 1/2, 1 + c^2*x^2])/Sqrt
[1 + c^2*x^2] - 5*b*(-c^2)^(3/2)*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sq
rt[-c^2]] + 5*b*(-c^2)^(3/2)*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x
])/c] + 5*b*(-c^2)^(3/2)*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 5*b*(-
c^2)^(3/2)*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(2*d^2)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.12

method	result
derivativedivides	$c^3\left(\frac{a\left(-\frac{1}{3c^3x^3}+\frac{2}{cx}+\frac{cx}{2c^2x^2+2}+\frac{5\arctan(cx)}{2}\right)}{d^2}+\frac{b\left(-\frac{\text{arcsinh}(cx)}{3c^3x^3}+\frac{2\,\text{arcsinh}(cx)}{cx}+\frac{\text{arcsinh}(cx)cx}{2c^2x^2+2}+\frac{5\,\text{arcsinh}(cx)\arctan(cx)}{2}+\right.}{}\right.$
default	$c^3\left(\frac{a\left(-\frac{1}{3c^3x^3}+\frac{2}{cx}+\frac{cx}{2c^2x^2+2}+\frac{5\arctan(cx)}{2}\right)}{d^2}+\frac{b\left(-\frac{\text{arcsinh}(cx)}{3c^3x^3}+\frac{2\,\text{arcsinh}(cx)}{cx}+\frac{\text{arcsinh}(cx)cx}{2c^2x^2+2}+\frac{5\,\text{arcsinh}(cx)\arctan(cx)}{2}+\right.}{}\right.$
parts	$\frac{a\left(c^4\left(\frac{x}{2c^2x^2+2}+\frac{5\arctan(cx)}{2c}\right)-\frac{1}{3x^3}+\frac{2c^2}{x}\right)}{d^2}+\frac{b\,c^3\left(-\frac{\text{arcsinh}(cx)}{3c^3x^3}+\frac{2\,\text{arcsinh}(cx)}{cx}+\frac{\text{arcsinh}(cx)cx}{2c^2x^2+2}+\frac{5\,\text{arcsinh}(cx)\arctan(cx)}{2}+\right.}{}$

```
[In] int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(a/d^2*(-1/3/c^3/x^3+2/c/x+1/2*c*x/(c^2*x^2+1)+5/2*arctan(c*x))+b/d^2*(
-1/3*arcsinh(c*x)/c^3/x^3+2*arcsinh(c*x)/c/x+1/2*c*x/(c^2*x^2+1)*arcsinh(c*
x)+5/2*arcsinh(c*x)*arctan(c*x)+1/3/(c^2*x^2+1)^(1/2)-1/6/c^2/x^2/(c^2*x^2+
```

```
1)^(1/2)+13/6*arctanh(1/(c^2*x^2+1)^(1/2))+5/2*arctan(c*x)*ln(1+I*(1+I*c*x)
/(c^2*x^2+1)^(1/2))-5/2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-5/2
*I*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+5/2*I*dilog(1-I*(1+I*c*x)/(c^2*x^
2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^2}\,dx=\int \frac{b\operatorname{arsinh}(cx)+a}{(c^2dx^2+d)^2x^4}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arcsinh(c*x) + a)/(c^4*d^2*x^8 + 2*c^2*d^2*x^6 + d^2*x^4), x)
```

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^2}\,dx=\frac{\int \frac{a}{c^4x^8+2c^2x^6+x^4}\,dx+\int \frac{b\operatorname{asinh}(cx)}{c^4x^8+2c^2x^6+x^4}\,dx}{d^2}$$

```
[In] integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d)**2,x)
[Out] (Integral(a/(c**4*x**8 + 2*c**2*x**6 + x**4), x) + Integral(b*asinh(c*x)/(c
**4*x**8 + 2*c**2*x**6 + x**4), x))/d**2
```

Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^2}\,dx=\int \frac{b\operatorname{arsinh}(cx)+a}{(c^2dx^2+d)^2x^4}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^2,x, algorithm="maxima")
[Out] 1/6*(15*c^3*arctan(c*x)/d^2 + (15*c^4*x^4 + 10*c^2*x^2 - 2)/(c^2*d^2*x^5 +
d^2*x^3))*a + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^8 + 2*c^2
*d^2*x^6 + d^2*x^4), x)
```

Giac [F]

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{x^4 (d + c^2 d x^2)^2} \, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(c^2 d x^2 + d)^2 x^4} \, dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^2*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{x^4 (d + c^2 d x^2)^2} \, dx = \int \frac{a + b \operatorname{asinh}(c\, x)}{x^4 \left(d\, c^2\, x^2 + d\right)^2} \, dx$$

```
[In] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^2),x)
[Out] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^2), x)
```

3.46 $\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 186

$$
\begin{aligned}
\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx &= \frac{b}{12c^5d^3(1+c^2x^2)^{3/2}} - \frac{5b}{8c^5d^3\sqrt{1+c^2x^2}} \\
&\quad - \frac{x^3(a+b\text{arcsinh}(cx))}{4c^2d^3(1+c^2x^2)^2} - \frac{3x(a+b\text{arcsinh}(cx))}{8c^4d^3(1+c^2x^2)} \\
&\quad + \frac{3(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4c^5d^3} \\
&\quad - \frac{3ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{8c^5d^3} + \frac{3ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{8c^5d^3}
\end{aligned}
$$

```
[Out] 1/12*b/c^5/d^3/(c^2*x^2+1)^(3/2)-1/4*x^3*(a+b*arcsinh(c*x))/c^2/d^3/(c^2*x^
2+1)^2-3/8*x*(a+b*arcsinh(c*x))/c^4/d^3/(c^2*x^2+1)+3/4*(a+b*arcsinh(c*x))*
arctan(c*x+(c^2*x^2+1)^(1/2))/c^5/d^3-3/8*I*b*polylog(2,-I*(c*x+(c^2*x^2+1)
^(1/2)))/c^5/d^3+3/8*I*b*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d^3-5/8*b
/c^5/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{5810, 5789, 4265, 2317, 2438, 267, 272, 45\}$

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \frac{3\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{4c^5d^3} - \frac{x^3(a+b\text{arcsinh}(cx))}{4c^2d^3(c^2x^2+1)^2} - \frac{3x(a+b\text{arcsinh}(cx))}{8c^4d^3(c^2x^2+1)} - \frac{3ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{8c^5d^3} + \frac{3ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{8c^5d^3} - \frac{5b}{8c^5d^3\sqrt{c^2x^2+1}} + \frac{b}{12c^5d^3(c^2x^2+1)^{3/2}}$$

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] b/(12*c^5*d^3*(1 + c^2*x^2)^(3/2)) - (5*b)/(8*c^5*d^3*Sqrt[1 + c^2*x^2]) -
(x^3*(a + b*ArcSinh[c*x]))/(4*c^2*d^3*(1 + c^2*x^2)^2) - (3*x*(a + b*ArcSin
h[c*x]))/(8*c^4*d^3*(1 + c^2*x^2)) + (3*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSi
nh[c*x]])/(4*c^5*d^3) - (((3*I)/8)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^5*
d^3) + (((3*I)/8)*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c^5*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^3(a+b\text{arcsinh}(cx))}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{b\int\frac{x^3}{(1+c^2x^2)^{5/2}}\,dx}{4cd^3}+\frac{3\int\frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx}{4c^2d}\\
&= -\frac{x^3(a+b\text{arcsinh}(cx))}{4c^2d^3\left(1+c^2x^2\right)^2}-\frac{3x(a+b\text{arcsinh}(cx))}{8c^4d^3\left(1+c^2x^2\right)}+\frac{(3b)\int\frac{x}{(1+c^2x^2)^{3/2}}\,dx}{8c^3d^3}\\
&\quad+\frac{b\text{Subst}\left(\int\frac{x}{(1+c^2x)^{5/2}}\,dx,x,x^2\right)}{8cd^3}+\frac{3\int\frac{a+b\text{arcsinh}(cx)}{d+c^2dx^2}\,dx}{8c^4d^2}\\
&= -\frac{3b}{8c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3(a+b\text{arcsinh}(cx))}{4c^2d^3\left(1+c^2x^2\right)^2}-\frac{3x(a+b\text{arcsinh}(cx))}{8c^4d^3\left(1+c^2x^2\right)}\\
&\quad+\frac{3\text{Subst}\left(\int(a+bx)\text{sech}(x)\,dx,x,\text{arcsinh}(cx)\right)}{8c^5d^3}\\
&\quad+\frac{b\text{Subst}\left(\int\left(-\frac{1}{c^2(1+c^2x)^{5/2}}+\frac{1}{c^2(1+c^2x)^{3/2}}\right)dx,x,x^2\right)}{8cd^3}
\end{aligned}
$$

$$= \frac{b}{12c^5d^3(1+c^2x^2)^{3/2}} - \frac{5b}{8c^5d^3\sqrt{1+c^2x^2}} - \frac{x^3(a+b\text{arcsinh}(cx))}{4c^2d^3(1+c^2x^2)^2} - \frac{3x(a+b\text{arcsinh}(cx))}{8c^4d^3(1+c^2x^2)} + \frac{3(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4c^5d^3} - \frac{(3ib)\text{Subst}\left(\int\log\left(1-ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{8c^5d^3} + \frac{(3ib)\text{Subst}\left(\int\log\left(1+ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{8c^5d^3}$$

$$= \frac{b}{12c^5d^3(1+c^2x^2)^{3/2}} - \frac{5b}{8c^5d^3\sqrt{1+c^2x^2}} - \frac{x^3(a+b\text{arcsinh}(cx))}{4c^2d^3(1+c^2x^2)^2} - \frac{3x(a+b\text{arcsinh}(cx))}{8c^4d^3(1+c^2x^2)} + \frac{3(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4c^5d^3} - \frac{(3ib)\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{8c^5d^3} + \frac{(3ib)\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{8c^5d^3}$$

$$= \frac{b}{12c^5d^3(1+c^2x^2)^{3/2}} - \frac{5b}{8c^5d^3\sqrt{1+c^2x^2}} - \frac{x^3(a+b\text{arcsinh}(cx))}{4c^2d^3(1+c^2x^2)^2} - \frac{3x(a+b\text{arcsinh}(cx))}{8c^4d^3(1+c^2x^2)} + \frac{3(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4c^5d^3} - \frac{3ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{8c^5d^3} + \frac{3ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{8c^5d^3}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.83

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = -\frac{9acx+15ac^3x^3+13b\sqrt{1+c^2x^2}+15bc^2x^2\sqrt{1+c^2x^2}+9bcx\text{arcsinh}(cx)+15bc^3x^3\text{arcsinh}(cx)-9a\,\text{arct}}{}$$

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] -1/24*(9*a*c*x + 15*a*c^3*x^3 + 13*b*Sqrt[1 + c^2*x^2] + 15*b*c^2*x^2*Sqrt[
1 + c^2*x^2] + 9*b*c*x*ArcSinh[c*x] + 15*b*c^3*x^3*ArcSinh[c*x] - 9*a*ArcTa
n[c*x] - 18*a*c^2*x^2*ArcTan[c*x] - 9*a*c^4*x^4*ArcTan[c*x] - (9*I)*b*ArcSi
nh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (18*I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 - I
*E^ArcSinh[c*x]] - (9*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] +
 (9*I)*b*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] + (18*I)*b*c^2*x^2*ArcSinh[
```

```
c*x]*Log[1 + I*E^ArcSinh[c*x]] + (9*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 + I*E^A
rcSinh[c*x]] + (9*I)*b*(1 + c^2*x^2)^2*PolyLog[2, (-I)*E^ArcSinh[c*x]] - (9
*I)*b*(1 + c^2*x^2)^2*PolyLog[2, I*E^ArcSinh[c*x]])/(c^5*d^3*(1 + c^2*x^2)^
2)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{\frac{a\left(\frac{-\frac{5}{8}c^3x^3-\frac{3}{8}cx}{\left(c^2x^2+1\right)^2}+\frac{3\arctan(cx)}{8}\right)}{d^3}+\frac{b\left(-\frac{5c^3x^3\operatorname{arcsinh}(cx)}{8\left(c^2x^2+1\right)^2}-\frac{3cx\operatorname{arcsinh}(cx)}{8\left(c^2x^2+1\right)^2}+\frac{3\operatorname{arcsinh}(cx)\arctan(cx)}{8}+\frac{3\arctan(cx)\ln\left(1+\frac{i(icx-}{\sqrt{c^2x^2}}\right.}{8}\right.}{}}{c^5}$
default	$\frac{\frac{a\left(\frac{-\frac{5}{8}c^3x^3-\frac{3}{8}cx}{\left(c^2x^2+1\right)^2}+\frac{3\arctan(cx)}{8}\right)}{d^3}+\frac{b\left(-\frac{5c^3x^3\operatorname{arcsinh}(cx)}{8\left(c^2x^2+1\right)^2}-\frac{3cx\operatorname{arcsinh}(cx)}{8\left(c^2x^2+1\right)^2}+\frac{3\operatorname{arcsinh}(cx)\arctan(cx)}{8}+\frac{3\arctan(cx)\ln\left(1+\frac{i(icx-}{\sqrt{c^2x^2}}\right.}{8}\right.}{}}{c^5}$
parts	$\frac{a\left(\frac{-\frac{5x^3}{8c^2}-\frac{3x}{8c^4}}{\left(c^2x^2+1\right)^2}+\frac{3\arctan(cx)}{8c^5}\right)}{d^3}+\frac{b\left(-\frac{5c^3x^3\operatorname{arcsinh}(cx)}{8\left(c^2x^2+1\right)^2}-\frac{3cx\operatorname{arcsinh}(cx)}{8\left(c^2x^2+1\right)^2}+\frac{3\operatorname{arcsinh}(cx)\arctan(cx)}{8}+\frac{3\arctan(cx)\ln\left(1+\frac{i(i}{\sqrt{c}}\right.}{8}\right.}{}$

```
[In] int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(a/d^3*((-5/8*c^3*x^3-3/8*c*x)/(c^2*x^2+1)^2+3/8*arctan(c*x))+b/d^3*(
-5/8*c^3*x^3/(c^2*x^2+1)^2*arcsinh(c*x)-3/8*c*x/(c^2*x^2+1)^2*arcsinh(c*x)+
3/8*arcsinh(c*x)*arctan(c*x)+3/8*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(
1/2))-3/8*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/8*I*dilog(1+I*(
1+I*c*x)/(c^2*x^2+1)^(1/2))+3/8*I*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-13
/24/(c^2*x^2+1)^(3/2)-5/8*c^2*x^2/(c^2*x^2+1)^(3/2)))
```

Fricas [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{\left(d+c^2dx^2\right)^3}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)x^4}{\left(c^2dx^2+d\right)^3}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arcsinh(c*x) + a*x^4)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*
d^3*x^2 + d^3), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \frac{\int \frac{ax^4}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{bx^4\operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

```
[In] integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a*x**4/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral
(b*x**4*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

Maxima [F]

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x^4}{(c^2dx^2+d)^3}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a*((5*c^2*x^3 + 3*x)/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) - 3*arcta
n(c*x)/(c^5*d^3)) + b*integrate(x^4*log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3*x
^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Giac [F]

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x^4}{(c^2dx^2+d)^3}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^4/(c^2*d*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \int \frac{x^4\,(a+b\operatorname{asinh}(c\,x))}{(d\,c^2\,x^2+d)^3}\,dx$$

```
[In] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3,x)
```

```
[Out] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3, x)
```

3.47 $\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 97

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3} dx = \frac{bx^3}{12cd^3(1+c^2x^2)^{3/2}} + \frac{bx}{4c^3d^3\sqrt{1+c^2x^2}} - \frac{b\text{arcsinh}(cx)}{4c^4d^3} + \frac{x^4(a+b\text{arcsinh}(cx))}{4d^3(1+c^2x^2)^2}$$

```
[Out] 1/12*b*x^3/c/d^3/(c^2*x^2+1)^(3/2)-1/4*b*arcsinh(c*x)/c^4/d^3+1/4*x^4*(a+b*
arcsinh(c*x))/d^3/(c^2*x^2+1)^2+1/4*b*x/c^3/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5800, 294, 221}

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3} dx = \frac{x^4(a+b\text{arcsinh}(cx))}{4d^3(c^2x^2+1)^2} - \frac{b\text{arcsinh}(cx)}{4c^4d^3} + \frac{bx^3}{12cd^3(c^2x^2+1)^{3/2}} + \frac{bx}{4c^3d^3\sqrt{c^2x^2+1}}$$

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] (b*x^3)/(12*c*d^3*(1 + c^2*x^2)^(3/2)) + (b*x)/(4*c^3*d^3*Sqrt[1 + c^2*x^2]
) - (b*ArcSinh[c*x])/(4*c^4*d^3) + (x^4*(a + b*ArcSinh[c*x]))/(4*d^3*(1 + c
^2*x^2)^2)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^4(a+b\operatorname{arcsinh}(cx))}{4d^3(1+c^2x^2)^2} - \frac{(bc)\int \frac{x^4}{(1+c^2x^2)^{5/2}}\,dx}{4d^3}\\
&= \frac{bx^3}{12cd^3(1+c^2x^2)^{3/2}} + \frac{x^4(a+b\operatorname{arcsinh}(cx))}{4d^3(1+c^2x^2)^2} - \frac{b\int \frac{x^2}{(1+c^2x^2)^{3/2}}\,dx}{4cd^3}\\
&= \frac{bx^3}{12cd^3(1+c^2x^2)^{3/2}} + \frac{bx}{4c^3d^3\sqrt{1+c^2x^2}} + \frac{x^4(a+b\operatorname{arcsinh}(cx))}{4d^3(1+c^2x^2)^2} - \frac{b\int \frac{1}{\sqrt{1+c^2x^2}}\,dx}{4c^3d^3}\\
&= \frac{bx^3}{12cd^3(1+c^2x^2)^{3/2}} + \frac{bx}{4c^3d^3\sqrt{1+c^2x^2}} - \frac{b\operatorname{arcsinh}(cx)}{4c^4d^3} + \frac{x^4(a+b\operatorname{arcsinh}(cx))}{4d^3(1+c^2x^2)^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \frac{-3a(1+2c^2x^2)+bcx\sqrt{1+c^2x^2}(3+4c^2x^2)-3(b+2bc^2x^2)\operatorname{arcsinh}(cx)}{12c^4d^3(1+c^2x^2)^2}$$

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
[Out] (-3*a*(1 + 2*c^2*x^2) + b*c*x*Sqrt[1 + c^2*x^2]*(3 + 4*c^2*x^2) - 3*(b + 2*
b*c^2*x^2)*ArcSinh[c*x])/(12*c^4*d^3*(1 + c^2*x^2)^2)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{\frac{a\left(\frac{1}{4\left(c^{2}x^{2}+1\right)^{2}}-\frac{1}{2\left(c^{2}x^{2}+1\right)}\right)}{d^{3}}+\frac{b\left(\frac{\operatorname{arcsinh}(cx)}{4\left(c^{2}x^{2}+1\right)^{2}}-\frac{\operatorname{arcsinh}(cx)}{2\left(c^{2}x^{2}+1\right)}-\frac{cx}{12\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}}+\frac{cx}{3\sqrt{c^{2}x^{2}+1}}\right)}{d^{3}}}{c^{4}}$	108
default	$\frac{\frac{a\left(\frac{1}{4\left(c^{2}x^{2}+1\right)^{2}}-\frac{1}{2\left(c^{2}x^{2}+1\right)}\right)}{d^{3}}+\frac{b\left(\frac{\operatorname{arcsinh}(cx)}{4\left(c^{2}x^{2}+1\right)^{2}}-\frac{\operatorname{arcsinh}(cx)}{2\left(c^{2}x^{2}+1\right)}-\frac{cx}{12\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}}+\frac{cx}{3\sqrt{c^{2}x^{2}+1}}\right)}{d^{3}}}{c^{4}}$	108
parts	$\frac{a\left(-\frac{1}{2c^{4}\left(c^{2}x^{2}+1\right)}+\frac{1}{4c^{4}\left(c^{2}x^{2}+1\right)^{2}}\right)}{d^{3}}+\frac{b\left(\frac{\operatorname{arcsinh}(cx)}{4\left(c^{2}x^{2}+1\right)^{2}}-\frac{\operatorname{arcsinh}(cx)}{2\left(c^{2}x^{2}+1\right)}-\frac{cx}{12\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}}+\frac{cx}{3\sqrt{c^{2}x^{2}+1}}\right)}{d^{3}c^{4}}$	113

```
[In] int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(a/d^3*(1/4/(c^2*x^2+1)^2-1/2/(c^2*x^2+1))+b/d^3*(1/4/(c^2*x^2+1)^2*a
rcsinh(c*x)-1/2/(c^2*x^2+1)*arcsinh(c*x)-1/12/(c^2*x^2+1)^(3/2)*c*x+1/3*c*x
/(c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.02

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{\left(d+c^2dx^2\right)^3}\,dx = \frac{3\,ac^4x^4-3\,\left(2\,bc^2x^2+b\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(4\,bc^3x^3+3\,bcx\right)\sqrt{c^2x^2+1}}{12\,\left(c^8d^3x^4+2\,c^6d^3x^2+c^4d^3\right)}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^4*x^4 - 3*(2*b*c^2*x^2 + b)*log(c*x + sqrt(c^2*x^2 + 1)) + (4*b
*c^3*x^3 + 3*b*c*x)*sqrt(c^2*x^2 + 1))/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d
^3)
```

Sympy [F]

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \frac{\int \frac{ax^3}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{bx^3\operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

```
[In] integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a*x**3/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral
(b*x**3*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

Maxima [F]

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x^3}{(c^2dx^2+d)^3}\,dx$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/16*b*((4*c^2*x^2 + 4*(2*c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2 + 1)) + 3)/(
c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) - 16*integrate(1/4*(2*c^2*x^2 + 1)/(
c^10*d^3*x^7 + 3*c^8*d^3*x^5 + 3*c^6*d^3*x^3 + c^4*d^3*x + (c^9*d^3*x^6 + 3
*c^7*d^3*x^4 + 3*c^5*d^3*x^2 + c^3*d^3)*sqrt(c^2*x^2 + 1)), x)) - 1/4*(2*c^
2*x^2 + 1)*a/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx=\int \frac{x^3\,(a+b\,\mathrm{asinh}(c\,x))}{{(d\,c^2\,x^2+d)}^3}\,dx$$

```
[In] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3,x)
[Out] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3, x)
```

3.48 $\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 184

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = -\frac{b}{12c^3d^3(1+c^2x^2)^{3/2}}+\frac{b}{8c^3d^3\sqrt{1+c^2x^2}}-\frac{x(a+b\text{arcsinh}(cx))}{4c^2d^3(1+c^2x^2)^2}+\frac{x(a+b\text{arcsinh}(cx))}{8c^2d^3(1+c^2x^2)}+\frac{(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4c^3d^3}-\frac{ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{8c^3d^3}+\frac{ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{8c^3d^3}$$

```
[Out] -1/12*b/c^3/d^3/(c^2*x^2+1)^(3/2)-1/4*x*(a+b*arcsinh(c*x))/c^2/d^3/(c^2*x^2
+1)^2+1/8*x*(a+b*arcsinh(c*x))/c^2/d^3/(c^2*x^2+1)+1/4*(a+b*arcsinh(c*x))*a
rctan(c*x+(c^2*x^2+1)^(1/2))/c^3/d^3-1/8*I*b*polylog(2,-I*(c*x+(c^2*x^2+1)^
(1/2)))/c^3/d^3+1/8*I*b*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^3+1/8*b/
c^3/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5810, 5788, 5789, 4265, 2317, 2438, 267}

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \frac{\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{4c^3d^3} + \frac{x(a+b\text{arcsinh}(cx))}{8c^2d^3(c^2x^2+1)} - \frac{x(a+b\text{arcsinh}(cx))}{4c^2d^3(c^2x^2+1)^2} - \frac{ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{8c^3d^3} + \frac{ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{8c^3d^3} + \frac{b}{8c^3d^3\sqrt{c^2x^2+1}} - \frac{b}{12c^3d^3(c^2x^2+1)^{3/2}}$$

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] -1/12*b/(c^3*d^3*(1 + c^2*x^2)^(3/2)) + b/(8*c^3*d^3*Sqrt[1 + c^2*x^2]) - (
x*(a + b*ArcSinh[c*x]))/(4*c^2*d^3*(1 + c^2*x^2)^2) + (x*(a + b*ArcSinh[c*x
]))/(8*c^2*d^3*(1 + c^2*x^2)) + ((a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]
])/(4*c^3*d^3) - ((I/8)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^3*d^3) + ((I/
8)*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c^3*d^3)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
```

```
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x(a+b\operatorname{arcsinh}(cx))}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{b\int\frac{x}{(1+c^2x^2)^{5/2}}\,dx}{4cd^3}+\frac{\int\frac{a+b\operatorname{arcsinh}(cx)}{\left(d+c^2dx^2\right)^2}\,dx}{4c^2d}\\
&= -\frac{b}{12c^3d^3\left(1+c^2x^2\right)^{3/2}}-\frac{x(a+b\operatorname{arcsinh}(cx))}{4c^2d^3\left(1+c^2x^2\right)^2}\\
&\quad+\frac{x(a+b\operatorname{arcsinh}(cx))}{8c^2d^3\left(1+c^2x^2\right)}-\frac{b\int\frac{x}{(1+c^2x^2)^{3/2}}\,dx}{8cd^3}+\frac{\int\frac{a+b\operatorname{arcsinh}(cx)}{d+c^2dx^2}\,dx}{8c^2d^2}\\
&= -\frac{b}{12c^3d^3\left(1+c^2x^2\right)^{3/2}}+\frac{b}{8c^3d^3\sqrt{1+c^2x^2}}-\frac{x(a+b\operatorname{arcsinh}(cx))}{4c^2d^3\left(1+c^2x^2\right)^2}\\
&\quad+\frac{x(a+b\operatorname{arcsinh}(cx))}{8c^2d^3\left(1+c^2x^2\right)}+\frac{\operatorname{Subst}\left(\int(a+bx)\operatorname{sech}(x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{8c^3d^3}
\end{aligned}$$

$$= -\frac{b}{12c^3d^3(1+c^2x^2)^{3/2}} + \frac{b}{8c^3d^3\sqrt{1+c^2x^2}} - \frac{x(a+b\text{arcsinh}(cx))}{4c^2d^3(1+c^2x^2)^2} + \frac{x(a+b\text{arcsinh}(cx))}{8c^2d^3(1+c^2x^2)} + \frac{(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4c^3d^3} - \frac{(ib)\text{Subst}\left(\int\log(1-ie^x)\,dx, x, \text{arcsinh}(cx)\right)}{8c^3d^3} + \frac{(ib)\text{Subst}\left(\int\log(1+ie^x)\,dx, x, \text{arcsinh}(cx)\right)}{8c^3d^3}$$

$$= -\frac{b}{12c^3d^3(1+c^2x^2)^{3/2}} + \frac{b}{8c^3d^3\sqrt{1+c^2x^2}} - \frac{x(a+b\text{arcsinh}(cx))}{4c^2d^3(1+c^2x^2)^2} + \frac{x(a+b\text{arcsinh}(cx))}{8c^2d^3(1+c^2x^2)} + \frac{(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4c^3d^3} - \frac{(ib)\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{8c^3d^3} + \frac{(ib)\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{8c^3d^3}$$

$$= -\frac{b}{12c^3d^3(1+c^2x^2)^{3/2}} + \frac{b}{8c^3d^3\sqrt{1+c^2x^2}} - \frac{x(a+b\text{arcsinh}(cx))}{4c^2d^3(1+c^2x^2)^2} + \frac{x(a+b\text{arcsinh}(cx))}{8c^2d^3(1+c^2x^2)} + \frac{(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4c^3d^3} - \frac{ib\,\text{PolyLog}\left(2, -ie^{\text{arcsinh}(cx)}\right)}{8c^3d^3} + \frac{ib\,\text{PolyLog}\left(2, ie^{\text{arcsinh}(cx)}\right)}{8c^3d^3}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.85

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \frac{-3acx + 3ac^3x^3 + b\sqrt{1+c^2x^2} + 3bc^2x^2\sqrt{1+c^2x^2} - 3bcx\text{arcsinh}(cx) + 3bc^3x^3\text{arcsinh}(cx) + 3a\arctan(cx)}{}$$

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] (-3*a*c*x + 3*a*c^3*x^3 + b*Sqrt[1 + c^2*x^2] + 3*b*c^2*x^2*Sqrt[1 + c^2*x^
2] - 3*b*c*x*ArcSinh[c*x] + 3*b*c^3*x^3*ArcSinh[c*x] + 3*a*ArcTan[c*x] + 6*
a*c^2*x^2*ArcTan[c*x] + 3*a*c^4*x^4*ArcTan[c*x] + (3*I)*b*ArcSinh[c*x]*Log[
1 - I*E^ArcSinh[c*x]] + (6*I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*
x]] + (3*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (3*I)*b*ArcS
inh[c*x]*Log[1 + I*E^ArcSinh[c*x]] - (6*I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 + I
*E^ArcSinh[c*x]] - (3*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] -
 (3*I)*b*(1 + c^2*x^2)^2*PolyLog[2, (-I)*E^ArcSinh[c*x]] + (3*I)*b*(1 + c^2
*x^2)^2*PolyLog[2, I*E^ArcSinh[c*x]])/(24*c^3*d^3*(1 + c^2*x^2)^2)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{\frac{a\left(\frac{\frac{1}{8}c^3x^3-\frac{1}{8}cx}{(c^2x^2+1)^2}+\frac{\arctan(cx)}{8}\right)}{d^3}+\frac{b\left(\frac{c^3x^3\,\mathrm{arcsinh}(cx)}{8(c^2x^2+1)^2}-\frac{cx\,\mathrm{arcsinh}(cx)}{8(c^2x^2+1)^2}+\frac{\mathrm{arcsinh}(cx)\arctan(cx)}{8}+\frac{\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{8}-\ldots}{}}{c^3}$
default	$\frac{\frac{a\left(\frac{\frac{1}{8}c^3x^3-\frac{1}{8}cx}{(c^2x^2+1)^2}+\frac{\arctan(cx)}{8}\right)}{d^3}+\frac{b\left(\frac{c^3x^3\,\mathrm{arcsinh}(cx)}{8(c^2x^2+1)^2}-\frac{cx\,\mathrm{arcsinh}(cx)}{8(c^2x^2+1)^2}+\frac{\mathrm{arcsinh}(cx)\arctan(cx)}{8}+\frac{\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{8}-\ldots}{}}{c^3}$
parts	$\frac{a\left(\frac{\frac{x^3}{8}-\frac{x}{8c^2}}{(c^2x^2+1)^2}+\frac{\arctan(cx)}{8c^3}\right)}{d^3}+\frac{b\left(\frac{c^3x^3\,\mathrm{arcsinh}(cx)}{8(c^2x^2+1)^2}-\frac{cx\,\mathrm{arcsinh}(cx)}{8(c^2x^2+1)^2}+\frac{\mathrm{arcsinh}(cx)\arctan(cx)}{8}+\frac{\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{8}-\ldots}{}$

```
[In] int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(a/d^3*((1/8*c^3*x^3-1/8*c*x)/(c^2*x^2+1)^2+1/8*arctan(c*x))+b/d^3*(1
/8*c^3*x^3/(c^2*x^2+1)^2*arcsinh(c*x)-1/8*c*x/(c^2*x^2+1)^2*arcsinh(c*x)+1/
8*arcsinh(c*x)*arctan(c*x)+1/8*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/
2))-1/8*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/8*I*dilog(1+I*(1+
I*c*x)/(c^2*x^2+1)^(1/2))+1/8*I*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/8*
c^2*x^2/(c^2*x^2+1)^(3/2)+1/24/(c^2*x^2+1)^(3/2)))
```

Fricas [F]

$$\int \frac{x^2(a+b\mathrm{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx=\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)x^2}{(c^2dx^2+d)^3}\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arcsinh(c*x) + a*x^2)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*
d^3*x^2 + d^3), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\mathrm{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx=\frac{\int \frac{ax^2}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx+\int \frac{bx^2\,\mathrm{asinh}\,(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

```
[In] integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a*x**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral
(b*x**2*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

Maxima [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^2}{(c^2dx^2+d)^3}\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a*((c^2*x^3 - x)/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3) + arctan(c*x)/
(c^3*d^3)) + b*integrate(x^2*log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3*x^6 + 3*
c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Giac [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^2}{(c^2dx^2+d)^3}\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^2/(c^2*d*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx=\int \frac{x^2\,(a+b\,\text{asinh}(c\,x))}{(d\,c^2\,x^2+d)^3}\,dx$$

```
[In] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3,x)
```

```
[Out] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3, x)
```

3.49 $\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 80

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3} dx = \frac{bx}{12cd^3(1+c^2x^2)^{3/2}} + \frac{bx}{6cd^3\sqrt{1+c^2x^2}} - \frac{a+b\text{arcsinh}(cx)}{4c^2d^3(1+c^2x^2)^2}$$

```
[Out] 1/12*b*x/c/d^3/(c^2*x^2+1)^(3/2)+1/4*(-a-b*arcsinh(c*x))/c^2/d^3/(c^2*x^2+1
)^2+1/6*b*x/c/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5798, 198, 197}

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3} dx = -\frac{a+b\text{arcsinh}(cx)}{4c^2d^3(c^2x^2+1)^2} + \frac{bx}{6cd^3\sqrt{c^2x^2+1}} + \frac{bx}{12cd^3(c^2x^2+1)^{3/2}}$$

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] (b*x)/(12*c*d^3*(1 + c^2*x^2)^(3/2)) + (b*x)/(6*c*d^3*Sqrt[1 + c^2*x^2]) -
(a + b*ArcSinh[c*x])/(4*c^2*d^3*(1 + c^2*x^2)^2)
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\text{arcsinh}(cx)}{4c^2d^3(1+c^2x^2)^2}+\frac{b\int\frac{1}{(1+c^2x^2)^{5/2}}\,dx}{4cd^3}\\
&= \frac{bx}{12cd^3(1+c^2x^2)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{4c^2d^3(1+c^2x^2)^2}+\frac{b\int\frac{1}{(1+c^2x^2)^{3/2}}\,dx}{6cd^3}\\
&= \frac{bx}{12cd^3(1+c^2x^2)^{3/2}}+\frac{bx}{6cd^3\sqrt{1+c^2x^2}}-\frac{a+b\text{arcsinh}(cx)}{4c^2d^3(1+c^2x^2)^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.70

$$\int\frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx=\frac{-3a+bcx\sqrt{1+c^2x^2}(3+2c^2x^2)-3b\text{arcsinh}(cx)}{12d^3(c+c^3x^2)^2}$$

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
[Out] (-3*a + b*c*x*Sqrt[1 + c^2*x^2]*(3 + 2*c^2*x^2) - 3*b*ArcSinh[c*x])/(12*d^3
*(c + c^3*x^2)^2)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{-\frac{a}{4d^3\left(c^2x^2+1\right)^2}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{4\left(c^2x^2+1\right)^2}+\frac{cx}{12\left(c^2x^2+1\right)^{\frac{3}{2}}}+\frac{cx}{6\sqrt{c^2x^2+1}}\right)}{d^3}}{c^2}$	76
default	$\frac{-\frac{a}{4d^3\left(c^2x^2+1\right)^2}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{4\left(c^2x^2+1\right)^2}+\frac{cx}{12\left(c^2x^2+1\right)^{\frac{3}{2}}}+\frac{cx}{6\sqrt{c^2x^2+1}}\right)}{d^3}}{c^2}$	76
parts	$-\frac{a}{4d^3c^2(c^2x^2+1)^2}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{4\left(c^2x^2+1\right)^2}+\frac{cx}{12\left(c^2x^2+1\right)^{\frac{3}{2}}}+\frac{cx}{6\sqrt{c^2x^2+1}}\right)}{d^3c^2}$	78

```
[In] int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-1/4*a/d^3/(c^2*x^2+1)^2+b/d^3*(-1/4/(c^2*x^2+1)^2*arcsinh(c*x)+1/12
/(c^2*x^2+1)^(3/2)*c*x+1/6*c*x/(c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.22

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{\left(d+c^2dx^2\right)^3}\,dx = \frac{3\,ac^4x^4+6\,ac^2x^2-3\,b\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(2\,bc^3x^3+3\,bcx\right)\sqrt{c^2x^2+1}}{12\,\left(c^6d^3x^4+2\,c^4d^3x^2+c^2d^3\right)}$$

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^4*x^4 + 6*a*c^2*x^2 - 3*b*log(c*x + sqrt(c^2*x^2 + 1)) + (2*b*c
^3*x^3 + 3*b*c*x)*sqrt(c^2*x^2 + 1))/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3
)
```

Sympy [F]

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \frac{\int \frac{ax}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{bx\operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

```
[In] integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
[Out] (Integral(a*x/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral(b*
x*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

Maxima [F]

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x}{(c^2dx^2+d)^3}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
[Out] -1/16*b*((4*log(c*x + sqrt(c^2*x^2 + 1)) + 1)/(c^6*d^3*x^4 + 2*c^4*d^3*x^2
+ c^2*d^3) - 16*integrate(1/4/(c^8*d^3*x^7 + 3*c^6*d^3*x^5 + 3*c^4*d^3*x^3
+ c^2*d^3*x + (c^7*d^3*x^6 + 3*c^5*d^3*x^4 + 3*c^3*d^3*x^2 + c*d^3)*sqrt(c^
2*x^2 + 1)), x)) - 1/4*a/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3)
```

Giac [F]

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x}{(c^2dx^2+d)^3}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)*x/(c^2*d*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \int \frac{x\,(a+b\operatorname{asinh}(c\,x))}{(d\,c^2\,x^2+d)^3}\,dx$$

```
[In] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3,x)
[Out] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3, x)
```

3.50 $\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 178

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^3}\,dx &= \frac{b}{12cd^3\left(1+c^2x^2\right)^{3/2}}+\frac{3b}{8cd^3\sqrt{1+c^2x^2}}+\frac{x(a+b\text{arcsinh}(cx))}{4d^3\left(1+c^2x^2\right)^2}\\&\quad+\frac{3x(a+b\text{arcsinh}(cx))}{8d^3\left(1+c^2x^2\right)}+\frac{3(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4cd^3}\\&\quad-\frac{3ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{8cd^3}+\frac{3ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{8cd^3}\end{aligned}$$

```
[Out] 1/12*b/c/d^3/(c^2*x^2+1)^(3/2)+1/4*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^2+3
/8*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)+3/4*(a+b*arcsinh(c*x))*arctan(c*x+(
c^2*x^2+1)^(1/2))/c/d^3-3/8*I*b*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^3
+3/8*I*b*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^3+3/8*b/c/d^3/(c^2*x^2+1)
^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5788, 5789, 4265, 2317, 2438, 267}

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^3}\,dx = \frac{3\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{4cd^3} + \frac{3x(a+b\text{arcsinh}(cx))}{8d^3(c^2x^2+1)} + \frac{x(a+b\text{arcsinh}(cx))}{4d^3(c^2x^2+1)^2} - \frac{3ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{8cd^3} + \frac{3ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{8cd^3} + \frac{3b}{8cd^3\sqrt{c^2x^2+1}} + \frac{b}{12cd^3(c^2x^2+1)^{3/2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^3,x]
```

```
[Out] b/(12*c*d^3*(1 + c^2*x^2)^(3/2)) + (3*b)/(8*c*d^3*Sqrt[1 + c^2*x^2]) + (x*(
a + b*ArcSinh[c*x]))/(4*d^3*(1 + c^2*x^2)^2) + (3*x*(a + b*ArcSinh[c*x]))/(
8*d^3*(1 + c^2*x^2)) + (3*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(4*c
*d^3) - (((3*I)/8)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*d^3) + (((3*I)/8)*
b*PolyLog[2, I*E^ArcSinh[c*x]])/(c*d^3)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
```

```
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(a+b\text{arcsinh}(cx))}{4d^3\left(1+c^2x^2\right)^2}-\frac{(bc)\int \frac{x}{\left(1+c^2x^2\right)^{5/2}}\,dx}{4d^3}+\frac{3\int \frac{a+b\text{arcsinh}(cx)}{\left(d+c^2dx^2\right)^2}\,dx}{4d} \\
&= \frac{b}{12cd^3\left(1+c^2x^2\right)^{3/2}}+\frac{x(a+b\text{arcsinh}(cx))}{4d^3\left(1+c^2x^2\right)^2}+\frac{3x(a+b\text{arcsinh}(cx))}{8d^3\left(1+c^2x^2\right)} \\
&\quad -\frac{(3bc)\int \frac{x}{\left(1+c^2x^2\right)^{3/2}}\,dx}{8d^3}+\frac{3\int \frac{a+b\text{arcsinh}(cx)}{d+c^2dx^2}\,dx}{8d^2} \\
&= \frac{b}{12cd^3\left(1+c^2x^2\right)^{3/2}}+\frac{3b}{8cd^3\sqrt{1+c^2x^2}}+\frac{x(a+b\text{arcsinh}(cx))}{4d^3\left(1+c^2x^2\right)^2} \\
&\quad +\frac{3x(a+b\text{arcsinh}(cx))}{8d^3\left(1+c^2x^2\right)}+\frac{3\text{Subst}\left(\int (a+bx)\text{sech}(x)\,dx,x,\text{arcsinh}(cx)\right)}{8cd^3} \\
&= \frac{b}{12cd^3\left(1+c^2x^2\right)^{3/2}}+\frac{3b}{8cd^3\sqrt{1+c^2x^2}}+\frac{x(a+b\text{arcsinh}(cx))}{4d^3\left(1+c^2x^2\right)^2} \\
&\quad +\frac{3x(a+b\text{arcsinh}(cx))}{8d^3\left(1+c^2x^2\right)}+\frac{3(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4cd^3} \\
&\quad -\frac{(3ib)\text{Subst}\left(\int \log\left(1-ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{8cd^3} \\
&\quad +\frac{(3ib)\text{Subst}\left(\int \log\left(1+ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{8cd^3}
\end{aligned}$$

$$= \frac{b}{12cd^3(1+c^2x^2)^{3/2}} + \frac{3b}{8cd^3\sqrt{1+c^2x^2}} + \frac{x(a+b\text{arcsinh}(cx))}{4d^3(1+c^2x^2)^2} + \frac{3x(a+b\text{arcsinh}(cx))}{8d^3(1+c^2x^2)} + \frac{3(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4cd^3} - \frac{(3ib)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{8cd^3} + \frac{(3ib)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{8cd^3}$$

$$= \frac{b}{12cd^3(1+c^2x^2)^{3/2}} + \frac{3b}{8cd^3\sqrt{1+c^2x^2}} + \frac{x(a+b\text{arcsinh}(cx))}{4d^3(1+c^2x^2)^2} + \frac{3x(a+b\text{arcsinh}(cx))}{8d^3(1+c^2x^2)} + \frac{3(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4cd^3} - \frac{3ib\,\text{PolyLog}\left(2, -ie^{\text{arcsinh}(cx)}\right)}{8cd^3} + \frac{3ib\,\text{PolyLog}\left(2, ie^{\text{arcsinh}(cx)}\right)}{8cd^3}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.92

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^3}\,dx = \frac{15acx + 9ac^3x^3 + 11b\sqrt{1+c^2x^2} + 9bc^2x^2\sqrt{1+c^2x^2} + 15bcx\text{arcsinh}(cx) + 9bc^3x^3\text{arcsinh}(cx) + 9a\arctan}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^3,x]
```

```
[Out] (15*a*c*x + 9*a*c^3*x^3 + 11*b*Sqrt[1 + c^2*x^2] + 9*b*c^2*x^2*Sqrt[1 + c^2
*x^2] + 15*b*c*x*ArcSinh[c*x] + 9*b*c^3*x^3*ArcSinh[c*x] + 9*a*ArcTan[c*x]
+ 18*a*c^2*x^2*ArcTan[c*x] + 9*a*c^4*x^4*ArcTan[c*x] + (9*I)*b*ArcSinh[c*x]
*Log[1 - I*E^ArcSinh[c*x]] + (18*I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 - I*E^ArcS
inh[c*x]] + (9*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (9*I)*
b*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] - (18*I)*b*c^2*x^2*ArcSinh[c*x]*Lo
g[1 + I*E^ArcSinh[c*x]] - (9*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[
c*x]] - (9*I)*b*(1 + c^2*x^2)^2*PolyLog[2, (-I)*E^ArcSinh[c*x]] + (9*I)*b*(
1 + c^2*x^2)^2*PolyLog[2, I*E^ArcSinh[c*x]])/(24*c*d^3*(1 + c^2*x^2)^2)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{\frac{a\left(\frac{cx}{4(c^2x^2+1)^2}+\frac{3cx}{8(c^2x^2+1)}+\frac{3\arctan(cx)}{8}\right)}{d^3}+\frac{b\left(\frac{cx\operatorname{arcsinh}(cx)}{4(c^2x^2+1)^2}+\frac{3cx\operatorname{arcsinh}(cx)}{8(c^2x^2+1)}+\frac{3\operatorname{arcsinh}(cx)\arctan(cx)}{8}+\frac{11}{24(c^2x^2+1)^{\frac{3}{2}}}+\frac{}{8(c}\right.}{}}{c}$
default	$\frac{\frac{a\left(\frac{cx}{4(c^2x^2+1)^2}+\frac{3cx}{8(c^2x^2+1)}+\frac{3\arctan(cx)}{8}\right)}{d^3}+\frac{b\left(\frac{cx\operatorname{arcsinh}(cx)}{4(c^2x^2+1)^2}+\frac{3cx\operatorname{arcsinh}(cx)}{8(c^2x^2+1)}+\frac{3\operatorname{arcsinh}(cx)\arctan(cx)}{8}+\frac{11}{24(c^2x^2+1)^{\frac{3}{2}}}+\frac{}{8(c}\right.}{}}{c}$
parts	$\frac{a\left(\frac{x}{4(c^2x^2+1)^2}+\frac{3x}{8(c^2x^2+1)}+\frac{3\arctan(cx)}{8c}\right)}{d^3}+\frac{b\left(\frac{cx\operatorname{arcsinh}(cx)}{4(c^2x^2+1)^2}+\frac{3cx\operatorname{arcsinh}(cx)}{8(c^2x^2+1)}+\frac{3\operatorname{arcsinh}(cx)\arctan(cx)}{8}+\frac{11}{24(c^2x^2+1)^{\frac{3}{2}}}\right.}{}$

```
[In] int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a/d^3*(1/4*c*x/(c^2*x^2+1)^2+3/8*c*x/(c^2*x^2+1)+3/8*arctan(c*x))+b/d^
3*(1/4*c*x/(c^2*x^2+1)^2*arcsinh(c*x)+3/8*c*x/(c^2*x^2+1)*arcsinh(c*x)+3/8*
arcsinh(c*x)*arctan(c*x)+11/24/(c^2*x^2+1)^(3/2)+3/8*c^2*x^2/(c^2*x^2+1)^(3
/2)+3/8*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/8*arctan(c*x)*ln(
1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/8*I*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2
))+3/8*I*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+c^2dx^2)^3}\,dx=\int \frac{b\operatorname{arsinh}(cx)+a}{(c^2dx^2+d)^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2
+ d^3), x)
```

Sympy [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+c^2dx^2)^3}\,dx=\frac{\int \frac{a}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx+\int \frac{b\operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

```
[In] integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral(b*as
inh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + c^2dx^2)^3} \, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(c^2dx^2 + d)^3} \, dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a*((3*c^2*x^3 + 5*x)/(c^4*d^3*x^4 + 2*c^2*d^3*x^2 + d^3) + 3*arctan(c*x
)/(c*d^3)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3*x^6 + 3*c^4*
d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + c^2dx^2)^3} \, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(c^2dx^2 + d)^3} \, dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + c^2dx^2)^3} \, dx = \int \frac{a + b \operatorname{asinh}(c\,x)}{(d\,c^2\,x^2 + d)^3} \, dx$$

```
[In] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^3,x)
```

```
[Out] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^3, x)
```

3.51 $\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 159

$$\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)^3} dx = -\frac{bcx}{12d^3(1+c^2x^2)^{3/2}} - \frac{2bcx}{3d^3\sqrt{1+c^2x^2}} + \frac{a+b\text{arcsinh}(cx)}{4d^3(1+c^2x^2)^2} + \frac{a+b\text{arcsinh}(cx)}{2d^3(1+c^2x^2)} - \frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^3} - \frac{b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{2d^3} + \frac{b\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{2d^3}$$

```
[Out] -1/12*b*c*x/d^3/(c^2*x^2+1)^(3/2)+1/4*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^2+
1/2*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)-2*(a+b*arcsinh(c*x))*arctanh((c*x+(c
^2*x^2+1)^(1/2))^2)/d^3-1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^3+1/2
*b*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d^3-2/3*b*c*x/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5811, 5799, 5569, 4267, 2317, 2438, 197, 198}

$$\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)^3} dx = -\frac{2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d^3} + \frac{a+b\text{arcsinh}(cx)}{2d^3(c^2x^2+1)} + \frac{a+b\text{arcsinh}(cx)}{4d^3(c^2x^2+1)^2} - \frac{b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{2d^3} + \frac{b\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{2d^3} - \frac{2bcx}{3d^3\sqrt{c^2x^2+1}} - \frac{bcx}{12d^3(c^2x^2+1)^{3/2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^3),x]
```

```
[Out] -1/12*(b*c*x)/(d^3*(1 + c^2*x^2)^(3/2)) - (2*b*c*x)/(3*d^3*Sqrt[1 + c^2*x^2
]) + (a + b*ArcSinh[c*x])/(4*d^3*(1 + c^2*x^2)^2) + (a + b*ArcSinh[c*x])/(2
*d^3*(1 + c^2*x^2)) - (2*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/
d^3 - (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/(2*d^3) + (b*PolyLog[2, E^(2*ArcS
inh[c*x])])/(2*d^3)
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
```

```
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{a+b\text{arcsinh}(cx)}{4d^3\left(1+c^2x^2\right)^2}-\frac{(bc)\int\frac{1}{(1+c^2x^2)^{5/2}}\,dx}{4d^3}+\frac{\int\frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)^2}\,dx}{d}\\
&=-\frac{bcx}{12d^3\left(1+c^2x^2\right)^{3/2}}+\frac{a+b\text{arcsinh}(cx)}{4d^3\left(1+c^2x^2\right)^2}+\frac{a+b\text{arcsinh}(cx)}{2d^3\left(1+c^2x^2\right)}\\
&\quad-\frac{(bc)\int\frac{1}{(1+c^2x^2)^{3/2}}\,dx}{6d^3}-\frac{(bc)\int\frac{1}{(1+c^2x^2)^{3/2}}\,dx}{2d^3}+\frac{\int\frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)}\,dx}{d^2}\\
&=-\frac{bcx}{12d^3\left(1+c^2x^2\right)^{3/2}}-\frac{2bcx}{3d^3\sqrt{1+c^2x^2}}+\frac{a+b\text{arcsinh}(cx)}{4d^3\left(1+c^2x^2\right)^2}\\
&\quad+\frac{a+b\text{arcsinh}(cx)}{2d^3\left(1+c^2x^2\right)}+\frac{\text{Subst}\left(\int(a+bx)\text{csch}(x)\text{sech}(x)\,dx,x,\text{arcsinh}(cx)\right)}{d^3}\\
&=-\frac{bcx}{12d^3\left(1+c^2x^2\right)^{3/2}}-\frac{2bcx}{3d^3\sqrt{1+c^2x^2}}+\frac{a+b\text{arcsinh}(cx)}{4d^3\left(1+c^2x^2\right)^2}\\
&\quad+\frac{a+b\text{arcsinh}(cx)}{2d^3\left(1+c^2x^2\right)}+\frac{2\text{Subst}\left(\int(a+bx)\text{csch}(2x)\,dx,x,\text{arcsinh}(cx)\right)}{d^3}\\
&=-\frac{bcx}{12d^3\left(1+c^2x^2\right)^{3/2}}-\frac{2bcx}{3d^3\sqrt{1+c^2x^2}}+\frac{a+b\text{arcsinh}(cx)}{4d^3\left(1+c^2x^2\right)^2}\\
&\quad+\frac{a+b\text{arcsinh}(cx)}{2d^3\left(1+c^2x^2\right)}-\frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^3}\\
&\quad-\frac{b\text{Subst}\left(\int\log\left(1-e^{2x}\right)\,dx,x,\text{arcsinh}(cx)\right)}{d^3}\\
&\quad+\frac{b\text{Subst}\left(\int\log\left(1+e^{2x}\right)\,dx,x,\text{arcsinh}(cx)\right)}{d^3}
\end{aligned}
$$

$$= -\frac{bcx}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{2bcx}{3d^3\sqrt{1+c^2x^2}} + \frac{a+b\text{arcsinh}(cx)}{4d^3\left(1+c^2x^2\right)^2} + \frac{a+b\text{arcsinh}(cx)}{2d^3\left(1+c^2x^2\right)} - \frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^3} - \frac{b\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\text{arcsinh}(cx)}\right)}{2d^3} + \frac{b\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\text{arcsinh}(cx)}\right)}{2d^3}$$

$$= -\frac{bcx}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{2bcx}{3d^3\sqrt{1+c^2x^2}} + \frac{a+b\text{arcsinh}(cx)}{4d^3\left(1+c^2x^2\right)^2} + \frac{a+b\text{arcsinh}(cx)}{2d^3\left(1+c^2x^2\right)} - \frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^3} - \frac{b\,\text{PolyLog}\left(2, -e^{2\text{arcsinh}(cx)}\right)}{2d^3} + \frac{b\,\text{PolyLog}\left(2, e^{2\text{arcsinh}(cx)}\right)}{2d^3}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.82

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^3}\,dx = \frac{-\frac{2a^2}{b} + \frac{a}{(1+c^2x^2)^2} - \frac{bcx}{3(1+c^2x^2)^{3/2}} + \frac{2a}{1+c^2x^2} - \frac{8bcx}{3\sqrt{1+c^2x^2}} - 4a\text{arcsinh}(cx) + \frac{b\text{arcsinh}(cx)}{(1+c^2x^2)^2} + \frac{2b\text{arcsinh}(cx)}{1+c^2x^2} - 4b\text{arc}}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^3),x]
```

```
[Out] ((-2*a^2)/b + a/(1 + c^2*x^2)^2 - (b*c*x)/(3*(1 + c^2*x^2)^(3/2)) + (2*a)/(
1 + c^2*x^2) - (8*b*c*x)/(3*Sqrt[1 + c^2*x^2]) - 4*a*ArcSinh[c*x] + (b*ArcS
inh[c*x])/(1 + c^2*x^2)^2 + (2*b*ArcSinh[c*x])/(1 + c^2*x^2) - 4*b*ArcSinh[
c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 4*b*ArcSinh[c*x]*Log[1 + (Sqr
t[-c^2]*E^ArcSinh[c*x])/c] + 4*a*Log[1 - E^(2*ArcSinh[c*x])] + 4*b*ArcSinh[
c*x]*Log[1 - E^(2*ArcSinh[c*x])] - 2*a*Log[1 + c^2*x^2] - 4*b*PolyLog[2, (c
*E^ArcSinh[c*x])/Sqrt[-c^2]] - 4*b*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c
] + 2*b*PolyLog[2, E^(2*ArcSinh[c*x])])/(4*d^3)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.76

method	result
derivativedivides	$\frac{a\left(\ln(cx)+\frac{1}{4(c^2x^2+1)^2}+\frac{1}{2c^2x^2+2}-\frac{\ln(c^2x^2+1)}{2}\right)}{d^3}+\frac{b\left(\frac{-8c^3x^3\sqrt{c^2x^2+1}+8c^4x^4+6\,\mathrm{arcsinh}(cx)c^2x^2-9cx\sqrt{c^2x^2+1}+16c^2x^2+}{12c^4x^4+24c^2x^2+12}\right.}{}$
default	$\frac{a\left(\ln(cx)+\frac{1}{4(c^2x^2+1)^2}+\frac{1}{2c^2x^2+2}-\frac{\ln(c^2x^2+1)}{2}\right)}{d^3}+\frac{b\left(\frac{-8c^3x^3\sqrt{c^2x^2+1}+8c^4x^4+6\,\mathrm{arcsinh}(cx)c^2x^2-9cx\sqrt{c^2x^2+1}+16c^2x^2+}{12c^4x^4+24c^2x^2+12}\right.}{}$
parts	$\frac{a\left(-\frac{c^2\left(-\frac{1}{c^2(c^2x^2+1)}-\frac{1}{2c^2(c^2x^2+1)^2}+\frac{\ln(c^2x^2+1)}{c^2}\right)}{2}+\ln(x)\right)}{d^3}+\frac{b\left(\frac{-8c^3x^3\sqrt{c^2x^2+1}+8c^4x^4+6\,\mathrm{arcsinh}(cx)c^2x^2-9cx\sqrt{c^2x}}{12c^4x^4+24c^2x^2+12}\right.}{}$

```
[In] int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a/d^3*(ln(c*x)+1/4/(c^2*x^2+1)^2+1/2/(c^2*x^2+1)-1/2*ln(c^2*x^2+1))+b/d^3*(
1/12*(-8*c^3*x^3*(c^2*x^2+1)^(1/2)+8*c^4*x^4+6*arcsinh(c*x)*c^2*x^2-9*c*x*(
c^2*x^2+1)^(1/2)+16*c^2*x^2+9*arcsinh(c*x)+8)/(c^4*x^4+2*c^2*x^2+1)+arcsinh
(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+polylog(2,-c*x-(c^2*x^2+1)^(1/2))-arcsinh
(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2)
)^2)+arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+polylog(2,c*x+(c^2*x^2+1)^(1/
2)))
```

Fricas [F]

$$\int \frac{a+b\mathrm{arcsinh}(cx)}{x\,(d+c^2dx^2)^3}\,dx=\int \frac{b\,\mathrm{arsinh}\,(cx)+a}{(c^2dx^2+d)^3x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3
+ d^3*x), x)
```

Sympy [F]

$$\int \frac{a+b\mathrm{arcsinh}(cx)}{x\,(d+c^2dx^2)^3}\,dx=\frac{\int \frac{a}{c^6x^7+3c^4x^5+3c^2x^3+x}\,dx+\int \frac{b\,\mathrm{asinh}\,(cx)}{c^6x^7+3c^4x^5+3c^2x^3+x}\,dx}{d^3}$$

```
[In] integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a/(c**6*x**7 + 3*c**4*x**5 + 3*c**2*x**3 + x), x) + Integral(b*as
inh(c*x)/(c**6*x**7 + 3*c**4*x**5 + 3*c**2*x**3 + x), x))/d**3
```

Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x\left(d + c^2dx^2\right)^3} \, dx = \int \frac{b \operatorname{arsinh}\left(cx\right) + a}{\left(c^2dx^2 + d\right)^3 x} \, dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*((2*c^2*x^2 + 3)/(c^4*d^3*x^4 + 2*c^2*d^3*x^2 + d^3) - 2*log(c^2*x^2
+ 1)/d^3 + 4*log(x)/d^3) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^
3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x), x)
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x\left(d + c^2dx^2\right)^3} \, dx = \int \frac{b \operatorname{arsinh}\left(cx\right) + a}{\left(c^2dx^2 + d\right)^3 x} \, dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^3*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{x\left(d + c^2dx^2\right)^3} \, dx = \int \frac{a + b \operatorname{asinh}(c\,x)}{x\left(d\,c^2\,x^2 + d\right)^3} \, dx$$

```
[In] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^3),x)
```

```
[Out] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^3), x)
```

3.52 $\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 222

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^3}\,dx =& -\frac{bc}{12d^3(1+c^2x^2)^{3/2}}-\frac{7bc}{8d^3\sqrt{1+c^2x^2}}-\frac{a+b\text{arcsinh}(cx)}{d^3x(1+c^2x^2)^2}\\ &-\frac{5c^2x(a+b\text{arcsinh}(cx))}{4d^3(1+c^2x^2)^2}-\frac{15c^2x(a+b\text{arcsinh}(cx))}{8d^3(1+c^2x^2)}\\ &-\frac{15c(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4d^3}\\ &-\frac{b\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{d^3}+\frac{15ibc\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{8d^3}\\ &-\frac{15ibc\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{8d^3}\end{aligned}$$

```
[Out] -1/12*b*c/d^3/(c^2*x^2+1)^(3/2)+(-a-b*arcsinh(c*x))/d^3/x/(c^2*x^2+1)^2-5/4
*c^2*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^2-15/8*c^2*x*(a+b*arcsinh(c*x))/d
^3/(c^2*x^2+1)-15/4*c*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/d^3-
b*c*arctanh((c^2*x^2+1)^(1/2))/d^3+15/8*I*b*c*polylog(2,-I*(c*x+(c^2*x^2+1)
^(1/2)))/d^3-15/8*I*b*c*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d^3-7/8*b*c/d^
3/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {5809, 5788, 5789, 4265, 2317, 2438, 267, 272, 53, 65, 214}

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^3}\,dx = -\frac{15c\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{4d^3} - \frac{15c^2x(a+b\text{arcsinh}(cx))}{8d^3(c^2x^2+1)} - \frac{5c^2x(a+b\text{arcsinh}(cx))}{4d^3(c^2x^2+1)^2} - \frac{a+b\text{arcsinh}(cx)}{d^3x(c^2x^2+1)^2} + \frac{15ibc\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{8d^3} - \frac{15ibc\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{8d^3} - \frac{b\text{carctanh}\left(\sqrt{c^2x^2+1}\right)}{d^3} - \frac{7bc}{8d^3\sqrt{c^2x^2+1}} - \frac{bc}{12d^3(c^2x^2+1)^{3/2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^3),x]
```

```
[Out] -1/12*(b*c)/(d^3*(1 + c^2*x^2)^(3/2)) - (7*b*c)/(8*d^3*Sqrt[1 + c^2*x^2]) -
 (a + b*ArcSinh[c*x])/(d^3*x*(1 + c^2*x^2)^2) - (5*c^2*x*(a + b*ArcSinh[c*x
]))/(4*d^3*(1 + c^2*x^2)^2) - (15*c^2*x*(a + b*ArcSinh[c*x]))/(8*d^3*(1 + c
^2*x^2)) - (15*c*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(4*d^3) - (b*
c*ArcTanh[Sqrt[1 + c^2*x^2]])/d^3 + (((15*I)/8)*b*c*PolyLog[2, (-I)*E^ArcSi
nh[c*x]])/d^3 - (((15*I)/8)*b*c*PolyLog[2, I*E^ArcSinh[c*x]])/d^3
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\operatorname{arcsinh}(cx)}{d^3x\left(1+c^2x^2\right)^2}-\left(5c^2\right)\int\frac{a+b\operatorname{arcsinh}(cx)}{\left(d+c^2dx^2\right)^3}\,dx+\frac{(bc)\int\frac{1}{x\left(1+c^2x^2\right)^{5/2}}\,dx}{d^3}\\
&= -\frac{a+b\operatorname{arcsinh}(cx)}{d^3x\left(1+c^2x^2\right)^2}-\frac{5c^2x(a+b\operatorname{arcsinh}(cx))}{4d^3\left(1+c^2x^2\right)^2}+\frac{(bc)\mathrm{Subst}\left(\int\frac{1}{x\left(1+c^2x\right)^{5/2}}\,dx,x,x^2\right)}{2d^3}\\
&\quad+\frac{\left(5bc^3\right)\int\frac{x}{\left(1+c^2x^2\right)^{5/2}}\,dx}{4d^3}-\frac{\left(15c^2\right)\int\frac{a+b\operatorname{arcsinh}(cx)}{\left(d+c^2dx^2\right)^2}\,dx}{4d}\\
&= -\frac{bc}{12d^3\left(1+c^2x^2\right)^{3/2}}-\frac{a+b\operatorname{arcsinh}(cx)}{d^3x\left(1+c^2x^2\right)^2}-\frac{5c^2x(a+b\operatorname{arcsinh}(cx))}{4d^3\left(1+c^2x^2\right)^2}\\
&\quad-\frac{15c^2x(a+b\operatorname{arcsinh}(cx))}{8d^3\left(1+c^2x^2\right)}+\frac{(bc)\mathrm{Subst}\left(\int\frac{1}{x\left(1+c^2x\right)^{3/2}}\,dx,x,x^2\right)}{2d^3}\\
&\quad+\frac{\left(15bc^3\right)\int\frac{x}{\left(1+c^2x^2\right)^{3/2}}\,dx}{8d^3}-\frac{\left(15c^2\right)\int\frac{a+b\operatorname{arcsinh}(cx)}{d+c^2dx^2}\,dx}{8d^2}\\
&= -\frac{bc}{12d^3\left(1+c^2x^2\right)^{3/2}}-\frac{7bc}{8d^3\sqrt{1+c^2x^2}}-\frac{a+b\operatorname{arcsinh}(cx)}{d^3x\left(1+c^2x^2\right)^2}-\frac{5c^2x(a+b\operatorname{arcsinh}(cx))}{4d^3\left(1+c^2x^2\right)^2}\\
&\quad-\frac{15c^2x(a+b\operatorname{arcsinh}(cx))}{8d^3\left(1+c^2x^2\right)}-\frac{(15c)\mathrm{Subst}\left(\int(a+bx)\operatorname{sech}(x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{8d^3}\\
&\quad+\frac{(bc)\mathrm{Subst}\left(\int\frac{1}{x\sqrt{1+c^2x}}\,dx,x,x^2\right)}{2d^3}
\end{aligned}$$

$$= -\frac{bc}{12d^3(1+c^2x^2)^{3/2}} - \frac{7bc}{8d^3\sqrt{1+c^2x^2}} - \frac{a+b\text{arcsinh}(cx)}{d^3x(1+c^2x^2)^2} - \frac{5c^2x(a+b\text{arcsinh}(cx))}{4d^3(1+c^2x^2)^2} - \frac{15c^2x(a+b\text{arcsinh}(cx))}{8d^3(1+c^2x^2)} - \frac{15c(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4d^3} + \frac{b\text{Subst}\left(\int \frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx, x, \sqrt{1+c^2x^2}\right)}{cd^3} + \frac{(15ibc)\text{Subst}\left(\int \log\left(1-ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{8d^3} - \frac{(15ibc)\text{Subst}\left(\int \log\left(1+ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{8d^3}$$

$$= -\frac{bc}{12d^3(1+c^2x^2)^{3/2}} - \frac{7bc}{8d^3\sqrt{1+c^2x^2}} - \frac{a+b\text{arcsinh}(cx)}{d^3x(1+c^2x^2)^2} - \frac{5c^2x(a+b\text{arcsinh}(cx))}{4d^3(1+c^2x^2)^2} - \frac{15c^2x(a+b\text{arcsinh}(cx))}{8d^3(1+c^2x^2)} - \frac{15c(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4d^3} - \frac{b\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{d^3} + \frac{(15ibc)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{8d^3} - \frac{(15ibc)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{8d^3}$$

$$= -\frac{bc}{12d^3(1+c^2x^2)^{3/2}} - \frac{7bc}{8d^3\sqrt{1+c^2x^2}} - \frac{a+b\text{arcsinh}(cx)}{d^3x(1+c^2x^2)^2} - \frac{5c^2x(a+b\text{arcsinh}(cx))}{4d^3(1+c^2x^2)^2} - \frac{15c^2x(a+b\text{arcsinh}(cx))}{8d^3(1+c^2x^2)} - \frac{15c(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4d^3} - \frac{b\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{d^3} + \frac{15ibc\,\text{PolyLog}\left(2, -ie^{\text{arcsinh}(cx)}\right)}{8d^3} - \frac{15ibc\,\text{PolyLog}\left(2, ie^{\text{arcsinh}(cx)}\right)}{8d^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.78 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.34

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^3}\,dx = -\frac{\frac{45(a+b\text{arcsinh}(cx))}{x} - \frac{6(a+b\text{arcsinh}(cx))}{x(1+c^2x^2)^2} - \frac{15(a+b\text{arcsinh}(cx))}{x+c^2x^3} + 45ac\arctan(cx) + 45b\text{carctanh}\left(\sqrt{1+c^2x^2}\right) +}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^3),x]
```

[Out] -1/24*((45*(a + b*ArcSinh[c*x]))/x - (6*(a + b*ArcSinh[c*x]))/(x*(1 + c^2*x^2)^2) - (15*(a + b*ArcSinh[c*x]))/(x + c^2*x^3) + 45*a*c*ArcTan[c*x] + 45*b*c*ArcTanh[Sqrt[1 + c^2*x^2]] + (2*b*c*Hypergeometric2F1[-3/2, 1, -1/2, 1 + c^2*x^2])/(1 + c^2*x^2)^(3/2) + (15*b*c*Hypergeometric2F1[-1/2, 1, 1/2, 1 + c^2*x^2])/Sqrt[1 + c^2*x^2] + 45*b*Sqrt[-c^2]*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 45*b*Sqrt[-c^2]*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 45*b*Sqrt[-c^2]*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 45*b*Sqrt[-c^2]*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/d^3

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.31

method	result
derivativedivides	$c\left(\frac{a\left(-\frac{1}{cx}-\frac{\frac{7}{8}c^3x^3+\frac{9}{8}cx}{\left(c^2x^2+1\right)^2}-\frac{15\arctan(cx)}{8}\right)}{d^3}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{cx}-\frac{7c^3x^3\operatorname{arcsinh}(cx)}{8\left(c^2x^2+1\right)^2}-\frac{9cx\operatorname{arcsinh}(cx)}{8\left(c^2x^2+1\right)^2}-\frac{15\operatorname{arcsinh}(cx)\arctan(\ldots}{8}\ldots\right.}{\ldots}\right.$
default	$c\left(\frac{a\left(-\frac{1}{cx}-\frac{\frac{7}{8}c^3x^3+\frac{9}{8}cx}{\left(c^2x^2+1\right)^2}-\frac{15\arctan(cx)}{8}\right)}{d^3}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{cx}-\frac{7c^3x^3\operatorname{arcsinh}(cx)}{8\left(c^2x^2+1\right)^2}-\frac{9cx\operatorname{arcsinh}(cx)}{8\left(c^2x^2+1\right)^2}-\frac{15\operatorname{arcsinh}(cx)\arctan(\ldots}{8}\ldots\right.}{\ldots}\right.$
parts	$\frac{a\left(-c^2\left(\frac{\frac{7}{8}x^3c^2+\frac{9}{8}x}{\left(c^2x^2+1\right)^2}+\frac{15\arctan(cx)}{8c}\right)-\frac{1}{x}\right)}{d^3}+\frac{bc\left(-\frac{\operatorname{arcsinh}(cx)}{cx}-\frac{7c^3x^3\operatorname{arcsinh}(cx)}{8\left(c^2x^2+1\right)^2}-\frac{9cx\operatorname{arcsinh}(cx)}{8\left(c^2x^2+1\right)^2}-\frac{15\operatorname{arcsinh}(cx)\arctan\ldots}{8}\ldots\right.}{\ldots}$

[In] int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)

[Out] c*(a/d^3*(-1/c/x-(7/8*c^3*x^3+9/8*c*x)/(c^2*x^2+1)^2-15/8*arctan(c*x))+b/d^3*(-arcsinh(c*x)/c/x-7/8*c^3*x^3/(c^2*x^2+1)^2*arcsinh(c*x)-9/8*c*x/(c^2*x^2+1)^2*arcsinh(c*x)-15/8*arcsinh(c*x)*arctan(c*x)-15/8*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+15/8*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+15/8*I*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-15/8*I*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-47/24/(c^2*x^2+1)^(3/2)+1/(c^2*x^2+1)^(1/2)-arctanh(1/(c^2*x^2+1)^(1/2))-15/8*c^2*x^2/(c^2*x^2+1)^(3/2)))

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(d+c^2dx^2\right)^3}\,dx=\int \frac{b\operatorname{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^3x^2}\,dx$$

[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")

[Out] integral((b*arcsinh(c*x) + a)/(c^6*d^3*x^8 + 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 + d^3*x^2), x)

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(d+c^2dx^2\right)^3}\,dx=\frac{\int \frac{a}{c^6x^8+3c^4x^6+3c^2x^4+x^2}\,dx+\int \frac{b\operatorname{asinh}\left(cx\right)}{c^6x^8+3c^4x^6+3c^2x^4+x^2}\,dx}{d^3}$$

[In] integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d)**3,x)

[Out] (Integral(a/(c**6*x**8 + 3*c**4*x**6 + 3*c**2*x**4 + x**2), x) + Integral(b*asinh(c*x)/(c**6*x**8 + 3*c**4*x**6 + 3*c**2*x**4 + x**2), x))/d**3

Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(d+c^2dx^2\right)^3}\,dx=\int \frac{b\operatorname{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^3x^2}\,dx$$

[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")

[Out] -1/8*a*((15*c^4*x^4 + 25*c^2*x^2 + 8)/(c^4*d^3*x^5 + 2*c^2*d^3*x^3 + d^3*x) + 15*c*arctan(c*x)/d^3) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3*x^8 + 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 + d^3*x^2), x)

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(d+c^2dx^2\right)^3}\,dx=\int \frac{b\operatorname{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^3x^2}\,dx$$

[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^3,x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^3*x^2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^2\left(d+c^2dx^2\right)^3}\,dx=\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x^2\,{\left(d\,c^2\,x^2+d\right)}^3}\,dx$$

```
[In] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^3),x)
[Out] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^3), x)
```

3.53 $\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 232

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^3}\,dx = &-\frac{bc}{2d^3x(1+c^2x^2)^{3/2}}-\frac{5bc^3x}{12d^3(1+c^2x^2)^{3/2}}\\&+\frac{2bc^3x}{3d^3\sqrt{1+c^2x^2}}-\frac{3c^2(a+b\text{arcsinh}(cx))}{4d^3(1+c^2x^2)^2}\\&-\frac{a+b\text{arcsinh}(cx)}{2d^3x^2(1+c^2x^2)^2}-\frac{3c^2(a+b\text{arcsinh}(cx))}{2d^3(1+c^2x^2)}\\&+\frac{6c^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^3}\\&+\frac{3bc^2\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{2d^3}-\frac{3bc^2\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{2d^3}\end{aligned}$$

```
[Out] -1/2*b*c/d^3/x/(c^2*x^2+1)^(3/2)-5/12*b*c^3*x/d^3/(c^2*x^2+1)^(3/2)-3/4*c^2
*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^2+1/2*(-a-b*arcsinh(c*x))/d^3/x^2/(c^2*
x^2+1)^2-3/2*c^2*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)+6*c^2*(a+b*arcsinh(c*x)
)*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)/d^3+3/2*b*c^2*polylog(2,-(c*x+(c^2*x^2
+1)^(1/2))^2)/d^3-3/2*b*c^2*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d^3+2/3*b*
c^3*x/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5809, 5811, 5799, 5569, 4267, 2317, 2438, 197, 198, 277}

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^3}\,dx = \frac{6c^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d^3} - \frac{3c^2(a+b\text{arcsinh}(cx))}{2d^3(c^2x^2+1)} - \frac{3c^2(a+b\text{arcsinh}(cx))}{4d^3(c^2x^2+1)^2} - \frac{a+b\text{arcsinh}(cx)}{2d^3x^2(c^2x^2+1)^2} + \frac{3bc^2\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{2d^3} - \frac{3bc^2\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{2d^3} - \frac{bc}{2d^3x(c^2x^2+1)^{3/2}} + \frac{2bc^3x}{3d^3\sqrt{c^2x^2+1}} - \frac{5bc^3x}{12d^3(c^2x^2+1)^{3/2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^3),x]
```

```
[Out] -1/2*(b*c)/(d^3*x*(1 + c^2*x^2)^(3/2)) - (5*b*c^3*x)/(12*d^3*(1 + c^2*x^2)^
(3/2)) + (2*b*c^3*x)/(3*d^3*Sqrt[1 + c^2*x^2]) - (3*c^2*(a + b*ArcSinh[c*x]
))/(4*d^3*(1 + c^2*x^2)^2) - (a + b*ArcSinh[c*x])/(2*d^3*x^2*(1 + c^2*x^2)^
2) - (3*c^2*(a + b*ArcSinh[c*x]))/(2*d^3*(1 + c^2*x^2)) + (6*c^2*(a + b*Arc
Sinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/d^3 + (3*b*c^2*PolyLog[2, -E^(2*Arc
Sinh[c*x])])/(2*d^3) - (3*b*c^2*PolyLog[2, E^(2*ArcSinh[c*x])])/(2*d^3)
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
```

```
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\text{arcsinh}(cx)}{2d^3x^2\left(1+c^2x^2\right)^2}-\left(3c^2\right)\int\frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^3}\,dx+\frac{(bc)\int\frac{1}{x^2\left(1+c^2x^2\right)^{5/2}}\,dx}{2d^3}\\
&= -\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}}-\frac{3c^2(a+b\text{arcsinh}(cx))}{4d^3\left(1+c^2x^2\right)^2}-\frac{a+b\text{arcsinh}(cx)}{2d^3x^2\left(1+c^2x^2\right)^2}\\
&\quad+\frac{\left(3bc^3\right)\int\frac{1}{\left(1+c^2x^2\right)^{5/2}}\,dx}{4d^3}-\frac{\left(2bc^3\right)\int\frac{1}{\left(1+c^2x^2\right)^{5/2}}\,dx}{d^3}-\frac{\left(3c^2\right)\int\frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^2}\,dx}{d}\\
&= -\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}}-\frac{5bc^3x}{12d^3\left(1+c^2x^2\right)^{3/2}}-\frac{3c^2(a+b\text{arcsinh}(cx))}{4d^3\left(1+c^2x^2\right)^2}\\
&\quad-\frac{a+b\text{arcsinh}(cx)}{2d^3x^2\left(1+c^2x^2\right)^2}-\frac{3c^2(a+b\text{arcsinh}(cx))}{2d^3\left(1+c^2x^2\right)}+\frac{\left(bc^3\right)\int\frac{1}{\left(1+c^2x^2\right)^{3/2}}\,dx}{2d^3}\\
&\quad-\frac{\left(4bc^3\right)\int\frac{1}{\left(1+c^2x^2\right)^{3/2}}\,dx}{3d^3}+\frac{\left(3bc^3\right)\int\frac{1}{\left(1+c^2x^2\right)^{3/2}}\,dx}{2d^3}-\frac{\left(3c^2\right)\int\frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)}\,dx}{d^2}\\
&= -\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}}-\frac{5bc^3x}{12d^3\left(1+c^2x^2\right)^{3/2}}+\frac{2bc^3x}{3d^3\sqrt{1+c^2x^2}}\\
&\quad-\frac{3c^2(a+b\text{arcsinh}(cx))}{4d^3\left(1+c^2x^2\right)^2}-\frac{a+b\text{arcsinh}(cx)}{2d^3x^2\left(1+c^2x^2\right)^2}-\frac{3c^2(a+b\text{arcsinh}(cx))}{2d^3\left(1+c^2x^2\right)}\\
&\quad-\frac{\left(3c^2\right)\text{Subst}\left(\int(a+bx)\text{csch}(x)\text{sech}(x)\,dx,x,\text{arcsinh}(cx)\right)}{d^3}\\
&= -\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}}-\frac{5bc^3x}{12d^3\left(1+c^2x^2\right)^{3/2}}+\frac{2bc^3x}{3d^3\sqrt{1+c^2x^2}}\\
&\quad-\frac{3c^2(a+b\text{arcsinh}(cx))}{4d^3\left(1+c^2x^2\right)^2}-\frac{a+b\text{arcsinh}(cx)}{2d^3x^2\left(1+c^2x^2\right)^2}-\frac{3c^2(a+b\text{arcsinh}(cx))}{2d^3\left(1+c^2x^2\right)}\\
&\quad-\frac{\left(6c^2\right)\text{Subst}\left(\int(a+bx)\text{csch}(2x)\,dx,x,\text{arcsinh}(cx)\right)}{d^3}
\end{aligned}$$

$$= -\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x}{12d^3\left(1+c^2x^2\right)^{3/2}} + \frac{2bc^3x}{3d^3\sqrt{1+c^2x^2}} - \frac{3c^2(a+b\operatorname{arcsinh}(cx))}{4d^3\left(1+c^2x^2\right)^2} - \frac{a+b\operatorname{arcsinh}(cx)}{2d^3x^2\left(1+c^2x^2\right)^2} - \frac{3c^2(a+b\operatorname{arcsinh}(cx))}{2d^3\left(1+c^2x^2\right)} + \frac{6c^2(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{2\operatorname{arcsinh}(cx)}\right)}{d^3} + \frac{(3bc^2)\operatorname{Subst}\left(\int\log\left(1-e^{2x}\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{d^3} - \frac{(3bc^2)\operatorname{Subst}\left(\int\log\left(1+e^{2x}\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{d^3}$$

$$= -\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x}{12d^3\left(1+c^2x^2\right)^{3/2}} + \frac{2bc^3x}{3d^3\sqrt{1+c^2x^2}} - \frac{3c^2(a+b\operatorname{arcsinh}(cx))}{4d^3\left(1+c^2x^2\right)^2} - \frac{a+b\operatorname{arcsinh}(cx)}{2d^3x^2\left(1+c^2x^2\right)^2} - \frac{3c^2(a+b\operatorname{arcsinh}(cx))}{2d^3\left(1+c^2x^2\right)} + \frac{6c^2(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{2\operatorname{arcsinh}(cx)}\right)}{d^3} + \frac{(3bc^2)\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2\operatorname{arcsinh}(cx)}\right)}{2d^3} - \frac{(3bc^2)\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\operatorname{arcsinh}(cx)}\right)}{2d^3}$$

$$= -\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x}{12d^3\left(1+c^2x^2\right)^{3/2}} + \frac{2bc^3x}{3d^3\sqrt{1+c^2x^2}} - \frac{3c^2(a+b\operatorname{arcsinh}(cx))}{4d^3\left(1+c^2x^2\right)^2} - \frac{a+b\operatorname{arcsinh}(cx)}{2d^3x^2\left(1+c^2x^2\right)^2} - \frac{3c^2(a+b\operatorname{arcsinh}(cx))}{2d^3\left(1+c^2x^2\right)} + \frac{6c^2(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{2\operatorname{arcsinh}(cx)}\right)}{d^3} + \frac{3bc^2\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{2d^3} - \frac{3bc^2\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(cx)}\right)}{2d^3}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.52

$$\int\frac{a+b\operatorname{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^3}\,dx = \frac{-\frac{18bc\sqrt{1+c^2x^2}}{x} + \frac{9bc(1+2c^2x^2)}{x\sqrt{1+c^2x^2}} + \frac{bc(3+12c^2x^2+8c^4x^4)}{x(1+c^2x^2)^{3/2}} - 18bc^2\operatorname{arcsinh}(cx)^2 - \frac{18(a+b\operatorname{arcsinh}(cx))}{x^2} + \frac{3(a+b\operatorname{arcsinh}(cx))}{(x+c^2x^3)^2} +}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^3),x]
```

```
[Out] ((-18*b*c*Sqrt[1 + c^2*x^2])/x + (9*b*c*(1 + 2*c^2*x^2))/(x*Sqrt[1 + c^2*x^
2]) + (b*c*(3 + 12*c^2*x^2 + 8*c^4*x^4))/(x*(1 + c^2*x^2)^(3/2)) - 18*b*c^2
*ArcSinh[c*x]^2 - (18*(a + b*ArcSinh[c*x]))/x^2 + (3*(a + b*ArcSinh[c*x]))/
(x + c^2*x^3)^2 + (9*(a + b*ArcSinh[c*x]))/(x^2 + c^2*x^4) + (18*c^2*(a + b
```

```
*ArcSinh[c*x])^2)/b + 36*b*c^2*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqrt
[-c^2]] + 36*b*c^2*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 18
*a*c^2*Log[1 + c^2*x^2] + 36*b*c^2*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]
] + 36*b*c^2*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 18*c^2*(2*(a + b*A
rcSinh[c*x])*Log[1 - E^(2*ArcSinh[c*x])] + b*PolyLog[2, E^(2*ArcSinh[c*x])]
))/(12*d^3)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.48

method	result
derivativedivides	$c^2\left(\frac{a\left(-\frac{1}{2c^2x^2}-3\ln(cx)-\frac{1}{4(c^2x^2+1)^2}-\frac{1}{c^2x^2+1}+\frac{3\ln(c^2x^2+1)}{2}\right)}{d^3}+\frac{b\left(-\frac{-8c^5x^5\sqrt{c^2x^2+1}+8c^6x^6+18\operatorname{arcsinh}(cx)c^4x^4-\cdots}{\cdots}\right.}{\cdots}\right.$
default	$c^2\left(\frac{a\left(-\frac{1}{2c^2x^2}-3\ln(cx)-\frac{1}{4(c^2x^2+1)^2}-\frac{1}{c^2x^2+1}+\frac{3\ln(c^2x^2+1)}{2}\right)}{d^3}+\frac{b\left(-\frac{-8c^5x^5\sqrt{c^2x^2+1}+8c^6x^6+18\operatorname{arcsinh}(cx)c^4x^4-\cdots}{\cdots}\right.}{\cdots}\right.$
parts	$\frac{a\left(\frac{c^4\left(-\frac{2}{c^2(c^2x^2+1)}-\frac{1}{2c^2(c^2x^2+1)^2}+\frac{3\ln(c^2x^2+1)}{c^2}\right)}{2}-\frac{1}{2x^2}-3c^2\ln(x)\right)}{d^3}+\frac{b\,c^2\left(-\frac{-8c^5x^5\sqrt{c^2x^2+1}+8c^6x^6+18\operatorname{arcsinh}(\cdots}{\cdots}\right.}{\cdots}$

```
[In] int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(a/d^3*(-1/2/c^2/x^2-3*ln(c*x)-1/4/(c^2*x^2+1)^2-1/(c^2*x^2+1)+3/2*ln(c
^2*x^2+1))+b/d^3*(-1/12/(c^4*x^4+2*c^2*x^2+1)/c^2/x^2*(-8*c^5*x^5*(c^2*x^2+
1)^(1/2)+8*c^6*x^6+18*arcsinh(c*x)*c^4*x^4-3*c^3*x^3*(c^2*x^2+1)^(1/2)+16*c
^4*x^4+27*arcsinh(c*x)*c^2*x^2+6*c*x*(c^2*x^2+1)^(1/2)+8*c^2*x^2+6*arcsinh(
c*x))-3*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-3*polylog(2,-c*x-(c^2*x^2+
1)^(1/2))+3*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+3/2*polylog(2,-(c*
x+(c^2*x^2+1)^(1/2))^2)-3*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-3*polylo
g(2,c*x+(c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^3 (d + c^2dx^2)^3} \, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(c^2dx^2 + d)^3x^3} \, dx$$

[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^3,x, algorithm="fricas")

[Out] integral((b*arcsinh(c*x) + a)/(c^6*d^3*x^9 + 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5 + d^3*x^3), x)

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^3 (d + c^2dx^2)^3} \, dx = \frac{\int \frac{a}{c^6x^9+3c^4x^7+3c^2x^5+x^3} \, dx + \int \frac{b \operatorname{asinh}(cx)}{c^6x^9+3c^4x^7+3c^2x^5+x^3} \, dx}{d^3}$$

[In] integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d)**3,x)

[Out] (Integral(a/(c**6*x**9 + 3*c**4*x**7 + 3*c**2*x**5 + x**3), x) + Integral(b*asinh(c*x)/(c**6*x**9 + 3*c**4*x**7 + 3*c**2*x**5 + x**3), x))/d**3

Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^3 (d + c^2dx^2)^3} \, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(c^2dx^2 + d)^3x^3} \, dx$$

[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^3,x, algorithm="maxima")

[Out] -1/4*a*((6*c^4*x^4 + 9*c^2*x^2 + 2)/(c^4*d^3*x^6 + 2*c^2*d^3*x^4 + d^3*x^2) - 6*c^2*log(c^2*x^2 + 1)/d^3 + 12*c^2*log(x)/d^3) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3*x^9 + 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5 + d^3*x^3), x)

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^3 (d + c^2dx^2)^3} \, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(c^2dx^2 + d)^3x^3} \, dx$$

[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^3,x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^3*x^3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^3}\,dx=\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x^3\,{\left(d\,c^2\,x^2+d\right)}^3}\,dx$$

```
[In] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^3),x)
[Out] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^3), x)
```

3.54 $\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 295

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^3}\,dx =& -\frac{bc^3}{12d^3(1+c^2x^2)^{3/2}}-\frac{bc}{6d^3x^2(1+c^2x^2)^{3/2}}+\frac{29bc^3}{24d^3\sqrt{1+c^2x^2}}\\&-\frac{a+b\text{arcsinh}(cx)}{3d^3x^3(1+c^2x^2)^2}+\frac{7c^2(a+b\text{arcsinh}(cx))}{3d^3x(1+c^2x^2)^2}\\&+\frac{35c^4x(a+b\text{arcsinh}(cx))}{12d^3(1+c^2x^2)^2}+\frac{35c^4x(a+b\text{arcsinh}(cx))}{8d^3(1+c^2x^2)}\\&+\frac{35c^3(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4d^3}\\&+\frac{19bc^3\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{6d^3}-\frac{35ibc^3\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{8d^3}\\&+\frac{35ibc^3\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{8d^3}\end{aligned}$$

```
[Out] -1/12*b*c^3/d^3/(c^2*x^2+1)^(3/2)-1/6*b*c/d^3/x^2/(c^2*x^2+1)^(3/2)+1/3*(-a
-b*arcsinh(c*x))/d^3/x^3/(c^2*x^2+1)^2+7/3*c^2*(a+b*arcsinh(c*x))/d^3/x/(c^
2*x^2+1)^2+35/12*c^4*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^2+35/8*c^4*x*(a+b
*arcsinh(c*x))/d^3/(c^2*x^2+1)+35/4*c^3*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*
x^2+1)^(1/2))/d^3+19/6*b*c^3*arctanh((c^2*x^2+1)^(1/2))/d^3-35/8*I*b*c^3*po
lylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^3+35/8*I*b*c^3*polylog(2,I*(c*x+(c^2*
x^2+1)^(1/2)))/d^3+29/24*b*c^3/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5809, 5788, 5789, 4265, 2317, 2438, 267, 272, 53, 65, 214, 44}

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^3}\,dx = \frac{35c^3\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{4d^3}+\frac{7c^2(a+b\text{arcsinh}(cx))}{3d^3x(c^2x^2+1)^2} - \frac{a+b\text{arcsinh}(cx)}{3d^3x^3(c^2x^2+1)^2}+\frac{35c^4x(a+b\text{arcsinh}(cx))}{8d^3(c^2x^2+1)} + \frac{35c^4x(a+b\text{arcsinh}(cx))}{12d^3(c^2x^2+1)^2}-\frac{35ibc^3\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{8d^3} + \frac{35ibc^3\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{8d^3}+\frac{19bc^3\text{arctanh}\left(\sqrt{c^2x^2+1}\right)}{6d^3} - \frac{bc}{6d^3x^2(c^2x^2+1)^{3/2}}+\frac{29bc^3}{24d^3\sqrt{c^2x^2+1}}-\frac{bc^3}{12d^3(c^2x^2+1)^{3/2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^3),x]
```

```
[Out] -1/12*(b*c^3)/(d^3*(1 + c^2*x^2)^(3/2)) - (b*c)/(6*d^3*x^2*(1 + c^2*x^2)^(3
/2)) + (29*b*c^3)/(24*d^3*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(3*d^3*
x^3*(1 + c^2*x^2)^2) + (7*c^2*(a + b*ArcSinh[c*x]))/(3*d^3*x*(1 + c^2*x^2)^
2) + (35*c^4*x*(a + b*ArcSinh[c*x]))/(12*d^3*(1 + c^2*x^2)^2) + (35*c^4*x*(
a + b*ArcSinh[c*x]))/(8*d^3*(1 + c^2*x^2)) + (35*c^3*(a + b*ArcSinh[c*x])*A
rcTan[E^ArcSinh[c*x]])/(4*d^3) + (19*b*c^3*ArcTanh[Sqrt[1 + c^2*x^2]])/(6*d
^3) - (((35*I)/8)*b*c^3*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^3 + (((35*I)/8)*
b*c^3*PolyLog[2, I*E^ArcSinh[c*x]])/d^3
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\text{arcsinh}(cx)}{3d^3x^3\left(1+c^2x^2\right)^2}-\frac{1}{3}\left(7c^2\right)\int\frac{a+b\text{arcsinh}(cx)}{x^2\left(d+c^2dx^2\right)^3}\,dx+\frac{(bc)\int\frac{1}{x^3(1+c^2x^2)^{5/2}}\,dx}{3d^3}\\
&= -\frac{a+b\text{arcsinh}(cx)}{3d^3x^3\left(1+c^2x^2\right)^2}+\frac{7c^2(a+b\text{arcsinh}(cx))}{3d^3x\left(1+c^2x^2\right)^2}+\frac{1}{3}\left(35c^4\right)\int\frac{a+b\text{arcsinh}(cx)}{\left(d+c^2dx^2\right)^3}\,dx\\
&\quad+\frac{(bc)\text{Subst}\left(\int\frac{1}{x^2(1+c^2x)^{5/2}}\,dx,x,x^2\right)}{6d^3}-\frac{\left(7bc^3\right)\int\frac{1}{x(1+c^2x^2)^{5/2}}\,dx}{3d^3}\\
&= -\frac{bc}{6d^3x^2\left(1+c^2x^2\right)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{3d^3x^3\left(1+c^2x^2\right)^2}\\
&\quad+\frac{7c^2(a+b\text{arcsinh}(cx))}{3d^3x\left(1+c^2x^2\right)^2}+\frac{35c^4x(a+b\text{arcsinh}(cx))}{12d^3\left(1+c^2x^2\right)^2}\\
&\quad-\frac{\left(5bc^3\right)\text{Subst}\left(\int\frac{1}{x(1+c^2x)^{5/2}}\,dx,x,x^2\right)}{12d^3}-\frac{\left(7bc^3\right)\text{Subst}\left(\int\frac{1}{x(1+c^2x)^{5/2}}\,dx,x,x^2\right)}{6d^3}\\
&\quad-\frac{\left(35bc^5\right)\int\frac{x}{(1+c^2x^2)^{5/2}}\,dx}{12d^3}+\frac{\left(35c^4\right)\int\frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^2}\,dx}{4d}
\end{aligned}$$

$$= -\frac{bc^3}{12d^3(1+c^2x^2)^{3/2}} - \frac{bc}{6d^3x^2(1+c^2x^2)^{3/2}} - \frac{a+b\operatorname{arcsinh}(cx)}{3d^3x^3(1+c^2x^2)^2} + \frac{7c^2(a+b\operatorname{arcsinh}(cx))}{3d^3x(1+c^2x^2)^2} + \frac{35c^4x(a+b\operatorname{arcsinh}(cx))}{12d^3(1+c^2x^2)^2} + \frac{35c^4x(a+b\operatorname{arcsinh}(cx))}{8d^3(1+c^2x^2)} - \frac{(5bc^3)\operatorname{Subst}\left(\int\frac{1}{x(1+c^2x)^{3/2}}\,dx,x,x^2\right)}{12d^3} - \frac{(7bc^3)\operatorname{Subst}\left(\int\frac{1}{x(1+c^2x)^{3/2}}\,dx,x,x^2\right)}{6d^3} - \frac{(35bc^5)\int\frac{x}{(1+c^2x^2)^{3/2}}\,dx}{8d^3} + \frac{(35c^4)\int\frac{a+b\operatorname{arcsinh}(cx)}{d+c^2dx^2}\,dx}{8d^2}$$

$$= -\frac{bc^3}{12d^3(1+c^2x^2)^{3/2}} - \frac{bc}{6d^3x^2(1+c^2x^2)^{3/2}} + \frac{29bc^3}{24d^3\sqrt{1+c^2x^2}} - \frac{a+b\operatorname{arcsinh}(cx)}{3d^3x^3(1+c^2x^2)^2} + \frac{7c^2(a+b\operatorname{arcsinh}(cx))}{3d^3x(1+c^2x^2)^2} + \frac{35c^4x(a+b\operatorname{arcsinh}(cx))}{12d^3(1+c^2x^2)^2} + \frac{35c^4x(a+b\operatorname{arcsinh}(cx))}{8d^3(1+c^2x^2)} + \frac{(35c^3)\operatorname{Subst}(\int(a+bx)\operatorname{sech}(x)\,dx,x,\operatorname{arcsinh}(cx))}{8d^3} - \frac{(5bc^3)\operatorname{Subst}\left(\int\frac{1}{x\sqrt{1+c^2x}}\,dx,x,x^2\right)}{12d^3} - \frac{(7bc^3)\operatorname{Subst}\left(\int\frac{1}{x\sqrt{1+c^2x}}\,dx,x,x^2\right)}{6d^3}$$

$$= -\frac{bc^3}{12d^3(1+c^2x^2)^{3/2}} - \frac{bc}{6d^3x^2(1+c^2x^2)^{3/2}} + \frac{29bc^3}{24d^3\sqrt{1+c^2x^2}} - \frac{a+b\operatorname{arcsinh}(cx)}{3d^3x^3(1+c^2x^2)^2} + \frac{7c^2(a+b\operatorname{arcsinh}(cx))}{3d^3x(1+c^2x^2)^2} + \frac{35c^4x(a+b\operatorname{arcsinh}(cx))}{12d^3(1+c^2x^2)^2} + \frac{35c^4x(a+b\operatorname{arcsinh}(cx))}{8d^3(1+c^2x^2)} + \frac{35c^3(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{4d^3} - \frac{(5bc)\operatorname{Subst}\left(\int\frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx,x,\sqrt{1+c^2x^2}\right)}{6d^3} - \frac{(7bc)\operatorname{Subst}\left(\int\frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx,x,\sqrt{1+c^2x^2}\right)}{3d^3} - \frac{(35ibc^3)\operatorname{Subst}(\int\log(1-ie^x)\,dx,x,\operatorname{arcsinh}(cx))}{8d^3} + \frac{(35ibc^3)\operatorname{Subst}(\int\log(1+ie^x)\,dx,x,\operatorname{arcsinh}(cx))}{8d^3}$$

$$= -\frac{bc^3}{12d^3(1+c^2x^2)^{3/2}} - \frac{bc}{6d^3x^2(1+c^2x^2)^{3/2}} + \frac{29bc^3}{24d^3\sqrt{1+c^2x^2}} - \frac{a+b\text{arcsinh}(cx)}{3d^3x^3(1+c^2x^2)^2} + \frac{7c^2(a+b\text{arcsinh}(cx))}{3d^3x(1+c^2x^2)^2} + \frac{35c^4x(a+b\text{arcsinh}(cx))}{12d^3(1+c^2x^2)^2} + \frac{35c^4x(a+b\text{arcsinh}(cx))}{8d^3(1+c^2x^2)} + \frac{35c^3(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4d^3} + \frac{19bc^3\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{6d^3} - \frac{(35ibc^3)\,\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{8d^3} + \frac{(35ibc^3)\,\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{8d^3}$$

$$= -\frac{bc^3}{12d^3(1+c^2x^2)^{3/2}} - \frac{bc}{6d^3x^2(1+c^2x^2)^{3/2}} + \frac{29bc^3}{24d^3\sqrt{1+c^2x^2}} - \frac{a+b\text{arcsinh}(cx)}{3d^3x^3(1+c^2x^2)^2} + \frac{7c^2(a+b\text{arcsinh}(cx))}{3d^3x(1+c^2x^2)^2} + \frac{35c^4x(a+b\text{arcsinh}(cx))}{12d^3(1+c^2x^2)^2} + \frac{35c^4x(a+b\text{arcsinh}(cx))}{8d^3(1+c^2x^2)} + \frac{35c^3(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4d^3} + \frac{19bc^3\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{6d^3} - \frac{35ibc^3\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{8d^3} + \frac{35ibc^3\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{8d^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.06 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.27

$$\int\frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^3}\,dx$$

$$= \frac{\frac{3(a+b\text{arcsinh}(cx))}{x^3(1+c^2x^2)^2} + \frac{21(a+b\text{arcsinh}(cx))}{2(x^3+c^2x^5)} + \frac{bc^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},2,-\frac{1}{2},1+c^2x^2\right)}{(1+c^2x^2)^{3/2}} + \frac{21bc^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},1+c^2x^2\right)}{2\sqrt{1+c^2x^2}}}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^3),x]
```

```
[Out] ((3*(a + b*ArcSinh[c*x]))/(x^3*(1 + c^2*x^2)^2) + (21*(a + b*ArcSinh[c*x]))
/(2*(x^3 + c^2*x^5)) + (b*c^3*Hypergeometric2F1[-3/2, 2, -1/2, 1 + c^2*x^2]
)/(1 + c^2*x^2)^(3/2) + (21*b*c^3*Hypergeometric2F1[-1/2, 2, 1/2, 1 + c^2*x
^2])/(2*Sqrt[1 + c^2*x^2]) + (35*(-2*a + 6*a*c^2*x^2 - b*c*x*Sqrt[1 + c^2*x
^2] - 2*b*ArcSinh[c*x] + 6*b*c^2*x^2*ArcSinh[c*x] + 6*a*c^3*x^3*ArcTan[c*x]
 + 7*b*c^3*x^3*ArcTanh[Sqrt[1 + c^2*x^2]] - 6*b*(-c^2)^(3/2)*x^3*ArcSinh[c*
x]*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 6*b*(-c^2)^(3/2)*x^3*ArcSinh[c*
x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 6*b*(-c^2)^(3/2)*x^3*PolyLog[2,
 (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 6*b*(-c^2)^(3/2)*x^3*PolyLog[2, (Sqrt[-c^
2]*E^ArcSinh[c*x])/c]))/(4*x^3))/(12*d^3)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.13

method	result
derivativedivides	$c^3\left(\frac{a\left(-\frac{1}{3c^3x^3}+\frac{3}{cx}+\frac{\frac{11}{8}c^3x^3+\frac{13}{8}cx}{(c^2x^2+1)^2}+\frac{35\arctan(cx)}{8}\right)}{d^3}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{3c^3x^3}+\frac{3\operatorname{arcsinh}(cx)}{cx}+\frac{11c^3x^3\operatorname{arcsinh}(cx)}{8(c^2x^2+1)^2}+\frac{13cx\operatorname{arcsinh}(}{8(c^2x^2+1)}\right.}{}\right.$
default	$c^3\left(\frac{a\left(-\frac{1}{3c^3x^3}+\frac{3}{cx}+\frac{\frac{11}{8}c^3x^3+\frac{13}{8}cx}{(c^2x^2+1)^2}+\frac{35\arctan(cx)}{8}\right)}{d^3}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{3c^3x^3}+\frac{3\operatorname{arcsinh}(cx)}{cx}+\frac{11c^3x^3\operatorname{arcsinh}(cx)}{8(c^2x^2+1)^2}+\frac{13cx\operatorname{arcsinh}(}{8(c^2x^2+1)}\right.}{}\right.$
parts	$\frac{a\left(c^4\left(\frac{\frac{11}{8}x^3c^2+\frac{13}{8}x}{(c^2x^2+1)^2}+\frac{35\arctan(cx)}{8c}\right)-\frac{1}{3x^3}+\frac{3c^2}{x}\right)}{d^3}+\frac{b\,c^3\left(-\frac{\operatorname{arcsinh}(cx)}{3c^3x^3}+\frac{3\operatorname{arcsinh}(cx)}{cx}+\frac{11c^3x^3\operatorname{arcsinh}(cx)}{8(c^2x^2+1)^2}+\frac{13cx\operatorname{arcsinh}(c}{8(c^2x^2+1)^2}\right.}{}$

```
[In] int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(a/d^3*(-1/3/c^3/x^3+3/c/x+(11/8*c^3*x^3+13/8*c*x)/(c^2*x^2+1)^2+35/8*a
rctan(c*x))+b/d^3*(-1/3*arcsinh(c*x)/c^3/x^3+3*arcsinh(c*x)/c/x+11/8*c^3*x^
3/(c^2*x^2+1)^2*arcsinh(c*x)+13/8*c*x/(c^2*x^2+1)^2*arcsinh(c*x)+35/8*arcsi
nh(c*x)*arctan(c*x)+103/24/(c^2*x^2+1)^(3/2)-1/6/c^2/x^2/(c^2*x^2+1)^(3/2)-
19/6/(c^2*x^2+1)^(1/2)+19/6*arctanh(1/(c^2*x^2+1)^(1/2))+35/8*c^2*x^2/(c^2*
x^2+1)^(3/2)+35/8*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-35/8*arct
an(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-35/8*I*dilog(1+I*(1+I*c*x)/(c^2
*x^2+1)^(1/2))+35/8*I*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^4\left(d+c^2dx^2\right)^3}\,dx=\int \frac{b\operatorname{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^3x^4}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^6*d^3*x^10 + 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6
 + d^3*x^4), x)
```

Sympy [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^4\left(d+c^2dx^2\right)^3}\,dx = \frac{\int \frac{a}{c^6x^{10}+3c^4x^8+3c^2x^6+x^4}\,dx + \int \frac{b\operatorname{asinh}(cx)}{c^6x^{10}+3c^4x^8+3c^2x^6+x^4}\,dx}{d^3}$$

```
[In] integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d)**3,x)
[Out] (Integral(a/(c**6*x**10 + 3*c**4*x**8 + 3*c**2*x**6 + x**4), x) + Integral(
b*asinh(c*x)/(c**6*x**10 + 3*c**4*x**8 + 3*c**2*x**6 + x**4), x))/d**3
```

Maxima [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^4\left(d+c^2dx^2\right)^3}\,dx = \int \frac{b\operatorname{arsinh}(cx)+a}{\left(c^2dx^2+d\right)^3x^4}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^3,x, algorithm="maxima")
[Out] 1/24*a*(105*c^3*arctan(c*x)/d^3 + (105*c^6*x^6 + 175*c^4*x^4 + 56*c^2*x^2 -
 8)/(c^4*d^3*x^7 + 2*c^2*d^3*x^5 + d^3*x^3)) + b*integrate(log(c*x + sqrt(c
^2*x^2 + 1))/(c^6*d^3*x^10 + 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6 + d^3*x^4), x)
```

Giac [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^4\left(d+c^2dx^2\right)^3}\,dx = \int \frac{b\operatorname{arsinh}(cx)+a}{\left(c^2dx^2+d\right)^3x^4}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^3,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^3*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^4\left(d+c^2dx^2\right)^3}\,dx = \int \frac{a+b\operatorname{asinh}(c\,x)}{x^4\left(d\,c^2\,x^2+d\right)^3}\,dx$$

```
[In] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^3),x)
[Out] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^3), x)
```

3.55 $\int x^3\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 109

$$\int x^3\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = \frac{2b\sqrt{\pi}x}{15c^3} - \frac{b\sqrt{\pi}x^3}{45c} - \frac{1}{25}bc\sqrt{\pi}x^5 - \frac{(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))}{3c^4\pi} + \frac{(\pi + c^2\pi x^2)^{5/2}(a + b\text{arcsinh}(cx))}{5c^4\pi^2}$$

```
[Out] -1/3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/c^4/Pi+1/5*(Pi*c^2*x^2+Pi)^(5
/2)*(a+b*arcsinh(c*x))/c^4/Pi^2+2/15*b*x*Pi^(1/2)/c^3-1/45*b*x^3*Pi^(1/2)/c
-1/25*b*c*x^5*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 45, 5804, 12}

$$\int x^3\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = \frac{(\pi c^2x^2 + \pi)^{5/2}(a + b\text{arcsinh}(cx))}{5\pi^2c^4} - \frac{(\pi c^2x^2 + \pi)^{3/2}(a + b\text{arcsinh}(cx))}{3\pi c^4} + \frac{2\sqrt{\pi}bx}{15c^3} - \frac{1}{25}\sqrt{\pi}bcx^5 - \frac{\sqrt{\pi}bx^3}{45c}$$

```
[In] Int[x^3*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*b*Sqrt[Pi]*x)/(15*c^3) - (b*Sqrt[Pi]*x^3)/(45*c) - (b*c*Sqrt[Pi]*x^5)/25
 - ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^4*Pi) + ((Pi + c^2*P
i*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^4*Pi^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c^4\pi} \\
&\quad+\frac{\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))}{5c^4\pi^2}-\left(bc\sqrt{\pi}\right)\int\frac{-2+c^2x^2+3c^4x^4}{15c^4}\,dx \\
&= -\frac{\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c^4\pi}+\frac{\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))}{5c^4\pi^2} \\
&\quad-\frac{\left(b\sqrt{\pi}\right)\int\left(-2+c^2x^2+3c^4x^4\right)\,dx}{15c^3} \\
&= \frac{2b\sqrt{\pi}x}{15c^3}-\frac{b\sqrt{\pi}x^3}{45c}-\frac{1}{25}bc\sqrt{\pi}x^5-\frac{\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c^4\pi} \\
&\quad+\frac{\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))}{5c^4\pi^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.97

$$\int x^3\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{\sqrt{\pi}\big(15a\sqrt{1+c^2x^2}(-2+c^2x^2+3c^4x^4)+b(30cx-5c^3x^3-9c^5x^5)+15b\sqrt{1+c^2x^2}(-2+c^2x^2+3c^4x^4)\,\text{arc}}{225c^4}$$

```
[In] Integrate[x^3*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Sqrt[Pi]*(15*a*Sqrt[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4) + b*(30*c*x -
5*c^3*x^3 - 9*c^5*x^5) + 15*b*Sqrt[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4)*
ArcSinh[c*x]))/(225*c^4)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.50

method	result
default	$a\left(\frac{x^2(\pi c^2x^2+\pi)^{\frac{3}{2}}}{5\pi c^2}-\frac{2(\pi c^2x^2+\pi)^{\frac{3}{2}}}{15\pi c^4}\right)+\frac{b\sqrt{\pi}\left(45\ \text{arcsinh}(cx)c^6x^6+60\ \text{arcsinh}(cx)c^4x^4-9c^5x^5\sqrt{c^2x^2+1}-15\ \text{arcsinh}(cx)c^2x^2-\right.}{225c^4\sqrt{c^2x^2+1}}$
parts	$a\left(\frac{x^2(\pi c^2x^2+\pi)^{\frac{3}{2}}}{5\pi c^2}-\frac{2(\pi c^2x^2+\pi)^{\frac{3}{2}}}{15\pi c^4}\right)+\frac{b\sqrt{\pi}\left(45\ \text{arcsinh}(cx)c^6x^6+60\ \text{arcsinh}(cx)c^4x^4-9c^5x^5\sqrt{c^2x^2+1}-15\ \text{arcsinh}(cx)c^2x^2-\right.}{225c^4\sqrt{c^2x^2+1}}$

```
[In] int(x^3*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/5*x^2*(Pi*c^2*x^2+Pi)^(3/2)/Pi/c^2-2/15/Pi/c^4*(Pi*c^2*x^2+Pi)^(3/2))+
1/225*b/c^4*Pi^(1/2)/(c^2*x^2+1)^(1/2)*(45*arcsinh(c*x)*c^6*x^6+60*arcsinh(
c*x)*c^4*x^4-9*c^5*x^5*(c^2*x^2+1)^(1/2)-15*arcsinh(c*x)*c^2*x^2-5*c^3*x^3*
(c^2*x^2+1)^(1/2)-30*arcsinh(c*x)+30*c*x*(c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.45

$$\int x^3\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{15\sqrt{\pi+\pi c^2x^2}(3\,bc^6x^6+4\,bc^4x^4-bc^2x^2-2\,b)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}\big(45\,ac^6x^6+60\,ac^4x^4-}{225\,(c^6x^2+c^4)}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/225*(15*sqrt(pi + pi*c^2*x^2)*(3*b*c^6*x^6 + 4*b*c^4*x^4 - b*c^2*x^2 - 2*
b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(45*a*c^6*x^6 + 60*
a*c^4*x^4 - 15*a*c^2*x^2 - (9*b*c^5*x^5 + 5*b*c^3*x^3 - 30*b*c*x)*sqrt(c^2*
x^2 + 1) - 30*a))/(c^6*x^2 + c^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(100) = 200$.

Time = 0.90 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.03

$$\int x^3\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx$$
$$=\begin{cases} \frac{\sqrt{\pi}ax^4\sqrt{c^2x^2+1}}{5}+\frac{\sqrt{\pi}ax^2\sqrt{c^2x^2+1}}{15c^2}-\frac{2\sqrt{\pi}a\sqrt{c^2x^2+1}}{15c^4}-\frac{\sqrt{\pi}bcx^5}{25}+\frac{\sqrt{\pi}bx^4\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{5}-\frac{\sqrt{\pi}bx^3}{45c}+\frac{\sqrt{\pi}bx^2\sqrt{c^2x^2+1}\operatorname{asinh}(}{15c^2} \\ \frac{\sqrt{\pi}ax^4}{4} \end{cases}$$

```
[In] integrate(x**3*(a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] Piecewise((sqrt(pi)*a*x**4*sqrt(c**2*x**2 + 1)/5 + sqrt(pi)*a*x**2*sqrt(c**
2*x**2 + 1)/(15*c**2) - 2*sqrt(pi)*a*sqrt(c**2*x**2 + 1)/(15*c**4) - sqrt(p
i)*b*c*x**5/25 + sqrt(pi)*b*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/5 - sqrt(pi
)*b*x**3/(45*c) + sqrt(pi)*b*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(15*c**2)
+ 2*sqrt(pi)*b*x/(15*c**3) - 2*sqrt(pi)*b*sqrt(c**2*x**2 + 1)*asinh(c*x)/(1
5*c**4), Ne(c, 0)), (sqrt(pi)*a*x**4/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.23

$$\int x^3\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{1}{15}b\left(\frac{3\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x^2}{\pi c^2}-\frac{2\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}{\pi c^4}\right)\operatorname{arsinh}(cx)+\frac{1}{15}a\left(\frac{3\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x^2}{\pi c^2}-\frac{2\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}{\pi c^4}\right)-\frac{\left(9\sqrt{\pi}c^4x^5+5\sqrt{\pi}c^2x^3-30\sqrt{\pi}x\right)b}{225\,c^3}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/15*b*(3*(pi + pi*c^2*x^2)^(3/2)*x^2/(pi*c^2) - 2*(pi + pi*c^2*x^2)^(3/2)/
(pi*c^4))*arcsinh(c*x) + 1/15*a*(3*(pi + pi*c^2*x^2)^(3/2)*x^2/(pi*c^2) - 2
*(pi + pi*c^2*x^2)^(3/2)/(pi*c^4)) - 1/225*(9*sqrt(pi)*c^4*x^5 + 5*sqrt(pi)
*c^2*x^3 - 30*sqrt(pi)*x)*b/c^3
```

Giac [F(-2)]

Exception generated.

$$\int x^3\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = \int x^3\,(a + b\,\text{asinh}(c\,x))\,\sqrt{\Pi\,c^2\,x^2 + \Pi}\,dx$$

```
[In] int(x^3*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2),x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2), x)
```

3.56 $\int x^2\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 119

$$\int x^2\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = -\frac{b\sqrt{\pi}x^2}{16c} - \frac{1}{16}bc\sqrt{\pi}x^4 + \frac{\sqrt{\pi}x\sqrt{1+c^2x^2}(a + b\text{arcsinh}(cx))}{8c^2} + \frac{1}{4}x^3\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx)) - \frac{\sqrt{\pi}(a + b\text{arcsinh}(cx))^2}{16bc^3}$$

```
[Out] -1/16*b*x^2*Pi^(1/2)/c-1/16*b*c*x^4*Pi^(1/2)-1/16*(a+b*arcsinh(c*x))^2*Pi^(
1/2)/b/c^3+1/8*x*(a+b*arcsinh(c*x))*Pi^(1/2)*(c^2*x^2+1)^(1/2)/c^2+1/4*x^3*
(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5806, 5812, 5783, 30}

$$\int x^2\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = -\frac{\sqrt{\pi}(a + b\text{arcsinh}(cx))^2}{16bc^3} + \frac{\sqrt{\pi}x\sqrt{c^2x^2+1}(a + b\text{arcsinh}(cx))}{8c^2} + \frac{1}{4}x^3\sqrt{\pi c^2x^2 + \pi}(a + b\text{arcsinh}(cx)) - \frac{1}{16}\sqrt{\pi}bcx^4 - \frac{\sqrt{\pi}bx^2}{16c}$$

```
[In] Int[x^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/16*(b*Sqrt[Pi]*x^2)/c - (b*c*Sqrt[Pi]*x^4)/16 + (Sqrt[Pi]*x*Sqrt[1 + c^2
*x^2]*(a + b*ArcSinh[c*x]))/(8*c^2) + (x^3*Sqrt[Pi + c^2*Pi*x^2]*(a + b*Arc
Sinh[c*x]))/4 - (Sqrt[Pi]*(a + b*ArcSinh[c*x])^2)/(16*b*c^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}x^3\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx)) \\ &\quad +\frac{1}{4}\sqrt{\pi}\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx-\frac{1}{4}\big(bc\sqrt{\pi}\big)\int x^3\,dx\end{aligned}$$

$$= -\frac{1}{16}bc\sqrt{\pi}x^4 + \frac{\sqrt{\pi}x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{8c^2} + \frac{1}{4}x^3\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx)) - \frac{\sqrt{\pi}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}dx}{8c^2} - \frac{(b\sqrt{\pi})\int x\,dx}{8c}$$

$$= -\frac{b\sqrt{\pi}x^2}{16c} - \frac{1}{16}bc\sqrt{\pi}x^4 + \frac{\sqrt{\pi}x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{8c^2} + \frac{1}{4}x^3\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx)) - \frac{\sqrt{\pi}(a+b\text{arcsinh}(cx))^2}{16bc^3}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.66

$$\int x^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx = \frac{\sqrt{\pi}\left(16acx\sqrt{1+c^2x^2}(1+2c^2x^2) - 8b\text{arcsinh}(cx)^2 - b\cosh(4\text{arcsinh}(cx)) + \text{arcsinh}(cx)(-16a+4b\sinh(\ldots\right.}{128c^3}$$

```
[In] Integrate[x^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Sqrt[Pi]*(16*a*c*x*Sqrt[1 + c^2*x^2]*(1 + 2*c^2*x^2) - 8*b*ArcSinh[c*x]^2
- b*Cosh[4*ArcSinh[c*x]] + ArcSinh[c*x]*(-16*a + 4*b*Sinh[4*ArcSinh[c*x]]))
)/(128*c^3)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.31

method	result
default	$\frac{ax(\pi c^2x^2+\pi)^{\frac{3}{2}}}{4\pi c^2} - \frac{ax\sqrt{\pi c^2x^2+\pi}}{8c^2} - \frac{a\pi\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{8c^2\sqrt{\pi c^2}} - \frac{b\sqrt{\pi}\left(-4\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+c^4x^4-2\,\text{arcsinh}(cx)c\ldots\right.}{16c^3}$
parts	$\frac{ax(\pi c^2x^2+\pi)^{\frac{3}{2}}}{4\pi c^2} - \frac{ax\sqrt{\pi c^2x^2+\pi}}{8c^2} - \frac{a\pi\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{8c^2\sqrt{\pi c^2}} - \frac{b\sqrt{\pi}\left(-4\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+c^4x^4-2\,\text{arcsinh}(cx)c\ldots\right.}{16c^3}$

```
[In] int(x^2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*x*(Pi*c^2*x^2+Pi)^(3/2)/Pi/c^2-1/8*a/c^2*x*(Pi*c^2*x^2+Pi)^(1/2)-1/8*
a/c^2*Pi*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)-1
/16*b*Pi^(1/2)*(-4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3+c^4*x^4-2*arcsinh
(c*x)*c*x*(c^2*x^2+1)^(1/2)+c^2*x^2+arcsinh(c*x)^2)/c^3
```

Fricas [F]

$$\int x^2\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = \int \sqrt{\pi + \pi c^2x^2}(b\,\text{arsinh}\,(cx) + a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*x^2*arcsinh(c*x) + a*x^2), x)
```

Sympy [F]

$$\int x^2\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = \sqrt{\pi}\left(\int ax^2\sqrt{c^2x^2 + 1}\,dx + \int bx^2\sqrt{c^2x^2 + 1}\,\text{asinh}\,(cx)\,dx\right)$$

```
[In] integrate(x**2*(a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2),x)
[Out] sqrt(pi)*(Integral(a*x**2*sqrt(c**2*x**2 + 1), x) + Integral(b*x**2*sqrt(c*
*2*x**2 + 1)*asinh(c*x), x))
```

Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int x^2\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = \int \sqrt{\pi + \pi c^2x^2}(b\,\text{arsinh}\,(cx) + a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = \int x^2\,(a + b\,\text{asinh}(c\,x))\,\sqrt{\Pi\,c^2\,x^2 + \Pi}\,dx$$

```
[In] int(x^2*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2),x)
[Out] int(x^2*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2), x)
```

3.57 $\int x\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 61

$$\int x\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = -\frac{b\sqrt{\pi}x}{3c} - \frac{1}{9}bc\sqrt{\pi}x^3 + \frac{(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))}{3c^2\pi}$$

```
[Out] 1/3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/c^2/Pi-1/3*b*x*Pi^(1/2)/c-1/9*
b*c*x^3*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {5798}

$$\int x\sqrt{\pi + c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx = \frac{(\pi c^2x^2 + \pi)^{3/2}(a + b\text{arcsinh}(cx))}{3\pi c^2} - \frac{1}{9}\sqrt{\pi}bcx^3 - \frac{\sqrt{\pi}bx}{3c}$$

```
[In] Int[x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/3*(b*Sqrt[Pi]*x)/c - (b*c*Sqrt[Pi]*x^3)/9 + ((Pi + c^2*Pi*x^2)^(3/2)*(a
+ b*ArcSinh[c*x]))/(3*c^2*Pi)
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))}{3c^2\pi} - \frac{(b\sqrt{\pi})\int (1 + c^2x^2)\,dx}{3c} \\ &= -\frac{b\sqrt{\pi}x}{3c} - \frac{1}{9}bc\sqrt{\pi}x^3 + \frac{(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))}{3c^2\pi}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

$$\int x\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = \frac{\sqrt{\pi}\left(3a(1 + c^2x^2)^{3/2} - bcx(3 + c^2x^2) + 3b(1 + c^2x^2)^{3/2}\,\text{arcsinh}(cx)\right)}{9c^2}$$

```
[In] Integrate[x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Sqrt[Pi]*(3*a*(1 + c^2*x^2)^(3/2) - b*c*x*(3 + c^2*x^2) + 3*b*(1 + c^2*x^2
)^(3/2)*ArcSinh[c*x]))/(9*c^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(49) = 98.

Time = 0.21 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.77

method	result	size
default	$\frac{a(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3\pi c^2}+\frac{b\sqrt{\pi}\left(3\,\text{arcsinh}(cx)c^4x^4+6\,\text{arcsinh}(cx)c^2x^2-c^3x^3\sqrt{c^2x^2+1}+3\,\text{arcsinh}(cx)-3cx\sqrt{c^2x^2+1}\right)}{9c^2\sqrt{c^2x^2+1}}$	108
parts	$\frac{a(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3\pi c^2}+\frac{b\sqrt{\pi}\left(3\,\text{arcsinh}(cx)c^4x^4+6\,\text{arcsinh}(cx)c^2x^2-c^3x^3\sqrt{c^2x^2+1}+3\,\text{arcsinh}(cx)-3cx\sqrt{c^2x^2+1}\right)}{9c^2\sqrt{c^2x^2+1}}$	108

```
[In] int(x*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*(Pi*c^2*x^2+Pi)^(3/2)/Pi/c^2+1/9*b/c^2*Pi^(1/2)/(c^2*x^2+1)^(1/2)*(3*
arcsinh(c*x)*c^4*x^4+6*arcsinh(c*x)*c^2*x^2-c^3*x^3*(c^2*x^2+1)^(1/2)+3*arc
sinh(c*x)-3*c*x*(c^2*x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(49) = 98$.

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.08

$$\int x\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx$$
$$= \frac{3\sqrt{\pi + \pi c^2x^2}(bc^4x^4 + 2\,bc^2x^2 + b)\log\left(cx + \sqrt{c^2x^2 + 1}\right) + \sqrt{\pi + \pi c^2x^2}\left(3\,ac^4x^4 + 6\,ac^2x^2 - (bc^3x^3 + 3\,bcx\right.}{9\,(c^4x^2 + c^2)}$$

```
[In] integrate(x*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(3*sqrt(pi + pi*c^2*x^2)*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*log(c*x + sqrt(c
^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(3*a*c^4*x^4 + 6*a*c^2*x^2 - (b*c^3*x^
3 + 3*b*c*x)*sqrt(c^2*x^2 + 1) + 3*a))/(c^4*x^2 + c^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(53) = 106$.

Time = 0.34 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.31

$$\int x\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx$$
$$= \begin{cases} \frac{\sqrt{\pi}ax^2\sqrt{c^2x^2+1}}{3} + \frac{\sqrt{\pi}a\sqrt{c^2x^2+1}}{3c^2} - \frac{\sqrt{\pi}bcx^3}{9} + \frac{\sqrt{\pi}bx^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{3} - \frac{\sqrt{\pi}bx}{3c} + \frac{\sqrt{\pi}b\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{3c^2} & \text{for}\ c \neq 0 \\ \frac{\sqrt{\pi}ax^2}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x*(a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] Piecewise((sqrt(pi)*a*x**2*sqrt(c**2*x**2 + 1)/3 + sqrt(pi)*a*sqrt(c**2*x**
2 + 1)/(3*c**2) - sqrt(pi)*b*c*x**3/9 + sqrt(pi)*b*x**2*sqrt(c**2*x**2 + 1)
*asinh(c*x)/3 - sqrt(pi)*b*x/(3*c) + sqrt(pi)*b*sqrt(c**2*x**2 + 1)*asinh(c
*x)/(3*c**2), Ne(c, 0)), (sqrt(pi)*a*x**2/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20

$$\int x\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = \frac{(\pi + \pi c^2x^2)^{\frac{3}{2}}b\,\text{arsinh}\,(cx)}{3\,\pi c^2} - \frac{\left(\pi^{\frac{3}{2}}c^2x^3 + 3\,\pi^{\frac{3}{2}}x\right)b}{9\,\pi c} + \frac{(\pi + \pi c^2x^2)^{\frac{3}{2}}a}{3\,\pi c^2}$$

```
[In] integrate(x*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(pi + pi*c^2*x^2)^(3/2)*b*arcsinh(c*x)/(pi*c^2) - 1/9*(pi^(3/2)*c^2*x^3
 + 3*pi^(3/2)*x)*b/(pi*c) + 1/3*(pi + pi*c^2*x^2)^(3/2)*a/(pi*c^2)
```

Giac [F(-2)]

Exception generated.

$$\int x\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = \int x\,(a + b\,\text{asinh}(c\,x))\,\sqrt{\Pi\,c^2\,x^2 + \Pi}\,dx$$

```
[In] int(x*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2), x)
```

3.58 $\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 67

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = -\frac{1}{4}bc\sqrt{\pi}x^2 + \frac{1}{2}x\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx)) + \frac{\sqrt{\pi}(a + b\text{arcsinh}(cx))^2}{4bc}$$

```
[Out] -1/4*b*c*x^2*Pi^(1/2)+1/4*(a+b*arcsinh(c*x))^2*Pi^(1/2)/b/c+1/2*x*(a+b*arcs
inh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5785, 5783, 30}

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = \frac{1}{2}x\sqrt{\pi c^2x^2 + \pi}(a + b\text{arcsinh}(cx)) + \frac{\sqrt{\pi}(a + b\text{arcsinh}(cx))^2}{4bc} - \frac{1}{4}\sqrt{\pi}bcx^2$$

```
[In] Int[Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/4*(b*c*Sqrt[Pi]*x^2) + (x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/2
+ (Sqrt[Pi]*(a + b*ArcSinh[c*x])^2)/(4*b*c)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))+\frac{1}{2}\sqrt{\pi}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx-\frac{1}{2}\left(bc\sqrt{\pi}\right)\int x\,dx$$

$$= -\frac{1}{4}bc\sqrt{\pi}x^2+\frac{1}{2}x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))+\frac{\sqrt{\pi}(a+b\text{arcsinh}(cx))^2}{4bc}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.03

$$\int\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx$$
$$= \frac{\sqrt{\pi}\left(4acx\sqrt{1+c^2x^2}+2b\text{arcsinh}(cx)^2-b\cosh(2\text{arcsinh}(cx))+2\text{arcsinh}(cx)(2a+b\sinh(2\text{arcsinh}(cx)))\right)}{8c}$$

```
[In] Integrate[Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
[Out] (Sqrt[Pi]*(4*a*c*x*Sqrt[1 + c^2*x^2] + 2*b*ArcSinh[c*x]^2 - b*Cosh[2*ArcSin
h[c*x]] + 2*ArcSinh[c*x]*(2*a + b*Sinh[2*ArcSinh[c*x]])))/(8*c)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.49

method	result	size
default	$\frac{ax\sqrt{\pi\,c^2x^2+\pi}}{2}+\frac{a\pi\ln\left(\frac{\pi\,c^2x}{\sqrt{\pi\,c^2}}+\sqrt{\pi\,c^2x^2+\pi}\right)}{2\sqrt{\pi\,c^2}}+\frac{b\sqrt{\pi}\,\left(2\,\mathrm{arcsinh}(cx)cx\sqrt{c^2x^2+1}-c^2x^2+\mathrm{arcsinh}(cx)^2-1\right)}{4c}$	100
parts	$\frac{ax\sqrt{\pi\,c^2x^2+\pi}}{2}+\frac{a\pi\ln\left(\frac{\pi\,c^2x}{\sqrt{\pi\,c^2}}+\sqrt{\pi\,c^2x^2+\pi}\right)}{2\sqrt{\pi\,c^2}}+\frac{b\sqrt{\pi}\,\left(2\,\mathrm{arcsinh}(cx)cx\sqrt{c^2x^2+1}-c^2x^2+\mathrm{arcsinh}(cx)^2-1\right)}{4c}$	100

```
[In] int((a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*a*x*(Pi*c^2*x^2+Pi)^(1/2)+1/2*a*Pi*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x
^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/4*b*Pi^(1/2)*(2*arcsinh(c*x)*c*x*(c^2*x^2+1)
^(1/2)-c^2*x^2+arcsinh(c*x)^2-1)/c
```

Fricas [F]

$$\int\sqrt{\pi+c^2\pi x^2}(a+b\mathrm{arcsinh}(cx))\,dx=\int\sqrt{\pi+\pi c^2x^2}(b\,\mathrm{arsinh}\,(cx)+a)\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a), x)
```

Sympy [F]

$$\int\sqrt{\pi+c^2\pi x^2}(a+b\mathrm{arcsinh}(cx))\,dx=\sqrt{\pi}\left(\int a\sqrt{c^2x^2+1}\,dx+\int b\sqrt{c^2x^2+1}\,\mathrm{asinh}\,(cx)\,dx\right)$$

```
[In] integrate((a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2),x)
[Out] sqrt(pi)*(Integral(a*sqrt(c**2*x**2 + 1), x) + Integral(b*sqrt(c**2*x**2 +
1)*asinh(c*x), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = \int (a + b\,\text{asinh}(c\,x))\,\sqrt{\Pi\,c^2\,x^2 + \Pi}\,dx$$

```
[In] int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2), x)
```

3.59 $\int \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x}\,dx = -bc\sqrt{\pi}x+\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx)) - 2\sqrt{\pi}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) - b\sqrt{\pi}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right) + b\sqrt{\pi}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)$$

```
[Out] -b*c*x*Pi^(1/2)-2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*Pi^(1/2
)-b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*Pi^(1/2)+b*polylog(2,c*x+(c^2*x^2+1)^
(1/2))*Pi^(1/2)+(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5806, 5816, 4267, 2317, 2438, 8}

$$\int \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x}\,dx = -2\sqrt{\pi}\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)) + \sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx)) - \sqrt{\pi}b\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right) + \sqrt{\pi}b\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right) + \sqrt{\pi}(-b)cx$$

```
[In] Int[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] -(b*c*Sqrt[Pi]*x) + Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]) - 2*Sqrt[Pi]
*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - b*Sqrt[Pi]*PolyLog[2, -E^Ar
cSinh[c*x]] + b*Sqrt[Pi]*PolyLog[2, E^ArcSinh[c*x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))+\sqrt{\pi}\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{1+c^2x^2}}\,dx-\left(bc\sqrt{\pi}\right)\int 1\,dx\\
&= -bc\sqrt{\pi}x+\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))+\sqrt{\pi}\text{Subst}\left(\int(a+bx)\text{csch}(x)\,dx,x,\text{arcsinh}(cx)\right)\\
&= -bc\sqrt{\pi}x+\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))\\
&\quad-2\sqrt{\pi}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)\\
&\quad-\left(b\sqrt{\pi}\right)\text{Subst}\left(\int\log\left(1-e^x\right)\,dx,x,\text{arcsinh}(cx)\right)\\
&\quad+\left(b\sqrt{\pi}\right)\text{Subst}\left(\int\log\left(1+e^x\right)\,dx,x,\text{arcsinh}(cx)\right)\\
&= -bc\sqrt{\pi}x+\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))\\
&\quad-2\sqrt{\pi}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)\\
&\quad-\left(b\sqrt{\pi}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)\\
&\quad+\left(b\sqrt{\pi}\right)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)\\
&= -bc\sqrt{\pi}x+\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))\\
&\quad-2\sqrt{\pi}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)\\
&\quad-b\sqrt{\pi}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)+b\sqrt{\pi}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.47

$$\begin{aligned}
\int\frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x}\,dx &= \sqrt{\pi}\Big(a\sqrt{1+c^2x^2}+a\log(x)\\
&\quad-a\log\left(\pi\left(1+\sqrt{1+c^2x^2}\right)\right)\\
&\quad+b\Big(-cx+\sqrt{1+c^2x^2}\text{arcsinh}(cx)\\
&\quad+\text{arcsinh}(cx)\log\left(1-e^{-\text{arcsinh}(cx)}\right)\\
&\quad-\text{arcsinh}(cx)\log\left(1+e^{-\text{arcsinh}(cx)}\right)\\
&\quad+\text{PolyLog}\left(2,-e^{-\text{arcsinh}(cx)}\right)\\
&\quad-\text{PolyLog}\left(2,e^{-\text{arcsinh}(cx)}\right)\Big)\Big)
\end{aligned}$$

```
[In] Integrate[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] Sqrt[Pi]*(a*Sqrt[1 + c^2*x^2] + a*Log[x] - a*Log[Pi*(1 + Sqrt[1 + c^2*x^2])
] + b*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1 - E^(-A
rcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2, -E^(-A
rcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.92

method	result
default	$a\left(\sqrt{\pi\, c^{2} x^{2}+\pi}-\sqrt{\pi}\, \operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi\, c^{2} x^{2}+\pi}}\right)\right)+\sqrt{c^{2} x^{2}+1}\, \operatorname{arcsinh}\left(c x\right) \sqrt{\pi}\, b+\operatorname{arcsinh}\left(c x\right) \ln\left(1\right.$
parts	$a\left(\sqrt{\pi\, c^{2} x^{2}+\pi}-\sqrt{\pi}\, \operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi\, c^{2} x^{2}+\pi}}\right)\right)+\sqrt{c^{2} x^{2}+1}\, \operatorname{arcsinh}\left(c x\right) \sqrt{\pi}\, b+\operatorname{arcsinh}\left(c x\right) \ln\left(1\right.$

```
[In] int((a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] a*((Pi*c^2*x^2+Pi)^(1/2)-Pi^(1/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2)))+
(c^2*x^2+1)^(1/2)*arcsinh(c*x)*Pi^(1/2)*b+arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)
^(1/2))*Pi^(1/2)*b-arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*Pi^(1/2)*b-b*c*
x*Pi^(1/2)+b*polylog(2,c*x+(c^2*x^2+1)^(1/2))*Pi^(1/2)-b*polylog(2,-c*x-(c^
2*x^2+1)^(1/2))*Pi^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\operatorname{arcsinh}(cx))}{x}\, dx = \int \frac{\sqrt{\pi + \pi c^{2} x^{2}}\left(b \operatorname{arsinh}\left(c x\right) + a\right)}{x}\, dx$$

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/x, x)
```

Sympy [F]

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\operatorname{arcsinh}(cx))}{x}\, dx = \sqrt{\pi}\left(\int \frac{a\sqrt{c^{2} x^{2} + 1}}{x}\, dx + \int \frac{b\sqrt{c^{2} x^{2} + 1}\operatorname{asinh}\left(c x\right)}{x}\, dx\right)$$

```
[In] integrate((a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2)/x,x)
```

```
[Out] sqrt(pi)*(Integral(a*sqrt(c**2*x**2 + 1)/x, x) + Integral(b*sqrt(c**2*x**2
+ 1)*asinh(c*x)/x, x))
```

Maxima [F]

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x}\, dx = \int \frac{\sqrt{\pi + \pi c^2x^2}(b\,\text{arsinh}\,(cx) + a)}{x}\, dx$$

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -(sqrt(pi)*arcsinh(1/(c*abs(x))) - sqrt(pi + pi*c^2*x^2))*a + b*integrate(s
qrt(pi + pi*c^2*x^2)*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x}\, dx = \int \frac{(a + b\,\text{asinh}(c\,x))\,\sqrt{\Pi\,c^2\,x^2 + \Pi}}{x}\, dx$$

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x, x)
```

3.60 $\int \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 61

$$\int \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x^2}\,dx = -\frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x} + \frac{c\sqrt{\pi}(a+b\text{arcsinh}(cx))^2}{2b} + bc\sqrt{\pi}\log(x)$$

```
[Out] 1/2*c*(a+b*arcsinh(c*x))^2*Pi^(1/2)/b+b*c*ln(x)*Pi^(1/2)-(a+b*arcsinh(c*x))
*(Pi*c^2*x^2+Pi)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5805, 29, 5783}

$$\int \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x^2}\,dx = -\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{x} + \frac{\sqrt{\pi}c(a+b\text{arcsinh}(cx))^2}{2b} + \sqrt{\pi}bc\log(x)$$

```
[In] Int[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] -((Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x) + (c*Sqrt[Pi]*(a + b*ArcS
inh[c*x])^2)/(2*b) + b*c*Sqrt[Pi]*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]], Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x]
, x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], In
t[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Free
Q[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
&\text{integral}\\
&= -\frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x} + \left(bc\sqrt{\pi}\right)\int \frac{1}{x}\,dx + \left(c^2\sqrt{\pi}\right)\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{1 + c^2x^2}}\,dx\\
&= -\frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x} + \frac{c\sqrt{\pi}(a + b\text{arcsinh}(cx))^2}{2b} + bc\sqrt{\pi}\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^2}\,dx = \frac{\sqrt{\pi}\left(-2a\sqrt{1 + c^2x^2} + 2\left(acx - b\sqrt{1 + c^2x^2}\right)\text{arcsinh}(cx) + bcx\text{arcsinh}(cx)^2 + 2bcx\log(cx)\right)}{2x}$$

```
[In] Integrate[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^2,x]
[Out] (Sqrt[Pi]*(-2*a*Sqrt[1 + c^2*x^2] + 2*(a*c*x - b*Sqrt[1 + c^2*x^2])*ArcSinh
[c*x] + b*c*x*ArcSinh[c*x]^2 + 2*b*c*x*Log[c*x]))/(2*x)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(53) = 106$.

Time = 0.17 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.54

method	result
default	$-\frac{a(\pi c^2x^2+\pi)^{\frac{3}{2}}}{\pi x}+a\,c^2x\sqrt{\pi\,c^2x^2+\pi}+\frac{a\,c^2\pi\ln\left(\frac{\pi\,c^2x}{\sqrt{\pi\,c^2}}+\sqrt{\pi\,c^2x^2+\pi}\right)}{\sqrt{\pi\,c^2}}+\frac{bc\sqrt{\pi}\,\operatorname{arcsinh}(cx)^2}{2}-bc\sqrt{\pi}\,\operatorname{arcsinh}(c$
parts	$-\frac{a(\pi c^2x^2+\pi)^{\frac{3}{2}}}{\pi x}+a\,c^2x\sqrt{\pi\,c^2x^2+\pi}+\frac{a\,c^2\pi\ln\left(\frac{\pi\,c^2x}{\sqrt{\pi\,c^2}}+\sqrt{\pi\,c^2x^2+\pi}\right)}{\sqrt{\pi\,c^2}}+\frac{bc\sqrt{\pi}\,\operatorname{arcsinh}(cx)^2}{2}-bc\sqrt{\pi}\,\operatorname{arcsinh}(c$

```
[In] int((a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/Pi/x*(Pi*c^2*x^2+Pi)^(3/2)+a*c^2*x*(Pi*c^2*x^2+Pi)^(1/2)+a*c^2*Pi*ln(Pi*
c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/2*b*c*Pi^(1/2)
*arcsinh(c*x)^2-b*c*Pi^(1/2)*arcsinh(c*x)-b*Pi^(1/2)*arcsinh(c*x)/x*(c^2*x^
2+1)^(1/2)+b*c*Pi^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)
```

Fricas [F]

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^2}\,dx = \int \frac{\sqrt{\pi + \pi c^2x^2}(b\,\text{arsinh}\,(cx) + a)}{x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^2,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/x^2, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(54) = 108$.

Time = 1.61 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^2}\,dx = -\frac{\sqrt{\pi}ac^2x}{\sqrt{c^2x^2+1}} + \sqrt{\pi}ac\,\text{asinh}\,(cx) - \frac{\sqrt{\pi}a}{x\sqrt{c^2x^2+1}} + \sqrt{\pi}bc\log{(x)} + \frac{\sqrt{\pi}bc\,\text{asinh}^2\,(cx)}{2} - \frac{\sqrt{\pi}b\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)}{x}$$

```
[In] integrate((a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2)/x**2,x)
```

```
[Out] -sqrt(pi)*a*c**2*x/sqrt(c**2*x**2 + 1) + sqrt(pi)*a*c*asinh(c*x) - sqrt(pi)
*a/(x*sqrt(c**2*x**2 + 1)) + sqrt(pi)*b*c*log(x) + sqrt(pi)*b*c*asinh(c*x)*
*2/2 - sqrt(pi)*b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x
```

Maxima [F]

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^2}\,dx = \int \frac{\sqrt{\pi + \pi c^2x^2}(b\,\text{arsinh}\,(cx) + a)}{x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^2,x, algorithm="maxima
")
```

```
[Out] (sqrt(pi)*c*arcsinh(c*x) - sqrt(pi + pi*c^2*x^2)/x)*a + b*integrate(sqrt(pi
 + pi*c^2*x^2)*log(c*x + sqrt(c^2*x^2 + 1))/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^2}\,dx = \int \frac{(a + b\,\text{asinh}(c\,x))\,\sqrt{\Pi\,c^2\,x^2 + \Pi}}{x^2}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x^2, x)
```

3.61 $\int \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 113

$$\int \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x^3}\,dx = -\frac{bc\sqrt{\pi}}{2x}-\frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{2x^2} - c^2\sqrt{\pi}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) - \frac{1}{2}bc^2\sqrt{\pi}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right) + \frac{1}{2}bc^2\sqrt{\pi}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)$$

```
[Out] -1/2*b*c*Pi^(1/2)/x-c^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*P
i^(1/2)-1/2*b*c^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*Pi^(1/2)+1/2*b*c^2*poly
log(2,c*x+(c^2*x^2+1)^(1/2))*Pi^(1/2)-1/2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi
)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5805, 30, 5816, 4267, 2317, 2438}

$$\int \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x^3}\,dx = -\sqrt{\pi}c^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)) - \frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2x^2} - \frac{1}{2}\sqrt{\pi}bc^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right) + \frac{1}{2}\sqrt{\pi}bc^2\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right) - \frac{\sqrt{\pi}bc}{2x}$$

```
[In] Int[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*Sqrt[Pi])/x - (Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(2*x^2
) - c^2*Sqrt[Pi]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - (b*c^2*Sqrt
[Pi]*PolyLog[2, -E^ArcSinh[c*x]])/2 + (b*c^2*Sqrt[Pi]*PolyLog[2, E^ArcSinh[
c*x]])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]], Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x]
, x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], In
t[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Free
Q[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{2x^2} + \frac{1}{2}\left(bc\sqrt{\pi}\right)\int \frac{1}{x^2}\,dx + \frac{1}{2}\left(c^2\sqrt{\pi}\right)\int \frac{a + b\text{arcsinh}(cx)}{x\sqrt{1 + c^2x^2}}\,dx$$

$$= -\frac{bc\sqrt{\pi}}{2x} - \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{2x^2} + \frac{1}{2}\left(c^2\sqrt{\pi}\right)\text{Subst}\left(\int (a + bx)\text{csch}(x)\,dx, x, \text{arcsinh}(cx)\right)$$

$$= -\frac{bc\sqrt{\pi}}{2x} - \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{2x^2} - c^2\sqrt{\pi}(a + b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) - \frac{1}{2}\left(bc^2\sqrt{\pi}\right)\text{Subst}\left(\int \log\left(1 - e^x\right)\,dx, x, \text{arcsinh}(cx)\right) + \frac{1}{2}\left(bc^2\sqrt{\pi}\right)\text{Subst}\left(\int \log\left(1 + e^x\right)\,dx, x, \text{arcsinh}(cx)\right)$$

$$= -\frac{bc\sqrt{\pi}}{2x} - \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{2x^2} - c^2\sqrt{\pi}(a + b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) - \frac{1}{2}\left(bc^2\sqrt{\pi}\right)\text{Subst}\left(\int \frac{\log(1 - x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right) + \frac{1}{2}\left(bc^2\sqrt{\pi}\right)\text{Subst}\left(\int \frac{\log(1 + x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)$$

$$= -\frac{bc\sqrt{\pi}}{2x} - \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{2x^2} - c^2\sqrt{\pi}(a + b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) - \frac{1}{2}bc^2\sqrt{\pi}\,\text{PolyLog}\left(2, -e^{\text{arcsinh}(cx)}\right) + \frac{1}{2}bc^2\sqrt{\pi}\,\text{PolyLog}\left(2, e^{\text{arcsinh}(cx)}\right)$$

Mathematica [A] (verified)

Time = 2.48 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x^3}\,dx = \frac{1}{8}\sqrt{\pi}\left(-\frac{4a\sqrt{1+c^2x^2}}{x^2}+4ac^2\log(x)-4ac^2\log\left(\pi\left(1+\sqrt{1+c^2x^2}\right)\right)+bc^2\left(-2\coth\left(\frac{1}{2}\text{arcsinh}(cx)\right)-\text{arcsinh}(cx)\text{csch}^2\left(\frac{1}{2}\text{arcsinh}(cx)\right)+4\text{arcsinh}(cx)\log\left(1-e^{-\text{arcsinh}(cx)}\right)-4\text{arcsinh}(cx)\log\left(1+e^{-\text{arcsinh}(cx)}\right)+4\,\text{PolyLog}\left(2,-e^{-\text{arcsinh}(cx)}\right)-4\,\text{PolyLog}\left(2,e^{-\text{arcsinh}(cx)}\right)-\text{arcsinh}(cx)\text{sech}^2\left(\frac{1}{2}\text{arcsinh}(cx)\right)+2\tanh\left(\frac{1}{2}\text{arcsinh}(cx)\right)\right)\right)$$

```
[In] Integrate[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (Sqrt[Pi]*((-4*a*Sqrt[1 + c^2*x^2])/x^2 + 4*a*c^2*Log[x] - 4*a*c^2*Log[Pi*(
1 + Sqrt[1 + c^2*x^2])] + b*c^2*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csc
h[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 4*ArcSinh
[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 4*PolyLog[2, -E^(-ArcSinh[c*x])] - 4*Pol
yLog[2, E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[A
rcSinh[c*x]/2])))/8
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.04

method	result
default	$a\left(-\frac{(\pi c^2x^2+\pi)^{\frac{3}{2}}}{2\pi x^2}+\frac{c^2\left(\sqrt{\pi c^2x^2+\pi}-\sqrt{\pi}\,\text{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)\right)}{2}\right)+b\left(-\frac{\sqrt{\pi}\left(\text{arcsinh}(cx)c^2x^2+cx\sqrt{c^2x^2+1}+\text{arcsinh}(cx)\right)}{2\sqrt{c^2x^2+1}\,x^2}\right.$
parts	$a\left(-\frac{(\pi c^2x^2+\pi)^{\frac{3}{2}}}{2\pi x^2}+\frac{c^2\left(\sqrt{\pi c^2x^2+\pi}-\sqrt{\pi}\,\text{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)\right)}{2}\right)+b\left(-\frac{\sqrt{\pi}\left(\text{arcsinh}(cx)c^2x^2+cx\sqrt{c^2x^2+1}+\text{arcsinh}(cx)\right)}{2\sqrt{c^2x^2+1}\,x^2}\right.$

```
[In] int((a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/2/Pi/x^2*(Pi*c^2*x^2+Pi)^(3/2)+1/2*c^2*((Pi*c^2*x^2+Pi)^(1/2)-Pi^(1/2
)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2))))+b*(-1/2*Pi^(1/2)/(c^2*x^2+1)^(1
/2)*(arcsinh(c*x)*c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x))/x^2-1/2*c^2*P
i^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-1/2*c^2*Pi^(1/2)*polylog(2
,-c*x-(c^2*x^2+1)^(1/2))+1/2*c^2*Pi^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)
^(1/2))+1/2*c^2*Pi^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^3}\,dx = \int \frac{\sqrt{\pi + \pi c^2x^2}(b\,\text{arsinh}\,(cx) + a)}{x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^3,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/x^3, x)
```

Sympy [F]

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^3}\,dx = \sqrt{\pi}\left(\int \frac{a\sqrt{c^2x^2+1}}{x^3}\,dx + \int \frac{b\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)}{x^3}\,dx\right)$$

```
[In] integrate((a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2)/x**3,x)
```

```
[Out] sqrt(pi)*(Integral(a*sqrt(c**2*x**2 + 1)/x**3, x) + Integral(b*sqrt(c**2*x*
*2 + 1)*asinh(c*x)/x**3, x))
```

Maxima [F]

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^3}\,dx = \int \frac{\sqrt{\pi + \pi c^2x^2}(b\,\text{arsinh}\,(cx) + a)}{x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(sqrt(pi)*c^2*arcsinh(1/(c*abs(x))) - sqrt(pi + pi*c^2*x^2)*c^2 + (pi
+ pi*c^2*x^2)^(3/2)/(pi*x^2))*a + b*integrate(sqrt(pi + pi*c^2*x^2)*log(c*x
 + sqrt(c^2*x^2 + 1))/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^3} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^3} dx = \int \frac{(a + b\,\text{asinh}(c\,x))\,\sqrt{\Pi\,c^2\,x^2 + \Pi}}{x^3} dx$$

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x^3, x)
```

3.62 $\int \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 62

$$\int \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x^4}\,dx=-\frac{bc\sqrt{\pi}}{6x^2}-\frac{(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3\pi x^3}+\frac{1}{3}bc^3\sqrt{\pi}\log(x)$$

```
[Out] -1/3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/Pi/x^3-1/6*b*c*Pi^(1/2)/x^2+1
/3*b*c^3*ln(x)*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5800, 14}

$$\int \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x^4}\,dx=-\frac{(\pi c^2x^2+\pi)^{3/2}(a+b\text{arcsinh}(cx))}{3\pi x^3}+\frac{1}{3}\sqrt{\pi}bc^3\log(x)-\frac{\sqrt{\pi}bc}{6x^2}$$

```
[In] Int[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*Sqrt[Pi])/x^2 - ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3
*Pi*x^3) + (b*c^3*Sqrt[Pi]*Log[x])/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3\pi x^3}+\frac{1}{3}\left(bc\sqrt{\pi}\right)\int\frac{1+c^2x^2}{x^3}\,dx\\ &= -\frac{(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3\pi x^3}+\frac{1}{3}\left(bc\sqrt{\pi}\right)\int\left(\frac{1}{x^3}+\frac{c^2}{x}\right)dx\\ &= -\frac{bc\sqrt{\pi}}{6x^2}-\frac{(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3\pi x^3}+\frac{1}{3}bc^3\sqrt{\pi}\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.55

$$\int\frac{\sqrt{\pi+c^2\pi x^2}(a+b\operatorname{arcsinh}(cx))}{x^4}\,dx = \frac{\sqrt{\pi}\left(-bcx-3bc^3x^3-2a\sqrt{1+c^2x^2}-2ac^2x^2\sqrt{1+c^2x^2}-2b(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)+2bc^3x^3\log(x)\right)}{6x^3}$$

```
[In] Integrate[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] (Sqrt[Pi]*(-(b*c*x) - 3*b*c^3*x^3 - 2*a*Sqrt[1 + c^2*x^2] - 2*a*c^2*x^2*Sqr
t[1 + c^2*x^2] - 2*b*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x] + 2*b*c^3*x^3*Log[x])
)/(6*x^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(50) = 100$.

Time = 0.16 (sec) , antiderivative size = 501, normalized size of antiderivative = 8.08

method	result
default	$-\frac{a(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3\pi x^3} - \frac{2bc^3\sqrt{\pi}\,\operatorname{arcsinh}(cx)}{3} + \frac{b\sqrt{\pi}\,x^4\operatorname{arcsinh}(cx)c^7}{3c^4x^4+3c^2x^2+1} - \frac{b\sqrt{\pi}\,x^3\sqrt{c^2x^2+1}\,\operatorname{arcsinh}(cx)c^6}{3c^4x^4+3c^2x^2+1} + \frac{b\sqrt{\pi}\,x^4c^7}{18c^4x^4+18c^2x^2+6} -$
parts	$-\frac{a(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3\pi x^3} - \frac{2bc^3\sqrt{\pi}\,\operatorname{arcsinh}(cx)}{3} + \frac{b\sqrt{\pi}\,x^4\operatorname{arcsinh}(cx)c^7}{3c^4x^4+3c^2x^2+1} - \frac{b\sqrt{\pi}\,x^3\sqrt{c^2x^2+1}\,\operatorname{arcsinh}(cx)c^6}{3c^4x^4+3c^2x^2+1} + \frac{b\sqrt{\pi}\,x^4c^7}{18c^4x^4+18c^2x^2+6} -$

```
[In] int((a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/Pi/x^3*(Pi*c^2*x^2+Pi)^(3/2)-2/3*b*c^3*Pi^(1/2)*arcsinh(c*x)+b*Pi^(1
/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^4*arcsinh(c*x)*c^7-b*Pi^(1/2)/(3*c^4*x^4+3*c^
2*x^2+1)*x^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^6+1/6*b*Pi^(1/2)/(3*c^4*x^4+3
*c^2*x^2+1)*x^4*c^7-1/6*b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^2*(c^2*x^2+1)*
c^5+b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^2*arcsinh(c*x)*c^5-2*b*Pi^(1/2)/(3
*c^4*x^4+3*c^2*x^2+1)*x*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^4-1/3*b*Pi^(1/2)/(
3*c^4*x^4+3*c^2*x^2+1)*(c^2*x^2+1)*c^3+1/3*b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+
1)*arcsinh(c*x)*c^3-4/3*b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x*(c^2*x^2+1)^(1
/2)*arcsinh(c*x)*c^2-1/6*b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x^2*(c^2*x^2+1)
*c-1/3*b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x
)+1/3*b*c^3*Pi^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(50) = 100$.

Time = 0.30 (sec) , antiderivative size = 217, normalized size of antiderivative = 3.50

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\operatorname{arcsinh}(cx))}{x^4}\,dx = -\frac{2\sqrt{\pi + \pi c^2x^2}(bc^4x^4 + 2\,bc^2x^2 + b)\log\left(cx + \sqrt{c^2x^2 + 1}\right) - \sqrt{\pi}(bc^5x^5 + bc^3x^3)\log\left(\frac{\pi+\pi c^2x^6+\pi c^2x^2+\pi x^4+\sqrt{}}{c^2}\right.}{6\left(c^2x^5 + x^3\right)}$$

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^4,x, algorithm="fricas
")
```

```
[Out] -1/6*(2*sqrt(pi + pi*c^2*x^2)*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*log(c*x + sqrt(
c^2*x^2 + 1)) - sqrt(pi)*(b*c^5*x^5 + b*c^3*x^3)*log((pi + pi*c^2*x^6 + pi*
c^2*x^2 + pi*x^4 + sqrt(pi)*sqrt(pi + pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*(x^4 -
1))/(c^2*x^4 + x^2)) + sqrt(pi + pi*c^2*x^2)*(2*a*c^4*x^4 + 4*a*c^2*x^2 - (
b*c*x^3 - b*c*x)*sqrt(c^2*x^2 + 1) + 2*a))/(c^2*x^5 + x^3)
```

Sympy [F]

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^4}\,dx = \sqrt{\pi}\left(\int \frac{a\sqrt{c^2x^2+1}}{x^4}\,dx + \int \frac{b\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)}{x^4}\,dx\right)$$

```
[In] integrate((a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2)/x**4,x)
```

```
[Out] sqrt(pi)*(Integral(a*sqrt(c**2*x**2 + 1)/x**4, x) + Integral(b*sqrt(c**2*x**
*2 + 1)*asinh(c*x)/x**4, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(50) = 100$.

Time = 0.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.15

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^4}\,dx = -\frac{\left(\pi^{\frac{3}{2}}(-1)^{2\,\pi+2\,\pi c^2x^2}\,c^2\log\left(2\,\pi c^2 + \frac{2\,\pi}{x^2}\right) - \pi^{\frac{3}{2}}c^2\log\left(x^2 + \frac{1}{c^2}\right) + \frac{\pi\sqrt{\pi+\pi c^4x^4+2\,\pi c^2x^2}}{x^2}\right)bc}{6\,\pi} - \frac{\left(\pi + \pi c^2x^2\right)^{\frac{3}{2}}b\,\text{arsinh}\,(cx)}{3\,\pi x^3} - \frac{\left(\pi + \pi c^2x^2\right)^{\frac{3}{2}}a}{3\,\pi x^3}$$

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^4,x, algorithm="maxima
")
```

```
[Out] -1/6*(pi^(3/2)*(-1)^(2*pi + 2*pi*c^2*x^2)*c^2*log(2*pi*c^2 + 2*pi/x^2) - pi
^(3/2)*c^2*log(x^2 + 1/c^2) + pi*sqrt(pi + pi*c^4*x^4 + 2*pi*c^2*x^2)/x^2)*
b*c/pi - 1/3*(pi + pi*c^2*x^2)^(3/2)*b*arcsinh(c*x)/(pi*x^3) - 1/3*(pi + pi
*c^2*x^2)^(3/2)*a/(pi*x^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\operatorname{arcsinh}(cx))}{x^4}\,dx = \int \frac{(a + b\,\operatorname{asinh}(c\,x))\,\sqrt{\Pi\,c^2\,x^2 + \Pi}}{x^4}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x^4,x)
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x^4, x)
```

3.63 $\int x^3(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 125

$$\int x^3(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))\,dx = \frac{2b\pi^{3/2}x}{35c^3} - \frac{b\pi^{3/2}x^3}{105c} - \frac{8}{175}bc\pi^{3/2}x^5 - \frac{1}{49}bc^3\pi^{3/2}x^7 - \frac{(\pi + c^2\pi x^2)^{5/2}(a + b\text{arcsinh}(cx))}{5c^4\pi} + \frac{(\pi + c^2\pi x^2)^{7/2}(a + b\text{arcsinh}(cx))}{7c^4\pi^2}$$

```
[Out] 2/35*b*Pi^(3/2)*x/c^3-1/105*b*Pi^(3/2)*x^3/c-8/175*b*c*Pi^(3/2)*x^5-1/49*b*
c^3*Pi^(3/2)*x^7-1/5*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/c^4/Pi+1/7*(P
i*c^2*x^2+Pi)^(7/2)*(a+b*arcsinh(c*x))/c^4/Pi^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {272, 45, 5804, 12, 380}

$$\int x^3(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))\,dx = \frac{(\pi c^2x^2 + \pi)^{7/2}(a + b\text{arcsinh}(cx))}{7\pi^2c^4} - \frac{(\pi c^2x^2 + \pi)^{5/2}(a + b\text{arcsinh}(cx))}{5\pi c^4} - \frac{1}{49}\pi^{3/2}bc^3x^7 + \frac{2\pi^{3/2}bx}{35c^3} - \frac{8}{175}\pi^{3/2}bcx^5 - \frac{\pi^{3/2}bx^3}{105c}$$

```
[In] Int[x^3*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*b*Pi^(3/2)*x)/(35*c^3) - (b*Pi^(3/2)*x^3)/(105*c) - (8*b*c*Pi^(3/2)*x^5)
/175 - (b*c^3*Pi^(3/2)*x^7)/49 - ((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*
x]))/(5*c^4*Pi) + ((Pi + c^2*Pi*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^4*Pi^
2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(\pi+c^2\pi x^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))}{5c^4\pi}+\frac{(\pi+c^2\pi x^2)^{7/2}\,(a+b\operatorname{arcsinh}(cx))}{7c^4\pi^2}\\ &\quad-\left(bc\sqrt{\pi}\right)\int\frac{\pi(1+c^2x^2)^2\,(-2+5c^2x^2)}{35c^4}\,dx\\ &= -\frac{(\pi+c^2\pi x^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))}{5c^4\pi}+\frac{(\pi+c^2\pi x^2)^{7/2}\,(a+b\operatorname{arcsinh}(cx))}{7c^4\pi^2}\\ &\quad-\frac{\left(b\pi^{3/2}\right)\int(1+c^2x^2)^2\,(-2+5c^2x^2)\,dx}{35c^3}\end{aligned}$$

$$= -\frac{(\pi + c^2\pi x^2)^{5/2}(a + b\mathrm{arcsinh}(cx))}{5c^4\pi} + \frac{(\pi + c^2\pi x^2)^{7/2}(a + b\mathrm{arcsinh}(cx))}{7c^4\pi^2} - \frac{(b\pi^{3/2})\int(-2 + c^2x^2 + 8c^4x^4 + 5c^6x^6)\,dx}{35c^3}$$

$$= \frac{2b\pi^{3/2}x}{35c^3} - \frac{b\pi^{3/2}x^3}{105c} - \frac{8}{175}bc\pi^{3/2}x^5 - \frac{1}{49}bc^3\pi^{3/2}x^7 - \frac{(\pi + c^2\pi x^2)^{5/2}(a + b\mathrm{arcsinh}(cx))}{5c^4\pi} + \frac{(\pi + c^2\pi x^2)^{7/2}(a + b\mathrm{arcsinh}(cx))}{7c^4\pi^2}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.80

$$\int x^3(\pi + c^2\pi x^2)^{3/2}(a + b\mathrm{arcsinh}(cx))\,dx = \frac{\pi^{3/2}\Big(105a(1 + c^2x^2)^{5/2}(-2 + 5c^2x^2) - bcx(-210 + 35c^2x^2 + 168c^4x^4 + 75c^6x^6) + 105b(}{3675c^4}$$

```
[In] Integrate[x^3*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Pi^(3/2)*(105*a*(1 + c^2*x^2)^(5/2)*(-2 + 5*c^2*x^2) - b*c*x*(-210 + 35*c^
2*x^2 + 168*c^4*x^4 + 75*c^6*x^6) + 105*b*(1 + c^2*x^2)^(5/2)*(-2 + 5*c^2*x
^2)*ArcSinh[c*x]))/(3675*c^4)
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.56

method	result
default	$a\left(\frac{x^2(\pi c^2x^2+\pi)^{\frac{5}{2}}}{7\pi c^2} - \frac{2(\pi c^2x^2+\pi)^{\frac{5}{2}}}{35\pi c^4}\right) + \frac{b\pi^{\frac{3}{2}}\left(525\ \mathrm{arcsinh}(cx)c^8x^8+1365\ \mathrm{arcsinh}(cx)c^6x^6-75c^7x^7\sqrt{c^2x^2+1}+945\ \mathrm{arcsinh}(cx)c\right.}{3}$
parts	$a\left(\frac{x^2(\pi c^2x^2+\pi)^{\frac{5}{2}}}{7\pi c^2} - \frac{2(\pi c^2x^2+\pi)^{\frac{5}{2}}}{35\pi c^4}\right) + \frac{b\pi^{\frac{3}{2}}\left(525\ \mathrm{arcsinh}(cx)c^8x^8+1365\ \mathrm{arcsinh}(cx)c^6x^6-75c^7x^7\sqrt{c^2x^2+1}+945\ \mathrm{arcsinh}(cx)c\right.}{3}$

```
[In] int(x^3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/7*x^2*(Pi*c^2*x^2+Pi)^(5/2)/Pi/c^2-2/35/Pi/c^4*(Pi*c^2*x^2+Pi)^(5/2))+
1/3675*b/c^4*Pi^(3/2)/(c^2*x^2+1)^(1/2)*(525*arcsinh(c*x)*c^8*x^8+1365*arcs
inh(c*x)*c^6*x^6-75*c^7*x^7*(c^2*x^2+1)^(1/2)+945*arcsinh(c*x)*c^4*x^4-168*
c^5*x^5*(c^2*x^2+1)^(1/2)-105*arcsinh(c*x)*c^2*x^2-35*c^3*x^3*(c^2*x^2+1)^(
1/2)-210*arcsinh(c*x)+210*c*x*(c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.59

$$\int x^3\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \frac{105\sqrt{\pi+\pi c^2x^2}(5\,\pi bc^8x^8+13\,\pi bc^6x^6+9\,\pi bc^4x^4-\pi bc^2x^2-2\,\pi b)\log\left(cx+\sqrt{c^2x^2+1}\right)\cdots}{\cdots}$$

```
[In] integrate(x^3*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas
")
```

```
[Out] 1/3675*(105*sqrt(pi + pi*c^2*x^2)*(5*pi*b*c^8*x^8 + 13*pi*b*c^6*x^6 + 9*pi*
b*c^4*x^4 - pi*b*c^2*x^2 - 2*pi*b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt(pi +
 pi*c^2*x^2)*(525*pi*a*c^8*x^8 + 1365*pi*a*c^6*x^6 + 945*pi*a*c^4*x^4 - 105
*pi*a*c^2*x^2 - 210*pi*a - (75*pi*b*c^7*x^7 + 168*pi*b*c^5*x^5 + 35*pi*b*c^
3*x^3 - 210*pi*b*c*x)*sqrt(c^2*x^2 + 1)))/(c^6*x^2 + c^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(117) = 234.

Time = 9.33 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.41

$$\int x^3\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \begin{cases} \frac{\pi^{\frac{3}{2}}ac^2x^6\sqrt{c^2x^2+1}}{7}+\frac{8\pi^{\frac{3}{2}}ax^4\sqrt{c^2x^2+1}}{35}+\frac{\pi^{\frac{3}{2}}ax^2\sqrt{c^2x^2+1}}{35c^2}-\frac{2\pi^{\frac{3}{2}}a\sqrt{c^2x^2+1}}{35c^4}-\frac{\pi^{\frac{3}{2}}bc^3x^7}{49}+\frac{\pi^{\frac{3}{2}}bc^2x^6\sqrt{c^2x^2+1}\cdots}{7}\cdots \\ \frac{\pi^{\frac{3}{2}}ax^4}{4} \end{cases}$$

```
[In] integrate(x**3*(pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((pi**(3/2)*a*c**2*x**6*sqrt(c**2*x**2 + 1)/7 + 8*pi**(3/2)*a*x**4
*sqrt(c**2*x**2 + 1)/35 + pi**(3/2)*a*x**2*sqrt(c**2*x**2 + 1)/(35*c**2) -
2*pi**(3/2)*a*sqrt(c**2*x**2 + 1)/(35*c**4) - pi**(3/2)*b*c**3*x**7/49 + pi
**(3/2)*b*c**2*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/7 - 8*pi**(3/2)*b*c*x**5
/175 + 8*pi**(3/2)*b*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/35 - pi**(3/2)*b*x
**3/(105*c) + pi**(3/2)*b*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(35*c**2) + 2
*pi**(3/2)*b*x/(35*c**3) - 2*pi**(3/2)*b*sqrt(c**2*x**2 + 1)*asinh(c*x)/(35
*c**4), Ne(c, 0)), (pi**(3/2)*a*x**4/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.16

$$\int x^3 (\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \frac{1}{35} \left(\frac{5 (\pi + \pi c^2x^2)^{\frac{5}{2}} x^2}{\pi c^2} - \frac{2 (\pi + \pi c^2x^2)^{\frac{5}{2}}}{\pi c^4} \right) b \operatorname{arsinh}(cx) + \frac{1}{35} \left(\frac{5 (\pi + \pi c^2x^2)^{\frac{5}{2}} x^2}{\pi c^2} - \frac{2 (\pi + \pi c^2x^2)^{\frac{5}{2}}}{\pi c^4} \right) a - \frac{\left(75 \pi^{\frac{3}{2}} c^6x^7 + 168 \pi^{\frac{3}{2}} c^4x^5 + 35 \pi^{\frac{3}{2}} c^2x^3 - 210 \pi^{\frac{3}{2}} x\right) b}{3675 c^3}$$

```
[In] integrate(x^3*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima
")
```

```
[Out] 1/35*(5*(pi + pi*c^2*x^2)^(5/2)*x^2/(pi*c^2) - 2*(pi + pi*c^2*x^2)^(5/2)/(p
i*c^4))*b*arcsinh(c*x) + 1/35*(5*(pi + pi*c^2*x^2)^(5/2)*x^2/(pi*c^2) - 2*(
pi + pi*c^2*x^2)^(5/2)/(pi*c^4))*a - 1/3675*(75*pi^(3/2)*c^6*x^7 + 168*pi^(
3/2)*c^4*x^5 + 35*pi^(3/2)*c^2*x^3 - 210*pi^(3/2)*x)*b/c^3
```

Giac [F(-2)]

Exception generated.

$$\int x^3 (\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(\pi + c^2 \pi x^2\right)^{3/2} (a + b \operatorname{arcsinh}(cx))\, dx = \int x^3 \, (a + b \operatorname{asinh}(c\, x)) \, \left(\Pi\, c^2\, x^2 + \Pi\right)^{3/2} dx$$

```
[In] int(x^3*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2),x)
[Out] int(x^3*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2), x)
```

3.64 $\int x^2 (\pi + c^2 \pi x^2)^{3/2} (a + b\,\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 165

$$\int x^2 (\pi + c^2 \pi x^2)^{3/2} (a + b\,\text{arcsinh}(cx))\,dx = -\frac{b\pi^{3/2}x^2}{32c} - \frac{7}{96}bc\pi^{3/2}x^4 - \frac{1}{36}bc^3\pi^{3/2}x^6 + \frac{\pi^{3/2}x\sqrt{1+c^2x^2}(a + b\,\text{arcsinh}(cx))}{16c^2} + \frac{1}{8}\pi x^3\sqrt{\pi + c^2\pi x^2}(a + b\,\text{arcsinh}(cx)) + \frac{1}{6}x^3(\pi + c^2\pi x^2)^{3/2}(a + b\,\text{arcsinh}(cx)) - \frac{\pi^{3/2}(a + b\,\text{arcsinh}(cx))^2}{32bc^3}$$

```
[Out] -1/32*b*Pi^(3/2)*x^2/c-7/96*b*c*Pi^(3/2)*x^4-1/36*b*c^3*Pi^(3/2)*x^6+1/6*x^
3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))-1/32*Pi^(3/2)*(a+b*arcsinh(c*x))
^2/b/c^3+1/16*Pi^(3/2)*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^2+1/8*Pi*x^
3*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5808, 5806, 5812, 5783, 30, 14}

$$\int x^2 (\pi + c^2 \pi x^2)^{3/2} (a + b\,\text{arcsinh}(cx))\,dx = -\frac{\pi^{3/2}(a + b\,\text{arcsinh}(cx))^2}{32bc^3} + \frac{\pi^{3/2}x\sqrt{c^2x^2+1}(a + b\,\text{arcsinh}(cx))}{16c^2} + \frac{1}{6}x^3(\pi c^2x^2 + \pi)^{3/2}(a + b\,\text{arcsinh}(cx)) + \frac{1}{8}\pi x^3\sqrt{\pi c^2x^2 + \pi}(a + b\,\text{arcsinh}(cx)) - \frac{1}{36}\pi^{3/2}bc^3x^6 - \frac{7}{96}\pi^{3/2}bcx^4 - \frac{\pi^{3/2}\cdots}{32\cdots}$$

```
[In] Int[x^2*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/32*(b*Pi^(3/2)*x^2)/c - (7*b*c*Pi^(3/2)*x^4)/96 - (b*c^3*Pi^(3/2)*x^6)/3
6 + (Pi^(3/2)*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(16*c^2) + (Pi*x^3*
Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/8 + (x^3*(Pi + c^2*Pi*x^2)^(3/2
)*(a + b*ArcSinh[c*x]))/6 - (Pi^(3/2)*(a + b*ArcSinh[c*x])^2)/(32*b*c^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
```

```
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{6}x^3\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{1}{2}\pi\int x^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx - \frac{1}{6}\left(bc\pi^{3/2}\right)\int x^3\left(1+c^2x^2\right)\,dx$$

$$= \frac{1}{8}\pi x^3\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))+\frac{1}{6}x^3\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{1}{8}\pi^{3/2}\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx - \frac{1}{8}\left(bc\pi^{3/2}\right)\int x^3\,dx-\frac{1}{6}\left(bc\pi^{3/2}\right)\int\left(x^3+c^2x^5\right)\,dx$$

$$= -\frac{7}{96}bc\pi^{3/2}x^4-\frac{1}{36}bc^3\pi^{3/2}x^6+\frac{\pi^{3/2}x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{16c^2} + \frac{1}{8}\pi x^3\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))+\frac{1}{6}x^3\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{\pi^{3/2}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}}{16c^2}$$

$$= -\frac{b\pi^{3/2}x^2}{32c}-\frac{7}{96}bc\pi^{3/2}x^4-\frac{1}{36}bc^3\pi^{3/2}x^6+\frac{\pi^{3/2}x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{16c^2} + \frac{1}{8}\pi x^3\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))+\frac{1}{6}x^3\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{\pi^{3/2}(a+b\text{arcsinh}(cx}{32bc^3}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.93

$$\int x^2\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \frac{\pi^{3/2}\left(144acx\sqrt{1+c^2x^2}+672ac^3x^3\sqrt{1+c^2x^2}+384ac^5x^5\sqrt{1+c^2x^2}-72b\text{arcsinh}(cx)^2+\right.}{}$$

```
[In] Integrate[x^2*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Pi^(3/2)*(144*a*c*x*Sqrt[1 + c^2*x^2] + 672*a*c^3*x^3*Sqrt[1 + c^2*x^2] +
384*a*c^5*x^5*Sqrt[1 + c^2*x^2] - 72*b*ArcSinh[c*x]^2 + 18*b*Cosh[2*ArcSinh
[c*x]] - 9*b*Cosh[4*ArcSinh[c*x]] - 2*b*Cosh[6*ArcSinh[c*x]] - 12*ArcSinh[c
*x]*(12*a + 3*b*Sinh[2*ArcSinh[c*x]] - 3*b*Sinh[4*ArcSinh[c*x]] - b*Sinh[6*
ArcSinh[c*x]])))/(2304*c^3)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.30

method	result
default	$\frac{ax(\pi c^2x^2+\pi)^{\frac{5}{2}}}{6\pi c^2}-\frac{ax(\pi c^2x^2+\pi)^{\frac{3}{2}}}{24c^2}-\frac{a\pi x\sqrt{\pi c^2x^2+\pi}}{16c^2}-\frac{a\pi^2\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{16c^2\sqrt{\pi c^2}}-\frac{b\pi^{\frac{3}{2}}\left(-48\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^5\right.}{}$
parts	$\frac{ax(\pi c^2x^2+\pi)^{\frac{5}{2}}}{6\pi c^2}-\frac{ax(\pi c^2x^2+\pi)^{\frac{3}{2}}}{24c^2}-\frac{a\pi x\sqrt{\pi c^2x^2+\pi}}{16c^2}-\frac{a\pi^2\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{16c^2\sqrt{\pi c^2}}-\frac{b\pi^{\frac{3}{2}}\left(-48\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^5\right.}{}$

```
[In] int(x^2*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a*x*(Pi*c^2*x^2+Pi)^(5/2)/Pi/c^2-1/24*a/c^2*x*(Pi*c^2*x^2+Pi)^(3/2)-1/1
6*a/c^2*Pi*x*(Pi*c^2*x^2+Pi)^(1/2)-1/16*a/c^2*Pi^2*ln(Pi*c^2*x/(Pi*c^2)^(1/
2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)-1/288*b*Pi^(3/2)*(-48*arcsinh(c*x)
*(c^2*x^2+1)^(1/2)*x^5*c^5+8*c^6*x^6-84*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*
c^3+21*c^4*x^4-18*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)+9*c^2*x^2+9*arcsinh(c*
x)^2-4)/c^3
```

Fricas [F]

$$\int x^2\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\mathrm{arcsinh}(cx))\,dx=\int\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx)+a)x^2\,dx$$

```
[In] integrate(x^2*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^4 + pi*a*x^2 + (pi*b*c^2*x^4 + p
i*b*x^2)*arcsinh(c*x)), x)
```

Sympy [A] (verification not implemented)

Time = 5.49 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.59

$$\int x^2\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\mathrm{arcsinh}(cx))\,dx=\begin{cases}\frac{\pi^{\frac{3}{2}}ac^2x^5\sqrt{c^2x^2+1}}{6}+\frac{7\pi^{\frac{3}{2}}ax^3\sqrt{c^2x^2+1}}{24}+\frac{\pi^{\frac{3}{2}}ax\sqrt{c^2x^2+1}}{16c^2}-\frac{\pi^{\frac{3}{2}}a\,\mathrm{asinh}\,(cx)}{16c^3}-\frac{\pi^{\frac{3}{2}}bc^3x^6}{36}+\frac{\pi^{\frac{3}{2}}bc^2x^5\sqrt{c^2x^2+}}{6}\\ \frac{\pi^{\frac{3}{2}}ax^3}{3}\end{cases}$$

```
[In] integrate(x**2*(pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((pi**(3/2)*a*c**2*x**5*sqrt(c**2*x**2 + 1)/6 + 7*pi**(3/2)*a*x**3
*sqrt(c**2*x**2 + 1)/24 + pi**(3/2)*a*x*sqrt(c**2*x**2 + 1)/(16*c**2) - pi*
```

```
*(3/2)*a*asinh(c*x)/(16*c**3) - pi**(3/2)*b*c**3*x**6/36 + pi**(3/2)*b*c**2
*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)/6 - 7*pi**(3/2)*b*c*x**4/96 + 7*pi**(3
/2)*b*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/24 - pi**(3/2)*b*x**2/(32*c) + pi
**(3/2)*b*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(16*c**2) - pi**(3/2)*b*asinh(c*
x)**2/(32*c**3), Ne(c, 0)), (pi**(3/2)*a*x**3/3, True))
```

Maxima [F(-2)]

Exception generated.

$$\int x^2(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int x^2(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx=\int(\pi+\pi c^2x^2)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)x^2\,dx$$

```
[In] integrate(x^2*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((pi + pi*c^2*x^2)^(3/2)*(b*arcsinh(c*x) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx=\int x^2\,(a+b\,\operatorname{asinh}(c\,x))\,\left(\Pi\,c^2\,x^2+\Pi\right)^{3/2}\,dx$$

```
[In] int(x^2*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2),x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2), x)
```

3.65 $\int x(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx)) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 77

$$\int x(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx)) \, dx = -\frac{b\pi^{3/2}x}{5c} - \frac{2}{15}bc\pi^{3/2}x^3 - \frac{1}{25}bc^3\pi^{3/2}x^5 + \frac{(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))}{5c^2\pi}$$

```
[Out] -1/5*b*Pi^(3/2)*x/c-2/15*b*c*Pi^(3/2)*x^3-1/25*b*c^3*Pi^(3/2)*x^5+1/5*(Pi*c
^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/c^2/Pi
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5798, 200}

$$\int x(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx)) \, dx = \frac{(\pi c^2x^2 + \pi)^{5/2} (a + b\text{arcsinh}(cx))}{5\pi c^2} - \frac{1}{25}\pi^{3/2}bc^3x^5 - \frac{2}{15}\pi^{3/2}bcx^3 - \frac{\pi^{3/2}bx}{5c}$$

```
[In] Int[x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/5*(b*Pi^(3/2)*x)/c - (2*b*c*Pi^(3/2)*x^3)/15 - (b*c^3*Pi^(3/2)*x^5)/25 +
 ((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^2*Pi)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(\pi+c^2\pi x^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))}{5c^2\pi} - \frac{(b\pi^{3/2})\int (1+c^2x^2)^2\,dx}{5c} \\
&= \frac{(\pi+c^2\pi x^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))}{5c^2\pi} - \frac{(b\pi^{3/2})\int (1+2c^2x^2+c^4x^4)\,dx}{5c} \\
&= -\frac{b\pi^{3/2}x}{5c} - \frac{2}{15}bc\pi^{3/2}x^3 - \frac{1}{25}bc^3\pi^{3/2}x^5 + \frac{(\pi+c^2\pi x^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))}{5c^2\pi}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int x\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \frac{\pi^{3/2}\left(15a(1+c^2x^2)^{5/2}-bcx(15+10c^2x^2+3c^4x^4)+15b(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)\right)}{75c^2}$$

```
[In] Integrate[x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
[Out] (Pi^(3/2)*(15*a*(1 + c^2*x^2)^(5/2) - b*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4) +
 15*b*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]))/(75*c^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(61) = 122.

Time = 0.17 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.81

method	result
default	$\frac{a(\pi c^2x^2+\pi)^{\frac{5}{2}}}{5\pi c^2}+\frac{b\pi^{\frac{3}{2}}\left(15\operatorname{arcsinh}(cx)c^6x^6+45\operatorname{arcsinh}(cx)c^4x^4-3c^5x^5\sqrt{c^2x^2+1}+45\operatorname{arcsinh}(cx)c^2x^2-10c^3x^3\sqrt{c^2x^2+1}+15\operatorname{arcsin}\right.}{75c^2\sqrt{c^2x^2+1}}$
parts	$\frac{a(\pi c^2x^2+\pi)^{\frac{5}{2}}}{5\pi c^2}+\frac{b\pi^{\frac{3}{2}}\left(15\operatorname{arcsinh}(cx)c^6x^6+45\operatorname{arcsinh}(cx)c^4x^4-3c^5x^5\sqrt{c^2x^2+1}+45\operatorname{arcsinh}(cx)c^2x^2-10c^3x^3\sqrt{c^2x^2+1}+15\operatorname{arcsin}\right.}{75c^2\sqrt{c^2x^2+1}}$

```
[In] int(x*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a*(Pi*c^2*x^2+Pi)^(5/2)/Pi/c^2+1/75*b/c^2*Pi^(3/2)/(c^2*x^2+1)^(1/2)*(1
5*arcsinh(c*x)*c^6*x^6+45*arcsinh(c*x)*c^4*x^4-3*c^5*x^5*(c^2*x^2+1)^(1/2)+
45*arcsinh(c*x)*c^2*x^2-10*c^3*x^3*(c^2*x^2+1)^(1/2)+15*arcsinh(c*x)-15*c*x
*(c^2*x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(61) = 122$.

Time = 0.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.17

$$\int x(\pi + c^2\pi x^2)^{3/2} (a + b\operatorname{arcsinh}(cx))\, dx = \frac{15\sqrt{\pi + \pi c^2x^2}(\pi bc^6x^6 + 3\,\pi bc^4x^4 + 3\,\pi bc^2x^2 + \pi b)\log\left(cx + \sqrt{c^2x^2+1}\right) + \sqrt{\pi + \pi c^2x}}{75\,(}$$

```
[In] integrate(x*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/75*(15*sqrt(pi + pi*c^2*x^2)*(pi*b*c^6*x^6 + 3*pi*b*c^4*x^4 + 3*pi*b*c^2*
x^2 + pi*b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(15*pi*a*c
^6*x^6 + 45*pi*a*c^4*x^4 + 45*pi*a*c^2*x^2 + 15*pi*a - (3*pi*b*c^5*x^5 + 10
*pi*b*c^3*x^3 + 15*pi*b*c*x)*sqrt(c^2*x^2 + 1)))/(c^4*x^2 + c^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(70) = 140$.

Time = 2.93 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.87

$$\int x(\pi + c^2\pi x^2)^{3/2} (a + b\operatorname{arcsinh}(cx))\, dx = \begin{cases} \frac{\pi^{\frac{3}{2}}ac^2x^4\sqrt{c^2x^2+1}}{5} + \frac{2\pi^{\frac{3}{2}}ax^2\sqrt{c^2x^2+1}}{5} + \frac{\pi^{\frac{3}{2}}a\sqrt{c^2x^2+1}}{5c^2} - \frac{\pi^{\frac{3}{2}}bc^3x^5}{25} + \frac{\pi^{\frac{3}{2}}bc^2x^4\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{5} - \frac{2\pi^{\frac{3}{2}}}{1} \\ \frac{\pi^{\frac{3}{2}}ax^2}{2} \end{cases}$$

```
[In] integrate(x*(pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((pi**(3/2)*a*c**2*x**4*sqrt(c**2*x**2 + 1)/5 + 2*pi**(3/2)*a*x**2
*sqrt(c**2*x**2 + 1)/5 + pi**(3/2)*a*sqrt(c**2*x**2 + 1)/(5*c**2) - pi**(3/
2)*b*c**3*x**5/25 + pi**(3/2)*b*c**2*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/5
- 2*pi**(3/2)*b*c*x**3/15 + 2*pi**(3/2)*b*x**2*sqrt(c**2*x**2 + 1)*asinh(c*
x)/5 - pi**(3/2)*b*x/(5*c) + pi**(3/2)*b*sqrt(c**2*x**2 + 1)*asinh(c*x)/(5*
c**2), Ne(c, 0)), (pi**(3/2)*a*x**2/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int x(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))\,dx = \frac{(\pi + \pi c^2x^2)^{\frac{5}{2}}b\,\text{arsinh}\,(cx)}{5\,\pi c^2} + \frac{(\pi + \pi c^2x^2)^{\frac{5}{2}}a}{5\,\pi c^2} - \frac{\left(3\,\pi^{\frac{5}{2}}c^4x^5 + 10\,\pi^{\frac{5}{2}}c^2x^3 + 15\,\pi^{\frac{5}{2}}x\right)b}{75\,\pi c}$$

```
[In] integrate(x*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] 1/5*(pi + pi*c^2*x^2)^(5/2)*b*arcsinh(c*x)/(pi*c^2) + 1/5*(pi + pi*c^2*x^2)
^(5/2)*a/(pi*c^2) - 1/75*(3*pi^(5/2)*c^4*x^5 + 10*pi^(5/2)*c^2*x^3 + 15*pi^
(5/2)*x)*b/(pi*c)
```

Giac [F(-2)]

Exception generated.

$$\int x(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))\,dx = \int x\,(a + b\,\text{asinh}(c\,x))\,\left(\Pi\,c^2\,x^2 + \Pi\right)^{3/2}\,dx$$

```
[In] int(x*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2),x)
[Out] int(x*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2), x)
```

3.66 $\int (\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))\, dx$

Optimal result

Integrand size = 23, antiderivative size = 111

$$\int (\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))\, dx = -\frac{5}{16} bc\pi^{3/2}x^2 - \frac{1}{16} bc^3\pi^{3/2}x^4 + \frac{3}{8}\pi x\sqrt{\pi + c^2\pi x^2}(a+b\text{arcsinh}(cx)) + \frac{1}{4}x(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{3\pi^{3/2}(a + b\text{arcsinh}(cx))^2}{16bc}$$

```
[Out] -5/16*b*c*Pi^(3/2)*x^2-1/16*b*c^3*Pi^(3/2)*x^4+1/4*x*(Pi*c^2*x^2+Pi)^(3/2)*
(a+b*arcsinh(c*x))+3/16*Pi^(3/2)*(a+b*arcsinh(c*x))^2/b/c+3/8*Pi*x*(a+b*arc
sinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5786, 5785, 5783, 30, 14}

$$\int (\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \frac{1}{4}x(\pi c^2x^2 + \pi)^{3/2}(a + b\text{arcsinh}(cx)) + \frac{3}{8}\pi x\sqrt{\pi c^2x^2 + \pi}(a + b\text{arcsinh}(cx)) + \frac{3\pi^{3/2}(a + b\text{arcsinh}(cx))^2}{16bc} - \frac{1}{16}\pi^{3/2}bc^3x^4 - \frac{5}{16}\pi^{3/2}bcx^2$$

```
[In] Int[(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-5*b*c*Pi^(3/2)*x^2)/16 - (b*c^3*Pi^(3/2)*x^4)/16 + (3*Pi*x*Sqrt[Pi + c^2*
Pi*x^2]*(a + b*ArcSinh[c*x]))/8 + (x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh
[c*x]))/4 + (3*Pi^(3/2)*(a + b*ArcSinh[c*x])^2)/(16*b*c)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{4}x\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{1}{4}(3\pi)\int\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx \\ -\frac{1}{4}\left(bc\pi^{3/2}\right)\int x\left(1+c^2x^2\right)\,dx$$

$$= \frac{3}{8}\pi x\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx)) + \frac{1}{4}x\left(\pi + c^2\pi x^2\right)^{3/2}(a + b\text{arcsinh}(cx)) + \frac{1}{8}\left(3\pi^{3/2}\right)\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{1 + c^2x^2}}\,dx - \frac{1}{4}\left(bc\pi^{3/2}\right)\int \left(x + c^2x^3\right)\,dx - \frac{1}{8}\left(3bc\pi^{3/2}\right)\int x\,dx$$

$$= -\frac{5}{16}bc\pi^{3/2}x^2 - \frac{1}{16}bc^3\pi^{3/2}x^4 + \frac{3}{8}\pi x\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx)) + \frac{1}{4}x\left(\pi + c^2\pi x^2\right)^{3/2}(a + b\text{arcsinh}(cx)) + \frac{3\pi^{3/2}(a + b\text{arcsinh}(cx))^2}{16bc}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00

$$\int \left(\pi + c^2\pi x^2\right)^{3/2}(a + b\text{arcsinh}(cx))\,dx = \frac{\pi^{3/2}\left(80acx\sqrt{1 + c^2x^2} + 32ac^3x^3\sqrt{1 + c^2x^2} + 24b\text{arcsinh}(cx)^2 - 16b\cosh(2\text{arcsinh}(cx))\right.}{}$$

```
[In] Integrate[(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Pi^(3/2)*(80*a*c*x*Sqrt[1 + c^2*x^2] + 32*a*c^3*x^3*Sqrt[1 + c^2*x^2] + 24
*b*ArcSinh[c*x]^2 - 16*b*Cosh[2*ArcSinh[c*x]] - b*Cosh[4*ArcSinh[c*x]] + 4*
ArcSinh[c*x]*(12*a + 8*b*Sinh[2*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]])))/(
128*c)
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.37

method	result
default	$\frac{x\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}a}{4} + \frac{3a\pi x\sqrt{\pi c^2x^2+\pi}}{8} + \frac{3a\pi^2\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{8\sqrt{\pi c^2}} + \frac{b\pi^{\frac{3}{2}}\left(4\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3-c^4x^4+10\,\text{arcsinh}(\right.}{16c}$
parts	$\frac{x\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}a}{4} + \frac{3a\pi x\sqrt{\pi c^2x^2+\pi}}{8} + \frac{3a\pi^2\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{8\sqrt{\pi c^2}} + \frac{b\pi^{\frac{3}{2}}\left(4\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3-c^4x^4+10\,\text{arcsinh}(\right.}{16c}$

```
[In] int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x*(Pi*c^2*x^2+Pi)^(3/2)*a+3/8*a*Pi*x*(Pi*c^2*x^2+Pi)^(1/2)+3/8*a*Pi^2*l
n(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/16*b*Pi^(
3/2)*(4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3-c^4*x^4+10*arcsinh(c*x)*c*x*
(c^2*x^2+1)^(1/2)-5*c^2*x^2+3*arcsinh(c*x)^2-4)/c
```

Fricas [F]

$$\int \left(\pi +c^2\pi x^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx=\int \left(\pi +\pi c^2x^2\right)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)\,dx$$

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^2 + pi*a + (pi*b*c^2*x^2 + pi*b)
*arcsinh(c*x)), x)
```

Sympy [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.67

$$\int \left(\pi +c^2\pi x^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx=\begin{cases}\frac{\pi^{\frac{3}{2}}ac^2x^3\sqrt{c^2x^2+1}}{4}+\frac{5\pi^{\frac{3}{2}}ax\sqrt{c^2x^2+1}}{8}+\frac{3\pi^{\frac{3}{2}}a\,\text{asinh}\,(cx)}{8c}-\frac{\pi^{\frac{3}{2}}bc^3x^4}{16}+\frac{\pi^{\frac{3}{2}}bc^2x^3\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)}{4}-\frac{5\pi^{\frac{3}{2}}bc}{16} \\ \pi^{\frac{3}{2}}ax\end{cases}$$

```
[In] integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((pi**(3/2)*a*c**2*x**3*sqrt(c**2*x**2 + 1)/4 + 5*pi**(3/2)*a*x*sq
rt(c**2*x**2 + 1)/8 + 3*pi**(3/2)*a*asinh(c*x)/(8*c) - pi**(3/2)*b*c**3*x**
4/16 + pi**(3/2)*b*c**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/4 - 5*pi**(3/2)
*b*c*x**2/16 + 5*pi**(3/2)*b*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/8 + 3*pi**(3/
2)*b*asinh(c*x)**2/(16*c), Ne(c, 0)), (pi**(3/2)*a*x, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(\pi +c^2\pi x^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))\,dx = \int (a + b\,\text{asinh}(c\,x))\,\left(\Pi\,c^2\,x^2 + \Pi\right)^{3/2}\,dx$$

```
[In] int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2), x)
```

3.67 $\int \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\,\mathbf{arcsinh}(cx))}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 134

$$\int \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))}{x}\,dx = -\frac{4}{3}bc\pi^{3/2}x - \frac{1}{9}bc^3\pi^{3/2}x^3$$
$$+\pi\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))+\frac{1}{3}\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))-2\pi^{3/2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}}\right.$$

```
[Out] -4/3*b*c*Pi^(3/2)*x-1/9*b*c^3*Pi^(3/2)*x^3+1/3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*a
rcsinh(c*x))-2*Pi^(3/2)*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))-b
*Pi^(3/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+b*Pi^(3/2)*polylog(2,c*x+(c^2*x
^2+1)^(1/2))+Pi*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5808, 5806, 5816, 4267, 2317, 2438, 8}

$$\int \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))}{x}\,dx = -2\pi^{3/2}\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))$$
$$+\frac{1}{3}\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))+\pi\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))-\pi^{3/2}b\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)+\pi^{3/2}b$$

```
[In] Int[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (-4*b*c*Pi^(3/2)*x)/3 - (b*c^3*Pi^(3/2)*x^3)/9 + Pi*Sqrt[Pi + c^2*Pi*x^2]*(
a + b*ArcSinh[c*x]) + ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/3 - 2*
Pi^(3/2)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - b*Pi^(3/2)*PolyLog[
2, -E^ArcSinh[c*x]] + b*Pi^(3/2)*PolyLog[2, E^ArcSinh[c*x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
```

```
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\text{arcsinh}(cx)\right) + \pi \int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x}\,dx - \frac{1}{3}\left(bc\pi^{3/2}\right)\int \left(1 + c^2x^2\right)\,dx$$

$$= -\frac{1}{3}bc\pi^{3/2}x - \frac{1}{9}bc^3\pi^{3/2}x^3 + \pi\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx)) + \frac{1}{3}\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\text{arcsinh}(cx)\right)+\pi^{3/2}\int \frac{a + b\text{arcsinh}(cx)}{x\sqrt{1 + c^2x^2}}\,dx-\left(bc\pi^{3/2}\right)\int 1\,dx$$

$$= -\frac{4}{3}bc\pi^{3/2}x - \frac{1}{9}bc^3\pi^{3/2}x^3 + \pi\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx)) + \frac{1}{3}\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\text{arcsinh}(cx)\right)+\pi^{3/2}\text{Subst}\left(\int (a+bx)\text{csch}(x)\,dx, x, \text{arcsinh}(cx)\right)$$

$$= -\frac{4}{3}bc\pi^{3/2}x - \frac{1}{9}bc^3\pi^{3/2}x^3 + \pi\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx)) + \frac{1}{3}\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\text{arcsinh}(cx)\right)-2\pi^{3/2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)-\left(b\pi^{3/2}\right)\text{Subst}\Big($$

$$= -\frac{4}{3}bc\pi^{3/2}x - \frac{1}{9}bc^3\pi^{3/2}x^3 + \pi\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx)) + \frac{1}{3}\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\text{arcsinh}(cx)\right)-2\pi^{3/2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)-\left(b\pi^{3/2}\right)\text{Subst}\Big($$

$$= -\frac{4}{3}bc\pi^{3/2}x - \frac{1}{9}bc^3\pi^{3/2}x^3 + \pi\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx)) + \frac{1}{3}\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\text{arcsinh}(cx)\right)-2\pi^{3/2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)-b\pi^{3/2}\,\text{PolyLog}\,($$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.34

$$\int \frac{\left(\pi + c^2\pi x^2\right)^{3/2}(a + b\text{arcsinh}(cx))}{x}\,dx = \frac{1}{9}\pi^{3/2}\Big(3a\sqrt{1 + c^2x^2}\left(4+c^2x^2\right)-b\Big(3cx+c^3x^3-3\left(1+c^2x^2\right)^{3/2}\text{arcsin}$$

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x,x]
[Out] (Pi^(3/2)*(3*a*Sqrt[1 + c^2*x^2]*(4 + c^2*x^2) - b*(3*c*x + c^3*x^3 - 3*(1
+ c^2*x^2)^(3/2)*ArcSinh[c*x]) + 9*a*Log[x] - 9*a*Log[Pi*(1 + Sqrt[1 + c^2*
x^2])] + 9*b*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1
- E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2,
 -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])])))/9
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.70

method	result
default	$a\left(\frac{(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3}+\pi\left(\sqrt{\pi c^2x^2+\pi}-\sqrt{\pi}\,\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)\right)\right)+\frac{b\sqrt{c^2x^2+1}\,\operatorname{arcsinh}(cx)\pi^{\frac{3}{2}}x^2c^2}{3}+\frac{4b\sqrt{c^2}}{}$
parts	$a\left(\frac{(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3}+\pi\left(\sqrt{\pi c^2x^2+\pi}-\sqrt{\pi}\,\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)\right)\right)+\frac{b\sqrt{c^2x^2+1}\,\operatorname{arcsinh}(cx)\pi^{\frac{3}{2}}x^2c^2}{3}+\frac{4b\sqrt{c^2}}{}$

```
[In] int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/3*(Pi*c^2*x^2+Pi)^(3/2)+Pi*((Pi*c^2*x^2+Pi)^(1/2)-Pi^(1/2)*arctanh(Pi^
(1/2)/(Pi*c^2*x^2+Pi)^(1/2))))+1/3*b*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*Pi^(3/2
)*x^2*c^2+4/3*b*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*Pi^(3/2)-4/3*b*c*Pi^(3/2)*x-
1/9*b*c^3*Pi^(3/2)*x^3+b*Pi^(3/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-
b*Pi^(3/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+b*Pi^(3/2)*polylog(2,c*
x+(c^2*x^2+1)^(1/2))-b*Pi^(3/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{(\pi+c^2\pi x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{x}\,dx=\int \frac{(\pi+\pi c^2x^2)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{x}\,dx$$

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^2 + pi*a + (pi*b*c^2*x^2 + pi*b)
*arcsinh(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(\pi+c^2\pi x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{x}\,dx=\pi^{\frac{3}{2}}\left(\int \frac{a\sqrt{c^2x^2+1}}{x}\,dx+\int ac^2x\sqrt{c^2x^2+1}\,dx+\int \frac{b\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)}{x}\,dx+\int bc^2x\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)\,dx\right)$$

```
[In] integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x))/x,x)
```

```
[Out] pi**(3/2)*(Integral(a*sqrt(c**2*x**2 + 1)/x, x) + Integral(a*c**2*x*sqrt(c*
*2*x**2 + 1), x) + Integral(b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x, x) + Integr
al(b*c**2*x*sqrt(c**2*x**2 + 1)*asinh(c*x), x))
```

Maxima [F]

$$\int \frac{(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))}{x} \, dx = \int \frac{(\pi + \pi c^2x^2)^{\frac{3}{2}} (b \operatorname{arsinh}(cx) + a)}{x} \, dx$$

[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")

[Out] -1/3*(3*pi^(3/2)*arcsinh(1/(c*abs(x))) - 3*pi*sqrt(pi + pi*c^2*x^2) - (pi + pi*c^2*x^2)^(3/2))*a + b*integrate((pi + pi*c^2*x^2)^(3/2)*log(c*x + sqrt(c^2*x^2 + 1))/x, x)

Giac [F(-2)]

Exception generated.

$$\int \frac{(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))}{x} \, dx = \text{Exception raised: TypeError}$$

[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))}{x} \, dx = \int \frac{(a + b \operatorname{asinh}(c\,x)) \left(\Pi\, c^2\, x^2 + \Pi\right)^{3/2}}{x} \, dx$$

[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x,x)

[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x, x)

3.68 $\int \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^2} dx$

Optimal result

Integrand size = 26, antiderivative size = 108

$$\int \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^2} dx = -\frac{1}{4}bc^3\pi^{3/2}x^2 + \frac{3}{2}c^2\pi x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx)) - \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))}{x} + \frac{3c\pi^{3/2}(a+b\text{arcsinh}(cx))^2}{4b} + bc\pi^{3/2}\log(x)$$

```
[Out] -1/4*b*c^3*Pi^(3/2)*x^2-(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/x+3/4*c*Pi
^(3/2)*(a+b*arcsinh(c*x))^2/b+b*c*Pi^(3/2)*ln(x)+3/2*c^2*Pi*x*(a+b*arcsinh(
c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5807, 5785, 5783, 30, 14}

$$\int \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^2} dx = \frac{3}{2}\pi c^2 x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx)) - \frac{(\pi c^2x^2+\pi)^{3/2}(a+b\text{arcsinh}(cx))}{x} + \frac{3\pi^{3/2}c(a+b\text{arcsinh}(cx))^2}{4b} - \frac{1}{4}\pi^{3/2}bc^3x^2 + \pi^{3/2}bc\log(x)$$

```
[In] Int[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] -1/4*(b*c^3*Pi^(3/2)*x^2) + (3*c^2*Pi*x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSin
h[c*x]))/2 - ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x + (3*c*Pi^(3/
2)*(a + b*ArcSinh[c*x])^2)/(4*b) + b*c*Pi^(3/2)*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))}{x} + (3c^2\pi)\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx + (bc\pi^{3/2})\int \frac{1 + c^2x^2}{x}\,dx$$

$$= \frac{3}{2}c^2\pi x\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx)) - \frac{(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))}{x}$$
$$+ (bc\pi^{3/2})\int\left(\frac{1}{x} + c^2x\right)dx + \frac{1}{2}(3c^2\pi^{3/2})\int\frac{a + b\text{arcsinh}(cx)}{\sqrt{1 + c^2x^2}}dx - \frac{1}{2}(3bc^3\pi^{3/2})\int x\,dx$$
$$= -\frac{1}{4}bc^3\pi^{3/2}x^2 + \frac{3}{2}c^2\pi x\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))$$
$$- \frac{(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))}{x} + \frac{3c\pi^{3/2}(a + b\text{arcsinh}(cx))^2}{4b} + bc\pi^{3/2}\log(x)$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.13

$$\int\frac{(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))}{x^2}dx = \frac{\pi^{3/2}\left(-8a\sqrt{1 + c^2x^2} + 4ac^2x^2\sqrt{1 + c^2x^2} + 6bcx\text{arcsinh}(cx)^2 - bc\right.}{}$$

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (Pi^(3/2)*(-8*a*Sqrt[1 + c^2*x^2] + 4*a*c^2*x^2*Sqrt[1 + c^2*x^2] + 6*b*c*x
*ArcSinh[c*x]^2 - b*c*x*Cosh[2*ArcSinh[c*x]] + 8*b*c*x*Log[c*x] + 2*ArcSinh
[c*x]*(6*a*c*x - 4*b*Sqrt[1 + c^2*x^2] + b*c*x*Sinh[2*ArcSinh[c*x]])))/(8*x
)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(92) = 184$.

Time = 0.17 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.93

method	result
default	$-\frac{a(\pi c^2x^2+\pi)^{\frac{5}{2}}}{\pi x} + a c^2 x(\pi c^2x^2 + \pi)^{\frac{3}{2}} + \frac{3a c^2\pi x\sqrt{\pi c^2x^2+\pi}}{2} + \frac{3a c^2\pi^2\ln\left(\frac{\pi c^2 x}{\sqrt{\pi c^2}} + \sqrt{\pi c^2x^2+\pi}\right)}{2\sqrt{\pi c^2}} + \frac{b\pi^{\frac{3}{2}}\left(4\ \text{arcsinh}(c\right.}{}$
parts	$-\frac{a(\pi c^2x^2+\pi)^{\frac{5}{2}}}{\pi x} + a c^2 x(\pi c^2x^2 + \pi)^{\frac{3}{2}} + \frac{3a c^2\pi x\sqrt{\pi c^2x^2+\pi}}{2} + \frac{3a c^2\pi^2\ln\left(\frac{\pi c^2 x}{\sqrt{\pi c^2}} + \sqrt{\pi c^2x^2+\pi}\right)}{2\sqrt{\pi c^2}} + \frac{b\pi^{\frac{3}{2}}\left(4\ \text{arcsinh}(c\right.}{}$

```
[In] int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/Pi/x*(Pi*c^2*x^2+Pi)^(5/2)+a*c^2*x*(Pi*c^2*x^2+Pi)^(3/2)+3/2*a*c^2*Pi*x*
(Pi*c^2*x^2+Pi)^(1/2)+3/2*a*c^2*Pi^2*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2
+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/8*b*Pi^(3/2)*(4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)
*x^2*c^2-2*c^3*x^3+6*arcsinh(c*x)^2*x*c-8*arcsinh(c*x)*c*x+8*ln((c*x+(c^2*x
^2+1)^(1/2))^2-1)*x*c-8*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-c*x)/x
```

Fricas [F]

$$\int \frac{(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))}{x^2} dx = \int \frac{(\pi + \pi c^2x^2)^{\frac{3}{2}} (b \operatorname{arsinh}(cx) + a)}{x^2} dx$$

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^2 + pi*a + (pi*b*c^2*x^2 + pi*b)
*arcsinh(c*x))/x^2, x)
```

Sympy [F]

$$\int \frac{(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))}{x^2} dx = \pi^{\frac{3}{2}} \left(\int ac^2\sqrt{c^2x^2 + 1}\, dx + \int \frac{a\sqrt{c^2x^2 + 1}}{x^2} dx + \int bc^2\sqrt{c^2x^2 + 1} \operatorname{asinh}(cx)\, dx + \int \frac{b\sqrt{c^2x^2 + 1} \operatorname{asinh}(cx)}{x^2} dx \right)$$

```
[In] integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x))/x**2,x)
```

```
[Out] pi**(3/2)*(Integral(a*c**2*sqrt(c**2*x**2 + 1), x) + Integral(a*sqrt(c**2*x
**2 + 1)/x**2, x) + Integral(b*c**2*sqrt(c**2*x**2 + 1)*asinh(c*x), x) + In
tegral(b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x**2, x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))}{x^2} dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))}{x^2} \, dx = \text{Exception raised: TypeError}$$

[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))}{x^2} \, dx = \int \frac{(a + b\,\text{asinh}(c\,x))\,\left(\Pi\,c^2\,x^2 + \Pi\right)^{3/2}}{x^2} \, dx$$

[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x^2,x)

[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x^2, x)

3.69 $\int \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\,\mathrm{arcsinh}(cx))}{x^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 155

$$\int \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{x^3}\,dx=-\frac{bc\pi^{3/2}}{2x}-bc^3\pi^{3/2}x$$
$$+\frac{3}{2}c^2\pi\sqrt{\pi+c^2\pi x^2}(a+b\operatorname{arcsinh}(cx))-\frac{(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{2x^2}-3c^2\pi^{3/2}(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}(e$$

```
[Out] -1/2*b*c*Pi^(3/2)/x-b*c^3*Pi^(3/2)*x-1/2*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh
(c*x))/x^2-3*c^2*Pi^(3/2)*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))
-3/2*b*c^2*Pi^(3/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+3/2*b*c^2*Pi^(3/2)*po
lylog(2,c*x+(c^2*x^2+1)^(1/2))+3/2*c^2*Pi*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi
)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5807, 5806, 5816, 4267, 2317, 2438, 8, 14}

$$\int \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{x^3}\,dx=-3\pi^{3/2}c^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx))$$
$$+\frac{3}{2}\pi c^2\sqrt{\pi c^2x^2+\pi}(a+b\operatorname{arcsinh}(cx))-\frac{(\pi c^2x^2+\pi)^{3/2}(a+b\operatorname{arcsinh}(cx))}{2x^2}$$
$$-\frac{3}{2}\pi^{3/2}bc^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)+\frac{3}{2}\pi^{3/2}bc^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)+\pi^{3/2}(-b)c^3x-\frac{\pi^{3/2}bc}{2x}$$

```
[In] Int[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*Pi^(3/2))/x - b*c^3*Pi^(3/2)*x + (3*c^2*Pi*Sqrt[Pi + c^2*Pi*x^2]*
(a + b*ArcSinh[c*x]))/2 - ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(2
*x^2) - 3*c^2*Pi^(3/2)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - (3*b*
c^2*Pi^(3/2)*PolyLog[2, -E^ArcSinh[c*x]])/2 + (3*b*c^2*Pi^(3/2)*PolyLog[2,
E^ArcSinh[c*x]])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\text{arcsinh}(cx)\right)}{2x^2} \\
&\quad+\frac{1}{2}\left(3c^2\pi\right)\int\frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\text{arcsinh}(cx)\right)}{x}\,dx+\frac{1}{2}\left(bc\pi^{3/2}\right)\int\frac{1+c^2x^2}{x^2}\,dx \\
&= \frac{3}{2}c^2\pi\sqrt{\pi+c^2\pi x^2}\left(a+b\text{arcsinh}(cx)\right)-\frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\text{arcsinh}(cx)\right)}{2x^2} \\
&\quad+\frac{1}{2}\left(bc\pi^{3/2}\right)\int\left(c^2+\frac{1}{x^2}\right)\,dx+\frac{1}{2}\left(3c^2\pi^{3/2}\right)\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{1+c^2x^2}}\,dx-\frac{1}{2}\left(3bc^3\pi^{3/2}\right)\int 1\,dx \\
&= -\frac{bc\pi^{3/2}}{2x}-bc^3\pi^{3/2}x \\
&\quad+\frac{3}{2}c^2\pi\sqrt{\pi+c^2\pi x^2}\left(a+b\text{arcsinh}(cx)\right)-\frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\text{arcsinh}(cx)\right)}{2x^2} \\
&\quad+\frac{1}{2}\left(3c^2\pi^{3/2}\right)\text{Subst}\left(\int(a+bx)\text{csch}(x)\,dx,x,\text{arcsinh}(cx)\right) \\
&= -\frac{bc\pi^{3/2}}{2x}-bc^3\pi^{3/2}x \\
&\quad+\frac{3}{2}c^2\pi\sqrt{\pi+c^2\pi x^2}\left(a+b\text{arcsinh}(cx)\right)-\frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\text{arcsinh}(cx)\right)}{2x^2} \\
&\quad-3c^2\pi^{3/2}\left(a+b\text{arcsinh}(cx)\right)\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)-\frac{1}{2}\left(3bc^2\pi^{3/2}\right)\text{Subst}\left(\int\log\left(1-e^x\right)\,dx,x,\text{arcsinh}(c
\end{aligned}$$

$$= -\frac{bc\pi^{3/2}}{2x} - bc^3\pi^{3/2}x + \frac{3}{2}c^2\pi\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx)) - \frac{(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))}{2x^2} - 3c^2\pi^{3/2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) - \frac{1}{2}\left(3bc^2\pi^{3/2}\right)\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}$$

$$= -\frac{bc\pi^{3/2}}{2x} - bc^3\pi^{3/2}x + \frac{3}{2}c^2\pi\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx)) - \frac{(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))}{2x^2} - 3c^2\pi^{3/2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) - \frac{3}{2}bc^2\pi^{3/2}\text{PolyLog}\left(2, -e^{\text{arcsinh}(cx)}\right) + \frac{3}{2}bc^2\pi^{3/2}\text{Pol}$$

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.88

$$\int \frac{(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))}{x^3}\,dx = \frac{\pi^{3/2}\left(-8bc^3x^3 - 4a\sqrt{1+c^2x^2} + 8ac^2x^2\sqrt{1+c^2x^2} + 8bc^2x^2\sqrt{1+\right.}{}$$

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (Pi^(3/2)*(-8*b*c^3*x^3 - 4*a*Sqrt[1 + c^2*x^2] + 8*a*c^2*x^2*Sqrt[1 + c^2*
x^2] + 8*b*c^2*x^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - b*c^3*x^3*Csch[ArcSinh[
c*x]/2]^2 - b*c^2*x^2*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 12*b*c^2*x^2*Ar
cSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 12*b*c^2*x^2*ArcSinh[c*x]*Log[1 + E
^(-ArcSinh[c*x])] + 12*a*c^2*x^2*Log[x] - 12*a*c^2*x^2*Log[Pi*(1 + Sqrt[1 +
 c^2*x^2])] + 12*b*c^2*x^2*PolyLog[2, -E^(-ArcSinh[c*x])] - 12*b*c^2*x^2*Po
lyLog[2, E^(-ArcSinh[c*x])] + 4*b*c*x*Sinh[ArcSinh[c*x]/2]^2 - 4*b*ArcSinh[
c*x]*Sinh[ArcSinh[c*x]/2]^2))/(8*x^2)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.88

method	result
default	$a\left(-\frac{(\pi c^2x^2+\pi)^{\frac{5}{2}}}{2\pi x^2}+\frac{3c^2\left(\frac{(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3}+\pi\left(\sqrt{\pi c^2x^2+\pi}-\sqrt{\pi}\,\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)\right)\right)}{2}\right)+b\sqrt{c^2x^2+1}\,\operatorname{arcsinh}(cx)$
parts	$a\left(-\frac{(\pi c^2x^2+\pi)^{\frac{5}{2}}}{2\pi x^2}+\frac{3c^2\left(\frac{(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3}+\pi\left(\sqrt{\pi c^2x^2+\pi}-\sqrt{\pi}\,\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)\right)\right)}{2}\right)+b\sqrt{c^2x^2+1}\,\operatorname{arcsinh}(cx)$

```
[In] int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/2/Pi/x^2*(Pi*c^2*x^2+Pi)^(5/2)+3/2*c^2*(1/3*(Pi*c^2*x^2+Pi)^(3/2)+Pi*
((Pi*c^2*x^2+Pi)^(1/2)-Pi^(1/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2)))))+
b*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*Pi^(3/2)*c^2-b*c^3*Pi^(3/2)*x-1/2*b*Pi^(3/
2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^2-1/2*b*c*Pi^(3/2)/x-1/2*b*Pi^(3/2)/(c^
2*x^2+1)^(1/2)/x^2*arcsinh(c*x)-3/2*b*c^2*Pi^(3/2)*arcsinh(c*x)*ln(1+c*x+(c
^2*x^2+1)^(1/2))-3/2*b*c^2*Pi^(3/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+3/2*b
*c^2*Pi^(3/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+3/2*b*c^2*Pi^(3/2)*p
olylog(2,c*x+(c^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{(\pi + c^2\pi x^2)^{3/2}\,(a + b\text{arcsinh}(cx))}{x^3}\,dx = \int \frac{(\pi + \pi c^2x^2)^{\frac{3}{2}}\,(b\,\text{arsinh}\,(cx) + a)}{x^3}\,dx$$

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^2 + pi*a + (pi*b*c^2*x^2 + pi*b)
*arcsinh(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(\pi + c^2\pi x^2)^{3/2}\,(a + b\text{arcsinh}(cx))}{x^3}\,dx = \pi^{\frac{3}{2}}\left(\int \frac{a\sqrt{c^2x^2+1}}{x^3}\,dx + \int \frac{ac^2\sqrt{c^2x^2+1}}{x}\,dx + \int \frac{b\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)}{x^3}\,dx + \int \frac{bc^2\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)}{x}\,dx\right)$$

```
[In] integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x))/x**3,x)
```

```
[Out] pi**(3/2)*(Integral(a*sqrt(c**2*x**2 + 1)/x**3, x) + Integral(a*c**2*sqrt(c
**2*x**2 + 1)/x, x) + Integral(b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x**3, x) +
Integral(b*c**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/x, x))
```

Maxima [F]

$$\int \frac{(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))}{x^3} \, dx = \int \frac{(\pi + \pi c^2x^2)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx) + a)}{x^3} \, dx$$

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(3*pi^(3/2)*c^2*arcsinh(1/(c*abs(x))) - 3*pi*sqrt(pi + pi*c^2*x^2)*c^2
 - (pi + pi*c^2*x^2)^(3/2)*c^2 + (pi + pi*c^2*x^2)^(5/2)/(pi*x^2))*a + b*in
tegrate((pi + pi*c^2*x^2)^(3/2)*log(c*x + sqrt(c^2*x^2 + 1))/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))}{x^3} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))}{x^3} \, dx = \int \frac{(a + b\,\text{asinh}(c\,x))\,\left(\Pi\,c^2\,x^2 + \Pi\right)^{3/2}}{x^3} \, dx$$

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x^3, x)
```

3.70 $\int \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^4} dx$

Optimal result

Integrand size = 26, antiderivative size = 115

$$\int \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx = -\frac{bc\pi^{3/2}}{6x^2} - \frac{c^2\pi\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x} - \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3x^3} + \frac{c^3\pi^{3/2}(a+b\text{arcsinh}(cx))^2}{2b} + \frac{4}{3}bc^3\pi^{3/2}\log(x)$$

```
[Out] -1/6*b*c*Pi^(3/2)/x^2-1/3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/x^3+1/2*
c^3*Pi^(3/2)*(a+b*arcsinh(c*x))^2/b+4/3*b*c^3*Pi^(3/2)*ln(x)-c^2*Pi*(a+b*ar
csinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5807, 5805, 29, 5783, 14}

$$\int \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx = \frac{\pi^{3/2}c^3(a+b\text{arcsinh}(cx))^2}{2b} - \frac{\pi c^2\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{x} - \frac{(\pi c^2x^2+\pi)^{3/2}(a+b\text{arcsinh}(cx))}{3x^3} + \frac{4}{3}\pi^{3/2}bc^3\log(x) - \frac{\pi^{3/2}bc}{6x^2}$$

```
[In] Int[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*Pi^(3/2))/x^2 - (c^2*Pi*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]
))/x - ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*x^3) + (c^3*Pi^(3/
2)*(a + b*ArcSinh[c*x])^2)/(2*b) + (4*b*c^3*Pi^(3/2)*Log[x])/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]], Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x]
, x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], In
t[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Free
Q[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(\pi + c^2\pi x^2)^{3/2}\,(a + b\text{arcsinh}(cx))}{3x^3}\\ &\quad + (c^2\pi)\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^2}\,dx + \frac{1}{3}(bc\pi^{3/2})\int \frac{1 + c^2x^2}{x^3}\,dx\end{aligned}$$

$$= -\frac{c^2\pi\sqrt{\pi+c^2\pi x^2}(a+b\operatorname{arcsinh}(cx))}{x} - \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3x^3} + \frac{1}{3}\left(bc\pi^{3/2}\right)\int\left(\frac{1}{x^3}+\frac{c^2}{x}\right)dx + \left(bc^3\pi^{3/2}\right)\int\frac{1}{x}\,dx + \left(c^4\pi^{3/2}\right)\int\frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx$$

$$= -\frac{bc\pi^{3/2}}{6x^2} - \frac{c^2\pi\sqrt{\pi+c^2\pi x^2}(a+b\operatorname{arcsinh}(cx))}{x} - \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3x^3} + \frac{c^3\pi^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{2b} + \frac{4}{3}bc^3\pi^{3/2}\log(x)$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.09

$$\int\frac{(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{x^4}\,dx = \frac{\pi^{3/2}\left(-bcx-2a\sqrt{1+c^2x^2}-8ac^2x^2\sqrt{1+c^2x^2}+\left(6ac^3x^3-2b\sqrt{1-\right.\right.}{}$$

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] (Pi^(3/2)*(-(b*c*x) - 2*a*Sqrt[1 + c^2*x^2] - 8*a*c^2*x^2*Sqrt[1 + c^2*x^2]
 + (6*a*c^3*x^3 - 2*b*Sqrt[1 + c^2*x^2]*(1 + 4*c^2*x^2))*ArcSinh[c*x] + 3*b
*c^3*x^3*ArcSinh[c*x]^2 + 8*b*c^3*x^3*Log[c*x]))/(6*x^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 621 vs. 2(97) = 194.

Time = 0.16 (sec) , antiderivative size = 622, normalized size of antiderivative = 5.41

method	result
default	$-\frac{a(\pi c^2x^2+\pi)^{\frac{5}{2}}}{3\pi x^3} - \frac{2ac^2(\pi c^2x^2+\pi)^{\frac{5}{2}}}{3\pi x} + \frac{2ac^4x(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3} + ac^4\pi x\sqrt{\pi c^2x^2+\pi} + \frac{ac^4\pi^2\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{\sqrt{\pi c^2}}$
parts	$-\frac{a(\pi c^2x^2+\pi)^{\frac{5}{2}}}{3\pi x^3} - \frac{2ac^2(\pi c^2x^2+\pi)^{\frac{5}{2}}}{3\pi x} + \frac{2ac^4x(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3} + ac^4\pi x\sqrt{\pi c^2x^2+\pi} + \frac{ac^4\pi^2\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{\sqrt{\pi c^2}}$

```
[In] int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/Pi/x^3*(Pi*c^2*x^2+Pi)^(5/2)-2/3*a*c^2/Pi/x*(Pi*c^2*x^2+Pi)^(5/2)+2/
3*a*c^4*x*(Pi*c^2*x^2+Pi)^(3/2)+a*c^4*Pi*x*(Pi*c^2*x^2+Pi)^(1/2)+a*c^4*Pi^2
*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/2*b*c^3
*Pi^(3/2)*arcsinh(c*x)^2-8/3*b*c^3*Pi^(3/2)*arcsinh(c*x)+32*b*Pi^(3/2)/(24*
c^4*x^4+9*c^2*x^2+1)*x^4*arcsinh(c*x)*c^7-32*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x
^2+1)*x^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^6+8/3*b*Pi^(3/2)/(24*c^4*x^4+9*c
^2*x^2+1)*x^4*c^7-8/3*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x^2+1)*x^2*(c^2*x^2+1)*c
^5+12*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x^2+1)*x^2*arcsinh(c*x)*c^5-20*b*Pi^(3/2
```

```
)/(24*c^4*x^4+9*c^2*x^2+1)*x*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^4-4/3*b*Pi^(3
/2)/(24*c^4*x^4+9*c^2*x^2+1)*(c^2*x^2+1)*c^3+4/3*b*Pi^(3/2)/(24*c^4*x^4+9*c
^2*x^2+1)*arcsinh(c*x)*c^3-13/3*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x^2+1)/x*(c^2*
x^2+1)^(1/2)*arcsinh(c*x)*c^2-1/6*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x^2+1)/x^2*(
c^2*x^2+1)*c-1/3*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x^2+1)/x^3*(c^2*x^2+1)^(1/2)*
arcsinh(c*x)+4/3*b*c^3*Pi^(3/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)
```

Fricas [F]

$$\int \frac{(\pi + c^2\pi x^2)^{3/2}\,(a + b\mathrm{arcsinh}(cx))}{x^4}\,dx = \int \frac{(\pi + \pi c^2x^2)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx) + a)}{x^4}\,dx$$

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas
")
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^2 + pi*a + (pi*b*c^2*x^2 + pi*b)
*arcsinh(c*x))/x^4, x)
```

Sympy [F]

$$\int \frac{(\pi + c^2\pi x^2)^{3/2}\,(a + b\mathrm{arcsinh}(cx))}{x^4}\,dx = \pi^{\frac{3}{2}}\left(\int \frac{a\sqrt{c^2x^2+1}}{x^4}\,dx + \int \frac{ac^2\sqrt{c^2x^2+1}}{x^2}\,dx + \int \frac{b\sqrt{c^2x^2+1}\,\mathrm{asinh}\,(cx)}{x^4}\,dx + \int \frac{bc^2\sqrt{c^2x^2+1}\,\mathrm{asinh}\,(cx)}{x^2}\,dx\right)$$

```
[In] integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x))/x**4,x)
[Out] pi**(3/2)*(Integral(a*sqrt(c**2*x**2 + 1)/x**4, x) + Integral(a*c**2*sqrt(c
**2*x**2 + 1)/x**2, x) + Integral(b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x**4, x)
 + Integral(b*c**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/x**2, x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(\pi + c^2\pi x^2)^{3/2}\,(a + b\mathrm{arcsinh}(cx))}{x^4}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima
")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\text{arcsinh}(cx)\right)}{x^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\text{arcsinh}(cx)\right)}{x^4}\,dx = \int \frac{\left(a + b\,\text{asinh}(c\,x)\right)\,\left(\Pi\,c^2\,x^2 + \Pi\right)^{3/2}}{x^4}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x^4,x)
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x^4, x)
```

3.71 $\int x^3(\pi + c^2\pi x^2)^{5/2}(a + b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 141

$$\int x^3\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\frac{2b\pi^{5/2}x}{63c^3}-\frac{b\pi^{5/2}x^3}{189c}-\frac{1}{21}bc\pi^{5/2}x^5-\frac{19}{441}bc^3\pi^{5/2}x^7-\frac{1}{81}bc^5\pi^{5/2}x^9-\frac{\left(\pi+c^2\pi x^2\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4\pi}+\frac{\left(\pi+c^2\pi x^2\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^4\pi^2}$$

```
[Out] 2/63*b*Pi^(5/2)*x/c^3-1/189*b*Pi^(5/2)*x^3/c-1/21*b*c*Pi^(5/2)*x^5-19/441*b
*c^3*Pi^(5/2)*x^7-1/81*b*c^5*Pi^(5/2)*x^9-1/7*(Pi*c^2*x^2+Pi)^(7/2)*(a+b*ar
csinh(c*x))/c^4/Pi+1/9*(Pi*c^2*x^2+Pi)^(9/2)*(a+b*arcsinh(c*x))/c^4/Pi^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {272, 45, 5804, 12, 380}

$$\int x^3\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\frac{\left(\pi c^2x^2+\pi\right)^{9/2}(a+b\text{arcsinh}(cx))}{9\pi^2c^4}-\frac{\left(\pi c^2x^2+\pi\right)^{7/2}(a+b\text{arcsinh}(cx))}{7\pi c^4}-\frac{1}{81}\pi^{5/2}bc^5x^9-\frac{19}{441}\pi^{5/2}bc^3x^7+\frac{2\pi^{5/2}bx}{63c^3}-\frac{1}{21}\pi^{5/2}bcx^5-\frac{\pi^{5/2}bx^3}{189c}$$

```
[In] Int[x^3*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*b*Pi^(5/2)*x)/(63*c^3) - (b*Pi^(5/2)*x^3)/(189*c) - (b*c*Pi^(5/2)*x^5)/2
1 - (19*b*c^3*Pi^(5/2)*x^7)/441 - (b*c^5*Pi^(5/2)*x^9)/81 - ((Pi + c^2*Pi*x
^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^4*Pi) + ((Pi + c^2*Pi*x^2)^(9/2)*(a +
b*ArcSinh[c*x]))/(9*c^4*Pi^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\left(\pi + c^2\pi x^2\right)^{7/2} (a + b\text{arcsinh}(cx))}{7c^4\pi} + \frac{\left(\pi + c^2\pi x^2\right)^{9/2} (a + b\text{arcsinh}(cx))}{9c^4\pi^2} \\ &\quad - \left(bc\sqrt{\pi}\right) \int \frac{\pi^2\left(1 + c^2x^2\right)^3 \left(-2 + 7c^2x^2\right)}{63c^4}\, dx\end{aligned}$$

$$= -\frac{(\pi + c^2\pi x^2)^{7/2} (a + b\text{arcsinh}(cx))}{7c^4\pi} + \frac{(\pi + c^2\pi x^2)^{9/2} (a + b\text{arcsinh}(cx))}{9c^4\pi^2} - \frac{(b\pi^{5/2}) \int (1 + c^2x^2)^3 (-2 + 7c^2x^2) \, dx}{63c^3}$$

$$= -\frac{(\pi + c^2\pi x^2)^{7/2} (a + b\text{arcsinh}(cx))}{7c^4\pi} + \frac{(\pi + c^2\pi x^2)^{9/2} (a + b\text{arcsinh}(cx))}{9c^4\pi^2} - \frac{(b\pi^{5/2}) \int (-2 + c^2x^2 + 15c^4x^4 + 19c^6x^6 + 7c^8x^8) \, dx}{63c^3}$$

$$= \frac{2b\pi^{5/2}x}{63c^3} - \frac{b\pi^{5/2}x^3}{189c} - \frac{1}{21}bc\pi^{5/2}x^5 - \frac{19}{441}bc^3\pi^{5/2}x^7 - \frac{1}{81}bc^5\pi^{5/2}x^9 - \frac{(\pi + c^2\pi x^2)^{7/2} (a + b\text{arcsinh}(cx))}{7c^4\pi} + \frac{(\pi + c^2\pi x^2)^{9/2} (a + b\text{arcsinh}(cx))}{9c^4\pi^2}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.77

$$\int x^3 (\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx)) \, dx = \frac{\pi^{5/2}\Big(63a(1 + c^2x^2)^{7/2} (-2 + 7c^2x^2) - bcx(-126 + 21c^2x^2 + 189c^4x^4 + 171c^6x^6 + 49c^8x^8) + \ldots}{3969c^4}$$

```
[In] Integrate[x^3*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Pi^(5/2)*(63*a*(1 + c^2*x^2)^(7/2)*(-2 + 7*c^2*x^2) - b*c*x*(-126 + 21*c^2
*x^2 + 189*c^4*x^4 + 171*c^6*x^6 + 49*c^8*x^8) + 63*b*(1 + c^2*x^2)^(7/2)*(
-2 + 7*c^2*x^2)*ArcSinh[c*x]))/(3969*c^4)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.60

method	result
default	$a\left(\frac{x^2(\pi c^2x^2+\pi)^{\frac{7}{2}}}{9\pi c^2} - \frac{2(\pi c^2x^2+\pi)^{\frac{7}{2}}}{63\pi c^4}\right) + \frac{b\pi^{\frac{5}{2}}\left(441 \text{ arcsinh}(cx)c^{10}x^{10}+1638 \text{ arcsinh}(cx)c^8x^8-49c^9x^9\sqrt{c^2x^2+1}+2142 \text{ arcsinh} \ldots\right.}{\ldots}$
parts	$a\left(\frac{x^2(\pi c^2x^2+\pi)^{\frac{7}{2}}}{9\pi c^2} - \frac{2(\pi c^2x^2+\pi)^{\frac{7}{2}}}{63\pi c^4}\right) + \frac{b\pi^{\frac{5}{2}}\left(441 \text{ arcsinh}(cx)c^{10}x^{10}+1638 \text{ arcsinh}(cx)c^8x^8-49c^9x^9\sqrt{c^2x^2+1}+2142 \text{ arcsinh} \ldots\right.}{\ldots}$

```
[In] int(x^3*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/9*x^2*(Pi*c^2*x^2+Pi)^(7/2)/Pi/c^2-2/63/Pi/c^4*(Pi*c^2*x^2+Pi)^(7/2))+
1/3969*b/c^4*Pi^(5/2)/(c^2*x^2+1)^(1/2)*(441*arcsinh(c*x)*c^10*x^10+1638*ar
```

```
csinh(c*x)*c^8*x^8-49*c^9*x^9*(c^2*x^2+1)^(1/2)+2142*arcsinh(c*x)*c^6*x^6-1
71*c^7*x^7*(c^2*x^2+1)^(1/2)+1008*arcsinh(c*x)*c^4*x^4-189*c^5*x^5*(c^2*x^2
+1)^(1/2)-63*arcsinh(c*x)*c^2*x^2-21*c^3*x^3*(c^2*x^2+1)^(1/2)-126*arcsinh(
c*x)+126*c*x*(c^2*x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(113) = 226$.

Time = 0.30 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.87

$$\int x^3(\pi + c^2\pi x^2)^{5/2}(a + b\text{arcsinh}(cx))\,dx = \frac{63\sqrt{\pi + \pi c^2x^2}(7\pi^2bc^{10}x^{10} + 26\pi^2bc^8x^8 + 34\pi^2bc^6x^6 + 16\pi^2bc^4x^4 - \pi^2bc^2x^2 - 2\pi^2b)\,\text{lo}}{}$$

```
[In] integrate(x^3*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas
")
```

```
[Out] 1/3969*(63*sqrt(pi + pi*c^2*x^2)*(7*pi^2*b*c^10*x^10 + 26*pi^2*b*c^8*x^8 +
34*pi^2*b*c^6*x^6 + 16*pi^2*b*c^4*x^4 - pi^2*b*c^2*x^2 - 2*pi^2*b)*log(c*x
+ sqrt(c^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(441*pi^2*a*c^10*x^10 + 1638*p
i^2*a*c^8*x^8 + 2142*pi^2*a*c^6*x^6 + 1008*pi^2*a*c^4*x^4 - 63*pi^2*a*c^2*x
^2 - 126*pi^2*a - (49*pi^2*b*c^9*x^9 + 171*pi^2*b*c^7*x^7 + 189*pi^2*b*c^5*
x^5 + 21*pi^2*b*c^3*x^3 - 126*pi^2*b*c*x)*sqrt(c^2*x^2 + 1)))/(c^6*x^2 + c^
4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(133) = 266$.

Time = 90.31 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.69

$$\int x^3(\pi + c^2\pi x^2)^{5/2}(a + b\text{arcsinh}(cx))\,dx = \begin{cases} \frac{\pi^{\frac{5}{2}}ac^4x^8\sqrt{c^2x^2+1}}{9} + \frac{19\pi^{\frac{5}{2}}ac^2x^6\sqrt{c^2x^2+1}}{63} + \frac{5\pi^{\frac{5}{2}}ax^4\sqrt{c^2x^2+1}}{21} + \frac{\pi^{\frac{5}{2}}ax^2\sqrt{c^2x^2+1}}{63c^2} - \frac{2\pi^{\frac{5}{2}}a\sqrt{c^2x^2+1}}{63c^4} - \frac{\pi^{\frac{5}{2}}bc}{81} \\ \frac{\pi^{\frac{5}{2}}ax^4}{4} \end{cases}$$

```
[In] integrate(x**3*(pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((pi**(5/2)*a*c**4*x**8*sqrt(c**2*x**2 + 1)/9 + 19*pi**(5/2)*a*c**
2*x**6*sqrt(c**2*x**2 + 1)/63 + 5*pi**(5/2)*a*x**4*sqrt(c**2*x**2 + 1)/21 +
 pi**(5/2)*a*x**2*sqrt(c**2*x**2 + 1)/(63*c**2) - 2*pi**(5/2)*a*sqrt(c**2*x
**2 + 1)/(63*c**4) - pi**(5/2)*b*c**5*x**9/81 + pi**(5/2)*b*c**4*x**8*sqrt(
```

```
c**2*x**2 + 1)*asinh(c*x)/9 - 19*pi**(5/2)*b*c**3*x**7/441 + 19*pi**(5/2)*b
*c**2*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/63 - pi**(5/2)*b*c*x**5/21 + 5*pi
**(5/2)*b*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/21 - pi**(5/2)*b*x**3/(189*c)
 + pi**(5/2)*b*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(63*c**2) + 2*pi**(5/2)*
b*x/(63*c**3) - 2*pi**(5/2)*b*sqrt(c**2*x**2 + 1)*asinh(c*x)/(63*c**4), Ne(
c, 0)), (pi**(5/2)*a*x**4/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.11

$$\int x^3\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx=\frac{1}{63}\left(\frac{7\left(\pi+\pi c^2x^2\right)^{\frac{7}{2}}x^2}{\pi c^2}-\frac{2\left(\pi+\pi c^2x^2\right)^{\frac{7}{2}}}{\pi c^4}\right)b\operatorname{arsinh}(cx)+\frac{1}{63}\left(\frac{7\left(\pi+\pi c^2x^2\right)^{\frac{7}{2}}x^2}{\pi c^2}-\frac{2\left(\pi+\pi c^2x^2\right)^{\frac{7}{2}}}{\pi c^4}\right)a-\frac{\left(49\,\pi^{\frac{5}{2}}c^8x^9+171\,\pi^{\frac{5}{2}}c^6x^7+189\,\pi^{\frac{5}{2}}c^4x^5+21\,\pi^{\frac{5}{2}}c^2x^3-126\,\pi^{\frac{5}{2}}x\right)b}{3969\,c^3}$$

```
[In] integrate(x^3*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima
")
```

```
[Out] 1/63*(7*(pi + pi*c^2*x^2)^(7/2)*x^2/(pi*c^2) - 2*(pi + pi*c^2*x^2)^(7/2)/(p
i*c^4))*b*arcsinh(c*x) + 1/63*(7*(pi + pi*c^2*x^2)^(7/2)*x^2/(pi*c^2) - 2*(
pi + pi*c^2*x^2)^(7/2)/(pi*c^4))*a - 1/3969*(49*pi^(5/2)*c^8*x^9 + 171*pi^(
5/2)*c^6*x^7 + 189*pi^(5/2)*c^4*x^5 + 21*pi^(5/2)*c^2*x^3 - 126*pi^(5/2)*x)
*b/c^3
```

Giac [F(-2)]

Exception generated.

$$\int x^3\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(\pi + c^2 \pi x^2\right)^{5/2} (a + b \text{arcsinh}(cx))\, dx = \int x^3 \left(a + b \,\text{asinh}(c\, x)\right) \left(\Pi\, c^2\, x^2 + \Pi\right)^{5/2} dx$$

```
[In] int(x^3*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2),x)
[Out] int(x^3*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2), x)
```

3.72 $\int x^2(\pi + c^2\pi x^2)^{5/2}\,(a + b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 213

$$\int x^2\left(\pi + c^2\pi x^2\right)^{5/2}(a + b\text{arcsinh}(cx))\,dx = -\frac{5b\pi^{5/2}x^2}{256c} - \frac{59}{768}bc\pi^{5/2}x^4$$
$$- \frac{17}{288}bc^3\pi^{5/2}x^6 - \frac{1}{64}bc^5\pi^{5/2}x^8 + \frac{5\pi^{5/2}x\sqrt{1+c^2x^2}(a + b\text{arcsinh}(cx))}{128c^2}$$
$$+\frac{5}{64}\pi^2x^3\sqrt{\pi + c^2\pi x^2}(a+b\text{arcsinh}(cx))+\frac{5}{48}\pi x^3\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{1}{8}x^3\left(\pi+c^2\pi x^2\right)^{5/2}(a+\text{bar}$$

```
[Out] -5/256*b*Pi^(5/2)*x^2/c-59/768*b*c*Pi^(5/2)*x^4-17/288*b*c^3*Pi^(5/2)*x^6-1
/64*b*c^5*Pi^(5/2)*x^8+5/48*Pi*x^3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))
+1/8*x^3*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))-5/256*Pi^(5/2)*(a+b*arcsi
nh(c*x))^2/b/c^3+5/128*Pi^(5/2)*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^2+
5/64*Pi^2*x^3*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5808, 5806, 5812, 5783, 30, 14, 272, 45}

$$\int x^2\left(\pi + c^2\pi x^2\right)^{5/2}(a + b\text{arcsinh}(cx))\,dx =$$
$$-\frac{5\pi^{5/2}(a + b\text{arcsinh}(cx))^2}{256bc^3} + \frac{5\pi^{5/2}x\sqrt{c^2x^2+1}(a + b\text{arcsinh}(cx))}{128c^2}$$
$$+\frac{1}{8}x^3\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5}{48}\pi x^3\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{5}{64}\pi^2x^3\sqrt{\pi c^2x^2+\pi}(a+\text{bar}$$

```
[In] Int[x^2*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-5*b*Pi^(5/2)*x^2)/(256*c) - (59*b*c*Pi^(5/2)*x^4)/768 - (17*b*c^3*Pi^(5/2
)*x^6)/288 - (b*c^5*Pi^(5/2)*x^8)/64 + (5*Pi^(5/2)*x*Sqrt[1 + c^2*x^2]*(a +
 b*ArcSinh[c*x]))/(128*c^2) + (5*Pi^2*x^3*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcS
inh[c*x]))/64 + (5*Pi*x^3*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/48
+ (x^3*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/8 - (5*Pi^(5/2)*(a + b
*ArcSinh[c*x])^2)/(256*b*c^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{8}x^3\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\text{arcsinh}(cx)) \\
&\quad+\frac{1}{8}(5\pi)\int x^2\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx-\frac{1}{8}\left(bc\pi^{5/2}\right)\int x^3\left(1+c^2x^2\right)^2\,dx \\
&= \frac{5}{48}\pi x^3\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{1}{8}x^3\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\text{arcsinh}(cx)) \\
&\quad+\frac{1}{16}\left(5\pi^2\right)\int x^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx \\
&\quad-\frac{1}{16}\left(bc\pi^{5/2}\right)\text{Subst}\left(\int x\left(1+c^2x\right)^2\,dx,x,x^2\right)-\frac{1}{48}\left(5bc\pi^{5/2}\right)\int x^3\left(1+c^2x^2\right)\,dx \\
&= \frac{5}{64}\pi^2x^3\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx)) \\
&\quad+\frac{5}{48}\pi x^3\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{1}{8}x^3\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\text{arcsinh}(cx)) \\
&\quad+\frac{1}{64}\left(5\pi^{5/2}\right)\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx-\frac{1}{16}\left(bc\pi^{5/2}\right)\text{Subst}\left(\int\left(x+2c^2x^2+c^4x^3\right)\,dx,x,x^2\right)-\frac{1}{64} \\
&= -\frac{59}{768}bc\pi^{5/2}x^4-\frac{17}{288}bc^3\pi^{5/2}x^6-\frac{1}{64}bc^5\pi^{5/2}x^8+\frac{5\pi^{5/2}x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{128c^2} \\
&\quad+\frac{5}{64}\pi^2x^3\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))+\frac{5}{48}\pi x^3\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{1}{8}x^3\left(\pi+c^2\pi x^2\right)
\end{aligned}$$

$$= -\frac{5b\pi^{5/2}x^2}{256c} - \frac{59}{768}bc\pi^{5/2}x^4 - \frac{17}{288}bc^3\pi^{5/2}x^6 - \frac{1}{64}bc^5\pi^{5/2}x^8 + \frac{5\pi^{5/2}x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{128c^2} + \frac{5}{64}\pi^2x^3\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx)) + \frac{5}{48}\pi x^3\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{1}{8}x^3\left(\pi+c^2\pi x^2\right)^{5/}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.92

$$\int x^2\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \frac{\pi^{5/2}\left(2880acx\sqrt{1+c^2x^2}+22656ac^3x^3\sqrt{1+c^2x^2}+26112ac^5x^5\sqrt{1+c^2x^2}+9216ac^7x^7\sqrt{\right.}{}$$

```
[In] Integrate[x^2*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Pi^(5/2)*(2880*a*c*x*Sqrt[1 + c^2*x^2] + 22656*a*c^3*x^3*Sqrt[1 + c^2*x^2]
 + 26112*a*c^5*x^5*Sqrt[1 + c^2*x^2] + 9216*a*c^7*x^7*Sqrt[1 + c^2*x^2] - 1
440*b*ArcSinh[c*x]^2 + 576*b*Cosh[2*ArcSinh[c*x]] - 144*b*Cosh[4*ArcSinh[c*
x]] - 64*b*Cosh[6*ArcSinh[c*x]] - 9*b*Cosh[8*ArcSinh[c*x]] - 24*ArcSinh[c*x
]*(120*a + 48*b*Sinh[2*ArcSinh[c*x]] - 24*b*Sinh[4*ArcSinh[c*x]] - 16*b*Sin
h[6*ArcSinh[c*x]] - 3*b*Sinh[8*ArcSinh[c*x]])))/(73728*c^3)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.25

method	result
default	$\frac{ax(\pi c^2x^2+\pi)^{\frac{7}{2}}}{8\pi c^2} - \frac{ax(\pi c^2x^2+\pi)^{\frac{5}{2}}}{48c^2} - \frac{5a\pi x(\pi c^2x^2+\pi)^{\frac{3}{2}}}{192c^2} - \frac{5a\pi^2x\sqrt{\pi c^2x^2+\pi}}{128c^2} - \frac{5a\pi^3\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{128c^2\sqrt{\pi c^2}} - \frac{b\pi^{\frac{5}{2}}(-2}{}$
parts	$\frac{ax(\pi c^2x^2+\pi)^{\frac{7}{2}}}{8\pi c^2} - \frac{ax(\pi c^2x^2+\pi)^{\frac{5}{2}}}{48c^2} - \frac{5a\pi x(\pi c^2x^2+\pi)^{\frac{3}{2}}}{192c^2} - \frac{5a\pi^2x\sqrt{\pi c^2x^2+\pi}}{128c^2} - \frac{5a\pi^3\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{128c^2\sqrt{\pi c^2}} - \frac{b\pi^{\frac{5}{2}}(-2}{}$

```
[In] int(x^2*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*a*x*(Pi*c^2*x^2+Pi)^(7/2)/Pi/c^2-1/48*a/c^2*x*(Pi*c^2*x^2+Pi)^(5/2)-5/1
92*a/c^2*Pi*x*(Pi*c^2*x^2+Pi)^(3/2)-5/128*a/c^2*Pi^2*x*(Pi*c^2*x^2+Pi)^(1/2
)-5/128*a/c^2*Pi^3*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^
2)^(1/2)-1/2304*b*Pi^(5/2)*(-288*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^7*c^7+36*
c^8*x^8-816*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^5*c^5+136*c^6*x^6-708*arcsinh(
c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3+177*c^4*x^4-90*arcsinh(c*x)*c*x*(c^2*x^2+1)^
(1/2)+45*c^2*x^2+45*arcsinh(c*x)^2-32)/c^3
```

Fricas [F]

$$\int x^2\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\int\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)x^2\,dx$$

```
[In] integrate(x^2*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^6 + 2*pi^2*a*c^2*x^4 + pi^2*a*
x^2 + (pi^2*b*c^4*x^6 + 2*pi^2*b*c^2*x^4 + pi^2*b*x^2)*arcsinh(c*x)), x)
```

Sympy [A] (verification not implemented)

Time = 49.52 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.64

$$\int x^2\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\begin{cases}\frac{\pi^{\frac{5}{2}}ac^4x^7\sqrt{c^2x^2+1}}{8}+\frac{17\pi^{\frac{5}{2}}ac^2x^5\sqrt{c^2x^2+1}}{48}+\frac{59\pi^{\frac{5}{2}}ax^3\sqrt{c^2x^2+1}}{192}+\frac{5\pi^{\frac{5}{2}}ax\sqrt{c^2x^2+1}}{128c^2}-\frac{5\pi^{\frac{5}{2}}a\,\text{asinh}\,(cx)}{128c^3}-\pi\\ \frac{\pi^{\frac{5}{2}}ax^3}{3}\end{cases}$$

```
[In] integrate(x**2*(pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((pi**(5/2)*a*c**4*x**7*sqrt(c**2*x**2 + 1)/8 + 17*pi**(5/2)*a*c**
2*x**5*sqrt(c**2*x**2 + 1)/48 + 59*pi**(5/2)*a*x**3*sqrt(c**2*x**2 + 1)/192
 + 5*pi**(5/2)*a*x*sqrt(c**2*x**2 + 1)/(128*c**2) - 5*pi**(5/2)*a*asinh(c*x
)/(128*c**3) - pi**(5/2)*b*c**5*x**8/64 + pi**(5/2)*b*c**4*x**7*sqrt(c**2*x
**2 + 1)*asinh(c*x)/8 - 17*pi**(5/2)*b*c**3*x**6/288 + 17*pi**(5/2)*b*c**2*
x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)/48 - 59*pi**(5/2)*b*c*x**4/768 + 59*pi*
*(5/2)*b*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/192 - 5*pi**(5/2)*b*x**2/(256*
c) + 5*pi**(5/2)*b*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(128*c**2) - 5*pi**(5/2
)*b*asinh(c*x)**2/(256*c**3), Ne(c, 0)), (pi**(5/2)*a*x**3/3, True))
```

Maxima [F(-2)]

Exception generated.

$$\int x^2\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int x^2(\pi + c^2\pi x^2)^{5/2}(a + b\text{arcsinh}(cx))\,dx = \int (\pi + \pi c^2x^2)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx) + a)x^2\,dx$$

```
[In] integrate(x^2*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((pi + pi*c^2*x^2)^(5/2)*(b*arcsinh(c*x) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(\pi + c^2\pi x^2)^{5/2}(a + b\text{arcsinh}(cx))\,dx = \int x^2\,(a + b\,\text{asinh}(c\,x))\,\left(\Pi\,c^2\,x^2 + \Pi\right)^{5/2}\,dx$$

```
[In] int(x^2*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2),x)
[Out] int(x^2*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2), x)
```

3.73 $\int x(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx)) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 93

$$\int x\left(\pi + c^2\pi x^2\right)^{5/2} (a + b\text{arcsinh}(cx)) \, dx = -\frac{b\pi^{5/2}x}{7c} - \frac{1}{7}bc\pi^{5/2}x^3 - \frac{3}{35}bc^3\pi^{5/2}x^5 - \frac{1}{49}bc^5\pi^{5/2}x^7 + \frac{\left(\pi + c^2\pi x^2\right)^{7/2} (a + b\text{arcsinh}(cx))}{7c^2\pi}$$

```
[Out] -1/7*b*Pi^(5/2)*x/c-1/7*b*c*Pi^(5/2)*x^3-3/35*b*c^3*Pi^(5/2)*x^5-1/49*b*c^5
*Pi^(5/2)*x^7+1/7*(Pi*c^2*x^2+Pi)^(7/2)*(a+b*arcsinh(c*x))/c^2/Pi
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5798, 200}

$$\int x\left(\pi + c^2\pi x^2\right)^{5/2} (a + b\text{arcsinh}(cx)) \, dx = \frac{\left(\pi c^2x^2 + \pi\right)^{7/2} (a + b\text{arcsinh}(cx))}{7\pi c^2} - \frac{1}{49}\pi^{5/2}bc^5x^7 - \frac{3}{35}\pi^{5/2}bc^3x^5 - \frac{1}{7}\pi^{5/2}bcx^3 - \frac{\pi^{5/2}bx}{7c}$$

```
[In] Int[x*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/7*(b*Pi^(5/2)*x)/c - (b*c*Pi^(5/2)*x^3)/7 - (3*b*c^3*Pi^(5/2)*x^5)/35 -
(b*c^5*Pi^(5/2)*x^7)/49 + ((Pi + c^2*Pi*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7
*c^2*Pi)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(\pi+c^2\pi x^2)^{7/2}(a+b\operatorname{arcsinh}(cx))}{7c^2\pi} - \frac{(b\pi^{5/2})\int (1+c^2x^2)^3\,dx}{7c} \\
&= \frac{(\pi+c^2\pi x^2)^{7/2}(a+b\operatorname{arcsinh}(cx))}{7c^2\pi} - \frac{(b\pi^{5/2})\int (1+3c^2x^2+3c^4x^4+c^6x^6)\,dx}{7c} \\
&= -\frac{b\pi^{5/2}x}{7c} - \frac{1}{7}bc\pi^{5/2}x^3 - \frac{3}{35}bc^3\pi^{5/2}x^5 - \frac{1}{49}bc^5\pi^{5/2}x^7 + \frac{(\pi+c^2\pi x^2)^{7/2}(a+b\operatorname{arcsinh}(cx))}{7c^2\pi}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int x(\pi+c^2\pi x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx = \frac{\pi^{5/2}\Big(35a(1+c^2x^2)^{7/2} - bcx(35+35c^2x^2+21c^4x^4+5c^6x^6) + 35b(1+c^2x^2)^{7/2}\operatorname{arcsinh}(cx)\Big)}{245c^2}$$

```
[In] Integrate[x*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
[Out] (Pi^(5/2)*(35*a*(1 + c^2*x^2)^(7/2) - b*c*x*(35 + 35*c^2*x^2 + 21*c^4*x^4 +
 5*c^6*x^6) + 35*b*(1 + c^2*x^2)^(7/2)*ArcSinh[c*x]))/(245*c^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(73) = 146.

Time = 0.20 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.83

method	result
default	$\frac{a(\pi c^2x^2+\pi)^{\frac{7}{2}}}{7\pi c^2}+\frac{b\pi^{\frac{5}{2}}\left(35\operatorname{arcsinh}(cx)c^8x^8+140\operatorname{arcsinh}(cx)c^6x^6-5c^7x^7\sqrt{c^2x^2+1}+210\operatorname{arcsinh}(cx)c^4x^4-21c^5x^5\sqrt{c^2x^2+1}+140\right.}{245c^2\sqrt{c^2x^2+1}}$
parts	$\frac{a(\pi c^2x^2+\pi)^{\frac{7}{2}}}{7\pi c^2}+\frac{b\pi^{\frac{5}{2}}\left(35\operatorname{arcsinh}(cx)c^8x^8+140\operatorname{arcsinh}(cx)c^6x^6-5c^7x^7\sqrt{c^2x^2+1}+210\operatorname{arcsinh}(cx)c^4x^4-21c^5x^5\sqrt{c^2x^2+1}+140\right.}{245c^2\sqrt{c^2x^2+1}}$

```
[In] int(x*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*a*(Pi*c^2*x^2+Pi)^(7/2)/Pi/c^2+1/245*b/c^2*Pi^(5/2)/(c^2*x^2+1)^(1/2)*(
35*arcsinh(c*x)*c^8*x^8+140*arcsinh(c*x)*c^6*x^6-5*c^7*x^7*(c^2*x^2+1)^(1/2
)+210*arcsinh(c*x)*c^4*x^4-21*c^5*x^5*(c^2*x^2+1)^(1/2)+140*arcsinh(c*x)*c^
2*x^2-35*c^3*x^3*(c^2*x^2+1)^(1/2)+35*arcsinh(c*x)-35*c*x*(c^2*x^2+1)^(1/2)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(73) = 146.

Time = 0.26 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.42

$$\int x\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx=\frac{35\sqrt{\pi+\pi c^2x^2}(\pi^2bc^8x^8+4\pi^2bc^6x^6+6\pi^2bc^4x^4+4\pi^2bc^2x^2+\pi^2b)\log\left(cx+\sqrt{c^2x^2+1}\right)}{}$$

```
[In] integrate(x*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/245*(35*sqrt(pi + pi*c^2*x^2)*(pi^2*b*c^8*x^8 + 4*pi^2*b*c^6*x^6 + 6*pi^2
*b*c^4*x^4 + 4*pi^2*b*c^2*x^2 + pi^2*b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt
(pi + pi*c^2*x^2)*(35*pi^2*a*c^8*x^8 + 140*pi^2*a*c^6*x^6 + 210*pi^2*a*c^4*
x^4 + 140*pi^2*a*c^2*x^2 + 35*pi^2*a - (5*pi^2*b*c^7*x^7 + 21*pi^2*b*c^5*x^
5 + 35*pi^2*b*c^3*x^3 + 35*pi^2*b*c*x)*sqrt(c^2*x^2 + 1)))/(c^4*x^2 + c^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(85) = 170.

Time = 28.36 (sec) , antiderivative size = 299, normalized size of antiderivative = 3.22

$$\int x\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx=\begin{cases}\frac{\pi^{\frac{5}{2}}ac^4x^6\sqrt{c^2x^2+1}}{7}+\frac{3\pi^{\frac{5}{2}}ac^2x^4\sqrt{c^2x^2+1}}{7}+\frac{3\pi^{\frac{5}{2}}ax^2\sqrt{c^2x^2+1}}{7}+\frac{\pi^{\frac{5}{2}}a\sqrt{c^2x^2+1}}{7c^2}-\frac{\pi^{\frac{5}{2}}bc^5x^7}{49}+\frac{\pi^{\frac{5}{2}}bc^4x^6\sqrt{c^2}}{} \\ \frac{\pi^{\frac{5}{2}}ax^2}{2}\end{cases}$$

```
[In] integrate(x*(pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((pi**(5/2)*a*c**4*x**6*sqrt(c**2*x**2 + 1)/7 + 3*pi**(5/2)*a*c**2
*x**4*sqrt(c**2*x**2 + 1)/7 + 3*pi**(5/2)*a*x**2*sqrt(c**2*x**2 + 1)/7 + pi
**(5/2)*a*sqrt(c**2*x**2 + 1)/(7*c**2) - pi**(5/2)*b*c**5*x**7/49 + pi**(5/
2)*b*c**4*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/7 - 3*pi**(5/2)*b*c**3*x**5/3
5 + 3*pi**(5/2)*b*c**2*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/7 - pi**(5/2)*b*
c*x**3/7 + 3*pi**(5/2)*b*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/7 - pi**(5/2)*
b*x/(7*c) + pi**(5/2)*b*sqrt(c**2*x**2 + 1)*asinh(c*x)/(7*c**2), Ne(c, 0)),
 (pi**(5/2)*a*x**2/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.03

$$\int x\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\frac{\left(\pi+\pi c^2x^2\right)^{\frac{7}{2}}b\,\text{arsinh}\,(cx)}{7\,\pi c^2}+\frac{\left(\pi+\pi c^2x^2\right)^{\frac{7}{2}}a}{7\,\pi c^2}-\frac{\left(5\,\pi^{\frac{7}{2}}c^6x^7+21\,\pi^{\frac{7}{2}}c^4x^5+35\,\pi^{\frac{7}{2}}c^2x^3+35\,\pi^{\frac{7}{2}}x\right)b}{245\,\pi c}$$

```
[In] integrate(x*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*(pi + pi*c^2*x^2)^(7/2)*b*arcsinh(c*x)/(pi*c^2) + 1/7*(pi + pi*c^2*x^2)
^(7/2)*a/(pi*c^2) - 1/245*(5*pi^(7/2)*c^6*x^7 + 21*pi^(7/2)*c^4*x^5 + 35*pi
^(7/2)*c^2*x^3 + 35*pi^(7/2)*x)*b/(pi*c)
```

Giac [F(-2)]

Exception generated.

$$\int x\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(\pi + c^2\pi x^2)^{5/2}(a + b\operatorname{arcsinh}(cx))\,dx = \int x\,(a + b\,\operatorname{asinh}(c\,x))\,\left(\Pi\,c^2\,x^2 + \Pi\right)^{5/2}\,dx$$

```
[In] int(x*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2),x)
[Out] int(x*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2), x)
```

3.74 $\int \left(\pi + c^2\pi x^2\right)^{5/2} (a + b\text{arcsinh}(cx))\, dx$

Optimal result

Integrand size = 23, antiderivative size = 165

$$\int \left(\pi + c^2\pi x^2\right)^{5/2} (a + b\text{arcsinh}(cx))\, dx = -\frac{25}{96} bc\pi^{5/2}x^2 - \frac{5}{96} bc^3\pi^{5/2}x^4 - \frac{b\pi^{5/2}(1+c^2x^2)^3}{36c}$$
$$+ \frac{5}{16}\pi^2 x\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx)) + \frac{5}{24}\pi x\left(\pi + c^2\pi x^2\right)^{3/2}(a + b\text{arcsinh}(cx)) + \frac{1}{6}x\left(\pi + c^2\pi x^2\right)^{5/2}(a + b\text{arcsinh}$$

```
[Out] -25/96*b*c*Pi^(5/2)*x^2-5/96*b*c^3*Pi^(5/2)*x^4-1/36*b*Pi^(5/2)*(c^2*x^2+1)
^3/c+5/24*Pi*x*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))+1/6*x*(Pi*c^2*x^2+P
i)^(5/2)*(a+b*arcsinh(c*x))+5/32*Pi^(5/2)*(a+b*arcsinh(c*x))^2/b/c+5/16*Pi^
2*x*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5786, 5785, 5783, 30, 14, 267}

$$\int \left(\pi + c^2\pi x^2\right)^{5/2} (a + b\text{arcsinh}(cx))\, dx = \frac{1}{6}x\left(\pi c^2x^2 + \pi\right)^{5/2}(a + b\text{arcsinh}(cx))$$
$$+ \frac{5}{24}\pi x\left(\pi c^2x^2 + \pi\right)^{3/2}(a + b\text{arcsinh}(cx)) + \frac{5}{16}\pi^2 x\sqrt{\pi c^2x^2 + \pi}(a + b\text{arcsinh}(cx)) + \frac{5\pi^{5/2}(a + b\text{arcsinh}(cx))^2}{32bc} - \frac{5}{96}$$

```
[In] Int[(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-25*b*c*Pi^(5/2)*x^2)/96 - (5*b*c^3*Pi^(5/2)*x^4)/96 - (b*Pi^(5/2)*(1 + c^
2*x^2)^3)/(36*c) + (5*Pi^2*x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/16
 + (5*Pi*x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/24 + (x*(Pi + c^2*
```

```
Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/6 + (5*Pi^(5/2)*(a + b*ArcSinh[c*x])^2)
/(32*b*c)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{6}x\left(\pi + c^2\pi x^2\right)^{5/2}(a + b\text{arcsinh}(cx)) + \frac{1}{6}(5\pi)\int\left(\pi + c^2\pi x^2\right)^{3/2}(a + b\text{arcsinh}(cx))\,dx - \frac{1}{6}\left(bc\pi^{5/2}\right)\int x\left(1 + c^2x^2\right)^2\,dx$$

$$= -\frac{b\pi^{5/2}\left(1 + c^2x^2\right)^3}{36c} + \frac{5}{24}\pi x\left(\pi + c^2\pi x^2\right)^{3/2}(a + b\text{arcsinh}(cx)) + \frac{1}{6}x\left(\pi + c^2\pi x^2\right)^{5/2}(a + b\text{arcsinh}(cx)) + \frac{1}{8}\left(5\pi^2\right)\int\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx - \frac{1}{24}\left(5bc\pi^{5/2}\right)\int$$

$$= -\frac{b\pi^{5/2}\left(1 + c^2x^2\right)^3}{36c} + \frac{5}{16}\pi^2x\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx)) + \frac{5}{24}\pi x\left(\pi + c^2\pi x^2\right)^{3/2}(a + b\text{arcsinh}(cx)) + \frac{1}{6}x\left(\pi + c^2\pi x^2\right)^{5/2}(a + b\text{arcsinh}(cx)) + \frac{1}{16}\left(5\pi^{5/2}\right)\int\frac{a + b\text{arcsinh}(cx)}{\sqrt{1 + c^2x^2}}\,dx - \frac{1}{24}\left(5bc\pi^{5/2}\right)\int\left(x + c^2x\right.$$

$$= -\frac{25}{96}bc\pi^{5/2}x^2 - \frac{5}{96}bc^3\pi^{5/2}x^4 - \frac{b\pi^{5/2}\left(1 + c^2x^2\right)^3}{36c} + \frac{5}{16}\pi^2x\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx)) + \frac{5}{24}\pi x\left(\pi + c^2\pi x^2\right)^{3/2}(a + b\text{arcsinh}(cx)) + \frac{1}{6}x\left(\pi + c^2\pi x^2\right)^{5/2}($$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.93

$$\int\left(\pi + c^2\pi x^2\right)^{5/2}(a + b\text{arcsinh}(cx))\,dx = \frac{\pi^{5/2}\left(1584acx\sqrt{1 + c^2x^2} + 1248ac^3x^3\sqrt{1 + c^2x^2} + 384ac^5x^5\sqrt{1 + c^2x^2} + 360b\text{arcsinh}(cx)\right.}{}$$

```
[In] Integrate[(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Pi^(5/2)*(1584*a*c*x*Sqrt[1 + c^2*x^2] + 1248*a*c^3*x^3*Sqrt[1 + c^2*x^2]
+ 384*a*c^5*x^5*Sqrt[1 + c^2*x^2] + 360*b*ArcSinh[c*x]^2 - 270*b*Cosh[2*Arc
Sinh[c*x]] - 27*b*Cosh[4*ArcSinh[c*x]] - 2*b*Cosh[6*ArcSinh[c*x]] + 12*ArcS
inh[c*x]*(60*a + 45*b*Sinh[2*ArcSinh[c*x]] + 9*b*Sinh[4*ArcSinh[c*x]] + b*S
inh[6*ArcSinh[c*x]])))/(2304*c)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.22

method	result
default	$\frac{x(\pi c^2x^2+\pi)^{\frac{5}{2}}a}{6}+\frac{5a\pi x(\pi c^2x^2+\pi)^{\frac{3}{2}}}{24}+\frac{5a\pi^2x\sqrt{\pi c^2x^2+\pi}}{16}+\frac{5a\pi^3\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{16\sqrt{\pi c^2}}+\frac{b\pi^{\frac{5}{2}}\left(48\operatorname{arcsinh}(cx)\sqrt{c^2x^2+}\right.}{}$
parts	$\frac{x(\pi c^2x^2+\pi)^{\frac{5}{2}}a}{6}+\frac{5a\pi x(\pi c^2x^2+\pi)^{\frac{3}{2}}}{24}+\frac{5a\pi^2x\sqrt{\pi c^2x^2+\pi}}{16}+\frac{5a\pi^3\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{16\sqrt{\pi c^2}}+\frac{b\pi^{\frac{5}{2}}\left(48\operatorname{arcsinh}(cx)\sqrt{c^2x^2+}\right.}{}$

```
[In] int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
[Out] 1/6*x*(Pi*c^2*x^2+Pi)^(5/2)*a+5/24*a*Pi*x*(Pi*c^2*x^2+Pi)^(3/2)+5/16*a*Pi^2
*x*(Pi*c^2*x^2+Pi)^(1/2)+5/16*a*Pi^3*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2
+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/288*b*Pi^(5/2)*(48*arcsinh(c*x)*(c^2*x^2+1)^(1
/2)*x^5*c^5-8*c^6*x^6+156*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3-39*c^4*x^4
+198*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)-99*c^2*x^2+45*arcsinh(c*x)^2-68)/c
```

Fricas [F]

$$\int\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx=\int\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)\,dx$$

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^4 + 2*pi^2*a*c^2*x^2 + pi^2*a
+ (pi^2*b*c^4*x^4 + 2*pi^2*b*c^2*x^2 + pi^2*b)*arcsinh(c*x)), x)
```

Sympy [A] (verification not implemented)

Time = 16.11 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.61

$$\int\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx=\begin{cases}\frac{\pi^{\frac{5}{2}}ac^4x^5\sqrt{c^2x^2+1}}{6}+\frac{13\pi^{\frac{5}{2}}ac^2x^3\sqrt{c^2x^2+1}}{24}+\frac{11\pi^{\frac{5}{2}}ax\sqrt{c^2x^2+1}}{16}+\frac{5\pi^{\frac{5}{2}}a\operatorname{asinh}(cx)}{16c}-\frac{\pi^{\frac{5}{2}}bc^5x^6}{36}+\frac{\pi^{\frac{5}{2}}bc^4x^5\sqrt{}}{} \\ \pi^{\frac{5}{2}}ax\end{cases}$$

```
[In] integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x)),x)
[Out] Piecewise((pi**(5/2)*a*c**4*x**5*sqrt(c**2*x**2 + 1)/6 + 13*pi**(5/2)*a*c**
2*x**3*sqrt(c**2*x**2 + 1)/24 + 11*pi**(5/2)*a*x*sqrt(c**2*x**2 + 1)/16 + 5
*pi**(5/2)*a*asinh(c*x)/(16*c) - pi**(5/2)*b*c**5*x**6/36 + pi**(5/2)*b*c**
4*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)/6 - 13*pi**(5/2)*b*c**3*x**4/96 + 13*
```

```
pi**(5/2)*b*c**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/24 - 11*pi**(5/2)*b*c*
x**2/32 + 11*pi**(5/2)*b*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/16 + 5*pi**(5/2)*
b*asinh(c*x)**2/(32*c), Ne(c, 0)), (pi**(5/2)*a*x, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(\pi + c^2\pi x^2\right)^{5/2} (a + b\text{arcsinh}(cx))\, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \left(\pi + c^2\pi x^2\right)^{5/2} (a + b\text{arcsinh}(cx))\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(\pi + c^2\pi x^2\right)^{5/2} (a + b\text{arcsinh}(cx))\, dx = \int (a + b\,\text{asinh}(c\,x))\, \left(\Pi\,c^2\,x^2 + \Pi\right)^{5/2} dx$$

```
[In] int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2), x)
```

3.75 $\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\mathbf{arcsinh}(cx))}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 179

$$\int \frac{(\pi + c^2\pi x^2)^{5/2}(a + b\text{arcsinh}(cx))}{x}\,dx = -\frac{23}{15}bc\pi^{5/2}x - \frac{11}{45}bc^3\pi^{5/2}x^3 - \frac{1}{25}bc^5\pi^{5/2}x^5 + \pi^2\sqrt{\pi + c^2\pi x^2}(a+b\text{arcsinh}(cx)) + \frac{1}{3}\pi(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{1}{5}(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))$$

```
[Out] -23/15*b*c*Pi^(5/2)*x-11/45*b*c^3*Pi^(5/2)*x^3-1/25*b*c^5*Pi^(5/2)*x^5+1/3*
Pi*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))+1/5*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*
arcsinh(c*x))-2*Pi^(5/2)*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))-
b*Pi^(5/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+b*Pi^(5/2)*polylog(2,c*x+(c^2*
x^2+1)^(1/2))+Pi^2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5808, 5806, 5816, 4267, 2317, 2438, 8, 200}

$$\int \frac{(\pi + c^2\pi x^2)^{5/2}(a + b\text{arcsinh}(cx))}{x}\,dx = -2\pi^{5/2}\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a + b\text{arcsinh}(cx)) + \frac{1}{5}(\pi c^2x^2+\pi)^{5/2}(a+b\text{arcsinh}(cx)) + \frac{1}{3}\pi(\pi c^2x^2+\pi)^{3/2}(a+b\text{arcsinh}(cx)) + \pi^2\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))$$

```
[In] Int[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (-23*b*c*Pi^(5/2)*x)/15 - (11*b*c^3*Pi^(5/2)*x^3)/45 - (b*c^5*Pi^(5/2)*x^5)
/25 + Pi^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]) + (Pi*(Pi + c^2*Pi*x^
2)^(3/2)*(a + b*ArcSinh[c*x]))/3 + ((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[
```

```
c*x]))/5 - 2*Pi^(5/2)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - b*Pi^(
5/2)*PolyLog[2, -E^ArcSinh[c*x]] + b*Pi^(5/2)*PolyLog[2, E^ArcSinh[c*x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
```

```
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{1}{5}\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\text{arcsinh}(cx))+\pi\int\frac{\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}\,dx-\frac{1}{5}\left(bc\pi^{5/2}\right)\int\left(1+c^2x^2\right)^2\,dx$$

$$=\frac{1}{3}\pi\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{1}{5}\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\text{arcsinh}(cx))+\pi^2\int\frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x}\,dx-\frac{1}{5}\left(bc\pi^{5/2}\right)\int\left(1+2c^2x^2+c^4x^4\right)\,dx-\frac{1}{3}\left(bc\pi^{5/2}\right)\int\left(1+c^2x^2\right)\,dx$$

$$=-\frac{8}{15}bc\pi^{5/2}x-\frac{11}{45}bc^3\pi^{5/2}x^3-\frac{1}{25}bc^5\pi^{5/2}x^5+\pi^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))+\frac{1}{3}\pi\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{1}{5}\left(\pi+c^2\pi x^2\right)^{5/2}(a+\text{bar}$$

$$=-\frac{23}{15}bc\pi^{5/2}x-\frac{11}{45}bc^3\pi^{5/2}x^3-\frac{1}{25}bc^5\pi^{5/2}x^5+\pi^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))+\frac{1}{3}\pi\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{1}{5}\left(\pi+c^2\pi x^2\right)^{5/2}(a+\text{bar}$$

$$=-\frac{23}{15}bc\pi^{5/2}x-\frac{11}{45}bc^3\pi^{5/2}x^3-\frac{1}{25}bc^5\pi^{5/2}x^5+\pi^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))+\frac{1}{3}\pi\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{1}{5}\left(\pi+c^2\pi x^2\right)^{5/2}(a+\text{bar}$$

$$=-\frac{23}{15}bc\pi^{5/2}x-\frac{11}{45}bc^3\pi^{5/2}x^3-\frac{1}{25}bc^5\pi^{5/2}x^5+\pi^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))+\frac{1}{3}\pi\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{1}{5}\left(\pi+c^2\pi x^2\right)^{5/2}(a+\text{bar}$$

$$=-\frac{23}{15}bc\pi^{5/2}x-\frac{11}{45}bc^3\pi^{5/2}x^3-\frac{1}{25}bc^5\pi^{5/2}x^5+\pi^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))+\frac{1}{3}\pi\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{1}{5}\left(\pi+c^2\pi x^2\right)^{5/2}(a+\text{bar}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.44

$$\int \frac{(\pi + c^2\pi x^2)^{5/2}(a + b\text{arcsinh}(cx))}{x}\,dx = \frac{1}{225}\pi^{5/2}\Big(-345bcx - 55bc^3x^3 - 9bc^5x^5 + 345a\sqrt{1+c^2x^2} + 165ac^2x^2\sqrt{}$$

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (Pi^(5/2)*(-345*b*c*x - 55*b*c^3*x^3 - 9*b*c^5*x^5 + 345*a*Sqrt[1 + c^2*x^2
] + 165*a*c^2*x^2*Sqrt[1 + c^2*x^2] + 45*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 345*
b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 165*b*c^2*x^2*Sqrt[1 + c^2*x^2]*ArcSinh[
c*x] + 45*b*c^4*x^4*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 225*b*ArcSinh[c*x]*Log
[1 - E^(-ArcSinh[c*x])] - 225*b*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 2
25*a*Log[x] - 225*a*Log[Pi*(1 + Sqrt[1 + c^2*x^2])] + 225*b*PolyLog[2, -E^(
-ArcSinh[c*x])] - 225*b*PolyLog[2, E^(-ArcSinh[c*x])]))/225
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.59

method	result
default	$a\left(\frac{(\pi c^2x^2+\pi)^{\frac{5}{2}}}{5} + \pi\left(\frac{(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3} + \pi\left(\sqrt{\pi c^2x^2+\pi} - \sqrt{\pi}\,\text{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)\right)\right)\right) - b\,\pi^{\frac{5}{2}}\,\text{arcsinh}\,($
parts	$a\left(\frac{(\pi c^2x^2+\pi)^{\frac{5}{2}}}{5} + \pi\left(\frac{(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3} + \pi\left(\sqrt{\pi c^2x^2+\pi} - \sqrt{\pi}\,\text{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)\right)\right)\right) - b\,\pi^{\frac{5}{2}}\,\text{arcsinh}\,($

```
[In] int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/5*(Pi*c^2*x^2+Pi)^(5/2)+Pi*(1/3*(Pi*c^2*x^2+Pi)^(3/2)+Pi*((Pi*c^2*x^2+
Pi)^(1/2)-Pi^(1/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2)))))-b*Pi^(5/2)*ar
csinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+b*Pi^(5/2)*arcsinh(c*x)*ln(1-c*x-(c^
2*x^2+1)^(1/2))-1/25*b*c^5*Pi^(5/2)*x^5-11/45*b*c^3*Pi^(5/2)*x^3+23/15*b*(c
^2*x^2+1)^(1/2)*arcsinh(c*x)*Pi^(5/2)-23/15*b*c*Pi^(5/2)*x+b*Pi^(5/2)*polyl
og(2,c*x+(c^2*x^2+1)^(1/2))-b*Pi^(5/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+1/
5*b*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*Pi^(5/2)*x^4*c^4+11/15*b*(c^2*x^2+1)^(1/
2)*arcsinh(c*x)*Pi^(5/2)*x^2*c^2
```

Fricas [F]

$$\int \frac{(\pi + c^2\pi x^2)^{5/2}(a + b\text{arcsinh}(cx))}{x}\,dx = \int \frac{(\pi + \pi c^2x^2)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx) + a)}{x}\,dx$$

[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")

[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^4 + 2*pi^2*a*c^2*x^2 + pi^2*a + (pi^2*b*c^4*x^4 + 2*pi^2*b*c^2*x^2 + pi^2*b)*arcsinh(c*x))/x, x)

Sympy [F]

$$\int \frac{(\pi + c^2\pi x^2)^{5/2}(a + b\text{arcsinh}(cx))}{x}\,dx = \pi^{\frac{5}{2}}\left(\int \frac{a\sqrt{c^2x^2+1}}{x}\,dx + \int 2ac^2x\sqrt{c^2x^2+1}\,dx + \int ac^4x^3\sqrt{c^2x^2+1}\,dx + \int \frac{b\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)}{x}\,dx + \int 2bc^2x\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)\,dx + \int bc^4x^3\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)\,dx\right)$$

[In] integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x))/x,x)

[Out] pi**(5/2)*(Integral(a*sqrt(c**2*x**2 + 1)/x, x) + Integral(2*a*c**2*x*sqrt(c**2*x**2 + 1), x) + Integral(a*c**4*x**3*sqrt(c**2*x**2 + 1), x) + Integral(b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x, x) + Integral(2*b*c**2*x*sqrt(c**2*x**2 + 1)*asinh(c*x), x) + Integral(b*c**4*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x), x))

Maxima [F]

$$\int \frac{(\pi + c^2\pi x^2)^{5/2}(a + b\text{arcsinh}(cx))}{x}\,dx = \int \frac{(\pi + \pi c^2x^2)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx) + a)}{x}\,dx$$

[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")

[Out] -1/15*(15*pi^(5/2)*arcsinh(1/(c*abs(x))) - 15*pi^2*sqrt(pi + pi*c^2*x^2) - 5*pi*(pi + pi*c^2*x^2)^(3/2) - 3*(pi + pi*c^2*x^2)^(5/2))*a + b*integrate((pi + pi*c^2*x^2)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1))/x, x)

Giac [F(-2)]

Exception generated.

$$\int \frac{(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))}{x} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))}{x} dx = \int \frac{(a + b\,\text{asinh}(c\,x))\ (\Pi\,c^2\,x^2 + \Pi)^{5/2}}{x} dx$$

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x, x)
```

3.76 $\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 157

$$\int \frac{(\pi + c^2\pi x^2)^{5/2}(a + b\text{arcsinh}(cx))}{x^2}\,dx = -\frac{9}{16}bc^3\pi^{5/2}x^2 - \frac{1}{16}bc^5\pi^{5/2}x^4 + \frac{15}{8}c^2\pi^2x\sqrt{\pi + c^2\pi x^2}(a+b\text{arcsinh}(cx)) + \frac{5}{4}c^2\pi x(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{(\pi + c^2\pi x^2)^{5/2}(a + b\text{arcs}$$

```
[Out] -9/16*b*c^3*Pi^(5/2)*x^2-1/16*b*c^5*Pi^(5/2)*x^4+5/4*c^2*Pi*x*(Pi*c^2*x^2+P
i)^(3/2)*(a+b*arcsinh(c*x))-(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/x+15/1
6*c*Pi^(5/2)*(a+b*arcsinh(c*x))^2/b+b*c*Pi^(5/2)*ln(x)+15/8*c^2*Pi^2*x*(a+b
*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5807, 5786, 5785, 5783, 30, 14, 272, 45}

$$\int \frac{(\pi + c^2\pi x^2)^{5/2}(a + b\text{arcsinh}(cx))}{x^2}\,dx = \frac{5}{4}\pi c^2x(\pi c^2x^2 + \pi)^{3/2}(a + b\text{arcsinh}(cx)) + \frac{15}{8}\pi^2c^2x\sqrt{\pi c^2x^2 + \pi}(a + b\text{arcsinh}(cx)) - \frac{(\pi c^2x^2 + \pi)^{5/2}(a + b\text{arcsinh}(cx))}{x} + \frac{15\pi^{5/2}c(a + b\text{arcsinh}(cx))^2}{16b} - \frac{1}{16}\pi^{5/2}bc^5x^4 - \frac{9}{16}\pi^{5/2}bc^3x^2 + \pi^{5/2}bc\log(x)$$

```
[In] Int[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (-9*b*c^3*Pi^(5/2)*x^2)/16 - (b*c^5*Pi^(5/2)*x^4)/16 + (15*c^2*Pi^2*x*Sqrt[
Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/8 + (5*c^2*Pi*x*(Pi + c^2*Pi*x^2)^(3
```

```
/2)*(a + b*ArcSinh[c*x]))/4 - ((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x])
)/x + (15*c*Pi^(5/2)*(a + b*ArcSinh[c*x])^2)/(16*b) + b*c*Pi^(5/2)*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
```

```
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x} + (5c^2\pi)\int(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))\,dx + (bc\pi^{5/2})\int\frac{(1+c^2x^2)^2}{x}\,dx$$

$$= \frac{5}{4}c^2\pi x(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x} + \frac{1}{4}(15c^2\pi^2)\int\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx + \frac{1}{2}(bc\pi^{5/2})\,\text{Subst}\left(\int\frac{(1+c^2x)^2}{x}\,dx,x,x^2\right) - \frac{1}{4}(5bc^3\pi^{5/2})\int x(1+c^2x^2)\,dx$$

$$= \frac{15}{8}c^2\pi^2x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx)) + \frac{5}{4}c^2\pi x(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x} + \frac{1}{2}(bc\pi^{5/2})\,\text{Subst}\left(\int\left(2c^2+\frac{1}{x}+c^4x\right)dx,x,x^2\right) + \frac{1}{8}(15c^2\pi^{5/2})\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx - \frac{1}{4}(5bc^3$$

$$= -\frac{9}{16}bc^3\pi^{5/2}x^2 - \frac{1}{16}bc^5\pi^{5/2}x^4 + \frac{15}{8}c^2\pi^2x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx)) + \frac{5}{4}c^2\pi x(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x} + \frac{15c\pi^{5/2}(a+b\text{arcsi}}{16b}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.07

$$\int \frac{(\pi + c^2\pi x^2)^{5/2}(a + b\text{arcsinh}(cx))}{x^2}\,dx = \frac{\pi^{5/2}\left(-128a\sqrt{1+c^2x^2} + 144ac^2x^2\sqrt{1+c^2x^2} + 32ac^4x^4\sqrt{1+c^2x^2}\right.}{}$$

[In] Integrate[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^2,x]

[Out] (Pi^(5/2)*(-128*a*Sqrt[1 + c^2*x^2] + 144*a*c^2*x^2*Sqrt[1 + c^2*x^2] + 32*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 120*b*c*x*ArcSinh[c*x]^2 - 32*b*c*x*Cosh[2*ArcSinh[c*x]] - b*c*x*Cosh[4*ArcSinh[c*x]] + 128*b*c*x*Log[c*x] + 4*ArcSinh[c*x]*(60*a*c*x - 32*b*Sqrt[1 + c^2*x^2] + 16*b*c*x*Sinh[2*ArcSinh[c*x]] + b*c*x*Sinh[4*ArcSinh[c*x]])))/(128*x)

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.66

method	result
default	$-\frac{a(\pi c^2x^2+\pi)^{\frac{7}{2}}}{\pi x} + a c^2 x(\pi c^2x^2+\pi)^{\frac{5}{2}} + \frac{5a c^2\pi x(\pi c^2x^2+\pi)^{\frac{3}{2}}}{4} + \frac{15a c^2\pi^2 x\sqrt{\pi c^2x^2+\pi}}{8} + \frac{15a c^2\pi^3\ln\left(\frac{\pi c^2 x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2}\right.}{8\sqrt{\pi c^2}}$
parts	$-\frac{a(\pi c^2x^2+\pi)^{\frac{7}{2}}}{\pi x} + a c^2 x(\pi c^2x^2+\pi)^{\frac{5}{2}} + \frac{5a c^2\pi x(\pi c^2x^2+\pi)^{\frac{3}{2}}}{4} + \frac{15a c^2\pi^2 x\sqrt{\pi c^2x^2+\pi}}{8} + \frac{15a c^2\pi^3\ln\left(\frac{\pi c^2 x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2}\right.}{8\sqrt{\pi c^2}}$

[In] int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/x^2,x,method=_RETURNVERBOSE)

[Out] -a/Pi/x*(Pi*c^2*x^2+Pi)^(7/2)+a*c^2*x*(Pi*c^2*x^2+Pi)^(5/2)+5/4*a*c^2*Pi*x*(Pi*c^2*x^2+Pi)^(3/2)+15/8*a*c^2*Pi^2*x*(Pi*c^2*x^2+Pi)^(1/2)+15/8*a*c^2*Pi^3*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/128*b*Pi^(5/2)*(32*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4*c^4-8*c^5*x^5+144*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2-72*c^3*x^3+120*arcsinh(c*x)^2*x*c-128*arcsinh(c*x)*c*x+128*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*x*c-128*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-33*c*x)/x

Fricas [F]

$$\int \frac{(\pi + c^2\pi x^2)^{5/2}\,(a + b\text{arcsinh}(cx))}{x^2}\,dx = \int \frac{(\pi + \pi c^2x^2)^{\frac{5}{2}}\,(b\,\text{arsinh}\,(cx) + a)}{x^2}\,dx$$

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^4 + 2*pi^2*a*c^2*x^2 + pi^2*a
+ (pi^2*b*c^4*x^4 + 2*pi^2*b*c^2*x^2 + pi^2*b)*arcsinh(c*x))/x^2, x)
```

Sympy [F]

$$\int \frac{(\pi + c^2\pi x^2)^{5/2}\,(a + b\text{arcsinh}(cx))}{x^2}\,dx = \pi^{\frac{5}{2}}\left(\int 2ac^2\sqrt{c^2x^2+1}\,dx + \int \frac{a\sqrt{c^2x^2+1}}{x^2}\,dx + \int ac^4x^2\sqrt{c^2x^2+1}\,dx + \int 2bc^2\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)\,dx + \int \frac{b\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)}{x^2}\,dx + \int bc^4x^2\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)\,dx\right)$$

```
[In] integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x))/x**2,x)
```

```
[Out] pi**(5/2)*(Integral(2*a*c**2*sqrt(c**2*x**2 + 1), x) + Integral(a*sqrt(c**2
*x**2 + 1)/x**2, x) + Integral(a*c**4*x**2*sqrt(c**2*x**2 + 1), x) + Integr
al(2*b*c**2*sqrt(c**2*x**2 + 1)*asinh(c*x), x) + Integral(b*sqrt(c**2*x**2
+ 1)*asinh(c*x)/x**2, x) + Integral(b*c**4*x**2*sqrt(c**2*x**2 + 1)*asinh(c
*x), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(\pi + c^2\pi x^2)^{5/2}\,(a + b\text{arcsinh}(cx))}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))}{x^2} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))}{x^2} dx = \int \frac{(a + b\,\text{asinh}(c\,x))\ (\Pi\,c^2\,x^2 + \Pi)^{5/2}}{x^2} dx$$

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x^2, x)
```

3.77 $\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 205

$$\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx=-\frac{bc\pi^{5/2}}{2x}-\frac{7}{3}bc^3\pi^{5/2}x-\frac{1}{9}bc^5\pi^{5/2}x^3+\frac{5}{2}c^2\pi^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))+\frac{5}{6}c^2\pi(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))-\frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}$$

```
[Out] -1/2*b*c*Pi^(5/2)/x-7/3*b*c^3*Pi^(5/2)*x-1/9*b*c^5*Pi^(5/2)*x^3+5/6*c^2*Pi*
(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))-1/2*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arc
sinh(c*x))/x^2-5*c^2*Pi^(5/2)*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1
/2))-5/2*b*c^2*Pi^(5/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+5/2*b*c^2*Pi^(5/2
)*polylog(2,c*x+(c^2*x^2+1)^(1/2))+5/2*c^2*Pi^2*(a+b*arcsinh(c*x))*(Pi*c^2*
x^2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5807, 5808, 5806, 5816, 4267, 2317, 2438, 8, 276}

$$\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx=-5\pi^{5/2}c^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+\frac{5}{6}\pi c^2(\pi c^2x^2+\pi)^{3/2}(a+b\text{arcsinh}(cx))+\frac{5}{2}\pi^2c^2\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))-\frac{(\pi c^2x^2+\pi)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}$$

```
[In] Int[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*Pi^(5/2))/x - (7*b*c^3*Pi^(5/2)*x)/3 - (b*c^5*Pi^(5/2)*x^3)/9 + (
5*c^2*Pi^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/2 + (5*c^2*Pi*(Pi +
```

```
c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/6 - ((Pi + c^2*Pi*x^2)^(5/2)*(a + b
*ArcSinh[c*x]))/(2*x^2) - 5*c^2*Pi^(5/2)*(a + b*ArcSinh[c*x])*ArcTanh[E^Arc
Sinh[c*x]] - (5*b*c^2*Pi^(5/2)*PolyLog[2, -E^ArcSinh[c*x]])/2 + (5*b*c^2*Pi
^(5/2)*PolyLog[2, E^ArcSinh[c*x]])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
```

```
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(\pi+c^2\pi x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{2x^2} \\
&\quad+\frac{1}{2}(5c^2\pi)\int\frac{(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{x}\,dx+\frac{1}{2}(bc\pi^{5/2})\int\frac{(1+c^2x^2)^2}{x^2}\,dx \\
&= \frac{5}{6}c^2\pi(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))-\frac{(\pi+c^2\pi x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{2x^2} \\
&\quad+\frac{1}{2}(5c^2\pi^2)\int\frac{\sqrt{\pi+c^2\pi x^2}(a+b\operatorname{arcsinh}(cx))}{x}\,dx \\
&\quad+\frac{1}{2}(bc\pi^{5/2})\int\left(2c^2+\frac{1}{x^2}+c^4x^2\right)dx-\frac{1}{6}(5bc^3\pi^{5/2})\int(1+c^2x^2)\,dx \\
&= -\frac{bc\pi^{5/2}}{2x}+\frac{1}{6}bc^3\pi^{5/2}x-\frac{1}{9}bc^5\pi^{5/2}x^3 \\
&\quad+\frac{5}{2}c^2\pi^2\sqrt{\pi+c^2\pi x^2}(a+b\operatorname{arcsinh}(cx))+\frac{5}{6}c^2\pi(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))-\frac{(\pi+c^2\pi x^2)^{5/2}(a}{2x} \\
&= -\frac{bc\pi^{5/2}}{2x}-\frac{7}{3}bc^3\pi^{5/2}x-\frac{1}{9}bc^5\pi^{5/2}x^3 \\
&\quad+\frac{5}{2}c^2\pi^2\sqrt{\pi+c^2\pi x^2}(a+b\operatorname{arcsinh}(cx))+\frac{5}{6}c^2\pi(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))-\frac{(\pi+c^2\pi x^2)^{5/2}(a}{2x}
\end{aligned}$$

$$= -\frac{bc\pi^{5/2}}{2x} - \frac{7}{3}bc^3\pi^{5/2}x - \frac{1}{9}bc^5\pi^{5/2}x^3 + \frac{5}{2}c^2\pi^2\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx)) + \frac{5}{6}c^2\pi(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx)) - \frac{(\pi + c^2\pi x^2)^{5/2}(a +}{2x^2}$$

$$= -\frac{bc\pi^{5/2}}{2x} - \frac{7}{3}bc^3\pi^{5/2}x - \frac{1}{9}bc^5\pi^{5/2}x^3 + \frac{5}{2}c^2\pi^2\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx)) + \frac{5}{6}c^2\pi(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx)) - \frac{(\pi + c^2\pi x^2)^{5/2}(a +}{2x^2}$$

$$= -\frac{bc\pi^{5/2}}{2x} - \frac{7}{3}bc^3\pi^{5/2}x - \frac{1}{9}bc^5\pi^{5/2}x^3 + \frac{5}{2}c^2\pi^2\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx)) + \frac{5}{6}c^2\pi(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx)) - \frac{(\pi + c^2\pi x^2)^{5/2}(a +}{2x^2}$$

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.70

$$\int \frac{(\pi + c^2\pi x^2)^{5/2}(a + b\text{arcsinh}(cx))}{x^3}\,dx = \frac{\pi^{5/2}\left(-168bc^3x^3 - 8bc^5x^5 - 36a\sqrt{1 + c^2x^2} + 168ac^2x^2\sqrt{1 + c^2x^2} +\right.}{}$$

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (Pi^(5/2)*(-168*b*c^3*x^3 - 8*b*c^5*x^5 - 36*a*Sqrt[1 + c^2*x^2] + 168*a*c^
2*x^2*Sqrt[1 + c^2*x^2] + 24*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 168*b*c^2*x^2*Sq
rt[1 + c^2*x^2]*ArcSinh[c*x] + 24*b*c^4*x^4*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
- 9*b*c^3*x^3*Csch[ArcSinh[c*x]/2]^2 - 9*b*c^2*x^2*ArcSinh[c*x]*Csch[ArcSin
h[c*x]/2]^2 + 180*b*c^2*x^2*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 180*b
*c^2*x^2*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 180*a*c^2*x^2*Log[x] - 1
80*a*c^2*x^2*Log[Pi*(1 + Sqrt[1 + c^2*x^2])] + 180*b*c^2*x^2*PolyLog[2, -E^
(-ArcSinh[c*x])] - 180*b*c^2*x^2*PolyLog[2, E^(-ArcSinh[c*x])] + 36*b*c*x*S
inh[ArcSinh[c*x]/2]^2 - 36*b*ArcSinh[c*x]*Sinh[ArcSinh[c*x]/2]^2))/(72*x^2)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.70

method	result
default	$a\left(-\frac{(\pi c^2x^2+\pi)^{\frac{7}{2}}}{2\pi x^2}+\frac{5c^2\left(\frac{(\pi c^2x^2+\pi)^{\frac{5}{2}}}{5}+\pi\left(\frac{(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3}+\pi\left(\sqrt{\pi c^2x^2+\pi}-\sqrt{\pi}\,\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)\right)\right)\right)}{2}\right)-\frac{b\pi^{\frac{5}{2}}\,\mathrm{arcsi}}{2\sqrt{c^2}}$
parts	$a\left(-\frac{(\pi c^2x^2+\pi)^{\frac{7}{2}}}{2\pi x^2}+\frac{5c^2\left(\frac{(\pi c^2x^2+\pi)^{\frac{5}{2}}}{5}+\pi\left(\frac{(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3}+\pi\left(\sqrt{\pi c^2x^2+\pi}-\sqrt{\pi}\,\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)\right)\right)\right)}{2}\right)-\frac{b\pi^{\frac{5}{2}}\,\mathrm{arcsi}}{2\sqrt{c^2}}$

```
[In] int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/2/Pi/x^2*(Pi*c^2*x^2+Pi)^(7/2)+5/2*c^2*(1/5*(Pi*c^2*x^2+Pi)^(5/2)+Pi*
(1/3*(Pi*c^2*x^2+Pi)^(3/2)+Pi*((Pi*c^2*x^2+Pi)^(1/2)-Pi^(1/2)*arctanh(Pi^(1
/2)/(Pi*c^2*x^2+Pi)^(1/2))))))-1/2*b*Pi^(5/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x
)*c^2+1/3*b*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*Pi^(5/2)*x^2*c^4+5/2*b*c^2*Pi^(5
/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-1/9*b*c^5*Pi^(5/2)*x^3-7/3*b*c
^3*Pi^(5/2)*x+5/2*b*c^2*Pi^(5/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-5/2*b*c^2
*Pi^(5/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-5/2*b*c^2*Pi^(5/2)*arcsinh(c*x)
*ln(1+c*x+(c^2*x^2+1)^(1/2))-1/2*b*c*Pi^(5/2)/x-1/2*b*Pi^(5/2)/(c^2*x^2+1)^
(1/2)/x^2*arcsinh(c*x)+7/3*b*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*Pi^(5/2)*c^2
```

Fricas [F]

$$\int \frac{(\pi+c^2\pi x^2)^{5/2}\,(a+b\mathrm{arcsinh}(cx))}{x^3}\,dx=\int \frac{(\pi+\pi c^2x^2)^{\frac{5}{2}}\,(b\,\mathrm{arsinh}\,(cx)+a)}{x^3}\,dx$$

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^4 + 2*pi^2*a*c^2*x^2 + pi^2*a
+ (pi^2*b*c^4*x^4 + 2*pi^2*b*c^2*x^2 + pi^2*b)*arcsinh(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(\pi+c^2\pi x^2)^{5/2}\,(a+b\mathrm{arcsinh}(cx))}{x^3}\,dx=\pi^{\frac{5}{2}}\left(\int \frac{a\sqrt{c^2x^2+1}}{x^3}\,dx+\int \frac{2ac^2\sqrt{c^2x^2+1}}{x}\,dx+\int ac^4x\sqrt{c^2x^2+1}\,dx+\int \frac{b\sqrt{c^2x^2+1}\,\mathrm{asinh}\,(cx)}{x^3}\,dx+\int \frac{2bc^2\sqrt{c^2x^2+1}\,\mathrm{asinh}\,(cx)}{x}\,dx+\int bc^4x\sqrt{c^2x^2+1}\,\mathrm{asinh}\,(cx)\,dx\right)$$

```
[In] integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x))/x**3,x)
```

```
[Out] pi**(5/2)*(Integral(a*sqrt(c**2*x**2 + 1)/x**3, x) + Integral(2*a*c**2*sqrt
(c**2*x**2 + 1)/x, x) + Integral(a*c**4*x*sqrt(c**2*x**2 + 1), x) + Integra
l(b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x**3, x) + Integral(2*b*c**2*sqrt(c**2*x
**2 + 1)*asinh(c*x)/x, x) + Integral(b*c**4*x*sqrt(c**2*x**2 + 1)*asinh(c*x
), x))
```

Maxima [F]

$$\int \frac{(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))}{x^3} dx = \int \frac{(\pi + \pi c^2x^2)^{\frac{5}{2}} (b \operatorname{arsinh}(cx) + a)}{x^3} dx$$

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima
")
```

```
[Out] -1/6*(15*pi^(5/2)*c^2*arcsinh(1/(c*abs(x))) - 15*pi^2*sqrt(pi + pi*c^2*x^2)
*c^2 - 5*pi*(pi + pi*c^2*x^2)^(3/2)*c^2 - 3*(pi + pi*c^2*x^2)^(5/2)*c^2 + 3
*(pi + pi*c^2*x^2)^(7/2)/(pi*x^2))*a + b*integrate((pi + pi*c^2*x^2)^(5/2)*
log(c*x + sqrt(c^2*x^2 + 1))/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))}{x^3} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))}{x^3} dx = \int \frac{(a + b \operatorname{asinh}(c x)) (\Pi c^2 x^2 + \Pi)^{5/2}}{x^3} dx$$

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x^3, x)
```

3.78 $\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\textbf{arcsinh}(cx))}{x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 166

$$\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx = -\frac{bc\pi^{5/2}}{6x^2}-\frac{1}{4}bc^5\pi^{5/2}x^2 + \frac{5}{2}c^4\pi^2x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx)) - \frac{5c^2\pi(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3x} - \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsi}}{3x^3}$$

```
[Out] -1/6*b*c*Pi^(5/2)/x^2-1/4*b*c^5*Pi^(5/2)*x^2-5/3*c^2*Pi*(Pi*c^2*x^2+Pi)^(3/
2)*(a+b*arcsinh(c*x))/x-1/3*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/x^3+5/
4*c^3*Pi^(5/2)*(a+b*arcsinh(c*x))^2/b+7/3*b*c^3*Pi^(5/2)*ln(x)+5/2*c^4*Pi^2
*x*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5807, 5785, 5783, 30, 14, 272, 45}

$$\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx = \frac{5\pi^{5/2}c^3(a+b\text{arcsinh}(cx))^2}{4b} - \frac{5\pi c^2(\pi c^2x^2+\pi)^{3/2}(a+b\text{arcsinh}(cx))}{3x} - \frac{(\pi c^2x^2+\pi)^{5/2}(a+b\text{arcsinh}(cx))}{3x^3} + \frac{5}{2}\pi^2c^4x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx)) - \frac{1}{4}\pi^{5/2}bc^5x^2 + \frac{7}{3}\pi^{5/2}bc^3\log(x) - \frac{\pi^{5/2}bc}{6x^2}$$

```
[In] Int[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*Pi^(5/2))/x^2 - (b*c^5*Pi^(5/2)*x^2)/4 + (5*c^4*Pi^2*x*Sqrt[Pi +
c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/2 - (5*c^2*Pi*(Pi + c^2*Pi*x^2)^(3/2)*(a
```

```
+ b*ArcSinh[c*x]))/(3*x) - ((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(
3*x^3) + (5*c^3*Pi^(5/2)*(a + b*ArcSinh[c*x])^2)/(4*b) + (7*b*c^3*Pi^(5/2)*
Log[x])/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
```

```
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{3x^3} \\
&\quad+\frac{1}{3}(5c^2\pi)\int\frac{(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx+\frac{1}{3}(bc\pi^{5/2})\int\frac{(1+c^2x^2)^2}{x^3}\,dx \\
&= -\frac{5c^2\pi(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3x}-\frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{3x^3} \\
&\quad+(5c^4\pi^2)\int\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx \\
&\quad+\frac{1}{6}(bc\pi^{5/2})\,\text{Subst}\left(\int\frac{(1+c^2x)^2}{x^2}\,dx,x,x^2\right)+\frac{1}{3}(5bc^3\pi^{5/2})\int\frac{1+c^2x^2}{x}\,dx \\
&= \frac{5}{2}c^4\pi^2x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx)) \\
&\quad-\frac{5c^2\pi(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3x}-\frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{3x^3} \\
&\quad+\frac{1}{6}(bc\pi^{5/2})\,\text{Subst}\left(\int\left(c^4+\frac{1}{x^2}+\frac{2c^2}{x}\right)dx,x,x^2\right)+\frac{1}{3}(5bc^3\pi^{5/2})\int\left(\frac{1}{x}+c^2x\right)dx+\frac{1}{2}(5c^4\pi^{5/2})\int \\
&= -\frac{bc\pi^{5/2}}{6x^2}-\frac{1}{4}bc^5\pi^{5/2}x^2 \\
&\quad+\frac{5}{2}c^4\pi^2x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))-\frac{5c^2\pi(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3x} \\
&\quad-\frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{3x^3}+\frac{5c^3\pi^{5/2}(a+b\text{arcsinh}(cx))^2}{4b}+\frac{7}{3}bc^3\pi^{5/2}\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.08

$$\int\frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx=\frac{\pi^{5/2}\left(-4bcx-8a\sqrt{1+c^2x^2}-56ac^2x^2\sqrt{1+c^2x^2}+12ac^4x^4\sqrt{1+\right.}{}$$

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^4,x]
[Out] (Pi^(5/2)*(-4*b*c*x - 8*a*Sqrt[1 + c^2*x^2] - 56*a*c^2*x^2*Sqrt[1 + c^2*x^2
] + 12*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 30*b*c^3*x^3*ArcSinh[c*x]^2 - 3*b*c^3*
```

```
x^3*Cosh[2*ArcSinh[c*x]] + 56*b*c^3*x^3*Log[c*x] + ArcSinh[c*x]*(60*a*c^3*x
^3 - 8*b*Sqrt[1 + c^2*x^2]*(1 + 7*c^2*x^2) + 6*b*c^3*x^3*Sinh[2*ArcSinh[c*x
]])))/(24*x^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 691 vs. $2(138) = 276$.

Time = 0.22 (sec) , antiderivative size = 692, normalized size of antiderivative = 4.17

method	result
default	$-\frac{a(\pi c^2x^2+\pi)^{\frac{7}{2}}}{3\pi x^3}-\frac{4a c^2(\pi c^2x^2+\pi)^{\frac{7}{2}}}{3\pi x}+\frac{4a c^4x(\pi c^2x^2+\pi)^{\frac{5}{2}}}{3}+\frac{5a c^4\pi x(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3}+\frac{5a c^4\pi^2x\sqrt{\pi c^2x^2+\pi}}{2}+\frac{5a c^4\pi^3\ln\Big($
parts	$-\frac{a(\pi c^2x^2+\pi)^{\frac{7}{2}}}{3\pi x^3}-\frac{4a c^2(\pi c^2x^2+\pi)^{\frac{7}{2}}}{3\pi x}+\frac{4a c^4x(\pi c^2x^2+\pi)^{\frac{5}{2}}}{3}+\frac{5a c^4\pi x(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3}+\frac{5a c^4\pi^2x\sqrt{\pi c^2x^2+\pi}}{2}+\frac{5a c^4\pi^3\ln\Big($

```
[In] int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/Pi/x^3*(Pi*c^2*x^2+Pi)^(7/2)-4/3*a*c^2/Pi/x*(Pi*c^2*x^2+Pi)^(7/2)+4/
3*a*c^4*x*(Pi*c^2*x^2+Pi)^(5/2)+5/3*a*c^4*Pi*x*(Pi*c^2*x^2+Pi)^(3/2)+5/2*a*
c^4*Pi^2*x*(Pi*c^2*x^2+Pi)^(1/2)+5/2*a*c^4*Pi^3*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+
(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)-1/8*b*c^3*Pi^(5/2)-14/3*b*c^3*Pi^(5/2
)*arcsinh(c*x)+5/4*b*c^3*Pi^(5/2)*arcsinh(c*x)^2+147*b*Pi^(5/2)/(63*c^4*x^4
+15*c^2*x^2+1)*x^4*arcsinh(c*x)*c^7-49/6*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+
1)*x^2*(c^2*x^2+1)*c^5+35*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)*x^2*arcsinh(
c*x)*c^5-7/3*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)*(c^2*x^2+1)*c^3+7/3*b*Pi^
(5/2)/(63*c^4*x^4+15*c^2*x^2+1)*arcsinh(c*x)*c^3-1/6*b*Pi^(5/2)/(63*c^4*x^4
+15*c^2*x^2+1)/x^2*(c^2*x^2+1)*c-1/3*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)/x
^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)+1/2*b*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*Pi^(
5/2)*x*c^4+7/3*b*c^3*Pi^(5/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)-1/4*b*c^5*Pi^
(5/2)*x^2+49/6*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)*x^4*c^7-147*b*Pi^(5/2)/
(63*c^4*x^4+15*c^2*x^2+1)*x^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^6-56*b*Pi^(5
/2)/(63*c^4*x^4+15*c^2*x^2+1)*x*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^4-22/3*b*P
i^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)/x*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^2
```

Fricas [F]

$$\int \frac{(\pi + c^2\pi x^2)^{5/2}(a + b\mathrm{arcsinh}(cx))}{x^4}\,dx = \int \frac{(\pi + \pi c^2x^2)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx) + a)}{x^4}\,dx$$

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^4 + 2*pi^2*a*c^2*x^2 + pi^2*a
+ (pi^2*b*c^4*x^4 + 2*pi^2*b*c^2*x^2 + pi^2*b)*arcsinh(c*x))/x^4, x)
```

Sympy [F]

$$\int \frac{(\pi + c^2\pi x^2)^{5/2}\,(a + b\operatorname{arcsinh}(cx))}{x^4}\,dx = \pi^{\frac{5}{2}}\left(\int ac^4\sqrt{c^2x^2+1}\,dx + \int \frac{a\sqrt{c^2x^2+1}}{x^4}\,dx + \int \frac{2ac^2\sqrt{c^2x^2+1}}{x^2}\,dx + \int bc^4\sqrt{c^2x^2+1}\operatorname{asinh}(cx)\,dx + \int \frac{b\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{x^4}\,dx + \int \frac{2bc^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{x^2}\,dx\right)$$

```
[In] integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x))/x**4,x)
```

```
[Out] pi**(5/2)*(Integral(a*c**4*sqrt(c**2*x**2 + 1), x) + Integral(a*sqrt(c**2*x
**2 + 1)/x**4, x) + Integral(2*a*c**2*sqrt(c**2*x**2 + 1)/x**2, x) + Integr
al(b*c**4*sqrt(c**2*x**2 + 1)*asinh(c*x), x) + Integral(b*sqrt(c**2*x**2 +
1)*asinh(c*x)/x**4, x) + Integral(2*b*c**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/x
**2, x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(\pi + c^2\pi x^2)^{5/2}\,(a + b\operatorname{arcsinh}(cx))}{x^4}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(\pi + c^2\pi x^2)^{5/2}\,(a + b\operatorname{arcsinh}(cx))}{x^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx=\int \frac{\left(a+b\,\text{asinh}(c\,x)\right)\,\left(\Pi\,c^2\,x^2+\Pi\right)^{5/2}}{x^4}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x^4,x)
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x^4, x)
```

3.79 $\int \sqrt{1+x^2}\operatorname{arcsinh}(x)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \sqrt{1+x^2}\operatorname{arcsinh}(x)\,dx=-\frac{x^2}{4}+\frac{1}{2}x\sqrt{1+x^2}\operatorname{arcsinh}(x)+\frac{\operatorname{arcsinh}(x)^2}{4}$$

```
[Out] -1/4*x^2+1/4*arcsinh(x)^2+1/2*x*arcsinh(x)*(x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5785, 5783, 30}

$$\int \sqrt{1+x^2}\operatorname{arcsinh}(x)\,dx=\frac{1}{2}\sqrt{x^2+1}x\operatorname{arcsinh}(x)+\frac{\operatorname{arcsinh}(x)^2}{4}-\frac{x^2}{4}$$

```
[In] Int[Sqrt[1 + x^2]*ArcSinh[x],x]
```

```
[Out] -1/4*x^2 + (x*Sqrt[1 + x^2]*ArcSinh[x])/2 + ArcSinh[x]^2/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
```

```
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x\sqrt{1+x^2}\text{arcsinh}(x) - \frac{\int x\,dx}{2} + \frac{1}{2}\int \frac{\text{arcsinh}(x)}{\sqrt{1+x^2}}\,dx\\ &= -\frac{x^2}{4} + \frac{1}{2}x\sqrt{1+x^2}\text{arcsinh}(x) + \frac{\text{arcsinh}(x)^2}{4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \sqrt{1+x^2}\text{arcsinh}(x)\,dx = \frac{1}{4}\left(-x^2 + 2x\sqrt{1+x^2}\text{arcsinh}(x) + \text{arcsinh}(x)^2\right)$$

```
[In] Integrate[Sqrt[1 + x^2]*ArcSinh[x],x]
[Out] (-x^2 + 2*x*Sqrt[1 + x^2]*ArcSinh[x] + ArcSinh[x]^2)/4
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{x\,\text{arcsinh}(x)\sqrt{x^2+1}}{2} + \frac{\text{arcsinh}(x)^2}{4} - \frac{x^2}{4} - \frac{1}{4}$	26

```
[In] int(arcsinh(x)*(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x*arcsinh(x)*(x^2+1)^(1/2)+1/4*arcsinh(x)^2-1/4*x^2-1/4
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \sqrt{1+x^2}\text{arcsinh}(x)\,dx = \frac{1}{2}\sqrt{x^2+1}x\log\left(x+\sqrt{x^2+1}\right) - \frac{1}{4}x^2 + \frac{1}{4}\log\left(x+\sqrt{x^2+1}\right)^2$$

```
[In] integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(x^2 + 1)*x*log(x + sqrt(x^2 + 1)) - 1/4*x^2 + 1/4*log(x + sqrt(x^2
 + 1))^2
```

Sympy [F]

$$\int \sqrt{1+x^2}\text{arcsinh}(x)\,dx = \int \sqrt{x^2+1}\,\text{asinh}\,(x)\,dx$$

```
[In] integrate(asinh(x)*(x**2+1)**(1/2),x)
[Out] Integral(sqrt(x**2 + 1)*asinh(x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \sqrt{1+x^2}\text{arcsinh}(x)\,dx = -\frac{1}{4}x^2 + \frac{1}{2}\left(\sqrt{x^2+1}x + \text{arsinh}\,(x)\right)\text{arsinh}\,(x) - \frac{1}{4}\,\text{arsinh}\,(x)^2$$

```
[In] integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/4*x^2 + 1/2*(sqrt(x^2 + 1)*x + arcsinh(x))*arcsinh(x) - 1/4*arcsinh(x)^2
```

Giac [F]

$$\int \sqrt{1+x^2}\text{arcsinh}(x)\,dx = \int \sqrt{x^2+1}\,\text{arsinh}\,(x)\,dx$$

```
[In] integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^2 + 1)*arcsinh(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+x^2}\text{arcsinh}(x)\,dx = \int \text{asinh}(x)\,\sqrt{x^2+1}\,dx$$

```
[In] int(asinh(x)*(x^2 + 1)^(1/2),x)
[Out] int(asinh(x)*(x^2 + 1)^(1/2), x)
```

3.80 $\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 149

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = -\frac{8bx}{15c^5\sqrt{\pi}}+\frac{4bx^3}{45c^3\sqrt{\pi}}-\frac{bx^5}{25c\sqrt{\pi}}+\frac{8\sqrt{\pi+c^2\pi x^2}(a+b\operatorname{arcsinh}(cx))}{15c^6\pi}-\frac{4x^2\sqrt{\pi+c^2\pi x^2}(a+b\operatorname{arcsinh}(cx))}{15c^4\pi}+\frac{x^4\sqrt{\pi+c^2\pi x^2}(a+b\operatorname{arcsinh}(cx))}{5c^2\pi}$$

```
[Out] -8/15*b*x/c^5/Pi^(1/2)+4/45*b*x^3/c^3/Pi^(1/2)-1/25*b*x^5/c/Pi^(1/2)+8/15*(
a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^6/Pi-4/15*x^2*(a+b*arcsinh(c*x))*
(Pi*c^2*x^2+Pi)^(1/2)/c^4/Pi+1/5*x^4*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/
2)/c^2/Pi
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used

$= \{5812, 5798, 8, 30\}$

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{x^4\sqrt{\pi c^2x^2+\pi}(a+b\operatorname{arcsinh}(cx))}{5\pi c^2} + \frac{8\sqrt{\pi c^2x^2+\pi}(a+b\operatorname{arcsinh}(cx))}{15\pi c^6} - \frac{4x^2\sqrt{\pi c^2x^2+\pi}(a+b\operatorname{arcsinh}(cx))}{15\pi c^4} - \frac{8bx}{15\sqrt{\pi}c^5} + \frac{4bx^3}{45\sqrt{\pi}c^3} - \frac{bx^5}{25\sqrt{\pi}c}$$

```
[In] Int[(x^5*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (-8*b*x)/(15*c^5*Sqrt[Pi]) + (4*b*x^3)/(45*c^3*Sqrt[Pi]) - (b*x^5)/(25*c*Sq
rt[Pi]) + (8*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(15*c^6*Pi) - (4*x
^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(15*c^4*Pi) + (x^4*Sqrt[Pi +
 c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2*Pi)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{x^4\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{5c^2\pi} - \frac{4\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx}{5c^2} - \frac{b\int x^4\,dx}{5c\sqrt{\pi}}$$

$$= -\frac{bx^5}{25c\sqrt{\pi}} - \frac{4x^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{15c^4\pi} + \frac{x^4\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{5c^2\pi} + \frac{8\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx}{15c^4} + \frac{(4b)\int x^2\,dx}{15c^3\sqrt{\pi}}$$

$$= \frac{4bx^3}{45c^3\sqrt{\pi}} - \frac{bx^5}{25c\sqrt{\pi}} + \frac{8\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{15c^6\pi} - \frac{4x^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{15c^4\pi} + \frac{x^4\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{5c^2\pi} - \frac{(8b)\int 1\,dx}{15c^5\sqrt{\pi}}$$

$$= -\frac{8bx}{15c^5\sqrt{\pi}} + \frac{4bx^3}{45c^3\sqrt{\pi}} - \frac{bx^5}{25c\sqrt{\pi}} + \frac{8\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{15c^6\pi} - \frac{4x^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{15c^4\pi} + \frac{x^4\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{5c^2\pi}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.72

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{15a\sqrt{1+c^2x^2}(8-4c^2x^2+3c^4x^4)+b(-120cx+20c^3x^3-9c^5x^5)+15b\sqrt{1+c^2x^2}(8-4c^2x^2+3c^4x^4)\,\text{arc}}{225c^6\sqrt{\pi}}$$

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
[Out] (15*a*Sqrt[1 + c^2*x^2]*(8 - 4*c^2*x^2 + 3*c^4*x^4) + b*(-120*c*x + 20*c^3*
x^3 - 9*c^5*x^5) + 15*b*Sqrt[1 + c^2*x^2]*(8 - 4*c^2*x^2 + 3*c^4*x^4)*ArcSi
nh[c*x])/(225*c^6*Sqrt[Pi])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.30

method	result
default	$a\left(\frac{x^4\sqrt{\pi c^2x^2+\pi}}{5\pi c^2}-\frac{4\left(\frac{x^2\sqrt{\pi c^2x^2+\pi}}{3\pi c^2}-\frac{2\sqrt{\pi c^2x^2+\pi}}{3\pi c^4}\right)}{5c^2}\right)+\frac{b\left(45\ \mathrm{arcsinh}(cx)c^6x^6-15\ \mathrm{arcsinh}(cx)c^4x^4-9c^5x^5\sqrt{c^2x^2+1}+60\ \mathrm{arcsin}\right.}{225c^6\sqrt{\pi}\sqrt{c}}$
parts	$a\left(\frac{x^4\sqrt{\pi c^2x^2+\pi}}{5\pi c^2}-\frac{4\left(\frac{x^2\sqrt{\pi c^2x^2+\pi}}{3\pi c^2}-\frac{2\sqrt{\pi c^2x^2+\pi}}{3\pi c^4}\right)}{5c^2}\right)+\frac{b\left(45\ \mathrm{arcsinh}(cx)c^6x^6-15\ \mathrm{arcsinh}(cx)c^4x^4-9c^5x^5\sqrt{c^2x^2+1}+60\ \mathrm{arcsin}\right.}{225c^6\sqrt{\pi}\sqrt{c}}$

```
[In] int(x^5*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/5*x^4/Pi/c^2*(Pi*c^2*x^2+Pi)^(1/2)-4/5/c^2*(1/3*x^2/Pi/c^2*(Pi*c^2*x^2
+Pi)^(1/2)-2/3/Pi/c^4*(Pi*c^2*x^2+Pi)^(1/2)))+1/225*b/c^6/Pi^(1/2)/(c^2*x^2
+1)^(1/2)*(45*arcsinh(c*x)*c^6*x^6-15*arcsinh(c*x)*c^4*x^4-9*c^5*x^5*(c^2*x
^2+1)^(1/2)+60*arcsinh(c*x)*c^2*x^2+20*c^3*x^3*(c^2*x^2+1)^(1/2)+120*arcsin
h(c*x)-120*c*x*(c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.08

$$\int \frac{x^5(a+b\mathrm{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx$$
$$=\frac{15\sqrt{\pi+\pi c^2x^2}(3\,bc^6x^6-bc^4x^4+4\,bc^2x^2+8\,b)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}\left(45\,ac^6x^6-15\,ac^4x^4+\right.}{225\,(\pi c^8x^2+\pi c^6)}$$

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/225*(15*sqrt(pi + pi*c^2*x^2)*(3*b*c^6*x^6 - b*c^4*x^4 + 4*b*c^2*x^2 + 8*
b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(45*a*c^6*x^6 - 15*
a*c^4*x^4 + 60*a*c^2*x^2 - (9*b*c^5*x^5 - 20*b*c^3*x^3 + 120*b*c*x)*sqrt(c^
2*x^2 + 1) + 120*a))/(pi*c^8*x^2 + pi*c^6)
```

Sympy [A] (verification not implemented)

Time = 3.15 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.23

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{a\left(\begin{cases}\frac{x^4\sqrt{c^2x^2+1}}{5c^2}-\frac{4x^2\sqrt{c^2x^2+1}}{15c^4}+\frac{8\sqrt{c^2x^2+1}}{15c^6} & \text{for } c^2\neq 0\\ \frac{x^6}{6} & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}} + \frac{b\left(\begin{cases}-\frac{x^5}{25c}+\frac{x^4\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{5c^2}+\frac{4x^3}{45c^3}-\frac{4x^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{15c^4}-\frac{8x}{15c^5}+\frac{8\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{15c^6} & \text{for } c\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}}$$

```
[In] integrate(x**5*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] a*Piecewise((x**4*sqrt(c**2*x**2 + 1)/(5*c**2) - 4*x**2*sqrt(c**2*x**2 + 1)
/(15*c**4) + 8*sqrt(c**2*x**2 + 1)/(15*c**6), Ne(c**2, 0)), (x**6/6, True))
/sqrt(pi) + b*Piecewise((-x**5/(25*c) + x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)
/(5*c**2) + 4*x**3/(45*c**3) - 4*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(15*c*
*4) - 8*x/(15*c**5) + 8*sqrt(c**2*x**2 + 1)*asinh(c*x)/(15*c**6), Ne(c, 0))
, (0, True))/sqrt(pi)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.17

$$\begin{aligned}\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx &= \frac{1}{15}\left(\frac{3\sqrt{\pi+\pi c^2x^2}x^4}{\pi c^2}-\frac{4\sqrt{\pi+\pi c^2x^2}x^2}{\pi c^4}+\frac{8\sqrt{\pi+\pi c^2x^2}}{\pi c^6}\right)b\operatorname{arsinh}(cx)\\&\quad+\frac{1}{15}\left(\frac{3\sqrt{\pi+\pi c^2x^2}x^4}{\pi c^2}-\frac{4\sqrt{\pi+\pi c^2x^2}x^2}{\pi c^4}+\frac{8\sqrt{\pi+\pi c^2x^2}}{\pi c^6}\right)a\\&\quad-\frac{(9\,c^4x^5-20\,c^2x^3+120\,x)b}{225\sqrt{\pi}c^5}\end{aligned}$$

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/15*(3*sqrt(pi + pi*c^2*x^2)*x^4/(pi*c^2) - 4*sqrt(pi + pi*c^2*x^2)*x^2/(p
i*c^4) + 8*sqrt(pi + pi*c^2*x^2)/(pi*c^6))*b*arcsinh(c*x) + 1/15*(3*sqrt(pi
 + pi*c^2*x^2)*x^4/(pi*c^2) - 4*sqrt(pi + pi*c^2*x^2)*x^2/(pi*c^4) + 8*sqrt
(pi + pi*c^2*x^2)/(pi*c^6))*a - 1/225*(9*c^4*x^5 - 20*c^2*x^3 + 120*x)*b/(s
qrt(pi)*c^5)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a + b\text{arcsinh}(cx))}{\sqrt{\pi + c^2\pi x^2}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a + b\text{arcsinh}(cx))}{\sqrt{\pi + c^2\pi x^2}}\,dx = \int \frac{x^5\,(a + b\,\text{asinh}(c\,x))}{\sqrt{\Pi\,c^2\,x^2 + \Pi}}\,dx$$

[In] int((x^5*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2),x)

[Out] int((x^5*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2), x)

3.81 $\int \frac{x^4(a+b\,\mathrm{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 126

$$\int \frac{x^4(a+b\,\mathrm{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{3bx^2}{16c^3\sqrt{\pi}}-\frac{bx^4}{16c\sqrt{\pi}}-\frac{3x\sqrt{\pi+c^2\pi x^2}(a+b\,\mathrm{arcsinh}(cx))}{8c^4\pi}+\frac{x^3\sqrt{\pi+c^2\pi x^2}(a+b\,\mathrm{arcsinh}(cx))}{4c^2\pi}+\frac{3(a+b\,\mathrm{arcsinh}(cx))^2}{16bc^5\sqrt{\pi}}$$

```
[Out] 3/16*b*x^2/c^3/Pi^(1/2)-1/16*b*x^4/c/Pi^(1/2)+3/16*(a+b*arcsinh(c*x))^2/b/c
^5/Pi^(1/2)-3/8*x*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^4/Pi+1/4*x^3*(
a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^2/Pi
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5812, 5783, 30}

$$\int \frac{x^4(a+b\,\mathrm{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{3(a+b\,\mathrm{arcsinh}(cx))^2}{16\sqrt{\pi}bc^5}+\frac{x^3\sqrt{\pi c^2x^2+\pi}(a+b\,\mathrm{arcsinh}(cx))}{4\pi c^2}-\frac{3x\sqrt{\pi c^2x^2+\pi}(a+b\,\mathrm{arcsinh}(cx))}{8\pi c^4}+\frac{3bx^2}{16\sqrt{\pi}c^3}-\frac{bx^4}{16\sqrt{\pi}c}$$

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (3*b*x^2)/(16*c^3*Sqrt[Pi]) - (b*x^4)/(16*c*Sqrt[Pi]) - (3*x*Sqrt[Pi + c^2*
Pi*x^2]*(a + b*ArcSinh[c*x]))/(8*c^4*Pi) + (x^3*Sqrt[Pi + c^2*Pi*x^2]*(a +
b*ArcSinh[c*x]))/(4*c^2*Pi) + (3*(a + b*ArcSinh[c*x])^2)/(16*b*c^5*Sqrt[Pi]
)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^3\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{4c^2\pi} - \frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx}{4c^2} - \frac{b\int x^3\,dx}{4c\sqrt{\pi}} \\
&= -\frac{bx^4}{16c\sqrt{\pi}} - \frac{3x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{8c^4\pi} \\
&\quad + \frac{x^3\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{4c^2\pi} + \frac{3\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx}{8c^4} + \frac{(3b)\int x\,dx}{8c^3\sqrt{\pi}} \\
&= \frac{3bx^2}{16c^3\sqrt{\pi}} - \frac{bx^4}{16c\sqrt{\pi}} - \frac{3x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{8c^4\pi} \\
&\quad + \frac{x^3\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{4c^2\pi} + \frac{3(a+b\text{arcsinh}(cx))^2}{16bc^5\sqrt{\pi}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.88

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx$$
$$= \frac{-48acx\sqrt{1+c^2x^2}+32ac^3x^3\sqrt{1+c^2x^2}+24b\text{arcsinh}(cx)^2+16b\cosh(2\text{arcsinh}(cx))-b\cosh(4\text{arcsinh}(cx}{128c^5\sqrt{\pi}}$$

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (-48*a*c*x*Sqrt[1 + c^2*x^2] + 32*a*c^3*x^3*Sqrt[1 + c^2*x^2] + 24*b*ArcSin
h[c*x]^2 + 16*b*Cosh[2*ArcSinh[c*x]] - b*Cosh[4*ArcSinh[c*x]] + 4*ArcSinh[c
*x]*(12*a - 8*b*Sinh[2*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]]))/(128*c^5*Sq
rt[Pi])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.31

method	result
default	$\frac{a\,x^3\sqrt{\pi\,c^2x^2+\pi}}{4\pi\,c^2}-\frac{3ax\sqrt{\pi\,c^2x^2+\pi}}{8c^4\pi}+\frac{3a\ln\left(\frac{\pi\,c^2x}{\sqrt{\pi\,c^2}}+\sqrt{\pi\,c^2x^2+\pi}\right)}{8c^4\sqrt{\pi\,c^2}}+\frac{b\left(4\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3-c^4x^4-6\,\text{arcsinh}(cx)cx\sqrt{c^2}\right.}{16c^5\sqrt{\pi}}$
parts	$\frac{a\,x^3\sqrt{\pi\,c^2x^2+\pi}}{4\pi\,c^2}-\frac{3ax\sqrt{\pi\,c^2x^2+\pi}}{8c^4\pi}+\frac{3a\ln\left(\frac{\pi\,c^2x}{\sqrt{\pi\,c^2}}+\sqrt{\pi\,c^2x^2+\pi}\right)}{8c^4\sqrt{\pi\,c^2}}+\frac{b\left(4\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3-c^4x^4-6\,\text{arcsinh}(cx)cx\sqrt{c^2}\right.}{16c^5\sqrt{\pi}}$

```
[In] int(x^4*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*x^3/Pi/c^2*(Pi*c^2*x^2+Pi)^(1/2)-3/8*a/c^4*x/Pi*(Pi*c^2*x^2+Pi)^(1/2)
+3/8*a/c^4*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)
+1/16*b*(4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3-c^4*x^4-6*arcsinh(c*x)*c*
x*(c^2*x^2+1)^(1/2)+3*c^2*x^2+3*arcsinh(c*x)^2+3)/c^5/Pi^(1/2)
```

Fricas [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^4}{\sqrt{\pi+\pi c^2x^2}}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral((b*x^4*arcsinh(c*x) + a*x^4)/sqrt(pi + pi*c^2*x^2), x)
```

Sympy [A] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.47

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{a\left(\begin{cases}\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3x\sqrt{c^2x^2+1}}{8c^4}+\frac{3\log\left(2c^2x+2\sqrt{c^2x^2+1}\sqrt{c^2}\right)}{8c^4\sqrt{c^2}} & \text{for } c^2\neq 0\\ \frac{x^5}{5} & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}} + \frac{b\left(\begin{cases}-\frac{x^4}{16c}+\frac{x^3\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{4c^2}+\frac{3x^2}{16c^3}-\frac{3x\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{8c^4}+\frac{3\operatorname{asinh}^2(cx)}{16c^5} & \text{for } c\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}}$$

```
[In] integrate(x**4*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] a*Piecewise((x**3*sqrt(c**2*x**2 + 1)/(4*c**2) - 3*x*sqrt(c**2*x**2 + 1)/(8
*c**4) + 3*log(2*c**2*x + 2*sqrt(c**2*x**2 + 1)*sqrt(c**2))/(8*c**4*sqrt(c*
*2)), Ne(c**2, 0)), (x**5/5, True))/sqrt(pi) + b*Piecewise((-x**4/(16*c) +
x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/(4*c**2) + 3*x**2/(16*c**3) - 3*x*sqrt(
c**2*x**2 + 1)*asinh(c*x)/(8*c**4) + 3*asinh(c*x)**2/(16*c**5), Ne(c, 0)),
(0, True))/sqrt(pi)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int \frac{x^4(a + b\text{arcsinh}(cx))}{\sqrt{\pi + c^2\pi x^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)x^4}{\sqrt{\pi + \pi c^2x^2}}\,dx$$

[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)*x^4/sqrt(pi + pi*c^2*x^2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b\text{arcsinh}(cx))}{\sqrt{\pi + c^2\pi x^2}}\,dx = \int \frac{x^4\,(a + b\,\text{asinh}(c\,x))}{\sqrt{\Pi\,c^2\,x^2 + \Pi}}\,dx$$

[In] int((x^4*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2),x)

[Out] int((x^4*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2), x)

3.82 $\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 98

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{2bx}{3c^3\sqrt{\pi}} - \frac{bx^3}{9c\sqrt{\pi}} - \frac{2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{3c^4\pi} + \frac{x^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{3c^2\pi}$$

```
[Out] 2/3*b*x/c^3/Pi^(1/2)-1/9*b*x^3/c/Pi^(1/2)-2/3*(a+b*arcsinh(c*x))*(Pi*c^2*x^
2+Pi)^(1/2)/c^4/Pi+1/3*x^2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^2/Pi
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5812, 5798, 8, 30}

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{x^2\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{3\pi c^2} - \frac{2\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{3\pi c^4} + \frac{2bx}{3\sqrt{\pi}c^3} - \frac{bx^3}{9\sqrt{\pi}c}$$

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (2*b*x)/(3*c^3*Sqrt[Pi]) - (b*x^3)/(9*c*Sqrt[Pi]) - (2*Sqrt[Pi + c^2*Pi*x^2
]*(a + b*ArcSinh[c*x]))/(3*c^4*Pi) + (x^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcS
inh[c*x]))/(3*c^2*Pi)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{3c^2\pi} - \frac{2\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx}{3c^2} - \frac{b\int x^2\,dx}{3c\sqrt{\pi}} \\
&= -\frac{bx^3}{9c\sqrt{\pi}} - \frac{2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{3c^4\pi} + \frac{x^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{3c^2\pi} + \frac{(2b)\int 1\,dx}{3c^3\sqrt{\pi}} \\
&= \frac{2bx}{3c^3\sqrt{\pi}} - \frac{bx^3}{9c\sqrt{\pi}} - \frac{2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{3c^4\pi} + \frac{x^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{3c^2\pi}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.84

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{3a(-2+c^2x^2)\sqrt{1+c^2x^2}+b(6cx-c^3x^3)+3b(-2+c^2x^2)\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{9c^4\sqrt{\pi}}$$

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (3*a*(-2 + c^2*x^2)*Sqrt[1 + c^2*x^2] + b*(6*c*x - c^3*x^3) + 3*b*(-2 + c^2
*x^2)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(9*c^4*Sqrt[Pi])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.36

method	result
default	$a\left(\frac{x^2\sqrt{\pi c^2x^2+\pi}}{3\pi c^2}-\frac{2\sqrt{\pi c^2x^2+\pi}}{3\pi c^4}\right)+\frac{b\left(3\,\text{arcsinh}(cx)c^4x^4-3\,\text{arcsinh}(cx)c^2x^2-c^3x^3\sqrt{c^2x^2+1}-6\,\text{arcsinh}(cx)+6cx\sqrt{c^2x^2+1}\right)}{9c^4\sqrt{\pi}\sqrt{c^2x^2+1}}$
parts	$a\left(\frac{x^2\sqrt{\pi c^2x^2+\pi}}{3\pi c^2}-\frac{2\sqrt{\pi c^2x^2+\pi}}{3\pi c^4}\right)+\frac{b\left(3\,\text{arcsinh}(cx)c^4x^4-3\,\text{arcsinh}(cx)c^2x^2-c^3x^3\sqrt{c^2x^2+1}-6\,\text{arcsinh}(cx)+6cx\sqrt{c^2x^2+1}\right)}{9c^4\sqrt{\pi}\sqrt{c^2x^2+1}}$

```
[In] int(x^3*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/3*x^2/Pi/c^2*(Pi*c^2*x^2+Pi)^(1/2)-2/3/Pi/c^4*(Pi*c^2*x^2+Pi)^(1/2))+1
/9*b/c^4/Pi^(1/2)/(c^2*x^2+1)^(1/2)*(3*arcsinh(c*x)*c^4*x^4-3*arcsinh(c*x)*
c^2*x^2-c^3*x^3*(c^2*x^2+1)^(1/2)-6*arcsinh(c*x)+6*c*x*(c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.35

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{3\sqrt{\pi+\pi c^2x^2}(bc^4x^4-bc^2x^2-2b)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}\left(3ac^4x^4-3ac^2x^2-(bc^3x^3-6bcx\right.}{9(\pi c^6x^2+\pi c^4)}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/9*(3*sqrt(pi + pi*c^2*x^2)*(b*c^4*x^4 - b*c^2*x^2 - 2*b)*log(c*x + sqrt(c
^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(3*a*c^4*x^4 - 3*a*c^2*x^2 - (b*c^3*x^
3 - 6*b*c*x)*sqrt(c^2*x^2 + 1) - 6*a))/(pi*c^6*x^2 + pi*c^4)
```

Sympy [A] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.27

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{a\left(\begin{cases}\frac{x^2\sqrt{c^2x^2+1}}{3c^2}-\frac{2\sqrt{c^2x^2+1}}{3c^4} & \text{for } c^2\neq 0\\ \frac{x^4}{4} & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}} + \frac{b\left(\begin{cases}-\frac{x^3}{9c}+\frac{x^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{3c^2}+\frac{2x}{3c^3}-\frac{2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{3c^4} & \text{for } c\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}}$$

```
[In] integrate(x**3*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] a*Piecewise((x**2*sqrt(c**2*x**2 + 1)/(3*c**2) - 2*sqrt(c**2*x**2 + 1)/(3*c
**4), Ne(c**2, 0)), (x**4/4, True))/sqrt(pi) + b*Piecewise((-x**3/(9*c) + x
**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3*c**2) + 2*x/(3*c**3) - 2*sqrt(c**2*x*
*2 + 1)*asinh(c*x)/(3*c**4), Ne(c, 0)), (0, True))/sqrt(pi)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.19

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{1}{3}\,b\left(\frac{\sqrt{\pi+\pi c^2x^2}x^2}{\pi c^2}-\frac{2\sqrt{\pi+\pi c^2x^2}}{\pi c^4}\right)\operatorname{arsinh}(cx) + \frac{1}{3}\,a\left(\frac{\sqrt{\pi+\pi c^2x^2}x^2}{\pi c^2}-\frac{2\sqrt{\pi+\pi c^2x^2}}{\pi c^4}\right)-\frac{(c^2x^3-6\,x)b}{9\sqrt{\pi}c^3}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/3*b*(sqrt(pi + pi*c^2*x^2)*x^2/(pi*c^2) - 2*sqrt(pi + pi*c^2*x^2)/(pi*c^4
))*arcsinh(c*x) + 1/3*a*(sqrt(pi + pi*c^2*x^2)*x^2/(pi*c^2) - 2*sqrt(pi + p
i*c^2*x^2)/(pi*c^4)) - 1/9*(c^2*x^3 - 6*x)*b/(sqrt(pi)*c^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\text{arcsinh}(cx))}{\sqrt{\pi + c^2\pi x^2}} dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\text{arcsinh}(cx))}{\sqrt{\pi + c^2\pi x^2}} dx = \int \frac{x^3 (a + b\,\text{asinh}(c\,x))}{\sqrt{\Pi\,c^2\,x^2 + \Pi}} dx$$

```
[In] int((x^3*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2), x)
```

3.83 $\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 75

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = -\frac{bx^2}{4c\sqrt{\pi}}+\frac{x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{2c^2\pi}-\frac{(a+b\text{arcsinh}(cx))^2}{4bc^3\sqrt{\pi}}$$

```
[Out] -1/4*b*x^2/c/Pi^(1/2)-1/4*(a+b*arcsinh(c*x))^2/b/c^3/Pi^(1/2)+1/2*x*(a+b*ar
csinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^2/Pi
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5812, 5783, 30}

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = -\frac{(a+b\text{arcsinh}(cx))^2}{4\sqrt{\pi}bc^3}+\frac{x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2\pi c^2}-\frac{bx^2}{4\sqrt{\pi}c}$$

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] -1/4*(b*x^2)/(c*Sqrt[Pi]) + (x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/
(2*c^2*Pi) - (a + b*ArcSinh[c*x])^2/(4*b*c^3*Sqrt[Pi])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{2c^2\pi} - \frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx}{2c^2} - \frac{b\int x\,dx}{2c\sqrt{\pi}} \\ &= -\frac{bx^2}{4c\sqrt{\pi}} + \frac{x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{2c^2\pi} - \frac{(a+b\text{arcsinh}(cx))^2}{4bc^3\sqrt{\pi}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{4acx\sqrt{1+c^2x^2} - 2b\text{arcsinh}(cx)^2 - b\cosh(2\text{arcsinh}(cx)) + \text{arcsinh}(cx)(-4a+2b\sinh(2\text{arcsinh}(cx)))}{8c^3\sqrt{\pi}}$$

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (4*a*c*x*Sqrt[1 + c^2*x^2] - 2*b*ArcSinh[c*x]^2 - b*Cosh[2*ArcSinh[c*x]] +
ArcSinh[c*x]*(-4*a + 2*b*Sinh[2*ArcSinh[c*x]]))/(8*c^3*Sqrt[Pi])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.43

method	result	size
default	$\frac{ax\sqrt{\pi c^2 x^2+\pi}}{2\pi c^2}-\frac{a\ln\left(\frac{\pi c^2 x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2 x^2+\pi}\right)}{2c^2\sqrt{\pi c^2}}-\frac{b\left(-2\operatorname{arcsinh}(cx)cx\sqrt{c^2x^2+1}+c^2x^2+\operatorname{arcsinh}(cx)^2+1\right)}{4\sqrt{\pi}\,c^3}$	107
parts	$\frac{ax\sqrt{\pi c^2 x^2+\pi}}{2\pi c^2}-\frac{a\ln\left(\frac{\pi c^2 x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2 x^2+\pi}\right)}{2c^2\sqrt{\pi c^2}}-\frac{b\left(-2\operatorname{arcsinh}(cx)cx\sqrt{c^2x^2+1}+c^2x^2+\operatorname{arcsinh}(cx)^2+1\right)}{4\sqrt{\pi}\,c^3}$	107

```
[In] int(x^2*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*a*x/Pi/c^2*(Pi*c^2*x^2+Pi)^(1/2)-1/2*a/c^2*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(
Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)-1/4*b/Pi^(1/2)*(-2*arcsinh(c*x)*c*x*(c
^2*x^2+1)^(1/2)+c^2*x^2+arcsinh(c*x)^2+1)/c^3
```

Fricas [F]

$$\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)x^2}{\sqrt{\pi+\pi c^2x^2}}\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
[Out] integral((b*x^2*arcsinh(c*x) + a*x^2)/sqrt(pi + pi*c^2*x^2), x)
```

Sympy [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.63

$$\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx=\frac{a\left(\begin{cases}\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\log\left(2c^2x+2\sqrt{c^2x^2+1}\sqrt{c^2}\right)}{2c^2\sqrt{c^2}} & \text{for } c^2\neq 0\\ \frac{x^3}{3} & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}}+\frac{b\left(\begin{cases}-\frac{x^2}{4c}+\frac{x\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{2c^2}-\frac{\operatorname{asinh}^2(cx)}{4c^3} & \text{for } c\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}}$$

```
[In] integrate(x**2*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
[Out] a*Piecewise((x*sqrt(c**2*x**2 + 1)/(2*c**2) - log(2*c**2*x + 2*sqrt(c**2*x*
*2 + 1)*sqrt(c**2))/(2*c**2*sqrt(c**2)), Ne(c**2, 0)), (x**3/3, True))/sqrt
(pi) + b*Piecewise((-x**2/(4*c) + x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(2*c**2)
 - asinh(c*x)**2/(4*c**3), Ne(c, 0)), (0, True))/sqrt(pi)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b\text{arcsinh}(cx))}{\sqrt{\pi + c^2\pi x^2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int \frac{x^2(a + b\text{arcsinh}(cx))}{\sqrt{\pi + c^2\pi x^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)x^2}{\sqrt{\pi + \pi c^2 x^2}}\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^2/sqrt(pi + pi*c^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b\text{arcsinh}(cx))}{\sqrt{\pi + c^2\pi x^2}}\,dx = \int \frac{x^2\,(a + b\,\text{asinh}(c\,x))}{\sqrt{\Pi\,c^2\,x^2 + \Pi}}\,dx$$

```
[In] int((x^2*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2),x)
```

```
[Out] int((x^2*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2), x)
```

3.84 $\int \frac{x(a+b\,\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 42

$$\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx=-\frac{bx}{c\sqrt{\pi}}+\frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{c^2\pi}$$

```
[Out] -b*x/c/Pi^(1/2)+(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^2/Pi
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5798, 8}

$$\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx=\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{\pi c^2}-\frac{bx}{\sqrt{\pi}c}$$

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] -((b*x)/(c*Sqrt[Pi])) + (Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(c^2*P
i)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
```

```
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{c^2\pi} - \frac{b\int 1\,dx}{c\sqrt{\pi}}$$
$$= -\frac{bx}{c\sqrt{\pi}} + \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{c^2\pi}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int \frac{x(a + b\text{arcsinh}(cx))}{\sqrt{\pi + c^2\pi x^2}}\,dx = \frac{-bcx + a\sqrt{1 + c^2x^2} + b\sqrt{1 + c^2x^2}\text{arcsinh}(cx)}{c^2\sqrt{\pi}}$$

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
[Out] (-(b*c*x) + a*Sqrt[1 + c^2*x^2] + b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(c^2*Sq
rt[Pi])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.71

method	result	size
default	$\frac{a\sqrt{\pi c^2x^2+\pi}}{\pi c^2}+\frac{b\left(\text{arcsinh}(cx)c^2x^2+\text{arcsinh}(cx)-cx\sqrt{c^2x^2+1}\right)}{c^2\sqrt{\pi}\sqrt{c^2x^2+1}}$	72
parts	$\frac{a\sqrt{\pi c^2x^2+\pi}}{\pi c^2}+\frac{b\left(\text{arcsinh}(cx)c^2x^2+\text{arcsinh}(cx)-cx\sqrt{c^2x^2+1}\right)}{c^2\sqrt{\pi}\sqrt{c^2x^2+1}}$	72

```
[In] int(x*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
[Out] a/Pi/c^2*(Pi*c^2*x^2+Pi)^(1/2)+b/c^2/Pi^(1/2)/(c^2*x^2+1)^(1/2)*(arcsinh(c*
x)*c^2*x^2+arcsinh(c*x)-c*x*(c^2*x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(38) = 76.

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.29

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{\sqrt{\pi+\pi c^2x^2}(bc^2x^2+b)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}\left(ac^2x^2-\sqrt{c^2x^2+1}bcx+a\right)}{\pi c^4x^2+\pi c^2}$$

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(pi + pi*c^2*x^2)*(b*c^2*x^2 + b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt(
pi + pi*c^2*x^2)*(a*c^2*x^2 - sqrt(c^2*x^2 + 1)*b*c*x + a))/(pi*c^4*x^2 + p
i*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.43

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{a\left(\begin{cases}\frac{\sqrt{c^2x^2+1}}{c^2} & \text{for } c^2\neq 0\\ \frac{x^2}{2} & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}} + \frac{b\left(\begin{cases}-\frac{x}{c}+\frac{\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{c^2} & \text{for } c\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}}$$

```
[In] integrate(x*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] a*Piecewise((sqrt(c**2*x**2 + 1)/c**2, Ne(c**2, 0)), (x**2/2, True))/sqrt(p
i) + b*Piecewise((-x/c + sqrt(c**2*x**2 + 1)*asinh(c*x)/c**2, Ne(c, 0)), (0
, True))/sqrt(pi)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.31

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = -\frac{bx}{\sqrt{\pi}c}+\frac{\sqrt{\pi+\pi c^2x^2}b\operatorname{arsinh}(cx)}{\pi c^2}+\frac{\sqrt{\pi+\pi c^2x^2}a}{\pi c^2}$$

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] -b*x/(sqrt(pi)*c) + sqrt(pi + pi*c^2*x^2)*b*arcsinh(c*x)/(pi*c^2) + sqrt(pi
 + pi*c^2*x^2)*a/(pi*c^2)
```

Giac [F]

$$\int \frac{x(a + b\text{arcsinh}(cx))}{\sqrt{\pi + c^2\pi x^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)x}{\sqrt{\pi + \pi c^2x^2}}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)*x/sqrt(pi + pi*c^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b\text{arcsinh}(cx))}{\sqrt{\pi + c^2\pi x^2}}\,dx = \int \frac{x\,(a + b\,\text{asinh}(c\,x))}{\sqrt{\Pi\,c^2\,x^2 + \Pi}}\,dx$$

```
[In] int((x*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2),x)
[Out] int((x*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2), x)
```

3.85 $\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 25

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx=\frac{(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{\pi}}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))^2/b/c/Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {5783}

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx=\frac{(a+b\text{arcsinh}(cx))^2}{2\sqrt{\pi}bc}$$

```
[In] Int[(a + b*ArcSinh[c*x])/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (a + b*ArcSinh[c*x])^2/(2*b*c*Sqrt[Pi])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\text{integral}=\frac{(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{\pi}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx=\frac{(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{\pi}}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/Sqrt[Pi + c^2*Pi*x^2],x]
[Out] (a + b*ArcSinh[c*x])^2/(2*b*c*Sqrt[Pi])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(21) = 42$.

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.12

method	result	size
default	$\frac{a\ln\left(\frac{\pi c^2 x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2 x^2+\pi}\right)}{\sqrt{\pi c^2}}+\frac{b\,\text{arcsinh}(cx)^2}{2c\sqrt{\pi}}$	53
parts	$\frac{a\ln\left(\frac{\pi c^2 x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2 x^2+\pi}\right)}{\sqrt{\pi c^2}}+\frac{b\,\text{arcsinh}(cx)^2}{2c\sqrt{\pi}}$	53

```
[In] int((a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
[Out] a*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/2*b/c/
Pi^(1/2)*arcsinh(c*x)^2
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{\pi+\pi c^2x^2}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas")
[Out] integral((b*arcsinh(c*x) + a)/sqrt(pi + pi*c^2*x^2), x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(19) = 38.

Time = 0.62 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.48

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx = \begin{cases} a\left(\begin{cases} \frac{\log\left(2\pi c^2x+2\sqrt{\pi}\sqrt{\pi c^2x^2+\pi}\sqrt{c^2}\right)}{\sqrt{\pi}\sqrt{c^2}} & \text{for}\ \pi c^2\neq 0 \\ \frac{x}{\sqrt{\pi}} & \text{otherwise} \end{cases}\right) & \text{for}\ b=0 \\ \frac{ax}{\sqrt{\pi}} & \text{for}\ c=0 \\ \frac{(a+b\,\text{asinh}\,(cx))^2}{2\sqrt{\pi}bc} & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] Piecewise((a*Piecewise((log(2*pi*c**2*x + 2*sqrt(pi)*sqrt(pi*c**2*x**2 + pi
)*sqrt(c**2))/(sqrt(pi)*sqrt(c**2)), Ne(pi*c**2, 0)), (x/sqrt(pi), True)),
Eq(b, 0)), (a*x/sqrt(pi), Eq(c, 0)), ((a + b*asinh(c*x))**2/(2*sqrt(pi)*b*c
), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{b\,\text{arsinh}\,(cx)^2}{2\,\sqrt{\pi}c} + \frac{a\,\text{arsinh}\,(cx)}{\sqrt{\pi}c}$$

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*arcsinh(c*x)^2/(sqrt(pi)*c) + a*arcsinh(c*x)/(sqrt(pi)*c)
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{\pi+\pi c^2x^2}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/sqrt(pi + pi*c^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{\pi + c^2\pi x^2}}\,dx = \int \frac{a + b\,\text{asinh}(c\,x)}{\sqrt{\Pi\,c^2\,x^2 + \Pi}}\,dx$$

```
[In] int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(1/2),x)
[Out] int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(1/2), x)
```

3.86 $\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 56

$$\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx = -\frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{\pi}} - \frac{b\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{\pi}} + \frac{b\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{\pi}}$$

```
[Out] -2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))/Pi^(1/2)-b*polylog(2,-
c*x-(c^2*x^2+1)^(1/2))/Pi^(1/2)+b*polylog(2,c*x+(c^2*x^2+1)^(1/2))/Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5816, 4267, 2317, 2438}

$$\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx = -\frac{2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\sqrt{\pi}} - \frac{b\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{\pi}} + \frac{b\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{\pi}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] (-2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[Pi] - (b*PolyLog[2,
-E^ArcSinh[c*x]])/Sqrt[Pi] + (b*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[Pi]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}(\int (a+bx)\text{csch}(x)\,dx, x, \text{arcsinh}(cx))}{\sqrt{\pi}} \\
&= -\frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{\pi}} \\
&\quad -\frac{b\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{\sqrt{\pi}} \\
&\quad +\frac{b\text{Subst}\left(\int \log\left(1+e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{\sqrt{\pi}} \\
&= -\frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{\pi}} \\
&\quad -\frac{b\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{\sqrt{\pi}} + \frac{b\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{\sqrt{\pi}} \\
&= -\frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{\pi}} \\
&\quad -\frac{b\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{\pi}} + \frac{b\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{\pi}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.71

$$\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{b\text{arcsinh}(cx)\log\left(1-e^{-\text{arcsinh}(cx)}\right)-b\text{arcsinh}(cx)\log\left(1+e^{-\text{arcsinh}(cx)}\right)+a\log(x)-a\log\left(\pi\left(1+\sqrt{1+c^2\right.\right.}{\sqrt{\pi}}$$

[In] Integrate[(a + b*ArcSinh[c*x])/(x*Sqrt[Pi + c^2*Pi*x^2]),x]

[Out] (b*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - b*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + a*Log[x] - a*Log[Pi*(1 + Sqrt[1 + c^2*x^2])] + b*PolyLog[2, -E^(-ArcSinh[c*x])] - b*PolyLog[2, E^(-ArcSinh[c*x])])/Sqrt[Pi]

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.11

method	result
default	$-\frac{a\,\text{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi\,c^2x^2+\pi}}\right)}{\sqrt{\pi}}+\frac{b\left(-\text{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)-\text{polylog}\left(2,-cx-\sqrt{c^2x^2+1}\right)+\text{arcsinh}(cx)\ln\left(1-cx-\sqrt{c^2x^2+\right.\right.}{\sqrt{\pi}}$
parts	$-\frac{a\,\text{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi\,c^2x^2+\pi}}\right)}{\sqrt{\pi}}+\frac{b\left(-\text{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)-\text{polylog}\left(2,-cx-\sqrt{c^2x^2+1}\right)+\text{arcsinh}(cx)\ln\left(1-cx-\sqrt{c^2x^2+\right.\right.}{\sqrt{\pi}}$

[In] int((a+b*arcsinh(c*x))/x/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)

[Out] -a/Pi^(1/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2))+b/Pi^(1/2)*(-arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-polylog(2,-c*x-(c^2*x^2+1)^(1/2))+arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+polylog(2,c*x+(c^2*x^2+1)^(1/2)))

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{\pi+\pi c^2x^2}x}\,dx$$

[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi*c^2*x^3 + pi*x), x)

Sympy [F]

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{x\sqrt{\pi + c^2\pi x^2}}\,dx = \frac{\int \frac{a}{x\sqrt{c^2x^2+1}}\,dx + \int \frac{b\operatorname{asinh}(cx)}{x\sqrt{c^2x^2+1}}\,dx}{\sqrt{\pi}}$$

```
[In] integrate((a+b*asinh(c*x))/x/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] (Integral(a/(x*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(x*sqrt(c**
2*x**2 + 1)), x))/sqrt(pi)
```

Maxima [F]

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{x\sqrt{\pi + c^2\pi x^2}}\,dx = \int \frac{b\operatorname{arsinh}(cx) + a}{\sqrt{\pi + \pi c^2x^2}x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(pi + pi*c^2*x^2)*x), x) - a*
arcsinh(1/(c*abs(x)))/sqrt(pi)
```

Giac [F]

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{x\sqrt{\pi + c^2\pi x^2}}\,dx = \int \frac{b\operatorname{arsinh}(cx) + a}{\sqrt{\pi + \pi c^2x^2}x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(pi + pi*c^2*x^2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{x\sqrt{\pi + c^2\pi x^2}}\,dx = \int \frac{a + b\operatorname{asinh}(c\,x)}{x\,\sqrt{\Pi\,c^2\,x^2 + \Pi}}\,dx$$

```
[In] int((a + b*asinh(c*x))/(x*(Pi + Pi*c^2*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x*(Pi + Pi*c^2*x^2)^(1/2)), x)
```

3.87 $\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 41

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{\pi+c^2\pi x^2}}\,dx=-\frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{\pi x}+\frac{bc\log(x)}{\sqrt{\pi}}$$

```
[Out] b*c*ln(x)/Pi^(1/2)-(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/Pi/x
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5800, 29}

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{\pi+c^2\pi x^2}}\,dx=\frac{bc\log(x)}{\sqrt{\pi}}-\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{\pi x}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] -((Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(Pi*x)) + (b*c*Log[x])/Sqrt[
Pi]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
```

```
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{\pi x} + \frac{(bc)\int \frac{1}{x}\,dx}{\sqrt{\pi}}$$
$$= -\frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{\pi x} + \frac{bc\log(x)}{\sqrt{\pi}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.27

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2\sqrt{\pi + c^2\pi x^2}}\,dx = \frac{-a\sqrt{1 + c^2x^2} - b\sqrt{1 + c^2x^2}\text{arcsinh}(cx) + bcx\log(x)}{\sqrt{\pi}x}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*Sqrt[Pi + c^2*Pi*x^2]),x]
[Out] (-(a*Sqrt[1 + c^2*x^2]) - b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + b*c*x*Log[x])/
(Sqrt[Pi]*x)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(37) = 74.

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.05

method	result	size
default	$-\frac{a\sqrt{\pi c^2x^2+\pi}}{\pi x}-\frac{bc\,\text{arcsinh}(cx)}{\sqrt{\pi}}-\frac{b\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}}{\sqrt{\pi}\,x}+\frac{bc\ln\left(\left(cx+\sqrt{c^2x^2+1}\right)^2-1\right)}{\sqrt{\pi}}$	84
parts	$-\frac{a\sqrt{\pi c^2x^2+\pi}}{\pi x}-\frac{bc\,\text{arcsinh}(cx)}{\sqrt{\pi}}-\frac{b\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}}{\sqrt{\pi}\,x}+\frac{bc\ln\left(\left(cx+\sqrt{c^2x^2+1}\right)^2-1\right)}{\sqrt{\pi}}$	84

```
[In] int((a+b*arcsinh(c*x))/x^2/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
[Out] -a/Pi/x*(Pi*c^2*x^2+Pi)^(1/2)-b*c/Pi^(1/2)*arcsinh(c*x)-b/Pi^(1/2)*arcsinh(
c*x)/x*(c^2*x^2+1)^(1/2)+b*c/Pi^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(37) = 74.

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 3.22

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{\sqrt{\pi}bcx\log\left(\frac{\pi+\pi c^2x^6+\pi c^2x^2+\pi x^4+\sqrt{\pi}\sqrt{\pi+\pi c^2x^2}\sqrt{c^2x^2+1}(x^4-1)}{c^2x^4+x^2}\right)-2\sqrt{\pi+\pi c^2x^2}b\log\left(cx+\sqrt{c^2x^2+1}\right)-2\sqrt{\pi+}}{2\pi x}$$

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/2*(sqrt(pi)*b*c*x*log((pi + pi*c^2*x^6 + pi*c^2*x^2 + pi*x^4 + sqrt(pi)*s
qrt(pi + pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*(x^4 - 1))/(c^2*x^4 + x^2)) - 2*sqrt
(pi + pi*c^2*x^2)*b*log(c*x + sqrt(c^2*x^2 + 1)) - 2*sqrt(pi + pi*c^2*x^2)*
a)/(pi*x)
```

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{\int \frac{a}{x^2\sqrt{c^2x^2+1}}\,dx+\int \frac{b\,\text{asinh}\,(cx)}{x^2\sqrt{c^2x^2+1}}\,dx}{\sqrt{\pi}}$$

```
[In] integrate((a+b*asinh(c*x))/x**2/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] (Integral(a/(x**2*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(x**2*sq
rt(c**2*x**2 + 1)), x))/sqrt(pi)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(37) = 74.

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.46

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{\pi+c^2\pi x^2}}\,dx = -\frac{\left(\sqrt{\pi}(-1)^{2\pi+2\pi c^2x^2}\log\left(2\pi c^2+\frac{2\pi}{x^2}\right)-\sqrt{\pi}\log\left(x^2+\frac{1}{c^2}\right)\right)bc}{2\pi} - \frac{\sqrt{\pi+\pi c^2x^2}b\,\text{arsinh}\,(cx)}{\pi x} - \frac{\sqrt{\pi+\pi c^2x^2}a}{\pi x}$$

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/2*(sqrt(pi)*(-1)^(2*pi + 2*pi*c^2*x^2)*log(2*pi*c^2 + 2*pi/x^2) - sqrt(p
i)*log(x^2 + 1/c^2))*b*c/pi - sqrt(pi + pi*c^2*x^2)*b*arcsinh(c*x)/(pi*x) -
 sqrt(pi + pi*c^2*x^2)*a/(pi*x)
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{\pi+c^2\pi x^2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{\pi+\pi c^2x^2}x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(pi + pi*c^2*x^2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{\pi+c^2\pi x^2}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{x^2\,\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

```
[In] int((a + b*asinh(c*x))/(x^2*(Pi + Pi*c^2*x^2)^(1/2)),x)
[Out] int((a + b*asinh(c*x))/(x^2*(Pi + Pi*c^2*x^2)^(1/2)), x)
```

3.88 $\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 115

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{\pi+c^2\pi x^2}}\,dx=-\frac{bc}{2\sqrt{\pi}x}-\frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{2\pi x^2}+\frac{c^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{\pi}}+\frac{bc^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2\sqrt{\pi}}-\frac{bc^2\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2\sqrt{\pi}}$$

```
[Out] -1/2*b*c/x/Pi^(1/2)+c^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))/P
i^(1/2)+1/2*b*c^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))/Pi^(1/2)-1/2*b*c^2*poly
log(2,c*x+(c^2*x^2+1)^(1/2))/Pi^(1/2)-1/2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi
)^(1/2)/Pi/x^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5809, 5816, 4267, 2317, 2438, 30}

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{\pi+c^2\pi x^2}}\,dx=\frac{c^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\sqrt{\pi}}-\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2\pi x^2}+\frac{bc^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2\sqrt{\pi}}-\frac{bc^2\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2\sqrt{\pi}}-\frac{bc}{2\sqrt{\pi}x}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] -1/2*(b*c)/(Sqrt[Pi]*x) - (Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(2*P
i*x^2) + (c^2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[Pi] + (b*c
^2*PolyLog[2, -E^ArcSinh[c*x]])/(2*Sqrt[Pi]) - (b*c^2*PolyLog[2, E^ArcSinh[
c*x]])/(2*Sqrt[Pi])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{2\pi x^2} - \frac{1}{2}c^2\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx + \frac{(bc)\int\frac{1}{x^2}\,dx}{2\sqrt{\pi}}$$

$$= -\frac{bc}{2\sqrt{\pi}x} - \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{2\pi x^2} - \frac{c^2\text{Subst}\left(\int(a+bx)\text{csch}(x)\,dx, x, \text{arcsinh}(cx)\right)}{2\sqrt{\pi}}$$

$$= -\frac{bc}{2\sqrt{\pi}x} - \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{2\pi x^2} + \frac{c^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{\pi}} + \frac{(bc^2)\,\text{Subst}\left(\int\log\left(1-e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{2\sqrt{\pi}} - \frac{(bc^2)\,\text{Subst}\left(\int\log\left(1+e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{2\sqrt{\pi}}$$

$$= -\frac{bc}{2\sqrt{\pi}x} - \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{2\pi x^2} + \frac{c^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{\pi}} + \frac{(bc^2)\,\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{2\sqrt{\pi}} - \frac{(bc^2)\,\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{2\sqrt{\pi}}$$

$$= -\frac{bc}{2\sqrt{\pi}x} - \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{2\pi x^2} + \frac{c^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{\pi}} + \frac{bc^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2\sqrt{\pi}} - \frac{bc^2\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2\sqrt{\pi}}$$

Mathematica [A] (verified)

Time = 1.95 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.61

$$\int\frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{-\frac{4a\sqrt{1+c^2x^2}}{x^2} - 4ac^2\log(x) + 4ac^2\log\left(\pi\left(1+\sqrt{1+c^2x^2}\right)\right) + bc^2\left(-2\coth\left(\frac{1}{2}\text{arcsinh}(cx)\right) - \text{arcsinh}(cx)\text{csch}\right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*Sqrt[Pi + c^2*Pi*x^2]),x]
[Out] ((-4*a*Sqrt[1 + c^2*x^2])/x^2 - 4*a*c^2*Log[x] + 4*a*c^2*Log[Pi*(1 + Sqrt[1
 + c^2*x^2])] + b*c^2*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[
c*x]/2]^2 - 4*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] + 4*ArcSinh[c*x]*Log[
1 + E^(-ArcSinh[c*x])] - 4*PolyLog[2, -E^(-ArcSinh[c*x])] + 4*PolyLog[2, E^
(-ArcSinh[c*x])] - ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x
]/2]))/(8*Sqrt[Pi])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.88

method	result
default	$a\left(-\frac{\sqrt{\pi\,c^2x^2+\pi}}{2\pi\,x^2}+\frac{c^2\,\text{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi\,c^2x^2+\pi}}\right)}{2\sqrt{\pi}}\right)+b\left(-\frac{\text{arcsinh}(cx)c^2x^2+cx\sqrt{c^2x^2+1}+\text{arcsinh}(cx)}{2\sqrt{\pi}\,\sqrt{c^2x^2+1}\,x^2}+\frac{c^2\,\text{arcsinh}(cx)\ln\left(1+cx\right.}{2\sqrt{\pi}}\right.$
parts	$a\left(-\frac{\sqrt{\pi\,c^2x^2+\pi}}{2\pi\,x^2}+\frac{c^2\,\text{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi\,c^2x^2+\pi}}\right)}{2\sqrt{\pi}}\right)+b\left(-\frac{\text{arcsinh}(cx)c^2x^2+cx\sqrt{c^2x^2+1}+\text{arcsinh}(cx)}{2\sqrt{\pi}\,\sqrt{c^2x^2+1}\,x^2}+\frac{c^2\,\text{arcsinh}(cx)\ln\left(1+cx\right.}{2\sqrt{\pi}}\right.$

```
[In] int((a+b*arcsinh(c*x))/x^3/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/2/Pi/x^2*(Pi*c^2*x^2+Pi)^(1/2)+1/2/Pi^(1/2)*c^2*arctanh(Pi^(1/2)/(Pi*
c^2*x^2+Pi)^(1/2)))+b*(-1/2/Pi^(1/2)/(c^2*x^2+1)^(1/2)*(arcsinh(c*x)*c^2*x^
2+c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x))/x^2+1/2*c^2/Pi^(1/2)*arcsinh(c*x)*ln(
1+c*x+(c^2*x^2+1)^(1/2))+1/2*c^2/Pi^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))
-1/2*c^2/Pi^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-1/2*c^2/Pi^(1/2)
*polylog(2,c*x+(c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{\pi+c^2\pi x^2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{\pi+\pi c^2x^2}x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi*c^2*x^5 + pi*x^3),
x)
```

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{\pi+c^2\pi x^2}}\,dx=\frac{\int \frac{a}{x^3\sqrt{c^2x^2+1}}\,dx+\int \frac{b\,\text{asinh}\,(cx)}{x^3\sqrt{c^2x^2+1}}\,dx}{\sqrt{\pi}}$$

```
[In] integrate((a+b*asinh(c*x))/x**3/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] (Integral(a/(x**3*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(x**3*sq
rt(c**2*x**2 + 1)), x))/sqrt(pi)
```

Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^3\sqrt{\pi + c^2\pi x^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{\sqrt{\pi + \pi c^2x^2}x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
[Out] 1/2*(c^2*arcsinh(1/(c*abs(x)))/sqrt(pi) - sqrt(pi + pi*c^2*x^2)/(pi*x^2))*a
 + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(pi + pi*c^2*x^2)*x^3), x)
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^3\sqrt{\pi + c^2\pi x^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{\sqrt{\pi + \pi c^2x^2}x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(pi + pi*c^2*x^2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{x^3\sqrt{\pi + c^2\pi x^2}}\,dx = \int \frac{a + b\,\text{asinh}(c\,x)}{x^3\,\sqrt{\Pi\,c^2\,x^2 + \Pi}}\,dx$$

```
[In] int((a + b*asinh(c*x))/(x^3*(Pi + Pi*c^2*x^2)^(1/2)),x)
[Out] int((a + b*asinh(c*x))/(x^3*(Pi + Pi*c^2*x^2)^(1/2)), x)
```

3.89 $\int \frac{a+b\text{arcsinh}(cx)}{x^4\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 97

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\sqrt{\pi+c^2\pi x^2}}\,dx=-\frac{bc}{6\sqrt{\pi}x^2}-\frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{3\pi x^3}+\frac{2c^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{3\pi x}-\frac{2bc^3\log(x)}{3\sqrt{\pi}}$$

```
[Out] -1/6*b*c/x^2/Pi^(1/2)-2/3*b*c^3*ln(x)/Pi^(1/2)-1/3*(a+b*arcsinh(c*x))*(Pi*c
^2*x^2+Pi)^(1/2)/Pi/x^3+2/3*c^2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/Pi
/x
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5809, 5800, 29, 30}

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\sqrt{\pi+c^2\pi x^2}}\,dx=\frac{2c^2\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{3\pi x}-\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{3\pi x^3}-\frac{2bc^3\log(x)}{3\sqrt{\pi}}-\frac{bc}{6\sqrt{\pi}x^2}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] -1/6*(b*c)/(Sqrt[Pi]*x^2) - (Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(3
*Pi*x^3) + (2*c^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(3*Pi*x) - (2
*b*c^3*Log[x])/(3*Sqrt[Pi])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{3\pi x^3}-\frac{1}{3}\left(2c^2\right)\int\frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{\pi+c^2\pi x^2}}\,dx+\frac{(bc)\int\frac{1}{x^3}\,dx}{3\sqrt{\pi}}\\
&= -\frac{bc}{6\sqrt{\pi}x^2}-\frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{3\pi x^3}\\
&\quad+\frac{2c^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{3\pi x}-\frac{(2bc^3)\int\frac{1}{x}\,dx}{3\sqrt{\pi}}\\
&= -\frac{bc}{6\sqrt{\pi}x^2}-\frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{3\pi x^3}+\frac{2c^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{3\pi x}-\frac{2bc^3\log(x)}{3\sqrt{\pi}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\sqrt{\pi+c^2\pi x^2}}\,dx$$
$$=\frac{-bcx+6bc^3x^3-2a\sqrt{1+c^2x^2}+4ac^2x^2\sqrt{1+c^2x^2}+2b\sqrt{1+c^2x^2}(-1+2c^2x^2)\operatorname{arcsinh}(cx)-4bc^3x^3\log(x)}{6\sqrt{\pi}x^3}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] (-(b*c*x) + 6*b*c^3*x^3 - 2*a*Sqrt[1 + c^2*x^2] + 4*a*c^2*x^2*Sqrt[1 + c^2*
x^2] + 2*b*Sqrt[1 + c^2*x^2]*(-1 + 2*c^2*x^2)*ArcSinh[c*x] - 4*b*c^3*x^3*Lo
g[x])/(6*Sqrt[Pi]*x^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(81) = 162.

Time = 0.19 (sec) , antiderivative size = 373, normalized size of antiderivative = 3.85

method	result
default	$a\left(-\frac{\sqrt{\pi c^2x^2+\pi}}{3\pi x^3}+\frac{2c^2\sqrt{\pi c^2x^2+\pi}}{3\pi x}\right)+\frac{4b c^3 \operatorname{arcsinh}(cx)}{3\sqrt{\pi}}-\frac{2b x^4 c^7}{3\sqrt{\pi}(3c^2x^2-1)}+\frac{2b x^2(c^2x^2+1)c^5}{3\sqrt{\pi}(3c^2x^2-1)}-\frac{2b x^2 \operatorname{arcsinh}(cx)c^5}{\sqrt{\pi}(3c^2x^2-1)}+2b$
parts	$a\left(-\frac{\sqrt{\pi c^2x^2+\pi}}{3\pi x^3}+\frac{2c^2\sqrt{\pi c^2x^2+\pi}}{3\pi x}\right)+\frac{4b c^3 \operatorname{arcsinh}(cx)}{3\sqrt{\pi}}-\frac{2b x^4 c^7}{3\sqrt{\pi}(3c^2x^2-1)}+\frac{2b x^2(c^2x^2+1)c^5}{3\sqrt{\pi}(3c^2x^2-1)}-\frac{2b x^2 \operatorname{arcsinh}(cx)c^5}{\sqrt{\pi}(3c^2x^2-1)}+2b$

```
[In] int((a+b*arcsinh(c*x))/x^4/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/3/Pi/x^3*(Pi*c^2*x^2+Pi)^(1/2)+2/3/Pi*c^2/x*(Pi*c^2*x^2+Pi)^(1/2))+4/
3*b*c^3/Pi^(1/2)*arcsinh(c*x)-2/3*b/Pi^(1/2)/(3*c^2*x^2-1)*x^4*c^7+2/3*b/Pi
^(1/2)/(3*c^2*x^2-1)*x^2*(c^2*x^2+1)*c^5-2*b/Pi^(1/2)/(3*c^2*x^2-1)*x^2*arc
sinh(c*x)*c^5+2*b/Pi^(1/2)/(3*c^2*x^2-1)*x*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c
^4-2/3*b/Pi^(1/2)/(3*c^2*x^2-1)*(c^2*x^2+1)*c^3+2/3*b/Pi^(1/2)/(3*c^2*x^2-1
)*arcsinh(c*x)*c^3-5/3*b/Pi^(1/2)/(3*c^2*x^2-1)/x*(c^2*x^2+1)^(1/2)*arcsinh
(c*x)*c^2+1/6*b/Pi^(1/2)/(3*c^2*x^2-1)/x^2*(c^2*x^2+1)*c+1/3*b/Pi^(1/2)/(3*
c^2*x^2-1)/x^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)-2/3*b*c^3/Pi^(1/2)*ln((c*x+(c
^2*x^2+1)^(1/2))^2-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(81) = 162$.

Time = 0.30 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.29

$$\int \frac{a + b\text{arcsinh}(cx)}{x^4\sqrt{\pi + c^2\pi x^2}}\,dx$$
$$= \frac{2\sqrt{\pi + \pi c^2x^2}(2\,bc^4x^4 + bc^2x^2 - b)\log\left(cx + \sqrt{c^2x^2+1}\right) + 2\sqrt{\pi}(bc^5x^5 + bc^3x^3)\log\left(\frac{\pi+\pi c^2x^6+\pi c^2x^2+\pi x^4-\sqrt{\pi}}{c^2x}\right.}{6\left(\pi c^2x^5 + \pi x^3\right)}$$

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/6*(2*sqrt(pi + pi*c^2*x^2)*(2*b*c^4*x^4 + b*c^2*x^2 - b)*log(c*x + sqrt(c
^2*x^2 + 1)) + 2*sqrt(pi)*(b*c^5*x^5 + b*c^3*x^3)*log((pi + pi*c^2*x^6 + pi
*c^2*x^2 + pi*x^4 - sqrt(pi)*sqrt(pi + pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*(x^4 -
 1))/(c^2*x^4 + x^2)) + sqrt(pi + pi*c^2*x^2)*(4*a*c^4*x^4 + 2*a*c^2*x^2 +
(b*c*x^3 - b*c*x)*sqrt(c^2*x^2 + 1) - 2*a))/(pi*c^2*x^5 + pi*x^3)
```

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^4\sqrt{\pi + c^2\pi x^2}}\,dx = \frac{\int \frac{a}{x^4\sqrt{c^2x^2+1}}\,dx + \int \frac{b\,\text{asinh}\,(cx)}{x^4\sqrt{c^2x^2+1}}\,dx}{\sqrt{\pi}}$$

```
[In] integrate((a+b*asinh(c*x))/x**4/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] (Integral(a/(x**4*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(x**4*sq
rt(c**2*x**2 + 1)), x))/sqrt(pi)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.25

$$\int \frac{a + b\text{arcsinh}(cx)}{x^4\sqrt{\pi + c^2\pi x^2}}\,dx = -\frac{1}{6}\left(\frac{4\,c^2\log(x)}{\sqrt{\pi}} + \frac{1}{\sqrt{\pi}x^2}\right)bc + \frac{1}{3}\,b\left(\frac{2\sqrt{\pi + \pi c^2x^2}c^2}{\pi x} - \frac{\sqrt{\pi + \pi c^2x^2}}{\pi x^3}\right)\text{arsinh}\,(cx) + \frac{1}{3}\,a\left(\frac{2\sqrt{\pi + \pi c^2x^2}c^2}{\pi x} - \frac{\sqrt{\pi + \pi c^2x^2}}{\pi x^3}\right)$$

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/6*(4*c^2*log(x)/sqrt(pi) + 1/(sqrt(pi)*x^2))*b*c + 1/3*b*(2*sqrt(pi + pi
*c^2*x^2)*c^2/(pi*x) - sqrt(pi + pi*c^2*x^2)/(pi*x^3))*arcsinh(c*x) + 1/3*a
*(2*sqrt(pi + pi*c^2*x^2)*c^2/(pi*x) - sqrt(pi + pi*c^2*x^2)/(pi*x^3))
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^4\sqrt{\pi + c^2\pi x^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{\sqrt{\pi + \pi c^2x^2}x^4}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(pi + pi*c^2*x^2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{x^4\sqrt{\pi + c^2\pi x^2}}\,dx = \int \frac{a + b\,\text{asinh}(c\,x)}{x^4\,\sqrt{\Pi\,c^2\,x^2 + \Pi}}\,dx$$

```
[In] int((a + b*asinh(c*x))/(x^4*(Pi + Pi*c^2*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^4*(Pi + Pi*c^2*x^2)^(1/2)), x)
```

3.90 $\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 137

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{5bx}{3c^5\pi^{3/2}}-\frac{bx^3}{9c^3\pi^{3/2}}-\frac{a+b\operatorname{arcsinh}(cx)}{c^6\pi\sqrt{\pi+c^2\pi x^2}} - \frac{2\sqrt{\pi+c^2\pi x^2}(a+b\operatorname{arcsinh}(cx))}{c^6\pi^2}+\frac{(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c^6\pi^3}+\frac{b\arctan(cx)}{c^6\pi^{3/2}}$$

```
[Out] 5/3*b*x/c^5/Pi^(3/2)-1/9*b*x^3/c^3/Pi^(3/2)+1/3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*
arcsinh(c*x))/c^6/Pi^3+b*arctan(c*x)/c^6/Pi^(3/2)+(-a-b*arcsinh(c*x))/c^6/P
i/(Pi*c^2*x^2+Pi)^(1/2)-2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^6/Pi^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {272, 45, 5804, 12, 1167, 209}

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{(\pi c^2x^2+\pi)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3\pi^3c^6} - \frac{2\sqrt{\pi c^2x^2+\pi}(a+b\operatorname{arcsinh}(cx))}{\pi^2c^6}-\frac{a+b\operatorname{arcsinh}(cx)}{\pi c^6\sqrt{\pi c^2x^2+\pi}}+\frac{b\arctan(cx)}{\pi^{3/2}c^6}+\frac{5bx}{3\pi^{3/2}c^5}-\frac{bx^3}{9\pi^{3/2}c^3}$$

```
[In] Int[(x^5*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (5*b*x)/(3*c^5*Pi^(3/2)) - (b*x^3)/(9*c^3*Pi^(3/2)) - (a + b*ArcSinh[c*x])/
(c^6*Pi*Sqrt[Pi + c^2*Pi*x^2]) - (2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*
x]))/(c^6*Pi^2) + ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^6*Pi^
3) + (b*ArcTan[c*x])/(c^6*Pi^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\text{integral} = -\frac{a+b\operatorname{arcsinh}(cx)}{c^6\pi\sqrt{\pi+c^2\pi x^2}} - \frac{2\sqrt{\pi+c^2\pi x^2}(a+b\operatorname{arcsinh}(cx))}{c^6\pi^2} + \frac{\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c^6\pi^3} - \left(bc\sqrt{\pi}\right)\int \frac{-8-4c^2x^2+c^4x^4}{3c^6\pi^2\left(1+c^2x^2\right)}\,dx$$

$$= -\frac{a+b\operatorname{arcsinh}(cx)}{c^6\pi\sqrt{\pi+c^2\pi x^2}} - \frac{2\sqrt{\pi+c^2\pi x^2}(a+b\operatorname{arcsinh}(cx))}{c^6\pi^2} + \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c^6\pi^3} - \frac{b\int\frac{-8-4c^2x^2+c^4x^4}{1+c^2x^2}\,dx}{3c^5\pi^{3/2}}$$

$$= -\frac{a+b\operatorname{arcsinh}(cx)}{c^6\pi\sqrt{\pi+c^2\pi x^2}} - \frac{2\sqrt{\pi+c^2\pi x^2}(a+b\operatorname{arcsinh}(cx))}{c^6\pi^2} + \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c^6\pi^3} - \frac{b\int\left(-5+c^2x^2-\frac{3}{1+c^2x^2}\right)\,dx}{3c^5\pi^{3/2}}$$

$$= \frac{5bx}{3c^5\pi^{3/2}} - \frac{bx^3}{9c^3\pi^{3/2}} - \frac{a+b\operatorname{arcsinh}(cx)}{c^6\pi\sqrt{\pi+c^2\pi x^2}} - \frac{2\sqrt{\pi+c^2\pi x^2}(a+b\operatorname{arcsinh}(cx))}{c^6\pi^2} + \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c^6\pi^3} + \frac{b\int\frac{1}{1+c^2x^2}\,dx}{c^5\pi^{3/2}}$$

$$= \frac{5bx}{3c^5\pi^{3/2}} - \frac{bx^3}{9c^3\pi^{3/2}} - \frac{a+b\operatorname{arcsinh}(cx)}{c^6\pi\sqrt{\pi+c^2\pi x^2}} - \frac{2\sqrt{\pi+c^2\pi x^2}(a+b\operatorname{arcsinh}(cx))}{c^6\pi^2} + \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c^6\pi^3} + \frac{b\arctan(cx)}{c^6\pi^{3/2}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int\frac{x^5(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{-24a-12ac^2x^2+3ac^4x^4+15bcx\sqrt{1+c^2x^2}-bc^3x^3\sqrt{1+c^2x^2}+3b(-8-4c}{9c^6\pi^{3/2}\sqrt{1+c^2x^2}}$$

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (-24*a - 12*a*c^2*x^2 + 3*a*c^4*x^4 + 15*b*c*x*Sqrt[1 + c^2*x^2] - b*c^3*x^
3*Sqrt[1 + c^2*x^2] + 3*b*(-8 - 4*c^2*x^2 + c^4*x^4)*ArcSinh[c*x] + 9*b*Sqr
t[1 + c^2*x^2]*ArcTan[c*x])/(9*c^6*Pi^(3/2)*Sqrt[1 + c^2*x^2])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.04

method	result
default	$a\left(\frac{x^4}{3\pi c^2\sqrt{\pi c^2x^2+\pi}}-\frac{4\left(\frac{x^2}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{2}{\pi c^4\sqrt{\pi c^2x^2+\pi}}\right)}{3c^2}\right)-\frac{ib\left(3i\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^4c^4-ix^5c^5-12i\operatorname{arcsinh}(cx)\sqrt{c^2x^2+\right.}{}$
parts	$a\left(\frac{x^4}{3\pi c^2\sqrt{\pi c^2x^2+\pi}}-\frac{4\left(\frac{x^2}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{2}{\pi c^4\sqrt{\pi c^2x^2+\pi}}\right)}{3c^2}\right)-\frac{ib\left(3i\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^4c^4-ix^5c^5-12i\operatorname{arcsinh}(cx)\sqrt{c^2x^2+\right.}{}$

```
[In] int(x^5*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/3*x^4/Pi/c^2/(Pi*c^2*x^2+Pi)^(1/2)-4/3/c^2*(x^2/Pi/c^2/(Pi*c^2*x^2+Pi)
^(1/2)+2/Pi/c^4/(Pi*c^2*x^2+Pi)^(1/2)))-1/9*I*b*(3*I*arcsinh(c*x)*(c^2*x^2+
1)^(1/2)*x^4*c^4-I*x^5*c^5-12*I*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2+14*I
*x^3*c^3-9*ln(c*x+(c^2*x^2+1)^(1/2)+I)*x^2*c^2+9*ln(c*x+(c^2*x^2+1)^(1/2)-I
)*x^2*c^2-24*I*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+15*I*c*x-9*ln(c*x+(c^2*x^2+1)
^(1/2)+I)+9*ln(c*x+(c^2*x^2+1)^(1/2)-I))/Pi^(3/2)/c^6/(c^2*x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.43

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx =$$

$$-\frac{9\sqrt{\pi}(bc^2x^2+b)\arctan\left(-\frac{2\sqrt{\pi}\sqrt{\pi+\pi c^2x^2}\sqrt{c^2x^2+1}cx}{\pi-\pi c^4x^4}\right)-6\sqrt{\pi+\pi c^2x^2}(bc^4x^4-4bc^2x^2-8b)\log\left(cx+\sqrt{c^2x^2+\right.}{18\left(\pi^2c^8x^2+\pi^2c^6\right)}$$

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/18*(9*sqrt(pi)*(b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi + pi*c^2*x^2)*
sqrt(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) - 6*sqrt(pi + pi*c^2*x^2)*(b*c^4*x
^4 - 4*b*c^2*x^2 - 8*b)*log(c*x + sqrt(c^2*x^2 + 1)) - 2*sqrt(pi + pi*c^2*x
^2)*(3*a*c^4*x^4 - 12*a*c^2*x^2 - (b*c^3*x^3 - 15*b*c*x)*sqrt(c^2*x^2 + 1)
- 24*a))/(pi^2*c^8*x^2 + pi^2*c^6)
```

Sympy [F]

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{\int \frac{ax^5}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{bx^5\operatorname{asinh}(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

```
[In] integrate(x**5*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a*x**5/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x)
+ Integral(b*x**5*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**
2 + 1)), x))/pi**(3/2)
```

Maxima [F]

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x^5}{(\pi+\pi c^2x^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/3*a*(x^4/(pi*sqrt(pi + pi*c^2*x^2)*c^2) - 4*x^2/(pi*sqrt(pi + pi*c^2*x^2)
*c^4) - 8/(pi*sqrt(pi + pi*c^2*x^2)*c^6)) + 1/3*b*((sqrt(pi)*c^4*x^4 - 4*sq
rt(pi)*c^2*x^2 - 8*sqrt(pi))*log(c*x + sqrt(c^2*x^2 + 1))/(pi^2*sqrt(c^2*x^
2 + 1)*c^6) - integrate((sqrt(pi)*c^4*x^4 - 4*sqrt(pi)*c^2*x^2 - 8*sqrt(pi)
)/(sqrt(c^2*x^2 + 1)*x), x)/(pi^2*c^6) + 3*integrate(1/3*(sqrt(pi)*c^4*x^4
- 4*sqrt(pi)*c^2*x^2 - 8*sqrt(pi))/(pi^2*c^9*x^4 + pi^2*c^7*x^2 + (pi^2*c^8
*x^3 + pi^2*c^6*x)*sqrt(c^2*x^2 + 1)), x))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\int \frac{x^5\,(a+b\,\text{asinh}(c\,x))}{(\Pi\,c^2\,x^2+\Pi)^{3/2}}\,dx$$

```
[In] int((x^5*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2),x)
[Out] int((x^5*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2), x)
```

3.91 $\int \frac{x^4(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 131

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = -\frac{bx^2}{4c^3\pi^{3/2}} - \frac{x^3(a+b\text{arcsinh}(cx))}{c^2\pi\sqrt{\pi+c^2\pi x^2}} + \frac{3x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{2c^4\pi^2} - \frac{3(a+b\text{arcsinh}(cx))^2}{4bc^5\pi^{3/2}} - \frac{b\log(1+c^2x^2)}{2c^5\pi^{3/2}}$$

```
[Out] -1/4*b*x^2/c^3/Pi^(3/2)-3/4*(a+b*arcsinh(c*x))^2/b/c^5/Pi^(3/2)-1/2*b*ln(c^
2*x^2+1)/c^5/Pi^(3/2)-x^3*(a+b*arcsinh(c*x))/c^2/Pi/(Pi*c^2*x^2+Pi)^(1/2)+3
/2*x*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^4/Pi^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5810, 5812, 5783, 30, 272, 45}

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = -\frac{3(a+b\text{arcsinh}(cx))^2}{4\pi^{3/2}bc^5} - \frac{x^3(a+b\text{arcsinh}(cx))}{\pi c^2\sqrt{\pi c^2x^2+\pi}} + \frac{3x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2\pi^2c^4} - \frac{bx^2}{4\pi^{3/2}c^3} - \frac{b\log(c^2x^2+1)}{2\pi^{3/2}c^5}$$

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] -1/4*(b*x^2)/(c^3*Pi^(3/2)) - (x^3*(a + b*ArcSinh[c*x]))/(c^2*Pi*Sqrt[Pi +
c^2*Pi*x^2]) + (3*x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(2*c^4*Pi^2
) - (3*(a + b*ArcSinh[c*x])^2)/(4*b*c^5*Pi^(3/2)) - (b*Log[1 + c^2*x^2])/(2
*c^5*Pi^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{x^3(a+b\text{arcsinh}(cx))}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{b\int\frac{x^3}{1+c^2x^2}\,dx}{c\pi^{3/2}}+\frac{3\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx}{c^2\pi}$$

$$=-\frac{x^3(a+b\text{arcsinh}(cx))}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{3x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{2c^4\pi^2}$$
$$-\frac{(3b)\int x\,dx}{2c^3\pi^{3/2}}+\frac{b\text{Subst}\left(\int\frac{x}{1+c^2x}\,dx,x,x^2\right)}{2c\pi^{3/2}}-\frac{3\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx}{2c^4\pi}$$

$$=-\frac{3bx^2}{4c^3\pi^{3/2}}-\frac{x^3(a+b\text{arcsinh}(cx))}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{3x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{2c^4\pi^2}$$
$$-\frac{3(a+b\text{arcsinh}(cx))^2}{4bc^5\pi^{3/2}}+\frac{b\text{Subst}\left(\int\left(\frac{1}{c^2}-\frac{1}{c^2(1+c^2x)}\right)dx,x,x^2\right)}{2c\pi^{3/2}}$$

$$=-\frac{bx^2}{4c^3\pi^{3/2}}-\frac{x^3(a+b\text{arcsinh}(cx))}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{3x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{2c^4\pi^2}$$
$$-\frac{3(a+b\text{arcsinh}(cx))^2}{4bc^5\pi^{3/2}}-\frac{b\log(1+c^2x^2)}{2c^5\pi^{3/2}}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.12

$$\int\frac{x^4(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\frac{12acx+4ac^3x^3-6b\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2-b\sqrt{1+c^2x^2}\cosh(2\text{arcsinh}(cx))}{8c}$$

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
[Out] (12*a*c*x + 4*a*c^3*x^3 - 6*b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 - b*Sqrt[1 +
 c^2*x^2]*Cosh[2*ArcSinh[c*x]] - 4*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] + A
rcSinh[c*x]*(9*b*c*x - 12*a*Sqrt[1 + c^2*x^2] + b*Sinh[3*ArcSinh[c*x]]))/(8
*c^5*Pi^(3/2)*Sqrt[1 + c^2*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(113) = 226.

Time = 0.29 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.99

method	result
default	$\frac{a\,x^{3}}{2\pi\,c^{2}\sqrt{\pi\,c^{2}x^{2}+\pi}}+\frac{3ax}{2c^{4}\pi\sqrt{\pi\,c^{2}x^{2}+\pi}}-\frac{3a\ln\left(\frac{\pi\,c^{2}x}{\sqrt{\pi\,c^{2}}}+\sqrt{\pi\,c^{2}x^{2}+\pi}\right)}{2c^{4}\pi\sqrt{\pi\,c^{2}}}-\frac{b\left(-4\,\mathrm{arcsinh}(cx)\sqrt{c^{2}x^{2}+1}\,x^{3}c^{3}+2c^{4}x^{4}+6\,\mathrm{arcsinh}(cx)^{2}x^{2}\right.}{}$
parts	$\frac{a\,x^{3}}{2\pi\,c^{2}\sqrt{\pi\,c^{2}x^{2}+\pi}}+\frac{3ax}{2c^{4}\pi\sqrt{\pi\,c^{2}x^{2}+\pi}}-\frac{3a\ln\left(\frac{\pi\,c^{2}x}{\sqrt{\pi\,c^{2}}}+\sqrt{\pi\,c^{2}x^{2}+\pi}\right)}{2c^{4}\pi\sqrt{\pi\,c^{2}}}-\frac{b\left(-4\,\mathrm{arcsinh}(cx)\sqrt{c^{2}x^{2}+1}\,x^{3}c^{3}+2c^{4}x^{4}+6\,\mathrm{arcsinh}(cx)^{2}x^{2}\right.}{}$

```
[In] int(x^4*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x^3/Pi/c^2/(Pi*c^2*x^2+Pi)^(1/2)+3/2*a/c^4*x/Pi/(Pi*c^2*x^2+Pi)^(1/2)
-3/2*a/c^4/Pi*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1
/2)-1/8*b*(-4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3+2*c^4*x^4+6*arcsinh(c*
x)^2*x^2*c^2-8*arcsinh(c*x)*c^2*x^2+8*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^2*c
^2-12*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)+3*c^2*x^2+6*arcsinh(c*x)^2-8*arcsi
nh(c*x)+8*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1)/Pi^(3/2)/c^5/(c^2*x^2+1)
```

Fricas [F]

$$\int \frac{x^4(a+b\mathrm{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)x^4}{(\pi+\pi c^2x^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*x^4*arcsinh(c*x) + a*x^4)/(pi^2*c^4*x^4 +
 2*pi^2*c^2*x^2 + pi^2), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\mathrm{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\frac{\int \frac{ax^4}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx^4\,\mathrm{asinh}\,(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

```
[In] integrate(x**4*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a*x**4/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x)
+ Integral(b*x**4*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**
2 + 1)), x))/pi**(3/2)
```

Maxima [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^4}{(\pi+\pi c^2x^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/2*a*(x^3/(pi*sqrt(pi + pi*c^2*x^2)*c^2) + 3*x/(pi*sqrt(pi + pi*c^2*x^2)*c
^4) - 3*arcsinh(c*x)/(pi^(3/2)*c^5)) + b*integrate(x^4*log(c*x + sqrt(c^2*x
^2 + 1))/(pi + pi*c^2*x^2)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \int \frac{x^4\,(a+b\,\text{asinh}(c\,x))}{(\Pi\,c^2\,x^2+\Pi)^{3/2}}\,dx$$

```
[In] int((x^4*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2),x)
```

```
[Out] int((x^4*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2), x)
```

3.92 $\int \frac{x^3(a+b\,\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{x^3(a+b\,\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = -\frac{bx}{c^3\pi^{3/2}} + \frac{a+b\,\text{arcsinh}(cx)}{c^4\pi\sqrt{\pi+c^2\pi x^2}} + \frac{\sqrt{\pi+c^2\pi x^2}(a+b\,\text{arcsinh}(cx))}{c^4\pi^2} - \frac{b\arctan(cx)}{c^4\pi^{3/2}}$$

```
[Out] -b*x/c^3/Pi^(3/2)-b*arctan(c*x)/c^4/Pi^(3/2)+(a+b*arcsinh(c*x))/c^4/Pi/(Pi*
c^2*x^2+Pi)^(1/2)+(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^4/Pi^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {272, 45, 5804, 12, 396, 209}

$$\int \frac{x^3(a+b\,\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{\sqrt{\pi c^2x^2+\pi}(a+b\,\text{arcsinh}(cx))}{\pi^2c^4} + \frac{a+b\,\text{arcsinh}(cx)}{\pi c^4\sqrt{\pi c^2x^2+\pi}} - \frac{b\arctan(cx)}{\pi^{3/2}c^4} - \frac{bx}{\pi^{3/2}c^3}$$

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] -((b*x)/(c^3*Pi^(3/2))) + (a + b*ArcSinh[c*x])/(c^4*Pi*Sqrt[Pi + c^2*Pi*x^2
]) + (Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(c^4*Pi^2) - (b*ArcTan[c*
x])/(c^4*Pi^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{a+b\operatorname{arcsinh}(cx)}{c^4\pi\sqrt{\pi+c^2\pi x^2}}+\frac{\sqrt{\pi+c^2\pi x^2}(a+b\operatorname{arcsinh}(cx))}{c^4\pi^2}-\left(bc\sqrt{\pi}\right)\int\frac{2+c^2x^2}{c^4\pi^2\left(1+c^2x^2\right)}\,dx\\
&= \frac{a+b\operatorname{arcsinh}(cx)}{c^4\pi\sqrt{\pi+c^2\pi x^2}}+\frac{\sqrt{\pi+c^2\pi x^2}(a+b\operatorname{arcsinh}(cx))}{c^4\pi^2}-\frac{b\int\frac{2+c^2x^2}{1+c^2x^2}\,dx}{c^3\pi^{3/2}}
\end{aligned}
$$

$$= -\frac{bx}{c^3\pi^{3/2}} + \frac{a + b\text{arcsinh}(cx)}{c^4\pi\sqrt{\pi + c^2\pi x^2}} + \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{c^4\pi^2} - \frac{b\int \frac{1}{1+c^2x^2}\,dx}{c^3\pi^{3/2}}$$

$$= -\frac{bx}{c^3\pi^{3/2}} + \frac{a + b\text{arcsinh}(cx)}{c^4\pi\sqrt{\pi + c^2\pi x^2}} + \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{c^4\pi^2} - \frac{b\arctan(cx)}{c^4\pi^{3/2}}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.01

$$\int \frac{x^3(a + b\text{arcsinh}(cx))}{(\pi + c^2\pi x^2)^{3/2}}\,dx = \frac{2a + ac^2x^2 - bcx\sqrt{1 + c^2x^2} + b(2 + c^2x^2)\,\text{arcsinh}(cx) - b\sqrt{1 + c^2x^2}\,\arctan(cx)}{c^4\pi^{3/2}\sqrt{1 + c^2x^2}}$$

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
[Out] (2*a + a*c^2*x^2 - b*c*x*Sqrt[1 + c^2*x^2] + b*(2 + c^2*x^2)*ArcSinh[c*x] -
 b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(c^4*Pi^(3/2)*Sqrt[1 + c^2*x^2])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.90

method	result
default	$a\left(\frac{x^2}{\pi c^2\sqrt{\pi c^2x^2+\pi}} + \frac{2}{\pi c^4\sqrt{\pi c^2x^2+\pi}}\right) + \frac{b\left(\text{arcsinh}(cx)c^2x^2+i\sqrt{c^2x^2+1}\,\ln\left(cx+\sqrt{c^2x^2+1}-i\right)-i\sqrt{c^2x^2+1}\,\ln\left(cx+\sqrt{c^2x^2+1}+i\right)\right.}{\pi^{\frac{3}{2}}\sqrt{c^2x^2+1}\,c^4}$
parts	$a\left(\frac{x^2}{\pi c^2\sqrt{\pi c^2x^2+\pi}} + \frac{2}{\pi c^4\sqrt{\pi c^2x^2+\pi}}\right) + \frac{b\left(\text{arcsinh}(cx)c^2x^2+i\sqrt{c^2x^2+1}\,\ln\left(cx+\sqrt{c^2x^2+1}-i\right)-i\sqrt{c^2x^2+1}\,\ln\left(cx+\sqrt{c^2x^2+1}+i\right)\right.}{\pi^{\frac{3}{2}}\sqrt{c^2x^2+1}\,c^4}$

```
[In] int(x^3*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
[Out] a*(x^2/Pi/c^2/(Pi*c^2*x^2+Pi)^(1/2)+2/Pi/c^4/(Pi*c^2*x^2+Pi)^(1/2))+b/Pi^(3
/2)/(c^2*x^2+1)^(1/2)*(arcsinh(c*x)*c^2*x^2+I*(c^2*x^2+1)^(1/2)*ln(c*x+(c^2
*x^2+1)^(1/2)-I)-I*(c^2*x^2+1)^(1/2)*ln(c*x+(c^2*x^2+1)^(1/2)+I)-c*x*(c^2*x
^2+1)^(1/2)+2*arcsinh(c*x))/c^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(78) = 156.

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.92

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{\sqrt{\pi}(bc^2x^2+b)\arctan\left(-\frac{2\sqrt{\pi}\sqrt{\pi+\pi c^2x^2}\sqrt{c^2x^2+1}cx}{\pi-\pi c^4x^4}\right)+2\sqrt{\pi+\pi c^2x^2}(bc^2x^2+2b)\,\mathrm{l}}{2\,(\pi^2c^6x^2+\pi^{}}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/2*(sqrt(pi)*(b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi + pi*c^2*x^2)*sqrt
(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) + 2*sqrt(pi + pi*c^2*x^2)*(b*c^2*x^2 +
 2*b)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*sqrt(pi + pi*c^2*x^2)*(a*c^2*x^2 - s
qrt(c^2*x^2 + 1)*b*c*x + 2*a))/(pi^2*c^6*x^2 + pi^2*c^4)
```

Sympy [F]

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{\int \frac{ax^3}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx^3\,\text{asinh}\,(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

```
[In] integrate(x**3*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a*x**3/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x)
+ Integral(b*x**3*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**
2 + 1)), x))/pi**(3/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.38

$$\begin{aligned}\int \frac{x^3(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx &= -bc\left(\frac{x}{\pi^{\frac{3}{2}}c^4}+\frac{\arctan\,(cx)}{\pi^{\frac{3}{2}}c^5}\right)\\ &\quad+b\left(\frac{x^2}{\pi\sqrt{\pi+\pi c^2x^2}c^2}+\frac{2}{\pi\sqrt{\pi+\pi c^2x^2}c^4}\right)\text{arsinh}\,(cx)\\ &\quad+a\left(\frac{x^2}{\pi\sqrt{\pi+\pi c^2x^2}c^2}+\frac{2}{\pi\sqrt{\pi+\pi c^2x^2}c^4}\right)\end{aligned}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima
")
```

```
[Out] -b*c*(x/(pi^(3/2)*c^4) + arctan(c*x)/(pi^(3/2)*c^5)) + b*(x^2/(pi*sqrt(pi +
 pi*c^2*x^2)*c^2) + 2/(pi*sqrt(pi + pi*c^2*x^2)*c^4))*arcsinh(c*x) + a*(x^2
/(pi*sqrt(pi + pi*c^2*x^2)*c^2) + 2/(pi*sqrt(pi + pi*c^2*x^2)*c^4))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \int \frac{x^3\,(a+b\,\operatorname{asinh}(c\,x))}{(\Pi\,c^2\,x^2+\Pi)^{3/2}}\,dx$$

```
[In] int((x^3*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2),x)
```

```
[Out] int((x^3*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2), x)
```

3.93 $\int \frac{x^2(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 80

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = -\frac{x(a+b\text{arcsinh}(cx))}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{(a+b\text{arcsinh}(cx))^2}{2bc^3\pi^{3/2}}+\frac{b\log(1+c^2x^2)}{2c^3\pi^{3/2}}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))^2/b/c^3/Pi^(3/2)+1/2*b*ln(c^2*x^2+1)/c^3/Pi^(3/2)-x*
(a+b*arcsinh(c*x))/c^2/Pi/(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5810, 5783, 266}

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{(a+b\text{arcsinh}(cx))^2}{2\pi^{3/2}bc^3}-\frac{x(a+b\text{arcsinh}(cx))}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{b\log(c^2x^2+1)}{2\pi^{3/2}c^3}$$

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] -((x*(a + b*ArcSinh[c*x]))/(c^2*Pi*Sqrt[Pi + c^2*Pi*x^2])) + (a + b*ArcSinh
[c*x])^2/(2*b*c^3*Pi^(3/2)) + (b*Log[1 + c^2*x^2])/(2*c^3*Pi^(3/2))
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x(a+b\text{arcsinh}(cx))}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{b\int\frac{x}{1+c^2x^2}\,dx}{c\pi^{3/2}}+\frac{\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx}{c^2\pi}\\ &= -\frac{x(a+b\text{arcsinh}(cx))}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{(a+b\text{arcsinh}(cx))^2}{2bc^3\pi^{3/2}}+\frac{b\log(1+c^2x^2)}{2c^3\pi^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int\frac{x^2(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\frac{-\frac{2acx}{\sqrt{1+c^2x^2}}+\left(2a-\frac{2bcx}{\sqrt{1+c^2x^2}}\right)\text{arcsinh}(cx)+b\text{arcsinh}(cx)^2+b\log(1+c^2x^2)}{2c^3\pi^{3/2}}$$

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
[Out] ((-2*a*c*x)/Sqrt[1 + c^2*x^2] + (2*a - (2*b*c*x)/Sqrt[1 + c^2*x^2])*ArcSinh
[c*x] + b*ArcSinh[c*x]^2 + b*Log[1 + c^2*x^2])/(2*c^3*Pi^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(70) = 140.

Time = 0.21 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.10

method	result
default	$-\frac{ax}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{a\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{\pi c^2\sqrt{\pi c^2}}+b\left(\frac{\operatorname{arcsinh}(cx)^2}{2c^3\pi^{\frac{3}{2}}}-\frac{2\operatorname{arcsinh}(cx)}{c^3\pi^{\frac{3}{2}}}+\frac{\left(c^2x^2-cx\sqrt{c^2x^2+1}+1\right)\operatorname{arcsinh}(cx)}{\pi^{\frac{3}{2}}c^3(c^2x^2+1)}\right.$
parts	$-\frac{ax}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{a\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{\pi c^2\sqrt{\pi c^2}}+b\left(\frac{\operatorname{arcsinh}(cx)^2}{2c^3\pi^{\frac{3}{2}}}-\frac{2\operatorname{arcsinh}(cx)}{c^3\pi^{\frac{3}{2}}}+\frac{\left(c^2x^2-cx\sqrt{c^2x^2+1}+1\right)\operatorname{arcsinh}(cx)}{\pi^{\frac{3}{2}}c^3(c^2x^2+1)}\right.$

```
[In] int(x^2*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a*x/Pi/c^2/(Pi*c^2*x^2+Pi)^(1/2)+a/Pi/c^2*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c
^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+b*(1/2/c^3/Pi^(3/2)*arcsinh(c*x)^2-2/c^3/P
i^(3/2)*arcsinh(c*x)+1/Pi^(3/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*arcsinh(c
*x)/c^3/(c^2*x^2+1)+1/c^3/Pi^(3/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2))
```

Fricas [F]

$$\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)x^2}{(\pi+\pi c^2x^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*x^2*arcsinh(c*x) + a*x^2)/(pi^2*c^4*x^4 +
 2*pi^2*c^2*x^2 + pi^2), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\frac{\int \frac{ax^2}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx^2\operatorname{asinh}(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

```
[In] integrate(x**2*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a*x**2/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x)
+ Integral(b*x**2*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**
2 + 1)), x))/pi**(3/2)
```

Maxima [F]

$$\int \frac{x^2(a+b\mathrm{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)x^2}{(\pi+\pi c^2x^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima
")
```

```
[Out] -a*(x/(pi*sqrt(pi + pi*c^2*x^2)*c^2) - arcsinh(c*x)/(pi^(3/2)*c^3)) + b*int
egrate(x^2*log(c*x + sqrt(c^2*x^2 + 1))/(pi + pi*c^2*x^2)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2(a+b\mathrm{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)x^2}{(\pi+\pi c^2x^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^2/(pi + pi*c^2*x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\mathrm{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\int \frac{x^2\,(a+b\,\mathrm{asinh}(c\,x))}{(\Pi\,c^2\,x^2+\Pi)^{3/2}}\,dx$$

```
[In] int((x^2*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2),x)
```

```
[Out] int((x^2*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2), x)
```

3.94 $\int \frac{x(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 45

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx=-\frac{a+b\text{arcsinh}(cx)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{b\arctan(cx)}{c^2\pi^{3/2}}$$

```
[Out] b*arctan(c*x)/c^2/Pi^(3/2)+(-a-b*arcsinh(c*x))/c^2/Pi/(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5798, 209}

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\frac{b\arctan(cx)}{\pi^{3/2}c^2}-\frac{a+b\text{arcsinh}(cx)}{\pi c^2\sqrt{\pi c^2x^2+\pi}}$$

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] -((a + b*ArcSinh[c*x])/(c^2*Pi*Sqrt[Pi + c^2*Pi*x^2])) + (b*ArcTan[c*x])/(c
^2*Pi^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
```

```
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{a+b\operatorname{arcsinh}(cx)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{b\int\frac{1}{1+c^2x^2}\,dx}{c\pi^{3/2}}\\ &= -\frac{a+b\operatorname{arcsinh}(cx)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{b\arctan(cx)}{c^2\pi^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16

$$\int\frac{x(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\frac{-a-b\operatorname{arcsinh}(cx)+b\sqrt{1+c^2x^2}\arctan(cx)}{c^2\pi^{3/2}\sqrt{1+c^2x^2}}$$

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
[Out] (-a - b*ArcSinh[c*x] + b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(c^2*Pi^(3/2)*Sqrt[
1 + c^2*x^2])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.29

method	result	size
default	$-\frac{a}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+b\left(-\frac{\operatorname{arcsinh}(cx)}{c^2\pi^{\frac{3}{2}}\sqrt{c^2x^2+1}}+\frac{i\ln\left(cx+\sqrt{c^2x^2+1}+i\right)}{c^2\pi^{\frac{3}{2}}}-\frac{i\ln\left(cx+\sqrt{c^2x^2+1}-i\right)}{c^2\pi^{\frac{3}{2}}}\right)$	103
parts	$-\frac{a}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+b\left(-\frac{\operatorname{arcsinh}(cx)}{c^2\pi^{\frac{3}{2}}\sqrt{c^2x^2+1}}+\frac{i\ln\left(cx+\sqrt{c^2x^2+1}+i\right)}{c^2\pi^{\frac{3}{2}}}-\frac{i\ln\left(cx+\sqrt{c^2x^2+1}-i\right)}{c^2\pi^{\frac{3}{2}}}\right)$	103

```
[In] int(x*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
[Out] -a/Pi/c^2/(Pi*c^2*x^2+Pi)^(1/2)+b*(-1/c^2/Pi^(3/2)*arcsinh(c*x)/(c^2*x^2+1)
^(1/2)+I/c^2/Pi^(3/2)*ln(c*x+(c^2*x^2+1)^(1/2)+I)-I/c^2/Pi^(3/2)*ln(c*x+(c^
2*x^2+1)^(1/2)-I))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(41) = 82$.

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.82

$$\int \frac{x(a + b\text{arcsinh}(cx))}{(\pi + c^2\pi x^2)^{3/2}}\,dx = -\frac{\sqrt{\pi}(bc^2x^2 + b)\arctan\left(-\frac{2\sqrt{\pi}\sqrt{\pi+\pi c^2x^2}\sqrt{c^2x^2+1}cx}{\pi-\pi c^4x^4}\right) + 2\sqrt{\pi + \pi c^2x^2}b\log\left(cx + \sqrt{c^2x^2+1}\right) + 2\sqrt{\pi+\pi c^2x^2}a}{2(\pi^2c^4x^2 + \pi^2c^2)}$$

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(pi)*(b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi + pi*c^2*x^2)*sqr
t(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) + 2*sqrt(pi + pi*c^2*x^2)*b*log(c*x +
 sqrt(c^2*x^2 + 1)) + 2*sqrt(pi + pi*c^2*x^2)*a)/(pi^2*c^4*x^2 + pi^2*c^2)
```

Sympy [F]

$$\int \frac{x(a + b\text{arcsinh}(cx))}{(\pi + c^2\pi x^2)^{3/2}}\,dx = \frac{\int \frac{ax}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{bx\operatorname{asinh}(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

```
[In] integrate(x*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a*x/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x) + I
ntegral(b*x*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)
), x))/pi**(3/2)
```

Maxima [F]

$$\int \frac{x(a + b\text{arcsinh}(cx))}{(\pi + c^2\pi x^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx) + a)x}{(\pi + \pi c^2x^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima")
```

```
[Out] b*(integrate(1/(sqrt(c^2*x^2 + 1)*x), x)/(pi^(3/2)*c^2) - log(c*x + sqrt(c^
2*x^2 + 1))/(pi^(3/2)*sqrt(c^2*x^2 + 1)*c^2) - integrate(1/(pi^(3/2)*c^5*x^
4 + pi^(3/2)*c^3*x^2 + (pi^(3/2)*c^4*x^3 + pi^(3/2)*c^2*x)*sqrt(c^2*x^2 + 1
)), x)) - a/(pi*sqrt(pi + pi*c^2*x^2)*c^2)
```

Giac [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x}{(\pi+\pi c^2x^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)*x/(pi + pi*c^2*x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\int \frac{x\,(a+b\,\text{asinh}(c\,x))}{(\Pi\,c^2\,x^2+\Pi)^{3/2}}\,dx$$

```
[In] int((x*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2),x)
[Out] int((x*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2), x)
```

3.95 $\int \frac{a+b\mathbf{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{3/2}} dx$

Optimal result

Integrand size = 23, antiderivative size = 51

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{3/2}} dx = \frac{x(a+b\text{arcsinh}(cx))}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{b\log(1+c^2x^2)}{2c\pi^{3/2}}$$

```
[Out] -1/2*b*ln(c^2*x^2+1)/c/Pi^(3/2)+x*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(1/
2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5787, 266}

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{3/2}} dx = \frac{x(a+b\text{arcsinh}(cx))}{\pi\sqrt{\pi c^2x^2+\pi}} - \frac{b\log(c^2x^2+1)}{2\pi^{3/2}c}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcSinh[c*x]))/(Pi*Sqrt[Pi + c^2*Pi*x^2]) - (b*Log[1 + c^2*x^2])/
(2*c*Pi^(3/2))
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
```

```
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x(a+b\text{arcsinh}(cx))}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{(bc)\int \frac{x}{1+c^2x^2}\,dx}{\pi^{3/2}} \\ &= \frac{x(a+b\text{arcsinh}(cx))}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{b\log(1+c^2x^2)}{2c\pi^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{2acx+2bcx\text{arcsinh}(cx)-b\sqrt{1+c^2x^2}\log(1+c^2x^2)}{2c\pi^{3/2}\sqrt{1+c^2x^2}}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(Pi + c^2*Pi*x^2)^(3/2),x]
[Out] (2*a*c*x + 2*b*c*x*ArcSinh[c*x] - b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(2*
c*Pi^(3/2)*Sqrt[1 + c^2*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(45) = 90.

Time = 0.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.16

method	result	size
default	$\frac{ax}{\pi\sqrt{\pi c^2x^2+\pi}}+b\left(\frac{2\,\text{arcsinh}(cx)}{c\,\pi^{\frac{3}{2}}}-\frac{\left(c^2x^2-cx\sqrt{c^2x^2+1}+1\right)\text{arcsinh}(cx)}{\pi^{\frac{3}{2}}c(c^2x^2+1)}-\frac{\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)}{c\,\pi^{\frac{3}{2}}}\right)$	110
parts	$\frac{ax}{\pi\sqrt{\pi c^2x^2+\pi}}+b\left(\frac{2\,\text{arcsinh}(cx)}{c\,\pi^{\frac{3}{2}}}-\frac{\left(c^2x^2-cx\sqrt{c^2x^2+1}+1\right)\text{arcsinh}(cx)}{\pi^{\frac{3}{2}}c(c^2x^2+1)}-\frac{\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)}{c\,\pi^{\frac{3}{2}}}\right)$	110

```
[In] int((a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
[Out] a/Pi*x/(Pi*c^2*x^2+Pi)^(1/2)+b*(2/c/Pi^(3/2)*arcsinh(c*x)-1/Pi^(3/2)*(c^2*x
^2-c*x*(c^2*x^2+1)^(1/2)+1)*arcsinh(c*x)/c/(c^2*x^2+1)-1/c/Pi^(3/2)*ln(1+(c
*x+(c^2*x^2+1)^(1/2))^2))
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{(\pi+\pi c^2x^2)^{\frac{3}{2}}}\,dx$$

[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^2*c^4*x^4 + 2*pi^2*
c^2*x^2 + pi^2), x)

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{\int \frac{a}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{b\,\text{asinh}\,(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

[In] integrate((a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)

[Out] (Integral(a/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x) + Int
egral(b*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x
))/pi**(3/2)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{bx\,\text{arsinh}\,(cx)}{\pi\sqrt{\pi+\pi c^2x^2}} + \frac{ax}{\pi\sqrt{\pi+\pi c^2x^2}} - \frac{b\log\left(x^2+\frac{1}{c^2}\right)}{2\,\pi^{\frac{3}{2}}c}$$

[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima")

[Out] b*x*arcsinh(c*x)/(pi*sqrt(pi + pi*c^2*x^2)) + a*x/(pi*sqrt(pi + pi*c^2*x^2)
) - 1/2*b*log(x^2 + 1/c^2)/(pi^(3/2)*c)

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(\pi + c^2\pi x^2)^{3/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{(\pi + \pi c^2x^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/(pi + pi*c^2*x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{(\pi + c^2\pi x^2)^{3/2}}\,dx = \int \frac{a + b\,\text{asinh}(c\,x)}{(\Pi\,c^2\,x^2 + \Pi)^{3/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(3/2),x)
[Out] int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(3/2), x)
```

3.96 $\int \frac{a+b\text{arcsinh}(cx)}{x(\pi+c^2\pi x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 94

$$\int \frac{a+b\text{arcsinh}(cx)}{x(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{b\arctan(cx)}{\pi^{3/2}} - \frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\pi^{3/2}} - \frac{b\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\pi^{3/2}} + \frac{b\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\pi^{3/2}}$$

```
[Out] -b*arctan(c*x)/Pi^(3/2)-2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))
/Pi^(3/2)-b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))/Pi^(3/2)+b*polylog(2,c*x+(c^2
*x^2+1)^(1/2))/Pi^(3/2)+(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5811, 5816, 4267, 2317, 2438, 209}

$$\int \frac{a+b\text{arcsinh}(cx)}{x(\pi+c^2\pi x^2)^{3/2}}\,dx = -\frac{2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\pi^{3/2}} + \frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}} - \frac{b\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\pi^{3/2}} + \frac{b\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\pi^{3/2}} - \frac{b\arctan(cx)}{\pi^{3/2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] (a + b*ArcSinh[c*x])/(Pi*Sqrt[Pi + c^2*Pi*x^2]) - (b*ArcTan[c*x])/Pi^(3/2)
- (2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Pi^(3/2) - (b*PolyLog[2,
 -E^ArcSinh[c*x]])/Pi^(3/2) + (b*PolyLog[2, E^ArcSinh[c*x]])/Pi^(3/2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{a + b\text{arcsinh}(cx)}{\pi\sqrt{\pi + c^2\pi x^2}} - \frac{(bc)\int \frac{1}{1+c^2x^2}\,dx}{\pi^{3/2}} + \frac{\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx}{\pi}$$

$$= \frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{b\arctan(cx)}{\pi^{3/2}} + \frac{\text{Subst}(\int(a+bx)\text{csch}(x)\,dx, x, \text{arcsinh}(cx))}{\pi^{3/2}}$$

$$= \frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{b\arctan(cx)}{\pi^{3/2}} - \frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\pi^{3/2}} - \frac{b\text{Subst}\left(\int\log\left(1-e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{\pi^{3/2}} + \frac{b\text{Subst}\left(\int\log\left(1+e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{\pi^{3/2}}$$

$$= \frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{b\arctan(cx)}{\pi^{3/2}} - \frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\pi^{3/2}} - \frac{b\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{\pi^{3/2}} + \frac{b\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{\pi^{3/2}}$$

$$= \frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{b\arctan(cx)}{\pi^{3/2}} - \frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\pi^{3/2}} - \frac{b\,\text{PolyLog}\left(2, -e^{\text{arcsinh}(cx)}\right)}{\pi^{3/2}} + \frac{b\,\text{PolyLog}\left(2, e^{\text{arcsinh}(cx)}\right)}{\pi^{3/2}}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.52

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx = \frac{\frac{a}{\sqrt{1+c^2x^2}} + \frac{b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}} - 2b\arctan\left(\tanh\left(\frac{1}{2}\text{arcsinh}(cx)\right)\right) + b\text{arcsinh}(cx)\log\left(1-\right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] (a/Sqrt[1 + c^2*x^2] + (b*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - 2*b*ArcTan[Tanh
[ArcSinh[c*x]/2]] + b*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - b*ArcSinh[c
*x]*Log[1 + E^(-ArcSinh[c*x])] + a*Log[x] - a*Log[Pi*(1 + Sqrt[1 + c^2*x^2]
)] + b*PolyLog[2, -E^(-ArcSinh[c*x])] - b*PolyLog[2, E^(-ArcSinh[c*x])])/Pi
^(3/2)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.65

method	result
default	$a\left(\frac{1}{\pi\sqrt{\pi c^2x^2+\pi}} - \frac{\text{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)}{\pi^{\frac{3}{2}}}\right) + b\left(\frac{\text{arcsinh}(cx)}{\pi^{\frac{3}{2}}\sqrt{c^2x^2+1}} - \frac{2\arctan\left(cx+\sqrt{c^2x^2+1}\right)}{\pi^{\frac{3}{2}}} - \frac{\text{dilog}\left(1+cx+\sqrt{c^2x^2+1}\right)}{\pi^{\frac{3}{2}}} - \right.$
parts	$a\left(\frac{1}{\pi\sqrt{\pi c^2x^2+\pi}} - \frac{\text{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)}{\pi^{\frac{3}{2}}}\right) + b\left(\frac{\text{arcsinh}(cx)}{\pi^{\frac{3}{2}}\sqrt{c^2x^2+1}} - \frac{2\arctan\left(cx+\sqrt{c^2x^2+1}\right)}{\pi^{\frac{3}{2}}} - \frac{\text{dilog}\left(1+cx+\sqrt{c^2x^2+1}\right)}{\pi^{\frac{3}{2}}} - \right.$

```
[In] int((a+b*arcsinh(c*x))/x/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/Pi/(Pi*c^2*x^2+Pi)^(1/2)-1/Pi^(3/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(
1/2)))+b*(1/Pi^(3/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)-2/Pi^(3/2)*arctan(c*x+(
c^2*x^2+1)^(1/2))-1/Pi^(3/2)*dilog(1+c*x+(c^2*x^2+1)^(1/2))-1/Pi^(3/2)*arcs
inh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-1/Pi^(3/2)*dilog(c*x+(c^2*x^2+1)^(1/2)
))
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\,(\pi+c^2\pi x^2)^{3/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(\pi+\pi c^2x^2)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^2*c^4*x^5 + 2*pi^2*
c^2*x^3 + pi^2*x), x)
```

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\,(\pi+c^2\pi x^2)^{3/2}}\,dx=\frac{\int \frac{a}{c^2x^3\sqrt{c^2x^2+1}+x\sqrt{c^2x^2+1}}\,dx+\int \frac{b\,\text{asinh}\,(cx)}{c^2x^3\sqrt{c^2x^2+1}+x\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

```
[In] integrate((a+b*asinh(c*x))/x/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a/(c**2*x**3*sqrt(c**2*x**2 + 1) + x*sqrt(c**2*x**2 + 1)), x) + I
ntegral(b*asinh(c*x)/(c**2*x**3*sqrt(c**2*x**2 + 1) + x*sqrt(c**2*x**2 + 1)
), x))/pi**(3/2)
```

Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\,(\pi+c^2\pi x^2)^{3/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(\pi+\pi c^2x^2)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*(arcsinh(1/(c*abs(x)))/pi^(3/2) - 1/(pi*sqrt(pi + pi*c^2*x^2))) + b*inte
grate(log(c*x + sqrt(c^2*x^2 + 1))/((pi + pi*c^2*x^2)^(3/2)*x), x)
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x\left(\pi + c^2\pi x^2\right)^{3/2}}\, dx = \int \frac{b\,\text{arsinh}\left(cx\right) + a}{\left(\pi + \pi c^2 x^2\right)^{\frac{3}{2}} x}\, dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{x\left(\pi + c^2\pi x^2\right)^{3/2}}\, dx = \int \frac{a + b\,\text{asinh}(c\,x)}{x\left(\Pi\,c^2\,x^2 + \Pi\right)^{3/2}}\, dx$$

```
[In] int((a + b*asinh(c*x))/(x*(Pi + Pi*c^2*x^2)^(3/2)),x)
[Out] int((a + b*asinh(c*x))/(x*(Pi + Pi*c^2*x^2)^(3/2)), x)
```

3.97 $\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 93

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx=-\frac{a+b\text{arcsinh}(cx)}{\pi x\sqrt{\pi+c^2\pi x^2}}-\frac{2c^2x(a+b\text{arcsinh}(cx))}{\pi\sqrt{\pi+c^2\pi x^2}}+\frac{bc\log(x)}{\pi^{3/2}}+\frac{bc\log\left(1+c^2x^2\right)}{2\pi^{3/2}}$$

```
[Out] b*c*ln(x)/Pi^(3/2)+1/2*b*c*ln(c^2*x^2+1)/Pi^(3/2)+(-a-b*arcsinh(c*x))/Pi/x/
(Pi*c^2*x^2+Pi)^(1/2)-2*c^2*x*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {277, 197, 5804, 12, 457, 78}

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx=-\frac{2c^2x(a+b\text{arcsinh}(cx))}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\text{arcsinh}(cx)}{\pi x\sqrt{\pi c^2x^2+\pi}}+\frac{bc\log\left(c^2x^2+1\right)}{2\pi^{3/2}}+\frac{bc\log(x)}{\pi^{3/2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] -((a + b*ArcSinh[c*x])/(Pi*x*Sqrt[Pi + c^2*Pi*x^2])) - (2*c^2*x*(a + b*ArcS
inh[c*x]))/(Pi*Sqrt[Pi + c^2*Pi*x^2]) + (b*c*Log[x])/Pi^(3/2) + (b*c*Log[1
+ c^2*x^2])/(2*Pi^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\text{integral} = -\frac{a+b\text{arcsinh}(cx)}{\pi x\sqrt{\pi+c^2\pi x^2}}-\frac{2c^2x(a+b\text{arcsinh}(cx))}{\pi\sqrt{\pi+c^2\pi x^2}}-\left(bc\sqrt{\pi}\right)\int\frac{-1-2c^2x^2}{\pi^2x\left(1+c^2x^2\right)}\,dx$$

$$=-\frac{a+b\text{arcsinh}(cx)}{\pi x\sqrt{\pi+c^2\pi x^2}}-\frac{2c^2x(a+b\text{arcsinh}(cx))}{\pi\sqrt{\pi+c^2\pi x^2}}-\frac{(bc)\int\frac{-1-2c^2x^2}{x(1+c^2x^2)}\,dx}{\pi^{3/2}}$$

$$= -\frac{a+b\text{arcsinh}(cx)}{\pi x\sqrt{\pi+c^2\pi x^2}} - \frac{2c^2x(a+b\text{arcsinh}(cx))}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{(bc)\text{Subst}\left(\int \frac{-1-2c^2x}{x(1+c^2x)}\,dx, x, x^2\right)}{2\pi^{3/2}}$$

$$= -\frac{a+b\text{arcsinh}(cx)}{\pi x\sqrt{\pi+c^2\pi x^2}} - \frac{2c^2x(a+b\text{arcsinh}(cx))}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{(bc)\text{Subst}\left(\int \left(-\frac{1}{x}-\frac{c^2}{1+c^2x}\right)\,dx, x, x^2\right)}{2\pi^{3/2}}$$

$$= -\frac{a+b\text{arcsinh}(cx)}{\pi x\sqrt{\pi+c^2\pi x^2}} - \frac{2c^2x(a+b\text{arcsinh}(cx))}{\pi\sqrt{\pi+c^2\pi x^2}} + \frac{bc\log(x)}{\pi^{3/2}} + \frac{bc\log(1+c^2x^2)}{2\pi^{3/2}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.10

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{-2a-4ac^2x^2-2(b+2bc^2x^2)\,\text{arcsinh}(cx)+2bcx\sqrt{1+c^2x^2}\log(x)+bcx\sqrt{1+c^2x^2}}{2\pi^{3/2}x\sqrt{1+c^2x^2}}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] (-2*a - 4*a*c^2*x^2 - 2*(b + 2*b*c^2*x^2)*ArcSinh[c*x] + 2*b*c*x*Sqrt[1 + c
^2*x^2]*Log[x] + b*c*x*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(2*Pi^(3/2)*x*Sq
rt[1 + c^2*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(85) = 170.

Time = 0.23 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.62

method	result
default	$a\left(-\frac{1}{\pi x\sqrt{\pi c^2x^2+\pi}}-\frac{2c^2x}{\pi\sqrt{\pi c^2x^2+\pi}}\right)-\frac{b\left(2\ln\left(\left(cx+\sqrt{c^2x^2+1}\right)^4-1\right)x^4c^4-2\sqrt{c^2x^2+1}\,\ln\left(\left(cx+\sqrt{c^2x^2+1}\right)^4-1\right)x^3c^3+2\ln\left(\left(c\right.\right.\right.}{}$
parts	$a\left(-\frac{1}{\pi x\sqrt{\pi c^2x^2+\pi}}-\frac{2c^2x}{\pi\sqrt{\pi c^2x^2+\pi}}\right)-\frac{b\left(2\ln\left(\left(cx+\sqrt{c^2x^2+1}\right)^4-1\right)x^4c^4-2\sqrt{c^2x^2+1}\,\ln\left(\left(cx+\sqrt{c^2x^2+1}\right)^4-1\right)x^3c^3+2\ln\left(\left(c\right.\right.\right.}{}$

```
[In] int((a+b*arcsinh(c*x))/x^2/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/Pi/x/(Pi*c^2*x^2+Pi)^(1/2)-2/Pi*c^2*x/(Pi*c^2*x^2+Pi)^(1/2))-b*(2*ln(
(c*x+(c^2*x^2+1)^(1/2))^4-1)*x^4*c^4-2*(c^2*x^2+1)^(1/2)*ln((c*x+(c^2*x^2+1
)^(1/2))^4-1)*x^3*c^3+2*ln((c*x+(c^2*x^2+1)^(1/2))^4-1)*x^2*c^2-(c^2*x^2+1)
^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^4-1)*x*c+arcsinh(c*x))*(2*c^3*x^3+2*c^2*x
^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))/Pi^(3/2)/x/(c^2*x^2+1)
```

Fricas [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2 \left(\pi + c^2\pi x^2\right)^{3/2}}\, dx = \int \frac{b \,\text{arsinh}\,(cx) + a}{\left(\pi + \pi c^2x^2\right)^{\frac{3}{2}}x^2}\, dx$$

[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas ")

[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^2*c^4*x^6 + 2*pi^2* c^2*x^4 + pi^2*x^2), x)

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2 \left(\pi + c^2\pi x^2\right)^{3/2}}\, dx = \frac{\int \frac{a}{c^2x^4\sqrt{c^2x^2+1}+x^2\sqrt{c^2x^2+1}}\, dx + \int \frac{b \,\text{asinh}\,(cx)}{c^2x^4\sqrt{c^2x^2+1}+x^2\sqrt{c^2x^2+1}}\, dx}{\pi^{\frac{3}{2}}}$$

[In] integrate((a+b*asinh(c*x))/x**2/(pi*c**2*x**2+pi)**(3/2),x)

[Out] (Integral(a/(c**2*x**4*sqrt(c**2*x**2 + 1) + x**2*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(c**2*x**4*sqrt(c**2*x**2 + 1) + x**2*sqrt(c**2*x** 2 + 1)), x))/pi**(3/2)

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.28

$$\begin{aligned}\int \frac{a + b\text{arcsinh}(cx)}{x^2 \left(\pi + c^2\pi x^2\right)^{3/2}}\, dx &= \frac{1}{2}\,bc\left(\frac{\log\left(c^2x^2+1\right)}{\pi^{\frac{3}{2}}} + \frac{2\,\log\left(x\right)}{\pi^{\frac{3}{2}}}\right) \\ &\quad - \left(\frac{2\,c^2x}{\pi\sqrt{\pi+\pi c^2x^2}} + \frac{1}{\pi\sqrt{\pi+\pi c^2x^2}x}\right)b\,\text{arsinh}\,(cx) \\ &\quad - \left(\frac{2\,c^2x}{\pi\sqrt{\pi+\pi c^2x^2}} + \frac{1}{\pi\sqrt{\pi+\pi c^2x^2}x}\right)a\end{aligned}$$

[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima ")

[Out] 1/2*b*c*(log(c^2*x^2 + 1)/pi^(3/2) + 2*log(x)/pi^(3/2)) - (2*c^2*x/(pi*sqrt (pi + pi*c^2*x^2)) + 1/(pi*sqrt(pi + pi*c^2*x^2)*x))*b*arcsinh(c*x) - (2*c^ 2*x/(pi*sqrt(pi + pi*c^2*x^2)) + 1/(pi*sqrt(pi + pi*c^2*x^2)*x))*a

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2 (\pi + c^2\pi x^2)^{3/2}}\, dx = \int \frac{b \text{arsinh}\,(cx) + a}{(\pi + \pi c^2 x^2)^{\frac{3}{2}} x^2}\, dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2 (\pi + c^2\pi x^2)^{3/2}}\, dx = \int \frac{a + b\,\text{asinh}(c\,x)}{x^2\,(\Pi\,c^2\,x^2 + \Pi)^{3/2}}\, dx$$

```
[In] int((a + b*asinh(c*x))/(x^2*(Pi + Pi*c^2*x^2)^(3/2)),x)
[Out] int((a + b*asinh(c*x))/(x^2*(Pi + Pi*c^2*x^2)^(3/2)), x)
```

3.98 $\int \frac{a+b\text{arcsinh}(cx)}{x^3(\pi+c^2\pi x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 162

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(\pi+c^2\pi x^2)^{3/2}}\,dx = -\frac{bc}{2\pi^{3/2}x}-\frac{3c^2(a+b\text{arcsinh}(cx))}{2\pi\sqrt{\pi+c^2\pi x^2}}-\frac{a+b\text{arcsinh}(cx)}{2\pi x^2\sqrt{\pi+c^2\pi x^2}}+\frac{bc^2\arctan(cx)}{\pi^{3/2}}+\frac{3c^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\pi^{3/2}}+\frac{3bc^2\operatorname{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2\pi^{3/2}}-\frac{3bc^2\operatorname{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2\pi^{3/2}}$$

```
[Out] -1/2*b*c/Pi^(3/2)/x+b*c^2*arctan(c*x)/Pi^(3/2)+3*c^2*(a+b*arcsinh(c*x))*arc
tanh(c*x+(c^2*x^2+1)^(1/2))/Pi^(3/2)+3/2*b*c^2*polylog(2,-c*x-(c^2*x^2+1)^(
1/2))/Pi^(3/2)-3/2*b*c^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))/Pi^(3/2)-3/2*c^2*
(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(1/2)+1/2*(-a-b*arcsinh(c*x))/Pi/x^2/
(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5809, 5811, 5816, 4267, 2317, 2438, 209, 331}

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{3c^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\pi^{3/2}}-\frac{3c^2(a+b\text{arcsinh}(cx))}{2\pi\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\text{arcsinh}(cx)}{2\pi x^2\sqrt{\pi c^2x^2+\pi}}+\frac{3bc^2\operatorname{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2\pi^{3/2}}-\frac{3bc^2\operatorname{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2\pi^{3/2}}+\frac{bc^2\arctan(cx)}{\pi^{3/2}}-\frac{bc}{2\pi^{3/2}x}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] -1/2*(b*c)/(Pi^(3/2)*x) - (3*c^2*(a + b*ArcSinh[c*x]))/(2*Pi*Sqrt[Pi + c^2*
Pi*x^2]) - (a + b*ArcSinh[c*x])/(2*Pi*x^2*Sqrt[Pi + c^2*Pi*x^2]) + (b*c^2*A
rcTan[c*x])/Pi^(3/2) + (3*c^2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])
/Pi^(3/2) + (3*b*c^2*PolyLog[2, -E^ArcSinh[c*x]])/(2*Pi^(3/2)) - (3*b*c^2*P
olyLog[2, E^ArcSinh[c*x]])/(2*Pi^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
```

```
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a+b\operatorname{arcsinh}(cx)}{2\pi x^2\sqrt{\pi+c^2\pi x^2}}-\frac{1}{2}\left(3c^2\right)\int\frac{a+b\operatorname{arcsinh}(cx)}{x\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx+\frac{(bc)\int\frac{1}{x^2(1+c^2x^2)}\,dx}{2\pi^{3/2}}\\
&= -\frac{bc}{2\pi^{3/2}x}-\frac{3c^2(a+b\operatorname{arcsinh}(cx))}{2\pi\sqrt{\pi+c^2\pi x^2}}-\frac{a+b\operatorname{arcsinh}(cx)}{2\pi x^2\sqrt{\pi+c^2\pi x^2}}\\
&\quad -\frac{\left(bc^3\right)\int\frac{1}{1+c^2x^2}\,dx}{2\pi^{3/2}}+\frac{\left(3bc^3\right)\int\frac{1}{1+c^2x^2}\,dx}{2\pi^{3/2}}-\frac{\left(3c^2\right)\int\frac{a+b\operatorname{arcsinh}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx}{2\pi}\\
&= -\frac{bc}{2\pi^{3/2}x}-\frac{3c^2(a+b\operatorname{arcsinh}(cx))}{2\pi\sqrt{\pi+c^2\pi x^2}}-\frac{a+b\operatorname{arcsinh}(cx)}{2\pi x^2\sqrt{\pi+c^2\pi x^2}}\\
&\quad +\frac{bc^2\arctan(cx)}{\pi^{3/2}}-\frac{\left(3c^2\right)\operatorname{Subst}\left(\int(a+bx)\operatorname{csch}(x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{2\pi^{3/2}}\\
&= -\frac{bc}{2\pi^{3/2}x}-\frac{3c^2(a+b\operatorname{arcsinh}(cx))}{2\pi\sqrt{\pi+c^2\pi x^2}}-\frac{a+b\operatorname{arcsinh}(cx)}{2\pi x^2\sqrt{\pi+c^2\pi x^2}}\\
&\quad +\frac{bc^2\arctan(cx)}{\pi^{3/2}}+\frac{3c^2(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{\pi^{3/2}}\\
&\quad +\frac{\left(3bc^2\right)\operatorname{Subst}\left(\int\log\left(1-e^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{2\pi^{3/2}}\\
&\quad -\frac{\left(3bc^2\right)\operatorname{Subst}\left(\int\log\left(1+e^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{2\pi^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bc}{2\pi^{3/2}x} - \frac{3c^2(a+b\operatorname{arcsinh}(cx))}{2\pi\sqrt{\pi+c^2\pi x^2}} - \frac{a+b\operatorname{arcsinh}(cx)}{2\pi x^2\sqrt{\pi+c^2\pi x^2}} \\
&\quad + \frac{bc^2\arctan(cx)}{\pi^{3/2}} + \frac{3c^2(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{\pi^{3/2}} \\
&\quad + \frac{(3bc^2)\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{2\pi^{3/2}} \\
&\quad - \frac{(3bc^2)\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{2\pi^{3/2}} \\
&= -\frac{bc}{2\pi^{3/2}x} - \frac{3c^2(a+b\operatorname{arcsinh}(cx))}{2\pi\sqrt{\pi+c^2\pi x^2}} - \frac{a+b\operatorname{arcsinh}(cx)}{2\pi x^2\sqrt{\pi+c^2\pi x^2}} \\
&\quad + \frac{bc^2\arctan(cx)}{\pi^{3/2}} + \frac{3c^2(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{\pi^{3/2}} \\
&\quad + \frac{3bc^2\operatorname{PolyLog}\left(2, -e^{\operatorname{arcsinh}(cx)}\right)}{2\pi^{3/2}} - \frac{3bc^2\operatorname{PolyLog}\left(2, e^{\operatorname{arcsinh}(cx)}\right)}{2\pi^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.93 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.66

$$
\int \frac{a+b\operatorname{arcsinh}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx = \frac{-\frac{8ac^2}{\sqrt{1+c^2x^2}} - \frac{4a\sqrt{1+c^2x^2}}{x^2} - \frac{8bc^2\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}} + 16bc^2\arctan\left(\tanh\left(\frac{1}{2}\operatorname{arcsinh}(cx)\right)\right) - 2bc
$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] ((-8*a*c^2)/Sqrt[1 + c^2*x^2] - (4*a*Sqrt[1 + c^2*x^2])/x^2 - (8*b*c^2*ArcS
inh[c*x])/Sqrt[1 + c^2*x^2] + 16*b*c^2*ArcTan[Tanh[ArcSinh[c*x]/2]] - 2*b*c
^2*Coth[ArcSinh[c*x]/2] - b*c^2*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 12*b*
c^2*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] + 12*b*c^2*ArcSinh[c*x]*Log[1 +
 E^(-ArcSinh[c*x])] - 12*a*c^2*Log[x] + 12*a*c^2*Log[Pi*(1 + Sqrt[1 + c^2*x
^2])] - 12*b*c^2*PolyLog[2, -E^(-ArcSinh[c*x])] + 12*b*c^2*PolyLog[2, E^(-A
rcSinh[c*x])] - b*c^2*ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*b*c^2*Tanh[Ar
cSinh[c*x]/2])/(8*Pi^(3/2))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.38

method	result
default	$a\left(-\frac{1}{2\pi\,x^{2}\sqrt{\pi\,c^{2}x^{2}+\pi}}-\frac{3c^{2}\left(\frac{1}{\pi\sqrt{\pi\,c^{2}x^{2}+\pi}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi\,c^{2}x^{2}+\pi}}\right)}{\pi^{\frac{3}{2}}}\right)}{2}\right)+b\left(-\frac{3\operatorname{arcsinh}(cx)c^{2}x^{2}+cx\sqrt{c^{2}x^{2}+1}+\operatorname{arcsinh}(cx)}{2\pi^{\frac{3}{2}}\sqrt{c^{2}x^{2}+1}\,x^{2}}\right.$
parts	$a\left(-\frac{1}{2\pi\,x^{2}\sqrt{\pi\,c^{2}x^{2}+\pi}}-\frac{3c^{2}\left(\frac{1}{\pi\sqrt{\pi\,c^{2}x^{2}+\pi}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi\,c^{2}x^{2}+\pi}}\right)}{\pi^{\frac{3}{2}}}\right)}{2}\right)+b\left(-\frac{3\operatorname{arcsinh}(cx)c^{2}x^{2}+cx\sqrt{c^{2}x^{2}+1}+\operatorname{arcsinh}(cx)}{2\pi^{\frac{3}{2}}\sqrt{c^{2}x^{2}+1}\,x^{2}}\right.$

```
[In] int((a+b*arcsinh(c*x))/x^3/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/2/Pi/x^2/(Pi*c^2*x^2+Pi)^(1/2)-3/2*c^2*(1/Pi/(Pi*c^2*x^2+Pi)^(1/2)-1/
Pi^(3/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2))))+b*(-1/2/Pi^(3/2)/(c^2*x^
2+1)^(1/2)*(3*arcsinh(c*x)*c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x))/x^2+
2*c^2/Pi^(3/2)*arctan(c*x+(c^2*x^2+1)^(1/2))+3/2*c^2/Pi^(3/2)*dilog(1+c*x+(
c^2*x^2+1)^(1/2))+3/2*c^2/Pi^(3/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))
+3/2*c^2/Pi^(3/2)*dilog(c*x+(c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{a+b\mathrm{arcsinh}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx=\int \frac{b\,\mathrm{arsinh}\left(cx\right)+a}{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^2*c^4*x^7 + 2*pi^2*
c^2*x^5 + pi^2*x^3), x)
```

Sympy [F]

$$\int \frac{a+b\mathrm{arcsinh}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx=\frac{\int \frac{a}{c^2x^5\sqrt{c^2x^2+1}+x^3\sqrt{c^2x^2+1}}\,dx+\int \frac{b\,\mathrm{asinh}\left(cx\right)}{c^2x^5\sqrt{c^2x^2+1}+x^3\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

```
[In] integrate((a+b*asinh(c*x))/x**3/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a/(c**2*x**5*sqrt(c**2*x**2 + 1) + x**3*sqrt(c**2*x**2 + 1)), x)
+ Integral(b*asinh(c*x)/(c**2*x**5*sqrt(c**2*x**2 + 1) + x**3*sqrt(c**2*x**
2 + 1)), x))/pi**(3/2)
```

Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx=\int \frac{b\operatorname{arsinh}\left(cx\right)+a}{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/2*(3*c^2*arcsinh(1/(c*abs(x)))/pi^(3/2) - 3*c^2/(pi*sqrt(pi + pi*c^2*x^2)
) - 1/(pi*sqrt(pi + pi*c^2*x^2)*x^2))*a + b*integrate(log(c*x + sqrt(c^2*x^
2 + 1))/((pi + pi*c^2*x^2)^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx=\int \frac{b\operatorname{arsinh}\left(cx\right)+a}{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x^3\,{\left(\Pi\,c^2\,x^2+\Pi\right)}^{3/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))/(x^3*(Pi + Pi*c^2*x^2)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^3*(Pi + Pi*c^2*x^2)^(3/2)), x)
```

3.99 $\int \frac{a+b\text{arcsinh}(cx)}{x^4(\pi+c^2\pi x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 153

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(\pi+c^2\pi x^2)^{3/2}}\,dx = -\frac{bc}{6\pi^{3/2}x^2} - \frac{a+b\text{arcsinh}(cx)}{3\pi x^3\sqrt{\pi+c^2\pi x^2}} + \frac{4c^2(a+b\text{arcsinh}(cx))}{3\pi x\sqrt{\pi+c^2\pi x^2}} + \frac{8c^4x(a+b\text{arcsinh}(cx))}{3\pi\sqrt{\pi+c^2\pi x^2}} - \frac{5bc^3\log(x)}{3\pi^{3/2}} - \frac{bc^3\log(1+c^2x^2)}{2\pi^{3/2}}$$

```
[Out] -1/6*b*c/Pi^(3/2)/x^2-5/3*b*c^3*ln(x)/Pi^(3/2)-1/2*b*c^3*ln(c^2*x^2+1)/Pi^(
3/2)+1/3*(-a-b*arcsinh(c*x))/Pi/x^3/(Pi*c^2*x^2+Pi)^(1/2)+4/3*c^2*(a+b*arcs
inh(c*x))/Pi/x/(Pi*c^2*x^2+Pi)^(1/2)+8/3*c^4*x*(a+b*arcsinh(c*x))/Pi/(Pi*c^
2*x^2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {277, 197, 5804, 12, 1265, 907}

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{4c^2(a+b\text{arcsinh}(cx))}{3\pi x\sqrt{\pi c^2x^2+\pi}} - \frac{a+b\text{arcsinh}(cx)}{3\pi x^3\sqrt{\pi c^2x^2+\pi}} + \frac{8c^4x(a+b\text{arcsinh}(cx))}{3\pi\sqrt{\pi c^2x^2+\pi}} - \frac{5bc^3\log(x)}{3\pi^{3/2}} - \frac{bc^3\log(c^2x^2+1)}{2\pi^{3/2}} - \frac{bc}{6\pi^{3/2}x^2}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] -1/6*(b*c)/(Pi^(3/2)*x^2) - (a + b*ArcSinh[c*x])/(3*Pi*x^3*Sqrt[Pi + c^2*Pi
*x^2]) + (4*c^2*(a + b*ArcSinh[c*x]))/(3*Pi*x*Sqrt[Pi + c^2*Pi*x^2]) + (8*c
```

```
^4*x*(a + b*ArcSinh[c*x]))/(3*Pi*Sqrt[Pi + c^2*Pi*x^2]) - (5*b*c^3*Log[x])/
(3*Pi^(3/2)) - (b*c^3*Log[1 + c^2*x^2])/(2*Pi^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a+b\text{arcsinh}(cx)}{3\pi x^3\sqrt{\pi+c^2\pi x^2}}+\frac{4c^2(a+b\text{arcsinh}(cx))}{3\pi x\sqrt{\pi+c^2\pi x^2}} \\
&\quad +\frac{8c^4x(a+b\text{arcsinh}(cx))}{3\pi\sqrt{\pi+c^2\pi x^2}}-\left(bc\sqrt{\pi}\right)\int\frac{-1+4c^2x^2+8c^4x^4}{3\pi^2x^3\left(1+c^2x^2\right)}\,dx \\
&= -\frac{a+b\text{arcsinh}(cx)}{3\pi x^3\sqrt{\pi+c^2\pi x^2}}+\frac{4c^2(a+b\text{arcsinh}(cx))}{3\pi x\sqrt{\pi+c^2\pi x^2}} \\
&\quad +\frac{8c^4x(a+b\text{arcsinh}(cx))}{3\pi\sqrt{\pi+c^2\pi x^2}}-\frac{(bc)\int\frac{-1+4c^2x^2+8c^4x^4}{x^3(1+c^2x^2)}\,dx}{3\pi^{3/2}} \\
&= -\frac{a+b\text{arcsinh}(cx)}{3\pi x^3\sqrt{\pi+c^2\pi x^2}}+\frac{4c^2(a+b\text{arcsinh}(cx))}{3\pi x\sqrt{\pi+c^2\pi x^2}} \\
&\quad +\frac{8c^4x(a+b\text{arcsinh}(cx))}{3\pi\sqrt{\pi+c^2\pi x^2}}-\frac{(bc)\text{Subst}\left(\int\frac{-1+4c^2x+8c^4x^2}{x^2(1+c^2x)}\,dx,x,x^2\right)}{6\pi^{3/2}} \\
&= -\frac{a+b\text{arcsinh}(cx)}{3\pi x^3\sqrt{\pi+c^2\pi x^2}}+\frac{4c^2(a+b\text{arcsinh}(cx))}{3\pi x\sqrt{\pi+c^2\pi x^2}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3\pi\sqrt{\pi+c^2\pi x^2}} \\
&\quad -\frac{(bc)\text{Subst}\left(\int\left(-\frac{1}{x^2}+\frac{5c^2}{x}+\frac{3c^4}{1+c^2x}\right)\,dx,x,x^2\right)}{6\pi^{3/2}} \\
&= -\frac{bc}{6\pi^{3/2}x^2}-\frac{a+b\text{arcsinh}(cx)}{3\pi x^3\sqrt{\pi+c^2\pi x^2}}+\frac{4c^2(a+b\text{arcsinh}(cx))}{3\pi x\sqrt{\pi+c^2\pi x^2}} \\
&\quad +\frac{8c^4x(a+b\text{arcsinh}(cx))}{3\pi\sqrt{\pi+c^2\pi x^2}}-\frac{5bc^3\log(x)}{3\pi^{3/2}}-\frac{bc^3\log\left(1+c^2x^2\right)}{2\pi^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.10

$$
\int\frac{a+b\text{arcsinh}(cx)}{x^4\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx=\frac{-2a+8ac^2x^2+16ac^4x^4-bcx\sqrt{1+c^2x^2}-16bc^3x^3\sqrt{1+c^2x^2}+2b(-1+4c^2x^2+\dots}{6\pi^{3/2}x^3\sqrt{\dots}}
$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] (-2*a + 8*a*c^2*x^2 + 16*a*c^4*x^4 - b*c*x*Sqrt[1 + c^2*x^2] - 16*b*c^3*x^3
*Sqrt[1 + c^2*x^2] + 2*b*(-1 + 4*c^2*x^2 + 8*c^4*x^4)*ArcSinh[c*x] - 10*b*c
^3*x^3*Sqrt[1 + c^2*x^2]*Log[x] - 3*b*c^3*x^3*Sqrt[1 + c^2*x^2]*Log[1 + c^2
*x^2])/(6*Pi^(3/2)*x^3*Sqrt[1 + c^2*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(132) = 264$.

Time = 0.17 (sec) , antiderivative size = 604, normalized size of antiderivative = 3.95

method	result
default	$a\left(-\frac{1}{3\pi x^3\sqrt{\pi c^2x^2+\pi}}-\frac{4c^2\left(-\frac{1}{\pi x\sqrt{\pi c^2x^2+\pi}}-\frac{2c^2x}{\pi\sqrt{\pi c^2x^2+\pi}}\right)}{3}\right)+\frac{16b\,c^3\operatorname{arcsinh}(cx)}{3\pi^{\frac{3}{2}}}-\frac{32b\,x^8c^{11}}{3\pi^{\frac{3}{2}}(8c^2x^2-1)(c^2x^2+1)}+\frac{32b\,x^6}{3\pi^{\frac{3}{2}}(8c^2}$
parts	$a\left(-\frac{1}{3\pi x^3\sqrt{\pi c^2x^2+\pi}}-\frac{4c^2\left(-\frac{1}{\pi x\sqrt{\pi c^2x^2+\pi}}-\frac{2c^2x}{\pi\sqrt{\pi c^2x^2+\pi}}\right)}{3}\right)+\frac{16b\,c^3\operatorname{arcsinh}(cx)}{3\pi^{\frac{3}{2}}}-\frac{32b\,x^8c^{11}}{3\pi^{\frac{3}{2}}(8c^2x^2-1)(c^2x^2+1)}+\frac{32b\,x^6}{3\pi^{\frac{3}{2}}(8c^2}$

```
[In] int((a+b*arcsinh(c*x))/x^4/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/3/Pi/x^3/(Pi*c^2*x^2+Pi)^(1/2)-4/3*c^2*(-1/Pi/x/(Pi*c^2*x^2+Pi)^(1/2)
-2/Pi*c^2*x/(Pi*c^2*x^2+Pi)^(1/2)))+16/3*b*c^3/Pi^(3/2)*arcsinh(c*x)-32/3*b
/Pi^(3/2)/(8*c^2*x^2-1)*x^8/(c^2*x^2+1)*c^11+32/3*b/Pi^(3/2)/(8*c^2*x^2-1)*
x^6*c^9-64/3*b/Pi^(3/2)/(8*c^2*x^2-1)*x^6/(c^2*x^2+1)*c^9+32/3*b/Pi^(3/2)/(
8*c^2*x^2-1)*x^4*c^7-64/3*b/Pi^(3/2)/(8*c^2*x^2-1)*x^4/(c^2*x^2+1)*arcsinh(
c*x)*c^7+64/3*b/Pi^(3/2)/(8*c^2*x^2-1)*x^3/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c
^6-32/3*b/Pi^(3/2)/(8*c^2*x^2-1)*x^4/(c^2*x^2+1)*c^7-56/3*b/Pi^(3/2)/(8*c^2
*x^2-1)*x^2/(c^2*x^2+1)*arcsinh(c*x)*c^5+8*b/Pi^(3/2)/(8*c^2*x^2-1)*x/(c^2*
x^2+1)^(1/2)*arcsinh(c*x)*c^4-4/3*b/Pi^(3/2)/(8*c^2*x^2-1)*c^3+8/3*b/Pi^(3/
2)/(8*c^2*x^2-1)/(c^2*x^2+1)*arcsinh(c*x)*c^3-4*b/Pi^(3/2)/(8*c^2*x^2-1)/x/
(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^2+1/6*b/Pi^(3/2)/(8*c^2*x^2-1)/x^2*c+1/3*b
/Pi^(3/2)/(8*c^2*x^2-1)/x^3/(c^2*x^2+1)^(1/2)*arcsinh(c*x)-5/3*b*c^3/Pi^(3/
2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)-b*c^3/Pi^(3/2)*ln(1+(c*x+(c^2*x^2+1)^(1/
2))^2)
```

Fricas [F]

$$\int\frac{a+b\text{arcsinh}(cx)}{x^4\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx=\int\frac{b\,\text{arsinh}\,(cx)+a}{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x^4}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^2*c^4*x^8 + 2*pi^2*
c^2*x^6 + pi^2*x^4), x)
```

Sympy [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^4(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{\int \frac{a}{c^2x^6\sqrt{c^2x^2+1}+x^4\sqrt{c^2x^2+1}}\,dx + \int \frac{b\operatorname{asinh}(cx)}{c^2x^6\sqrt{c^2x^2+1}+x^4\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

```
[In] integrate((a+b*asinh(c*x))/x**4/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a/(c**2*x**6*sqrt(c**2*x**2 + 1) + x**4*sqrt(c**2*x**2 + 1)), x)
+ Integral(b*asinh(c*x)/(c**2*x**6*sqrt(c**2*x**2 + 1) + x**4*sqrt(c**2*x**
2 + 1)), x))/pi**(3/2)
```

Maxima [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^4(\pi+c^2\pi x^2)^{3/2}}\,dx = \int \frac{b\operatorname{arsinh}(cx)+a}{(\pi+\pi c^2x^2)^{\frac{3}{2}}x^4}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/3*(8*c^4*x/(pi*sqrt(pi + pi*c^2*x^2)) + 4*c^2/(pi*sqrt(pi + pi*c^2*x^2)*x
) - 1/(pi*sqrt(pi + pi*c^2*x^2)*x^3))*a + b*integrate(log(c*x + sqrt(c^2*x^
2 + 1))/((pi + pi*c^2*x^2)^(3/2)*x^4), x)
```

Giac [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^4(\pi+c^2\pi x^2)^{3/2}}\,dx = \int \frac{b\operatorname{arsinh}(cx)+a}{(\pi+\pi c^2x^2)^{\frac{3}{2}}x^4}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^4(\pi+c^2\pi x^2)^{3/2}}\,dx = \int \frac{a+b\operatorname{asinh}(c\,x)}{x^4\,(\Pi\,c^2\,x^2+\Pi)^{3/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))/(x^4*(Pi + Pi*c^2*x^2)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^4*(Pi + Pi*c^2*x^2)^(3/2)), x)
```

3.100 $\int \frac{x^6(a+b\mathbf{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 192

$$\begin{aligned}\int \frac{x^6(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx &= -\frac{bx^2}{4c^5\pi^{5/2}}-\frac{b}{6c^7\pi^{5/2}(1+c^2x^2)} \\ &-\frac{x^5(a+b\text{arcsinh}(cx))}{3c^2\pi(\pi+c^2\pi x^2)^{3/2}}-\frac{5x^3(a+b\text{arcsinh}(cx))}{3c^4\pi^2\sqrt{\pi+c^2\pi x^2}} \\ &+\frac{5x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{2c^6\pi^3}-\frac{5(a+b\text{arcsinh}(cx))^2}{4bc^7\pi^{5/2}}-\frac{7b\log(1+c^2x^2)}{6c^7\pi^{5/2}}\end{aligned}$$

```
[Out] -1/4*b*x^2/c^5/Pi^(5/2)-1/6*b/c^7/Pi^(5/2)/(c^2*x^2+1)-1/3*x^5*(a+b*arcsinh
(c*x))/c^2/Pi/(Pi*c^2*x^2+Pi)^(3/2)-5/4*(a+b*arcsinh(c*x))^2/b/c^7/Pi^(5/2)
-7/6*b*ln(c^2*x^2+1)/c^7/Pi^(5/2)-5/3*x^3*(a+b*arcsinh(c*x))/c^4/Pi^2/(Pi*c
^2*x^2+Pi)^(1/2)+5/2*x*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^6/Pi^3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5810, 5812, 5783, 30, 272, 45}

$$\begin{aligned}\int \frac{x^6(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx &= -\frac{5(a+b\text{arcsinh}(cx))^2}{4\pi^{5/2}bc^7} \\ &-\frac{x^5(a+b\text{arcsinh}(cx))}{3\pi c^2(\pi c^2x^2+\pi)^{3/2}}+\frac{5x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2\pi^3c^6} \\ &-\frac{5x^3(a+b\text{arcsinh}(cx))}{3\pi^2c^4\sqrt{\pi c^2x^2+\pi}}-\frac{bx^2}{4\pi^{5/2}c^5}-\frac{b}{6\pi^{5/2}c^7(c^2x^2+1)}-\frac{7b\log(c^2x^2+1)}{6\pi^{5/2}c^7}\end{aligned}$$

```
[In] Int[(x^6*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] -1/4*(b*x^2)/(c^5*Pi^(5/2)) - b/(6*c^7*Pi^(5/2)*(1 + c^2*x^2)) - (x^5*(a +
b*ArcSinh[c*x]))/(3*c^2*Pi*(Pi + c^2*Pi*x^2)^(3/2)) - (5*x^3*(a + b*ArcSinh
[c*x]))/(3*c^4*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) + (5*x*Sqrt[Pi + c^2*Pi*x^2]*(a
+ b*ArcSinh[c*x]))/(2*c^6*Pi^3) - (5*(a + b*ArcSinh[c*x])^2)/(4*b*c^7*Pi^(5
/2)) - (7*b*Log[1 + c^2*x^2])/(6*c^7*Pi^(5/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
```

```
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^5(a+b\text{arcsinh}(cx))}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{b\int \frac{x^5}{(1+c^2x^2)^2}\,dx}{3c\pi^{5/2}}+\frac{5\int \frac{x^4(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx}{3c^2\pi} \\
&= -\frac{x^5(a+b\text{arcsinh}(cx))}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{5x^3(a+b\text{arcsinh}(cx))}{3c^4\pi^2\sqrt{\pi+c^2\pi x^2}}+\frac{(5b)\int \frac{x^3}{1+c^2x^2}\,dx}{3c^3\pi^{5/2}} \\
&\quad +\frac{b\text{Subst}\left(\int \frac{x^2}{(1+c^2x)^2}\,dx,x,x^2\right)}{6c\pi^{5/2}}+\frac{5\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx}{c^4\pi^2} \\
&= -\frac{x^5(a+b\text{arcsinh}(cx))}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{5x^3(a+b\text{arcsinh}(cx))}{3c^4\pi^2\sqrt{\pi+c^2\pi x^2}} \\
&\quad +\frac{5x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{2c^6\pi^3}-\frac{(5b)\int x\,dx}{2c^5\pi^{5/2}}+\frac{(5b)\text{Subst}\left(\int \frac{x}{1+c^2x}\,dx,x,x^2\right)}{6c^3\pi^{5/2}} \\
&\quad +\frac{b\text{Subst}\left(\int \left(\frac{1}{c^4}+\frac{1}{c^4(1+c^2x)^2}-\frac{2}{c^4(1+c^2x)}\right)dx,x,x^2\right)}{6c\pi^{5/2}}-\frac{5\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx}{2c^6\pi^2} \\
&= -\frac{13bx^2}{12c^5\pi^{5/2}}-\frac{b}{6c^7\pi^{5/2}\left(1+c^2x^2\right)}-\frac{x^5(a+b\text{arcsinh}(cx))}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{5x^3(a+b\text{arcsinh}(cx))}{3c^4\pi^2\sqrt{\pi+c^2\pi x^2}} \\
&\quad +\frac{5x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{2c^6\pi^3}-\frac{5(a+b\text{arcsinh}(cx))^2}{4bc^7\pi^{5/2}} \\
&\quad -\frac{b\log\left(1+c^2x^2\right)}{3c^7\pi^{5/2}}+\frac{(5b)\text{Subst}\left(\int \left(\frac{1}{c^2}-\frac{1}{c^2(1+c^2x)}\right)dx,x,x^2\right)}{6c^3\pi^{5/2}} \\
&= -\frac{bx^2}{4c^5\pi^{5/2}}-\frac{b}{6c^7\pi^{5/2}\left(1+c^2x^2\right)}-\frac{x^5(a+b\text{arcsinh}(cx))}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{5x^3(a+b\text{arcsinh}(cx))}{3c^4\pi^2\sqrt{\pi+c^2\pi x^2}} \\
&\quad +\frac{5x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{2c^6\pi^3}-\frac{5(a+b\text{arcsinh}(cx))^2}{4bc^7\pi^{5/2}}-\frac{7b\log\left(1+c^2x^2\right)}{6c^7\pi^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.05

$$\int \frac{x^6(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{60acx+80ac^3x^3+12ac^5x^5-7b\sqrt{1+c^2x^2}-9bc^2x^2\sqrt{1+c^2x^2}-6bc^4x^4\sqrt{1+}}{}$$

```
[In] Integrate[(x^6*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (60*a*c*x + 80*a*c^3*x^3 + 12*a*c^5*x^5 - 7*b*Sqrt[1 + c^2*x^2] - 9*b*c^2*x
^2*Sqrt[1 + c^2*x^2] - 6*b*c^4*x^4*Sqrt[1 + c^2*x^2] + 4*(-15*a*(1 + c^2*x^
2)^(3/2) + b*c*x*(15 + 20*c^2*x^2 + 3*c^4*x^4))*ArcSinh[c*x] - 30*b*(1 + c^
2*x^2)^(3/2)*ArcSinh[c*x]^2 - 28*b*(1 + c^2*x^2)^(3/2)*Log[1 + c^2*x^2])/(2
4*c^7*Pi^(5/2)*(1 + c^2*x^2)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 969 vs. 2(164) = 328.

Time = 0.33 (sec) , antiderivative size = 970, normalized size of antiderivative = 5.05

method	result	size
default	Expression too large to display	970
parts	Expression too large to display	970

```
[In] int(x^6*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 49/6*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)/c*x^6+14*b/Pi^(5/2)
/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)/c^3*x^4+6*b/Pi^(5/2)/(63*c^4*x^4+1
11*c^2*x^2+49)/(c^2*x^2+1)/c^5*x^2+147*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+4
9)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)*x^7+1/2*b/c^6/Pi^(5/2)*(c^2*x^2+1)^(1/2)*
arcsinh(c*x)*x-49/6*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^2*c*
x^8-98/3*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^2/c*x^6-49*b/Pi
^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^2/c^3*x^4-98/3*b/Pi^(5/2)/(6
3*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^2/c^5*x^2-343/3*b/Pi^(5/2)/(63*c^4*x^
4+111*c^2*x^2+49)/(c^2*x^2+1)^2/c^7*arcsinh(c*x)+5/6*a/c^4*x^3/Pi/(Pi*c^2*x
^2+Pi)^(3/2)+5/2*a/c^6/Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2)-5/2*a/c^6/Pi^2*ln(Pi*c^
2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/2*a*x^5/Pi/c^2/(
Pi*c^2*x^2+Pi)^(3/2)-1/8*b/c^7/Pi^(5/2)-1/4*b*x^2/c^5/Pi^(5/2)-5/4*b/c^7/Pi
^(5/2)*arcsinh(c*x)^2-7/3*b/c^7/Pi^(5/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+14
/3*b/c^7/Pi^(5/2)*arcsinh(c*x)-49/6*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/
(c^2*x^2+1)^2/c^7-147*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^2*
c*arcsinh(c*x)*x^8-553*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^2
/c*arcsinh(c*x)*x^6-2338/3*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+
1)^2/c^3*arcsinh(c*x)*x^4-1463/3*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^
```

```
2*x^2+1)^2/c^5*arcsinh(c*x)*x^2+385*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/
(c^2*x^2+1)^(3/2)/c^2*arcsinh(c*x)*x^5+1009/3*b/Pi^(5/2)/(63*c^4*x^4+111*c^
2*x^2+49)/(c^2*x^2+1)^(3/2)/c^4*arcsinh(c*x)*x^3+98*b/Pi^(5/2)/(63*c^4*x^4+
111*c^2*x^2+49)/(c^2*x^2+1)^(3/2)/c^6*arcsinh(c*x)*x
```

Fricas [F]

$$\int \frac{x^6(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^6}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^6*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*x^6*arcsinh(c*x) + a*x^6)/(pi^3*c^6*x^6 +
 3*pi^3*c^4*x^4 + 3*pi^3*c^2*x^2 + pi^3), x)
```

Sympy [F]

$$\int \frac{x^6(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\frac{\int \frac{ax^6}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx^6\,\text{asinh}\,(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

```
[In] integrate(x**6*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a*x**6/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**
2 + 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*x**6*asinh(c*x)/(c**4*x**4*s
qrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1))
, x))/pi**(5/2)
```

Maxima [F]

$$\int \frac{x^6(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^6}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^6*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] 1/6*a*(3*x^5/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) + 5*x*(3*x^2/(pi*(pi + pi*c^2
*x^2)^(3/2)*c^2) + 2/(pi*(pi + pi*c^2*x^2)^(3/2)*c^4))/c^2 + 5*x/(pi^2*sqrt
(pi + pi*c^2*x^2)*c^6) - 15*arcsinh(c*x)/(pi^(5/2)*c^7)) + b*integrate(x^6*
log(c*x + sqrt(c^2*x^2 + 1))/(pi + pi*c^2*x^2)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^6(a + b\text{arcsinh}(cx))}{(\pi + c^2\pi x^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^6*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(a + b\text{arcsinh}(cx))}{(\pi + c^2\pi x^2)^{5/2}}\,dx = \int \frac{x^6\,(a + b\,\text{asinh}(c\,x))}{(\Pi\,c^2\,x^2 + \Pi)^{5/2}}\,dx$$

```
[In] int((x^6*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2),x)
```

```
[Out] int((x^6*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

3.101 $\int \frac{x^5(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}} dx$

Optimal result

Integrand size = 26, antiderivative size = 146

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}} dx = -\frac{bx}{c^5\pi^{5/2}} + \frac{bx}{6c^5\pi^{5/2}(1+c^2x^2)} - \frac{a+b\text{arcsinh}(cx)}{3c^6\pi(\pi+c^2\pi x^2)^{3/2}} + \frac{2(a+b\text{arcsinh}(cx))}{c^6\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{c^6\pi^3} - \frac{11b\arctan(cx)}{6c^6\pi^{5/2}}$$

```
[Out] -b*x/c^5/Pi^(5/2)+1/6*b*x/c^5/Pi^(5/2)/(c^2*x^2+1)+1/3*(-a-b*arcsinh(c*x))/
c^6/Pi/(Pi*c^2*x^2+Pi)^(3/2)-11/6*b*arctan(c*x)/c^6/Pi^(5/2)+2*(a+b*arcsinh
(c*x))/c^6/Pi^2/(Pi*c^2*x^2+Pi)^(1/2)+(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1
/2)/c^6/Pi^3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {272, 45, 5804, 12, 1171, 396, 209}

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}} dx = \frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{\pi^3c^6} + \frac{2(a+b\text{arcsinh}(cx))}{\pi^2c^6\sqrt{\pi c^2x^2+\pi}} - \frac{a+b\text{arcsinh}(cx)}{3\pi c^6(\pi c^2x^2+\pi)^{3/2}} - \frac{11b\arctan(cx)}{6\pi^{5/2}c^6} - \frac{bx}{\pi^{5/2}c^5} + \frac{bx}{6\pi^{5/2}c^5(c^2x^2+1)}$$

```
[In] Int[(x^5*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] -((b*x)/(c^5*Pi^(5/2))) + (b*x)/(6*c^5*Pi^(5/2)*(1 + c^2*x^2)) - (a + b*Arc
Sinh[c*x])/(3*c^6*Pi*(Pi + c^2*Pi*x^2)^(3/2)) + (2*(a + b*ArcSinh[c*x]))/(c
```

```
^6*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) + (Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]
))/(c^6*Pi^3) - (11*b*ArcTan[c*x])/(6*c^6*Pi^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
```

```
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\text{arcsinh}(cx)}{3c^6\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{2(a+b\text{arcsinh}(cx))}{c^6\pi^2\sqrt{\pi+c^2\pi x^2}} \\
&\quad+\frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{c^6\pi^3}-\left(bc\sqrt{\pi}\right)\int\frac{8+12c^2x^2+3c^4x^4}{3c^6\pi^3\left(1+c^2x^2\right)^2}\,dx \\
&= -\frac{a+b\text{arcsinh}(cx)}{3c^6\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{2(a+b\text{arcsinh}(cx))}{c^6\pi^2\sqrt{\pi+c^2\pi x^2}} \\
&\quad+\frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{c^6\pi^3}-\frac{b\int\frac{8+12c^2x^2+3c^4x^4}{\left(1+c^2x^2\right)^2}\,dx}{3c^5\pi^{5/2}} \\
&= \frac{bx}{6c^5\pi^{5/2}\left(1+c^2x^2\right)}-\frac{a+b\text{arcsinh}(cx)}{3c^6\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{2(a+b\text{arcsinh}(cx))}{c^6\pi^2\sqrt{\pi+c^2\pi x^2}} \\
&\quad+\frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{c^6\pi^3}+\frac{b\int\frac{-17-6c^2x^2}{1+c^2x^2}\,dx}{6c^5\pi^{5/2}} \\
&= -\frac{bx}{c^5\pi^{5/2}}+\frac{bx}{6c^5\pi^{5/2}\left(1+c^2x^2\right)}-\frac{a+b\text{arcsinh}(cx)}{3c^6\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{2(a+b\text{arcsinh}(cx))}{c^6\pi^2\sqrt{\pi+c^2\pi x^2}} \\
&\quad+\frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{c^6\pi^3}-\frac{(11b)\int\frac{1}{1+c^2x^2}\,dx}{6c^5\pi^{5/2}} \\
&= -\frac{bx}{c^5\pi^{5/2}}+\frac{bx}{6c^5\pi^{5/2}\left(1+c^2x^2\right)}-\frac{a+b\text{arcsinh}(cx)}{3c^6\pi\left(\pi+c^2\pi x^2\right)^{3/2}} \\
&\quad+\frac{2(a+b\text{arcsinh}(cx))}{c^6\pi^2\sqrt{\pi+c^2\pi x^2}}+\frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{c^6\pi^3}-\frac{11b\arctan(cx)}{6c^6\pi^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.90

$$\int\frac{x^5(a+b\text{arcsinh}(cx))}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx=\frac{16a+24ac^2x^2+6ac^4x^4-5bcx\sqrt{1+c^2x^2}-6bc^3x^3\sqrt{1+c^2x^2}+2b(8+12c^2x^2}{6c^6\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}$$

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (16*a + 24*a*c^2*x^2 + 6*a*c^4*x^4 - 5*b*c*x*Sqrt[1 + c^2*x^2] - 6*b*c^3*x^
3*Sqrt[1 + c^2*x^2] + 2*b*(8 + 12*c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x] - 11*b*
(1 + c^2*x^2)^(3/2)*ArcTan[c*x])/(6*c^6*Pi^(5/2)*(1 + c^2*x^2)^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.62

method	result
default	$a\left(\frac{x^4}{\pi c^2(\pi c^2x^2+\pi)^{\frac{3}{2}}}-\frac{4\left(-\frac{x^2}{\pi c^2\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}-\frac{2}{3\pi c^4\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}\right)}{c^2}\right)+\frac{b\sqrt{c^2x^2+1}\,\mathrm{arcsinh}(cx)}{c^6\pi^{\frac{5}{2}}}-\frac{bx}{c^5\pi^{\frac{5}{2}}}+\frac{2b\,\mathrm{arcsinh}(cx)x}{\pi^{\frac{5}{2}}(c^2x^2+1)^{\frac{3}{2}}c}$
parts	$a\left(\frac{x^4}{\pi c^2(\pi c^2x^2+\pi)^{\frac{3}{2}}}-\frac{4\left(-\frac{x^2}{\pi c^2\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}-\frac{2}{3\pi c^4\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}\right)}{c^2}\right)+\frac{b\sqrt{c^2x^2+1}\,\mathrm{arcsinh}(cx)}{c^6\pi^{\frac{5}{2}}}-\frac{bx}{c^5\pi^{\frac{5}{2}}}+\frac{2b\,\mathrm{arcsinh}(cx)x}{\pi^{\frac{5}{2}}(c^2x^2+1)^{\frac{3}{2}}c}$

```
[In] int(x^5*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(x^4/Pi/c^2/(Pi*c^2*x^2+Pi)^(3/2)-4/c^2*(-x^2/Pi/c^2/(Pi*c^2*x^2+Pi)^(3/2
)-2/3/Pi/c^4/(Pi*c^2*x^2+Pi)^(3/2)))+b/c^6/Pi^(5/2)*(c^2*x^2+1)^(1/2)*arcsi
nh(c*x)-b*x/c^5/Pi^(5/2)+2*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)/c^4*arcsinh(c*x)*x^
2+1/6*b*x/c^5/Pi^(5/2)/(c^2*x^2+1)+5/3*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)/c^6*arc
sinh(c*x)+11/6*I*b/c^6/Pi^(5/2)*ln(c*x+(c^2*x^2+1)^(1/2)-I)-11/6*I*b/c^6/Pi
^(5/2)*ln(c*x+(c^2*x^2+1)^(1/2)+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.49

$$\int \frac{x^5(a+b\mathrm{arcsinh}(cx))}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx=\frac{11\sqrt{\pi}(bc^4x^4+2\,bc^2x^2+b)\arctan\left(-\frac{2\sqrt{\pi}\sqrt{\pi+\pi c^2x^2}\sqrt{c^2x^2+1}cx}{\pi-\pi c^4x^4}\right)+4\sqrt{\pi+\pi c^2x^2}(}{}$$

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] 1/12*(11*sqrt(pi)*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi
+ pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) + 4*sqrt(pi + pi*c^2
*x^2)*(3*b*c^4*x^4 + 12*b*c^2*x^2 + 8*b)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*s
qrt(pi + pi*c^2*x^2)*(6*a*c^4*x^4 + 24*a*c^2*x^2 - (6*b*c^3*x^3 + 5*b*c*x)*
sqrt(c^2*x^2 + 1) + 16*a))/(pi^3*c^10*x^4 + 2*pi^3*c^8*x^2 + pi^3*c^6)
```

Sympy [F]

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{\int \frac{ax^5}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{bx^5\operatorname{asinh}(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

```
[In] integrate(x**5*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a*x**5/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**
2 + 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*x**5*asinh(c*x)/(c**4*x**4*s
qrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1))
, x))/pi**(5/2)
```

Maxima [F]

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x^5}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] 1/3*b*((3*sqrt(pi)*c^4*x^4 + 12*sqrt(pi)*c^2*x^2 + 8*sqrt(pi))*log(c*x + sq
rt(c^2*x^2 + 1))/((pi^3*c^8*x^2 + pi^3*c^6)*sqrt(c^2*x^2 + 1)) + 3*integrat
e(1/3*(3*sqrt(pi)*c^4*x^4 + 12*sqrt(pi)*c^2*x^2 + 8*sqrt(pi))/(pi^3*c^11*x^
6 + 2*pi^3*c^9*x^4 + pi^3*c^7*x^2 + (pi^3*c^10*x^5 + 2*pi^3*c^8*x^3 + pi^3*
c^6*x)*sqrt(c^2*x^2 + 1)), x) - 3*integrate(1/3*(3*sqrt(pi)*c^4*x^4 + 12*sq
rt(pi)*c^2*x^2 + 8*sqrt(pi))/((pi^3*c^8*x^3 + pi^3*c^6*x)*sqrt(c^2*x^2 + 1)
), x)) + 1/3*a*(3*x^4/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) + 12*x^2/(pi*(pi + p
i*c^2*x^2)^(3/2)*c^4) + 8/(pi*(pi + pi*c^2*x^2)^(3/2)*c^6))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a + b\text{arcsinh}(cx))}{(\pi + c^2\pi x^2)^{5/2}}\,dx = \int \frac{x^5\,(a + b\,\text{asinh}(c\,x))}{(\Pi\,c^2\,x^2 + \Pi)^{5/2}}\,dx$$

```
[In] int((x^5*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2),x)
[Out] int((x^5*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

3.102 $\int \frac{x^4(a+b\,\mathbf{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 139

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\frac{b}{6c^5\pi^{5/2}(1+c^2x^2)}-\frac{x^3(a+b\operatorname{arcsinh}(cx))}{3c^2\pi(\pi+c^2\pi x^2)^{3/2}}-\frac{x(a+b\operatorname{arcsinh}(cx))}{c^4\pi^2\sqrt{\pi+c^2\pi x^2}}+\frac{(a+b\operatorname{arcsinh}(cx))^2}{2bc^5\pi^{5/2}}+\frac{2b\log(1+c^2x^2)}{3c^5\pi^{5/2}}$$

```
[Out] 1/6*b/c^5/Pi^(5/2)/(c^2*x^2+1)-1/3*x^3*(a+b*arcsinh(c*x))/c^2/Pi/(Pi*c^2*x^
2+Pi)^(3/2)+1/2*(a+b*arcsinh(c*x))^2/b/c^5/Pi^(5/2)+2/3*b*ln(c^2*x^2+1)/c^5
/Pi^(5/2)-x*(a+b*arcsinh(c*x))/c^4/Pi^2/(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5810, 5783, 266, 272, 45}

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\frac{(a+b\operatorname{arcsinh}(cx))^2}{2\pi^{5/2}bc^5}-\frac{x^3(a+b\operatorname{arcsinh}(cx))}{3\pi c^2(\pi c^2x^2+\pi)^{3/2}}-\frac{x(a+b\operatorname{arcsinh}(cx))}{\pi^2c^4\sqrt{\pi c^2x^2+\pi}}+\frac{b}{6\pi^{5/2}c^5(c^2x^2+1)}+\frac{2b\log(c^2x^2+1)}{3\pi^{5/2}c^5}$$

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] b/(6*c^5*Pi^(5/2)*(1 + c^2*x^2)) - (x^3*(a + b*ArcSinh[c*x]))/(3*c^2*Pi*(Pi
 + c^2*Pi*x^2)^(3/2)) - (x*(a + b*ArcSinh[c*x]))/(c^4*Pi^2*Sqrt[Pi + c^2*Pi
*x^2]) + (a + b*ArcSinh[c*x])^2/(2*b*c^5*Pi^(5/2)) + (2*b*Log[1 + c^2*x^2])
/(3*c^5*Pi^(5/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{b\int\frac{x^3}{(1+c^2x^2)^2}\,dx}{3c\pi^{5/2}}+\frac{\int\frac{x^2(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx}{c^2\pi}\\
&= -\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{x(a+b\text{arcsinh}(cx))}{c^4\pi^2\sqrt{\pi+c^2\pi x^2}}+\frac{b\int\frac{x}{1+c^2x^2}\,dx}{c^3\pi^{5/2}}\\
&\quad+\frac{b\text{Subst}\left(\int\frac{x}{(1+c^2x)^2}\,dx,x,x^2\right)}{6c\pi^{5/2}}+\frac{\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx}{c^4\pi^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{x(a+b\text{arcsinh}(cx))}{c^4\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{(a+b\text{arcsinh}(cx))^2}{2bc^5\pi^{5/2}} \\
&\quad + \frac{b\log\left(1+c^2x^2\right)}{2c^5\pi^{5/2}} + \frac{b\text{Subst}\left(\int\left(-\frac{1}{c^2(1+c^2x)^2}+\frac{1}{c^2(1+c^2x)}\right)dx,x,x^2\right)}{6c\pi^{5/2}} \\
&= \frac{b}{6c^5\pi^{5/2}\left(1+c^2x^2\right)} - \frac{x^3(a+b\text{arcsinh}(cx))}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{x(a+b\text{arcsinh}(cx))}{c^4\pi^2\sqrt{\pi+c^2\pi x^2}} \\
&\quad + \frac{(a+b\text{arcsinh}(cx))^2}{2bc^5\pi^{5/2}} + \frac{2b\log\left(1+c^2x^2\right)}{3c^5\pi^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.19

$$
\int \frac{x^4(a+b\text{arcsinh}(cx))}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = \frac{b+bc^2x^2-6acx\sqrt{1+c^2x^2}-8ac^3x^3\sqrt{1+c^2x^2}+2\left(3a(1+c^2x^2)^2-bcx\sqrt{1+\cdots}\right.}{6c^5\pi\cdots}
$$

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (b + b*c^2*x^2 - 6*a*c*x*Sqrt[1 + c^2*x^2] - 8*a*c^3*x^3*Sqrt[1 + c^2*x^2]
+ 2*(3*a*(1 + c^2*x^2)^2 - b*c*x*Sqrt[1 + c^2*x^2]*(3 + 4*c^2*x^2))*ArcSinh
[c*x] + 3*b*(1 + c^2*x^2)^2*ArcSinh[c*x]^2 + 4*b*(1 + c^2*x^2)^2*Log[1 + c^
2*x^2])/(6*c^5*Pi^(5/2)*(1 + c^2*x^2)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 896 vs. $2(121) = 242$.

Time = 0.17 (sec) , antiderivative size = 897, normalized size of antiderivative = 6.45

method	result
default	$-\frac{a\,x^3}{3\pi\,c^2(\pi\,c^2x^2+\pi)^{\frac{3}{2}}}-\frac{ax}{\pi^2c^4\sqrt{\pi\,c^2x^2+\pi}}+\frac{a\ln\left(\frac{\pi\,c^2x}{\sqrt{\pi\,c^2}}+\sqrt{\pi\,c^2x^2+\pi}\right)}{\pi^2c^4\sqrt{\pi\,c^2}}+\frac{b\,\text{arcsinh}(cx)^2}{2c^5\pi^{\frac{5}{2}}}-\frac{8b\,\text{arcsinh}(cx)}{3c^5\pi^{\frac{5}{2}}}+\frac{32b\,c^3\,\text{arc}\cdots}{\pi^{\frac{5}{2}}(24c^4x^4+39c^2\cdots}$
parts	$-\frac{a\,x^3}{3\pi\,c^2(\pi\,c^2x^2+\pi)^{\frac{3}{2}}}-\frac{ax}{\pi^2c^4\sqrt{\pi\,c^2x^2+\pi}}+\frac{a\ln\left(\frac{\pi\,c^2x}{\sqrt{\pi\,c^2}}+\sqrt{\pi\,c^2x^2+\pi}\right)}{\pi^2c^4\sqrt{\pi\,c^2}}+\frac{b\,\text{arcsinh}(cx)^2}{2c^5\pi^{\frac{5}{2}}}-\frac{8b\,\text{arcsinh}(cx)}{3c^5\pi^{\frac{5}{2}}}+\frac{32b\,c^3\,\text{arc}\cdots}{\pi^{\frac{5}{2}}(24c^4x^4+39c^2\cdots}$

```
[In] int(x^4*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a*x^3/Pi/c^2/(Pi*c^2*x^2+Pi)^(3/2)-a/Pi^2/c^4*x/(Pi*c^2*x^2+Pi)^(1/2)+
a/Pi^2/c^4*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)
+1/2*b/c^5/Pi^(5/2)*arcsinh(c*x)^2-8/3*b/c^5/Pi^(5/2)*arcsinh(c*x)+32*b/Pi^
(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2*c^3*arcsinh(c*x)*x^8-32*b/Pi
^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^(3/2)*c^2*arcsinh(c*x)*x^7+8/
```

```
3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2*c^3*x^8-8/3*b/Pi^(5/2
)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)*c*x^6+116*b/Pi^(5/2)/(24*c^4*x^4+3
9*c^2*x^2+16)/(c^2*x^2+1)^2*c*arcsinh(c*x)*x^6-76*b/Pi^(5/2)/(24*c^4*x^4+39
*c^2*x^2+16)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)*x^5+32/3*b/Pi^(5/2)/(24*c^4*x^4
+39*c^2*x^2+16)/(c^2*x^2+1)^2*c*x^6-4*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)
/(c^2*x^2+1)/c*x^4+472/3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^
2/c*arcsinh(c*x)*x^4-181/3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1
)^(3/2)/c^2*arcsinh(c*x)*x^3+16*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*
x^2+1)^2/c*x^4-3/2*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)/c^3*x^
2+284/3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2/c^3*arcsinh(c*x
)*x^2-16*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^(3/2)/c^4*arcsin
h(c*x)*x+32/3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2/c^3*x^2+6
4/3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2/c^5*arcsinh(c*x)+8/
3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2/c^5+4/3*b/c^5/Pi^(5/2
)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

Fricas [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^4}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*x^4*arcsinh(c*x) + a*x^4)/(pi^3*c^6*x^6 +
 3*pi^3*c^4*x^4 + 3*pi^3*c^2*x^2 + pi^3), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\frac{\int \frac{ax^4}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx^4\,\text{asinh}\,(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

```
[In] integrate(x**4*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a*x**4/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**
2 + 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*x**4*asinh(c*x)/(c**4*x**4*s
qrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1))
, x))/pi**(5/2)
```

Maxima [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^4}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/3*(x*(3*x^2/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) + 2/(pi*(pi + pi*c^2*x^2)^(
3/2)*c^4)) + x/(pi^2*sqrt(pi + pi*c^2*x^2)*c^4) - 3*arcsinh(c*x)/(pi^(5/2)*
c^5))*a + b*integrate(x^4*log(c*x + sqrt(c^2*x^2 + 1))/(pi + pi*c^2*x^2)^(5
/2), x)
```

Giac [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^4}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^4/(pi + pi*c^2*x^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \int \frac{x^4\,(a+b\,\text{asinh}(c\,x))}{(\Pi\,c^2\,x^2+\Pi)^{5/2}}\,dx$$

```
[In] int((x^4*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2),x)
```

```
[Out] int((x^4*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

3.103 $\int \frac{x^3(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 105

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = -\frac{bx}{6c^3\pi^{5/2}(1+c^2x^2)} + \frac{a+b\text{arcsinh}(cx)}{3c^4\pi(\pi+c^2\pi x^2)^{3/2}} - \frac{a+b\text{arcsinh}(cx)}{c^4\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{5b\arctan(cx)}{6c^4\pi^{5/2}}$$

```
[Out] -1/6*b*x/c^3/Pi^(5/2)/(c^2*x^2+1)+1/3*(a+b*arcsinh(c*x))/c^4/Pi/(Pi*c^2*x^2
+Pi)^(3/2)+5/6*b*arctan(c*x)/c^4/Pi^(5/2)+(-a-b*arcsinh(c*x))/c^4/Pi^2/(Pi*
c^2*x^2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {272, 45, 5804, 12, 393, 209}

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = -\frac{a+b\text{arcsinh}(cx)}{\pi^2c^4\sqrt{\pi c^2x^2+\pi}} + \frac{a+b\text{arcsinh}(cx)}{3\pi c^4(\pi c^2x^2+\pi)^{3/2}} + \frac{5b\arctan(cx)}{6\pi^{5/2}c^4} - \frac{bx}{6\pi^{5/2}c^3(c^2x^2+1)}$$

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] -1/6*(b*x)/(c^3*Pi^(5/2)*(1 + c^2*x^2)) + (a + b*ArcSinh[c*x])/(3*c^4*Pi*(P
i + c^2*Pi*x^2)^(3/2)) - (a + b*ArcSinh[c*x])/(c^4*Pi^2*Sqrt[Pi + c^2*Pi*x^
2]) + (5*b*ArcTan[c*x])/(6*c^4*Pi^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\text{integral} = \frac{a + b\operatorname{arcsinh}(cx)}{3c^4\pi\left(\pi + c^2\pi x^2\right)^{3/2}} - \frac{a + b\operatorname{arcsinh}(cx)}{c^4\pi^2\sqrt{\pi + c^2\pi x^2}} - \left(bc\sqrt{\pi}\right)\int \frac{-2 - 3c^2x^2}{3c^4\pi^3\left(1 + c^2x^2\right)^2}\,dx$$

$$= \frac{a+b\operatorname{arcsinh}(cx)}{3c^4\pi\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{a+b\operatorname{arcsinh}(cx)}{c^4\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{b\int\frac{-2-3c^2x^2}{(1+c^2x^2)^2}\,dx}{3c^3\pi^{5/2}}$$

$$= -\frac{bx}{6c^3\pi^{5/2}\left(1+c^2x^2\right)} + \frac{a+b\operatorname{arcsinh}(cx)}{3c^4\pi\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{a+b\operatorname{arcsinh}(cx)}{c^4\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{(5b)\int\frac{1}{1+c^2x^2}\,dx}{6c^3\pi^{5/2}}$$

$$= -\frac{bx}{6c^3\pi^{5/2}\left(1+c^2x^2\right)} + \frac{a+b\operatorname{arcsinh}(cx)}{3c^4\pi\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{a+b\operatorname{arcsinh}(cx)}{c^4\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{5b\arctan(cx)}{6c^4\pi^{5/2}}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int\frac{x^3(a+b\operatorname{arcsinh}(cx))}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = \frac{-4a-6ac^2x^2-bcx\sqrt{1+c^2x^2}-2b(2+3c^2x^2)\operatorname{arcsinh}(cx)+5b(1+c^2x^2)^{3/2}}{6c^4\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}$$

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (-4*a - 6*a*c^2*x^2 - b*c*x*Sqrt[1 + c^2*x^2] - 2*b*(2 + 3*c^2*x^2)*ArcSinh
[c*x] + 5*b*(1 + c^2*x^2)^(3/2)*ArcTan[c*x])/(6*c^4*Pi^(5/2)*(1 + c^2*x^2)^
(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.50

method	result
default	$a\left(-\frac{x^2}{\pi c^2(\pi c^2x^2+\pi)^{\frac{3}{2}}}-\frac{2}{3\pi c^4(\pi c^2x^2+\pi)^{\frac{3}{2}}}\right)+b\left(-\frac{6\operatorname{arcsinh}(cx)c^2x^2+cx\sqrt{c^2x^2+1}+4\operatorname{arcsinh}(cx)}{6\pi^{\frac{5}{2}}(c^2x^2+1)^{\frac{3}{2}}c^4}+\frac{5i\ln\left(cx+\sqrt{c^2x^2+1}\right.}{6c^4\pi^{\frac{5}{2}}}\right.$
parts	$a\left(-\frac{x^2}{\pi c^2(\pi c^2x^2+\pi)^{\frac{3}{2}}}-\frac{2}{3\pi c^4(\pi c^2x^2+\pi)^{\frac{3}{2}}}\right)+b\left(-\frac{6\operatorname{arcsinh}(cx)c^2x^2+cx\sqrt{c^2x^2+1}+4\operatorname{arcsinh}(cx)}{6\pi^{\frac{5}{2}}(c^2x^2+1)^{\frac{3}{2}}c^4}+\frac{5i\ln\left(cx+\sqrt{c^2x^2+1}\right.}{6c^4\pi^{\frac{5}{2}}}\right.$

```
[In] int(x^3*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-x^2/Pi/c^2/(Pi*c^2*x^2+Pi)^(3/2)-2/3/Pi/c^4/(Pi*c^2*x^2+Pi)^(3/2))+b*(-
1/6/Pi^(5/2)/(c^2*x^2+1)^(3/2)*(6*arcsinh(c*x)*c^2*x^2+c*x*(c^2*x^2+1)^(1/2
)+4*arcsinh(c*x))/c^4+5/6*I/c^4/Pi^(5/2)*ln(c*x+(c^2*x^2+1)^(1/2)+I)-5/6*I/
c^4/Pi^(5/2)*ln(c*x+(c^2*x^2+1)^(1/2)-I))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(91) = 182.

Time = 0.30 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.78

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = -\frac{5\sqrt{\pi}(bc^4x^4+2\,bc^2x^2+b)\arctan\left(-\frac{2\sqrt{\pi}\sqrt{\pi+\pi c^2x^2}\sqrt{c^2x^2+1}cx}{\pi-\pi c^4x^4}\right)+4\sqrt{\pi+\pi c^2x^2}(3\,bc^2x^2+2\,b)\log\left(cx+\sqrt{c^2x^2}\right.}{12\,(\pi^3c^8x^4+2\,\pi^3c^6x^2+\pi^3c^4)}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
[Out] -1/12*(5*sqrt(pi)*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi
+ pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) + 4*sqrt(pi + pi*c^2
*x^2)*(3*b*c^2*x^2 + 2*b)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*sqrt(pi + pi*c^2
*x^2)*(6*a*c^2*x^2 + sqrt(c^2*x^2 + 1)*b*c*x + 4*a))/(pi^3*c^8*x^4 + 2*pi^3
*c^6*x^2 + pi^3*c^4)
```

Sympy [F]

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{\int \frac{ax^3}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx^3\operatorname{asinh}(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

```
[In] integrate(x**3*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
[Out] (Integral(a*x**3/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**
2 + 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*x**3*asinh(c*x)/(c**4*x**4*s
qrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1))
, x))/pi**(5/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.31

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = -\frac{1}{6}\,bc\left(\frac{x}{\pi^{\frac{5}{2}}c^6x^2+\pi^{\frac{5}{2}}c^4}-\frac{5\,\arctan(cx)}{\pi^{\frac{5}{2}}c^5}\right) - \frac{1}{3}\,b\left(\frac{3\,x^2}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}c^2}+\frac{2}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}c^4}\right)\operatorname{arsinh}(cx) - \frac{1}{3}\,a\left(\frac{3\,x^2}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}c^2}+\frac{2}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}c^4}\right)$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/6*b*c*(x/(pi^(5/2)*c^6*x^2 + pi^(5/2)*c^4) - 5*arctan(c*x)/(pi^(5/2)*c^5
)) - 1/3*b*(3*x^2/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) + 2/(pi*(pi + pi*c^2*x^2
)^(3/2)*c^4))*arcsinh(c*x) - 1/3*a*(3*x^2/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2)
+ 2/(pi*(pi + pi*c^2*x^2)^(3/2)*c^4))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\text{arcsinh}(cx))}{(\pi + c^2\pi x^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\text{arcsinh}(cx))}{(\pi + c^2\pi x^2)^{5/2}}\,dx = \int \frac{x^3\,(a + b\,\text{asinh}(c\,x))}{(\Pi\,c^2\,x^2 + \Pi)^{5/2}}\,dx$$

```
[In] int((x^3*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2),x)
```

```
[Out] int((x^3*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

3.104 $\int \frac{x^2(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 80

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = -\frac{b}{6c^3\pi^{5/2}(1+c^2x^2)}+\frac{x^3(a+b\text{arcsinh}(cx))}{3\pi(\pi+c^2\pi x^2)^{3/2}}-\frac{b\log(1+c^2x^2)}{6c^3\pi^{5/2}}$$

```
[Out] -1/6*b/c^3/Pi^(5/2)/(c^2*x^2+1)+1/3*x^3*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+P
i)^(3/2)-1/6*b*ln(c^2*x^2+1)/c^3/Pi^(5/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5800, 272, 45}

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{x^3(a+b\text{arcsinh}(cx))}{3\pi(\pi c^2x^2+\pi)^{3/2}}-\frac{b}{6\pi^{5/2}c^3(c^2x^2+1)}-\frac{b\log(c^2x^2+1)}{6\pi^{5/2}c^3}$$

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] -1/6*b/(c^3*Pi^(5/2)*(1 + c^2*x^2)) + (x^3*(a + b*ArcSinh[c*x]))/(3*Pi*(Pi
+ c^2*Pi*x^2)^(3/2)) - (b*Log[1 + c^2*x^2])/(6*c^3*Pi^(5/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^3(a+b\text{arcsinh}(cx))}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{(bc)\int\frac{x^3}{(1+c^2x^2)^2}\,dx}{3\pi^{5/2}}\\
&= \frac{x^3(a+b\text{arcsinh}(cx))}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{(bc)\text{Subst}\left(\int\frac{x}{(1+c^2x)^2}\,dx,x,x^2\right)}{6\pi^{5/2}}\\
&= \frac{x^3(a+b\text{arcsinh}(cx))}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{(bc)\text{Subst}\left(\int\left(-\frac{1}{c^2(1+c^2x)^2}+\frac{1}{c^2(1+c^2x)}\right)\,dx,x,x^2\right)}{6\pi^{5/2}}\\
&= -\frac{b}{6c^3\pi^{5/2}\left(1+c^2x^2\right)}+\frac{x^3(a+b\text{arcsinh}(cx))}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{b\log\left(1+c^2x^2\right)}{6c^3\pi^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.10

$$\int\frac{x^2(a+b\text{arcsinh}(cx))}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = -\frac{-2ac^3x^3+b\sqrt{1+c^2x^2}-2bc^3x^3\text{arcsinh}(cx)+b(1+c^2x^2)^{3/2}\log\left(1+c^2x^2\right)}{6c^3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}$$

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] -1/6*(-2*a*c^3*x^3 + b*Sqrt[1 + c^2*x^2] - 2*b*c^3*x^3*ArcSinh[c*x] + b*(1
+ c^2*x^2)^(3/2)*Log[1 + c^2*x^2])/(c^3*Pi^(5/2)*(1 + c^2*x^2)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(68) = 136$.

Time = 0.19 (sec) , antiderivative size = 729, normalized size of antiderivative = 9.11

method	result
default	$a\left(-\frac{x}{2\pi c^2(\pi c^2x^2+\pi)^{\frac{3}{2}}}+\frac{\frac{x}{3\pi(\pi c^2x^2+\pi)^{\frac{3}{2}}}+\frac{2x}{3\pi^2\sqrt{\pi c^2x^2+\pi}}}{2c^2}\right)+\frac{2b\,\mathrm{arcsinh}(cx)}{3c^3\pi^{\frac{5}{2}}}-\frac{b\,c^5\,\mathrm{arcsinh}(cx)x^8}{\pi^{\frac{5}{2}}(3c^4x^4+3c^2x^2+1)(c^2x^2+1)^2}+\frac{}{\pi^{\frac{5}{2}}(3c^4}$
parts	$a\left(-\frac{x}{2\pi c^2(\pi c^2x^2+\pi)^{\frac{3}{2}}}+\frac{\frac{x}{3\pi(\pi c^2x^2+\pi)^{\frac{3}{2}}}+\frac{2x}{3\pi^2\sqrt{\pi c^2x^2+\pi}}}{2c^2}\right)+\frac{2b\,\mathrm{arcsinh}(cx)}{3c^3\pi^{\frac{5}{2}}}-\frac{b\,c^5\,\mathrm{arcsinh}(cx)x^8}{\pi^{\frac{5}{2}}(3c^4x^4+3c^2x^2+1)(c^2x^2+1)^2}+\frac{}{\pi^{\frac{5}{2}}(3c^4}$

```
[In] int(x^2*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/2*x/Pi/c^2/(Pi*c^2*x^2+Pi)^(3/2)+1/2/c^2*(1/3/Pi*x/(Pi*c^2*x^2+Pi)^(3
/2)+2/3/Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2)))+2/3*b/c^3/Pi^(5/2)*arcsinh(c*x)-b/Pi
^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2*c^5*arcsinh(c*x)*x^8+b/Pi^(5/2
)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^(3/2)*c^4*arcsinh(c*x)*x^7-1/6*b/Pi^(
5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2*c^5*x^8+1/6*b/Pi^(5/2)/(3*c^4*x^
4+3*c^2*x^2+1)/(c^2*x^2+1)*c^3*x^6-3*b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^
2*x^2+1)^2*c^3*arcsinh(c*x)*x^6+b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2
+1)^(3/2)*c^2*arcsinh(c*x)*x^5-2/3*b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*
x^2+1)^2*c^3*x^6-10/3*b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2*c*ar
csinh(c*x)*x^4+1/3*b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^(3/2)*arc
sinh(c*x)*x^3-b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2*c*x^4-5/3*b/
Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2/c*arcsinh(c*x)*x^2-2/3*b/Pi^
(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2/c*x^2-1/3*b/Pi^(5/2)/(3*c^4*x^4
+3*c^2*x^2+1)/(c^2*x^2+1)^2/c^3*arcsinh(c*x)-1/6*b/Pi^(5/2)/(3*c^4*x^4+3*c^
2*x^2+1)/(c^2*x^2+1)^2/c^3-1/3*b/c^3/Pi^(5/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^
2)
```

Fricas [F]

$$\int \frac{x^2(a+b\mathrm{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)x^2}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*x^2*arcsinh(c*x) + a*x^2)/(pi^3*c^6*x^6 +
 3*pi^3*c^4*x^4 + 3*pi^3*c^2*x^2 + pi^3), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{\int \frac{ax^2}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{bx^2\operatorname{asinh}(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

```
[In] integrate(x**2*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
[Out] (Integral(a*x**2/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**
2 + 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*x**2*asinh(c*x)/(c**4*x**4*s
qrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1))
, x))/pi**(5/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(68) = 136.

Time = 0.22 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.71

$$\begin{aligned}\int \frac{x^2(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx &= -\frac{1}{6}\,bc\left(\frac{1}{\pi^{\frac{5}{2}}c^6x^2+\pi^{\frac{5}{2}}c^4}+\frac{\log\left(c^2x^2+1\right)}{\pi^{\frac{5}{2}}c^4}\right)\\ &- \frac{1}{3}\,b\left(\frac{x}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}c^2}-\frac{x}{\pi^2\sqrt{\pi+\pi c^2x^2}c^2}\right)\operatorname{arsinh}(cx)\\ &- \frac{1}{3}\,a\left(\frac{x}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}c^2}-\frac{x}{\pi^2\sqrt{\pi+\pi c^2x^2}c^2}\right)\end{aligned}$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
[Out] -1/6*b*c*(1/(pi^(5/2)*c^6*x^2 + pi^(5/2)*c^4) + log(c^2*x^2 + 1)/(pi^(5/2)*
c^4)) - 1/3*b*(x/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) - x/(pi^2*sqrt(pi + pi*c^
2*x^2)*c^2))*arcsinh(c*x) - 1/3*a*(x/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) - x/(
pi^2*sqrt(pi + pi*c^2*x^2)*c^2))
```

Giac [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x^2}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)*x^2/(pi + pi*c^2*x^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\int \frac{x^2\,(a+b\,\mathrm{asinh}(c\,x))}{(\Pi\,c^2\,x^2+\Pi)^{5/2}}\,dx$$

```
[In] int((x^2*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2),x)
[Out] int((x^2*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

3.105 $\int \frac{x(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 75

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\frac{bx}{6c\pi^{5/2}(1+c^2x^2)}-\frac{a+b\text{arcsinh}(cx)}{3c^2\pi(\pi+c^2\pi x^2)^{3/2}}+\frac{b\arctan(cx)}{6c^2\pi^{5/2}}$$

```
[Out] 1/6*b*x/c/Pi^(5/2)/(c^2*x^2+1)+1/3*(-a-b*arcsinh(c*x))/c^2/Pi/(Pi*c^2*x^2+P
i)^(3/2)+1/6*b*arctan(c*x)/c^2/Pi^(5/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5798, 205, 209}

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx=-\frac{a+b\text{arcsinh}(cx)}{3\pi c^2(\pi c^2x^2+\pi)^{3/2}}+\frac{b\arctan(cx)}{6\pi^{5/2}c^2}+\frac{bx}{6\pi^{5/2}c(c^2x^2+1)}$$

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (b*x)/(6*c*Pi^(5/2)*(1 + c^2*x^2)) - (a + b*ArcSinh[c*x])/(3*c^2*Pi*(Pi + c
^2*Pi*x^2)^(3/2)) + (b*ArcTan[c*x])/(6*c^2*Pi^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\text{arcsinh}(cx)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{b\int\frac{1}{(1+c^2x^2)^2}\,dx}{3c\pi^{5/2}}\\
&= \frac{bx}{6c\pi^{5/2}\left(1+c^2x^2\right)}-\frac{a+b\text{arcsinh}(cx)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{b\int\frac{1}{1+c^2x^2}\,dx}{6c\pi^{5/2}}\\
&= \frac{bx}{6c\pi^{5/2}\left(1+c^2x^2\right)}-\frac{a+b\text{arcsinh}(cx)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{b\arctan(cx)}{6c^2\pi^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

$$\int\frac{x(a+b\text{arcsinh}(cx))}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx=\frac{-2a+bcx\sqrt{1+c^2x^2}-2b\text{arcsinh}(cx)+b\left(1+c^2x^2\right)^{3/2}\arctan(cx)}{6c^2\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}$$

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
[Out] (-2*a + b*c*x*Sqrt[1 + c^2*x^2] - 2*b*ArcSinh[c*x] + b*(1 + c^2*x^2)^(3/2)*
ArcTan[c*x])/(6*c^2*Pi^(5/2)*(1 + c^2*x^2)^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.61

method	result	size
default	$-\frac{a}{3\pi\, c^{2}(\pi\, c^{2}x^{2}+\pi)^{\frac{3}{2}}}+b\left(-\frac{-cx\sqrt{c^{2}x^{2}+1}+2\,\operatorname{arcsinh}(cx)}{6\pi^{\frac{5}{2}}(c^{2}x^{2}+1)^{\frac{3}{2}}c^{2}}+\frac{i\ln\left(cx+\sqrt{c^{2}x^{2}+1}+i\right)}{6c^{2}\pi^{\frac{5}{2}}}-\frac{i\ln\left(cx+\sqrt{c^{2}x^{2}+1}-i\right)}{6c^{2}\pi^{\frac{5}{2}}}\right)$	121
parts	$-\frac{a}{3\pi\, c^{2}(\pi\, c^{2}x^{2}+\pi)^{\frac{3}{2}}}+b\left(-\frac{-cx\sqrt{c^{2}x^{2}+1}+2\,\operatorname{arcsinh}(cx)}{6\pi^{\frac{5}{2}}(c^{2}x^{2}+1)^{\frac{3}{2}}c^{2}}+\frac{i\ln\left(cx+\sqrt{c^{2}x^{2}+1}+i\right)}{6c^{2}\pi^{\frac{5}{2}}}-\frac{i\ln\left(cx+\sqrt{c^{2}x^{2}+1}-i\right)}{6c^{2}\pi^{\frac{5}{2}}}\right)$	121

```
[In] int(x*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/Pi/c^2/(Pi*c^2*x^2+Pi)^(3/2)+b*(-1/6/Pi^(5/2)/(c^2*x^2+1)^(3/2)*(-c*
x*(c^2*x^2+1)^(1/2)+2*arcsinh(c*x))/c^2+1/6*I/c^2/Pi^(5/2)*ln(c*x+(c^2*x^2+
1)^(1/2)+I)-1/6*I/c^2/Pi^(5/2)*ln(c*x+(c^2*x^2+1)^(1/2)-I))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(63) = 126.

Time = 0.31 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.20

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx =$$

$$-\frac{\sqrt{\pi}(bc^4x^4+2\,bc^2x^2+b)\arctan\left(-\frac{2\sqrt{\pi}\sqrt{\pi+\pi c^2x^2}\sqrt{c^2x^2+1}cx}{\pi-\pi c^4x^4}\right)+4\sqrt{\pi+\pi c^2x^2}b\log\left(cx+\sqrt{c^2x^2+1}\right)-2\sqrt{\pi}}{12\left(\pi^3c^6x^4+2\,\pi^3c^4x^2+\pi^3c^2\right)}$$

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(sqrt(pi)*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi +
pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) + 4*sqrt(pi + pi*c^2*x
^2)*b*log(c*x + sqrt(c^2*x^2 + 1)) - 2*sqrt(pi + pi*c^2*x^2)*(sqrt(c^2*x^2
+ 1)*b*c*x - 2*a))/(pi^3*c^6*x^4 + 2*pi^3*c^4*x^2 + pi^3*c^2)
```

Sympy [F]

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{\int \frac{ax}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{bx\operatorname{asinh}(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

```
[In] integrate(x*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a*x/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 +
 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*x*asinh(c*x)/(c**4*x**4*sqrt(c*
*2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x))/
pi**(5/2)
```

Maxima [F]

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima")
```

```
[Out] b*integrate(x*log(c*x + sqrt(c^2*x^2 + 1))/(pi + pi*c^2*x^2)^(5/2), x) - 1/
3*a/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2)
```

Giac [F]

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x/(pi + pi*c^2*x^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \int \frac{x\,(a+b\operatorname{asinh}(c\,x))}{(\Pi\,c^2\,x^2+\Pi)^{5/2}}\,dx$$

```
[In] int((x*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2),x)
```

```
[Out] int((x*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

3.106 $\int \frac{a+b\,\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \frac{a+b\,\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\frac{b}{6c\pi^{5/2}(1+c^2x^2)}+\frac{x(a+b\,\text{arcsinh}(cx))}{3\pi(\pi+c^2\pi x^2)^{3/2}}+\frac{2x(a+b\,\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi+c^2\pi x^2}}-\frac{b\log(1+c^2x^2)}{3c\pi^{5/2}}$$

```
[Out] 1/6*b/c/Pi^(5/2)/(c^2*x^2+1)+1/3*x*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(3
/2)-1/3*b*ln(c^2*x^2+1)/c/Pi^(5/2)+2/3*x*(a+b*arcsinh(c*x))/Pi^2/(Pi*c^2*x^
2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5788, 5787, 266, 267}

$$\int \frac{a+b\,\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\frac{2x(a+b\,\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi c^2x^2+\pi}}+\frac{x(a+b\,\text{arcsinh}(cx))}{3\pi(\pi c^2x^2+\pi)^{3/2}}+\frac{b}{6\pi^{5/2}c(c^2x^2+1)}-\frac{b\log(c^2x^2+1)}{3\pi^{5/2}c}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] b/(6*c*Pi^(5/2)*(1 + c^2*x^2)) + (x*(a + b*ArcSinh[c*x]))/(3*Pi*(Pi + c^2*P
i*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x]))/(3*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) -
 (b*Log[1 + c^2*x^2])/(3*c*Pi^(5/2))
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x(a+b\text{arcsinh}(cx))}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{(bc)\int \frac{x}{\left(1+c^2x^2\right)^2}\,dx}{3\pi^{5/2}} + \frac{2\int \frac{a+b\text{arcsinh}(cx)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx}{3\pi} \\
&= \frac{b}{6c\pi^{5/2}\left(1+c^2x^2\right)} + \frac{x(a+b\text{arcsinh}(cx))}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{2x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{(2bc)\int \frac{x}{1+c^2x^2}\,dx}{3\pi^{5/2}} \\
&= \frac{b}{6c\pi^{5/2}\left(1+c^2x^2\right)} + \frac{x(a+b\text{arcsinh}(cx))}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{2x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{b\log\left(1+c^2x^2\right)}{3c\pi^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{6acx+4ac^3x^3+b\sqrt{1+c^2x^2}+2bcx(3+2c^2x^2)\operatorname{arcsinh}(cx)-2b(1+c^2x^2)^{3/2}\log(}{6c\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (6*a*c*x + 4*a*c^3*x^3 + b*Sqrt[1 + c^2*x^2] + 2*b*c*x*(3 + 2*c^2*x^2)*ArcS
inh[c*x] - 2*b*(1 + c^2*x^2)^(3/2)*Log[1 + c^2*x^2])/(6*c*Pi^(5/2)*(1 + c^2
*x^2)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(92) = 184$.

Time = 0.20 (sec) , antiderivative size = 619, normalized size of antiderivative = 5.73

method	result
default	$a\left(\frac{x}{3\pi(\pi c^2x^2+\pi)^{\frac{3}{2}}}+\frac{2x}{3\pi^2\sqrt{\pi c^2x^2+\pi}}\right)+\frac{4b\operatorname{arcsinh}(cx)}{3c\pi^{\frac{5}{2}}}+\frac{2bc^7x^8}{3\pi^{\frac{5}{2}}(3c^2x^2+4)(c^2x^2+1)^2}-\frac{2bc^5x^6}{3\pi^{\frac{5}{2}}(3c^2x^2+4)(c^2x^2+1)}-\frac{2b}{\pi^{\frac{5}{2}}(3}$
parts	$a\left(\frac{x}{3\pi(\pi c^2x^2+\pi)^{\frac{3}{2}}}+\frac{2x}{3\pi^2\sqrt{\pi c^2x^2+\pi}}\right)+\frac{4b\operatorname{arcsinh}(cx)}{3c\pi^{\frac{5}{2}}}+\frac{2bc^7x^8}{3\pi^{\frac{5}{2}}(3c^2x^2+4)(c^2x^2+1)^2}-\frac{2bc^5x^6}{3\pi^{\frac{5}{2}}(3c^2x^2+4)(c^2x^2+1)}-\frac{2b}{\pi^{\frac{5}{2}}(3}$

```
[In] int((a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/3/Pi*x/(Pi*c^2*x^2+Pi)^(3/2)+2/3/Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2))+4/3*b/c
/Pi^(5/2)*arcsinh(c*x)+2/3*b/Pi^(5/2)*c^7/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^8-2
/3*b/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)*x^6-2*b/Pi^(5/2)*c^5/(3*c^2*x^2
+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^6+2*b/Pi^(5/2)*c^4/(3*c^2*x^2+4)/(c^2*x^2+
1)^(3/2)*arcsinh(c*x)*x^5+8/3*b/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^
6-2*b/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)*x^4-20/3*b/Pi^(5/2)*c^3/(3*c^2
*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^4+17/3*b/Pi^(5/2)*c^2/(3*c^2*x^2+4)/(c
^2*x^2+1)^(3/2)*arcsinh(c*x)*x^3+4*b/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)
^2*x^4-3/2*b/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)*x^2-22/3*b/Pi^(5/2)*c/(3*
c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^2+4*b/Pi^(5/2)/(3*c^2*x^2+4)/(c^2*x
^2+1)^(3/2)*arcsinh(c*x)*x+8/3*b/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^2
-8/3*b/Pi^(5/2)/c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)+2/3*b/Pi^(5/2)/c
/(3*c^2*x^2+4)/(c^2*x^2+1)^2-2/3*b/c/Pi^(5/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^
2)
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^3*c^6*x^6 + 3*pi^3*
c^4*x^4 + 3*pi^3*c^2*x^2 + pi^3), x)
```

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{\int \frac{a}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{b\,\text{asinh}\,(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

```
[In] integrate((a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1
) + sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(c**4*x**4*sqrt(c**2*x
**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x))/pi**
(5/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.17

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{1}{6}\,bc\left(\frac{1}{\pi^{\frac{5}{2}}c^4x^2+\pi^{\frac{5}{2}}c^2} - \frac{2\,\log\left(c^2x^2+1\right)}{\pi^{\frac{5}{2}}c^2}\right) + \frac{1}{3}\,b\left(\frac{x}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}} + \frac{2\,x}{\pi^2\sqrt{\pi+\pi c^2x^2}}\right)\text{arsinh}\,(cx) + \frac{1}{3}\,a\left(\frac{x}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}} + \frac{2\,x}{\pi^2\sqrt{\pi+\pi c^2x^2}}\right)$$

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*b*c*(1/(pi^(5/2)*c^4*x^2 + pi^(5/2)*c^2) - 2*log(c^2*x^2 + 1)/(pi^(5/2)
*c^2)) + 1/3*b*(x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 2*x/(pi^2*sqrt(pi + pi*c^2
*x^2)))*arcsinh(c*x) + 1/3*a*(x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 2*x/(pi^2*sq
rt(pi + pi*c^2*x^2)))
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(\pi + c^2\pi x^2)^{5/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{(\pi + \pi c^2 x^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/(pi + pi*c^2*x^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{(\pi + c^2\pi x^2)^{5/2}}\,dx = \int \frac{a + b\,\text{asinh}(c\,x)}{(\Pi\,c^2\,x^2 + \Pi)^{5/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(5/2),x)
[Out] int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

3.107 $\int \frac{a+b\text{arcsinh}(cx)}{x(\pi+c^2\pi x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 148

$$\int \frac{a+b\text{arcsinh}(cx)}{x(\pi+c^2\pi x^2)^{5/2}}\,dx = -\frac{bcx}{6\pi^{5/2}(1+c^2x^2)}+\frac{a+b\text{arcsinh}(cx)}{3\pi(\pi+c^2\pi x^2)^{3/2}}+\frac{a+b\text{arcsinh}(cx)}{\pi^2\sqrt{\pi+c^2\pi x^2}}-\frac{7b\arctan(cx)}{6\pi^{5/2}}-\frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\pi^{5/2}}-\frac{b\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\pi^{5/2}}+\frac{b\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\pi^{5/2}}$$

```
[Out] -1/6*b*c*x/Pi^(5/2)/(c^2*x^2+1)+1/3*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(
3/2)-7/6*b*arctan(c*x)/Pi^(5/2)-2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1
)^(1/2))/Pi^(5/2)-b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))/Pi^(5/2)+b*polylog(2,
c*x+(c^2*x^2+1)^(1/2))/Pi^(5/2)+(a+b*arcsinh(c*x))/Pi^2/(Pi*c^2*x^2+Pi)^(1/
2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5811, 5816, 4267, 2317, 2438, 209, 205}

$$\int \frac{a+b\text{arcsinh}(cx)}{x(\pi+c^2\pi x^2)^{5/2}}\,dx = -\frac{2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\pi^{5/2}}+\frac{a+b\text{arcsinh}(cx)}{\pi^2\sqrt{\pi c^2x^2+\pi}}+\frac{a+b\text{arcsinh}(cx)}{3\pi(\pi c^2x^2+\pi)^{3/2}}-\frac{b\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\pi^{5/2}}+\frac{b\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\pi^{5/2}}-\frac{7b\arctan(cx)}{6\pi^{5/2}}-\frac{bcx}{6\pi^{5/2}(c^2x^2+1)}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] -1/6*(b*c*x)/(Pi^(5/2)*(1 + c^2*x^2)) + (a + b*ArcSinh[c*x])/(3*Pi*(Pi + c^
2*Pi*x^2)^(3/2)) + (a + b*ArcSinh[c*x])/(Pi^2*Sqrt[Pi + c^2*Pi*x^2]) - (7*b
*ArcTan[c*x])/(6*Pi^(5/2)) - (2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]
])/Pi^(5/2) - (b*PolyLog[2, -E^ArcSinh[c*x]])/Pi^(5/2) + (b*PolyLog[2, E^Ar
cSinh[c*x]])/Pi^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
```

```
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a+b\text{arcsinh}(cx)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{(bc)\int\frac{1}{(1+c^2x^2)^2}\,dx}{3\pi^{5/2}}+\frac{\int\frac{a+b\text{arcsinh}(cx)}{x(\pi+c^2\pi x^2)^{3/2}}\,dx}{\pi}\\
&= -\frac{bcx}{6\pi^{5/2}\left(1+c^2x^2\right)}+\frac{a+b\text{arcsinh}(cx)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{a+b\text{arcsinh}(cx)}{\pi^2\sqrt{\pi+c^2\pi x^2}}\\
&\quad-\frac{(bc)\int\frac{1}{1+c^2x^2}\,dx}{6\pi^{5/2}}-\frac{(bc)\int\frac{1}{1+c^2x^2}\,dx}{\pi^{5/2}}+\frac{\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx}{\pi^2}\\
&= -\frac{bcx}{6\pi^{5/2}\left(1+c^2x^2\right)}+\frac{a+b\text{arcsinh}(cx)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{a+b\text{arcsinh}(cx)}{\pi^2\sqrt{\pi+c^2\pi x^2}}\\
&\quad-\frac{7b\arctan(cx)}{6\pi^{5/2}}+\frac{\text{Subst}\left(\int(a+bx)\text{csch}(x)\,dx,x,\text{arcsinh}(cx)\right)}{\pi^{5/2}}\\
&= -\frac{bcx}{6\pi^{5/2}\left(1+c^2x^2\right)}+\frac{a+b\text{arcsinh}(cx)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{a+b\text{arcsinh}(cx)}{\pi^2\sqrt{\pi+c^2\pi x^2}}\\
&\quad-\frac{7b\arctan(cx)}{6\pi^{5/2}}-\frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\pi^{5/2}}\\
&\quad-\frac{b\text{Subst}\left(\int\log\left(1-e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{\pi^{5/2}}\\
&\quad+\frac{b\text{Subst}\left(\int\log\left(1+e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{\pi^{5/2}}\\
&= -\frac{bcx}{6\pi^{5/2}\left(1+c^2x^2\right)}+\frac{a+b\text{arcsinh}(cx)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{a+b\text{arcsinh}(cx)}{\pi^2\sqrt{\pi+c^2\pi x^2}}\\
&\quad-\frac{7b\arctan(cx)}{6\pi^{5/2}}-\frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\pi^{5/2}}\\
&\quad-\frac{b\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{\pi^{5/2}}+\frac{b\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{\pi^{5/2}}
\end{aligned}$$

$$= -\frac{bcx}{6\pi^{5/2}(1+c^2x^2)} + \frac{a+b\text{arcsinh}(cx)}{3\pi(\pi+c^2\pi x^2)^{3/2}} + \frac{a+b\text{arcsinh}(cx)}{\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{7b\arctan(cx)}{6\pi^{5/2}} - \frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\pi^{5/2}} - \frac{b\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\pi^{5/2}} + \frac{b\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\pi^{5/2}}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.41

$$\int \frac{a+b\text{arcsinh}(cx)}{x(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{\frac{2a}{(1+c^2x^2)^{3/2}} - \frac{bcx}{1+c^2x^2} + \frac{6a}{\sqrt{1+c^2x^2}} + \frac{8b\text{arcsinh}(cx)}{(1+c^2x^2)^{3/2}} + \frac{6bc^2x^2\text{arcsinh}(cx)}{(1+c^2x^2)^{3/2}} - 14b\arctan\left(\tanh\right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] ((2*a)/(1 + c^2*x^2)^(3/2) - (b*c*x)/(1 + c^2*x^2) + (6*a)/Sqrt[1 + c^2*x^2
] + (8*b*ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) + (6*b*c^2*x^2*ArcSinh[c*x])/(1
+ c^2*x^2)^(3/2) - 14*b*ArcTan[Tanh[ArcSinh[c*x]/2]] + 6*b*ArcSinh[c*x]*Log
[1 - E^(-ArcSinh[c*x])] - 6*b*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 6*a
*Log[x] - 6*a*Log[Pi*(1 + Sqrt[1 + c^2*x^2])] + 6*b*PolyLog[2, -E^(-ArcSinh
[c*x])] - 6*b*PolyLog[2, E^(-ArcSinh[c*x])])/(6*Pi^(5/2))
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.41

method	result
default	$a\left(\frac{1}{3\pi(\pi c^2x^2+\pi)^{\frac{3}{2}}} + \frac{\frac{1}{\pi\sqrt{\pi c^2x^2+\pi}} - \frac{\text{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)}{\pi^{\frac{3}{2}}}}{\pi}\right) + b\left(\frac{6\,\text{arcsinh}(cx)c^2x^2 - cx\sqrt{c^2x^2+1} + 8\,\text{arcsinh}(cx)}{6\pi^{\frac{5}{2}}(c^2x^2+1)^{\frac{3}{2}}} - \frac{7\,\text{arctan}}{}\right.$
parts	$a\left(\frac{1}{3\pi(\pi c^2x^2+\pi)^{\frac{3}{2}}} + \frac{\frac{1}{\pi\sqrt{\pi c^2x^2+\pi}} - \frac{\text{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)}{\pi^{\frac{3}{2}}}}{\pi}\right) + b\left(\frac{6\,\text{arcsinh}(cx)c^2x^2 - cx\sqrt{c^2x^2+1} + 8\,\text{arcsinh}(cx)}{6\pi^{\frac{5}{2}}(c^2x^2+1)^{\frac{3}{2}}} - \frac{7\,\text{arctan}}{}\right.$

```
[In] int((a+b*arcsinh(c*x))/x/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/3/Pi/(Pi*c^2*x^2+Pi)^(3/2)+1/Pi*(1/Pi/(Pi*c^2*x^2+Pi)^(1/2)-1/Pi^(3/2)
*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2))))+b*(1/6/Pi^(5/2)/(c^2*x^2+1)^(3/2
)*(6*arcsinh(c*x)*c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+8*arcsinh(c*x))-7/3/Pi^(5/2
)*arctan(c*x+(c^2*x^2+1)^(1/2))-1/Pi^(5/2)*dilog(1+c*x+(c^2*x^2+1)^(1/2))-1
```

```
/Pi^(5/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-1/Pi^(5/2)*dilog(c*x+(c^
2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x (\pi + c^2\pi x^2)^{5/2}} \, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(\pi + \pi c^2 x^2)^{\frac{5}{2}} x} \, dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^3*c^6*x^7 + 3*pi^3*
c^4*x^5 + 3*pi^3*c^2*x^3 + pi^3*x), x)
```

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x (\pi + c^2\pi x^2)^{5/2}} \, dx = \frac{\int \frac{a}{c^4x^5\sqrt{c^2x^2+1}+2c^2x^3\sqrt{c^2x^2+1}+x\sqrt{c^2x^2+1}} \, dx + \int \frac{b \operatorname{asinh}(cx)}{c^4x^5\sqrt{c^2x^2+1}+2c^2x^3\sqrt{c^2x^2+1}+x\sqrt{c^2x^2+1}} \, dx}{\pi^{\frac{5}{2}}}$$

```
[In] integrate((a+b*asinh(c*x))/x/(pi*c**2*x**2+pi)**(5/2),x)
[Out] (Integral(a/(c**4*x**5*sqrt(c**2*x**2 + 1) + 2*c**2*x**3*sqrt(c**2*x**2 + 1
) + x*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(c**4*x**5*sqrt(c**2
*x**2 + 1) + 2*c**2*x**3*sqrt(c**2*x**2 + 1) + x*sqrt(c**2*x**2 + 1)), x))/
pi**(5/2)
```

Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x (\pi + c^2\pi x^2)^{5/2}} \, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(\pi + \pi c^2 x^2)^{\frac{5}{2}} x} \, dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima")
[Out] -1/3*a*(3*arcsinh(1/(c*abs(x)))/pi^(5/2) - 1/(pi*(pi + pi*c^2*x^2)^(3/2)) -
 3/(pi^2*sqrt(pi + pi*c^2*x^2))) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))
/((pi + pi*c^2*x^2)^(5/2)*x), x)
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x\,(\pi + c^2\pi x^2)^{5/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{(\pi + \pi c^2x^2)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(5/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{x\,(\pi + c^2\pi x^2)^{5/2}}\,dx = \int \frac{a + b\,\text{asinh}(c\,x)}{x\,(\Pi\,c^2\,x^2 + \Pi)^{5/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))/(x*(Pi + Pi*c^2*x^2)^(5/2)),x)
[Out] int((a + b*asinh(c*x))/(x*(Pi + Pi*c^2*x^2)^(5/2)), x)
```

3.108 $\int \frac{a+b\text{arcsinh}(cx)}{x^2(\pi+c^2\pi x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 150

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2(\pi+c^2\pi x^2)^{5/2}}\,dx = -\frac{bc}{6\pi^{5/2}(1+c^2x^2)}-\frac{a+b\text{arcsinh}(cx)}{\pi x(\pi+c^2\pi x^2)^{3/2}} -\frac{4c^2x(a+b\text{arcsinh}(cx))}{3\pi(\pi+c^2\pi x^2)^{3/2}}-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi+c^2\pi x^2}}+\frac{bc\log(x)}{\pi^{5/2}}+\frac{5bc\log(1+c^2x^2)}{6\pi^{5/2}}$$

```
[Out] -1/6*b*c/Pi^(5/2)/(c^2*x^2+1)+(-a-b*arcsinh(c*x))/Pi/x/(Pi*c^2*x^2+Pi)^(3/2
)-4/3*c^2*x*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(3/2)+b*c*ln(x)/Pi^(5/2)+
5/6*b*c*ln(c^2*x^2+1)/Pi^(5/2)-8/3*c^2*x*(a+b*arcsinh(c*x))/Pi^2/(Pi*c^2*x^
2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {277, 198, 197, 5804, 12, 1265, 907}

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2(\pi+c^2\pi x^2)^{5/2}}\,dx = -\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi c^2x^2+\pi}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3\pi(\pi c^2x^2+\pi)^{3/2}} -\frac{a+b\text{arcsinh}(cx)}{\pi x(\pi c^2x^2+\pi)^{3/2}}-\frac{bc}{6\pi^{5/2}(c^2x^2+1)}+\frac{5bc\log(c^2x^2+1)}{6\pi^{5/2}}+\frac{bc\log(x)}{\pi^{5/2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] -1/6*(b*c)/(Pi^(5/2)*(1 + c^2*x^2)) - (a + b*ArcSinh[c*x])/(Pi*x*(Pi + c^2*
Pi*x^2)^(3/2)) - (4*c^2*x*(a + b*ArcSinh[c*x]))/(3*Pi*(Pi + c^2*Pi*x^2)^(3/
```

```
2)) - (8*c^2*x*(a + b*ArcSinh[c*x]))/(3*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) + (b*c*
Log[x])/Pi^(5/2) + (5*b*c*Log[1 + c^2*x^2])/(6*Pi^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
```

```
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\text{arcsinh}(cx)}{\pi x\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} \\
&\quad -\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi+c^2\pi x^2}}-\left(bc\sqrt{\pi}\right)\int\frac{-3-12c^2x^2-8c^4x^4}{3\pi^3x\left(1+c^2x^2\right)^2}\,dx \\
&= -\frac{a+b\text{arcsinh}(cx)}{\pi x\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} \\
&\quad -\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi+c^2\pi x^2}}-\frac{(bc)\int\frac{-3-12c^2x^2-8c^4x^4}{x\left(1+c^2x^2\right)^2}\,dx}{3\pi^{5/2}} \\
&= -\frac{a+b\text{arcsinh}(cx)}{\pi x\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} \\
&\quad -\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi+c^2\pi x^2}}-\frac{(bc)\text{Subst}\left(\int\frac{-3-12c^2x-8c^4x^2}{x\left(1+c^2x\right)^2}\,dx,x,x^2\right)}{6\pi^{5/2}} \\
&= -\frac{a+b\text{arcsinh}(cx)}{\pi x\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi+c^2\pi x^2}} \\
&\quad -\frac{(bc)\text{Subst}\left(\int\left(-\frac{3}{x}-\frac{c^2}{\left(1+c^2x\right)^2}-\frac{5c^2}{1+c^2x}\right)\,dx,x,x^2\right)}{6\pi^{5/2}} \\
&= -\frac{bc}{6\pi^{5/2}\left(1+c^2x^2\right)}-\frac{a+b\text{arcsinh}(cx)}{\pi x\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} \\
&\quad -\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi+c^2\pi x^2}}+\frac{bc\log(x)}{\pi^{5/2}}+\frac{5bc\log\left(1+c^2x^2\right)}{6\pi^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.83

$$\int\frac{a+b\text{arcsinh}(cx)}{x^2\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx=\frac{-bcx\sqrt{1+c^2x^2}-2a\left(3+12c^2x^2+8c^4x^4\right)-2b\left(3+12c^2x^2+8c^4x^4\right)\text{arcsinh}(cx)+}{6\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] (-(b*c*x*Sqrt[1 + c^2*x^2]) - 2*a*(3 + 12*c^2*x^2 + 8*c^4*x^4) - 2*b*(3 + 1
2*c^2*x^2 + 8*c^4*x^4)*ArcSinh[c*x] + b*c*x*(1 + c^2*x^2)^(3/2)*(16 + 6*Log
[x] + 5*Log[1 + c^2*x^2]))/(6*Pi^(5/2)*x*(1 + c^2*x^2)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 777 vs. $2(132) = 264$.

Time = 0.18 (sec) , antiderivative size = 778, normalized size of antiderivative = 5.19

method	result
default	$a\left(-\frac{1}{\pi x(\pi c^2x^2+\pi)^{\frac{3}{2}}}-4c^2\left(\frac{x}{3\pi(\pi c^2x^2+\pi)^{\frac{3}{2}}}+\frac{2x}{3\pi^2\sqrt{\pi c^2x^2+\pi}}\right)\right)-\frac{16bc\,\mathrm{arcsinh}(cx)}{3\pi^{\frac{5}{2}}}+\frac{32b\,x^{10}c^{11}}{3\pi^{\frac{5}{2}}(8c^2x^2+9)(c^2x^2+1)^2}-$
parts	$a\left(-\frac{1}{\pi x(\pi c^2x^2+\pi)^{\frac{3}{2}}}-4c^2\left(\frac{x}{3\pi(\pi c^2x^2+\pi)^{\frac{3}{2}}}+\frac{2x}{3\pi^2\sqrt{\pi c^2x^2+\pi}}\right)\right)-\frac{16bc\,\mathrm{arcsinh}(cx)}{3\pi^{\frac{5}{2}}}+\frac{32b\,x^{10}c^{11}}{3\pi^{\frac{5}{2}}(8c^2x^2+9)(c^2x^2+1)^2}-$

```
[In] int((a+b*arcsinh(c*x))/x^2/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/Pi/x/(Pi*c^2*x^2+Pi)^(3/2)-4*c^2*(1/3/Pi*x/(Pi*c^2*x^2+Pi)^(3/2)+2/3/
Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2)))-16/3*b*c/Pi^(5/2)*arcsinh(c*x)+32/3*b/Pi^(5/
2)/(8*c^2*x^2+9)/(c^2*x^2+1)^2*x^10*c^11-32/3*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2
*x^2+1)*x^8*c^9+128/3*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)^2*x^8*c^9-32*b/P
i^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)*x^6*c^7+64/3*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^
2*x^2+1)^2*x^6*arcsinh(c*x)*c^7-64/3*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)^(
3/2)*x^5*arcsinh(c*x)*c^6+64*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)^2*x^6*c^7
-32*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)*x^4*c^5+200/3*b/Pi^(5/2)/(8*c^2*x^
2+9)/(c^2*x^2+1)^2*x^4*arcsinh(c*x)*c^5-56*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^
2+1)^(3/2)*x^3*arcsinh(c*x)*c^4+128/3*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)^
2*x^4*c^5-12*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)*x^2*c^3+208/3*b/Pi^(5/2)/
(8*c^2*x^2+9)/(c^2*x^2+1)^2*x^2*arcsinh(c*x)*c^3-44*b/Pi^(5/2)/(8*c^2*x^2+9
)/(c^2*x^2+1)^(3/2)*x*arcsinh(c*x)*c^2+32/3*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x
^2+1)^2*x^2*c^3-3/2*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)*c+24*b/Pi^(5/2)/(8
*c^2*x^2+9)/(c^2*x^2+1)^2*arcsinh(c*x)*c-9*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^
2+1)^(3/2)/x*arcsinh(c*x)+5/3*b*c/Pi^(5/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+
b*c/Pi^(5/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)
```

Fricas [F]

$$\int \frac{a+b\mathrm{arcsinh}(cx)}{x^2\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx=\int \frac{b\,\mathrm{arsinh}\,(cx)+a}{(\pi+\pi c^2x^2)^{\frac{5}{2}}x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^3*c^6*x^8 + 3*pi^3*
c^4*x^6 + 3*pi^3*c^2*x^4 + pi^3*x^2), x)
```

Sympy [F]

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{x^2 (\pi + c^2\pi x^2)^{5/2}}\,dx = \frac{\int \frac{a}{c^4x^6\sqrt{c^2x^2+1}+2c^2x^4\sqrt{c^2x^2+1}+x^2\sqrt{c^2x^2+1}}\,dx + \int \frac{b\,\operatorname{asinh}(cx)}{c^4x^6\sqrt{c^2x^2+1}+2c^2x^4\sqrt{c^2x^2+1}+x^2\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

```
[In] integrate((a+b*asinh(c*x))/x**2/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a/(c**4*x**6*sqrt(c**2*x**2 + 1) + 2*c**2*x**4*sqrt(c**2*x**2 + 1
) + x**2*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(c**4*x**6*sqrt(c
**2*x**2 + 1) + 2*c**2*x**4*sqrt(c**2*x**2 + 1) + x**2*sqrt(c**2*x**2 + 1))
, x))/pi**(5/2)
```

Maxima [F]

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{x^2 (\pi + c^2\pi x^2)^{5/2}}\,dx = \int \frac{b\,\operatorname{arsinh}(cx) + a}{(\pi + \pi c^2x^2)^{\frac{5}{2}}x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/3*a*(4*c^2*x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 8*c^2*x/(pi^2*sqrt(pi + pi*c
^2*x^2)) + 3/(pi*(pi + pi*c^2*x^2)^(3/2)*x)) + b*integrate(log(c*x + sqrt(c
^2*x^2 + 1))/((pi + pi*c^2*x^2)^(5/2)*x^2), x)
```

Giac [F]

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{x^2 (\pi + c^2\pi x^2)^{5/2}}\,dx = \int \frac{b\,\operatorname{arsinh}(cx) + a}{(\pi + \pi c^2x^2)^{\frac{5}{2}}x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(5/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{x^2\,\left(\Pi\,c^2\,x^2+\Pi\right)^{5/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))/(x^2*(Pi + Pi*c^2*x^2)^(5/2)),x)
[Out] int((a + b*asinh(c*x))/(x^2*(Pi + Pi*c^2*x^2)^(5/2)), x)
```

3.109 $\int \frac{a+b\text{arcsinh}(cx)}{x^3(\pi+c^2\pi x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 247

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{x^3(\pi+c^2\pi x^2)^{5/2}}\,dx &= -\frac{3bc}{4\pi^{5/2}x}+\frac{bc}{4\pi^{5/2}x(1+c^2x^2)}+\frac{5bc^3x}{12\pi^{5/2}(1+c^2x^2)}\\ &-\frac{5c^2(a+b\text{arcsinh}(cx))}{6\pi(\pi+c^2\pi x^2)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{2\pi x^2(\pi+c^2\pi x^2)^{3/2}}-\frac{5c^2(a+b\text{arcsinh}(cx))}{2\pi^2\sqrt{\pi+c^2\pi x^2}}\\ &+\frac{13bc^2\arctan(cx)}{6\pi^{5/2}}+\frac{5c^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\pi^{5/2}}\\ &+\frac{5bc^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2\pi^{5/2}}-\frac{5bc^2\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2\pi^{5/2}}\end{aligned}$$

```
[Out] -3/4*b*c/Pi^(5/2)/x+1/4*b*c/Pi^(5/2)/x/(c^2*x^2+1)+5/12*b*c^3*x/Pi^(5/2)/(c
^2*x^2+1)-5/6*c^2*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(3/2)+1/2*(-a-b*arc
sinh(c*x))/Pi/x^2/(Pi*c^2*x^2+Pi)^(3/2)+13/6*b*c^2*arctan(c*x)/Pi^(5/2)+5*c
^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))/Pi^(5/2)+5/2*b*c^2*pol
ylog(2,-c*x-(c^2*x^2+1)^(1/2))/Pi^(5/2)-5/2*b*c^2*polylog(2,c*x+(c^2*x^2+1)
^(1/2))/Pi^(5/2)-5/2*c^2*(a+b*arcsinh(c*x))/Pi^2/(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules

used = {5809, 5811, 5816, 4267, 2317, 2438, 209, 205, 296, 331}

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = \frac{5c^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\pi^{5/2}} - \frac{5c^2(a+b\text{arcsinh}(cx))}{2\pi^2\sqrt{\pi c^2x^2+\pi}} - \frac{5c^2(a+b\text{arcsinh}(cx))}{6\pi\left(\pi c^2x^2+\pi\right)^{3/2}} - \frac{a+b\text{arcsinh}(cx)}{2\pi x^2\left(\pi c^2x^2+\pi\right)^{3/2}} + \frac{5bc^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2\pi^{5/2}} - \frac{5bc^2\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2\pi^{5/2}} + \frac{13bc^2\arctan(cx)}{6\pi^{5/2}} + \frac{bc}{4\pi^{5/2}x\left(c^2x^2+1\right)} + \frac{5bc^3x}{12\pi^{5/2}\left(c^2x^2+1\right)} - \frac{3bc}{4\pi^{5/2}x}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] (-3*b*c)/(4*Pi^(5/2)*x) + (b*c)/(4*Pi^(5/2)*x*(1 + c^2*x^2)) + (5*b*c^3*x)/
(12*Pi^(5/2)*(1 + c^2*x^2)) - (5*c^2*(a + b*ArcSinh[c*x]))/(6*Pi*(Pi + c^2*
Pi*x^2)^(3/2)) - (a + b*ArcSinh[c*x])/(2*Pi*x^2*(Pi + c^2*Pi*x^2)^(3/2)) -
(5*c^2*(a + b*ArcSinh[c*x]))/(2*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) + (13*b*c^2*Arc
Tan[c*x])/(6*Pi^(5/2)) + (5*c^2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]
])/Pi^(5/2) + (5*b*c^2*PolyLog[2, -E^ArcSinh[c*x]])/(2*Pi^(5/2)) - (5*b*c^2
*PolyLog[2, E^ArcSinh[c*x]])/(2*Pi^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\frac{a+b\text{arcsinh}(cx)}{2\pi x^2\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{1}{2}\left(5c^2\right)\int \frac{a+b\text{arcsinh}(cx)}{x\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx + \frac{(bc)\int \frac{1}{x^2(1+c^2x^2)^2}\,dx}{2\pi^{5/2}}$$

$$= \frac{bc}{4\pi^{5/2}x\left(1+c^2x^2\right)} - \frac{5c^2(a+b\text{arcsinh}(cx))}{6\pi\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{a+b\text{arcsinh}(cx)}{2\pi x^2\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{(3bc)\int \frac{1}{x^2(1+c^2x^2)}\,dx}{4\pi^{5/2}} + \frac{\left(5bc^3\right)\int \frac{1}{(1+c^2x^2)^2}\,dx}{6\pi^{5/2}} - \frac{\left(5c^2\right)\int \frac{a+b\text{arcsinh}(cx)}{x(\pi+c^2\pi x^2)^{3/2}}\,dx}{2\pi}$$

$$= -\frac{3bc}{4\pi^{5/2}x} + \frac{bc}{4\pi^{5/2}x\left(1+c^2x^2\right)} + \frac{5bc^3x}{12\pi^{5/2}\left(1+c^2x^2\right)} - \frac{5c^2(a+b\text{arcsinh}(cx))}{6\pi\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{a+b\text{arcsinh}(cx)}{2\pi x^2\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{5c^2(a+b\text{arcsinh}(cx))}{2\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{\left(5bc^3\right)\int \frac{1}{1+c^2x^2}\,dx}{12\pi^{5/2}} - \frac{\left(3bc^3\right)\int \frac{1}{1+c^2x^2}\,dx}{4\pi^{5/2}} + \frac{\left(5bc^3\right)\int \frac{1}{1+c^2x^2}\,dx}{2\pi^{5/2}} - \frac{\left(5c^2\right)\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx}{2\pi^2}$$

$$= -\frac{3bc}{4\pi^{5/2}x} + \frac{bc}{4\pi^{5/2}x\left(1+c^2x^2\right)} + \frac{5bc^3x}{12\pi^{5/2}\left(1+c^2x^2\right)} - \frac{5c^2(a+b\text{arcsinh}(cx))}{6\pi\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{a+b\text{arcsinh}(cx)}{2\pi x^2\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{5c^2(a+b\text{arcsinh}(cx))}{2\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{13bc^2\arctan(cx)}{6\pi^{5/2}} - \frac{\left(5c^2\right)\text{Subst}\left(\int (a+bx)\text{csch}(x)\,dx, x, \text{arcsinh}(cx)\right)}{2\pi^{5/2}}$$

$$= -\frac{3bc}{4\pi^{5/2}x} + \frac{bc}{4\pi^{5/2}x\left(1+c^2x^2\right)} + \frac{5bc^3x}{12\pi^{5/2}\left(1+c^2x^2\right)} - \frac{5c^2(a+b\text{arcsinh}(cx))}{6\pi\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{a+b\text{arcsinh}(cx)}{2\pi x^2\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{5c^2(a+b\text{arcsinh}(cx))}{2\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{13bc^2\arctan(cx)}{6\pi^{5/2}} + \frac{5c^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\pi^{5/2}} + \frac{\left(5bc^2\right)\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{2\pi^{5/2}} - \frac{\left(5bc^2\right)\text{Subst}\left(\int \log\left(1+e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{2\pi^{5/2}}$$

$$= -\frac{3bc}{4\pi^{5/2}x} + \frac{bc}{4\pi^{5/2}x(1+c^2x^2)} + \frac{5bc^3x}{12\pi^{5/2}(1+c^2x^2)} - \frac{5c^2(a+b\text{arcsinh}(cx))}{6\pi(\pi+c^2\pi x^2)^{3/2}} - \frac{a+b\text{arcsinh}(cx)}{2\pi x^2(\pi+c^2\pi x^2)^{3/2}} - \frac{5c^2(a+b\text{arcsinh}(cx))}{2\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{13bc^2\arctan(cx)}{6\pi^{5/2}} + \frac{5c^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\pi^{5/2}} + \frac{(5bc^2)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{2\pi^{5/2}} - \frac{(5bc^2)\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{2\pi^{5/2}}$$

$$= -\frac{3bc}{4\pi^{5/2}x} + \frac{bc}{4\pi^{5/2}x(1+c^2x^2)} + \frac{5bc^3x}{12\pi^{5/2}(1+c^2x^2)} - \frac{5c^2(a+b\text{arcsinh}(cx))}{6\pi(\pi+c^2\pi x^2)^{3/2}} - \frac{a+b\text{arcsinh}(cx)}{2\pi x^2(\pi+c^2\pi x^2)^{3/2}} - \frac{5c^2(a+b\text{arcsinh}(cx))}{2\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{13bc^2\arctan(cx)}{6\pi^{5/2}} + \frac{5c^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\pi^{5/2}} + \frac{5bc^2\,\text{PolyLog}\left(2, -e^{\text{arcsinh}(cx)}\right)}{2\pi^{5/2}} - \frac{5bc^2\,\text{PolyLog}\left(2, e^{\text{arcsinh}(cx)}\right)}{2\pi^{5/2}}$$

Mathematica [A] (verified)

Time = 4.11 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.34

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{-\frac{8ac^2}{(1+c^2x^2)^{3/2}} + \frac{4bc^3x}{1+c^2x^2} - \frac{48ac^2}{\sqrt{1+c^2x^2}} - \frac{12a\sqrt{1+c^2x^2}}{x^2} - \frac{56bc^2\text{arcsinh}(cx)}{(1+c^2x^2)^{3/2}} - \frac{48bc^4x^2\text{arcsinh}(cx)}{(1+c^2x^2)^{3/2}} - \cdots}{\cdots}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] ((-8*a*c^2)/(1 + c^2*x^2)^(3/2) + (4*b*c^3*x)/(1 + c^2*x^2) - (48*a*c^2)/Sq
rt[1 + c^2*x^2] - (12*a*Sqrt[1 + c^2*x^2])/x^2 - (56*b*c^2*ArcSinh[c*x])/(1
 + c^2*x^2)^(3/2) - (48*b*c^4*x^2*ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) + 104*b
*c^2*ArcTan[Tanh[ArcSinh[c*x]/2]] - 6*b*c^2*Coth[ArcSinh[c*x]/2] - 3*b*c^2*
ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 60*b*c^2*ArcSinh[c*x]*Log[1 - E^(-Arc
Sinh[c*x])] + 60*b*c^2*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 60*a*c^2*L
og[x] + 60*a*c^2*Log[Pi*(1 + Sqrt[1 + c^2*x^2])] - 60*b*c^2*PolyLog[2, -E^(
-ArcSinh[c*x])] + 60*b*c^2*PolyLog[2, E^(-ArcSinh[c*x])] + 6*b*c^2*Tanh[Arc
Sinh[c*x]/2] - (6*b*c*ArcSinh[c*x]*Tanh[ArcSinh[c*x]/2])/x)/(24*Pi^(5/2))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.27

method	result
default	$a\left(-\frac{1}{2\pi x^2(\pi c^2x^2+\pi)^{\frac{3}{2}}}-\frac{5c^2\left(\frac{1}{3\pi\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}+\frac{\frac{1}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)}{\pi^{\frac{3}{2}}}}{\pi}\right)}{2}\right)-\frac{5b\,x^2\operatorname{arcsinh}(cx)c^4}{2\pi^{\frac{5}{2}}(c^2x^2+1)^{\frac{3}{2}}}-\frac{b\,c^3}{3\pi^{\frac{5}{2}}(c^2}$
parts	$a\left(-\frac{1}{2\pi x^2(\pi c^2x^2+\pi)^{\frac{3}{2}}}-\frac{5c^2\left(\frac{1}{3\pi\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}+\frac{\frac{1}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)}{\pi^{\frac{3}{2}}}}{\pi}\right)}{2}\right)-\frac{5b\,x^2\operatorname{arcsinh}(cx)c^4}{2\pi^{\frac{5}{2}}(c^2x^2+1)^{\frac{3}{2}}}-\frac{b\,c^3}{3\pi^{\frac{5}{2}}(c^2}$

```
[In] int((a+b*arcsinh(c*x))/x^3/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/2/Pi/x^2/(Pi*c^2*x^2+Pi)^(3/2)-5/2*c^2*(1/3/Pi/(Pi*c^2*x^2+Pi)^(3/2)+
1/Pi*(1/Pi/(Pi*c^2*x^2+Pi)^(1/2)-1/Pi^(3/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi
)^(1/2)))))-5/2*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)*x^2*arcsinh(c*x)*c^4-1/3*b*c^3
*x/Pi^(5/2)/(c^2*x^2+1)-10/3*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)*c^2-
1/2*b*c/Pi^(5/2)/x/(c^2*x^2+1)-1/2*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)/x^2*arcsinh
(c*x)+13/3*b*c^2/Pi^(5/2)*arctan(c*x+(c^2*x^2+1)^(1/2))+5/2*b*c^2/Pi^(5/2)*
dilog(1+c*x+(c^2*x^2+1)^(1/2))+5/2*b*c^2/Pi^(5/2)*arcsinh(c*x)*ln(1+c*x+(c^
2*x^2+1)^(1/2))+5/2*b*c^2/Pi^(5/2)*dilog(c*x+(c^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx=\int \frac{b\operatorname{arsinh}\left(cx\right)+a}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^3*c^6*x^9 + 3*pi^3*
c^4*x^7 + 3*pi^3*c^2*x^5 + pi^3*x^3), x)
```

Sympy [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = \frac{\int \frac{a}{c^4x^7\sqrt{c^2x^2+1}+2c^2x^5\sqrt{c^2x^2+1}+x^3\sqrt{c^2x^2+1}}\,dx + \int \frac{b\operatorname{asinh}(cx)}{c^4x^7\sqrt{c^2x^2+1}+2c^2x^5\sqrt{c^2x^2+1}+x^3\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

```
[In] integrate((a+b*asinh(c*x))/x**3/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a/(c**4*x**7*sqrt(c**2*x**2 + 1) + 2*c**2*x**5*sqrt(c**2*x**2 + 1
) + x**3*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(c**4*x**7*sqrt(c
**2*x**2 + 1) + 2*c**2*x**5*sqrt(c**2*x**2 + 1) + x**3*sqrt(c**2*x**2 + 1))
, x))/pi**(5/2)
```

Maxima [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = \int \frac{b\operatorname{arsinh}(cx)+a}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] 1/6*a*(15*c^2*arcsinh(1/(c*abs(x)))/pi^(5/2) - 5*c^2/(pi*(pi + pi*c^2*x^2)^
(3/2)) - 15*c^2/(pi^2*sqrt(pi + pi*c^2*x^2)) - 3/(pi*(pi + pi*c^2*x^2)^(3/2
)*x^2)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/((pi + pi*c^2*x^2)^(5/2)
*x^3), x)
```

Giac [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = \int \frac{b\operatorname{arsinh}(cx)+a}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(5/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx=\int \frac{a+b\,\operatorname{asinh}(c\,x)}{x^3\,\left(\Pi\,c^2\,x^2+\Pi\right)^{5/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))/(x^3*(Pi + Pi*c^2*x^2)^(5/2)),x)
[Out] int((a + b*asinh(c*x))/(x^3*(Pi + Pi*c^2*x^2)^(5/2)), x)
```

3.110 $\int \frac{a+b\text{arcsinh}(cx)}{x^4(\pi+c^2\pi x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 208

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(\pi+c^2\pi x^2)^{5/2}}\,dx = -\frac{bc}{6\pi^{5/2}x^2}+\frac{bc^3}{6\pi^{5/2}(1+c^2x^2)} - \frac{a+b\text{arcsinh}(cx)}{3\pi x^3(\pi+c^2\pi x^2)^{3/2}}+\frac{2c^2(a+b\text{arcsinh}(cx))}{\pi x(\pi+c^2\pi x^2)^{3/2}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3\pi(\pi+c^2\pi x^2)^{3/2}} + \frac{16c^4x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi+c^2\pi x^2}}-\frac{8bc^3\log(x)}{3\pi^{5/2}}-\frac{4bc^3\log(1+c^2x^2)}{3\pi^{5/2}}$$

```
[Out] -1/6*b*c/Pi^(5/2)/x^2+1/6*b*c^3/Pi^(5/2)/(c^2*x^2+1)+1/3*(-a-b*arcsinh(c*x)
)/Pi/x^3/(Pi*c^2*x^2+Pi)^(3/2)+2*c^2*(a+b*arcsinh(c*x))/Pi/x/(Pi*c^2*x^2+Pi
)^(3/2)+8/3*c^4*x*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(3/2)-8/3*b*c^3*ln(
x)/Pi^(5/2)-4/3*b*c^3*ln(c^2*x^2+1)/Pi^(5/2)+16/3*c^4*x*(a+b*arcsinh(c*x))/
Pi^2/(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {277, 198, 197, 5804, 12, 1813, 1634}

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{2c^2(a+b\text{arcsinh}(cx))}{\pi x(\pi c^2x^2+\pi)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{3\pi x^3(\pi c^2x^2+\pi)^{3/2}} + \frac{16c^4x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi c^2x^2+\pi}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3\pi(\pi c^2x^2+\pi)^{3/2}} - \frac{8bc^3\log(x)}{3\pi^{5/2}}+\frac{bc^3}{6\pi^{5/2}(c^2x^2+1)}-\frac{4bc^3\log(c^2x^2+1)}{3\pi^{5/2}}-\frac{bc}{6\pi^{5/2}x^2}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] -1/6*(b*c)/(Pi^(5/2)*x^2) + (b*c^3)/(6*Pi^(5/2)*(1 + c^2*x^2)) - (a + b*Arc
Sinh[c*x])/(3*Pi*x^3*(Pi + c^2*Pi*x^2)^(3/2)) + (2*c^2*(a + b*ArcSinh[c*x])
)/(Pi*x*(Pi + c^2*Pi*x^2)^(3/2)) + (8*c^4*x*(a + b*ArcSinh[c*x]))/(3*Pi*(Pi
 + c^2*Pi*x^2)^(3/2)) + (16*c^4*x*(a + b*ArcSinh[c*x]))/(3*Pi^2*Sqrt[Pi + c
^2*Pi*x^2]) - (8*b*c^3*Log[x])/(3*Pi^(5/2)) - (4*b*c^3*Log[1 + c^2*x^2])/(3
*Pi^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\text{arcsinh}(cx)}{3\pi x^3\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{2c^2(a+b\text{arcsinh}(cx))}{\pi x\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} \\
&\quad+\frac{16c^4x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi+c^2\pi x^2}}-\left(bc\sqrt{\pi}\right)\int\frac{-1+6c^2x^2+24c^4x^4+16c^6x^6}{3\pi^3x^3\left(1+c^2x^2\right)^2}\,dx \\
&= -\frac{a+b\text{arcsinh}(cx)}{3\pi x^3\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{2c^2(a+b\text{arcsinh}(cx))}{\pi x\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} \\
&\quad+\frac{16c^4x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi+c^2\pi x^2}}-\frac{(bc)\int\frac{-1+6c^2x^2+24c^4x^4+16c^6x^6}{x^3\left(1+c^2x^2\right)^2}\,dx}{3\pi^{5/2}} \\
&= -\frac{a+b\text{arcsinh}(cx)}{3\pi x^3\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{2c^2(a+b\text{arcsinh}(cx))}{\pi x\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} \\
&\quad+\frac{16c^4x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi+c^2\pi x^2}}-\frac{(bc)\text{Subst}\left(\int\frac{-1+6c^2x+24c^4x^2+16c^6x^3}{x^2\left(1+c^2x\right)^2}\,dx,x,x^2\right)}{6\pi^{5/2}} \\
&= -\frac{a+b\text{arcsinh}(cx)}{3\pi x^3\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{2c^2(a+b\text{arcsinh}(cx))}{\pi x\left(\pi+c^2\pi x^2\right)^{3/2}} \\
&\quad+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{16c^4x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi+c^2\pi x^2}} \\
&\quad-\frac{(bc)\text{Subst}\left(\int\left(-\frac{1}{x^2}+\frac{8c^2}{x}+\frac{c^4}{\left(1+c^2x\right)^2}+\frac{8c^4}{1+c^2x}\right)dx,x,x^2\right)}{6\pi^{5/2}} \\
&= -\frac{bc}{6\pi^{5/2}x^2}+\frac{bc^3}{6\pi^{5/2}\left(1+c^2x^2\right)}-\frac{a+b\text{arcsinh}(cx)}{3\pi x^3\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{2c^2(a+b\text{arcsinh}(cx))}{\pi x\left(\pi+c^2\pi x^2\right)^{3/2}} \\
&\quad+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{16c^4x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi+c^2\pi x^2}}-\frac{8bc^3\log(x)}{3\pi^{5/2}} \\
&\quad-\frac{4bc^3\log\left(1+c^2x^2\right)}{3\pi^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.15

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^4\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = \frac{-2a+12ac^2x^2+48ac^4x^4+32ac^6x^6-bcx\sqrt{1+c^2x^2}-32bc^3x^3\sqrt{1+c^2x^2}-32bc}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] (-2*a + 12*a*c^2*x^2 + 48*a*c^4*x^4 + 32*a*c^6*x^6 - b*c*x*Sqrt[1 + c^2*x^2
] - 32*b*c^3*x^3*Sqrt[1 + c^2*x^2] - 32*b*c^5*x^5*Sqrt[1 + c^2*x^2] + 2*b*(
-1 + 6*c^2*x^2 + 24*c^4*x^4 + 16*c^6*x^6)*ArcSinh[c*x] - 16*b*c^3*x^3*(1 +
c^2*x^2)^(3/2)*Log[x] - 8*b*c^3*x^3*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] - 8*
b*c^5*x^5*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(6*Pi^(5/2)*x^3*(1 + c^2*x^2)
^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1154 vs. 2(181) = 362.

Time = 0.17 (sec) , antiderivative size = 1155, normalized size of antiderivative = 5.55

method	result	size
default	Expression too large to display	1155
parts	Expression too large to display	1155

```
[In] int((a+b*arcsinh(c*x))/x^4/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)*c^3+128/3*b/Pi^(5/2)/(1
2*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)*x^4*c^7-2*b/Pi^(5/2)/(12*c^4*x^4+12*c^2
*x^2-1)/(c^2*x^2+1)*x^2*c^5+1/6*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x
^2+1)/x^2*c+1/3*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^(3/2)/x^3*
arcsinh(c*x)+128/3*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)*x^10*c^
13+128*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)*x^8*c^11+128*b/Pi^(
5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)*x^6*c^9+16/3*b/Pi^(5/2)/(12*c^4*
x^4+12*c^2*x^2-1)/(c^2*x^2+1)^2*arcsinh(c*x)*c^3-128/3*b/Pi^(5/2)/(12*c^4*x
^4+12*c^2*x^2-1)/(c^2*x^2+1)^2*x^4*c^7-128/3*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*
x^2-1)/(c^2*x^2+1)^2*x^12*c^15-512/3*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(
c^2*x^2+1)^2*x^10*c^13-256*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)
^2*x^8*c^11-512/3*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^2*x^6*c^
9+a*(-1/3/Pi/x^3/(Pi*c^2*x^2+Pi)^(3/2)-2*c^2*(-1/Pi/x/(Pi*c^2*x^2+Pi)^(3/2)
-4*c^2*(1/3/Pi*x/(Pi*c^2*x^2+Pi)^(3/2)+2/3/Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2))))+
32/3*b*c^3/Pi^(5/2)*arcsinh(c*x)-8/3*b*c^3/Pi^(5/2)*ln((c*x+(c^2*x^2+1)^(1/
2))^4-1)-560/3*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^2*x^4*arcsi
nh(c*x)*c^7-160/3*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^2*x^2*ar
```

```
csinh(c*x)*c^5-64*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^2*x^8*ar
csinh(c*x)*c^11-192*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^2*x^6*
arcsinh(c*x)*c^9+64*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^(3/2)*
x^7*arcsinh(c*x)*c^10+160*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^
(3/2)*x^5*arcsinh(c*x)*c^8+344/3*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*
x^2+1)^(3/2)*x^3*arcsinh(c*x)*c^6+12*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(
c^2*x^2+1)^(3/2)*x*arcsinh(c*x)*c^4-6*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/
(c^2*x^2+1)^(3/2)/x*arcsinh(c*x)*c^2
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\,(\pi+c^2\pi x^2)^{5/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{(\pi+\pi c^2x^2)^{\frac{5}{2}}x^4}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^3*c^6*x^10 + 3*pi^3
*c^4*x^8 + 3*pi^3*c^2*x^6 + pi^3*x^4), x)
```

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\,(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{\int \frac{a}{c^4x^8\sqrt{c^2x^2+1}+2c^2x^6\sqrt{c^2x^2+1}+x^4\sqrt{c^2x^2+1}}\,dx + \int \frac{b\,\text{asinh}\,(cx)}{c^4x^8\sqrt{c^2x^2+1}+2c^2x^6\sqrt{c^2x^2+1}+x^4\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

```
[In] integrate((a+b*asinh(c*x))/x**4/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a/(c**4*x**8*sqrt(c**2*x**2 + 1) + 2*c**2*x**6*sqrt(c**2*x**2 + 1
) + x**4*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(c**4*x**8*sqrt(c
**2*x**2 + 1) + 2*c**2*x**6*sqrt(c**2*x**2 + 1) + x**4*sqrt(c**2*x**2 + 1))
, x))/pi**(5/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.13

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{x^4\,(\pi+c^2\pi x^2)^{5/2}}\,dx &= -\frac{1}{6}bc\left(\frac{8\,c^2\log\left(c^2x^2+1\right)}{\pi^{\frac{5}{2}}}+\frac{16\,c^2\log\left(x\right)}{\pi^{\frac{5}{2}}}+\frac{1}{\pi^{\frac{5}{2}}c^2x^4+\pi^{\frac{5}{2}}x^2}\right)\\ &\quad+\frac{1}{3}\left(\frac{8\,c^4x}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}}+\frac{16\,c^4x}{\pi^2\sqrt{\pi+\pi c^2x^2}}+\frac{6\,c^2}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}x}-\frac{1}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}x^3}\right)b\,\text{arsinh}\,(cx)\\ &\quad+\frac{1}{3}\left(\frac{8\,c^4x}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}}+\frac{16\,c^4x}{\pi^2\sqrt{\pi+\pi c^2x^2}}+\frac{6\,c^2}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}x}-\frac{1}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}x^3}\right)a\end{aligned}$$

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/6*b*c*(8*c^2*log(c^2*x^2 + 1)/pi^(5/2) + 16*c^2*log(x)/pi^(5/2) + 1/(pi^
(5/2)*c^2*x^4 + pi^(5/2)*x^2)) + 1/3*(8*c^4*x/(pi*(pi + pi*c^2*x^2)^(3/2))
+ 16*c^4*x/(pi^2*sqrt(pi + pi*c^2*x^2)) + 6*c^2/(pi*(pi + pi*c^2*x^2)^(3/2)
*x) - 1/(pi*(pi + pi*c^2*x^2)^(3/2)*x^3))*b*arcsinh(c*x) + 1/3*(8*c^4*x/(pi
*(pi + pi*c^2*x^2)^(3/2)) + 16*c^4*x/(pi^2*sqrt(pi + pi*c^2*x^2)) + 6*c^2/(
pi*(pi + pi*c^2*x^2)^(3/2)*x) - 1/(pi*(pi + pi*c^2*x^2)^(3/2)*x^3))*a
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^4 (\pi + c^2\pi x^2)^{5/2}}\, dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{(\pi + \pi c^2x^2)^{\frac{5}{2}}x^4}\, dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(5/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{x^4 (\pi + c^2\pi x^2)^{5/2}}\, dx = \int \frac{a + b\,\text{asinh}(c\,x)}{x^4\,(\Pi\,c^2\,x^2 + \Pi)^{5/2}}\, dx$$

```
[In] int((a + b*asinh(c*x))/(x^4*(Pi + Pi*c^2*x^2)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^4*(Pi + Pi*c^2*x^2)^(5/2)), x)
```

3.111 $\int \frac{\operatorname{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 200

$$\int \frac{\operatorname{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx=\frac{1}{20ac^3(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}}+\frac{2}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}+\frac{x\operatorname{arcsinh}(ax)}{5c(c+a^2cx^2)^{5/2}}+\frac{4x\operatorname{arcsinh}(ax)}{15c^2(c+a^2cx^2)^{3/2}}+\frac{8x\operatorname{arcsinh}(ax)}{15c^3\sqrt{c+a^2cx^2}}-\frac{4\sqrt{1+a^2x^2}\log(1+a^2x^2)}{15ac^3\sqrt{c+a^2cx^2}}$$

```
[Out] 1/5*x*arcsinh(a*x)/c/(a^2*c*x^2+c)^(5/2)+4/15*x*arcsinh(a*x)/c^2/(a^2*c*x^2
+c)^(3/2)+1/20/a/c^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2)+8/15*x*arcsinh(a
*x)/c^3/(a^2*c*x^2+c)^(1/2)+2/15/a/c^3/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2
)-4/15*ln(a^2*x^2+1)*(a^2*x^2+1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {5788, 5787, 266, 267}

$$\int \frac{\operatorname{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx=\frac{8x\operatorname{arcsinh}(ax)}{15c^3\sqrt{a^2cx^2+c}}+\frac{4x\operatorname{arcsinh}(ax)}{15c^2(a^2cx^2+c)^{3/2}}+\frac{x\operatorname{arcsinh}(ax)}{5c(a^2cx^2+c)^{5/2}}+\frac{2}{15ac^3\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}+\frac{1}{20ac^3(a^2x^2+1)^{3/2}\sqrt{a^2cx^2+c}}-\frac{4\sqrt{a^2x^2+1}\log(a^2x^2+1)}{15ac^3\sqrt{a^2cx^2+c}}$$

```
[In] Int[ArcSinh[a*x]/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] 1/(20*a*c^3*(1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*x^2]) + 2/(15*a*c^3*Sqrt[1 +
 a^2*x^2]*Sqrt[c + a^2*c*x^2]) + (x*ArcSinh[a*x])/(5*c*(c + a^2*c*x^2)^(5/2
)) + (4*x*ArcSinh[a*x])/(15*c^2*(c + a^2*c*x^2)^(3/2)) + (8*x*ArcSinh[a*x])
/(15*c^3*Sqrt[c + a^2*c*x^2]) - (4*Sqrt[1 + a^2*x^2]*Log[1 + a^2*x^2])/(15*
a*c^3*Sqrt[c + a^2*c*x^2])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x\text{arcsinh}(ax)}{5c\,(c+a^2cx^2)^{5/2}} + \frac{4\int \frac{\text{arcsinh}(ax)}{(c+a^2cx^2)^{5/2}}\,dx}{5c} - \frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x}{(1+a^2x^2)^3}\,dx}{5c^3\sqrt{c+a^2cx^2}} \\
&= \frac{1}{20ac^3\,(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} + \frac{x\text{arcsinh}(ax)}{5c\,(c+a^2cx^2)^{5/2}} + \frac{4x\text{arcsinh}(ax)}{15c^2\,(c+a^2cx^2)^{3/2}} \\
&\quad + \frac{8\int \frac{\text{arcsinh}(ax)}{(c+a^2cx^2)^{3/2}}\,dx}{15c^2} - \frac{\left(4a\sqrt{1+a^2x^2}\right)\int \frac{x}{(1+a^2x^2)^2}\,dx}{15c^3\sqrt{c+a^2cx^2}}
\end{aligned}
$$

$$=\frac{1}{20ac^3(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}}+\frac{2}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}+\frac{x\text{arcsinh}(ax)}{5c(c+a^2cx^2)^{5/2}}+\frac{4x\text{arcsinh}(ax)}{15c^2(c+a^2cx^2)^{3/2}}+\frac{8x\text{arcsinh}(ax)}{15c^3\sqrt{c+a^2cx^2}}-\frac{\left(8a\sqrt{1+a^2x^2}\right)\int\frac{x}{1+a^2x^2}\,dx}{15c^3\sqrt{c+a^2cx^2}}$$

$$=\frac{1}{20ac^3(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}}+\frac{2}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}+\frac{x\text{arcsinh}(ax)}{5c(c+a^2cx^2)^{5/2}}+\frac{4x\text{arcsinh}(ax)}{15c^2(c+a^2cx^2)^{3/2}}+\frac{8x\text{arcsinh}(ax)}{15c^3\sqrt{c+a^2cx^2}}-\frac{4\sqrt{1+a^2x^2}\log(1+a^2x^2)}{15ac^3\sqrt{c+a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.60

$$\int\frac{\text{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx=\frac{\sqrt{c+a^2cx^2}\left(4ax\sqrt{1+a^2x^2}(15+20a^2x^2+8a^4x^4)\,\text{arcsinh}(ax)-(1+a^2x^2)\left(-11-8a\right.\right.}{60ac^4(1+a^2x^2)^{7/2}}$$

```
[In] Integrate[ArcSinh[a*x]/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(4*a*x*Sqrt[1 + a^2*x^2]*(15 + 20*a^2*x^2 + 8*a^4*x^4)
*ArcSinh[a*x] - (1 + a^2*x^2)*(-11 - 8*a^2*x^2 + 16*(1 + a^2*x^2)^2*Log[1 +
 a^2*x^2])))/(60*a*c^4*(1 + a^2*x^2)^(7/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(170) = 340.

Time = 0.28 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.82

method	result
default	$\frac{16\sqrt{c(a^2x^2+1)}\,\text{arcsinh}(ax)}{15\sqrt{a^2x^2+1}\,a\,c^4}+\frac{\sqrt{c(a^2x^2+1)}\left(8a^5x^5-8a^4x^4\sqrt{a^2x^2+1}+20a^3x^3-16a^2x^2\sqrt{a^2x^2+1}+15ax-8\sqrt{a^2x^2+1}\right)\left(-64a^8x^8-6\right.}{}$

```
[In] int(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 16/15*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^4*arcsinh(a*x)+1/60*(c*(a
^2*x^2+1))^(1/2)*(8*a^5*x^5-8*a^4*x^4*(a^2*x^2+1)^(1/2)+20*a^3*x^3-16*a^2*x
^2*(a^2*x^2+1)^(1/2)+15*a*x-8*(a^2*x^2+1)^(1/2))*(-64*a^8*x^8-64*(a^2*x^2+1
)^(1/2)*a^7*x^7-280*a^6*x^6-248*x^5*a^5*(a^2*x^2+1)^(1/2)+160*a^4*x^4*arcsi
nh(a*x)-456*a^4*x^4-340*a^3*x^3*(a^2*x^2+1)^(1/2)+380*a^2*x^2*arcsinh(a*x)-
328*a^2*x^2-165*a*x*(a^2*x^2+1)^(1/2)+256*arcsinh(a*x)-88)/(40*a^10*x^10+21
5*a^8*x^8+469*a^6*x^6+517*a^4*x^4+287*a^2*x^2+64)/a/c^4-8/15*(c*(a^2*x^2+1)
)^(1/2)/(a^2*x^2+1)^(1/2)/a/c^4*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)
```

Fricas [F]

$$\int \frac{\text{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx=\int \frac{\text{arsinh}\,(ax)}{(a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

[In] integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")

[Out] integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)/(a^8*c^4*x^8 + 4*a^6*c^4*x^6 + 6*a^4*c^4*x^4 + 4*a^2*c^4*x^2 + c^4), x)

Sympy [F]

$$\int \frac{\text{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx=\int \frac{\text{asinh}\,(ax)}{(c\,(a^2x^2+1))^{\frac{7}{2}}}\,dx$$

[In] integrate(asinh(a*x)/(a**2*c*x**2+c)**(7/2),x)

[Out] Integral(asinh(a*x)/(c*(a**2*x**2 + 1))**(7/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.72

$$\int \frac{\text{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx=\frac{1}{60}\,a\left(\frac{3}{\left(a^6c^{\frac{5}{2}}x^4+2\,a^4c^{\frac{5}{2}}x^2+a^2c^{\frac{5}{2}}\right)c}+\frac{8}{\left(a^4c^{\frac{3}{2}}x^2+a^2c^{\frac{3}{2}}\right)c^2}-\frac{16\,\log\left(x^2+\frac{1}{a^2}\right)}{a^2c^{\frac{7}{2}}}\right)+\frac{1}{15}\left(\frac{8\,x}{\sqrt{a^2cx^2+c}c^3}+\frac{4\,x}{(a^2cx^2+c)^{\frac{3}{2}}c^2}+\frac{3\,x}{(a^2cx^2+c)^{\frac{5}{2}}c}\right)\text{arsinh}\,(ax)$$

[In] integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")

[Out] 1/60*a*(3/((a^6*c^(5/2)*x^4 + 2*a^4*c^(5/2)*x^2 + a^2*c^(5/2))*c) + 8/((a^4*c^(3/2)*x^2 + a^2*c^(3/2))*c^2) - 16*log(x^2 + 1/a^2)/(a^2*c^(7/2))) + 1/15*(8*x/(sqrt(a^2*c*x^2 + c)*c^3) + 4*x/((a^2*c*x^2 + c)^(3/2)*c^2) + 3*x/((a^2*c*x^2 + c)^(5/2)*c))*arcsinh(a*x)

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.62

$$\int \frac{\operatorname{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx = -\frac{1}{60}\,\sqrt{c}\left(\frac{16\,\log\left(a^2x^2+1\right)}{ac^4} - \frac{24\,a^4x^4+56\,a^2x^2+35}{\left(a^2x^2+1\right)^2ac^4}\right) + \frac{\left(4\left(\frac{2\,a^4x^2}{c}+\frac{5\,a^2}{c}\right)x^2+\frac{15}{c}\right)x\log\left(ax+\sqrt{a^2x^2+1}\right)}{15\left(a^2cx^2+c\right)^{\frac{5}{2}}}$$

```
[In] integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
[Out] -1/60*sqrt(c)*(16*log(a^2*x^2 + 1)/(a*c^4) - (24*a^4*x^4 + 56*a^2*x^2 + 35)
/((a^2*x^2 + 1)^2*a*c^4)) + 1/15*(4*(2*a^4*x^2/c + 5*a^2/c)*x^2 + 15/c)*x*l
og(a*x + sqrt(a^2*x^2 + 1))/(a^2*c*x^2 + c)^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)}{\left(c\,a^2\,x^2+c\right)^{7/2}}\,dx$$

```
[In] int(asinh(a*x)/(c + a^2*c*x^2)^(7/2),x)
[Out] int(asinh(a*x)/(c + a^2*c*x^2)^(7/2), x)
```

3.112 $\int \frac{x^4 \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 86

$$\int \frac{x^4 \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{3x^2}{16a^3} - \frac{x^4}{16a} - \frac{3x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{4a^2} + \frac{3\text{arcsinh}(ax)^2}{16a^5}$$

```
[Out] 3/16*x^2/a^3-1/16*x^4/a+3/16*arcsinh(a*x)^2/a^5-3/8*x*arcsinh(a*x)*(a^2*x^2
+1)^(1/2)/a^4+1/4*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5812, 5783, 30}

$$\int \frac{x^4 \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{3\text{arcsinh}(ax)^2}{16a^5} + \frac{3x^2}{16a^3} + \frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{4a^2} - \frac{3x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{8a^4} - \frac{x^4}{16a}$$

```
[In] Int[(x^4*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (3*x^2)/(16*a^3) - x^4/(16*a) - (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(8*a^4
) + (x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(4*a^2) + (3*ArcSinh[a*x]^2)/(16*a
^5)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{4a^2} - \frac{3\int \frac{x^2\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{4a^2} - \frac{\int x^3\,dx}{4a} \\ &= -\frac{x^4}{16a} - \frac{3x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{4a^2} + \frac{3\int \frac{\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{8a^4} \\ &\quad + \frac{3\int x\,dx}{8a^3} \\ &= \frac{3x^2}{16a^3} - \frac{x^4}{16a} - \frac{3x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{4a^2} + \frac{3\operatorname{arcsinh}(ax)^2}{16a^5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.73

$$\int \frac{x^4\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{3a^2x^2 - a^4x^4 + 2ax\sqrt{1+a^2x^2}\,(-3+2a^2x^2)\operatorname{arcsinh}(ax) + 3\operatorname{arcsinh}(ax)^2}{16a^5}$$

```
[In] Integrate[(x^4*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
[Out] (3*a^2*x^2 - a^4*x^4 + 2*a*x*Sqrt[1 + a^2*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[a*x
] + 3*ArcSinh[a*x]^2)/(16*a^5)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{4a^3x^3\,\mathrm{arcsinh}(ax)\sqrt{a^2x^2+1}-a^4x^4-6\,\mathrm{arcsinh}(ax)\sqrt{a^2x^2+1}\,ax+3a^2x^2+3\,\mathrm{arcsinh}(ax)^2+3}{16a^5}$	74

```
[In] int(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/16*(4*a^3*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)-a^4*x^4-6*arcsinh(a*x)*(a^2*
x^2+1)^(1/2)*a*x+3*a^2*x^2+3*arcsinh(a*x)^2+3)/a^5
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int \frac{x^4\mathrm{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{a^4x^4-3\,a^2x^2-2\,(2\,a^3x^3-3\,ax)\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)-3\,\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{16\,a^5}$$

```
[In] integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/16*(a^4*x^4 - 3*a^2*x^2 - 2*(2*a^3*x^3 - 3*a*x)*sqrt(a^2*x^2 + 1)*log(a*
x + sqrt(a^2*x^2 + 1)) - 3*log(a*x + sqrt(a^2*x^2 + 1))^2)/a^5
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int \frac{x^4\mathrm{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} -\frac{x^4}{16a}+\frac{x^3\sqrt{a^2x^2+1}\,\mathrm{asinh}\,(ax)}{4a^2}+\frac{3x^2}{16a^3}-\frac{3x\sqrt{a^2x^2+1}\,\mathrm{asinh}\,(ax)}{8a^4}+\frac{3\,\mathrm{asinh}^2\,(ax)}{16a^5} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(x**4*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
[Out] Piecewise((-x**4/(16*a) + x**3*sqrt(a**2*x**2 + 1)*asinh(a*x)/(4*a**2) + 3*
x**2/(16*a**3) - 3*x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(8*a**4) + 3*asinh(a*x)
**2/(16*a**5), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int \frac{x^4\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{1}{16}\left(\frac{x^4}{a^2} - \frac{3\,x^2}{a^4} + \frac{3\,\text{arsinh}\,(ax)^2}{a^6}\right)a + \frac{1}{8}\left(\frac{2\,\sqrt{a^2x^2+1}x^3}{a^2} - \frac{3\,\sqrt{a^2x^2+1}x}{a^4} + \frac{3\,\text{arsinh}\,(ax)}{a^5}\right)\text{arsinh}\,(ax)$$

```
[In] integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/16*(x^4/a^2 - 3*x^2/a^4 + 3*arcsinh(a*x)^2/a^6)*a + 1/8*(2*sqrt(a^2*x^2
+ 1)*x^3/a^2 - 3*sqrt(a^2*x^2 + 1)*x/a^4 + 3*arcsinh(a*x)/a^5)*arcsinh(a*x)
```

Giac [F]

$$\int \frac{x^4\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^4\,\text{arsinh}\,(ax)}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^4*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^4\,\text{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int((x^4*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^4*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

3.113 $\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx=\frac{2x}{3a^3}-\frac{x^3}{9a}-\frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{3a^4}+\frac{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{3a^2}$$

```
[Out] 2/3*x/a^3-1/9*x^3/a-2/3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^4+1/3*x^2*arcsinh(
a*x)*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5812, 5798, 8, 30}

$$\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx=\frac{2x}{3a^3}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3a^2}-\frac{2\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3a^4}-\frac{x^3}{9a}$$

```
[In] Int[(x^3*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (2*x)/(3*a^3) - x^3/(9*a) - (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(3*a^4) + (x
^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(3*a^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{3a^2} - \frac{2\int \frac{x\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{3a^2} - \frac{\int x^2\,dx}{3a}\\
&= -\frac{x^3}{9a} - \frac{2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{3a^4} + \frac{x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{3a^2} + \frac{2\int 1\,dx}{3a^3}\\
&= \frac{2x}{3a^3} - \frac{x^3}{9a} - \frac{2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{3a^4} + \frac{x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{3a^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.69

$$\int \frac{x^3\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{6ax - a^3x^3 + 3(-2+a^2x^2)\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{9a^4}$$

```
[In] Integrate[(x^3*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
[Out] (6*a*x - a^3*x^3 + 3*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(9*a^4)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{3a^4x^4 \operatorname{arcsinh}(ax)-3a^2x^2 \operatorname{arcsinh}(ax)-a^3x^3\sqrt{a^2x^2+1}-6 \operatorname{arcsinh}(ax)+6ax\sqrt{a^2x^2+1}}{9a^4\sqrt{a^2x^2+1}}$	82

```
[In] int(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9/a^4/(a^2*x^2+1)^(1/2)*(3*a^4*x^4*arcsinh(a*x)-3*a^2*x^2*arcsinh(a*x)-a^
3*x^3*(a^2*x^2+1)^(1/2)-6*arcsinh(a*x)+6*a*x*(a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.79

$$\int \frac{x^3 \operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{a^3x^3 - 3\sqrt{a^2x^2+1}(a^2x^2-2)\log\left(ax+\sqrt{a^2x^2+1}\right) - 6\,ax}{9\,a^4}$$

```
[In] integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*(a^3*x^3 - 3*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 2)*log(a*x + sqrt(a^2*x^2 +
1)) - 6*a*x)/a^4
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int \frac{x^3 \operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} -\frac{x^3}{9a} + \frac{x^2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{3a^2} + \frac{2x}{3a^3} - \frac{2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{3a^4} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(x**3*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**3/(9*a) + x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)/(3*a**2) + 2*x
/(3*a**3) - 2*sqrt(a**2*x**2 + 1)*asinh(a*x)/(3*a**4), Ne(a, 0)), (0, True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{1}{9}\,a\left(\frac{x^3}{a^2}-\frac{6\,x}{a^4}\right)+\frac{1}{3}\left(\frac{\sqrt{a^2x^2+1}x^2}{a^2}-\frac{2\,\sqrt{a^2x^2+1}}{a^4}\right)\text{arsinh}\,(ax)$$

```
[In] integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/9*a*(x^3/a^2 - 6*x/a^4) + 1/3*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^
2 + 1)/a^4)*arcsinh(a*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^3\,\text{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int((x^3*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^3*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

3.114 $\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}} \, dx = -\frac{x^2}{4a}+\frac{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{2a^2}-\frac{\text{arcsinh}(ax)^2}{4a^3}$$

```
[Out] -1/4*x^2/a-1/4*arcsinh(a*x)^2/a^3+1/2*x*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5812, 5783, 30}

$$\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}} \, dx = -\frac{\text{arcsinh}(ax)^2}{4a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}$$

```
[In] Int[(x^2*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -1/4*x^2/a + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a^2) - ArcSinh[a*x]^2/(4
*a^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
```

```
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)}{2a^2} - \frac{\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{2a^2} - \frac{\int x\,dx}{2a}\\ &= -\frac{x^2}{4a} + \frac{x\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)}{2a^2} - \frac{\text{arcsinh}(ax)^2}{4a^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{a^2x^2 - 2ax\sqrt{1+a^2x^2}\,\text{arcsinh}(ax) + \text{arcsinh}(ax)^2}{4a^3}$$

```
[In] Integrate[(x^2*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
[Out] -1/4*(a^2*x^2 - 2*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + ArcSinh[a*x]^2)/a^3
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{-2\,\text{arcsinh}(ax)\sqrt{a^2x^2+1}\,ax+a^2x^2+\text{arcsinh}(ax)^2+1}{4a^3}$	40

```
[In] int(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/4*(-2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x+a^2*x^2+arcsinh(a*x)^2+1)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.27

$$\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{a^2x^2-2\,\sqrt{a^2x^2+1}ax\log\left(ax+\sqrt{a^2x^2+1}\right)+\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{4\,a^3}$$

```
[In] integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(a^2*x^2 - 2*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1)) + log(
a*x + sqrt(a^2*x^2 + 1))^2)/a^3
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} -\frac{x^2}{4a}+\frac{x\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{2a^2}-\frac{\operatorname{asinh}^2(ax)}{4a^3} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**2/(4*a) + x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(2*a**2) - asinh(
a*x)**2/(4*a**3), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

$$\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{1}{4}\,a\left(\frac{x^2}{a^2}-\frac{\operatorname{arsinh}(ax)^2}{a^4}\right)+\frac{1}{2}\left(\frac{\sqrt{a^2x^2+1}x}{a^2}-\frac{\operatorname{arsinh}(ax)}{a^3}\right)\operatorname{arsinh}(ax)$$

```
[In] integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*a*(x^2/a^2 - arcsinh(a*x)^2/a^4) + 1/2*(sqrt(a^2*x^2 + 1)*x/a^2 - arcs
inh(a*x)/a^3)*arcsinh(a*x)
```

Giac [F]

$$\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2\,\text{arsinh}\,(ax)}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2\,\text{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int((x^2*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^2*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

3.115 $\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx=-\frac{x}{a}+\frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{a^2}$$

```
[Out] -x/a+arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5798, 8}

$$\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx=\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{x}{a}$$

```
[In] Int[(x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
```

```
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{a^2} - \frac{\int 1\,dx}{a} \\ &= -\frac{x}{a} + \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{a^2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{x}{a} + \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{a^2}$$

```
[In] Integrate[(x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
[Out] -(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.68

method	result	size
default	$\frac{a^2x^2\,\text{arcsinh}(ax)+\text{arcsinh}(ax)-ax\sqrt{a^2x^2+1}}{a^2\sqrt{a^2x^2+1}}$	47

```
[In] int(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/a^2/(a^2*x^2+1)^(1/2)*(a^2*x^2*arcsinh(a*x)+arcsinh(a*x)-a*x*(a^2*x^2+1)^
(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{ax - \sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}$$

```
[In] integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -(a*x - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} -\frac{x}{a} + \frac{\sqrt{a^2x^2+1}\,\text{asinh}\,(ax)}{a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(x*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
[Out] Piecewise((-x/a + sqrt(a**2*x**2 + 1)*asinh(a*x)/a**2, Ne(a, 0)), (0, True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{x}{a} + \frac{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}{a^2}$$

```
[In] integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -x/a + sqrt(a^2*x^2 + 1)*arcsinh(a*x)/a^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{x}{a} + \frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}$$

```
[In] integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] -x/a + sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))/a^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x\,\text{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int((x*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

3.116 $\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}} dx$

Optimal result

Integrand size = 18, antiderivative size = 13

$$\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}} dx = \frac{\text{arcsinh}(ax)^2}{2a}$$

```
[Out] 1/2*arcsinh(a*x)^2/a
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {5783}

$$\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}} dx = \frac{\text{arcsinh}(ax)^2}{2a}$$

```
[In] Int[ArcSinh[a*x]/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ArcSinh[a*x]^2/(2*a)
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{\text{arcsinh}(ax)^2}{2a}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx=\frac{\operatorname{arcsinh}(ax)^2}{2a}$$

```
[In] Integrate[ArcSinh[a*x]/Sqrt[1 + a^2*x^2],x]
[Out] ArcSinh[a*x]^2/(2*a)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\operatorname{arcsinh}(ax)^2}{2a}$	12
default	$\frac{\operatorname{arcsinh}(ax)^2}{2a}$	12

```
[In] int(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arcsinh(a*x)^2/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx=\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{2\,a}$$

```
[In] integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*log(a*x + sqrt(a^2*x^2 + 1))^2/a
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} \frac{\operatorname{asinh}^2(ax)}{2a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(asinh(a*x)/(a**2*x**2+1)**(1/2),x)
[Out] Piecewise((asinh(a*x)**2/(2*a), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{\operatorname{arsinh}(ax)^2}{2a}$$

```
[In] integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*arcsinh(a*x)^2/a
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{\operatorname{asinh}(a\,x)^2}{2\,a}$$

```
[In] int(asinh(a*x)/(a^2*x^2 + 1)^(1/2),x)
[Out] asinh(a*x)^2/(2*a)
```

3.117 $\int \frac{\operatorname{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 34

$$\int \frac{\operatorname{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx = -2\operatorname{arcsinh}(ax)\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) - \operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right) + \operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right)$$

```
[Out] -2*arcsinh(a*x)*arctanh(a*x+(a^2*x^2+1)^(1/2))-polylog(2,-a*x-(a^2*x^2+1)^(
1/2))+polylog(2,a*x+(a^2*x^2+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5816, 4267, 2317, 2438}

$$\int \frac{\operatorname{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx = -2\operatorname{arcsinh}(ax)\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) - \operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right) + \operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right)$$

```
[In] Int[ArcSinh[a*x]/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -2*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]] - PolyLog[2, -E^ArcSinh[a*x]] + Pol
yLog[2, E^ArcSinh[a*x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int x\text{csch}(x)\,dx, x, \text{arcsinh}(ax)\right)\\
&= -2\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) - \text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \text{arcsinh}(ax)\right)\\
&\quad + \text{Subst}\left(\int \log\left(1+e^x\right)\,dx, x, \text{arcsinh}(ax)\right)\\
&= -2\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) - \text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\text{arcsinh}(ax)}\right)\\
&\quad + \text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{\text{arcsinh}(ax)}\right)\\
&= -2\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) - \text{PolyLog}\left(2, -e^{\text{arcsinh}(ax)}\right) + \text{PolyLog}\left(2, e^{\text{arcsinh}(ax)}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.68

$$\begin{aligned}
\int \frac{\text{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx &= \text{arcsinh}(ax)\left(\log\left(1-e^{-\text{arcsinh}(ax)}\right) - \log\left(1+e^{-\text{arcsinh}(ax)}\right)\right)\\
&\quad + \text{PolyLog}\left(2, -e^{-\text{arcsinh}(ax)}\right) - \text{PolyLog}\left(2, e^{-\text{arcsinh}(ax)}\right)
\end{aligned}$$

```
[In] Integrate[ArcSinh[a*x]/(x*Sqrt[1 + a^2*x^2]),x]
[Out] ArcSinh[a*x]*(Log[1 - E^(-ArcSinh[a*x])] - Log[1 + E^(-ArcSinh[a*x])]) + Po
lyLog[2, -E^(-ArcSinh[a*x])] - PolyLog[2, E^(-ArcSinh[a*x])]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.62

method	result
default	$\operatorname{arcsinh}(ax)\ln\left(1-ax-\sqrt{a^{2}x^{2}+1}\right)+\operatorname{polylog}\left(2,ax+\sqrt{a^{2}x^{2}+1}\right)-\operatorname{arcsinh}(ax)\ln\left(1+ax+\right.$

[In] int(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))+polylog(2,a*x+(a^2*x^2+1)^(1/2))-a
rcsinh(a*x)*ln(1+a*x+(a^2*x^2+1)^(1/2))-polylog(2,-a*x-(a^2*x^2+1)^(1/2))

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx=\int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}x}\,dx$$

[In] integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)/(a^2*x^3 + x), x)

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx=\int \frac{\operatorname{asinh}(ax)}{x\sqrt{a^2x^2+1}}\,dx$$

[In] integrate(asinh(a*x)/x/(a**2*x**2+1)**(1/2),x)

[Out] Integral(asinh(a*x)/(x*sqrt(a**2*x**2 + 1)), x)

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx=\int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}x}\,dx$$

[In] integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x), x)

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)}{\sqrt{a^2x^2+1}x}\,dx$$

```
[In] integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)}{x\,\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int(asinh(a*x)/(x*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(asinh(a*x)/(x*(a^2*x^2 + 1)^(1/2)), x)
```

3.118 $\int \frac{\operatorname{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 27

$$\int \frac{\operatorname{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = -\frac{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{x}+a\log(x)$$

```
[Out] a*ln(x)-arcsinh(a*x)*(a^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5800, 29}

$$\int \frac{\operatorname{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = a\log(x)-\frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)}{x}$$

```
[In] Int[ArcSinh[a*x]/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -((Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) + a*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
```

```
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{x} + a\int \frac{1}{x}\,dx\\ &= -\frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{x} + a\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\text{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = -\frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{x} + a\log(ax)$$

```
[In] Integrate[ArcSinh[a*x]/(x^2*Sqrt[1 + a^2*x^2]),x]
[Out] -((Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) + a*Log[a*x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(25) = 50.

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.07

method	result	size
default	$-2a\,\text{arcsinh}\,(ax)+\frac{\left(ax-\sqrt{a^2x^2+1}\right)\text{arcsinh}(ax)}{x}+a\ln\left(\left(ax+\sqrt{a^2x^2+1}\right)^2-1\right)$	56

```
[In] int(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*a*arcsinh(a*x)+(a*x-(a^2*x^2+1)^(1/2))/x*arcsinh(a*x)+a*ln((a*x+(a^2*x^2
+1)^(1/2))^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \frac{\operatorname{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = \frac{ax\log(x) - \sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{x}$$

```
[In] integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] (a*x*log(x) - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/x
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(ax)}{x^2\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(asinh(a*x)/x**2/(a**2*x**2+1)**(1/2),x)
[Out] Integral(asinh(a*x)/(x**2*sqrt(a**2*x**2 + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = a\log(x) - \frac{\sqrt{a^2x^2+1}\operatorname{arsinh}(ax)}{x}$$

```
[In] integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] a*log(x) - sqrt(a^2*x^2 + 1)*arcsinh(a*x)/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(25) = 50.

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.63

$$\int \frac{\operatorname{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = -a\log\left(-x|a|+\sqrt{a^2x^2+1}\right) + a\log(|x|) + \frac{2|a|\log\left(ax+\sqrt{a^2x^2+1}\right)}{\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1}$$

```
[In] integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] -a*log(-x*abs(a) + sqrt(a^2*x^2 + 1)) + a*log(abs(x)) + 2*abs(a)*log(a*x +
sqrt(a^2*x^2 + 1))/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)}{x^2\,\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int(asinh(a*x)/(x^2*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(asinh(a*x)/(x^2*(a^2*x^2 + 1)^(1/2)), x)
```

3.119 $\int \frac{\text{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{\text{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = -\frac{a}{2x} - \frac{\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)}{2x^2} + a^2\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) + \frac{1}{2}a^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right) - \frac{1}{2}a^2\,\text{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right)$$

```
[Out] -1/2*a/x+a^2*arcsinh(a*x)*arctanh(a*x+(a^2*x^2+1)^(1/2))+1/2*a^2*polylog(2,
-a*x-(a^2*x^2+1)^(1/2))-1/2*a^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))-1/2*arcsin
h(a*x)*(a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5809, 5816, 4267, 2317, 2438, 30}

$$\int \frac{\text{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = a^2\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) + \frac{1}{2}a^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right) - \frac{1}{2}a^2\,\text{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right) - \frac{\sqrt{a^2x^2+1}\,\text{arcsinh}(ax)}{2x^2} - \frac{a}{2x}$$

```
[In] Int[ArcSinh[a*x]/(x^3*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -1/2*a/x - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*x^2) + a^2*ArcSinh[a*x]*ArcT
anh[E^ArcSinh[a*x]] + (a^2*PolyLog[2, -E^ArcSinh[a*x]])/2 - (a^2*PolyLog[2,
 E^ArcSinh[a*x]])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{2x^2} + \frac{1}{2}a\int \frac{1}{x^2}\,dx - \frac{1}{2}a^2\int \frac{\operatorname{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{a}{2x} - \frac{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{2x^2} - \frac{1}{2}a^2\text{Subst}\left(\int x\operatorname{csch}(x)\,dx, x, \operatorname{arcsinh}(ax)\right)
\end{aligned}$$

$$= -\frac{a}{2x} - \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{2x^2} + a^2\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right)$$
$$+ \frac{1}{2}a^2\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \text{arcsinh}(ax)\right)$$
$$- \frac{1}{2}a^2\text{Subst}\left(\int \log\left(1+e^x\right)\,dx, x, \text{arcsinh}(ax)\right)$$

$$= -\frac{a}{2x} - \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{2x^2} + a^2\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right)$$
$$+ \frac{1}{2}a^2\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\text{arcsinh}(ax)}\right)$$
$$- \frac{1}{2}a^2\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{\text{arcsinh}(ax)}\right)$$

$$= -\frac{a}{2x} - \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{2x^2} + a^2\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right)$$
$$+ \frac{1}{2}a^2\,\text{PolyLog}\left(2, -e^{\text{arcsinh}(ax)}\right) - \frac{1}{2}a^2\,\text{PolyLog}\left(2, e^{\text{arcsinh}(ax)}\right)$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.58

$$\int \frac{\text{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = \frac{1}{8}a^2\left(-2\coth\left(\frac{1}{2}\text{arcsinh}(ax)\right) - \text{arcsinh}(ax)\text{csch}^2\left(\frac{1}{2}\text{arcsinh}(ax)\right)\right.$$
$$- 4\text{arcsinh}(ax)\log\left(1-e^{-\text{arcsinh}(ax)}\right)$$
$$+ 4\text{arcsinh}(ax)\log\left(1+e^{-\text{arcsinh}(ax)}\right) - 4\,\text{PolyLog}\left(2, -e^{-\text{arcsinh}(ax)}\right)$$
$$+ 4\,\text{PolyLog}\left(2, e^{-\text{arcsinh}(ax)}\right) - \text{arcsinh}(ax)\text{sech}^2\left(\frac{1}{2}\text{arcsinh}(ax)\right)$$
$$\left. + 2\tanh\left(\frac{1}{2}\text{arcsinh}(ax)\right)\right)$$

```
[In] Integrate[ArcSinh[a*x]/(x^3*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (a^2*(-2*Coth[ArcSinh[a*x]/2] - ArcSinh[a*x]*Csch[ArcSinh[a*x]/2]^2 - 4*Arc
Sinh[a*x]*Log[1 - E^(-ArcSinh[a*x])] + 4*ArcSinh[a*x]*Log[1 + E^(-ArcSinh[a
*x])] - 4*PolyLog[2, -E^(-ArcSinh[a*x])] + 4*PolyLog[2, E^(-ArcSinh[a*x])]
- ArcSinh[a*x]*Sech[ArcSinh[a*x]/2]^2 + 2*Tanh[ArcSinh[a*x]/2]))/8
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.88

method	result
default	$-\frac{a^2x^2 \operatorname{arcsinh}(ax)+ax\sqrt{a^2x^2+1}+\operatorname{arcsinh}(ax)}{2\sqrt{a^2x^2+1}\,x^2}-\frac{a^2 \operatorname{arcsinh}(ax)\ln\left(1-ax-\sqrt{a^2x^2+1}\right)}{2}-\frac{a^2 \operatorname{polylog}\left(2,ax+\sqrt{a^2x^2+1}\right)}{2}+\frac{a^2 \mathrm{a}}{}$

```
[In] int(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2/(a^2*x^2+1)^(1/2)*(a^2*x^2*arcsinh(a*x)+a*x*(a^2*x^2+1)^(1/2)+arcsinh(
a*x))/x^2-1/2*a^2*arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))-1/2*a^2*polylog(
2,a*x+(a^2*x^2+1)^(1/2))+1/2*a^2*arcsinh(a*x)*ln(1+a*x+(a^2*x^2+1)^(1/2))+1
/2*a^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}x^3}\,dx$$

```
[In] integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)/(a^2*x^5 + x^3), x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(ax)}{x^3\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(asinh(a*x)/x**3/(a**2*x**2+1)**(1/2),x)
[Out] Integral(asinh(a*x)/(x**3*sqrt(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}x^3}\,dx$$

```
[In] integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x^3), x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}x^3}\,dx$$

```
[In] integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)}{x^3\,\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int(asinh(a*x)/(x^3*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(asinh(a*x)/(x^3*(a^2*x^2 + 1)^(1/2)), x)
```

3.120 $\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 175

$$\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\frac{2bx\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}}-\frac{bx^3\sqrt{d+c^2dx^2}}{45c\sqrt{1+c^2x^2}}-\frac{bcx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}}-\frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c^4d}+\frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4d^2}$$

```
[Out] -1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/c^4/d+1/5*(c^2*d*x^2+d)^(5/2)*(
a+b*arcsinh(c*x))/c^4/d^2+2/15*b*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2
)-1/45*b*x^3*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/25*b*c*x^5*(c^2*d*x^
2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 45, 5804, 12}

$$\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\frac{(c^2dx^2+d)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4d^2}-\frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))}{3c^4d}-\frac{bcx^5\sqrt{c^2dx^2+d}}{25\sqrt{c^2x^2+1}}-\frac{bx^3\sqrt{c^2dx^2+d}}{45c\sqrt{c^2x^2+1}}+\frac{2bx\sqrt{c^2dx^2+d}}{15c^3\sqrt{c^2x^2+1}}$$

```
[In] Int[x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*b*x*Sqrt[d + c^2*d*x^2])/(15*c^3*Sqrt[1 + c^2*x^2]) - (b*x^3*Sqrt[d + c^
2*d*x^2])/(45*c*Sqrt[1 + c^2*x^2]) - (b*c*x^5*Sqrt[d + c^2*d*x^2])/(25*Sqrt
[1 + c^2*x^2]) - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^4*d) + (
(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^4*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(d+c^2dx^2)^{3/2}\,(a+b\operatorname{arcsinh}(cx))}{3c^4d}+\frac{(d+c^2dx^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))}{5c^4d^2}\\
&\quad-\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int\frac{-2+c^2x^2+3c^4x^4}{15c^4}\,dx}{\sqrt{1+c^2x^2}}\\
&= -\frac{(d+c^2dx^2)^{3/2}\,(a+b\operatorname{arcsinh}(cx))}{3c^4d}+\frac{(d+c^2dx^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))}{5c^4d^2}\\
&\quad-\frac{\left(b\sqrt{d+c^2dx^2}\right)\int\left(-2+c^2x^2+3c^4x^4\right)\,dx}{15c^3\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$= \frac{2bx\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}} - \frac{bx^3\sqrt{d+c^2dx^2}}{45c\sqrt{1+c^2x^2}} - \frac{bcx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}} - \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c^4d} + \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4d^2}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.69

$$\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$$
$$= \frac{\sqrt{d+c^2dx^2}\Big(15a(1+c^2x^2)^2(-2+3c^2x^2)+bcx\sqrt{1+c^2x^2}(30-5c^2x^2-9c^4x^4)+15b(1+c^2x^2)^2(-2+3}{225c^4(1+c^2x^2)}$$

```
[In] Integrate[x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(15*a*(1 + c^2*x^2)^2*(-2 + 3*c^2*x^2) + b*c*x*Sqrt[1
+ c^2*x^2]*(30 - 5*c^2*x^2 - 9*c^4*x^4) + 15*b*(1 + c^2*x^2)^2*(-2 + 3*c^2*
x^2)*ArcSinh[c*x]))/(225*c^4*(1 + c^2*x^2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. 2(149) = 298.

Time = 0.27 (sec) , antiderivative size = 578, normalized size of antiderivative = 3.30

method	result
default	$a\left(\frac{x^2(c^2dx^2+d)^{\frac{3}{2}}}{5c^2d} - \frac{2(c^2dx^2+d)^{\frac{3}{2}}}{15dc^4}\right) + b\Bigg(\frac{\sqrt{d(c^2x^2+1)}\,\big(16c^6x^6+16c^5x^5\sqrt{c^2x^2+1}+28c^4x^4+20c^3x^3\sqrt{c^2x^2+1}+13c^2x^2+5cx}{800c^4(c^2x^2+1)}$
parts	$a\left(\frac{x^2(c^2dx^2+d)^{\frac{3}{2}}}{5c^2d} - \frac{2(c^2dx^2+d)^{\frac{3}{2}}}{15dc^4}\right) + b\Bigg(\frac{\sqrt{d(c^2x^2+1)}\,\big(16c^6x^6+16c^5x^5\sqrt{c^2x^2+1}+28c^4x^4+20c^3x^3\sqrt{c^2x^2+1}+13c^2x^2+5cx}{800c^4(c^2x^2+1)}$

```
[In] int(x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/5*x^2*(c^2*d*x^2+d)^(3/2)/c^2/d-2/15/d/c^4*(c^2*d*x^2+d)^(3/2))+b*(1/8
00*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^
4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+5*
arcsinh(c*x))/c^4/(c^2*x^2+1)-1/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*
x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*
x))/c^4/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/
2)+1)*(-1+arcsinh(c*x))/c^4/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2
-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)+1)/c^4/(c^2*x^2+1)-1/288*(d*(c^2*x^
2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2
+1)^(1/2)+1)*(3*arcsinh(c*x)+1)/c^4/(c^2*x^2+1)+1/800*(d*(c^2*x^2+1))^(1/2)
```

```
*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)
^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*(1+5*arcsinh(c*x))/c^4/(c^2*x^
2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.90

$$\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \frac{15\,(3\,bc^6x^6+4\,bc^4x^4-bc^2x^2-2\,b)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(45\,ac^6x^6+60\,ac^4x^4-15\,ac^2x^2-\left(\right.\right.}{225\,(c^6x^2+c^4)}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
[Out] 1/225*(15*(3*b*c^6*x^6 + 4*b*c^4*x^4 - b*c^2*x^2 - 2*b)*sqrt(c^2*d*x^2 + d)
*log(c*x + sqrt(c^2*x^2 + 1)) + (45*a*c^6*x^6 + 60*a*c^4*x^4 - 15*a*c^2*x^2
 - (9*b*c^5*x^5 + 5*b*c^3*x^3 - 30*b*c*x)*sqrt(c^2*x^2 + 1) - 30*a)*sqrt(c^
2*d*x^2 + d))/(c^6*x^2 + c^4)
```

Sympy [F]

$$\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \int x^{3}\sqrt{d\left(c^{2}x^{2}+1\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)\,dx$$

```
[In] integrate(x**3*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
[Out] Integral(x**3*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.77

$$\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \frac{1}{15}\,b\left(\frac{3\,(c^2dx^2+d)^{\frac{3}{2}}x^2}{c^2d}-\frac{2\,(c^2dx^2+d)^{\frac{3}{2}}}{c^4d}\right)\operatorname{arsinh}\left(cx\right) + \frac{1}{15}\,a\left(\frac{3\,(c^2dx^2+d)^{\frac{3}{2}}x^2}{c^2d}-\frac{2\,(c^2dx^2+d)^{\frac{3}{2}}}{c^4d}\right)-\frac{\left(9\,c^4\sqrt{d}x^5+5\,c^2\sqrt{d}x^3-30\,\sqrt{d}x\right)b}{225\,c^3}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*b*(3*(c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(3/2)/(c^4*
d))*arcsinh(c*x) + 1/15*a*(3*(c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) - 2*(c^2*d*x
^2 + d)^(3/2)/(c^4*d)) - 1/225*(9*c^4*sqrt(d)*x^5 + 5*c^2*sqrt(d)*x^3 - 30*
sqrt(d)*x)*b/c^3
```

Giac [F(-2)]

Exception generated.

$$\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \int x^3\,(a+b\,\text{asinh}(c\,x))\,\sqrt{d\,c^2\,x^2+d}\,dx$$

```
[In] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

3.121 $\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 181

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=-\frac{bx^2\sqrt{d+c^2dx^2}}{16c\sqrt{1+c^2x^2}}-\frac{bcx^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}+\frac{x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8c^2}+\frac{1}{4}x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))-\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{16bc^3\sqrt{1+c^2x^2}}$$

```
[Out] 1/8*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2+1/4*x^3*(a+b*arcsinh(c*x))
*(c^2*d*x^2+d)^(1/2)-1/16*b*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/1
6*b*c*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*(a+b*arcsinh(c*x))^2*(
c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5806, 5812, 5783, 30}

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{8c^2}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{16bc^3\sqrt{c^2x^2+1}}-\frac{bx^2\sqrt{c^2dx^2+d}}{16c\sqrt{c^2x^2+1}}-\frac{bcx^4\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}$$

```
[In] Int[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/16*(b*x^2*Sqrt[d + c^2*d*x^2])/(c*Sqrt[1 + c^2*x^2]) - (b*c*x^4*Sqrt[d +
 c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) + (x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x]))/(8*c^2) + (x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/4 - (Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\ &\quad+\frac{\sqrt{d+c^2dx^2}\int\frac{x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{4\sqrt{1+c^2x^2}}-\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int x^3\,dx}{4\sqrt{1+c^2x^2}}\end{aligned}$$

$$= -\frac{bcx^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} + \frac{x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8c^2}$$
$$+ \frac{1}{4}x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))$$
$$- \frac{\sqrt{d+c^2dx^2}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{8c^2\sqrt{1+c^2x^2}} - \frac{\left(b\sqrt{d+c^2dx^2}\right)\int x\,dx}{8c\sqrt{1+c^2x^2}}$$
$$= -\frac{bx^2\sqrt{d+c^2dx^2}}{16c\sqrt{1+c^2x^2}} - \frac{bcx^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} + \frac{x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8c^2}$$
$$+ \frac{1}{4}x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) - \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{16bc^3\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.71

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx =$$
$$-\frac{-16acx(1+2c^2x^2)\sqrt{d+c^2dx^2} + 16a\sqrt{d}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right) + \frac{b\sqrt{d+c^2dx^2}\left(8\text{arcsinh}(cx)^2+\cosh(4\text{arcs}\right.}{\sqrt{}}}{128c^3}$$

```
[In] Integrate[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/128*(-16*a*c*x*(1 + 2*c^2*x^2)*Sqrt[d + c^2*d*x^2] + 16*a*Sqrt[d]*Log[c*
d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]
^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/Sqrt[1 +
c^2*x^2])/c^3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(155) = 310.

Time = 0.18 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.87

method	result
default	$\frac{ax(c^2dx^2+d)^{\frac{3}{2}}}{4c^2d} - \frac{ax\sqrt{c^2dx^2+d}}{8c^2} - \frac{ad\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{8c^2\sqrt{c^2d}} + b\left(-\frac{\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)^2}{16\sqrt{c^2x^2+1}\,c^3} + \frac{\sqrt{d(c^2x^2+1)}\left(8c^5x^5+8c^4\right.}{}\right.$
parts	$\frac{ax(c^2dx^2+d)^{\frac{3}{2}}}{4c^2d} - \frac{ax\sqrt{c^2dx^2+d}}{8c^2} - \frac{ad\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{8c^2\sqrt{c^2d}} + b\left(-\frac{\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)^2}{16\sqrt{c^2x^2+1}\,c^3} + \frac{\sqrt{d(c^2x^2+1)}\left(8c^5x^5+8c^4\right.}{}\right.$

```
[In] int(x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*x*(c^2*d*x^2+d)^(3/2)/c^2/d-1/8*a/c^2*x*(c^2*d*x^2+d)^(1/2)-1/8*a/c^2
*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b*(-1/16*(d*
(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^2+1/256*(d*(c^2*x^2+1
))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x
^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(c*x))/c^3/(c^2*x^2+1)+1/
256*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3
-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(1+4*arcsinh(c*x))/c^
3/(c^2*x^2+1))
```

Fricas [F]

$$\int x^2\sqrt{d+c^2dx^2}(a+b\mathrm{arcsinh}(cx))\,dx=\int\sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*x^2*arcsinh(c*x) + a*x^2), x)
```

Sympy [F]

$$\int x^2\sqrt{d+c^2dx^2}(a+b\mathrm{arcsinh}(cx))\,dx=\int x^2\sqrt{d\left(c^2x^2+1\right)}(a+b\operatorname{asinh}(cx))\,dx$$

```
[In] integrate(x**2*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
[Out] Integral(x**2*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{d+c^2dx^2}(a+b\mathrm{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \int \sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \int x^2\,(a+b\,\text{asinh}(c\,x))\,\sqrt{d\,c^2\,x^2+d}\,dx$$

```
[In] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
[Out] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

3.122 $\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 105

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = -\frac{bx\sqrt{d+c^2dx^2}}{3c\sqrt{1+c^2x^2}} - \frac{bcx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}} + \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2d}$$

```
[Out] 1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/c^2/d-1/3*b*x*(c^2*d*x^2+d)^(1/2
)/c/(c^2*x^2+1)^(1/2)-1/9*b*c*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {5798}

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2d} - \frac{bx\sqrt{c^2dx^2+d}}{3c\sqrt{c^2x^2+1}} - \frac{bcx^3\sqrt{c^2dx^2+d}}{9\sqrt{c^2x^2+1}}$$

```
[In] Int[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/3*(b*x*Sqrt[d + c^2*d*x^2])/(c*Sqrt[1 + c^2*x^2]) - (b*c*x^3*Sqrt[d + c^
2*d*x^2])/(9*Sqrt[1 + c^2*x^2]) + ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x
]))/(3*c^2*d)
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2d} - \frac{\left(b\sqrt{d+c^2dx^2}\right)\int(1+c^2x^2)\,dx}{3c\sqrt{1+c^2x^2}}\\ &= -\frac{bx\sqrt{d+c^2dx^2}}{3c\sqrt{1+c^2x^2}} - \frac{bcx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}} + \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \frac{\sqrt{d+c^2dx^2}\left(3a(1+c^2x^2)^2 - bcx\sqrt{1+c^2x^2}(3+c^2x^2) + 3b(1+c^2x^2)^2\,\text{arcsinh}(cx)\right)}{9c^2(1+c^2x^2)}$$

```
[In] Integrate[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
[Out] (Sqrt[d + c^2*d*x^2]*(3*a*(1 + c^2*x^2)^2 - b*c*x*Sqrt[1 + c^2*x^2]*(3 + c^
2*x^2) + 3*b*(1 + c^2*x^2)^2*ArcSinh[c*x]))/(9*c^2*(1 + c^2*x^2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(89) = 178.

Time = 0.23 (sec) , antiderivative size = 321, normalized size of antiderivative = 3.06

method	result
default	$\frac{a(c^2dx^2+d)^{\frac{3}{2}}}{3c^2d} + b\left(\frac{\sqrt{d(c^2x^2+1)}\left(4c^4x^4+4c^3x^3\sqrt{c^2x^2+1}+5c^2x^2+3cx\sqrt{c^2x^2+1}+1\right)(-1+3\,\text{arcsinh}(cx))}{72c^2(c^2x^2+1)} + \frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2+\right.}{}\right.$
parts	$\frac{a(c^2dx^2+d)^{\frac{3}{2}}}{3c^2d} + b\left(\frac{\sqrt{d(c^2x^2+1)}\left(4c^4x^4+4c^3x^3\sqrt{c^2x^2+1}+5c^2x^2+3cx\sqrt{c^2x^2+1}+1\right)(-1+3\,\text{arcsinh}(cx))}{72c^2(c^2x^2+1)} + \frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2+\right.}{}\right.$

```
[In] int(x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*(c^2*d*x^2+d)^(3/2)/c^2/d+b*(1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*
c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsin
h(c*x))/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^
(1/2)+1)*(-1+arcsinh(c*x))/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x
^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)+1)/c^2/(c^2*x^2+1)+1/72*(d*(c^2*x
^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^
2+1)^(1/2)+1)*(3*arcsinh(c*x)+1)/c^2/(c^2*x^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.21

$$\int x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\frac{3\left(bc^4x^4+2\,bc^2x^2+b\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(3\,ac^4x^4+6\,ac^2x^2-\left(bc^3x^3+3\,bcx\right)\sqrt{c^2x^2+1}\right.}{9\left(c^4x^2+c^2\right)}$$

```
[In] integrate(x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(3*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2
*x^2 + 1)) + (3*a*c^4*x^4 + 6*a*c^2*x^2 - (b*c^3*x^3 + 3*b*c*x)*sqrt(c^2*x^
2 + 1) + 3*a)*sqrt(c^2*d*x^2 + d))/(c^4*x^2 + c^2)
```

Sympy [F]

$$\int x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx=\int x\sqrt{d\left(c^2x^2+1\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)\,dx$$

```
[In] integrate(x*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.70

$$\int x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx=\frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}b\operatorname{arsinh}\left(cx\right)}{3\,c^2d}-\frac{\left(c^2d^{\frac{3}{2}}x^3+3\,d^{\frac{3}{2}}x\right)b}{9\,cd}+\frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}a}{3\,c^2d}$$

```
[In] integrate(x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(c^2*d*x^2 + d)^(3/2)*b*arcsinh(c*x)/(c^2*d) - 1/9*(c^2*d^(3/2)*x^3 + 3
*d^(3/2)*x)*b/(c*d) + 1/3*(c^2*d*x^2 + d)^(3/2)*a/(c^2*d)
```

Giac [F(-2)]

Exception generated.

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \int x\,(a+b\,\text{asinh}(c\,x))\,\sqrt{d\,c^2\,x^2+d}\,dx$$

```
[In] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

3.123 $\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 111

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=-\frac{bcx^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}}+\frac{1}{2}x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))+\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}}$$

```
[Out] 1/2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-1/4*b*c*x^2*(c^2*d*x^2+d)^(1/2
)/(c^2*x^2+1)^(1/2)+1/4*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x
^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5785, 5783, 30}

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{c^2x^2+1}}-\frac{bcx^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}$$

```
[In] Int[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/4*(b*c*x^2*Sqrt[d + c^2*d*x^2])/Sqrt[1 + c^2*x^2] + (x*Sqrt[d + c^2*d*x^
2]*(a + b*ArcSinh[c*x]))/2 + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(
4*b*c*Sqrt[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) \\ &\quad+\frac{\sqrt{d+c^2dx^2}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}}-\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int x\,dx}{2\sqrt{1+c^2x^2}} \\ &=-\frac{bcx^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}}+\frac{1}{2}x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))+\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.08

$$\int\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\frac{1}{8}\left(4ax\sqrt{d+c^2dx^2}+\frac{4a\sqrt{d}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)}{c}\right.$$
$$\left.+\frac{b\sqrt{d+c^2dx^2}(-\cosh(2\text{arcsinh}(cx))+2\text{arcsinh}(cx)(\text{arcsinh}(cx)+\sinh(2\text{arcsinh}(cx))))}{c\sqrt{1+c^2x^2}}\right)$$

```
[In] Integrate[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (4*a*x*Sqrt[d + c^2*d*x^2] + (4*a*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*
d*x^2]])/c + (b*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*x]
*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/(c*Sqrt[1 + c^2*x^2]))/8
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(95) = 190$.

Time = 0.17 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.31

method	result
default	$\frac{ax\sqrt{c^2d\,x^2+d}}{2}+\frac{ad\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2\sqrt{c^2d}}+b\left(\frac{\sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(cx)^2}{4\sqrt{c^2x^2+1}\,c}+\frac{\sqrt{d(c^2x^2+1)}\,\left(2c^3x^3+2c^2x^2\sqrt{c^2x^2+1}+2cx+\sqrt{}\right.}{16c(c^2x^2+1)}\right.$
parts	$\frac{ax\sqrt{c^2d\,x^2+d}}{2}+\frac{ad\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2\sqrt{c^2d}}+b\left(\frac{\sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(cx)^2}{4\sqrt{c^2x^2+1}\,c}+\frac{\sqrt{d(c^2x^2+1)}\,\left(2c^3x^3+2c^2x^2\sqrt{c^2x^2+1}+2cx+\sqrt{}\right.}{16c(c^2x^2+1)}\right.$

```
[In] int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*a*x*(c^2*d*x^2+d)^(1/2)+1/2*a*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^
(1/2))/(c^2*d)^(1/2)+b*(1/4*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsi
nh(c*x)^2+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)
+2*c*x+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*x))/c/(c^2*x^2+1)+1/16*(d*(c^2*x^
2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))
*(1+2*arcsinh(c*x))/c/(c^2*x^2+1))
```

Fricas [F]

$$\int\sqrt{d+c^2dx^2}(a+b\mathrm{arcsinh}(cx))\,dx=\int\sqrt{c^2dx^2+d}(b\,\mathrm{arsinh}\,(cx)+a)\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a), x)
```

Sympy [F]

$$\int\sqrt{d+c^2dx^2}(a+b\mathrm{arcsinh}(cx))\,dx=\int\sqrt{d\,(c^2x^2+1)}(a+b\,\mathrm{asinh}\,(cx))\,dx$$

```
[In] integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d + c^2dx^2}(a + b\text{arcsinh}(cx))\, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d + c^2dx^2}(a + b\text{arcsinh}(cx))\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d + c^2dx^2}(a + b\text{arcsinh}(cx))\, dx = \int (a + b\,\text{asinh}(c\,x))\, \sqrt{d\,c^2\,x^2 + d}\, dx$$

```
[In] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

3.124 $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 177

$$\begin{aligned}\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x}\,dx &= -\frac{bcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\\ &\quad-\frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}\\ &\quad-\frac{b\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}\\ &\quad+\frac{b\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}\end{aligned}$$

```
[Out] (a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-b*c*x*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1
)^(1/2)-2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(
1/2)/(c^2*x^2+1)^(1/2)-b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1
/2)/(c^2*x^2+1)^(1/2)+b*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2
)/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used

$= \{5806, 5816, 4267, 2317, 2438, 8\}$

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x}\,dx = -\frac{2\sqrt{c^2dx^2+d}\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}} + \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx)) - \frac{b\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{c^2x^2+1}} + \frac{b\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{c^2x^2+1}} - \frac{bcx\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}$$

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] -((b*c*x*Sqrt[d + c^2*d*x^2])/Sqrt[1 + c^2*x^2]) + Sqrt[d + c^2*d*x^2]*(a +
 b*ArcSinh[c*x]) - (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^Ar
cSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b*Sqrt[d + c^2*d*x^2]*PolyLog[2, -E^ArcSi
nh[c*x]])/Sqrt[1 + c^2*x^2] + (b*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^ArcSinh[c
*x]])/Sqrt[1 + c^2*x^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
```

```
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) \\
&\quad+\frac{\sqrt{d+c^2dx^2}\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{\sqrt{1+c^2x^2}}-\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int 1\,dx}{\sqrt{1+c^2x^2}} \\
&= -\frac{bcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) \\
&\quad+\frac{\sqrt{d+c^2dx^2}\text{Subst}\left(\int(a+bx)\text{csch}(x)\,dx,x,\text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}} \\
&= -\frac{bcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) \\
&\quad-\frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad-\frac{\left(b\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\log\left(1-e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}} \\
&\quad+\frac{\left(b\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\log\left(1+e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}} \\
&= -\frac{bcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) \\
&\quad-\frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad-\frac{\left(b\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad+\frac{\left(b\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}
\end{aligned}$$

$$= -\frac{bcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) - \frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{b\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{b\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x}\,dx$$
$$= a\sqrt{d+c^2dx^2} + a\sqrt{d}\log(x) - a\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right) + \frac{b\sqrt{d+c^2dx^2}\left(-cx+\sqrt{1+c^2x^2}\text{arcsinh}(cx)+\text{arcsinh}(cx)\log\left(1-e^{-\text{arcsinh}(cx)}\right)-\text{arcsinh}(cx)\log\left(1+e^{-}\right.\right.}{\sqrt{1+c^2x^2}}$$

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] a*Sqrt[d + c^2*d*x^2] + a*Sqrt[d]*Log[x] - a*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d
 + c^2*d*x^2]] + (b*Sqrt[d + c^2*d*x^2]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh
[c*x] + ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-
ArcSinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*
x])]))/Sqrt[1 + c^2*x^2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.87

method	result
default	$-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)a+a\sqrt{c^2d\,x^2+d}+\frac{b\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)x^2c^2}{c^2x^2+1}-\frac{b\sqrt{d(c^2x^2+1)}\,cx}{\sqrt{c^2x^2+1}}+\frac{b\sqrt{d(c^2x^2+}}{c^2}$
parts	$-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)a+a\sqrt{c^2d\,x^2+d}+\frac{b\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)x^2c^2}{c^2x^2+1}-\frac{b\sqrt{d(c^2x^2+1)}\,cx}{\sqrt{c^2x^2+1}}+\frac{b\sqrt{d(c^2x^2+}}{c^2}$

```
[In] int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)*a+a*(c^2*d*x^2+d)^(1/2)+
b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(c*x)*x^2*c^2-b*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)*c*x+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(c*x
)+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+
1)^(1/2))+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+
1)^(1/2))-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(
```

```
c^2*x^2+1)^(1/2))-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-
(c^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x}\,dx=\int \frac{\sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)}{x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="fricas")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/x, x)
```

Sympy [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x}\,dx=\int \frac{\sqrt{d(c^2x^2+1)}(a+b\operatorname{asinh}(cx))}{x}\,dx$$

```
[In] integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/x,x)
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x}\,dx=\int \frac{\sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)}{x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="maxima")
[Out] -(sqrt(d)*arcsinh(1/(c*abs(x))) - sqrt(c^2*d*x^2 + d))*a + b*integrate(sqrt
(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x}\,dx=\int \frac{\left(a+b\,\text{asinh}(c\,x)\right)\,\sqrt{d\,c^2\,x^2+d}}{x}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x,x)
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x, x)
```

3.125 $\int \frac{\sqrt{d+c^2dx^2}(a+b\,\mathrm{arcsinh}(cx))}{x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 105

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^2}\,dx=-\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x}+\frac{c\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2b\sqrt{1+c^2x^2}}+\frac{bc\sqrt{d+c^2dx^2}\log(x)}{\sqrt{1+c^2x^2}}$$

```
[Out] -(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x+1/2*c*(a+b*arcsinh(c*x))^2*(c^2*d
*x^2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+b*c*ln(x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)
^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5805, 29, 5783}

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^2}\,dx=\frac{c\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2b\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{x}+\frac{bc\log(x)\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}$$

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] -((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x) + (c*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])^2)/(2*b*Sqrt[1 + c^2*x^2]) + (b*c*Sqrt[d + c^2*d*x^2]*Lo
g[x])/Sqrt[1 + c^2*x^2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]], Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x]
, x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], In
t[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Free
Q[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x}+\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int\frac{1}{x}\,dx}{\sqrt{1+c^2x^2}} \\
&\quad+\frac{\left(c^2\sqrt{d+c^2dx^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{1+c^2x^2}} \\
&= -\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x}+\frac{c\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2b\sqrt{1+c^2x^2}}+\frac{bc\sqrt{d+c^2dx^2}\log(x)}{\sqrt{1+c^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.23

$$\begin{aligned}
&\int\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^2}\,dx \\
&= -\frac{a\sqrt{d(1+c^2x^2)}}{x}+\frac{bc\sqrt{d(1+c^2x^2)}\left(-\frac{2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{cx}+\text{arcsinh}(cx)^2+2\log(cx)\right)}{2\sqrt{1+c^2x^2}} \\
&\quad+ac\sqrt{d}\log\left(cdx+\sqrt{d}\sqrt{d(1+c^2x^2)}\right)
\end{aligned}$$

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] -((a*Sqrt[d*(1 + c^2*x^2)])/x) + (b*c*Sqrt[d*(1 + c^2*x^2)]*((-2*Sqrt[1 + c
^2*x^2]*ArcSinh[c*x])/(c*x) + ArcSinh[c*x]^2 + 2*Log[c*x]))/(2*Sqrt[1 + c^2
*x^2]) + a*c*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(93) = 186$.

Time = 0.21 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.39

method	result
default	$-\frac{a(c^2 d\,x^2+d)^{\frac{3}{2}}}{dx}+a\,c^2 x\sqrt{c^2 d\,x^2+d}+\frac{a\,c^2 d\ln\left(\frac{c^2 dx}{\sqrt{c^2 d}}+\sqrt{c^2 d\,x^2+d}\right)}{\sqrt{c^2 d}}+b\left(\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2 c}{2\sqrt{c^2x^2+1}}-\frac{2\sqrt{d(c^2x^2+}}{\sqrt{c}}\right.$
parts	$-\frac{a(c^2 d\,x^2+d)^{\frac{3}{2}}}{dx}+a\,c^2 x\sqrt{c^2 d\,x^2+d}+\frac{a\,c^2 d\ln\left(\frac{c^2 dx}{\sqrt{c^2 d}}+\sqrt{c^2 d\,x^2+d}\right)}{\sqrt{c^2 d}}+b\left(\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2 c}{2\sqrt{c^2x^2+1}}-\frac{2\sqrt{d(c^2x^2+}}{\sqrt{c}}\right.$

```
[In] int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/d/x*(c^2*d*x^2+d)^(3/2)+a*c^2*x*(c^2*d*x^2+d)^(1/2)+a*c^2*d*ln(c^2*d*x/(
c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b*(1/2*(d*(c^2*x^2+1))^(1/2
)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c-2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1
/2)*arcsinh(c*x)*c-(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*
arcsinh(c*x)/x/(c^2*x^2+1)+(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*ln((c*x+
(c^2*x^2+1)^(1/2))^2-1)*c)
```

Fricas [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^2}\,dx=\int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)}{x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{d + c^2 d x^2}(a + b\operatorname{arcsinh}(cx))}{x^2}\,dx = \int \frac{\sqrt{d\left(c^2x^2 + 1\right)}(a + b\operatorname{asinh}(cx))}{x^2}\,dx$$

```
[In] integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/x**2,x)
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{d + c^2 d x^2}(a + b\operatorname{arcsinh}(cx))}{x^2}\,dx = \int \frac{\sqrt{c^2 d x^2 + d}(b\operatorname{arsinh}(cx) + a)}{x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="maxima")
[Out] (c*sqrt(d)*arcsinh(c*x) - sqrt(c^2*d*x^2 + d)/x)*a + b*integrate(sqrt(c^2*d
*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d + c^2 d x^2}(a + b\operatorname{arcsinh}(cx))}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d + c^2 d x^2}(a + b\operatorname{arcsinh}(cx))}{x^2}\,dx = \int \frac{(a + b\operatorname{asinh}(c\,x))\,\sqrt{d\,c^2\,x^2 + d}}{x^2}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x^2,x)
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x^2, x)
```

3.126 $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 201

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^3}\,dx = -\frac{bc\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}} - \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2x^2} - \frac{c^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{bc^2\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2\sqrt{1+c^2x^2}} + \frac{bc^2\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2\sqrt{1+c^2x^2}}$$

```
[Out] -1/2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^2-1/2*b*c*(c^2*d*x^2+d)^(1/2)
/x/(c^2*x^2+1)^(1/2)-c^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*
(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/2*b*c^2*polylog(2,-c*x-(c^2*x^2+1)^
(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/2*b*c^2*polylog(2,c*x+(c^2*x
^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used

$= \{5805, 30, 5816, 4267, 2317, 2438\}$

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{x^3}\,dx = -\frac{c^2\sqrt{c^2dx^2+d}\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2x^2+1}} - \frac{\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{2x^2} - \frac{bc^2\sqrt{c^2dx^2+d}\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)}{2\sqrt{c^2x^2+1}} + \frac{bc^2\sqrt{c^2dx^2+d}\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)}{2\sqrt{c^2x^2+1}} - \frac{bc\sqrt{c^2dx^2+d}}{2x\sqrt{c^2x^2+1}}$$

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*Sqrt[d + c^2*d*x^2])/(x*Sqrt[1 + c^2*x^2]) - (Sqrt[d + c^2*d*x^2]
*(a + b*ArcSinh[c*x]))/(2*x^2) - (c^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b*c^2*Sqrt[d + c^2*d*x^2]
*PolyLog[2, -E^ArcSinh[c*x]])/(2*Sqrt[1 + c^2*x^2]) + (b*c^2*Sqrt[d + c^2*d
*x^2]*PolyLog[2, E^ArcSinh[c*x]])/(2*Sqrt[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]], Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x]
, x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], In
t[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Free
Q[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int\frac{1}{x^2}\,dx}{2\sqrt{1+c^2x^2}} \\
&\quad+\frac{\left(c^2\sqrt{d+c^2dx^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}} \\
&= -\frac{bc\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2x^2} \\
&\quad+\frac{\left(c^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)\text{csch}(x)\,dx,x,\text{arcsinh}(cx)\right)}{2\sqrt{1+c^2x^2}} \\
&= -\frac{bc\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2x^2} \\
&\quad-\frac{c^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad-\frac{\left(bc^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\log\left(1-e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{2\sqrt{1+c^2x^2}} \\
&\quad+\frac{\left(bc^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\log\left(1+e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{2\sqrt{1+c^2x^2}}
\end{aligned}$$

$$= -\frac{bc\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}} - \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2x^2} - \frac{c^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{\left(bc^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{2\sqrt{1+c^2x^2}} + \frac{\left(bc^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{2\sqrt{1+c^2x^2}}$$

$$= -\frac{bc\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}} - \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2x^2} - \frac{c^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{bc^2\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2\sqrt{1+c^2x^2}} + \frac{bc^2\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 2.25 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^3}\,dx$$

$$= \frac{1}{8}\left(-\frac{4a\sqrt{d+c^2dx^2}}{x^2} + 4ac^2\sqrt{d}\log(x) - 4ac^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right) + \frac{bc^2\sqrt{d+c^2dx^2}\left(-2\coth\left(\frac{1}{2}\text{arcsinh}(cx)\right) - \text{arcsinh}(cx)\text{csch}^2\left(\frac{1}{2}\text{arcsinh}(cx)\right) + 4\text{arcsinh}(cx)\log\left(1-e^{-\text{arcs}}\right.\right.}{}\right.$$

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] ((-4*a*Sqrt[d + c^2*d*x^2])/x^2 + 4*a*c^2*Sqrt[d]*Log[x] - 4*a*c^2*Sqrt[d]*
Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*c^2*Sqrt[d + c^2*d*x^2]*(-2*Coth[
ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]*Log[
1 - E^(-ArcSinh[c*x])] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 4*Poly
Log[2, -E^(-ArcSinh[c*x])] - 4*PolyLog[2, E^(-ArcSinh[c*x])] - ArcSinh[c*x]
*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/Sqrt[1 + c^2*x^2])/8
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.66

method	result
default	$a\left(-\frac{\left(c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{2 d\,x^{2}}+\frac{c^{2} \left(\sqrt{c^{2} d\,x^{2}+d}-\sqrt{d}\,\ln \left(\frac{2 d+2 \sqrt{d}\,\sqrt{c^{2} d\,x^{2}+d}}{x}\right)\right)}{2}\right)+b\left(-\frac{\left(\operatorname{arcsinh}(cx) c^{2} x^{2}+c x \sqrt{c^{2} x^{2}+1}+\operatorname{arcsinh}(cx)\right)}{2 x^{2} \left(c^{2} x^{2}+1\right)}\right.$
parts	$a\left(-\frac{\left(c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{2 d\,x^{2}}+\frac{c^{2} \left(\sqrt{c^{2} d\,x^{2}+d}-\sqrt{d}\,\ln \left(\frac{2 d+2 \sqrt{d}\,\sqrt{c^{2} d\,x^{2}+d}}{x}\right)\right)}{2}\right)+b\left(-\frac{\left(\operatorname{arcsinh}(cx) c^{2} x^{2}+c x \sqrt{c^{2} x^{2}+1}+\operatorname{arcsinh}(cx)\right)}{2 x^{2} \left(c^{2} x^{2}+1\right)}\right.$

```
[In] int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/2/d/x^2*(c^2*d*x^2+d)^(3/2)+1/2*c^2*((c^2*d*x^2+d)^(1/2)-d^(1/2)*ln((
2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)))+b*(-1/2*(arcsinh(c*x)*c^2*x^2+c*x*(
c^2*x^2+1)^(1/2)+arcsinh(c*x))*(d*(c^2*x^2+1))^(1/2)/x^2/(c^2*x^2+1)-1/2*(d
*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/
2))*c^2-1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2
+1)^(1/2))*c^2+1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(
1-c*x-(c^2*x^2+1)^(1/2))*c^2+1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*po
lylog(2,c*x+(c^2*x^2+1)^(1/2))*c^2)
```

Fricas [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{x^3}\,dx=\int \frac{\sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)}{x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/x^3, x)
```

Sympy [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{x^3}\,dx=\int \frac{\sqrt{d(c^2x^2+1)}(a+b\operatorname{asinh}(cx))}{x^3}\,dx$$

```
[In] integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{d + c^2 d x^2}(a + b \text{arcsinh}(cx))}{x^3} \, dx = \int \frac{\sqrt{c^2 d x^2 + d}(b \,\text{arsinh}\,(cx) + a)}{x^3} \, dx$$

[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="maxima")

[Out] -1/2*(c^2*sqrt(d)*arcsinh(1/(c*abs(x))) - sqrt(c^2*d*x^2 + d)*c^2 + (c^2*d*x^2 + d)^(3/2)/(d*x^2))*a + b*integrate(sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))/x^3, x)

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d + c^2 d x^2}(a + b \text{arcsinh}(cx))}{x^3} \, dx = \text{Exception raised: TypeError}$$

[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d + c^2 d x^2}(a + b \text{arcsinh}(cx))}{x^3} \, dx = \int \frac{(a + b \,\text{asinh}(c\,x)) \, \sqrt{d\,c^2\,x^2 + d}}{x^3} \, dx$$

[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x^3,x)

[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x^3, x)

3.127 $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 106

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^4}\,dx = -\frac{bc\sqrt{d+c^2dx^2}}{6x^2\sqrt{1+c^2x^2}} - \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{3dx^3} + \frac{bc^3\sqrt{d+c^2dx^2}\log(x)}{3\sqrt{1+c^2x^2}}$$

```
[Out] -1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/d/x^3-1/6*b*c*(c^2*d*x^2+d)^(1/
2)/x^2/(c^2*x^2+1)^(1/2)+1/3*b*c^3*ln(x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1
/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5800, 14}

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^4}\,dx = -\frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))}{3dx^3} - \frac{bc\sqrt{c^2dx^2+d}}{6x^2\sqrt{c^2x^2+1}} + \frac{bc^3\log(x)\sqrt{c^2dx^2+d}}{3\sqrt{c^2x^2+1}}$$

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*Sqrt[d + c^2*d*x^2])/(x^2*Sqrt[1 + c^2*x^2]) - ((d + c^2*d*x^2)^(
3/2)*(a + b*ArcSinh[c*x]))/(3*d*x^3) + (b*c^3*Sqrt[d + c^2*d*x^2]*Log[x])/(
3*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{3dx^3}+\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int\frac{1+c^2x^2}{x^3}\,dx}{3\sqrt{1+c^2x^2}}\\ &= -\frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{3dx^3}+\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int\left(\frac{1}{x^3}+\frac{c^2}{x}\right)dx}{3\sqrt{1+c^2x^2}}\\ &= -\frac{bc\sqrt{d+c^2dx^2}}{6x^2\sqrt{1+c^2x^2}}-\frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{3dx^3}+\frac{bc^3\sqrt{d+c^2dx^2}\log(x)}{3\sqrt{1+c^2x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.10

$$\int\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^4}\,dx = -\frac{\sqrt{d+c^2dx^2}\left(bcx+3bc^3x^3+2a\sqrt{1+c^2x^2}+2ac^2x^2\sqrt{1+c^2x^2}+2b(1+c^2x^2)^{3/2}\text{arcsinh}(cx)-2bc^3x^3\log\right.}{6x^3\sqrt{1+c^2x^2}}$$

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -1/6*(Sqrt[d + c^2*d*x^2]*(b*c*x + 3*b*c^3*x^3 + 2*a*Sqrt[1 + c^2*x^2] + 2*
a*c^2*x^2*Sqrt[1 + c^2*x^2] + 2*b*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x] - 2*b*c^
3*x^3*Log[x]))/(x^3*Sqrt[1 + c^2*x^2])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.29

method	result
default	$-\frac{a(c^2 d x^2+d)^{\frac{3}{2}}}{3d x^3}-\frac{b\sqrt{d(c^2x^2+1)}\left(2\,\text{arcsinh}(cx)c^3x^3-2\ln\left(\left(cx+\sqrt{c^2x^2+1}\right)^2-1\right)x^3c^3+2\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^2c^2+2\,\text{arcsinh}(}{6\sqrt{c^2x^2+1}\,x^3}$
parts	$-\frac{a(c^2 d x^2+d)^{\frac{3}{2}}}{3d x^3}-\frac{b\sqrt{d(c^2x^2+1)}\left(2\,\text{arcsinh}(cx)c^3x^3-2\ln\left(\left(cx+\sqrt{c^2x^2+1}\right)^2-1\right)x^3c^3+2\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^2c^2+2\,\text{arcsinh}(}{6\sqrt{c^2x^2+1}\,x^3}$

```
[In] int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/d/x^3*(c^2*d*x^2+d)^(3/2)-1/6*b*(d*(c^2*x^2+1))^(1/2)*(2*arcsinh(c*x
)*c^3*x^3-2*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*x^3*c^3+2*arcsinh(c*x)*(c^2*x^2
+1)^(1/2)*x^2*c^2+2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+c*x)/(c^2*x^2+1)^(1/2)/x
^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(90) = 180.

Time = 0.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^4}\,dx =$$
$$-\frac{2\left(bc^4x^4+2\,bc^2x^2+b\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(bc^5x^5+bc^3x^3\right)\sqrt{d}\log\left(\frac{c^2dx^6+c^2dx^2+dx^4+\sqrt{c^2dx}}{c^2x^4}\right.}{6\left(c^2x^5+x^3\right)}$$

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^
2*x^2 + 1)) - (b*c^5*x^5 + b*c^3*x^3)*sqrt(d)*log((c^2*d*x^6 + c^2*d*x^2 +
d*x^4 + sqrt(c^2*d*x^2 + d)*sqrt(c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) + d)/(c^2*x
^4 + x^2)) + (2*a*c^4*x^4 + 4*a*c^2*x^2 - (b*c*x^3 - b*c*x)*sqrt(c^2*x^2 +
1) + 2*a)*sqrt(c^2*d*x^2 + d))/(c^2*x^5 + x^3)
```

Sympy [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{x^4}\,dx = \int \frac{\sqrt{d\left(c^2x^2+1\right)}(a+b\operatorname{asinh}(cx))}{x^4}\,dx$$

```
[In] integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/x**4,x)
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{x^4}\,dx = -\frac{\left((-1)^{2c^2dx^2+2d}c^2d^{\frac{3}{2}}\log\left(2c^2d+\frac{2d}{x^2}\right)-c^2d^{\frac{3}{2}}\log\left(x^2+\frac{1}{c^2}\right)+\frac{\sqrt{c^4dx^4+2c^2dx^2+d}d}{x^2}\right)bc}{6d} - \frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}b\operatorname{arsinh}(cx)}{3dx^3} - \frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}a}{3dx^3}$$

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="maxima")
[Out] -1/6*((-1)^(2*c^2*d*x^2 + 2*d)*c^2*d^(3/2)*log(2*c^2*d + 2*d/x^2) - c^2*d^(
3/2)*log(x^2 + 1/c^2) + sqrt(c^4*d*x^4 + 2*c^2*d*x^2 + d)*d/x^2)*b*c/d - 1/
3*(c^2*d*x^2 + d)^(3/2)*b*arcsinh(c*x)/(d*x^3) - 1/3*(c^2*d*x^2 + d)^(3/2)*
a/(d*x^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{x^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^4}\,dx=\int \frac{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{d\,c^2\,x^2+d}}{x^4}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x^4,x)
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x^4, x)
```

3.128 $\int x^3(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 217

$$\int x^3\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\frac{2bdx\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}}-\frac{bdx^3\sqrt{d+c^2dx^2}}{105c\sqrt{1+c^2x^2}}-\frac{8bcdx^5\sqrt{d+c^2dx^2}}{175\sqrt{1+c^2x^2}}-\frac{bc^3dx^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}}-\frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4d}+\frac{\left(d+c^2dx^2\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4d^2}$$

```
[Out] -1/5*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/c^4/d+1/7*(c^2*d*x^2+d)^(7/2)*(
a+b*arcsinh(c*x))/c^4/d^2+2/35*b*d*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1
/2)-1/105*b*d*x^3*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-8/175*b*c*d*x^5*(
c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/49*b*c^3*d*x^7*(c^2*d*x^2+d)^(1/2)/(
c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {272, 45, 5804, 12, 380}

$$\int x^3\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\frac{\left(c^2dx^2+d\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4d^2}-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4d}-\frac{8bcdx^5\sqrt{c^2dx^2+d}}{175\sqrt{c^2x^2+1}}-\frac{bdx^3\sqrt{c^2dx^2+d}}{105c\sqrt{c^2x^2+1}}+\frac{2bdx\sqrt{c^2dx^2+d}}{35c^3\sqrt{c^2x^2+1}}-\frac{bc^3dx^7\sqrt{c^2dx^2+d}}{49\sqrt{c^2x^2+1}}$$

```
[In] Int[x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*b*d*x*Sqrt[d + c^2*d*x^2])/(35*c^3*Sqrt[1 + c^2*x^2]) - (b*d*x^3*Sqrt[d
+ c^2*d*x^2])/(105*c*Sqrt[1 + c^2*x^2]) - (8*b*c*d*x^5*Sqrt[d + c^2*d*x^2])
/(175*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt[1 + c
^2*x^2]) - ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^4*d) + ((d + c
^2*d*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^4*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\text{integral} = -\frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4d}+\frac{\left(d+c^2dx^2\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4d^2} - \frac{\left(bc\sqrt{d+c^2dx^2}\right)\int \frac{d(1+c^2x^2)^2(-2+5c^2x^2)}{35c^4}\,dx}{\sqrt{1+c^2x^2}}$$

$$= -\frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4d} + \frac{(d+c^2dx^2)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4d^2} - \frac{\left(bd\sqrt{d+c^2dx^2}\right)\int(1+c^2x^2)^2(-2+5c^2x^2)\,dx}{35c^3\sqrt{1+c^2x^2}}$$

$$= -\frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4d} + \frac{(d+c^2dx^2)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4d^2} - \frac{\left(bd\sqrt{d+c^2dx^2}\right)\int(-2+c^2x^2+8c^4x^4+5c^6x^6)\,dx}{35c^3\sqrt{1+c^2x^2}}$$

$$= \frac{2bdx\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}} - \frac{bdx^3\sqrt{d+c^2dx^2}}{105c\sqrt{1+c^2x^2}} - \frac{8bcdx^5\sqrt{d+c^2dx^2}}{175\sqrt{1+c^2x^2}} - \frac{bc^3dx^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}} - \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4d} + \frac{(d+c^2dx^2)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4d^2}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.60

$$\int x^3(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \frac{d\sqrt{d+c^2dx^2}\left(105a(1+c^2x^2)^3(-2+5c^2x^2) - bcx\sqrt{1+c^2x^2}(-210+35c^2x^2+168c^4x^4 \right.}{3675c^4(1+c^2x^2)}$$

```
[In] Integrate[x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*Sqrt[d + c^2*d*x^2]*(105*a*(1 + c^2*x^2)^3*(-2 + 5*c^2*x^2) - b*c*x*Sqrt
[1 + c^2*x^2]*(-210 + 35*c^2*x^2 + 168*c^4*x^4 + 75*c^6*x^6) + 105*b*(1 + c
^2*x^2)^3*(-2 + 5*c^2*x^2)*ArcSinh[c*x]))/(3675*c^4*(1 + c^2*x^2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 871 vs. 2(185) = 370.

Time = 0.22 (sec) , antiderivative size = 872, normalized size of antiderivative = 4.02

method	result
default	$a\left(\frac{x^2(c^2dx^2+d)^{\frac{5}{2}}}{7c^2d} - \frac{2(c^2dx^2+d)^{\frac{5}{2}}}{35dc^4}\right) + b\left(\frac{\sqrt{d(c^2x^2+1)}\left(64c^8x^8+64c^7x^7\sqrt{c^2x^2+1}+144c^6x^6+112c^5x^5\sqrt{c^2x^2+1}+104c^4x^4+56\right.}{6272c^4(c^2x^2+1)}\right.$
parts	$a\left(\frac{x^2(c^2dx^2+d)^{\frac{5}{2}}}{7c^2d} - \frac{2(c^2dx^2+d)^{\frac{5}{2}}}{35dc^4}\right) + b\left(\frac{\sqrt{d(c^2x^2+1)}\left(64c^8x^8+64c^7x^7\sqrt{c^2x^2+1}+144c^6x^6+112c^5x^5\sqrt{c^2x^2+1}+104c^4x^4+56\right.}{6272c^4(c^2x^2+1)}\right.$

```
[In] int(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/7*x^2*(c^2*d*x^2+d)^(5/2)/c^2/d-2/35/d/c^4*(c^2*d*x^2+d)^(5/2))+b*(1/6
272*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*
x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1/2)+
25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+7*arcsinh(c*x))*d/c^4/(c^2*x^2+1)
+1/3200*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c
^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(
-1+5*arcsinh(c*x))*d/c^4/(c^2*x^2+1)-1/384*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4
+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arc
sinh(c*x))*d/c^4/(c^2*x^2+1)-3/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*
x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))*d/c^4/(c^2*x^2+1)-3/128*(d*(c^2*x^2+1))^(
1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)+1)*d/c^4/(c^2*x^2+1)-1
/384*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2
-3*c*x*(c^2*x^2+1)^(1/2)+1)*(3*arcsinh(c*x)+1)*d/c^4/(c^2*x^2+1)+1/3200*(d*
(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c
^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*(1+5*arcsinh
(c*x))*d/c^4/(c^2*x^2+1)+1/6272*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8-64*c^7*x^
7*(c^2*x^2+1)^(1/2)+144*c^6*x^6-112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4-5
6*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2-7*c*x*(c^2*x^2+1)^(1/2)+1)*(1+7*arcs
inh(c*x))*d/c^4/(c^2*x^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.92

$$\int x^3\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx=\frac{105\left(5\,bc^8dx^8+13\,bc^6dx^6+9\,bc^4dx^4-bc^2dx^2-2\,bd\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)}{}$$

```
[In] integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3675*(105*(5*b*c^8*d*x^8 + 13*b*c^6*d*x^6 + 9*b*c^4*d*x^4 - b*c^2*d*x^2 -
 2*b*d)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (525*a*c^8*d*x^8
 + 1365*a*c^6*d*x^6 + 945*a*c^4*d*x^4 - 105*a*c^2*d*x^2 - 210*a*d - (75*b*c
^7*d*x^7 + 168*b*c^5*d*x^5 + 35*b*c^3*d*x^3 - 210*b*c*d*x)*sqrt(c^2*x^2 + 1
))*sqrt(c^2*d*x^2 + d))/(c^6*x^2 + c^4)
```

Sympy [F]

$$\int x^3 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \int x^3 \left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}} (a+b\operatorname{asinh}(cx))\,dx$$

```
[In] integrate(x**3*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
[Out] Integral(x**3*(d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.67

$$\int x^3 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \frac{1}{35}\left(\frac{5\left(c^2dx^2+d\right)^{\frac{5}{2}}x^2}{c^2d} - \frac{2\left(c^2dx^2+d\right)^{\frac{5}{2}}}{c^4d}\right)b\operatorname{arsinh}(cx) + \frac{1}{35}\left(\frac{5\left(c^2dx^2+d\right)^{\frac{5}{2}}x^2}{c^2d} - \frac{2\left(c^2dx^2+d\right)^{\frac{5}{2}}}{c^4d}\right)a - \frac{\left(75\,c^6d^{\frac{3}{2}}x^7 + 168\,c^4d^{\frac{3}{2}}x^5 + 35\,c^2d^{\frac{3}{2}}x^3 - 210\,d^{\frac{3}{2}}x\right)b}{3675\,c^3}$$

```
[In] integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] 1/35*(5*(c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(5/2)/(c^4*d)
)*b*arcsinh(c*x) + 1/35*(5*(c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) - 2*(c^2*d*x^2
 + d)^(5/2)/(c^4*d))*a - 1/3675*(75*c^6*d^(3/2)*x^7 + 168*c^4*d^(3/2)*x^5 +
 35*c^2*d^(3/2)*x^3 - 210*d^(3/2)*x)*b/c^3
```

Giac [F(-2)]

Exception generated.

$$\int x^3 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx=\int x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^{3/2}\,dx$$

```
[In] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
[Out] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

3.129 $\int x^2(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 254

$$\int x^2\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=-\frac{bdx^2\sqrt{d+c^2dx^2}}{32c\sqrt{1+c^2x^2}}-\frac{7bcdx^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{bc^3dx^6\sqrt{d+c^2dx^2}}{36\sqrt{1+c^2x^2}}+\frac{dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{16c^2}+\frac{1}{8}dx^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))+\frac{1}{6}x^3\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{32bc^3\sqrt{1+c^2x^2}}$$

```
[Out] 1/6*x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+1/16*d*x*(a+b*arcsinh(c*x))*
(c^2*d*x^2+d)^(1/2)/c^2+1/8*d*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-1/
32*b*d*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-7/96*b*c*d*x^4*(c^2*d*x^
2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/36*b*c^3*d*x^6*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+
1)^(1/2)-1/32*d*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^
(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used

$= \{5808, 5806, 5812, 5783, 30, 14\}$

$$\int x^2\left(d+c^2dx^2\right)^{3/2}\left(a+b\text{arcsinh}(cx)\right)dx=\frac{dx\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{16c^2}$$
$$+\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}\left(a+b\text{arcsinh}(cx)\right)+\frac{1}{8}dx^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))$$
$$-\frac{d\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{32bc^3\sqrt{c^2x^2+1}}-\frac{bdx^2\sqrt{c^2dx^2+d}}{32c\sqrt{c^2x^2+1}}$$
$$-\frac{7bcdx^4\sqrt{c^2dx^2+d}}{96\sqrt{c^2x^2+1}}-\frac{bc^3dx^6\sqrt{c^2dx^2+d}}{36\sqrt{c^2x^2+1}}$$

```
[In] Int[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/32*(b*d*x^2*Sqrt[d + c^2*d*x^2])/(c*Sqrt[1 + c^2*x^2]) - (7*b*c*d*x^4*Sq
rt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^6*Sqrt[d + c^2*d*x^2
])/(36*Sqrt[1 + c^2*x^2]) + (d*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/
(16*c^2) + (d*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (x^3*(d + c
^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/6 - (d*Sqrt[d + c^2*d*x^2]*(a + b*Arc
Sinh[c*x])^2)/(32*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
```

```
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{6}x^3\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx)) + \frac{1}{2}d\int x^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx - \frac{\left(bcd\sqrt{d+c^2dx^2}\right)\int x^3\left(1+c^2x^2\right)\,dx}{6\sqrt{1+c^2x^2}}$$

$$= \frac{1}{8}dx^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{1}{6}x^3\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx)) + \frac{\left(d\sqrt{d+c^2dx^2}\right)\int\frac{x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{8\sqrt{1+c^2x^2}} - \frac{\left(bcd\sqrt{d+c^2dx^2}\right)\int x^3\,dx}{8\sqrt{1+c^2x^2}} - \frac{\left(bcd\sqrt{d+c^2dx^2}\right)\int\left(x^3+c^2x^5\right)\,dx}{6\sqrt{1+c^2x^2}}$$

$$= -\frac{7bcdx^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{bc^3dx^6\sqrt{d+c^2dx^2}}{36\sqrt{1+c^2x^2}} + \frac{dx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{16c^2} + \frac{1}{8}dx^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{1}{6}x^3\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx)) - \frac{\left(d\sqrt{d+c^2dx^2}\right)\int\frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{16c^2\sqrt{1+c^2x^2}} - \frac{\left(bd\sqrt{d+c^2dx^2}\right)\int x\,dx}{16c\sqrt{1+c^2x^2}}$$

$$
\begin{aligned}
&= -\frac{bdx^2\sqrt{d+c^2dx^2}}{32c\sqrt{1+c^2x^2}} - \frac{7bcdx^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{bc^3dx^6\sqrt{d+c^2dx^2}}{36\sqrt{1+c^2x^2}} \\
&\quad + \frac{dx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{16c^2} + \frac{1}{8}dx^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{1}{6}x^3\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx)) - \frac{d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{32bc^3\sqrt{1+c^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.99

$$
\int x^2\left(d+c^2dx^2\right)^{3/2}(a +b\operatorname{arcsinh}(cx))\,dx = \frac{48acdx\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(3+14c^2x^2+8c^4x^4) - 144ad^{3/2}\sqrt{1+c^2x^2}\log\left(cdx+\sqrt{d}\right.}{}
$$

```
[In] Integrate[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (48*a*c*d*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(3 + 14*c^2*x^2 + 8*c^4*x
^4) - 144*a*d^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^
2]] - 18*b*d*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] -
 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]) + b*d*Sqrt[d + c^2*d*x^2]*(72*ArcSinh
[c*x]^2 + 18*Cosh[2*ArcSinh[c*x]] + 9*Cosh[4*ArcSinh[c*x]] - 2*Cosh[6*ArcSi
nh[c*x]] + 12*ArcSinh[c*x]*(-3*Sinh[2*ArcSinh[c*x]] - 3*Sinh[4*ArcSinh[c*x]
] + Sinh[6*ArcSinh[c*x]])))/(2304*c^3*Sqrt[1 + c^2*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 798 vs. 2(218) = 436.

Time = 0.22 (sec) , antiderivative size = 799, normalized size of antiderivative = 3.15

method	result
default	$\frac{ax(c^2dx^2+d)^{\frac{5}{2}}}{6c^2d} - \frac{ax(c^2dx^2+d)^{\frac{3}{2}}}{24c^2} - \frac{adx\sqrt{c^2dx^2+d}}{16c^2} - \frac{ad^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{16c^2\sqrt{c^2d}} + b\left(-\frac{\sqrt{d(c^2x^2+1)}\operatorname{arcsinh}(cx)^2d}{32\sqrt{c^2x^2+1}\,c^3} + \right.$
parts	$\frac{ax(c^2dx^2+d)^{\frac{5}{2}}}{6c^2d} - \frac{ax(c^2dx^2+d)^{\frac{3}{2}}}{24c^2} - \frac{adx\sqrt{c^2dx^2+d}}{16c^2} - \frac{ad^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{16c^2\sqrt{c^2d}} + b\left(-\frac{\sqrt{d(c^2x^2+1)}\operatorname{arcsinh}(cx)^2d}{32\sqrt{c^2x^2+1}\,c^3} + \right.$

```
[In] int(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a*x*(c^2*d*x^2+d)^(5/2)/c^2/d-1/24*a/c^2*x*(c^2*d*x^2+d)^(3/2)-1/16*a/c
^2*d*x*(c^2*d*x^2+d)^(1/2)-1/16*a/c^2*d^2*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x
^2+d)^(1/2))/(c^2*d)^(1/2)+b*(-1/32*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)
```

```
/c^3*arcsinh(c*x)^2*d+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7+32*c^6*x^6*(
c^2*x^2+1)^(1/2)+64*c^5*x^5+48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3+18*c^2*
x^2*(c^2*x^2+1)^(1/2)+6*c*x+(c^2*x^2+1)^(1/2))*(-1+6*arcsinh(c*x))*d/c^3/(c
^2*x^2+1)+1/512*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2
)+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(-1+4*arc
sinh(c*x))*d/c^3/(c^2*x^2+1)-1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x
^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*x))*d/c^3/(c^
2*x^2+1)-1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)
+2*c*x-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(c*x))*d/c^3/(c^2*x^2+1)+1/512*(d*(c^
2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3-8*c^2*x^2
*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(1+4*arcsinh(c*x))*d/c^3/(c^2*x
^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7-32*c^6*x^6*(c^2*x^2+1)^(1/2)
+64*c^5*x^5-48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3-18*c^2*x^2*(c^2*x^2+1)^
(1/2)+6*c*x-(c^2*x^2+1)^(1/2))*(1+6*arcsinh(c*x))*d/c^3/(c^2*x^2+1))
```

Fricas [F]

$$\int x^2 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \int \left(c^2dx^2+d\right)^{\frac{3}{2}} (b\,\text{arsinh}\,(cx)+a)x^2\,dx$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((a*c^2*d*x^4 + a*d*x^2 + (b*c^2*d*x^4 + b*d*x^2)*arcsinh(c*x))*sqr
t(c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int x^2 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \int x^2 \left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}} (a+b\,\text{asinh}\,(cx))\,\,dx$$

```
[In] integrate(x**2*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
[Out] Integral(x**2*(d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^2 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int x^2 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \int \left(c^2dx^2+d\right)^{\frac{3}{2}} (b\,\text{arsinh}\,(cx)+a)x^2\,dx$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \int x^2\,(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

```
[In] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
[Out] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

3.130 $\int x(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 146

$$\int x\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=-\frac{bdx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}}-\frac{2bcdx^3\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}}-\frac{bc^3dx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}}+\frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^2d}$$

```
[Out] 1/5*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/c^2/d-1/5*b*d*x*(c^2*d*x^2+d)^(1
/2)/c/(c^2*x^2+1)^(1/2)-2/15*b*c*d*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2
)-1/25*b*c^3*d*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5798, 200}

$$\int x\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^2d}-\frac{bdx\sqrt{c^2dx^2+d}}{5c\sqrt{c^2x^2+1}}-\frac{2bcdx^3\sqrt{c^2dx^2+d}}{15\sqrt{c^2x^2+1}}-\frac{bc^3dx^5\sqrt{c^2dx^2+d}}{25\sqrt{c^2x^2+1}}$$

```
[In] Int[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/5*(b*d*x*Sqrt[d + c^2*d*x^2])/(c*Sqrt[1 + c^2*x^2]) - (2*b*c*d*x^3*Sqrt[
d + c^2*d*x^2])/(15*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^5*Sqrt[d + c^2*d*x^2])/
(25*Sqrt[1 + c^2*x^2]) + ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^
2*d)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{5c^2d}-\frac{\left(bd\sqrt{d+c^2dx^2}\right)\int(1+c^2x^2)^2\,dx}{5c\sqrt{1+c^2x^2}}\\ &= \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{5c^2d}-\frac{\left(bd\sqrt{d+c^2dx^2}\right)\int(1+2c^2x^2+c^4x^4)\,dx}{5c\sqrt{1+c^2x^2}}\\ &= -\frac{bdx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}}-\frac{2bcdx^3\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}}\\ &\quad-\frac{bc^3dx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}}+\frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{5c^2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.70

$$\int x(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\frac{d\sqrt{d+c^2dx^2}\left(15a(1+c^2x^2)^3-bcx\sqrt{1+c^2x^2}(15+10c^2x^2+3c^4x^4)+15b(1+c^2x^2)^3\,\text{a}\right.}{75c^2(1+c^2x^2)}$$

```
[In] Integrate[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*Sqrt[d + c^2*d*x^2]*(15*a*(1 + c^2*x^2)^3 - b*c*x*Sqrt[1 + c^2*x^2]*(15
+ 10*c^2*x^2 + 3*c^4*x^4) + 15*b*(1 + c^2*x^2)^3*ArcSinh[c*x]))/(75*c^2*(1
+ c^2*x^2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. 2(124) = 248.

Time = 0.20 (sec) , antiderivative size = 559, normalized size of antiderivative = 3.83

method	result
default	$\frac{a(c^2dx^2+d)^{\frac{5}{2}}}{5c^2d}+b\left(\frac{\sqrt{d(c^2x^2+1)}\left(16c^6x^6+16c^5x^5\sqrt{c^2x^2+1}+28c^4x^4+20c^3x^3\sqrt{c^2x^2+1}+13c^2x^2+5cx\sqrt{c^2x^2+1}+1\right)(-1+5\operatorname{arcsinh}}{800c^2(c^2x^2+1)}\right.$
parts	$\frac{a(c^2dx^2+d)^{\frac{5}{2}}}{5c^2d}+b\left(\frac{\sqrt{d(c^2x^2+1)}\left(16c^6x^6+16c^5x^5\sqrt{c^2x^2+1}+28c^4x^4+20c^3x^3\sqrt{c^2x^2+1}+13c^2x^2+5cx\sqrt{c^2x^2+1}+1\right)(-1+5\operatorname{arcsinh}}{800c^2(c^2x^2+1)}\right.$

```
[In] int(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a*(c^2*d*x^2+d)^(5/2)/c^2/d+b*(1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+
16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2
*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/96*
(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*
x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/16*(d*(c^2*x
^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))*d/c^2/(c^2
*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsi
nh(c*x)+1)*d/c^2/(c^2*x^2+1)+1/96*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^
3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(3*arcsinh(c*x)+1)
*d/c^2/(c^2*x^2+1)+1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*
x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*
x^2+1)^(1/2)+1)*(1+5*arcsinh(c*x))*d/c^2/(c^2*x^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.14

$$\int x\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx=\frac{15\,(bc^6dx^6+3\,bc^4dx^4+3\,bc^2dx^2+bd)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(15\,ac^6dx^6+\cdots\right.}{75\,(c^4x\cdots}$$

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/75*(15*(b*c^6*d*x^6 + 3*b*c^4*d*x^4 + 3*b*c^2*d*x^2 + b*d)*sqrt(c^2*d*x^2
 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (15*a*c^6*d*x^6 + 45*a*c^4*d*x^4 + 45*
a*c^2*d*x^2 + 15*a*d - (3*b*c^5*d*x^5 + 10*b*c^3*d*x^3 + 15*b*c*d*x)*sqrt(c
^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^4*x^2 + c^2)
```

Sympy [F]

$$\int x\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\int x\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}(a+b\operatorname{asinh}(cx))\,dx$$

```
[In] integrate(x*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
[Out] Integral(x*(d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.58

$$\int x\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}b\,\text{arsinh}\,(cx)}{5\,c^2d}+\frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}a}{5\,c^2d}-\frac{\left(3\,c^4d^{\frac{5}{2}}x^5+10\,c^2d^{\frac{5}{2}}x^3+15\,d^{\frac{5}{2}}x\right)b}{75\,cd}$$

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] 1/5*(c^2*d*x^2 + d)^(5/2)*b*arcsinh(c*x)/(c^2*d) + 1/5*(c^2*d*x^2 + d)^(5/2
)*a/(c^2*d) - 1/75*(3*c^4*d^(5/2)*x^5 + 10*c^2*d^(5/2)*x^3 + 15*d^(5/2)*x)*
b/(c*d)
```

Giac [F(-2)]

Exception generated.

$$\int x\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\int x\,(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

```
[In] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
[Out] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

3.131 $\int (d+c^2dx^2)^{3/2} (a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 180

$$\int \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = -\frac{5bcdx^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{bc^3dx^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} + \frac{3}{8}dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{1}{4}x\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{3d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{16bc\sqrt{1+c^2x^2}}$$

```
[Out] 1/4*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+3/8*d*x*(a+b*arcsinh(c*x))*(c^
2*d*x^2+d)^(1/2)-5/16*b*c*d*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*
b*c^3*d*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+3/16*d*(a+b*arcsinh(c*x))
^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5786, 5785, 5783, 30, 14}

$$\int \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{3}{8}dx\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx)) + \frac{3d\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{16bc\sqrt{c^2x^2+1}} - \frac{5bcdx^2\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}} - \frac{bc^3dx^4\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}$$

```
[In] Int[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-5*b*c*d*x^2*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^4*Sq
rt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) + (3*d*x*Sqrt[d + c^2*d*x^2]*(a +
 b*ArcSinh[c*x]))/8 + (x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/4 + (3
*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{4}x\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))+\frac{1}{4}(3d)\int\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx-\frac{\left(bcd\sqrt{d+c^2dx^2}\right)\int x\left(1+c^2x^2\right)\,dx}{4\sqrt{1+c^2x^2}}$$

$$= \frac{3}{8}dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{1}{4}x\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{\left(3d\sqrt{d+c^2dx^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{8\sqrt{1+c^2x^2}} - \frac{\left(bcd\sqrt{d+c^2dx^2}\right)\int\left(x+c^2x^3\right)\,dx}{4\sqrt{1+c^2x^2}} - \frac{\left(3bcd\sqrt{d+c^2dx^2}\right)\int x\,dx}{8\sqrt{1+c^2x^2}}$$

$$= -\frac{5bcdx^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{bc^3dx^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} + \frac{3}{8}dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{1}{4}x\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{3d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{16bc\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.11

$$\int\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \frac{1}{8}adx\left(5+2c^2x^2\right)\sqrt{d+c^2dx^2} + \frac{3ad^{3/2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)}{8c} + \frac{bd\sqrt{d+c^2dx^2}(-\cosh(2\text{arcsinh}(cx))+2\text{arcsinh}(cx)(\text{arcsinh}(cx)+\sinh(2\text{arcsinh}(cx))))}{8c\sqrt{1+c^2x^2}} - \frac{bd\sqrt{d+c^2dx^2}(8\text{arcsinh}(cx)^2+\cosh(4\text{arcsinh}(cx))-4\text{arcsinh}(cx)\sinh(4\text{arcsinh}(cx)))}{128c\sqrt{1+c^2x^2}}$$

```
[In] Integrate[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (a*d*x*(5 + 2*c^2*x^2)*Sqrt[d + c^2*d*x^2])/8 + (3*a*d^(3/2)*Log[c*d*x + Sq
rt[d]*Sqrt[d + c^2*d*x^2]])/(8*c) + (b*d*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSi
nh[c*x]] + 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/(8*c*Sqrt
[1 + c^2*x^2]) - (b*d*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSin
h[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/(128*c*Sqrt[1 + c^2*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(154) = 308$.

Time = 0.18 (sec) , antiderivative size = 496, normalized size of antiderivative = 2.76

method	result
default	$\frac{x\left(c^{2} d \,x^{2}+d \right)^{\frac{3}{2}} a}{4}+\frac{3 a d x \sqrt{c^{2} d \,x^{2}+d}}{8}+\frac{3 a \,d^{2} \ln \left(\frac{c^{2} d x}{\sqrt{c^{2} d}}+\sqrt{c^{2} d \,x^{2}+d}\right)}{8 \sqrt{c^{2} d}}+b \left(\frac{3 \sqrt{d \left(c^{2} x^{2}+1\right)}\, \operatorname{arcsinh}\left(c x \right)^{2} d}{16 \sqrt{c^{2} x^{2}+1}\, c}+\frac{\sqrt{d \left(c^{2} x^{2}+1\right)}\, \left(8 c^{5} x^{5}\right.}{}\right.$
parts	$\frac{x\left(c^{2} d \,x^{2}+d \right)^{\frac{3}{2}} a}{4}+\frac{3 a d x \sqrt{c^{2} d \,x^{2}+d}}{8}+\frac{3 a \,d^{2} \ln \left(\frac{c^{2} d x}{\sqrt{c^{2} d}}+\sqrt{c^{2} d \,x^{2}+d}\right)}{8 \sqrt{c^{2} d}}+b \left(\frac{3 \sqrt{d \left(c^{2} x^{2}+1\right)}\, \operatorname{arcsinh}\left(c x \right)^{2} d}{16 \sqrt{c^{2} x^{2}+1}\, c}+\frac{\sqrt{d \left(c^{2} x^{2}+1\right)}\, \left(8 c^{5} x^{5}\right.}{}\right.$

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x*(c^2*d*x^2+d)^(3/2)*a+3/8*a*d*x*(c^2*d*x^2+d)^(1/2)+3/8*a*d^2*ln(c^2*
d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b*(3/16*(d*(c^2*x^2+1)
)^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(c*x)^2*d+1/256*(d*(c^2*x^2+1))^(1/2)*(8
*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)
+4*c*x+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(c*x))*d/c/(c^2*x^2+1)+1/16*(d*(c^2*
x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2
))*(-1+2*arcsinh(c*x))*d/c/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^
3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(c*x))*d
/c/(c^2*x^2+1)+1/256*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)
^(1/2)+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(1+4
*arcsinh(c*x))*d/c/(c^2*x^2+1))
```

Fricas [F]

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\int \left(c^{2} d x^{2} + d\right)^{\frac{3}{2}} {\left(b \operatorname{arsinh}\left(c x\right) + a\right)}\,{d x}$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d*
x^2 + d), x)
```

Sympy [F]

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\int \left(d \left(c^{2} x^{2} + 1\right)\right)^{\frac{3}{2}} \left(a + b \operatorname{asinh}{\left(c x \right)}\right)\, dx$$

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx=\int (a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

```
[In] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

3.132 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 249

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x}\,dx = -\frac{4bcdx\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}} - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}} + d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{1}{3}(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{bd\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{bd\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}$$

```
[Out] 1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+d*(a+b*arcsinh(c*x))*(c^2*d*x^2+
d)^(1/2)-4/3*b*c*d*x*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/9*b*c^3*d*x^3*
(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*d*(a+b*arcsinh(c*x))*arctanh(c*x+(c
^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b*d*polylog(2,-c*x-(
c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+b*d*polylog(2,c*x+(
c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used

$= \{5808, 5806, 5816, 4267, 2317, 2438, 8\}$

$$\int \frac{(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{x}\,dx = -\frac{2d\sqrt{c^2dx^2+d}\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}} + \frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx)) + d\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx)) - \frac{bd\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{c^2x^2+1}} + \frac{bd\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{c^2x^2+1}} - \frac{4bcdx\sqrt{c^2dx^2+d}}{3\sqrt{c^2x^2+1}} - \frac{bc^3dx^3\sqrt{c^2dx^2+d}}{9\sqrt{c^2x^2+1}}$$

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (-4*b*c*d*x*Sqrt[d + c^2*d*x^2])/(3*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^3*Sqrt[
d + c^2*d*x^2])/(9*Sqrt[1 + c^2*x^2]) + d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
h[c*x]) + ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/3 - (2*d*Sqrt[d + c^
2*d*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] -
(b*d*Sqrt[d + c^2*d*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (
b*d*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))+d\int\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x}\,dx \\
&\quad -\frac{\left(bcd\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)\,dx}{3\sqrt{1+c^2x^2}} \\
&= -\frac{bcdx\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}-\frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) \\
&\quad +\frac{1}{3}\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{\left(d\sqrt{d+c^2dx^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{\sqrt{1+c^2x^2}} \\
&\quad -\frac{\left(bcd\sqrt{d+c^2dx^2}\right)\int 1\,dx}{\sqrt{1+c^2x^2}} \\
&= -\frac{4bcdx\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}-\frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))+\frac{1}{3}\left(d+c^2dx^2\right)^{3/2}(a \\
&\quad +b\text{arcsinh}(cx))+\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)\text{csch}(x)\,dx,x,\text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}}
\end{aligned}$$

$$= -\frac{4bcdx\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}} - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}} + d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{1}{3}\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx)) - \frac{2d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{\left(bd\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\log\left(1-e^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}} + \frac{\left(bd\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\log\left(1+e^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}}$$

$$= -\frac{4bcdx\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}} - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}} + d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{1}{3}\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx)) - \frac{2d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{\left(bd\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{\left(bd\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}$$

$$= -\frac{4bcdx\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}} - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}} + d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{1}{3}\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx)) - \frac{2d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{bd\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{bd\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00

$$\int\frac{\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))}{x}\,dx = \frac{1}{3}ad\left(4+c^2x^2\right)\sqrt{d+c^2dx^2} + \frac{bd\sqrt{d+c^2dx^2}\left(-cx(3+c^2x^2)+3(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)\right)}{9\sqrt{1+c^2x^2}} + ad^{3/2}\log(x) - ad^{3/2}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right) + \frac{bd\sqrt{d+c^2dx^2}\left(-cx+\sqrt{1+c^2x^2}\operatorname{arcsinh}(cx)+\operatorname{arcsinh}(cx)\operatorname{l}\right.}{}$$

[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x,x]

[Out] (a*d*(4 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/3 + (b*d*Sqrt[d + c^2*d*x^2]*(-(c*x*(3 + c^2*x^2)) + 3*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]))/(9*Sqrt[1 + c^2*x^2]) + a*d^(3/2)*Log[x] - a*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*d*Sqrt[d + c^2*d*x^2]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2]

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.72

method	result
default	$\frac{(c^2 d\,x^2+d)^{\frac{3}{2}} a}{3} - a\,d^{\frac{3}{2}} \ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2 d\,x^2+d}}{x}\right) + ad\sqrt{c^2 d\,x^2+d} + \frac{b\sqrt{d(c^2x^2+1)}\ \mathrm{polylog}\left(2,cx+\sqrt{c^2x^2+1}\right)d}{\sqrt{c^2x^2+1}} + \frac{4b\sqrt{d\cdots}}{\cdots}$
parts	$\frac{(c^2 d\,x^2+d)^{\frac{3}{2}} a}{3} - a\,d^{\frac{3}{2}} \ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2 d\,x^2+d}}{x}\right) + ad\sqrt{c^2 d\,x^2+d} + \frac{b\sqrt{d(c^2x^2+1)}\ \mathrm{polylog}\left(2,cx+\sqrt{c^2x^2+1}\right)d}{\sqrt{c^2x^2+1}} + \frac{4b\sqrt{d\cdots}}{\cdots}$

[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x,x,method=_RETURNVERBOSE)

[Out] 1/3*(c^2*d*x^2+d)^(3/2)*a-a*d^(3/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)+a*d*(c^2*d*x^2+d)^(1/2)+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*d+4/3*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*d-4/3*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)^(1/2)*c*x-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*d+1/3*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)*x^4*c^4-1/9*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)^(1/2)*c^3*x^3+5/3*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)*x^2*c^2+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*d

Fricas [F]

$$\int \frac{(d+c^2dx^2)^{3/2}\,(a+b\mathrm{arcsinh}(cx))}{x}\,dx = \int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx)+a)}{x}\,dx$$

[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")

[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x, x)

Sympy [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x}\,dx = \int \frac{(d(c^2x^2+1))^{\frac{3}{2}}(a+b\,\text{asinh}\,(cx))}{x}\,dx$$

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))/x,x)
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))/x, x)
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x}\,dx = \int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{x}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
[Out] -1/3*(3*d^(3/2)*arcsinh(1/(c*abs(x))) - (c^2*d*x^2 + d)^(3/2) - 3*sqrt(c^2*
d*x^2 + d)*d)*a + b*integrate((c^2*d*x^2 + d)^(3/2)*log(c*x + sqrt(c^2*x^2
+ 1))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,(d\,c^2\,x^2+d)^{3/2}}{x}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x,x)
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x, x)
```

3.133 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 177

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx = -\frac{bc^3dx^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}} + \frac{3}{2}c^2dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) - \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x} + \frac{3cd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{4b\sqrt{1+c^2x^2}} + \frac{bcd\sqrt{d+c^2dx^2}\log(x)}{\sqrt{1+c^2x^2}}$$

```
[Out] -(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x+3/2*c^2*d*x*(a+b*arcsinh(c*x))*(c
^2*d*x^2+d)^(1/2)-1/4*b*c^3*d*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+3/4
*c*d*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+b*c*d*ln(
x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5807, 5785, 5783, 30, 14}

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx = \frac{3}{2}c^2dx\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx)) + \frac{3cd\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{4b\sqrt{c^2x^2+1}} - \frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))}{x} + \frac{bcd\log(x)\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}} - \frac{bc^3dx^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}$$

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] -1/4*(b*c^3*d*x^2*Sqrt[d + c^2*d*x^2])/Sqrt[1 + c^2*x^2] + (3*c^2*d*x*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/2 - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcS
inh[c*x]))/x + (3*c*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*Sqrt
[1 + c^2*x^2]) + (b*c*d*Sqrt[d + c^2*d*x^2]*Log[x])/Sqrt[1 + c^2*x^2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))}{x} \\
&\quad + \left(3c^2d\right)\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx + \frac{\left(bcd\sqrt{d+c^2dx^2}\right)\int \frac{1+c^2x^2}{x}\,dx}{\sqrt{1+c^2x^2}} \\
&= \frac{3}{2}c^2dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) - \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))}{x} \\
&\quad + \frac{\left(bcd\sqrt{d+c^2dx^2}\right)\int \left(\frac{1}{x}+c^2x\right)\,dx}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(3c^2d\sqrt{d+c^2dx^2}\right)\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}} - \frac{\left(3bc^3d\sqrt{d+c^2dx^2}\right)\int x\,dx}{2\sqrt{1+c^2x^2}} \\
&= -\frac{bc^3dx^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}} + \frac{3}{2}c^2dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) \\
&\quad - \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))}{x} \\
&\quad + \frac{3cd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{4b\sqrt{1+c^2x^2}} + \frac{bcd\sqrt{d+c^2dx^2}\log(x)}{\sqrt{1+c^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.13

$$
\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx = \frac{1}{8}\left(\frac{4ad(-2+c^2x^2)\sqrt{d+c^2dx^2}}{x}\right.
$$

$$
+\frac{4bd\sqrt{d+c^2dx^2}\left(-2\sqrt{1+c^2x^2}\text{arcsinh}(cx)+cx\text{arcsinh}(cx)^2+2cx\log(cx)\right)}{x\sqrt{1+c^2x^2}}
$$

$$
+12acd^{3/2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)+\frac{bcd\sqrt{d+c^2dx^2}(-\cosh(2\text{arcsinh}(cx))+2\text{arcsinh}(cx)(\text{arcsinh}(cx)+\text{si}}{\sqrt{1+c^2x^2}}
$$

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] ((4*a*d*(-2 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/x + (4*b*d*Sqrt[d + c^2*d*x^2]*
(-2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + c*x*ArcSinh[c*x]^2 + 2*c*x*Log[c*x]))/
(x*Sqrt[1 + c^2*x^2]) + 12*a*c*d^(3/2)*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x
^2]] + (b*c*d*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*x]*(
ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/Sqrt[1 + c^2*x^2])/8
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.30

method	result
default	$-\frac{a(c^2 d\,x^2+d)^{\frac{5}{2}}}{dx}+a\,c^2 x(c^2 d\,x^2+d)^{\frac{3}{2}}+\frac{3\sqrt{c^2 d\,x^2+d}\,a\,c^2 dx}{2}+\frac{3a\,c^2 d^2 \ln\left(\frac{c^2 dx}{\sqrt{c^2 d}}+\sqrt{c^2 d\,x^2+d}\right)}{2\sqrt{c^2 d}}+\frac{b\sqrt{d(c^2x^2+1)}\,\left(4\,\mathrm{arc}\right.}{}$
parts	$-\frac{a(c^2 d\,x^2+d)^{\frac{5}{2}}}{dx}+a\,c^2 x(c^2 d\,x^2+d)^{\frac{3}{2}}+\frac{3\sqrt{c^2 d\,x^2+d}\,a\,c^2 dx}{2}+\frac{3a\,c^2 d^2 \ln\left(\frac{c^2 dx}{\sqrt{c^2 d}}+\sqrt{c^2 d\,x^2+d}\right)}{2\sqrt{c^2 d}}+\frac{b\sqrt{d(c^2x^2+1)}\,\left(4\,\mathrm{arc}\right.}{}$

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/d/x*(c^2*d*x^2+d)^(5/2)+a*c^2*x*(c^2*d*x^2+d)^(3/2)+3/2*(c^2*d*x^2+d)^(1
/2)*a*c^2*d*x+3/2*a*c^2*d^2*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(
c^2*d)^(1/2)+1/8*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/x*(4*arcsinh(c*x
)*(c^2*x^2+1)^(1/2)*x^2*c^2-2*c^3*x^3+6*arcsinh(c*x)^2*x*c-8*arcsinh(c*x)*c
*x+8*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*x*c-8*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-c
*x)*d
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)^{3/2}\,(a+b\mathrm{arcsinh}(cx))}{x^2}\,dx=\int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx)+a)}{x^2}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d*
x^2 + d)/x^2, x)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^{3/2}\,(a+b\mathrm{arcsinh}(cx))}{x^2}\,dx=\int \frac{(d(c^2x^2+1))^{\frac{3}{2}}\,(a+b\,\mathrm{asinh}\,(cx))}{x^2}\,dx$$

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))/x**2,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{x^2}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{3/2}}{x^2}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x^2, x)
```

3.134 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 270

$$\begin{aligned}\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx &= -\frac{bcd\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{bc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}\\
&+\frac{3}{2}c^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))-\frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}\\
&-\frac{3c^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}\\
&-\frac{3bc^2d\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2\sqrt{1+c^2x^2}}\\
&+\frac{3bc^2d\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2\sqrt{1+c^2x^2}}\end{aligned}$$

```
[Out] -1/2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^2+3/2*c^2*d*(a+b*arcsinh(c*x)
)*(c^2*d*x^2+d)^(1/2)-1/2*b*c*d*(c^2*d*x^2+d)^(1/2)/x/(c^2*x^2+1)^(1/2)-b*c
^3*d*x*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3*c^2*d*(a+b*arcsinh(c*x))*arc
tanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3/2*b*c^2
*d*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+
3/2*b*c^2*d*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1
)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5807, 5806, 5816, 4267, 2317, 2438, 8, 14}

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx = -\frac{3c^2d\sqrt{c^2dx^2+d}\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}} + \frac{3}{2}c^2d\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx)) - \frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2} - \frac{3bc^2d\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2\sqrt{c^2x^2+1}} + \frac{3bc^2d\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2\sqrt{c^2x^2+1}} - \frac{bcd\sqrt{c^2dx^2+d}}{2x\sqrt{c^2x^2+1}} - \frac{bc^3dx\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}$$

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*d*Sqrt[d + c^2*d*x^2])/(x*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x*Sqrt[d
+ c^2*d*x^2])/Sqrt[1 + c^2*x^2] + (3*c^2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSi
nh[c*x]))/2 - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(2*x^2) - (3*c^2
*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1
 + c^2*x^2] - (3*b*c^2*d*Sqrt[d + c^2*d*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/(
2*Sqrt[1 + c^2*x^2]) + (3*b*c^2*d*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^ArcSinh[
c*x]])/(2*Sqrt[1 + c^2*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2} \\ &\quad + \frac{1}{2}(3c^2d)\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x}\,dx + \frac{\left(bcd\sqrt{d+c^2dx^2}\right)\int \frac{1+c^2x^2}{x^2}\,dx}{2\sqrt{1+c^2x^2}}\end{aligned}$$

$$= \frac{3}{2}c^2d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) - \frac{(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{2x^2} + \frac{\left(bcd\sqrt{d+c^2dx^2}\right)\int\left(c^2+\frac{1}{x^2}\right)dx}{2\sqrt{1+c^2x^2}} + \frac{\left(3c^2d\sqrt{d+c^2dx^2}\right)\int\frac{a+b\operatorname{arcsinh}(cx)}{x\sqrt{1+c^2x^2}}dx}{2\sqrt{1+c^2x^2}} - \frac{\left(3bc^3d\sqrt{d+c^2dx^2}\right)\int 1\,dx}{2\sqrt{1+c^2x^2}}$$

$$= -\frac{bcd\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}} - \frac{bc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{3}{2}c^2d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) - \frac{(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{2x^2} + \frac{\left(3c^2d\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int(a+bx)\operatorname{csch}(x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{2\sqrt{1+c^2x^2}}$$

$$= -\frac{bcd\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}} - \frac{bc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{3}{2}c^2d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) - \frac{(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{2x^2} - \frac{3c^2d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{\left(3bc^2d\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\log(1-e^x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{2\sqrt{1+c^2x^2}} + \frac{\left(3bc^2d\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\log(1+e^x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{2\sqrt{1+c^2x^2}}$$

$$= -\frac{bcd\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}} - \frac{bc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{3}{2}c^2d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) - \frac{(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{2x^2} - \frac{3c^2d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{\left(3bc^2d\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{2\sqrt{1+c^2x^2}} + \frac{\left(3bc^2d\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{2\sqrt{1+c^2x^2}}$$

$$= -\frac{bcd\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}} - \frac{bc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{3}{2}c^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) - \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2} - \frac{3c^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{3bc^2d\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2\sqrt{1+c^2x^2}} + \frac{3bc^2d\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 3.47 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.30

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx = a\left(c^2d-\frac{d}{2x^2}\right)\sqrt{d+c^2dx^2} + \frac{3}{2}ac^2d^{3/2}\log(x) - \frac{3}{2}ac^2d^{3/2}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right) + \frac{bc^2d\sqrt{d+c^2dx^2}\left(-cx+\sqrt{1+c^2x^2}\text{arcsinh}(cx)+\text{arcsinh}(cx)\log\left(1-\right.\right.}{}$$

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] a*(c^2*d - d/(2*x^2))*Sqrt[d + c^2*d*x^2] + (3*a*c^2*d^(3/2)*Log[x])/2 - (3
*a*c^2*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/2 + (b*c^2*d*Sqrt[d +
c^2*d*x^2]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1 -
E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2, -
E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2] + (b
*c^2*d*Sqrt[d + c^2*d*x^2]*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[Arc
Sinh[c*x]/2]^2 + 4*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 4*ArcSinh[c*x]
*Log[1 + E^(-ArcSinh[c*x])] + 4*PolyLog[2, -E^(-ArcSinh[c*x])] - 4*PolyLog[
2, E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSin
h[c*x]/2]))/(8*Sqrt[1 + c^2*x^2])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.07

method	result
default	$a\left(-\frac{\left(c^{2} d\,x^{2}+d\right)^{\frac{5}{2}}}{2 d\,x^{2}}+\frac{3 c^{2}\left(\frac{\left(c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{3}+d\left(\sqrt{c^{2} d\,x^{2}+d}-\sqrt{d}\,\ln\left(\frac{2 d+2 \sqrt{d}\,\sqrt{c^{2} d\,x^{2}+d}}{x}\right)\right)\right)}{2}\right)+\frac{b\sqrt{d\left(c^{2} x^{2}+1\right)}\,\left(2\,\mathrm{arcsinh}\left(c x\right)\sqrt{c^{2}\cdots}\right.}{\cdots}$
parts	$a\left(-\frac{\left(c^{2} d\,x^{2}+d\right)^{\frac{5}{2}}}{2 d\,x^{2}}+\frac{3 c^{2}\left(\frac{\left(c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{3}+d\left(\sqrt{c^{2} d\,x^{2}+d}-\sqrt{d}\,\ln\left(\frac{2 d+2 \sqrt{d}\,\sqrt{c^{2} d\,x^{2}+d}}{x}\right)\right)\right)}{2}\right)+\frac{b\sqrt{d\left(c^{2} x^{2}+1\right)}\,\left(2\,\mathrm{arcsinh}\left(c x\right)\sqrt{c^{2}\cdots}\right.}{\cdots}$

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/2/d/x^2*(c^2*d*x^2+d)^(5/2)+3/2*c^2*(1/3*(c^2*d*x^2+d)^(3/2)+d*((c^2*
d*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x))))+1/2*b*(
d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/x^2*(2*arcsinh(c*x)*(c^2*x^2+1)^(1/2
)*x^2*c^2+3*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*x^2*c^2-3*arcsinh(c*x)
*ln(1+c*x+(c^2*x^2+1)^(1/2))*x^2*c^2-2*c^3*x^3+3*polylog(2,c*x+(c^2*x^2+1)^
(1/2))*x^2*c^2-3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*x^2*c^2-arcsinh(c*x)*(c^
2*x^2+1)^(1/2)-c*x)*d
```

Fricas [F]

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\mathrm{arcsinh}(cx))}{x^3}\,dx=\int \frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)}{x^3}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d*
x^2 + d)/x^3, x)
```

Sympy [F]

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\mathrm{arcsinh}(cx))}{x^3}\,dx=\int \frac{\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}\left(a+b\,\mathrm{asinh}\left(cx\right)\right)}{x^3}\,dx$$

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))/x**3,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))/x**3, x)
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx = \int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{x^3}\,dx$$

[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima")

[Out] -1/2*(3*c^2*d^(3/2)*arcsinh(1/(c*abs(x))) - (c^2*d*x^2 + d)^(3/2)*c^2 - 3*sqrt(c^2*d*x^2 + d)*c^2*d + (c^2*d*x^2 + d)^(5/2)/(d*x^2))*a + b*integrate((c^2*d*x^2 + d)^(3/2)*log(c*x + sqrt(c^2*x^2 + 1))/x^3, x)

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx = \text{Exception raised: TypeError}$$

[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{3/2}}{x^3}\,dx$$

[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x^3,x)

[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x^3, x)

3.135 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 184

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx = -\frac{bcd\sqrt{d+c^2dx^2}}{6x^2\sqrt{1+c^2x^2}} - \frac{c^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x} - \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{3x^3} + \frac{c^3d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2b\sqrt{1+c^2x^2}} + \frac{4bc^3d\sqrt{d+c^2dx^2}\log(x)}{3\sqrt{1+c^2x^2}}$$

```
[Out] -1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^3-c^2*d*(a+b*arcsinh(c*x))*(c
^2*d*x^2+d)^(1/2)/x-1/6*b*c*d*(c^2*d*x^2+d)^(1/2)/x^2/(c^2*x^2+1)^(1/2)+1/2
*c^3*d*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+4/3*b*c
^3*d*ln(x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5807, 5805, 29, 5783, 14}

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx = -\frac{c^2d\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{x} - \frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))}{3x^3} + \frac{c^3d\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2b\sqrt{c^2x^2+1}} - \frac{bcd\sqrt{c^2dx^2+d}}{6x^2\sqrt{c^2x^2+1}} + \frac{4bc^3d\log(x)\sqrt{c^2dx^2+d}}{3\sqrt{c^2x^2+1}}$$

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*d*Sqrt[d + c^2*d*x^2])/(x^2*Sqrt[1 + c^2*x^2]) - (c^2*d*Sqrt[d +
c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[
c*x]))/(3*x^3) + (c^3*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*Sq
rt[1 + c^2*x^2]) + (4*b*c^3*d*Sqrt[d + c^2*d*x^2]*Log[x])/(3*Sqrt[1 + c^2*x
^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]], Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x]
, x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], In
t[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Free
Q[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{3x^3} + (c^2d)\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^2}\,dx + \frac{\left(bcd\sqrt{d+c^2dx^2}\right)\int \frac{1+c^2x^2}{x^3}\,dx}{3\sqrt{1+c^2x^2}}$$

$$= -\frac{c^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x} - \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{3x^3} + \frac{\left(bcd\sqrt{d+c^2dx^2}\right)\int\left(\frac{1}{x^3}+\frac{c^2}{x}\right)dx}{3\sqrt{1+c^2x^2}} + \frac{\left(bc^3d\sqrt{d+c^2dx^2}\right)\int\frac{1}{x}\,dx}{\sqrt{1+c^2x^2}} + \frac{\left(c^4d\sqrt{d+c^2dx^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{1+c^2x^2}}$$

$$= -\frac{bcd\sqrt{d+c^2dx^2}}{6x^2\sqrt{1+c^2x^2}} - \frac{c^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x} - \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{3x^3} + \frac{c^3d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2b\sqrt{1+c^2x^2}} + \frac{4bc^3d\sqrt{d+c^2dx^2}\log(x)}{3\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.18

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx = \frac{1}{6}\left(-\frac{2ad(1+4c^2x^2)\sqrt{d+c^2dx^2}}{x^3} - \frac{2b(d+c^2dx^2)^{3/2}\text{arcsinh}(cx)}{x^3} + \frac{3bc^3d\sqrt{d+c^2dx^2}\left(-\frac{2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{cx}+\text{arcsinh}(cx)^2+2\log(cx)\right)}{\sqrt{1+c^2x^2}} + \frac{bcd\sqrt{d+c^2dx^2}(-1+2c^2x^2\log(cx))}{x^2\sqrt{1+c^2x^2}} + 6ac^3d^{3/2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)\right)$$

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] ((-2*a*d*(1 + 4*c^2*x^2)*Sqrt[d + c^2*d*x^2])/x^3 - (2*b*(d + c^2*d*x^2)^(3
/2)*ArcSinh[c*x])/x^3 + (3*b*c^3*d*Sqrt[d + c^2*d*x^2]*((-2*Sqrt[1 + c^2*x^
2]*ArcSinh[c*x])/(c*x) + ArcSinh[c*x]^2 + 2*Log[c*x]))/Sqrt[1 + c^2*x^2] +
(b*c*d*Sqrt[d + c^2*d*x^2]*(-1 + 2*c^2*x^2*Log[c*x]))/(x^2*Sqrt[1 + c^2*x^2
]) + 6*a*c^3*d^(3/2)*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/6
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.40

method	result
default	$-\frac{a\left(c^{2} d\,x^{2}+d\right)^{\frac{5}{2}}}{3 d\,x^{3}}-\frac{2 a\,c^{2} \left(c^{2} d\,x^{2}+d\right)^{\frac{5}{2}}}{3 d x}+\frac{2 a\,c^{4} x \left(c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{3}+a\,c^{4} d x \sqrt{c^{2} d\,x^{2}+d}+\frac{a\,c^{4} d^{2} \ln\left(\frac{c^{2} d x}{\sqrt{c^{2} d}}+\sqrt{c^{2} d\,x^{2}+d}\right)}{\sqrt{c^{2} d}}+$
parts	$-\frac{a\left(c^{2} d\,x^{2}+d\right)^{\frac{5}{2}}}{3 d\,x^{3}}-\frac{2 a\,c^{2} \left(c^{2} d\,x^{2}+d\right)^{\frac{5}{2}}}{3 d x}+\frac{2 a\,c^{4} x \left(c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{3}+a\,c^{4} d x \sqrt{c^{2} d\,x^{2}+d}+\frac{a\,c^{4} d^{2} \ln\left(\frac{c^{2} d x}{\sqrt{c^{2} d}}+\sqrt{c^{2} d\,x^{2}+d}\right)}{\sqrt{c^{2} d}}+$

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/d/x^3*(c^2*d*x^2+d)^(5/2)-2/3*a*c^2/d/x*(c^2*d*x^2+d)^(5/2)+2/3*a*c^
4*x*(c^2*d*x^2+d)^(3/2)+a*c^4*d*x*(c^2*d*x^2+d)^(1/2)+a*c^4*d^2*ln(c^2*d*x/
(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/6*b*(d*(c^2*x^2+1))^(1/2
)/(c^2*x^2+1)^(1/2)/x^3*(3*arcsinh(c*x)^2*x^3*c^3-8*arcsinh(c*x)*c^3*x^3+8*
ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*x^3*c^3-8*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^
2*c^2-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-c*x)*d
```

Fricas [F]

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx=\int \frac{\left(c^{2} d x^{2}+d\right)^{\frac{3}{2}}\left(b \operatorname{arsinh}\left(c x\right)+a\right)}{x^{4}}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d*
x^2 + d)/x^4, x)
```

Sympy [F]

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx=\int \frac{\left(d\left(c^{2} x^{2}+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{asinh}\left(c x\right)\right)}{x^{4}}\,dx$$

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))/x**4,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))/x**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{3/2}\,(a+b\operatorname{arcsinh}(cx))}{x^4}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{3/2}\,(a+b\operatorname{arcsinh}(cx))}{x^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{3/2}\,(a+b\operatorname{arcsinh}(cx))}{x^4}\,dx = \int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{3/2}}{x^4}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x^4, x)
```

3.136 $\int x^3(d+c^2dx^2)^{5/2}(a+b\,\mathrm{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 266

$$\int x^3\left(d+c^2dx^2\right)^{5/2}(a+b\,\mathrm{arcsinh}(cx))\,dx=\frac{2bd^2x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}}-\frac{bd^2x^3\sqrt{d+c^2dx^2}}{189c\sqrt{1+c^2x^2}}-\frac{bcd^2x^5\sqrt{d+c^2dx^2}}{21\sqrt{1+c^2x^2}}-\frac{19bc^3d^2x^7\sqrt{d+c^2dx^2}}{441\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^9\sqrt{d+c^2dx^2}}{81\sqrt{1+c^2x^2}}-\frac{\left(d+c^2dx^2\right)^{7/2}(a+b\,\mathrm{arcsinh}(cx))}{7c^4d}+\frac{\left(d+c^2dx^2\right)^{9/2}(a+b\,\mathrm{arcsinh}(cx))}{9c^4d^2}$$

```
[Out] -1/7*(c^2*d*x^2+d)^(7/2)*(a+b*arcsinh(c*x))/c^4/d+1/9*(c^2*d*x^2+d)^(9/2)*(
a+b*arcsinh(c*x))/c^4/d^2+2/63*b*d^2*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^
(1/2)-1/189*b*d^2*x^3*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/21*b*c*d^2*
x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-19/441*b*c^3*d^2*x^7*(c^2*d*x^2+d
)^(1/2)/(c^2*x^2+1)^(1/2)-1/81*b*c^5*d^2*x^9*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1
)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {272, 45, 5804, 12, 380}

$$\int x^3\left(d+c^2dx^2\right)^{5/2}(a+b\,\mathrm{arcsinh}(cx))\,dx=\frac{\left(c^2dx^2+d\right)^{9/2}(a+b\,\mathrm{arcsinh}(cx))}{9c^4d^2}-\frac{\left(c^2dx^2+d\right)^{7/2}(a+b\,\mathrm{arcsinh}(cx))}{7c^4d}-\frac{bcd^2x^5\sqrt{c^2dx^2+d}}{21\sqrt{c^2x^2+1}}-\frac{bd^2x^3\sqrt{c^2dx^2+d}}{189c\sqrt{c^2x^2+1}}-\frac{bc^5d^2x^9\sqrt{c^2dx^2+d}}{81\sqrt{c^2x^2+1}}+\frac{2bd^2x\sqrt{c^2dx^2+d}}{63c^3\sqrt{c^2x^2+1}}-\frac{19bc^3d^2x^7\sqrt{c^2dx^2+d}}{441\sqrt{c^2x^2+1}}$$

```
[In] Int[x^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*b*d^2*x*Sqrt[d + c^2*d*x^2])/(63*c^3*Sqrt[1 + c^2*x^2]) - (b*d^2*x^3*Sqr
t[d + c^2*d*x^2])/(189*c*Sqrt[1 + c^2*x^2]) - (b*c*d^2*x^5*Sqrt[d + c^2*d*x
^2])/(21*Sqrt[1 + c^2*x^2]) - (19*b*c^3*d^2*x^7*Sqrt[d + c^2*d*x^2])/(441*S
qrt[1 + c^2*x^2]) - (b*c^5*d^2*x^9*Sqrt[d + c^2*d*x^2])/(81*Sqrt[1 + c^2*x^
2]) - ((d + c^2*d*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^4*d) + ((d + c^2*d*
x^2)^(9/2)*(a + b*ArcSinh[c*x]))/(9*c^4*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\text{integral} = -\frac{(d+c^2dx^2)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4d} + \frac{(d+c^2dx^2)^{9/2}(a+b\text{arcsinh}(cx))}{9c^4d^2} - \frac{\left(bc\sqrt{d+c^2dx^2}\right)\int \frac{d^2(1+c^2x^2)^3(-2+7c^2x^2)}{63c^4}\,dx}{\sqrt{1+c^2x^2}}$$

$$= -\frac{(d+c^2dx^2)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4d} + \frac{(d+c^2dx^2)^{9/2}(a+b\text{arcsinh}(cx))}{9c^4d^2} - \frac{\left(bd^2\sqrt{d+c^2dx^2}\right)\int(1+c^2x^2)^3(-2+7c^2x^2)\,dx}{63c^3\sqrt{1+c^2x^2}}$$

$$= -\frac{(d+c^2dx^2)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4d} + \frac{(d+c^2dx^2)^{9/2}(a+b\text{arcsinh}(cx))}{9c^4d^2} - \frac{\left(bd^2\sqrt{d+c^2dx^2}\right)\int(-2+c^2x^2+15c^4x^4+19c^6x^6+7c^8x^8)\,dx}{63c^3\sqrt{1+c^2x^2}}$$

$$= \frac{2bd^2x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}} - \frac{bd^2x^3\sqrt{d+c^2dx^2}}{189c\sqrt{1+c^2x^2}} - \frac{bcd^2x^5\sqrt{d+c^2dx^2}}{21\sqrt{1+c^2x^2}} - \frac{19bc^3d^2x^7\sqrt{d+c^2dx^2}}{441\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^9\sqrt{d+c^2dx^2}}{81\sqrt{1+c^2x^2}} - \frac{(d+c^2dx^2)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4d} + \frac{(d+c^2dx^2)^{9/2}(a+b\text{arcsinh}(cx))}{9c^4d^2}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.53

$$\int x^3(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \frac{d^2\sqrt{d+c^2dx^2}\Big(63a(1+c^2x^2)^4(-2+7c^2x^2) - bcx\sqrt{1+c^2x^2}(-126+21c^2x^2+189c^4x^4}{3969c^4(1+c^2x^2)}$$

```
[In] Integrate[x^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*Sqrt[d + c^2*d*x^2]*(63*a*(1 + c^2*x^2)^4*(-2 + 7*c^2*x^2) - b*c*x*Sqr
t[1 + c^2*x^2]*(-126 + 21*c^2*x^2 + 189*c^4*x^4 + 171*c^6*x^6 + 49*c^8*x^8)
 + 63*b*(1 + c^2*x^2)^4*(-2 + 7*c^2*x^2)*ArcSinh[c*x]))/(3969*c^4*(1 + c^2*
x^2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 995 vs. 2(228) = 456.

Time = 0.23 (sec) , antiderivative size = 996, normalized size of antiderivative = 3.74

method	result
default	$a\left(\frac{x^2(c^2d\,x^2+d)^{\frac{7}{2}}}{9c^2d}-\frac{2(c^2d\,x^2+d)^{\frac{7}{2}}}{63d\,c^4}\right)+b\left(\frac{\sqrt{d(c^2x^2+1)}\,\left(256c^{10}x^{10}+256c^9x^9\sqrt{c^2x^2+1}+704c^8x^8+576c^7x^7\sqrt{c^2x^2+1}+688c^6x^6\right.}{4}\right.$
parts	$a\left(\frac{x^2(c^2d\,x^2+d)^{\frac{7}{2}}}{9c^2d}-\frac{2(c^2d\,x^2+d)^{\frac{7}{2}}}{63d\,c^4}\right)+b\left(\frac{\sqrt{d(c^2x^2+1)}\,\left(256c^{10}x^{10}+256c^9x^9\sqrt{c^2x^2+1}+704c^8x^8+576c^7x^7\sqrt{c^2x^2+1}+688c^6x^6\right.}{4}\right.$

```
[In] int(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/9*x^2*(c^2*d*x^2+d)^(7/2)/c^2/d-2/63/d/c^4*(c^2*d*x^2+d)^(7/2))+b*(1/4
1472*(d*(c^2*x^2+1))^(1/2)*(256*c^10*x^10+256*c^9*x^9*(c^2*x^2+1)^(1/2)+704
*c^8*x^8+576*c^7*x^7*(c^2*x^2+1)^(1/2)+688*c^6*x^6+432*c^5*x^5*(c^2*x^2+1)^
(1/2)+280*c^4*x^4+120*c^3*x^3*(c^2*x^2+1)^(1/2)+41*c^2*x^2+9*c*x*(c^2*x^2+1
)^(1/2)+1)*(-1+9*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)+3/25088*(d*(c^2*x^2+1))^
(1/2)*(64*c^8*x^8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6+112*c^5*x^5*(c^2
*x^2+1)^(1/2)+104*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2+7*c*x*(c^
2*x^2+1)^(1/2)+1)*(-1+7*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)-1/576*(d*(c^2*x^2
+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+
1)^(1/2)+1)*(-1+3*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)-3/256*(d*(c^2*x^2+1))^(
1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))*d^2/c^4/(c^2*x^2+1
)-3/256*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*
x)+1)*d^2/c^4/(c^2*x^2+1)-1/576*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*
(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(3*arcsinh(c*x)+1)*d
^2/c^4/(c^2*x^2+1)+3/25088*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8-64*c^7*x^7*(c^
2*x^2+1)^(1/2)+144*c^6*x^6-112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4-56*c^3
*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2-7*c*x*(c^2*x^2+1)^(1/2)+1)*(1+7*arcsinh(c
*x))*d^2/c^4/(c^2*x^2+1)+1/41472*(d*(c^2*x^2+1))^(1/2)*(256*c^10*x^10-256*c
^9*x^9*(c^2*x^2+1)^(1/2)+704*c^8*x^8-576*c^7*x^7*(c^2*x^2+1)^(1/2)+688*c^6*
x^6-432*c^5*x^5*(c^2*x^2+1)^(1/2)+280*c^4*x^4-120*c^3*x^3*(c^2*x^2+1)^(1/2)
+41*c^2*x^2-9*c*x*(c^2*x^2+1)^(1/2)+1)*(1+9*arcsinh(c*x))*d^2/c^4/(c^2*x^2+
1))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.99

$$\int x^3\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\frac{63\left(7\,bc^{10}d^2x^{10}+26\,bc^8d^2x^8+34\,bc^6d^2x^6+16\,bc^4d^2x^4-bc^2d^2x^2-2\,bd^2\right)\sqrt{c^2dx^2+d}\log}{}$$

```
[In] integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3969*(63*(7*b*c^10*d^2*x^10 + 26*b*c^8*d^2*x^8 + 34*b*c^6*d^2*x^6 + 16*b*
c^4*d^2*x^4 - b*c^2*d^2*x^2 - 2*b*d^2)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c
^2*x^2 + 1)) + (441*a*c^10*d^2*x^10 + 1638*a*c^8*d^2*x^8 + 2142*a*c^6*d^2*x
^6 + 1008*a*c^4*d^2*x^4 - 63*a*c^2*d^2*x^2 - 126*a*d^2 - (49*b*c^9*d^2*x^9
+ 171*b*c^7*d^2*x^7 + 189*b*c^5*d^2*x^5 + 21*b*c^3*d^2*x^3 - 126*b*c*d^2*x)
*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^6*x^2 + c^4)
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \left(d + c^2 d x^2\right)^{5/2} (a + b\operatorname{arcsinh}(cx))\, dx = \text{Timed out}$$

```
[In] integrate(x**3*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.59

$$\int x^3 \left(d + c^2 d x^2\right)^{5/2} (a + b\operatorname{arcsinh}(cx))\, dx = \frac{1}{63} \left(\frac{7 \left(c^2 d x^2 + d\right)^{\frac{7}{2}} x^2}{c^2 d} - \frac{2 \left(c^2 d x^2 + d\right)^{\frac{7}{2}}}{c^4 d}\right) b \operatorname{arsinh}(cx) + \frac{1}{63} \left(\frac{7 \left(c^2 d x^2 + d\right)^{\frac{7}{2}} x^2}{c^2 d} - \frac{2 \left(c^2 d x^2 + d\right)^{\frac{7}{2}}}{c^4 d}\right) a - \frac{\left(49\, c^8 d^{\frac{5}{2}} x^9 + 171\, c^6 d^{\frac{5}{2}} x^7 + 189\, c^4 d^{\frac{5}{2}} x^5 + 21\, c^2 d^{\frac{5}{2}} x^3 - 126\, d^{\frac{5}{2}} x\right) b}{3969\, c^3}$$

```
[In] integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/63*(7*(c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(7/2)/(c^4*d)
)*b*arcsinh(c*x) + 1/63*(7*(c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) - 2*(c^2*d*x^2
 + d)^(7/2)/(c^4*d))*a - 1/3969*(49*c^8*d^(5/2)*x^9 + 171*c^6*d^(5/2)*x^7 +
 189*c^4*d^(5/2)*x^5 + 21*c^2*d^(5/2)*x^3 - 126*d^(5/2)*x)*b/c^3
```

Giac [F(-2)]

Exception generated.

$$\int x^3 (d + c^2dx^2)^{5/2} (a + b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (d + c^2dx^2)^{5/2} (a + b\text{arcsinh}(cx))\,dx = \int x^3\,(a + b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2 + d\right)^{5/2}\,dx$$

```
[In] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

3.137 $\int x^2(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 337

$$\begin{aligned}\int x^2(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx=&-\frac{5bd^2x^2\sqrt{d+c^2dx^2}}{256c\sqrt{1+c^2x^2}}\\&-\frac{59bcd^2x^4\sqrt{d+c^2dx^2}}{768\sqrt{1+c^2x^2}}-\frac{17bc^3d^2x^6\sqrt{d+c^2dx^2}}{288\sqrt{1+c^2x^2}}\\&-\frac{bc^5d^2x^8\sqrt{d+c^2dx^2}}{64\sqrt{1+c^2x^2}}+\frac{5d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{128c^2}\\&+\frac{5}{64}d^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))+\frac{5}{48}dx^3(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))\\&+\frac{1}{8}x^3(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))-\frac{5d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{256bc^3\sqrt{1+c^2x^2}}\end{aligned}$$

```
[Out] 5/48*d*x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+1/8*x^3*(c^2*d*x^2+d)^(5/
2)*(a+b*arcsinh(c*x))+5/128*d^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^
2+5/64*d^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-5/256*b*d^2*x^2*(c^2*
d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-59/768*b*c*d^2*x^4*(c^2*d*x^2+d)^(1/2)/(
c^2*x^2+1)^(1/2)-17/288*b*c^3*d^2*x^6*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
-1/64*b*c^5*d^2*x^8*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-5/256*d^2*(a+b*ar
csinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5808, 5806, 5812, 5783, 30, 14, 272, 45}

$$\int x^2\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\frac{5d^2x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{128c^2}+\frac{5}{64}d^2x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))+\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5}{48}dx^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{5d^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{256bc^3\sqrt{c^2x^2+1}}-\frac{5bd^2x^2\sqrt{c^2dx^2+d}}{256c\sqrt{c^2x^2+1}}-\frac{59bcd^2x^4}{768\sqrt{}}$$

```
[In] Int[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-5*b*d^2*x^2*Sqrt[d + c^2*d*x^2])/(256*c*Sqrt[1 + c^2*x^2]) - (59*b*c*d^2*
x^4*Sqrt[d + c^2*d*x^2])/(768*Sqrt[1 + c^2*x^2]) - (17*b*c^3*d^2*x^6*Sqrt[d
 + c^2*d*x^2])/(288*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^8*Sqrt[d + c^2*d*x^2]
)/(64*Sqrt[1 + c^2*x^2]) + (5*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
))/(128*c^2) + (5*d^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/64 + (5
*d*x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/48 + (x^3*(d + c^2*d*x^2
)^(5/2)*(a + b*ArcSinh[c*x]))/8 - (5*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x])^2)/(256*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{8}x^3\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{1}{8}(5d)\int x^2\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx-\frac{\left(bcd^2\sqrt{d+c^2dx^2}\right)\int x^3\left(1+c^2x^2\right)^2\,dx}{8\sqrt{1+c^2x^2}}$$

$$= \frac{5}{48} d x^3 \left(d+c^2 d x^2\right)^{3/2} (a+b \text{arcsinh}(c x))+\frac{1}{8} x^3 \left(d+c^2 d x^2\right)^{5/2} (a+b \text{arcsinh}(c x)) + \frac{1}{16}\left(5 d^2\right) \int x^2 \sqrt{d+c^2 d x^2}(a+b \text{arcsinh}(c x)) \, dx - \frac{\left(b c d^2 \sqrt{d+c^2 d x^2}\right) \text{Subst}\left(\int x\left(1+c^2 x\right)^2 \, dx, x, x^2\right)}{16 \sqrt{1+c^2 x^2}} - \frac{\left(5 b c d^2 \sqrt{d+c^2 d x^2}\right) \int x^3\left(1+c^2 x^2\right) \, dx}{48 \sqrt{1+c^2 x^2}}$$

$$= \frac{5}{64} d^2 x^3 \sqrt{d+c^2 d x^2}(a+b \text{arcsinh}(c x))+\frac{5}{48} d x^3 \left(d+c^2 d x^2\right)^{3/2} (a+b \text{arcsinh}(c x)) + \frac{1}{8} x^3 \left(d+c^2 d x^2\right)^{5/2} (a+b \text{arcsinh}(c x))+\frac{\left(5 d^2 \sqrt{d+c^2 d x^2}\right) \int \frac{x^2(a+b \text{arcsinh}(c x))}{\sqrt{1+c^2 x^2}} \, dx}{64 \sqrt{1+c^2 x^2}} - \frac{\left(b c d^2 \sqrt{d+c^2 d x^2}\right) \text{Subst}\left(\int \left(x+2 c^2 x^2+c^4 x^3\right) \, dx, x, x^2\right)}{16 \sqrt{1+c^2 x^2}} - \frac{\left(5 b c d^2 \sqrt{d+c^2 d x^2}\right) \int x^3 \, dx}{64 \sqrt{1+c^2 x^2}}-\frac{\left(5 b c d^2 \sqrt{d+c^2 d x^2}\right) \int \left(x^3+c^2 x^5\right) \, dx}{48 \sqrt{1+c^2 x^2}}$$

$$= -\frac{59 b c d^2 x^4 \sqrt{d+c^2 d x^2}}{768 \sqrt{1+c^2 x^2}}-\frac{17 b c^3 d^2 x^6 \sqrt{d+c^2 d x^2}}{288 \sqrt{1+c^2 x^2}}-\frac{b c^5 d^2 x^8 \sqrt{d+c^2 d x^2}}{64 \sqrt{1+c^2 x^2}} + \frac{5 d^2 x \sqrt{d+c^2 d x^2}(a+b \text{arcsinh}(c x))}{128 c^2}+\frac{5}{64} d^2 x^3 \sqrt{d+c^2 d x^2}(a+b \text{arcsinh}(c x)) + \frac{5}{48} d x^3 \left(d+c^2 d x^2\right)^{3/2} (a+b \text{arcsinh}(c x))+\frac{1}{8} x^3 \left(d+c^2 d x^2\right)^{5/2} (a+b \text{arcsinh}(c x)) - \frac{\left(5 d^2 \sqrt{d+c^2 d x^2}\right) \int \frac{a+b \text{arcsinh}(c x)}{\sqrt{1+c^2 x^2}} \, dx}{128 c^2 \sqrt{1+c^2 x^2}}-\frac{\left(5 b d^2 \sqrt{d+c^2 d x^2}\right) \int x \, dx}{128 c \sqrt{1+c^2 x^2}}$$

$$= -\frac{5 b d^2 x^2 \sqrt{d+c^2 d x^2}}{256 c \sqrt{1+c^2 x^2}}-\frac{59 b c d^2 x^4 \sqrt{d+c^2 d x^2}}{768 \sqrt{1+c^2 x^2}}-\frac{17 b c^3 d^2 x^6 \sqrt{d+c^2 d x^2}}{288 \sqrt{1+c^2 x^2}} - \frac{b c^5 d^2 x^8 \sqrt{d+c^2 d x^2}}{64 \sqrt{1+c^2 x^2}}+\frac{5 d^2 x \sqrt{d+c^2 d x^2}(a+b \text{arcsinh}(c x))}{128 c^2} + \frac{5}{64} d^2 x^3 \sqrt{d+c^2 d x^2}(a+b \text{arcsinh}(c x))+\frac{5}{48} d x^3 \left(d+c^2 d x^2\right)^{3/2} (a+b \text{arcsinh}(c x)) + \frac{1}{8} x^3 \left(d+c^2 d x^2\right)^{5/2} (a+b \text{arcsinh}(c x))-\frac{5 d^2 \sqrt{d+c^2 d x^2}(a+b \text{arcsinh}(c x))^2}{256 b c^3 \sqrt{1+c^2 x^2}}$$

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.15

$$\int x^2(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \frac{d^2\Big(2880acx\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+22656ac^3x^3\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+26112ac^5x^5\sqrt{}}{}$$

```
[In] Integrate[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(2880*a*c*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 22656*a*c^3*x^3*Sq
rt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 26112*a*c^5*x^5*Sqrt[1 + c^2*x^2]*Sqr
t[d + c^2*d*x^2] + 9216*a*c^7*x^7*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] - 1
440*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2 + 576*b*Sqrt[d + c^2*d*x^2]*Cosh[2
*ArcSinh[c*x]] - 144*b*Sqrt[d + c^2*d*x^2]*Cosh[4*ArcSinh[c*x]] - 64*b*Sqrt
[d + c^2*d*x^2]*Cosh[6*ArcSinh[c*x]] - 9*b*Sqrt[d + c^2*d*x^2]*Cosh[8*ArcSi
nh[c*x]] - 2880*a*Sqrt[d]*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^
2*d*x^2]] + 24*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(-48*Sinh[2*ArcSinh[c*x]]
 + 24*Sinh[4*ArcSinh[c*x]] + 16*Sinh[6*ArcSinh[c*x]] + 3*Sinh[8*ArcSinh[c*x
]])))/(73728*c^3*Sqrt[1 + c^2*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1164 vs. 2(291) = 582.

Time = 0.22 (sec) , antiderivative size = 1165, normalized size of antiderivative = 3.46

method	result	size
default	Expression too large to display	1165
parts	Expression too large to display	1165

```
[In] int(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*a*x*(c^2*d*x^2+d)^(7/2)/c^2/d-1/48*a/c^2*x*(c^2*d*x^2+d)^(5/2)-5/192*a/
c^2*d*x*(c^2*d*x^2+d)^(3/2)-5/128*a/c^2*d^2*x*(c^2*d*x^2+d)^(1/2)-5/128*a/c
^2*d^3*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b*(-5/25
6*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^2*d^2+1/16384*(d
*(c^2*x^2+1))^(1/2)*(128*c^9*x^9+128*c^8*x^8*(c^2*x^2+1)^(1/2)+320*c^7*x^7+
256*c^6*x^6*(c^2*x^2+1)^(1/2)+272*c^5*x^5+160*c^4*x^4*(c^2*x^2+1)^(1/2)+88*
c^3*x^3+32*c^2*x^2*(c^2*x^2+1)^(1/2)+8*c*x+(c^2*x^2+1)^(1/2))*(-1+8*arcsinh
(c*x))*d^2/c^3/(c^2*x^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7+32*c^6*
x^6*(c^2*x^2+1)^(1/2)+64*c^5*x^5+48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3+18
*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x+(c^2*x^2+1)^(1/2))*(-1+6*arcsinh(c*x))*d^2
/c^3/(c^2*x^2+1)+1/1024*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2
```

```
+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(
-1+4*arcsinh(c*x))*d^2/c^3/(c^2*x^2+1)-1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x
^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*x))
*d^2/c^3/(c^2*x^2+1)-1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*
x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(c*x))*d^2/c^3/(c^2*x^2+1
)+1/1024*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^
3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(1+4*arcsinh(c*x
))*d^2/c^3/(c^2*x^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7-32*c^6*x^6*
(c^2*x^2+1)^(1/2)+64*c^5*x^5-48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3-18*c^2
*x^2*(c^2*x^2+1)^(1/2)+6*c*x-(c^2*x^2+1)^(1/2))*(1+6*arcsinh(c*x))*d^2/c^3/
(c^2*x^2+1)+1/16384*(d*(c^2*x^2+1))^(1/2)*(128*c^9*x^9-128*c^8*x^8*(c^2*x^2
+1)^(1/2)+320*c^7*x^7-256*c^6*x^6*(c^2*x^2+1)^(1/2)+272*c^5*x^5-160*c^4*x^4
*(c^2*x^2+1)^(1/2)+88*c^3*x^3-32*c^2*x^2*(c^2*x^2+1)^(1/2)+8*c*x-(c^2*x^2+1
)^(1/2))*(1+8*arcsinh(c*x))*d^2/c^3/(c^2*x^2+1))
```

Fricas [F]

$$\int x^2 (d + c^2 d x^2)^{5/2} (a + b \operatorname{arcsinh}(cx)) \, dx = \int (c^2 d x^2 + d)^{\frac{5}{2}} (b \operatorname{arsinh}(cx) + a) x^2 \, dx$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((a*c^4*d^2*x^6 + 2*a*c^2*d^2*x^4 + a*d^2*x^2 + (b*c^4*d^2*x^6 + 2*
b*c^2*d^2*x^4 + b*d^2*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 (d + c^2 d x^2)^{5/2} (a + b \operatorname{arcsinh}(cx)) \, dx = \text{Timed out}$$

```
[In] integrate(x**2*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^2 (d + c^2 dx^2)^{5/2} (a + b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int x^2 (d + c^2 dx^2)^{5/2} (a + b\text{arcsinh}(cx))\,dx = \int (c^2 dx^2 + d)^{\frac{5}{2}} (b \operatorname{arsinh}(cx) + a) x^2\,dx$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (d + c^2 dx^2)^{5/2} (a + b\text{arcsinh}(cx))\,dx = \int x^2 (a + b \operatorname{asinh}(c\,x)) \left(d\,c^2\,x^2 + d\right)^{5/2} dx$$

```
[In] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
[Out] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

3.138 $\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 193

$$\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx = -\frac{bd^2x\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}} - \frac{bcd^2x^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}} - \frac{3bc^3d^2x^5\sqrt{d+c^2dx^2}}{35\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}} + \frac{(d+c^2dx^2)^{7/2}(a+b\text{arcsinh}(cx))}{7c^2d}$$

```
[Out] 1/7*(c^2*d*x^2+d)^(7/2)*(a+b*arcsinh(c*x))/c^2/d-1/7*b*d^2*x*(c^2*d*x^2+d)^
(1/2)/c/(c^2*x^2+1)^(1/2)-1/7*b*c*d^2*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(
1/2)-3/35*b*c^3*d^2*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/49*b*c^5*d^
2*x^7*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5798, 200}

$$\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \frac{(c^2dx^2+d)^{7/2}(a+b\text{arcsinh}(cx))}{7c^2d} - \frac{bd^2x\sqrt{c^2dx^2+d}}{7c\sqrt{c^2x^2+1}} - \frac{bcd^2x^3\sqrt{c^2dx^2+d}}{7\sqrt{c^2x^2+1}} - \frac{bc^5d^2x^7\sqrt{c^2dx^2+d}}{49\sqrt{c^2x^2+1}} - \frac{3bc^3d^2x^5\sqrt{c^2dx^2+d}}{35\sqrt{c^2x^2+1}}$$

```
[In] Int[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/7*(b*d^2*x*Sqrt[d + c^2*d*x^2])/(c*Sqrt[1 + c^2*x^2]) - (b*c*d^2*x^3*Sqr
t[d + c^2*d*x^2])/(7*Sqrt[1 + c^2*x^2]) - (3*b*c^3*d^2*x^5*Sqrt[d + c^2*d*x
^2])/(35*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt[
1 + c^2*x^2]) + ((d + c^2*d*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^2*d)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(d+c^2dx^2)^{7/2}\,(a+b\operatorname{arcsinh}(cx))}{7c^2d} - \frac{\left(bd^2\sqrt{d+c^2dx^2}\right)\int (1+c^2x^2)^3\,dx}{7c\sqrt{1+c^2x^2}}\\
&= \frac{(d+c^2dx^2)^{7/2}\,(a+b\operatorname{arcsinh}(cx))}{7c^2d} - \frac{\left(bd^2\sqrt{d+c^2dx^2}\right)\int (1+3c^2x^2+3c^4x^4+c^6x^6)\,dx}{7c\sqrt{1+c^2x^2}}\\
&= -\frac{bd^2x\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}} - \frac{bcd^2x^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}} - \frac{3bc^3d^2x^5\sqrt{d+c^2dx^2}}{35\sqrt{1+c^2x^2}}\\
&\quad - \frac{bc^5d^2x^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}} + \frac{(d+c^2dx^2)^{7/2}\,(a+b\operatorname{arcsinh}(cx))}{7c^2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.58

$$\int x\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx = \frac{d^2\sqrt{d+c^2dx^2}\Big(35a(1+c^2x^2)^4 - bcx\sqrt{1+c^2x^2}(35+35c^2x^2+21c^4x^4+5c^6x^6) + 35b(1}{245c^2\,(1+c^2x^2)}$$

```
[In] Integrate[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*Sqrt[d + c^2*d*x^2]*(35*a*(1 + c^2*x^2)^4 - b*c*x*Sqrt[1 + c^2*x^2]*(3
5 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) + 35*b*(1 + c^2*x^2)^4*ArcSinh[c*x
]))/(245*c^2*(1 + c^2*x^2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 862 vs. 2(165) = 330.

Time = 0.22 (sec) , antiderivative size = 863, normalized size of antiderivative = 4.47

method	result
default	$\frac{a(c^2dx^2+d)^{\frac{7}{2}}}{7c^2d}+b\left(\frac{\sqrt{d(c^2x^2+1)}\,\left(64c^8x^8+64c^7x^7\sqrt{c^2x^2+1}+144c^6x^6+112c^5x^5\sqrt{c^2x^2+1}+104c^4x^4+56c^3x^3\sqrt{c^2x^2+1}+25c^2x^2+7c\right.}{6272c^2(c^2x^2+1)}\right.$
parts	$\frac{a(c^2dx^2+d)^{\frac{7}{2}}}{7c^2d}+b\left(\frac{\sqrt{d(c^2x^2+1)}\,\left(64c^8x^8+64c^7x^7\sqrt{c^2x^2+1}+144c^6x^6+112c^5x^5\sqrt{c^2x^2+1}+104c^4x^4+56c^3x^3\sqrt{c^2x^2+1}+25c^2x^2+7c\right.}{6272c^2(c^2x^2+1)}\right.$

```
[In] int(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*a*(c^2*d*x^2+d)^(7/2)/c^2/d+b*(1/6272*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8
+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2)+104
*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1)
*(-1+7*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/640*(d*(c^2*x^2+1))^(1/2)*(16*c^
6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+
13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))*d^2/c^2/(c^2*x^2+
1)+1/128*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2
*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+5/1
28*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x)
)*d^2/c^2/(c^2*x^2+1)+5/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^
(1/2)+1)*(arcsinh(c*x)+1)*d^2/c^2/(c^2*x^2+1)+1/128*(d*(c^2*x^2+1))^(1/2)*(
4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*
(3*arcsinh(c*x)+1)*d^2/c^2/(c^2*x^2+1)+1/640*(d*(c^2*x^2+1))^(1/2)*(16*c^6*
x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13
*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*(1+5*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+
1/6272*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8-64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c
^6*x^6-112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4-56*c^3*x^3*(c^2*x^2+1)^(1/
2)+25*c^2*x^2-7*c*x*(c^2*x^2+1)^(1/2)+1)*(1+7*arcsinh(c*x))*d^2/c^2/(c^2*x^
2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.17

$$\int x\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx=\frac{35\left(bc^8d^2x^8+4\,bc^6d^2x^6+6\,bc^4d^2x^4+4\,bc^2d^2x^2+bd^2\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)+}{}$$

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/245*(35*(b*c^8*d^2*x^8 + 4*b*c^6*d^2*x^6 + 6*b*c^4*d^2*x^4 + 4*b*c^2*d^2*
x^2 + b*d^2)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (35*a*c^8*d
^2*x^8 + 140*a*c^6*d^2*x^6 + 210*a*c^4*d^2*x^4 + 140*a*c^2*d^2*x^2 + 35*a*d
^2 - (5*b*c^7*d^2*x^7 + 21*b*c^5*d^2*x^5 + 35*b*c^3*d^2*x^3 + 35*b*c*d^2*x)
*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^4*x^2 + c^2)
```

Sympy [F]

$$\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\int x\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)\,dx$$

```
[In] integrate(x*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
[Out] Integral(x*(d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.50

$$\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\frac{(c^2dx^2+d)^{\frac{7}{2}}b\,\text{arsinh}\,(cx)}{7\,c^2d}+\frac{(c^2dx^2+d)^{\frac{7}{2}}a}{7\,c^2d}-\frac{\left(5\,c^6d^{\frac{7}{2}}x^7+21\,c^4d^{\frac{7}{2}}x^5+35\,c^2d^{\frac{7}{2}}x^3+35\,d^{\frac{7}{2}}x\right)b}{245\,cd}$$

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] 1/7*(c^2*d*x^2 + d)^(7/2)*b*arcsinh(c*x)/(c^2*d) + 1/7*(c^2*d*x^2 + d)^(7/2
)*a/(c^2*d) - 1/245*(5*c^6*d^(7/2)*x^7 + 21*c^4*d^(7/2)*x^5 + 35*c^2*d^(7/2
)*x^3 + 35*d^(7/2)*x)*b/(c*d)
```

Giac [F(-2)]

Exception generated.

$$\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx=\int x\,(a+b\,\operatorname{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

```
[In] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
[Out] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

3.139 $\int \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 254

$$\int \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx = -\frac{25bcd^2x^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{5bc^3d^2x^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}$$
$$-\frac{bd^2\left(1+c^2x^2\right)^{5/2}\sqrt{d+c^2dx^2}}{36c} + \frac{5}{16}d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))$$
$$+\frac{5}{24}dx\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{1}{6}x\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx)) + \frac{5d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(}{32bc\sqrt{1+c^2x^2}}$$

```
[Out] 5/24*d*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+1/6*x*(c^2*d*x^2+d)^(5/2)*(
a+b*arcsinh(c*x))-1/36*b*d^2*(c^2*x^2+1)^(5/2)*(c^2*d*x^2+d)^(1/2)/c+5/16*d
^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-25/96*b*c*d^2*x^2*(c^2*d*x^2+d)
^(1/2)/(c^2*x^2+1)^(1/2)-5/96*b*c^3*d^2*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)
^(1/2)+5/32*d^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1
/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5786, 5785, 5783, 30, 14, 267}

$$\int \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx = \frac{5}{16}d^2x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))$$
$$+\frac{5d^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{32bc\sqrt{c^2x^2+1}} + \frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))$$
$$+\frac{5}{24}dx\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{25bcd^2x^2\sqrt{c^2dx^2+d}}{96\sqrt{c^2x^2+1}} - \frac{bd^2\left(c^2x^2+1\right)^{5/2}\sqrt{c^2dx^2+d}}{36c} - \frac{5bc^3d^2x^4\sqrt{c^2}}{96\sqrt{c^2x^2}}$$

```
[In] Int[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-25*b*c*d^2*x^2*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (5*b*c^3*d^2
*x^4*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (b*d^2*(1 + c^2*x^2)^(5/
2)*Sqrt[d + c^2*d*x^2])/(36*c) + (5*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
h[c*x]))/16 + (5*d*x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/24 + (x*(d
 + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/6 + (5*d^2*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])^2)/(32*b*c*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
```

```
+ c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{6}x\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{1}{6}(5d)\int\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx - \frac{\left(bcd^2\sqrt{d+c^2dx^2}\right)\int x\left(1+c^2x^2\right)^2\,dx}{6\sqrt{1+c^2x^2}}$$

$$= -\frac{bd^2(1+c^2x^2)^{5/2}\sqrt{d+c^2dx^2}}{36c}+\frac{5}{24}dx\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{1}{6}x\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{1}{8}\left(5d^2\right)\int\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx - \frac{\left(5bcd^2\sqrt{d+c\right.}{2}$$

$$= -\frac{bd^2(1+c^2x^2)^{5/2}\sqrt{d+c^2dx^2}}{36c} + \frac{5}{16}d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))+\frac{5}{24}dx\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{1}{6}x\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{\left(5d^2\sqrt{d+c^2dx^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{16\sqrt{1+c^2x^2}} - \frac{\left(5bcd^2\sqrt{d+c^2dx^2}\right)}{24\sqrt{1+}}$$

$$= -\frac{25bcd^2x^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{5bc^3d^2x^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{bd^2(1+c^2x^2)^{5/2}\sqrt{d+c^2dx^2}}{36c} + \frac{5}{16}d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))+\frac{5}{24}dx\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{1}{6}x\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{32bc\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.25

$$\int\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \frac{d^2\left(1584acx\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+1248ac^3x^3\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+384ac^5x^5\sqrt{1+\right.}{}$$

```
[In] Integrate[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(1584*a*c*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 1248*a*c^3*x^3*Sqr
t[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 384*a*c^5*x^5*Sqrt[1 + c^2*x^2]*Sqrt[d
 + c^2*d*x^2] + 360*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2 - 270*b*Sqrt[d + c
```

^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 27*b*Sqrt[d + c^2*d*x^2]*Cosh[4*ArcSinh[c*x]] - 2*b*Sqrt[d + c^2*d*x^2]*Cosh[6*ArcSinh[c*x]] + 720*a*Sqrt[d]*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 12*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(45*Sinh[2*ArcSinh[c*x]] + 9*Sinh[4*ArcSinh[c*x]] + Sinh[6*ArcSinh[c*x]])))/(2304*c*Sqrt[1 + c^2*x^2])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 800 vs. $2(218) = 436$.

Time = 0.18 (sec) , antiderivative size = 801, normalized size of antiderivative = 3.15

method	result
default	$\frac{x(c^2dx^2+d)^{\frac{5}{2}}a}{6}+\frac{5adx(c^2dx^2+d)^{\frac{3}{2}}}{24}+\frac{5ad^2x\sqrt{c^2dx^2+d}}{16}+\frac{5ad^3\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{16\sqrt{c^2d}}+b\left(\frac{5\sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(cx)^2d^2}{32\sqrt{c^2x^2+1}\,c}\right.$
parts	$\frac{x(c^2dx^2+d)^{\frac{5}{2}}a}{6}+\frac{5adx(c^2dx^2+d)^{\frac{3}{2}}}{24}+\frac{5ad^2x\sqrt{c^2dx^2+d}}{16}+\frac{5ad^3\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{16\sqrt{c^2d}}+b\left(\frac{5\sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(cx)^2d^2}{32\sqrt{c^2x^2+1}\,c}\right.$

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)

[Out] 1/6*x*(c^2*d*x^2+d)^(5/2)*a+5/24*a*d*x*(c^2*d*x^2+d)^(3/2)+5/16*a*d^2*x*(c^
2*d*x^2+d)^(1/2)+5/16*a*d^3*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(
c^2*d)^(1/2)+b*(5/32*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(c*x)
^2*d^2+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7+32*c^6*x^6*(c^2*x^2+1)^(1/2
)+64*c^5*x^5+48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3+18*c^2*x^2*(c^2*x^2+1)
^(1/2)+6*c*x+(c^2*x^2+1)^(1/2))*(-1+6*arcsinh(c*x))*d^2/c/(c^2*x^2+1)+3/512
*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*
c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(c*x))*d^2/
c/(c^2*x^2+1)+15/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)
^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*x))*d^2/c/(c^2*x^2+1)+15/25
6*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x
^2+1)^(1/2))*(1+2*arcsinh(c*x))*d^2/c/(c^2*x^2+1)+3/512*(d*(c^2*x^2+1))^(1/
2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^
(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(1+4*arcsinh(c*x))*d^2/c/(c^2*x^2+1)+1/2304*
(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7-32*c^6*x^6*(c^2*x^2+1)^(1/2)+64*c^5*x^5-4
8*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3-18*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x-(
c^2*x^2+1)^(1/2))*(1+6*arcsinh(c*x))*d^2/c/(c^2*x^2+1))
```

Fricas [F]

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\int \left(c^2dx^2+d\right)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\int \left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}\left(a+b\,\text{asinh}\,(cx)\right)\,dx$$

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx=\int (a+b\operatorname{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

```
[In] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
[Out] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

3.140 $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 329

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x}\,dx = -\frac{23bcd^2x\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}} - \frac{11bc^3d^2x^3\sqrt{d+c^2dx^2}}{45\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}} + d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{1}{3}d(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{1}{5}(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx)) - \frac{2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{bd^2\sqrt{d+c^2dx^2}}{}$$

```
[Out] 1/3*d*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+1/5*(c^2*d*x^2+d)^(5/2)*(a+b*a
rcsinh(c*x))+d^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-23/15*b*c*d^2*x*(c^
2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-11/45*b*c^3*d^2*x^3*(c^2*d*x^2+d)^(1/2)/
(c^2*x^2+1)^(1/2)-1/25*b*c^5*d^2*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-
2*d^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)
/(c^2*x^2+1)^(1/2)-b*d^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1
/2)/(c^2*x^2+1)^(1/2)+b*d^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^
(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used

$= \{5808, 5806, 5816, 4267, 2317, 2438, 8, 200\}$

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x}\,dx =$$
$$-\frac{2d^2\sqrt{c^2dx^2+d}\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}$$
$$+d^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))+\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))$$
$$+\frac{1}{3}d\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{bd^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{bd^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e\right.}{\sqrt{c^2x^2+1}}$$

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (-23*b*c*d^2*x*Sqrt[d + c^2*d*x^2])/(15*Sqrt[1 + c^2*x^2]) - (11*b*c^3*d^2*
x^3*Sqrt[d + c^2*d*x^2])/(45*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^5*Sqrt[d + c
^2*d*x^2])/(25*Sqrt[1 + c^2*x^2]) + d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[
c*x]) + (d*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/3 + ((d + c^2*d*x^2)
^(5/2)*(a + b*ArcSinh[c*x]))/5 - (2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[
c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b*d^2*Sqrt[d + c^2*d*x^
2]*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (b*d^2*Sqrt[d + c^2*d*x
^2]*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
```

```
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))+d\int\frac{\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))}{x}\,dx \\
&\quad -\frac{\left(bcd^2\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)^2\,dx}{5\sqrt{1+c^2x^2}} \\
&= \frac{1}{3}d\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad +\frac{1}{5}\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))+d^2\int\frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{x}\,dx \\
&\quad -\frac{\left(bcd^2\sqrt{d+c^2dx^2}\right)\int\left(1+2c^2x^2+c^4x^4\right)\,dx}{5\sqrt{1+c^2x^2}}-\frac{\left(bcd^2\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)\,dx}{3\sqrt{1+c^2x^2}}
\end{aligned}$$

$$= -\frac{8bcd^2x\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}} - \frac{11bc^3d^2x^3\sqrt{d+c^2dx^2}}{45\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}} + d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{1}{3}d\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{1}{5}\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx)) + \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{\sqrt{1+c^2x^2}} - \frac{\left(bcd^2\sqrt{d+c^2dx^2}\right)\int 1\,dx}{\sqrt{1+c^2x^2}}$$

$$= -\frac{23bcd^2x\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}} - \frac{11bc^3d^2x^3\sqrt{d+c^2dx^2}}{45\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}} + d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{1}{3}d\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{1}{5}\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx)) + \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)\text{csch}(x)\,dx,x,\text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}}$$

$$= -\frac{23bcd^2x\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}} - \frac{11bc^3d^2x^3\sqrt{d+c^2dx^2}}{45\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}} + d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{1}{3}d\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{1}{5}\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx)) - \frac{2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{\left(bd^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\log\left(1-e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}} + \frac{\left(bd^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\log\left(1+e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}}$$

$$= -\frac{23bcd^2x\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}} - \frac{11bc^3d^2x^3\sqrt{d+c^2dx^2}}{45\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}} + d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{1}{3}d(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx)) + \frac{1}{5}(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx)) - \frac{2d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{\left(bd^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{\left(bd^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}$$

$$= -\frac{23bcd^2x\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}} - \frac{11bc^3d^2x^3\sqrt{d+c^2dx^2}}{45\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}} + d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{1}{3}d(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx)) + \frac{1}{5}(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx)) - \frac{2d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{bd^2\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{bd^2\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.03

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{x}\,dx = \frac{-40bcd^3x\sqrt{1+c^2x^2}(3+c^2x^2) - 3bc^3d^3x^3\sqrt{1+c^2x^2}(5+3c^2x^2) +}{}$$

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (-40*b*c*d^3*x*Sqrt[1 + c^2*x^2]*(3 + c^2*x^2) - 3*b*c^3*d^3*x^3*Sqrt[1 + c
^2*x^2]*(5 + 3*c^2*x^2) + 15*a*d^3*(1 + c^2*x^2)*(23 + 11*c^2*x^2 + 3*c^4*x
^4) + 150*b*d^3*(1 + c^2*x^2)^2*ArcSinh[c*x] + 15*b*d^3*(1 + c^2*x^2)^2*(-2
 + 3*c^2*x^2)*ArcSinh[c*x] + 225*a*d^(5/2)*Sqrt[d + c^2*d*x^2]*Log[x] - 225
*a*d^(5/2)*Sqrt[d + c^2*d*x^2]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 225*b
*d^3*Sqrt[1 + c^2*x^2]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c
```

*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))/(225*Sqrt[d + c^2*d*x^2])

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.64

method	result
default	$\frac{(c^2 d x^2+d)^{\frac{5}{2}} a}{5}+\frac{a d (c^2 d x^2+d)^{\frac{3}{2}}}{3}-a d^{\frac{5}{2}} \ln\left(\frac{2d+2\sqrt{d}\sqrt{c^2 d x^2+d}}{x}\right)+a d^2\sqrt{c^2 d x^2+d}-\frac{b\sqrt{d(c^2x^2+1)}\,\operatorname{polylog}\left(2,-cx\right.}{\sqrt{c^2x^2+1}}$
parts	$\frac{(c^2 d x^2+d)^{\frac{5}{2}} a}{5}+\frac{a d (c^2 d x^2+d)^{\frac{3}{2}}}{3}-a d^{\frac{5}{2}} \ln\left(\frac{2d+2\sqrt{d}\sqrt{c^2 d x^2+d}}{x}\right)+a d^2\sqrt{c^2 d x^2+d}-\frac{b\sqrt{d(c^2x^2+1)}\,\operatorname{polylog}\left(2,-cx\right.}{\sqrt{c^2x^2+1}}$

[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x,x,method=_RETURNVERBOSE)

[Out] 1/5*(c^2*d*x^2+d)^(5/2)*a+1/3*a*d*(c^2*d*x^2+d)^(3/2)-a*d^(5/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)+a*d^2*(c^2*d*x^2+d)^(1/2)-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*d^2+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*d^2+23/15*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*d^2-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*d^2+1/5*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^6*c^6-1/25*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*c^5*x^5+14/15*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^4*c^4-11/45*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*c^3*x^3+34/15*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^2*c^2-23/15*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*c*x

Fricas [F]

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))}{x}\,dx=\int \frac{(c^2dx^2+d)^{\frac{5}{2}}\,(b\operatorname{arsinh}(cx)+a)}{x}\,dx$$

[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")

[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x, x)

Sympy [F]

$$\int \frac{(d + c^2dx^2)^{5/2}\,(a + b\text{arcsinh}(cx))}{x}\,dx = \int \frac{(d(c^2x^2 + 1))^{\frac{5}{2}}\,(a + b\,\text{asinh}\,(cx))}{x}\,dx$$

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))/x,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))/x, x)
```

Maxima [F]

$$\int \frac{(d + c^2dx^2)^{5/2}\,(a + b\text{arcsinh}(cx))}{x}\,dx = \int \frac{(c^2dx^2 + d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx) + a)}{x}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
```

```
[Out] -1/15*(15*d^(5/2)*arcsinh(1/(c*abs(x))) - 3*(c^2*d*x^2 + d)^(5/2) - 5*(c^2*
d*x^2 + d)^(3/2)*d - 15*sqrt(c^2*d*x^2 + d)*d^2)*a + b*integrate((c^2*d*x^2
 + d)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + c^2dx^2)^{5/2}\,(a + b\text{arcsinh}(cx))}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + c^2dx^2)^{5/2}\,(a + b\text{arcsinh}(cx))}{x}\,dx = \int \frac{(a + b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2 + d\right)^{5/2}}{x}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x, x)
```

3.141 $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 257

$$\begin{aligned}\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx &= -\frac{9bc^3d^2x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}\\ &-\frac{bc^5d^2x^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}+\frac{15}{8}c^2d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\\ &+\frac{5}{4}c^2dx(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))-\frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x}\\ &+\frac{15cd^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{16b\sqrt{1+c^2x^2}}+\frac{bcd^2\sqrt{d+c^2dx^2}\log(x)}{\sqrt{1+c^2x^2}}\end{aligned}$$

```
[Out] 5/4*c^2*d*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))-(c^2*d*x^2+d)^(5/2)*(a+b
*arcsinh(c*x))/x+15/8*c^2*d^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-9/16
*b*c^3*d^2*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*b*c^5*d^2*x^4*(c^
2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+15/16*c*d^2*(a+b*arcsinh(c*x))^2*(c^2*d*
x^2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+b*c*d^2*ln(x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2
+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used

$= \{5807, 5786, 5785, 5783, 30, 14, 272, 45\}$

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx = \frac{15}{8}c^2d^2x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx)) + \frac{15cd^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{16b\sqrt{c^2x^2+1}} + \frac{5}{4}c^2dx(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{(c^2dx^2+d)^{5/2}(a+b\text{arcsinh}(cx))}{x} + \frac{bcd^2\log(x)\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}} - \frac{bc^5d^2x^4\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}} - \frac{9bc^3d^2x^2\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}$$

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (-9*b*c^3*d^2*x^2*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*
x^4*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) + (15*c^2*d^2*x*Sqrt[d + c^
2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (5*c^2*d*x*(d + c^2*d*x^2)^(3/2)*(a + b*
ArcSinh[c*x]))/4 - ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x + (15*c*d
^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*Sqrt[1 + c^2*x^2]) + (
b*c*d^2*Sqrt[d + c^2*d*x^2]*Log[x])/Sqrt[1 + c^2*x^2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{x} \\
&\quad +(5c^2d)\int (d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx+\frac{\left(bcd^2\sqrt{d+c^2dx^2}\right)\int \frac{(1+c^2x^2)^2}{x}\,dx}{\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$= \frac{5}{4}c^2dx\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx)) - \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))}{x} + \frac{1}{4}\left(15c^2d^2\right)\int\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx + \frac{\left(bcd^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\frac{(1+c^2x)^2}{x}\,dx,x,x^2\right)}{2\sqrt{1+c^2x^2}} - \frac{\left(5bc^3d^2\sqrt{d+c^2dx^2}\right)\int x\left(1+c^2x^2\right)\,dx}{4\sqrt{1+c^2x^2}}$$

$$= \frac{15}{8}c^2d^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{5}{4}c^2dx\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx)) - \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))}{x} + \frac{\left(bcd^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\left(2c^2+\frac{1}{x}+c^4x\right)\,dx,x,x^2\right)}{2\sqrt{1+c^2x^2}} + \frac{\left(15c^2d^2\sqrt{d+c^2dx^2}\right)\int\frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{8\sqrt{1+c^2x^2}} - \frac{\left(5bc^3d^2\sqrt{d+c^2dx^2}\right)\int\left(x+c^2x^3\right)\,dx}{4\sqrt{1+c^2x^2}} - \frac{\left(15bc^3d^2\sqrt{d+c^2dx^2}\right)\int x\,dx}{8\sqrt{1+c^2x^2}}$$

$$= -\frac{9bc^3d^2x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} + \frac{15}{8}c^2d^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{5}{4}c^2dx\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx)) - \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))}{x} + \frac{15cd^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{16b\sqrt{1+c^2x^2}} + \frac{bcd^2\sqrt{d+c^2dx^2}\log(x)}{\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.05

$$\int\frac{\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))}{x^2}\,dx = \frac{1}{128}d^2\left(\frac{16a\sqrt{d+c^2dx^2}\left(-8+9c^2x^2+2c^4x^4\right)}{x} + \frac{64b\sqrt{d+c^2dx^2}\left(-2\sqrt{1+c^2x^2}\operatorname{arcsinh}(cx)+cx\operatorname{arcsinh}(cx)^2+2cx\log(cx)\right)}{x\sqrt{1+c^2x^2}} + 240ac\sqrt{d}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right) + \frac{32bc\sqrt{d+c^2dx^2}(-\cosh(2\operatorname{arcsinh}(cx))+2\operatorname{arcsinh}(cx)(\operatorname{arcsinh}(cx)+\sinh(2\operatorname{arcsinh}(cx))))}{\sqrt{1+c^2x^2}} - \frac{bc\sqrt{d+c^2dx^2}\left(8\operatorname{arcsinh}(cx)^2+\cosh(4\operatorname{arcsinh}(cx))-4\operatorname{arcsinh}(cx)\sinh(4\operatorname{arcsinh}(cx))\right)}{\sqrt{1+c^2x^2}}\right)$$

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (d^2*((16*a*Sqrt[d + c^2*d*x^2]*(-8 + 9*c^2*x^2 + 2*c^4*x^4))/x + (64*b*Sqr
t[d + c^2*d*x^2]*(-2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + c*x*ArcSinh[c*x]^2 +
2*c*x*Log[c*x]))/(x*Sqrt[1 + c^2*x^2]) + 240*a*c*Sqrt[d]*Log[c*d*x + Sqrt[d
]*Sqrt[d + c^2*d*x^2]] + (32*b*c*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSinh[c*x]]
 + 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/Sqrt[1 + c^2*x^2]
 - (b*c*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*Ar
cSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/Sqrt[1 + c^2*x^2]))/128
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.11

method	result
default	$-\frac{a(c^2 d\,x^2+d)^{\frac{7}{2}}}{dx}+a\,c^2 x(c^2 d\,x^2+d)^{\frac{5}{2}}+\frac{5(c^2 d\,x^2+d)^{\frac{3}{2}}a\,c^2 dx}{4}+\frac{15a\,d^2\sqrt{c^2 d\,x^2+d}\,c^2 x}{8}+\frac{15a\,c^2 d^3\ln\left(\frac{c^2 dx}{\sqrt{c^2 d}}+\sqrt{c^2 d\,x^2+d}\right)}{8\sqrt{c^2 d}}$
parts	$-\frac{a(c^2 d\,x^2+d)^{\frac{7}{2}}}{dx}+a\,c^2 x(c^2 d\,x^2+d)^{\frac{5}{2}}+\frac{5(c^2 d\,x^2+d)^{\frac{3}{2}}a\,c^2 dx}{4}+\frac{15a\,d^2\sqrt{c^2 d\,x^2+d}\,c^2 x}{8}+\frac{15a\,c^2 d^3\ln\left(\frac{c^2 dx}{\sqrt{c^2 d}}+\sqrt{c^2 d\,x^2+d}\right)}{8\sqrt{c^2 d}}$

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/d/x*(c^2*d*x^2+d)^(7/2)+a*c^2*x*(c^2*d*x^2+d)^(5/2)+5/4*(c^2*d*x^2+d)^(3
/2)*a*c^2*d*x+15/8*a*d^2*(c^2*d*x^2+d)^(1/2)*c^2*x+15/8*a*c^2*d^3*ln(c^2*d*
x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/128*b*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)/x*(32*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4*c^4-8*c^5*
x^5+144*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2-72*c^3*x^3+120*arcsinh(c*x)^
2*x*c-128*arcsinh(c*x)*c*x+128*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*x*c-128*arcs
inh(c*x)*(c^2*x^2+1)^(1/2)-33*c*x)*d^2
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))}{x^2}\,dx=\int \frac{(c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}{x^2}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^2, x)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\,\mathrm{arcsinh}(cx))}{x^2}\,dx = \int \frac{(d(c^2x^2+1))^{\frac{5}{2}}\,(a+b\,\mathrm{asinh}\,(cx))}{x^2}\,dx$$

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))/x**2,x)
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\,\mathrm{arcsinh}(cx))}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\,\mathrm{arcsinh}(cx))}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\,\mathrm{arcsinh}(cx))}{x^2}\,dx = \int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{5/2}}{x^2}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x^2,x)
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x^2, x)
```

3.142 $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 355

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx = -\frac{bcd^2\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}} - \frac{7bc^3d^2x\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}} + \frac{5}{2}c^2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{5}{6}c^2d(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2} - \frac{5c^2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{5bc^2d^2\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2\sqrt{1+c^2x^2}} + \frac{5bc^2d^2\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2\sqrt{1+c^2x^2}}$$

```
[Out] 5/6*c^2*d*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))-1/2*(c^2*d*x^2+d)^(5/2)*(a
+b*arcsinh(c*x))/x^2+5/2*c^2*d^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-1/2
*b*c*d^2*(c^2*d*x^2+d)^(1/2)/x/(c^2*x^2+1)^(1/2)-7/3*b*c^3*d^2*x*(c^2*d*x^2
+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/9*b*c^5*d^2*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+
1)^(1/2)-5*c^2*d^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*d
*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-5/2*b*c^2*d^2*polylog(2,-c*x-(c^2*x^2+1)^(1
/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5/2*b*c^2*d^2*polylog(2,c*x+(c^2
*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5807, 5808, 5806, 5816, 4267, 2317, 2438, 8, 276}

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{x^3}\,dx = -\frac{5c^2d^2\sqrt{c^2dx^2+d}\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2x^2+1}} + \frac{5}{2}c^2d^2\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx)) + \frac{5}{6}c^2d\left(c^2dx^2+d\right)^{3/2}(a+b\operatorname{arcsinh}(cx)) - \frac{(c^2dx^2+d)^{5/2}(a+b\operatorname{arcsinh}(cx))}{2x^2} - \frac{5bc^2d^2\sqrt{c^2dx^2+d}\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)}{2\sqrt{c^2x^2+1}} + \frac{5bc^2d^2\sqrt{c^2dx^2+d}\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)}{2\sqrt{c^2x^2+1}} - \frac{bcd^2\sqrt{c^2dx^2+d}}{2x\sqrt{c^2x^2+1}} - \frac{bc^5d^2x^3\sqrt{c^2dx^2+d}}{9\sqrt{c^2x^2+1}} - \frac{7bc^3d^2x\sqrt{c^2dx^2+d}}{3\sqrt{c^2x^2+1}}$$

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*d^2*Sqrt[d + c^2*d*x^2])/(x*Sqrt[1 + c^2*x^2]) - (7*b*c^3*d^2*x*S
qrt[d + c^2*d*x^2])/(3*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^3*Sqrt[d + c^2*d*x
^2])/(9*Sqrt[1 + c^2*x^2]) + (5*c^2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[
c*x]))/2 + (5*c^2*d*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/6 - ((d + c
^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(2*x^2) - (5*c^2*d^2*Sqrt[d + c^2*d*x
^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (5*b*
c^2*d^2*Sqrt[d + c^2*d*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/(2*Sqrt[1 + c^2*x^
2]) + (5*b*c^2*d^2*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^ArcSinh[c*x]])/(2*Sqrt[
1 + c^2*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2} \\
&\quad +\frac{1}{2}(5c^2d)\int\frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x}\,dx \\
&\quad +\frac{\left(bcd^2\sqrt{d+c^2dx^2}\right)\int\frac{(1+c^2x^2)^2}{x^2}\,dx}{2\sqrt{1+c^2x^2}} \\
&= \frac{5}{6}c^2d(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))-\frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2} \\
&\quad +\frac{1}{2}(5c^2d^2)\int\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x}\,dx \\
&\quad +\frac{\left(bcd^2\sqrt{d+c^2dx^2}\right)\int\left(2c^2+\frac{1}{x^2}+c^4x^2\right)dx}{2\sqrt{1+c^2x^2}} \\
&\quad -\frac{\left(5bc^3d^2\sqrt{d+c^2dx^2}\right)\int(1+c^2x^2)\,dx}{6\sqrt{1+c^2x^2}} \\
&= -\frac{bcd^2\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}+\frac{bc^3d^2x\sqrt{d+c^2dx^2}}{6\sqrt{1+c^2x^2}} \\
&\quad -\frac{bc^5d^2x^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+\frac{5}{2}c^2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) \\
&\quad +\frac{5}{6}c^2d(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))-\frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2} \\
&\quad +\frac{\left(5c^2d^2\sqrt{d+c^2dx^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}}-\frac{\left(5bc^3d^2\sqrt{d+c^2dx^2}\right)\int 1\,dx}{2\sqrt{1+c^2x^2}} \\
&= -\frac{bcd^2\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{7bc^3d^2x\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}} \\
&\quad -\frac{bc^5d^2x^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+\frac{5}{2}c^2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) \\
&\quad +\frac{5}{6}c^2d(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))-\frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2} \\
&\quad +\frac{\left(5c^2d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)\text{csch}(x)\,dx,x,\text{arcsinh}(cx)\right)}{2\sqrt{1+c^2x^2}}
\end{aligned}$$

$$= -\frac{bcd^2\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}} - \frac{7bc^3d^2x\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}} + \frac{5}{2}c^2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{5}{6}c^2d\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2} - \frac{5c^2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{\left(5bc^2d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\log\left(1-e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{2\sqrt{1+c^2x^2}} + \frac{\left(5bc^2d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\log\left(1+e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{2\sqrt{1+c^2x^2}}$$

$$= -\frac{bcd^2\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}} - \frac{7bc^3d^2x\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}} + \frac{5}{2}c^2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{5}{6}c^2d\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2} - \frac{5c^2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{\left(5bc^2d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{2\sqrt{1+c^2x^2}} + \frac{\left(5bc^2d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{2\sqrt{1+c^2x^2}}$$

$$= -\frac{bcd^2\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}} - \frac{7bc^3d^2x\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}} + \frac{5}{2}c^2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{5}{6}c^2d\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2} - \frac{5c^2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{5bc^2d^2\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2\sqrt{1+c^2x^2}} + \frac{5bc^2d^2\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 6.41 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.19

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx = \frac{1}{72}d^2\left(\frac{12a\sqrt{d+c^2dx^2}(-3+14c^2x^2+2c^4x^4)}{x^2}\right.$$

$$-\frac{8bc^2\sqrt{d+c^2dx^2}\left(3cx+c^3x^3-3(1+c^2x^2)^{3/2}\text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}}$$

$$+180ac^2\sqrt{d}\log(x)-180ac^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right)+\frac{144bc^2\sqrt{d+c^2dx^2}\left(-cx+\sqrt{1+c^2x^2}\text{arcsinh}(cx)+\right.}{}$$

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (d^2*((12*a*Sqrt[d + c^2*d*x^2]*(-3 + 14*c^2*x^2 + 2*c^4*x^4))/x^2 - (8*b*c
^2*Sqrt[d + c^2*d*x^2]*(3*c*x + c^3*x^3 - 3*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x
]))/Sqrt[1 + c^2*x^2] + 180*a*c^2*Sqrt[d]*Log[x] - 180*a*c^2*Sqrt[d]*Log[d
+ Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (144*b*c^2*Sqrt[d + c^2*d*x^2]*(-(c*x) + S
qrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - A
rcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - P
olyLog[2, E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2] + (9*b*c^2*Sqrt[d + c^2*d*
x^2]*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 4*Arc
Sinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c
*x])] + 4*PolyLog[2, -E^(-ArcSinh[c*x])] - 4*PolyLog[2, E^(-ArcSinh[c*x])]
- ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/Sqrt[1 + c
^2*x^2]))/72
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.95

method	result
default	$a\left(-\frac{(c^2dx^2+d)^{\frac{7}{2}}}{2dx^2}+\frac{5c^2\left(\frac{(c^2dx^2+d)^{\frac{5}{2}}}{5}+d\left(\frac{(c^2dx^2+d)^{\frac{3}{2}}}{3}+d\left(\sqrt{c^2dx^2+d}-\sqrt{d}\ln\left(\frac{2d+2\sqrt{d}\sqrt{c^2dx^2+d}}{x}\right)\right)\right)\right)}{2}\right)+\frac{b\sqrt{d(c^2x^2}}{}$
parts	$a\left(-\frac{(c^2dx^2+d)^{\frac{7}{2}}}{2dx^2}+\frac{5c^2\left(\frac{(c^2dx^2+d)^{\frac{5}{2}}}{5}+d\left(\frac{(c^2dx^2+d)^{\frac{3}{2}}}{3}+d\left(\sqrt{c^2dx^2+d}-\sqrt{d}\ln\left(\frac{2d+2\sqrt{d}\sqrt{c^2dx^2+d}}{x}\right)\right)\right)\right)}{2}\right)+\frac{b\sqrt{d(c^2x^2}}{}$

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/2/d/x^2*(c^2*d*x^2+d)^(7/2)+5/2*c^2*(1/5*(c^2*d*x^2+d)^(5/2)+d*(1/3*(
c^2*d*x^2+d)^(3/2)+d*((c^2*d*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d*
x^2+d)^(1/2))/x)))))+1/18*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/x^2*(6*
arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4*c^4-2*c^5*x^5+42*arcsinh(c*x)*(c^2*x^2+1
)^(1/2)*x^2*c^2+45*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*x^2*c^2-45*arcs
inh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*x^2*c^2-42*c^3*x^3+45*polylog(2,c*x+(c
^2*x^2+1)^(1/2))*x^2*c^2-45*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*x^2*c^2-9*arc
sinh(c*x)*(c^2*x^2+1)^(1/2)-9*c*x)*d^2
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))}{x^3}\,dx=\int \frac{(c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}{x^3}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^3, x)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))}{x^3}\,dx=\int \frac{(d(c^2x^2+1))^{\frac{5}{2}}\,(a+b\,\text{asinh}\,(cx))}{x^3}\,dx$$

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))/x**3,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))/x**3, x)
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))}{x^3}\,dx=\int \frac{(c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}{x^3}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/6*(15*c^2*d^(5/2)*arcsinh(1/(c*abs(x))) - 3*(c^2*d*x^2 + d)^(5/2)*c^2 -
5*(c^2*d*x^2 + d)^(3/2)*c^2*d - 15*sqrt(c^2*d*x^2 + d)*c^2*d^2 + 3*(c^2*d*x
^2 + d)^(7/2)/(d*x^2))*a + b*integrate((c^2*d*x^2 + d)^(5/2)*log(c*x + sqrt
(c^2*x^2 + 1))/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{5/2}}{x^3}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x^3, x)
```

3.143 $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 266

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx = -\frac{bcd^2\sqrt{d+c^2dx^2}}{6x^2\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}} + \frac{5}{2}c^4d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) - \frac{5c^2d(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{3x} - \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{3x^3} + \frac{5c^3d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{4b\sqrt{1+c^2x^2}} + \frac{7bc^3d^2\sqrt{d+c^2dx^2}\log(x)}{3\sqrt{1+c^2x^2}}$$

```
[Out] -5/3*c^2*d*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x-1/3*(c^2*d*x^2+d)^(5/2)
*(a+b*arcsinh(c*x))/x^3+5/2*c^4*d^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2
)-1/6*b*c*d^2*(c^2*d*x^2+d)^(1/2)/x^2/(c^2*x^2+1)^(1/2)-1/4*b*c^5*d^2*x^2*(
c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5/4*c^3*d^2*(a+b*arcsinh(c*x))^2*(c^2*
d*x^2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+7/3*b*c^3*d^2*ln(x)*(c^2*d*x^2+d)^(1/2)/
(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used

$= \{5807, 5785, 5783, 30, 14, 272, 45\}$

$$\int \frac{(d + c^2 dx^2)^{5/2} (a + b\text{arcsinh}(cx))}{x^4} dx =$$
$$-\frac{5c^2 d(c^2 dx^2 + d)^{3/2} (a + b\text{arcsinh}(cx))}{3x} - \frac{(c^2 dx^2 + d)^{5/2} (a + b\text{arcsinh}(cx))}{3x^3}$$
$$+\frac{5}{2}c^4 d^2 x\sqrt{c^2 dx^2 + d}(a + b\text{arcsinh}(cx)) + \frac{5c^3 d^2 \sqrt{c^2 dx^2 + d}(a + b\text{arcsinh}(cx))^2}{4b\sqrt{c^2x^2 + 1}} - \frac{bcd^2\sqrt{c^2 dx^2 + d}}{6x^2\sqrt{c^2x^2 + 1}} - \frac{bc^5 d^2 x^2 \sqrt{\cdots}}{4\sqrt{c^2 x \cdots}}$$

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*d^2*Sqrt[d + c^2*d*x^2])/(x^2*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^2
*Sqrt[d + c^2*d*x^2])/(4*Sqrt[1 + c^2*x^2]) + (5*c^4*d^2*x*Sqrt[d + c^2*d*x
^2]*(a + b*ArcSinh[c*x]))/2 - (5*c^2*d*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh
[c*x]))/(3*x) - ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(3*x^3) + (5*c
^3*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*Sqrt[1 + c^2*x^2])
+ (7*b*c^3*d^2*Sqrt[d + c^2*d*x^2]*Log[x])/(3*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
```

```
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{3x^3} \\
&\quad+\frac{1}{3}(5c^2d)\int\frac{(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{x^2}\,dx \\
&\quad+\frac{\left(bcd^2\sqrt{d+c^2dx^2}\right)\int\frac{(1+c^2x^2)^2}{x^3}\,dx}{3\sqrt{1+c^2x^2}} \\
&= -\frac{5c^2d(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3x}-\frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{3x^3} \\
&\quad+(5c^4d^2)\int\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx \\
&\quad+\frac{\left(bcd^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\frac{(1+c^2x)^2}{x^2}\,dx,x,x^2\right)}{6\sqrt{1+c^2x^2}}+\frac{\left(5bc^3d^2\sqrt{d+c^2dx^2}\right)\int\frac{1+c^2x^2}{x}\,dx}{3\sqrt{1+c^2x^2}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{5}{2}c^4d^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) - \frac{5c^2d(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3x} \\
&\quad - \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{3x^3} \\
&\quad + \frac{\left(bcd^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\left(c^4+\frac{1}{x^2}+\frac{2c^2}{x}\right)dx,x,x^2\right)}{6\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(5bc^3d^2\sqrt{d+c^2dx^2}\right)\int\left(\frac{1}{x}+c^2x\right)dx}{3\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(5c^4d^2\sqrt{d+c^2dx^2}\right)\int\frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}dx}{2\sqrt{1+c^2x^2}} - \frac{\left(5bc^5d^2\sqrt{d+c^2dx^2}\right)\int x\,dx}{2\sqrt{1+c^2x^2}} \\
&= -\frac{bcd^2\sqrt{d+c^2dx^2}}{6x^2\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}} + \frac{5}{2}c^4d^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad - \frac{5c^2d(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3x} - \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{3x^3} \\
&\quad + \frac{5c^3d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4b\sqrt{1+c^2x^2}} + \frac{7bc^3d^2\sqrt{d+c^2dx^2}\log(x)}{3\sqrt{1+c^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.08

$$
\int \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{x^4}\,dx = \frac{d^2\Big(4a\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(-2-14c^2x^2+3c^4x^4)+24bc^2x^2\sqrt{d}
$$

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] (d^2*(4*a*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(-2 - 14*c^2*x^2 + 3*c^4*x^
4) + 24*b*c^2*x^2*Sqrt[d + c^2*d*x^2]*(-2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] +
c*x*ArcSinh[c*x]^2 + 2*c*x*Log[c*x]) - 4*b*Sqrt[d + c^2*d*x^2]*(c*x + 2*(1
+ c^2*x^2)^(3/2)*ArcSinh[c*x] - 2*c^3*x^3*Log[c*x]) + 60*a*c^3*Sqrt[d]*x^3*
Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 3*b*c^3*x^3*Sq
rt[d + c^2*d*x^2]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Si
nh[2*ArcSinh[c*x]]))))/(24*x^3*Sqrt[1 + c^2*x^2])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.21

method	result
default	$-\frac{a(c^2 d\,x^2+d)^{\frac{7}{2}}}{3d\,x^3}-\frac{4a\,c^2(c^2 d\,x^2+d)^{\frac{7}{2}}}{3dx}+\frac{4a\,c^4 x(c^2 d\,x^2+d)^{\frac{5}{2}}}{3}+\frac{5a\,c^4 dx(c^2 d\,x^2+d)^{\frac{3}{2}}}{3}+\frac{5a\,c^4 d^2 x\sqrt{c^2 d\,x^2+d}}{2}+\frac{5a\,c^4 d^3\ln\left(\frac{c^2}{\sqrt{c}}\right.}{2}$
parts	$-\frac{a(c^2 d\,x^2+d)^{\frac{7}{2}}}{3d\,x^3}-\frac{4a\,c^2(c^2 d\,x^2+d)^{\frac{7}{2}}}{3dx}+\frac{4a\,c^4 x(c^2 d\,x^2+d)^{\frac{5}{2}}}{3}+\frac{5a\,c^4 dx(c^2 d\,x^2+d)^{\frac{3}{2}}}{3}+\frac{5a\,c^4 d^2 x\sqrt{c^2 d\,x^2+d}}{2}+\frac{5a\,c^4 d^3\ln\left(\frac{c^2}{\sqrt{c}}\right.}{2}$

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/d/x^3*(c^2*d*x^2+d)^(7/2)-4/3*a*c^2/d/x*(c^2*d*x^2+d)^(7/2)+4/3*a*c^
4*x*(c^2*d*x^2+d)^(5/2)+5/3*a*c^4*d*x*(c^2*d*x^2+d)^(3/2)+5/2*a*c^4*d^2*x*(
c^2*d*x^2+d)^(1/2)+5/2*a*c^4*d^3*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/
2))/(c^2*d)^(1/2)+1/24*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/x^3*(12*ar
csinh(c*x)*(c^2*x^2+1)^(1/2)*x^4*c^4-6*c^5*x^5+30*arcsinh(c*x)^2*x^3*c^3-56
*arcsinh(c*x)*c^3*x^3+56*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*x^3*c^3-56*arcsinh
(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2-3*c^3*x^3-8*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-
4*c*x)*d^2
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))}{x^4}\,dx=\int \frac{(c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}{x^4}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^4, x)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))}{x^4}\,dx=\int \frac{(d(c^2x^2+1))^{\frac{5}{2}}\,(a+b\,\text{asinh}\,(cx))}{x^4}\,dx$$

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))/x**4,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))/x**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d + c^2dx^2)^{5/2} (a + b\text{arcsinh}(cx))}{x^4} dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + c^2dx^2)^{5/2} (a + b\text{arcsinh}(cx))}{x^4} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + c^2dx^2)^{5/2} (a + b\text{arcsinh}(cx))}{x^4} dx = \int \frac{(a + b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2 + d\right)^{5/2}}{x^4} dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x^4, x)
```

3.144 $\int \sqrt{1+x^2}\text{arcsinh}(x)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \sqrt{1+x^2}\text{arcsinh}(x)\,dx = -\frac{x^2}{4}+\frac{1}{2}x\sqrt{1+x^2}\text{arcsinh}(x)+\frac{\text{arcsinh}(x)^2}{4}$$

```
[Out] -1/4*x^2+1/4*arcsinh(x)^2+1/2*x*arcsinh(x)*(x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5785, 5783, 30}

$$\int \sqrt{1+x^2}\text{arcsinh}(x)\,dx = \frac{1}{2}\sqrt{x^2+1}x\text{arcsinh}(x)+\frac{\text{arcsinh}(x)^2}{4}-\frac{x^2}{4}$$

```
[In] Int[Sqrt[1 + x^2]*ArcSinh[x],x]
```

```
[Out] -1/4*x^2 + (x*Sqrt[1 + x^2]*ArcSinh[x])/2 + ArcSinh[x]^2/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
```

```
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2}x\sqrt{1+x^2}\text{arcsinh}(x) - \frac{\int x\,dx}{2} + \frac{1}{2}\int \frac{\text{arcsinh}(x)}{\sqrt{1+x^2}}\,dx \\ &= -\frac{x^2}{4} + \frac{1}{2}x\sqrt{1+x^2}\text{arcsinh}(x) + \frac{\text{arcsinh}(x)^2}{4} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \sqrt{1+x^2}\text{arcsinh}(x)\,dx = \frac{1}{4}\left(-x^2 + 2x\sqrt{1+x^2}\text{arcsinh}(x) + \text{arcsinh}(x)^2\right)$$

```
[In] Integrate[Sqrt[1 + x^2]*ArcSinh[x],x]
[Out] (-x^2 + 2*x*Sqrt[1 + x^2]*ArcSinh[x] + ArcSinh[x]^2)/4
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{x\ \text{arcsinh}(x)\sqrt{x^2+1}}{2} + \frac{\text{arcsinh}(x)^2}{4} - \frac{x^2}{4} - \frac{1}{4}$	26

```
[In] int(arcsinh(x)*(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x*arcsinh(x)*(x^2+1)^(1/2)+1/4*arcsinh(x)^2-1/4*x^2-1/4
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \sqrt{1+x^2}\text{arcsinh}(x)\,dx = \frac{1}{2}\sqrt{x^2+1}x\log\left(x+\sqrt{x^2+1}\right) - \frac{1}{4}x^2 + \frac{1}{4}\log\left(x+\sqrt{x^2+1}\right)^2$$

```
[In] integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(x^2 + 1)*x*log(x + sqrt(x^2 + 1)) - 1/4*x^2 + 1/4*log(x + sqrt(x^2
 + 1))^2
```

Sympy [F]

$$\int \sqrt{1+x^2}\text{arcsinh}(x)\,dx = \int \sqrt{x^2+1}\,\text{asinh}\,(x)\,dx$$

```
[In] integrate(asinh(x)*(x**2+1)**(1/2),x)
[Out] Integral(sqrt(x**2 + 1)*asinh(x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \sqrt{1+x^2}\text{arcsinh}(x)\,dx = -\frac{1}{4}x^2 + \frac{1}{2}\left(\sqrt{x^2+1}x + \text{arsinh}\,(x)\right)\text{arsinh}\,(x) - \frac{1}{4}\,\text{arsinh}\,(x)^2$$

```
[In] integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/4*x^2 + 1/2*(sqrt(x^2 + 1)*x + arcsinh(x))*arcsinh(x) - 1/4*arcsinh(x)^2
```

Giac [F]

$$\int \sqrt{1+x^2}\text{arcsinh}(x)\,dx = \int \sqrt{x^2+1}\,\text{arsinh}\,(x)\,dx$$

```
[In] integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^2 + 1)*arcsinh(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+x^2}\operatorname{arcsinh}(x)\,dx = \int \operatorname{asinh}(x)\,\sqrt{x^2+1}\,dx$$

```
[In] int(asinh(x)*(x^2 + 1)^(1/2),x)
[Out] int(asinh(x)*(x^2 + 1)^(1/2), x)
```

3.145 $\int \frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 215

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = -\frac{8bx\sqrt{1+c^2x^2}}{15c^5\sqrt{d+c^2dx^2}}+\frac{4bx^3\sqrt{1+c^2x^2}}{45c^3\sqrt{d+c^2dx^2}} -\frac{bx^5\sqrt{1+c^2x^2}}{25c\sqrt{d+c^2dx^2}}+\frac{8\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{15c^6d} -\frac{4x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{15c^4d} +\frac{x^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{5c^2d}$$

```
[Out] -8/15*b*x*(c^2*x^2+1)^(1/2)/c^5/(c^2*d*x^2+d)^(1/2)+4/45*b*x^3*(c^2*x^2+1)^
(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-1/25*b*x^5*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^
(1/2)+8/15*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^6/d-4/15*x^2*(a+b*arcsi
nh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4/d+1/5*x^4*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)
^(1/2)/c^2/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used

= {5812, 5798, 8, 30}

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{5c^2d} + \frac{8\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{15c^6d} - \frac{4x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{15c^4d} - \frac{bx^5\sqrt{c^2x^2+1}}{25c\sqrt{c^2dx^2+d}} - \frac{8bx\sqrt{c^2x^2+1}}{15c^5\sqrt{c^2dx^2+d}} + \frac{4bx^3\sqrt{c^2x^2+1}}{45c^3\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x^5*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (-8*b*x*Sqrt[1 + c^2*x^2])/(15*c^5*Sqrt[d + c^2*d*x^2]) + (4*b*x^3*Sqrt[1 +
 c^2*x^2])/(45*c^3*Sqrt[d + c^2*d*x^2]) - (b*x^5*Sqrt[1 + c^2*x^2])/(25*c*S
qrt[d + c^2*d*x^2]) + (8*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(15*c^6*
d) - (4*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(15*c^4*d) + (x^4*Sqr
t[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{x^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{5c^2d} - \frac{4\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx}{5c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int x^4\,dx}{5c\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx^5\sqrt{1+c^2x^2}}{25c\sqrt{d+c^2dx^2}} - \frac{4x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{15c^4d} + \frac{x^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{5c^2d} + \frac{8\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx}{15c^4} + \frac{\left(4b\sqrt{1+c^2x^2}\right)\int x^2\,dx}{15c^3\sqrt{d+c^2dx^2}}$$

$$= \frac{4bx^3\sqrt{1+c^2x^2}}{45c^3\sqrt{d+c^2dx^2}} - \frac{bx^5\sqrt{1+c^2x^2}}{25c\sqrt{d+c^2dx^2}} + \frac{8\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{15c^6d} - \frac{4x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{15c^4d} + \frac{x^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{5c^2d} - \frac{\left(8b\sqrt{1+c^2x^2}\right)\int 1\,dx}{15c^5\sqrt{d+c^2dx^2}}$$

$$= -\frac{8bx\sqrt{1+c^2x^2}}{15c^5\sqrt{d+c^2dx^2}} + \frac{4bx^3\sqrt{1+c^2x^2}}{45c^3\sqrt{d+c^2dx^2}} - \frac{bx^5\sqrt{1+c^2x^2}}{25c\sqrt{d+c^2dx^2}} + \frac{8\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{15c^6d} - \frac{4x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{15c^4d} + \frac{x^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{5c^2d}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.55

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{bcx\sqrt{1+c^2x^2}(-120+20c^2x^2-9c^4x^4)+15a(8+4c^2x^2-c^4x^4+3c^6x^6)+15b(8+4c^2x^2-c^4x^4+3c^6x^6)\,a}{225c^6\sqrt{d+c^2dx^2}}$$

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (b*c*x*Sqrt[1 + c^2*x^2]*(-120 + 20*c^2*x^2 - 9*c^4*x^4) + 15*a*(8 + 4*c^2*
x^2 - c^4*x^4 + 3*c^6*x^6) + 15*b*(8 + 4*c^2*x^2 - c^4*x^4 + 3*c^6*x^6)*Arc
Sinh[c*x])/(225*c^6*Sqrt[d + c^2*d*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(185) = 370$.

Time = 0.22 (sec) , antiderivative size = 625, normalized size of antiderivative = 2.91

method	result
default	$a\left(\frac{x^4\sqrt{c^2 d\,x^2+d}}{5c^2 d}-\frac{4\left(\frac{x^2\sqrt{c^2 d\,x^2+d}}{3c^2 d}-\frac{2\sqrt{c^2 d\,x^2+d}}{3d\,c^4}\right)}{5c^2}\right)+b\left(\frac{\sqrt{d(c^2x^2+1)}\,\left(16c^6x^6+16c^5x^5\sqrt{c^2x^2+1}+28c^4x^4+20c^3x^3\sqrt{c^2x^2+}\right.}{800c^6 d(c^2x^2+1)}\right.$
parts	$a\left(\frac{x^4\sqrt{c^2 d\,x^2+d}}{5c^2 d}-\frac{4\left(\frac{x^2\sqrt{c^2 d\,x^2+d}}{3c^2 d}-\frac{2\sqrt{c^2 d\,x^2+d}}{3d\,c^4}\right)}{5c^2}\right)+b\left(\frac{\sqrt{d(c^2x^2+1)}\,\left(16c^6x^6+16c^5x^5\sqrt{c^2x^2+1}+28c^4x^4+20c^3x^3\sqrt{c^2x^2+}\right.}{800c^6 d(c^2x^2+1)}\right.$

```
[In] int(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/5*x^4/c^2/d*(c^2*d*x^2+d)^(1/2)-4/5/c^2*(1/3*x^2/c^2/d*(c^2*d*x^2+d)^(
1/2)-2/3/d/c^4*(c^2*d*x^2+d)^(1/2)))+b*(1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6
*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+1
3*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))/c^6/d/(c^2*x^2+1)-
5/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^
2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))/c^6/d/(c^2*x^2+1)+5/16*(d*
(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))/c^6/
d/(c^2*x^2+1)+5/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*
(arcsinh(c*x)+1)/c^6/d/(c^2*x^2+1)-5/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4
*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(3*arcsinh(
c*x)+1)/c^6/d/(c^2*x^2+1)+1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^
5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*
x*(c^2*x^2+1)^(1/2)+1)*(1+5*arcsinh(c*x))/c^6/d/(c^2*x^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.75

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$$
$$=\frac{15\,(3\,bc^6x^6-bc^4x^4+4\,bc^2x^2+8\,b)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(45\,ac^6x^6-15\,ac^4x^4+60\,ac^2x^2-\right.}{225\,(c^8dx^2+c^6d)}$$

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/225*(15*(3*b*c^6*x^6 - b*c^4*x^4 + 4*b*c^2*x^2 + 8*b)*sqrt(c^2*d*x^2 + d)
*log(c*x + sqrt(c^2*x^2 + 1)) + (45*a*c^6*x^6 - 15*a*c^4*x^4 + 60*a*c^2*x^2
 - (9*b*c^5*x^5 - 20*b*c^3*x^3 + 120*b*c*x)*sqrt(c^2*x^2 + 1) + 120*a)*sqrt
(c^2*d*x^2 + d))/(c^8*d*x^2 + c^6*d)
```

Sympy [F]

$$\int \frac{x^5(a + b\text{arcsinh}(cx))}{\sqrt{d + c^2dx^2}}\,dx = \int \frac{x^5(a + b\,\text{asinh}\,(cx))}{\sqrt{d\,(c^2x^2 + 1)}}\,dx$$

```
[In] integrate(x**5*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
[Out] Integral(x**5*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.81

$$\begin{aligned}
&\int \frac{x^5(a + b\text{arcsinh}(cx))}{\sqrt{d + c^2dx^2}}\,dx \\
&= \frac{1}{15}\left(\frac{3\sqrt{c^2dx^2 + d}x^4}{c^2d} - \frac{4\sqrt{c^2dx^2 + d}x^2}{c^4d} + \frac{8\sqrt{c^2dx^2 + d}}{c^6d}\right)b\,\text{arsinh}\,(cx) \\
&\quad + \frac{1}{15}\left(\frac{3\sqrt{c^2dx^2 + d}x^4}{c^2d} - \frac{4\sqrt{c^2dx^2 + d}x^2}{c^4d} + \frac{8\sqrt{c^2dx^2 + d}}{c^6d}\right)a \\
&\quad - \frac{(9\,c^4x^5 - 20\,c^2x^3 + 120\,x)b}{225\,c^5\sqrt{d}}
\end{aligned}$$

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
[Out] 1/15*(3*sqrt(c^2*d*x^2 + d)*x^4/(c^2*d) - 4*sqrt(c^2*d*x^2 + d)*x^2/(c^4*d)
 + 8*sqrt(c^2*d*x^2 + d)/(c^6*d))*b*arcsinh(c*x) + 1/15*(3*sqrt(c^2*d*x^2 +
 d)*x^4/(c^2*d) - 4*sqrt(c^2*d*x^2 + d)*x^2/(c^4*d) + 8*sqrt(c^2*d*x^2 + d)
/(c^6*d))*a - 1/225*(9*c^4*x^5 - 20*c^2*x^3 + 120*x)*b/(c^5*sqrt(d))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a + b\text{arcsinh}(cx))}{\sqrt{d + c^2dx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{x^5\,(a+b\operatorname{asinh}(c\,x))}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

```
[In] int((x^5*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
[Out] int((x^5*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

3.146 $\int \frac{x^4(a+b\,\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 192

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{3bx^2\sqrt{1+c^2x^2}}{16c^3\sqrt{d+c^2dx^2}} - \frac{bx^4\sqrt{1+c^2x^2}}{16c\sqrt{d+c^2dx^2}} - \frac{3x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8c^4d} + \frac{x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{4c^2d} + \frac{3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{16bc^5\sqrt{d+c^2dx^2}}$$

```
[Out] 3/16*b*x^2*(c^2*x^2+1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-1/16*b*x^4*(c^2*x^2+1)
^(1/2)/c/(c^2*d*x^2+d)^(1/2)+3/16*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/
c^5/(c^2*d*x^2+d)^(1/2)-3/8*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4/d+
1/4*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used

= {5812, 5783, 30}

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{x^3\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{4c^2d} + \frac{3\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^2}{16bc^5\sqrt{c^2dx^2+d}} - \frac{3x\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{8c^4d} - \frac{bx^4\sqrt{c^2x^2+1}}{16c\sqrt{c^2dx^2+d}} + \frac{3bx^2\sqrt{c^2x^2+1}}{16c^3\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (3*b*x^2*Sqrt[1 + c^2*x^2])/(16*c^3*Sqrt[d + c^2*d*x^2]) - (b*x^4*Sqrt[1 +
c^2*x^2])/(16*c*Sqrt[d + c^2*d*x^2]) - (3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcS
inh[c*x]))/(8*c^4*d) + (x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(4*c^
2*d) + (3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c^5*Sqrt[d + c^2*
d*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{4c^2d} - \frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx}{4c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int x^3\,dx}{4c\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx^4\sqrt{1+c^2x^2}}{16c\sqrt{d+c^2dx^2}} - \frac{3x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8c^4d} + \frac{x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{4c^2d} + \frac{3\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx}{8c^4} + \frac{\left(3b\sqrt{1+c^2x^2}\right)\int x\,dx}{8c^3\sqrt{d+c^2dx^2}}$$

$$= \frac{3bx^2\sqrt{1+c^2x^2}}{16c^3\sqrt{d+c^2dx^2}} - \frac{bx^4\sqrt{1+c^2x^2}}{16c\sqrt{d+c^2dx^2}} - \frac{3x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8c^4d} + \frac{x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{4c^2d} + \frac{3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{16bc^5\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.79

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{\frac{16acx(-3+2c^2x^2)\sqrt{d+c^2dx^2}}{d} + \frac{48a\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)}{\sqrt{d}} + \frac{b\sqrt{1+c^2x^2}(16\cosh(2\text{arcsinh}(cx))-\cosh(4\text{arcsinh}(cx))+4\text{arcsinh}(cx)(}{\sqrt{d+c^2dx^2}}}{128c^5}$$

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] ((16*a*c*x*(-3 + 2*c^2*x^2)*Sqrt[d + c^2*d*x^2])/d + (48*a*Log[c*d*x + Sqrt
[d]*Sqrt[d + c^2*d*x^2]])/Sqrt[d] + (b*Sqrt[1 + c^2*x^2]*(16*Cosh[2*ArcSinh
[c*x]] - Cosh[4*ArcSinh[c*x]] + 4*ArcSinh[c*x]*(6*ArcSinh[c*x] - 8*Sinh[2*A
rcSinh[c*x]] + Sinh[4*ArcSinh[c*x]])))/Sqrt[d + c^2*d*x^2])/(128*c^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. 2(166) = 332.

Time = 0.20 (sec) , antiderivative size = 519, normalized size of antiderivative = 2.70

method	result
default	$\frac{a\,x^{3}\sqrt{c^{2}d\,x^{2}+d}}{4c^{2}d}-\frac{3ax\sqrt{c^{2}d\,x^{2}+d}}{8c^{4}d}+\frac{3a\ln\left(\frac{c^{2}dx}{\sqrt{c^{2}d}}+\sqrt{c^{2}d\,x^{2}+d}\right)}{8c^{4}\sqrt{c^{2}d}}+b\left(\frac{3\sqrt{d(c^{2}x^{2}+1)}\,\operatorname{arcsinh}(cx)^{2}}{16\sqrt{c^{2}x^{2}+1}\,c^{5}d}+\frac{\sqrt{d(c^{2}x^{2}+1)}\,\left(8c^{5}x^{5}+8c\right.}{}\right.$
parts	$\frac{a\,x^{3}\sqrt{c^{2}d\,x^{2}+d}}{4c^{2}d}-\frac{3ax\sqrt{c^{2}d\,x^{2}+d}}{8c^{4}d}+\frac{3a\ln\left(\frac{c^{2}dx}{\sqrt{c^{2}d}}+\sqrt{c^{2}d\,x^{2}+d}\right)}{8c^{4}\sqrt{c^{2}d}}+b\left(\frac{3\sqrt{d(c^{2}x^{2}+1)}\,\operatorname{arcsinh}(cx)^{2}}{16\sqrt{c^{2}x^{2}+1}\,c^{5}d}+\frac{\sqrt{d(c^{2}x^{2}+1)}\,\left(8c^{5}x^{5}+8c\right.}{}\right.$

```
[In] int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*x^3/c^2/d*(c^2*d*x^2+d)^(1/2)-3/8*a/c^4*x/d*(c^2*d*x^2+d)^(1/2)+3/8*a
/c^4*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b*(3/16*(d
*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d*arcsinh(c*x)^2+1/256*(d*(c^2*x^
2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^
2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(c*x))/c^5/d/(c^2*x^2+
1)-1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+
(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*x))/c^5/d/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1
))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(1
+2*arcsinh(c*x))/c^5/d/(c^2*x^2+1)+1/256*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8
*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^
2*x^2+1)^(1/2))*(1+4*arcsinh(c*x))/c^5/d/(c^2*x^2+1))
```

Fricas [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)x^4}{\sqrt{c^2dx^2+d}}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arcsinh(c*x) + a*x^4)/sqrt(c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{x^4(a+b\operatorname{asinh}(cx))}{\sqrt{d(c^2x^2+1)}}\,dx$$

```
[In] integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**4*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a + b\text{arcsinh}(cx))}{\sqrt{d + c^2dx^2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int \frac{x^4(a + b\text{arcsinh}(cx))}{\sqrt{d + c^2dx^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)x^4}{\sqrt{c^2dx^2 + d}}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^4/sqrt(c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b\text{arcsinh}(cx))}{\sqrt{d + c^2dx^2}}\,dx = \int \frac{x^4\,(a + b\,\text{asinh}(c\,x))}{\sqrt{d\,c^2\,x^2 + d}}\,dx$$

```
[In] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

3.147 $\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 142

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{2bx\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}}-\frac{bx^3\sqrt{1+c^2x^2}}{9c\sqrt{d+c^2dx^2}}-\frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3c^4d}+\frac{x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3c^2d}$$

```
[Out] 2/3*b*x*(c^2*x^2+1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-1/9*b*x^3*(c^2*x^2+1)^(1/
2)/c/(c^2*d*x^2+d)^(1/2)-2/3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4/d+1
/3*x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5812, 5798, 8, 30}

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{3c^2d}-\frac{2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{3c^4d}-\frac{bx^3\sqrt{c^2x^2+1}}{9c\sqrt{c^2dx^2+d}}+\frac{2bx\sqrt{c^2x^2+1}}{3c^3\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (2*b*x*Sqrt[1 + c^2*x^2])/(3*c^3*Sqrt[d + c^2*d*x^2]) - (b*x^3*Sqrt[1 + c^2
*x^2])/(9*c*Sqrt[d + c^2*d*x^2]) - (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x]))/(3*c^4*d) + (x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*c^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3c^2d} - \frac{2\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx}{3c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int x^2\,dx}{3c\sqrt{d+c^2dx^2}} \\
&= -\frac{bx^3\sqrt{1+c^2x^2}}{9c\sqrt{d+c^2dx^2}} - \frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3c^4d} \\
&\quad + \frac{x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3c^2d} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int 1\,dx}{3c^3\sqrt{d+c^2dx^2}} \\
&= \frac{2bx\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}} - \frac{bx^3\sqrt{1+c^2x^2}}{9c\sqrt{d+c^2dx^2}} - \frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3c^4d} \\
&\quad + \frac{x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3c^2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.65

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{bcx(6-c^2x^2)\sqrt{1+c^2x^2}+3a(-2-c^2x^2+c^4x^4)+3b(-2-c^2x^2+c^4x^4)\operatorname{arcsinh}(cx)}{9c^4\sqrt{d+c^2dx^2}}$$

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (b*c*x*(6 - c^2*x^2)*Sqrt[1 + c^2*x^2] + 3*a*(-2 - c^2*x^2 + c^4*x^4) + 3*b
*(-2 - c^2*x^2 + c^4*x^4)*ArcSinh[c*x])/(9*c^4*Sqrt[d + c^2*d*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(122) = 244.

Time = 0.20 (sec) , antiderivative size = 358, normalized size of antiderivative = 2.52

method	result
default	$a\left(\frac{x^2\sqrt{c^2d\,x^2+d}}{3c^2d}-\frac{2\sqrt{c^2d\,x^2+d}}{3d\,c^4}\right)+b\left(\frac{\sqrt{d(c^2x^2+1)}\left(4c^4x^4+4c^3x^3\sqrt{c^2x^2+1}+5c^2x^2+3cx\sqrt{c^2x^2+1}+1\right)(-1+3\operatorname{arcsinh}(cx))}{72c^4d(c^2x^2+1)}\right.$
parts	$a\left(\frac{x^2\sqrt{c^2d\,x^2+d}}{3c^2d}-\frac{2\sqrt{c^2d\,x^2+d}}{3d\,c^4}\right)+b\left(\frac{\sqrt{d(c^2x^2+1)}\left(4c^4x^4+4c^3x^3\sqrt{c^2x^2+1}+5c^2x^2+3cx\sqrt{c^2x^2+1}+1\right)(-1+3\operatorname{arcsinh}(cx))}{72c^4d(c^2x^2+1)}\right.$

```
[In] int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/3*x^2/c^2/d*(c^2*d*x^2+d)^(1/2)-2/3/d/c^4*(c^2*d*x^2+d)^(1/2))+b*(1/72
*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c
*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))/c^4/d/(c^2*x^2+1)-3/8*(d*(c^2*x
^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))/c^4/d/(c^2
*x^2+1)-3/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsin
h(c*x)+1)/c^4/d/(c^2*x^2+1)+1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3
*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(3*arcsinh(c*x)+1)/
c^4/d/(c^2*x^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.93

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{3\left(bc^4x^4-bc^2x^2-2\,b\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(3\,ac^4x^4-3\,ac^2x^2-\left(bc^3x^3-6\,bcx\right)\sqrt{c^2x^2+1}\right.}{9\left(c^6dx^2+c^4d\right)}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(3*(b*c^4*x^4 - b*c^2*x^2 - 2*b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2
*x^2 + 1)) + (3*a*c^4*x^4 - 3*a*c^2*x^2 - (b*c^3*x^3 - 6*b*c*x)*sqrt(c^2*x^
2 + 1) - 6*a)*sqrt(c^2*d*x^2 + d))/(c^6*d*x^2 + c^4*d)
```

Sympy [F]

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{x^3\left(a+b\operatorname{asinh}\left(cx\right)\right)}{\sqrt{d\left(c^2x^2+1\right)}}\,dx$$

```
[In] integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.82

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{1}{3}\,b\left(\frac{\sqrt{c^2dx^2+d}x^2}{c^2d}-\frac{2\sqrt{c^2dx^2+d}}{c^4d}\right)\operatorname{arsinh}\left(cx\right) + \frac{1}{3}\,a\left(\frac{\sqrt{c^2dx^2+d}x^2}{c^2d}-\frac{2\sqrt{c^2dx^2+d}}{c^4d}\right)-\frac{\left(c^2x^3-6\,x\right)b}{9\,c^3\sqrt{d}}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*b*(sqrt(c^2*d*x^2 + d)*x^2/(c^2*d) - 2*sqrt(c^2*d*x^2 + d)/(c^4*d))*arc
sinh(c*x) + 1/3*a*(sqrt(c^2*d*x^2 + d)*x^2/(c^2*d) - 2*sqrt(c^2*d*x^2 + d)/
(c^4*d)) - 1/9*(c^2*x^3 - 6*x)*b/(c^3*sqrt(d))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\text{arcsinh}(cx))}{\sqrt{d + c^2dx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\text{arcsinh}(cx))}{\sqrt{d + c^2dx^2}}\,dx = \int \frac{x^3\,(a + b\,\text{asinh}(c\,x))}{\sqrt{d\,c^2\,x^2 + d}}\,dx$$

```
[In] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

3.148 $\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 119

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = -\frac{bx^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}}+\frac{x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2c^2d} - \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{4bc^3\sqrt{d+c^2dx^2}}$$

```
[Out] -1/4*b*x^2*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)-1/4*(a+b*arcsinh(c*x))^2
*(c^2*x^2+1)^(1/2)/b/c^3/(c^2*d*x^2+d)^(1/2)+1/2*x*(a+b*arcsinh(c*x))*(c^2*
d*x^2+d)^(1/2)/c^2/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5812, 5783, 30}

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2c^2d} - \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{4bc^3\sqrt{c^2dx^2+d}} - \frac{bx^2\sqrt{c^2x^2+1}}{4c\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] -1/4*(b*x^2*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2]) + (x*Sqrt[d + c^2*d*
x^2]*(a + b*ArcSinh[c*x]))/(2*c^2*d) - (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*
x])^2)/(4*b*c^3*Sqrt[d + c^2*d*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2c^2d} - \frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx}{2c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int x\,dx}{2c\sqrt{d+c^2dx^2}}\\ &= -\frac{bx^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}} + \frac{x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2c^2d} - \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{4bc^3\sqrt{d+c^2dx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.02

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = -\frac{-\frac{4acx\sqrt{d+c^2dx^2}}{d} + \frac{4a\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)}{\sqrt{d}} + \frac{b\sqrt{1+c^2x^2}(\cosh(2\text{arcsinh}(cx))+2\text{arcsinh}(cx)(\text{arcsinh}(cx)-\sinh(2\text{arcsinh}(cx\ldots}{\sqrt{d+c^2dx^2}}}{8c^3}$$

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] -1/8*((-4*a*c*x*Sqrt[d + c^2*d*x^2])/d + (4*a*Log[c*d*x + Sqrt[d]*Sqrt[d +
c^2*d*x^2]])/Sqrt[d] + (b*Sqrt[1 + c^2*x^2]*(Cosh[2*ArcSinh[c*x]] + 2*ArcSi
nh[c*x]*(ArcSinh[c*x] - Sinh[2*ArcSinh[c*x]])))/Sqrt[d + c^2*d*x^2])/c^3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(103) = 206$.

Time = 0.21 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.29

method	result
default	$\frac{ax\sqrt{c^2dx^2+d}}{2c^2d} - \frac{a\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{2c^2\sqrt{c^2d}} + b\left(-\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2}{4\sqrt{c^2x^2+1}\,c^3d} + \frac{\sqrt{d(c^2x^2+1)}\,\left(2c^3x^3+2c^2x^2\sqrt{c^2x^2+1}+2cx+\sqrt{\cdots}\right.}{16c^3d(c^2x^2+1)}\right.$
parts	$\frac{ax\sqrt{c^2dx^2+d}}{2c^2d} - \frac{a\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{2c^2\sqrt{c^2d}} + b\left(-\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2}{4\sqrt{c^2x^2+1}\,c^3d} + \frac{\sqrt{d(c^2x^2+1)}\,\left(2c^3x^3+2c^2x^2\sqrt{c^2x^2+1}+2cx+\sqrt{\cdots}\right.}{16c^3d(c^2x^2+1)}\right.$

```
[In] int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x/c^2/d*(c^2*d*x^2+d)^(1/2)-1/2*a/c^2*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d
*x^2+d)^(1/2))/(c^2*d)^(1/2)+b*(-1/4*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2
)/c^3/d*arcsinh(c*x)^2+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2
*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*x))/c^3/d/(c^2*x^2+1
)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(
c^2*x^2+1)^(1/2))*(1+2*arcsinh(c*x))/c^3/d/(c^2*x^2+1))
```

Fricas [F]

$$\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x^2}{\sqrt{c^2dx^2+d}}\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arcsinh(c*x) + a*x^2)/sqrt(c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{x^2(a+b\operatorname{asinh}(cx))}{\sqrt{d(c^2x^2+1)}}\,dx$$

```
[In] integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^2}{\sqrt{c^2dx^2+d}}\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)*x^2/sqrt(c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{x^2\,(a+b\,\text{asinh}(c\,x))}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

```
[In] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
[Out] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

3.149 $\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=-\frac{bx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}}+\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{c^2d}$$

```
[Out] -b*x*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)+(a+b*arcsinh(c*x))*(c^2*d*x^2+
d)^(1/2)/c^2/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5798, 8}

$$\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{c^2d}-\frac{bx\sqrt{c^2x^2+1}}{c\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] -((b*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2])) + (Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x]))/(c^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
```

```
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{c^2d} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int 1\,dx}{c\sqrt{d+c^2dx^2}}$$
$$= -\frac{bx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} + \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{c^2d}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.16

$$\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{d+c^2dx^2}\left(-bcx+a\sqrt{1+c^2x^2}+b\sqrt{1+c^2x^2}\text{arcsinh}(cx)\right)}{c^2d\sqrt{1+c^2x^2}}$$

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
[Out] (Sqrt[d + c^2*d*x^2]*(-(b*c*x) + a*Sqrt[1 + c^2*x^2] + b*Sqrt[1 + c^2*x^2]*
ArcSinh[c*x]))/(c^2*d*Sqrt[1 + c^2*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(58) = 116.

Time = 0.24 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.31

method	result
default	$\frac{a\sqrt{c^2d\,x^2+d}}{c^2d}+b\left(\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2+cx\sqrt{c^2x^2+1}+1\right)(-1+\text{arcsinh}(cx))}{2c^2d(c^2x^2+1)}+\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2-cx\sqrt{c^2x^2+1}+1\right)(\text{arcsinh}(cx)}{2c^2d(c^2x^2+1)}\right.$
parts	$\frac{a\sqrt{c^2d\,x^2+d}}{c^2d}+b\left(\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2+cx\sqrt{c^2x^2+1}+1\right)(-1+\text{arcsinh}(cx))}{2c^2d(c^2x^2+1)}+\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2-cx\sqrt{c^2x^2+1}+1\right)(\text{arcsinh}(cx)}{2c^2d(c^2x^2+1)}\right.$

```
[In] int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
[Out] a/c^2/d*(c^2*d*x^2+d)^(1/2)+b*(1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*
x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))/c^2/d/(c^2*x^2+1)+1/2*(d*(c^2*x^2+1))^(1/
2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)+1)/c^2/d/(c^2*x^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.50

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{(bc^2x^2+b)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(ac^2x^2-\sqrt{c^2x^2+1}bcx+a\right)\sqrt{c^2dx^2+d}}{c^4dx^2+c^2d}$$

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
[Out] ((b*c^2*x^2 + b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*c^2*
x^2 - sqrt(c^2*x^2 + 1)*b*c*x + a)*sqrt(c^2*d*x^2 + d))/(c^4*d*x^2 + c^2*d)
```

Sympy [F]

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{x(a+b\operatorname{asinh}(cx))}{\sqrt{d(c^2x^2+1)}}\,dx$$

```
[In] integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
[Out] Integral(x*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = -\frac{bx}{c\sqrt{d}} + \frac{\sqrt{c^2dx^2+d}\,b\operatorname{arsinh}(cx)}{c^2d} + \frac{\sqrt{c^2dx^2+d}\,a}{c^2d}$$

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
[Out] -b*x/(c*sqrt(d)) + sqrt(c^2*d*x^2 + d)*b*arcsinh(c*x)/(c^2*d) + sqrt(c^2*d*
x^2 + d)*a/(c^2*d)
```

Giac [F]

$$\int \frac{x(a + b\text{arcsinh}(cx))}{\sqrt{d + c^2dx^2}}\, dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)x}{\sqrt{c^2dx^2 + d}}\, dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)*x/sqrt(c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b\text{arcsinh}(cx))}{\sqrt{d + c^2dx^2}}\, dx = \int \frac{x\,(a + b\,\text{asinh}(c\,x))}{\sqrt{d\,c^2\,x^2 + d}}\, dx$$

```
[In] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
[Out] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

3.150 $\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 47

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx=\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+c^2dx^2}}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {5783}

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx=\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{c^2dx^2+d}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + c^2*d*x^2])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\text{integral}=\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx=\frac{b\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2}{2c\sqrt{d(1+c^2x^2)}}+\frac{a\text{arctanh}\left(\frac{c\sqrt{d}x}{\sqrt{d+c^2dx^2}}\right)}{c\sqrt{d}}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/(2*c*Sqrt[d*(1 + c^2*x^2)]) + (a*ArcTa
nh[(c*Sqrt[d]*x)/Sqrt[d + c^2*d*x^2]])/(c*Sqrt[d])
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.64

method	result	size
default	$\frac{a\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{\sqrt{c^2d}}+\frac{b\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)^2}{2\sqrt{c^2x^2+1}\,cd}$	77
parts	$\frac{a\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{\sqrt{c^2d}}+\frac{b\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)^2}{2\sqrt{c^2x^2+1}\,cd}$	77

```
[In] int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/2*b*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d*arcsinh(c*x)^2
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+d}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{a+b\operatorname{asinh}{(cx)}}{\sqrt{d\,(c^2x^2+1)}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
[Out] Integral((a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.60

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx=\frac{b\,\text{arsinh}\,(cx)^2}{2\,c\sqrt{d}}+\frac{a\,\text{arsinh}\,(cx)}{c\sqrt{d}}$$

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
[Out] 1/2*b*arcsinh(c*x)^2/(c*sqrt(d)) + a*arcsinh(c*x)/(c*sqrt(d))
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+d}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{a+b\,\mathrm{asinh}(c\,x)}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

```
[In] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(1/2),x)
[Out] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(1/2), x)
```

3.151 $\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 122

$$\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{d+c^2dx^2}}\,dx = -\frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} + \frac{b\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}}$$

```
[Out] -2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2
*d*x^2+d)^(1/2)-b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*
d*x^2+d)^(1/2)+b*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*
x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5816, 4267, 2317, 2438}

$$\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{d+c^2dx^2}}\,dx = -\frac{2\sqrt{c^2x^2+1}\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}} - \frac{b\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{c^2dx^2+d}} + \frac{b\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{c^2dx^2+d}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (-2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[d
+ c^2*d*x^2] - (b*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt[d + c
^2*d*x^2] + (b*Sqrt[1 + c^2*x^2]*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[d + c^2*d
*x^2]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1+c^2x^2}\text{Subst}\left(\int (a+bx)\text{csch}(x)\,dx, x, \text{arcsinh}(cx)\right)}{\sqrt{d+c^2dx^2}} \\
&= -\frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\left(b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \log\left(1+e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{\sqrt{d+c^2dx^2}}
\end{aligned}
$$

$$= -\frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{\left(b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} + \frac{\left(b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}}$$

$$= -\frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} + \frac{b\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.06

$$\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{d+c^2dx^2}}\,dx = \frac{a\log(x)}{\sqrt{d}} - \frac{a\log\left(d+\sqrt{d}\sqrt{d(1+c^2x^2)}\right)}{\sqrt{d}} + \frac{b\sqrt{1+c^2x^2}\left(\text{arcsinh}(cx)\left(\log\left(1-e^{-\text{arcsinh}(cx)}\right)-\log\left(1+e^{-\text{arcsinh}(cx)}\right)\right)+\text{PolyLog}\left(2,-e^{-\text{arcsinh}(cx)}\right)-\right.}{\sqrt{d(1+c^2x^2)}}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (a*Log[x])/Sqrt[d] - (a*Log[d + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]])/Sqrt[d] + (
b*Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(Log[1 - E^(-ArcSinh[c*x])] - Log[1 + E^(
-ArcSinh[c*x])]) + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[
c*x])]))/Sqrt[d*(1 + c^2*x^2)]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.91

method	result
default	$-\frac{a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)}{\sqrt{d}}+b\left(\frac{\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)\ln\left(1-cx-\sqrt{c^2x^2+1}\right)}{\sqrt{c^2x^2+1}\,d}+\frac{\sqrt{d(c^2x^2+1)}\,\text{polylog}\left(2,cx+\sqrt{c^2x^2+1}\right)}{\sqrt{c^2x^2+1}\,d}-\right.$
parts	$-\frac{a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)}{\sqrt{d}}+b\left(\frac{\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)\ln\left(1-cx-\sqrt{c^2x^2+1}\right)}{\sqrt{c^2x^2+1}\,d}+\frac{\sqrt{d(c^2x^2+1)}\,\text{polylog}\left(2,cx+\sqrt{c^2x^2+1}\right)}{\sqrt{c^2x^2+1}\,d}-\right.$

```
[In] int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] -a/d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)+b*((d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,c*x+(c^2*x^2+1)^(1/2))-(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,-c*x-(c^2*x^2+1)^(1/2)))

Fricas [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x\sqrt{d + c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{\sqrt{c^2dx^2 + d}x}\,dx$$

[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^2*d*x^3 + d*x), x)

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x\sqrt{d + c^2dx^2}}\,dx = \int \frac{a + b\,\text{asinh}\,(cx)}{x\sqrt{d\,(c^2x^2 + 1)}}\,dx$$

[In] integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d)**(1/2),x)

[Out] Integral((a + b*asinh(c*x))/(x*sqrt(d*(c**2*x**2 + 1))), x)

Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x\sqrt{d + c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{\sqrt{c^2dx^2 + d}x}\,dx$$

[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")

[Out] b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(c^2*d*x^2 + d)*x), x) - a*arcsinh(1/(c*abs(x)))/sqrt(d)

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x\sqrt{d + c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{\sqrt{c^2dx^2 + d}x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{x\sqrt{d + c^2dx^2}}\,dx = \int \frac{a + b\,\text{asinh}(c\,x)}{x\,\sqrt{d\,c^2\,x^2 + d}}\,dx$$

```
[In] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^(1/2)),x)
[Out] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^(1/2)), x)
```

3.152 $\int \frac{a+b\,\text{arcsinh}(cx)}{x^2\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 63

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{d+c^2dx^2}}\,dx = -\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{dx}+\frac{bc\sqrt{1+c^2x^2}\log(x)}{\sqrt{d+c^2dx^2}}$$

```
[Out] b*c*ln(x)*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-(a+b*arcsinh(c*x))*(c^2*d*x
^2+d)^(1/2)/d/x
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5800, 29}

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{d+c^2dx^2}}\,dx = \frac{bc\sqrt{c^2x^2+1}\log(x)}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{dx}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] -((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(d*x)) + (b*c*Sqrt[1 + c^2*x^2
]*Log[x])/Sqrt[d + c^2*d*x^2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
```

```
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{dx}+\frac{\left(bc\sqrt{1+c^2x^2}\right)\int\frac{1}{x}\,dx}{\sqrt{d+c^2dx^2}}\\ &= -\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{dx}+\frac{bc\sqrt{1+c^2x^2}\log(x)}{\sqrt{d+c^2dx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.22

$$\int\frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{d+c^2dx^2}}\,dx=\frac{\sqrt{d+c^2dx^2}\left(-a\sqrt{1+c^2x^2}-b\sqrt{1+c^2x^2}\text{arcsinh}(cx)+bcx\log(x)\right)}{dx\sqrt{1+c^2x^2}}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*Sqrt[d + c^2*d*x^2]),x]
[Out] (Sqrt[d + c^2*d*x^2]*(-(a*Sqrt[1 + c^2*x^2]) - b*Sqrt[1 + c^2*x^2]*ArcSinh[
c*x] + b*c*x*Log[x]))/(d*x*Sqrt[1 + c^2*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(57) = 114.

Time = 0.24 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.68

method	result
default	$-\frac{a\sqrt{c^2d\,x^2+d}}{dx}+b\left(-\frac{2\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)c}{\sqrt{c^2x^2+1}\,d}-\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2-cx\sqrt{c^2x^2+1}+1\right)\text{arcsinh}(cx)}{(c^2x^2+1)dx}+\frac{\sqrt{d(c^2x^2+1)}\,\ln\left(\left(\right.\right.}{\sqrt{c^2}}\right.$
parts	$-\frac{a\sqrt{c^2d\,x^2+d}}{dx}+b\left(-\frac{2\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)c}{\sqrt{c^2x^2+1}\,d}-\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2-cx\sqrt{c^2x^2+1}+1\right)\text{arcsinh}(cx)}{(c^2x^2+1)dx}+\frac{\sqrt{d(c^2x^2+1)}\,\ln\left(\left(\right.\right.}{\sqrt{c^2}}\right.$

```
[In] int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
[Out] -a/d/x*(c^2*d*x^2+d)^(1/2)+b*(-2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*
arcsinh(c*x)*c-(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*arcs
inh(c*x)/(c^2*x^2+1)/d/x+(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*ln((c*x+
(c^2*x^2+1)^(1/2))^2-1)*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(57) = 114$.

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.10

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2\sqrt{d + c^2dx^2}}\,dx = \frac{bc\sqrt{d}x\log\left(\frac{c^2dx^6+c^2dx^2+dx^4+\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}(x^4-1)\sqrt{d}+d}{c^2x^4+x^2}\right) - 2\sqrt{c^2dx^2+d}b\log\left(cx+\sqrt{c^2x^2+1}\right) - 2\sqrt{c^2dx^2+\cdots}}{2\,dx}$$

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*c*sqrt(d)*x*log((c^2*d*x^6 + c^2*d*x^2 + d*x^4 + sqrt(c^2*d*x^2 + d)
*sqrt(c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) + d)/(c^2*x^4 + x^2)) - 2*sqrt(c^2*d*x
^2 + d)*b*log(c*x + sqrt(c^2*x^2 + 1)) - 2*sqrt(c^2*d*x^2 + d)*a)/(d*x)
```

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2\sqrt{d + c^2dx^2}}\,dx = \int \frac{a + b\operatorname{asinh}(cx)}{x^2\sqrt{d\,(c^2x^2+1)}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(x**2*sqrt(d*(c**2*x**2 + 1))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.60

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2\sqrt{d + c^2dx^2}}\,dx = -\frac{\left((-1)^{2\,c^2dx^2+2\,d}\sqrt{d}\log\left(2\,c^2d + \frac{2\,d}{x^2}\right) - \sqrt{d}\log\left(x^2 + \frac{1}{c^2}\right)\right)bc}{2\,d} - \frac{\sqrt{c^2dx^2+d}b\operatorname{arsinh}(cx)}{dx} - \frac{\sqrt{c^2dx^2+d}a}{dx}$$

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*((-1)^(2*c^2*d*x^2 + 2*d)*sqrt(d)*log(2*c^2*d + 2*d/x^2) - sqrt(d)*log
(x^2 + 1/c^2))*b*c/d - sqrt(c^2*d*x^2 + d)*b*arcsinh(c*x)/(d*x) - sqrt(c^2*
d*x^2 + d)*a/(d*x)
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{d+c^2dx^2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+d}x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{d+c^2dx^2}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{x^2\,\sqrt{d\,c^2\,x^2+d}}\,dx$$

```
[In] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^(1/2)),x)
[Out] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^(1/2)), x)
```

3.153 $\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 203

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{d+c^2dx^2}}\,dx = -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2dx^2} + \frac{c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} + \frac{bc^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2\sqrt{d+c^2dx^2}} - \frac{bc^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2\sqrt{d+c^2dx^2}}$$

```
[Out] -1/2*b*c*(c^2*x^2+1)^(1/2)/x/(c^2*d*x^2+d)^(1/2)+c^2*(a+b*arcsinh(c*x))*arc
tanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+1/2*b*c^2
*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-1/
2*b*c^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1
/2)-1/2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/d/x^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used

= {5809, 5816, 4267, 2317, 2438, 30}

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{d+c^2dx^2}}\,dx = \frac{c^2\sqrt{c^2x^2+1}\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}} - \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2dx^2} + \frac{bc^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2\sqrt{c^2dx^2+d}} - \frac{bc^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2\sqrt{c^2dx^2+d}} - \frac{bc\sqrt{c^2x^2+1}}{2x\sqrt{c^2dx^2+d}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] -1/2*(b*c*Sqrt[1 + c^2*x^2])/(x*Sqrt[d + c^2*d*x^2]) - (Sqrt[d + c^2*d*x^2]
*(a + b*ArcSinh[c*x]))/(2*d*x^2) + (c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*
x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] + (b*c^2*Sqrt[1 + c^2*x^2]
*PolyLog[2, -E^ArcSinh[c*x]])/(2*Sqrt[d + c^2*d*x^2]) - (b*c^2*Sqrt[1 + c^2
*x^2]*PolyLog[2, E^ArcSinh[c*x]])/(2*Sqrt[d + c^2*d*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
```

```
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2dx^2} \\
&\quad -\frac{1}{2}c^2\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{d+c^2dx^2}}\,dx+\frac{\left(bc\sqrt{1+c^2x^2}\right)\int\frac{1}{x^2}\,dx}{2\sqrt{d+c^2dx^2}} \\
&= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2dx^2} \\
&\quad -\frac{\left(c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\text{csch}(x)\,dx,x,\text{arcsinh}(cx)\right)}{2\sqrt{d+c^2dx^2}} \\
&= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2dx^2} \\
&\quad +\frac{c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} \\
&\quad +\frac{\left(bc^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1-e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{2\sqrt{d+c^2dx^2}} \\
&\quad -\frac{\left(bc^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1+e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{2\sqrt{d+c^2dx^2}} \\
&= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2dx^2} \\
&\quad +\frac{c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} \\
&\quad +\frac{\left(bc^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{2\sqrt{d+c^2dx^2}} \\
&\quad -\frac{\left(bc^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{2\sqrt{d+c^2dx^2}}
\end{aligned}
$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2dx^2} + \frac{c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} + \frac{bc^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2\sqrt{d+c^2dx^2}} - \frac{bc^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 2.28 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.13

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{d+c^2dx^2}}\,dx$$

$$= \frac{-\frac{4a\sqrt{d+c^2dx^2}}{x^2} - 4ac^2\sqrt{d}\log(x) + 4ac^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right) + \frac{bc^2d^2(1+c^2x^2)^{3/2}\left(-2\coth\left(\frac{1}{2}\text{arcsinh}(cx)\right)-\text{a}\right.}{}}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] ((-4*a*Sqrt[d + c^2*d*x^2])/x^2 - 4*a*c^2*Sqrt[d]*Log[x] + 4*a*c^2*Sqrt[d]*
Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*c^2*d^2*(1 + c^2*x^2)^(3/2)*(-2*C
oth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 4*ArcSinh[c*x]*
Log[1 - E^(-ArcSinh[c*x])] + 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 4*
PolyLog[2, -E^(-ArcSinh[c*x])] + 4*PolyLog[2, E^(-ArcSinh[c*x])] - ArcSinh[
c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/(d + c^2*d*x^2)^(3/2
))/(8*d)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.64

method	result
default	$-\frac{a\sqrt{c^2d\,x^2+d}}{2d\,x^2}+\frac{a\,c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)}{2\sqrt{d}}+b\left(-\frac{\left(\text{arcsinh}(cx)c^2x^2+cx\sqrt{c^2x^2+1}+\text{arcsinh}(cx)\right)\sqrt{d(c^2x^2+1)}}{2x^2d(c^2x^2+1)}-\frac{\sqrt{d(c^2x^2}}{}\right.$
parts	$-\frac{a\sqrt{c^2d\,x^2+d}}{2d\,x^2}+\frac{a\,c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)}{2\sqrt{d}}+b\left(-\frac{\left(\text{arcsinh}(cx)c^2x^2+cx\sqrt{c^2x^2+1}+\text{arcsinh}(cx)\right)\sqrt{d(c^2x^2+1)}}{2x^2d(c^2x^2+1)}-\frac{\sqrt{d(c^2x^2}}{}\right.$

```
[In] int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d/x^2*(c^2*d*x^2+d)^(1/2)+1/2*a*c^2/d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d
*x^2+d)^(1/2))/x)+b*(-1/2*(arcsinh(c*x)*c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+arcsi
nh(c*x))*(d*(c^2*x^2+1))^(1/2)/x^2/d/(c^2*x^2+1)-1/2*(d*(c^2*x^2+1))^(1/2)/
(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^2-1/2*(d*(c^
```

```
2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^2+1/
2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+
1)^(1/2))*c^2+1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,-c*x-
(c^2*x^2+1)^(1/2))*c^2)
```

Fricas [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^3\sqrt{d + c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{\sqrt{c^2dx^2 + dx^3}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^2*d*x^5 + d*x^3), x)
```

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^3\sqrt{d + c^2dx^2}}\,dx = \int \frac{a + b\,\text{asinh}\,(cx)}{x^3\sqrt{d\,(c^2x^2 + 1)}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d)**(1/2),x)
[Out] Integral((a + b*asinh(c*x))/(x**3*sqrt(d*(c**2*x**2 + 1))), x)
```

Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^3\sqrt{d + c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{\sqrt{c^2dx^2 + dx^3}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
[Out] 1/2*(c^2*arcsinh(1/(c*abs(x)))/sqrt(d) - sqrt(c^2*d*x^2 + d)/(d*x^2))*a + b
*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(c^2*d*x^2 + d)*x^3), x)
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^3\sqrt{d + c^2dx^2}}\, dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{\sqrt{c^2dx^2 + d}x^3}\, dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{x^3\sqrt{d + c^2dx^2}}\, dx = \int \frac{a + b\,\text{asinh}(c\,x)}{x^3\,\sqrt{d\,c^2\,x^2 + d}}\, dx$$

```
[In] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^(1/2)),x)
[Out] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^(1/2)), x)
```

3.154 $\int \frac{a+b\text{arcsinh}(cx)}{x^4\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 141

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\sqrt{d+c^2dx^2}}\,dx = -\frac{bc\sqrt{1+c^2x^2}}{6x^2\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3dx^3} + \frac{2c^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3dx} - \frac{2bc^3\sqrt{1+c^2x^2}\log(x)}{3\sqrt{d+c^2dx^2}}$$

```
[Out] -1/6*b*c*(c^2*x^2+1)^(1/2)/x^2/(c^2*d*x^2+d)^(1/2)-2/3*b*c^3*ln(x)*(c^2*x^2
+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-1/3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/d/
x^3+2/3*c^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/d/x
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5809, 5800, 29, 30}

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\sqrt{d+c^2dx^2}}\,dx = \frac{2c^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{3dx} - \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{3dx^3} - \frac{bc\sqrt{c^2x^2+1}}{6x^2\sqrt{c^2dx^2+d}} - \frac{2bc^3\sqrt{c^2x^2+1}\log(x)}{3\sqrt{c^2dx^2+d}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] -1/6*(b*c*Sqrt[1 + c^2*x^2])/(x^2*Sqrt[d + c^2*d*x^2]) - (Sqrt[d + c^2*d*x^
2]*(a + b*ArcSinh[c*x]))/(3*d*x^3) + (2*c^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcS
inh[c*x]))/(3*d*x) - (2*b*c^3*Sqrt[1 + c^2*x^2]*Log[x])/(3*Sqrt[d + c^2*d*x
^2])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3dx^3} \\
&\quad -\frac{1}{3}\left(2c^2\right)\int\frac{a+b\operatorname{arcsinh}(cx)}{x^2\sqrt{d+c^2dx^2}}\,dx+\frac{\left(bc\sqrt{1+c^2x^2}\right)\int\frac{1}{x^3}\,dx}{3\sqrt{d+c^2dx^2}} \\
&= -\frac{bc\sqrt{1+c^2x^2}}{6x^2\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3dx^3} \\
&\quad +\frac{2c^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3dx}-\frac{\left(2bc^3\sqrt{1+c^2x^2}\right)\int\frac{1}{x}\,dx}{3\sqrt{d+c^2dx^2}} \\
&= -\frac{bc\sqrt{1+c^2x^2}}{6x^2\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3dx^3} \\
&\quad +\frac{2c^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3dx}-\frac{2bc^3\sqrt{1+c^2x^2}\log(x)}{3\sqrt{d+c^2dx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.93

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{d+c^2dx^2}\left(-bcx+6bc^3x^3-2a\sqrt{1+c^2x^2}+4ac^2x^2\sqrt{1+c^2x^2}+2b\sqrt{1+c^2x^2}(-1+2c^2x^2)\,\text{arcsinh}(cx)-\right.}{6dx^3\sqrt{1+c^2x^2}}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(-(b*c*x) + 6*b*c^3*x^3 - 2*a*Sqrt[1 + c^2*x^2] + 4*a*
c^2*x^2*Sqrt[1 + c^2*x^2] + 2*b*Sqrt[1 + c^2*x^2]*(-1 + 2*c^2*x^2)*ArcSinh[
c*x] - 4*b*c^3*x^3*Log[x]))/(6*d*x^3*Sqrt[1 + c^2*x^2])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.18

method	result
default	$a\left(-\frac{\sqrt{c^2dx^2+d}}{3dx^3}+\frac{2c^2\sqrt{c^2dx^2+d}}{3dx}\right)+\frac{b\sqrt{d(c^2x^2+1)}\left(4\,\text{arcsinh}(cx)c^3x^3-4\ln\left(\left(cx+\sqrt{c^2x^2+1}\right)^2-1\right)x^3c^3+4\,\text{arcsinh}(cx)\sqrt{c^2x^2}\right.}{6\sqrt{c^2x^2+1}\,dx^3}$
parts	$a\left(-\frac{\sqrt{c^2dx^2+d}}{3dx^3}+\frac{2c^2\sqrt{c^2dx^2+d}}{3dx}\right)+\frac{b\sqrt{d(c^2x^2+1)}\left(4\,\text{arcsinh}(cx)c^3x^3-4\ln\left(\left(cx+\sqrt{c^2x^2+1}\right)^2-1\right)x^3c^3+4\,\text{arcsinh}(cx)\sqrt{c^2x^2}\right.}{6\sqrt{c^2x^2+1}\,dx^3}$

```
[In] int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/3/d/x^3*(c^2*d*x^2+d)^(1/2)+2/3*c^2/d/x*(c^2*d*x^2+d)^(1/2))+1/6*b*(d
*(c^2*x^2+1))^(1/2)*(4*arcsinh(c*x)*c^3*x^3-4*ln((c*x+(c^2*x^2+1)^(1/2))^2-
1)*x^3*c^3+4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2-2*arcsinh(c*x)*(c^2*x^2
+1)^(1/2)-c*x)/(c^2*x^2+1)^(1/2)/d/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.57

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\sqrt{d+c^2dx^2}}\,dx = \frac{2\left(2\,bc^4x^4+bc^2x^2-b\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)+2\left(bc^5x^5+bc^3x^3\right)\sqrt{d}\log\left(\frac{c^2dx^6+c^2dx^2+dx^4-\sqrt{c^2dx^2+}}{c^2x^4+}\right.}{6\left(c^2dx^5+dx^3\right)}$$

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(2*b*c^4*x^4 + b*c^2*x^2 - b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2
*x^2 + 1)) + 2*(b*c^5*x^5 + b*c^3*x^3)*sqrt(d)*log((c^2*d*x^6 + c^2*d*x^2 +
 d*x^4 - sqrt(c^2*d*x^2 + d)*sqrt(c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) + d)/(c^2*
x^4 + x^2)) + (4*a*c^4*x^4 + 2*a*c^2*x^2 + (b*c*x^3 - b*c*x)*sqrt(c^2*x^2 +
 1) - 2*a)*sqrt(c^2*d*x^2 + d))/(c^2*d*x^5 + d*x^3)
```

Sympy [F]

$$\int \frac{a + b\mathrm{arcsinh}(cx)}{x^4\sqrt{d + c^2dx^2}}\,dx = \int \frac{a + b\,\mathrm{asinh}\,(cx)}{x^4\sqrt{d\,(c^2x^2 + 1)}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(x**4*sqrt(d*(c**2*x**2 + 1))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86

$$\int \frac{a + b\mathrm{arcsinh}(cx)}{x^4\sqrt{d + c^2dx^2}}\,dx = -\frac{1}{6}\left(\frac{4\,c^2\log(x)}{\sqrt{d}} + \frac{1}{\sqrt{d}x^2}\right)bc + \frac{1}{3}\,b\left(\frac{2\,\sqrt{c^2dx^2 + d}c^2}{dx} - \frac{\sqrt{c^2dx^2 + d}}{dx^3}\right)\mathrm{arsinh}\,(cx) + \frac{1}{3}\,a\left(\frac{2\,\sqrt{c^2dx^2 + d}c^2}{dx} - \frac{\sqrt{c^2dx^2 + d}}{dx^3}\right)$$

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*(4*c^2*log(x)/sqrt(d) + 1/(sqrt(d)*x^2))*b*c + 1/3*b*(2*sqrt(c^2*d*x^2
 + d)*c^2/(d*x) - sqrt(c^2*d*x^2 + d)/(d*x^3))*arcsinh(c*x) + 1/3*a*(2*sqrt
(c^2*d*x^2 + d)*c^2/(d*x) - sqrt(c^2*d*x^2 + d)/(d*x^3))
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^4\sqrt{d + c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{\sqrt{c^2dx^2 + dx^4}}\,dx$$

[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*x^4), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{x^4\sqrt{d + c^2dx^2}}\,dx = \int \frac{a + b\,\text{asinh}(c\,x)}{x^4\,\sqrt{d\,c^2\,x^2 + d}}\,dx$$

[In] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^(1/2)),x)

[Out] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^(1/2)), x)

3.155 $\int \frac{x^5(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 212

$$\begin{aligned}\int \frac{x^5(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx &= \frac{5bx\sqrt{d+c^2dx^2}}{3c^5d^2\sqrt{1+c^2x^2}}-\frac{bx^3\sqrt{d+c^2dx^2}}{9c^3d^2\sqrt{1+c^2x^2}}\\ &\quad-\frac{a+b\text{arcsinh}(cx)}{c^6d\sqrt{d+c^2dx^2}}-\frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{c^6d^2}\\ &\quad+\frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c^6d^3}+\frac{b\sqrt{d+c^2dx^2}\arctan(cx)}{c^6d^2\sqrt{1+c^2x^2}}\end{aligned}$$

```
[Out] 1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/c^6/d^3+(-a-b*arcsinh(c*x))/c^6/
d/(c^2*d*x^2+d)^(1/2)-2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^6/d^2+5/3*
b*x*(c^2*d*x^2+d)^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2)-1/9*b*x^3*(c^2*d*x^2+d)^(
1/2)/c^3/d^2/(c^2*x^2+1)^(1/2)+b*arctan(c*x)*(c^2*d*x^2+d)^(1/2)/c^6/d^2/(c
^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {272, 45, 5804, 12, 1167, 209}

$$\begin{aligned}\int \frac{x^5(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx &= \frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))}{3c^6d^3}\\ &\quad-\frac{2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{c^6d^2}-\frac{a+b\text{arcsinh}(cx)}{c^6d\sqrt{c^2dx^2+d}}\\ &\quad+\frac{b\arctan(cx)\sqrt{c^2dx^2+d}}{c^6d^2\sqrt{c^2x^2+1}}+\frac{5bx\sqrt{c^2dx^2+d}}{3c^5d^2\sqrt{c^2x^2+1}}-\frac{bx^3\sqrt{c^2dx^2+d}}{9c^3d^2\sqrt{c^2x^2+1}}\end{aligned}$$

```
[In] Int[(x^5*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (5*b*x*Sqrt[d + c^2*d*x^2])/(3*c^5*d^2*Sqrt[1 + c^2*x^2]) - (b*x^3*Sqrt[d +
 c^2*d*x^2])/(9*c^3*d^2*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(c^6*d*Sq
rt[d + c^2*d*x^2]) - (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(c^6*d^2)
 + ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^6*d^3) + (b*Sqrt[d + c
^2*d*x^2]*ArcTan[c*x])/(c^6*d^2*Sqrt[1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a+b\text{arcsinh}(cx)}{c^6d\sqrt{d+c^2dx^2}} - \frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{c^6d^2} \\
&\quad + \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c^6d^3} - \frac{\left(bc\sqrt{d+c^2dx^2}\right)\int \frac{-8-4c^2x^2+c^4x^4}{3c^6d^2(1+c^2x^2)}\,dx}{\sqrt{1+c^2x^2}} \\
&= -\frac{a+b\text{arcsinh}(cx)}{c^6d\sqrt{d+c^2dx^2}} - \frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{c^6d^2} \\
&\quad + \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c^6d^3} - \frac{\left(b\sqrt{d+c^2dx^2}\right)\int \frac{-8-4c^2x^2+c^4x^4}{1+c^2x^2}\,dx}{3c^5d^2\sqrt{1+c^2x^2}} \\
&= -\frac{a+b\text{arcsinh}(cx)}{c^6d\sqrt{d+c^2dx^2}} - \frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{c^6d^2} \\
&\quad + \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c^6d^3} - \frac{\left(b\sqrt{d+c^2dx^2}\right)\int \left(-5+c^2x^2-\frac{3}{1+c^2x^2}\right)dx}{3c^5d^2\sqrt{1+c^2x^2}} \\
&= \frac{5bx\sqrt{d+c^2dx^2}}{3c^5d^2\sqrt{1+c^2x^2}} - \frac{bx^3\sqrt{d+c^2dx^2}}{9c^3d^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{a+b\text{arcsinh}(cx)}{c^6d\sqrt{d+c^2dx^2}} - \frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{c^6d^2} \\
&\quad + \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c^6d^3} + \frac{\left(b\sqrt{d+c^2dx^2}\right)\int \frac{1}{1+c^2x^2}\,dx}{c^5d^2\sqrt{1+c^2x^2}} \\
&= \frac{5bx\sqrt{d+c^2dx^2}}{3c^5d^2\sqrt{1+c^2x^2}} - \frac{bx^3\sqrt{d+c^2dx^2}}{9c^3d^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{a+b\text{arcsinh}(cx)}{c^6d\sqrt{d+c^2dx^2}} - \frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{c^6d^2} \\
&\quad + \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c^6d^3} + \frac{b\sqrt{d+c^2dx^2}\arctan(cx)}{c^6d^2\sqrt{1+c^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.82

$$
\int \frac{x^5(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \frac{\sqrt{d+c^2dx^2}\left(15bcx+14bc^3x^3-bc^5x^5-24a\sqrt{1+c^2x^2}-12ac^2x^2\sqrt{1+c^2x^2}+\right.}{9c}
$$

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
[Out] (Sqrt[d + c^2*d*x^2]*(15*b*c*x + 14*b*c^3*x^3 - b*c^5*x^5 - 24*a*Sqrt[1 + c
^2*x^2] - 12*a*c^2*x^2*Sqrt[1 + c^2*x^2] + 3*a*c^4*x^4*Sqrt[1 + c^2*x^2] +
3*b*Sqrt[1 + c^2*x^2]*(-8 - 4*c^2*x^2 + c^4*x^4)*ArcSinh[c*x] + 9*(b + b*c^
2*x^2)*ArcTan[c*x]))/(9*c^6*d^2*(1 + c^2*x^2)^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.12

method	result
default	$a\left(\frac{x^4}{3c^2d\sqrt{c^2dx^2+d}}-\frac{4\left(\frac{x^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2}{d\,c^4\sqrt{c^2dx^2+d}}\right)}{3c^2}\right)+\frac{b\sqrt{d(c^2x^2+1)}\left(3\,\mathrm{arcsinh}(cx)c^4x^4-c^3x^3\sqrt{c^2x^2+1}-12\,\mathrm{arcsinh}(cx)\right.}{}$
parts	$a\left(\frac{x^4}{3c^2d\sqrt{c^2dx^2+d}}-\frac{4\left(\frac{x^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2}{d\,c^4\sqrt{c^2dx^2+d}}\right)}{3c^2}\right)+\frac{b\sqrt{d(c^2x^2+1)}\left(3\,\mathrm{arcsinh}(cx)c^4x^4-c^3x^3\sqrt{c^2x^2+1}-12\,\mathrm{arcsinh}(cx)\right.}{}$

```
[In] int(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/3*x^4/c^2/d/(c^2*d*x^2+d)^(1/2)-4/3/c^2*(x^2/c^2/d/(c^2*d*x^2+d)^(1/2)
+2/d/c^4/(c^2*d*x^2+d)^(1/2)))+1/9*b*(d*(c^2*x^2+1))^(1/2)*(3*arcsinh(c*x)*
c^4*x^4-c^3*x^3*(c^2*x^2+1)^(1/2)-12*arcsinh(c*x)*c^2*x^2-9*I*(c^2*x^2+1)^(
1/2)*ln(c*x+(c^2*x^2+1)^(1/2)-I)+9*I*(c^2*x^2+1)^(1/2)*ln(c*x+(c^2*x^2+1)^(
1/2)+I)+15*c*x*(c^2*x^2+1)^(1/2)-24*arcsinh(c*x))/c^6/d^2/(c^2*x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.93

$$\int \frac{x^5(a+b\mathrm{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx =$$

$$-\frac{9\,(bc^2x^2+b)\sqrt{d}\arctan\left(\frac{2\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}c\sqrt{d}x}{c^4dx^4-d}\right)-6\,(bc^4x^4-4\,bc^2x^2-8\,b)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)}{18\,(c^8d^2x^2+c^6d^2)}$$

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/18*(9*(b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d)*sqrt(c^2*x^2
+ 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) - 6*(b*c^4*x^4 - 4*b*c^2*x^2 - 8*b)*sqrt(
c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) - 2*(3*a*c^4*x^4 - 12*a*c^2*x^2
 - (b*c^3*x^3 - 15*b*c*x)*sqrt(c^2*x^2 + 1) - 24*a)*sqrt(c^2*d*x^2 + d))/(c
^8*d^2*x^2 + c^6*d^2)
```

Sympy [F]

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{x^5(a+b\operatorname{asinh}(cx))}{(d(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**5*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
[Out] Integral(x**5*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x^5}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
[Out] 1/3*a*(x^4/(sqrt(c^2*d*x^2 + d)*c^2*d) - 4*x^2/(sqrt(c^2*d*x^2 + d)*c^4*d)
- 8/(sqrt(c^2*d*x^2 + d)*c^6*d)) + 1/3*b*((c^4*sqrt(d)*x^4 - 4*c^2*sqrt(d)*
x^2 - 8*sqrt(d))*log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(c^2*x^2 + 1)*c^6*d^2) -
 integrate((c^4*sqrt(d)*x^4 - 4*c^2*sqrt(d)*x^2 - 8*sqrt(d))/(sqrt(c^2*x^2
+ 1)*x), x)/(c^6*d^2) + 3*integrate(1/3*(c^4*sqrt(d)*x^4 - 4*c^2*sqrt(d)*x^
2 - 8*sqrt(d))/(c^9*d^2*x^4 + c^7*d^2*x^2 + (c^8*d^2*x^3 + c^6*d^2*x)*sqrt(
c^2*x^2 + 1)), x))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{x^5\,(a+b\,\text{asinh}(c\,x))}{(d\,c^2\,x^2+d)^{3/2}}\,dx$$

```
[In] int((x^5*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2),x)
[Out] int((x^5*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2), x)
```

3.156 $\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 206

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = -\frac{bx^2\sqrt{1+c^2x^2}}{4c^3d\sqrt{d+c^2dx^2}} - \frac{x^3(a+b\operatorname{arcsinh}(cx))}{c^2d\sqrt{d+c^2dx^2}} + \frac{3x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2c^4d^2} - \frac{3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{4bc^5d\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\log(1+c^2x^2)}{2c^5d\sqrt{d+c^2dx^2}}$$

```
[Out] -x^3*(a+b*arcsinh(c*x))/c^2/d/(c^2*d*x^2+d)^(1/2)-1/4*b*x^2*(c^2*x^2+1)^(1/
2)/c^3/d/(c^2*d*x^2+d)^(1/2)-3/4*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c
^5/d/(c^2*d*x^2+d)^(1/2)-1/2*b*ln(c^2*x^2+1)*(c^2*x^2+1)^(1/2)/c^5/d/(c^2*d
*x^2+d)^(1/2)+3/2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4/d^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5810, 5812, 5783, 30, 272, 45}

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = -\frac{x^3(a+b\operatorname{arcsinh}(cx))}{c^2d\sqrt{c^2dx^2+d}} - \frac{3\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^2}{4bc^5d\sqrt{c^2dx^2+d}} + \frac{3x\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{2c^4d^2} - \frac{b\sqrt{c^2x^2+1}\log(c^2x^2+1)}{2c^5d\sqrt{c^2dx^2+d}} - \frac{bx^2\sqrt{c^2x^2+1}}{4c^3d\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -1/4*(b*x^2*Sqrt[1 + c^2*x^2])/(c^3*d*Sqrt[d + c^2*d*x^2]) - (x^3*(a + b*Ar
cSinh[c*x]))/(c^2*d*Sqrt[d + c^2*d*x^2]) + (3*x*Sqrt[d + c^2*d*x^2]*(a + b*
ArcSinh[c*x]))/(2*c^4*d^2) - (3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(
4*b*c^5*d*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(2*
c^5*d*Sqrt[d + c^2*d*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
```

```
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^3(a+b\text{arcsinh}(cx))}{c^2d\sqrt{d+c^2dx^2}}+\frac{3\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx}{c^2d}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{x^3}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}}\\
&= -\frac{x^3(a+b\text{arcsinh}(cx))}{c^2d\sqrt{d+c^2dx^2}}+\frac{3x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2c^4d^2}-\frac{3\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx}{2c^4d}\\
&\quad-\frac{\left(3b\sqrt{1+c^2x^2}\right)\int x\,dx}{2c^3d\sqrt{d+c^2dx^2}}+\frac{\left(b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{x}{1+c^2x}\,dx,x,x^2\right)}{2cd\sqrt{d+c^2dx^2}}\\
&= -\frac{3bx^2\sqrt{1+c^2x^2}}{4c^3d\sqrt{d+c^2dx^2}}-\frac{x^3(a+b\text{arcsinh}(cx))}{c^2d\sqrt{d+c^2dx^2}}+\frac{3x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2c^4d^2}\\
&\quad-\frac{3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{4bc^5d\sqrt{d+c^2dx^2}}+\frac{\left(b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\left(\frac{1}{c^2}-\frac{1}{c^2(1+c^2x)}\right)\,dx,x,x^2\right)}{2cd\sqrt{d+c^2dx^2}}\\
&= -\frac{bx^2\sqrt{1+c^2x^2}}{4c^3d\sqrt{d+c^2dx^2}}-\frac{x^3(a+b\text{arcsinh}(cx))}{c^2d\sqrt{d+c^2dx^2}}+\frac{3x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2c^4d^2}\\
&\quad-\frac{3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{4bc^5d\sqrt{d+c^2dx^2}}-\frac{b\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)}{2c^5d\sqrt{d+c^2dx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.78

$$\int\frac{x^4(a+b\text{arcsinh}(cx))}{\left(d+c^2dx^2\right)^{3/2}}\,dx=\frac{4ac\sqrt{d}x(3+c^2x^2)-12a\sqrt{d+c^2dx^2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)+b\sqrt{d}(8cxa$$

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
[Out] (4*a*c*Sqrt[d]*x*(3 + c^2*x^2) - 12*a*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[
d]*Sqrt[d + c^2*d*x^2]] + b*Sqrt[d]*(8*c*x*ArcSinh[c*x] - Sqrt[1 + c^2*x^2]
*(6*ArcSinh[c*x]^2 + Cosh[2*ArcSinh[c*x]] + 4*Log[1 + c^2*x^2] - 2*ArcSinh[
c*x]*Sinh[2*ArcSinh[c*x]])))/(8*c^5*d^(3/2)*Sqrt[d + c^2*d*x^2])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.33

method	result
default	$\frac{a\,x^3}{2c^2d\sqrt{c^2d\,x^2+d}}+\frac{3ax}{2c^4d\sqrt{c^2d\,x^2+d}}-\frac{3a\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2c^4d\sqrt{c^2d}}-\frac{b\sqrt{d(c^2x^2+1)}\left(-4\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+2c^4x^4+6\,\mathrm{arcs}\right.}{}$
parts	$\frac{a\,x^3}{2c^2d\sqrt{c^2d\,x^2+d}}+\frac{3ax}{2c^4d\sqrt{c^2d\,x^2+d}}-\frac{3a\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2c^4d\sqrt{c^2d}}-\frac{b\sqrt{d(c^2x^2+1)}\left(-4\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+2c^4x^4+6\,\mathrm{arcs}\right.}{}$

```
[In] int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x^3/c^2/d/(c^2*d*x^2+d)^(1/2)+3/2*a/c^4*x/d/(c^2*d*x^2+d)^(1/2)-3/2*a
/c^4/d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-1/8*b/(c
^2*x^2+1)^(3/2)*(d*(c^2*x^2+1))^(1/2)*(-4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^
3*c^3+2*c^4*x^4+6*arcsinh(c*x)^2*x^2*c^2-8*arcsinh(c*x)*c^2*x^2+8*ln(1+(c*x
+(c^2*x^2+1)^(1/2))^2)*x^2*c^2-12*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)+3*c^2*
x^2+6*arcsinh(c*x)^2-8*arcsinh(c*x)+8*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1)/c^
5/d^2
```

Fricas [F]

$$\int \frac{x^4(a+b\mathrm{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)x^4}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arcsinh(c*x) + a*x^4)*sqrt(c^2*d*x^2 + d)/(c^4*d^2*x^4 + 2*
c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\mathrm{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{x^4(a+b\,\mathrm{asinh}\,(cx))}{(d\,(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**4*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x^4}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*(x^3/(sqrt(c^2*d*x^2 + d)*c^2*d) + 3*x/(sqrt(c^2*d*x^2 + d)*c^4*d) -
3*arcsinh(c*x)/(c^5*d^(3/2))) + b*integrate(x^4*log(c*x + sqrt(c^2*x^2 + 1)
)/(c^2*d*x^2 + d)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{x^4\,(a+b\operatorname{asinh}(c\,x))}{(d\,c^2\,x^2+d)^{3/2}}\,dx$$

```
[In] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2), x)
```

3.157 $\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 136

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = -\frac{bx\sqrt{d+c^2dx^2}}{c^3d^2\sqrt{1+c^2x^2}} + \frac{a+b\text{arcsinh}(cx)}{c^4d\sqrt{d+c^2dx^2}} + \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{c^4d^2} - \frac{b\sqrt{d+c^2dx^2}\arctan(cx)}{c^4d^2\sqrt{1+c^2x^2}}$$

```
[Out] (a+b*arcsinh(c*x))/c^4/d/(c^2*d*x^2+d)^(1/2)+(a+b*arcsinh(c*x))*(c^2*d*x^2+
d)^(1/2)/c^4/d^2-b*x*(c^2*d*x^2+d)^(1/2)/c^3/d^2/(c^2*x^2+1)^(1/2)-b*arctan
(c*x)*(c^2*d*x^2+d)^(1/2)/c^4/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {272, 45, 5804, 12, 396, 209}

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{c^4d^2} + \frac{a+b\text{arcsinh}(cx)}{c^4d\sqrt{c^2dx^2+d}} - \frac{b\arctan(cx)\sqrt{c^2dx^2+d}}{c^4d^2\sqrt{c^2x^2+1}} - \frac{bx\sqrt{c^2dx^2+d}}{c^3d^2\sqrt{c^2x^2+1}}$$

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((b*x*Sqrt[d + c^2*d*x^2])/(c^3*d^2*Sqrt[1 + c^2*x^2])) + (a + b*ArcSinh[c
*x])/(c^4*d*Sqrt[d + c^2*d*x^2]) + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
))/(c^4*d^2) - (b*Sqrt[d + c^2*d*x^2]*ArcTan[c*x])/(c^4*d^2*Sqrt[1 + c^2*x^
2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

integral

$$= \frac{a + b\text{arcsinh}(cx)}{c^4 d\sqrt{d + c^2 dx^2}} + \frac{\sqrt{d + c^2 dx^2}(a + b\text{arcsinh}(cx))}{c^4 d^2} - \frac{\left(bc\sqrt{d + c^2 dx^2}\right)\int \frac{2+c^2x^2}{c^4d^2(1+c^2x^2)}\,dx}{\sqrt{1 + c^2x^2}}$$

$$= \frac{a+b\operatorname{arcsinh}(cx)}{c^4d\sqrt{d+c^2dx^2}} + \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{c^4d^2} - \frac{\left(b\sqrt{d+c^2dx^2}\right)\int\frac{2+c^2x^2}{1+c^2x^2}\,dx}{c^3d^2\sqrt{1+c^2x^2}}$$

$$= -\frac{bx\sqrt{d+c^2dx^2}}{c^3d^2\sqrt{1+c^2x^2}} + \frac{a+b\operatorname{arcsinh}(cx)}{c^4d\sqrt{d+c^2dx^2}} + \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{c^4d^2} - \frac{\left(b\sqrt{d+c^2dx^2}\right)\int\frac{1}{1+c^2x^2}\,dx}{c^3d^2\sqrt{1+c^2x^2}}$$

$$= -\frac{bx\sqrt{d+c^2dx^2}}{c^3d^2\sqrt{1+c^2x^2}} + \frac{a+b\operatorname{arcsinh}(cx)}{c^4d\sqrt{d+c^2dx^2}} + \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{c^4d^2} - \frac{b\sqrt{d+c^2dx^2}\arctan(cx)}{c^4d^2\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.96

$$\int\frac{x^3(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \frac{\sqrt{d+c^2dx^2}\left(-bcx-bc^3x^3+2a\sqrt{1+c^2x^2}+ac^2x^2\sqrt{1+c^2x^2}+b\sqrt{1+c^2x^2}(2+\right.}{c^4d^2\left(1+c^2x^2\right)^{3/2}}$$

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(-(b*c*x) - b*c^3*x^3 + 2*a*Sqrt[1 + c^2*x^2] + a*c^2*
x^2*Sqrt[1 + c^2*x^2] + b*Sqrt[1 + c^2*x^2]*(2 + c^2*x^2)*ArcSinh[c*x] - (b
 + b*c^2*x^2)*ArcTan[c*x]))/(c^4*d^2*(1 + c^2*x^2)^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.29

method	result
default	$a\left(\frac{x^2}{c^2d\sqrt{c^2d\,x^2+d}}+\frac{2}{d\,c^4\sqrt{c^2d\,x^2+d}}\right)+\frac{b\sqrt{d(c^2x^2+1)}\left(\operatorname{arcsinh}(cx)c^2x^2+i\sqrt{c^2x^2+1}\,\ln\left(cx+\sqrt{c^2x^2+1}-i\right)-i\sqrt{c^2x^2+1}\,\ln\left(cx+\sqrt{\right.\right.}}{c^4d^2(c^2x^2+1)}$
parts	$a\left(\frac{x^2}{c^2d\sqrt{c^2d\,x^2+d}}+\frac{2}{d\,c^4\sqrt{c^2d\,x^2+d}}\right)+\frac{b\sqrt{d(c^2x^2+1)}\left(\operatorname{arcsinh}(cx)c^2x^2+i\sqrt{c^2x^2+1}\,\ln\left(cx+\sqrt{c^2x^2+1}-i\right)-i\sqrt{c^2x^2+1}\,\ln\left(cx+\sqrt{\right.\right.}}{c^4d^2(c^2x^2+1)}$

```
[In] int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(x^2/c^2/d/(c^2*d*x^2+d)^(1/2)+2/d/c^4/(c^2*d*x^2+d)^(1/2))+b*(d*(c^2*x^2
+1))^(1/2)*(arcsinh(c*x)*c^2*x^2+I*(c^2*x^2+1)^(1/2)*ln(c*x+(c^2*x^2+1)^(1/
2)-I)-I*(c^2*x^2+1)^(1/2)*ln(c*x+(c^2*x^2+1)^(1/2)+I)-c*x*(c^2*x^2+1)^(1/2)
+2*arcsinh(c*x))/c^4/d^2/(c^2*x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.22

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \frac{(bc^2x^2+b)\sqrt{d}\arctan\left(\frac{2\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}c\sqrt{d}x}{c^4dx^4-d}\right)+2\,(bc^2x^2+2\,b)\sqrt{c^2dx^2+d}\log}{2\,(c^6d^2x^2+c^4d}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d)*sqrt(c^2*x^2 + 1)
*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 2*(b*c^2*x^2 + 2*b)*sqrt(c^2*d*x^2 + d)*log
(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*c^2*x^2 - sqrt(c^2*x^2 + 1)*b*c*x + 2*a)*s
qrt(c^2*d*x^2 + d))/(c^6*d^2*x^2 + c^4*d^2)
```

Sympy [F]

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{x^3(a+b\operatorname{asinh}(cx))}{(d\,(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx &= -bc\left(\frac{x}{c^4d^{\frac{3}{2}}}+\frac{\arctan(cx)}{c^5d^{\frac{3}{2}}}\right)\\ &+ b\left(\frac{x^2}{\sqrt{c^2dx^2+d}c^2d}+\frac{2}{\sqrt{c^2dx^2+d}c^4d}\right)\operatorname{arsinh}(cx)\\ &+ a\left(\frac{x^2}{\sqrt{c^2dx^2+d}c^2d}+\frac{2}{\sqrt{c^2dx^2+d}c^4d}\right)\end{aligned}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -b*c*(x/(c^4*d^(3/2)) + arctan(c*x)/(c^5*d^(3/2))) + b*(x^2/(sqrt(c^2*d*x^2
 + d)*c^2*d) + 2/(sqrt(c^2*d*x^2 + d)*c^4*d))*arcsinh(c*x) + a*(x^2/(sqrt(c
^2*d*x^2 + d)*c^2*d) + 2/(sqrt(c^2*d*x^2 + d)*c^4*d))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^{3/2}}\,dx = \int \frac{x^3\,(a + b\,\text{asinh}(c\,x))}{(d\,c^2\,x^2 + d)^{3/2}}\,dx$$

```
[In] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2), x)
```

3.158 $\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 130

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = -\frac{x(a+b\text{arcsinh}(cx))}{c^2d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc^3d\sqrt{d+c^2dx^2}} + \frac{b\sqrt{1+c^2x^2}\log(1+c^2x^2)}{2c^3d\sqrt{d+c^2dx^2}}$$

```
[Out] -x*(a+b*arcsinh(c*x))/c^2/d/(c^2*d*x^2+d)^(1/2)+1/2*(a+b*arcsinh(c*x))^2*(c
^2*x^2+1)^(1/2)/b/c^3/d/(c^2*d*x^2+d)^(1/2)+1/2*b*ln(c^2*x^2+1)*(c^2*x^2+1)
^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5810, 5783, 266}

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = -\frac{x(a+b\text{arcsinh}(cx))}{c^2d\sqrt{c^2dx^2+d}} + \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{2bc^3d\sqrt{c^2dx^2+d}} + \frac{b\sqrt{c^2x^2+1}\log(c^2x^2+1)}{2c^3d\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((x*(a + b*ArcSinh[c*x]))/(c^2*d*Sqrt[d + c^2*d*x^2])) + (Sqrt[1 + c^2*x^2
]*(a + b*ArcSinh[c*x])^2)/(2*b*c^3*d*Sqrt[d + c^2*d*x^2]) + (b*Sqrt[1 + c^2
*x^2]*Log[1 + c^2*x^2])/(2*c^3*d*Sqrt[d + c^2*d*x^2])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x(a+b\text{arcsinh}(cx))}{c^2d\sqrt{d+c^2dx^2}}+\frac{\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx}{c^2d}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{x}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}}\\ &= -\frac{x(a+b\text{arcsinh}(cx))}{c^2d\sqrt{d+c^2dx^2}}+\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc^3d\sqrt{d+c^2dx^2}}+\frac{b\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)}{2c^3d\sqrt{d+c^2dx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.12

$$\begin{aligned}\int\frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx &= -\frac{ax\sqrt{d\left(1+c^2x^2\right)}}{c^2d^2\left(1+c^2x^2\right)}\\ &\quad+\frac{b\left(-2cx\text{arcsinh}(cx)+\sqrt{1+c^2x^2}\left(\text{arcsinh}(cx)^2+2\log\left(\sqrt{1+c^2x^2}\right)\right)\right)}{2c^3d\sqrt{d\left(1+c^2x^2\right)}}\\ &\quad+\frac{a\log\left(cdx+\sqrt{d}\sqrt{d\left(1+c^2x^2\right)}\right)}{c^3d^{3/2}}\end{aligned}$$

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

[Out] -((a*x*Sqrt[d*(1 + c^2*x^2)])/(c^2*d^2*(1 + c^2*x^2))) + (b*(-2*c*x*ArcSinh[c*x] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]^2 + 2*Log[Sqrt[1 + c^2*x^2]])))/(2*c^3*d*Sqrt[d*(1 + c^2*x^2)]) + (a*Log[c*d*x + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]])/(c^3*d^(3/2))

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.78

method	result
default	$-\frac{ax}{c^2 d\sqrt{c^2 d\,x^2+d}}+\frac{a\ln\left(\frac{c^2 dx}{\sqrt{c^2 d}}+\sqrt{c^2 d\,x^2+d}\right)}{c^2 d\sqrt{c^2 d}}+\frac{b\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2}{2\sqrt{c^2x^2+1}\,c^3 d^2}-\frac{b\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)}{\sqrt{c^2x^2+1}\,c^3 d^2}-\frac{b\sqrt{d(c^2x^2+1)}\,\mathrm{a}}{c^2 d^2(c^2x}$
parts	$-\frac{ax}{c^2 d\sqrt{c^2 d\,x^2+d}}+\frac{a\ln\left(\frac{c^2 dx}{\sqrt{c^2 d}}+\sqrt{c^2 d\,x^2+d}\right)}{c^2 d\sqrt{c^2 d}}+\frac{b\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2}{2\sqrt{c^2x^2+1}\,c^3 d^2}-\frac{b\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)}{\sqrt{c^2x^2+1}\,c^3 d^2}-\frac{b\sqrt{d(c^2x^2+1)}\,\mathrm{a}}{c^2 d^2(c^2x}$

[In] int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)

[Out] -a*x/c^2/d/(c^2*d*x^2+d)^(1/2)+a/c^2/d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d^2*arcsinh(c*x)^2-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d^2*arcsinh(c*x)-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/c^2/d^2/(c^2*x^2+1)*x+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)

Fricas [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)x^2}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(c^2*d*x^2 + d)*(b*x^2*arcsinh(c*x) + a*x^2)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)

Sympy [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{x^2(a+b\operatorname{asinh}(cx))}{(d(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

[In] integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)

[Out] Integral(x**2*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)

Maxima [F]

$$\int \frac{x^2(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)x^2}{(c^2dx^2 + d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
[Out] -a*(x/(sqrt(c^2*d*x^2 + d)*c^2*d) - arcsinh(c*x)/(c^3*d^(3/2))) + b*integra
te(x^2*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)x^2}{(c^2dx^2 + d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)*x^2/(c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^{3/2}}\,dx = \int \frac{x^2\,(a + b\,\text{asinh}(c\,x))}{(d\,c^2\,x^2 + d)^{3/2}}\,dx$$

```
[In] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2),x)
[Out] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2), x)
```

3.159 $\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 70

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = -\frac{a+b\text{arcsinh}(cx)}{c^2d\sqrt{d+c^2dx^2}}+\frac{b\sqrt{1+c^2x^2}\arctan(cx)}{c^2d\sqrt{d+c^2dx^2}}$$

```
[Out] (-a-b*arcsinh(c*x))/c^2/d/(c^2*d*x^2+d)^(1/2)+b*arctan(c*x)*(c^2*x^2+1)^(1/
2)/c^2/d/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5798, 209}

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \frac{b\sqrt{c^2x^2+1}\arctan(cx)}{c^2d\sqrt{c^2dx^2+d}}-\frac{a+b\text{arcsinh}(cx)}{c^2d\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((a + b*ArcSinh[c*x])/(c^2*d*Sqrt[d + c^2*d*x^2])) + (b*Sqrt[1 + c^2*x^2]*
ArcTan[c*x])/(c^2*d*Sqrt[d + c^2*d*x^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{a+b\text{arcsinh}(cx)}{c^2d\sqrt{d+c^2dx^2}}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{1}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}}\\ &= -\frac{a+b\text{arcsinh}(cx)}{c^2d\sqrt{d+c^2dx^2}}+\frac{b\sqrt{1+c^2x^2}\arctan(cx)}{c^2d\sqrt{d+c^2dx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.23

$$\int\frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx=\frac{\sqrt{d+c^2dx^2}\left(-a\sqrt{1+c^2x^2}-b\sqrt{1+c^2x^2}\text{arcsinh}(cx)+(b+bc^2x^2)\arctan(cx)\right)}{c^2d^2(1+c^2x^2)^{3/2}}$$

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
[Out] (Sqrt[d + c^2*d*x^2]*(-(a*Sqrt[1 + c^2*x^2]) - b*Sqrt[1 + c^2*x^2]*ArcSinh[
c*x] + (b + b*c^2*x^2)*ArcTan[c*x]))/(c^2*d^2*(1 + c^2*x^2)^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.79

method	result	size
default	$-\frac{a}{c^2d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{d(c^2x^2+1)}\left(-i\sqrt{c^2x^2+1}\ln\left(cx+\sqrt{c^2x^2+1}+i\right)+i\sqrt{c^2x^2+1}\ln\left(cx+\sqrt{c^2x^2+1}-i\right)+\text{arcsinh}(cx)\right)}{c^2d^2(c^2x^2+1)}$	125
parts	$-\frac{a}{c^2d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{d(c^2x^2+1)}\left(-i\sqrt{c^2x^2+1}\ln\left(cx+\sqrt{c^2x^2+1}+i\right)+i\sqrt{c^2x^2+1}\ln\left(cx+\sqrt{c^2x^2+1}-i\right)+\text{arcsinh}(cx)\right)}{c^2d^2(c^2x^2+1)}$	125

```
[In] int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
[Out] -a/c^2/d/(c^2*d*x^2+d)^(1/2)-b*(d*(c^2*x^2+1))^(1/2)*(-I*(c^2*x^2+1)^(1/2)*
ln(c*x+(c^2*x^2+1)^(1/2)+I)+I*(c^2*x^2+1)^(1/2)*ln(c*x+(c^2*x^2+1)^(1/2)-I)
+arcsinh(c*x))/c^2/d^2/(c^2*x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.83

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = -\frac{(bc^2x^2+b)\sqrt{d}\arctan\left(\frac{2\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}c\sqrt{d}x}{c^4dx^4-d}\right)+2\sqrt{c^2dx^2+d}b\log\left(cx+\sqrt{c^2x^2+1}\right)+2\sqrt{c^2dx^2+d}a}{2\left(c^4d^2x^2+c^2d^2\right)}$$

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*((b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d)*sqrt(c^2*x^2 + 1
)*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 2*sqrt(c^2*d*x^2 + d)*b*log(c*x + sqrt(c^2
*x^2 + 1)) + 2*sqrt(c^2*d*x^2 + d)*a)/(c^4*d^2*x^2 + c^2*d^2)
```

Sympy [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{x(a+b\,\text{asinh}\,(cx))}{\left(d\,(c^2x^2+1)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] b*(integrate(1/(sqrt(c^2*x^2 + 1)*x), x)/(c^2*d^(3/2)) - log(c*x + sqrt(c^2
*x^2 + 1))/(sqrt(c^2*x^2 + 1)*c^2*d^(3/2)) - integrate(1/(c^5*d^(3/2)*x^4 +
 c^3*d^(3/2)*x^2 + (c^4*d^(3/2)*x^3 + c^2*d^(3/2)*x)*sqrt(c^2*x^2 + 1)), x)
) - a/(sqrt(c^2*d*x^2 + d)*c^2*d)
```

Giac [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)*x/(c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{x\,(a+b\,\text{asinh}(c\,x))}{(d\,c^2\,x^2+d)^{3/2}}\,dx$$

```
[In] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2),x)
[Out] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2), x)
```

3.160 $\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+c^2dx^2)^{3/2}}\,dx=\frac{x(a+b\operatorname{arcsinh}(cx))}{d\sqrt{d+c^2dx^2}}-\frac{b\sqrt{1+c^2x^2}\log(1+c^2x^2)}{2cd\sqrt{d+c^2dx^2}}$$

```
[Out] x*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(1/2)-1/2*b*ln(c^2*x^2+1)*(c^2*x^2+1)^
(1/2)/c/d/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5787, 266}

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+c^2dx^2)^{3/2}}\,dx=\frac{x(a+b\operatorname{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\log(c^2x^2+1)}{2cd\sqrt{c^2dx^2+d}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcSinh[c*x]))/(d*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*Log
[1 + c^2*x^2])/(2*c*d*Sqrt[d + c^2*d*x^2])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x(a+b\operatorname{arcsinh}(cx))}{d\sqrt{d+c^2dx^2}}-\frac{\left(bc\sqrt{1+c^2x^2}\right)\int\frac{x}{1+c^2x^2}\,dx}{d\sqrt{d+c^2dx^2}}\\ &= \frac{x(a+b\operatorname{arcsinh}(cx))}{d\sqrt{d+c^2dx^2}}-\frac{b\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)}{2cd\sqrt{d+c^2dx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.32

$$\int\frac{a+b\operatorname{arcsinh}(cx)}{\left(d+c^2dx^2\right)^{3/2}}\,dx=\frac{\sqrt{d+c^2dx^2}\left(2acx\sqrt{1+c^2x^2}+2bcx\sqrt{1+c^2x^2}\operatorname{arcsinh}(cx)-\left(b+bc^2x^2\right)\log\left(1+c^2x\right.\right.}{2cd^2\left(1+c^2x^2\right)^{3/2}}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(2*a*c*x*Sqrt[1 + c^2*x^2] + 2*b*c*x*Sqrt[1 + c^2*x^2]
*ArcSinh[c*x] - (b + b*c^2*x^2)*Log[1 + c^2*x^2]))/(2*c*d^2*(1 + c^2*x^2)^(
3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(68) = 136.

Time = 0.18 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.88

method	result	size
default	$\frac{ax}{d\sqrt{c^2d\,x^2+d}}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)}{\sqrt{c^2x^2+1}\,c\,d^2}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)x}{d^2\left(c^2x^2+1\right)}-\frac{b\sqrt{d\left(c^2x^2+1\right)}\,\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)}{\sqrt{c^2x^2+1}\,c\,d^2}$	143
parts	$\frac{ax}{d\sqrt{c^2d\,x^2+d}}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)}{\sqrt{c^2x^2+1}\,c\,d^2}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)x}{d^2\left(c^2x^2+1\right)}-\frac{b\sqrt{d\left(c^2x^2+1\right)}\,\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)}{\sqrt{c^2x^2+1}\,c\,d^2}$	143

```
[In] int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a/d*x/(c^2*d*x^2+d)^(1/2)+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d^2*a
rcsinh(c*x)+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d^2/(c^2*x^2+1)*x-b*(d*(c^
2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{a+b\,\text{asinh}\,(cx)}{(d\,(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

[In] integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)

[Out] Integral((a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.76

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{3/2}}\,dx=\frac{bx\,\text{arsinh}\,(cx)}{\sqrt{c^2dx^2+d}d}+\frac{ax}{\sqrt{c^2dx^2+d}d}-\frac{b\log\left(x^2+\frac{1}{c^2}\right)}{2\,cd^{\frac{3}{2}}}$$

[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")

[Out] b*x*arcsinh(c*x)/(sqrt(c^2*d*x^2 + d)*d) + a*x/(sqrt(c^2*d*x^2 + d)*d) - 1/2*b*log(x^2 + 1/c^2)/(c*d^(3/2))

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{a+b\,\text{asinh}(c\,x)}{(d\,c^2\,x^2+d)^{3/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(3/2),x)
[Out] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(3/2), x)
```

3.161 $\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 194

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^{3/2}}\,dx = \frac{a+b\text{arcsinh}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\arctan(cx)}{d\sqrt{d+c^2dx^2}} - \frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} + \frac{b\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$

```
[Out] (a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(1/2)-b*arctan(c*x)*(c^2*x^2+1)^(1/2)/d/
(c^2*d*x^2+d)^(1/2)-2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^
2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c
^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+b*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c
^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5811, 5816, 4267, 2317, 2438, 209}

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{2\sqrt{c^2x^2+1}\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}} + \frac{a+b\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}} - \frac{b\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d\sqrt{c^2dx^2+d}} + \frac{b\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d\sqrt{c^2dx^2+d}} - \frac{b\sqrt{c^2x^2+1}\arctan(cx)}{d\sqrt{c^2dx^2+d}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] (a + b*ArcSinh[c*x])/(d*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*ArcTan[
c*x])/(d*Sqrt[d + c^2*d*x^2]) - (2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*A
rcTanh[E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*Poly
Log[2, -E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + (b*Sqrt[1 + c^2*x^2]*Pol
yLog[2, E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
```

```
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{a+b\text{arcsinh}(cx)}{d\sqrt{d+c^2dx^2}} + \frac{\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{d+c^2dx^2}}\,dx}{d} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{1+c^2x^2}\,dx}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{a+b\text{arcsinh}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\arctan(cx)}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\text{Subst}\left(\int(a+bx)\text{csch}(x)\,dx, x, \text{arcsinh}(cx)\right)}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{a+b\text{arcsinh}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\arctan(cx)}{d\sqrt{d+c^2dx^2}} - \frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} - \frac{\left(b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1-e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{d\sqrt{d+c^2dx^2}} + \frac{\left(b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1+e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{a+b\text{arcsinh}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\arctan(cx)}{d\sqrt{d+c^2dx^2}} - \frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} - \frac{\left(b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} + \frac{\left(b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{a+b\text{arcsinh}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\arctan(cx)}{d\sqrt{d+c^2dx^2}} - \frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} + \frac{b\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.19

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^{3/2}}\,dx = \frac{\frac{a\sqrt{d+c^2dx^2}}{1+c^2x^2}+a\sqrt{d}\log(x)-a\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right)+\frac{bd\left(\text{arcsinh}(cx)-2\sqrt{1+c^2x^2}\,\text{a}\right.}{}}{}$$

[In] Integrate[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^(3/2)),x]

[Out] ((a*Sqrt[d + c^2*d*x^2])/(1 + c^2*x^2) + a*Sqrt[d]*Log[x] - a*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*d*(ArcSinh[c*x] - 2*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-ArcSinh[c*x])] - Sqrt[1 + c^2*x^2]*PolyLog[2, E^(-ArcSinh[c*x])]))/Sqrt[d + c^2*d*x^2])/d^2

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.40

method	result
default	$\frac{a}{d\sqrt{c^2d\,x^2+d}}-\frac{a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)}{d^{\frac{3}{2}}}-\frac{b\sqrt{d(c^2x^2+1)}\left(\text{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)x^2c^2+2\arctan\left(cx+\sqrt{c^2x^2+1}\right)x^2c^2+\right.}{}$
parts	$\frac{a}{d\sqrt{c^2d\,x^2+d}}-\frac{a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)}{d^{\frac{3}{2}}}-\frac{b\sqrt{d(c^2x^2+1)}\left(\text{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)x^2c^2+2\arctan\left(cx+\sqrt{c^2x^2+1}\right)x^2c^2+\right.}{}$

[In] int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)

[Out] a/d/(c^2*d*x^2+d)^(1/2)-a/d^(3/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)-b/(c^2*x^2+1)^(3/2)*(d*(c^2*x^2+1))^(1/2)/d^2*(arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*x^2*c^2+2*arctan(c*x+(c^2*x^2+1)^(1/2))*x^2*c^2+dilog(1+c*x+(c^2*x^2+1)^(1/2))*x^2*c^2+dilog(c*x+(c^2*x^2+1)^(1/2))*x^2*c^2-arcsinh(c*x)*(c^2*x^2+1)^(1/2)+arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*arctan(c*x+(c^2*x^2+1)^(1/2))+dilog(1+c*x+(c^2*x^2+1)^(1/2))+dilog(c*x+(c^2*x^2+1)^(1/2)))

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\,(d+c^2dx^2)^{3/2}}\,dx=\int \frac{b\operatorname{arsinh}(cx)+a}{(c^2dx^2+d)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^4*d^2*x^5 + 2*c^2*d^2*
x^3 + d^2*x), x)
```

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\,(d+c^2dx^2)^{3/2}}\,dx=\int \frac{a+b\operatorname{asinh}(cx)}{x\,(d\,(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d)**(3/2),x)
[Out] Integral((a + b*asinh(c*x))/(x*(d*(c**2*x**2 + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\,(d+c^2dx^2)^{3/2}}\,dx=\int \frac{b\operatorname{arsinh}(cx)+a}{(c^2dx^2+d)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
[Out] -a*(arcsinh(1/(c*abs(x)))/d^(3/2) - 1/(sqrt(c^2*d*x^2 + d)*d)) + b*integrat
e(log(c*x + sqrt(c^2*x^2 + 1))/((c^2*d*x^2 + d)^(3/2)*x), x)
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\,(d+c^2dx^2)^{3/2}}\,dx=\int \frac{b\operatorname{arsinh}(cx)+a}{(c^2dx^2+d)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x\,{\left(d\,c^2\,x^2+d\right)}^{3/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^(3/2)),x)
[Out] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^(3/2)), x)
```

3.162 $\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 143

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^{3/2}}\,dx = -\frac{a+b\text{arcsinh}(cx)}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x(a+b\text{arcsinh}(cx))}{d\sqrt{d+c^2dx^2}} + \frac{bc\sqrt{d+c^2dx^2}\log(x)}{d^2\sqrt{1+c^2x^2}} + \frac{bc\sqrt{d+c^2dx^2}\log(1+c^2x^2)}{2d^2\sqrt{1+c^2x^2}}$$

```
[Out] (-a-b*arcsinh(c*x))/d/x/(c^2*d*x^2+d)^(1/2)-2*c^2*x*(a+b*arcsinh(c*x))/d/(c
^2*d*x^2+d)^(1/2)+b*c*ln(x)*(c^2*d*x^2+d)^(1/2)/d^2/(c^2*x^2+1)^(1/2)+1/2*b
*c*ln(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {277, 197, 5804, 12, 457, 78}

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^{3/2}}\,dx = -\frac{2c^2x(a+b\text{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}} - \frac{a+b\text{arcsinh}(cx)}{dx\sqrt{c^2dx^2+d}} + \frac{bc\log(x)\sqrt{c^2dx^2+d}}{d^2\sqrt{c^2x^2+1}} + \frac{bc\sqrt{c^2dx^2+d}\log(c^2x^2+1)}{2d^2\sqrt{c^2x^2+1}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -((a + b*ArcSinh[c*x])/(d*x*Sqrt[d + c^2*d*x^2])) - (2*c^2*x*(a + b*ArcSinh
[c*x]))/(d*Sqrt[d + c^2*d*x^2]) + (b*c*Sqrt[d + c^2*d*x^2]*Log[x])/(d^2*Sqr
t[1 + c^2*x^2]) + (b*c*Sqrt[d + c^2*d*x^2]*Log[1 + c^2*x^2])/(2*d^2*Sqrt[1
+ c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\text{integral} = -\frac{a+b\text{arcsinh}(cx)}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x(a+b\text{arcsinh}(cx))}{d\sqrt{d+c^2dx^2}} - \frac{\left(bc\sqrt{d+c^2dx^2}\right)\int \frac{-1-2c^2x^2}{d^2x(1+c^2x^2)}\,dx}{\sqrt{1+c^2x^2}}$$

$$= -\frac{a+b\operatorname{arcsinh}(cx)}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x(a+b\operatorname{arcsinh}(cx))}{d\sqrt{d+c^2dx^2}} - \frac{\left(bc\sqrt{d+c^2dx^2}\right)\int \frac{-1-2c^2x^2}{x(1+c^2x^2)}\,dx}{d^2\sqrt{1+c^2x^2}}$$

$$= -\frac{a+b\operatorname{arcsinh}(cx)}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x(a+b\operatorname{arcsinh}(cx))}{d\sqrt{d+c^2dx^2}} - \frac{\left(bc\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{-1-2c^2x}{x(1+c^2x)}\,dx,x,x^2\right)}{2d^2\sqrt{1+c^2x^2}}$$

$$= -\frac{a+b\operatorname{arcsinh}(cx)}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x(a+b\operatorname{arcsinh}(cx))}{d\sqrt{d+c^2dx^2}} - \frac{\left(bc\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \left(-\frac{1}{x}-\frac{c^2}{1+c^2x}\right)\,dx,x,x^2\right)}{2d^2\sqrt{1+c^2x^2}}$$

$$= -\frac{a+b\operatorname{arcsinh}(cx)}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x(a+b\operatorname{arcsinh}(cx))}{d\sqrt{d+c^2dx^2}} + \frac{bc\sqrt{d+c^2dx^2}\log(x)}{d^2\sqrt{1+c^2x^2}} + \frac{bc\sqrt{d+c^2dx^2}\log(1+c^2x^2)}{2d^2\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.14

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^2(d+c^2dx^2)^{3/2}}\,dx = -\frac{\sqrt{d+c^2dx^2}\left(2a\sqrt{1+c^2x^2}+4ac^2x^2\sqrt{1+c^2x^2}+2b\sqrt{1+c^2x^2}(1+2c^2x^2)\operatorname{arcsinh}(cx)+bcx(1+c^2x^2)\log(\right.}{2d^2x(1+c^2x^2)^{3/2}}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -1/2*(Sqrt[d + c^2*d*x^2]*(2*a*Sqrt[1 + c^2*x^2] + 4*a*c^2*x^2*Sqrt[1 + c^2
*x^2] + 2*b*Sqrt[1 + c^2*x^2]*(1 + 2*c^2*x^2)*ArcSinh[c*x] + b*c*x*(1 + c^2
*x^2)*Log[1 + 1/(c^2*x^2)] - 2*b*c*x*Log[1 + c^2*x^2] - 2*b*c^3*x^3*Log[1 +
 c^2*x^2]))/(d^2*x*(1 + c^2*x^2)^(3/2))
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.67

method	result
default	$a\left(-\frac{1}{dx\sqrt{c^2d\,x^2+d}}-\frac{2c^2x}{d\sqrt{c^2d\,x^2+d}}\right)-\frac{b\left(2\ln\left(\left(cx+\sqrt{c^2x^2+1}\right)^4-1\right)x^4c^4-2\sqrt{c^2x^2+1}\,\ln\left(\left(cx+\sqrt{c^2x^2+1}\right)^4-1\right)x^3c^3+2\ln\left(\right.\right.}{}$
parts	$a\left(-\frac{1}{dx\sqrt{c^2d\,x^2+d}}-\frac{2c^2x}{d\sqrt{c^2d\,x^2+d}}\right)-\frac{b\left(2\ln\left(\left(cx+\sqrt{c^2x^2+1}\right)^4-1\right)x^4c^4-2\sqrt{c^2x^2+1}\,\ln\left(\left(cx+\sqrt{c^2x^2+1}\right)^4-1\right)x^3c^3+2\ln\left(\right.\right.}{}$

```
[In] int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/d/x/(c^2*d*x^2+d)^(1/2)-2*c^2/d*x/(c^2*d*x^2+d)^(1/2))-b*(2*ln((c*x+(
c^2*x^2+1)^(1/2))^4-1)*x^4*c^4-2*(c^2*x^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2
))^4-1)*x^3*c^3+2*ln((c*x+(c^2*x^2+1)^(1/2))^4-1)*x^2*c^2-(c^2*x^2+1)^(1/2)
*ln((c*x+(c^2*x^2+1)^(1/2))^4-1)*x*c+arcsinh(c*x))*(2*c^2*x^2+1+2*c*x*(c^2*
x^2+1)^(1/2))*(d*(c^2*x^2+1))^(1/2)/x/d^2/(c^2*x^2+1)
```

Fricas [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2 (d + c^2dx^2)^{3/2}}\,dx = \int \frac{b\,\text{arsinh}\left(cx\right) + a}{\left(c^2dx^2 + d\right)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^4*d^2*x^6 + 2*c^2*d^2*
x^4 + d^2*x^2), x)
```

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2 (d + c^2dx^2)^{3/2}}\,dx = \int \frac{a + b\,\text{asinh}\left(cx\right)}{x^2\left(d\left(c^2x^2 + 1\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(x**2*(d*(c**2*x**2 + 1))**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.83

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2 (d + c^2dx^2)^{3/2}}\,dx = \frac{1}{2}bc\left(\frac{\log\left(c^2x^2 + 1\right)}{d^{\frac{3}{2}}} + \frac{2\,\log\left(x\right)}{d^{\frac{3}{2}}}\right) - \left(\frac{2\,c^2x}{\sqrt{c^2dx^2 + d}d} + \frac{1}{\sqrt{c^2dx^2 + d}dx}\right)b\,\text{arsinh}\left(cx\right) - \left(\frac{2\,c^2x}{\sqrt{c^2dx^2 + d}d} + \frac{1}{\sqrt{c^2dx^2 + d}dx}\right)a$$

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*c*(log(c^2*x^2 + 1)/d^(3/2) + 2*log(x)/d^(3/2)) - (2*c^2*x/(sqrt(c^2*
d*x^2 + d)*d) + 1/(sqrt(c^2*d*x^2 + d)*d*x))*b*arcsinh(c*x) - (2*c^2*x/(sqr
t(c^2*d*x^2 + d)*d) + 1/(sqrt(c^2*d*x^2 + d)*d*x))*a
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2\left(d + c^2dx^2\right)^{3/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{\left(c^2dx^2 + d\right)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2\left(d + c^2dx^2\right)^{3/2}}\,dx = \int \frac{a + b\,\text{asinh}(c\,x)}{x^2\left(d\,c^2\,x^2 + d\right)^{3/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^(3/2)),x)
[Out] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^(3/2)), x)
```

3.163 $\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 287

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^{3/2}}\,dx &= -\frac{bc\sqrt{1+c^2x^2}}{2dx\sqrt{d+c^2dx^2}}-\frac{3c^2(a+b\text{arcsinh}(cx))}{2d\sqrt{d+c^2dx^2}}\\ &\quad-\frac{a+b\text{arcsinh}(cx)}{2dx^2\sqrt{d+c^2dx^2}}+\frac{bc^2\sqrt{1+c^2x^2}\arctan(cx)}{d\sqrt{d+c^2dx^2}}+\frac{3c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}\\ &\quad+\frac{3bc^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2d\sqrt{d+c^2dx^2}}-\frac{3bc^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2d\sqrt{d+c^2dx^2}}\end{aligned}$$

```
[Out] -3/2*c^2*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(1/2)+1/2*(-a-b*arcsinh(c*x))/d
/x^2/(c^2*d*x^2+d)^(1/2)-1/2*b*c*(c^2*x^2+1)^(1/2)/d/x/(c^2*d*x^2+d)^(1/2)+
b*c^2*arctan(c*x)*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+3*c^2*(a+b*arcsin
h(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1
/2)+3/2*b*c^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*
x^2+d)^(1/2)-3/2*b*c^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d
/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used

$= \{5809, 5811, 5816, 4267, 2317, 2438, 209, 331\}$

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^{3/2}}\,dx = \frac{3c^2\sqrt{c^2x^2+1}\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}} - \frac{3c^2(a+b\text{arcsinh}(cx))}{2d\sqrt{c^2dx^2+d}} - \frac{a+b\text{arcsinh}(cx)}{2dx^2\sqrt{c^2dx^2+d}} + \frac{3bc^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2d\sqrt{c^2dx^2+d}} - \frac{3bc^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2d\sqrt{c^2dx^2+d}} + \frac{bc^2\sqrt{c^2x^2+1}\,\text{arctan}(cx)}{d\sqrt{c^2dx^2+d}} - \frac{bc\sqrt{c^2x^2+1}}{2dx\sqrt{c^2dx^2+d}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -1/2*(b*c*Sqrt[1 + c^2*x^2])/(d*x*Sqrt[d + c^2*d*x^2]) - (3*c^2*(a + b*ArcS
inh[c*x]))/(2*d*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x])/(2*d*x^2*Sqrt[d
 + c^2*d*x^2]) + (b*c^2*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(d*Sqrt[d + c^2*d*x^
2]) + (3*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]]
)/(d*Sqrt[d + c^2*d*x^2]) + (3*b*c^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^ArcSin
h[c*x]])/(2*d*Sqrt[d + c^2*d*x^2]) - (3*b*c^2*Sqrt[1 + c^2*x^2]*PolyLog[2,
E^ArcSinh[c*x]])/(2*d*Sqrt[d + c^2*d*x^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\operatorname{arcsinh}(cx)}{2dx^2\sqrt{d+c^2dx^2}}-\frac{1}{2}\left(3c^2\right)\int\frac{a+b\operatorname{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^{3/2}}\,dx+\frac{\left(bc\sqrt{1+c^2x^2}\right)\int\frac{1}{x^2\left(1+c^2x^2\right)}\,dx}{2d\sqrt{d+c^2dx^2}}\\
&= -\frac{bc\sqrt{1+c^2x^2}}{2dx\sqrt{d+c^2dx^2}}-\frac{3c^2(a+b\operatorname{arcsinh}(cx))}{2d\sqrt{d+c^2dx^2}}\\
&\quad-\frac{a+b\operatorname{arcsinh}(cx)}{2dx^2\sqrt{d+c^2dx^2}}-\frac{\left(3c^2\right)\int\frac{a+b\operatorname{arcsinh}(cx)}{x\sqrt{d+c^2dx^2}}\,dx}{2d}\\
&\quad-\frac{\left(bc^3\sqrt{1+c^2x^2}\right)\int\frac{1}{1+c^2x^2}\,dx}{2d\sqrt{d+c^2dx^2}}+\frac{\left(3bc^3\sqrt{1+c^2x^2}\right)\int\frac{1}{1+c^2x^2}\,dx}{2d\sqrt{d+c^2dx^2}}
\end{aligned}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx\sqrt{d+c^2dx^2}} - \frac{3c^2(a+b\text{arcsinh}(cx))}{2d\sqrt{d+c^2dx^2}} - \frac{a+b\text{arcsinh}(cx)}{2dx^2\sqrt{d+c^2dx^2}} + \frac{bc^2\sqrt{1+c^2x^2}\arctan(cx)}{d\sqrt{d+c^2dx^2}} - \frac{\left(3c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\text{csch}(x)\,dx, x, \text{arcsinh}(cx)\right)}{2d\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx\sqrt{d+c^2dx^2}} - \frac{3c^2(a+b\text{arcsinh}(cx))}{2d\sqrt{d+c^2dx^2}} - \frac{a+b\text{arcsinh}(cx)}{2dx^2\sqrt{d+c^2dx^2}} + \frac{bc^2\sqrt{1+c^2x^2}\arctan(cx)}{d\sqrt{d+c^2dx^2}} + \frac{3c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} + \frac{\left(3bc^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1-e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{2d\sqrt{d+c^2dx^2}} - \frac{\left(3bc^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1+e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{2d\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx\sqrt{d+c^2dx^2}} - \frac{3c^2(a+b\text{arcsinh}(cx))}{2d\sqrt{d+c^2dx^2}} - \frac{a+b\text{arcsinh}(cx)}{2dx^2\sqrt{d+c^2dx^2}} + \frac{bc^2\sqrt{1+c^2x^2}\arctan(cx)}{d\sqrt{d+c^2dx^2}} + \frac{3c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} + \frac{\left(3bc^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{2d\sqrt{d+c^2dx^2}} - \frac{\left(3bc^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{2d\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx\sqrt{d+c^2dx^2}} - \frac{3c^2(a+b\text{arcsinh}(cx))}{2d\sqrt{d+c^2dx^2}} - \frac{a+b\text{arcsinh}(cx)}{2dx^2\sqrt{d+c^2dx^2}} + \frac{bc^2\sqrt{1+c^2x^2}\arctan(cx)}{d\sqrt{d+c^2dx^2}} + \frac{3c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} + \frac{3bc^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2d\sqrt{d+c^2dx^2}} - \frac{3bc^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2d\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 6.18 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.29

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx = \frac{-\frac{4a(1+3c^2x^2)\sqrt{d+c^2dx^2}}{x^2+c^2x^4}-12ac^2\sqrt{d}\log(x)+12ac^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right)+\frac{bc^2d(}{}}{}$$

[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^(3/2)),x]

[Out] ((-4*a*(1 + 3*c^2*x^2)*Sqrt[d + c^2*d*x^2])/(x^2 + c^2*x^4) - 12*a*c^2*Sqrt[d]*Log[x] + 12*a*c^2*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*c^2*d*(-8*ArcSinh[c*x] + 16*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] - 2*Sqrt[1 + c^2*x^2]*Coth[ArcSinh[c*x]/2] - Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] + 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 12*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-ArcSinh[c*x])] + 12*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(-ArcSinh[c*x])] - Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Sqrt[1 + c^2*x^2]*Tanh[ArcSinh[c*x]/2]))/Sqrt[d + c^2*d*x^2])/(8*d^2)

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.19

method	result
default	$a\left(-\frac{1}{2d\,x^2\sqrt{c^2d\,x^2+d}}-\frac{3c^2\left(\frac{1}{d\sqrt{c^2d\,x^2+d}}-\frac{\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)}{d^{\frac{3}{2}}}\right)}{2}\right)+b\left(-\frac{\sqrt{d(c^2x^2+1)}\left(3\,\text{arcsinh}(cx)c^2x^2+cx\sqrt{c^2x^2+1}\right.}{2d^2(c^2x^2+1)x^2}\right.$
parts	$a\left(-\frac{1}{2d\,x^2\sqrt{c^2d\,x^2+d}}-\frac{3c^2\left(\frac{1}{d\sqrt{c^2d\,x^2+d}}-\frac{\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)}{d^{\frac{3}{2}}}\right)}{2}\right)+b\left(-\frac{\sqrt{d(c^2x^2+1)}\left(3\,\text{arcsinh}(cx)c^2x^2+cx\sqrt{c^2x^2+1}\right.}{2d^2(c^2x^2+1)x^2}\right.$

[In] int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)

[Out] a*(-1/2/d/x^2/(c^2*d*x^2+d)^(1/2)-3/2*c^2*(1/d/(c^2*d*x^2+d)^(1/2)-1/d^(3/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)))+b*(-1/2*(d*(c^2*x^2+1))^(1/2)*(3*arcsinh(c*x)*c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x))/d^2/(c^2*x^2+1)/x^2+2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arctan(c*x+(c^2*x^2+1)^(1/2))*c^2+3/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*dilog(1+c*x+(c^2*x^2+1)^(1/2))*c^2+3/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arcsinh(

c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2+3/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*dilog(c*x+(c^2*x^2+1)^(1/2))*c^2)

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{b\,\text{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^4*d^2*x^7 + 2*c^2*d^2*x^5 + d^2*x^3), x)

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{a+b\,\text{asinh}\left(cx\right)}{x^3\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

[In] integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d)**(3/2),x)

[Out] Integral((a + b*asinh(c*x))/(x**3*(d*(c**2*x**2 + 1))**(3/2)), x)

Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{b\,\text{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")

[Out] 1/2*(3*c^2*arcsinh(1/(c*abs(x)))/d^(3/2) - 3*c^2/(sqrt(c^2*d*x^2 + d)*d) - 1/(sqrt(c^2*d*x^2 + d)*d*x^2))*a + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/((c^2*d*x^2 + d)^(3/2)*x^3), x)

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(3/2)*x^3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{x^3\,\left(d\,c^2\,x^2+d\right)^{3/2}}\,dx$$

[In] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^(3/2)),x)

[Out] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^(3/2)), x)

3.164 $\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 228

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^{3/2}}\,dx = -\frac{bc\sqrt{d+c^2dx^2}}{6d^2x^2\sqrt{1+c^2x^2}}-\frac{a+b\text{arcsinh}(cx)}{3dx^3\sqrt{d+c^2dx^2}}+\frac{4c^2(a+b\text{arcsinh}(cx))}{3dx\sqrt{d+c^2dx^2}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3d\sqrt{d+c^2dx^2}}-\frac{5bc^3\sqrt{d+c^2dx^2}\log(x)}{3d^2\sqrt{1+c^2x^2}}-\frac{bc^3\sqrt{d+c^2dx^2}\log(1+c^2x^2)}{2d^2\sqrt{1+c^2x^2}}$$

```
[Out] 1/3*(-a-b*arcsinh(c*x))/d/x^3/(c^2*d*x^2+d)^(1/2)+4/3*c^2*(a+b*arcsinh(c*x)
)/d/x/(c^2*d*x^2+d)^(1/2)+8/3*c^4*x*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(1/2
)-1/6*b*c*(c^2*d*x^2+d)^(1/2)/d^2/x^2/(c^2*x^2+1)^(1/2)-5/3*b*c^3*ln(x)*(c^
2*d*x^2+d)^(1/2)/d^2/(c^2*x^2+1)^(1/2)-1/2*b*c^3*ln(c^2*x^2+1)*(c^2*d*x^2+d
)^(1/2)/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {277, 197, 5804, 12, 1265, 907}

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^{3/2}}\,dx = \frac{4c^2(a+b\text{arcsinh}(cx))}{3dx\sqrt{c^2dx^2+d}}-\frac{a+b\text{arcsinh}(cx)}{3dx^3\sqrt{c^2dx^2+d}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3d\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2dx^2+d}}{6d^2x^2\sqrt{c^2x^2+1}}-\frac{5bc^3\log(x)\sqrt{c^2dx^2+d}}{3d^2\sqrt{c^2x^2+1}}-\frac{bc^3\sqrt{c^2dx^2+d}\log(c^2x^2+1)}{2d^2\sqrt{c^2x^2+1}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -1/6*(b*c*Sqrt[d + c^2*d*x^2])/(d^2*x^2*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh
[c*x])/(3*d*x^3*Sqrt[d + c^2*d*x^2]) + (4*c^2*(a + b*ArcSinh[c*x]))/(3*d*x*
Sqrt[d + c^2*d*x^2]) + (8*c^4*x*(a + b*ArcSinh[c*x]))/(3*d*Sqrt[d + c^2*d*x
^2]) - (5*b*c^3*Sqrt[d + c^2*d*x^2]*Log[x])/(3*d^2*Sqrt[1 + c^2*x^2]) - (b*
c^3*Sqrt[d + c^2*d*x^2]*Log[1 + c^2*x^2])/(2*d^2*Sqrt[1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\operatorname{arcsinh}(cx)}{3dx^3\sqrt{d+c^2dx^2}}+\frac{4c^2(a+b\operatorname{arcsinh}(cx))}{3dx\sqrt{d+c^2dx^2}} \\
&\quad +\frac{8c^4x(a+b\operatorname{arcsinh}(cx))}{3d\sqrt{d+c^2dx^2}}-\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int\frac{-1+4c^2x^2+8c^4x^4}{3d^2x^3(1+c^2x^2)}\,dx}{\sqrt{1+c^2x^2}} \\
&= -\frac{a+b\operatorname{arcsinh}(cx)}{3dx^3\sqrt{d+c^2dx^2}}+\frac{4c^2(a+b\operatorname{arcsinh}(cx))}{3dx\sqrt{d+c^2dx^2}} \\
&\quad +\frac{8c^4x(a+b\operatorname{arcsinh}(cx))}{3d\sqrt{d+c^2dx^2}}-\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int\frac{-1+4c^2x^2+8c^4x^4}{x^3(1+c^2x^2)}\,dx}{3d^2\sqrt{1+c^2x^2}} \\
&= -\frac{a+b\operatorname{arcsinh}(cx)}{3dx^3\sqrt{d+c^2dx^2}}+\frac{4c^2(a+b\operatorname{arcsinh}(cx))}{3dx\sqrt{d+c^2dx^2}}+\frac{8c^4x(a+b\operatorname{arcsinh}(cx))}{3d\sqrt{d+c^2dx^2}} \\
&\quad -\frac{\left(bc\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\frac{-1+4c^2x+8c^4x^2}{x^2(1+c^2x)}\,dx,x,x^2\right)}{6d^2\sqrt{1+c^2x^2}} \\
&= -\frac{a+b\operatorname{arcsinh}(cx)}{3dx^3\sqrt{d+c^2dx^2}}+\frac{4c^2(a+b\operatorname{arcsinh}(cx))}{3dx\sqrt{d+c^2dx^2}}+\frac{8c^4x(a+b\operatorname{arcsinh}(cx))}{3d\sqrt{d+c^2dx^2}} \\
&\quad -\frac{\left(bc\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\left(-\frac{1}{x^2}+\frac{5c^2}{x}+\frac{3c^4}{1+c^2x}\right)dx,x,x^2\right)}{6d^2\sqrt{1+c^2x^2}} \\
&= -\frac{bc\sqrt{d+c^2dx^2}}{6d^2x^2\sqrt{1+c^2x^2}}-\frac{a+b\operatorname{arcsinh}(cx)}{3dx^3\sqrt{d+c^2dx^2}}+\frac{4c^2(a+b\operatorname{arcsinh}(cx))}{3dx\sqrt{d+c^2dx^2}} \\
&\quad +\frac{8c^4x(a+b\operatorname{arcsinh}(cx))}{3d\sqrt{d+c^2dx^2}}-\frac{5bc^3\sqrt{d+c^2dx^2}\log(x)}{3d^2\sqrt{1+c^2x^2}}-\frac{bc^3\sqrt{d+c^2dx^2}\log\left(1+c^2x^2\right)}{2d^2\sqrt{1+c^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.95

$$\int\frac{a+b\operatorname{arcsinh}(cx)}{x^4\left(d+c^2dx^2\right)^{3/2}}\,dx=\frac{\sqrt{d+c^2dx^2}\left(-bcx-bc^3x^3-2a\sqrt{1+c^2x^2}+8ac^2x^2\sqrt{1+c^2x^2}+16ac^4x^4\sqrt{1+c^2x\right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^(3/2)),x]
[Out] (Sqrt[d + c^2*d*x^2]*(-(b*c*x) - b*c^3*x^3 - 2*a*Sqrt[1 + c^2*x^2] + 8*a*c^
2*x^2*Sqrt[1 + c^2*x^2] + 16*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 2*b*Sqrt[1 + c^2
*x^2]*(-1 + 4*c^2*x^2 + 8*c^4*x^4)*ArcSinh[c*x] + 5*b*c^3*x^3*(1 + c^2*x^2)
*Log[1 + 1/(c^2*x^2)] - 8*b*c^3*x^3*Log[1 + c^2*x^2] - 8*b*c^5*x^5*Log[1 +
c^2*x^2]))/(6*d^2*x^3*(1 + c^2*x^2)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 967 vs. 2(201) = 402.

Time = 0.22 (sec) , antiderivative size = 968, normalized size of antiderivative = 4.25

method	result
default	$a\left(-\frac{1}{3dx^3\sqrt{c^2dx^2+d}}-\frac{4c^2\left(-\frac{1}{dx\sqrt{c^2dx^2+d}}-\frac{2c^2x}{d\sqrt{c^2dx^2+d}}\right)}{3}\right)+\frac{16b\sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(cx)c^3}{3\sqrt{c^2x^2+1}\,d^2}+\frac{32b\sqrt{d(c^2x^2+1)}\,x^7c^{10}}{3(8c^4x^4+7c^2x^2-1)d^2}-$
parts	$a\left(-\frac{1}{3dx^3\sqrt{c^2dx^2+d}}-\frac{4c^2\left(-\frac{1}{dx\sqrt{c^2dx^2+d}}-\frac{2c^2x}{d\sqrt{c^2dx^2+d}}\right)}{3}\right)+\frac{16b\sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(cx)c^3}{3\sqrt{c^2x^2+1}\,d^2}+\frac{32b\sqrt{d(c^2x^2+1)}\,x^7c^{10}}{3(8c^4x^4+7c^2x^2-1)d^2}-$

```
[In] int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/3/d/x^3/(c^2*d*x^2+d)^(1/2)-4/3*c^2*(-1/d/x/(c^2*d*x^2+d)^(1/2)-2*c^2
/d*x/(c^2*d*x^2+d)^(1/2)))+16/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d
^2*arcsinh(c*x)*c^3+32/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^
2*x^7*c^10-32/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^5*(c^
2*x^2+1)*c^8+16*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^5*c^8
-16/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*(c^2*x^2+1)*c
^6+64/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*arcsinh(c*x
)*c^6-64/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^2*(c^2*x^2
+1)^(1/2)*arcsinh(c*x)*c^5+4*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1
)/d^2*x^3*c^6+4/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x*(c^
2*x^2+1)*c^4+8*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x*arcsin
h(c*x)*c^4+8/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*(c^2*x^2
+1)^(1/2)*arcsinh(c*x)*c^3-4/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2
-1)/d^2*x*c^4-4/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*c^3*(
c^2*x^2+1)^(1/2)-4*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2/x*ar
csinh(c*x)*c^2+1/6*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2/x^2*
c*(c^2*x^2+1)^(1/2)+1/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2
/x^3*arcsinh(c*x)-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*ln(1+(c*x+(
c^2*x^2+1)^(1/2))^2)*c^3-5/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*
ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c^3
```

Fricas [F]

$$\int \frac{a+b\mathrm{arcsinh}(cx)}{x^4\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{b\,\mathrm{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x^4}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^4*d^2*x^8 + 2*c^2*d^2*
x^6 + d^2*x^4), x)
```

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^4 (d + c^2dx^2)^{3/2}}\,dx = \int \frac{a + b\operatorname{asinh}(cx)}{x^4 (d(c^2x^2 + 1))^{\frac{3}{2}}}\,dx$$

[In] integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d)**(3/2),x)

[Out] Integral((a + b*asinh(c*x))/(x**4*(d*(c**2*x**2 + 1))**(3/2)), x)

Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^4 (d + c^2dx^2)^{3/2}}\,dx = \int \frac{b\,\text{arsinh}(cx) + a}{(c^2dx^2 + d)^{\frac{3}{2}}x^4}\,dx$$

[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")

[Out] 1/3*(8*c^4*x/(sqrt(c^2*d*x^2 + d)*d) + 4*c^2/(sqrt(c^2*d*x^2 + d)*d*x) - 1/(sqrt(c^2*d*x^2 + d)*d*x^3))*a + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/((c^2*d*x^2 + d)^(3/2)*x^4), x)

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^4 (d + c^2dx^2)^{3/2}}\,dx = \int \frac{b\,\text{arsinh}(cx) + a}{(c^2dx^2 + d)^{\frac{3}{2}}x^4}\,dx$$

[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(3/2)*x^4), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{x^4 (d + c^2dx^2)^{3/2}}\,dx = \int \frac{a + b\operatorname{asinh}(c\,x)}{x^4\,(d\,c^2\,x^2 + d)^{3/2}}\,dx$$

[In] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^(3/2)),x)

[Out] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^(3/2)), x)

3.165 $\int \frac{x^6(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 281

$$\int \frac{x^6(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = -\frac{b}{6c^7d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}}{4c^5d^2\sqrt{d+c^2dx^2}} - \frac{x^5(a+b\text{arcsinh}(cx))}{3c^2d\,(d+c^2dx^2)^{3/2}} - \frac{5x^3(a+b\text{arcsinh}(cx))}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{5x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2c^6d^3} - \frac{5\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{4bc^7d^2\sqrt{d+c^2dx^2}} - \frac{7b\sqrt{1+c^2x^2}\log(1+c^2x^2)}{6c^7d^2\sqrt{d+c^2dx^2}}$$

```
[Out] -1/3*x^5*(a+b*arcsinh(c*x))/c^2/d/(c^2*d*x^2+d)^(3/2)-5/3*x^3*(a+b*arcsinh(
c*x))/c^4/d^2/(c^2*d*x^2+d)^(1/2)-1/6*b/c^7/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^
2+d)^(1/2)-1/4*b*x^2*(c^2*x^2+1)^(1/2)/c^5/d^2/(c^2*d*x^2+d)^(1/2)-5/4*(a+b
*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c^7/d^2/(c^2*d*x^2+d)^(1/2)-7/6*b*ln(c
^2*x^2+1)*(c^2*x^2+1)^(1/2)/c^7/d^2/(c^2*d*x^2+d)^(1/2)+5/2*x*(a+b*arcsinh(
c*x))*(c^2*d*x^2+d)^(1/2)/c^6/d^3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used

$= \{5810, 5812, 5783, 30, 272, 45\}$

$$\int \frac{x^6(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = -\frac{x^5(a+b\operatorname{arcsinh}(cx))}{3c^2d\,(c^2dx^2+d)^{3/2}} - \frac{5\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^2}{4bc^7d^2\sqrt{c^2dx^2+d}} + \frac{5x\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{2c^6d^3} - \frac{5x^3(a+b\operatorname{arcsinh}(cx))}{3c^4d^2\sqrt{c^2dx^2+d}} - \frac{b}{6c^7d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}} - \frac{7b\sqrt{c^2x^2+1}\log(c^2x^2+1)}{6c^7d^2\sqrt{c^2dx^2+d}} - \frac{bx^2\sqrt{c^2x^2+1}}{4c^5d^2\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x^6*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -1/6*b/(c^7*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (b*x^2*Sqrt[1 + c^
2*x^2])/(4*c^5*d^2*Sqrt[d + c^2*d*x^2]) - (x^5*(a + b*ArcSinh[c*x]))/(3*c^2
*d*(d + c^2*d*x^2)^(3/2)) - (5*x^3*(a + b*ArcSinh[c*x]))/(3*c^4*d^2*Sqrt[d
+ c^2*d*x^2]) + (5*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(2*c^6*d^3)
- (5*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c^7*d^2*Sqrt[d + c^2*d*
x^2]) - (7*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(6*c^7*d^2*Sqrt[d + c^2*d*
x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^5(a+b\text{arcsinh}(cx))}{3c^2d\left(d+c^2dx^2\right)^{3/2}}+\frac{5\int\frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx}{3c^2d}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{x^5}{(1+c^2x^2)^2}\,dx}{3cd^2\sqrt{d+c^2dx^2}}\\
&= -\frac{x^5(a+b\text{arcsinh}(cx))}{3c^2d\left(d+c^2dx^2\right)^{3/2}}-\frac{5x^3(a+b\text{arcsinh}(cx))}{3c^4d^2\sqrt{d+c^2dx^2}}+\frac{5\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx}{c^4d^2}\\
&\quad+\frac{\left(5b\sqrt{1+c^2x^2}\right)\int\frac{x^3}{1+c^2x^2}\,dx}{3c^3d^2\sqrt{d+c^2dx^2}}+\frac{\left(b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{x^2}{(1+c^2x)^2}\,dx,x,x^2\right)}{6cd^2\sqrt{d+c^2dx^2}}\\
&= -\frac{x^5(a+b\text{arcsinh}(cx))}{3c^2d\left(d+c^2dx^2\right)^{3/2}}-\frac{5x^3(a+b\text{arcsinh}(cx))}{3c^4d^2\sqrt{d+c^2dx^2}}\\
&\quad+\frac{5x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2c^6d^3}-\frac{5\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx}{2c^6d^2}\\
&\quad-\frac{\left(5b\sqrt{1+c^2x^2}\right)\int x\,dx}{2c^5d^2\sqrt{d+c^2dx^2}}+\frac{\left(5b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{x}{1+c^2x}\,dx,x,x^2\right)}{6c^3d^2\sqrt{d+c^2dx^2}}\\
&\quad+\frac{\left(b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\left(\frac{1}{c^4}+\frac{1}{c^4(1+c^2x)^2}-\frac{2}{c^4(1+c^2x)}\right)\,dx,x,x^2\right)}{6cd^2\sqrt{d+c^2dx^2}}
\end{aligned}
$$

$$= -\frac{b}{6c^7d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{13bx^2\sqrt{1+c^2x^2}}{12c^5d^2\sqrt{d+c^2dx^2}} - \frac{x^5(a+b\operatorname{arcsinh}(cx))}{3c^2d\,(d+c^2dx^2)^{3/2}} - \frac{5x^3(a+b\operatorname{arcsinh}(cx))}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{5x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2c^6d^3} - \frac{5\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{4bc^7d^2\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\log(1+c^2x^2)}{3c^7d^2\sqrt{d+c^2dx^2}} + \frac{\left(5b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\left(\frac{1}{c^2}-\frac{1}{c^2(1+c^2x)}\right)dx,x,x^2\right)}{6c^3d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{b}{6c^7d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}}{4c^5d^2\sqrt{d+c^2dx^2}} - \frac{x^5(a+b\operatorname{arcsinh}(cx))}{3c^2d\,(d+c^2dx^2)^{3/2}} - \frac{5x^3(a+b\operatorname{arcsinh}(cx))}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{5x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2c^6d^3} - \frac{5\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{4bc^7d^2\sqrt{d+c^2dx^2}} - \frac{7b\sqrt{1+c^2x^2}\log(1+c^2x^2)}{6c^7d^2\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.79

$$\int \frac{x^6(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \frac{4acdx(15+20c^2x^2+3c^4x^4)+bd\Big(4cx(15+20c^2x^2+3c^4x^4)\operatorname{arcsinh}(cx)-30$$

```
[In] Integrate[(x^6*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (4*a*c*d*x*(15 + 20*c^2*x^2 + 3*c^4*x^4) + b*d*(4*c*x*(15 + 20*c^2*x^2 + 3*
c^4*x^4)*ArcSinh[c*x] - 30*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]^2 - Sqrt[1 + c^
2*x^2]*(7 + 9*c^2*x^2 + 6*c^4*x^4 + 28*(1 + c^2*x^2)*Log[1 + c^2*x^2])) - 6
0*a*Sqrt[d]*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d +
c^2*d*x^2]])/(24*c^7*d^3*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.46

method	result
default	$\frac{a\,x^5}{2c^2d(c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{5a\,x^3}{6c^4d(c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{5ax}{2c^6d^2\sqrt{c^2d\,x^2+d}}-\frac{5a\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2c^6d^2\sqrt{c^2d}}-\frac{b\sqrt{d(c^2x^2+1)}\,\sqrt{c^2x^2+1}\,\Big(-12\operatorname{arc}$
parts	$\frac{a\,x^5}{2c^2d(c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{5a\,x^3}{6c^4d(c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{5ax}{2c^6d^2\sqrt{c^2d\,x^2+d}}-\frac{5a\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2c^6d^2\sqrt{c^2d}}-\frac{b\sqrt{d(c^2x^2+1)}\,\sqrt{c^2x^2+1}\,\Big(-12\operatorname{arc}$

[In] int(x^6*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/2*a*x^5/c^2/d/(c^2*d*x^2+d)^(3/2)+5/6*a/c^4*x^3/d/(c^2*d*x^2+d)^(3/2)+5/2*a/c^6/d^2*x/(c^2*d*x^2+d)^(1/2)-5/2*a/c^6/d^2*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-1/24*b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*(-12*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^5*c^5+6*c^6*x^6+30*arcsinh(c*x)^2*x^4*c^4-56*arcsinh(c*x)*c^4*x^4+56*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^4*c^4-80*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3+15*c^4*x^4+60*arcsinh(c*x)^2*x^2*c^2-112*arcsinh(c*x)*c^2*x^2+112*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^2*c^2-60*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)+16*c^2*x^2+30*arcsinh(c*x)^2-56*arcsinh(c*x)+56*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+7)/(c^6*x^6+3*c^4*x^4+3*c^2*x^2+1)/c^7/d^3

Fricas [F]

$$\int \frac{x^6(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^6}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

[In] integrate(x^6*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")

[Out] integral((b*x^6*arcsinh(c*x) + a*x^6)*sqrt(c^2*d*x^2 + d)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)

Sympy [F]

$$\int \frac{x^6(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{x^6(a+b\,\text{asinh}\,(cx))}{(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

[In] integrate(x**6*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)

[Out] Integral(x**6*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)

Maxima [F]

$$\int \frac{x^6(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^6}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

[In] integrate(x^6*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")

[Out] 1/6*a*(3*x^5/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 5*x*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2/((c^2*d*x^2 + d)^(3/2)*c^4*d))/c^2 + 5*x/(sqrt(c^2*d*x^2 + d)*c^6*d^2) - 15*arcsinh(c*x)/(c^7*d^(5/2))) + b*integrate(x^6*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d)^(5/2), x)

Giac [F(-2)]

Exception generated.

$$\int \frac{x^6(a + b\operatorname{arcsinh}(cx))}{(d + c^2dx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^6*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(a + b\operatorname{arcsinh}(cx))}{(d + c^2dx^2)^{5/2}}\,dx = \int \frac{x^6\,(a + b\operatorname{asinh}(c\,x))}{(d\,c^2\,x^2 + d)^{5/2}}\,dx$$

```
[In] int((x^6*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^6*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2), x)
```

3.166 $\int \frac{x^5(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 210

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx=\frac{bx\sqrt{d+c^2dx^2}}{6c^5d^3(1+c^2x^2)^{3/2}}-\frac{bx\sqrt{d+c^2dx^2}}{c^5d^3\sqrt{1+c^2x^2}}-\frac{a+b\text{arcsinh}(cx)}{3c^6d(d+c^2dx^2)^{3/2}}+\frac{2(a+b\text{arcsinh}(cx))}{c^6d^2\sqrt{d+c^2dx^2}}+\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{c^6d^3}-\frac{11b\sqrt{d+c^2dx^2}\arctan(cx)}{6c^6d^3\sqrt{1+c^2x^2}}$$

```
[Out] 1/3*(-a-b*arcsinh(c*x))/c^6/d/(c^2*d*x^2+d)^(3/2)+2*(a+b*arcsinh(c*x))/c^6/
d^2/(c^2*d*x^2+d)^(1/2)+1/6*b*x*(c^2*d*x^2+d)^(1/2)/c^5/d^3/(c^2*x^2+1)^(3/
2)+(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^6/d^3-b*x*(c^2*d*x^2+d)^(1/2)/c
^5/d^3/(c^2*x^2+1)^(1/2)-11/6*b*arctan(c*x)*(c^2*d*x^2+d)^(1/2)/c^6/d^3/(c^
2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {272, 45, 5804, 12, 1171, 396, 209}

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx=\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{c^6d^3}+\frac{2(a+b\text{arcsinh}(cx))}{c^6d^2\sqrt{c^2dx^2+d}}-\frac{a+b\text{arcsinh}(cx)}{3c^6d(c^2dx^2+d)^{3/2}}-\frac{11b\arctan(cx)\sqrt{c^2dx^2+d}}{6c^6d^3\sqrt{c^2x^2+1}}-\frac{bx\sqrt{c^2dx^2+d}}{c^5d^3\sqrt{c^2x^2+1}}+\frac{bx\sqrt{c^2dx^2+d}}{6c^5d^3(c^2x^2+1)^{3/2}}$$

```
[In] Int[(x^5*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (b*x*Sqrt[d + c^2*d*x^2])/(6*c^5*d^3*(1 + c^2*x^2)^(3/2)) - (b*x*Sqrt[d + c
^2*d*x^2])/(c^5*d^3*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(3*c^6*d*(d +
 c^2*d*x^2)^(3/2)) + (2*(a + b*ArcSinh[c*x]))/(c^6*d^2*Sqrt[d + c^2*d*x^2])
 + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(c^6*d^3) - (11*b*Sqrt[d + c^
2*d*x^2]*ArcTan[c*x])/(6*c^6*d^3*Sqrt[1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\operatorname{arcsinh}(cx)}{3c^6d\left(d+c^2dx^2\right)^{3/2}}+\frac{2(a+b\operatorname{arcsinh}(cx))}{c^6d^2\sqrt{d+c^2dx^2}} \\
&\quad+\frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{c^6d^3}-\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int\frac{8+12c^2x^2+3c^4x^4}{3c^6d^3(1+c^2x^2)^2}\,dx}{\sqrt{1+c^2x^2}} \\
&= -\frac{a+b\operatorname{arcsinh}(cx)}{3c^6d\left(d+c^2dx^2\right)^{3/2}}+\frac{2(a+b\operatorname{arcsinh}(cx))}{c^6d^2\sqrt{d+c^2dx^2}} \\
&\quad+\frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{c^6d^3}-\frac{\left(b\sqrt{d+c^2dx^2}\right)\int\frac{8+12c^2x^2+3c^4x^4}{(1+c^2x^2)^2}\,dx}{3c^5d^3\sqrt{1+c^2x^2}} \\
&= \frac{bx\sqrt{d+c^2dx^2}}{6c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{a+b\operatorname{arcsinh}(cx)}{3c^6d\left(d+c^2dx^2\right)^{3/2}}+\frac{2(a+b\operatorname{arcsinh}(cx))}{c^6d^2\sqrt{d+c^2dx^2}} \\
&\quad+\frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{c^6d^3}+\frac{\left(b\sqrt{d+c^2dx^2}\right)\int\frac{-17-6c^2x^2}{1+c^2x^2}\,dx}{6c^5d^3\sqrt{1+c^2x^2}} \\
&= \frac{bx\sqrt{d+c^2dx^2}}{6c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{bx\sqrt{d+c^2dx^2}}{c^5d^3\sqrt{1+c^2x^2}}-\frac{a+b\operatorname{arcsinh}(cx)}{3c^6d\left(d+c^2dx^2\right)^{3/2}}+\frac{2(a+b\operatorname{arcsinh}(cx))}{c^6d^2\sqrt{d+c^2dx^2}} \\
&\quad+\frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{c^6d^3}-\frac{\left(11b\sqrt{d+c^2dx^2}\right)\int\frac{1}{1+c^2x^2}\,dx}{6c^5d^3\sqrt{1+c^2x^2}} \\
&= \frac{bx\sqrt{d+c^2dx^2}}{6c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{bx\sqrt{d+c^2dx^2}}{c^5d^3\sqrt{1+c^2x^2}}-\frac{a+b\operatorname{arcsinh}(cx)}{3c^6d\left(d+c^2dx^2\right)^{3/2}}+\frac{2(a+b\operatorname{arcsinh}(cx))}{c^6d^2\sqrt{d+c^2dx^2}} \\
&\quad+\frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{c^6d^3}-\frac{11b\sqrt{d+c^2dx^2}\arctan(cx)}{6c^6d^3\sqrt{1+c^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.84

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \frac{\sqrt{d+c^2dx^2}\Big(-5bcx-11bc^3x^3-6bc^5x^5+16a\sqrt{1+c^2x^2}+24ac^2x^2\sqrt{1+c^2x^2}}{}$$

[In] Integrate[(x^5*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]

[Out] (Sqrt[d + c^2*d*x^2]*(-5*b*c*x - 11*b*c^3*x^3 - 6*b*c^5*x^5 + 16*a*Sqrt[1 + c^2*x^2] + 24*a*c^2*x^2*Sqrt[1 + c^2*x^2] + 6*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 2*b*Sqrt[1 + c^2*x^2]*(8 + 12*c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x] - 11*b*(1 + c^2*x^2)^2*ArcTan[c*x]))/(6*c^6*d^3*(1 + c^2*x^2)^(5/2))

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.90

method	result
default	$a\left(\frac{x^4}{c^2d(c^2dx^2+d)^{\frac{3}{2}}}-\frac{4\left(-\frac{x^2}{c^2d(c^2dx^2+d)^{\frac{3}{2}}}-\frac{2}{3dc^4(c^2dx^2+d)^{\frac{3}{2}}}\right)}{c^2}\right)+\frac{b\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)x^2}{c^4d^3(c^2x^2+1)}-\frac{b\sqrt{d(c^2x^2+1)}\,x}{c^5d^3\sqrt{c^2x^2+1}}+$
parts	$a\left(\frac{x^4}{c^2d(c^2dx^2+d)^{\frac{3}{2}}}-\frac{4\left(-\frac{x^2}{c^2d(c^2dx^2+d)^{\frac{3}{2}}}-\frac{2}{3dc^4(c^2dx^2+d)^{\frac{3}{2}}}\right)}{c^2}\right)+\frac{b\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)x^2}{c^4d^3(c^2x^2+1)}-\frac{b\sqrt{d(c^2x^2+1)}\,x}{c^5d^3\sqrt{c^2x^2+1}}+$

[In] int(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)

[Out] a*(x^4/c^2/d/(c^2*d*x^2+d)^(3/2)-4/c^2*(-x^2/c^2/d/(c^2*d*x^2+d)^(3/2)-2/3/d/c^4/(c^2*d*x^2+d)^(3/2)))+b*(d*(c^2*x^2+1))^(1/2)/c^4/d^3/(c^2*x^2+1)*arcsinh(c*x)*x^2-b*(d*(c^2*x^2+1))^(1/2)/c^5/d^3/(c^2*x^2+1)^(1/2)*x+b*(d*(c^2*x^2+1))^(1/2)/c^6/d^3/(c^2*x^2+1)*arcsinh(c*x)+2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/c^4/d^3*arcsinh(c*x)*x^2+1/6*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(3/2)/c^5/d^3*x+5/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/c^6/d^3*arcsinh(c*x)+11/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^3*ln(c*x+(c^2*x^2+1)^(1/2)-I)-11/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^3*ln(c*x+(c^2*x^2+1)^(1/2)+I)

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.04

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \frac{11\left(bc^4x^4+2\,bc^2x^2+b\right)\sqrt{d}\arctan\left(\frac{2\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}c\sqrt{d}x}{c^4dx^4-d}\right)+4\left(3\,bc^4x^4+12\,bc^2\right.}{}$$

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(11*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d)
*sqrt(c^2*x^2 + 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 4*(3*b*c^4*x^4 + 12*b*c^2
*x^2 + 8*b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(6*a*c^4*x
^4 + 24*a*c^2*x^2 - (6*b*c^3*x^3 + 5*b*c*x)*sqrt(c^2*x^2 + 1) + 16*a)*sqrt(
c^2*d*x^2 + d))/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3)
```

Sympy [F]

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x^5(a+b\operatorname{asinh}(cx))}{(d(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**5*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**5*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x^5}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*b*((3*c^4*sqrt(d)*x^4 + 12*c^2*sqrt(d)*x^2 + 8*sqrt(d))*log(c*x + sqrt(
c^2*x^2 + 1))/((c^8*d^3*x^2 + c^6*d^3)*sqrt(c^2*x^2 + 1)) + 3*integrate(1/3
*(3*c^4*sqrt(d)*x^4 + 12*c^2*sqrt(d)*x^2 + 8*sqrt(d))/(c^11*d^3*x^6 + 2*c^9
*d^3*x^4 + c^7*d^3*x^2 + (c^10*d^3*x^5 + 2*c^8*d^3*x^3 + c^6*d^3*x)*sqrt(c^
2*x^2 + 1)), x) - 3*integrate(1/3*(3*c^4*sqrt(d)*x^4 + 12*c^2*sqrt(d)*x^2 +
 8*sqrt(d))/((c^8*d^3*x^3 + c^6*d^3*x)*sqrt(c^2*x^2 + 1)), x)) + 1/3*a*(3*x
^4/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 12*x^2/((c^2*d*x^2 + d)^(3/2)*c^4*d) + 8
/((c^2*d*x^2 + d)^(3/2)*c^6*d))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^{5/2}}\,dx = \int \frac{x^5\,(a + b\,\text{asinh}(c\,x))}{(d\,c^2\,x^2 + d)^{5/2}}\,dx$$

```
[In] int((x^5*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^5*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2), x)
```

3.167 $\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 203

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \frac{b}{6c^5d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d\,(d+c^2dx^2)^{3/2}} - \frac{x(a+b\text{arcsinh}(cx))}{c^4d^2\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc^5d^2\sqrt{d+c^2dx^2}} + \frac{2b\sqrt{1+c^2x^2}\log(1+c^2x^2)}{3c^5d^2\sqrt{d+c^2dx^2}}$$

```
[Out] -1/3*x^3*(a+b*arcsinh(c*x))/c^2/d/(c^2*d*x^2+d)^(3/2)-x*(a+b*arcsinh(c*x))/
c^4/d^2/(c^2*d*x^2+d)^(1/2)+1/6*b/c^5/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(
1/2)+1/2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c^5/d^2/(c^2*d*x^2+d)^(1/
2)+2/3*b*ln(c^2*x^2+1)*(c^2*x^2+1)^(1/2)/c^5/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5810, 5783, 266, 272, 45}

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = -\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d\,(c^2dx^2+d)^{3/2}} + \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{2bc^5d^2\sqrt{c^2dx^2+d}} - \frac{x(a+b\text{arcsinh}(cx))}{c^4d^2\sqrt{c^2dx^2+d}} + \frac{b}{6c^5d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}} + \frac{2b\sqrt{c^2x^2+1}\log(c^2x^2+1)}{3c^5d^2\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] b/(6*c^5*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (x^3*(a + b*ArcSinh[c
*x]))/(3*c^2*d*(d + c^2*d*x^2)^(3/2)) - (x*(a + b*ArcSinh[c*x]))/(c^4*d^2*S
```

```
qrt[d + c^2*d*x^2]) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c^5*d
^2*Sqrt[d + c^2*d*x^2]) + (2*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(3*c^5*d
^2*Sqrt[d + c^2*d*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rubi steps

$$\text{integral} = -\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d\left(d+c^2dx^2\right)^{3/2}}+\frac{\int\frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx}{c^2d}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{x^3}{(1+c^2x^2)^2}\,dx}{3cd^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{x(a+b\text{arcsinh}(cx))}{c^4d^2\sqrt{d+c^2dx^2}} + \frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx}{c^4d^2} + \frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{x}{1+c^2x^2}\,dx}{c^3d^2\sqrt{d+c^2dx^2}} + \frac{\left(b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{x}{(1+c^2x)^2}\,dx, x, x^2\right)}{6cd^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{x(a+b\text{arcsinh}(cx))}{c^4d^2\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc^5d^2\sqrt{d+c^2dx^2}} + \frac{b\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)}{2c^5d^2\sqrt{d+c^2dx^2}} + \frac{\left(b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \left(-\frac{1}{c^2(1+c^2x)^2}+\frac{1}{c^2(1+c^2x)}\right)\,dx, x, x^2\right)}{6cd^2\sqrt{d+c^2dx^2}}$$

$$= \frac{b}{6c^5d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{x(a+b\text{arcsinh}(cx))}{c^4d^2\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc^5d^2\sqrt{d+c^2dx^2}} + \frac{2b\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)}{3c^5d^2\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.94

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{\left(d+c^2dx^2\right)^{5/2}}\,dx = \frac{-2ac\sqrt{d}x(3+4c^2x^2)+b\sqrt{d}\left(\sqrt{1+c^2x^2}+2cx\text{arcsinh}(cx)-8cx(1+c^2x^2)\,\text{arcs}\right.}{}$$

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (-2*a*c*Sqrt[d]*x*(3 + 4*c^2*x^2) + b*Sqrt[d]*(Sqrt[1 + c^2*x^2] + 2*c*x*Ar
cSinh[c*x] - 8*c*x*(1 + c^2*x^2)*ArcSinh[c*x] + (1 + c^2*x^2)^(3/2)*(3*ArcS
inh[c*x]^2 + 4*Log[1 + c^2*x^2])) + 6*a*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*L
og[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/(6*c^5*d^(5/2)*(1 + c^2*x^2)*Sqrt[
d + c^2*d*x^2])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.70

method	result
default	$-\frac{a\,x^3}{3c^2d(c^2d\,x^2+d)^{\frac{3}{2}}}-\frac{ax}{c^4d^2\sqrt{c^2d\,x^2+d}}+\frac{a\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{c^4d^2\sqrt{c^2d}}+\frac{b\sqrt{d(c^2x^2+1)}\,\sqrt{c^2x^2+1}\,\left(3\,\text{arcsinh}(cx)^2x^4c^4-8\,\text{arcsinh}(c\right.}{}$
parts	$-\frac{a\,x^3}{3c^2d(c^2d\,x^2+d)^{\frac{3}{2}}}-\frac{ax}{c^4d^2\sqrt{c^2d\,x^2+d}}+\frac{a\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{c^4d^2\sqrt{c^2d}}+\frac{b\sqrt{d(c^2x^2+1)}\,\sqrt{c^2x^2+1}\,\left(3\,\text{arcsinh}(cx)^2x^4c^4-8\,\text{arcsinh}(c\right.}{}$

```
[In] int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a*x^3/c^2/d/(c^2*d*x^2+d)^(3/2)-a/c^4/d^2*x/(c^2*d*x^2+d)^(1/2)+a/c^4/
d^2*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/6*b*(d*(c
^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/(c^6*x^6+3*c^4*x^4+3*c^2*x^2+1)/c^5/d^3*
(3*arcsinh(c*x)^2*x^4*c^4-8*arcsinh(c*x)*c^4*x^4+8*ln(1+(c*x+(c^2*x^2+1)^(1
/2))^2)*x^4*c^4-8*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3+6*arcsinh(c*x)^2*x
^2*c^2-16*arcsinh(c*x)*c^2*x^2+16*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^2*c^2-6
*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)+c^2*x^2+3*arcsinh(c*x)^2-8*arcsinh(c*x)
+8*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1)
```

Fricas [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^4}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arcsinh(c*x) + a*x^4)*sqrt(c^2*d*x^2 + d)/(c^6*d^3*x^6 + 3*
c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{x^4(a+b\,\text{asinh}\,(cx))}{(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**4*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^4}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(x*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2/((c^2*d*x^2 + d)^(3/2)*c^4
*d)) + x/(sqrt(c^2*d*x^2 + d)*c^4*d^2) - 3*arcsinh(c*x)/(c^5*d^(5/2)))*a +
b*integrate(x^4*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d)^(5/2), x)
```

Giac [F]

$$\int \frac{x^4(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)x^4}{(c^2dx^2 + d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)*x^4/(c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^{5/2}}\,dx = \int \frac{x^4\,(a + b\,\text{asinh}(c\,x))}{(d\,c^2\,x^2 + d)^{5/2}}\,dx$$

```
[In] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2),x)
[Out] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2), x)
```

3.168 $\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 144

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = -\frac{bx\sqrt{d+c^2dx^2}}{6c^3d^3(1+c^2x^2)^{3/2}} + \frac{a+b\text{arcsinh}(cx)}{3c^4d(d+c^2dx^2)^{3/2}} - \frac{a+b\text{arcsinh}(cx)}{c^4d^2\sqrt{d+c^2dx^2}} + \frac{5b\sqrt{d+c^2dx^2}\arctan(cx)}{6c^4d^3\sqrt{1+c^2x^2}}$$

```
[Out] 1/3*(a+b*arcsinh(c*x))/c^4/d/(c^2*d*x^2+d)^(3/2)+(-a-b*arcsinh(c*x))/c^4/d^
2/(c^2*d*x^2+d)^(1/2)-1/6*b*x*(c^2*d*x^2+d)^(1/2)/c^3/d^3/(c^2*x^2+1)^(3/2)
+5/6*b*arctan(c*x)*(c^2*d*x^2+d)^(1/2)/c^4/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {272, 45, 5804, 12, 393, 209}

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = -\frac{a+b\text{arcsinh}(cx)}{c^4d^2\sqrt{c^2dx^2+d}} + \frac{a+b\text{arcsinh}(cx)}{3c^4d(c^2dx^2+d)^{3/2}} + \frac{5b\arctan(cx)\sqrt{c^2dx^2+d}}{6c^4d^3\sqrt{c^2x^2+1}} - \frac{bx\sqrt{c^2dx^2+d}}{6c^3d^3(c^2x^2+1)^{3/2}}$$

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -1/6*(b*x*Sqrt[d + c^2*d*x^2])/(c^3*d^3*(1 + c^2*x^2)^(3/2)) + (a + b*ArcSi
nh[c*x])/(3*c^4*d*(d + c^2*d*x^2)^(3/2)) - (a + b*ArcSinh[c*x])/(c^4*d^2*Sq
```

```
rt[d + c^2*d*x^2]) + (5*b*Sqrt[d + c^2*d*x^2]*ArcTan[c*x])/(6*c^4*d^3*Sqrt[
1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\text{integral} = \frac{a+b\text{arcsinh}(cx)}{3c^4d\left(d+c^2dx^2\right)^{3/2}} - \frac{a+b\text{arcsinh}(cx)}{c^4d^2\sqrt{d+c^2dx^2}} - \frac{\left(bc\sqrt{d+c^2dx^2}\right)\int \frac{-2-3c^2x^2}{3c^4d^3(1+c^2x^2)^2}\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{a+b\text{arcsinh}(cx)}{3c^4d\left(d+c^2dx^2\right)^{3/2}} - \frac{a+b\text{arcsinh}(cx)}{c^4d^2\sqrt{d+c^2dx^2}} - \frac{\left(b\sqrt{d+c^2dx^2}\right)\int \frac{-2-3c^2x^2}{(1+c^2x^2)^2}\,dx}{3c^3d^3\sqrt{1+c^2x^2}}$$

$$= -\frac{bx\sqrt{d+c^2dx^2}}{6c^3d^3\left(1+c^2x^2\right)^{3/2}} + \frac{a+b\text{arcsinh}(cx)}{3c^4d\left(d+c^2dx^2\right)^{3/2}} - \frac{a+b\text{arcsinh}(cx)}{c^4d^2\sqrt{d+c^2dx^2}} + \frac{\left(5b\sqrt{d+c^2dx^2}\right)\int \frac{1}{1+c^2x^2}\,dx}{6c^3d^3\sqrt{1+c^2x^2}}$$

$$= -\frac{bx\sqrt{d+c^2dx^2}}{6c^3d^3\left(1+c^2x^2\right)^{3/2}} + \frac{a+b\text{arcsinh}(cx)}{3c^4d\left(d+c^2dx^2\right)^{3/2}} - \frac{a+b\text{arcsinh}(cx)}{c^4d^2\sqrt{d+c^2dx^2}} + \frac{5b\sqrt{d+c^2dx^2}\arctan(cx)}{6c^4d^3\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.94

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{\left(d+c^2dx^2\right)^{5/2}}\,dx =$$
$$-\frac{\sqrt{d+c^2dx^2}\left(bcx+bc^3x^3+4a\sqrt{1+c^2x^2}+6ac^2x^2\sqrt{1+c^2x^2}+2b\sqrt{1+c^2x^2}(2+3c^2x^2)\,\text{arcsinh}(cx)-5b(\ldots\right.}{6c^4d^3\left(1+c^2x^2\right)^{5/2}}$$

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -1/6*(Sqrt[d + c^2*d*x^2]*(b*c*x + b*c^3*x^3 + 4*a*Sqrt[1 + c^2*x^2] + 6*a*
c^2*x^2*Sqrt[1 + c^2*x^2] + 2*b*Sqrt[1 + c^2*x^2]*(2 + 3*c^2*x^2)*ArcSinh[c
*x] - 5*b*(1 + c^2*x^2)^2*ArcTan[c*x]))/(c^4*d^3*(1 + c^2*x^2)^(5/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.83

method	result
default	$a\left(-\frac{x^2}{c^2d(c^2dx^2+d)^{\frac{3}{2}}}-\frac{2}{3dc^4(c^2dx^2+d)^{\frac{3}{2}}}\right)-\frac{b\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)x^2}{(c^2x^2+1)^2d^3c^2}-\frac{b\sqrt{d(c^2x^2+1)}\,x}{6(c^2x^2+1)^{\frac{3}{2}}d^3c^3}-\frac{2b\sqrt{d(c^2x^2+1)}\,\text{arcsinh}\ldots}{3(c^2x^2+1)^2d^3c^4}$
parts	$a\left(-\frac{x^2}{c^2d(c^2dx^2+d)^{\frac{3}{2}}}-\frac{2}{3dc^4(c^2dx^2+d)^{\frac{3}{2}}}\right)-\frac{b\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)x^2}{(c^2x^2+1)^2d^3c^2}-\frac{b\sqrt{d(c^2x^2+1)}\,x}{6(c^2x^2+1)^{\frac{3}{2}}d^3c^3}-\frac{2b\sqrt{d(c^2x^2+1)}\,\text{arcsinh}\ldots}{3(c^2x^2+1)^2d^3c^4}$

```
[In] int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-x^2/c^2/d/(c^2*d*x^2+d)^(3/2)-2/3/d/c^4/(c^2*d*x^2+d)^(3/2))-b*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^2/d^3/c^2*arcsinh(c*x)*x^2-1/6*b*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(3/2)/d^3/c^3*x-2/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2
/d^3/c^4*arcsinh(c*x)+5/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d
^3*ln(c*x+(c^2*x^2+1)^(1/2)+I)-5/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1
/2)/c^4/d^3*ln(c*x+(c^2*x^2+1)^(1/2)-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.31

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx =$$
$$-\frac{5\left(bc^4x^4+2\,bc^2x^2+b\right)\sqrt{d}\arctan\left(\frac{2\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}c\sqrt{d}x}{c^4dx^4-d}\right)+4\left(3\,bc^2x^2+2\,b\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right.}{12\left(c^8d^3x^4+2\,c^6d^3x^2+c^4d^3\right)}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(5*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d)
*sqrt(c^2*x^2 + 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 4*(3*b*c^2*x^2 + 2*b)*sqr
t(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(6*a*c^2*x^2 + sqrt(c^2*x
^2 + 1)*b*c*x + 4*a)*sqrt(c^2*d*x^2 + d))/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^
4*d^3)
```

Sympy [F]

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x^3(a+b\operatorname{asinh}(cx))}{(d(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.96

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = -\frac{1}{6}\,bc\left(\frac{x}{c^6d^{\frac{5}{2}}x^2+c^4d^{\frac{5}{2}}}-\frac{5\,\arctan\left(cx\right)}{c^5d^{\frac{5}{2}}}\right) - \frac{1}{3}\,b\left(\frac{3\,x^2}{(c^2dx^2+d)^{\frac{3}{2}}c^2d}+\frac{2}{(c^2dx^2+d)^{\frac{3}{2}}c^4d}\right)\text{arsinh}\left(cx\right) - \frac{1}{3}\,a\left(\frac{3\,x^2}{(c^2dx^2+d)^{\frac{3}{2}}c^2d}+\frac{2}{(c^2dx^2+d)^{\frac{3}{2}}c^4d}\right)$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/6*b*c*(x/(c^6*d^(5/2)*x^2 + c^4*d^(5/2)) - 5*arctan(c*x)/(c^5*d^(5/2)))
- 1/3*b*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2/((c^2*d*x^2 + d)^(3/2)*c^4
*d))*arcsinh(c*x) - 1/3*a*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2/((c^2*d*
x^2 + d)^(3/2)*c^4*d))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x^3\,\left(a+b\,\mathrm{asinh}(c\,x)\right)}{{\left(d\,c^2\,x^2+d\right)}^{5/2}} \,d x$$

```
[In] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2), x)
```

3.169 $\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 119

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = -\frac{b}{6c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x^3(a+b\text{arcsinh}(cx))}{3d(d+c^2dx^2)^{3/2}} - \frac{b\sqrt{1+c^2x^2}\log(1+c^2x^2)}{6c^3d^2\sqrt{d+c^2dx^2}}$$

```
[Out] 1/3*x^3*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(3/2)-1/6*b/c^3/d^2/(c^2*x^2+1)^
(1/2)/(c^2*d*x^2+d)^(1/2)-1/6*b*ln(c^2*x^2+1)*(c^2*x^2+1)^(1/2)/c^3/d^2/(c^
2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {5800, 272, 45}

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \frac{x^3(a+b\text{arcsinh}(cx))}{3d(c^2dx^2+d)^{3/2}} - \frac{b}{6c^3d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}} - \frac{b\sqrt{c^2x^2+1}\log(c^2x^2+1)}{6c^3d^2\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -1/6*b/(c^3*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (x^3*(a + b*ArcSin
h[c*x]))/(3*d*(d + c^2*d*x^2)^(3/2)) - (b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2
])/(6*c^3*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^3(a+b\text{arcsinh}(cx))}{3d\,(d+c^2dx^2)^{3/2}} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{x^3}{(1+c^2x^2)^2}\,dx}{3d^2\sqrt{d+c^2dx^2}} \\
&= \frac{x^3(a+b\text{arcsinh}(cx))}{3d\,(d+c^2dx^2)^{3/2}} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{x}{(1+c^2x)^2}\,dx, x, x^2\right)}{6d^2\sqrt{d+c^2dx^2}} \\
&= \frac{x^3(a+b\text{arcsinh}(cx))}{3d\,(d+c^2dx^2)^{3/2}} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \left(-\frac{1}{c^2(1+c^2x)^2}+\frac{1}{c^2(1+c^2x)}\right)dx, x, x^2\right)}{6d^2\sqrt{d+c^2dx^2}} \\
&= -\frac{b}{6c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x^3(a+b\text{arcsinh}(cx))}{3d\,(d+c^2dx^2)^{3/2}} - \frac{b\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)}{6c^3d^2\sqrt{d+c^2dx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx =$$
$$-\frac{\sqrt{d+c^2dx^2}\left(b+bc^2x^2-2ac^3x^3\sqrt{1+c^2x^2}-2bc^3x^3\sqrt{1+c^2x^2}\operatorname{arcsinh}(cx)+b(1+c^2x^2)^2\log(1+c^2x^2)\right)}{6c^3d^3(1+c^2x^2)^{5/2}}$$

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -1/6*(Sqrt[d + c^2*d*x^2]*(b + b*c^2*x^2 - 2*a*c^3*x^3*Sqrt[1 + c^2*x^2] -
2*b*c^3*x^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + b*(1 + c^2*x^2)^2*Log[1 + c^2*
x^2]))/(c^3*d^3*(1 + c^2*x^2)^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(103) = 206.

Time = 0.26 (sec) , antiderivative size = 413, normalized size of antiderivative = 3.47

method	result
default	$a\left(-\frac{x}{2c^2d(c^2dx^2+d)^{\frac{3}{2}}}+\frac{\frac{x}{3d(c^2dx^2+d)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{c^2dx^2+d}}}{2c^2}\right)+\frac{b\sqrt{d(c^2x^2+1)}\left(c^3x^3+c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}\right)\left(-2\ln\left(1+(cx\right.\right.}{}$
parts	$a\left(-\frac{x}{2c^2d(c^2dx^2+d)^{\frac{3}{2}}}+\frac{\frac{x}{3d(c^2dx^2+d)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{c^2dx^2+d}}}{2c^2}\right)+\frac{b\sqrt{d(c^2x^2+1)}\left(c^3x^3+c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}\right)\left(-2\ln\left(1+(cx\right.\right.}{}$

```
[In] int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/2*x/c^2/d/(c^2*d*x^2+d)^(3/2)+1/2/c^2*(1/3/d*x/(c^2*d*x^2+d)^(3/2)+2/
3/d^2*x/(c^2*d*x^2+d)^(1/2)))+1/6*b*(d*(c^2*x^2+1))^(1/2)*(c^3*x^3+c^2*x^2*
(c^2*x^2+1)^(1/2)+(c^2*x^2+1)^(1/2))*(-2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^
6*c^6+2*(c^2*x^2+1)^(1/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^5*c^5+6*arcsinh
(c*x)*c^4*x^4-6*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^4*c^4+2*(c^2*x^2+1)^(1/2)
*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^3*c^3-c^4*x^4+c^3*x^3*(c^2*x^2+1)^(1/2)+
6*arcsinh(c*x)*c^2*x^2-6*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^2*c^2-2*c^2*x^2+
2*arcsinh(c*x)-2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-1)/(3*c^8*x^8+9*c^6*x^6+10
*c^4*x^4+5*c^2*x^2+1)/c^3/d^3
```

Fricas [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^2}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(c^2*d*x^2 + d)*(b*x^2*arcsinh(c*x) + a*x^2)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)

Sympy [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x^2(a+b\,\text{asinh}\,(cx))}{(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

[In] integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)

[Out] Integral(x**2*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.15

$$\begin{aligned}\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx =& -\frac{1}{6}bc\left(\frac{1}{c^6d^{\frac{5}{2}}x^2+c^4d^{\frac{5}{2}}}+\frac{\log\left(c^2x^2+1\right)}{c^4d^{\frac{5}{2}}}\right) \\ &+ \frac{1}{3}b\left(\frac{x}{\sqrt{c^2dx^2+d}c^2d^2}-\frac{x}{(c^2dx^2+d)^{\frac{3}{2}}c^2d}\right)\text{arsinh}\,(cx) \\ &+ \frac{1}{3}a\left(\frac{x}{\sqrt{c^2dx^2+d}c^2d^2}-\frac{x}{(c^2dx^2+d)^{\frac{3}{2}}c^2d}\right)\end{aligned}$$

[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")

[Out] -1/6*b*c*(1/(c^6*d^(5/2)*x^2 + c^4*d^(5/2)) + log(c^2*x^2 + 1)/(c^4*d^(5/2))) + 1/3*b*(x/(sqrt(c^2*d*x^2 + d)*c^2*d^2) - x/((c^2*d*x^2 + d)^(3/2)*c^2*d))*arcsinh(c*x) + 1/3*a*(x/(sqrt(c^2*d*x^2 + d)*c^2*d^2) - x/((c^2*d*x^2 + d)^(3/2)*c^2*d))

Giac [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^2}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)*x^2/(c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{x^2\,(a+b\,\text{asinh}(c\,x))}{(d\,c^2\,x^2+d)^{5/2}}\,dx$$

```
[In] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2),x)
[Out] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2), x)
```

3.170 $\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 114

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \frac{bx}{6cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{a+b\text{arcsinh}(cx)}{3c^2d\,(d+c^2dx^2)^{3/2}} + \frac{b\sqrt{1+c^2x^2}\arctan(cx)}{6c^2d^2\sqrt{d+c^2dx^2}}$$

```
[Out] 1/3*(-a-b*arcsinh(c*x))/c^2/d/(c^2*d*x^2+d)^(3/2)+1/6*b*x/c/d^2/(c^2*x^2+1)
^(1/2)/(c^2*d*x^2+d)^(1/2)+1/6*b*arctan(c*x)*(c^2*x^2+1)^(1/2)/c^2/d^2/(c^2
*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5798, 205, 209}

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = -\frac{a+b\text{arcsinh}(cx)}{3c^2d\,(c^2dx^2+d)^{3/2}} + \frac{b\sqrt{c^2x^2+1}\arctan(cx)}{6c^2d^2\sqrt{c^2dx^2+d}} + \frac{bx}{6cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (b*x)/(6*c*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x]
)/(3*c^2*d*(d + c^2*d*x^2)^(3/2)) + (b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(6*c^
2*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a+b\operatorname{arcsinh}(cx)}{3c^2d\,(d+c^2dx^2)^{3/2}}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{1}{(1+c^2x^2)^2}\,dx}{3cd^2\sqrt{d+c^2dx^2}}\\
&= \frac{bx}{6cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{a+b\operatorname{arcsinh}(cx)}{3c^2d\,(d+c^2dx^2)^{3/2}}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{1}{1+c^2x^2}\,dx}{6cd^2\sqrt{d+c^2dx^2}}\\
&= \frac{bx}{6cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{a+b\operatorname{arcsinh}(cx)}{3c^2d\,(d+c^2dx^2)^{3/2}}+\frac{b\sqrt{1+c^2x^2}\arctan(cx)}{6c^2d^2\sqrt{d+c^2dx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

$$
\int\frac{x(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx=\frac{\sqrt{d+c^2dx^2}\Big(bcx+bc^3x^3-2a\sqrt{1+c^2x^2}-2b\sqrt{1+c^2x^2}\operatorname{arcsinh}(cx)+b(1+c^2x^2}{6c^2d^3\,(1+c^2x^2)^{5/2}}
$$

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(b*c*x + b*c^3*x^3 - 2*a*Sqrt[1 + c^2*x^2] - 2*b*Sqrt[
1 + c^2*x^2]*ArcSinh[c*x] + b*(1 + c^2*x^2)^2*ArcTan[c*x]))/(6*c^2*d^3*(1 +
 c^2*x^2)^(5/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.74

method	result
default	$-\frac{a}{3c^2d(c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{b\sqrt{d(c^2x^2+1)}\,x}{6(c^2x^2+1)^{\frac{3}{2}}d^3c}-\frac{b\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)}{3(c^2x^2+1)^2d^3c^2}+\frac{ib\sqrt{d(c^2x^2+1)}\,\ln\left(cx+\sqrt{c^2x^2+1}+i\right)}{6\sqrt{c^2x^2+1}\,c^2d^3}-\frac{ib\sqrt{d(c^2x^2+1}}{6\sqrt{}}$
parts	$-\frac{a}{3c^2d(c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{b\sqrt{d(c^2x^2+1)}\,x}{6(c^2x^2+1)^{\frac{3}{2}}d^3c}-\frac{b\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)}{3(c^2x^2+1)^2d^3c^2}+\frac{ib\sqrt{d(c^2x^2+1)}\,\ln\left(cx+\sqrt{c^2x^2+1}+i\right)}{6\sqrt{c^2x^2+1}\,c^2d^3}-\frac{ib\sqrt{d(c^2x^2+1}}{6\sqrt{}}$

```
[In] int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/c^2/d/(c^2*d*x^2+d)^(3/2)+1/6*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(3
/2)/d^3/c*x-1/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/d^3/c^2*arcsinh(c*x)+
1/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*ln(c*x+(c^2*x^2+1)^
(1/2)+I)-1/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*ln(c*x+(c^
2*x^2+1)^(1/2)-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.46

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx =$$
$$-\frac{(bc^4x^4+2\,bc^2x^2+b)\sqrt{d}\arctan\left(\frac{2\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}c\sqrt{d}x}{c^4dx^4-d}\right)+4\sqrt{c^2dx^2+d}b\log\left(cx+\sqrt{c^2x^2+1}\right)-2\sqrt{c^2dx^2}}{12\left(c^6d^3x^4+2\,c^4d^3x^2+c^2d^3\right)}$$

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*((b*c^4*x^4 + 2*b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d)*s
qrt(c^2*x^2 + 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 4*sqrt(c^2*d*x^2 + d)*b*log
(c*x + sqrt(c^2*x^2 + 1)) - 2*sqrt(c^2*d*x^2 + d)*(sqrt(c^2*x^2 + 1)*b*c*x
- 2*a))/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3)
```

Sympy [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x(a+b\,\text{asinh}\,(cx))}{(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
[Out] Integral(x*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
[Out] b*integrate(x*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d)^(5/2), x) - 1/3*
a/((c^2*d*x^2 + d)^(3/2)*c^2*d)
```

Giac [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)*x/(c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x\,(a+b\,\text{asinh}(c\,x))}{(d\,c^2\,x^2+d)^{5/2}}\,dx$$

```
[In] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2),x)
[Out] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2), x)
```

3.171 $\int \frac{a+b\text{arcsinh}(cx)}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{5/2}}\,dx = \frac{b}{6cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x(a+b\text{arcsinh}(cx))}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)}{3cd^2\sqrt{d+c^2dx^2}}$$

```
[Out] 1/3*x*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(3/2)+2/3*x*(a+b*arcsinh(c*x))/d^2
/(c^2*d*x^2+d)^(1/2)+1/6*b/c/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-1/3*
b*ln(c^2*x^2+1)*(c^2*x^2+1)^(1/2)/c/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5788, 5787, 266, 267}

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{5/2}}\,dx = \frac{2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{c^2dx^2+d}} + \frac{x(a+b\text{arcsinh}(cx))}{3d\left(c^2dx^2+d\right)^{3/2}} + \frac{b}{6cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}} - \frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{3cd^2\sqrt{c^2dx^2+d}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] b/(6*c*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (x*(a + b*ArcSinh[c*x])
)/(3*d*(d + c^2*d*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt[d +
c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(3*c*d^2*Sqrt[d + c^2*
d*x^2])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(a+b\text{arcsinh}(cx))}{3d\left(d+c^2dx^2\right)^{3/2}}+\frac{2\int\frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{3/2}}\,dx}{3d}-\frac{\left(bc\sqrt{1+c^2x^2}\right)\int\frac{x}{(1+c^2x^2)^2}\,dx}{3d^2\sqrt{d+c^2dx^2}}\\
&= \frac{b}{6cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}+\frac{x(a+b\text{arcsinh}(cx))}{3d\left(d+c^2dx^2\right)^{3/2}}\\
&\quad+\frac{2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+c^2dx^2}}-\frac{\left(2bc\sqrt{1+c^2x^2}\right)\int\frac{x}{1+c^2x^2}\,dx}{3d^2\sqrt{d+c^2dx^2}}\\
&= \frac{b}{6cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}+\frac{x(a+b\text{arcsinh}(cx))}{3d\left(d+c^2dx^2\right)^{3/2}}\\
&\quad+\frac{2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+c^2dx^2}}-\frac{b\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)}{3cd^2\sqrt{d+c^2dx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.97

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{5/2}}\,dx = \frac{\sqrt{d+c^2dx^2}\Big(b+bc^2x^2+6acx\sqrt{1+c^2x^2}+4ac^3x^3\sqrt{1+c^2x^2}+2bcx\sqrt{1+c^2x^2}(3+\ldots}{6cd^3\,(1+c^2x^2)^{5/2}}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(b + b*c^2*x^2 + 6*a*c*x*Sqrt[1 + c^2*x^2] + 4*a*c^3*x
^3*Sqrt[1 + c^2*x^2] + 2*b*c*x*Sqrt[1 + c^2*x^2]*(3 + 2*c^2*x^2)*ArcSinh[c*
x] - 2*b*(1 + c^2*x^2)^2*Log[1 + c^2*x^2]))/(6*c*d^3*(1 + c^2*x^2)^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. 2(127) = 254.

Time = 0.28 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.89

method	result
default	$a\left(\frac{x}{3d(c^2dx^2+d)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{c^2dx^2+d}}\right)+\frac{b\sqrt{d(c^2x^2+1)}\left(2c^3x^3+2c^2x^2\sqrt{c^2x^2+1}+3cx+2\sqrt{c^2x^2+1}\right)\left(-8\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right.\right.\right.\ldots}{\ldots}$
parts	$a\left(\frac{x}{3d(c^2dx^2+d)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{c^2dx^2+d}}\right)+\frac{b\sqrt{d(c^2x^2+1)}\left(2c^3x^3+2c^2x^2\sqrt{c^2x^2+1}+3cx+2\sqrt{c^2x^2+1}\right)\left(-8\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right.\right.\right.\ldots}{\ldots}$

```
[In] int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/3/d*x/(c^2*d*x^2+d)^(3/2)+2/3/d^2*x/(c^2*d*x^2+d)^(1/2))+1/6*b*(d*(c^2
*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x+2*(c^2*x^2+1)^(
1/2))*(-8*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^6*c^6+8*(c^2*x^2+1)^(1/2)*ln(1+
(c*x+(c^2*x^2+1)^(1/2))^2)*x^5*c^5-24*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^4*c
^4+20*(c^2*x^2+1)^(1/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^3*c^3+2*c^4*x^4-2
*c^3*x^3*(c^2*x^2+1)^(1/2)+6*arcsinh(c*x)*c^2*x^2-24*ln(1+(c*x+(c^2*x^2+1)^
(1/2))^2)*x^2*c^2+12*(c^2*x^2+1)^(1/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x*c+
4*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+8*arcsinh(c*x)-8*ln(1+(c*x+(c^2*x^2+1)^(1
/2))^2)+2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/c/d^3
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^6*d^3*x^6 + 3*c^4*d^3*
x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{a+b\,\text{asinh}\,(cx)}{(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
[Out] Integral((a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{5/2}}\,dx &= \frac{1}{6}\,bc\left(\frac{1}{c^4d^{\frac{5}{2}}x^2+c^2d^{\frac{5}{2}}}-\frac{2\,\log\left(c^2x^2+1\right)}{c^2d^{\frac{5}{2}}}\right)\\&+\frac{1}{3}\,b\left(\frac{2\,x}{\sqrt{c^2dx^2+d}d^2}+\frac{x}{(c^2dx^2+d)^{\frac{3}{2}}d}\right)\text{arsinh}\,(cx)\\&+\frac{1}{3}\,a\left(\frac{2\,x}{\sqrt{c^2dx^2+d}d^2}+\frac{x}{(c^2dx^2+d)^{\frac{3}{2}}d}\right)\end{aligned}$$

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
[Out] 1/6*b*c*(1/(c^4*d^(5/2)*x^2 + c^2*d^(5/2)) - 2*log(c^2*x^2 + 1)/(c^2*d^(5/2
))) + 1/3*b*(2*x/(sqrt(c^2*d*x^2 + d)*d^2) + x/((c^2*d*x^2 + d)^(3/2)*d))*a
rcsinh(c*x) + 1/3*a*(2*x/(sqrt(c^2*d*x^2 + d)*d^2) + x/((c^2*d*x^2 + d)^(3/
2)*d))
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + c^2dx^2)^{5/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{(c^2dx^2 + d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + c^2dx^2)^{5/2}}\,dx = \int \frac{a + b\,\text{asinh}(c\,x)}{(d\,c^2\,x^2 + d)^{5/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(5/2),x)
[Out] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(5/2), x)
```

3.172 $\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 262

$$\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)^{5/2}}\,dx = -\frac{bcx}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{a+b\text{arcsinh}(cx)}{3d(d+c^2dx^2)^{3/2}} + \frac{a+b\text{arcsinh}(cx)}{d^2\sqrt{d+c^2dx^2}} - \frac{7b\sqrt{1+c^2x^2}\arctan(cx)}{6d^2\sqrt{d+c^2dx^2}} - \frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} + \frac{b\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}$$

```
[Out] 1/3*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(3/2)+(a+b*arcsinh(c*x))/d^2/(c^2*d*
x^2+d)^(1/2)-1/6*b*c*x/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-7/6*b*arct
an(c*x)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-2*(a+b*arcsinh(c*x))*arct
anh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-b*poly
log(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+b*p
olylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5811, 5816, 4267, 2317, 2438, 209, 205}

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^{5/2}}\,dx=-\frac{2\sqrt{c^2x^2+1}\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d^2\sqrt{c^2dx^2+d}}+\frac{a+b\text{arcsinh}(cx)}{d^2\sqrt{c^2dx^2+d}}+\frac{a+b\text{arcsinh}(cx)}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{b\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{c^2dx^2+d}}-\frac{7b\sqrt{c^2x^2+1}\arctan(cx)}{6d^2\sqrt{c^2dx^2+d}}-\frac{bcx}{6d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] -1/6*(b*c*x)/(d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (a + b*ArcSinh[c
*x])/(3*d*(d + c^2*d*x^2)^(3/2)) + (a + b*ArcSinh[c*x])/(d^2*Sqrt[d + c^2*d
*x^2]) - (7*b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(6*d^2*Sqrt[d + c^2*d*x^2]) -
(2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/(d^2*Sqr
t[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/(d^2*
Sqrt[d + c^2*d*x^2]) + (b*Sqrt[1 + c^2*x^2]*PolyLog[2, E^ArcSinh[c*x]])/(d^
2*Sqrt[d + c^2*d*x^2])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a+b\text{arcsinh}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}}+\frac{\int\frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)^{3/2}}\,dx}{d}-\frac{\left(bc\sqrt{1+c^2x^2}\right)\int\frac{1}{(1+c^2x^2)^2}\,dx}{3d^2\sqrt{d+c^2dx^2}}\\
&=-\frac{bcx}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}+\frac{a+b\text{arcsinh}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}}+\frac{a+b\text{arcsinh}(cx)}{d^2\sqrt{d+c^2dx^2}}\\
&\quad+\frac{\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{d+c^2dx^2}}\,dx}{d^2}-\frac{\left(bc\sqrt{1+c^2x^2}\right)\int\frac{1}{1+c^2x^2}\,dx}{6d^2\sqrt{d+c^2dx^2}}-\frac{\left(bc\sqrt{1+c^2x^2}\right)\int\frac{1}{1+c^2x^2}\,dx}{d^2\sqrt{d+c^2dx^2}}\\
&=-\frac{bcx}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}+\frac{a+b\text{arcsinh}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}}+\frac{a+b\text{arcsinh}(cx)}{d^2\sqrt{d+c^2dx^2}}\\
&\quad-\frac{7b\sqrt{1+c^2x^2}\arctan(cx)}{6d^2\sqrt{d+c^2dx^2}}+\frac{\sqrt{1+c^2x^2}\,\text{Subst}\left(\int(a+bx)\text{csch}(x)\,dx,x,\text{arcsinh}(cx)\right)}{d^2\sqrt{d+c^2dx^2}}
\end{aligned}$$

$$= -\frac{bcx}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{a+b\text{arcsinh}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{a+b\text{arcsinh}(cx)}{d^2\sqrt{d+c^2dx^2}} - \frac{7b\sqrt{1+c^2x^2}\arctan(cx)}{6d^2\sqrt{d+c^2dx^2}} - \frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} - \frac{\left(b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1-e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{d^2\sqrt{d+c^2dx^2}} + \frac{\left(b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1+e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bcx}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{a+b\text{arcsinh}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{a+b\text{arcsinh}(cx)}{d^2\sqrt{d+c^2dx^2}} - \frac{7b\sqrt{1+c^2x^2}\arctan(cx)}{6d^2\sqrt{d+c^2dx^2}} - \frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} - \frac{\left(b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} + \frac{\left(b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bcx}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{a+b\text{arcsinh}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{a+b\text{arcsinh}(cx)}{d^2\sqrt{d+c^2dx^2}} - \frac{7b\sqrt{1+c^2x^2}\arctan(cx)}{6d^2\sqrt{d+c^2dx^2}} - \frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} + \frac{b\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.94

$$\int\frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^{5/2}}\,dx = \frac{\frac{2a(4+3c^2x^2)\sqrt{d+c^2dx^2}}{(1+c^2x^2)^2} + 6a\sqrt{d}\log(x) - 6a\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right) + \frac{bd^2(1+c^2x^2)^{3/}}{}}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^(5/2)),x]
[Out] ((2*a*(4 + 3*c^2*x^2)*Sqrt[d + c^2*d*x^2])/(1 + c^2*x^2)^2 + 6*a*Sqrt[d]*Lo
g[x] - 6*a*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*d^2*(1 + c^2*x
^2)^(3/2)*(-((c*x)/(1 + c^2*x^2)) + (2*ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) +
(6*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - 14*ArcTan[Tanh[ArcSinh[c*x]/2]] + 6*Ar
cSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 6*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[
c*x])] + 6*PolyLog[2, -E^(-ArcSinh[c*x])] - 6*PolyLog[2, E^(-ArcSinh[c*x])]
))/(d + c^2*d*x^2)^(3/2))/(6*d^3)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.39

method	result
default	$\frac{a}{3d(c^2dx^2+d)^{\frac{3}{2}}}+\frac{a}{d^2\sqrt{c^2dx^2+d}}-\frac{a\ln\left(\frac{2d+2\sqrt{d}\sqrt{c^2dx^2+d}}{x}\right)}{d^{\frac{5}{2}}}+\frac{b\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)x^2c^2}{(c^2x^2+1)^2d^3}-\frac{b\sqrt{d(c^2x^2+1)}\,cx}{6(c^2x^2+1)^{\frac{3}{2}}d^3}+\frac{4b\sqrt{d(c}{3}$
parts	$\frac{a}{3d(c^2dx^2+d)^{\frac{3}{2}}}+\frac{a}{d^2\sqrt{c^2dx^2+d}}-\frac{a\ln\left(\frac{2d+2\sqrt{d}\sqrt{c^2dx^2+d}}{x}\right)}{d^{\frac{5}{2}}}+\frac{b\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)x^2c^2}{(c^2x^2+1)^2d^3}-\frac{b\sqrt{d(c^2x^2+1)}\,cx}{6(c^2x^2+1)^{\frac{3}{2}}d^3}+\frac{4b\sqrt{d(c}{3}$

```
[In] int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a/d/(c^2*d*x^2+d)^(3/2)+a/d^2/(c^2*d*x^2+d)^(1/2)-a/d^(5/2)*ln((2*d+2*d
^(1/2)*(c^2*d*x^2+d)^(1/2))/x)+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/d^3*ar
csinh(c*x)*x^2*c^2-1/6*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(3/2)/d^3*c*x+4/
3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/d^3*arcsinh(c*x)-7/3*b*(d*(c^2*x^2+
1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arctan(c*x+(c^2*x^2+1)^(1/2))-b*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*dilog(1+c*x+(c^2*x^2+1)^(1/2))-b*(d*(c^2*x
^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))
-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*dilog(c*x+(c^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x\,(d+c^2dx^2)^{5/2}}\,dx=\int \frac{b\operatorname{arsinh}(cx)+a}{(c^2dx^2+d)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^6*d^3*x^7 + 3*c^4*d^3*
x^5 + 3*c^2*d^3*x^3 + d^3*x), x)
```

Sympy [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x\,(d+c^2dx^2)^{5/2}}\,dx=\int \frac{a+b\operatorname{asinh}(cx)}{x\,(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(x*(d*(c**2*x**2 + 1))**(5/2)), x)
```

Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x\left(d + c^2dx^2\right)^{5/2}} \, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{\left(c^2dx^2 + d\right)^{\frac{5}{2}}x} \, dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(3*arcsinh(1/(c*abs(x)))/d^(5/2) - 3/(sqrt(c^2*d*x^2 + d)*d^2) - 1/(
(c^2*d*x^2 + d)^(3/2)*d)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/((c^2*
d*x^2 + d)^(5/2)*x), x)
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x\left(d + c^2dx^2\right)^{5/2}} \, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{\left(c^2dx^2 + d\right)^{\frac{5}{2}}x} \, dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(5/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{x\left(d + c^2dx^2\right)^{5/2}} \, dx = \int \frac{a + b \operatorname{asinh}(c\,x)}{x\left(d\,c^2\,x^2 + d\right)^{5/2}} \, dx$$

```
[In] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^(5/2)), x)
```

3.173 $\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 214

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^{5/2}}\,dx &= -\frac{bc\sqrt{d+c^2dx^2}}{6d^3(1+c^2x^2)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{dx(d+c^2dx^2)^{3/2}}\\ &\quad-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3d(d+c^2dx^2)^{3/2}}-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+c^2dx^2}}\\ &\quad+\frac{bc\sqrt{d+c^2dx^2}\log(x)}{d^3\sqrt{1+c^2x^2}}+\frac{5bc\sqrt{d+c^2dx^2}\log(1+c^2x^2)}{6d^3\sqrt{1+c^2x^2}}\end{aligned}$$

```
[Out] (-a-b*arcsinh(c*x))/d/x/(c^2*d*x^2+d)^(3/2)-4/3*c^2*x*(a+b*arcsinh(c*x))/d/
(c^2*d*x^2+d)^(3/2)-8/3*c^2*x*(a+b*arcsinh(c*x))/d^2/(c^2*d*x^2+d)^(1/2)-1/
6*b*c*(c^2*d*x^2+d)^(1/2)/d^3/(c^2*x^2+1)^(3/2)+b*c*ln(x)*(c^2*d*x^2+d)^(1/
2)/d^3/(c^2*x^2+1)^(1/2)+5/6*b*c*ln(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/d^3/(c^2
*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {277, 198, 197, 5804, 12, 1265, 907}

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^{5/2}}\,dx &= -\frac{8c^2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{c^2dx^2+d}}\\ &\quad-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3d(c^2dx^2+d)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{dx(c^2dx^2+d)^{3/2}}-\frac{bc\sqrt{c^2dx^2+d}}{6d^3(c^2x^2+1)^{3/2}}\\ &\quad+\frac{bc\log(x)\sqrt{c^2dx^2+d}}{d^3\sqrt{c^2x^2+1}}+\frac{5bc\sqrt{c^2dx^2+d}\log(c^2x^2+1)}{6d^3\sqrt{c^2x^2+1}}\end{aligned}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] -1/6*(b*c*Sqrt[d + c^2*d*x^2])/(d^3*(1 + c^2*x^2)^(3/2)) - (a + b*ArcSinh[c
*x])/(d*x*(d + c^2*d*x^2)^(3/2)) - (4*c^2*x*(a + b*ArcSinh[c*x]))/(3*d*(d +
 c^2*d*x^2)^(3/2)) - (8*c^2*x*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt[d + c^2*d*x
^2]) + (b*c*Sqrt[d + c^2*d*x^2]*Log[x])/(d^3*Sqrt[1 + c^2*x^2]) + (5*b*c*Sq
rt[d + c^2*d*x^2]*Log[1 + c^2*x^2])/(6*d^3*Sqrt[1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\text{integral} = -\frac{a+b\text{arcsinh}(cx)}{dx\left(d+c^2dx^2\right)^{3/2}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3d\left(d+c^2dx^2\right)^{3/2}}-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+c^2dx^2}}-\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int\frac{-3-12c^2x^2-8c^4x^4}{3d^3x(1+c^2x^2)^2}\,dx}{\sqrt{1+c^2x^2}}$$

$$=-\frac{a+b\text{arcsinh}(cx)}{dx\left(d+c^2dx^2\right)^{3/2}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3d\left(d+c^2dx^2\right)^{3/2}}-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+c^2dx^2}}-\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int\frac{-3-12c^2x^2-8c^4x^4}{x(1+c^2x^2)^2}\,dx}{3d^3\sqrt{1+c^2x^2}}$$

$$=-\frac{a+b\text{arcsinh}(cx)}{dx\left(d+c^2dx^2\right)^{3/2}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3d\left(d+c^2dx^2\right)^{3/2}}-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+c^2dx^2}}-\frac{\left(bc\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{-3-12c^2x-8c^4x^2}{x(1+c^2x)^2}\,dx,x,x^2\right)}{6d^3\sqrt{1+c^2x^2}}$$

$$=-\frac{a+b\text{arcsinh}(cx)}{dx\left(d+c^2dx^2\right)^{3/2}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3d\left(d+c^2dx^2\right)^{3/2}}-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+c^2dx^2}}-\frac{\left(bc\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(-\frac{3}{x}-\frac{c^2}{(1+c^2x)^2}-\frac{5c^2}{1+c^2x}\right)dx,x,x^2\right)}{6d^3\sqrt{1+c^2x^2}}$$

$$=-\frac{bc\sqrt{d+c^2dx^2}}{6d^3\left(1+c^2x^2\right)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{dx\left(d+c^2dx^2\right)^{3/2}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3d\left(d+c^2dx^2\right)^{3/2}}-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+c^2dx^2}}+\frac{bc\sqrt{d+c^2dx^2}\log(x)}{d^3\sqrt{1+c^2x^2}}+\frac{5bc\sqrt{d+c^2dx^2}\log\left(1+c^2x^2\right)}{6d^3\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.06

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^2(d+c^2dx^2)^{5/2}}\,dx =$$
$$-\frac{\sqrt{d+c^2dx^2}\Big(bcx+bc^3x^3+6a\sqrt{1+c^2x^2}+24ac^2x^2\sqrt{1+c^2x^2}+16ac^4x^4\sqrt{1+c^2x^2}+2b\sqrt{1+c^2x^2}(3+12}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] -1/6*(Sqrt[d + c^2*d*x^2]*(b*c*x + b*c^3*x^3 + 6*a*Sqrt[1 + c^2*x^2] + 24*a
*c^2*x^2*Sqrt[1 + c^2*x^2] + 16*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 2*b*Sqrt[1 +
c^2*x^2]*(3 + 12*c^2*x^2 + 8*c^4*x^4)*ArcSinh[c*x] + 3*b*c*x*(1 + c^2*x^2)^
2*Log[1 + 1/(c^2*x^2)] - 8*b*c*x*Log[1 + c^2*x^2] - 16*b*c^3*x^3*Log[1 + c^
2*x^2] - 8*b*c^5*x^5*Log[1 + c^2*x^2]))/(d^3*x*(1 + c^2*x^2)^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1256 vs. 2(190) = 380.

Time = 0.25 (sec) , antiderivative size = 1257, normalized size of antiderivative = 5.87

method	result	size
default	Expression too large to display	1257
parts	Expression too large to display	1257

```
[In] int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/d/x/(c^2*d*x^2+d)^(3/2)-4*c^2*(1/3/d*x/(c^2*d*x^2+d)^(3/2)+2/3/d^2*x/
(c^2*d*x^2+d)^(1/2)))-16/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*ar
csinh(c*x)*c-32/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+
9)/d^3*x^9*c^10+32/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x
^2+9)/d^3*x^7*(c^2*x^2+1)*c^8-112/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c
^4*x^4+26*c^2*x^2+9)/d^3*x^7*c^8+80/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25
*c^4*x^4+26*c^2*x^2+9)/d^3*x^5*(c^2*x^2+1)*c^6-64/3*b*(d*(c^2*x^2+1))^(1/2)
/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^5*arcsinh(c*x)*c^6+64/3*b*(d*(c^
2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^4*(c^2*x^2+1)^(1/
2)*arcsinh(c*x)*c^5-140/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*
c^2*x^2+9)/d^3*x^5*c^6+20*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*
c^2*x^2+9)/d^3*x^3*(c^2*x^2+1)*c^4-56*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25
*c^4*x^4+26*c^2*x^2+9)/d^3*x^3*arcsinh(c*x)*c^4+136/3*b*(d*(c^2*x^2+1))^(1/
2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^2*(c^2*x^2+1)^(1/2)*arcsinh(c*
x)*c^3-24*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x
^3*c^4-4/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*
```

```
x^2*c^3*(c^2*x^2+1)^(1/2)+4*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+2
6*c^2*x^2+9)/d^3*x*(c^2*x^2+1)*c^2-44*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25
*c^4*x^4+26*c^2*x^2+9)/d^3*x*arcsinh(c*x)*c^2+24*b*(d*(c^2*x^2+1))^(1/2)/(8
*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c-4*b*
(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x*c^2-3/2*b*(
d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*c*(c^2*x^2+1)^
(1/2)-9*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3/x*a
rcsinh(c*x)+5/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*ln(1+(c*x+(c^
2*x^2+1)^(1/2))^2)*c+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*ln((c*x+
(c^2*x^2+1)^(1/2))^2-1)*c
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(d+c^2dx^2\right)^{5/2}}\,dx=\int \frac{b\,\text{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^6*d^3*x^8 + 3*c^4*d^3*
x^6 + 3*c^2*d^3*x^4 + d^3*x^2), x)
```

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(d+c^2dx^2\right)^{5/2}}\,dx=\int \frac{a+b\,\text{asinh}\left(cx\right)}{x^2\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d)**(5/2),x)
[Out] Integral((a + b*asinh(c*x))/(x**2*(d*(c**2*x**2 + 1))**(5/2)), x)
```

Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(d+c^2dx^2\right)^{5/2}}\,dx=\int \frac{b\,\text{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
[Out] -1/3*a*(8*c^2*x/(sqrt(c^2*d*x^2 + d)*d^2) + 4*c^2*x/((c^2*d*x^2 + d)^(3/2)*
d) + 3/((c^2*d*x^2 + d)^(3/2)*d*x)) + b*integrate(log(c*x + sqrt(c^2*x^2 +
1))/((c^2*d*x^2 + d)^(5/2)*x^2), x)
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2 (d + c^2dx^2)^{5/2}} \, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(c^2dx^2 + d)^{\frac{5}{2}} x^2} \, dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(5/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2 (d + c^2dx^2)^{5/2}} \, dx = \int \frac{a + b \operatorname{asinh}(c\,x)}{x^2 \left(d\,c^2\,x^2 + d\right)^{5/2}} \, dx$$

```
[In] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^(5/2)),x)
[Out] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^(5/2)), x)
```

3.174 $\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 400

$$\begin{aligned}
\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^{5/2}}\,dx &= \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}+\frac{5bc^3x}{12d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} \\
&-\frac{3bc\sqrt{1+c^2x^2}}{4d^2x\sqrt{d+c^2dx^2}}-\frac{5c^2(a+b\text{arcsinh}(cx))}{6d(d+c^2dx^2)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{2dx^2(d+c^2dx^2)^{3/2}} \\
&-\frac{5c^2(a+b\text{arcsinh}(cx))}{2d^2\sqrt{d+c^2dx^2}}+\frac{13bc^2\sqrt{1+c^2x^2}\arctan(cx)}{6d^2\sqrt{d+c^2dx^2}} \\
&+\frac{5c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} \\
&+\frac{5bc^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2d^2\sqrt{d+c^2dx^2}}-\frac{5bc^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2d^2\sqrt{d+c^2dx^2}}
\end{aligned}$$

```
[Out] -5/6*c^2*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(3/2)+1/2*(-a-b*arcsinh(c*x))/d
/x^2/(c^2*d*x^2+d)^(3/2)-5/2*c^2*(a+b*arcsinh(c*x))/d^2/(c^2*d*x^2+d)^(1/2)
+1/4*b*c/d^2/x/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+5/12*b*c^3*x/d^2/(c^2*
x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-3/4*b*c*(c^2*x^2+1)^(1/2)/d^2/x/(c^2*d*x^2
+d)^(1/2)+13/6*b*c^2*arctan(c*x)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+
5*c^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d
^2/(c^2*d*x^2+d)^(1/2)+5/2*b*c^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2
+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-5/2*b*c^2*polylog(2,c*x+(c^2*x^2+1)^(1/2)
)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5809, 5811, 5816, 4267, 2317, 2438, 209, 205, 296, 331}

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^{5/2}}\,dx = \frac{5c^2\sqrt{c^2x^2+1}\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d^2\sqrt{c^2dx^2+d}} - \frac{5c^2(a+b\text{arcsinh}(cx))}{2d^2\sqrt{c^2dx^2+d}} - \frac{5c^2(a+b\text{arcsinh}(cx))}{6d(c^2dx^2+d)^{3/2}} - \frac{a+b\text{arcsinh}(cx)}{2dx^2(c^2dx^2+d)^{3/2}} + \frac{5bc^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2d^2\sqrt{c^2dx^2+d}} - \frac{5bc^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2d^2\sqrt{c^2dx^2+d}} + \frac{13bc^2\sqrt{c^2x^2+1}\arctan(cx)}{6d^2\sqrt{c^2dx^2+d}} - \frac{3bc\sqrt{c^2x^2+1}}{4d^2x\sqrt{c^2dx^2+d}} + \frac{bc}{4d^2x\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}} + \frac{5bc^3x}{12d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] (b*c)/(4*d^2*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (5*b*c^3*x)/(12*d^2
*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (3*b*c*Sqrt[1 + c^2*x^2])/(4*d^2*
x*Sqrt[d + c^2*d*x^2]) - (5*c^2*(a + b*ArcSinh[c*x]))/(6*d*(d + c^2*d*x^2)^
(3/2)) - (a + b*ArcSinh[c*x])/(2*d*x^2*(d + c^2*d*x^2)^(3/2)) - (5*c^2*(a +
 b*ArcSinh[c*x]))/(2*d^2*Sqrt[d + c^2*d*x^2]) + (13*b*c^2*Sqrt[1 + c^2*x^2]
*ArcTan[c*x])/(6*d^2*Sqrt[d + c^2*d*x^2]) + (5*c^2*Sqrt[1 + c^2*x^2]*(a + b
*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) + (5*b*c^
2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/(2*d^2*Sqrt[d + c^2*d*x^2]
) - (5*b*c^2*Sqrt[1 + c^2*x^2]*PolyLog[2, E^ArcSinh[c*x]])/(2*d^2*Sqrt[d +
c^2*d*x^2])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
```

```
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\text{arcsinh}(cx)}{2dx^2\left(d+c^2dx^2\right)^{3/2}} \\
&\quad -\frac{1}{2}\left(5c^2\right)\int\frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^{5/2}}\,dx+\frac{\left(bc\sqrt{1+c^2x^2}\right)\int\frac{1}{x^2\left(1+c^2x^2\right)^2}\,dx}{2d^2\sqrt{d+c^2dx^2}} \\
&= \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{5c^2(a+b\text{arcsinh}(cx))}{6d\left(d+c^2dx^2\right)^{3/2}} \\
&\quad -\frac{a+b\text{arcsinh}(cx)}{2dx^2\left(d+c^2dx^2\right)^{3/2}}-\frac{\left(5c^2\right)\int\frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^{3/2}}\,dx}{2d} \\
&\quad +\frac{\left(3bc\sqrt{1+c^2x^2}\right)\int\frac{1}{x^2\left(1+c^2x^2\right)}\,dx}{4d^2\sqrt{d+c^2dx^2}}+\frac{\left(5bc^3\sqrt{1+c^2x^2}\right)\int\frac{1}{\left(1+c^2x^2\right)^2}\,dx}{6d^2\sqrt{d+c^2dx^2}} \\
&= \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}+\frac{5bc^3x}{12d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{3bc\sqrt{1+c^2x^2}}{4d^2x\sqrt{d+c^2dx^2}} \\
&\quad -\frac{5c^2(a+b\text{arcsinh}(cx))}{6d\left(d+c^2dx^2\right)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{2dx^2\left(d+c^2dx^2\right)^{3/2}}-\frac{5c^2(a+b\text{arcsinh}(cx))}{2d^2\sqrt{d+c^2dx^2}} \\
&\quad -\frac{\left(5c^2\right)\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{d+c^2dx^2}}\,dx}{2d^2}+\frac{\left(5bc^3\sqrt{1+c^2x^2}\right)\int\frac{1}{1+c^2x^2}\,dx}{12d^2\sqrt{d+c^2dx^2}} \\
&\quad -\frac{\left(3bc^3\sqrt{1+c^2x^2}\right)\int\frac{1}{1+c^2x^2}\,dx}{4d^2\sqrt{d+c^2dx^2}}+\frac{\left(5bc^3\sqrt{1+c^2x^2}\right)\int\frac{1}{1+c^2x^2}\,dx}{2d^2\sqrt{d+c^2dx^2}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{5bc^3x}{12d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} \\
&\quad - \frac{3bc\sqrt{1+c^2x^2}}{4d^2x\sqrt{d+c^2dx^2}} - \frac{5c^2(a+b\text{arcsinh}(cx))}{6d\left(d+c^2dx^2\right)^{3/2}} - \frac{a+b\text{arcsinh}(cx)}{2dx^2\left(d+c^2dx^2\right)^{3/2}} \\
&\quad - \frac{5c^2(a+b\text{arcsinh}(cx))}{2d^2\sqrt{d+c^2dx^2}} + \frac{13bc^2\sqrt{1+c^2x^2}\arctan(cx)}{6d^2\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\left(5c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\text{csch}(x)\,dx, x, \text{arcsinh}(cx)\right)}{2d^2\sqrt{d+c^2dx^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{5bc^3x}{12d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} \\
&\quad - \frac{3bc\sqrt{1+c^2x^2}}{4d^2x\sqrt{d+c^2dx^2}} - \frac{5c^2(a+b\text{arcsinh}(cx))}{6d\left(d+c^2dx^2\right)^{3/2}} - \frac{a+b\text{arcsinh}(cx)}{2dx^2\left(d+c^2dx^2\right)^{3/2}} \\
&\quad - \frac{5c^2(a+b\text{arcsinh}(cx))}{2d^2\sqrt{d+c^2dx^2}} + \frac{13bc^2\sqrt{1+c^2x^2}\arctan(cx)}{6d^2\sqrt{d+c^2dx^2}} \\
&\quad + \frac{5c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(5bc^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{2d^2\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\left(5bc^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \log\left(1+e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{2d^2\sqrt{d+c^2dx^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{5bc^3x}{12d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} \\
&\quad - \frac{3bc\sqrt{1+c^2x^2}}{4d^2x\sqrt{d+c^2dx^2}} - \frac{5c^2(a+b\text{arcsinh}(cx))}{6d\left(d+c^2dx^2\right)^{3/2}} - \frac{a+b\text{arcsinh}(cx)}{2dx^2\left(d+c^2dx^2\right)^{3/2}} \\
&\quad - \frac{5c^2(a+b\text{arcsinh}(cx))}{2d^2\sqrt{d+c^2dx^2}} + \frac{13bc^2\sqrt{1+c^2x^2}\arctan(cx)}{6d^2\sqrt{d+c^2dx^2}} \\
&\quad + \frac{5c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(5bc^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{2d^2\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\left(5bc^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{2d^2\sqrt{d+c^2dx^2}}
\end{aligned}
$$

$$= \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{5bc^3x}{12d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4d^2x\sqrt{d+c^2dx^2}} - \frac{5c^2(a+b\operatorname{arcsinh}(cx))}{6d\left(d+c^2dx^2\right)^{3/2}} - \frac{a+b\operatorname{arcsinh}(cx)}{2dx^2\left(d+c^2dx^2\right)^{3/2}} - \frac{5c^2(a+b\operatorname{arcsinh}(cx))}{2d^2\sqrt{d+c^2dx^2}} + \frac{13bc^2\sqrt{1+c^2x^2}\arctan(cx)}{6d^2\sqrt{d+c^2dx^2}} + \frac{5c^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} + \frac{5bc^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)}{2d^2\sqrt{d+c^2dx^2}} - \frac{5bc^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)}{2d^2\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 5.36 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.02

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx = \frac{-\frac{4a\sqrt{d+c^2dx^2}\left(3+20c^2x^2+15c^4x^4\right)}{\left(x+c^2x^3\right)^2} - 60ac^2\sqrt{d}\log(x) + 60ac^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] ((-4*a*Sqrt[d + c^2*d*x^2]*(3 + 20*c^2*x^2 + 15*c^4*x^4))/(x + c^2*x^3)^2 -
 60*a*c^2*Sqrt[d]*Log[x] + 60*a*c^2*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*
x^2]] + (b*c^2*d*((4*c*x)/Sqrt[1 + c^2*x^2] - 48*ArcSinh[c*x] - (8*ArcSinh[
c*x])/(1 + c^2*x^2) + 104*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] -
6*Sqrt[1 + c^2*x^2]*Coth[ArcSinh[c*x]/2] - 3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
*Csch[ArcSinh[c*x]/2]^2 - 60*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - E^(-Arc
Sinh[c*x])] + 60*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])]
- 60*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-ArcSinh[c*x])] + 60*Sqrt[1 + c^2*x^2
]*PolyLog[2, E^(-ArcSinh[c*x])] - 3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Sech[Arc
Sinh[c*x]/2]^2 + 6*Sqrt[1 + c^2*x^2]*Tanh[ArcSinh[c*x]/2]))/Sqrt[d + c^2*d*
x^2])/(24*d^3)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.02

method	result
default	$-\frac{a}{2d\,x^{2}(c^{2}d\,x^{2}+d)^{\frac{3}{2}}}-\frac{5a\,c^{2}}{6d(c^{2}d\,x^{2}+d)^{\frac{3}{2}}}-\frac{5a\,c^{2}}{2d^{2}\sqrt{c^{2}d\,x^{2}+d}}+\frac{5a\,c^{2}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^{2}d\,x^{2}+d}}{x}\right)}{2d^{\frac{5}{2}}}+b\left(-\frac{\sqrt{d(c^{2}x^{2}+1)}\,\left(15\,\mathrm{arcsinh}(cx\right.}{}\right.$
parts	$-\frac{a}{2d\,x^{2}(c^{2}d\,x^{2}+d)^{\frac{3}{2}}}-\frac{5a\,c^{2}}{6d(c^{2}d\,x^{2}+d)^{\frac{3}{2}}}-\frac{5a\,c^{2}}{2d^{2}\sqrt{c^{2}d\,x^{2}+d}}+\frac{5a\,c^{2}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^{2}d\,x^{2}+d}}{x}\right)}{2d^{\frac{5}{2}}}+b\left(-\frac{\sqrt{d(c^{2}x^{2}+1)}\,\left(15\,\mathrm{arcsinh}(cx\right.}{}\right.$

```
[In] int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d/x^2/(c^2*d*x^2+d)^(3/2)-5/6*a*c^2/d/(c^2*d*x^2+d)^(3/2)-5/2*a*c^2/
d^2/(c^2*d*x^2+d)^(1/2)+5/2*a*c^2/d^(5/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(
1/2))/x)+b*(-1/6*(d*(c^2*x^2+1))^(1/2)*(15*arcsinh(c*x)*c^4*x^4+2*c^3*x^3*(
c^2*x^2+1)^(1/2)+20*arcsinh(c*x)*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+3*arcsinh(
c*x))/(c^4*x^4+2*c^2*x^2+1)/d^3/x^2+13/3*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)/d^3*arctan(c*x+(c^2*x^2+1)^(1/2))*c^2+5/2*(d*(c^2*x^2+1))^(1/2)/(c^2*
x^2+1)^(1/2)/d^3*dilog(1+c*x+(c^2*x^2+1)^(1/2))*c^2+5/2*(d*(c^2*x^2+1))^(1/
2)/(c^2*x^2+1)^(1/2)/d^3*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2+5/2*(
d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*dilog(c*x+(c^2*x^2+1)^(1/2))*c^2
)
```

Fricas [F]

$$\int \frac{a+b\mathrm{arcsinh}(cx)}{x^3\,(d+c^2dx^2)^{5/2}}\,dx=\int \frac{b\,\mathrm{arsinh}\,(cx)+a}{(c^2dx^2+d)^{\frac{5}{2}}x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^6*d^3*x^9 + 3*c^4*d^3*
x^7 + 3*c^2*d^3*x^5 + d^3*x^3), x)
```

Sympy [F]

$$\int \frac{a+b\mathrm{arcsinh}(cx)}{x^3\,(d+c^2dx^2)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{asinh}\,(cx)}{x^3\,(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(x**3*(d*(c**2*x**2 + 1))**(5/2)), x)
```

Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx = \int \frac{b\,\text{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
[Out] 1/6*a*(15*c^2*arcsinh(1/(c*abs(x)))/d^(5/2) - 15*c^2/(sqrt(c^2*d*x^2 + d)*d
^2) - 5*c^2/((c^2*d*x^2 + d)^(3/2)*d) - 3/((c^2*d*x^2 + d)^(3/2)*d*x^2)) +
b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/((c^2*d*x^2 + d)^(5/2)*x^3), x)
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx = \int \frac{b\,\text{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(5/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx = \int \frac{a+b\,\text{asinh}(c\,x)}{x^3\,\left(d\,c^2\,x^2+d\right)^{5/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^(5/2)),x)
[Out] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^(5/2)), x)
```

3.175 $\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 297

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^{5/2}}\,dx = \frac{bc^3\sqrt{d+c^2dx^2}}{6d^3(1+c^2x^2)^{3/2}} - \frac{bc\sqrt{d+c^2dx^2}}{6d^3x^2\sqrt{1+c^2x^2}} - \frac{a+b\text{arcsinh}(cx)}{3dx^3(d+c^2dx^2)^{3/2}} + \frac{2c^2(a+b\text{arcsinh}(cx))}{dx(d+c^2dx^2)^{3/2}} + \frac{8c^4x(a+b\text{arcsinh}(cx))}{3d(d+c^2dx^2)^{3/2}} + \frac{16c^4x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+c^2dx^2}} - \frac{8bc^3\sqrt{d+c^2dx^2}\log(x)}{3d^3\sqrt{1+c^2x^2}} - \frac{4bc^3\sqrt{d+c^2dx^2}\log(1+c^2x^2)}{3d^3\sqrt{1+c^2x^2}}$$

```
[Out] 1/3*(-a-b*arcsinh(c*x))/d/x^3/(c^2*d*x^2+d)^(3/2)+2*c^2*(a+b*arcsinh(c*x))/
d/x/(c^2*d*x^2+d)^(3/2)+8/3*c^4*x*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(3/2)+
16/3*c^4*x*(a+b*arcsinh(c*x))/d^2/(c^2*d*x^2+d)^(1/2)+1/6*b*c^3*(c^2*d*x^2+
d)^(1/2)/d^3/(c^2*x^2+1)^(3/2)-1/6*b*c*(c^2*d*x^2+d)^(1/2)/d^3/x^2/(c^2*x^2
+1)^(1/2)-8/3*b*c^3*ln(x)*(c^2*d*x^2+d)^(1/2)/d^3/(c^2*x^2+1)^(1/2)-4/3*b*c
^3*ln(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used

$= \{277, 198, 197, 5804, 12, 1813, 1634\}$

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^4\left(d+c^2dx^2\right)^{5/2}}\,dx = \frac{2c^2(a+b\operatorname{arcsinh}(cx))}{dx\left(c^2dx^2+d\right)^{3/2}} - \frac{a+b\operatorname{arcsinh}(cx)}{3dx^3\left(c^2dx^2+d\right)^{3/2}} + \frac{16c^4x(a+b\operatorname{arcsinh}(cx))}{3d^2\sqrt{c^2dx^2+d}} + \frac{8c^4x(a+b\operatorname{arcsinh}(cx))}{3d\left(c^2dx^2+d\right)^{3/2}} - \frac{bc\sqrt{c^2dx^2+d}}{6d^3x^2\sqrt{c^2x^2+1}} + \frac{bc^3\sqrt{c^2dx^2+d}}{6d^3\left(c^2x^2+1\right)^{3/2}} - \frac{8bc^3\log(x)\sqrt{c^2dx^2+d}}{3d^3\sqrt{c^2x^2+1}} - \frac{4bc^3\sqrt{c^2dx^2+d}\log\left(c^2x^2+1\right)}{3d^3\sqrt{c^2x^2+1}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] (b*c^3*Sqrt[d + c^2*d*x^2])/(6*d^3*(1 + c^2*x^2)^(3/2)) - (b*c*Sqrt[d + c^2
*d*x^2])/(6*d^3*x^2*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(3*d*x^3*(d +
 c^2*d*x^2)^(3/2)) + (2*c^2*(a + b*ArcSinh[c*x]))/(d*x*(d + c^2*d*x^2)^(3/2
)) + (8*c^4*x*(a + b*ArcSinh[c*x]))/(3*d*(d + c^2*d*x^2)^(3/2)) + (16*c^4*x
*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt[d + c^2*d*x^2]) - (8*b*c^3*Sqrt[d + c^2*
d*x^2]*Log[x])/(3*d^3*Sqrt[1 + c^2*x^2]) - (4*b*c^3*Sqrt[d + c^2*d*x^2]*Log
[1 + c^2*x^2])/(3*d^3*Sqrt[1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
```

```
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\text{arcsinh}(cx)}{3dx^3\left(d+c^2dx^2\right)^{3/2}}+\frac{2c^2(a+b\text{arcsinh}(cx))}{dx\left(d+c^2dx^2\right)^{3/2}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3d\left(d+c^2dx^2\right)^{3/2}} \\
&\quad+\frac{16c^4x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+c^2dx^2}}-\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int\frac{-1+6c^2x^2+24c^4x^4+16c^6x^6}{3d^3x^3(1+c^2x^2)^2}\,dx}{\sqrt{1+c^2x^2}} \\
&= -\frac{a+b\text{arcsinh}(cx)}{3dx^3\left(d+c^2dx^2\right)^{3/2}}+\frac{2c^2(a+b\text{arcsinh}(cx))}{dx\left(d+c^2dx^2\right)^{3/2}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3d\left(d+c^2dx^2\right)^{3/2}} \\
&\quad+\frac{16c^4x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+c^2dx^2}}-\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int\frac{-1+6c^2x^2+24c^4x^4+16c^6x^6}{x^3(1+c^2x^2)^2}\,dx}{3d^3\sqrt{1+c^2x^2}} \\
&= -\frac{a+b\text{arcsinh}(cx)}{3dx^3\left(d+c^2dx^2\right)^{3/2}}+\frac{2c^2(a+b\text{arcsinh}(cx))}{dx\left(d+c^2dx^2\right)^{3/2}} \\
&\quad+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3d\left(d+c^2dx^2\right)^{3/2}}+\frac{16c^4x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+c^2dx^2}} \\
&\quad-\frac{\left(bc\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{-1+6c^2x+24c^4x^2+16c^6x^3}{x^2(1+c^2x)^2}\,dx,x,x^2\right)}{6d^3\sqrt{1+c^2x^2}} \\
&= -\frac{a+b\text{arcsinh}(cx)}{3dx^3\left(d+c^2dx^2\right)^{3/2}}+\frac{2c^2(a+b\text{arcsinh}(cx))}{dx\left(d+c^2dx^2\right)^{3/2}} \\
&\quad+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3d\left(d+c^2dx^2\right)^{3/2}}+\frac{16c^4x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+c^2dx^2}} \\
&\quad-\frac{\left(bc\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(-\frac{1}{x^2}+\frac{8c^2}{x}+\frac{c^4}{(1+c^2x)^2}+\frac{8c^4}{1+c^2x}\right)\,dx,x,x^2\right)}{6d^3\sqrt{1+c^2x^2}}
\end{aligned}$$

$$= \frac{bc^3\sqrt{d+c^2dx^2}}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{bc\sqrt{d+c^2dx^2}}{6d^3x^2\sqrt{1+c^2x^2}} - \frac{a+b\operatorname{arcsinh}(cx)}{3dx^3\left(d+c^2dx^2\right)^{3/2}}$$
$$+ \frac{2c^2(a+b\operatorname{arcsinh}(cx))}{dx\left(d+c^2dx^2\right)^{3/2}} + \frac{8c^4x(a+b\operatorname{arcsinh}(cx))}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{16c^4x(a+b\operatorname{arcsinh}(cx))}{3d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{8bc^3\sqrt{d+c^2dx^2}\log(x)}{3d^3\sqrt{1+c^2x^2}} - \frac{4bc^3\sqrt{d+c^2dx^2}\log\left(1+c^2x^2\right)}{3d^3\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.90

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^4\left(d+c^2dx^2\right)^{5/2}}\,dx = \frac{\sqrt{d+c^2dx^2}\left(-bcx-bc^3x^3-2a\sqrt{1+c^2x^2}+12ac^2x^2\sqrt{1+c^2x^2}+48ac^4x^4\sqrt{1+c\right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(-(b*c*x) - b*c^3*x^3 - 2*a*Sqrt[1 + c^2*x^2] + 12*a*c
^2*x^2*Sqrt[1 + c^2*x^2] + 48*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 32*a*c^6*x^6*Sq
rt[1 + c^2*x^2] + 2*b*Sqrt[1 + c^2*x^2]*(-1 + 6*c^2*x^2 + 24*c^4*x^4 + 16*c
^6*x^6)*ArcSinh[c*x] + 8*b*c^3*x^3*(1 + c^2*x^2)^2*Log[1 + 1/(c^2*x^2)] - 1
6*b*c^3*x^3*Log[1 + c^2*x^2] - 32*b*c^5*x^5*Log[1 + c^2*x^2] - 16*b*c^7*x^7
*Log[1 + c^2*x^2]))/(6*d^3*x^3*(1 + c^2*x^2)^(5/2))
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.25

method	result
default	$a\left(-\frac{1}{3d\,x^3(c^2d\,x^2+d)^{\frac{3}{2}}}-2c^2\left(-\frac{1}{dx(c^2d\,x^2+d)^{\frac{3}{2}}}-4c^2\left(\frac{x}{3d(c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{c^2d\,x^2+d}}\right)\right)\right)+\frac{b\sqrt{d(c^2x^2+1)}\,\sqrt{c^2}}{}$
parts	$a\left(-\frac{1}{3d\,x^3(c^2d\,x^2+d)^{\frac{3}{2}}}-2c^2\left(-\frac{1}{dx(c^2d\,x^2+d)^{\frac{3}{2}}}-4c^2\left(\frac{x}{3d(c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{c^2d\,x^2+d}}\right)\right)\right)+\frac{b\sqrt{d(c^2x^2+1)}\,\sqrt{c^2}}{}$

```
[In] int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/3/d/x^3/(c^2*d*x^2+d)^(3/2)-2*c^2*(-1/d/x/(c^2*d*x^2+d)^(3/2)-4*c^2*(
1/3/d*x/(c^2*d*x^2+d)^(3/2)+2/3/d^2*x/(c^2*d*x^2+d)^(1/2))))+1/6*b*(d*(c^2*
x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*(32*arcsinh(c*x)*c^7*x^7-16*ln((c*x+(c^2*x^
2+1)^(1/2))^4-1)*x^7*c^7+32*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^6*x^6+64*arcsi
nh(c*x)*c^5*x^5-32*ln((c*x+(c^2*x^2+1)^(1/2))^4-1)*x^5*c^5+48*arcsinh(c*x)*
(c^2*x^2+1)^(1/2)*x^4*c^4+32*arcsinh(c*x)*c^3*x^3-16*ln((c*x+(c^2*x^2+1)^(1
```

```
/2))^4-1)*x^3*c^3+12*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2-c^3*x^3-2*arcsi
nh(c*x)*(c^2*x^2+1)^(1/2)-c*x)/(c^6*x^6+3*c^4*x^4+3*c^2*x^2+1)/d^3/x^3
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^{5/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(c^2dx^2+d)^{\frac{5}{2}}x^4}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^6*d^3*x^10 + 3*c^4*d^3
*x^8 + 3*c^2*d^3*x^6 + d^3*x^4), x)
```

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^{5/2}}\,dx=\int \frac{a+b\,\text{asinh}\,(cx)}{x^4\,(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d)**(5/2),x)
[Out] Integral((a + b*asinh(c*x))/(x**4*(d*(c**2*x**2 + 1))**(5/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.79

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^{5/2}}\,dx &= -\frac{1}{6}\,bc\left(\frac{8\,c^2\log\left(c^2x^2+1\right)}{d^{\frac{5}{2}}}+\frac{16\,c^2\log\left(x\right)}{d^{\frac{5}{2}}}+\frac{1}{c^2d^{\frac{5}{2}}x^4+d^{\frac{5}{2}}x^2}\right)\\ &+\frac{1}{3}\left(\frac{16\,c^4x}{\sqrt{c^2dx^2+d}d^2}+\frac{8\,c^4x}{(c^2dx^2+d)^{\frac{3}{2}}d}+\frac{6\,c^2}{(c^2dx^2+d)^{\frac{3}{2}}dx}-\frac{1}{(c^2dx^2+d)^{\frac{3}{2}}dx^3}\right)b\,\text{arsinh}\,(cx)\\ &+\frac{1}{3}\left(\frac{16\,c^4x}{\sqrt{c^2dx^2+d}d^2}+\frac{8\,c^4x}{(c^2dx^2+d)^{\frac{3}{2}}d}+\frac{6\,c^2}{(c^2dx^2+d)^{\frac{3}{2}}dx}-\frac{1}{(c^2dx^2+d)^{\frac{3}{2}}dx^3}\right)a\end{aligned}$$

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
[Out] -1/6*b*c*(8*c^2*log(c^2*x^2 + 1)/d^(5/2) + 16*c^2*log(x)/d^(5/2) + 1/(c^2*d
^(5/2)*x^4 + d^(5/2)*x^2)) + 1/3*(16*c^4*x/(sqrt(c^2*d*x^2 + d)*d^2) + 8*c^
4*x/((c^2*d*x^2 + d)^(3/2)*d) + 6*c^2/((c^2*d*x^2 + d)^(3/2)*d*x) - 1/((c^2
*d*x^2 + d)^(3/2)*d*x^3))*b*arcsinh(c*x) + 1/3*(16*c^4*x/(sqrt(c^2*d*x^2 +
d)*d^2) + 8*c^4*x/((c^2*d*x^2 + d)^(3/2)*d) + 6*c^2/((c^2*d*x^2 + d)^(3/2)*
d*x) - 1/((c^2*d*x^2 + d)^(3/2)*d*x^3))*a
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^4 (d + c^2dx^2)^{5/2}} \, dx = \int \frac{b \,\text{arsinh}\,(cx) + a}{(c^2dx^2 + d)^{\frac{5}{2}} x^4} \, dx$$

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(5/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{x^4 (d + c^2dx^2)^{5/2}} \, dx = \int \frac{a + b \,\text{asinh}(c\,x)}{x^4 \,(d\,c^2\,x^2 + d)^{5/2}} \, dx$$

```
[In] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^(5/2)),x)
[Out] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^(5/2)), x)
```

3.176 $\int \frac{\text{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 200

$$\int \frac{\text{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx = \frac{1}{20ac^3(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} + \frac{2}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\text{arcsinh}(ax)}{5c(c+a^2cx^2)^{5/2}} + \frac{4x\text{arcsinh}(ax)}{15c^2(c+a^2cx^2)^{3/2}} + \frac{8x\text{arcsinh}(ax)}{15c^3\sqrt{c+a^2cx^2}} - \frac{4\sqrt{1+a^2x^2}\log(1+a^2x^2)}{15ac^3\sqrt{c+a^2cx^2}}$$

```
[Out] 1/5*x*arcsinh(a*x)/c/(a^2*c*x^2+c)^(5/2)+4/15*x*arcsinh(a*x)/c^2/(a^2*c*x^2
+c)^(3/2)+1/20/a/c^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2)+8/15*x*arcsinh(a
*x)/c^3/(a^2*c*x^2+c)^(1/2)+2/15/a/c^3/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2
)-4/15*ln(a^2*x^2+1)*(a^2*x^2+1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {5788, 5787, 266, 267}

$$\int \frac{\text{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx = \frac{8x\text{arcsinh}(ax)}{15c^3\sqrt{a^2cx^2+c}} + \frac{4x\text{arcsinh}(ax)}{15c^2(a^2cx^2+c)^{3/2}} + \frac{x\text{arcsinh}(ax)}{5c(a^2cx^2+c)^{5/2}} + \frac{2}{15ac^3\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}} + \frac{1}{20ac^3(a^2x^2+1)^{3/2}\sqrt{a^2cx^2+c}} - \frac{4\sqrt{a^2x^2+1}\log(a^2x^2+1)}{15ac^3\sqrt{a^2cx^2+c}}$$

```
[In] Int[ArcSinh[a*x]/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] 1/(20*a*c^3*(1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*x^2]) + 2/(15*a*c^3*Sqrt[1 +
 a^2*x^2]*Sqrt[c + a^2*c*x^2]) + (x*ArcSinh[a*x])/(5*c*(c + a^2*c*x^2)^(5/2
)) + (4*x*ArcSinh[a*x])/(15*c^2*(c + a^2*c*x^2)^(3/2)) + (8*x*ArcSinh[a*x])
/(15*c^3*Sqrt[c + a^2*c*x^2]) - (4*Sqrt[1 + a^2*x^2]*Log[1 + a^2*x^2])/(15*
a*c^3*Sqrt[c + a^2*c*x^2])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x\text{arcsinh}(ax)}{5c\left(c+a^2cx^2\right)^{5/2}}+\frac{4\int\frac{\text{arcsinh}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{5c}-\frac{\left(a\sqrt{1+a^2x^2}\right)\int\frac{x}{\left(1+a^2x^2\right)^3}\,dx}{5c^3\sqrt{c+a^2cx^2}}\\
&=\frac{1}{20ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}}+\frac{x\text{arcsinh}(ax)}{5c\left(c+a^2cx^2\right)^{5/2}}+\frac{4x\text{arcsinh}(ax)}{15c^2\left(c+a^2cx^2\right)^{3/2}}\\
&\quad+\frac{8\int\frac{\text{arcsinh}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{15c^2}-\frac{\left(4a\sqrt{1+a^2x^2}\right)\int\frac{x}{\left(1+a^2x^2\right)^2}\,dx}{15c^3\sqrt{c+a^2cx^2}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{20ac^3(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} + \frac{2}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\text{arcsinh}(ax)}{5c(c+a^2cx^2)^{5/2}} \\
&\quad + \frac{4x\text{arcsinh}(ax)}{15c^2(c+a^2cx^2)^{3/2}} + \frac{8x\text{arcsinh}(ax)}{15c^3\sqrt{c+a^2cx^2}} - \frac{\left(8a\sqrt{1+a^2x^2}\right)\int \frac{x}{1+a^2x^2}\,dx}{15c^3\sqrt{c+a^2cx^2}} \\
&= \frac{1}{20ac^3(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} + \frac{2}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\text{arcsinh}(ax)}{5c(c+a^2cx^2)^{5/2}} \\
&\quad + \frac{4x\text{arcsinh}(ax)}{15c^2(c+a^2cx^2)^{3/2}} + \frac{8x\text{arcsinh}(ax)}{15c^3\sqrt{c+a^2cx^2}} - \frac{4\sqrt{1+a^2x^2}\log(1+a^2x^2)}{15ac^3\sqrt{c+a^2cx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.60

$$
\int \frac{\text{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx = \frac{\sqrt{c+a^2cx^2}\left(4ax\sqrt{1+a^2x^2}(15+20a^2x^2+8a^4x^4)\,\text{arcsinh}(ax) - (1+a^2x^2)\left(-11-8a\right.\right.}{60ac^4(1+a^2x^2)^{7/2}}
$$

```
[In] Integrate[ArcSinh[a*x]/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(4*a*x*Sqrt[1 + a^2*x^2]*(15 + 20*a^2*x^2 + 8*a^4*x^4)
*ArcSinh[a*x] - (1 + a^2*x^2)*(-11 - 8*a^2*x^2 + 16*(1 + a^2*x^2)^2*Log[1 +
 a^2*x^2])))/(60*a*c^4*(1 + a^2*x^2)^(7/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(170) = 340.

Time = 0.18 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.82

method	result
default	$\frac{16\sqrt{c(a^2x^2+1)}\,\text{arcsinh}(ax)}{15\sqrt{a^2x^2+1}\,a\,c^4} + \frac{\sqrt{c(a^2x^2+1)}\left(8a^5x^5-8a^4x^4\sqrt{a^2x^2+1}+20a^3x^3-16a^2x^2\sqrt{a^2x^2+1}+15ax-8\sqrt{a^2x^2+1}\right)\left(-64a^8x^8-6\right.}{}$

```
[In] int(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 16/15*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^4*arcsinh(a*x)+1/60*(c*(a
^2*x^2+1))^(1/2)*(8*a^5*x^5-8*a^4*x^4*(a^2*x^2+1)^(1/2)+20*a^3*x^3-16*a^2*x
^2*(a^2*x^2+1)^(1/2)+15*a*x-8*(a^2*x^2+1)^(1/2))*(-64*a^8*x^8-64*(a^2*x^2+1
)^(1/2)*a^7*x^7-280*a^6*x^6-248*x^5*a^5*(a^2*x^2+1)^(1/2)+160*a^4*x^4*arcsi
nh(a*x)-456*a^4*x^4-340*a^3*x^3*(a^2*x^2+1)^(1/2)+380*a^2*x^2*arcsinh(a*x)-
328*a^2*x^2-165*a*x*(a^2*x^2+1)^(1/2)+256*arcsinh(a*x)-88)/(40*a^10*x^10+21
5*a^8*x^8+469*a^6*x^6+517*a^4*x^4+287*a^2*x^2+64)/a/c^4-8/15*(c*(a^2*x^2+1)
)^(1/2)/(a^2*x^2+1)^(1/2)/a/c^4*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{\operatorname{arsinh}(ax)}{(a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)/(a^8*c^4*x^8 + 4*a^6*c^4*x^6 + 6*
a^4*c^4*x^4 + 4*a^2*c^4*x^2 + c^4), x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{\operatorname{asinh}(ax)}{(c(a^2x^2+1))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(asinh(a*x)/(a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral(asinh(a*x)/(c*(a**2*x**2 + 1))**(7/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.72

$$\int \frac{\operatorname{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx = \frac{1}{60}\,a\left(\frac{3}{\left(a^6c^{\frac{5}{2}}x^4+2\,a^4c^{\frac{5}{2}}x^2+a^2c^{\frac{5}{2}}\right)c}+\frac{8}{\left(a^4c^{\frac{3}{2}}x^2+a^2c^{\frac{3}{2}}\right)c^2}-\frac{16\,\log\left(x^2+\frac{1}{a^2}\right)}{a^2c^{\frac{7}{2}}}\right)+\frac{1}{15}\left(\frac{8\,x}{\sqrt{a^2cx^2+c}c^3}+\frac{4\,x}{(a^2cx^2+c)^{\frac{3}{2}}c^2}+\frac{3\,x}{(a^2cx^2+c)^{\frac{5}{2}}c}\right)\operatorname{arsinh}(ax)$$

```
[In] integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/60*a*(3/((a^6*c^(5/2)*x^4 + 2*a^4*c^(5/2)*x^2 + a^2*c^(5/2))*c) + 8/((a^4
*c^(3/2)*x^2 + a^2*c^(3/2))*c^2) - 16*log(x^2 + 1/a^2)/(a^2*c^(7/2))) + 1/1
5*(8*x/(sqrt(a^2*c*x^2 + c)*c^3) + 4*x/((a^2*c*x^2 + c)^(3/2)*c^2) + 3*x/((
a^2*c*x^2 + c)^(5/2)*c))*arcsinh(a*x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.62

$$\int \frac{\operatorname{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx = -\frac{1}{60}\sqrt{c}\left(\frac{16\log\left(a^2x^2+1\right)}{ac^4} - \frac{24\,a^4x^4+56\,a^2x^2+35}{\left(a^2x^2+1\right)^2ac^4}\right) + \frac{\left(4\left(\frac{2\,a^4x^2}{c}+\frac{5\,a^2}{c}\right)x^2+\frac{15}{c}\right)x\log\left(ax+\sqrt{a^2x^2+1}\right)}{15\left(a^2cx^2+c\right)^{\frac{5}{2}}}$$

```
[In] integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/60*sqrt(c)*(16*log(a^2*x^2 + 1)/(a*c^4) - (24*a^4*x^4 + 56*a^2*x^2 + 35)
/((a^2*x^2 + 1)^2*a*c^4)) + 1/15*(4*(2*a^4*x^2/c + 5*a^2/c)*x^2 + 15/c)*x*l
og(a*x + sqrt(a^2*x^2 + 1))/(a^2*c*x^2 + c)^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)}{\left(c\,a^2\,x^2+c\right)^{7/2}}\,dx$$

```
[In] int(asinh(a*x)/(c + a^2*c*x^2)^(7/2),x)
```

```
[Out] int(asinh(a*x)/(c + a^2*c*x^2)^(7/2), x)
```

3.177 $\int \frac{x^4 \operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 86

$$\int \frac{x^4 \operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{3x^2}{16a^3} - \frac{x^4}{16a} - \frac{3x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{4a^2} + \frac{3\operatorname{arcsinh}(ax)^2}{16a^5}$$

```
[Out] 3/16*x^2/a^3-1/16*x^4/a+3/16*arcsinh(a*x)^2/a^5-3/8*x*arcsinh(a*x)*(a^2*x^2
+1)^(1/2)/a^4+1/4*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5812, 5783, 30}

$$\int \frac{x^4 \operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{3\operatorname{arcsinh}(ax)^2}{16a^5} + \frac{3x^2}{16a^3} + \frac{x^3\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)}{4a^2} - \frac{3x\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)}{8a^4} - \frac{x^4}{16a}$$

```
[In] Int[(x^4*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (3*x^2)/(16*a^3) - x^4/(16*a) - (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(8*a^4
) + (x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(4*a^2) + (3*ArcSinh[a*x]^2)/(16*a
^5)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{4a^2} - \frac{3\int \frac{x^2\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{4a^2} - \frac{\int x^3\,dx}{4a} \\ &= -\frac{x^4}{16a} - \frac{3x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{4a^2} + \frac{3\int \frac{\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{8a^4} \\ &\quad + \frac{3\int x\,dx}{8a^3} \\ &= \frac{3x^2}{16a^3} - \frac{x^4}{16a} - \frac{3x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{4a^2} + \frac{3\operatorname{arcsinh}(ax)^2}{16a^5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.73

$$\int \frac{x^4\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{3a^2x^2 - a^4x^4 + 2ax\sqrt{1+a^2x^2}(-3+2a^2x^2)\operatorname{arcsinh}(ax) + 3\operatorname{arcsinh}(ax)^2}{16a^5}$$

```
[In] Integrate[(x^4*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
[Out] (3*a^2*x^2 - a^4*x^4 + 2*a*x*Sqrt[1 + a^2*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[a*x
] + 3*ArcSinh[a*x]^2)/(16*a^5)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{4a^3x^3 \operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}-a^4x^4-6 \operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}\, ax+3a^2x^2+3 \operatorname{arcsinh}(ax)^2+3}{16a^5}$	74

```
[In] int(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/16*(4*a^3*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)-a^4*x^4-6*arcsinh(a*x)*(a^2*
x^2+1)^(1/2)*a*x+3*a^2*x^2+3*arcsinh(a*x)^2+3)/a^5
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int \frac{x^4\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{a^4x^4-3\,a^2x^2-2\,(2\,a^3x^3-3\,ax)\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)-3\,\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{16\,a^5}$$

```
[In] integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/16*(a^4*x^4 - 3*a^2*x^2 - 2*(2*a^3*x^3 - 3*a*x)*sqrt(a^2*x^2 + 1)*log(a*
x + sqrt(a^2*x^2 + 1)) - 3*log(a*x + sqrt(a^2*x^2 + 1))^2)/a^5
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int \frac{x^4\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} -\frac{x^4}{16a}+\frac{x^3\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{4a^2}+\frac{3x^2}{16a^3}-\frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{8a^4}+\frac{3\operatorname{asinh}^2(ax)}{16a^5} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(x**4*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
[Out] Piecewise((-x**4/(16*a) + x**3*sqrt(a**2*x**2 + 1)*asinh(a*x)/(4*a**2) + 3*
x**2/(16*a**3) - 3*x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(8*a**4) + 3*asinh(a*x)
**2/(16*a**5), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int \frac{x^4 \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{1}{16}\left(\frac{x^4}{a^2}-\frac{3\,x^2}{a^4}+\frac{3\,\text{arsinh}\,(ax)^2}{a^6}\right)a + \frac{1}{8}\left(\frac{2\,\sqrt{a^2x^2+1}x^3}{a^2}-\frac{3\,\sqrt{a^2x^2+1}x}{a^4}+\frac{3\,\text{arsinh}\,(ax)}{a^5}\right)\text{arsinh}\,(ax)$$

```
[In] integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/16*(x^4/a^2 - 3*x^2/a^4 + 3*arcsinh(a*x)^2/a^6)*a + 1/8*(2*sqrt(a^2*x^2
+ 1)*x^3/a^2 - 3*sqrt(a^2*x^2 + 1)*x/a^4 + 3*arcsinh(a*x)/a^5)*arcsinh(a*x)
```

Giac [F]

$$\int \frac{x^4 \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^4\,\text{arsinh}\,(ax)}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^4*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^4\,\text{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int((x^4*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^4*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

3.178 $\int \frac{x^3\textbf{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx=\frac{2x}{3a^3}-\frac{x^3}{9a}-\frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{3a^4}+\frac{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{3a^2}$$

```
[Out] 2/3*x/a^3-1/9*x^3/a-2/3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^4+1/3*x^2*arcsinh(
a*x)*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5812, 5798, 8, 30}

$$\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx=\frac{2x}{3a^3}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3a^2}-\frac{2\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3a^4}-\frac{x^3}{9a}$$

```
[In] Int[(x^3*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (2*x)/(3*a^3) - x^3/(9*a) - (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(3*a^4) + (x
^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(3*a^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{3a^2} - \frac{2\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{3a^2} - \frac{\int x^2\,dx}{3a} \\
&= -\frac{x^3}{9a} - \frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{3a^4} + \frac{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{3a^2} + \frac{2\int 1\,dx}{3a^3} \\
&= \frac{2x}{3a^3} - \frac{x^3}{9a} - \frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{3a^4} + \frac{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{3a^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.69

$$\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{6ax - a^3x^3 + 3(-2+a^2x^2)\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{9a^4}$$

```
[In] Integrate[(x^3*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
[Out] (6*a*x - a^3*x^3 + 3*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(9*a^4)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{3a^4x^4 \operatorname{arcsinh}(ax)-3a^2x^2 \operatorname{arcsinh}(ax)-a^3x^3\sqrt{a^2x^2+1}-6 \operatorname{arcsinh}(ax)+6ax\sqrt{a^2x^2+1}}{9a^4\sqrt{a^2x^2+1}}$	82

```
[In] int(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9/a^4/(a^2*x^2+1)^(1/2)*(3*a^4*x^4*arcsinh(a*x)-3*a^2*x^2*arcsinh(a*x)-a^
3*x^3*(a^2*x^2+1)^(1/2)-6*arcsinh(a*x)+6*a*x*(a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.79

$$\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{a^3x^3 - 3\,\sqrt{a^2x^2+1}(a^2x^2-2)\log\left(ax+\sqrt{a^2x^2+1}\right) - 6\,ax}{9\,a^4}$$

```
[In] integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*(a^3*x^3 - 3*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 2)*log(a*x + sqrt(a^2*x^2 +
1)) - 6*a*x)/a^4
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} -\frac{x^3}{9a} + \frac{x^2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{3a^2} + \frac{2x}{3a^3} - \frac{2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{3a^4} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(x**3*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**3/(9*a) + x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)/(3*a**2) + 2*x
/(3*a**3) - 2*sqrt(a**2*x**2 + 1)*asinh(a*x)/(3*a**4), Ne(a, 0)), (0, True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{1}{9}\,a\left(\frac{x^3}{a^2}-\frac{6\,x}{a^4}\right)+\frac{1}{3}\left(\frac{\sqrt{a^2x^2+1}x^2}{a^2}-\frac{2\,\sqrt{a^2x^2+1}}{a^4}\right)\text{arsinh}\,(ax)$$

```
[In] integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/9*a*(x^3/a^2 - 6*x/a^4) + 1/3*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^
2 + 1)/a^4)*arcsinh(a*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^3\,\text{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int((x^3*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((x^3*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

3.179 $\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{x^2}{4a}+\frac{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{2a^2}-\frac{\text{arcsinh}(ax)^2}{4a^3}$$

```
[Out] -1/4*x^2/a-1/4*arcsinh(a*x)^2/a^3+1/2*x*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5812, 5783, 30}

$$\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{\text{arcsinh}(ax)^2}{4a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}$$

```
[In] Int[(x^2*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -1/4*x^2/a + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a^2) - ArcSinh[a*x]^2/(4
*a^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
```

```
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{2a^2} - \frac{\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{2a^2} - \frac{\int x\,dx}{2a} \\ &= -\frac{x^2}{4a} + \frac{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{2a^2} - \frac{\text{arcsinh}(ax)^2}{4a^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{a^2x^2 - 2ax\sqrt{1+a^2x^2}\text{arcsinh}(ax) + \text{arcsinh}(ax)^2}{4a^3}$$

```
[In] Integrate[(x^2*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
[Out] -1/4*(a^2*x^2 - 2*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + ArcSinh[a*x]^2)/a^3
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{-2\,\text{arcsinh}(ax)\sqrt{a^2x^2+1}\,ax+a^2x^2+\text{arcsinh}(ax)^2+1}{4a^3}$	40

```
[In] int(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/4*(-2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x+a^2*x^2+arcsinh(a*x)^2+1)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.27

$$\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{a^2x^2-2\sqrt{a^2x^2+1}ax\log\left(ax+\sqrt{a^2x^2+1}\right)+\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{4\,a^3}$$

```
[In] integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(a^2*x^2 - 2*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1)) + log(
a*x + sqrt(a^2*x^2 + 1))^2)/a^3
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} -\frac{x^2}{4a}+\frac{x\sqrt{a^2x^2+1}\,\text{asinh}\,(ax)}{2a^2}-\frac{\text{asinh}^2\,(ax)}{4a^3} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**2/(4*a) + x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(2*a**2) - asinh(
a*x)**2/(4*a**3), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

$$\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{1}{4}\,a\left(\frac{x^2}{a^2}-\frac{\text{arsinh}\,(ax)^2}{a^4}\right) + \frac{1}{2}\left(\frac{\sqrt{a^2x^2+1}x}{a^2}-\frac{\text{arsinh}\,(ax)}{a^3}\right)\text{arsinh}\,(ax)$$

```
[In] integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*a*(x^2/a^2 - arcsinh(a*x)^2/a^4) + 1/2*(sqrt(a^2*x^2 + 1)*x/a^2 - arcs
inh(a*x)/a^3)*arcsinh(a*x)
```

Giac [F]

$$\int \frac{x^2 \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2\,\text{arsinh}\,(ax)}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2\,\text{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int((x^2*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^2*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

3.180 $\int \frac{x\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{x\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{x}{a} + \frac{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{a^2}$$

```
[Out] -x/a+arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5798, 8}

$$\int \frac{x\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)}{a^2} - \frac{x}{a}$$

```
[In] Int[(x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
```

```
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{a^2} - \frac{\int 1\,dx}{a}$$
$$= -\frac{x}{a} + \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{a^2}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{x}{a} + \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{a^2}$$

```
[In] Integrate[(x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
[Out] -(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.68

method	result	size
default	$\frac{a^2x^2\,\text{arcsinh}(ax)+\text{arcsinh}(ax)-ax\sqrt{a^2x^2+1}}{a^2\sqrt{a^2x^2+1}}$	47

```
[In] int(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/a^2/(a^2*x^2+1)^(1/2)*(a^2*x^2*arcsinh(a*x)+arcsinh(a*x)-a*x*(a^2*x^2+1)^
(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{ax - \sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}$$

```
[In] integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -(a*x - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} -\frac{x}{a} + \frac{\sqrt{a^2x^2+1}\,\text{asinh}\,(ax)}{a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(x*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
[Out] Piecewise((-x/a + sqrt(a**2*x**2 + 1)*asinh(a*x)/a**2, Ne(a, 0)), (0, True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{x}{a} + \frac{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}{a^2}$$

```
[In] integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -x/a + sqrt(a^2*x^2 + 1)*arcsinh(a*x)/a^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{x}{a} + \frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}$$

```
[In] integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] -x/a + sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))/a^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x\,\text{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int((x*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

3.181 $\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 13

$$\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{\text{arcsinh}(ax)^2}{2a}$$

```
[Out] 1/2*arcsinh(a*x)^2/a
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {5783}

$$\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{\text{arcsinh}(ax)^2}{2a}$$

```
[In] Int[ArcSinh[a*x]/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ArcSinh[a*x]^2/(2*a)
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{\text{arcsinh}(ax)^2}{2a}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{\operatorname{arcsinh}(ax)^2}{2a}$$

```
[In] Integrate[ArcSinh[a*x]/Sqrt[1 + a^2*x^2],x]
[Out] ArcSinh[a*x]^2/(2*a)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\operatorname{arcsinh}(ax)^2}{2a}$	12
default	$\frac{\operatorname{arcsinh}(ax)^2}{2a}$	12

```
[In] int(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arcsinh(a*x)^2/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{2\,a}$$

```
[In] integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*log(a*x + sqrt(a^2*x^2 + 1))^2/a
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} \frac{\operatorname{asinh}^2(ax)}{2a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(asinh(a*x)/(a**2*x**2+1)**(1/2),x)
[Out] Piecewise((asinh(a*x)**2/(2*a), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{\operatorname{arsinh}(ax)^2}{2a}$$

```
[In] integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*arcsinh(a*x)^2/a
```

Giac [F]

$$\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{\operatorname{asinh}(a\,x)^2}{2\,a}$$

```
[In] int(asinh(a*x)/(a^2*x^2 + 1)^(1/2),x)
[Out] asinh(a*x)^2/(2*a)
```

3.182 $\int \frac{\text{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 34

$$\int \frac{\text{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx = -2\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) - \text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right) + \text{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right)$$

```
[Out] -2*arcsinh(a*x)*arctanh(a*x+(a^2*x^2+1)^(1/2))-polylog(2,-a*x-(a^2*x^2+1)^(
1/2))+polylog(2,a*x+(a^2*x^2+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5816, 4267, 2317, 2438}

$$\int \frac{\text{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx = -2\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) - \text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right) + \text{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right)$$

```
[In] Int[ArcSinh[a*x]/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -2*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]] - PolyLog[2, -E^ArcSinh[a*x]] + Pol
yLog[2, E^ArcSinh[a*x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int x\text{csch}(x)\,dx, x, \text{arcsinh}(ax)\right)\\
&= -2\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) - \text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \text{arcsinh}(ax)\right)\\
&\quad + \text{Subst}\left(\int \log\left(1+e^x\right)\,dx, x, \text{arcsinh}(ax)\right)\\
&= -2\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) - \text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\text{arcsinh}(ax)}\right)\\
&\quad + \text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{\text{arcsinh}(ax)}\right)\\
&= -2\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) - \text{PolyLog}\left(2, -e^{\text{arcsinh}(ax)}\right) + \text{PolyLog}\left(2, e^{\text{arcsinh}(ax)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.68

$$
\begin{aligned}
\int \frac{\text{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx &= \text{arcsinh}(ax)\left(\log\left(1-e^{-\text{arcsinh}(ax)}\right) - \log\left(1+e^{-\text{arcsinh}(ax)}\right)\right)\\
&\quad + \text{PolyLog}\left(2, -e^{-\text{arcsinh}(ax)}\right) - \text{PolyLog}\left(2, e^{-\text{arcsinh}(ax)}\right)
\end{aligned}
$$

```
[In] Integrate[ArcSinh[a*x]/(x*Sqrt[1 + a^2*x^2]),x]
[Out] ArcSinh[a*x]*(Log[1 - E^(-ArcSinh[a*x])] - Log[1 + E^(-ArcSinh[a*x])]) + Po
lyLog[2, -E^(-ArcSinh[a*x])] - PolyLog[2, E^(-ArcSinh[a*x])]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.62

method	result
default	$\operatorname{arcsinh}(ax)\ln\left(1-ax-\sqrt{a^2x^2+1}\right)+\operatorname{polylog}\left(2,ax+\sqrt{a^2x^2+1}\right)-\operatorname{arcsinh}(ax)\ln\left(1+ax+\right.$

[In] int(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))+polylog(2,a*x+(a^2*x^2+1)^(1/2))-arcsinh(a*x)*ln(1+a*x+(a^2*x^2+1)^(1/2))-polylog(2,-a*x-(a^2*x^2+1)^(1/2))

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx=\int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}x}\,dx$$

[In] integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)/(a^2*x^3 + x), x)

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx=\int \frac{\operatorname{asinh}(ax)}{x\sqrt{a^2x^2+1}}\,dx$$

[In] integrate(asinh(a*x)/x/(a**2*x**2+1)**(1/2),x)

[Out] Integral(asinh(a*x)/(x*sqrt(a**2*x**2 + 1)), x)

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx=\int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}x}\,dx$$

[In] integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x), x)

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}x}\,dx$$

```
[In] integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)}{x\,\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int(asinh(a*x)/(x*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(asinh(a*x)/(x*(a^2*x^2 + 1)^(1/2)), x)
```

3.183 $\int \frac{\operatorname{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 27

$$\int \frac{\operatorname{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = -\frac{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{x} + a\log(x)$$

```
[Out] a*ln(x)-arcsinh(a*x)*(a^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5800, 29}

$$\int \frac{\operatorname{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = a\log(x) - \frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)}{x}$$

```
[In] Int[ArcSinh[a*x]/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -((Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) + a*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
```

```
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{x} + a\int \frac{1}{x}\,dx \\ &= -\frac{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{x} + a\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = -\frac{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{x} + a\log(ax)$$

```
[In] Integrate[ArcSinh[a*x]/(x^2*Sqrt[1 + a^2*x^2]),x]
[Out] -((Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) + a*Log[a*x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(25) = 50.

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.07

method	result	size
default	$-2a\operatorname{arcsinh}(ax)+\frac{\left(ax-\sqrt{a^2x^2+1}\right)\operatorname{arcsinh}(ax)}{x}+a\ln\left(\left(ax+\sqrt{a^2x^2+1}\right)^2-1\right)$	56

```
[In] int(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*a*arcsinh(a*x)+(a*x-(a^2*x^2+1)^(1/2))/x*arcsinh(a*x)+a*ln((a*x+(a^2*x^2
+1)^(1/2))^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \frac{\text{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = \frac{ax\log(x) - \sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{x}$$

```
[In] integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] (a*x*log(x) - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/x
```

Sympy [F]

$$\int \frac{\text{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(ax)}{x^2\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(asinh(a*x)/x**2/(a**2*x**2+1)**(1/2),x)
[Out] Integral(asinh(a*x)/(x**2*sqrt(a**2*x**2 + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\text{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = a\log(x) - \frac{\sqrt{a^2x^2+1}\,\text{arsinh}(ax)}{x}$$

```
[In] integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] a*log(x) - sqrt(a^2*x^2 + 1)*arcsinh(a*x)/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(25) = 50.

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.63

$$\int \frac{\text{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = -a\log\left(-x|a|+\sqrt{a^2x^2+1}\right) + a\log(|x|) + \frac{2\,|a|\log\left(ax+\sqrt{a^2x^2+1}\right)}{\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1}$$

```
[In] integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] -a*log(-x*abs(a) + sqrt(a^2*x^2 + 1)) + a*log(abs(x)) + 2*abs(a)*log(a*x +
sqrt(a^2*x^2 + 1))/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx=\int \frac{\operatorname{asinh}(a\,x)}{x^2\,\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int(asinh(a*x)/(x^2*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(asinh(a*x)/(x^2*(a^2*x^2 + 1)^(1/2)), x)
```

3.184 $\int \frac{\operatorname{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = -\frac{a}{2x} - \frac{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{2x^2} + a^2\operatorname{arcsinh}(ax)\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) + \frac{1}{2}a^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right) - \frac{1}{2}a^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right)$$

```
[Out] -1/2*a/x+a^2*arcsinh(a*x)*arctanh(a*x+(a^2*x^2+1)^(1/2))+1/2*a^2*polylog(2,
-a*x-(a^2*x^2+1)^(1/2))-1/2*a^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))-1/2*arcsin
h(a*x)*(a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5809, 5816, 4267, 2317, 2438, 30}

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = a^2\operatorname{arcsinh}(ax)\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) + \frac{1}{2}a^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right) - \frac{1}{2}a^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right) - \frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)}{2x^2} - \frac{a}{2x}$$

```
[In] Int[ArcSinh[a*x]/(x^3*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -1/2*a/x - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*x^2) + a^2*ArcSinh[a*x]*ArcT
anh[E^ArcSinh[a*x]] + (a^2*PolyLog[2, -E^ArcSinh[a*x]])/2 - (a^2*PolyLog[2,
 E^ArcSinh[a*x]])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{2x^2}+\frac{1}{2}a\int\frac{1}{x^2}\,dx-\frac{1}{2}a^2\int\frac{\text{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx\\ &= -\frac{a}{2x}-\frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{2x^2}-\frac{1}{2}a^2\text{Subst}(\int x\text{csch}(x)\,dx,x,\text{arcsinh}(ax))\end{aligned}$$

$$= -\frac{a}{2x} - \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{2x^2} + a^2\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) + \frac{1}{2}a^2\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \text{arcsinh}(ax)\right) - \frac{1}{2}a^2\text{Subst}\left(\int \log\left(1+e^x\right)\,dx, x, \text{arcsinh}(ax)\right)$$

$$= -\frac{a}{2x} - \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{2x^2} + a^2\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) + \frac{1}{2}a^2\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\text{arcsinh}(ax)}\right) - \frac{1}{2}a^2\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{\text{arcsinh}(ax)}\right)$$

$$= -\frac{a}{2x} - \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{2x^2} + a^2\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) + \frac{1}{2}a^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right) - \frac{1}{2}a^2\,\text{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right)$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.58

$$\int \frac{\text{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = \frac{1}{8}a^2\left(-2\coth\left(\frac{1}{2}\text{arcsinh}(ax)\right) - \text{arcsinh}(ax)\text{csch}^2\left(\frac{1}{2}\text{arcsinh}(ax)\right) - 4\text{arcsinh}(ax)\log\left(1-e^{-\text{arcsinh}(ax)}\right) + 4\text{arcsinh}(ax)\log\left(1+e^{-\text{arcsinh}(ax)}\right) - 4\,\text{PolyLog}\left(2,-e^{-\text{arcsinh}(ax)}\right) + 4\,\text{PolyLog}\left(2,e^{-\text{arcsinh}(ax)}\right) - \text{arcsinh}(ax)\text{sech}^2\left(\frac{1}{2}\text{arcsinh}(ax)\right) + 2\tanh\left(\frac{1}{2}\text{arcsinh}(ax)\right)\right)$$

```
[In] Integrate[ArcSinh[a*x]/(x^3*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (a^2*(-2*Coth[ArcSinh[a*x]/2] - ArcSinh[a*x]*Csch[ArcSinh[a*x]/2]^2 - 4*Arc
Sinh[a*x]*Log[1 - E^(-ArcSinh[a*x])] + 4*ArcSinh[a*x]*Log[1 + E^(-ArcSinh[a
*x])] - 4*PolyLog[2, -E^(-ArcSinh[a*x])] + 4*PolyLog[2, E^(-ArcSinh[a*x])]
- ArcSinh[a*x]*Sech[ArcSinh[a*x]/2]^2 + 2*Tanh[ArcSinh[a*x]/2]))/8
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.88

method	result
default	$-\frac{a^{2} x^{2} \operatorname{arcsinh}(a x)+a x \sqrt{a^{2} x^{2}+1}+\operatorname{arcsinh}(a x)}{2 \sqrt{a^{2} x^{2}+1}\, x^{2}}-\frac{a^{2} \operatorname{arcsinh}(a x) \ln \left(1-a x-\sqrt{a^{2} x^{2}+1}\right)}{2}-\frac{a^{2} \operatorname{polylog}\left(2, a x+\sqrt{a^{2} x^{2}+1}\right)}{2}+\frac{a^{2} \mathrm{a}}{}$

```
[In] int(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2/(a^2*x^2+1)^(1/2)*(a^2*x^2*arcsinh(a*x)+a*x*(a^2*x^2+1)^(1/2)+arcsinh(
a*x))/x^2-1/2*a^2*arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))-1/2*a^2*polylog(
2,a*x+(a^2*x^2+1)^(1/2))+1/2*a^2*arcsinh(a*x)*ln(1+a*x+(a^2*x^2+1)^(1/2))+1
/2*a^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}x^3}\,dx$$

```
[In] integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)/(a^2*x^5 + x^3), x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(ax)}{x^3\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(asinh(a*x)/x**3/(a**2*x**2+1)**(1/2),x)
[Out] Integral(asinh(a*x)/(x**3*sqrt(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}x^3}\,dx$$

```
[In] integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x^3), x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}x^3}\,dx$$

```
[In] integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)}{x^3\,\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int(asinh(a*x)/(x^3*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(asinh(a*x)/(x^3*(a^2*x^2 + 1)^(1/2)), x)
```

3.185 $\int x^m(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 313

$$\begin{aligned}\int x^m(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))\,dx &= -\frac{bcd^3(2271+1329m+284m^2+27m^3+m^4)\,x^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2(5+m)^2(7+m)^2} \\ &\quad -\frac{bc^3d^3(9+m)(13+2m)x^{4+m}\sqrt{1+c^2x^2}}{(5+m)^2(7+m)^2}-\frac{bc^5d^3x^{6+m}\sqrt{1+c^2x^2}}{(7+m)^2} \\ &\quad +\frac{d^3x^{1+m}(a+b\text{arcsinh}(cx))}{1+m}+\frac{3c^2d^3x^{3+m}(a+b\text{arcsinh}(cx))}{3+m} \\ &\quad +\frac{3c^4d^3x^{5+m}(a+b\text{arcsinh}(cx))}{5+m}+\frac{c^6d^3x^{7+m}(a+b\text{arcsinh}(cx))}{7+m} \\ &\quad -\frac{3bcd^3(2161+1813m+455m^2+35m^3)\,x^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{(1+m)(2+m)(3+m)^2(5+m)^2(7+m)^2}\end{aligned}$$

```
[Out] d^3*x^(1+m)*(a+b*arcsinh(c*x))/(1+m)+3*c^2*d^3*x^(3+m)*(a+b*arcsinh(c*x))/(
3+m)+3*c^4*d^3*x^(5+m)*(a+b*arcsinh(c*x))/(5+m)+c^6*d^3*x^(7+m)*(a+b*arcsin
h(c*x))/(7+m)-3*b*c*d^3*(35*m^3+455*m^2+1813*m+2161)*x^(2+m)*hypergeom([1/2
, 1+1/2*m],[2+1/2*m],-c^2*x^2)/(m^2+3*m+2)/(m^3+15*m^2+71*m+105)^2-b*c*d^3*
(m^4+27*m^3+284*m^2+1329*m+2271)*x^(2+m)*(c^2*x^2+1)^(1/2)/(7+m)^2/(m^2+8*m
+15)^2-b*c^3*d^3*(9+m)*(13+2*m)*x^(4+m)*(c^2*x^2+1)^(1/2)/(5+m)^2/(7+m)^2-b
*c^5*d^3*x^(6+m)*(c^2*x^2+1)^(1/2)/(7+m)^2
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {276, 5803, 12, 1823, 1281, 470, 371}

$$\int x^m\left(d+c^2dx^2\right)^3(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\frac{c^6d^3x^{m+7}(a+b\operatorname{arcsinh}(cx))}{m+7}+\frac{3c^4d^3x^{m+5}(a+b\operatorname{arcsinh}(cx))}{m+5}+\frac{3c^2d^3x^{m+3}(a+b\operatorname{arcsinh}(cx))}{m+3}+\frac{d^3x^{m+1}(a+b\operatorname{arcsinh}(cx))}{m+1}-\frac{3bcd^3(35m^3+455m^2+1813m+2161)\,x^{m+2}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{(m+1)(m+2)(m+3)^2(m+5)^2(m+7)^2}-\frac{bcd^3(m^4+27m^3+284m^2+1329m+2271)\sqrt{c^2x^2+1}x^{m+2}}{(m+3)^2(m+5)^2(m+7)^2}-\frac{bc^5d^3\sqrt{c^2x^2+1}x^{m+6}}{(m+7)^2}-\frac{bc^3d^3(m+9)(2m+13)\sqrt{c^2x^2+1}x^{m+4}}{(m+5)^2(m+7)^2}$$

```
[In] Int[x^m*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -((b*c*d^3*(2271 + 1329*m + 284*m^2 + 27*m^3 + m^4)*x^(2 + m)*Sqrt[1 + c^2*
x^2])/((3 + m)^2*(5 + m)^2*(7 + m)^2)) - (b*c^3*d^3*(9 + m)*(13 + 2*m)*x^(4
 + m)*Sqrt[1 + c^2*x^2])/((5 + m)^2*(7 + m)^2) - (b*c^5*d^3*x^(6 + m)*Sqrt[
1 + c^2*x^2])/(7 + m)^2 + (d^3*x^(1 + m)*(a + b*ArcSinh[c*x]))/(1 + m) + (3
*c^2*d^3*x^(3 + m)*(a + b*ArcSinh[c*x]))/(3 + m) + (3*c^4*d^3*x^(5 + m)*(a
+ b*ArcSinh[c*x]))/(5 + m) + (c^6*d^3*x^(7 + m)*(a + b*ArcSinh[c*x]))/(7 +
m) - (3*b*c*d^3*(2161 + 1813*m + 455*m^2 + 35*m^3)*x^(2 + m)*Hypergeometric
2F1[1/2, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/((1 + m)*(2 + m)*(3 + m)^2*(5 +
 m)^2*(7 + m)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{d^3x^{1+m}(a+b\text{arcsinh}(cx))}{1+m}+\frac{3c^2d^3x^{3+m}(a+b\text{arcsinh}(cx))}{3+m} \\
&\quad+\frac{3c^4d^3x^{5+m}(a+b\text{arcsinh}(cx))}{5+m}+\frac{c^6d^3x^{7+m}(a+b\text{arcsinh}(cx))}{7+m} \\
&\quad-(bc)\int\frac{d^3x^{1+m}\left(\frac{1}{1+m}+\frac{3c^2x^2}{3+m}+\frac{3c^4x^4}{5+m}+\frac{c^6x^6}{7+m}\right)}{\sqrt{1+c^2x^2}}\,dx \\
&= \frac{d^3x^{1+m}(a+b\text{arcsinh}(cx))}{1+m}+\frac{3c^2d^3x^{3+m}(a+b\text{arcsinh}(cx))}{3+m}+\frac{3c^4d^3x^{5+m}(a+b\text{arcsinh}(cx))}{5+m} \\
&\quad+\frac{c^6d^3x^{7+m}(a+b\text{arcsinh}(cx))}{7+m}-\left(bcd^3\right)\int\frac{x^{1+m}\left(\frac{1}{1+m}+\frac{3c^2x^2}{3+m}+\frac{3c^4x^4}{5+m}+\frac{c^6x^6}{7+m}\right)}{\sqrt{1+c^2x^2}}\,dx \\
&= -\frac{bc^5d^3x^{6+m}\sqrt{1+c^2x^2}}{(7+m)^2}+\frac{d^3x^{1+m}(a+b\text{arcsinh}(cx))}{1+m}+\frac{3c^2d^3x^{3+m}(a+b\text{arcsinh}(cx))}{3+m} \\
&\quad+\frac{3c^4d^3x^{5+m}(a+b\text{arcsinh}(cx))}{5+m}+\frac{c^6d^3x^{7+m}(a+b\text{arcsinh}(cx))}{7+m} \\
&\quad-\frac{\left(bd^3\right)\int\frac{x^{1+m}\left(\frac{c^2(7+m)}{1+m}+\frac{3c^4(7+m)x^2}{3+m}+\frac{c^6(9+m)(13+2m)x^4}{(5+m)(7+m)}\right)}{\sqrt{1+c^2x^2}}\,dx}{c(7+m)} \\
&= -\frac{bc^3d^3(9+m)(13+2m)x^{4+m}\sqrt{1+c^2x^2}}{(5+m)^2(7+m)^2}-\frac{bc^5d^3x^{6+m}\sqrt{1+c^2x^2}}{(7+m)^2} \\
&\quad+\frac{d^3x^{1+m}(a+b\text{arcsinh}(cx))}{1+m}+\frac{3c^2d^3x^{3+m}(a+b\text{arcsinh}(cx))}{3+m} \\
&\quad+\frac{3c^4d^3x^{5+m}(a+b\text{arcsinh}(cx))}{5+m}+\frac{c^6d^3x^{7+m}(a+b\text{arcsinh}(cx))}{7+m} \\
&\quad-\frac{\left(bd^3\right)\int\frac{x^{1+m}\left(\frac{c^4(5+m)(7+m)}{1+m}+\frac{c^6\left(2271+1329m+284m^2+27m^3+m^4\right)x^2}{(3+m)(5+m)(7+m)}\right)}{\sqrt{1+c^2x^2}}\,dx}{c^3(5+m)(7+m)} \\
&= -\frac{bcd^3\left(2271+1329m+284m^2+27m^3+m^4\right)x^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2(5+m)^2(7+m)^2} \\
&\quad-\frac{bc^3d^3(9+m)(13+2m)x^{4+m}\sqrt{1+c^2x^2}}{(5+m)^2(7+m)^2}-\frac{bc^5d^3x^{6+m}\sqrt{1+c^2x^2}}{(7+m)^2} \\
&\quad+\frac{d^3x^{1+m}(a+b\text{arcsinh}(cx))}{1+m}+\frac{3c^2d^3x^{3+m}(a+b\text{arcsinh}(cx))}{3+m} \\
&\quad+\frac{3c^4d^3x^{5+m}(a+b\text{arcsinh}(cx))}{5+m}+\frac{c^6d^3x^{7+m}(a+b\text{arcsinh}(cx))}{7+m} \\
&\quad-\frac{\left(3bcd^3\left(2161+1813m+455m^2+35m^3\right)\right)\int\frac{x^{1+m}}{\sqrt{1+c^2x^2}}\,dx}{(1+m)(3+m)^2(5+m)^2(7+m)^2}
\end{aligned}
$$

$$= -\frac{bcd^3(2271 + 1329m + 284m^2 + 27m^3 + m^4)\,x^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2(5+m)^2(7+m)^2} - \frac{bc^3d^3(9+m)(13+2m)x^{4+m}\sqrt{1+c^2x^2}}{(5+m)^2(7+m)^2} - \frac{bc^5d^3x^{6+m}\sqrt{1+c^2x^2}}{(7+m)^2} + \frac{d^3x^{1+m}(a+b\text{arcsinh}(cx))}{1+m} + \frac{3c^2d^3x^{3+m}(a+b\text{arcsinh}(cx))}{3+m} + \frac{3c^4d^3x^{5+m}(a+b\text{arcsinh}(cx))}{5+m} + \frac{c^6d^3x^{7+m}(a+b\text{arcsinh}(cx))}{7+m} - \frac{3bcd^3(2161 + 1813m + 455m^2 + 35m^3)\,x^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -c^2x^2\right)}{(1+m)(2+m)(3+m)^2(5+m)^2(7+m)^2}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.82

$$\int x^m \left(d + c^2dx^2\right)^3 (a + b\text{arcsinh}(cx))\,dx$$

$$= \frac{x^{1+m}\left(\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx)) - \frac{bcd^3x\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1+\frac{m}{2},2+\frac{m}{2},-c^2x^2\right)}{2+m} + \frac{6d\left(\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}\right.}{}\right.}{}$$

```
[In] Integrate[x^m*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (x^(1 + m)*((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]) - (b*c*d^3*x*Hypergeomet
ric2F1[-5/2, 1 + m/2, 2 + m/2, -(c^2*x^2)])/(2 + m) + (6*d*((d + c^2*d*x^2)
^2*(a + b*ArcSinh[c*x]) - (b*c*d^2*x*Hypergeometric2F1[-3/2, 1 + m/2, 2 + m
/2, -(c^2*x^2)])/(2 + m) + (4*d^2*((2 + m)*(3 + m + c^2*x^2 + c^2*m*x^2)*(a
 + b*ArcSinh[c*x]) - b*c*(1 + m)*x*Hypergeometric2F1[-1/2, 1 + m/2, 2 + m/2
, -(c^2*x^2)] - 2*b*c*x*Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, -(c^2*x^2)
]))/((1 + m)*(2 + m)*(3 + m))))/(5 + m)))/(7 + m)
```

Maple [F]

$$\int x^m \left(c^2d\,x^2 + d\right)^3 (a + b\ \text{arcsinh}\,(cx))\,dx$$

```
[In] int(x^m*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x)
```

Fricas [F]

$$\int x^m (d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))\,dx = \int (c^2dx^2+d)^3(b\,\text{arsinh}\,(cx)+a)x^m\,dx$$

```
[In] integrate(x^m*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^6*d^3*x^6 + 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 + a*d^3 + (b*c^
6*d^3*x^6 + 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 + b*d^3)*arcsinh(c*x))*x^m, x
)
```

Sympy [F]

$$\int x^m (d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))\,dx = d^3\left(\int ax^m\,dx+\int bx^m\,\text{asinh}\,(cx)\,dx + \int 3ac^2x^2x^m\,dx+\int 3ac^4x^4x^m\,dx + \int ac^6x^6x^m\,dx+\int 3bc^2x^2x^m\,\text{asinh}\,(cx)\,dx + \int 3bc^4x^4x^m\,\text{asinh}\,(cx)\,dx + \int bc^6x^6x^m\,\text{asinh}\,(cx)\,dx\right)$$

```
[In] integrate(x**m*(c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] d**3*(Integral(a*x**m, x) + Integral(b*x**m*asinh(c*x), x) + Integral(3*a*c
**2*x**2*x**m, x) + Integral(3*a*c**4*x**4*x**m, x) + Integral(a*c**6*x**6*
x**m, x) + Integral(3*b*c**2*x**2*x**m*asinh(c*x), x) + Integral(3*b*c**4*x
**4*x**m*asinh(c*x), x) + Integral(b*c**6*x**6*x**m*asinh(c*x), x))
```

Maxima [F]

$$\int x^m (d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))\,dx = \int (c^2dx^2+d)^3(b\,\text{arsinh}\,(cx)+a)x^m\,dx$$

```
[In] integrate(x^m*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] a*c^6*d^3*x^(m + 7)/(m + 7) + 3*a*c^4*d^3*x^(m + 5)/(m + 5) + 3*a*c^2*d^3*x
^(m + 3)/(m + 3) + a*d^3*x^(m + 1)/(m + 1) + ((m^3 + 9*m^2 + 23*m + 15)*b*c
^6*d^3*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)*b*c^4*d^3*x^5 + 3*(m^3 + 13*m^2 +
 47*m + 35)*b*c^2*d^3*x^3 + (m^3 + 15*m^2 + 71*m + 105)*b*d^3*x)*x^m*log(c*
```

```
x + sqrt(c^2*x^2 + 1))/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105) - integrate(((
m^3 + 9*m^2 + 23*m + 15)*b*c^7*d^3*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)*b*c^5
*d^3*x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*b*c^3*d^3*x^3 + (m^3 + 15*m^2 + 71*
m + 105)*b*c*d^3*x)*x^m/((m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c^3*x^3 + (m
^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c*x + ((m^4 + 16*m^3 + 86*m^2 + 176*m +
 105)*c^2*x^2 + m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*sqrt(c^2*x^2 + 1)), x)
 - integrate(((m^3 + 9*m^2 + 23*m + 15)*b*c^8*d^3*x^8 + 3*(m^3 + 11*m^2 + 3
1*m + 21)*b*c^6*d^3*x^6 + 3*(m^3 + 13*m^2 + 47*m + 35)*b*c^4*d^3*x^4 + (m^3
 + 15*m^2 + 71*m + 105)*b*c^2*d^3*x^2)*x^m/((m^4 + 16*m^3 + 86*m^2 + 176*m
+ 105)*c^2*x^2 + m^4 + 16*m^3 + 86*m^2 + 176*m + 105), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^m (d + c^2 d x^2)^3 (a + b\text{arcsinh}(cx)) \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^m*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^m (d + c^2 d x^2)^3 (a + b\text{arcsinh}(cx)) \, dx = \int x^m \, (a + b \, \text{asinh}(c \, x)) \, \left(d \, c^2 \, x^2 + d\right)^3 \, dx$$

```
[In] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3,x)
```

```
[Out] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3, x)
```

3.186 $\int x^m(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 217

$$\int x^m\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))\,dx = -\frac{bcd^2(38+13m+m^2)\,x^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2(5+m)^2} - \frac{bc^3d^2x^{4+m}\sqrt{1+c^2x^2}}{(5+m)^2} + \frac{d^2x^{1+m}(a+b\text{arcsinh}(cx))}{1+m} + \frac{2c^2d^2x^{3+m}(a+b\text{arcsinh}(cx))}{3+m} + \frac{c^4d^2x^{5+m}(a+b\text{arcsinh}(cx))}{5+m} - \frac{bcd^2(149+100m+15m^2)\,x^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{(1+m)(2+m)(3+m)^2(5+m)^2}$$

```
[Out] d^2*x^(1+m)*(a+b*arcsinh(c*x))/(1+m)+2*c^2*d^2*x^(3+m)*(a+b*arcsinh(c*x))/(
3+m)+c^4*d^2*x^(5+m)*(a+b*arcsinh(c*x))/(5+m)-b*c*d^2*(15*m^2+100*m+149)*x^
(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],-c^2*x^2)/(m^2+3*m+2)/(m^2+8*m+15)
^2-b*c*d^2*(m^2+13*m+38)*x^(2+m)*(c^2*x^2+1)^(1/2)/(3+m)^2/(5+m)^2-b*c^3*d^
2*x^(4+m)*(c^2*x^2+1)^(1/2)/(5+m)^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

$= \{276, 5803, 12, 1281, 470, 371\}$

$$\int x^m \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{c^4d^2x^{m+5}(a+b\text{arcsinh}(cx))}{m+5}+\frac{2c^2d^2x^{m+3}(a+b\text{arcsinh}(cx))}{m+3}+\frac{d^2x^{m+1}(a+b\text{arcsinh}(cx))}{m+1}$$
$$-\frac{bcd^2(15m^2+100m+149)\,x^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{(m+1)(m+2)(m+3)^2(m+5)^2}$$
$$-\frac{bcd^2(m^2+13m+38)\sqrt{c^2x^2+1}x^{m+2}}{(m+3)^2(m+5)^2}-\frac{bc^3d^2\sqrt{c^2x^2+1}x^{m+4}}{(m+5)^2}$$

```
[In] Int[x^m*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -((b*c*d^2*(38 + 13*m + m^2)*x^(2 + m)*Sqrt[1 + c^2*x^2])/((3 + m)^2*(5 + m
)^2)) - (b*c^3*d^2*x^(4 + m)*Sqrt[1 + c^2*x^2])/(5 + m)^2 + (d^2*x^(1 + m)*
(a + b*ArcSinh[c*x]))/(1 + m) + (2*c^2*d^2*x^(3 + m)*(a + b*ArcSinh[c*x]))/
(3 + m) + (c^4*d^2*x^(5 + m)*(a + b*ArcSinh[c*x]))/(5 + m) - (b*c*d^2*(149
+ 100*m + 15*m^2)*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(
c^2*x^2)])/((1 + m)*(2 + m)*(3 + m)^2*(5 + m)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d^2x^{1+m}(a+b\text{arcsinh}(cx))}{1+m}+\frac{2c^2d^2x^{3+m}(a+b\text{arcsinh}(cx))}{3+m}\\
&\quad+\frac{c^4d^2x^{5+m}(a+b\text{arcsinh}(cx))}{5+m}-(bc)\int\frac{d^2x^{1+m}\left(\frac{1}{1+m}+\frac{2c^2x^2}{3+m}+\frac{c^4x^4}{5+m}\right)}{\sqrt{1+c^2x^2}}\,dx\\
&= \frac{d^2x^{1+m}(a+b\text{arcsinh}(cx))}{1+m}+\frac{2c^2d^2x^{3+m}(a+b\text{arcsinh}(cx))}{3+m}\\
&\quad+\frac{c^4d^2x^{5+m}(a+b\text{arcsinh}(cx))}{5+m}-\left(bcd^2\right)\int\frac{x^{1+m}\left(\frac{1}{1+m}+\frac{2c^2x^2}{3+m}+\frac{c^4x^4}{5+m}\right)}{\sqrt{1+c^2x^2}}\,dx\\
&= -\frac{bc^3d^2x^{4+m}\sqrt{1+c^2x^2}}{(5+m)^2}+\frac{d^2x^{1+m}(a+b\text{arcsinh}(cx))}{1+m}+\frac{2c^2d^2x^{3+m}(a+b\text{arcsinh}(cx))}{3+m}\\
&\quad+\frac{c^4d^2x^{5+m}(a+b\text{arcsinh}(cx))}{5+m}-\frac{\left(bd^2\right)\int\frac{x^{1+m}\left(\frac{c^2(5+m)}{1+m}+\frac{c^4\left(38+13m+m^2\right)x^2}{(3+m)(5+m)}\right)}{\sqrt{1+c^2x^2}}\,dx}{c(5+m)}\\
&= -\frac{bcd^2\left(38+13m+m^2\right)x^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2(5+m)^2}-\frac{bc^3d^2x^{4+m}\sqrt{1+c^2x^2}}{(5+m)^2}\\
&\quad+\frac{d^2x^{1+m}(a+b\text{arcsinh}(cx))}{1+m}+\frac{2c^2d^2x^{3+m}(a+b\text{arcsinh}(cx))}{3+m}\\
&\quad+\frac{c^4d^2x^{5+m}(a+b\text{arcsinh}(cx))}{5+m}-\frac{\left(bcd^2\left(149+100m+15m^2\right)\right)\int\frac{x^{1+m}}{\sqrt{1+c^2x^2}}\,dx}{(1+m)(3+m)^2(5+m)^2}
\end{aligned}$$

$$= -\frac{bcd^2(38+13m+m^2)\,x^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2(5+m)^2} - \frac{bc^3d^2x^{4+m}\sqrt{1+c^2x^2}}{(5+m)^2} + \frac{d^2x^{1+m}(a+b\text{arcsinh}(cx))}{1+m} + \frac{2c^2d^2x^{3+m}(a+b\text{arcsinh}(cx))}{3+m} + \frac{c^4d^2x^{5+m}(a+b\text{arcsinh}(cx))}{5+m} - \frac{bcd^2(149+100m+15m^2)\,x^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{(1+m)(2+m)(3+m)^2(5+m)^2}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.87

$$\int x^m\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))\,dx = \frac{x^{1+m}\left(\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx)) - \frac{bcd^2x\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1+\frac{m}{2},2+\frac{m}{2},-c^2x^2\right)}{2+m} + \frac{4d^2\left((2+m)(3+m+c^2x^2+c^2mx\right.}{}\right.}{5+m}$$

```
[In] Integrate[x^m*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (x^(1 + m)*((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]) - (b*c*d^2*x*Hypergeomet
ric2F1[-3/2, 1 + m/2, 2 + m/2, -(c^2*x^2)])/(2 + m) + (4*d^2*((2 + m)*(3 +
m + c^2*x^2 + c^2*m*x^2)*(a + b*ArcSinh[c*x]) - b*c*(1 + m)*x*Hypergeometri
c2F1[-1/2, 1 + m/2, 2 + m/2, -(c^2*x^2)] - 2*b*c*x*Hypergeometric2F1[1/2, 1
 + m/2, 2 + m/2, -(c^2*x^2)]))/((1 + m)*(2 + m)*(3 + m))))/(5 + m)
```

Maple [F]

$$\int x^m\left(c^2d\,x^2+d\right)^2(a+b\,\text{arcsinh}\,(cx))\,dx$$

```
[In] int(x^m*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x)
```

Fricas [F]

$$\int x^m \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))\,dx = \int \left(c^2dx^2+d\right)^2 (b\operatorname{arsinh}(cx)+a)x^m\,dx$$

[In] integrate(x^m*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")

[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^2*d^2*x^2 + b*d^2)*arcsinh(c*x))*x^m, x)

Sympy [F]

$$\int x^m \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))\,dx = d^2\left(\int ax^m\,dx + \int bx^m \operatorname{asinh}(cx)\,dx + \int 2ac^2x^2x^m\,dx + \int ac^4x^4x^m\,dx + \int 2bc^2x^2x^m \operatorname{asinh}(cx)\,dx + \int bc^4x^4x^m \operatorname{asinh}(cx)\,dx\right)$$

[In] integrate(x**m*(c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)

[Out] d**2*(Integral(a*x**m, x) + Integral(b*x**m*asinh(c*x), x) + Integral(2*a*c**2*x**2*x**m, x) + Integral(a*c**4*x**4*x**m, x) + Integral(2*b*c**2*x**2*x**m*asinh(c*x), x) + Integral(b*c**4*x**4*x**m*asinh(c*x), x))

Maxima [F]

$$\int x^m \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))\,dx = \int \left(c^2dx^2+d\right)^2 (b\operatorname{arsinh}(cx)+a)x^m\,dx$$

[In] integrate(x^m*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")

[Out] a*c^4*d^2*x^(m + 5)/(m + 5) + 2*a*c^2*d^2*x^(m + 3)/(m + 3) + a*d^2*x^(m + 1)/(m + 1) + ((m^2 + 4*m + 3)*b*c^4*d^2*x^5 + 2*(m^2 + 6*m + 5)*b*c^2*d^2*x^3 + (m^2 + 8*m + 15)*b*d^2*x)*x^m*log(c*x + sqrt(c^2*x^2 + 1))/(m^3 + 9*m^2 + 23*m + 15) - integrate(((m^2 + 4*m + 3)*b*c^5*d^2*x^5 + 2*(m^2 + 6*m + 5)*b*c^3*d^2*x^3 + (m^2 + 8*m + 15)*b*c*d^2*x)*x^m/((m^3 + 9*m^2 + 23*m + 15)*c^3*x^3 + (m^3 + 9*m^2 + 23*m + 15)*c*x + ((m^3 + 9*m^2 + 23*m + 15)*c^2*x^2 + m^3 + 9*m^2 + 23*m + 15)*sqrt(c^2*x^2 + 1)), x) - integrate(((m^2 + 4*m + 3)*b*c^6*d^2*x^6 + 2*(m^2 + 6*m + 5)*b*c^4*d^2*x^4 + (m^2 + 8*m + 15)*b*c^2*d^2*x^2)*x^m/((m^3 + 9*m^2 + 23*m + 15)*c^2*x^2 + m^3 + 9*m^2 + 23*m + 15), x)

Giac [F(-2)]

Exception generated.

$$\int x^m \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^m*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^m \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))\,dx = \int x^m\,(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^2\,dx$$

```
[In] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2,x)
```

```
[Out] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2, x)
```

3.187 $\int x^m(d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 128

$$\int x^m(d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx = -\frac{bcdx^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2}+\frac{dx^{1+m}(a+b\text{arcsinh}(cx))}{1+m}+\frac{c^2dx^{3+m}(a+b\text{arcsinh}(cx))}{3+m} - \frac{bcd(7+3m)x^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{(1+m)(2+m)(3+m)^2}$$

```
[Out] d*x^(1+m)*(a+b*arcsinh(c*x))/(1+m)+c^2*d*x^(3+m)*(a+b*arcsinh(c*x))/(3+m)-b
*c*d*(7+3*m)*x^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],-c^2*x^2)/(3+m)^2/(
m^2+3*m+2)-b*c*d*x^(2+m)*(c^2*x^2+1)^(1/2)/(3+m)^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {14, 5803, 12, 470, 371}

$$\int x^m(d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx = \frac{c^2dx^{m+3}(a+b\text{arcsinh}(cx))}{m+3}+\frac{dx^{m+1}(a+b\text{arcsinh}(cx))}{m+1} - \frac{bcd(3m+7)x^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{(m+1)(m+2)(m+3)^2}-\frac{bcd\sqrt{c^2x^2+1}x^{m+2}}{(m+3)^2}$$

```
[In] Int[x^m*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -((b*c*d*x^(2 + m)*Sqrt[1 + c^2*x^2])/(3 + m)^2) + (d*x^(1 + m)*(a + b*ArcS
inh[c*x]))/(1 + m) + (c^2*d*x^(3 + m)*(a + b*ArcSinh[c*x]))/(3 + m) - (b*c*
d*(7 + 3*m)*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(c^2*x^
2)])/((1 + m)*(2 + m)*(3 + m)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{dx^{1+m}(a + b\text{arcsinh}(cx))}{1+m} + \frac{c^2dx^{3+m}(a + b\text{arcsinh}(cx))}{3+m} - (bc)\int \frac{dx^{1+m}\left(\frac{1}{1+m} + \frac{c^2x^2}{3+m}\right)}{\sqrt{1+c^2x^2}}\,dx$$

$$= \frac{dx^{1+m}(a + b\text{arcsinh}(cx))}{1+m} + \frac{c^2dx^{3+m}(a + b\text{arcsinh}(cx))}{3+m} - (bcd)\int \frac{x^{1+m}\left(\frac{1}{1+m} + \frac{c^2x^2}{3+m}\right)}{\sqrt{1+c^2x^2}}dx$$

$$= -\frac{bcdx^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2} + \frac{dx^{1+m}(a + b\text{arcsinh}(cx))}{1+m} + \frac{c^2dx^{3+m}(a + b\text{arcsinh}(cx))}{3+m} - \frac{(bcd(7+3m))\int \frac{x^{1+m}}{\sqrt{1+c^2x^2}}dx}{(1+m)(3+m)^2}$$

$$= -\frac{bcdx^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2} + \frac{dx^{1+m}(a + b\text{arcsinh}(cx))}{1+m} + \frac{c^2dx^{3+m}(a + b\text{arcsinh}(cx))}{3+m} - \frac{bcd(7+3m)x^{2+m}\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -c^2x^2\right)}{(1+m)(2+m)(3+m)^2}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.92

$$\int x^m\left(d + c^2dx^2\right)(a + b\text{arcsinh}(cx))\,dx = \frac{dx^{1+m}\left((2+m)\left(3+m+c^2x^2+c^2mx^2\right)(a + b\text{arcsinh}(cx)) - bc(1+m)x\,\text{Hypergeometric2F1}\left(-\frac{1}{2}, 1+\frac{m}{2},\right.\right.}{(1+m)(2+m)(3+m)}$$

```
[In] Integrate[x^m*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*x^(1 + m)*((2 + m)*(3 + m + c^2*x^2 + c^2*m*x^2)*(a + b*ArcSinh[c*x]) -
b*c*(1 + m)*x*Hypergeometric2F1[-1/2, 1 + m/2, 2 + m/2, -(c^2*x^2)] - 2*b*c
*x*Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, -(c^2*x^2)]))/((1 + m)*(2 + m)*
(3 + m))
```

Maple [F]

$$\int x^m\left(c^2d\,x^2 + d\right)(a + b\,\text{arcsinh}\,(cx))\,dx$$

```
[In] int(x^m*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x)
```

Fricas [F]

$$\int x^m (d + c^2dx^2) (a + b\text{arcsinh}(cx)) \, dx = \int (c^2dx^2 + d)(b \operatorname{arsinh}(cx) + a)x^m \, dx$$

```
[In] integrate(x^m*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*x^m, x)
```

Sympy [F]

$$\int x^m (d + c^2dx^2) (a + b\text{arcsinh}(cx)) \, dx = d\left(\int ax^m \, dx + \int bx^m \operatorname{asinh}(cx) \, dx + \int ac^2x^2x^m \, dx + \int bc^2x^2x^m \operatorname{asinh}(cx) \, dx\right)$$

```
[In] integrate(x**m*(c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
[Out] d*(Integral(a*x**m, x) + Integral(b*x**m*asinh(c*x), x) + Integral(a*c**2*x
**2*x**m, x) + Integral(b*c**2*x**2*x**m*asinh(c*x), x))
```

Maxima [F]

$$\int x^m (d + c^2dx^2) (a + b\text{arcsinh}(cx)) \, dx = \int (c^2dx^2 + d)(b \operatorname{arsinh}(cx) + a)x^m \, dx$$

```
[In] integrate(x^m*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] a*c^2*d*x^(m + 3)/(m + 3) + a*d*x^(m + 1)/(m + 1) + (b*c^2*d*(m + 1)*x^3 +
b*d*(m + 3)*x)*x^m*log(c*x + sqrt(c^2*x^2 + 1))/(m^2 + 4*m + 3) - integrate
((b*c^3*d*(m + 1)*x^3 + b*c*d*(m + 3)*x)*x^m/((m^2 + 4*m + 3)*c^3*x^3 + (m^
2 + 4*m + 3)*c*x + ((m^2 + 4*m + 3)*c^2*x^2 + m^2 + 4*m + 3)*sqrt(c^2*x^2 +
 1)), x) - integrate((b*c^4*d*(m + 1)*x^4 + b*c^2*d*(m + 3)*x^2)*x^m/((m^2
+ 4*m + 3)*c^2*x^2 + m^2 + 4*m + 3), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^m \left(d + c^2dx^2\right) (a + b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^m*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^m \left(d + c^2dx^2\right) (a + b\text{arcsinh}(cx))\,dx = \int x^m\,(a + b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2 + d\right)\,dx$$

```
[In] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2),x)
```

```
[Out] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2), x)
```

3.188 $\int \frac{x^m(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx=\text{Int}\left(\frac{x^m(a+b\text{arcsinh}(cx))}{d+c^2dx^2},x\right)$$

```
[Out] Unintegrable(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx=\int \frac{x^m(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx$$

```
[In] Int[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] Defer[Int][(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{x^m(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx=\int \frac{x^m(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx$$

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
[Out] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\,\text{arcsinh}\,(cx))}{c^2d\,x^2+d}dx$$

```
[In] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x)
[Out] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^m}{c^2dx^2+d}\,dx$$

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
[Out] integral((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 2.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx=\frac{\int \frac{ax^m}{c^2x^2+1}\,dx+\int \frac{bx^m\,\text{asinh}\,(cx)}{c^2x^2+1}\,dx}{d}$$

```
[In] integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d),x)
[Out] (Integral(a*x**m/(c**2*x**2 + 1), x) + Integral(b*x**m*asinh(c*x)/(c**2*x**
2 + 1), x))/d
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^m}{c^2dx^2+d}\,dx$$

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^m}{c^2dx^2+d}\,dx$$

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 2.58 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx = \int \frac{x^m\,(a+b\,\text{asinh}(c\,x))}{d\,c^2\,x^2+d}\,dx$$

```
[In] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2),x)
[Out] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2), x)
```

3.189 $\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2} dx = \frac{x^{1+m}(a+b\text{arcsinh}(cx))}{2d^2(1+c^2x^2)} - \frac{bcx^{2+m}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{2d^2(2+m)} + \frac{(1-m)\text{Int}\left(\frac{x^m(a+b\text{arcsinh}(cx))}{d+c^2dx^2},x\right)}{2d}$$

```
[Out] 1/2*x^(1+m)*(a+b*arcsinh(c*x))/d^2/(c^2*x^2+1)-1/2*b*c*x^(2+m)*hypergeom([3
/2, 1+1/2*m],[2+1/2*m],-c^2*x^2)/d^2/(2+m)+1/2*(1-m)*Unintegrable(x^m*(a+b*
arcsinh(c*x))/(c^2*d*x^2+d),x)/d
```

Rubi [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2} dx = \int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2} dx$$

```
[In] Int[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] (x^(1 + m)*(a + b*ArcSinh[c*x]))/(2*d^2*(1 + c^2*x^2)) - (b*c*x^(2 + m)*Hyp
ergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(2*d^2*(2 + m)) + ((
1 - m)*Defer[Int][(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2), x])/(2*d)
```

Rubi steps

$$\text{integral} = \frac{x^{1+m}(a+b\text{arcsinh}(cx))}{2d^2(1+c^2x^2)} - \frac{(bc)\int \frac{x^{1+m}}{(1+c^2x^2)^{3/2}}\,dx}{2d^2} + \frac{(1-m)\int \frac{x^m(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx}{2d}$$

$$= \frac{x^{1+m}(a+b\text{arcsinh}(cx))}{2d^2(1+c^2x^2)} - \frac{bcx^{2+m}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{2d^2(2+m)} + \frac{(1-m)\int \frac{x^m(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx}{2d}$$

Mathematica [N/A]

Not integrable

Time = 4.91 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx$$

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
[Out] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\,\text{arcsinh}\,(cx))}{(c^2d\,x^2+d)^2}dx$$

```
[In] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x)
[Out] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^m}{(c^2dx^2+d)^2}\,dx$$

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arcsinh(c*x) + a)*x^m/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [N/A]

Not integrable

Time = 16.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \frac{x^m(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx=\frac{\int \frac{ax^m}{c^4x^4+2c^2x^2+1}\,dx+\int \frac{bx^m\operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

```
[In] integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a*x**m/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*x**m*asinh(
c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)x^m}{(c^2dx^2+d)^2}\,dx$$

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)x^m}{(c^2dx^2+d)^2}\,dx$$

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.74 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\operatorname{arcsinh}(cx))}{\left(d+c^2dx^2\right)^2}\,dx=\int \frac{x^m\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(d\,c^2\,x^2+d\right)}^2} \,d x$$

```
[In] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2,x)
[Out] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2, x)
```

3.190 $\int \frac{x^m(a+b\text{arcsinh}(cx))}{\left(d+c^2dx^2\right)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{\left(d+c^2dx^2\right)^3}\,dx = \frac{x^{1+m}(a+b\text{arcsinh}(cx))}{4d^3\left(1+c^2x^2\right)^2}+\frac{(3-m)x^{1+m}(a+b\text{arcsinh}(cx))}{8d^3\left(1+c^2x^2\right)} - \frac{bc(3-m)x^{2+m}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{8d^3(2+m)} - \frac{bcx^{2+m}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{4d^3(2+m)} + \frac{(1-m)(3-m)\text{Int}\left(\frac{x^m(a+b\text{arcsinh}(cx))}{d+c^2dx^2},x\right)}{8d^2}$$

```
[Out] 1/4*x^(1+m)*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^2+1/8*(3-m)*x^(1+m)*(a+b*arc
sinh(c*x))/d^3/(c^2*x^2+1)-1/8*b*c*(3-m)*x^(2+m)*hypergeom([3/2, 1+1/2*m],[
2+1/2*m],-c^2*x^2)/d^3/(2+m)-1/4*b*c*x^(2+m)*hypergeom([5/2, 1+1/2*m],[2+1/
2*m],-c^2*x^2)/d^3/(2+m)+1/8*(1-m)*(3-m)*Unintegrable(x^m*(a+b*arcsinh(c*x)
)/(c^2*d*x^2+d),x)/d^2
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{\left(d+c^2dx^2\right)^3}\,dx = \int \frac{x^m(a+b\text{arcsinh}(cx))}{\left(d+c^2dx^2\right)^3}\,dx$$

```
[In] Int[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] (x^(1 + m)*(a + b*ArcSinh[c*x]))/(4*d^3*(1 + c^2*x^2)^2) + ((3 - m)*x^(1 +
m)*(a + b*ArcSinh[c*x]))/(8*d^3*(1 + c^2*x^2)) - (b*c*(3 - m)*x^(2 + m)*Hyp
ergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(8*d^3*(2 + m)) - (b
*c*x^(2 + m)*Hypergeometric2F1[5/2, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(4*d
^3*(2 + m)) + ((1 - m)*(3 - m)*Defer[Int][(x^m*(a + b*ArcSinh[c*x]))/(d + c
^2*d*x^2), x])/(8*d^2)
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^{1+m}(a+b\text{arcsinh}(cx))}{4d^3(1+c^2x^2)^2} - \frac{(bc)\int \frac{x^{1+m}}{(1+c^2x^2)^{5/2}}\,dx}{4d^3} + \frac{(3-m)\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx}{4d} \\
&= \frac{x^{1+m}(a+b\text{arcsinh}(cx))}{4d^3(1+c^2x^2)^2} + \frac{(3-m)x^{1+m}(a+b\text{arcsinh}(cx))}{8d^3(1+c^2x^2)} \\
&\quad - \frac{bcx^{2+m}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{4d^3(2+m)} \\
&\quad - \frac{(bc(3-m))\int \frac{x^{1+m}}{(1+c^2x^2)^{3/2}}\,dx}{8d^3} + \frac{((1-m)(3-m))\int \frac{x^m(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx}{8d^2} \\
&= \frac{x^{1+m}(a+b\text{arcsinh}(cx))}{4d^3(1+c^2x^2)^2} + \frac{(3-m)x^{1+m}(a+b\text{arcsinh}(cx))}{8d^3(1+c^2x^2)} \\
&\quad - \frac{bc(3-m)x^{2+m}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{8d^3(2+m)} \\
&\quad - \frac{bcx^{2+m}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{4d^3(2+m)} \\
&\quad + \frac{((1-m)(3-m))\int \frac{x^m(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx}{8d^2}\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 5.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx$$

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
[Out] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\,\text{arcsinh}\,(cx))}{(c^2d\,x^2+d)^3}dx$$

```
[In] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x)
```

```
[Out] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^m}{(c^2dx^2+d)^3}\,dx$$

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)*x^m/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*
x^2 + d^3), x)
```

Sympy [N/A]

Not integrable

Time = 117.92 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.96

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \frac{\int \frac{ax^m}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{bx^m\,\text{asinh}\,(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

```
[In] integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a*x**m/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral
(b*x**m*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^m}{(c^2dx^2+d)^3}\,dx$$

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^m}{(c^2dx^2+d)^3}\,dx$$

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^3, x)
```

Mupad [N/A]

Not integrable

Time = 2.71 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \int \frac{x^m\,(a+b\,\text{asinh}(c\,x))}{(d\,c^2\,x^2+d)^3}\,dx$$

```
[In] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3,x)
[Out] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3, x)
```

3.191 $\int x^m(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 618

$$\int x^m\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=-\frac{15bcd^2x^{2+m}\sqrt{d+c^2dx^2}}{(2+m)^2(4+m)(6+m)\sqrt{1+c^2x^2}}$$
$$-\frac{5bcd^2x^{2+m}\sqrt{d+c^2dx^2}}{(6+m)\left(8+6m+m^2\right)\sqrt{1+c^2x^2}}-\frac{bcd^2x^{2+m}\sqrt{d+c^2dx^2}}{\left(12+8m+m^2\right)\sqrt{1+c^2x^2}}$$
$$-\frac{5bc^3d^2x^{4+m}\sqrt{d+c^2dx^2}}{(4+m)^2(6+m)\sqrt{1+c^2x^2}}-\frac{2bc^3d^2x^{4+m}\sqrt{d+c^2dx^2}}{(4+m)(6+m)\sqrt{1+c^2x^2}}$$
$$-\frac{bc^5d^2x^{6+m}\sqrt{d+c^2dx^2}}{(6+m)^2\sqrt{1+c^2x^2}}+\frac{15d^2x^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(6+m)\left(8+6m+m^2\right)}$$
$$+\frac{5dx^{1+m}\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))}{(4+m)(6+m)}+\frac{x^{1+m}\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{6+m}$$
$$+\frac{15d^2x^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)}{(6+m)\left(8+14m+7m^2+m^3\right)\sqrt{1+c^2x^2}}$$
$$-\frac{15bcd^2x^{2+m}\sqrt{d+c^2dx^2}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-c^2x^2\right)}{(1+m)(2+m)^2(4+m)(6+m)\sqrt{1+c^2x^2}}$$

```
[Out] 5*d*x^(1+m)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(4+m)/(6+m)+x^(1+m)*(c^2
*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/(6+m)+15*d^2*x^(1+m)*(a+b*arcsinh(c*x))*
(c^2*d*x^2+d)^(1/2)/(6+m)/(m^2+6*m+8)-15*b*c*d^2*x^(2+m)*(c^2*d*x^2+d)^(1/2
)/(2+m)^2/(4+m)/(6+m)/(c^2*x^2+1)^(1/2)-5*b*c*d^2*x^(2+m)*(c^2*d*x^2+d)^(1/
2)/(6+m)/(m^2+6*m+8)/(c^2*x^2+1)^(1/2)-b*c*d^2*x^(2+m)*(c^2*d*x^2+d)^(1/2)/
(m^2+8*m+12)/(c^2*x^2+1)^(1/2)-5*b*c^3*d^2*x^(4+m)*(c^2*d*x^2+d)^(1/2)/(4+m
)^2/(6+m)/(c^2*x^2+1)^(1/2)-2*b*c^3*d^2*x^(4+m)*(c^2*d*x^2+d)^(1/2)/(4+m)/(
6+m)/(c^2*x^2+1)^(1/2)-b*c^5*d^2*x^(6+m)*(c^2*d*x^2+d)^(1/2)/(6+m)^2/(c^2*x
^2+1)^(1/2)+15*d^2*x^(1+m)*(a+b*arcsinh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3
```

```
/2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(6+m)/(m^3+7*m^2+14*m+8)/(c^2*x^2+1
)^(1/2)-15*b*c*d^2*x^(2+m)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/
2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(2+m)^2/(6+m)/(m^2+5*m+4)/(c^2*x^2+1)^(1
/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 618, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5808, 5806, 5817, 30, 14, 276}

$$\int x^m \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx = -\frac{15bcd^2x^{m+2}\sqrt{c^2dx^2+d}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{(m+1)(m+2)^2(m+4)(m+6)\sqrt{c^2x^2+1}} + \frac{15d^2x^{m+1}\sqrt{c^2dx^2+d}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-c^2x^2\right)(a+b\text{arcsinh}(cx))}{(m+6)\left(m^3+7m^2+14m+8\right)\sqrt{c^2x^2+1}} + \frac{15d^2x^{m+1}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{(m+6)\left(m^2+6m+8\right)} + \frac{x^{m+1}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{m+6} + \frac{5dx^{m+1}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{(m+4)(m+6)} - \frac{5bcd^2x^{m+2}\sqrt{c^2dx^2+d}}{(m+6)\left(m^2+6m+8\right)\sqrt{c^2x^2+1}} - \frac{bcd^2x^{m+2}\sqrt{c^2dx^2+d}}{\left(m^2+8m+12\right)\sqrt{c^2x^2+1}} - \frac{15bcd^2x^{m+2}\sqrt{c^2dx^2+d}}{(m+2)^2(m+4)(m+6)\sqrt{c^2x^2+1}} - \frac{bc^5d^2x^{m+6}\sqrt{c^2dx^2+d}}{(m+6)^2\sqrt{c^2x^2+1}} - \frac{2bc^3d^2x^{m+4}\sqrt{c^2dx^2+d}}{(m+4)(m+6)\sqrt{c^2x^2+1}} - \frac{5bc^3d^2x^{m+4}\sqrt{c^2dx^2+d}}{(m+4)^2(m+6)\sqrt{c^2x^2+1}}$$

```
[In] Int[x^m*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-15*b*c*d^2*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((2 + m)^2*(4 + m)*(6 + m)*Sqrt
[1 + c^2*x^2]) - (5*b*c*d^2*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((6 + m)*(8 + 6*
m + m^2)*Sqrt[1 + c^2*x^2]) - (b*c*d^2*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((12
+ 8*m + m^2)*Sqrt[1 + c^2*x^2]) - (5*b*c^3*d^2*x^(4 + m)*Sqrt[d + c^2*d*x^2
])/((4 + m)^2*(6 + m)*Sqrt[1 + c^2*x^2]) - (2*b*c^3*d^2*x^(4 + m)*Sqrt[d +
c^2*d*x^2])/((4 + m)*(6 + m)*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^(6 + m)*Sqrt
[d + c^2*d*x^2])/((6 + m)^2*Sqrt[1 + c^2*x^2]) + (15*d^2*x^(1 + m)*Sqrt[d +
 c^2*d*x^2]*(a + b*ArcSinh[c*x]))/((6 + m)*(8 + 6*m + m^2)) + (5*d*x^(1 + m
)*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/((4 + m)*(6 + m)) + (x^(1 + m
)*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(6 + m) + (15*d^2*x^(1 + m)*S
qrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (
3 + m)/2, -(c^2*x^2)])/((6 + m)*(8 + 14*m + 7*m^2 + m^3)*Sqrt[1 + c^2*x^2])
 - (15*b*c*d^2*x^(2 + m)*Sqrt[d + c^2*d*x^2]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])/((1 + m)*(2 + m)^2*(4 + m)*(6
 + m)*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^{1+m}(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{6+m} \\
&\quad + \frac{(5d)\int x^m(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))\,dx}{6+m} \\
&\quad - \frac{\left(bcd^2\sqrt{d+c^2dx^2}\right)\int x^{1+m}(1+c^2x^2)^2\,dx}{(6+m)\sqrt{1+c^2x^2}} \\
&= \frac{5dx^{1+m}(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{(4+m)(6+m)} + \frac{x^{1+m}(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{6+m} \\
&\quad + \frac{(15d^2)\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx}{(4+m)(6+m)} \\
&\quad - \frac{\left(bcd^2\sqrt{d+c^2dx^2}\right)\int \left(x^{1+m}+2c^2x^{3+m}+c^4x^{5+m}\right)\,dx}{(6+m)\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(5bcd^2\sqrt{d+c^2dx^2}\right)\int x^{1+m}(1+c^2x^2)\,dx}{(4+m)(6+m)\sqrt{1+c^2x^2}} \\
&= -\frac{bcd^2x^{2+m}\sqrt{d+c^2dx^2}}{(12+8m+m^2)\sqrt{1+c^2x^2}} - \frac{2bc^3d^2x^{4+m}\sqrt{d+c^2dx^2}}{(4+m)(6+m)\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^{6+m}\sqrt{d+c^2dx^2}}{(6+m)^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{15d^2x^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(2+m)(4+m)(6+m)} + \frac{5dx^{1+m}(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{(4+m)(6+m)} \\
&\quad + \frac{x^{1+m}(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{6+m} - \frac{\left(5bcd^2\sqrt{d+c^2dx^2}\right)\int \left(x^{1+m}+c^2x^{3+m}\right)\,dx}{(4+m)(6+m)\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(15d^2\sqrt{d+c^2dx^2}\right)\int \frac{x^m(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{(2+m)(4+m)(6+m)\sqrt{1+c^2x^2}} - \frac{\left(15bcd^2\sqrt{d+c^2dx^2}\right)\int x^{1+m}\,dx}{(2+m)(4+m)(6+m)\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$= -\frac{15bcd^2x^{2+m}\sqrt{d+c^2dx^2}}{(2+m)^2(4+m)(6+m)\sqrt{1+c^2x^2}} - \frac{5bcd^2x^{2+m}\sqrt{d+c^2dx^2}}{(2+m)(4+m)(6+m)\sqrt{1+c^2x^2}} - \frac{bcd^2x^{2+m}\sqrt{d+c^2dx^2}}{(12+8m+m^2)\sqrt{1+c^2x^2}} - \frac{5bc^3d^2x^{4+m}\sqrt{d+c^2dx^2}}{(4+m)^2(6+m)\sqrt{1+c^2x^2}} - \frac{2bc^3d^2x^{4+m}\sqrt{d+c^2dx^2}}{(4+m)(6+m)\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^{6+m}\sqrt{d+c^2dx^2}}{(6+m)^2\sqrt{1+c^2x^2}} + \frac{15d^2x^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(2+m)(4+m)(6+m)} + \frac{5dx^{1+m}(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{(4+m)(6+m)} + \frac{x^{1+m}(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{6+m} + \frac{15d^2x^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)}{(1+m)(2+m)(4+m)(6+m)\sqrt{1+c^2x^2}} - \frac{15bcd^2x^{2+m}\sqrt{d+c^2dx^2}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-c^2x^2\right)}{(1+m)(2+m)^2(4+m)(6+m)\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.54

$$\int x^m\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \frac{d^2x^{1+m}\sqrt{d+c^2dx^2}\left(-\frac{bcx((4+m)(6+m)+2c^2(2+m)(6+m)x^2+c^4(2+m)(4+m)x^4)}{(2+m)(4+m)(6+m)\sqrt{1+c^2x^2}} + (1+c^2x^2)^2(a+b\ldots\right.}{}$$

```
[In] Integrate[x^m*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*x^(1 + m)*Sqrt[d + c^2*d*x^2]*(-((b*c*x*((4 + m)*(6 + m) + 2*c^2*(2 +
m)*(6 + m)*x^2 + c^4*(2 + m)*(4 + m)*x^4))/((2 + m)*(4 + m)*(6 + m)*Sqrt[1
+ c^2*x^2])) + (1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]) - (5*(b*c*(1 + m)*(2 +
m)*x*(4 + m + c^2*(2 + m)*x^2) - (1 + m)*(2 + m)^2*(4 + m)*(1 + c^2*x^2)^(3
/2)*(a + b*ArcSinh[c*x]) + 3*(4 + m)*(b*c*(1 + m)*x - (1 + m)*(2 + m)*Sqrt[
1 + c^2*x^2]*(a + b*ArcSinh[c*x]) - (2 + m)*(a + b*ArcSinh[c*x])*Hypergeome
tric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)] + b*c*x*HypergeometricPFQ[{1
, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])))/((1 + m)*(2 + m)^
2*(4 + m)^2*Sqrt[1 + c^2*x^2])))/(6 + m)
```

Maple [F]

$$\int x^m (c^2 d\,x^2 + d)^{\frac{5}{2}} (a + b \operatorname{arcsinh}(cx))\,dx$$

```
[In] int(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x)
[Out] int(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x)
```

Fricas [F]

$$\int x^m (d + c^2 dx^2)^{5/2} (a + b\mathrm{arcsinh}(cx))\,dx = \int (c^2 dx^2 + d)^{\frac{5}{2}} (b \operatorname{arsinh}(cx) + a) x^m\,dx$$

```
[In] integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)*x^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^m (d + c^2 dx^2)^{5/2} (a + b\mathrm{arcsinh}(cx))\,dx = \text{Timed out}$$

```
[In] integrate(x**m*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
[Out] Timed out
```

Maxima [F]

$$\int x^m (d + c^2 dx^2)^{5/2} (a + b\mathrm{arcsinh}(cx))\,dx = \int (c^2 dx^2 + d)^{\frac{5}{2}} (b \operatorname{arsinh}(cx) + a) x^m\,dx$$

```
[In] integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)*x^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int x^m \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^m \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))\, dx = \int x^m\, (a+b\,\text{asinh}(c\,x))\, \left(d\,c^2\,x^2+d\right)^{5/2}\, dx$$

```
[In] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

3.192 $\int x^m(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 390

$$\begin{aligned}\int x^m\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx &= -\frac{3bcdx^{2+m}\sqrt{d+c^2dx^2}}{(2+m)^2(4+m)\sqrt{1+c^2x^2}} \\ &- \frac{bcdx^{2+m}\sqrt{d+c^2dx^2}}{(8+6m+m^2)\sqrt{1+c^2x^2}} - \frac{bc^3dx^{4+m}\sqrt{d+c^2dx^2}}{(4+m)^2\sqrt{1+c^2x^2}} \\ &+ \frac{3dx^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8+6m+m^2} + \frac{x^{1+m}\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))}{4+m} \\ &+ \frac{3dx^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)}{(8+14m+7m^2+m^3)\sqrt{1+c^2x^2}} \\ &- \frac{3bcdx^{2+m}\sqrt{d+c^2dx^2}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-c^2x^2\right)}{(1+m)(2+m)^2(4+m)\sqrt{1+c^2x^2}}\end{aligned}$$

```
[Out] x^(1+m)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(4+m)+3*d*x^(1+m)*(a+b*arcsi
nh(c*x))*(c^2*d*x^2+d)^(1/2)/(m^2+6*m+8)-3*b*c*d*x^(2+m)*(c^2*d*x^2+d)^(1/2
)/(2+m)^2/(4+m)/(c^2*x^2+1)^(1/2)-b*c*d*x^(2+m)*(c^2*d*x^2+d)^(1/2)/(m^2+6*
m+8)/(c^2*x^2+1)^(1/2)-b*c^3*d*x^(4+m)*(c^2*d*x^2+d)^(1/2)/(4+m)^2/(c^2*x^2
+1)^(1/2)+3*d*x^(1+m)*(a+b*arcsinh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/
2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(m^3+7*m^2+14*m+8)/(c^2*x^2+1)^(1/2)-3*b
*c*d*x^(2+m)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],-c^2*x^2)
*(c^2*d*x^2+d)^(1/2)/(2+m)^2/(m^2+5*m+4)/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5808, 5806, 5817, 30, 14}

$$\int x^m (d+c^2dx^2)^{3/2} (a+b\operatorname{arcsinh}(cx))\,dx =$$
$$-\frac{3bcdx^{m+2}\sqrt{c^2dx^2+d}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{(m+1)(m+2)^2(m+4)\sqrt{c^2x^2+1}}$$
$$+\frac{3dx^{m+1}\sqrt{c^2dx^2+d}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-c^2x^2\right)(a+b\operatorname{arcsinh}(cx))}{(m^3+7m^2+14m+8)\sqrt{c^2x^2+1}}$$
$$+\frac{3dx^{m+1}\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{m^2+6m+8}+\frac{x^{m+1}(c^2dx^2+d)^{3/2}(a+b\operatorname{arcsinh}(cx))}{m+4}$$
$$-\frac{bcdx^{m+2}\sqrt{c^2dx^2+d}}{(m^2+6m+8)\sqrt{c^2x^2+1}}-\frac{3bcdx^{m+2}\sqrt{c^2dx^2+d}}{(m+2)^2(m+4)\sqrt{c^2x^2+1}}-\frac{bc^3dx^{m+4}\sqrt{c^2dx^2+d}}{(m+4)^2\sqrt{c^2x^2+1}}$$

```
[In] Int[x^m*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-3*b*c*d*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((2 + m)^2*(4 + m)*Sqrt[1 + c^2*x^
2]) - (b*c*d*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((8 + 6*m + m^2)*Sqrt[1 + c^2*x
^2]) - (b*c^3*d*x^(4 + m)*Sqrt[d + c^2*d*x^2])/((4 + m)^2*Sqrt[1 + c^2*x^2]
) + (3*d*x^(1 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8 + 6*m + m^2
) + (x^(1 + m)*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(4 + m) + (3*d*x
^(1 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1
 + m)/2, (3 + m)/2, -(c^2*x^2)])/((8 + 14*m + 7*m^2 + m^3)*Sqrt[1 + c^2*x^2
]) - (3*b*c*d*x^(2 + m)*Sqrt[d + c^2*d*x^2]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])/((1 + m)*(2 + m)^2*(4 + m)*Sqr
t[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
```

```
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^{1+m}(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{4+m} \\
&\quad + \frac{(3d)\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx}{4+m} \\
&\quad - \frac{\left(bcd\sqrt{d+c^2dx^2}\right)\int x^{1+m}(1+c^2x^2)\,dx}{(4+m)\sqrt{1+c^2x^2}} \\
&= \frac{3dx^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8+6m+m^2} + \frac{x^{1+m}(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{4+m} \\
&\quad - \frac{\left(bcd\sqrt{d+c^2dx^2}\right)\int \left(x^{1+m}+c^2x^{3+m}\right)dx}{(4+m)\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(3d\sqrt{d+c^2dx^2}\right)\int \frac{x^m(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{(2+m)(4+m)\sqrt{1+c^2x^2}} - \frac{\left(3bcd\sqrt{d+c^2dx^2}\right)\int x^{1+m}\,dx}{(2+m)(4+m)\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$= -\frac{3bcdx^{2+m}\sqrt{d+c^2dx^2}}{(2+m)^2(4+m)\sqrt{1+c^2x^2}} - \frac{bcdx^{2+m}\sqrt{d+c^2dx^2}}{(8+6m+m^2)\sqrt{1+c^2x^2}} - \frac{bc^3dx^{4+m}\sqrt{d+c^2dx^2}}{(4+m)^2\sqrt{1+c^2x^2}} + \frac{3dx^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8+6m+m^2} + \frac{x^{1+m}(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{4+m} + \frac{3dx^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)}{(8+14m+7m^2+m^3)\sqrt{1+c^2x^2}} - \frac{3bcdx^{2+m}\sqrt{d+c^2dx^2}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-c^2x^2\right)}{(1+m)(2+m)^2(4+m)\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.60

$$\int x^m\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \frac{dx^{1+m}\sqrt{d+c^2dx^2}\left(-\frac{bcx(4+m+c^2(2+m)x^2)}{(2+m)(4+m)\sqrt{1+c^2x^2}} + (1+c^2x^2)(a+b\text{arcsinh}(cx)) - \frac{3\left(bc(1+m)x-(1+\right.}{}\right.}{}$$

```
[In] Integrate[x^m*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*x^(1 + m)*Sqrt[d + c^2*d*x^2]*(-((b*c*x*(4 + m + c^2*(2 + m)*x^2))/((2 +
 m)*(4 + m)*Sqrt[1 + c^2*x^2])) + (1 + c^2*x^2)*(a + b*ArcSinh[c*x]) - (3*(
b*c*(1 + m)*x - (1 + m)*(2 + m)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]) - (2
 + m)*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c
^2*x^2)] + b*c*x*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m
/2}, -(c^2*x^2)]))/((1 + m)*(2 + m)^2*Sqrt[1 + c^2*x^2])))/(4 + m)
```

Maple [F]

$$\int x^m\left(c^2d\,x^2+d\right)^{\frac{3}{2}}(a+b\,\text{arcsinh}\,(cx))\,dx$$

```
[In] int(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x)
```

Fricas [F]

$$\int x^m (d + c^2 dx^2)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \int (c^2 dx^2 + d)^{\frac{3}{2}} (b \operatorname{arsinh}(cx) + a) x^m\, dx$$

```
[In] integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d*
x^2 + d)*x^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^m (d + c^2 dx^2)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \text{Timed out}$$

```
[In] integrate(x**m*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
[Out] Timed out
```

Maxima [F]

$$\int x^m (d + c^2 dx^2)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \int (c^2 dx^2 + d)^{\frac{3}{2}} (b \operatorname{arsinh}(cx) + a) x^m\, dx$$

```
[In] integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)*x^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int x^m (d + c^2 dx^2)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^m (d + c^2dx^2)^{3/2} (a + b\text{arcsinh}(cx))\,dx = \int x^m\,(a + b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2 + d\right)^{3/2}\,dx$$

```
[In] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
[Out] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

3.193 $\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 240

$$\begin{aligned}\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx &= -\frac{bcx^{2+m}\sqrt{d+c^2dx^2}}{(2+m)^2\sqrt{1+c^2x^2}}+\frac{x^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2+m}\\&\quad+\frac{x^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)}{(2+3m+m^2)\sqrt{1+c^2x^2}}\\&\quad-\frac{bcx^{2+m}\sqrt{d+c^2dx^2}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-c^2x^2\right)}{(1+m)(2+m)^2\sqrt{1+c^2x^2}}\end{aligned}$$

```
[Out] x^(1+m)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(2+m)-b*c*x^(2+m)*(c^2*d*x^2
+d)^(1/2)/(2+m)^2/(c^2*x^2+1)^(1/2)+x^(1+m)*(a+b*arcsinh(c*x))*hypergeom([1
/2, 1/2+1/2*m],[3/2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(m^2+3*m+2)/(c^2*x
^2+1)^(1/2)-b*c*x^(2+m)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m
],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(1+m)/(2+m)^2/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used

$= \{5806, 5817, 30\}$

$$\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$$
$$= -\frac{bcx^{m+2}\sqrt{c^2dx^2+d}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{(m+1)(m+2)^2\sqrt{c^2x^2+1}}$$
$$+\frac{x^{m+1}\sqrt{c^2dx^2+d}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-c^2x^2\right)(a+b\text{arcsinh}(cx))}{(m^2+3m+2)\sqrt{c^2x^2+1}}$$
$$+\frac{x^{m+1}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{m+2}-\frac{bcx^{m+2}\sqrt{c^2dx^2+d}}{(m+2)^2\sqrt{c^2x^2+1}}$$

```
[In] Int[x^m*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -((b*c*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((2 + m)^2*Sqrt[1 + c^2*x^2])) + (x^(
1 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(2 + m) + (x^(1 + m)*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
m)/2, -(c^2*x^2)])/((2 + 3*m + m^2)*Sqrt[1 + c^2*x^2]) - (b*c*x^(2 + m)*Sqr
t[d + c^2*d*x^2]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m
/2}, -(c^2*x^2)])/((1 + m)*(2 + m)^2*Sqrt[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{x^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2+m} + \frac{\sqrt{d+c^2dx^2}\int\frac{x^m(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{(2+m)\sqrt{1+c^2x^2}} - \frac{\left(bc\sqrt{d+c^2dx^2}\right)\int x^{1+m}\,dx}{(2+m)\sqrt{1+c^2x^2}}$$

$$= -\frac{bcx^{2+m}\sqrt{d+c^2dx^2}}{(2+m)^2\sqrt{1+c^2x^2}} + \frac{x^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2+m} + \frac{x^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)}{(2+3m+m^2)\sqrt{1+c^2x^2}} - \frac{bcx^{2+m}\sqrt{d+c^2dx^2}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-c^2x^2\right)}{(1+m)(2+m)^2\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.75

$$\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \frac{x^{1+m}\sqrt{d+c^2dx^2}\left((1+m)\left(-bcx+a(2+m)\sqrt{1+c^2x^2}+b(2+m)\sqrt{1+c^2x^2}\text{arcsinh}(cx)\right)+(2+m)(a+\cdots\right.}{(1+m)(2+\cdots}$$

```
[In] Integrate[x^m*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (x^(1 + m)*Sqrt[d + c^2*d*x^2]*((1 + m)*(-(b*c*x) + a*(2 + m)*Sqrt[1 + c^2*
x^2] + b*(2 + m)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]) + (2 + m)*(a + b*ArcSinh[c
*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)] - b*c*x*Hyper
geometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)]))/((1
 + m)*(2 + m)^2*Sqrt[1 + c^2*x^2])
```

Maple [F]

$$\int x^m\sqrt{c^2d\,x^2+d}\,(a+b\,\text{arcsinh}\,(cx))\,dx$$

```
[In] int(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x)
```

Fricas [F]

$$\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\int \sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)x^m\,dx$$

[In] integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")

[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)*x^m, x)

Sympy [F]

$$\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\int x^m\sqrt{d(c^2x^2+1)}(a+b\operatorname{asinh}(cx))\,dx$$

[In] integrate(x**m*(c**2*d*x**2+d)**(1/2)*(a+b*asinh(c*x)),x)

[Out] Integral(x**m*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x)), x)

Maxima [F]

$$\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\int \sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)x^m\,dx$$

[In] integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")

[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)*x^m, x)

Giac [F(-2)]

Exception generated.

$$\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: TypeError}$$

[In] integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int x^m \sqrt{d + c^2 d x^2} (a + b \operatorname{arcsinh}(cx)) \, dx = \int x^m \left(a + b \operatorname{asinh}(c x)\right) \sqrt{d c^2 x^2 + d} \, dx$$

```
[In] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
[Out] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

3.194 $\int \frac{x^m(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 161

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{x^{1+m}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)}{(1+m)\sqrt{d+c^2dx^2}} - \frac{bcx^{2+m}\sqrt{1+c^2x^2}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-c^2x^2\right)}{(2+3m+m^2)\sqrt{d+c^2dx^2}}$$

```
[Out] x^(1+m)*(a+b*arcsinh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-c^2*x^2)
*(c^2*x^2+1)^(1/2)/(1+m)/(c^2*d*x^2+d)^(1/2)-b*c*x^(2+m)*hypergeom([1, 1+1/
2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/(m^2+3*m+2)/
(c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {5817}

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{c^2x^2+1}x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-c^2x^2\right)(a+b\text{arcsinh}(cx))}{(m+1)\sqrt{c^2dx^2+d}} - \frac{bc\sqrt{c^2x^2+1}x^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{(m^2+3m+2)\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x^m*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1
 + m)/2, (3 + m)/2, -(c^2*x^2)])/((1 + m)*Sqrt[d + c^2*d*x^2]) - (b*c*x^(2
+ m)*Sqrt[1 + c^2*x^2]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2,
 2 + m/2}, -(c^2*x^2)])/((2 + 3*m + m^2)*Sqrt[d + c^2*d*x^2])
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{x^{1+m}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)}{(1+m)\sqrt{d+c^2dx^2}} - \frac{bcx^{2+m}\sqrt{1+c^2x^2}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-c^2x^2\right)}{(2+3m+m^2)\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.80

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{x^{1+m}\sqrt{1+c^2x^2}\left((2+m)(a+b\text{arcsinh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)-bcx\,{}_3F_2\left(1,1+\frac{m}{2}\right.\right.}{(1+m)(2+m)\sqrt{d+c^2dx^2}}$$

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + c^2*x^2]*((2 + m)*(a + b*ArcSinh[c*x])*Hypergeometric2F
1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)] - b*c*x*HypergeometricPFQ[{1, 1 +
m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)]))/((1 + m)*(2 + m)*Sqrt[d
+ c^2*d*x^2])
```

Maple [F]

$$\int \frac{x^m(a + b \operatorname{arcsinh}(cx))}{\sqrt{c^2 d\, x^2 + d}} dx$$

```
[In] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
[Out] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^m(a + b\text{arcsinh}(cx))}{\sqrt{d + c^2 dx^2}}\, dx = \int \frac{(b \operatorname{arsinh}(cx) + a)x^m}{\sqrt{c^2 dx^2 + d}}\, dx$$

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*arcsinh(c*x) + a)*x^m/sqrt(c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x^m(a + b\text{arcsinh}(cx))}{\sqrt{d + c^2 dx^2}}\, dx = \int \frac{x^m(a + b \operatorname{asinh}(cx))}{\sqrt{d(c^2x^2 + 1)}}\, dx$$

```
[In] integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
[Out] Integral(x**m*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{x^m(a + b\text{arcsinh}(cx))}{\sqrt{d + c^2 dx^2}}\, dx = \int \frac{(b \operatorname{arsinh}(cx) + a)x^m}{\sqrt{c^2 dx^2 + d}}\, dx$$

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(c*x) + a)*x^m/sqrt(c^2*d*x^2 + d), x)
```

Giac [F]

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)x^m}{\sqrt{c^2dx^2+d}}\,dx$$

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)*x^m/sqrt(c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{x^m\,(a+b\,\mathrm{asinh}(c\,x))}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

```
[In] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
[Out] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

3.195 $\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 268

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \frac{x^{1+m}(a+b\text{arcsinh}(cx))}{d\sqrt{d+c^2dx^2}} - \frac{mx^{1+m}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)}{d(1+m)\sqrt{d+c^2dx^2}} - \frac{bcx^{2+m}\sqrt{1+c^2x^2}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{d(2+m)\sqrt{d+c^2dx^2}} + \frac{bcmx^{2+m}\sqrt{1+c^2x^2}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-c^2x^2\right)}{d\,(2+3m+m^2)\,\sqrt{d+c^2dx^2}}$$

```
[Out] x^(1+m)*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(1/2)-m*x^(1+m)*(a+b*arcsinh(c*x
))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/d/(1+
m)/(c^2*d*x^2+d)^(1/2)-b*c*x^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-c^2*x^
2)*(c^2*x^2+1)^(1/2)/d/(2+m)/(c^2*d*x^2+d)^(1/2)+b*c*m*x^(2+m)*hypergeom([1
, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/d/(m^2
+3*m+2)/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5811, 5817, 371}

$$\int \frac{x^m(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \frac{bcm\sqrt{c^2x^2+1}x^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{d\,(m^2+3m+2)\sqrt{c^2dx^2+d}}$$
$$-\frac{m\sqrt{c^2x^2+1}x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-c^2x^2\right)(a+b\operatorname{arcsinh}(cx))}{d(m+1)\sqrt{c^2dx^2+d}}$$
$$+\frac{x^{m+1}(a+b\operatorname{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}x^{m+2}\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{d(m+2)\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (x^(1 + m)*(a + b*ArcSinh[c*x]))/(d*Sqrt[d + c^2*d*x^2]) - (m*x^(1 + m)*Sqr
t[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
m)/2, -(c^2*x^2)])/(d*(1 + m)*Sqrt[d + c^2*d*x^2]) - (b*c*x^(2 + m)*Sqrt[1
+ c^2*x^2]*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(d*(2 +
m)*Sqrt[d + c^2*d*x^2]) + (b*c*m*x^(2 + m)*Sqrt[1 + c^2*x^2]*Hypergeometric
PFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])/(d*(2 + 3*m +
 m^2)*Sqrt[d + c^2*d*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
```

```
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{x^{1+m}(a+b\text{arcsinh}(cx))}{d\sqrt{d+c^2dx^2}} - \frac{m\int\frac{x^m(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx}{d} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\int\frac{x^{1+m}}{1+c^2x^2}\,dx}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{x^{1+m}(a+b\text{arcsinh}(cx))}{d\sqrt{d+c^2dx^2}} - \frac{mx^{1+m}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)}{d(1+m)\sqrt{d+c^2dx^2}} - \frac{bcx^{2+m}\sqrt{1+c^2x^2}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{d(2+m)\sqrt{d+c^2dx^2}} + \frac{bcmx^{2+m}\sqrt{1+c^2x^2}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-c^2x^2\right)}{d\left(2+3m+m^2\right)\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.77

$$\int\frac{x^m(a+b\text{arcsinh}(cx))}{\left(d+c^2dx^2\right)^{3/2}}\,dx = \frac{x^{1+m}\left(-m(2+m)\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},3\right.\right.}{}$$

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (x^(1 + m)*(-(m*(2 + m)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Hypergeometr
ic2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)]) + (1 + m)*((2 + m)*(a + b*Arc
Sinh[c*x]) - b*c*x*Sqrt[1 + c^2*x^2]*Hypergeometric2F1[1, 1 + m/2, 2 + m/2,
 -(c^2*x^2)]) + b*c*m*x*Sqrt[1 + c^2*x^2]*HypergeometricPFQ[{1, 1 + m/2, 1
+ m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)]))/(d*(1 + m)*(2 + m)*Sqrt[d + c^2
*d*x^2])
```

Maple [F]

$$\int \frac{x^m(a + b \operatorname{arcsinh}(cx))}{(c^2 d\, x^2 + d)^{\frac{3}{2}}}dx$$

```
[In] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x)
[Out] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^m(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^{3/2}}\, dx = \int \frac{(b \operatorname{arsinh}(cx) + a)x^m}{(c^2dx^2 + d)^{\frac{3}{2}}}\, dx$$

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)*x^m/(c^4*d^2*x^4 + 2*c^2*
d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^m(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^{3/2}}\, dx = \int \frac{x^m(a + b \operatorname{asinh}(cx))}{(d(c^2x^2 + 1))^{\frac{3}{2}}}\, dx$$

```
[In] integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
[Out] Integral(x**m*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^m(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^{3/2}}\, dx = \int \frac{(b \operatorname{arsinh}(cx) + a)x^m}{(c^2dx^2 + d)^{\frac{3}{2}}}\, dx$$

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^(3/2), x)
```

Giac [F]

$$\int \frac{x^m(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^{3/2}} \, dx = \int \frac{(b \operatorname{arsinh}(cx) + a)x^m}{(c^2dx^2 + d)^{\frac{3}{2}}} \, dx$$

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^{3/2}} \, dx = \int \frac{x^m \, (a + b \operatorname{asinh}(c \, x))}{(d \, c^2 \, x^2 + d)^{3/2}} \, dx$$

```
[In] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2),x)
[Out] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2), x)
```

3.196 $\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 402

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \frac{x^{1+m}(a+b\text{arcsinh}(cx))}{3d\,(d+c^2dx^2)^{3/2}}+\frac{(2-m)x^{1+m}(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+c^2dx^2}}-\frac{(2-m)mx^{1+m}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)}{3d^2(1+m)\sqrt{d+c^2dx^2}}-\frac{bc(2-m)x^{2+m}\sqrt{1+c^2x^2}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{3d^2(2+m)\sqrt{d+c^2dx^2}}-\frac{bcx^{2+m}\sqrt{1+c^2x^2}\,\text{Hypergeometric2F1}\left(2,\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{3d^2(2+m)\sqrt{d+c^2dx^2}}+\frac{bc(2-m)mx^{2+m}\sqrt{1+c^2x^2}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-c^2x^2\right)}{3d^2\,(2+3m+m^2)\sqrt{d+c^2dx^2}}$$

```
[Out] 1/3*x^(1+m)*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(3/2)+1/3*(2-m)*x^(1+m)*(a+b
*arcsinh(c*x))/d^2/(c^2*d*x^2+d)^(1/2)-1/3*(2-m)*m*x^(1+m)*(a+b*arcsinh(c*x
))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/d^2/(
1+m)/(c^2*d*x^2+d)^(1/2)-1/3*b*c*(2-m)*x^(2+m)*hypergeom([1, 1+1/2*m],[2+1/
2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/d^2/(2+m)/(c^2*d*x^2+d)^(1/2)-1/3*b*c*x^(2
+m)*hypergeom([2, 1+1/2*m],[2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/d^2/(2+m)/
(c^2*d*x^2+d)^(1/2)+1/3*b*c*(2-m)*m*x^(2+m)*hypergeom([1, 1+1/2*m, 1+1/2*m]
,[2+1/2*m, 3/2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/d^2/(m^2+3*m+2)/(c^2*d*x^
2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5811, 5817, 371}

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \frac{bc(2-m)m\sqrt{c^2x^2+1}x^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{3d^2\left(m^2+3m+2\right)\sqrt{c^2dx^2+d}}$$
$$-\frac{(2-m)m\sqrt{c^2x^2+1}x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-c^2x^2\right)(a+b\text{arcsinh}(cx))}{3d^2(m+1)\sqrt{c^2dx^2+d}}$$
$$+\frac{(2-m)x^{m+1}(a+b\text{arcsinh}(cx))}{3d^2\sqrt{c^2dx^2+d}}+\frac{x^{m+1}(a+b\text{arcsinh}(cx))}{3d\left(c^2dx^2+d\right)^{3/2}}$$
$$-\frac{bc(2-m)\sqrt{c^2x^2+1}x^{m+2}\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{3d^2(m+2)\sqrt{c^2dx^2+d}}$$
$$-\frac{bc\sqrt{c^2x^2+1}x^{m+2}\,\text{Hypergeometric2F1}\left(2,\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{3d^2(m+2)\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (x^(1 + m)*(a + b*ArcSinh[c*x]))/(3*d*(d + c^2*d*x^2)^(3/2)) + ((2 - m)*x^(
1 + m)*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt[d + c^2*d*x^2]) - ((2 - m)*m*x^(1
+ m)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, -(c^2*x^2)])/(3*d^2*(1 + m)*Sqrt[d + c^2*d*x^2]) - (b*c*(2 -
m)*x^(2 + m)*Sqrt[1 + c^2*x^2]*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -
(c^2*x^2)])/(3*d^2*(2 + m)*Sqrt[d + c^2*d*x^2]) - (b*c*x^(2 + m)*Sqrt[1 + c
^2*x^2]*Hypergeometric2F1[2, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(3*d^2*(2 +
 m)*Sqrt[d + c^2*d*x^2]) + (b*c*(2 - m)*m*x^(2 + m)*Sqrt[1 + c^2*x^2]*Hyper
geometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])/(3*d
^2*(2 + 3*m + m^2)*Sqrt[d + c^2*d*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
```

```
b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
m}, x] && EqQ[e, c^2*d] && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^{1+m}(a+b\text{arcsinh}(cx))}{3d\left(d+c^2dx^2\right)^{3/2}}+\frac{(2-m)\int\frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx}{3d} \\
&\quad-\frac{\left(bc\sqrt{1+c^2x^2}\right)\int\frac{x^{1+m}}{(1+c^2x^2)^2}\,dx}{3d^2\sqrt{d+c^2dx^2}} \\
&= \frac{x^{1+m}(a+b\text{arcsinh}(cx))}{3d\left(d+c^2dx^2\right)^{3/2}}+\frac{(2-m)x^{1+m}(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+c^2dx^2}} \\
&\quad-\frac{bcx^{2+m}\sqrt{1+c^2x^2}\,\text{Hypergeometric2F1}\left(2,\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{3d^2(2+m)\sqrt{d+c^2dx^2}} \\
&\quad-\frac{((2-m)m)\int\frac{x^m(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx}{3d^2}-\frac{\left(bc(2-m)\sqrt{1+c^2x^2}\right)\int\frac{x^{1+m}}{1+c^2x^2}\,dx}{3d^2\sqrt{d+c^2dx^2}} \\
&= \frac{x^{1+m}(a+b\text{arcsinh}(cx))}{3d\left(d+c^2dx^2\right)^{3/2}}+\frac{(2-m)x^{1+m}(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+c^2dx^2}} \\
&\quad-\frac{(2-m)mx^{1+m}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)}{3d^2(1+m)\sqrt{d+c^2dx^2}} \\
&\quad-\frac{bc(2-m)x^{2+m}\sqrt{1+c^2x^2}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{3d^2(2+m)\sqrt{d+c^2dx^2}} \\
&\quad-\frac{bcx^{2+m}\sqrt{1+c^2x^2}\,\text{Hypergeometric2F1}\left(2,\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{3d^2(2+m)\sqrt{d+c^2dx^2}} \\
&\quad+\frac{bc(2-m)mx^{2+m}\sqrt{1+c^2x^2}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-c^2x^2\right)}{3d^2\left(2+3m+m^2\right)\sqrt{d+c^2dx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.71

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \frac{x^{1+m}\Big((1+m)(2+m)(a+b\text{arcsinh}(cx)) - bc(1+m)x(1+c^2x^2)^{3/2}\,\text{Hypergeo}}{}$$

[In] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]

[Out] (x^(1 + m)*((1 + m)*(2 + m)*(a + b*ArcSinh[c*x]) - b*c*(1 + m)*x*(1 + c^2*x^2)^(3/2)*Hypergeometric2F1[2, 1 + m/2, 2 + m/2, -(c^2*x^2)] + (2 - m)*(1 + c^2*x^2)*((1 + m)*(2 + m)*(a + b*ArcSinh[c*x]) - b*c*(1 + m)*x*Sqrt[1 + c^2*x^2]*Hypergeometric2F1[1, 1 + m/2, 2 + m/2, -(c^2*x^2)] - m*Sqrt[1 + c^2*x^2]*((2 + m)*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)] - b*c*x*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)]))))/(3*d^2*(1 + m)*(2 + m)*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])

Maple [F]

$$\int \frac{x^m(a+b\,\text{arcsinh}\,(cx))}{(c^2d\,x^2+d)^{\frac{5}{2}}}dx$$

[In] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x)

[Out] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x)

Fricas [F]

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^m}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)*x^m/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)

Sympy [F]

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x^m(a+b\,\text{asinh}\,(cx))}{(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
[Out] Integral(x**m*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^m}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^(5/2), x)
```

Giac [F]

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^m}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x^m\,(a+b\,\text{asinh}(c\,x))}{(d\,c^2\,x^2+d)^{5/2}}\,dx$$

```
[In] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2),x)
[Out] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2), x)
```

3.197 $\int \frac{x^m \mathbf{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{x^m \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{x^{1+m}\text{arcsinh}(ax)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-a^2x^2\right)}{1+m} - \frac{ax^{2+m}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-a^2x^2\right)}{2+3m+m^2}$$

```
[Out] x^(1+m)*arcsinh(a*x)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-a^2*x^2)/(1+m)
-a*x^(2+m)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],-a^2*x^2)/(
m^2+3*m+2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {5817}

$$\int \frac{x^m \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{x^{m+1}\text{arcsinh}(ax)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1} - \frac{ax^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-a^2x^2\right)}{m^2+3m+2}$$

```
[In] Int[(x^m*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (x^(1 + m)*ArcSinh[a*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(a^2*
x^2)])/(1 + m) - (a*x^(2 + m)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2
 + m/2, 2 + m/2}, -(a^2*x^2)])/(2 + 3*m + m^2)
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^{1+m}\text{arcsinh}(ax)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-a^2x^2\right)}{1+m}\\&\quad-\frac{ax^{2+m}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-a^2x^2\right)}{2+3m+m^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.95

$$\int \frac{x^m\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{x^{1+m}\left((2+m)\text{arcsinh}(ax)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-a^2x^2\right)-ax\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2\right.\right.}{(1+m)(2+m)}$$

```
[In] Integrate[(x^m*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (x^(1 + m)*((2 + m)*ArcSinh[a*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/
2, -(a^2*x^2)] - a*x*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2
 + m/2}, -(a^2*x^2)]))/((1 + m)*(2 + m))
```

Maple [F]

$$\int \frac{x^m\,\text{arcsinh}\,(ax)}{\sqrt{a^2x^2+1}}dx$$

```
[In] int(x^m*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^m*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^m \operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x^m*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^m*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

Sympy [F]

$$\int \frac{x^m \operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{asinh}(ax)}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x**m*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
[Out] Integral(x**m*asinh(a*x)/sqrt(a**2*x**2 + 1), x)
```

Maxima [F]

$$\int \frac{x^m \operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x^m*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^m*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{x^m \operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x^m*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^m*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m \operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m\,\operatorname{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int((x^m*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^m*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

3.198 $\int x^4(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 283

$$\begin{aligned}
&\int x^4\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))^2\,dx \\
&=\frac{304b^2dx}{3675c^4}-\frac{152b^2dx^3}{11025c^2}+\frac{38b^2dx^5}{6125}+\frac{2}{343}b^2c^2dx^7 \\
&\quad-\frac{32bd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{525c^5}+\frac{16bdx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{525c^3} \\
&\quad-\frac{4bdx^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{175c}-\frac{2bd(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{21c^5} \\
&\quad+\frac{4bd(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))}{35c^5}-\frac{2bd(1+c^2x^2)^{7/2}\,(a+b\text{arcsinh}(cx))}{49c^5} \\
&\quad+\frac{2}{35}dx^5(a+b\text{arcsinh}(cx))^2+\frac{1}{7}dx^5\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2
\end{aligned}$$

```
[Out] 304/3675*b^2*d*x/c^4-152/11025*b^2*d*x^3/c^2+38/6125*b^2*d*x^5+2/343*b^2*c^
2*d*x^7-2/21*b*d*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c^5+4/35*b*d*(c^2*x^2
+1)^(5/2)*(a+b*arcsinh(c*x))/c^5-2/49*b*d*(c^2*x^2+1)^(7/2)*(a+b*arcsinh(c*
x))/c^5+2/35*d*x^5*(a+b*arcsinh(c*x))^2+1/7*d*x^5*(c^2*x^2+1)*(a+b*arcsinh(
c*x))^2-32/525*b*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^5+16/525*b*d*x^2*
(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-4/175*b*d*x^4*(a+b*arcsinh(c*x))*(
c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5808, 5776, 5812, 5798, 8, 30, 272, 45, 5804, 12}

$$\begin{aligned}&\int x^4\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))^2dx\\&=\frac{1}{7}dx^5\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2-\frac{4bdx^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{175c}\\&-\frac{2bd(c^2x^2+1)^{7/2}(a+b\text{arcsinh}(cx))}{49c^5}+\frac{4bd(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))}{35c^5}\\&-\frac{2bd(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{21c^5}-\frac{32bd\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{525c^5}\\&+\frac{16bdx^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{525c^3}\\&+\frac{2}{35}dx^5(a+b\text{arcsinh}(cx))^2+\frac{304b^2dx}{3675c^4}+\frac{2}{343}b^2c^2dx^7-\frac{152b^2dx^3}{11025c^2}+\frac{38b^2dx^5}{6125}\end{aligned}$$

```
[In] Int[x^4*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (304*b^2*d*x)/(3675*c^4) - (152*b^2*d*x^3)/(11025*c^2) + (38*b^2*d*x^5)/612
5 + (2*b^2*c^2*d*x^7)/343 - (32*b*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))
/(525*c^5) + (16*b*d*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(525*c^3)
- (4*b*d*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(175*c) - (2*b*d*(1 +
c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(21*c^5) + (4*b*d*(1 + c^2*x^2)^(5/2)*
(a + b*ArcSinh[c*x]))/(35*c^5) - (2*b*d*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[
c*x]))/(49*c^5) + (2*d*x^5*(a + b*ArcSinh[c*x])^2)/35 + (d*x^5*(1 + c^2*x^2
)*(a + b*ArcSinh[c*x])^2)/7
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{7}dx^5(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 + \frac{1}{7}(2d)\int x^4(a+b\text{arcsinh}(cx))^2\,dx - \frac{1}{7}(2bcd)\int x^5\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx$$

$$= -\frac{2bd(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{21c^5} + \frac{4bd(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{35c^5} - \frac{2bd(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{49c^5} + \frac{2}{35}dx^5(a+b\text{arcsinh}(cx))^2 + \frac{1}{7}dx^5(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 - \frac{1}{35}(4bcd)\int \frac{x^5(a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}$$

$$= -\frac{4bdx^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{175c} - \frac{2bd(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{21c^5} + \frac{4bd(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{35c^5} - \frac{2bd(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{49c^5} + \frac{2}{35}dx^5(a+b\text{arcsinh}(cx))^2 + \frac{1}{7}dx^5(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 + \frac{1}{175}(4b^2d)\int x^4\,dx + \frac{(2b^2d)\int(8}{}$$

$$= \frac{16b^2dx}{735c^4} - \frac{8b^2dx^3}{2205c^2} + \frac{38b^2dx^5}{6125} + \frac{2}{343}b^2c^2dx^7 + \frac{16bdx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{525c^3} - \frac{4bdx^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{175c} - \frac{2bd(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{21c^5} + \frac{4bd(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{35c^5} - \frac{2bd(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{49c^5} + \frac{2}{35}dx^5(a+b\text{arcsinh}(cx))^2 + \frac{1}{7}dx^5(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 - \frac{(32bd)\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{525c^3} - \frac{(}{}$$

$$
\begin{aligned}
&= \frac{16b^2dx}{735c^4} - \frac{152b^2dx^3}{11025c^2} + \frac{38b^2dx^5}{6125} + \frac{2}{343}b^2c^2dx^7 \\
&- \frac{32bd\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{525c^5} + \frac{16bdx^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{525c^3} \\
&- \frac{4bdx^4\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{175c} - \frac{2bd(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{21c^5} \\
&+ \frac{4bd(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{35c^5} - \frac{2bd(1+c^2x^2)^{7/2}(a+b\operatorname{arcsinh}(cx))}{49c^5} \\
&+ \frac{2}{35}dx^5(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{7}dx^5(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2 + \frac{(32b^2d)\int 1\,dx}{525c^4} \\
&= \frac{304b^2dx}{3675c^4} - \frac{152b^2dx^3}{11025c^2} + \frac{38b^2dx^5}{6125} + \frac{2}{343}b^2c^2dx^7 \\
&- \frac{32bd\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{525c^5} + \frac{16bdx^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{525c^3} \\
&- \frac{4bdx^4\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{175c} - \frac{2bd(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{21c^5} \\
&+ \frac{4bd(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{35c^5} - \frac{2bd(1+c^2x^2)^{7/2}(a+b\operatorname{arcsinh}(cx))}{49c^5} \\
&+ \frac{2}{35}dx^5(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{7}dx^5(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.71

$$
\int x^4(d+c^2dx^2)(a+b\operatorname{arcsinh}(cx))^2\,dx
$$
$$
= \frac{d\left(11025a^2c^5x^5(7+5c^2x^2) - 210ab\sqrt{1+c^2x^2}(152-76c^2x^2+57c^4x^4+75c^6x^6) + b^2(31920cx - 5320c^3x^3 \right.}{}
$$

```
[In] Integrate[x^4*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d*(11025*a^2*c^5*x^5*(7 + 5*c^2*x^2) - 210*a*b*Sqrt[1 + c^2*x^2]*(152 - 76
*c^2*x^2 + 57*c^4*x^4 + 75*c^6*x^6) + b^2*(31920*c*x - 5320*c^3*x^3 + 2394*
c^5*x^5 + 2250*c^7*x^7) - 210*b*(-105*a*c^5*x^5*(7 + 5*c^2*x^2) + b*Sqrt[1
+ c^2*x^2]*(152 - 76*c^2*x^2 + 57*c^4*x^4 + 75*c^6*x^6))*ArcSinh[c*x] + 110
25*b^2*c^5*x^5*(7 + 5*c^2*x^2)*ArcSinh[c*x]^2))/(385875*c^5)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.16

method	result
parts	$d\,a^2\left(\frac{1}{7}c^2x^7+\frac{1}{5}x^5\right)+\frac{d\,b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^3x^3\left(c^2x^2+1\right)^2}{7}-\frac{3\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{35}+\frac{2\operatorname{arcsinh}(cx)^2xc}{35}+\frac{\operatorname{arcsinh}(cx)^2}{}\cdots\right.}{}$
derivativedivides	$\frac{d\,a^2\left(\frac{1}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d\,b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^3x^3\left(c^2x^2+1\right)^2}{7}-\frac{3\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{35}+\frac{2\operatorname{arcsinh}(cx)^2xc}{35}+\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2\cdots\right.}{35}\right.}{}$
default	$\frac{d\,a^2\left(\frac{1}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d\,b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^3x^3\left(c^2x^2+1\right)^2}{7}-\frac{3\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{35}+\frac{2\operatorname{arcsinh}(cx)^2xc}{35}+\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2\cdots\right.}{35}\right.}{}$

```
[In] int(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] d*a^2*(1/7*c^2*x^7+1/5*x^5)+d*b^2/c^5*(1/7*arcsinh(c*x)^2*c^3*x^3*(c^2*x^2+
1)^2-3/35*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^2+2/35*arcsinh(c*x)^2*x*c+1/35*arc
sinh(c*x)^2*c*x*(c^2*x^2+1)-2/49*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^(5/2)+62/
1225*arcsinh(c*x)*(c^2*x^2+1)^(5/2)+2/343*c*x*(c^2*x^2+1)^3+37384/385875*c*
x-484/42875*c*x*(c^2*x^2+1)^2-3358/385875*c*x*(c^2*x^2+1)-4/35*arcsinh(c*x)
*(c^2*x^2+1)^(1/2)-2/105*arcsinh(c*x)*(c^2*x^2+1)^(3/2))+2*d*a*b/c^5*(1/7*a
rcsinh(c*x)*c^7*x^7+1/5*arcsinh(c*x)*c^5*x^5-1/49*c^6*x^6*(c^2*x^2+1)^(1/2)
-19/1225*c^4*x^4*(c^2*x^2+1)^(1/2)+76/3675*c^2*x^2*(c^2*x^2+1)^(1/2)-152/36
75*(c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.92

$$\int x^4\left(d+c^2dx^2\right)(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$=\frac{1125\,(49\,a^2+2\,b^2)c^7dx^7+63\,(1225\,a^2+38\,b^2)c^5dx^5-5320\,b^2c^3dx^3+31920\,b^2cdx+11025\,(5\,b^2c^7dx^7+7\cdots}{}$$

```
[In] integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/385875*(1125*(49*a^2 + 2*b^2)*c^7*d*x^7 + 63*(1225*a^2 + 38*b^2)*c^5*d*x^
5 - 5320*b^2*c^3*d*x^3 + 31920*b^2*c*d*x + 11025*(5*b^2*c^7*d*x^7 + 7*b^2*c
^5*d*x^5)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 210*(525*a*b*c^7*d*x^7 + 735*a*b
*c^5*d*x^5 - (75*b^2*c^6*d*x^6 + 57*b^2*c^4*d*x^4 - 76*b^2*c^2*d*x^2 + 152*
b^2*d)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 210*(75*a*b*c^6*d*
x^6 + 57*a*b*c^4*d*x^4 - 76*a*b*c^2*d*x^2 + 152*a*b*d)*sqrt(c^2*x^2 + 1))/c
^5
```

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.37

$$\int x^4(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\begin{cases}\frac{a^2c^2dx^7}{7}+\frac{a^2dx^5}{5}+\frac{2abc^2dx^7\operatorname{asinh}(cx)}{7}-\frac{2abcdx^6\sqrt{c^2x^2+1}}{49}+\frac{2abdx^5\operatorname{asinh}(cx)}{5}-\frac{38abdx^4\sqrt{c^2x^2+1}}{1225c}+\frac{152abdx^2\sqrt{c^2x^2+1}}{3675c^3}-\frac{304a\cdots}{}\\ \frac{a^2dx^5}{5}\end{cases}$$

```
[In] integrate(x**4*(c**2*d*x**2+d)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**2*d*x**7/7 + a**2*d*x**5/5 + 2*a*b*c**2*d*x**7*asinh(c*x
)/7 - 2*a*b*c*d*x**6*sqrt(c**2*x**2 + 1)/49 + 2*a*b*d*x**5*asinh(c*x)/5 - 3
8*a*b*d*x**4*sqrt(c**2*x**2 + 1)/(1225*c) + 152*a*b*d*x**2*sqrt(c**2*x**2 +
 1)/(3675*c**3) - 304*a*b*d*sqrt(c**2*x**2 + 1)/(3675*c**5) + b**2*c**2*d*x
**7*asinh(c*x)**2/7 + 2*b**2*c**2*d*x**7/343 - 2*b**2*c*d*x**6*sqrt(c**2*x*
*2 + 1)*asinh(c*x)/49 + b**2*d*x**5*asinh(c*x)**2/5 + 38*b**2*d*x**5/6125 -
 38*b**2*d*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/(1225*c) - 152*b**2*d*x**3/(
11025*c**2) + 152*b**2*d*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3675*c**3) +
304*b**2*d*x/(3675*c**4) - 304*b**2*d*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3675*
c**5), Ne(c, 0)), (a**2*d*x**5/5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.56

$$\int x^4(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{1}{7}\,b^2c^2dx^7\,\text{arsinh}\,(cx)^2+\frac{1}{7}\,a^2c^2dx^7+\frac{1}{5}\,b^2dx^5\,\text{arsinh}\,(cx)^2+\frac{1}{5}\,a^2dx^5$$
$$+\frac{2}{245}\left(35\,x^7\,\text{arsinh}\,(cx)-\left(\frac{5\sqrt{c^2x^2+1}x^6}{c^2}-\frac{6\sqrt{c^2x^2+1}x^4}{c^4}+\frac{8\sqrt{c^2x^2+1}x^2}{c^6}-\frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\right)abc^2d\cdots$$
$$-\frac{2}{25725}\left(105\left(\frac{5\sqrt{c^2x^2+1}x^6}{c^2}-\frac{6\sqrt{c^2x^2+1}x^4}{c^4}+\frac{8\sqrt{c^2x^2+1}x^2}{c^6}-\frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\,\text{arsinh}\,(cx)-\frac{75\,c^6x\cdots}{}\right.$$
$$+\frac{2}{75}\left(15\,x^5\,\text{arsinh}\,(cx)-\left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2}-\frac{4\sqrt{c^2x^2+1}x^2}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)abd$$
$$-\frac{2}{1125}\left(15\left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2}-\frac{4\sqrt{c^2x^2+1}x^2}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\,\text{arsinh}\,(cx)-\frac{9\,c^4x^5-20\,c^2x^3+120\,x}{c^4}\right)b\cdots$$

```
[In] integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

[Out] 1/7*b^2*c^2*d*x^7*arcsinh(c*x)^2 + 1/7*a^2*c^2*d*x^7 + 1/5*b^2*d*x^5*arcsinh(c*x)^2 + 1/5*a^2*d*x^5 + 2/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*a*b*c^2*d - 2/25725*(105*(5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsinh(c*x) - (75*c^6*x^7 - 126*c^4*x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*c^2*d + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*d - 2/1125*(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^2*d

Giac [F(-2)]

Exception generated.

$$\int x^4(d+c^2dx^2)\left(a+b\text{arcsinh}(cx)\right)^2dx = \text{Exception raised: RuntimeError}$$

[In] integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")

[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int x^4(d+c^2dx^2)\left(a+b\text{arcsinh}(cx)\right)^2dx = \int x^4\,\left(a+b\,\mathrm{asinh}(c\,x)\right)^2\,\left(d\,c^2\,x^2+d\right)\,dx$$

[In] int(x^4*(a + b*asinh(c*x))^2*(d + c^2*d*x^2),x)

[Out] int(x^4*(a + b*asinh(c*x))^2*(d + c^2*d*x^2), x)

3.199 $\int x^3(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 198

$$\begin{aligned}\int x^3(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx =& -\frac{b^2dx^2}{24c^2}+\frac{1}{72}b^2dx^4+\frac{1}{108}b^2c^2dx^6\\&+\frac{bdx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{12c^3}\\&-\frac{bdx^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{18c}\\&-\frac{1}{18}bcdx^5\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\\&-\frac{d(a+b\text{arcsinh}(cx))^2}{24c^4}\\&+\frac{1}{12}dx^4(a+b\text{arcsinh}(cx))^2\\&+\frac{1}{6}dx^4(1+c^2x^2)(a+b\text{arcsinh}(cx))^2\end{aligned}$$

```
[Out] -1/24*b^2*d*x^2/c^2+1/72*b^2*d*x^4+1/108*b^2*c^2*d*x^6-1/24*d*(a+b*arcsinh(
c*x))^2/c^4+1/12*d*x^4*(a+b*arcsinh(c*x))^2+1/6*d*x^4*(c^2*x^2+1)*(a+b*arcs
inh(c*x))^2+1/12*b*d*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-1/18*b*d*x^
3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c-1/18*b*c*d*x^5*(a+b*arcsinh(c*x))*
(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5808, 5776, 5812, 5783, 30, 5806}

$$\int x^3(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx = -\frac{d(a+b\text{arcsinh}(cx))^2}{24c^4} - \frac{1}{18}bcdx^5\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx)) + \frac{1}{6}dx^4(c^2x^2+1)(a+b\text{arcsinh}(cx))^2 - \frac{bdx^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{18c} + \frac{bdx\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{12c^3} + \frac{1}{12}dx^4(a+b\text{arcsinh}(cx))^2 + \frac{1}{108}b^2c^2dx^6 - \frac{b^2dx^2}{24c^2} + \frac{1}{72}b^2dx^4$$

```
[In] Int[x^3*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/24*(b^2*d*x^2)/c^2 + (b^2*d*x^4)/72 + (b^2*c^2*d*x^6)/108 + (b*d*x*Sqrt[
1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(12*c^3) - (b*d*x^3*Sqrt[1 + c^2*x^2]*(a
 + b*ArcSinh[c*x]))/(18*c) - (b*c*d*x^5*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*
x]))/18 - (d*(a + b*ArcSinh[c*x])^2)/(24*c^4) + (d*x^4*(a + b*ArcSinh[c*x])
^2)/12 + (d*x^4*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{6}dx^4\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2+\frac{1}{3}d\int x^3(a+b\text{arcsinh}(cx))^2\,dx \\
&\quad -\frac{1}{3}(bcd)\int x^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx \\
&= -\frac{1}{18}bcdx^5\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))+\frac{1}{12}dx^4(a+b\text{arcsinh}(cx))^2 \\
&\quad +\frac{1}{6}dx^4\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2-\frac{1}{18}(bcd)\int\frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx \\
&\quad -\frac{1}{6}(bcd)\int\frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx+\frac{1}{18}\left(b^2c^2d\right)\int x^5\,dx
\end{aligned}$$

$$= \frac{1}{108}b^2c^2dx^6 - \frac{bdx^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{18c} - \frac{1}{18}bcdx^5\sqrt{1+c^2x^2}(a + b\text{arcsinh}(cx)) + \frac{1}{12}dx^4(a+b\text{arcsinh}(cx))^2 + \frac{1}{6}dx^4(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 + \frac{1}{72}(b^2d)\int x^3\,dx + \frac{1}{24}(b^2d)\int x^3\,dx + \frac{(bd)\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{24c} + \frac{(bd)\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{8c}$$

$$= \frac{1}{72}b^2dx^4 + \frac{1}{108}b^2c^2dx^6 + \frac{bdx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{12c^3} - \frac{bdx^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{18c} - \frac{1}{18}bcdx^5\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) + \frac{1}{12}dx^4(a+b\text{arcsinh}(cx))^2 + \frac{1}{6}dx^4(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 - \frac{(bd)\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{48c^3} - \frac{(bd)\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{16c^3} - \frac{(b^2d)\int x\,dx}{48c^2} - \frac{(b^2d)\int x\,dx}{16c^2}$$

$$= -\frac{b^2dx^2}{24c^2} + \frac{1}{72}b^2dx^4 + \frac{1}{108}b^2c^2dx^6 + \frac{bdx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{12c^3} - \frac{bdx^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{18c} - \frac{1}{18}bcdx^5\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) - \frac{d(a+b\text{arcsinh}(cx))^2}{24c^4} + \frac{1}{12}dx^4(a+b\text{arcsinh}(cx))^2 + \frac{1}{6}dx^4(1+c^2x^2)(a+b\text{arcsinh}(cx))^2$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.94

$$\int x^3(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx = \frac{d\left(cx\left(18a^2c^3x^3(3+2c^2x^2) - 6ab\sqrt{1+c^2x^2}(-3+2c^2x^2+2c^4x^4) + b^2cx(-9+3c^2x^2+2c^4x^4)\right) + 6b\left(bcx\sqrt{\right.\right.}{}$$

```
[In] Integrate[x^3*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d*(c*x*(18*a^2*c^3*x^3*(3 + 2*c^2*x^2) - 6*a*b*Sqrt[1 + c^2*x^2]*(-3 + 2*c
^2*x^2 + 2*c^4*x^4) + b^2*c*x*(-9 + 3*c^2*x^2 + 2*c^4*x^4)) + 6*b*(b*c*x*Sq
rt[1 + c^2*x^2]*(3 - 2*c^2*x^2 - 2*c^4*x^4) + 3*a*(-1 + 6*c^4*x^4 + 4*c^6*x
^6))*ArcSinh[c*x] + 9*b^2*(-1 + 6*c^4*x^4 + 4*c^6*x^6)*ArcSinh[c*x]^2))/(21
6*c^4)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.34

method	result
parts	$d\,a^{2}\left(\frac{1}{6}c^{2}x^{6}+\frac{1}{4}x^{4}\right)+\frac{d\,b^{2}\left(\frac{\operatorname{arcsinh}(cx)^{2}c^{2}x^{2}\left(c^{2}x^{2}+1\right)^{2}}{6}-\frac{\operatorname{arcsinh}(cx)^{2}\left(c^{2}x^{2}+1\right)^{2}}{12}-\frac{\operatorname{arcsinh}(cx)cx\left(c^{2}x^{2}+1\right)^{\frac{5}{2}}}{18}+\frac{\operatorname{arcsinh}(\right.}{c}$
derivativedivides	$\frac{d\,a^{2}\left(\frac{1}{6}c^{6}x^{6}+\frac{1}{4}c^{4}x^{4}\right)+d\,b^{2}\left(\frac{\operatorname{arcsinh}(cx)^{2}c^{2}x^{2}\left(c^{2}x^{2}+1\right)^{2}}{6}-\frac{\operatorname{arcsinh}(cx)^{2}\left(c^{2}x^{2}+1\right)^{2}}{12}-\frac{\operatorname{arcsinh}(cx)cx\left(c^{2}x^{2}+1\right)^{\frac{5}{2}}}{18}+\frac{\operatorname{arcsinh}(cx)cx\left(c\right.}{18}\right.}{}$
default	$\frac{d\,a^{2}\left(\frac{1}{6}c^{6}x^{6}+\frac{1}{4}c^{4}x^{4}\right)+d\,b^{2}\left(\frac{\operatorname{arcsinh}(cx)^{2}c^{2}x^{2}\left(c^{2}x^{2}+1\right)^{2}}{6}-\frac{\operatorname{arcsinh}(cx)^{2}\left(c^{2}x^{2}+1\right)^{2}}{12}-\frac{\operatorname{arcsinh}(cx)cx\left(c^{2}x^{2}+1\right)^{\frac{5}{2}}}{18}+\frac{\operatorname{arcsinh}(cx)cx\left(c\right.}{18}\right.}{}$

```
[In] int(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] d*a^2*(1/6*c^2*x^6+1/4*x^4)+d*b^2/c^4*(1/6*arcsinh(c*x)^2*c^2*x^2*(c^2*x^2+
1)^2-1/12*arcsinh(c*x)^2*(c^2*x^2+1)^2-1/18*arcsinh(c*x)*c*x*(c^2*x^2+1)^(5
/2)+1/18*arcsinh(c*x)*c*x*(c^2*x^2+1)^(3/2)+1/12*arcsinh(c*x)*c*x*(c^2*x^2+
1)^(1/2)+1/24*arcsinh(c*x)^2+1/108*(c^2*x^2+1)^3-1/72*(c^2*x^2+1)^2-1/24*c^
2*x^2-1/24)+2*d*a*b/c^4*(1/6*arcsinh(c*x)*c^6*x^6+1/4*arcsinh(c*x)*c^4*x^4-
1/36*c^5*x^5*(c^2*x^2+1)^(1/2)-1/36*c^3*x^3*(c^2*x^2+1)^(1/2)+1/24*c*x*(c^2
*x^2+1)^(1/2)-1/24*arcsinh(c*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.21

$$\int x^{3}\left(d+c^{2}dx^{2}\right)\left(a+b\operatorname{arcsinh}(cx)\right)^{2}dx$$
$$=\frac{2\left(18a^{2}+b^{2}\right)c^{6}dx^{6}+3\left(18a^{2}+b^{2}\right)c^{4}dx^{4}-9b^{2}c^{2}dx^{2}+9\left(4b^{2}c^{6}dx^{6}+6b^{2}c^{4}dx^{4}-b^{2}d\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right.}{}$$

```
[In] integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/216*(2*(18*a^2 + b^2)*c^6*d*x^6 + 3*(18*a^2 + b^2)*c^4*d*x^4 - 9*b^2*c^2*
d*x^2 + 9*(4*b^2*c^6*d*x^6 + 6*b^2*c^4*d*x^4 - b^2*d)*log(c*x + sqrt(c^2*x^
2 + 1))^2 + 6*(12*a*b*c^6*d*x^6 + 18*a*b*c^4*d*x^4 - 3*a*b*d - (2*b^2*c^5*d
*x^5 + 2*b^2*c^3*d*x^3 - 3*b^2*c*d*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2
*x^2 + 1)) - 6*(2*a*b*c^5*d*x^5 + 2*a*b*c^3*d*x^3 - 3*a*b*c*d*x)*sqrt(c^2*x
^2 + 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.68

$$\int x^3(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\begin{cases}\frac{a^2c^2dx^6}{6}+\frac{a^2dx^4}{4}+\frac{abc^2dx^6\operatorname{asinh}(cx)}{3}-\frac{abcdx^5\sqrt{c^2x^2+1}}{18}+\frac{abdx^4\operatorname{asinh}(cx)}{2}-\frac{abdx^3\sqrt{c^2x^2+1}}{18c}+\frac{abdx\sqrt{c^2x^2+1}}{12c^3}-\frac{abd\operatorname{asinh}(cx)}{12c^4} \\ \frac{a^2dx^4}{4}\end{cases}$$

```
[In] integrate(x**3*(c**2*d*x**2+d)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**2*d*x**6/6 + a**2*d*x**4/4 + a*b*c**2*d*x**6*asinh(c*x)/
3 - a*b*c*d*x**5*sqrt(c**2*x**2 + 1)/18 + a*b*d*x**4*asinh(c*x)/2 - a*b*d*x
**3*sqrt(c**2*x**2 + 1)/(18*c) + a*b*d*x*sqrt(c**2*x**2 + 1)/(12*c**3) - a*
b*d*asinh(c*x)/(12*c**4) + b**2*c**2*d*x**6*asinh(c*x)**2/6 + b**2*c**2*d*x
**6/108 - b**2*c*d*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)/18 + b**2*d*x**4*asi
nh(c*x)**2/4 + b**2*d*x**4/72 - b**2*d*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/
(18*c) - b**2*d*x**2/(24*c**2) + b**2*d*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(1
2*c**3) - b**2*d*asinh(c*x)**2/(24*c**4), Ne(c, 0)), (a**2*d*x**4/4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(174) = 348$.

Time = 0.22 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.23

$$\int x^3(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{1}{6}b^2c^2dx^6\operatorname{arsinh}(cx)^2+\frac{1}{6}a^2c^2dx^6+\frac{1}{4}b^2dx^4\operatorname{arsinh}(cx)^2+\frac{1}{4}a^2dx^4$$
$$+\frac{1}{144}\left(48x^6\operatorname{arsinh}(cx)-\left(\frac{8\sqrt{c^2x^2+1}x^5}{c^2}-\frac{10\sqrt{c^2x^2+1}x^3}{c^4}+\frac{15\sqrt{c^2x^2+1}x}{c^6}-\frac{15\operatorname{arsinh}(cx)}{c^7}\right)c\right)ab$$
$$+\frac{1}{864}\left(\left(\frac{8x^6}{c^2}-\frac{15x^4}{c^4}+\frac{45x^2}{c^6}-\frac{45\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^8}\right)c^2-6\left(\frac{8\sqrt{c^2x^2+1}x^5}{c^2}-\frac{10\sqrt{c^2x^2+1}x^3}{c^4}\right.\right.$$
$$+\frac{1}{16}\left(8x^4\operatorname{arsinh}(cx)-\left(\frac{2\sqrt{c^2x^2+1}x^3}{c^2}-\frac{3\sqrt{c^2x^2+1}x}{c^4}+\frac{3\operatorname{arsinh}(cx)}{c^5}\right)c\right)abd$$
$$+\frac{1}{32}\left(\left(\frac{x^4}{c^2}-\frac{3x^2}{c^4}+\frac{3\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^6}\right)c^2-2\left(\frac{2\sqrt{c^2x^2+1}x^3}{c^2}-\frac{3\sqrt{c^2x^2+1}x}{c^4}+\frac{3\operatorname{arsinh}(cx)}{c^5}\right.\right.$$

```
[In] integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*c^2*d*x^6*arcsinh(c*x)^2 + 1/6*a^2*c^2*d*x^6 + 1/4*b^2*d*x^4*arcsin
h(c*x)^2 + 1/4*a^2*d*x^4 + 1/144*(48*x^6*arcsinh(c*x) - (8*sqrt(c^2*x^2 + 1
```

```
)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*
arcsinh(c*x)/c^7)*c)*a*b*c^2*d + 1/864*((8*x^6/c^2 - 15*x^4/c^4 + 45*x^2/c^
6 - 45*log(c*x + sqrt(c^2*x^2 + 1))^2/c^8)*c^2 - 6*(8*sqrt(c^2*x^2 + 1)*x^5
/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*arcsi
nh(c*x)/c^7)*c*arcsinh(c*x))*b^2*c^2*d + 1/16*(8*x^4*arcsinh(c*x) - (2*sqrt
(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*
a*b*d + 1/32*((x^4/c^2 - 3*x^2/c^4 + 3*log(c*x + sqrt(c^2*x^2 + 1))^2/c^6)*
c^2 - 2*(2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsin
h(c*x)/c^5)*c*arcsinh(c*x))*b^2*d
```

Giac [F(-2)]

Exception generated.

$$\int x^3 \left(d + c^2 d x^2\right) (a + b \text{arcsinh}(cx))^2 \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(d + c^2 d x^2\right) (a + b \text{arcsinh}(cx))^2 \, dx = \int x^3 \left(a + b \operatorname{asinh}(c x)\right)^2 \left(d c^2 x^2 + d\right) \, dx$$

```
[In] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2),x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2), x)
```

3.200 $\int x^2(d+c^2dx^2)(a+b\operatorname{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 206

$$\begin{aligned}\int x^2(d+c^2dx^2)(a+b\operatorname{arcsinh}(cx))^2\,dx =& -\frac{52b^2dx}{225c^2}+\frac{26}{675}b^2dx^3+\frac{2}{125}b^2c^2dx^5\\ &+\frac{8bd\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{45c^3}\\ &-\frac{4bdx^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{45c}\\ &+\frac{2bd(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{15c^3}\\ &-\frac{2bd(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{25c^3}\\ &+\frac{2}{15}dx^3(a+b\operatorname{arcsinh}(cx))^2\\ &+\frac{1}{5}dx^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2\end{aligned}$$

```
[Out] -52/225*b^2*d*x/c^2+26/675*b^2*d*x^3+2/125*b^2*c^2*d*x^5+2/15*b*d*(c^2*x^2+
1)^(3/2)*(a+b*arcsinh(c*x))/c^3-2/25*b*d*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x
))/c^3+2/15*d*x^3*(a+b*arcsinh(c*x))^2+1/5*d*x^3*(c^2*x^2+1)*(a+b*arcsinh(c
*x))^2+8/45*b*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-4/45*b*d*x^2*(a+b*
arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5808, 5776, 5812, 5798, 8, 30, 272, 45, 5804, 12}

$$\int x^2\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))^2\,dx = -\frac{4bdx^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{45c} + \frac{1}{5}dx^3\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2 - \frac{2bd(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))}{25c^3} + \frac{2bd(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{15c^3} + \frac{8bd\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{45c^3} + \frac{2}{15}dx^3(a+b\text{arcsinh}(cx))^2 + \frac{2}{125}b^2c^2dx^5 - \frac{52b^2dx}{225c^2} + \frac{26}{675}b^2dx^3$$

```
[In] Int[x^2*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-52*b^2*d*x)/(225*c^2) + (26*b^2*d*x^3)/675 + (2*b^2*c^2*d*x^5)/125 + (8*b
*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(45*c^3) - (4*b*d*x^2*Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x]))/(45*c) + (2*b*d*(1 + c^2*x^2)^(3/2)*(a + b*Ar
cSinh[c*x]))/(15*c^3) - (2*b*d*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(2
5*c^3) + (2*d*x^3*(a + b*ArcSinh[c*x])^2)/15 + (d*x^3*(1 + c^2*x^2)*(a + b*
ArcSinh[c*x])^2)/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}dx^3\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))^2+\frac{1}{5}(2d)\int x^2(a+b\operatorname{arcsinh}(cx))^2\,dx\\
&\quad-\frac{1}{5}(2bcd)\int x^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx\\
&=\frac{2bd(1+c^2x^2)^{3/2}\,(a+b\operatorname{arcsinh}(cx))}{15c^3}-\frac{2bd(1+c^2x^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))}{25c^3}\\
&\quad+\frac{2}{15}dx^3(a+b\operatorname{arcsinh}(cx))^2+\frac{1}{5}dx^3\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))^2\\
&\quad-\frac{1}{15}(4bcd)\int\frac{x^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx+\frac{1}{5}\left(2b^2c^2d\right)\int\frac{-2+c^2x^2+3c^4x^4}{15c^4}\,dx\\
&=-\frac{4bdx^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{45c}+\frac{2bd(1+c^2x^2)^{3/2}\,(a+b\operatorname{arcsinh}(cx))}{15c^3}\\
&\quad-\frac{2bd(1+c^2x^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))}{25c^3}+\frac{2}{15}dx^3(a+b\operatorname{arcsinh}(cx))^2\\
&\quad+\frac{1}{5}dx^3\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))^2+\frac{1}{45}\left(4b^2d\right)\int x^2\,dx\\
&\quad+\frac{(2b^2d)\int\left(-2+c^2x^2+3c^4x^4\right)\,dx}{75c^2}+\frac{(8bd)\int\frac{x(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{45c}\\
&=-\frac{4b^2dx}{75c^2}+\frac{26}{675}b^2dx^3+\frac{2}{125}b^2c^2dx^5+\frac{8bd\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{45c^3}\\
&\quad-\frac{4bdx^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{45c}+\frac{2bd(1+c^2x^2)^{3/2}\,(a+b\operatorname{arcsinh}(cx))}{15c^3}\\
&\quad-\frac{2bd(1+c^2x^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))}{25c^3}\\
&\quad+\frac{2}{15}dx^3(a+b\operatorname{arcsinh}(cx))^2+\frac{1}{5}dx^3\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))^2-\frac{(8b^2d)\int 1\,dx}{45c^2}
\end{aligned}$$

$$= -\frac{52b^2dx}{225c^2} + \frac{26}{675}b^2dx^3 + \frac{2}{125}b^2c^2dx^5 + \frac{8bd\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{45c^3} - \frac{4bdx^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{45c} + \frac{2bd(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{15c^3} - \frac{2bd(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{25c^3} + \frac{2}{15}dx^3(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{5}dx^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.86

$$\int x^2(d+c^2dx^2)(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$= \frac{d(225a^2c^3x^3(5+3c^2x^2) - 30ab\sqrt{1+c^2x^2}(-26+13c^2x^2+9c^4x^4) + 2b^2cx(-390+65c^2x^2+27c^4x^4) - 30}{}$$

```
[In] Integrate[x^2*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d*(225*a^2*c^3*x^3*(5 + 3*c^2*x^2) - 30*a*b*Sqrt[1 + c^2*x^2]*(-26 + 13*c^
2*x^2 + 9*c^4*x^4) + 2*b^2*c*x*(-390 + 65*c^2*x^2 + 27*c^4*x^4) - 30*b*(-15
*a*c^3*x^3*(5 + 3*c^2*x^2) + b*Sqrt[1 + c^2*x^2]*(-26 + 13*c^2*x^2 + 9*c^4*
x^4))*ArcSinh[c*x] + 225*b^2*c^3*x^3*(5 + 3*c^2*x^2)*ArcSinh[c*x]^2))/(3375
*c^3)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.20

method	result
parts	$d\,a^2\left(\frac{1}{5}c^2x^5+\frac{1}{3}x^3\right)+\frac{d\,b^2\left(\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{5}-\frac{2\operatorname{arcsinh}(cx)^2xc}{15}-\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)}{15}-\frac{2\operatorname{arcsinh}(cx)\left(c^2\right.}{25}\right.}{}$
derivativedivides	$\frac{d\,a^2\left(\frac{1}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d\,b^2\left(\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{5}-\frac{2\operatorname{arcsinh}(cx)^2xc}{15}-\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)}{15}-\frac{2\operatorname{arcsinh}(cx)\left(c^2x^2+1\right)^{\frac{5}{2}}}{25}\right.}{}$
default	$\frac{d\,a^2\left(\frac{1}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d\,b^2\left(\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{5}-\frac{2\operatorname{arcsinh}(cx)^2xc}{15}-\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)}{15}-\frac{2\operatorname{arcsinh}(cx)\left(c^2x^2+1\right)^{\frac{5}{2}}}{25}\right.}{}$

```
[In] int(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] d*a^2*(1/5*c^2*x^5+1/3*x^3)+d*b^2/c^3*(1/5*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^2
-2/15*arcsinh(c*x)^2*x*c-1/15*arcsinh(c*x)^2*c*x*(c^2*x^2+1)-2/25*arcsinh(c
```

```
*x)*(c^2*x^2+1)^(5/2)-856/3375*c*x+2/125*c*x*(c^2*x^2+1)^2+22/3375*c*x*(c^2
*x^2+1)+4/15*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2/45*arcsinh(c*x)*(c^2*x^2+1)^(
3/2))+2*d*a*b/c^3*(1/5*arcsinh(c*x)*c^5*x^5+1/3*arcsinh(c*x)*c^3*x^3-1/25*c
^4*x^4*(c^2*x^2+1)^(1/2)-13/225*c^2*x^2*(c^2*x^2+1)^(1/2)+26/225*(c^2*x^2+1
)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.09

$$\int x^2(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{27(25a^2+2b^2)c^5dx^5+5(225a^2+26b^2)c^3dx^3-780b^2cdx+225(3b^2c^5dx^5+5b^2c^3dx^3)\log\left(cx+\sqrt{c^2x^2\cdots}\right.}{}$$

```
[In] integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/3375*(27*(25*a^2 + 2*b^2)*c^5*d*x^5 + 5*(225*a^2 + 26*b^2)*c^3*d*x^3 - 78
0*b^2*c*d*x + 225*(3*b^2*c^5*d*x^5 + 5*b^2*c^3*d*x^3)*log(c*x + sqrt(c^2*x^
2 + 1))^2 + 30*(45*a*b*c^5*d*x^5 + 75*a*b*c^3*d*x^3 - (9*b^2*c^4*d*x^4 + 13
*b^2*c^2*d*x^2 - 26*b^2*d)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))
- 30*(9*a*b*c^4*d*x^4 + 13*a*b*c^2*d*x^2 - 26*a*b*d)*sqrt(c^2*x^2 + 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.52

$$\int x^2(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\begin{cases}\frac{a^2c^2dx^5}{5}+\frac{a^2dx^3}{3}+\frac{2abc^2dx^5\operatorname{asinh}(cx)}{5}-\frac{2abcdx^4\sqrt{c^2x^2+1}}{25}+\frac{2abdx^3\operatorname{asinh}(cx)}{3}-\frac{26abdx^2\sqrt{c^2x^2+1}}{225c}+\frac{52abd\sqrt{c^2x^2+1}}{225c^3}+\frac{b^2c^2dx^5}{\cdots} \\ \frac{a^2dx^3}{3}\end{cases}$$

```
[In] integrate(x**2*(c**2*d*x**2+d)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**2*d*x**5/5 + a**2*d*x**3/3 + 2*a*b*c**2*d*x**5*asinh(c*x
)/5 - 2*a*b*c*d*x**4*sqrt(c**2*x**2 + 1)/25 + 2*a*b*d*x**3*asinh(c*x)/3 - 2
6*a*b*d*x**2*sqrt(c**2*x**2 + 1)/(225*c) + 52*a*b*d*sqrt(c**2*x**2 + 1)/(22
5*c**3) + b**2*c**2*d*x**5*asinh(c*x)**2/5 + 2*b**2*c**2*d*x**5/125 - 2*b**
2*c*d*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/25 + b**2*d*x**3*asinh(c*x)**2/3
+ 26*b**2*d*x**3/675 - 26*b**2*d*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(225*c
) - 52*b**2*d*x/(225*c**2) + 52*b**2*d*sqrt(c**2*x**2 + 1)*asinh(c*x)/(225*
c**3), Ne(c, 0)), (a**2*d*x**3/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.68

$$\int x^2(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{1}{5}b^2c^2dx^5\operatorname{arsinh}(cx)^2 + \frac{1}{5}a^2c^2dx^5 + \frac{1}{3}b^2dx^3\operatorname{arsinh}(cx)^2 + \frac{2}{75}\left(15x^5\operatorname{arsinh}(cx) - \left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)abc^2d - \frac{2}{1125}\left(15\left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\operatorname{arsinh}(cx) - \frac{9c^4x^5-20c^2x^3+120x}{c^4}\right) + \frac{1}{3}a^2dx^3 + \frac{2}{9}\left(3x^3\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)abd - \frac{2}{27}\left(3c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\operatorname{arsinh}(cx) - \frac{c^2x^3-6x}{c^2}\right)b^2d$$

```
[In] integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*c^2*d*x^5*arcsinh(c*x)^2 + 1/5*a^2*c^2*d*x^5 + 1/3*b^2*d*x^3*arcsin
h(c*x)^2 + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqr
t(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^2*d - 2/1125*(15
*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^
2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^2*c^2*
d + 1/3*a^2*d*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2
- 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d - 2/27*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 -
2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*d
```

Giac [F(-2)]

Exception generated.

$$\int x^2(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2\,dx=\int x^2\,(a+b\,\text{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)\,dx$$

```
[In] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2),x)
[Out] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2), x)
```

3.201 $\int x(d+c^2dx^2)\,(a+b\mathbf{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 135

$$\int x\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))^2\,dx=\frac{5}{32}b^2dx^2+\frac{1}{32}b^2c^2dx^4-\frac{3bdx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{16c}-\frac{bdx(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{8c}-\frac{3d(a+b\text{arcsinh}(cx))^2}{32c^2}+\frac{d(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))^2}{4c^2}$$

```
[Out] 5/32*b^2*d*x^2+1/32*b^2*c^2*d*x^4-1/8*b*d*x*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(
c*x))/c-3/32*d*(a+b*arcsinh(c*x))^2/c^2+1/4*d*(c^2*x^2+1)^2*(a+b*arcsinh(c*
x))^2/c^2-3/16*b*d*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used

= {5798, 5786, 5785, 5783, 30, 14}

$$\int x\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))^2\,dx = -\frac{bdx(c^2x^2+1)^{3/2}\,(a+b\text{arcsinh}(cx))}{8c} - \frac{3bdx\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{16c} + \frac{d(c^2x^2+1)^2\,(a+b\text{arcsinh}(cx))^2}{4c^2} - \frac{3d(a+b\text{arcsinh}(cx))^2}{32c^2} + \frac{1}{32}b^2c^2dx^4 + \frac{5}{32}b^2dx^2$$

```
[In] Int[x*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (5*b^2*d*x^2)/32 + (b^2*c^2*d*x^4)/32 - (3*b*d*x*Sqrt[1 + c^2*x^2]*(a + b*A
rcSinh[c*x]))/(16*c) - (b*d*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(8*
c) - (3*d*(a + b*ArcSinh[c*x])^2)/(32*c^2) + (d*(1 + c^2*x^2)^2*(a + b*ArcS
inh[c*x])^2)/(4*c^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))^2}{4c^2}-\frac{(bd)\int (1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\,dx}{2c}\\
&= -\frac{bdx(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{8c}+\frac{d(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))^2}{4c^2}\\
&\quad+\frac{1}{8}\left(b^2d\right)\int x\left(1+c^2x^2\right)\,dx-\frac{(3bd)\int \sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx}{8c}\\
&= -\frac{3bdx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{16c}\\
&\quad-\frac{bdx(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{8c}+\frac{d(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))^2}{4c^2}\\
&\quad+\frac{1}{8}\left(b^2d\right)\int \left(x+c^2x^3\right)\,dx+\frac{1}{16}\left(3b^2d\right)\int x\,dx-\frac{(3bd)\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{16c}\\
&= \frac{5}{32}b^2dx^2+\frac{1}{32}b^2c^2dx^4-\frac{3bdx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{16c}\\
&\quad-\frac{bdx(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{8c}\\
&\quad-\frac{3d(a+b\text{arcsinh}(cx))^2}{32c^2}+\frac{d(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))^2}{4c^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.15

$$\int x(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{d\left(cx\left(8a^2cx(2+c^2x^2)+b^2cx(5+c^2x^2)-2ab\sqrt{1+c^2x^2}(5+2c^2x^2)\right)+2b\left(-bcx\sqrt{1+c^2x^2}(5+2c^2x^2)+a(\right.\right.}{32c^2}$$

[In] Integrate[x*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]

[Out] (d*(c*x*(8*a^2*c*x*(2 + c^2*x^2) + b^2*c*x*(5 + c^2*x^2) - 2*a*b*Sqrt[1 + c^2*x^2]*(5 + 2*c^2*x^2)) + 2*b*(-(b*c*x*Sqrt[1 + c^2*x^2]*(5 + 2*c^2*x^2)) + a*(5 + 16*c^2*x^2 + 8*c^4*x^4))*ArcSinh[c*x] + b^2*(5 + 16*c^2*x^2 + 8*c^4*x^4)*ArcSinh[c*x]^2))/(32*c^2)

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.35

method	result
derivativedivides	$\frac{\frac{d\,a^2\left(c^2x^2+1\right)^2}{4}+d\,b^2\left(\frac{\text{arcsinh}(cx)^2\left(c^2x^2+1\right)^2}{4}-\frac{\text{arcsinh}(cx)cx\left(c^2x^2+1\right)^{\frac{3}{2}}}{8}-\frac{3\,\text{arcsinh}(cx)cx\sqrt{c^2x^2+1}}{16}-\frac{3\,\text{arcsinh}(cx)^2}{32}+\frac{\left(c^2x^2+1\right.}{32}\right.}{c^2}$
default	$\frac{\frac{d\,a^2\left(c^2x^2+1\right)^2}{4}+d\,b^2\left(\frac{\text{arcsinh}(cx)^2\left(c^2x^2+1\right)^2}{4}-\frac{\text{arcsinh}(cx)cx\left(c^2x^2+1\right)^{\frac{3}{2}}}{8}-\frac{3\,\text{arcsinh}(cx)cx\sqrt{c^2x^2+1}}{16}-\frac{3\,\text{arcsinh}(cx)^2}{32}+\frac{\left(c^2x^2+1\right.}{32}\right.}{c^2}$
parts	$\frac{d\,a^2\left(c^2x^2+1\right)^2}{4c^2}+\frac{d\,b^2\left(\frac{\text{arcsinh}(cx)^2\left(c^2x^2+1\right)^2}{4}-\frac{\text{arcsinh}(cx)cx\left(c^2x^2+1\right)^{\frac{3}{2}}}{8}-\frac{3\,\text{arcsinh}(cx)cx\sqrt{c^2x^2+1}}{16}-\frac{3\,\text{arcsinh}(cx)^2}{32}+\frac{\left(c^2x^2\right.}{3}\right.}{c^2}$

[In] int(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)

[Out] 1/c^2*(1/4*d*a^2*(c^2*x^2+1)^2+d*b^2*(1/4*arcsinh(c*x)^2*(c^2*x^2+1)^2-1/8*arcsinh(c*x)*c*x*(c^2*x^2+1)^(3/2)-3/16*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)-3/32*arcsinh(c*x)^2+1/32*(c^2*x^2+1)^2+3/32*c^2*x^2+3/32)+2*d*a*b*(1/4*arcsinh(c*x)*c^4*x^4+1/2*arcsinh(c*x)*c^2*x^2+5/32*arcsinh(c*x)-1/16*c*x*(c^2*x^2+1)^(3/2)-3/32*c*x*(c^2*x^2+1)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.51

$$\int x\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{(8\,a^2+b^2)c^4dx^4+(16\,a^2+5\,b^2)c^2dx^2+(8\,b^2c^4dx^4+16\,b^2c^2dx^2+5\,b^2d)\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\left(8\,abc\right.}{}$$

```
[In] integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/32*((8*a^2 + b^2)*c^4*d*x^4 + (16*a^2 + 5*b^2)*c^2*d*x^2 + (8*b^2*c^4*d*x
^4 + 16*b^2*c^2*d*x^2 + 5*b^2*d)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(8*a*b*
c^4*d*x^4 + 16*a*b*c^2*d*x^2 + 5*a*b*d - (2*b^2*c^3*d*x^3 + 5*b^2*c*d*x)*sq
rt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 2*(2*a*b*c^3*d*x^3 + 5*a*b*
c*d*x)*sqrt(c^2*x^2 + 1))/c^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(129) = 258.

Time = 0.43 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.99

$$\int x\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\begin{cases}\frac{a^2c^2dx^4}{4}+\frac{a^2dx^2}{2}+\frac{abc^2dx^4\operatorname{asinh}(cx)}{2}-\frac{abcdx^3\sqrt{c^2x^2+1}}{8}+abdx^2\operatorname{asinh}(cx)-\frac{5abdx\sqrt{c^2x^2+1}}{16c}+\frac{5abd\operatorname{asinh}(cx)}{16c^2}+\frac{b^2c^2dx^4}{} \\ \frac{a^2dx^2}{2}\end{cases}$$

```
[In] integrate(x*(c**2*d*x**2+d)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**2*d*x**4/4 + a**2*d*x**2/2 + a*b*c**2*d*x**4*asinh(c*x)/
2 - a*b*c*d*x**3*sqrt(c**2*x**2 + 1)/8 + a*b*d*x**2*asinh(c*x) - 5*a*b*d*x*
sqrt(c**2*x**2 + 1)/(16*c) + 5*a*b*d*asinh(c*x)/(16*c**2) + b**2*c**2*d*x**
4*asinh(c*x)**2/4 + b**2*c**2*d*x**4/32 - b**2*c*d*x**3*sqrt(c**2*x**2 + 1)
*asinh(c*x)/8 + b**2*d*x**2*asinh(c*x)**2/2 + 5*b**2*d*x**2/32 - 5*b**2*d*x
*sqrt(c**2*x**2 + 1)*asinh(c*x)/(16*c) + 5*b**2*d*asinh(c*x)**2/(32*c**2),
Ne(c, 0)), (a**2*d*x**2/2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(119) = 238.

Time = 0.22 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.57

$$\int x(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{1}{4}\,b^2c^2dx^4\,\text{arsinh}\,(cx)^2+\frac{1}{4}\,a^2c^2dx^4+\frac{1}{2}\,b^2dx^2\,\text{arsinh}\,(cx)^2$$
$$+\frac{1}{16}\left(8\,x^4\,\text{arsinh}\,(cx)-\left(\frac{2\,\sqrt{c^2x^2+1}x^3}{c^2}-\frac{3\,\sqrt{c^2x^2+1}x}{c^4}+\frac{3\,\text{arsinh}\,(cx)}{c^5}\right)c\right)abc^2d$$
$$+\frac{1}{32}\left(\left(\frac{x^4}{c^2}-\frac{3\,x^2}{c^4}+\frac{3\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^6}\right)c^2-2\left(\frac{2\,\sqrt{c^2x^2+1}x^3}{c^2}-\frac{3\,\sqrt{c^2x^2+1}x}{c^4}+\frac{3\,\text{arsinh}\,(cx)}{c^5}\right)\right.$$
$$+\frac{1}{2}\,a^2dx^2+\frac{1}{2}\left(2\,x^2\,\text{arsinh}\,(cx)-c\left(\frac{\sqrt{c^2x^2+1}x}{c^2}-\frac{\text{arsinh}\,(cx)}{c^3}\right)\right)abd$$
$$+\frac{1}{4}\left(c^2\left(\frac{x^2}{c^2}-\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^4}\right)-2\,c\left(\frac{\sqrt{c^2x^2+1}x}{c^2}-\frac{\text{arsinh}\,(cx)}{c^3}\right)\text{arsinh}\,(cx)\right)b^2d$$

```
[In] integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*c^2*d*x^4*arcsinh(c*x)^2 + 1/4*a^2*c^2*d*x^4 + 1/2*b^2*d*x^2*arcsin
h(c*x)^2 + 1/16*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt
(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*a*b*c^2*d + 1/32*((x^4/c^2 - 3
*x^2/c^4 + 3*log(c*x + sqrt(c^2*x^2 + 1))^2/c^6)*c^2 - 2*(2*sqrt(c^2*x^2 +
1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c*arcsinh(c*x)
)*b^2*c^2*d + 1/2*a^2*d*x^2 + 1/2*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1
)*x/c^2 - arcsinh(c*x)/c^3))*a*b*d + 1/4*(c^2*(x^2/c^2 - log(c*x + sqrt(c^2
*x^2 + 1))^2/c^4) - 2*c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3)*arcsin
h(c*x))*b^2*d
```

Giac [F(-2)]

Exception generated.

$$\int x(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(d + c^2dx^2)(a + b\text{arcsinh}(cx))^2 dx = \int x(a + b\,\text{asinh}(c\,x))^2 \left(d\,c^2\,x^2 + d\right) dx$$

```
[In] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2),x)
[Out] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2), x)
```

3.202 $\int (d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 125

$$\int (d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx = \frac{14}{9}b^2dx+\frac{2}{27}b^2c^2dx^3 - \frac{4bd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3c} - \frac{2bd(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{9c} + \frac{2}{3}dx(a+b\text{arcsinh}(cx))^2 + \frac{1}{3}dx(1+c^2x^2)(a+b\text{arcsinh}(cx))^2$$

```
[Out] 14/9*b^2*d*x+2/27*b^2*c^2*d*x^3-2/9*b*d*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x)
)/c+2/3*d*x*(a+b*arcsinh(c*x))^2+1/3*d*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2-4
/3*b*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5786, 5772, 5798, 8}

$$\int (d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx = \frac{1}{3}dx(c^2x^2+1)(a+b\text{arcsinh}(cx))^2 - \frac{2bd(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{9c} - \frac{4bd\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c} + \frac{2}{3}dx(a+b\text{arcsinh}(cx))^2+\frac{2}{27}b^2c^2dx^3+\frac{14}{9}b^2dx$$

```
[In] Int[(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (14*b^2*d*x)/9 + (2*b^2*c^2*d*x^3)/27 - (4*b*d*Sqrt[1 + c^2*x^2]*(a + b*Arc
Sinh[c*x]))/(3*c) - (2*b*d*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(9*c)
+ (2*d*x*(a + b*ArcSinh[c*x])^2)/3 + (d*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
)^2)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}dx\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))^2+\frac{1}{3}(2d)\int(a+b\operatorname{arcsinh}(cx))^2\,dx \\
&\quad-\frac{1}{3}(2bcd)\int x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx \\
&=-\frac{2bd(1+c^2x^2)^{3/2}\,(a+b\operatorname{arcsinh}(cx))}{9c} \\
&\quad+\frac{2}{3}dx(a+b\operatorname{arcsinh}(cx))^2+\frac{1}{3}dx\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))^2 \\
&\quad+\frac{1}{9}\left(2b^2d\right)\int\left(1+c^2x^2\right)\,dx-\frac{1}{3}(4bcd)\int\frac{x(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx
\end{aligned}
$$

$$=\frac{2}{9}b^2dx+\frac{2}{27}b^2c^2dx^3-\frac{4bd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3c}-\frac{2bd(1+c^2x^2)^{3/2}\left(a+b\text{arcsinh}(cx)\right)}{9c}+\frac{2}{3}dx(a+b\text{arcsinh}(cx))^2+\frac{1}{3}dx\left(1+c^2x^2\right)\left(a+b\text{arcsinh}(cx)\right)^2+\frac{1}{3}\left(4b^2d\right)\int 1\,dx$$

$$=\frac{14}{9}b^2dx+\frac{2}{27}b^2c^2dx^3-\frac{4bd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3c}-\frac{2bd(1+c^2x^2)^{3/2}\left(a+b\text{arcsinh}(cx)\right)}{9c}+\frac{2}{3}dx(a+b\text{arcsinh}(cx))^2+\frac{1}{3}dx\left(1+c^2x^2\right)\left(a+b\text{arcsinh}(cx)\right)^2$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.08

$$\int \left(d+c^2dx^2\right)\left(a+b\text{arcsinh}(cx)\right)^2dx =\frac{d\left(9a^2cx(3+c^2x^2)-6ab\sqrt{1+c^2x^2}(7+c^2x^2)+2b^2cx(21+c^2x^2)-6b\left(-3acx(3+c^2x^2)+b\sqrt{1+c^2x^2}(7+\right.\right.}{27c}$$

```
[In] Integrate[(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d*(9*a^2*c*x*(3 + c^2*x^2) - 6*a*b*Sqrt[1 + c^2*x^2]*(7 + c^2*x^2) + 2*b^2
*c*x*(21 + c^2*x^2) - 6*b*(-3*a*c*x*(3 + c^2*x^2) + b*Sqrt[1 + c^2*x^2]*(7
+ c^2*x^2))*ArcSinh[c*x] + 9*b^2*c*x*(3 + c^2*x^2)*ArcSinh[c*x]^2))/(27*c)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{d\,a^2\left(\frac{1}{3}c^3x^3+cx\right)+d\,b^2\left(\frac{2\,\text{arcsinh}(cx)^2xc}{3}+\frac{\text{arcsinh}(cx)^2cx\left(c^2x^2+1\right)}{3}-\frac{4\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}}{3}+\frac{40cx}{27}-\frac{2\,\text{arcsinh}(cx)\left(c^2x^2+1\right)^{\frac{3}{2}}}{9}+\right.}{c}$
default	$\frac{d\,a^2\left(\frac{1}{3}c^3x^3+cx\right)+d\,b^2\left(\frac{2\,\text{arcsinh}(cx)^2xc}{3}+\frac{\text{arcsinh}(cx)^2cx\left(c^2x^2+1\right)}{3}-\frac{4\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}}{3}+\frac{40cx}{27}-\frac{2\,\text{arcsinh}(cx)\left(c^2x^2+1\right)^{\frac{3}{2}}}{9}+\right.}{c}$
parts	$d\,a^2\left(\frac{1}{3}x^3c^2+x\right)+\frac{d\,b^2\left(\frac{2\,\text{arcsinh}(cx)^2xc}{3}+\frac{\text{arcsinh}(cx)^2cx\left(c^2x^2+1\right)}{3}-\frac{4\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}}{3}+\frac{40cx}{27}-\frac{2\,\text{arcsinh}(cx)\left(c^2x^2\right.}{9}\right.}{c}$

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(d*a^2*(1/3*c^3*x^3+c*x)+d*b^2*(2/3*arcsinh(c*x)^2*x*c+1/3*arcsinh(c*x)
^2*c*x*(c^2*x^2+1)-4/3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+40/27*c*x-2/9*arcsinh
```

```
(c*x)*(c^2*x^2+1)^(3/2)+2/27*c*x*(c^2*x^2+1))+2*d*a*b*(1/3*arcsinh(c*x)*c^3
*x^3+arcsinh(c*x)*c*x-1/9*c^2*x^2*(c^2*x^2+1)^(1/2)-7/9*(c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.42

$$\int (d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{(9a^2+2b^2)c^3dx^3+3(9a^2+14b^2)cdx+9(b^2c^3dx^3+3b^2cdx)\log\left(cx+\sqrt{c^2x^2+1}\right)^2+6\left(3abc^3dx^3+9\right.}{27c}$$

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/27*((9*a^2 + 2*b^2)*c^3*d*x^3 + 3*(9*a^2 + 14*b^2)*c*d*x + 9*(b^2*c^3*d*x
^3 + 3*b^2*c*d*x)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 6*(3*a*b*c^3*d*x^3 + 9*a
*b*c*d*x - (b^2*c^2*d*x^2 + 7*b^2*d)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*
x^2 + 1)) - 6*(a*b*c^2*d*x^2 + 7*a*b*d)*sqrt(c^2*x^2 + 1))/c
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.79

$$\int (d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\begin{cases}\frac{a^2c^2dx^3}{3}+a^2dx+\frac{2abc^2dx^3\operatorname{asinh}(cx)}{3}-\frac{2abcdx^2\sqrt{c^2x^2+1}}{9}+2abdx\operatorname{asinh}(cx)-\frac{14abd\sqrt{c^2x^2+1}}{9c}+\frac{b^2c^2dx^3\operatorname{asinh}^2(cx)}{3}+\cdots\\ a^2dx\end{cases}$$

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**2*d*x**3/3 + a**2*d*x + 2*a*b*c**2*d*x**3*asinh(c*x)/3 -
 2*a*b*c*d*x**2*sqrt(c**2*x**2 + 1)/9 + 2*a*b*d*x*asinh(c*x) - 14*a*b*d*sqr
t(c**2*x**2 + 1)/(9*c) + b**2*c**2*d*x**3*asinh(c*x)**2/3 + 2*b**2*c**2*d*x
**3/27 - 2*b**2*c*d*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/9 + b**2*d*x*asinh(
c*x)**2 + 14*b**2*d*x/9 - 14*b**2*d*sqrt(c**2*x**2 + 1)*asinh(c*x)/(9*c), N
e(c, 0)), (a**2*d*x, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(109) = 218.

Time = 0.21 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.84

$$\begin{aligned}
&\int (d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2\,dx \\
&= \frac{1}{3}\,b^2c^2dx^3\,\text{arsinh}\,(cx)^2 + \frac{1}{3}\,a^2c^2dx^3 \\
&\quad + \frac{2}{9}\left(3\,x^3\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)abc^2d \\
&\quad - \frac{2}{27}\left(3\,c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\text{arsinh}\,(cx) - \frac{c^2x^3-6\,x}{c^2}\right)b^2c^2d \\
&\quad + b^2dx\,\text{arsinh}\,(cx)^2 + 2\,b^2d\left(x - \frac{\sqrt{c^2x^2+1}\,\text{arsinh}\,(cx)}{c}\right) \\
&\quad + a^2dx + \frac{2\left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)abd}{c}
\end{aligned}$$

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*c^2*d*x^3*arcsinh(c*x)^2 + 1/3*a^2*c^2*d*x^3 + 2/9*(3*x^3*arcsinh(c
*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^2*d -
2/27*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x
) - (c^2*x^3 - 6*x)/c^2)*b^2*c^2*d + b^2*d*x*arcsinh(c*x)^2 + 2*b^2*d*(x -
sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d*x + 2*(c*x*arcsinh(c*x) - sqrt(c^
2*x^2 + 1))*a*b*d/c
```

Giac [F(-2)]

Exception generated.

$$\int (d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d + c^2dx^2) (a + b\text{arcsinh}(cx))^2 dx = \int (a + b \operatorname{asinh}(c x))^2 (d c^2 x^2 + d) dx$$

[In] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2),x)

[Out] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2), x)

3.203 $\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 166

$$\begin{aligned}\int \frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2}{x}\,dx &= \frac{1}{4}b^2c^2dx^2-\frac{1}{2}bcdx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\\&\quad-\frac{1}{4}d(a+b\text{arcsinh}(cx))^2\\&\quad+\frac{1}{2}d\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2\\&\quad+\frac{d(a+b\text{arcsinh}(cx))^3}{3b}\\&\quad+d(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right)\\&\quad-bd(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)\\&\quad-\frac{1}{2}b^2d\,\text{PolyLog}\left(3,e^{-2\text{arcsinh}(cx)}\right)\end{aligned}$$

```
[Out] 1/4*b^2*c^2*d*x^2-1/4*d*(a+b*arcsinh(c*x))^2+1/2*d*(c^2*x^2+1)*(a+b*arcsinh
(c*x))^2+1/3*d*(a+b*arcsinh(c*x))^3/b+d*(a+b*arcsinh(c*x))^2*ln(1-1/(c*x+(c
^2*x^2+1)^(1/2))^2)-b*d*(a+b*arcsinh(c*x))*polylog(2,1/(c*x+(c^2*x^2+1)^(1/
2))^2)-1/2*b^2*d*polylog(3,1/(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b*c*d*x*(a+b*ar
csinh(c*x))*(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5808, 5775, 3797, 2221, 2611, 2320, 6724, 5785, 5783, 30}

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x}\,dx = \frac{1}{2}d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2 - \frac{1}{2}bcdx\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx)) - bd\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)) + \frac{d(a+b\text{arcsinh}(cx))^3}{3b} - \frac{1}{4}d(a+b\text{arcsinh}(cx))^2 + d\log\left(1-e^{-2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2 - \frac{1}{2}b^2d\,\text{PolyLog}\left(3,e^{-2\text{arcsinh}(cx)}\right) + \frac{1}{4}b^2c^2dx^2$$

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (b^2*c^2*d*x^2)/4 - (b*c*d*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 - (d
*(a + b*ArcSinh[c*x])^2)/4 + (d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/2 + (
d*(a + b*ArcSinh[c*x])^3)/(3*b) + d*(a + b*ArcSinh[c*x])^2*Log[1 - E^(-2*Ar
cSinh[c*x])] - b*d*(a + b*ArcSinh[c*x])*PolyLog[2, E^(-2*ArcSinh[c*x])] - (
b^2*d*PolyLog[3, E^(-2*ArcSinh[c*x])])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
```

```
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}d\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2+d\int \frac{(a+b\text{arcsinh}(cx))^2}{x}\,dx \\
&\quad -(bcd)\int \sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx \\
&= -\frac{1}{2}bcdx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))+\frac{1}{2}d\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2 \\
&\quad -\frac{d\text{Subst}\left(\int x^2\coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{b} \\
&\quad -\frac{1}{2}(bcd)\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx+\frac{1}{2}\left(b^2c^2d\right)\int x\,dx \\
&= \frac{1}{4}b^2c^2dx^2-\frac{1}{2}bcdx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))-\frac{1}{4}d(a+b\text{arcsinh}(cx))^2 \\
&\quad +\frac{1}{2}d\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2+\frac{d(a+b\text{arcsinh}(cx))^3}{3b} \\
&\quad +\frac{(2d)\text{Subst}\left(\int \frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x^2}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b} \\
&= \frac{1}{4}b^2c^2dx^2-\frac{1}{2}bcdx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))-\frac{1}{4}d(a+b\text{arcsinh}(cx))^2 \\
&\quad +\frac{1}{2}d\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2+\frac{d(a+b\text{arcsinh}(cx))^3}{3b} \\
&\quad +d(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right) \\
&\quad -(2d)\text{Subst}\left(\int x\log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx,x,a+b\text{arcsinh}(cx)\right) \\
&= \frac{1}{4}b^2c^2dx^2-\frac{1}{2}bcdx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) \\
&\quad -\frac{1}{4}d(a+b\text{arcsinh}(cx))^2+\frac{1}{2}d\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2 \\
&\quad +\frac{d(a+b\text{arcsinh}(cx))^3}{3b}+d(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right) \\
&\quad -bd(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right) \\
&\quad +(bd)\text{Subst}\left(\int \text{PolyLog}\left(2,e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{4}b^2c^2dx^2 - \frac{1}{2}bcdx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) \\
&\quad - \frac{1}{4}d(a+b\text{arcsinh}(cx))^2 + \frac{1}{2}d(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2 \\
&\quad + \frac{d(a+b\text{arcsinh}(cx))^3}{3b} + d(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right) \\
&\quad - bd(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right) \\
&\quad - \frac{1}{2}(b^2d)\,\text{Subst}\left(\int\frac{\text{PolyLog}(2,x)}{x}\,dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right) \\
&= \frac{1}{4}b^2c^2dx^2 - \frac{1}{2}bcdx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) - \frac{1}{4}d(a+b\text{arcsinh}(cx))^2 \\
&\quad + \frac{1}{2}d(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2 + \frac{d(a+b\text{arcsinh}(cx))^3}{3b} \\
&\quad + d(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right) \\
&\quad - bd(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right) \\
&\quad - \frac{1}{2}b^2d\,\text{PolyLog}\left(3,e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.36

$$
\begin{aligned}
\int\frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2}{x}\,dx &= \frac{1}{2}d\Bigg(a^2c^2x^2 + 2abc^2x^2\text{arcsinh}(cx) \\
&\quad + \frac{1}{4}b^2\left(1+2\text{arcsinh}(cx)^2\right)\cosh(2\text{arcsinh}(cx)) \\
&\quad - 2ab\text{arcsinh}(cx)\left(\text{arcsinh}(cx)\right. \\
&\quad \left. - 2\log\left(1-e^{2\text{arcsinh}(cx)}\right)\right) + 2a^2\log(x) \\
&\quad - ab\left(cx\sqrt{1+c^2x^2} + \log\left(-cx+\sqrt{1+c^2x^2}\right)\right) \\
&\quad + 2ab\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right) \\
&\quad + 2b^2\Bigg(-\frac{1}{3}\text{arcsinh}(cx)^3 \\
&\quad + \text{arcsinh}(cx)^2\log\left(1-e^{2\text{arcsinh}(cx)}\right) \\
&\quad + \text{arcsinh}(cx)\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right) \\
&\quad - \frac{1}{2}\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(cx)}\right)\Bigg) \\
&\quad - \frac{1}{2}b^2\text{arcsinh}(cx)\sinh(2\text{arcsinh}(cx))\Bigg)
\end{aligned}
$$

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (d*(a^2*c^2*x^2 + 2*a*b*c^2*x^2*ArcSinh[c*x] + (b^2*(1 + 2*ArcSinh[c*x]^2)*
Cosh[2*ArcSinh[c*x]])/4 - 2*a*b*ArcSinh[c*x]*(ArcSinh[c*x] - 2*Log[1 - E^(2
*ArcSinh[c*x])]) + 2*a^2*Log[x] - a*b*(c*x*Sqrt[1 + c^2*x^2] + Log[-(c*x) +
 Sqrt[1 + c^2*x^2]]) + 2*a*b*PolyLog[2, E^(2*ArcSinh[c*x])] + 2*b^2*(-1/3*A
rcSinh[c*x]^3 + ArcSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + ArcSinh[c*x]*P
olyLog[2, E^(2*ArcSinh[c*x])] - PolyLog[3, E^(2*ArcSinh[c*x])]/2) - (b^2*Ar
cSinh[c*x]*Sinh[2*ArcSinh[c*x]])/2))/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(179) = 358$.

Time = 0.17 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.50

method	result
derivativedivides	$d\,a^2\left(\frac{c^2x^2}{2}+\ln(cx)\right)+d\,b^2\left(-\frac{\operatorname{arcsinh}(cx)^3}{3}+\frac{\left(2\operatorname{arcsinh}(cx)^2-2\operatorname{arcsinh}(cx)+1\right)\left(2c^2x^2+1+2cx\sqrt{c^2x^2+1}\right)}{16}\right.$
default	$d\,a^2\left(\frac{c^2x^2}{2}+\ln(cx)\right)+d\,b^2\left(-\frac{\operatorname{arcsinh}(cx)^3}{3}+\frac{\left(2\operatorname{arcsinh}(cx)^2-2\operatorname{arcsinh}(cx)+1\right)\left(2c^2x^2+1+2cx\sqrt{c^2x^2+1}\right)}{16}\right.$
parts	$d\,a^2\left(\frac{c^2x^2}{2}+\ln(x)\right)-\frac{d\,b^2\operatorname{arcsinh}(cx)^3}{3}+\frac{d\,b^2\operatorname{arcsinh}(cx)^2x^2c^2}{2}-\frac{d\,b^2\sqrt{c^2x^2+1}\,\operatorname{arcsinh}(cx)xc}{2}+\frac{b^2c^2d\,x^2}{4}$

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] d*a^2*(1/2*c^2*x^2+ln(c*x))+d*b^2*(-1/3*arcsinh(c*x)^3+1/16*(2*arcsinh(c*x)
^2-2*arcsinh(c*x)+1)*(2*c^2*x^2+1+2*c*x*(c^2*x^2+1)^(1/2))+1/16*(-2*c*x*(c^
2*x^2+1)^(1/2)+2*c^2*x^2+1)*(2*arcsinh(c*x)^2+2*arcsinh(c*x)+1)+arcsinh(c*x
)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(
1/2))-2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))+arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+
1)^(1/2))+2*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*polylog(3,c*x+(
c^2*x^2+1)^(1/2)))-d*a*b*arcsinh(c*x)^2+d*a*b*arcsinh(c*x)*c^2*x^2-1/2*d*a*
b*c*x*(c^2*x^2+1)^(1/2)+1/2*d*a*b*arcsinh(c*x)+2*d*a*b*arcsinh(c*x)*ln(1+c*
x+(c^2*x^2+1)^(1/2))+2*d*a*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*d*a*b*arcs
inh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*d*a*b*polylog(2,c*x+(c^2*x^2+1)^(1/2
))
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2}{x}\,dx = \int \frac{(c^2dx^2+d)(b\,\text{arsinh}\,(cx)+a)^2}{x}\,dx$$

[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x,x, algorithm="fricas")

[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 + 2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))/x, x)

Sympy [F]

$$\int \frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2}{x}\,dx = d\left(\int \frac{a^2}{x}\,dx + \int a^2c^2x\,dx + \int \frac{b^2\,\text{asinh}^2\,(cx)}{x}\,dx + \int \frac{2ab\,\text{asinh}\,(cx)}{x}\,dx + \int b^2c^2x\,\text{asinh}^2\,(cx)\,dx + \int 2abc^2x\,\text{asinh}\,(cx)\,dx\right)$$

[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**2/x,x)

[Out] d*(Integral(a**2/x, x) + Integral(a**2*c**2*x, x) + Integral(b**2*asinh(c*x)**2/x, x) + Integral(2*a*b*asinh(c*x)/x, x) + Integral(b**2*c**2*x*asinh(c*x)**2, x) + Integral(2*a*b*c**2*x*asinh(c*x), x))

Maxima [F]

$$\int \frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2}{x}\,dx = \int \frac{(c^2dx^2+d)(b\,\text{arsinh}\,(cx)+a)^2}{x}\,dx$$

[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x,x, algorithm="maxima")

[Out] 1/2*a^2*c^2*d*x^2 + a^2*d*log(x) + integrate(b^2*c^2*d*x*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*a*b*c^2*d*x*log(c*x + sqrt(c^2*x^2 + 1)) + b^2*d*log(c*x + sqrt(c^2*x^2 + 1))^2/x + 2*a*b*d*log(c*x + sqrt(c^2*x^2 + 1))/x, x)

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + c^2dx^2)(a + b\text{arcsinh}(cx))^2}{x} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + c^2dx^2)(a + b\text{arcsinh}(cx))^2}{x} dx = \int \frac{(a + b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2 + d)}{x} dx$$

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x, x)
```

3.204 $\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 131

$$\begin{aligned}\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^2}\,dx &= 2b^2c^2dx-2bcd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\\ &\quad+2c^2dx(a+b\text{arcsinh}(cx))^2\\ &\quad-\frac{d(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{x}\\ &\quad-4bcd(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)\\ &\quad-2b^2cd\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)\\ &\quad+2b^2cd\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\end{aligned}$$

```
[Out] 2*b^2*c^2*d*x+2*c^2*d*x*(a+b*arcsinh(c*x))^2-d*(c^2*x^2+1)*(a+b*arcsinh(c*x
))^2/x-4*b*c*d*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))-2*b^2*c*d*
polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*b^2*c*d*polylog(2,c*x+(c^2*x^2+1)^(1/2)
)-2*b*c*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used

$= \{5807, 5772, 5798, 8, 5806, 5816, 4267, 2317, 2438\}$

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = -4bcd\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)) - 2bcd\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx)) - \frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x} + 2c^2dx(a+b\text{arcsinh}(cx))^2 - 2b^2cd\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right) + 2b^2cd\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right) + 2b^2c^2dx$$

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] 2*b^2*c^2*d*x - 2*b*c*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]) + 2*c^2*d*x*
(a + b*ArcSinh[c*x])^2 - (d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/x - 4*b*c
*d*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - 2*b^2*c*d*PolyLog[2, -E^A
rcSinh[c*x]] + 2*b^2*c*d*PolyLog[2, E^ArcSinh[c*x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))^2}{x}+(2bcd)\int\frac{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{x}\,dx\\
&\quad+\left(2c^2d\right)\int(a+b\operatorname{arcsinh}(cx))^2\,dx\\
&=2bcd\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))+2c^2dx(a+b\operatorname{arcsinh}(cx))^2\\
&\quad-\frac{d(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))^2}{x}+(2bcd)\int\frac{a+b\operatorname{arcsinh}(cx)}{x\sqrt{1+c^2x^2}}\,dx\\
&\quad-\left(2b^2c^2d\right)\int 1\,dx-\left(4bc^3d\right)\int\frac{x(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx
\end{aligned}$$

$$= -2b^2c^2dx - 2bcd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) + 2c^2dx(a+b\text{arcsinh}(cx))^2 - \frac{d(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{x} + (2bcd)\text{Subst}\left(\int (a+bx)\text{csch}(x)\,dx, x, \text{arcsinh}(cx)\right) + \left(4b^2c^2d\right)\int 1\,dx$$

$$= 2b^2c^2dx - 2bcd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) + 2c^2dx(a+b\text{arcsinh}(cx))^2 - \frac{d(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{x} - 4bcd(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) - \left(2b^2cd\right)\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \text{arcsinh}(cx)\right) + \left(2b^2cd\right)\text{Subst}\left(\int \log\left(1+e^x\right)\,dx, x, \text{arcsinh}(cx)\right)$$

$$= 2b^2c^2dx - 2bcd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) + 2c^2dx(a+b\text{arcsinh}(cx))^2 - \frac{d(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{x} - 4bcd(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) - \left(2b^2cd\right)\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right) + \left(2b^2cd\right)\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)$$

$$= 2b^2c^2dx - 2bcd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) + 2c^2dx(a+b\text{arcsinh}(cx))^2 - \frac{d(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{x} - 4bcd(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) - 2b^2cd\,\text{PolyLog}\left(2, -e^{\text{arcsinh}(cx)}\right) + 2b^2cd\,\text{PolyLog}\left(2, e^{\text{arcsinh}(cx)}\right)$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.47

$$\int \frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \frac{d\left(-a^2+a^2c^2x^2+2abcx\left(-\sqrt{1+c^2x^2}+cx\text{arcsinh}(cx)\right)+b^2cx\left(2cx-2\sqrt{1+c^2x^2}\text{arcsinh}(cx)+cx\text{arcsin}\right.\right.}{}$$

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (d*(-a^2 + a^2*c^2*x^2 + 2*a*b*c*x*(-Sqrt[1 + c^2*x^2] + c*x*ArcSinh[c*x])
+ b^2*c*x*(2*c*x - 2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + c*x*ArcSinh[c*x]^2) -
 2*a*b*(ArcSinh[c*x] + c*x*ArcTanh[Sqrt[1 + c^2*x^2]]) - b^2*(ArcSinh[c*x]*
(ArcSinh[c*x] + 2*c*x*(-Log[1 - E^(-ArcSinh[c*x])] + Log[1 + E^(-ArcSinh[c*
x])])) - 2*c*x*PolyLog[2, -E^(-ArcSinh[c*x])] + 2*c*x*PolyLog[2, E^(-ArcSin
h[c*x])]))))/x
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.82

method	result
derivativedivides	$c\left(d\,a^2\left(cx-\frac{1}{cx}\right)+d\,b^2\,\text{arcsinh}\,(cx)^2\,cx-2d\,b^2\,\text{arcsinh}\,(cx)\,\sqrt{c^2x^2+1}+2d\,b^2cx-\frac{d\,b^2\,\text{arc}}{}\right.$
default	$c\left(d\,a^2\left(cx-\frac{1}{cx}\right)+d\,b^2\,\text{arcsinh}\,(cx)^2\,cx-2d\,b^2\,\text{arcsinh}\,(cx)\,\sqrt{c^2x^2+1}+2d\,b^2cx-\frac{d\,b^2\,\text{arc}}{}\right.$
parts	$d\,a^2\left(c^2x-\frac{1}{x}\right)+d\,b^2c^2\,\text{arcsinh}\,(cx)^2\,x-2d\,b^2c\,\text{arcsinh}\,(cx)\,\sqrt{c^2x^2+1}+2b^2c^2dx-\frac{d\,b^2\,\text{ar}}{}$

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(d*a^2*(c*x-1/c/x)+d*b^2*arcsinh(c*x)^2*c*x-2*d*b^2*arcsinh(c*x)*(c^2*x^2
+1)^(1/2)+2*d*b^2*c*x-d*b^2*arcsinh(c*x)^2/c/x-2*d*b^2*arcsinh(c*x)*ln(1+c*
x+(c^2*x^2+1)^(1/2))-2*d*b^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*d*b^2*arcs
inh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*d*b^2*polylog(2,c*x+(c^2*x^2+1)^(1/2
))+2*d*a*b*(arcsinh(c*x)*c*x-arcsinh(c*x)/c/x-(c^2*x^2+1)^(1/2)-arctanh(1/(
c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2}{x^2}\,dx=\int \frac{(c^2dx^2+d)(b\,\text{arsinh}\,(cx)+a)^2}{x^2}\,dx$$

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))/x^2, x)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2}{x^2}\,dx=d\left(\int a^2c^2\,dx+\int \frac{a^2}{x^2}\,dx+\int b^2c^2\,\text{asinh}^2\,(cx)\,dx+\int \frac{b^2\,\text{asinh}^2\,(cx)}{x^2}\,dx+\int 2abc^2\,\text{asinh}\,(cx)\,dx+\int \frac{2ab\,\text{asinh}\,(cx)}{x^2}\,dx\right)$$

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**2/x**2,x)
```

```
[Out] d*(Integral(a**2*c**2, x) + Integral(a**2/x**2, x) + Integral(b**2*c**2*asi
nh(c*x)**2, x) + Integral(b**2*asinh(c*x)**2/x**2, x) + Integral(2*a*b*c**2
*asinh(c*x), x) + Integral(2*a*b*asinh(c*x)/x**2, x))
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)\,(a+b\operatorname{arcsinh}(cx))^2}{x^2}\,dx = \int \frac{(c^2dx^2+d)(b\operatorname{arsinh}(cx)+a)^2}{x^2}\,dx$$

[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="maxima")

[Out] b^2*c^2*d*x*arcsinh(c*x)^2 + 2*b^2*c^2*d*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*c^2*d*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*c*d - 2*(c*arcsinh(1/(c*abs(x))) + arcsinh(c*x)/x)*a*b*d - b^2*d*(log(c*x + sqrt(c^2*x^2 + 1))^2/x - integrate(2*(c^3*x^2 + sqrt(c^2*x^2 + 1)*c^2*x + c)*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*x^4 + c*x^2 + (c^2*x^3 + x)*sqrt(c^2*x^2 + 1)), x)) - a^2*d/x

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)\,(a+b\operatorname{arcsinh}(cx))^2}{x^2}\,dx = \text{Exception raised: TypeError}$$

[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)\,(a+b\operatorname{arcsinh}(cx))^2}{x^2}\,dx = \int \frac{(a+b\operatorname{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)}{x^2}\,dx$$

[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x^2,x)

[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x^2, x)

3.205 $\int \frac{(d+c^2dx^2)(a+b\,\mathbf{arcsinh}(cx))^2}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 180

$$\begin{aligned}\int \frac{(d+c^2dx^2)\,(a+b\,\mathrm{arcsinh}(cx))^2}{x^3}\,dx = &-\frac{bcd\sqrt{1+c^2x^2}(a+b\,\mathrm{arcsinh}(cx))}{x} \\ &+\frac{1}{2}c^2d(a+b\,\mathrm{arcsinh}(cx))^2 \\ &-\frac{d(1+c^2x^2)\,(a+b\,\mathrm{arcsinh}(cx))^2}{2x^2} \\ &+\frac{c^2d(a+b\,\mathrm{arcsinh}(cx))^3}{3b} \\ &+c^2d(a+b\,\mathrm{arcsinh}(cx))^2\log\left(1-e^{-2\mathrm{arcsinh}(cx)}\right) \\ &+b^2c^2d\log(x) \\ &-bc^2d(a+b\,\mathrm{arcsinh}(cx))\,\mathrm{PolyLog}\left(2,e^{-2\mathrm{arcsinh}(cx)}\right) \\ &-\frac{1}{2}b^2c^2d\,\mathrm{PolyLog}\left(3,e^{-2\mathrm{arcsinh}(cx)}\right)\end{aligned}$$

```
[Out] 1/2*c^2*d*(a+b*arcsinh(c*x))^2-1/2*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/x^2+1
/3*c^2*d*(a+b*arcsinh(c*x))^3/b+c^2*d*(a+b*arcsinh(c*x))^2*ln(1-1/(c*x+(c^2
*x^2+1)^(1/2))^2)+b^2*c^2*d*ln(x)-b*c^2*d*(a+b*arcsinh(c*x))*polylog(2,1/(c
*x+(c^2*x^2+1)^(1/2))^2)-1/2*b^2*c^2*d*polylog(3,1/(c*x+(c^2*x^2+1)^(1/2))^
2)-b*c*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5807, 5775, 3797, 2221, 2611, 2320, 6724, 5805, 29, 5783}

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^3}\,dx = -bc^2d\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)) - \frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{2x^2} - \frac{bcd\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x} + \frac{c^2d(a+b\text{arcsinh}(cx))^3}{3b} + \frac{1}{2}c^2d(a+b\text{arcsinh}(cx))^2 + c^2d\log\left(1-e^{-2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2 - \frac{1}{2}b^2c^2d\,\text{PolyLog}\left(3,e^{-2\text{arcsinh}(cx)}\right) + b^2c^2d\log(x)$$

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] -((b*c*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/x) + (c^2*d*(a + b*ArcSinh
[c*x])^2)/2 - (d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(2*x^2) + (c^2*d*(a
+ b*ArcSinh[c*x])^3)/(3*b) + c^2*d*(a + b*ArcSinh[c*x])^2*Log[1 - E^(-2*Arc
Sinh[c*x])] + b^2*c^2*d*Log[x] - b*c^2*d*(a + b*ArcSinh[c*x])*PolyLog[2, E^
(-2*ArcSinh[c*x])] - (b^2*c^2*d*PolyLog[3, E^(-2*ArcSinh[c*x])])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]], Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x]
, x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], In
t[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Free
Q[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
```

```
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{2x^2}+(bcd)\int\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{x^2}\,dx \\
&\quad+\left(c^2d\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{x}\,dx \\
&= -\frac{bcd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{x}-\frac{d(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{2x^2} \\
&\quad-\frac{(c^2d)\,\text{Subst}\left(\int x^2\coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{b} \\
&\quad+\left(b^2c^2d\right)\int\frac{1}{x}\,dx+\left(bc^3d\right)\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx \\
&= -\frac{bcd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{x}+\frac{1}{2}c^2d(a+b\text{arcsinh}(cx))^2 \\
&\quad-\frac{d(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{2x^2}+\frac{c^2d(a+b\text{arcsinh}(cx))^3}{3b} \\
&\quad+b^2c^2d\log(x)+\frac{(2c^2d)\,\text{Subst}\left(\int\frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x^2}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b} \\
&= -\frac{bcd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{x}+\frac{1}{2}c^2d(a+b\text{arcsinh}(cx))^2 \\
&\quad-\frac{d(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{2x^2}+\frac{c^2d(a+b\text{arcsinh}(cx))^3}{3b} \\
&\quad+c^2d(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right)+b^2c^2d\log(x) \\
&\quad-\left(2c^2d\right)\text{Subst}\left(\int x\log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{bcd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{x} + \frac{1}{2}c^2d(a+b\text{arcsinh}(cx))^2 \\
&\quad - \frac{d(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{2x^2} + \frac{c^2d(a+b\text{arcsinh}(cx))^3}{3b} \\
&\quad + c^2d(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right) + b^2c^2d\log(x) \\
&\quad - bc^2d(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right) \\
&\quad + \left(bc^2d\right)\text{Subst}\left(\int \text{PolyLog}\left(2,e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx,x,a+b\text{arcsinh}(cx)\right) \\
&= -\frac{bcd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{x} + \frac{1}{2}c^2d(a+b\text{arcsinh}(cx))^2 \\
&\quad - \frac{d(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{2x^2} + \frac{c^2d(a+b\text{arcsinh}(cx))^3}{3b} \\
&\quad + c^2d(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right) + b^2c^2d\log(x) \\
&\quad - bc^2d(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right) \\
&\quad - \frac{1}{2}\left(b^2c^2d\right)\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right) \\
&= -\frac{bcd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{x} + \frac{1}{2}c^2d(a+b\text{arcsinh}(cx))^2 \\
&\quad - \frac{d(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{2x^2} + \frac{c^2d(a+b\text{arcsinh}(cx))^3}{3b} \\
&\quad + c^2d(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right) + b^2c^2d\log(x) \\
&\quad - bc^2d(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right) \\
&\quad - \frac{1}{2}b^2c^2d\,\text{PolyLog}\left(3,e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.34

$$\int \frac{(d+c^2dx^2)(a+b\operatorname{arcsinh}(cx))^2}{x^3}\,dx = -\frac{a^2d}{2x^2}+2abc^2d\left(-\frac{\sqrt{1+c^2x^2}}{2cx}-\frac{\operatorname{arcsinh}(cx)}{2c^2x^2}\right)+a^2c^2d\log(x)+b^2c^2d\left(-\frac{\sqrt{1+c^2x^2}\operatorname{arcsinh}(cx)}{cx}-\frac{\operatorname{arcsinh}(cx)^2}{2c^2x^2}+\log(cx)\right)+2abc^2d\left(-\frac{1}{2}\operatorname{arcsinh}(cx)^2+\operatorname{arcsinh}(cx)\log\left(1-e^{2\operatorname{arcsinh}(cx)}\right)+\frac{1}{2}\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(cx)}\right)\right)+b^2c^2d\left(-\frac{1}{3}\operatorname{arcsinh}(cx)^3+\operatorname{arcsinh}(cx)^2\log\left(1-e^{2\operatorname{arcsinh}(cx)}\right)+\operatorname{arcsinh}(cx)\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(cx)}\right)-\frac{1}{2}\operatorname{PolyLog}\left(3,e^{2\operatorname{arcsinh}(cx)}\right)\right)$$

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] -1/2*(a^2*d)/x^2 + 2*a*b*c^2*d*(-1/2*Sqrt[1 + c^2*x^2]/(c*x) - ArcSinh[c*x]
/(2*c^2*x^2)) + a^2*c^2*d*Log[x] + b^2*c^2*d*(-((Sqrt[1 + c^2*x^2]*ArcSinh[
c*x])/(c*x)) - ArcSinh[c*x]^2/(2*c^2*x^2) + Log[c*x]) + 2*a*b*c^2*d*(-1/2*A
rcSinh[c*x]^2 + ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])] + PolyLog[2, E^(2*
ArcSinh[c*x])]/2) + b^2*c^2*d*(-1/3*ArcSinh[c*x]^3 + ArcSinh[c*x]^2*Log[1 -
 E^(2*ArcSinh[c*x])] + ArcSinh[c*x]*PolyLog[2, E^(2*ArcSinh[c*x])] - PolyLo
g[3, E^(2*ArcSinh[c*x])]/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(197) = 394.

Time = 0.20 (sec) , antiderivative size = 404, normalized size of antiderivative = 2.24

method	result
derivativedivides	$c^2\left(d\,a^2\left(\ln(cx)-\frac{1}{2c^2x^2}\right)+d\,b^2\left(-\frac{\operatorname{arcsinh}(cx)^3}{3}-\frac{\operatorname{arcsinh}(cx)\left(-2c^2x^2+2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)\right)}{2c^2x^2}+\ln\right.\right.$
default	$c^2\left(d\,a^2\left(\ln(cx)-\frac{1}{2c^2x^2}\right)+d\,b^2\left(-\frac{\operatorname{arcsinh}(cx)^3}{3}-\frac{\operatorname{arcsinh}(cx)\left(-2c^2x^2+2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)\right)}{2c^2x^2}+\ln\right.\right.$
parts	$d\,a^2\left(-\frac{1}{2x^2}+c^2\ln(x)\right)+d\,b^2c^2\left(-\frac{\operatorname{arcsinh}(cx)^3}{3}-\frac{\operatorname{arcsinh}(cx)\left(-2c^2x^2+2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)\right)}{2c^2x^2}+\ln\right.$

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(d*a^2*(ln(c*x)-1/2/c^2/x^2)+d*b^2*(-1/3*arcsinh(c*x)^3-1/2*arcsinh(c*x
)*(-2*c^2*x^2+2*c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x))/c^2/x^2+ln(1+c*x+(c^2*x
^2+1)^(1/2))-2*ln(c*x+(c^2*x^2+1)^(1/2))+ln(c*x+(c^2*x^2+1)^(1/2)-1)+arcsin
h(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2
+1)^(1/2))-2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))+arcsinh(c*x)^2*ln(1-c*x-(c^2
*x^2+1)^(1/2))+2*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*polylog(3,
c*x+(c^2*x^2+1)^(1/2)))+2*d*a*b*(-1/2*arcsinh(c*x)^2-1/2*(c*x*(c^2*x^2+1)^(
1/2)-c^2*x^2+arcsinh(c*x))/c^2/x^2+arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))
+polylog(2,-c*x-(c^2*x^2+1)^(1/2))+arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))
+polylog(2,c*x+(c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)\,(a+b\mathrm{arcsinh}(cx))^2}{x^3}\,dx=\int \frac{(c^2dx^2+d)(b\operatorname{arsinh}(cx)+a)^2}{x^3}\,dx$$

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)\,(a+b\mathrm{arcsinh}(cx))^2}{x^3}\,dx=d\left(\int \frac{a^2}{x^3}\,dx+\int \frac{a^2c^2}{x}\,dx+\int \frac{b^2\operatorname{asinh}^2(cx)}{x^3}\,dx+\int \frac{2ab\operatorname{asinh}(cx)}{x^3}\,dx+\int \frac{b^2c^2\operatorname{asinh}^2(cx)}{x}\,dx+\int \frac{2abc^2\operatorname{asinh}(cx)}{x}\,dx\right)$$

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**2/x**3,x)
```

[Out] d*(Integral(a**2/x**3, x) + Integral(a**2*c**2/x, x) + Integral(b**2*asinh(c*x)**2/x**3, x) + Integral(2*a*b*asinh(c*x)/x**3, x) + Integral(b**2*c**2*asinh(c*x)**2/x, x) + Integral(2*a*b*c**2*asinh(c*x)/x, x))

Maxima [F]

$$\int \frac{(d+c^2dx^2)\,(a+b\mathrm{arcsinh}(cx))^2}{x^3}\,dx = \int \frac{(c^2dx^2+d)(b\,\mathrm{arsinh}\,(cx)+a)^2}{x^3}\,dx$$

[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="maxima")

[Out] a^2*c^2*d*log(x) - a*b*d*(sqrt(c^2*x^2 + 1)*c/x + arcsinh(c*x)/x^2) - 1/2*a^2*d/x^2 + integrate(b^2*c^2*d*log(c*x + sqrt(c^2*x^2 + 1))^2/x + 2*a*b*c^2*d*log(c*x + sqrt(c^2*x^2 + 1))/x + b^2*d*log(c*x + sqrt(c^2*x^2 + 1))^2/x^3, x)

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)\,(a+b\mathrm{arcsinh}(cx))^2}{x^3}\,dx = \text{Exception raised: TypeError}$$

[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)\,(a+b\mathrm{arcsinh}(cx))^2}{x^3}\,dx = \int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)}{x^3}\,dx$$

[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x^3,x)

[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x^3, x)

3.206 $\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 158

$$\begin{aligned}\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = &-\frac{b^2c^2d}{3x}-\frac{bcd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3x^2}\\&-\frac{2c^2d(a+b\text{arcsinh}(cx))^2}{3x}\\&-\frac{d(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3x^3}\\&-\frac{10}{3}bc^3d(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)\\&-\frac{5}{3}b^2c^3d\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)\\&+\frac{5}{3}b^2c^3d\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\end{aligned}$$

```
[Out] -1/3*b^2*c^2*d/x-2/3*c^2*d*(a+b*arcsinh(c*x))^2/x-1/3*d*(c^2*x^2+1)*(a+b*ar
csinh(c*x))^2/x^3-10/3*b*c^3*d*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(
1/2))-5/3*b^2*c^3*d*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+5/3*b^2*c^3*d*polylog
(2,c*x+(c^2*x^2+1)^(1/2))-1/3*b*c*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/x^
2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5807, 5776, 5816, 4267, 2317, 2438, 5805, 30}

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = -\frac{10}{3}bc^3d\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)) - \frac{bcd\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3x^2} - \frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{3x^3} - \frac{2c^2d(a+b\text{arcsinh}(cx))^2}{3x} - \frac{5}{3}b^2c^3d\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right) + \frac{5}{3}b^2c^3d\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right) - \frac{b^2c^2d}{3x}$$

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] -1/3*(b^2*c^2*d)/x - (b*c*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*x^2)
 - (2*c^2*d*(a + b*ArcSinh[c*x])^2)/(3*x) - (d*(1 + c^2*x^2)*(a + b*ArcSinh
[c*x])^2)/(3*x^3) - (10*b*c^3*d*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]
])/3 - (5*b^2*c^3*d*PolyLog[2, -E^ArcSinh[c*x]])/3 + (5*b^2*c^3*d*PolyLog[2
, E^ArcSinh[c*x]])/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
```

```
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]], Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x]
, x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], In
t[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Free
Q[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\frac{d(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3x^3}+\frac{1}{3}(2bcd)\int\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{x^3}\,dx \\ +\frac{1}{3}\left(2c^2d\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$$

$$= -\frac{bcd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3x^2} - \frac{2c^2d(a+b\text{arcsinh}(cx))^2}{3x} - \frac{d(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3x^3} + \frac{1}{3}(b^2c^2d)\int \frac{1}{x^2}\,dx + \frac{1}{3}(bc^3d)\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{1+c^2x^2}}\,dx + \frac{1}{3}(4bc^3d)\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{1+c^2x^2}}\,dx$$

$$= -\frac{b^2c^2d}{3x} - \frac{bcd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3x^2} - \frac{2c^2d(a+b\text{arcsinh}(cx))^2}{3x} - \frac{d(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3x^3} + \frac{1}{3}(bc^3d)\,\text{Subst}\left(\int (a+bx)\text{csch}(x)\,dx, x, \text{arcsinh}(cx)\right) + \frac{1}{3}(4bc^3d)\,\text{Subst}\left(\int (a+bx)\text{csch}(x)\,dx, x, \text{arcsinh}(cx)\right)$$

$$= -\frac{b^2c^2d}{3x} - \frac{bcd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3x^2} - \frac{2c^2d(a+b\text{arcsinh}(cx))^2}{3x} - \frac{d(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3x^3} - \frac{10}{3}bc^3d(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) - \frac{1}{3}(b^2c^3d)\,\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \text{arcsinh}(cx)\right) + \frac{1}{3}(b^2c^3d)\,\text{Subst}\left(\int \log\left(1+e^x\right)\,dx, x, \text{arcsinh}(cx)\right) - \frac{1}{3}(4b^2c^3d)\,\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \text{arcsinh}(cx)\right) + \frac{1}{3}(4b^2c^3d)\,\text{Subst}\left(\int \log\left(1+e^x\right)\,dx, x, \text{arcsinh}(cx)\right)$$

$$= -\frac{b^2c^2d}{3x} - \frac{bcd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3x^2} - \frac{2c^2d(a+b\text{arcsinh}(cx))^2}{3x} - \frac{d(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3x^3} - \frac{10}{3}bc^3d(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) - \frac{1}{3}(b^2c^3d)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right) + \frac{1}{3}(b^2c^3d)\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right) - \frac{1}{3}(4b^2c^3d)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right) + \frac{1}{3}(4b^2c^3d)\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)$$

$$= -\frac{b^2c^2d}{3x} - \frac{bcd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3x^2} - \frac{2c^2d(a+b\text{arcsinh}(cx))^2}{3x} - \frac{d(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3x^3} - \frac{10}{3}bc^3d(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) - \frac{5}{3}b^2c^3d\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right) + \frac{5}{3}b^2c^3d\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.55

$$\int \frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = -\frac{d\big(a^2+3a^2c^2x^2+b^2c^2x^2+abcx\sqrt{1+c^2x^2}+2ab\text{arcsinh}(cx)+6abc^2x^2\text{arcsinh}(cx)+b^2cx\sqrt{1+c^2x^2}\text{arcs}}{}$$

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] -1/3*(d*(a^2 + 3*a^2*c^2*x^2 + b^2*c^2*x^2 + a*b*c*x*Sqrt[1 + c^2*x^2] + 2*
a*b*ArcSinh[c*x] + 6*a*b*c^2*x^2*ArcSinh[c*x] + b^2*c*x*Sqrt[1 + c^2*x^2]*A
rcSinh[c*x] + b^2*ArcSinh[c*x]^2 + 3*b^2*c^2*x^2*ArcSinh[c*x]^2 + 5*a*b*c^3
*x^3*ArcTanh[Sqrt[1 + c^2*x^2]] - 5*b^2*c^3*x^3*ArcSinh[c*x]*Log[1 - E^(-Ar
cSinh[c*x])] + 5*b^2*c^3*x^3*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 5*b^
2*c^3*x^3*PolyLog[2, -E^(-ArcSinh[c*x])] + 5*b^2*c^3*x^3*PolyLog[2, E^(-Arc
Sinh[c*x])]))/x^3
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.52

method	result
parts	$d\,a^2\left(-\frac{c^2}{x}-\frac{1}{3x^3}\right)+d\,b^2c^3\left(-\frac{3\,\text{arcsinh}(cx)^2x^2c^2+\text{arcsinh}(cx)cx\sqrt{c^2x^2+1}+\text{arcsinh}(cx)^2+c^2x^2}{3c^3x^3}-\frac{5\,\text{arcsinh}(c}{}\right.$
derivativedivides	$c^3\left(d\,a^2\left(-\frac{1}{3c^3x^3}-\frac{1}{cx}\right)+d\,b^2\left(-\frac{3\,\text{arcsinh}(cx)^2x^2c^2+\text{arcsinh}(cx)cx\sqrt{c^2x^2+1}+\text{arcsinh}(cx)^2+c^2x^2}{3c^3x^3}-\frac{5\,\text{arcsin}}{}\right.\right.$
default	$c^3\left(d\,a^2\left(-\frac{1}{3c^3x^3}-\frac{1}{cx}\right)+d\,b^2\left(-\frac{3\,\text{arcsinh}(cx)^2x^2c^2+\text{arcsinh}(cx)cx\sqrt{c^2x^2+1}+\text{arcsinh}(cx)^2+c^2x^2}{3c^3x^3}-\frac{5\,\text{arcsin}}{}\right.\right.$

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] d*a^2*(-c^2/x-1/3/x^3)+d*b^2*c^3*(-1/3*(3*arcsinh(c*x)^2*x^2*c^2+arcsinh(c*
x)*c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x)^2+c^2*x^2)/c^3/x^3-5/3*arcsinh(c*x)*l
```

```
n(1+c*x+(c^2*x^2+1)^(1/2))-5/3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+5/3*arcsin
h(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+5/3*polylog(2,c*x+(c^2*x^2+1)^(1/2)))+2*
d*a*b*c^3*(-1/3*arcsinh(c*x)/c^3/x^3-arcsinh(c*x)/c/x-1/6/c^2/x^2*(c^2*x^2+
1)^(1/2)-5/6*arctanh(1/(c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^4}\,dx=\int \frac{(c^2dx^2+d)(b\,\text{arsinh}\,(cx)+a)^2}{x^4}\,dx$$

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="fricas")
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))/x^4, x)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^4}\,dx=d\left(\int \frac{a^2}{x^4}\,dx+\int \frac{a^2c^2}{x^2}\,dx+\int \frac{b^2\,\text{asinh}^2\,(cx)}{x^4}\,dx+\int \frac{2ab\,\text{asinh}\,(cx)}{x^4}\,dx+\int \frac{b^2c^2\,\text{asinh}^2\,(cx)}{x^2}\,dx+\int \frac{2abc^2\,\text{asinh}\,(cx)}{x^2}\,dx\right)$$

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**2/x**4,x)
[Out] d*(Integral(a**2/x**4, x) + Integral(a**2*c**2/x**2, x) + Integral(b**2*asi
nh(c*x)**2/x**4, x) + Integral(2*a*b*asinh(c*x)/x**4, x) + Integral(b**2*c*
*2*asinh(c*x)**2/x**2, x) + Integral(2*a*b*c**2*asinh(c*x)/x**2, x))
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^4}\,dx=\int \frac{(c^2dx^2+d)(b\,\text{arsinh}\,(cx)+a)^2}{x^4}\,dx$$

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="maxima")
[Out] -2*(c*arcsinh(1/(c*abs(x))) + arcsinh(c*x)/x)*a*b*c^2*d + 1/3*((c^2*arcsinh
(1/(c*abs(x))) - sqrt(c^2*x^2 + 1)/x^2)*c - 2*arcsinh(c*x)/x^3)*a*b*d - a^2
*c^2*d/x - 1/3*a^2*d/x^3 - 1/3*(3*b^2*c^2*d*x^2 + b^2*d)*log(c*x + sqrt(c^2
*x^2 + 1))^2/x^3 + integrate(2/3*(3*b^2*c^5*d*x^4 + 4*b^2*c^3*d*x^2 + b^2*c
*d + (3*b^2*c^4*d*x^3 + b^2*c^2*d*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*
x^2 + 1))/(c^3*x^6 + c*x^4 + (c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + c^2dx^2)(a + b\text{arcsinh}(cx))^2}{x^4} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + c^2dx^2)(a + b\text{arcsinh}(cx))^2}{x^4} dx = \int \frac{(a + b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2 + d)}{x^4} dx$$

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x^4, x)
```

3.207 $\int x^4(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2 dx$

Optimal result

Integrand size = 26, antiderivative size = 386

$$\begin{aligned}\int x^4(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2 dx &= \frac{4208b^2d^2x}{99225c^4}-\frac{2104b^2d^2x^3}{297675c^2}+\frac{526b^2d^2x^5}{165375}\\
&+\frac{212b^2c^2d^2x^7}{27783}+\frac{2}{729}b^2c^4d^2x^9-\frac{128bd^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{4725c^5}\\
&+\frac{64bd^2x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{4725c^3}-\frac{16bd^2x^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{1575c}\\
&-\frac{8bd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{189c^5}+\frac{2bd^2(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{315c^5}\\
&+\frac{20bd^2(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{441c^5}-\frac{2bd^2(1+c^2x^2)^{9/2}(a+b\text{arcsinh}(cx))}{81c^5}\\
&+\frac{8}{315}d^2x^5(a+b\text{arcsinh}(cx))^2+\frac{4}{63}d^2x^5(1+c^2x^2)(a+b\text{arcsinh}(cx))^2+\frac{1}{9}d^2x^5(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2\end{aligned}$$

```
[Out] 4208/99225*b^2*d^2*x/c^4-2104/297675*b^2*d^2*x^3/c^2+526/165375*b^2*d^2*x^5
+212/27783*b^2*c^2*d^2*x^7+2/729*b^2*c^4*d^2*x^9-8/189*b*d^2*(c^2*x^2+1)^(3
/2)*(a+b*arcsinh(c*x))/c^5+2/315*b*d^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))
/c^5+20/441*b*d^2*(c^2*x^2+1)^(7/2)*(a+b*arcsinh(c*x))/c^5-2/81*b*d^2*(c^2*
x^2+1)^(9/2)*(a+b*arcsinh(c*x))/c^5+8/315*d^2*x^5*(a+b*arcsinh(c*x))^2+4/63
*d^2*x^5*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+1/9*d^2*x^5*(c^2*x^2+1)^2*(a+b*ar
csinh(c*x))^2-128/4725*b*d^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^5+64/47
25*b*d^2*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-16/1575*b*d^2*x^4*(a+
b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {5808, 5776, 5812, 5798, 8, 30, 272, 45, 5804, 12, 1167}

$$\int x^4 \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx=\frac{1}{9}d^2x^5\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2+\frac{4}{63}d^2x^5\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2-\frac{16bd^2x^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{1575c}-\frac{2bd^2(c^2x^2+1)^{9/2}(a+b\text{arcsinh}(cx))}{81c^5}+\frac{20bd^2(c^2x^2+1)^{7/2}(a+b\text{arcsinh}(cx))}{441c^5}+\frac{2bd^2(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))}{315c^5}-\frac{8bd^2(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{189c^5}-\frac{128bd^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{4725c^5}+\frac{64bd^2x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{4725c^3}+\frac{8}{315}d^2x^5(a+b\text{arcsinh}(cx))^2+\frac{2}{729}b^2c^4d^2x^9+\frac{4208b^2d^2x}{99225c^4}+\frac{212b^2c^2d^2x^7}{27783}-\frac{2104b^2d^2x^3}{297675c^2}+\frac{526b^2d^2x^5}{165375}$$

```
[In] Int[x^4*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (4208*b^2*d^2*x)/(99225*c^4) - (2104*b^2*d^2*x^3)/(297675*c^2) + (526*b^2*d
^2*x^5)/165375 + (212*b^2*c^2*d^2*x^7)/27783 + (2*b^2*c^4*d^2*x^9)/729 - (1
28*b*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(4725*c^5) + (64*b*d^2*x^2
*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(4725*c^3) - (16*b*d^2*x^4*Sqrt[1
+ c^2*x^2]*(a + b*ArcSinh[c*x]))/(1575*c) - (8*b*d^2*(1 + c^2*x^2)^(3/2)*(a
 + b*ArcSinh[c*x]))/(189*c^5) + (2*b*d^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh
[c*x]))/(315*c^5) + (20*b*d^2*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(44
1*c^5) - (2*b*d^2*(1 + c^2*x^2)^(9/2)*(a + b*ArcSinh[c*x]))/(81*c^5) + (8*d
^2*x^5*(a + b*ArcSinh[c*x])^2)/315 + (4*d^2*x^5*(1 + c^2*x^2)*(a + b*ArcSin
h[c*x])^2)/63 + (d^2*x^5*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/9
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{9}d^2x^5\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2+\frac{1}{9}(4d)\int x^4\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))^2\,dx$$
$$-\frac{1}{9}\left(2bcd^2\right)\int x^5\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx$$
$$=-\frac{2bd^2(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{45c^5}$$
$$+\frac{4bd^2(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{63c^5}-\frac{2bd^2(1+c^2x^2)^{9/2}(a+b\text{arcsinh}(cx))}{81c^5}$$
$$+\frac{4}{63}d^2x^5\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2+\frac{1}{9}d^2x^5\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2+\frac{1}{63}\left(8d^2\right)\int x^4(a+ba$$
$$=-\frac{8bd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{189c^5}+\frac{2bd^2(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{315c^5}$$
$$+\frac{20bd^2(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{441c^5}-\frac{2bd^2(1+c^2x^2)^{9/2}(a+b\text{arcsinh}(cx))}{81c^5}$$
$$+\frac{8}{315}d^2x^5(a+b\text{arcsinh}(cx))^2+\frac{4}{63}d^2x^5\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2+\frac{1}{9}d^2x^5\left(1+c^2x^2\right)^2(a+b\text{arcsinh}$$

$$= -\frac{16bd^2x^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{1575c} - \frac{8bd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{189c^5} + \frac{2bd^2(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{315c^5} + \frac{20bd^2(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{441c^5} - \frac{2bd^2(1+c^2x^2)^{9/2}(a+b\text{arcsinh}(cx))}{81c^5} + \frac{8}{315}d^2x^5(a+b\text{arcsinh}(cx))^2 + \frac{4}{63}d^2x^5(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 + \frac{1}{9}d^2x^5(1+c^2x^2)^2(a+b\text{arcsin}$$

$$= \frac{304b^2d^2x}{19845c^4} - \frac{152b^2d^2x^3}{59535c^2} + \frac{526b^2d^2x^5}{165375} + \frac{212b^2c^2d^2x^7}{27783} + \frac{2}{729}b^2c^4d^2x^9 + \frac{64bd^2x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{4725c^3} - \frac{16bd^2x^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{1575c} - \frac{8bd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{189c^5} + \frac{2bd^2(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{315c^5} + \frac{20bd^2(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{441c^5} - \frac{2bd^2(1+c^2x^2)^{9/2}(a+b\text{arcsinh}(cx))}{81c^5} + \frac{8}{315}d^2x^5(a+b\text{arcsinh}(cx))^2 + \frac{4}{63}d^2x^5(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 + \frac{1}{9}d^2x^5(1+c^2x^2)^2(a+b\text{arcsin}$$

$$= \frac{304b^2d^2x}{19845c^4} - \frac{2104b^2d^2x^3}{297675c^2} + \frac{526b^2d^2x^5}{165375} + \frac{212b^2c^2d^2x^7}{27783} + \frac{2}{729}b^2c^4d^2x^9 - \frac{128bd^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{4725c^5} + \frac{64bd^2x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{4725c^3} - \frac{16bd^2x^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{1575c} - \frac{8bd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{189c^5} + \frac{2bd^2(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{315c^5} + \frac{20bd^2(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{441c^5} - \frac{2bd^2(1+c^2x^2)^{9/2}(a+b\text{arcsinh}(cx))}{81c^5} + \frac{8}{315}d^2x^5(a+b\text{arcsinh}(cx))^2 + \frac{4}{63}d^2x^5(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 + \frac{1}{9}d^2x^5(1+c^2x^2)^2(a+b\text{arcsin}$$

$$
\begin{aligned}
&= \frac{4208b^2d^2x}{99225c^4} - \frac{2104b^2d^2x^3}{297675c^2} + \frac{526b^2d^2x^5}{165375} + \frac{212b^2c^2d^2x^7}{27783} \\
&\quad + \frac{2}{729}b^2c^4d^2x^9 - \frac{128bd^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{4725c^5} \\
&\quad + \frac{64bd^2x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{4725c^3} - \frac{16bd^2x^4\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{1575c} \\
&\quad - \frac{8bd^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{189c^5} + \frac{2bd^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{315c^5} \\
&\quad + \frac{20bd^2(1+c^2x^2)^{7/2}(a+b\operatorname{arcsinh}(cx))}{441c^5} - \frac{2bd^2(1+c^2x^2)^{9/2}(a+b\operatorname{arcsinh}(cx))}{81c^5} \\
&\quad + \frac{8}{315}d^2x^5(a+b\operatorname{arcsinh}(cx))^2 + \frac{4}{63}d^2x^5(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{9}d^2x^5(1+c^2x^2)^2(a+b\operatorname{arcsinh}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.65

$$
\int x^4(d+c^2dx^2)^2(a+b\operatorname{arcsinh}(cx))^2\,dx
$$

$$
= \frac{d^2\left(99225a^2c^5x^5(63+90c^2x^2+35c^4x^4) - 630ab\sqrt{1+c^2x^2}(2104-1052c^2x^2+789c^4x^4+2650c^6x^6+1225c\right.}{}
$$

```
[In] Integrate[x^4*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*(99225*a^2*c^5*x^5*(63 + 90*c^2*x^2 + 35*c^4*x^4) - 630*a*b*Sqrt[1 + c
^2*x^2]*(2104 - 1052*c^2*x^2 + 789*c^4*x^4 + 2650*c^6*x^6 + 1225*c^8*x^8) +
 2*b^2*c*x*(662760 - 110460*c^2*x^2 + 49707*c^4*x^4 + 119250*c^6*x^6 + 4287
5*c^8*x^8) - 630*b*(-315*a*c^5*x^5*(63 + 90*c^2*x^2 + 35*c^4*x^4) + b*Sqrt[
1 + c^2*x^2]*(2104 - 1052*c^2*x^2 + 789*c^4*x^4 + 2650*c^6*x^6 + 1225*c^8*x
^8))*ArcSinh[c*x] + 99225*b^2*c^5*x^5*(63 + 90*c^2*x^2 + 35*c^4*x^4)*ArcSin
h[c*x]^2))/(31255875*c^5)
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.11

method	result
parts	$d^2a^2\left(\frac{1}{9}c^4x^9+\frac{2}{7}c^2x^7+\frac{1}{5}x^5\right)+\frac{d^2b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^3x^3\left(c^2x^2+1\right)^3}{9}-\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^3}{21}+\frac{8\operatorname{arcsinh}(cx)^2xc}{315}\right.}{}$
derivativedivides	$\frac{d^2a^2\left(\frac{1}{9}c^9x^9+\frac{2}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^2b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^3x^3\left(c^2x^2+1\right)^3}{9}-\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^3}{21}+\frac{8\operatorname{arcsinh}(cx)^2xc}{315}+\frac{\operatorname{arcsinh}(c}{}\right.}{}$
default	$\frac{d^2a^2\left(\frac{1}{9}c^9x^9+\frac{2}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^2b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^3x^3\left(c^2x^2+1\right)^3}{9}-\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^3}{21}+\frac{8\operatorname{arcsinh}(cx)^2xc}{315}+\frac{\operatorname{arcsinh}(c}{}\right.}{}$

```
[In] int(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*a^2*(1/9*c^4*x^9+2/7*c^2*x^7+1/5*x^5)+d^2*b^2/c^5*(1/9*arcsinh(c*x)^2*c
^3*x^3*(c^2*x^2+1)^3-1/21*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^3+8/315*arcsinh(c*
x)^2*x*c+1/105*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^2+4/315*arcsinh(c*x)^2*c*x*(c
^2*x^2+1)-2/81*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^(7/2)+82/3969*arcsinh(c*x)*
(c^2*x^2+1)^(7/2)+2/729*c*x*(c^2*x^2+1)^4+1493104/31255875*c*x-836/250047*c
*x*(c^2*x^2+1)^3-33862/10418625*c*x*(c^2*x^2+1)^2-47248/31255875*c*x*(c^2*x
^2+1)-16/315*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-2/525*arcsinh(c*x)*(c^2*x^2+1)^
(5/2)-8/945*arcsinh(c*x)*(c^2*x^2+1)^(3/2))+2*d^2*a*b/c^5*(1/9*arcsinh(c*x)
*c^9*x^9+2/7*arcsinh(c*x)*c^7*x^7+1/5*arcsinh(c*x)*c^5*x^5-1/81*c^8*x^8*(c^
2*x^2+1)^(1/2)-106/3969*c^6*x^6*(c^2*x^2+1)^(1/2)-263/33075*c^4*x^4*(c^2*x^
2+1)^(1/2)+1052/99225*c^2*x^2*(c^2*x^2+1)^(1/2)-2104/99225*(c^2*x^2+1)^(1/2
))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 368, normalized size of antiderivative = 0.95

$$\int x^4\left(d+c^2dx^2\right)^2\left(a+b\operatorname{arcsinh}(cx)\right)^2dx$$

$$=\frac{42875\left(81a^2+2b^2\right)c^9d^2x^9+2250\left(3969a^2+106b^2\right)c^7d^2x^7+189\left(33075a^2+526b^2\right)c^5d^2x^5-220920b^2c^3}{}$$

```
[In] integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/31255875*(42875*(81*a^2 + 2*b^2)*c^9*d^2*x^9 + 2250*(3969*a^2 + 106*b^2)*
c^7*d^2*x^7 + 189*(33075*a^2 + 526*b^2)*c^5*d^2*x^5 - 220920*b^2*c^3*d^2*x^
3 + 1325520*b^2*c*d^2*x + 99225*(35*b^2*c^9*d^2*x^9 + 90*b^2*c^7*d^2*x^7 +
63*b^2*c^5*d^2*x^5)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 630*(11025*a*b*c^9*d^2
*x^9 + 28350*a*b*c^7*d^2*x^7 + 19845*a*b*c^5*d^2*x^5 - (1225*b^2*c^8*d^2*x^
8 + 2650*b^2*c^6*d^2*x^6 + 789*b^2*c^4*d^2*x^4 - 1052*b^2*c^2*d^2*x^2 + 210
```

```
4*b^2*d^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 630*(1225*a*b*
c^8*d^2*x^8 + 2650*a*b*c^6*d^2*x^6 + 789*a*b*c^4*d^2*x^4 - 1052*a*b*c^2*d^2
*x^2 + 2104*a*b*d^2)*sqrt(c^2*x^2 + 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.46

$$\int x^4(d+c^2dx^2)^2(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$=\begin{cases}\frac{a^2c^4d^2x^9}{9}+\frac{2a^2c^2d^2x^7}{7}+\frac{a^2d^2x^5}{5}+\frac{2abc^4d^2x^9\operatorname{asinh}(cx)}{9}-\frac{2abc^3d^2x^8\sqrt{c^2x^2+1}}{81}+\frac{4abc^2d^2x^7\operatorname{asinh}(cx)}{7}-\frac{212abcd^2x^6\sqrt{c^2x^2+1}}{3969}+ \\ \frac{a^2d^2x^5}{5}\end{cases}$$

```
[In] integrate(x**4*(c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**4*d**2*x**9/9 + 2*a**2*c**2*d**2*x**7/7 + a**2*d**2*x**5
/5 + 2*a*b*c**4*d**2*x**9*asinh(c*x)/9 - 2*a*b*c**3*d**2*x**8*sqrt(c**2*x**
2 + 1)/81 + 4*a*b*c**2*d**2*x**7*asinh(c*x)/7 - 212*a*b*c*d**2*x**6*sqrt(c*
*2*x**2 + 1)/3969 + 2*a*b*d**2*x**5*asinh(c*x)/5 - 526*a*b*d**2*x**4*sqrt(c
**2*x**2 + 1)/(33075*c) + 2104*a*b*d**2*x**2*sqrt(c**2*x**2 + 1)/(99225*c**
3) - 4208*a*b*d**2*sqrt(c**2*x**2 + 1)/(99225*c**5) + b**2*c**4*d**2*x**9*a
sinh(c*x)**2/9 + 2*b**2*c**4*d**2*x**9/729 - 2*b**2*c**3*d**2*x**8*sqrt(c**
2*x**2 + 1)*asinh(c*x)/81 + 2*b**2*c**2*d**2*x**7*asinh(c*x)**2/7 + 212*b**
2*c**2*d**2*x**7/27783 - 212*b**2*c*d**2*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x
)/3969 + b**2*d**2*x**5*asinh(c*x)**2/5 + 526*b**2*d**2*x**5/165375 - 526*b
**2*d**2*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/(33075*c) - 2104*b**2*d**2*x**
3/(297675*c**2) + 2104*b**2*d**2*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(99225
*c**3) + 4208*b**2*d**2*x/(99225*c**4) - 4208*b**2*d**2*sqrt(c**2*x**2 + 1)
*asinh(c*x)/(99225*c**5), Ne(c, 0)), (a**2*d**2*x**5/5, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 760 vs. 2(342) = 684.

Time = 0.22 (sec) , antiderivative size = 760, normalized size of antiderivative = 1.97

$$\int x^4(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2\,dx = \frac{1}{9}b^2c^4d^2x^9\,\text{arsinh}(cx)^2$$
$$+\frac{1}{9}a^2c^4d^2x^9+\frac{2}{7}b^2c^2d^2x^7\,\text{arsinh}(cx)^2+\frac{2}{7}a^2c^2d^2x^7+\frac{1}{5}b^2d^2x^5\,\text{arsinh}(cx)^2$$
$$+\frac{2}{2835}\left(315\,x^9\,\text{arsinh}(cx)-\left(\frac{35\sqrt{c^2x^2+1}x^8}{c^2}-\frac{40\sqrt{c^2x^2+1}x^6}{c^4}+\frac{48\sqrt{c^2x^2+1}x^4}{c^6}-\frac{64\sqrt{c^2x^2+1}x^2}{c^8}\right.\right.$$
$$-\frac{2}{893025}\left(315\left(\frac{35\sqrt{c^2x^2+1}x^8}{c^2}-\frac{40\sqrt{c^2x^2+1}x^6}{c^4}+\frac{48\sqrt{c^2x^2+1}x^4}{c^6}-\frac{64\sqrt{c^2x^2+1}x^2}{c^8}+\frac{128\sqrt{c^2x^2}}{c^{10}}\right.\right.$$
$$+\frac{1}{5}a^2d^2x^5$$
$$+\frac{4}{245}\left(35\,x^7\,\text{arsinh}(cx)-\left(\frac{5\sqrt{c^2x^2+1}x^6}{c^2}-\frac{6\sqrt{c^2x^2+1}x^4}{c^4}+\frac{8\sqrt{c^2x^2+1}x^2}{c^6}-\frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\right)abc^2$$
$$-\frac{4}{25725}\left(105\left(\frac{5\sqrt{c^2x^2+1}x^6}{c^2}-\frac{6\sqrt{c^2x^2+1}x^4}{c^4}+\frac{8\sqrt{c^2x^2+1}x^2}{c^6}-\frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\,\text{arsinh}(cx)-\frac{75\,c^6}{}\right.$$
$$+\frac{2}{75}\left(15\,x^5\,\text{arsinh}(cx)-\left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2}-\frac{4\sqrt{c^2x^2+1}x^2}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)abd^2$$
$$-\frac{2}{1125}\left(15\left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2}-\frac{4\sqrt{c^2x^2+1}x^2}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\,\text{arsinh}(cx)-\frac{9\,c^4x^5-20\,c^2x^3+120\,x}{c^4}\right)$$

```
[In] integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*c^4*d^2*x^9*arcsinh(c*x)^2 + 1/9*a^2*c^4*d^2*x^9 + 2/7*b^2*c^2*d^2*
x^7*arcsinh(c*x)^2 + 2/7*a^2*c^2*d^2*x^7 + 1/5*b^2*d^2*x^5*arcsinh(c*x)^2 +
 2/2835*(315*x^9*arcsinh(c*x) - (35*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2
*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2
/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c)*a*b*c^4*d^2 - 2/893025*(315*(35*sqrt(
c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*
x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c*arcs
inh(c*x) - (1225*c^8*x^9 - 1800*c^6*x^7 + 3024*c^4*x^5 - 6720*c^2*x^3 + 403
20*x)/c^8)*b^2*c^4*d^2 + 1/5*a^2*d^2*x^5 + 4/245*(35*x^7*arcsinh(c*x) - (5*
sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 +
1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*a*b*c^2*d^2 - 4/25725*(105*(5*sqr
t(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*
x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsinh(c*x) - (75*c^6*x^7 - 126*c^4*
x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*c^2*d^2 + 2/75*(15*x^5*arcsinh(c*x) -
(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2
```

```
 + 1)/c^6)*c)*a*b*d^2 - 2/1125*(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^
2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 -
 20*c^2*x^3 + 120*x)/c^4)*b^2*d^2
```

Giac [F(-2)]

Exception generated.

$$\int x^4 (d + c^2 d x^2)^2 (a + b\text{arcsinh}(cx))^2 \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (d + c^2 d x^2)^2 (a + b\text{arcsinh}(cx))^2 \, dx = \int x^4 (a + b \operatorname{asinh}(c x))^2 \left(d c^2 x^2 + d\right)^2 \, dx$$

```
[In] int(x^4*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2,x)
```

```
[Out] int(x^4*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2, x)
```

3.208 $\int x^3(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2 dx$

Optimal result

Integrand size = 26, antiderivative size = 296

$$\begin{aligned}\int x^3\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2\,dx=&-\frac{73b^2d^2x^2}{3072c^2}+\frac{73b^2d^2x^4}{9216}\\&+\frac{43b^2c^2d^2x^6}{3456}+\frac{1}{256}b^2c^4d^2x^8\\&+\frac{73bd^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{1536c^3}\\&-\frac{73bd^2x^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2304c}\\&-\frac{25}{576}bcd^2x^5\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\\&-\frac{1}{32}bcd^2x^5\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))\\&-\frac{73d^2(a+b\text{arcsinh}(cx))^2}{3072c^4}\\&+\frac{1}{24}d^2x^4(a+b\text{arcsinh}(cx))^2\\&+\frac{1}{12}d^2x^4\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2\\&+\frac{1}{8}d^2x^4\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2\end{aligned}$$

```
[Out] -73/3072*b^2*d^2*x^2/c^2+73/9216*b^2*d^2*x^4+43/3456*b^2*c^2*d^2*x^6+1/256*
b^2*c^4*d^2*x^8-1/32*b*c*d^2*x^5*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))-73/30
72*d^2*(a+b*arcsinh(c*x))^2/c^4+1/24*d^2*x^4*(a+b*arcsinh(c*x))^2+1/12*d^2*
x^4*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+1/8*d^2*x^4*(c^2*x^2+1)^2*(a+b*arcsinh
(c*x))^2+73/1536*b*d^2*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-73/2304*b
*d^2*x^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c-25/576*b*c*d^2*x^5*(a+b*arc
sinh(c*x))*(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5808, 5776, 5812, 5783, 30, 5806, 14}

$$\int x^3\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2\,dx=-\frac{73d^2(a+b\text{arcsinh}(cx))^2}{3072c^4}-\frac{1}{32}bcd^2x^5\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{25}{576}bcd^2x^5\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{1}{8}d^2x^4\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2+\frac{1}{12}d^2x^4\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2-\frac{73bd^2x^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2304c}+\frac{73bd^2x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{1536c^3}+\frac{1}{24}d^2x^4(a+b\text{arcsinh}(cx))^2+\frac{1}{256}b^2c^4d^2x^8+\frac{43b^2c^2d^2x^6}{3456}-\frac{73b^2d^2x^2}{3072c^2}+\frac{73b^2d^2x^4}{9216}$$

```
[In] Int[x^3*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-73*b^2*d^2*x^2)/(3072*c^2) + (73*b^2*d^2*x^4)/9216 + (43*b^2*c^2*d^2*x^6)
/3456 + (b^2*c^4*d^2*x^8)/256 + (73*b*d^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSin
h[c*x]))/(1536*c^3) - (73*b*d^2*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))
/(2304*c) - (25*b*c*d^2*x^5*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/576 - (
b*c*d^2*x^5*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/32 - (73*d^2*(a + b*A
rcSinh[c*x])^2)/(3072*c^4) + (d^2*x^4*(a + b*ArcSinh[c*x])^2)/24 + (d^2*x^4
*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/12 + (d^2*x^4*(1 + c^2*x^2)^2*(a + b
*ArcSinh[c*x])^2)/8
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{8}d^2x^4\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2+\frac{1}{2}d\int x^3\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))^2\,dx \\
&\quad -\frac{1}{4}\left(bcd^2\right)\int x^4\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx \\
&= -\frac{1}{32}bcd^2x^5\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{1}{12}d^2x^4\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2 \\
&\quad +\frac{1}{8}d^2x^4\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2+\frac{1}{6}d^2\int x^3(a+b\text{arcsinh}(cx))^2\,dx \\
&\quad -\frac{1}{32}\left(3bcd^2\right)\int x^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx \\
&\quad -\frac{1}{6}\left(bcd^2\right)\int x^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx+\frac{1}{32}\left(b^2c^2d^2\right)\int x^5\left(1+c^2x^2\right)\,dx \\
&= -\frac{25}{576}bcd^2x^5\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))-\frac{1}{32}bcd^2x^5\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx)) \\
&\quad +\frac{1}{24}d^2x^4(a+b\text{arcsinh}(cx))^2+\frac{1}{12}d^2x^4\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2 \\
&\quad +\frac{1}{8}d^2x^4\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2-\frac{1}{64}\left(bcd^2\right)\int\frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx \\
&\quad -\frac{1}{36}\left(bcd^2\right)\int\frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx-\frac{1}{12}\left(bcd^2\right)\int\frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx \\
&\quad +\frac{1}{64}\left(b^2c^2d^2\right)\int x^5\,dx+\frac{1}{36}\left(b^2c^2d^2\right)\int x^5\,dx+\frac{1}{32}\left(b^2c^2d^2\right)\int\left(x^5+c^2x^7\right)\,dx \\
&= \frac{43b^2c^2d^2x^6}{3456}+\frac{1}{256}b^2c^4d^2x^8-\frac{73bd^2x^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2304c} \\
&\quad -\frac{25}{576}bcd^2x^5\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))-\frac{1}{32}bcd^2x^5\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx)) \\
&\quad +\frac{1}{24}d^2x^4(a+b\text{arcsinh}(cx))^2+\frac{1}{12}d^2x^4\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2 \\
&\quad +\frac{1}{8}d^2x^4\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2+\frac{1}{256}\left(b^2d^2\right)\int x^3\,dx \\
&\quad +\frac{1}{144}\left(b^2d^2\right)\int x^3\,dx+\frac{1}{48}\left(b^2d^2\right)\int x^3\,dx+\frac{\left(3bd^2\right)\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{256c} \\
&\quad +\frac{\left(bd^2\right)\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{48c}+\frac{\left(bd^2\right)\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{16c}
\end{aligned}
$$

$$= \frac{73b^2d^2x^4}{9216} + \frac{43b^2c^2d^2x^6}{3456} + \frac{1}{256}b^2c^4d^2x^8 + \frac{73bd^2x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{1536c^3} - \frac{73bd^2x^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{2304c} - \frac{25}{576}bcd^2x^5\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx)) - \frac{1}{32}bcd^2x^5(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx)) + \frac{1}{24}d^2x^4(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{12}d^2x^4(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{8}d^2x^4(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2 - \frac{(3bd^2)\int\frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}dx}{512c^3} - \frac{(bd^2)\int\frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}dx}{96c^3} - \frac{(bd^2)\int\frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}dx}{32c^3} - \frac{(3b^2d^2)\int x\,dx}{512c^2} - \frac{(b^2d^2)\int x\,dx}{96c^2} - \frac{(b^2d^2)\int x\,dx}{32c^2}$$

$$= -\frac{73b^2d^2x^2}{3072c^2} + \frac{73b^2d^2x^4}{9216} + \frac{43b^2c^2d^2x^6}{3456} + \frac{1}{256}b^2c^4d^2x^8 + \frac{73bd^2x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{1536c^3} - \frac{73bd^2x^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{2304c} - \frac{25}{576}bcd^2x^5\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx)) - \frac{1}{32}bcd^2x^5(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx)) - \frac{73d^2(a+b\operatorname{arcsinh}(cx))^2}{3072c^4} + \frac{1}{24}d^2x^4(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{12}d^2x^4(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{8}d^2x^4(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.80

$$\int x^3(d+c^2dx^2)^2(a+b\operatorname{arcsinh}(cx))^2\,dx = \frac{d^2\left(cx\left(1152a^2c^3x^3(6+8c^2x^2+3c^4x^4)+b^2cx(-657+219c^2x^2+344c^4x^4+108c^6x^6)-6ab\sqrt{1+c^2x^2}(-2\right.\right.}{}$$

```
[In] Integrate[x^3*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*(c*x*(1152*a^2*c^3*x^3*(6 + 8*c^2*x^2 + 3*c^4*x^4) + b^2*c*x*(-657 + 2
19*c^2*x^2 + 344*c^4*x^4 + 108*c^6*x^6) - 6*a*b*Sqrt[1 + c^2*x^2]*(-219 + 1
46*c^2*x^2 + 344*c^4*x^4 + 144*c^6*x^6)) + 6*b*(-(b*c*x*Sqrt[1 + c^2*x^2]*(
-219 + 146*c^2*x^2 + 344*c^4*x^4 + 144*c^6*x^6)) + 3*a*(-73 + 768*c^4*x^4 +
 1024*c^6*x^6 + 384*c^8*x^8))*ArcSinh[c*x] + 9*b^2*(-73 + 768*c^4*x^4 + 102
4*c^6*x^6 + 384*c^8*x^8)*ArcSinh[c*x]^2))/(27648*c^4)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.16

method	result
parts	$d^2a^2\left(\frac{1}{8}c^4x^8+\frac{1}{3}c^2x^6+\frac{1}{4}x^4\right)+\frac{d^2b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^2x^2\left(c^2x^2+1\right)^3}{8}-\frac{\operatorname{arcsinh}(cx)^2\left(c^2x^2+1\right)^3}{24}-\frac{\operatorname{arcsinh}(cx)cx\left(c^2x^2+1\right.}{32}\right.}{}$
derivativedivides	$\frac{d^2a^2\left(\frac{1}{8}c^8x^8+\frac{1}{3}c^6x^6+\frac{1}{4}c^4x^4\right)+d^2b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^2x^2\left(c^2x^2+1\right)^3}{8}-\frac{\operatorname{arcsinh}(cx)^2\left(c^2x^2+1\right)^3}{24}-\frac{\operatorname{arcsinh}(cx)cx\left(c^2x^2+1\right)^{\frac{7}{2}}}{32}+\frac{11\ \operatorname{ar}}{}\right.}{}$
default	$\frac{d^2a^2\left(\frac{1}{8}c^8x^8+\frac{1}{3}c^6x^6+\frac{1}{4}c^4x^4\right)+d^2b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^2x^2\left(c^2x^2+1\right)^3}{8}-\frac{\operatorname{arcsinh}(cx)^2\left(c^2x^2+1\right)^3}{24}-\frac{\operatorname{arcsinh}(cx)cx\left(c^2x^2+1\right)^{\frac{7}{2}}}{32}+\frac{11\ \operatorname{ar}}{}\right.}{}$

```
[In] int(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*a^2*(1/8*c^4*x^8+1/3*c^2*x^6+1/4*x^4)+d^2*b^2/c^4*(1/8*arcsinh(c*x)^2*c
^2*x^2*(c^2*x^2+1)^3-1/24*arcsinh(c*x)^2*(c^2*x^2+1)^3-1/32*arcsinh(c*x)*c*
x*(c^2*x^2+1)^(7/2)+11/576*arcsinh(c*x)*c*x*(c^2*x^2+1)^(5/2)+55/2304*arcsi
nh(c*x)*c*x*(c^2*x^2+1)^(3/2)+55/1536*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)+55
/3072*arcsinh(c*x)^2+1/256*(c^2*x^2+1)^4-11/3456*(c^2*x^2+1)^3-55/9216*(c^2
*x^2+1)^2-55/3072*c^2*x^2-55/3072)+2*d^2*a*b/c^4*(1/8*arcsinh(c*x)*c^8*x^8+
1/3*arcsinh(c*x)*c^6*x^6+1/4*arcsinh(c*x)*c^4*x^4-1/64*c^7*x^7*(c^2*x^2+1)^
(1/2)-43/1152*c^5*x^5*(c^2*x^2+1)^(1/2)-73/4608*c^3*x^3*(c^2*x^2+1)^(1/2)+7
3/3072*c*x*(c^2*x^2+1)^(1/2)-73/3072*arcsinh(c*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.18

$$\int x^3\left(d+c^2dx^2\right)^2(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$=\frac{108\,(32\,a^2+b^2)c^8d^2x^8+8\,(1152\,a^2+43\,b^2)c^6d^2x^6+3\,(2304\,a^2+73\,b^2)c^4d^2x^4-657\,b^2c^2d^2x^2+9\,(384\,b^2c^8}{}$$

```
[In] integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/27648*(108*(32*a^2 + b^2)*c^8*d^2*x^8 + 8*(1152*a^2 + 43*b^2)*c^6*d^2*x^6
 + 3*(2304*a^2 + 73*b^2)*c^4*d^2*x^4 - 657*b^2*c^2*d^2*x^2 + 9*(384*b^2*c^8
*d^2*x^8 + 1024*b^2*c^6*d^2*x^6 + 768*b^2*c^4*d^2*x^4 - 73*b^2*d^2)*log(c*x
 + sqrt(c^2*x^2 + 1))^2 + 6*(1152*a*b*c^8*d^2*x^8 + 3072*a*b*c^6*d^2*x^6 +
2304*a*b*c^4*d^2*x^4 - 219*a*b*d^2 - (144*b^2*c^7*d^2*x^7 + 344*b^2*c^5*d^2
*x^5 + 146*b^2*c^3*d^2*x^3 - 219*b^2*c*d^2*x)*sqrt(c^2*x^2 + 1))*log(c*x +
sqrt(c^2*x^2 + 1)) - 6*(144*a*b*c^7*d^2*x^7 + 344*a*b*c^5*d^2*x^5 + 146*a*b
*c^3*d^2*x^3 - 219*a*b*c*d^2*x)*sqrt(c^2*x^2 + 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.74

$$\int x^3(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\begin{cases} \frac{a^2c^4d^2x^8}{8}+\frac{a^2c^2d^2x^6}{3}+\frac{a^2d^2x^4}{4}+\frac{abc^4d^2x^8\,\text{asinh}\,(cx)}{4}-\frac{abc^3d^2x^7\sqrt{c^2x^2+1}}{32}+\frac{2abc^2d^2x^6\,\text{asinh}\,(cx)}{3}-\frac{43abcd^2x^5\sqrt{c^2x^2+1}}{576}+\frac{abd \\ \frac{a^2d^2x^4}{4} \end{cases}$$

```
[In] integrate(x**3*(c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**4*d**2*x**8/8 + a**2*c**2*d**2*x**6/3 + a**2*d**2*x**4/4
 + a*b*c**4*d**2*x**8*asinh(c*x)/4 - a*b*c**3*d**2*x**7*sqrt(c**2*x**2 + 1)
/32 + 2*a*b*c**2*d**2*x**6*asinh(c*x)/3 - 43*a*b*c*d**2*x**5*sqrt(c**2*x**2
 + 1)/576 + a*b*d**2*x**4*asinh(c*x)/2 - 73*a*b*d**2*x**3*sqrt(c**2*x**2 +
1)/(2304*c) + 73*a*b*d**2*x*sqrt(c**2*x**2 + 1)/(1536*c**3) - 73*a*b*d**2*a
sinh(c*x)/(1536*c**4) + b**2*c**4*d**2*x**8*asinh(c*x)**2/8 + b**2*c**4*d**
2*x**8/256 - b**2*c**3*d**2*x**7*sqrt(c**2*x**2 + 1)*asinh(c*x)/32 + b**2*c
**2*d**2*x**6*asinh(c*x)**2/3 + 43*b**2*c**2*d**2*x**6/3456 - 43*b**2*c*d**
2*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)/576 + b**2*d**2*x**4*asinh(c*x)**2/4
+ 73*b**2*d**2*x**4/9216 - 73*b**2*d**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)
/(2304*c) - 73*b**2*d**2*x**2/(3072*c**2) + 73*b**2*d**2*x*sqrt(c**2*x**2 +
 1)*asinh(c*x)/(1536*c**3) - 73*b**2*d**2*asinh(c*x)**2/(3072*c**4), Ne(c,
0)), (a**2*d**2*x**4/4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(264) = 528$.

Time = 0.25 (sec) , antiderivative size = 762, normalized size of antiderivative = 2.57

$$\int x^3(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2\,dx = \frac{1}{8}\,b^2c^4d^2x^8\,\text{arsinh}\,(cx)^2$$
$$+\frac{1}{8}\,a^2c^4d^2x^8+\frac{1}{3}\,b^2c^2d^2x^6\,\text{arsinh}\,(cx)^2+\frac{1}{3}\,a^2c^2d^2x^6+\frac{1}{4}\,b^2d^2x^4\,\text{arsinh}\,(cx)^2$$
$$+\frac{1}{1536}\left(384\,x^8\,\text{arsinh}\,(cx)-\left(\frac{48\sqrt{c^2x^2+1}x^7}{c^2}-\frac{56\sqrt{c^2x^2+1}x^5}{c^4}+\frac{70\sqrt{c^2x^2+1}x^3}{c^6}-\frac{105\sqrt{c^2x^2+1}x}{c^8}+\right.\right.$$
$$+\frac{1}{9216}\left(\left(\frac{36\,x^8}{c^2}-\frac{56\,x^6}{c^4}+\frac{105\,x^4}{c^6}-\frac{315\,x^2}{c^8}+\frac{315\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^{10}}\right)c^2-6\left(\frac{48\sqrt{c^2x^2+1}x^7}{c^2}-56\right.\right.$$
$$+\frac{1}{4}\,a^2d^2x^4$$
$$+\frac{1}{72}\left(48\,x^6\,\text{arsinh}\,(cx)-\left(\frac{8\sqrt{c^2x^2+1}x^5}{c^2}-\frac{10\sqrt{c^2x^2+1}x^3}{c^4}+\frac{15\sqrt{c^2x^2+1}x}{c^6}-\frac{15\,\text{arsinh}\,(cx)}{c^7}\right)c\right)abc^2$$
$$+\frac{1}{432}\left(\left(\frac{8\,x^6}{c^2}-\frac{15\,x^4}{c^4}+\frac{45\,x^2}{c^6}-\frac{45\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^8}\right)c^2-6\left(\frac{8\sqrt{c^2x^2+1}x^5}{c^2}-\frac{10\sqrt{c^2x^2+1}x^3}{c^4}+\right.\right.$$
$$+\frac{1}{16}\left(8\,x^4\,\text{arsinh}\,(cx)-\left(\frac{2\sqrt{c^2x^2+1}x^3}{c^2}-\frac{3\sqrt{c^2x^2+1}x}{c^4}+\frac{3\,\text{arsinh}\,(cx)}{c^5}\right)c\right)abd^2$$
$$+\frac{1}{32}\left(\left(\frac{x^4}{c^2}-\frac{3\,x^2}{c^4}+\frac{3\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^6}\right)c^2-2\left(\frac{2\sqrt{c^2x^2+1}x^3}{c^2}-\frac{3\sqrt{c^2x^2+1}x}{c^4}+\frac{3\,\text{arsinh}\,(cx)}{c^5}\right)\right.$$

```
[In] integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*c^4*d^2*x^8*arcsinh(c*x)^2 + 1/8*a^2*c^4*d^2*x^8 + 1/3*b^2*c^2*d^2*
x^6*arcsinh(c*x)^2 + 1/3*a^2*c^2*d^2*x^6 + 1/4*b^2*d^2*x^4*arcsinh(c*x)^2 +
 1/1536*(384*x^8*arcsinh(c*x) - (48*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2
*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/
c^8 + 105*arcsinh(c*x)/c^9)*c)*a*b*c^4*d^2 + 1/9216*((36*x^8/c^2 - 56*x^6/c
^4 + 105*x^4/c^6 - 315*x^2/c^8 + 315*log(c*x + sqrt(c^2*x^2 + 1))^2/c^10)*c
^2 - 6*(48*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 70*sq
rt(c^2*x^2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/c^8 + 105*arcsinh(c*x)/c^
9)*c*arcsinh(c*x))*b^2*c^4*d^2 + 1/4*a^2*d^2*x^4 + 1/72*(48*x^6*arcsinh(c*x
) - (8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c
^2*x^2 + 1)*x/c^6 - 15*arcsinh(c*x)/c^7)*c)*a*b*c^2*d^2 + 1/432*((8*x^6/c^2
 - 15*x^4/c^4 + 45*x^2/c^6 - 45*log(c*x + sqrt(c^2*x^2 + 1))^2/c^8)*c^2 - 6
*(8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*
x^2 + 1)*x/c^6 - 15*arcsinh(c*x)/c^7)*c*arcsinh(c*x))*b^2*c^2*d^2 + 1/16*(8
*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^
4 + 3*arcsinh(c*x)/c^5)*c)*a*b*d^2 + 1/32*((x^4/c^2 - 3*x^2/c^4 + 3*log(c*x
 + sqrt(c^2*x^2 + 1))^2/c^6)*c^2 - 2*(2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(
c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c*arcsinh(c*x))*b^2*d^2
```

Giac [F(-2)]

Exception generated.

$$\int x^3 \left(d+c^2 d x^2\right)^2 (a+b \text{arcsinh}(cx))^2 \, dx = \text{Exception raised: TypeError}$$

[In] integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(d+c^2 d x^2\right)^2 (a+b \text{arcsinh}(cx))^2 \, dx = \int x^3 \, (a+b \, \text{asinh}(c \, x))^2 \, \left(d \, c^2 \, x^2+d\right)^2 \, dx$$

[In] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2,x)

[Out] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2, x)

3.209 $\int x^2(d+c^2dx^2)^2\,(a+b\mathbf{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 303

$$\int x^2\left(d+c^2dx^2\right)^2(a+b\operatorname{arcsinh}(cx))^2\,dx=-\frac{1636b^2d^2x}{11025c^2}+\frac{818b^2d^2x^3}{33075}+\frac{136b^2c^2d^2x^5}{6125}+\frac{2}{343}b^2c^4d^2x^7+\frac{32bd^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{315c^3}-\frac{16bd^2x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{315c}+\frac{8bd^2(1+c^2x^2)^{3/2}\,(a+b\operatorname{arcsinh}(cx))}{105c^3}+\frac{2bd^2(1+c^2x^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))}{175c^3}-\frac{2bd^2(1+c^2x^2)^{7/2}\,(a+b\operatorname{arcsinh}(cx))}{49c^3}+\frac{8}{105}d^2x^3(a+b\operatorname{arcsinh}(cx))^2+\frac{4}{35}d^2x^3\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))^2+\frac{1}{7}d^2x^3\left(1+c^2x^2\right)^2(a+b\operatorname{arcsinh}(cx))^2$$

```
[Out] -1636/11025*b^2*d^2*x/c^2+818/33075*b^2*d^2*x^3+136/6125*b^2*c^2*d^2*x^5+2/
343*b^2*c^4*d^2*x^7+8/105*b*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c^3+2/
175*b*d^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))/c^3-2/49*b*d^2*(c^2*x^2+1)^(
7/2)*(a+b*arcsinh(c*x))/c^3+8/105*d^2*x^3*(a+b*arcsinh(c*x))^2+4/35*d^2*x^3
*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+1/7*d^2*x^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*
x))^2+32/315*b*d^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-16/315*b*d^2*x^
2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {5808, 5776, 5812, 5798, 8, 30, 272, 45, 5804, 12, 380}

$$\int x^2(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2\,dx$$
$$= -\frac{16bd^2x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{315c}+\frac{1}{7}d^2x^3(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2$$
$$+\frac{4}{35}d^2x^3(c^2x^2+1)(a+b\text{arcsinh}(cx))^2$$
$$-\frac{2bd^2(c^2x^2+1)^{7/2}(a+b\text{arcsinh}(cx))}{49c^3}+\frac{2bd^2(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))}{175c^3}$$
$$+\frac{8bd^2(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{105c^3}+\frac{32bd^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{315c^3}$$
$$+\frac{8}{105}d^2x^3(a+b\text{arcsinh}(cx))^2+\frac{2}{343}b^2c^4d^2x^7+\frac{136b^2c^2d^2x^5}{6125}-\frac{1636b^2d^2x}{11025c^2}+\frac{818b^2d^2x^3}{33075}$$

```
[In] Int[x^2*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-1636*b^2*d^2*x)/(11025*c^2) + (818*b^2*d^2*x^3)/33075 + (136*b^2*c^2*d^2*
x^5)/6125 + (2*b^2*c^4*d^2*x^7)/343 + (32*b*d^2*Sqrt[1 + c^2*x^2]*(a + b*Ar
cSinh[c*x]))/(315*c^3) - (16*b*d^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x
]))/(315*c) + (8*b*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(105*c^3)
+ (2*b*d^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(175*c^3) - (2*b*d^2*(
1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(49*c^3) + (8*d^2*x^3*(a + b*ArcSi
nh[c*x])^2)/105 + (4*d^2*x^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/35 + (d^
2*x^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/7
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
```

```
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{7}d^2x^3\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2+\frac{1}{7}(4d)\int x^2\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))^2\,dx \\
&\quad -\frac{1}{7}\left(2bcd^2\right)\int x^3\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx \\
&= \frac{2bd^2\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{35c^3}-\frac{2bd^2\left(1+c^2x^2\right)^{7/2}(a+b\text{arcsinh}(cx))}{49c^3} \\
&\quad +\frac{4}{35}d^2x^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2 \\
&\quad +\frac{1}{7}d^2x^3\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2+\frac{1}{35}\left(8d^2\right)\int x^2(a+b\text{arcsinh}(cx))^2\,dx \\
&\quad -\frac{1}{35}\left(8bcd^2\right)\int x^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx \\
&\quad +\frac{1}{7}\left(2b^2c^2d^2\right)\int \frac{\left(1+c^2x^2\right)^2\left(-2+5c^2x^2\right)}{35c^4}\,dx \\
&= \frac{8bd^2\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))}{105c^3} \\
&\quad +\frac{2bd^2\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{175c^3}-\frac{2bd^2\left(1+c^2x^2\right)^{7/2}(a+b\text{arcsinh}(cx))}{49c^3} \\
&\quad +\frac{8}{105}d^2x^3(a+b\text{arcsinh}(cx))^2+\frac{4}{35}d^2x^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2+\frac{1}{7}d^2x^3\left(1+c^2x^2\right)^2(a+b\text{arcsin} \\
&= -\frac{16bd^2x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{315c}+\frac{8bd^2\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))}{105c^3} \\
&\quad +\frac{2bd^2\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{175c^3}-\frac{2bd^2\left(1+c^2x^2\right)^{7/2}(a+b\text{arcsinh}(cx))}{49c^3} \\
&\quad +\frac{8}{105}d^2x^3(a+b\text{arcsinh}(cx))^2+\frac{4}{35}d^2x^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2+\frac{1}{7}d^2x^3\left(1+c^2x^2\right)^2(a+b\text{arcsin}
\end{aligned}$$

$$=-\frac{172b^2d^2x}{3675c^2}+\frac{818b^2d^2x^3}{33075}+\frac{136b^2c^2d^2x^5}{6125}+\frac{2}{343}b^2c^4d^2x^7+\frac{32bd^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{315c^3}$$
$$-\frac{16bd^2x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{315c}+\frac{8bd^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{105c^3}$$
$$+\frac{2bd^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{175c^3}-\frac{2bd^2(1+c^2x^2)^{7/2}(a+b\operatorname{arcsinh}(cx))}{49c^3}$$
$$+\frac{8}{105}d^2x^3(a+b\operatorname{arcsinh}(cx))^2+\frac{4}{35}d^2x^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2+\frac{1}{7}d^2x^3(1+c^2x^2)^2(a+b\operatorname{arcsinh}$$

$$=-\frac{1636b^2d^2x}{11025c^2}+\frac{818b^2d^2x^3}{33075}+\frac{136b^2c^2d^2x^5}{6125}+\frac{2}{343}b^2c^4d^2x^7+\frac{32bd^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{315c^3}$$
$$-\frac{16bd^2x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{315c}+\frac{8bd^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{105c^3}$$
$$+\frac{2bd^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{175c^3}-\frac{2bd^2(1+c^2x^2)^{7/2}(a+b\operatorname{arcsinh}(cx))}{49c^3}$$
$$+\frac{8}{105}d^2x^3(a+b\operatorname{arcsinh}(cx))^2+\frac{4}{35}d^2x^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2+\frac{1}{7}d^2x^3(1+c^2x^2)^2(a+b\operatorname{arcsinh}$$

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.75

$$\int x^2(d+c^2dx^2)^2(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$=\frac{d^2\left(11025a^2c^3x^3(35+42c^2x^2+15c^4x^4)-210ab\sqrt{1+c^2x^2}(-818+409c^2x^2+612c^4x^4+225c^6x^6)+2b^2cx(\right.}{}$$

```
[In] Integrate[x^2*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*(11025*a^2*c^3*x^3*(35 + 42*c^2*x^2 + 15*c^4*x^4) - 210*a*b*Sqrt[1 + c
^2*x^2]*(-818 + 409*c^2*x^2 + 612*c^4*x^4 + 225*c^6*x^6) + 2*b^2*c*x*(-8589
0 + 14315*c^2*x^2 + 12852*c^4*x^4 + 3375*c^6*x^6) - 210*b*(-105*a*c^3*x^3*(
35 + 42*c^2*x^2 + 15*c^4*x^4) + b*Sqrt[1 + c^2*x^2]*(-818 + 409*c^2*x^2 + 6
12*c^4*x^4 + 225*c^6*x^6))*ArcSinh[c*x] + 11025*b^2*c^3*x^3*(35 + 42*c^2*x^
2 + 15*c^4*x^4)*ArcSinh[c*x]^2))/(1157625*c^3)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.14

method	result
parts	$d^2a^2\left(\frac{1}{7}c^4x^7+\frac{2}{5}c^2x^5+\frac{1}{3}x^3\right)+\frac{d^2b^2\left(\frac{\operatorname{arcsinh}(cx)^2 cx\left(c^2x^2+1\right)^3}{7}-\frac{8\operatorname{arcsinh}(cx)^2 xc}{105}-\frac{\operatorname{arcsinh}(cx)^2 cx\left(c^2x^2+1\right)^2}{35}-\frac{4}{}\right.}{}$
derivativedivides	$\frac{d^2a^2\left(\frac{1}{7}c^7x^7+\frac{2}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^2b^2\left(\frac{\operatorname{arcsinh}(cx)^2 cx\left(c^2x^2+1\right)^3}{7}-\frac{8\operatorname{arcsinh}(cx)^2 xc}{105}-\frac{\operatorname{arcsinh}(cx)^2 cx\left(c^2x^2+1\right)^2}{35}-\frac{4\operatorname{arcsinh}(cx)}{1}\right.}{}$
default	$\frac{d^2a^2\left(\frac{1}{7}c^7x^7+\frac{2}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^2b^2\left(\frac{\operatorname{arcsinh}(cx)^2 cx\left(c^2x^2+1\right)^3}{7}-\frac{8\operatorname{arcsinh}(cx)^2 xc}{105}-\frac{\operatorname{arcsinh}(cx)^2 cx\left(c^2x^2+1\right)^2}{35}-\frac{4\operatorname{arcsinh}(cx)}{1}\right.}{}$

```
[In] int(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*a^2*(1/7*c^4*x^7+2/5*c^2*x^5+1/3*x^3)+d^2*b^2/c^3*(1/7*arcsinh(c*x)^2*c
*x*(c^2*x^2+1)^3-8/105*arcsinh(c*x)^2*x*c-1/35*arcsinh(c*x)^2*c*x*(c^2*x^2+
1)^2-4/105*arcsinh(c*x)^2*c*x*(c^2*x^2+1)-2/49*arcsinh(c*x)*(c^2*x^2+1)^(7/
2)-181456/1157625*c*x+2/343*c*x*(c^2*x^2+1)^3+202/42875*c*x*(c^2*x^2+1)^2-2
528/1157625*c*x*(c^2*x^2+1)+16/105*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2/175*arc
sinh(c*x)*(c^2*x^2+1)^(5/2)+8/315*arcsinh(c*x)*(c^2*x^2+1)^(3/2))+2*d^2*a*b
/c^3*(1/7*arcsinh(c*x)*c^7*x^7+2/5*arcsinh(c*x)*c^5*x^5+1/3*arcsinh(c*x)*c^
3*x^3-1/49*c^6*x^6*(c^2*x^2+1)^(1/2)-68/1225*c^4*x^4*(c^2*x^2+1)^(1/2)-409/
11025*c^2*x^2*(c^2*x^2+1)^(1/2)+818/11025*(c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.08

$$\int x^2\left(d+c^2dx^2\right)^2(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$=\frac{3375\,(49\,a^2+2\,b^2)c^7d^2x^7+378\,(1225\,a^2+68\,b^2)c^5d^2x^5+35\,(11025\,a^2+818\,b^2)c^3d^2x^3-171780\,b^2cd^2x+}{}$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/1157625*(3375*(49*a^2 + 2*b^2)*c^7*d^2*x^7 + 378*(1225*a^2 + 68*b^2)*c^5*
d^2*x^5 + 35*(11025*a^2 + 818*b^2)*c^3*d^2*x^3 - 171780*b^2*c*d^2*x + 11025
*(15*b^2*c^7*d^2*x^7 + 42*b^2*c^5*d^2*x^5 + 35*b^2*c^3*d^2*x^3)*log(c*x + s
qrt(c^2*x^2 + 1))^2 + 210*(1575*a*b*c^7*d^2*x^7 + 4410*a*b*c^5*d^2*x^5 + 36
75*a*b*c^3*d^2*x^3 - (225*b^2*c^6*d^2*x^6 + 612*b^2*c^4*d^2*x^4 + 409*b^2*c
^2*d^2*x^2 - 818*b^2*d^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) -
 210*(225*a*b*c^6*d^2*x^6 + 612*a*b*c^4*d^2*x^4 + 409*a*b*c^2*d^2*x^2 - 818
*a*b*d^2)*sqrt(c^2*x^2 + 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.59

$$\int x^2 (d + c^2 d x^2)^2 (a + b\text{arcsinh}(cx))^2 \, dx$$
$$= \begin{cases} \frac{a^2 c^4 d^2 x^7}{7} + \frac{2 a^2 c^2 d^2 x^5}{5} + \frac{a^2 d^2 x^3}{3} + \frac{2 a b c^4 d^2 x^7 \operatorname{asinh}(cx)}{7} - \frac{2 a b c^3 d^2 x^6 \sqrt{c^2 x^2 + 1}}{49} + \frac{4 a b c^2 d^2 x^5 \operatorname{asinh}(cx)}{5} - \frac{136 a b c d^2 x^4 \sqrt{c^2 x^2 + 1}}{1225} + 2 \\ \frac{a^2 d^2 x^3}{3} \end{cases}$$

```
[In] integrate(x**2*(c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**4*d**2*x**7/7 + 2*a**2*c**2*d**2*x**5/5 + a**2*d**2*x**3
/3 + 2*a*b*c**4*d**2*x**7*asinh(c*x)/7 - 2*a*b*c**3*d**2*x**6*sqrt(c**2*x**
2 + 1)/49 + 4*a*b*c**2*d**2*x**5*asinh(c*x)/5 - 136*a*b*c*d**2*x**4*sqrt(c*
*2*x**2 + 1)/1225 + 2*a*b*d**2*x**3*asinh(c*x)/3 - 818*a*b*d**2*x**2*sqrt(c
**2*x**2 + 1)/(11025*c) + 1636*a*b*d**2*sqrt(c**2*x**2 + 1)/(11025*c**3) +
b**2*c**4*d**2*x**7*asinh(c*x)**2/7 + 2*b**2*c**4*d**2*x**7/343 - 2*b**2*c*
*3*d**2*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/49 + 2*b**2*c**2*d**2*x**5*asin
h(c*x)**2/5 + 136*b**2*c**2*d**2*x**5/6125 - 136*b**2*c*d**2*x**4*sqrt(c**2
*x**2 + 1)*asinh(c*x)/1225 + b**2*d**2*x**3*asinh(c*x)**2/3 + 818*b**2*d**2
*x**3/33075 - 818*b**2*d**2*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(11025*c) -
 1636*b**2*d**2*x/(11025*c**2) + 1636*b**2*d**2*sqrt(c**2*x**2 + 1)*asinh(c
*x)/(11025*c**3), Ne(c, 0)), (a**2*d**2*x**3/3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. 2(269) = 538.

Time = 0.22 (sec) , antiderivative size = 619, normalized size of antiderivative = 2.04

$$\int x^2(d+c^2dx^2)^2\,(a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{1}{7}\,b^2c^4d^2x^7\,\text{arsinh}\,(cx)^2+\frac{1}{7}\,a^2c^4d^2x^7+\frac{2}{5}\,b^2c^2d^2x^5\,\text{arsinh}\,(cx)^2+\frac{2}{5}\,a^2c^2d^2x^5$$
$$+\frac{2}{245}\left(35\,x^7\,\text{arsinh}\,(cx)-\left(\frac{5\,\sqrt{c^2x^2+1}x^6}{c^2}-\frac{6\,\sqrt{c^2x^2+1}x^4}{c^4}+\frac{8\,\sqrt{c^2x^2+1}x^2}{c^6}-\frac{16\,\sqrt{c^2x^2+1}}{c^8}\right)c\right)abc^4$$
$$-\frac{2}{25725}\left(105\left(\frac{5\,\sqrt{c^2x^2+1}x^6}{c^2}-\frac{6\,\sqrt{c^2x^2+1}x^4}{c^4}+\frac{8\,\sqrt{c^2x^2+1}x^2}{c^6}-\frac{16\,\sqrt{c^2x^2+1}}{c^8}\right)c\,\text{arsinh}\,(cx)-\frac{75\,c^6}{}\right.$$
$$+\frac{1}{3}\,b^2d^2x^3\,\text{arsinh}\,(cx)^2$$
$$+\frac{4}{75}\left(15\,x^5\,\text{arsinh}\,(cx)-\left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2}-\frac{4\,\sqrt{c^2x^2+1}x^2}{c^4}+\frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\right)abc^2d^2$$
$$-\frac{4}{1125}\left(15\left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2}-\frac{4\,\sqrt{c^2x^2+1}x^2}{c^4}+\frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\,\text{arsinh}\,(cx)-\frac{9\,c^4x^5-20\,c^2x^3+120\,x}{c^4}\right)$$
$$+\frac{1}{3}\,a^2d^2x^3+\frac{2}{9}\left(3\,x^3\,\text{arsinh}\,(cx)-c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)abd^2$$
$$-\frac{2}{27}\left(3\,c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\text{arsinh}\,(cx)-\frac{c^2x^3-6\,x}{c^2}\right)b^2d^2$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*c^4*d^2*x^7*arcsinh(c*x)^2 + 1/7*a^2*c^4*d^2*x^7 + 2/5*b^2*c^2*d^2*
x^5*arcsinh(c*x)^2 + 2/5*a^2*c^2*d^2*x^5 + 2/245*(35*x^7*arcsinh(c*x) - (5*
sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 +
1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*a*b*c^4*d^2 - 2/25725*(105*(5*sqr
t(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*
x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsinh(c*x) - (75*c^6*x^7 - 126*c^4*
x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*c^4*d^2 + 1/3*b^2*d^2*x^3*arcsinh(c*x)
^2 + 4/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*
x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^2*d^2 - 4/1125*(15*(3*
sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 +
1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^2*c^2*d^2
+ 1/3*a^2*d^2*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2
- 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d^2 - 2/27*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2
- 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*d^2
```

Giac [F(-2)]

Exception generated.

$$\int x^2 (d + c^2 d x^2)^2 (a + b \operatorname{arcsinh}(cx))^2 \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (d + c^2 d x^2)^2 (a + b \operatorname{arcsinh}(cx))^2 \, dx = \int x^2 (a + b \operatorname{asinh}(c x))^2 (d c^2 x^2 + d)^2 \, dx$$

```
[In] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2,x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2, x)
```

3.210 $\int x(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2 dx$

Optimal result

Integrand size = 24, antiderivative size = 204

$$\int x(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2 dx = \frac{25}{288}b^2d^2x^2 + \frac{5}{288}b^2c^2d^2x^4 + \frac{b^2d^2(1+c^2x^2)^3}{108c^2} - \frac{5bd^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{48c} - \frac{5bd^2x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{72c} - \frac{bd^2x(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{18c} - \frac{5d^2(a+b\text{arcsinh}(cx))^2}{96c^2} + \frac{d^2(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2}{6c^2}$$

```
[Out] 25/288*b^2*d^2*x^2+5/288*b^2*c^2*d^2*x^4+1/108*b^2*d^2*(c^2*x^2+1)^3/c^2-5/
72*b*d^2*x*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c-1/18*b*d^2*x*(c^2*x^2+1)^
(5/2)*(a+b*arcsinh(c*x))/c-5/96*d^2*(a+b*arcsinh(c*x))^2/c^2+1/6*d^2*(c^2*x
^2+1)^3*(a+b*arcsinh(c*x))^2/c^2-5/48*b*d^2*x*(a+b*arcsinh(c*x))*(c^2*x^2+1
)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5798, 5786, 5785, 5783, 30, 14, 267}

$$\int x(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2\,dx = -\frac{bd^2x(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))}{18c} - \frac{5bd^2x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{72c} - \frac{5bd^2x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{48c} + \frac{d^2(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2}{6c^2} - \frac{5d^2(a+b\text{arcsinh}(cx))^2}{96c^2} + \frac{5}{288}b^2c^2d^2x^4 + \frac{b^2d^2(c^2x^2+1)^3}{108c^2} + \frac{25}{288}b^2d^2x^2$$

```
[In] Int[x*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (25*b^2*d^2*x^2)/288 + (5*b^2*c^2*d^2*x^4)/288 + (b^2*d^2*(1 + c^2*x^2)^3)/
(108*c^2) - (5*b*d^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(48*c) - (5*
b*d^2*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(72*c) - (b*d^2*x*(1 + c^
2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(18*c) - (5*d^2*(a + b*ArcSinh[c*x])^2)/
(96*c^2) + (d^2*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/(6*c^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d^2(1+c^2x^2)^3\,(a+b\text{arcsinh}(cx))^2}{6c^2} - \frac{(bd^2)\int (1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))\,dx}{3c} \\
&= -\frac{bd^2x(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))}{18c} + \frac{d^2(1+c^2x^2)^3\,(a+b\text{arcsinh}(cx))^2}{6c^2} \\
&\quad + \frac{1}{18}(b^2d^2)\int x\left(1+c^2x^2\right)^2\,dx - \frac{(5bd^2)\int (1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\,dx}{18c}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{b^2d^2(1+c^2x^2)^3}{108c^2} - \frac{5bd^2x(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{72c} \\
&\quad - \frac{bd^2x(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{18c} + \frac{d^2(1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))^2}{6c^2} \\
&\quad + \frac{1}{72}(5b^2d^2)\int x(1+c^2x^2)\,dx - \frac{(5bd^2)\int\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{24c} \\
&= \frac{b^2d^2(1+c^2x^2)^3}{108c^2} - \frac{5bd^2x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{48c} \\
&\quad - \frac{5bd^2x(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{72c} \\
&\quad - \frac{bd^2x(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{18c} + \frac{d^2(1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))^2}{6c^2} \\
&\quad + \frac{1}{72}(5b^2d^2)\int(x+c^2x^3)\,dx + \frac{1}{48}(5b^2d^2)\int x\,dx - \frac{(5bd^2)\int\frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{48c} \\
&= \frac{25}{288}b^2d^2x^2 + \frac{5}{288}b^2c^2d^2x^4 + \frac{b^2d^2(1+c^2x^2)^3}{108c^2} - \frac{5bd^2x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{48c} \\
&\quad - \frac{5bd^2x(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{72c} - \frac{bd^2x(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{18c} \\
&\quad - \frac{5d^2(a+b\operatorname{arcsinh}(cx))^2}{96c^2} + \frac{d^2(1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))^2}{6c^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.02

$$
\int x(d+c^2dx^2)^2(a+b\operatorname{arcsinh}(cx))^2\,dx
$$
$$
= \frac{d^2\left(cx\left(144a^2cx(3+3c^2x^2+c^4x^4)-6ab\sqrt{1+c^2x^2}(33+26c^2x^2+8c^4x^4)+b^2cx(99+39c^2x^2+8c^4x^4)\right)+6\right.}{}
$$

```
[In] Integrate[x*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*(c*x*(144*a^2*c*x*(3 + 3*c^2*x^2 + c^4*x^4) - 6*a*b*Sqrt[1 + c^2*x^2]*
(33 + 26*c^2*x^2 + 8*c^4*x^4) + b^2*c*x*(99 + 39*c^2*x^2 + 8*c^4*x^4)) + 6*
b*(-(b*c*x*Sqrt[1 + c^2*x^2]*(33 + 26*c^2*x^2 + 8*c^4*x^4)) + 3*a*(11 + 48*
c^2*x^2 + 48*c^4*x^4 + 16*c^6*x^6))*ArcSinh[c*x] + 9*b^2*(11 + 48*c^2*x^2 +
 48*c^4*x^4 + 16*c^6*x^6)*ArcSinh[c*x]^2))/(864*c^2)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{\frac{d^2a^2(c^2x^2+1)^3}{6}+d^2b^2\left(\frac{\operatorname{arcsinh}(cx)^2(c^2x^2+1)^3}{6}-\frac{\operatorname{arcsinh}(cx)cx(c^2x^2+1)^{\frac{5}{2}}}{18}-\frac{5\operatorname{arcsinh}(cx)cx(c^2x^2+1)^{\frac{3}{2}}}{72}-\frac{5\operatorname{arcsinh}(cx)cx\sqrt{c^2}}{48}\right.}{}$
default	$\frac{\frac{d^2a^2(c^2x^2+1)^3}{6}+d^2b^2\left(\frac{\operatorname{arcsinh}(cx)^2(c^2x^2+1)^3}{6}-\frac{\operatorname{arcsinh}(cx)cx(c^2x^2+1)^{\frac{5}{2}}}{18}-\frac{5\operatorname{arcsinh}(cx)cx(c^2x^2+1)^{\frac{3}{2}}}{72}-\frac{5\operatorname{arcsinh}(cx)cx\sqrt{c^2}}{48}\right.}{}$
parts	$\frac{d^2a^2(c^2x^2+1)^3}{6c^2}+\frac{d^2b^2\left(\frac{\operatorname{arcsinh}(cx)^2(c^2x^2+1)^3}{6}-\frac{\operatorname{arcsinh}(cx)cx(c^2x^2+1)^{\frac{5}{2}}}{18}-\frac{5\operatorname{arcsinh}(cx)cx(c^2x^2+1)^{\frac{3}{2}}}{72}-\frac{5\operatorname{arcsinh}(cx)cx}{48}\right.}{c^2}$

```
[In] int(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/6*d^2*a^2*(c^2*x^2+1)^3+d^2*b^2*(1/6*arcsinh(c*x)^2*(c^2*x^2+1)^3-
1/18*arcsinh(c*x)*c*x*(c^2*x^2+1)^(5/2)-5/72*arcsinh(c*x)*c*x*(c^2*x^2+1)^(
3/2)-5/48*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)-5/96*arcsinh(c*x)^2+1/108*(c^2
*x^2+1)^3+5/288*(c^2*x^2+1)^2+5/96*c^2*x^2+5/96)+2*d^2*a*b*(1/6*arcsinh(c*x
)*c^6*x^6+1/2*arcsinh(c*x)*c^4*x^4+1/2*arcsinh(c*x)*c^2*x^2+11/96*arcsinh(c
*x)-1/36*c*x*(c^2*x^2+1)^(5/2)-5/144*c*x*(c^2*x^2+1)^(3/2)-5/96*c*x*(c^2*x^
2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.50

$$\int x(d+c^2dx^2)^2(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$=\frac{8(18a^2+b^2)c^6d^2x^6+3(144a^2+13b^2)c^4d^2x^4+9(48a^2+11b^2)c^2d^2x^2+9(16b^2c^6d^2x^6+48b^2c^4d^2x^4+}{}$$

```
[In] integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/864*(8*(18*a^2 + b^2)*c^6*d^2*x^6 + 3*(144*a^2 + 13*b^2)*c^4*d^2*x^4 + 9*
(48*a^2 + 11*b^2)*c^2*d^2*x^2 + 9*(16*b^2*c^6*d^2*x^6 + 48*b^2*c^4*d^2*x^4
+ 48*b^2*c^2*d^2*x^2 + 11*b^2*d^2)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 6*(48*a
*b*c^6*d^2*x^6 + 144*a*b*c^4*d^2*x^4 + 144*a*b*c^2*d^2*x^2 + 33*a*b*d^2 - (
8*b^2*c^5*d^2*x^5 + 26*b^2*c^3*d^2*x^3 + 33*b^2*c*d^2*x)*sqrt(c^2*x^2 + 1))
*log(c*x + sqrt(c^2*x^2 + 1)) - 6*(8*a*b*c^5*d^2*x^5 + 26*a*b*c^3*d^2*x^3 +
 33*a*b*c*d^2*x)*sqrt(c^2*x^2 + 1))/c^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(196) = 392.

Time = 0.79 (sec) , antiderivative size = 430, normalized size of antiderivative = 2.11

$$\int x(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2 \, dx$$
$$= \begin{cases} \frac{a^2c^4d^2x^6}{6} + \frac{a^2c^2d^2x^4}{2} + \frac{a^2d^2x^2}{2} + \frac{abc^4d^2x^6\,\text{asinh}\,(cx)}{3} - \frac{abc^3d^2x^5\sqrt{c^2x^2+1}}{18} + abc^2d^2x^4\,\text{asinh}\,(cx) - \frac{13abcd^2x^3\sqrt{c^2x^2+1}}{72} + c \\ \frac{a^2d^2x^2}{2} \end{cases}$$

```
[In] integrate(x*(c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**4*d**2*x**6/6 + a**2*c**2*d**2*x**4/2 + a**2*d**2*x**2/2
 + a*b*c**4*d**2*x**6*asinh(c*x)/3 - a*b*c**3*d**2*x**5*sqrt(c**2*x**2 + 1)
/18 + a*b*c**2*d**2*x**4*asinh(c*x) - 13*a*b*c*d**2*x**3*sqrt(c**2*x**2 + 1
)/72 + a*b*d**2*x**2*asinh(c*x) - 11*a*b*d**2*x*sqrt(c**2*x**2 + 1)/(48*c)
+ 11*a*b*d**2*asinh(c*x)/(48*c**2) + b**2*c**4*d**2*x**6*asinh(c*x)**2/6 +
b**2*c**4*d**2*x**6/108 - b**2*c**3*d**2*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x
)/18 + b**2*c**2*d**2*x**4*asinh(c*x)**2/2 + 13*b**2*c**2*d**2*x**4/288 - 1
3*b**2*c*d**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/72 + b**2*d**2*x**2*asinh
(c*x)**2/2 + 11*b**2*d**2*x**2/96 - 11*b**2*d**2*x*sqrt(c**2*x**2 + 1)*asin
h(c*x)/(48*c) + 11*b**2*d**2*asinh(c*x)**2/(96*c**2), Ne(c, 0)), (a**2*d**2
*x**2/2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 621 vs. 2(182) = 364.

Time = 0.24 (sec) , antiderivative size = 621, normalized size of antiderivative = 3.04

$$\int x\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{1}{6}b^2c^4d^2x^6\operatorname{arsinh}(cx)^2 + \frac{1}{6}a^2c^4d^2x^6 + \frac{1}{2}b^2c^2d^2x^4\operatorname{arsinh}(cx)^2 + \frac{1}{2}a^2c^2d^2x^4$$
$$+\frac{1}{144}\left(48\,x^6\operatorname{arsinh}(cx) - \left(\frac{8\sqrt{c^2x^2+1}x^5}{c^2} - \frac{10\sqrt{c^2x^2+1}x^3}{c^4} + \frac{15\sqrt{c^2x^2+1}x}{c^6} - \frac{15\operatorname{arsinh}(cx)}{c^7}\right)c\right)ab$$
$$+\frac{1}{864}\left(\left(\frac{8\,x^6}{c^2} - \frac{15\,x^4}{c^4} + \frac{45\,x^2}{c^6} - \frac{45\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^8}\right)c^2 - 6\left(\frac{8\sqrt{c^2x^2+1}x^5}{c^2} - \frac{10\sqrt{c^2x^2+1}x^3}{c^4}\right.\right.$$
$$+\frac{1}{2}b^2d^2x^2\operatorname{arsinh}(cx)^2$$
$$+\frac{1}{8}\left(8\,x^4\operatorname{arsinh}(cx) - \left(\frac{2\sqrt{c^2x^2+1}x^3}{c^2} - \frac{3\sqrt{c^2x^2+1}x}{c^4} + \frac{3\operatorname{arsinh}(cx)}{c^5}\right)c\right)abc^2d^2$$
$$+\frac{1}{16}\left(\left(\frac{x^4}{c^2} - \frac{3\,x^2}{c^4} + \frac{3\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^6}\right)c^2 - 2\left(\frac{2\sqrt{c^2x^2+1}x^3}{c^2} - \frac{3\sqrt{c^2x^2+1}x}{c^4} + \frac{3\operatorname{arsinh}(cx)}{c^5}\right.\right.$$
$$+\frac{1}{2}a^2d^2x^2 + \frac{1}{2}\left(2\,x^2\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\operatorname{arsinh}(cx)}{c^3}\right)\right)abd^2$$
$$+\frac{1}{4}\left(c^2\left(\frac{x^2}{c^2} - \frac{\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^4}\right) - 2\,c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\operatorname{arsinh}(cx)}{c^3}\right)\operatorname{arsinh}(cx)\right)b^2d^2$$

```
[In] integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*c^4*d^2*x^6*arcsinh(c*x)^2 + 1/6*a^2*c^4*d^2*x^6 + 1/2*b^2*c^2*d^2*
x^4*arcsinh(c*x)^2 + 1/2*a^2*c^2*d^2*x^4 + 1/144*(48*x^6*arcsinh(c*x) - (8*
sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2
+ 1)*x/c^6 - 15*arcsinh(c*x)/c^7)*c)*a*b*c^4*d^2 + 1/864*((8*x^6/c^2 - 15*x
^4/c^4 + 45*x^2/c^6 - 45*log(c*x + sqrt(c^2*x^2 + 1))^2/c^8)*c^2 - 6*(8*sqr
t(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1
)*x/c^6 - 15*arcsinh(c*x)/c^7)*c*arcsinh(c*x))*b^2*c^4*d^2 + 1/2*b^2*d^2*x^
2*arcsinh(c*x)^2 + 1/8*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 -
 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*a*b*c^2*d^2 + 1/16*((x^
4/c^2 - 3*x^2/c^4 + 3*log(c*x + sqrt(c^2*x^2 + 1))^2/c^6)*c^2 - 2*(2*sqrt(c
^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c*arc
sinh(c*x))*b^2*c^2*d^2 + 1/2*a^2*d^2*x^2 + 1/2*(2*x^2*arcsinh(c*x) - c*(sqr
t(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3))*a*b*d^2 + 1/4*(c^2*(x^2/c^2 - log
(c*x + sqrt(c^2*x^2 + 1))^2/c^4) - 2*c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c
*x)/c^3)*arcsinh(c*x))*b^2*d^2
```

Giac [F(-2)]

Exception generated.

$$\int x(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2\,dx = \int x\,(a+b\,\text{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^2\,dx$$

```
[In] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2,x)
```

```
[Out] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2, x)
```

3.211 $\int (d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 214

$$\begin{aligned}\int (d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2 \, dx = {} & \frac{298}{225}b^2d^2x + \frac{76}{675}b^2c^2d^2x^3 + \frac{2}{125}b^2c^4d^2x^5 \\ & - \frac{16bd^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{15c} \\ & - \frac{8bd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{45c} \\ & - \frac{2bd^2(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{25c} \\ & + \frac{8}{15}d^2x(a+b\text{arcsinh}(cx))^2 \\ & + \frac{4}{15}d^2x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 \\ & + \frac{1}{5}d^2x(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2\end{aligned}$$

```
[Out] 298/225*b^2*d^2*x+76/675*b^2*c^2*d^2*x^3+2/125*b^2*c^4*d^2*x^5-8/45*b*d^2*(
c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c-2/25*b*d^2*(c^2*x^2+1)^(5/2)*(a+b*arc
sinh(c*x))/c+8/15*d^2*x*(a+b*arcsinh(c*x))^2+4/15*d^2*x*(c^2*x^2+1)*(a+b*ar
csinh(c*x))^2+1/5*d^2*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2-16/15*b*d^2*(a+b
*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5786, 5772, 5798, 8, 200}

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))^2 \, dx = \frac{1}{5}d^2x\left(c^2x^2+1\right)^2 (a+b\text{arcsinh}(cx))^2 + \frac{4}{15}d^2x\left(c^2x^2+1\right) (a+b\text{arcsinh}(cx))^2 - \frac{2bd^2(c^2x^2+1)^{5/2} (a+b\text{arcsinh}(cx))}{25c} - \frac{8bd^2(c^2x^2+1)^{3/2} (a+b\text{arcsinh}(cx))}{45c} - \frac{16bd^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{15c} + \frac{8}{15}d^2x(a+b\text{arcsinh}(cx))^2 + \frac{2}{125}b^2c^4d^2x^5 + \frac{76}{675}b^2c^2d^2x^3 + \frac{298}{225}b^2d^2x$$

```
[In] Int[(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (298*b^2*d^2*x)/225 + (76*b^2*c^2*d^2*x^3)/675 + (2*b^2*c^4*d^2*x^5)/125 -
(16*b*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(15*c) - (8*b*d^2*(1 + c^
2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(45*c) - (2*b*d^2*(1 + c^2*x^2)^(5/2)*(a
 + b*ArcSinh[c*x]))/(25*c) + (8*d^2*x*(a + b*ArcSinh[c*x])^2)/15 + (4*d^2*x
*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/15 + (d^2*x*(1 + c^2*x^2)^2*(a + b*A
rcSinh[c*x])^2)/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}d^2x\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2+\frac{1}{5}(4d)\int\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))^2\,dx\\
&\quad-\frac{1}{5}\left(2bcd^2\right)\int x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx\\
&= -\frac{2bd^2\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{25c}+\frac{4}{15}d^2x\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2\\
&\quad+\frac{1}{5}d^2x\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2+\frac{1}{15}\left(8d^2\right)\int(a+b\text{arcsinh}(cx))^2\,dx\\
&\quad+\frac{1}{25}\left(2b^2d^2\right)\int\left(1+c^2x^2\right)^2\,dx-\frac{1}{15}\left(8bcd^2\right)\int x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx\\
&= -\frac{8bd^2\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))}{45c}-\frac{2bd^2\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{25c}\\
&\quad+\frac{8}{15}d^2x(a+b\text{arcsinh}(cx))^2+\frac{4}{15}d^2x\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2\\
&\quad+\frac{1}{5}d^2x\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2+\frac{1}{25}\left(2b^2d^2\right)\int\left(1+2c^2x^2+c^4x^4\right)\,dx\\
&\quad+\frac{1}{45}\left(8b^2d^2\right)\int\left(1+c^2x^2\right)\,dx-\frac{1}{15}\left(16bcd^2\right)\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx\\
&= \frac{58}{225}b^2d^2x+\frac{76}{675}b^2c^2d^2x^3+\frac{2}{125}b^2c^4d^2x^5-\frac{16bd^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{15c}\\
&\quad-\frac{8bd^2\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))}{45c}-\frac{2bd^2\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{25c}\\
&\quad+\frac{8}{15}d^2x(a+b\text{arcsinh}(cx))^2+\frac{4}{15}d^2x\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2\\
&\quad+\frac{1}{5}d^2x\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2+\frac{1}{15}\left(16b^2d^2\right)\int 1\,dx
\end{aligned}$$

$$= \frac{298}{225}b^2d^2x + \frac{76}{675}b^2c^2d^2x^3 + \frac{2}{125}b^2c^4d^2x^5 - \frac{16bd^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{15c} - \frac{8bd^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{45c} - \frac{2bd^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{25c} + \frac{8}{15}d^2x(a+b\operatorname{arcsinh}(cx))^2 + \frac{4}{15}d^2x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{5}d^2x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2$$

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.89

$$\int (d+c^2dx^2)^2(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$= \frac{d^2\left(225a^2cx(15+10c^2x^2+3c^4x^4) - 30ab\sqrt{1+c^2x^2}(149+38c^2x^2+9c^4x^4) + 2b^2cx(2235+190c^2x^2+27c^4\right.}{}$$

[In] Integrate[(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]

[Out] (d^2*(225*a^2*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4) - 30*a*b*Sqrt[1 + c^2*x^2]*(149 + 38*c^2*x^2 + 9*c^4*x^4) + 2*b^2*c*x*(2235 + 190*c^2*x^2 + 27*c^4*x^4) - 30*b*(-15*a*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4) + b*Sqrt[1 + c^2*x^2]*(149 + 38*c^2*x^2 + 9*c^4*x^4))*ArcSinh[c*x] + 225*b^2*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x]^2))/(3375*c)

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{d^2a^2\left(\frac{1}{5}c^5x^5+\frac{2}{3}c^3x^3+cx\right)+d^2b^2\left(\frac{8\operatorname{arcsinh}(cx)^2xc}{15}+\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{5}+\frac{4\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)}{15}-\frac{16\operatorname{arcsinh}(cx)\sqrt{c^2}}{15}\right.}{}$
default	$\frac{d^2a^2\left(\frac{1}{5}c^5x^5+\frac{2}{3}c^3x^3+cx\right)+d^2b^2\left(\frac{8\operatorname{arcsinh}(cx)^2xc}{15}+\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{5}+\frac{4\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)}{15}-\frac{16\operatorname{arcsinh}(cx)\sqrt{c^2}}{15}\right.}{}$
parts	$d^2a^2\left(\frac{1}{5}c^4x^5+\frac{2}{3}x^3c^2+x\right)+\frac{d^2b^2\left(\frac{8\operatorname{arcsinh}(cx)^2xc}{15}+\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{5}+\frac{4\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)}{15}-\frac{16\operatorname{arcs}}{}\right.}{}$

[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)

[Out] 1/c*(d^2*a^2*(1/5*c^5*x^5+2/3*c^3*x^3+c*x)+d^2*b^2*(8/15*arcsinh(c*x)^2*x*c+1/5*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^2+4/15*arcsinh(c*x)^2*c*x*(c^2*x^2+1)-1

```
6/15*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+4144/3375*c*x-2/25*arcsinh(c*x)*(c^2*x^
2+1)^(5/2)+2/125*c*x*(c^2*x^2+1)^2+272/3375*c*x*(c^2*x^2+1)-8/45*arcsinh(c*
x)*(c^2*x^2+1)^(3/2))+2*d^2*a*b*(1/5*arcsinh(c*x)*c^5*x^5+2/3*arcsinh(c*x)*
c^3*x^3+arcsinh(c*x)*c*x-149/225*(c^2*x^2+1)^(1/2)-1/25*c^4*x^4*(c^2*x^2+1)
^(1/2)-38/225*c^2*x^2*(c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.30

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{27\,(25\,a^2+2\,b^2)c^5d^2x^5+10\,(225\,a^2+38\,b^2)c^3d^2x^3+15\,(225\,a^2+298\,b^2)cd^2x+225\,(3\,b^2c^5d^2x^5+10\,b^2c^3d$$

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/3375*(27*(25*a^2 + 2*b^2)*c^5*d^2*x^5 + 10*(225*a^2 + 38*b^2)*c^3*d^2*x^3
 + 15*(225*a^2 + 298*b^2)*c*d^2*x + 225*(3*b^2*c^5*d^2*x^5 + 10*b^2*c^3*d^2
*x^3 + 15*b^2*c*d^2*x)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 30*(45*a*b*c^5*d^2*
x^5 + 150*a*b*c^3*d^2*x^3 + 225*a*b*c*d^2*x - (9*b^2*c^4*d^2*x^4 + 38*b^2*c
^2*d^2*x^2 + 149*b^2*d^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) -
 30*(9*a*b*c^4*d^2*x^4 + 38*a*b*c^2*d^2*x^2 + 149*a*b*d^2)*sqrt(c^2*x^2 + 1
))/c
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.82

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx$$
$$= \begin{cases} \frac{a^2c^4d^2x^5}{5}+\frac{2a^2c^2d^2x^3}{3}+a^2d^2x+\frac{2abc^4d^2x^5\operatorname{asinh}(cx)}{5}-\frac{2abc^3d^2x^4\sqrt{c^2x^2+1}}{25}+\frac{4abc^2d^2x^3\operatorname{asinh}(cx)}{3}-\frac{76abcd^2x^2\sqrt{c^2x^2+1}}{225}+ \\ a^2d^2x \end{cases}$$

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**4*d**2*x**5/5 + 2*a**2*c**2*d**2*x**3/3 + a**2*d**2*x +
2*a*b*c**4*d**2*x**5*asinh(c*x)/5 - 2*a*b*c**3*d**2*x**4*sqrt(c**2*x**2 + 1
)/25 + 4*a*b*c**2*d**2*x**3*asinh(c*x)/3 - 76*a*b*c*d**2*x**2*sqrt(c**2*x**
2 + 1)/225 + 2*a*b*d**2*x*asinh(c*x) - 298*a*b*d**2*sqrt(c**2*x**2 + 1)/(22
5*c) + b**2*c**4*d**2*x**5*asinh(c*x)**2/5 + 2*b**2*c**4*d**2*x**5/125 - 2*
b**2*c**3*d**2*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/25 + 2*b**2*c**2*d**2*x*
```

```
*3*asinh(c*x)**2/3 + 76*b**2*c**2*d**2*x**3/675 - 76*b**2*c*d**2*x**2*sqrt(
c**2*x**2 + 1)*asinh(c*x)/225 + b**2*d**2*x*asinh(c*x)**2 + 298*b**2*d**2*x
/225 - 298*b**2*d**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(225*c), Ne(c, 0)), (a*
*2*d**2*x, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. 2(190) = 380.

Time = 0.23 (sec) , antiderivative size = 457, normalized size of antiderivative = 2.14

$$\int \left(d+c^2dx^2\right)^2(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$= \frac{1}{5}\,b^2c^4d^2x^5\operatorname{arsinh}(cx)^2 + \frac{1}{5}\,a^2c^4d^2x^5 + \frac{2}{3}\,b^2c^2d^2x^3\operatorname{arsinh}(cx)^2$$
$$+ \frac{2}{75}\left(15\,x^5\operatorname{arsinh}(cx) - \left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)abc^4d^2$$
$$- \frac{2}{1125}\left(15\left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\operatorname{arsinh}(cx) - \frac{9\,c^4x^5 - 20\,c^2x^3 + 120\,x}{c^4}\right)b$$
$$+ \frac{2}{3}\,a^2c^2d^2x^3 + \frac{4}{9}\left(3\,x^3\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)abc^2d^2$$
$$- \frac{4}{27}\left(3\,c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\operatorname{arsinh}(cx) - \frac{c^2x^3 - 6\,x}{c^2}\right)b^2c^2d^2$$
$$+ b^2d^2x\operatorname{arsinh}(cx)^2 + 2\,b^2d^2\left(x - \frac{\sqrt{c^2x^2+1}\operatorname{arsinh}(cx)}{c}\right)$$
$$+ a^2d^2x + \frac{2\left(cx\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)abd^2}{c}$$

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*c^4*d^2*x^5*arcsinh(c*x)^2 + 1/5*a^2*c^4*d^2*x^5 + 2/3*b^2*c^2*d^2*
x^3*arcsinh(c*x)^2 + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c
^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^4*d^2
- 2/1125*(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8
*sqrt(c^2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c
^4)*b^2*c^4*d^2 + 2/3*a^2*c^2*d^2*x^3 + 4/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c
^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^2*d^2 - 4/27*(3*c*(sq
rt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3
- 6*x)/c^2)*b^2*c^2*d^2 + b^2*d^2*x*arcsinh(c*x)^2 + 2*b^2*d^2*(x - sqrt(c^
2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d^2*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2
 + 1))*a*b*d^2/c
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^2\,dx$$

[In] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2,x)

[Out] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2, x)

3.212 $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 257

$$
\begin{aligned}
\int \frac{(d+c^2dx^2)^2\,(a+b\text{arcsinh}(cx))^2}{x}\,dx &= \frac{13}{32}b^2c^2d^2x^2+\frac{1}{32}b^2c^4d^2x^4 \\
&-\frac{11}{16}bcd^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) \\
&-\frac{1}{8}bcd^2x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx)) \\
&-\frac{11}{32}d^2(a+b\text{arcsinh}(cx))^2 \\
&+\frac{1}{2}d^2\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2 \\
&+\frac{1}{4}d^2\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2 \\
&+\frac{d^2(a+b\text{arcsinh}(cx))^3}{3b} \\
&+d^2(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right) \\
&-bd^2(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right) \\
&-\frac{1}{2}b^2d^2\,\text{PolyLog}\left(3,e^{-2\text{arcsinh}(cx)}\right)
\end{aligned}
$$

```
[Out] 13/32*b^2*c^2*d^2*x^2+1/32*b^2*c^4*d^2*x^4-1/8*b*c*d^2*x*(c^2*x^2+1)^(3/2)*
(a+b*arcsinh(c*x))-11/32*d^2*(a+b*arcsinh(c*x))^2+1/2*d^2*(c^2*x^2+1)*(a+b*
arcsinh(c*x))^2+1/4*d^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2+1/3*d^2*(a+b*arc
sinh(c*x))^3/b+d^2*(a+b*arcsinh(c*x))^2*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)-b
*d^2*(a+b*arcsinh(c*x))*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b^2*d^2*
polylog(3,1/(c*x+(c^2*x^2+1)^(1/2))^2)-11/16*b*c*d^2*x*(a+b*arcsinh(c*x))*(
c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {5808, 5775, 3797, 2221, 2611, 2320, 6724, 5785, 5783, 30, 5786, 14}

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x} dx = -\frac{1}{8}bcd^2x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{11}{16}bcd^2x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx)) + \frac{1}{4}d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2 + \frac{1}{2}d^2(c^2x^2+1)(a+b\text{arcsinh}(cx))^2 - bd^2\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)) + \frac{d^2(a+b\text{arcsinh}(cx))^3}{3b} - \frac{11}{32}d^2(a+b\text{arcsinh}(cx))^2 + d^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2 - \frac{1}{2}b^2d^2\,\text{PolyLog}\left(3,e^{-2\text{arcsinh}(cx)}\right) + \frac{1}{32}b^2c^4d^2x^4 + \frac{13}{32}b^2c^2d^2x^2$$

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (13*b^2*c^2*d^2*x^2)/32 + (b^2*c^4*d^2*x^4)/32 - (11*b*c*d^2*x*Sqrt[1 + c^2
*x^2]*(a + b*ArcSinh[c*x]))/16 - (b*c*d^2*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcS
inh[c*x]))/8 - (11*d^2*(a + b*ArcSinh[c*x])^2)/32 + (d^2*(1 + c^2*x^2)*(a +
 b*ArcSinh[c*x])^2)/2 + (d^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/4 + (d
^2*(a + b*ArcSinh[c*x])^3)/(3*b) + d^2*(a + b*ArcSinh[c*x])^2*Log[1 - E^(-2
*ArcSinh[c*x])] - b*d^2*(a + b*ArcSinh[c*x])*PolyLog[2, E^(-2*ArcSinh[c*x])
] - (b^2*d^2*PolyLog[3, E^(-2*ArcSinh[c*x])])/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
```

```
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}d^2\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2+d\int\frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2}{x}\,dx \\
&\quad -\frac{1}{2}\left(bcd^2\right)\int\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx \\
&= -\frac{1}{8}bcd^2x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{1}{2}d^2\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2 \\
&\quad +\frac{1}{4}d^2\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2+d^2\int\frac{(a+b\text{arcsinh}(cx))^2}{x}\,dx \\
&\quad -\frac{1}{8}\left(3bcd^2\right)\int\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx \\
&\quad -\left(bcd^2\right)\int\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx+\frac{1}{8}\left(b^2c^2d^2\right)\int x\left(1+c^2x^2\right)\,dx
\end{aligned}$$

$$= -\frac{11}{16}bcd^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) - \frac{1}{8}bcd^2x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{1}{2}d^2\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2 + \frac{1}{4}d^2\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2 - \frac{d^2\text{Subst}\left(\int x^2\coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)}{b} - \frac{1}{16}\left(3bcd^2\right)\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx - \frac{1}{2}\left(bcd^2\right)\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx + \frac{1}{8}\left(b^2c^2d^2\right)\int\left(x+c^2x^3\right)\,dx + \frac{1}{16}\left(3b^2c^2d^2\right)\int x\,dx + \frac{1}{2}\left(b^2c^2d^2\right)\int x\,dx$$

$$= \frac{13}{32}b^2c^2d^2x^2 + \frac{1}{32}b^2c^4d^2x^4 - \frac{11}{16}bcd^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) - \frac{1}{8}bcd^2x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{11}{32}d^2(a+b\text{arcsinh}(cx))^2 + \frac{1}{2}d^2\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2 + \frac{1}{4}d^2\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2 + \frac{d^2(a+b\text{arcsinh}(cx))^3}{3b} + \frac{\left(2d^2\right)\text{Subst}\left(\int\frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x^2}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b}$$

$$= \frac{13}{32}b^2c^2d^2x^2 + \frac{1}{32}b^2c^4d^2x^4 - \frac{11}{16}bcd^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) - \frac{1}{8}bcd^2x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{11}{32}d^2(a+b\text{arcsinh}(cx))^2 + \frac{1}{2}d^2\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2 + \frac{1}{4}d^2\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2 + \frac{d^2(a+b\text{arcsinh}(cx))^3}{3b} + d^2(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right) - \left(2d^2\right)\text{Subst}\left(\int x\log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)$$

$$= \frac{13}{32}b^2c^2d^2x^2 + \frac{1}{32}b^2c^4d^2x^4 - \frac{11}{16}bcd^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) - \frac{1}{8}bcd^2x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{11}{32}d^2(a+b\text{arcsinh}(cx))^2 + \frac{1}{2}d^2\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2 + \frac{1}{4}d^2\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2 + \frac{d^2(a+b\text{arcsinh}(cx))^3}{3b} + d^2(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right) - bd^2(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2, e^{-2\text{arcsinh}(cx)}\right) + \left(bd^2\right)\text{Subst}\left(\int\text{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)$$

$$
\begin{aligned}
&= \frac{13}{32}b^2c^2d^2x^2 + \frac{1}{32}b^2c^4d^2x^4 - \frac{11}{16}bcd^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) \\
&\quad - \frac{1}{8}bcd^2x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{11}{32}d^2(a+b\text{arcsinh}(cx))^2 \\
&\quad + \frac{1}{2}d^2\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2 + \frac{1}{4}d^2\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2 \\
&\quad + \frac{d^2(a+b\text{arcsinh}(cx))^3}{3b} + d^2(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right) \\
&\quad - bd^2(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right) \\
&\quad - \frac{1}{2}\left(b^2d^2\right)\text{Subst}\left(\int\frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{13}{32}b^2c^2d^2x^2 + \frac{1}{32}b^2c^4d^2x^4 - \frac{11}{16}bcd^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) \\
&\quad - \frac{1}{8}bcd^2x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{11}{32}d^2(a+b\text{arcsinh}(cx))^2 \\
&\quad + \frac{1}{2}d^2\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2 + \frac{1}{4}d^2\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2 \\
&\quad + \frac{d^2(a+b\text{arcsinh}(cx))^3}{3b} + d^2(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right) \\
&\quad - bd^2(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right) \\
&\quad - \frac{1}{2}b^2d^2\,\text{PolyLog}\left(3,e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.27

$$
\begin{aligned}
\int\frac{\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2}{x}\,dx &= \frac{1}{768}d^2\Big(768a^2c^2x^2 + 192a^2c^4x^4 - 624abcx\sqrt{1+c^2x^2} \\
&\quad - 96abc^3x^3\sqrt{1+c^2x^2} + 1536abc^2x^2\text{arcsinh}(cx) \\
&\quad + 384abc^4x^4\text{arcsinh}(cx) - 768ab\text{arcsinh}(cx)^2 \\
&\quad - 256b^2\text{arcsinh}(cx)^3 + 144b^2\cosh(2\text{arcsinh}(cx)) \\
&\quad + 288b^2\text{arcsinh}(cx)^2\cosh(2\text{arcsinh}(cx)) \\
&\quad + 3b^2\cosh(4\text{arcsinh}(cx)) \\
&\quad + 24b^2\text{arcsinh}(cx)^2\cosh(4\text{arcsinh}(cx)) \\
&\quad + 1536ab\text{arcsinh}(cx)\log\left(1-e^{2\text{arcsinh}(cx)}\right) \\
&\quad + 768b^2\text{arcsinh}(cx)^2\log\left(1-e^{2\text{arcsinh}(cx)}\right) \\
&\quad + 768a^2\log(cx) - 624ab\log\left(-cx+\sqrt{1+c^2x^2}\right) \\
&\quad + 768b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right) \\
&\quad - 384b^2\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(cx)}\right) \\
&\quad - 288b^2\text{arcsinh}(cx)\sinh(2\text{arcsinh}(cx)) \\
&\quad - 12b^2\text{arcsinh}(cx)\sinh(4\text{arcsinh}(cx))\Big)
\end{aligned}
$$

[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x,x]

[Out] (d^2*(768*a^2*c^2*x^2 + 192*a^2*c^4*x^4 - 624*a*b*c*x*Sqrt[1 + c^2*x^2] - 96*a*b*c^3*x^3*Sqrt[1 + c^2*x^2] + 1536*a*b*c^2*x^2*ArcSinh[c*x] + 384*a*b*c^4*x^4*ArcSinh[c*x] - 768*a*b*ArcSinh[c*x]^2 - 256*b^2*ArcSinh[c*x]^3 + 144*b^2*Cosh[2*ArcSinh[c*x]] + 288*b^2*ArcSinh[c*x]^2*Cosh[2*ArcSinh[c*x]] + 3*b^2*Cosh[4*ArcSinh[c*x]] + 24*b^2*ArcSinh[c*x]^2*Cosh[4*ArcSinh[c*x]] + 1536*a*b*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])] + 768*b^2*ArcSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + 768*a^2*Log[c*x] - 624*a*b*Log[-(c*x) + Sqrt[1 + c^2*x^2]] + 768*b*(a + b*ArcSinh[c*x])*PolyLog[2, E^(2*ArcSinh[c*x])] - 384*b^2*PolyLog[3, E^(2*ArcSinh[c*x])] - 288*b^2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]] - 12*b^2*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/768

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(262) = 524$.

Time = 0.31 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.23

method	result
parts	$d^2a^2\left(\frac{c^4x^4}{4}+c^2x^2+\ln\left(x\right)\right)-\frac{d^2ab\,c^3x^3\sqrt{c^2x^2+1}}{8}+\frac{13d^2b^2\operatorname{arcsinh}(cx)^2}{32}-\frac{d^2b^2\operatorname{arcsinh}(cx)^3}{3}-2d^2b^2\,\mathrm{p}$
derivativedivides	$-\frac{d^2ab\,c^3x^3\sqrt{c^2x^2+1}}{8}+d^2a^2\left(\frac{c^4x^4}{4}+c^2x^2+\ln\left(cx\right)\right)+\frac{13d^2b^2\operatorname{arcsinh}(cx)^2}{32}-\frac{d^2b^2\operatorname{arcsinh}(cx)^3}{3}-2d^2b$
default	$-\frac{d^2ab\,c^3x^3\sqrt{c^2x^2+1}}{8}+d^2a^2\left(\frac{c^4x^4}{4}+c^2x^2+\ln\left(cx\right)\right)+\frac{13d^2b^2\operatorname{arcsinh}(cx)^2}{32}-\frac{d^2b^2\operatorname{arcsinh}(cx)^3}{3}-2d^2b$

[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x,x,method=_RETURNVERBOSE)

[Out] d^2*a^2*(1/4*c^4*x^4+c^2*x^2+ln(x))-1/8*d^2*a*b*c^3*x^3*(c^2*x^2+1)^(1/2)+13/32*d^2*b^2*arcsinh(c*x)^2-1/3*d^2*b^2*arcsinh(c*x)^3-2*d^2*b^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))-2*d^2*b^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))+49/256*d^2*b^2+13/32*b^2*c^2*d^2*x^2+1/32*b^2*c^4*d^2*x^4+d^2*b^2*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*d^2*b^2*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))+2*d^2*b^2*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+d^2*b^2*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))-d^2*a*b*arcsinh(c*x)^2+13/16*d^2*a*b*arcsinh(c*x)+2*d^2*a*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*d^2*a*b*polylog(2,c*x+(c^2*x^2+1)^(1/2))-13/16*d^2*a*b*c*x*(c^2*x^2+1)^(1/2)+1/2*d^2*a*b*arcsinh(c*x)*c^4*x^4+2*d^2*a*b*arcsinh(c*x)*c^2*x^2-1/8*d^2*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3*x^3-13/16*d^2*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x+1/4*d^2*b^2*arcsinh(c*x)^2*c^4*x^4+d^2*b^2*arcsinh(c*x)^2*c^2*x^2+2*d^2*a*b*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*d^2*a*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))

Fricas [F]

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x} \, dx = \int \frac{(c^2dx^2+d)^2(b \operatorname{arsinh}(cx)+a)^2}{x} \, dx$$

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x,x, algorithm="fricas")
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x} \, dx = d^2 \left(\int \frac{a^2}{x} \, dx + \int 2a^2c^2x \, dx + \int a^2c^4x^3 \, dx + \int \frac{b^2 \operatorname{asinh}^2(cx)}{x} \, dx + \int \frac{2ab \operatorname{asinh}(cx)}{x} \, dx + \int 2b^2c^2x \operatorname{asinh}^2(cx) \, dx + \int b^2c^4x^3 \operatorname{asinh}^2(cx) \, dx + \int 4abc^2x \operatorname{asinh}(cx) \, dx + \int 2abc^4x^3 \operatorname{asinh}(cx) \, dx \right)$$

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2/x,x)
[Out] d**2*(Integral(a**2/x, x) + Integral(2*a**2*c**2*x, x) + Integral(a**2*c**4
*x**3, x) + Integral(b**2*asinh(c*x)**2/x, x) + Integral(2*a*b*asinh(c*x)/x
, x) + Integral(2*b**2*c**2*x*asinh(c*x)**2, x) + Integral(b**2*c**4*x**3*a
sinh(c*x)**2, x) + Integral(4*a*b*c**2*x*asinh(c*x), x) + Integral(2*a*b*c*
*4*x**3*asinh(c*x), x))
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x} \, dx = \int \frac{(c^2dx^2+d)^2(b \operatorname{arsinh}(cx)+a)^2}{x} \, dx$$

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x,x, algorithm="maxima")
[Out] 1/4*a^2*c^4*d^2*x^4 + a^2*c^2*d^2*x^2 + a^2*d^2*log(x) + integrate(b^2*c^4*
d^2*x^3*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*a*b*c^4*d^2*x^3*log(c*x + sqrt(c
^2*x^2 + 1)) + 2*b^2*c^2*d^2*x*log(c*x + sqrt(c^2*x^2 + 1))^2 + 4*a*b*c^2*d
^2*x*log(c*x + sqrt(c^2*x^2 + 1)) + b^2*d^2*log(c*x + sqrt(c^2*x^2 + 1))^2/
x + 2*a*b*d^2*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x} \, dx = \text{Exception raised: TypeError}$$

[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x} \, dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)^2}{x} \, dx$$

[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x,x)

[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x, x)

3.213 $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 229

$$\begin{aligned}\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x^2}\,dx &= \frac{32}{9}b^2c^2d^2x+\frac{2}{27}b^2c^4d^2x^3\\ &\quad-\frac{10}{3}bcd^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\\ &\quad-\frac{2}{9}bcd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\\ &\quad+\frac{8}{3}c^2d^2x(a+b\text{arcsinh}(cx))^2\\ &\quad+\frac{4}{3}c^2d^2x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2\\ &\quad-\frac{d^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{x}\\ &\quad-4bcd^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)\\ &\quad-2b^2cd^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)\\ &\quad+2b^2cd^2\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\end{aligned}$$

```
[Out] 32/9*b^2*c^2*d^2*x+2/27*b^2*c^4*d^2*x^3-2/9*b*c*d^2*(c^2*x^2+1)^(3/2)*(a+b*
arcsinh(c*x))+8/3*c^2*d^2*x*(a+b*arcsinh(c*x))^2+4/3*c^2*d^2*x*(c^2*x^2+1)*
(a+b*arcsinh(c*x))^2-d^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/x-4*b*c*d^2*(a+
b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))-2*b^2*c*d^2*polylog(2,-c*x-(
c^2*x^2+1)^(1/2))+2*b^2*c*d^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))-10/3*b*c*d^2
*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {5807, 5786, 5772, 5798, 8, 5808, 5806, 5816, 4267, 2317, 2438}

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x^2} dx = -4bcd^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)) + \frac{4}{3}c^2d^2x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2 - \frac{2}{9}bcd^2\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{10}{3}bcd^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx)) - \frac{d^2(c^2x^2+1)^2 (a+b\text{arcsinh}(cx))^2}{x} + \frac{8}{3}c^2d^2x(a+b\text{arcsinh}(cx))^2 - 2b^2cd^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right) + 2b^2cd^2\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right) + \frac{2}{27}b^2c^4d^2x^3 + \frac{32}{9}b^2c^2d^2x$$

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (32*b^2*c^2*d^2*x)/9 + (2*b^2*c^4*d^2*x^3)/27 - (10*b*c*d^2*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x]))/3 - (2*b*c*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[
c*x]))/9 + (8*c^2*d^2*x*(a + b*ArcSinh[c*x])^2)/3 + (4*c^2*d^2*x*(1 + c^2*x
^2)*(a + b*ArcSinh[c*x])^2)/3 - (d^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2
)/x - 4*b*c*d^2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - 2*b^2*c*d^2*
PolyLog[2, -E^ArcSinh[c*x]] + 2*b^2*c*d^2*PolyLog[2, E^ArcSinh[c*x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
```

```
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))^2}{x} + (4c^2d)\int (d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2\,dx \\
&\quad + (2bcd^2)\int \frac{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{x}\,dx \\
&= \frac{2}{3}bcd^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx)) + \frac{4}{3}c^2d^2x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2 \\
&\quad - \frac{d^2(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))^2}{x} + (2bcd^2)\int \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{x}\,dx \\
&\quad + \frac{1}{3}(8c^2d^2)\int (a+b\text{arcsinh}(cx))^2\,dx - \frac{1}{3}(2b^2c^2d^2)\int (1+c^2x^2)\,dx \\
&\quad - \frac{1}{3}(8bc^3d^2)\int x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx
\end{aligned}$$

$$= -\frac{2}{3}b^2c^2d^2x - \frac{2}{9}b^2c^4d^2x^3 + 2bcd^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) - \frac{2}{9}bcd^2\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{8}{3}c^2d^2x(a+b\text{arcsinh}(cx))^2 + \frac{4}{3}c^2d^2x\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2 - \frac{d^2\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2}{x} + \left(2bcd^2\right)\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{1+c^2x^2}}\,dx + \frac{1}{9}\left(8b^2c^2d^2\right)\int \left(1+c^2x^2\right)\,dx - \left(2b^2c^2d^2\right)\int 1\,dx - \frac{1}{3}\left(16bc^3d^2\right)\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx$$

$$= -\frac{16}{9}b^2c^2d^2x + \frac{2}{27}b^2c^4d^2x^3 - \frac{10}{3}bcd^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) - \frac{2}{9}bcd^2\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{8}{3}c^2d^2x(a+b\text{arcsinh}(cx))^2 + \frac{4}{3}c^2d^2x\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2 - \frac{d^2\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2}{x} + \left(2bcd^2\right)\text{Subst}\left(\int (a+bx)\text{csch}(x)\,dx, x, \text{arcsinh}(cx)\right) + \frac{1}{3}\left(16b^2c^2d^2\right)\int 1\,dx$$

$$= \frac{32}{9}b^2c^2d^2x + \frac{2}{27}b^2c^4d^2x^3 - \frac{10}{3}bcd^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) - \frac{2}{9}bcd^2\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{8}{3}c^2d^2x(a+b\text{arcsinh}(cx))^2 + \frac{4}{3}c^2d^2x\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2 - \frac{d^2\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2}{x} - 4bcd^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) - \left(2b^2cd^2\right)\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \text{arcsinh}(cx)\right) + \left(2b^2cd^2\right)\text{Subst}\left(\int \log\left(1+e^x\right)\,dx, x, \text{arcsinh}(cx)\right)$$

$$= \frac{32}{9}b^2c^2d^2x + \frac{2}{27}b^2c^4d^2x^3 - \frac{10}{3}bcd^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) - \frac{2}{9}bcd^2\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{8}{3}c^2d^2x(a+b\text{arcsinh}(cx))^2 + \frac{4}{3}c^2d^2x\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2 - \frac{d^2\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2}{x} - 4bcd^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) - \left(2b^2cd^2\right)\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right) + \left(2b^2cd^2\right)\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)$$

$$= \frac{32}{9}b^2c^2d^2x + \frac{2}{27}b^2c^4d^2x^3 - \frac{10}{3}bcd^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx)) - \frac{2}{9}bcd^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx)) + \frac{8}{3}c^2d^2x(a+b\operatorname{arcsinh}(cx))^2 + \frac{4}{3}c^2d^2x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2 - \frac{d^2(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{x} - 4bcd^2(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right) - 2b^2cd^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right) + 2b^2cd^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)$$

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.34

$$\int \frac{(d+c^2dx^2)^2(a+b\operatorname{arcsinh}(cx))^2}{x^2}\,dx$$
$$= \frac{1}{54}d^2\left(-\frac{54a^2}{x} + 108a^2c^2x + 18a^2c^4x^3 - 12abc(-2+c^2x^2)\sqrt{1+c^2x^2} + 36abc^4x^3\operatorname{arcsinh}(cx) - 189b^2c\sqrt{1+c^2x^2}\operatorname{arcsinh}(cx) + 216abc\left(-\sqrt{1+c^2x^2} + cx\operatorname{arcsinh}(cx)\right) + 108b^2c^2x(2+\operatorname{arcsinh}(cx)^2) + 2b^2c^2x(-12+2c^2x^2+9c^2x^2\operatorname{arcsinh}(cx)^2) - \frac{108ab\left(\operatorname{arcsinh}(cx) + cx\operatorname{arctanh}\left(\sqrt{1+c^2x^2}\right)\right)}{x} - 3b^2c\operatorname{arcsinh}(cx)\cosh(3\operatorname{arcsinh}(cx)) - \frac{54b^2\operatorname{arcsinh}(cx)\left(\operatorname{arcsinh}(cx) + 2cx\left(-\log\left(1-e^{-\operatorname{arcsinh}(cx)}\right) + \log\left(1+e^{-\operatorname{arcsinh}(cx)}\right)\right)\right)}{x} + 108b^2c\operatorname{PolyLog}\left(2,-e^{-\operatorname{arcsinh}(cx)}\right) - 108b^2c\operatorname{PolyLog}\left(2,e^{-\operatorname{arcsinh}(cx)}\right)\right)$$

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (d^2*((-54*a^2)/x + 108*a^2*c^2*x + 18*a^2*c^4*x^3 - 12*a*b*c*(-2 + c^2*x^2
)*Sqrt[1 + c^2*x^2] + 36*a*b*c^4*x^3*ArcSinh[c*x] - 189*b^2*c*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x] + 216*a*b*c*(-Sqrt[1 + c^2*x^2] + c*x*ArcSinh[c*x]) + 108
*b^2*c^2*x*(2 + ArcSinh[c*x]^2) + 2*b^2*c^2*x*(-12 + 2*c^2*x^2 + 9*c^2*x^2*
ArcSinh[c*x]^2) - (108*a*b*(ArcSinh[c*x] + c*x*ArcTanh[Sqrt[1 + c^2*x^2]]))
/x - 3*b^2*c*ArcSinh[c*x]*Cosh[3*ArcSinh[c*x]] - (54*b^2*ArcSinh[c*x]*(ArcS
inh[c*x] + 2*c*x*(-Log[1 - E^(-ArcSinh[c*x])] + Log[1 + E^(-ArcSinh[c*x])])
))/x + 108*b^2*c*PolyLog[2, -E^(-ArcSinh[c*x])] - 108*b^2*c*PolyLog[2, E^(-
ArcSinh[c*x])]))/54
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.59

method	result
derivativedivides	$c\left(d^2a^2\left(\frac{c^3x^3}{3}+2cx-\frac{1}{cx}\right)+\frac{2d^2b^2c^3x^3}{27}+\frac{32d^2b^2cx}{9}+2d^2b^2\operatorname{polylog}\left(2,cx+\sqrt{c^2x^2+1}\right)-2\right.$
default	$c\left(d^2a^2\left(\frac{c^3x^3}{3}+2cx-\frac{1}{cx}\right)+\frac{2d^2b^2c^3x^3}{27}+\frac{32d^2b^2cx}{9}+2d^2b^2\operatorname{polylog}\left(2,cx+\sqrt{c^2x^2+1}\right)-2\right.$
parts	$d^2a^2\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)+2d^2b^2\operatorname{arcsinh}(cx)^2c^2x-\frac{d^2b^2\operatorname{arcsinh}(cx)^2}{x}+\frac{d^2b^2\operatorname{arcsinh}(cx)^2c^4x^3}{3}+$

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(d^2*a^2*(1/3*c^3*x^3+2*c*x-1/c/x)+2/27*d^2*b^2*c^3*x^3+32/9*d^2*b^2*c*x+
2*d^2*b^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*d^2*b^2*polylog(2,-c*x-(c^2*x^
2+1)^(1/2))-2/9*d^2*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^2*x^2+1/3*d^2*b^2*
arcsinh(c*x)^2*c^3*x^3+2*d^2*b^2*arcsinh(c*x)^2*c*x-32/9*d^2*b^2*arcsinh(c*
x)*(c^2*x^2+1)^(1/2)+2*d^2*b^2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-2*d
^2*b^2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-d^2*b^2*arcsinh(c*x)^2/c/x+
2*d^2*a*b*(1/3*arcsinh(c*x)*c^3*x^3+2*arcsinh(c*x)*c*x-arcsinh(c*x)/c/x-1/9
*c^2*x^2*(c^2*x^2+1)^(1/2)-16/9*(c^2*x^2+1)^(1/2)-arctanh(1/(c^2*x^2+1)^(1/
2))))
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)^2\,(a+b\operatorname{arcsinh}(cx))^2}{x^2}\,dx=\int \frac{(c^2dx^2+d)^2(b\operatorname{arsinh}(cx)+a)^2}{x^2}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))/x^2, x)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^2\,(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = d^2\left(\int 2a^2c^2\,dx + \int \frac{a^2}{x^2}\,dx + \int a^2c^4x^2\,dx + \int 2b^2c^2\operatorname{asinh}^2(cx)\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{x^2}\,dx + \int 4abc^2\operatorname{asinh}(cx)\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{x^2}\,dx + \int b^2c^4x^2\operatorname{asinh}^2(cx)\,dx + \int 2abc^4x^2\operatorname{asinh}(cx)\,dx\right)$$

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2/x**2,x)
```

```
[Out] d**2*(Integral(2*a**2*c**2, x) + Integral(a**2/x**2, x) + Integral(a**2*c**
4*x**2, x) + Integral(2*b**2*c**2*asinh(c*x)**2, x) + Integral(b**2*asinh(c
*x)**2/x**2, x) + Integral(4*a*b*c**2*asinh(c*x), x) + Integral(2*a*b*asinh
(c*x)/x**2, x) + Integral(b**2*c**4*x**2*asinh(c*x)**2, x) + Integral(2*a*b
*c**4*x**2*asinh(c*x), x))
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)^2\,(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \int \frac{(c^2dx^2+d)^2(b\operatorname{arsinh}(cx)+a)^2}{x^2}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*c^4*d^2*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^
2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^4*d^2 + 2*b^2*c^2*d^2*x*arcsinh(c*x)^2
+ 4*b^2*c^2*d^2*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + 2*a^2*c^2*d^2*x +
4*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*c*d^2 - 2*(c*arcsinh(1/(c*abs(
x))) + arcsinh(c*x)/x)*a*b*d^2 - a^2*d^2/x + 1/3*(b^2*c^4*d^2*x^4 - 3*b^2*d
^2)*log(c*x + sqrt(c^2*x^2 + 1))^2/x - integrate(2/3*(b^2*c^7*d^2*x^6 + b^2
*c^5*d^2*x^4 - 3*b^2*c^3*d^2*x^2 - 3*b^2*c*d^2 + (b^2*c^6*d^2*x^5 - 3*b^2*c
^2*d^2*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*x^4 + c*x^2
+ (c^2*x^3 + x)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x^2} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x^2} dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2 (d\,c^2\,x^2+d)^2}{x^2} dx$$

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x^2, x)
```

3.214 $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 272

$$\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x^3}\,dx = \frac{1}{4}b^2c^4d^2x^2+\frac{1}{2}bc^3d^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) - \frac{bcd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{x} + \frac{1}{4}c^2d^2(a+b\text{arcsinh}(cx))^2 + c^2d^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 - \frac{d^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{2x^2} + \frac{2c^2d^2(a+b\text{arcsinh}(cx))^3}{3b} + 2c^2d^2(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right) + b^2c^2d^2\log(x) - 2bc^2d^2(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right) - b^2c^2d^2\,\text{PolyLog}\left(3,e^{-2\text{arcsinh}(cx)}\right)$$

```
[Out] 1/4*b^2*c^4*d^2*x^2-b*c*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/x+1/4*c^2*
d^2*(a+b*arcsinh(c*x))^2+c^2*d^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2-1/2*d^2*(
c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/x^2+2/3*c^2*d^2*(a+b*arcsinh(c*x))^3/b+2*
c^2*d^2*(a+b*arcsinh(c*x))^2*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)+b^2*c^2*d^2*
ln(x)-2*b*c^2*d^2*(a+b*arcsinh(c*x))*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)
-b^2*c^2*d^2*polylog(3,1/(c*x+(c^2*x^2+1)^(1/2))^2)+1/2*b*c^3*d^2*x*(a+b*ar
csinh(c*x))*(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {5807, 5808, 5775, 3797, 2221, 2611, 2320, 6724, 5785, 5783, 30, 14}

$$\int \frac{(d+c^2dx^2)^2(a+b\operatorname{arcsinh}(cx))^2}{x^3}\,dx = -2bc^2d^2\operatorname{PolyLog}\left(2,e^{-2\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx)) + c^2d^2(c^2x^2+1)(a+b\operatorname{arcsinh}(cx))^2 - \frac{bcd^2(c^2x^2+1)^{3/2}(a+b\operatorname{arcsinh}(cx))}{x} - \frac{d^2(c^2x^2+1)^2(a+b\operatorname{arcsinh}(cx))^2}{2x^2} + \frac{2c^2d^2(a+b\operatorname{arcsinh}(cx))^3}{3b} + \frac{1}{4}c^2d^2(a+b\operatorname{arcsinh}(cx))^2 + 2c^2d^2\log\left(1-e^{-2\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{2}bc^3d^2x\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx)) - b^2c^2d^2\operatorname{PolyLog}\left(3,e^{-2\operatorname{arcsinh}(cx)}\right) + \frac{1}{4}b^2c^4d^2x^2 + b^2c^2d^2\log(x)$$

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] (b^2*c^4*d^2*x^2)/4 + (b*c^3*d^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/
2 - (b*c*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x + (c^2*d^2*(a + b*
ArcSinh[c*x])^2)/4 + c^2*d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2 - (d^2*(1
 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(2*x^2) + (2*c^2*d^2*(a + b*ArcSinh[c
*x])^3)/(3*b) + 2*c^2*d^2*(a + b*ArcSinh[c*x])^2*Log[1 - E^(-2*ArcSinh[c*x]
)] + b^2*c^2*d^2*Log[x] - 2*b*c^2*d^2*(a + b*ArcSinh[c*x])*PolyLog[2, E^(-2
*ArcSinh[c*x])] - b^2*c^2*d^2*PolyLog[3, E^(-2*ArcSinh[c*x])]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
```

```
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))^2}{2x^2} + (2c^2d)\int\frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2}{x}\,dx \\
&\quad + (bcd^2)\int\frac{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{x^2}\,dx \\
&= -\frac{bcd^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{x} + c^2d^2(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2 \\
&\quad - \frac{d^2(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))^2}{2x^2} + (2c^2d^2)\int\frac{(a+b\text{arcsinh}(cx))^2}{x}\,dx \\
&\quad + (b^2c^2d^2)\int\frac{1+c^2x^2}{x}\,dx - (2bc^3d^2)\int\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx \\
&\quad + (3bc^3d^2)\int\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx
\end{aligned}$$

$$= \frac{1}{2}bc^3d^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) - \frac{bcd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{x} + c^2d^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 - \frac{d^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{2x^2} - \frac{(2c^2d^2)\text{Subst}\left(\int x^2\coth\left(\frac{a}{b}-\frac{x}{b}\right)dx, x, a+b\text{arcsinh}(cx)\right)}{b} + (b^2c^2d^2)\int\left(\frac{1}{x}+c^2x\right)dx - (bc^3d^2)\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}dx + \frac{1}{2}(3bc^3d^2)\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}dx + (b^2c^4d^2)\int x\,dx - \frac{1}{2}(3b^2c^4d^2)\int x\,dx$$

$$= \frac{1}{4}b^2c^4d^2x^2 + \frac{1}{2}bc^3d^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) - \frac{bcd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{x} + \frac{1}{4}c^2d^2(a+b\text{arcsinh}(cx))^2 + c^2d^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 - \frac{d^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{2x^2} + \frac{2c^2d^2(a+b\text{arcsinh}(cx))^3}{3b} + b^2c^2d^2\log(x) + \frac{(4c^2d^2)\text{Subst}\left(\int\frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x^2}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}dx, x, a+b\text{arcsinh}(cx)\right)}{b}$$

$$= \frac{1}{4}b^2c^4d^2x^2 + \frac{1}{2}bc^3d^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) - \frac{bcd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{x} + \frac{1}{4}c^2d^2(a+b\text{arcsinh}(cx))^2 + c^2d^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 - \frac{d^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{2x^2} + \frac{2c^2d^2(a+b\text{arcsinh}(cx))^3}{3b} + 2c^2d^2(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right) + b^2c^2d^2\log(x) - (4c^2d^2)\text{Subst}\left(\int x\log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)dx, x, a+b\text{arcsinh}(cx)\right)$$

$$= \frac{1}{4}b^2c^4d^2x^2 + \frac{1}{2}bc^3d^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) - \frac{bcd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{x} + \frac{1}{4}c^2d^2(a+b\text{arcsinh}(cx))^2 + c^2d^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 - \frac{d^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{2x^2} + \frac{2c^2d^2(a+b\text{arcsinh}(cx))^3}{3b} + 2c^2d^2(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right) + b^2c^2d^2\log(x) - 2bc^2d^2(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2, e^{-2\text{arcsinh}(cx)}\right) + (2bc^2d^2)\text{Subst}\left(\int\text{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)dx, x, a+b\text{arcsinh}(cx)\right)$$

$$= \frac{1}{4}b^2c^4d^2x^2 + \frac{1}{2}bc^3d^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) - \frac{bcd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{x} + \frac{1}{4}c^2d^2(a+b\text{arcsinh}(cx))^2 + c^2d^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 - \frac{d^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{2x^2} + \frac{2c^2d^2(a+b\text{arcsinh}(cx))^3}{3b} + 2c^2d^2(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right) + b^2c^2d^2\log(x) - 2bc^2d^2(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right) - (b^2c^2d^2)\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right)$$

$$= \frac{1}{4}b^2c^4d^2x^2 + \frac{1}{2}bc^3d^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) - \frac{bcd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{x} + \frac{1}{4}c^2d^2(a+b\text{arcsinh}(cx))^2 + c^2d^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 - \frac{d^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{2x^2} + \frac{2c^2d^2(a+b\text{arcsinh}(cx))^3}{3b} + 2c^2d^2(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right) + b^2c^2d^2\log(x) - 2bc^2d^2(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right) - b^2c^2d^2\,\text{PolyLog}\left(3,e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right)$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.21

$$\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$$
$$= \frac{1}{2}d^2\left(-\frac{a^2}{x^2} + a^2c^4x^2 + 2abc^4x^2\text{arcsinh}(cx) - \frac{2ab\left(cx\sqrt{1+c^2x^2}+\text{arcsinh}(cx)\right)}{x^2} + \frac{1}{4}b^2c^2\left(1+2\text{arcsinh}(cx)^2\right)\cosh(2\text{arcsinh}(cx)) + 4a^2c^2\log(x) - \frac{b^2\left(2cx\sqrt{1+c^2x^2}\text{arcsinh}(cx)+\text{arcsinh}(cx)^2-2c^2x^2\log(cx)\right)}{x^2} - abc^2\left(cx\sqrt{1+c^2x^2}+\log\left(-cx+\sqrt{1+c^2x^2}\right)\right) - 4abc^2\left(\text{arcsinh}(cx)\left(\text{arcsinh}(cx)-2\log\left(1-e^{2\text{arcsinh}(cx)}\right)\right)-\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)\right) - \frac{2}{3}b^2c^2\left(2\text{arcsinh}(cx)^2\left(\text{arcsinh}(cx)-3\log\left(1-e^{2\text{arcsinh}(cx)}\right)\right) - 6\text{arcsinh}(cx)\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)+3\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(cx)}\right)\right) - \frac{1}{2}b^2c^2\text{arcsinh}(cx)\sinh(2\text{arcsinh}(cx))\right)$$

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] (d^2*(-(a^2/x^2) + a^2*c^4*x^2 + 2*a*b*c^4*x^2*ArcSinh[c*x] - (2*a*b*(c*x*S
qrt[1 + c^2*x^2] + ArcSinh[c*x]))/x^2 + (b^2*c^2*(1 + 2*ArcSinh[c*x]^2)*Cos
h[2*ArcSinh[c*x]])/4 + 4*a^2*c^2*Log[x] - (b^2*(2*c*x*Sqrt[1 + c^2*x^2]*Arc
Sinh[c*x] + ArcSinh[c*x]^2 - 2*c^2*x^2*Log[c*x]))/x^2 - a*b*c^2*(c*x*Sqrt[1
 + c^2*x^2] + Log[-(c*x) + Sqrt[1 + c^2*x^2]]) - 4*a*b*c^2*(ArcSinh[c*x]*(A
rcSinh[c*x] - 2*Log[1 - E^(2*ArcSinh[c*x])]) - PolyLog[2, E^(2*ArcSinh[c*x]
)]) - (2*b^2*c^2*(2*ArcSinh[c*x]^2*(ArcSinh[c*x] - 3*Log[1 - E^(2*ArcSinh[c
*x])]) - 6*ArcSinh[c*x]*PolyLog[2, E^(2*ArcSinh[c*x])] + 3*PolyLog[3, E^(2*
ArcSinh[c*x])]))/3 - (b^2*c^2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]])/2))/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(285) = 570$.

Time = 0.28 (sec) , antiderivative size = 584, normalized size of antiderivative = 2.15

method	result
derivativedivides	$c^2\left(d^2a^2\left(\frac{c^2x^2}{2}+2\ln(cx)-\frac{1}{2c^2x^2}\right)+d^2b^2\left(-\frac{2\,\mathrm{arcsinh}(cx)^3}{3}+\frac{\left(2\,\mathrm{arcsinh}(cx)^2-2\,\mathrm{arcsinh}(cx)+1\right)\left(2c^2x\right.}{16}\right.\right.$
default	$c^2\left(d^2a^2\left(\frac{c^2x^2}{2}+2\ln(cx)-\frac{1}{2c^2x^2}\right)+d^2b^2\left(-\frac{2\,\mathrm{arcsinh}(cx)^3}{3}+\frac{\left(2\,\mathrm{arcsinh}(cx)^2-2\,\mathrm{arcsinh}(cx)+1\right)\left(2c^2x\right.}{16}\right.\right.$
parts	$d^2a^2\left(\frac{c^4x^2}{2}-\frac{1}{2x^2}+2c^2\ln(x)\right)+d^2b^2c^2\left(-\frac{2\,\mathrm{arcsinh}(cx)^3}{3}+\frac{\left(2\,\mathrm{arcsinh}(cx)^2-2\,\mathrm{arcsinh}(cx)+1\right)\left(2c^2x^2+\right.}{16}\right.$

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(d^2*a^2*(1/2*c^2*x^2+2*ln(c*x)-1/2/c^2/x^2)+d^2*b^2*(-2/3*arcsinh(c*x)
^3+1/16*(2*arcsinh(c*x)^2-2*arcsinh(c*x)+1)*(2*c^2*x^2+1+2*c*x*(c^2*x^2+1)^
(1/2))+1/16*(-2*c*x*(c^2*x^2+1)^(1/2)+2*c^2*x^2+1)*(2*arcsinh(c*x)^2+2*arcs
inh(c*x)+1)-1/2*arcsinh(c*x)*(-2*c^2*x^2+2*c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*
x))/c^2/x^2+ln(1+c*x+(c^2*x^2+1)^(1/2))-2*ln(c*x+(c^2*x^2+1)^(1/2))+ln(c*x+
(c^2*x^2+1)^(1/2)-1)+2*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))+4*arcsinh
(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-4*polylog(3,-c*x-(c^2*x^2+1)^(1/2))
+2*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))+4*arcsinh(c*x)*polylog(2,c*x+
(c^2*x^2+1)^(1/2))-4*polylog(3,c*x+(c^2*x^2+1)^(1/2)))+2*d^2*a*b*(-arcsinh(
c*x)^2+1/16*(-1+2*arcsinh(c*x))*(2*c^2*x^2+1+2*c*x*(c^2*x^2+1)^(1/2))+1/16*
(-2*c*x*(c^2*x^2+1)^(1/2)+2*c^2*x^2+1)*(1+2*arcsinh(c*x))-1/2*(c*x*(c^2*x^2
+1)^(1/2)-c^2*x^2+arcsinh(c*x))/c^2/x^2+2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)
^(1/2))+2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*arcsinh(c*x)*ln(1-c*x-(c^2*x^
2+1)^(1/2))+2*polylog(2,c*x+(c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x^3}\,dx = \int \frac{(c^2dx^2+d)^2(b\operatorname{arsinh}(cx)+a)^2}{x^3}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="fricas")
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x^3}\,dx = d^2\left(\int \frac{a^2}{x^3}\,dx + \int \frac{2a^2c^2}{x}\,dx + \int a^2c^4x\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{x^3}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{x^3}\,dx + \int \frac{2b^2c^2\operatorname{asinh}^2(cx)}{x}\,dx + \int b^2c^4x\operatorname{asinh}^2(cx)\,dx + \int \frac{4abc^2\operatorname{asinh}(cx)}{x}\,dx + \int 2abc^4x\operatorname{asinh}(cx)\,dx\right)$$

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2/x**3,x)
[Out] d**2*(Integral(a**2/x**3, x) + Integral(2*a**2*c**2/x, x) + Integral(a**2*c
**4*x, x) + Integral(b**2*asinh(c*x)**2/x**3, x) + Integral(2*a*b*asinh(c*x
)/x**3, x) + Integral(2*b**2*c**2*asinh(c*x)**2/x, x) + Integral(b**2*c**4*
x*asinh(c*x)**2, x) + Integral(4*a*b*c**2*asinh(c*x)/x, x) + Integral(2*a*b
*c**4*x*asinh(c*x), x))
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x^3}\,dx = \int \frac{(c^2dx^2+d)^2(b\operatorname{arsinh}(cx)+a)^2}{x^3}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="maxima")
[Out] 1/2*a^2*c^4*d^2*x^2 + 2*a^2*c^2*d^2*log(x) - a*b*d^2*(sqrt(c^2*x^2 + 1)*c/x
 + arcsinh(c*x)/x^2) - 1/2*a^2*d^2/x^2 + integrate(b^2*c^4*d^2*x*log(c*x +
sqrt(c^2*x^2 + 1))^2 + 2*a*b*c^4*d^2*x*log(c*x + sqrt(c^2*x^2 + 1)) + 2*b^2
*c^2*d^2*log(c*x + sqrt(c^2*x^2 + 1))^2/x + 4*a*b*c^2*d^2*log(c*x + sqrt(c^
2*x^2 + 1))/x + b^2*d^2*log(c*x + sqrt(c^2*x^2 + 1))^2/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x^3} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x^3} \, dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)^2}{x^3} \, dx$$

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x^3, x)
```

3.215 $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 248

$$\begin{aligned}\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x^4}\,dx &= -\frac{b^2c^2d^2}{3x}+2b^2c^4d^2x \\ &\quad -\frac{5}{3}bc^3d^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) \\ &\quad -\frac{bcd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3x^2} \\ &\quad +\frac{8}{3}c^4d^2x(a+b\text{arcsinh}(cx))^2 \\ &\quad -\frac{4c^2d^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3x} \\ &\quad -\frac{d^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{3x^3} \\ &\quad -\frac{22}{3}bc^3d^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) \\ &\quad -\frac{11}{3}b^2c^3d^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right) \\ &\quad +\frac{11}{3}b^2c^3d^2\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\end{aligned}$$

```
[Out] -1/3*b^2*c^2*d^2/x+2*b^2*c^4*d^2*x-1/3*b*c*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsi
nh(c*x))/x^2+8/3*c^4*d^2*x*(a+b*arcsinh(c*x))^2-4/3*c^2*d^2*(c^2*x^2+1)*(a+
b*arcsinh(c*x))^2/x-1/3*d^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/x^3-22/3*b*c
^3*d^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))-11/3*b^2*c^3*d^2*p
olylog(2,-c*x-(c^2*x^2+1)^(1/2))+11/3*b^2*c^3*d^2*polylog(2,c*x+(c^2*x^2+1)
^(1/2))-5/3*b*c^3*d^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5807, 5772, 5798, 8, 5806, 5816, 4267, 2317, 2438, 14}

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x^4} \, dx = -\frac{22}{3}bc^3d^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)) + \frac{8}{3}c^4d^2x(a+b\text{arcsinh}(cx))^2 - \frac{4c^2d^2(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{3x} - \frac{bcd^2(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{3x^2} - \frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{3x^3} - \frac{5}{3}bc^3d^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx)) - \frac{11}{3}b^2c^3d^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right) + \frac{11}{3}b^2c^3d^2\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right) + 2b^2c^4d^2x - \frac{b^2c^2d^2}{3x}$$

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] -1/3*(b^2*c^2*d^2)/x + 2*b^2*c^4*d^2*x - (5*b*c^3*d^2*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x]))/3 - (b*c*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(
3*x^2) + (8*c^4*d^2*x*(a + b*ArcSinh[c*x])^2)/3 - (4*c^2*d^2*(1 + c^2*x^2)*
(a + b*ArcSinh[c*x])^2)/(3*x) - (d^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2
)/(3*x^3) - (22*b*c^3*d^2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/3 -
 (11*b^2*c^3*d^2*PolyLog[2, -E^ArcSinh[c*x]])/3 + (11*b^2*c^3*d^2*PolyLog[2
, E^ArcSinh[c*x]])/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
```

```
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))^2}{3x^3} \\
&\quad+\frac{1}{3}\left(4c^2d\right)\int\frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2}{x^2}\,dx \\
&\quad+\frac{1}{3}\left(2bcd^2\right)\int\frac{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{x^3}\,dx \\
&= -\frac{bcd^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{3x^2} \\
&\quad-\frac{4c^2d^2(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3x}-\frac{d^2(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))^2}{3x^3} \\
&\quad+\frac{1}{3}\left(b^2c^2d^2\right)\int\frac{1+c^2x^2}{x^2}\,dx+\left(bc^3d^2\right)\int\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{x}\,dx \\
&\quad+\frac{1}{3}\left(8bc^3d^2\right)\int\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{x}\,dx \\
&\quad+\frac{1}{3}\left(8c^4d^2\right)\int(a+b\text{arcsinh}(cx))^2\,dx \\
&= \frac{11}{3}bc^3d^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))-\frac{bcd^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{3x^2} \\
&\quad+\frac{8}{3}c^4d^2x(a+b\text{arcsinh}(cx))^2-\frac{4c^2d^2(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3x} \\
&\quad-\frac{d^2(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))^2}{3x^3}+\frac{1}{3}\left(b^2c^2d^2\right)\int\left(c^2+\frac{1}{x^2}\right)\,dx \\
&\quad+\left(bc^3d^2\right)\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{1+c^2x^2}}\,dx+\frac{1}{3}\left(8bc^3d^2\right)\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{1+c^2x^2}}\,dx \\
&\quad-\left(b^2c^4d^2\right)\int 1\,dx-\frac{1}{3}\left(8b^2c^4d^2\right)\int 1\,dx-\frac{1}{3}\left(16bc^5d^2\right)\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx
\end{aligned}$$

$$= -\frac{b^2c^2d^2}{3x} - \frac{10}{3}b^2c^4d^2x - \frac{5}{3}bc^3d^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) - \frac{bcd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3x^2} + \frac{8}{3}c^4d^2x(a+b\text{arcsinh}(cx))^2 - \frac{4c^2d^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3x} - \frac{d^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{3x^3} + (bc^3d^2)\,\text{Subst}\left(\int(a+bx)\text{csch}(x)\,dx, x, \text{arcsinh}(cx)\right) + \frac{1}{3}(8bc^3d^2)\,\text{Subst}\left(\int(a+bx)\text{csch}(x)\,dx, x, \text{arcsinh}(cx)\right) + \frac{1}{3}(16b^2c^4d^2)\int 1\,dx$$

$$= -\frac{b^2c^2d^2}{3x} + 2b^2c^4d^2x - \frac{5}{3}bc^3d^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) - \frac{bcd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3x^2} + \frac{8}{3}c^4d^2x(a+b\text{arcsinh}(cx))^2 - \frac{4c^2d^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3x} - \frac{d^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{3x^3} - \frac{22}{3}bc^3d^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) - (b^2c^3d^2)\,\text{Subst}\left(\int\log(1-e^x)\,dx, x, \text{arcsinh}(cx)\right) + (b^2c^3d^2)\,\text{Subst}\left(\int\log(1+e^x)\,dx, x, \text{arcsinh}(cx)\right) - \frac{1}{3}(8b^2c^3d^2)\,\text{Subst}\left(\int\log(1-e^x)\,dx, x, \text{arcsinh}(cx)\right) + \frac{1}{3}(8b^2c^3d^2)\,\text{Subst}\left(\int\log(1+e^x)\,dx, x, \text{arcsinh}(cx)\right)$$

$$= -\frac{b^2c^2d^2}{3x} + 2b^2c^4d^2x - \frac{5}{3}bc^3d^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) - \frac{bcd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3x^2} + \frac{8}{3}c^4d^2x(a+b\text{arcsinh}(cx))^2 - \frac{4c^2d^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3x} - \frac{d^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{3x^3} - \frac{22}{3}bc^3d^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) - (b^2c^3d^2)\,\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right) + (b^2c^3d^2)\,\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right) - \frac{1}{3}(8b^2c^3d^2)\,\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right) + \frac{1}{3}(8b^2c^3d^2)\,\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)$$

$$= -\frac{b^2c^2d^2}{3x} + 2b^2c^4d^2x - \frac{5}{3}bc^3d^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) - \frac{bcd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3x^2} + \frac{8}{3}c^4d^2x(a+b\text{arcsinh}(cx))^2 - \frac{4c^2d^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3x} - \frac{d^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{3x^3} - \frac{22}{3}bc^3d^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) - \frac{11}{3}b^2c^3d^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right) + \frac{11}{3}b^2c^3d^2\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.44

$$\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = \frac{d^2\left(-a^2-6a^2c^2x^2-b^2c^2x^2+3a^2c^4x^4+6b^2c^4x^4-abcx\sqrt{1+c^2x^2}-6abc^3x^3\sqrt{1+c^2x^2}-2ab\text{arcsinh}(cx)-\right.}{}$$

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] (d^2*(-a^2 - 6*a^2*c^2*x^2 - b^2*c^2*x^2 + 3*a^2*c^4*x^4 + 6*b^2*c^4*x^4 -
a*b*c*x*Sqrt[1 + c^2*x^2] - 6*a*b*c^3*x^3*Sqrt[1 + c^2*x^2] - 2*a*b*ArcSinh
[c*x] - 12*a*b*c^2*x^2*ArcSinh[c*x] + 6*a*b*c^4*x^4*ArcSinh[c*x] - b^2*c*x*
Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - 6*b^2*c^3*x^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*
x] - b^2*ArcSinh[c*x]^2 - 6*b^2*c^2*x^2*ArcSinh[c*x]^2 + 3*b^2*c^4*x^4*ArcS
inh[c*x]^2 - 11*a*b*c^3*x^3*ArcTanh[Sqrt[1 + c^2*x^2]] + 11*b^2*c^3*x^3*Arc
Sinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 11*b^2*c^3*x^3*ArcSinh[c*x]*Log[1 +
E^(-ArcSinh[c*x])] + 11*b^2*c^3*x^3*PolyLog[2, -E^(-ArcSinh[c*x])] - 11*b^2
*c^3*x^3*PolyLog[2, E^(-ArcSinh[c*x])]))/(3*x^3)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.46

method	result
derivativedivides	$c^3\left(d^2a^2\left(cx-\frac{1}{3c^3x^3}-\frac{2}{cx}\right)+d^2b^2\operatorname{arcsinh}(cx)^2cx-2d^2b^2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2d^2\right.$
default	$c^3\left(d^2a^2\left(cx-\frac{1}{3c^3x^3}-\frac{2}{cx}\right)+d^2b^2\operatorname{arcsinh}(cx)^2cx-2d^2b^2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2d^2\right.$
parts	$d^2a^2\left(c^4x-\frac{2c^2}{x}-\frac{1}{3x^3}\right)+d^2b^2c^4\operatorname{arcsinh}(cx)^2x-2d^2b^2c^3\sqrt{c^2x^2+1}\operatorname{arcsinh}(cx)+2b^2c$

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^4,x,method=_RETURNVERBOSE)
[Out] c^3*(d^2*a^2*(c*x-1/3/c^3/x^3-2/c/x)+d^2*b^2*arcsinh(c*x)^2*c*x-2*d^2*b^2*a
rcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*d^2*b^2*c*x-2*d^2*b^2*arcsinh(c*x)^2/c/x-1/
3*d^2*b^2/c^2/x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-1/3*d^2*b^2/c^3/x^3*arcsin
h(c*x)^2-1/3*d^2*b^2/c/x-11/3*d^2*b^2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/
2))-11/3*d^2*b^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+11/3*d^2*b^2*arcsinh(c*x
)*ln(1-c*x-(c^2*x^2+1)^(1/2))+11/3*d^2*b^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))
+2*d^2*a*b*(arcsinh(c*x)*c*x-1/3*arcsinh(c*x)/c^3/x^3-2*arcsinh(c*x)/c/x-(c
^2*x^2+1)^(1/2)-1/6/c^2/x^2*(c^2*x^2+1)^(1/2)-11/6*arctanh(1/(c^2*x^2+1)^(1
/2))))
```

Fricas [F]

$$\int\frac{(d+c^2dx^2)^2\,(a+b\operatorname{arcsinh}(cx))^2}{x^4}\,dx=\int\frac{(c^2dx^2+d)^2(b\operatorname{arsinh}(cx)+a)^2}{x^4}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="fricas")
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))/x^4, x)
```

Sympy [F]

$$\int\frac{(d+c^2dx^2)^2\,(a+b\operatorname{arcsinh}(cx))^2}{x^4}\,dx=d^2\left(\int a^2c^4\,dx+\int\frac{a^2}{x^4}\,dx+\int\frac{2a^2c^2}{x^2}\,dx+\int b^2c^4\operatorname{asinh}^2(cx)\,dx+\int\frac{b^2\operatorname{asinh}^2(cx)}{x^4}\,dx+\int 2abc^4\operatorname{asinh}(cx)\,dx+\int\frac{2ab\operatorname{asinh}(cx)}{x^4}\,dx+\int\frac{2b^2c^2\operatorname{asinh}^2(cx)}{x^2}\,dx+\int\frac{4abc^2\operatorname{asinh}(cx)}{x^2}\,dx\right)$$

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2/x**4,x)
```

```
[Out] d**2*(Integral(a**2*c**4, x) + Integral(a**2/x**4, x) + Integral(2*a**2*c**
2/x**2, x) + Integral(b**2*c**4*asinh(c*x)**2, x) + Integral(b**2*asinh(c*x
)**2/x**4, x) + Integral(2*a*b*c**4*asinh(c*x), x) + Integral(2*a*b*asinh(c
*x)/x**4, x) + Integral(2*b**2*c**2*asinh(c*x)**2/x**2, x) + Integral(4*a*b
*c**2*asinh(c*x)/x**2, x))
```

Maxima [F]

$$\int \frac{(d + c^2dx^2)^2 (a + b\text{arcsinh}(cx))^2}{x^4} dx = \int \frac{(c^2dx^2 + d)^2(b \operatorname{arsinh}(cx) + a)^2}{x^4} dx$$

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] b^2*c^4*d^2*x*arcsinh(c*x)^2 + 2*b^2*c^4*d^2*(x - sqrt(c^2*x^2 + 1)*arcsinh
(c*x)/c) + a^2*c^4*d^2*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*c^3
*d^2 - 4*(c*arcsinh(1/(c*abs(x))) + arcsinh(c*x)/x)*a*b*c^2*d^2 + 1/3*((c^2
*arcsinh(1/(c*abs(x))) - sqrt(c^2*x^2 + 1)/x^2)*c - 2*arcsinh(c*x)/x^3)*a*b
*d^2 - 2*a^2*c^2*d^2/x - 1/3*a^2*d^2/x^3 - 1/3*(6*b^2*c^2*d^2*x^2 + b^2*d^2
)*log(c*x + sqrt(c^2*x^2 + 1))^2/x^3 + integrate(2/3*(6*b^2*c^5*d^2*x^4 + 7
*b^2*c^3*d^2*x^2 + b^2*c*d^2 + (6*b^2*c^4*d^2*x^3 + b^2*c^2*d^2*x)*sqrt(c^2
*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*x^6 + c*x^4 + (c^2*x^5 + x^3)*
sqrt(c^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + c^2dx^2)^2 (a + b\text{arcsinh}(cx))^2}{x^4} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^2 (a+b\operatorname{arcsinh}(cx))^2}{x^4} \, dx = \int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)^2}{x^4} \, dx$$

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x^4,x)
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x^4, x)
```

3.216 $\int x^4 (d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2 dx$

Optimal result

Integrand size = 26, antiderivative size = 465

$$\int x^4 (d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2 dx = \frac{100976b^2d^3x}{4002075c^4} - \frac{50488b^2d^3x^3}{12006225c^2} + \frac{12622b^2d^3x^5}{6670125} + \frac{9410b^2c^2d^3x^7}{1120581} + \frac{182b^2c^4d^3x^9}{29403} + \frac{2b^2c^6d^3x^{11}}{1331} - \frac{256bd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{17325c^5} + \frac{128bd^3x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{17325c^3} - \frac{32bd^3x^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{5775c} - \frac{16bd^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{693c^5} + \frac{4bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{1155c^5} - \frac{2bd^3(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{1617c^5} + \frac{8bd^3(1+c^2x^2)^{9/2}(a+b\text{arcsinh}(cx))}{297c^5} - \frac{2bd^3(1+c^2x^2)^{11/2}(a+b\text{arcsinh}(cx))}{121c^5} + \frac{16d^3x^5(a+b\text{arcsinh}(cx))^2}{1155} + \frac{8}{231}d^3x^5(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 + \frac{2}{33}d^3x^5(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2 + \frac{1}{11}d^3x^5(1+c^2x^2)^3(a+b\text{arc}$$

```
[Out] 100976/4002075*b^2*d^3*x/c^4-50488/12006225*b^2*d^3*x^3/c^2+12622/6670125*b
^2*d^3*x^5+9410/1120581*b^2*c^2*d^3*x^7+182/29403*b^2*c^4*d^3*x^9+2/1331*b^
2*c^6*d^3*x^11-16/693*b*d^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c^5+4/1155
*b*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))/c^5-2/1617*b*d^3*(c^2*x^2+1)^(7
/2)*(a+b*arcsinh(c*x))/c^5+8/297*b*d^3*(c^2*x^2+1)^(9/2)*(a+b*arcsinh(c*x))
/c^5-2/121*b*d^3*(c^2*x^2+1)^(11/2)*(a+b*arcsinh(c*x))/c^5+16/1155*d^3*x^5*
(a+b*arcsinh(c*x))^2+8/231*d^3*x^5*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+2/33*d^
3*x^5*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2+1/11*d^3*x^5*(c^2*x^2+1)^3*(a+b*ar
csinh(c*x))^2-256/17325*b*d^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^5+128/
17325*b*d^3*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-32/5775*b*d^3*x^4*
(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {5808, 5776, 5812, 5798, 8, 30, 272, 45, 5804, 12, 1167}

$$\int x^4 \left(d+c^2 d x^2\right)^3 (a+b\text{arcsinh}(cx))^2 \, dx = \frac{1}{11} d^3 x^5 \left(c^2 x^2+1\right)^3 (a+b\text{arcsinh}(cx))^2 + \frac{2}{33} d^3 x^5 \left(c^2 x^2+1\right)^2 (a+b\text{arcsinh}(cx))^2 + \frac{8}{231} d^3 x^5 \left(c^2 x^2+1\right) (a+b\text{arcsinh}(cx))^2 - \frac{32 b d^3 x^4 \sqrt{c^2 x^2+1} (a+b\text{arcsinh}(cx))}{5775 c} - \frac{2 b d^3 \left(c^2 x^2+1\right)^{11/2} (a+b\text{arcsinh}(cx))}{121 c^5} + \frac{8 b d^3 \left(c^2 x^2+1\right)^{9/2} (a+b\text{arcsinh}(cx))}{297 c^5} - \frac{2 b d^3 \left(c^2 x^2+1\right)^{7/2} (a+b\text{arcsinh}(cx))}{1617 c^5} + \frac{4 b d^3 \left(c^2 x^2+1\right)^{5/2} (a+b\text{arcsinh}(cx))}{1155 c^5} - \frac{16 b d^3 \left(c^2 x^2+1\right)^{3/2} (a+b\text{arcsinh}(cx))}{693 c^5} - \frac{256 b d^3 \sqrt{c^2 x^2+1} (a+b\text{arcsinh}(cx))}{17325 c^5} + \frac{128 b d^3 x^2 \sqrt{c^2 x^2+1} (a+b\text{arcsinh}(cx))}{17325 c^3} + \frac{16 d^3 x^5 (a+b\text{arcsinh}(cx))^2}{1155} + \frac{2 b^2 c^6 d^3 x^{11}}{1331} + \frac{182 b^2 c^4 d^3 x^9}{29403} + \frac{100976 b^2 d^3 x}{4002075 c^4} + \frac{9410 b^2 c^2 d^3 x^7}{1120581} - \frac{50488 b^2 d^3 x^3}{12006225 c^2} + \frac{12622 b^2 d^3 x^5}{6670125}$$

```
[In] Int[x^4*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (100976*b^2*d^3*x)/(4002075*c^4) - (50488*b^2*d^3*x^3)/(12006225*c^2) + (12
622*b^2*d^3*x^5)/6670125 + (9410*b^2*c^2*d^3*x^7)/1120581 + (182*b^2*c^4*d^
3*x^9)/29403 + (2*b^2*c^6*d^3*x^11)/1331 - (256*b*d^3*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x]))/(17325*c^5) + (128*b*d^3*x^2*Sqrt[1 + c^2*x^2]*(a + b*Ar
cSinh[c*x]))/(17325*c^3) - (32*b*d^3*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c
*x]))/(5775*c) - (16*b*d^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(693*c
^5) + (4*b*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(1155*c^5) - (2*b*
```

```
d^3*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(1617*c^5) + (8*b*d^3*(1 + c^
2*x^2)^(9/2)*(a + b*ArcSinh[c*x]))/(297*c^5) - (2*b*d^3*(1 + c^2*x^2)^(11/2
)*(a + b*ArcSinh[c*x]))/(121*c^5) + (16*d^3*x^5*(a + b*ArcSinh[c*x])^2)/115
5 + (8*d^3*x^5*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/231 + (2*d^3*x^5*(1 +
c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/33 + (d^3*x^5*(1 + c^2*x^2)^3*(a + b*Arc
Sinh[c*x])^2)/11
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{11}d^3x^5\left(1+c^2x^2\right)^3(a+b\operatorname{arcsinh}(cx))^2 \\
&\quad+\frac{1}{11}(6d)\int x^4\left(d+c^2dx^2\right)^2(a+b\operatorname{arcsinh}(cx))^2\,dx \\
&\quad-\frac{1}{11}\left(2bcd^3\right)\int x^5\left(1+c^2x^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx
\end{aligned}
$$

$$= -\frac{2bd^3(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{77c^5} + \frac{4bd^3(1+c^2x^2)^{9/2}(a+b\text{arcsinh}(cx))}{99c^5} - \frac{2bd^3(1+c^2x^2)^{11/2}(a+b\text{arcsinh}(cx))}{121c^5} + \frac{2}{33}d^3x^5(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2 + \frac{1}{11}d^3x^5(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2 + \frac{1}{33}(8d^2)\int x^4(d+$$

$$= -\frac{4bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{165c^5} + \frac{2bd^3(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{231c^5} + \frac{8bd^3(1+c^2x^2)^{9/2}(a+b\text{arcsinh}(cx))}{297c^5} - \frac{2bd^3(1+c^2x^2)^{11/2}(a+b\text{arcsinh}(cx))}{121c^5} + \frac{8}{231}d^3x^5(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 + \frac{2}{33}d^3x^5(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2 + \frac{1}{11}d^3x^5(1+c^2x^2)^3$$

$$= -\frac{16bd^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{693c^5} + \frac{4bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{1155c^5} - \frac{2bd^3(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{1617c^5} + \frac{8bd^3(1+c^2x^2)^{9/2}(a+b\text{arcsinh}(cx))}{297c^5} - \frac{2bd^3(1+c^2x^2)^{11/2}(a+b\text{arcsinh}(cx))}{121c^5} + \frac{16d^3x^5(a+b\text{arcsinh}(cx))^2}{1155} + \frac{8}{231}d^3x^5(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 + \frac{2}{33}d^3x^5(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2 + \frac{1}{11}d^3x^5(1+c^2x^2)^3$$

$$= \frac{16b^2d^3x}{7623c^4} - \frac{8b^2d^3x^3}{22869c^2} + \frac{2b^2d^3x^5}{12705} + \frac{226b^2c^2d^3x^7}{53361} + \frac{46b^2c^4d^3x^9}{9801} + \frac{2b^2c^6d^3x^{11}}{1331} - \frac{32bd^3x^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{5775c} - \frac{16bd^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{693c^5} + \frac{4bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{1155c^5} - \frac{2bd^3(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{1617c^5} + \frac{8bd^3(1+c^2x^2)^{9/2}(a+b\text{arcsinh}(cx))}{297c^5} - \frac{2bd^3(1+c^2x^2)^{11/2}(a+b\text{arcsinh}(cx))}{121c^5} + \frac{16d^3x^5(a+b\text{arcsinh}(cx))^2}{1155} + \frac{8}{231}d^3x^5(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 + \frac{2}{33}d^3x^5(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2 + \frac{1}{11}d^3x^5(1+c^2x^2)^3$$

$$=\frac{8368b^2d^3x}{800415c^4}-\frac{4184b^2d^3x^3}{2401245c^2}+\frac{12622b^2d^3x^5}{6670125}+\frac{9410b^2c^2d^3x^7}{1120581}+\frac{182b^2c^4d^3x^9}{29403}+\frac{2b^2c^6d^3x^{11}}{1331}$$
$$+\frac{128bd^3x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{17325c^3}-\frac{32bd^3x^4\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{5775c}$$
$$-\frac{16bd^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{693c^5}+\frac{4bd^3(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{1155c^5}$$
$$-\frac{2bd^3(1+c^2x^2)^{7/2}(a+b\operatorname{arcsinh}(cx))}{1617c^5}+\frac{8bd^3(1+c^2x^2)^{9/2}(a+b\operatorname{arcsinh}(cx))}{297c^5}$$
$$-\frac{2bd^3(1+c^2x^2)^{11/2}(a+b\operatorname{arcsinh}(cx))}{121c^5}+\frac{16d^3x^5(a+b\operatorname{arcsinh}(cx))^2}{1155}$$
$$+\frac{8}{231}d^3x^5\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))^2+\frac{2}{33}d^3x^5\left(1+c^2x^2\right)^2(a+b\operatorname{arcsinh}(cx))^2+\frac{1}{11}d^3x^5\left(1+c^2x^2\right)$$

$$=\frac{8368b^2d^3x}{800415c^4}-\frac{50488b^2d^3x^3}{12006225c^2}+\frac{12622b^2d^3x^5}{6670125}+\frac{9410b^2c^2d^3x^7}{1120581}$$
$$+\frac{182b^2c^4d^3x^9}{29403}+\frac{2b^2c^6d^3x^{11}}{1331}-\frac{256bd^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{17325c^5}$$
$$+\frac{128bd^3x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{17325c^3}-\frac{32bd^3x^4\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{5775c}$$
$$-\frac{16bd^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{693c^5}+\frac{4bd^3(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{1155c^5}$$
$$-\frac{2bd^3(1+c^2x^2)^{7/2}(a+b\operatorname{arcsinh}(cx))}{1617c^5}+\frac{8bd^3(1+c^2x^2)^{9/2}(a+b\operatorname{arcsinh}(cx))}{297c^5}$$
$$-\frac{2bd^3(1+c^2x^2)^{11/2}(a+b\operatorname{arcsinh}(cx))}{121c^5}+\frac{16d^3x^5(a+b\operatorname{arcsinh}(cx))^2}{1155}$$
$$+\frac{8}{231}d^3x^5\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))^2+\frac{2}{33}d^3x^5\left(1+c^2x^2\right)^2(a+b\operatorname{arcsinh}(cx))^2+\frac{1}{11}d^3x^5\left(1+c^2x^2\right)$$

$$=\frac{100976b^2d^3x}{4002075c^4}-\frac{50488b^2d^3x^3}{12006225c^2}+\frac{12622b^2d^3x^5}{6670125}+\frac{9410b^2c^2d^3x^7}{1120581}$$
$$+\frac{182b^2c^4d^3x^9}{29403}+\frac{2b^2c^6d^3x^{11}}{1331}-\frac{256bd^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{17325c^5}$$
$$+\frac{128bd^3x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{17325c^3}-\frac{32bd^3x^4\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{5775c}$$
$$-\frac{16bd^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{693c^5}+\frac{4bd^3(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{1155c^5}$$
$$-\frac{2bd^3(1+c^2x^2)^{7/2}(a+b\operatorname{arcsinh}(cx))}{1617c^5}+\frac{8bd^3(1+c^2x^2)^{9/2}(a+b\operatorname{arcsinh}(cx))}{297c^5}$$
$$-\frac{2bd^3(1+c^2x^2)^{11/2}(a+b\operatorname{arcsinh}(cx))}{121c^5}+\frac{16d^3x^5(a+b\operatorname{arcsinh}(cx))^2}{1155}$$
$$+\frac{8}{231}d^3x^5\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))^2+\frac{2}{33}d^3x^5\left(1+c^2x^2\right)^2(a+b\operatorname{arcsinh}(cx))^2+\frac{1}{11}d^3x^5\left(1+c^2x^2\right)$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.64

$$\int x^4 \left(d+c^2 d x^2\right)^3 (a+b \operatorname{arcsinh}(c x))^2 \, dx$$
$$= \frac{d^3 \left(12006225 a^2 c^5 x^5 (231+495 c^2 x^2+385 c^4 x^4+105 c^6 x^6)-6930 a b \sqrt{1+c^2 x^2} (50488-25244 c^2 x^2+18933 c\right.}{}$$

```
[In] Integrate[x^4*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^3*(12006225*a^2*c^5*x^5*(231 + 495*c^2*x^2 + 385*c^4*x^4 + 105*c^6*x^6)
- 6930*a*b*Sqrt[1 + c^2*x^2]*(50488 - 25244*c^2*x^2 + 18933*c^4*x^4 + 11762
5*c^6*x^6 + 111475*c^8*x^8 + 33075*c^10*x^10) + 2*b^2*c*x*(174940920 - 2915
6820*c^2*x^2 + 13120569*c^4*x^4 + 58224375*c^6*x^6 + 42917875*c^8*x^8 + 104
18625*c^10*x^10) - 6930*b*(-3465*a*c^5*x^5*(231 + 495*c^2*x^2 + 385*c^4*x^4
 + 105*c^6*x^6) + b*Sqrt[1 + c^2*x^2]*(50488 - 25244*c^2*x^2 + 18933*c^4*x^
4 + 117625*c^6*x^6 + 111475*c^8*x^8 + 33075*c^10*x^10))*ArcSinh[c*x] + 1200
6225*b^2*c^5*x^5*(231 + 495*c^2*x^2 + 385*c^4*x^4 + 105*c^6*x^6)*ArcSinh[c*
x]^2))/(13867189875*c^5)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.12

method	result
parts	$d^3 a^2 \left(\frac{1}{11} c^6 x^{11}+\frac{1}{3} c^4 x^9+\frac{3}{7} c^2 x^7+\frac{1}{5} x^5\right)+\frac{d^3 b^2 \left(\frac{\operatorname{arcsinh}(cx)^2 c^3 x^3 \left(c^2 x^2+1\right)^4}{11}-\frac{\operatorname{arcsinh}(cx)^2 c x \left(c^2 x^2+1\right)^4}{33}+\frac{16 \operatorname{a}}{}\right.}{}$
derivativedivides	$\frac{d^3 a^2 \left(\frac{1}{11} c^{11} x^{11}+\frac{1}{3} c^9 x^9+\frac{3}{7} c^7 x^7+\frac{1}{5} c^5 x^5\right)+d^3 b^2 \left(\frac{\operatorname{arcsinh}(cx)^2 c^3 x^3 \left(c^2 x^2+1\right)^4}{11}-\frac{\operatorname{arcsinh}(cx)^2 c x \left(c^2 x^2+1\right)^4}{33}+\frac{16 \operatorname{arcsinh}(cx)^2 x c}{1155}\right.}{}$
default	$\frac{d^3 a^2 \left(\frac{1}{11} c^{11} x^{11}+\frac{1}{3} c^9 x^9+\frac{3}{7} c^7 x^7+\frac{1}{5} c^5 x^5\right)+d^3 b^2 \left(\frac{\operatorname{arcsinh}(cx)^2 c^3 x^3 \left(c^2 x^2+1\right)^4}{11}-\frac{\operatorname{arcsinh}(cx)^2 c x \left(c^2 x^2+1\right)^4}{33}+\frac{16 \operatorname{arcsinh}(cx)^2 x c}{1155}\right.}{}$

```
[In] int(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*a^2*(1/11*c^6*x^11+1/3*c^4*x^9+3/7*c^2*x^7+1/5*x^5)+d^3*b^2/c^5*(1/11*a
rcsinh(c*x)^2*c^3*x^3*(c^2*x^2+1)^4-1/33*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^4+1
6/1155*arcsinh(c*x)^2*x*c+1/231*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^3+2/385*arcs
inh(c*x)^2*c*x*(c^2*x^2+1)^2+8/1155*arcsinh(c*x)^2*c*x*(c^2*x^2+1)-2/1617*a
rcsinh(c*x)*(c^2*x^2+1)^(7/2)-428/323433*c*x*(c^2*x^2+1)^4-16/3465*arcsinh(
c*x)*(c^2*x^2+1)^(3/2)-606416/13867189875*c*x*(c^2*x^2+1)-5487704/462239662
5*c*x*(c^2*x^2+1)^2-148174/110937519*c*x*(c^2*x^2+1)^3-4/1925*arcsinh(c*x)*
```

```
(c^2*x^2+1)^(5/2)-2/121*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^(9/2)+382986368/13
867189875*c*x-32/1155*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+34/3267*arcsinh(c*x)*(
c^2*x^2+1)^(9/2)+2/1331*c*x*(c^2*x^2+1)^5)+2*d^3*a*b/c^5*(1/11*arcsinh(c*x)
*c^11*x^11+1/3*arcsinh(c*x)*c^9*x^9+3/7*arcsinh(c*x)*c^7*x^7+1/5*arcsinh(c*
x)*c^5*x^5-91/3267*c^8*x^8*(c^2*x^2+1)^(1/2)-4705/160083*c^6*x^6*(c^2*x^2+1
)^(1/2)-6311/1334025*c^4*x^4*(c^2*x^2+1)^(1/2)+25244/4002075*c^2*x^2*(c^2*x
^2+1)^(1/2)-50488/4002075*(c^2*x^2+1)^(1/2)-1/121*c^10*x^10*(c^2*x^2+1)^(1/
2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 444, normalized size of antiderivative = 0.95

$$\int x^4(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{10418625\,(121\,a^2+2\,b^2)c^{11}d^3x^{11}+471625\,(9801\,a^2+182\,b^2)c^9d^3x^9+12375\,(480249\,a^2+9410\,b^2)c^7d^3x^7}{}$$

```
[In] integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/13867189875*(10418625*(121*a^2 + 2*b^2)*c^11*d^3*x^11 + 471625*(9801*a^2
+ 182*b^2)*c^9*d^3*x^9 + 12375*(480249*a^2 + 9410*b^2)*c^7*d^3*x^7 + 2079*(
1334025*a^2 + 12622*b^2)*c^5*d^3*x^5 - 58313640*b^2*c^3*d^3*x^3 + 349881840
*b^2*c*d^3*x + 12006225*(105*b^2*c^11*d^3*x^11 + 385*b^2*c^9*d^3*x^9 + 495*
b^2*c^7*d^3*x^7 + 231*b^2*c^5*d^3*x^5)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 693
0*(363825*a*b*c^11*d^3*x^11 + 1334025*a*b*c^9*d^3*x^9 + 1715175*a*b*c^7*d^3
*x^7 + 800415*a*b*c^5*d^3*x^5 - (33075*b^2*c^10*d^3*x^10 + 111475*b^2*c^8*d
^3*x^8 + 117625*b^2*c^6*d^3*x^6 + 18933*b^2*c^4*d^3*x^4 - 25244*b^2*c^2*d^3
*x^2 + 50488*b^2*d^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 693
0*(33075*a*b*c^10*d^3*x^10 + 111475*a*b*c^8*d^3*x^8 + 117625*a*b*c^6*d^3*x^
6 + 18933*a*b*c^4*d^3*x^4 - 25244*a*b*c^2*d^3*x^2 + 50488*a*b*d^3)*sqrt(c^2
*x^2 + 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 3.48 (sec) , antiderivative size = 702, normalized size of antiderivative = 1.51

$$\int x^4(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\begin{cases}\frac{a^2c^6d^3x^{11}}{11}+\frac{a^2c^4d^3x^9}{3}+\frac{3a^2c^2d^3x^7}{7}+\frac{a^2d^3x^5}{5}+\frac{2abc^6d^3x^{11}\operatorname{asinh}(cx)}{11}-\frac{2abc^5d^3x^{10}\sqrt{c^2x^2+1}}{121}+\frac{2abc^4d^3x^9\operatorname{asinh}(cx)}{3}-\frac{182abc^3}{} \\ \frac{a^2d^3x^5}{5}\end{cases}$$

```
[In] integrate(x**4*(c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**6*d**3*x**11/11 + a**2*c**4*d**3*x**9/3 + 3*a**2*c**2*d*
*3*x**7/7 + a**2*d**3*x**5/5 + 2*a*b*c**6*d**3*x**11*asinh(c*x)/11 - 2*a*b*
c**5*d**3*x**10*sqrt(c**2*x**2 + 1)/121 + 2*a*b*c**4*d**3*x**9*asinh(c*x)/3
 - 182*a*b*c**3*d**3*x**8*sqrt(c**2*x**2 + 1)/3267 + 6*a*b*c**2*d**3*x**7*a
sinh(c*x)/7 - 9410*a*b*c*d**3*x**6*sqrt(c**2*x**2 + 1)/160083 + 2*a*b*d**3*
x**5*asinh(c*x)/5 - 12622*a*b*d**3*x**4*sqrt(c**2*x**2 + 1)/(1334025*c) + 5
0488*a*b*d**3*x**2*sqrt(c**2*x**2 + 1)/(4002075*c**3) - 100976*a*b*d**3*sqr
t(c**2*x**2 + 1)/(4002075*c**5) + b**2*c**6*d**3*x**11*asinh(c*x)**2/11 + 2
*b**2*c**6*d**3*x**11/1331 - 2*b**2*c**5*d**3*x**10*sqrt(c**2*x**2 + 1)*asi
nh(c*x)/121 + b**2*c**4*d**3*x**9*asinh(c*x)**2/3 + 182*b**2*c**4*d**3*x**9
/29403 - 182*b**2*c**3*d**3*x**8*sqrt(c**2*x**2 + 1)*asinh(c*x)/3267 + 3*b*
*2*c**2*d**3*x**7*asinh(c*x)**2/7 + 9410*b**2*c**2*d**3*x**7/1120581 - 9410
*b**2*c*d**3*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/160083 + b**2*d**3*x**5*as
inh(c*x)**2/5 + 12622*b**2*d**3*x**5/6670125 - 12622*b**2*d**3*x**4*sqrt(c*
*2*x**2 + 1)*asinh(c*x)/(1334025*c) - 50488*b**2*d**3*x**3/(12006225*c**2)
+ 50488*b**2*d**3*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(4002075*c**3) + 1009
76*b**2*d**3*x/(4002075*c**4) - 100976*b**2*d**3*sqrt(c**2*x**2 + 1)*asinh(
c*x)/(4002075*c**5), Ne(c, 0)), (a**2*d**3*x**5/5, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1109 vs. $2(413) = 826$.

Time = 0.23 (sec) , antiderivative size = 1109, normalized size of antiderivative = 2.38

$$\int x^4 (d + c^2 d x^2)^3 (a + b\text{arcsinh}(cx))^2 \, dx = \text{Too large to display}$$

```
[In] integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^2*c^6*d^3*x^11*arcsinh(c*x)^2 + 1/11*a^2*c^6*d^3*x^11 + 1/3*b^2*c^4*
d^3*x^9*arcsinh(c*x)^2 + 1/3*a^2*c^4*d^3*x^9 + 3/7*b^2*c^2*d^3*x^7*arcsinh(
c*x)^2 + 3/7*a^2*c^2*d^3*x^7 + 2/7623*(693*x^11*arcsinh(c*x) - (63*sqrt(c^2
*x^2 + 1)*x^10/c^2 - 70*sqrt(c^2*x^2 + 1)*x^8/c^4 + 80*sqrt(c^2*x^2 + 1)*x^
6/c^6 - 96*sqrt(c^2*x^2 + 1)*x^4/c^8 + 128*sqrt(c^2*x^2 + 1)*x^2/c^10 - 256
*sqrt(c^2*x^2 + 1)/c^12)*c)*a*b*c^6*d^3 - 2/26413695*(3465*(63*sqrt(c^2*x^2
 + 1)*x^10/c^2 - 70*sqrt(c^2*x^2 + 1)*x^8/c^4 + 80*sqrt(c^2*x^2 + 1)*x^6/c^
6 - 96*sqrt(c^2*x^2 + 1)*x^4/c^8 + 128*sqrt(c^2*x^2 + 1)*x^2/c^10 - 256*sqr
t(c^2*x^2 + 1)/c^12)*c*arcsinh(c*x) - (19845*c^10*x^11 - 26950*c^8*x^9 + 39
600*c^6*x^7 - 66528*c^4*x^5 + 147840*c^2*x^3 - 887040*x)/c^10)*b^2*c^6*d^3
+ 1/5*b^2*d^3*x^5*arcsinh(c*x)^2 + 2/945*(315*x^9*arcsinh(c*x) - (35*sqrt(c
^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*x
^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c)*a*b*
c^4*d^3 - 2/297675*(315*(35*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1
```

```
)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 1
28*sqrt(c^2*x^2 + 1)/c^10)*c*arcsinh(c*x) - (1225*c^8*x^9 - 1800*c^6*x^7 +
3024*c^4*x^5 - 6720*c^2*x^3 + 40320*x)/c^8)*b^2*c^4*d^3 + 1/5*a^2*d^3*x^5 +
 6/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2
 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*
a*b*c^2*d^3 - 2/8575*(105*(5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1
)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsi
nh(c*x) - (75*c^6*x^7 - 126*c^4*x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*c^2*d^
3 + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x
^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*d^3 - 2/1125*(15*(3*sqrt(
c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^
6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^2*d^3
```

Giac [F(-2)]

Exception generated.

$$\int x^4(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^4(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2\,dx = \int x^4\,(a+b\,\text{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^3\,dx$$

```
[In] int(x^4*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3,x)
[Out] int(x^4*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3, x)
```

3.217 $\int x^3(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 376

$$\begin{aligned}\int x^3(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2\,dx &= -\frac{79b^2d^3x^2}{5120c^2}+\frac{79b^2d^3x^4}{15360}+\frac{401b^2c^2d^3x^6}{28800}\\&\quad+\frac{57b^2c^4d^3x^8}{6400}+\frac{1}{500}b^2c^6d^3x^{10}\\&\quad+\frac{79bd^3x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2560c^3}\\&\quad-\frac{79bd^3x^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3840c}\\&\quad-\frac{31}{960}bcd^3x^5\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\\&\quad-\frac{1}{32}bcd^3x^5(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\\&\quad-\frac{1}{50}bcd^3x^5(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\\&\quad-\frac{79d^3(a+b\text{arcsinh}(cx))^2}{5120c^4}\\&\quad+\frac{1}{40}d^3x^4(a+b\text{arcsinh}(cx))^2\\&\quad+\frac{1}{20}d^3x^4(1+c^2x^2)(a+b\text{arcsinh}(cx))^2\\&\quad+\frac{3}{40}d^3x^4(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2\\&\quad+\frac{1}{10}d^3x^4(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2\end{aligned}$$

```
[Out] -79/5120*b^2*d^3*x^2/c^2+79/15360*b^2*d^3*x^4+401/28800*b^2*c^2*d^3*x^6+57/
6400*b^2*c^4*d^3*x^8+1/500*b^2*c^6*d^3*x^10-1/32*b*c*d^3*x^5*(c^2*x^2+1)^(3
/2)*(a+b*arcsinh(c*x))-1/50*b*c*d^3*x^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x)
```

```
)-79/5120*d^3*(a+b*arcsinh(c*x))^2/c^4+1/40*d^3*x^4*(a+b*arcsinh(c*x))^2+1/
20*d^3*x^4*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+3/40*d^3*x^4*(c^2*x^2+1)^2*(a+b
*arcsinh(c*x))^2+1/10*d^3*x^4*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2+79/2560*b*
d^3*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-79/3840*b*d^3*x^3*(a+b*arcsi
nh(c*x))*(c^2*x^2+1)^(1/2)/c-31/960*b*c*d^3*x^5*(a+b*arcsinh(c*x))*(c^2*x^2
+1)^(1/2)
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5808, 5776, 5812, 5783, 30, 5806, 14, 272, 45}

$$\begin{aligned}\int x^3\left(d+c^2dx^2\right)^3(a+b\operatorname{arcsinh}(cx))^2\,dx = & -\frac{79d^3(a+b\operatorname{arcsinh}(cx))^2}{5120c^4} \\ & -\frac{1}{50}bcd^3x^5\left(c^2x^2+1\right)^{5/2}(a+b\operatorname{arcsinh}(cx)) \\ & -\frac{1}{32}bcd^3x^5\left(c^2x^2+1\right)^{3/2}(a+b\operatorname{arcsinh}(cx)) \\ & -\frac{31}{960}bcd^3x^5\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx)) \\ & +\frac{1}{10}d^3x^4\left(c^2x^2+1\right)^3(a+b\operatorname{arcsinh}(cx))^2 \\ & +\frac{3}{40}d^3x^4\left(c^2x^2+1\right)^2(a+b\operatorname{arcsinh}(cx))^2 \\ & +\frac{1}{20}d^3x^4\left(c^2x^2+1\right)(a+b\operatorname{arcsinh}(cx))^2 \\ & -\frac{79bd^3x^3\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{3840c} \\ & +\frac{79bd^3x\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{2560c^3} \\ & +\frac{1}{40}d^3x^4(a+b\operatorname{arcsinh}(cx))^2+\frac{1}{500}b^2c^6d^3x^{10} \\ & +\frac{57b^2c^4d^3x^8}{6400}+\frac{401b^2c^2d^3x^6}{28800}-\frac{79b^2d^3x^2}{5120c^2} \\ & +\frac{79b^2d^3x^4}{15360}\end{aligned}$$

```
[In] Int[x^3*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-79*b^2*d^3*x^2)/(5120*c^2) + (79*b^2*d^3*x^4)/15360 + (401*b^2*c^2*d^3*x^
6)/28800 + (57*b^2*c^4*d^3*x^8)/6400 + (b^2*c^6*d^3*x^10)/500 + (79*b*d^3*x
*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2560*c^3) - (79*b*d^3*x^3*Sqrt[1
+ c^2*x^2]*(a + b*ArcSinh[c*x]))/(3840*c) - (31*b*c*d^3*x^5*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x]))/960 - (b*c*d^3*x^5*(1 + c^2*x^2)^(3/2)*(a + b*ArcS
inh[c*x]))/32 - (b*c*d^3*x^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/50 -
```

```
(79*d^3*(a + b*ArcSinh[c*x])^2)/(5120*c^4) + (d^3*x^4*(a + b*ArcSinh[c*x])
^2)/40 + (d^3*x^4*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/20 + (3*d^3*x^4*(1
+ c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/40 + (d^3*x^4*(1 + c^2*x^2)^3*(a + b*A
rcSinh[c*x])^2)/10
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
```

```
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{10}d^3x^4\left(1+c^2x^2\right)^3(a+b\text{arcsinh}(cx))^2 \\
&\quad+\frac{1}{5}(3d)\int x^3\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2\,dx \\
&\quad-\frac{1}{5}\left(bcd^3\right)\int x^4\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx \\
&= -\frac{1}{50}bcd^3x^5\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{3}{40}d^3x^4\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2 \\
&\quad+\frac{1}{10}d^3x^4\left(1+c^2x^2\right)^3(a+b\text{arcsinh}(cx))^2 \\
&\quad+\frac{1}{10}\left(3d^2\right)\int x^3\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))^2\,dx \\
&\quad-\frac{1}{10}\left(bcd^3\right)\int x^4\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx \\
&\quad-\frac{1}{20}\left(3bcd^3\right)\int x^4\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx+\frac{1}{50}\left(b^2c^2d^3\right)\int x^5\left(1+c^2x^2\right)^2\,dx
\end{aligned}$$

$$=-\frac{1}{32}bcd^3x^5\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{50}bcd^3x^5\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))$$
$$+\frac{1}{20}d^3x^4\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2+\frac{3}{40}d^3x^4\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2$$
$$+\frac{1}{10}d^3x^4\left(1+c^2x^2\right)^3(a+b\text{arcsinh}(cx))^2+\frac{1}{10}d^3\int x^3(a+b\text{arcsinh}(cx))^2\,dx$$
$$-\frac{1}{80}\left(3bcd^3\right)\int x^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx$$
$$-\frac{1}{160}\left(9bcd^3\right)\int x^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx$$
$$-\frac{1}{10}\left(bcd^3\right)\int x^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx$$
$$+\frac{1}{100}\left(b^2c^2d^3\right)\text{Subst}\left(\int x^2\left(1+c^2x\right)^2\,dx,x,x^2\right)$$
$$+\frac{1}{80}\left(b^2c^2d^3\right)\int x^5\left(1+c^2x^2\right)\,dx+\frac{1}{160}\left(3b^2c^2d^3\right)\int x^5\left(1+c^2x^2\right)\,dx$$

$$=-\frac{31}{960}bcd^3x^5\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))-\frac{1}{32}bcd^3x^5\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))$$
$$-\frac{1}{50}bcd^3x^5\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{1}{40}d^3x^4(a+b\text{arcsinh}(cx))^2$$
$$+\frac{1}{20}d^3x^4\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2+\frac{3}{40}d^3x^4\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2$$
$$+\frac{1}{10}d^3x^4\left(1+c^2x^2\right)^3(a+b\text{arcsinh}(cx))^2-\frac{1}{160}\left(bcd^3\right)\int\frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx$$
$$-\frac{1}{320}\left(3bcd^3\right)\int\frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx-\frac{1}{60}\left(bcd^3\right)\int\frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx$$
$$-\frac{1}{20}\left(bcd^3\right)\int\frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx+\frac{1}{160}\left(b^2c^2d^3\right)\int x^5\,dx$$
$$+\frac{1}{320}\left(3b^2c^2d^3\right)\int x^5\,dx+\frac{1}{100}\left(b^2c^2d^3\right)\text{Subst}\left(\int\left(x^2+2c^2x^3+c^4x^4\right)\,dx,x,x^2\right)$$
$$+\frac{1}{80}\left(b^2c^2d^3\right)\int\left(x^5+c^2x^7\right)\,dx+\frac{1}{60}\left(b^2c^2d^3\right)\int x^5\,dx$$
$$+\frac{1}{160}\left(3b^2c^2d^3\right)\int\left(x^5+c^2x^7\right)\,dx$$

$$= \frac{401b^2c^2d^3x^6}{28800} + \frac{57b^2c^4d^3x^8}{6400} + \frac{1}{500}b^2c^6d^3x^{10} - \frac{79bd^3x^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3840c} - \frac{31}{960}bcd^3x^5\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx)) - \frac{1}{32}bcd^3x^5\left(1+c^2x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx)) - \frac{1}{50}bcd^3x^5\left(1+c^2x^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx)) + \frac{1}{40}d^3x^4(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{20}d^3x^4\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))^2 + \frac{3}{40}d^3x^4\left(1+c^2x^2\right)^2(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{10}d^3x^4\left(1+c^2x^2\right)^3(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{640}\left(b^2d^3\right)\int x^3\,dx + \frac{\left(3b^2d^3\right)\int x^3\,dx}{1280} + \frac{1}{240}\left(b^2d^3\right)\int x^3\,dx + \frac{1}{80}\left(b^2d^3\right)\int x^3\,dx + \frac{\left(3bd^3\right)\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{640c} + \frac{\left(9bd^3\right)\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{1280c} + \frac{\left(bd^3\right)\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{80c} + \frac{\left(3bd^3\right)\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{80c}$$

$$= \frac{79b^2d^3x^4}{15360} + \frac{401b^2c^2d^3x^6}{28800} + \frac{57b^2c^4d^3x^8}{6400} + \frac{1}{500}b^2c^6d^3x^{10} + \frac{79bd^3x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{2560c^3} - \frac{79bd^3x^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3840c} - \frac{31}{960}bcd^3x^5\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx)) - \frac{1}{32}bcd^3x^5\left(1+c^2x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx)) - \frac{1}{50}bcd^3x^5\left(1+c^2x^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx)) + \frac{1}{40}d^3x^4(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{20}d^3x^4\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))^2 + \frac{3}{40}d^3x^4\left(1+c^2x^2\right)^2(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{10}d^3x^4\left(1+c^2x^2\right)^3(a+b\operatorname{arcsinh}(cx))^2 - \frac{\left(3bd^3\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{1280c^3} - \frac{\left(9bd^3\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{2560c^3} - \frac{\left(bd^3\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{160c^3} - \frac{\left(3bd^3\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{160c^3} - \frac{\left(3b^2d^3\right)\int x\,dx}{1280c^2} - \frac{\left(9b^2d^3\right)\int x\,dx}{2560c^2} - \frac{\left(b^2d^3\right)\int x\,dx}{160c^2} - \frac{\left(3b^2d^3\right)\int x\,dx}{160c^2}$$

$$= -\frac{79b^2d^3x^2}{5120c^2} + \frac{79b^2d^3x^4}{15360} + \frac{401b^2c^2d^3x^6}{28800} + \frac{57b^2c^4d^3x^8}{6400} + \frac{1}{500}b^2c^6d^3x^{10} + \frac{79bd^3x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2560c^3} - \frac{79bd^3x^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3840c} - \frac{31}{960}bcd^3x^5\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) - \frac{1}{32}bcd^3x^5(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{1}{50}bcd^3x^5(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx)) - \frac{79d^3(a+b\text{arcsinh}(cx))^2}{5120c^4} + \frac{1}{40}d^3x^4(a+b\text{arcsinh}(cx))^2 + \frac{1}{20}d^3x^4(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 + \frac{3}{40}d^3x^4(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2 + \frac{1}{10}d^3x^4(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.76

$$\int x^3(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2\,dx = \frac{d^3(cx(28800a^2c^3x^3(10+20c^2x^2+15c^4x^4+4c^6x^6) - 30ab\sqrt{1+c^2x^2}(-1185+790c^2x^2+3208c^4x^4+2736}{}$$

```
[In] Integrate[x^3*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^3*(c*x*(28800*a^2*c^3*x^3*(10 + 20*c^2*x^2 + 15*c^4*x^4 + 4*c^6*x^6) - 3
0*a*b*Sqrt[1 + c^2*x^2]*(-1185 + 790*c^2*x^2 + 3208*c^4*x^4 + 2736*c^6*x^6
+ 768*c^8*x^8) + b^2*c*x*(-17775 + 5925*c^2*x^2 + 16040*c^4*x^4 + 10260*c^6
*x^6 + 2304*c^8*x^8)) + 30*b*(-(b*c*x*Sqrt[1 + c^2*x^2]*(-1185 + 790*c^2*x^
2 + 3208*c^4*x^4 + 2736*c^6*x^6 + 768*c^8*x^8)) + 15*a*(-79 + 1280*c^4*x^4
+ 2560*c^6*x^6 + 1920*c^8*x^8 + 512*c^10*x^10))*ArcSinh[c*x] + 225*b^2*(-79
 + 1280*c^4*x^4 + 2560*c^6*x^6 + 1920*c^8*x^8 + 512*c^10*x^10)*ArcSinh[c*x]
^2))/(1152000*c^4)
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.10

method	result
parts	$d^3a^2\left(\frac{1}{10}c^6x^{10}+\frac{3}{8}c^4x^8+\frac{1}{2}c^2x^6+\frac{1}{4}x^4\right)+\frac{d^3b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^2x^2\left(c^2x^2+1\right)^4}{10}-\frac{\operatorname{arcsinh}(cx)^2\left(c^2x^2+1\right)^4}{40}-\frac{\operatorname{arcs}}{}\right.}{}$
derivativedivides	$\frac{d^3a^2\left(\frac{1}{10}c^{10}x^{10}+\frac{3}{8}c^8x^8+\frac{1}{2}c^6x^6+\frac{1}{4}c^4x^4\right)+d^3b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^2x^2\left(c^2x^2+1\right)^4}{10}-\frac{\operatorname{arcsinh}(cx)^2\left(c^2x^2+1\right)^4}{40}-\frac{\operatorname{arcsinh}(cx)cx\left(c^2x\right.}{50}\right.}{}$
default	$\frac{d^3a^2\left(\frac{1}{10}c^{10}x^{10}+\frac{3}{8}c^8x^8+\frac{1}{2}c^6x^6+\frac{1}{4}c^4x^4\right)+d^3b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^2x^2\left(c^2x^2+1\right)^4}{10}-\frac{\operatorname{arcsinh}(cx)^2\left(c^2x^2+1\right)^4}{40}-\frac{\operatorname{arcsinh}(cx)cx\left(c^2x\right.}{50}\right.}{}$

```
[In] int(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*a^2*(1/10*c^6*x^10+3/8*c^4*x^8+1/2*c^2*x^6+1/4*x^4)+d^3*b^2/c^4*(1/10*a
rcsinh(c*x)^2*c^2*x^2*(c^2*x^2+1)^4-1/40*arcsinh(c*x)^2*(c^2*x^2+1)^4-1/50*
arcsinh(c*x)*c*x*(c^2*x^2+1)^(9/2)+7/800*arcsinh(c*x)*c*x*(c^2*x^2+1)^(7/2)
+49/4800*arcsinh(c*x)*c*x*(c^2*x^2+1)^(5/2)+49/3840*arcsinh(c*x)*c*x*(c^2*x
^2+1)^(3/2)+49/2560*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)+49/5120*arcsinh(c*x)
^2+1/500*(c^2*x^2+1)^5-7/6400*(c^2*x^2+1)^4-49/28800*(c^2*x^2+1)^3-49/15360
*(c^2*x^2+1)^2-49/5120*c^2*x^2-49/5120)+2*d^3*a*b/c^4*(1/10*arcsinh(c*x)*c^
10*x^10+3/8*arcsinh(c*x)*c^8*x^8+1/2*arcsinh(c*x)*c^6*x^6+1/4*arcsinh(c*x)*
c^4*x^4-1/100*c^9*x^9*(c^2*x^2+1)^(1/2)-57/1600*c^7*x^7*(c^2*x^2+1)^(1/2)-4
01/9600*c^5*x^5*(c^2*x^2+1)^(1/2)-79/7680*c^3*x^3*(c^2*x^2+1)^(1/2)+79/5120
*c*x*(c^2*x^2+1)^(1/2)-79/5120*arcsinh(c*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.13

$$\int x^3\left(d+c^2dx^2\right)^3(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$=\frac{2304\,(50\,a^2+b^2)c^{10}d^3x^{10}+540\,(800\,a^2+19\,b^2)c^8d^3x^8+40\,(14400\,a^2+401\,b^2)c^6d^3x^6+75\,(3840\,a^2+79\,b}{}$$

```
[In] integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/1152000*(2304*(50*a^2 + b^2)*c^10*d^3*x^10 + 540*(800*a^2 + 19*b^2)*c^8*d
^3*x^8 + 40*(14400*a^2 + 401*b^2)*c^6*d^3*x^6 + 75*(3840*a^2 + 79*b^2)*c^4*
d^3*x^4 - 17775*b^2*c^2*d^3*x^2 + 225*(512*b^2*c^10*d^3*x^10 + 1920*b^2*c^8
*d^3*x^8 + 2560*b^2*c^6*d^3*x^6 + 1280*b^2*c^4*d^3*x^4 - 79*b^2*d^3)*log(c*
x + sqrt(c^2*x^2 + 1))^2 + 30*(7680*a*b*c^10*d^3*x^10 + 28800*a*b*c^8*d^3*x
^8 + 38400*a*b*c^6*d^3*x^6 + 19200*a*b*c^4*d^3*x^4 - 1185*a*b*d^3 - (768*b^
2*c^9*d^3*x^9 + 2736*b^2*c^7*d^3*x^7 + 3208*b^2*c^5*d^3*x^5 + 790*b^2*c^3*d
```

```
^3*x^3 - 1185*b^2*c*d^3*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))
- 30*(768*a*b*c^9*d^3*x^9 + 2736*a*b*c^7*d^3*x^7 + 3208*a*b*c^5*d^3*x^5 + 7
90*a*b*c^3*d^3*x^3 - 1185*a*b*c*d^3*x)*sqrt(c^2*x^2 + 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 654, normalized size of antiderivative = 1.74

$$\int x^3\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2\,dx = \begin{cases} \frac{a^2c^6d^3x^{10}}{10} + \frac{3a^2c^4d^3x^8}{8} + \frac{a^2c^2d^3x^6}{2} + \frac{a^2d^3x^4}{4} + \frac{abc^6d^3x^{10}\operatorname{asinh}(cx)}{5} - \frac{abc^5d^3x^9\sqrt{c^2x^2+1}}{50} + \frac{3abc^4d^3x^8\operatorname{asinh}(cx)}{4} - \frac{57abc^3d^3x^7\sqrt{}}{800} \\ \frac{a^2d^3x^4}{4} \end{cases}$$

```
[In] integrate(x**3*(c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**6*d**3*x**10/10 + 3*a**2*c**4*d**3*x**8/8 + a**2*c**2*d*
*3*x**6/2 + a**2*d**3*x**4/4 + a*b*c**6*d**3*x**10*asinh(c*x)/5 - a*b*c**5*
d**3*x**9*sqrt(c**2*x**2 + 1)/50 + 3*a*b*c**4*d**3*x**8*asinh(c*x)/4 - 57*a
*b*c**3*d**3*x**7*sqrt(c**2*x**2 + 1)/800 + a*b*c**2*d**3*x**6*asinh(c*x) -
 401*a*b*c*d**3*x**5*sqrt(c**2*x**2 + 1)/4800 + a*b*d**3*x**4*asinh(c*x)/2
- 79*a*b*d**3*x**3*sqrt(c**2*x**2 + 1)/(3840*c) + 79*a*b*d**3*x*sqrt(c**2*x
**2 + 1)/(2560*c**3) - 79*a*b*d**3*asinh(c*x)/(2560*c**4) + b**2*c**6*d**3*
x**10*asinh(c*x)**2/10 + b**2*c**6*d**3*x**10/500 - b**2*c**5*d**3*x**9*sqr
t(c**2*x**2 + 1)*asinh(c*x)/50 + 3*b**2*c**4*d**3*x**8*asinh(c*x)**2/8 + 57
*b**2*c**4*d**3*x**8/6400 - 57*b**2*c**3*d**3*x**7*sqrt(c**2*x**2 + 1)*asin
h(c*x)/800 + b**2*c**2*d**3*x**6*asinh(c*x)**2/2 + 401*b**2*c**2*d**3*x**6/
28800 - 401*b**2*c*d**3*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)/4800 + b**2*d**
3*x**4*asinh(c*x)**2/4 + 79*b**2*d**3*x**4/15360 - 79*b**2*d**3*x**3*sqrt(c
**2*x**2 + 1)*asinh(c*x)/(3840*c) - 79*b**2*d**3*x**2/(5120*c**2) + 79*b**2
*d**3*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(2560*c**3) - 79*b**2*d**3*asinh(c*x
)**2/(5120*c**4), Ne(c, 0)), (a**2*d**3*x**4/4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1112 vs. 2(336) = 672.

Time = 0.28 (sec) , antiderivative size = 1112, normalized size of antiderivative = 2.96

$$\int x^3\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*c^6*d^3*x^10*arcsinh(c*x)^2 + 1/10*a^2*c^6*d^3*x^10 + 3/8*b^2*c^4*
d^3*x^8*arcsinh(c*x)^2 + 3/8*a^2*c^4*d^3*x^8 + 1/2*b^2*c^2*d^3*x^6*arcsinh(
```

```
c*x)^2 + 1/2*a^2*c^2*d^3*x^6 + 1/6400*(1280*x^10*arcsinh(c*x) - (128*sqrt(c
^2*x^2 + 1)*x^9/c^2 - 144*sqrt(c^2*x^2 + 1)*x^7/c^4 + 168*sqrt(c^2*x^2 + 1)
*x^5/c^6 - 210*sqrt(c^2*x^2 + 1)*x^3/c^8 + 315*sqrt(c^2*x^2 + 1)*x/c^10 - 3
15*arcsinh(c*x)/c^11)*c)*a*b*c^6*d^3 + 1/64000*((128*x^10/c^2 - 180*x^8/c^4
 + 280*x^6/c^6 - 525*x^4/c^8 + 1575*x^2/c^10 - 1575*log(c*x + sqrt(c^2*x^2
+ 1))^2/c^12)*c^2 - 10*(128*sqrt(c^2*x^2 + 1)*x^9/c^2 - 144*sqrt(c^2*x^2 +
1)*x^7/c^4 + 168*sqrt(c^2*x^2 + 1)*x^5/c^6 - 210*sqrt(c^2*x^2 + 1)*x^3/c^8
+ 315*sqrt(c^2*x^2 + 1)*x/c^10 - 315*arcsinh(c*x)/c^11)*c*arcsinh(c*x))*b^2
*c^6*d^3 + 1/4*b^2*d^3*x^4*arcsinh(c*x)^2 + 1/512*(384*x^8*arcsinh(c*x) - (
48*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x
^2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/c^8 + 105*arcsinh(c*x)/c^9)*c)*a*
b*c^4*d^3 + 1/3072*((36*x^8/c^2 - 56*x^6/c^4 + 105*x^4/c^6 - 315*x^2/c^8 +
315*log(c*x + sqrt(c^2*x^2 + 1))^2/c^10)*c^2 - 6*(48*sqrt(c^2*x^2 + 1)*x^7/
c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^2 + 1)*x^3/c^6 - 105*sqr
t(c^2*x^2 + 1)*x/c^8 + 105*arcsinh(c*x)/c^9)*c*arcsinh(c*x))*b^2*c^4*d^3 +
1/4*a^2*d^3*x^4 + 1/48*(48*x^6*arcsinh(c*x) - (8*sqrt(c^2*x^2 + 1)*x^5/c^2
- 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*arcsinh(c*
x)/c^7)*c)*a*b*c^2*d^3 + 1/288*((8*x^6/c^2 - 15*x^4/c^4 + 45*x^2/c^6 - 45*l
og(c*x + sqrt(c^2*x^2 + 1))^2/c^8)*c^2 - 6*(8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 1
0*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*arcsinh(c*x)/
c^7)*c*arcsinh(c*x))*b^2*c^2*d^3 + 1/16*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x
^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*a*b*d^
3 + 1/32*((x^4/c^2 - 3*x^2/c^4 + 3*log(c*x + sqrt(c^2*x^2 + 1))^2/c^6)*c^2
- 2*(2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*
x)/c^5)*c*arcsinh(c*x))*b^2*d^3
```

Giac [F(-2)]

Exception generated.

$$\int x^3 \left(d + c^2 d x^2\right)^3 (a + b\text{arcsinh}(cx))^2 \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))^2\,dx=\int x^3\,\left(a+b\,\mathrm{asinh}(c\,x)\right)^2\,\left(d\,c^2\,x^2+d\right)^3\,dx$$

```
[In] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3,x)
[Out] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3, x)
```

3.218 $\int x^2(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 382

$$\begin{aligned}&\int x^2(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2\,dx\\&=-\frac{10516b^2d^3x}{99225c^2}+\frac{5258b^2d^3x^3}{297675}+\frac{4198b^2c^2d^3x^5}{165375}+\frac{374b^2c^4d^3x^7}{27783}+\frac{2}{729}b^2c^6d^3x^9\\&\quad+\frac{64bd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{945c^3}-\frac{32bd^3x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{945c}\\&\quad+\frac{16bd^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{315c^3}+\frac{4bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{525c^3}\\&\quad+\frac{2bd^3(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{441c^3}-\frac{2bd^3(1+c^2x^2)^{9/2}(a+b\text{arcsinh}(cx))}{81c^3}\\&\quad+\frac{16}{315}d^3x^3(a+b\text{arcsinh}(cx))^2+\frac{8}{105}d^3x^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2+\frac{2}{21}d^3x^3(1+c^2x^2)^2(a+b\text{arcsinh}(cx\end{aligned}$$

```
[Out] -10516/99225*b^2*d^3*x/c^2+5258/297675*b^2*d^3*x^3+4198/165375*b^2*c^2*d^3*
x^5+374/27783*b^2*c^4*d^3*x^7+2/729*b^2*c^6*d^3*x^9+16/315*b*d^3*(c^2*x^2+1
)^(3/2)*(a+b*arcsinh(c*x))/c^3+4/525*b*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c
*x))/c^3+2/441*b*d^3*(c^2*x^2+1)^(7/2)*(a+b*arcsinh(c*x))/c^3-2/81*b*d^3*(c
^2*x^2+1)^(9/2)*(a+b*arcsinh(c*x))/c^3+16/315*d^3*x^3*(a+b*arcsinh(c*x))^2+
8/105*d^3*x^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+2/21*d^3*x^3*(c^2*x^2+1)^2*(
a+b*arcsinh(c*x))^2+1/9*d^3*x^3*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2+64/945*b
*d^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-32/945*b*d^3*x^2*(a+b*arcsinh
(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {5808, 5776, 5812, 5798, 8, 30, 272, 45, 5804, 12, 380}

$$\int x^2 \left(d+c^2 d x^2\right)^3 (a+b\operatorname{arcsinh}(cx))^2 \, dx = -\frac{32 b d^3 x^2 \sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{945c} + \frac{1}{9} d^3 x^3 \left(c^2x^2+1\right)^3 (a+b\operatorname{arcsinh}(cx))^2 + \frac{2}{21} d^3 x^3 \left(c^2x^2+1\right)^2 (a+b\operatorname{arcsinh}(cx))^2 + \frac{8}{105} d^3 x^3 \left(c^2x^2+1\right) (a+b\operatorname{arcsinh}(cx))^2 - \frac{2 b d^3 \left(c^2x^2+1\right)^{9/2} (a+b\operatorname{arcsinh}(cx))}{81c^3} + \frac{2 b d^3 \left(c^2x^2+1\right)^{7/2} (a+b\operatorname{arcsinh}(cx))}{441c^3} + \frac{4 b d^3 \left(c^2x^2+1\right)^{5/2} (a+b\operatorname{arcsinh}(cx))}{525c^3} + \frac{16 b d^3 \left(c^2x^2+1\right)^{3/2} (a+b\operatorname{arcsinh}(cx))}{315c^3} + \frac{64 b d^3 \sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{945c^3} + \frac{16}{315} d^3 x^3 (a+b\operatorname{arcsinh}(cx))^2 + \frac{2}{729} b^2 c^6 d^3 x^9 + \frac{374 b^2 c^4 d^3 x^7}{27783} + \frac{4198 b^2 c^2 d^3 x^5}{165375} - \frac{10516 b^2 d^3 x}{99225c^2} + \frac{5258 b^2 d^3 x^3}{297675}$$

```
[In] Int[x^2*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-10516*b^2*d^3*x)/(99225*c^2) + (5258*b^2*d^3*x^3)/297675 + (4198*b^2*c^2*
d^3*x^5)/165375 + (374*b^2*c^4*d^3*x^7)/27783 + (2*b^2*c^6*d^3*x^9)/729 + (
64*b*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(945*c^3) - (32*b*d^3*x^2*
Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(945*c) + (16*b*d^3*(1 + c^2*x^2)^(
3/2)*(a + b*ArcSinh[c*x]))/(315*c^3) + (4*b*d^3*(1 + c^2*x^2)^(5/2)*(a + b*
ArcSinh[c*x]))/(525*c^3) + (2*b*d^3*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]
))/(441*c^3) - (2*b*d^3*(1 + c^2*x^2)^(9/2)*(a + b*ArcSinh[c*x]))/(81*c^3)
+ (16*d^3*x^3*(a + b*ArcSinh[c*x])^2)/315 + (8*d^3*x^3*(1 + c^2*x^2)*(a + b
*ArcSinh[c*x])^2)/105 + (2*d^3*x^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/
21 + (d^3*x^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/9
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
```

```
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{9}d^3x^3\left(1+c^2x^2\right)^3(a+b\text{arcsinh}(cx))^2 \\
&\quad+\frac{1}{3}(2d)\int x^2\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2\,dx \\
&\quad-\frac{1}{9}\left(2bcd^3\right)\int x^3\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx \\
&= \frac{2bd^3(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{63c^3}-\frac{2bd^3(1+c^2x^2)^{9/2}(a+b\text{arcsinh}(cx))}{81c^3} \\
&\quad+\frac{2}{21}d^3x^3\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2+\frac{1}{9}d^3x^3\left(1+c^2x^2\right)^3(a+b\text{arcsinh}(cx))^2 \\
&\quad+\frac{1}{21}\left(8d^2\right)\int x^2\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))^2\,dx \\
&\quad-\frac{1}{21}\left(4bcd^3\right)\int x^3\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx+\frac{1}{9}\left(2b^2c^2d^3\right)\int\frac{(1+c^2x^2)^3\left(-2+7c^2x^2\right)}{63c^4}\,dx \\
&= \frac{4bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{105c^3} \\
&\quad+\frac{2bd^3(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{441c^3}-\frac{2bd^3(1+c^2x^2)^{9/2}(a+b\text{arcsinh}(cx))}{81c^3} \\
&\quad+\frac{8}{105}d^3x^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{21}d^3x^3\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2+\frac{1}{9}d^3x^3\left(1+c^2x^2\right)^3(
\end{aligned}$$

$$= \frac{16bd^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{315c^3} + \frac{4bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{525c^3} + \frac{2bd^3(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{441c^3} - \frac{2bd^3(1+c^2x^2)^{9/2}(a+b\text{arcsinh}(cx))}{81c^3} + \frac{16}{315}d^3x^3(a+b\text{arcsinh}(cx))^2 + \frac{8}{105}d^3x^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 + \frac{2}{21}d^3x^3(1+c^2x^2)^2(a+b\text{arc}$$

$$= -\frac{4b^2d^3x}{567c^2} + \frac{2b^2d^3x^3}{1701} + \frac{2}{189}b^2c^2d^3x^5 + \frac{38b^2c^4d^3x^7}{3969} + \frac{2}{729}b^2c^6d^3x^9 - \frac{32bd^3x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{945c} + \frac{16bd^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{315c^3} + \frac{4bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{525c^3} + \frac{2bd^3(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{441c^3} - \frac{2bd^3(1+c^2x^2)^{9/2}(a+b\text{arcsinh}(cx))}{81c^3} + \frac{16}{315}d^3x^3(a+b\text{arcsinh}(cx))^2 + \frac{8}{105}d^3x^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 + \frac{2}{21}d^3x^3(1+c^2x^2)^2(a+b\text{arc}$$

$$= -\frac{3796b^2d^3x}{99225c^2} + \frac{5258b^2d^3x^3}{297675} + \frac{4198b^2c^2d^3x^5}{165375} + \frac{374b^2c^4d^3x^7}{27783} + \frac{2}{729}b^2c^6d^3x^9 + \frac{64bd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{945c^3} - \frac{32bd^3x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{945c} + \frac{16bd^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{315c^3} + \frac{4bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{525c^3} + \frac{2bd^3(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{441c^3} - \frac{2bd^3(1+c^2x^2)^{9/2}(a+b\text{arcsinh}(cx))}{81c^3} + \frac{16}{315}d^3x^3(a+b\text{arcsinh}(cx))^2 + \frac{8}{105}d^3x^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 + \frac{2}{21}d^3x^3(1+c^2x^2)^2(a+b\text{arc}$$

$$= -\frac{10516b^2d^3x}{99225c^2} + \frac{5258b^2d^3x^3}{297675} + \frac{4198b^2c^2d^3x^5}{165375} + \frac{374b^2c^4d^3x^7}{27783} + \frac{2}{729}b^2c^6d^3x^9 + \frac{64bd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{945c^3} - \frac{32bd^3x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{945c} + \frac{16bd^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{315c^3} + \frac{4bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{525c^3} + \frac{2bd^3(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{441c^3} - \frac{2bd^3(1+c^2x^2)^{9/2}(a+b\text{arcsinh}(cx))}{81c^3} + \frac{16}{315}d^3x^3(a+b\text{arcsinh}(cx))^2 + \frac{8}{105}d^3x^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 + \frac{2}{21}d^3x^3(1+c^2x^2)^2(a+b\text{arc}$$

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.72

$$\int x^2(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{d^3\left(99225a^2c^3x^3(105+189c^2x^2+135c^4x^4+35c^6x^6)-630ab\sqrt{1+c^2x^2}(-5258+2629c^2x^2+6297c^4x^4+4\right.}{}$$

```
[In] Integrate[x^2*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^3*(99225*a^2*c^3*x^3*(105 + 189*c^2*x^2 + 135*c^4*x^4 + 35*c^6*x^6) - 63
0*a*b*Sqrt[1 + c^2*x^2]*(-5258 + 2629*c^2*x^2 + 6297*c^4*x^4 + 4675*c^6*x^6
 + 1225*c^8*x^8) + b^2*(-3312540*c*x + 552090*c^3*x^3 + 793422*c^5*x^5 + 42
0750*c^7*x^7 + 85750*c^9*x^9) - 630*b*(-315*a*c^3*x^3*(105 + 189*c^2*x^2 +
135*c^4*x^4 + 35*c^6*x^6) + b*Sqrt[1 + c^2*x^2]*(-5258 + 2629*c^2*x^2 + 629
7*c^4*x^4 + 4675*c^6*x^6 + 1225*c^8*x^8))*ArcSinh[c*x] + 99225*b^2*c^3*x^3*
(105 + 189*c^2*x^2 + 135*c^4*x^4 + 35*c^6*x^6)*ArcSinh[c*x]^2))/(31255875*c
^3)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.14

method	result
parts	$d^3a^2\left(\frac{1}{9}c^6x^9+\frac{3}{7}c^4x^7+\frac{3}{5}c^2x^5+\frac{1}{3}x^3\right)+\frac{d^3b^2\left(\frac{\text{arcsinh}(cx)^2cx(c^2x^2+1)^4}{9}-\frac{16\,\text{arcsinh}(cx)^2xc}{315}-\frac{\text{arcsinh}(cx)^2cx(c^2\right.}{}$
derivativedivides	$\frac{d^3a^2\left(\frac{1}{9}c^9x^9+\frac{3}{7}c^7x^7+\frac{3}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^3b^2\left(\frac{\text{arcsinh}(cx)^2cx(c^2x^2+1)^4}{9}-\frac{16\,\text{arcsinh}(cx)^2xc}{315}-\frac{\text{arcsinh}(cx)^2cx(c^2x^2+1)^3}{63}-\frac{2\,\text{arc}}{}\right.}{}$
default	$\frac{d^3a^2\left(\frac{1}{9}c^9x^9+\frac{3}{7}c^7x^7+\frac{3}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^3b^2\left(\frac{\text{arcsinh}(cx)^2cx(c^2x^2+1)^4}{9}-\frac{16\,\text{arcsinh}(cx)^2xc}{315}-\frac{\text{arcsinh}(cx)^2cx(c^2x^2+1)^3}{63}-\frac{2\,\text{arc}}{}\right.}{}$

```
[In] int(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*a^2*(1/9*c^6*x^9+3/7*c^4*x^7+3/5*c^2*x^5+1/3*x^3)+d^3*b^2/c^3*(1/9*arcs
inh(c*x)^2*c*x*(c^2*x^2+1)^4-16/315*arcsinh(c*x)^2*x*c-1/63*arcsinh(c*x)^2*
c*x*(c^2*x^2+1)^3-2/105*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^2-8/315*arcsinh(c*x)
^2*c*x*(c^2*x^2+1)-2/81*arcsinh(c*x)*(c^2*x^2+1)^(9/2)-3406208/31255875*c*x
+2/729*c*x*(c^2*x^2+1)^4+622/250047*c*x*(c^2*x^2+1)^3+15224/10418625*c*x*(c
^2*x^2+1)^2-115504/31255875*c*x*(c^2*x^2+1)+32/315*arcsinh(c*x)*(c^2*x^2+1)
^(1/2)+2/441*arcsinh(c*x)*(c^2*x^2+1)^(7/2)+4/525*arcsinh(c*x)*(c^2*x^2+1)^
(5/2)+16/945*arcsinh(c*x)*(c^2*x^2+1)^(3/2))+2*d^3*a*b/c^3*(1/9*arcsinh(c*x
```

```
)*c^9*x^9+3/7*arcsinh(c*x)*c^7*x^7+3/5*arcsinh(c*x)*c^5*x^5+1/3*arcsinh(c*x
)*c^3*x^3-1/81*c^8*x^8*(c^2*x^2+1)^(1/2)-187/3969*c^6*x^6*(c^2*x^2+1)^(1/2)
-2099/33075*c^4*x^4*(c^2*x^2+1)^(1/2)-2629/99225*c^2*x^2*(c^2*x^2+1)^(1/2)+
5258/99225*(c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.05

$$\int x^2\left(d+c^2dx^2\right)^3(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$=\frac{42875\,(81\,a^2+2\,b^2)c^9d^3x^9+1125\,(11907\,a^2+374\,b^2)c^7d^3x^7+189\,(99225\,a^2+4198\,b^2)c^5d^3x^5+105\,(9922}{}$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/31255875*(42875*(81*a^2 + 2*b^2)*c^9*d^3*x^9 + 1125*(11907*a^2 + 374*b^2)
*c^7*d^3*x^7 + 189*(99225*a^2 + 4198*b^2)*c^5*d^3*x^5 + 105*(99225*a^2 + 52
58*b^2)*c^3*d^3*x^3 - 3312540*b^2*c*d^3*x + 99225*(35*b^2*c^9*d^3*x^9 + 135
*b^2*c^7*d^3*x^7 + 189*b^2*c^5*d^3*x^5 + 105*b^2*c^3*d^3*x^3)*log(c*x + sqr
t(c^2*x^2 + 1))^2 + 630*(11025*a*b*c^9*d^3*x^9 + 42525*a*b*c^7*d^3*x^7 + 59
535*a*b*c^5*d^3*x^5 + 33075*a*b*c^3*d^3*x^3 - (1225*b^2*c^8*d^3*x^8 + 4675*
b^2*c^6*d^3*x^6 + 6297*b^2*c^4*d^3*x^4 + 2629*b^2*c^2*d^3*x^2 - 5258*b^2*d^
3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 630*(1225*a*b*c^8*d^3*
x^8 + 4675*a*b*c^6*d^3*x^6 + 6297*a*b*c^4*d^3*x^4 + 2629*a*b*c^2*d^3*x^2 -
5258*a*b*d^3)*sqrt(c^2*x^2 + 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 626, normalized size of antiderivative = 1.64

$$\int x^2\left(d+c^2dx^2\right)^3(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$=\begin{cases}\frac{a^2c^6d^3x^9}{9}+\frac{3a^2c^4d^3x^7}{7}+\frac{3a^2c^2d^3x^5}{5}+\frac{a^2d^3x^3}{3}+\frac{2abc^6d^3x^9\operatorname{asinh}(cx)}{9}-\frac{2abc^5d^3x^8\sqrt{c^2x^2+1}}{81}+\frac{6abc^4d^3x^7\operatorname{asinh}(cx)}{7}-\frac{374abc^3d}{3} \\ \frac{a^2d^3x^3}{3}\end{cases}$$

```
[In] integrate(x**2*(c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**6*d**3*x**9/9 + 3*a**2*c**4*d**3*x**7/7 + 3*a**2*c**2*d*
*3*x**5/5 + a**2*d**3*x**3/3 + 2*a*b*c**6*d**3*x**9*asinh(c*x)/9 - 2*a*b*c*
*5*d**3*x**8*sqrt(c**2*x**2 + 1)/81 + 6*a*b*c**4*d**3*x**7*asinh(c*x)/7 - 3
74*a*b*c**3*d**3*x**6*sqrt(c**2*x**2 + 1)/3969 + 6*a*b*c**2*d**3*x**5*asinh
```

```
(c*x)/5 - 4198*a*b*c*d**3*x**4*sqrt(c**2*x**2 + 1)/33075 + 2*a*b*d**3*x**3*
asinh(c*x)/3 - 5258*a*b*d**3*x**2*sqrt(c**2*x**2 + 1)/(99225*c) + 10516*a*b
*d**3*sqrt(c**2*x**2 + 1)/(99225*c**3) + b**2*c**6*d**3*x**9*asinh(c*x)**2/
9 + 2*b**2*c**6*d**3*x**9/729 - 2*b**2*c**5*d**3*x**8*sqrt(c**2*x**2 + 1)*a
sinh(c*x)/81 + 3*b**2*c**4*d**3*x**7*asinh(c*x)**2/7 + 374*b**2*c**4*d**3*x
**7/27783 - 374*b**2*c**3*d**3*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/3969 + 3
*b**2*c**2*d**3*x**5*asinh(c*x)**2/5 + 4198*b**2*c**2*d**3*x**5/165375 - 41
98*b**2*c*d**3*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/33075 + b**2*d**3*x**3*a
sinh(c*x)**2/3 + 5258*b**2*d**3*x**3/297675 - 5258*b**2*d**3*x**2*sqrt(c**2
*x**2 + 1)*asinh(c*x)/(99225*c) - 10516*b**2*d**3*x/(99225*c**2) + 10516*b*
*2*d**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/(99225*c**3), Ne(c, 0)), (a**2*d**3*
x**3/3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 922 vs. $2(340) = 680$.

Time = 0.25 (sec) , antiderivative size = 922, normalized size of antiderivative = 2.41

$$\int x^2(d+c^2dx^2)^3(a+b\operatorname{arcsinh}(cx))^2\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*c^6*d^3*x^9*arcsinh(c*x)^2 + 1/9*a^2*c^6*d^3*x^9 + 3/7*b^2*c^4*d^3*
x^7*arcsinh(c*x)^2 + 3/7*a^2*c^4*d^3*x^7 + 3/5*b^2*c^2*d^3*x^5*arcsinh(c*x)
^2 + 2/2835*(315*x^9*arcsinh(c*x) - (35*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt
(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)
*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c)*a*b*c^6*d^3 - 2/893025*(315*(35*s
qrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2 +
 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c*
arcsinh(c*x) - (1225*c^8*x^9 - 1800*c^6*x^7 + 3024*c^4*x^5 - 6720*c^2*x^3 +
 40320*x)/c^8)*b^2*c^6*d^3 + 3/5*a^2*c^2*d^3*x^5 + 6/245*(35*x^7*arcsinh(c*
x) - (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^
2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*a*b*c^4*d^3 - 2/8575*(105
*(5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^
2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsinh(c*x) - (75*c^6*x^7 - 1
26*c^4*x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*c^4*d^3 + 1/3*b^2*d^3*x^3*arcsi
nh(c*x)^2 + 2/25*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sq
rt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^2*d^3 - 2/375*(
15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*
x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^2*c^
2*d^3 + 1/3*a^2*d^3*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^
2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d^3 - 2/27*(3*c*(sqrt(c^2*x^2 + 1)*x^
2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*d^
3
```

Giac [F(-2)]

Exception generated.

$$\int x^2 (d + c^2dx^2)^3 (a + b\text{arcsinh}(cx))^2 \, dx = \text{Exception raised: RuntimeError}$$

[In] integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")

[Out] Exception raised: RuntimeError >> an error occurred running a Giac command: INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int x^2 (d + c^2dx^2)^3 (a + b\text{arcsinh}(cx))^2 \, dx = \int x^2 (a + b \operatorname{asinh}(c x))^2 (d c^2 x^2 + d)^3 \, dx$$

[In] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3,x)

[Out] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3, x)

3.219 $\int x(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2 dx$

Optimal result

Integrand size = 24, antiderivative size = 261

$$\begin{aligned}\int x(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2 dx &= \frac{175b^2d^3x^2}{3072}+\frac{35b^2c^2d^3x^4}{3072} \\ &+ \frac{7b^2d^3(1+c^2x^2)^3}{1152c^2}+\frac{b^2d^3(1+c^2x^2)^4}{256c^2} \\ &- \frac{35bd^3x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{512c} \\ &- \frac{35bd^3x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{768c} \\ &- \frac{7bd^3x(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{192c} \\ &- \frac{bd^3x(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{32c} \\ &- \frac{35d^3(a+b\text{arcsinh}(cx))^2}{1024c^2} \\ &+ \frac{d^3(1+c^2x^2)^4(a+b\text{arcsinh}(cx))^2}{8c^2}\end{aligned}$$

```
[Out] 175/3072*b^2*d^3*x^2+35/3072*b^2*c^2*d^3*x^4+7/1152*b^2*d^3*(c^2*x^2+1)^3/c
^2+1/256*b^2*d^3*(c^2*x^2+1)^4/c^2-35/768*b*d^3*x*(c^2*x^2+1)^(3/2)*(a+b*ar
csinh(c*x))/c-7/192*b*d^3*x*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))/c-1/32*b*d
^3*x*(c^2*x^2+1)^(7/2)*(a+b*arcsinh(c*x))/c-35/1024*d^3*(a+b*arcsinh(c*x))^
2/c^2+1/8*d^3*(c^2*x^2+1)^4*(a+b*arcsinh(c*x))^2/c^2-35/512*b*d^3*x*(a+b*ar
csinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5798, 5786, 5785, 5783, 30, 14, 267}

$$\int x\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2\,dx = -\frac{bd^3x(c^2x^2+1)^{7/2}(a+b\text{arcsinh}(cx))}{32c} - \frac{7bd^3x(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))}{192c} - \frac{35bd^3x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{768c} - \frac{35bd^3x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{512c} + \frac{d^3(c^2x^2+1)^4(a+b\text{arcsinh}(cx))^2}{8c^2} - \frac{35d^3(a+b\text{arcsinh}(cx))^2}{1024c^2} + \frac{35b^2c^2d^3x^4}{3072} + \frac{b^2d^3(c^2x^2+1)^4}{256c^2} + \frac{7b^2d^3(c^2x^2+1)^3}{1152c^2} + \frac{175b^2d^3x^2}{3072}$$

```
[In] Int[x*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (175*b^2*d^3*x^2)/3072 + (35*b^2*c^2*d^3*x^4)/3072 + (7*b^2*d^3*(1 + c^2*x^
2)^3)/(1152*c^2) + (b^2*d^3*(1 + c^2*x^2)^4)/(256*c^2) - (35*b*d^3*x*Sqrt[1
 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(512*c) - (35*b*d^3*x*(1 + c^2*x^2)^(3/2)
*(a + b*ArcSinh[c*x]))/(768*c) - (7*b*d^3*x*(1 + c^2*x^2)^(5/2)*(a + b*ArcS
inh[c*x]))/(192*c) - (b*d^3*x*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(32
*c) - (35*d^3*(a + b*ArcSinh[c*x])^2)/(1024*c^2) + (d^3*(1 + c^2*x^2)^4*(a
+ b*ArcSinh[c*x])^2)/(8*c^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{d^3(1+c^2x^2)^4\,(a+b\text{arcsinh}(cx))^2}{8c^2} - \frac{(bd^3)\int (1+c^2x^2)^{7/2}\,(a+b\text{arcsinh}(cx))\,dx}{4c}\\ &= -\frac{bd^3x(1+c^2x^2)^{7/2}\,(a+b\text{arcsinh}(cx))}{32c} + \frac{d^3(1+c^2x^2)^4\,(a+b\text{arcsinh}(cx))^2}{8c^2}\\ &\quad + \frac{1}{32}(b^2d^3)\int x\left(1+c^2x^2\right)^3\,dx - \frac{(7bd^3)\int (1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))\,dx}{32c}\end{aligned}$$

$$=\frac{b^2d^3(1+c^2x^2)^4}{256c^2}-\frac{7bd^3x(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{192c}$$
$$-\frac{bd^3x(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{32c}+\frac{d^3(1+c^2x^2)^4(a+b\text{arcsinh}(cx))^2}{8c^2}$$
$$+\frac{1}{192}(7b^2d^3)\int x(1+c^2x^2)^2\,dx-\frac{(35bd^3)\int(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\,dx}{192c}$$
$$=\frac{7b^2d^3(1+c^2x^2)^3}{1152c^2}+\frac{b^2d^3(1+c^2x^2)^4}{256c^2}-\frac{35bd^3x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{768c}$$
$$-\frac{7bd^3x(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{192c}$$
$$-\frac{bd^3x(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{32c}+\frac{d^3(1+c^2x^2)^4(a+b\text{arcsinh}(cx))^2}{8c^2}$$
$$+\frac{1}{768}(35b^2d^3)\int x(1+c^2x^2)\,dx-\frac{(35bd^3)\int\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx}{256c}$$
$$=\frac{7b^2d^3(1+c^2x^2)^3}{1152c^2}+\frac{b^2d^3(1+c^2x^2)^4}{256c^2}-\frac{35bd^3x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{512c}$$
$$-\frac{35bd^3x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{768c}-\frac{7bd^3x(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{192c}$$
$$-\frac{bd^3x(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{32c}+\frac{d^3(1+c^2x^2)^4(a+b\text{arcsinh}(cx))^2}{8c^2}$$
$$+\frac{1}{768}(35b^2d^3)\int(x+c^2x^3)\,dx+\frac{1}{512}(35b^2d^3)\int x\,dx-\frac{(35bd^3)\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{512c}$$
$$=\frac{175b^2d^3x^2}{3072}+\frac{35b^2c^2d^3x^4}{3072}+\frac{7b^2d^3(1+c^2x^2)^3}{1152c^2}+\frac{b^2d^3(1+c^2x^2)^4}{256c^2}$$
$$-\frac{35bd^3x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{512c}-\frac{35bd^3x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{768c}$$
$$-\frac{7bd^3x(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{192c}-\frac{bd^3x(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{32c}$$
$$-\frac{35d^3(a+b\text{arcsinh}(cx))^2}{1024c^2}+\frac{d^3(1+c^2x^2)^4(a+b\text{arcsinh}(cx))^2}{8c^2}$$

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.98

$$\int x\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{d^3\left(cx\left(1152a^2cx(4+6c^2x^2+4c^4x^4+c^6x^6)+b^2cx(837+489c^2x^2+200c^4x^4+36c^6x^6)-6ab\sqrt{1+c^2x^2}(27\right.\right.}{}$$

[In] Integrate[x*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]

[Out] (d^3*(c*x*(1152*a^2*c*x*(4 + 6*c^2*x^2 + 4*c^4*x^4 + c^6*x^6) + b^2*c*x*(837 + 489*c^2*x^2 + 200*c^4*x^4 + 36*c^6*x^6) - 6*a*b*Sqrt[1 + c^2*x^2]*(279 + 326*c^2*x^2 + 200*c^4*x^4 + 48*c^6*x^6)) + 6*b*(-(b*c*x*Sqrt[1 + c^2*x^2]*(279 + 326*c^2*x^2 + 200*c^4*x^4 + 48*c^6*x^6)) + 3*a*(93 + 512*c^2*x^2 + 768*c^4*x^4 + 512*c^6*x^6 + 128*c^8*x^8))*ArcSinh[c*x] + 9*b^2*(93 + 512*c^2*x^2 + 768*c^4*x^4 + 512*c^6*x^6 + 128*c^8*x^8)*ArcSinh[c*x]^2))/(9216*c^2)

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{\frac{d^3a^2\left(c^2x^2+1\right)^4}{8}+d^3b^2\left(\frac{\text{arcsinh}(cx)^2\left(c^2x^2+1\right)^4}{8}-\frac{\text{arcsinh}(cx)cx\left(c^2x^2+1\right)^{\frac{7}{2}}}{32}-\frac{7\,\text{arcsinh}(cx)cx\left(c^2x^2+1\right)^{\frac{5}{2}}}{192}-\frac{35\,\text{arcsinh}(cx)cx\left(c^2x\right.}{768}\right.}{}$
default	$\frac{\frac{d^3a^2\left(c^2x^2+1\right)^4}{8}+d^3b^2\left(\frac{\text{arcsinh}(cx)^2\left(c^2x^2+1\right)^4}{8}-\frac{\text{arcsinh}(cx)cx\left(c^2x^2+1\right)^{\frac{7}{2}}}{32}-\frac{7\,\text{arcsinh}(cx)cx\left(c^2x^2+1\right)^{\frac{5}{2}}}{192}-\frac{35\,\text{arcsinh}(cx)cx\left(c^2x\right.}{768}\right.}{}$
parts	$\frac{d^3a^2\left(c^2x^2+1\right)^4}{8c^2}+\frac{d^3b^2\left(\frac{\text{arcsinh}(cx)^2\left(c^2x^2+1\right)^4}{8}-\frac{\text{arcsinh}(cx)cx\left(c^2x^2+1\right)^{\frac{7}{2}}}{32}-\frac{7\,\text{arcsinh}(cx)cx\left(c^2x^2+1\right)^{\frac{5}{2}}}{192}-\frac{35\,\text{arcsinh}(cx)cx\left(\right.}{768}\right.}{}$

[In] int(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)

[Out] 1/c^2*(1/8*d^3*a^2*(c^2*x^2+1)^4+d^3*b^2*(1/8*arcsinh(c*x)^2*(c^2*x^2+1)^4-1/32*arcsinh(c*x)*c*x*(c^2*x^2+1)^(7/2)-7/192*arcsinh(c*x)*c*x*(c^2*x^2+1)^(5/2)-35/768*arcsinh(c*x)*c*x*(c^2*x^2+1)^(3/2)-35/512*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)-35/1024*arcsinh(c*x)^2+1/256*(c^2*x^2+1)^4+7/1152*(c^2*x^2+1)^3+35/3072*(c^2*x^2+1)^2+35/1024*c^2*x^2+35/1024)+2*d^3*a*b*(1/8*arcsinh(c*x)*c^8*x^8+1/2*arcsinh(c*x)*c^6*x^6+3/4*arcsinh(c*x)*c^4*x^4+1/2*arcsinh(c*x)*c^2*x^2+93/1024*arcsinh(c*x)-1/64*c*x*(c^2*x^2+1)^(7/2)-7/384*c*x*(c^2*x^2+1)^(5/2)-35/1536*c*x*(c^2*x^2+1)^(3/2)-35/1024*c*x*(c^2*x^2+1)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.47

$$\int x(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{36\,(32\,a^2+b^2)c^8d^3x^8+8\,(576\,a^2+25\,b^2)c^6d^3x^6+3\,(2304\,a^2+163\,b^2)c^4d^3x^4+9\,(512\,a^2+93\,b^2)c^2d^3x^2+\cdots}{}$$

```
[In] integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/9216*(36*(32*a^2 + b^2)*c^8*d^3*x^8 + 8*(576*a^2 + 25*b^2)*c^6*d^3*x^6 +
3*(2304*a^2 + 163*b^2)*c^4*d^3*x^4 + 9*(512*a^2 + 93*b^2)*c^2*d^3*x^2 + 9*(
128*b^2*c^8*d^3*x^8 + 512*b^2*c^6*d^3*x^6 + 768*b^2*c^4*d^3*x^4 + 512*b^2*c
^2*d^3*x^2 + 93*b^2*d^3)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 6*(384*a*b*c^8*d^
3*x^8 + 1536*a*b*c^6*d^3*x^6 + 2304*a*b*c^4*d^3*x^4 + 1536*a*b*c^2*d^3*x^2
+ 279*a*b*d^3 - (48*b^2*c^7*d^3*x^7 + 200*b^2*c^5*d^3*x^5 + 326*b^2*c^3*d^3
*x^3 + 279*b^2*c*d^3*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 6
*(48*a*b*c^7*d^3*x^7 + 200*a*b*c^5*d^3*x^5 + 326*a*b*c^3*d^3*x^3 + 279*a*b*
c*d^3*x)*sqrt(c^2*x^2 + 1))/c^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. 2(252) = 504.

Time = 1.39 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.20

$$\int x(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\begin{cases}\frac{a^2c^6d^3x^8}{8}+\frac{a^2c^4d^3x^6}{2}+\frac{3a^2c^2d^3x^4}{4}+\frac{a^2d^3x^2}{2}+\frac{abc^6d^3x^8\operatorname{asinh}(cx)}{4}-\frac{abc^5d^3x^7\sqrt{c^2x^2+1}}{32}+abc^4d^3x^6\operatorname{asinh}(cx)-\frac{25abc^3d\cdots}{}\\ \frac{a^2d^3x^2}{2}\end{cases}$$

```
[In] integrate(x*(c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**6*d**3*x**8/8 + a**2*c**4*d**3*x**6/2 + 3*a**2*c**2*d**3
*x**4/4 + a**2*d**3*x**2/2 + a*b*c**6*d**3*x**8*asinh(c*x)/4 - a*b*c**5*d**
3*x**7*sqrt(c**2*x**2 + 1)/32 + a*b*c**4*d**3*x**6*asinh(c*x) - 25*a*b*c**3
*d**3*x**5*sqrt(c**2*x**2 + 1)/192 + 3*a*b*c**2*d**3*x**4*asinh(c*x)/2 - 16
3*a*b*c*d**3*x**3*sqrt(c**2*x**2 + 1)/768 + a*b*d**3*x**2*asinh(c*x) - 93*a
*b*d**3*x*sqrt(c**2*x**2 + 1)/(512*c) + 93*a*b*d**3*asinh(c*x)/(512*c**2) +
 b**2*c**6*d**3*x**8*asinh(c*x)**2/8 + b**2*c**6*d**3*x**8/256 - b**2*c**5*
d**3*x**7*sqrt(c**2*x**2 + 1)*asinh(c*x)/32 + b**2*c**4*d**3*x**6*asinh(c*x
)**2/2 + 25*b**2*c**4*d**3*x**6/1152 - 25*b**2*c**3*d**3*x**5*sqrt(c**2*x**
```

```
2 + 1)*asinh(c*x)/192 + 3*b**2*c**2*d**3*x**4*asinh(c*x)**2/4 + 163*b**2*c*
*2*d**3*x**4/3072 - 163*b**2*c*d**3*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/768
 + b**2*d**3*x**2*asinh(c*x)**2/2 + 93*b**2*d**3*x**2/1024 - 93*b**2*d**3*x
*sqrt(c**2*x**2 + 1)*asinh(c*x)/(512*c) + 93*b**2*d**3*asinh(c*x)**2/(1024*
c**2), Ne(c, 0)), (a**2*d**3*x**2/2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(233) = 466$.

Time = 0.25 (sec) , antiderivative size = 925, normalized size of antiderivative = 3.54

$$\int x(d + c^2dx^2)^3 (a + b\text{arcsinh}(cx))^2 \, dx = \text{Too large to display}$$

```
[In] integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*c^6*d^3*x^8*arcsinh(c*x)^2 + 1/8*a^2*c^6*d^3*x^8 + 1/2*b^2*c^4*d^3*
x^6*arcsinh(c*x)^2 + 1/2*a^2*c^4*d^3*x^6 + 3/4*b^2*c^2*d^3*x^4*arcsinh(c*x)
^2 + 1/1536*(384*x^8*arcsinh(c*x) - (48*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt
(c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1
)*x/c^8 + 105*arcsinh(c*x)/c^9)*c)*a*b*c^6*d^3 + 1/9216*((36*x^8/c^2 - 56*x
^6/c^4 + 105*x^4/c^6 - 315*x^2/c^8 + 315*log(c*x + sqrt(c^2*x^2 + 1))^2/c^1
0)*c^2 - 6*(48*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 7
0*sqrt(c^2*x^2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/c^8 + 105*arcsinh(c*x
)/c^9)*c*arcsinh(c*x))*b^2*c^6*d^3 + 3/4*a^2*c^2*d^3*x^4 + 1/48*(48*x^6*arc
sinh(c*x) - (8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 1
5*sqrt(c^2*x^2 + 1)*x/c^6 - 15*arcsinh(c*x)/c^7)*c)*a*b*c^4*d^3 + 1/288*((8
*x^6/c^2 - 15*x^4/c^4 + 45*x^2/c^6 - 45*log(c*x + sqrt(c^2*x^2 + 1))^2/c^8)
*c^2 - 6*(8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*s
qrt(c^2*x^2 + 1)*x/c^6 - 15*arcsinh(c*x)/c^7)*c*arcsinh(c*x))*b^2*c^4*d^3 +
 1/2*b^2*d^3*x^2*arcsinh(c*x)^2 + 3/16*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^
2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*a*b*c^2
*d^3 + 3/32*((x^4/c^2 - 3*x^2/c^4 + 3*log(c*x + sqrt(c^2*x^2 + 1))^2/c^6)*c
^2 - 2*(2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh
(c*x)/c^5)*c*arcsinh(c*x))*b^2*c^2*d^3 + 1/2*a^2*d^3*x^2 + 1/2*(2*x^2*arcsi
nh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3))*a*b*d^3 + 1/4*(c^
2*(x^2/c^2 - log(c*x + sqrt(c^2*x^2 + 1))^2/c^4) - 2*c*(sqrt(c^2*x^2 + 1)*x
/c^2 - arcsinh(c*x)/c^3)*arcsinh(c*x))*b^2*d^3
```

Giac [F(-2)]

Exception generated.

$$\int x(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2\,dx = \int x\,(a+b\,\text{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^3\,dx$$

```
[In] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3,x)
```

```
[Out] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3, x)
```

3.220 $\int (d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 291

$$\begin{aligned}
&\int (d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2 \, dx \\
&= \frac{4322b^2d^3x}{3675} + \frac{1514b^2c^2d^3x^3}{11025} + \frac{234b^2c^4d^3x^5}{6125} + \frac{2}{343}b^2c^6d^3x^7 \\
&\quad - \frac{32bd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{35c} - \frac{16bd^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{105c} \\
&\quad - \frac{12bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{175c} - \frac{2bd^3(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{49c} \\
&\quad + \frac{16}{35}d^3x(a+b\text{arcsinh}(cx))^2 + \frac{8}{35}d^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 + \frac{6}{35}d^3x(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2 + \frac{1}{7}
\end{aligned}$$

```
[Out] 4322/3675*b^2*d^3*x+1514/11025*b^2*c^2*d^3*x^3+234/6125*b^2*c^4*d^3*x^5+2/3
43*b^2*c^6*d^3*x^7-16/105*b*d^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c-12/1
75*b*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))/c-2/49*b*d^3*(c^2*x^2+1)^(7/2
)*(a+b*arcsinh(c*x))/c+16/35*d^3*x*(a+b*arcsinh(c*x))^2+8/35*d^3*x*(c^2*x^2
+1)*(a+b*arcsinh(c*x))^2+6/35*d^3*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2+1/7*
d^3*x*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2-32/35*b*d^3*(a+b*arcsinh(c*x))*(c^
2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5786, 5772, 5798, 8, 200}

$$\int (d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2 dx$$
$$= \frac{1}{7}d^3x(c^2x^2+1)^3 (a+b\text{arcsinh}(cx))^2 + \frac{6}{35}d^3x(c^2x^2+1)^2 (a+b\text{arcsinh}(cx))^2$$
$$+ \frac{8}{35}d^3x(c^2x^2+1) (a+b\text{arcsinh}(cx))^2 - \frac{2bd^3(c^2x^2+1)^{7/2} (a+b\text{arcsinh}(cx))}{49c}$$
$$- \frac{12bd^3(c^2x^2+1)^{5/2} (a+b\text{arcsinh}(cx))}{175c}$$
$$- \frac{16bd^3(c^2x^2+1)^{3/2} (a+b\text{arcsinh}(cx))}{105c} - \frac{32bd^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{35c}$$
$$+ \frac{16}{35}d^3x(a+b\text{arcsinh}(cx))^2 + \frac{2}{343}b^2c^6d^3x^7 + \frac{234b^2c^4d^3x^5}{6125} + \frac{1514b^2c^2d^3x^3}{11025} + \frac{4322b^2d^3x}{3675}$$

```
[In] Int[(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (4322*b^2*d^3*x)/3675 + (1514*b^2*c^2*d^3*x^3)/11025 + (234*b^2*c^4*d^3*x^5
)/6125 + (2*b^2*c^6*d^3*x^7)/343 - (32*b*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSi
nh[c*x]))/(35*c) - (16*b*d^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(105
*c) - (12*b*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(175*c) - (2*b*d^
3*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(49*c) + (16*d^3*x*(a + b*ArcSi
nh[c*x])^2)/35 + (8*d^3*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/35 + (6*d^3
*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/35 + (d^3*x*(1 + c^2*x^2)^3*(a +
 b*ArcSinh[c*x])^2)/7
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{7}d^3x\left(1+c^2x^2\right)^3(a+b\operatorname{arcsinh}(cx))^2+\frac{1}{7}(6d)\int\left(d+c^2dx^2\right)^2(a+b\operatorname{arcsinh}(cx))^2\,dx \\
&\quad -\frac{1}{7}\left(2bcd^3\right)\int x\left(1+c^2x^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx \\
&= -\frac{2bd^3\left(1+c^2x^2\right)^{7/2}(a+b\operatorname{arcsinh}(cx))}{49c}+\frac{6}{35}d^3x\left(1+c^2x^2\right)^2(a+b\operatorname{arcsinh}(cx))^2 \\
&\quad +\frac{1}{7}d^3x\left(1+c^2x^2\right)^3(a+b\operatorname{arcsinh}(cx))^2 \\
&\quad +\frac{1}{35}\left(24d^2\right)\int\left(d+c^2dx^2\right)(a+b\operatorname{arcsinh}(cx))^2\,dx+\frac{1}{49}\left(2b^2d^3\right)\int\left(1+c^2x^2\right)^3\,dx \\
&\quad -\frac{1}{35}\left(12bcd^3\right)\int x\left(1+c^2x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx \\
&= -\frac{12bd^3\left(1+c^2x^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))}{175c}-\frac{2bd^3\left(1+c^2x^2\right)^{7/2}(a+b\operatorname{arcsinh}(cx))}{49c} \\
&\quad +\frac{8}{35}d^3x\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))^2+\frac{6}{35}d^3x\left(1+c^2x^2\right)^2(a+b\operatorname{arcsinh}(cx))^2 \\
&\quad +\frac{1}{7}d^3x\left(1+c^2x^2\right)^3(a+b\operatorname{arcsinh}(cx))^2+\frac{1}{35}\left(16d^3\right)\int(a+b\operatorname{arcsinh}(cx))^2\,dx \\
&\quad +\frac{1}{49}\left(2b^2d^3\right)\int\left(1+3c^2x^2+3c^4x^4+c^6x^6\right)\,dx+\frac{1}{175}\left(12b^2d^3\right)\int\left(1+c^2x^2\right)^2\,dx \\
&\quad -\frac{1}{35}\left(16bcd^3\right)\int x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx
\end{aligned}$$

$$= \frac{2}{49}b^2d^3x + \frac{2}{49}b^2c^2d^3x^3 + \frac{6}{245}b^2c^4d^3x^5 + \frac{2}{343}b^2c^6d^3x^7 - \frac{16bd^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{105c} - \frac{12bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{175c} - \frac{2bd^3(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{49c} + \frac{16}{35}d^3x(a+b\text{arcsinh}(cx))^2 + \frac{8}{35}d^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 + \frac{6}{35}d^3x(1+c^2x^2)^2(a+b\text{arcsinh}($$

$$= \frac{962b^2d^3x}{3675} + \frac{1514b^2c^2d^3x^3}{11025} + \frac{234b^2c^4d^3x^5}{6125} + \frac{2}{343}b^2c^6d^3x^7 - \frac{32bd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{35c} - \frac{16bd^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{105c} - \frac{12bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{175c} - \frac{2bd^3(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{49c} + \frac{16}{35}d^3x(a+b\text{arcsinh}(cx))^2 + \frac{8}{35}d^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 + \frac{6}{35}d^3x(1+c^2x^2)^2(a+b\text{arcsinh}($$

$$= \frac{4322b^2d^3x}{3675} + \frac{1514b^2c^2d^3x^3}{11025} + \frac{234b^2c^4d^3x^5}{6125} + \frac{2}{343}b^2c^6d^3x^7 - \frac{32bd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{35c} - \frac{16bd^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{105c} - \frac{12bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{175c} - \frac{2bd^3(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{49c} + \frac{16}{35}d^3x(a+b\text{arcsinh}(cx))^2 + \frac{8}{35}d^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 + \frac{6}{35}d^3x(1+c^2x^2)^2(a+b\text{arcsinh}($$

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.82

$$\int (d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2 dx = \frac{d^3(11025a^2cx(35+35c^2x^2+21c^4x^4+5c^6x^6) - 210ab\sqrt{1+c^2x^2}(2161+757c^2x^2+351c^4x^4+75c^6x^6) + }{}$$

```
[In] Integrate[(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^3*(11025*a^2*c*x*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) - 210*a*b*Sq
rt[1 + c^2*x^2]*(2161 + 757*c^2*x^2 + 351*c^4*x^4 + 75*c^6*x^6) + 2*b^2*c*x
*(226905 + 26495*c^2*x^2 + 7371*c^4*x^4 + 1125*c^6*x^6) - 210*b*(-105*a*c*x
*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) + b*Sqrt[1 + c^2*x^2]*(2161 + 7
57*c^2*x^2 + 351*c^4*x^4 + 75*c^6*x^6))*ArcSinh[c*x] + 11025*b^2*c*x*(35 +
35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6)*ArcSinh[c*x]^2))/(385875*c)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{d^3a^2\left(\frac{1}{7}c^7x^7+\frac{3}{5}c^5x^5+c^3x^3+cx\right)+d^3b^2\left(\frac{16\,\mathrm{arcsinh}(cx)^2xc}{35}+\frac{\mathrm{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^3}{7}+\frac{6\,\mathrm{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{35}+\frac{8\,\mathrm{arcsinh}(}{}\right.}{}$
default	$\frac{d^3a^2\left(\frac{1}{7}c^7x^7+\frac{3}{5}c^5x^5+c^3x^3+cx\right)+d^3b^2\left(\frac{16\,\mathrm{arcsinh}(cx)^2xc}{35}+\frac{\mathrm{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^3}{7}+\frac{6\,\mathrm{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{35}+\frac{8\,\mathrm{arcsinh}(}{}\right.}{}$
parts	$d^3a^2\left(\frac{1}{7}c^6x^7+\frac{3}{5}c^4x^5+x^3c^2+x\right)+\frac{d^3b^2\left(\frac{16\,\mathrm{arcsinh}(cx)^2xc}{35}+\frac{\mathrm{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^3}{7}+\frac{6\,\mathrm{arcsinh}(cx)^2cx\left(c^2x^2+\right.}{35}\right.}{}$

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(d^3*a^2*(1/7*c^7*x^7+3/5*c^5*x^5+c^3*x^3+c*x)+d^3*b^2*(16/35*arcsinh(c
*x)^2*x*c+1/7*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^3+6/35*arcsinh(c*x)^2*c*x*(c^2
*x^2+1)^2+8/35*arcsinh(c*x)^2*c*x*(c^2*x^2+1)-32/35*arcsinh(c*x)*(c^2*x^2+1
)^(1/2)+413312/385875*c*x-2/49*arcsinh(c*x)*(c^2*x^2+1)^(7/2)+2/343*c*x*(c^
2*x^2+1)^3+888/42875*c*x*(c^2*x^2+1)^2+30256/385875*c*x*(c^2*x^2+1)-12/175*
arcsinh(c*x)*(c^2*x^2+1)^(5/2)-16/105*arcsinh(c*x)*(c^2*x^2+1)^(3/2))+2*d^3
*a*b*(1/7*arcsinh(c*x)*c^7*x^7+3/5*arcsinh(c*x)*c^5*x^5+arcsinh(c*x)*c^3*x^
3+arcsinh(c*x)*c*x-2161/3675*(c^2*x^2+1)^(1/2)-1/49*c^6*x^6*(c^2*x^2+1)^(1/
2)-117/1225*c^4*x^4*(c^2*x^2+1)^(1/2)-757/3675*c^2*x^2*(c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.22

$$\int \left(d+c^2dx^2\right)^3\left(a+b\,\mathrm{arcsinh}(cx)\right)^2dx$$
$$=\frac{1125\,(49\,a^2+2\,b^2)c^7d^3x^7+189\,(1225\,a^2+78\,b^2)c^5d^3x^5+35\,(11025\,a^2+1514\,b^2)c^3d^3x^3+105\,(3675\,a^2+4}{}$$

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/385875*(1125*(49*a^2 + 2*b^2)*c^7*d^3*x^7 + 189*(1225*a^2 + 78*b^2)*c^5*d
^3*x^5 + 35*(11025*a^2 + 1514*b^2)*c^3*d^3*x^3 + 105*(3675*a^2 + 4322*b^2)*
c*d^3*x + 11025*(5*b^2*c^7*d^3*x^7 + 21*b^2*c^5*d^3*x^5 + 35*b^2*c^3*d^3*x^
3 + 35*b^2*c*d^3*x)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 210*(525*a*b*c^7*d^3*x
^7 + 2205*a*b*c^5*d^3*x^5 + 3675*a*b*c^3*d^3*x^3 + 3675*a*b*c*d^3*x - (75*b
^2*c^6*d^3*x^6 + 351*b^2*c^4*d^3*x^4 + 757*b^2*c^2*d^3*x^2 + 2161*b^2*d^3)*
```

```
sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 210*(75*a*b*c^6*d^3*x^6 +
 351*a*b*c^4*d^3*x^4 + 757*a*b*c^2*d^3*x^2 + 2161*a*b*d^3)*sqrt(c^2*x^2 + 1
))/c
```

Sympy [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.80

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))^2\,dx$$

$$= \begin{cases} \frac{a^2c^6d^3x^7}{7} + \frac{3a^2c^4d^3x^5}{5} + a^2c^2d^3x^3 + a^2d^3x + \frac{2abc^6d^3x^7\operatorname{asinh}(cx)}{7} - \frac{2abc^5d^3x^6\sqrt{c^2x^2+1}}{49} + \frac{6abc^4d^3x^5\operatorname{asinh}(cx)}{5} - \frac{234abc^3d}{} \\ a^2d^3x \end{cases}$$

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**6*d**3*x**7/7 + 3*a**2*c**4*d**3*x**5/5 + a**2*c**2*d**3
*x**3 + a**2*d**3*x + 2*a*b*c**6*d**3*x**7*asinh(c*x)/7 - 2*a*b*c**5*d**3*x
**6*sqrt(c**2*x**2 + 1)/49 + 6*a*b*c**4*d**3*x**5*asinh(c*x)/5 - 234*a*b*c*
*3*d**3*x**4*sqrt(c**2*x**2 + 1)/1225 + 2*a*b*c**2*d**3*x**3*asinh(c*x) - 1
514*a*b*c*d**3*x**2*sqrt(c**2*x**2 + 1)/3675 + 2*a*b*d**3*x*asinh(c*x) - 43
22*a*b*d**3*sqrt(c**2*x**2 + 1)/(3675*c) + b**2*c**6*d**3*x**7*asinh(c*x)**
2/7 + 2*b**2*c**6*d**3*x**7/343 - 2*b**2*c**5*d**3*x**6*sqrt(c**2*x**2 + 1)
*asinh(c*x)/49 + 3*b**2*c**4*d**3*x**5*asinh(c*x)**2/5 + 234*b**2*c**4*d**3
*x**5/6125 - 234*b**2*c**3*d**3*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/1225 +
b**2*c**2*d**3*x**3*asinh(c*x)**2 + 1514*b**2*c**2*d**3*x**3/11025 - 1514*b
**2*c*d**3*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/3675 + b**2*d**3*x*asinh(c*x
)**2 + 4322*b**2*d**3*x/3675 - 4322*b**2*d**3*sqrt(c**2*x**2 + 1)*asinh(c*x
)/(3675*c), Ne(c, 0)), (a**2*d**3*x, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 712 vs. 2(259) = 518.

Time = 0.22 (sec) , antiderivative size = 712, normalized size of antiderivative = 2.45

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{1}{7}b^2c^6d^3x^7\operatorname{arsinh}(cx)^2 + \frac{1}{7}a^2c^6d^3x^7 + \frac{3}{5}b^2c^4d^3x^5\operatorname{arsinh}(cx)^2 + \frac{3}{5}a^2c^4d^3x^5$$
$$+ \frac{2}{245}\left(35\,x^7\operatorname{arsinh}(cx) - \left(\frac{5\sqrt{c^2x^2+1}x^6}{c^2} - \frac{6\sqrt{c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{c^2x^2+1}x^2}{c^6} - \frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\right)abc^6d$$
$$- \frac{2}{25725}\left(105\left(\frac{5\sqrt{c^2x^2+1}x^6}{c^2} - \frac{6\sqrt{c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{c^2x^2+1}x^2}{c^6} - \frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\operatorname{arsinh}(cx) - \frac{75\,c^6x}{}\right.$$
$$+ b^2c^2d^3x^3\operatorname{arsinh}(cx)^2$$
$$+ \frac{2}{25}\left(15\,x^5\operatorname{arsinh}(cx) - \left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)abc^4d^3$$
$$- \frac{2}{375}\left(15\left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\operatorname{arsinh}(cx) - \frac{9\,c^4x^5 - 20\,c^2x^3 + 120\,x}{c^4}\right)b^2$$
$$+ a^2c^2d^3x^3 + \frac{2}{3}\left(3\,x^3\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)abc^2d^3$$
$$- \frac{2}{9}\left(3\,c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\operatorname{arsinh}(cx) - \frac{c^2x^3 - 6\,x}{c^2}\right)b^2c^2d^3$$
$$+ b^2d^3x\operatorname{arsinh}(cx)^2 + 2\,b^2d^3\left(x - \frac{\sqrt{c^2x^2+1}\operatorname{arsinh}(cx)}{c}\right)$$
$$+ a^2d^3x + \frac{2\left(cx\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)abd^3}{c}$$

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*c^6*d^3*x^7*arcsinh(c*x)^2 + 1/7*a^2*c^6*d^3*x^7 + 3/5*b^2*c^4*d^3*
x^5*arcsinh(c*x)^2 + 3/5*a^2*c^4*d^3*x^5 + 2/245*(35*x^7*arcsinh(c*x) - (5*
sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 +
1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*a*b*c^6*d^3 - 2/25725*(105*(5*sqr
t(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*
x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsinh(c*x) - (75*c^6*x^7 - 126*c^4*
x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*c^6*d^3 + b^2*c^2*d^3*x^3*arcsinh(c*x)
^2 + 2/25*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*
x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^4*d^3 - 2/375*(15*(3*s
qrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1
)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^2*c^4*d^3 +
```

```
a^2*c^2*d^3*x^3 + 2/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 -
 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^2*d^3 - 2/9*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^
2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*c^2*d^
3 + b^2*d^3*x*arcsinh(c*x)^2 + 2*b^2*d^3*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x
)/c) + a^2*d^3*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*d^3/c
```

Giac [F(-2)]

Exception generated.

$$\int (d + c^2dx^2)^3 (a + b\text{arcsinh}(cx))^2 \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d + c^2dx^2)^3 (a + b\text{arcsinh}(cx))^2 \, dx = \int (a + b\,\text{asinh}(c\,x))^2 \left(d\,c^2\,x^2 + d\right)^3 \, dx$$

```
[In] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3,x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3, x)
```

3.221 $\int \frac{(d+c^2dx^2)^3(a+b\mathbf{arcsinh}(cx))^2}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 337

$$\begin{aligned}\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2}{x}\,dx &= \frac{71}{144}b^2c^2d^3x^2+\frac{7}{144}b^2c^4d^3x^4+\frac{1}{108}b^2d^3(1+c^2x^2)^3 \\ &-\frac{19}{24}bcd^3x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) \\ &-\frac{7}{36}bcd^3x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx)) \\ &-\frac{1}{18}bcd^3x(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx)) \\ &-\frac{19}{48}d^3(a+b\text{arcsinh}(cx))^2 \\ &+\frac{1}{2}d^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 \\ &+\frac{1}{4}d^3(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2 \\ &+\frac{1}{6}d^3(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2 \\ &+\frac{d^3(a+b\text{arcsinh}(cx))^3}{3b} \\ &+d^3(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right) \\ &-bd^3(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right) \\ &-\frac{1}{2}b^2d^3\,\text{PolyLog}\left(3,e^{-2\text{arcsinh}(cx)}\right)\end{aligned}$$

```
[Out] 71/144*b^2*c^2*d^3*x^2+7/144*b^2*c^4*d^3*x^4+1/108*b^2*d^3*(c^2*x^2+1)^3-7/
36*b*c*d^3*x*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))-1/18*b*c*d^3*x*(c^2*x^2+1
)^(5/2)*(a+b*arcsinh(c*x))-19/48*d^3*(a+b*arcsinh(c*x))^2+1/2*d^3*(c^2*x^2+
1)*(a+b*arcsinh(c*x))^2+1/4*d^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2+1/6*d^3*
```

```
(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2+1/3*d^3*(a+b*arcsinh(c*x))^3/b+d^3*(a+b*
arcsinh(c*x))^2*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)-b*d^3*(a+b*arcsinh(c*x))*
polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b^2*d^3*polylog(3,1/(c*x+(c^2*x^
2+1)^(1/2))^2)-19/24*b*c*d^3*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5808, 5775, 3797, 2221, 2611, 2320, 6724, 5785, 5783, 30, 5786, 14, 267}

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2}{x}\,dx = -\frac{1}{18}bcd^3x(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx)) - \frac{7}{36}bcd^3x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{19}{24}bcd^3x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx)) + \frac{1}{6}d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2 + \frac{1}{4}d^3(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2 + \frac{1}{2}d^3(c^2x^2+1)(a+b\text{arcsinh}(cx))^2 - bd^3\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)) + \frac{d^3(a+b\text{arcsinh}(cx))^3}{3b} - \frac{19}{48}d^3(a+b\text{arcsinh}(cx))^2 + d^3\log\left(1-e^{-2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2 - \frac{1}{2}b^2d^3\,\text{PolyLog}\left(3,e^{-2\text{arcsinh}(cx)}\right) + \frac{7}{144}b^2c^4d^3x^4 + \frac{71}{144}b^2c^2d^3x^2 + \frac{1}{108}b^2d^3(c^2x^2+1)^3$$

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (71*b^2*c^2*d^3*x^2)/144 + (7*b^2*c^4*d^3*x^4)/144 + (b^2*d^3*(1 + c^2*x^2)
^3)/108 - (19*b*c*d^3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/24 - (7*b*c
*d^3*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/36 - (b*c*d^3*x*(1 + c^2*x
^2)^(5/2)*(a + b*ArcSinh[c*x]))/18 - (19*d^3*(a + b*ArcSinh[c*x])^2)/48 + (
d^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/2 + (d^3*(1 + c^2*x^2)^2*(a + b*A
rcSinh[c*x])^2)/4 + (d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/6 + (d^3*(
a + b*ArcSinh[c*x])^3)/(3*b) + d^3*(a + b*ArcSinh[c*x])^2*Log[1 - E^(-2*Arc
Sinh[c*x])] - b*d^3*(a + b*ArcSinh[c*x])*PolyLog[2, E^(-2*ArcSinh[c*x])] -
(b^2*d^3*PolyLog[3, E^(-2*ArcSinh[c*x])])/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{1}{6}d^3\left(1+c^2x^2\right)^3(a+b\text{arcsinh}(cx))^2 + d\int\frac{\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2}{x}\,dx - \frac{1}{3}\left(bcd^3\right)\int\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx$$

$$= -\frac{1}{18}bcd^3x\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx)) + \frac{1}{4}d^3\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2 + \frac{1}{6}d^3\left(1+c^2x^2\right)^3(a+b\text{arcsinh}(cx))^2 + d^2\int\frac{\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))^2}{x}\,dx - \frac{1}{18}\left(5bcd^3\right)\int\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx - \frac{1}{2}\left(bcd^3\right)\int\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx + \frac{1}{18}\left(b^2c^2d^3\right)\int x\left(1+c^2x^2\right)^2\,dx$$

$$= \frac{1}{108}b^2d^3\left(1+c^2x^2\right)^3 - \frac{7}{36}bcd^3x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{1}{18}bcd^3x\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx)) + \frac{1}{2}d^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2 + \frac{1}{4}d^3\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2 + \frac{1}{6}d^3\left(1+c^2x^2\right)^3(a+b\text{arcsinh}(cx))^2 + d^3\int\frac{(a+b\text{arcsinh}(cx))^2}{x}\,dx - \frac{1}{24}\left(5bcd^3\right)\int\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx - \frac{1}{8}\left(3bcd^3\right)\int\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx - \left(bcd^3\right)\int\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx + \frac{1}{72}\left(5b^2c^2d^3\right)\int x\left(1+c^2x^2\right)\,dx + \frac{1}{8}\left(b^2c^2d^3\right)\int x\left(1+c^2x^2\right)\,dx$$

$$= \frac{1}{108} b^2 d^3 \left(1 + c^2 x^2\right)^3 - \frac{19}{24} b c d^3 x \sqrt{1 + c^2 x^2} (a + b \operatorname{arcsinh}(cx)) - \frac{7}{36} b c d^3 x \left(1 + c^2 x^2\right)^{3/2} (a + b \operatorname{arcsinh}(cx)) - \frac{1}{18} b c d^3 x \left(1 + c^2 x^2\right)^{5/2} (a + b \operatorname{arcsinh}(cx)) + \frac{1}{2} d^3 \left(1 + c^2 x^2\right) (a + b \operatorname{arcsinh}(cx))^2 + \frac{1}{4} d^3 \left(1 + c^2 x^2\right)^2 (a + b \operatorname{arcsinh}(cx))^2 + \frac{1}{6} d^3 \left(1 + c^2 x^2\right)^3 (a + b \operatorname{arcsinh}(cx))^2 - \frac{d^3 \operatorname{Subst}\left(\int x^2 \coth\left(\frac{a}{b} - \frac{x}{b}\right) dx, x, a + b \operatorname{arcsinh}(cx)\right)}{b} - \frac{1}{48} \left(5 b c d^3\right) \int \frac{a + b \operatorname{arcsinh}(cx)}{\sqrt{1 + c^2 x^2}} dx - \frac{1}{16} \left(3 b c d^3\right) \int \frac{a + b \operatorname{arcsinh}(cx)}{\sqrt{1 + c^2 x^2}} dx - \frac{1}{2} \left(b c d^3\right) \int \frac{a + b \operatorname{arcsinh}(cx)}{\sqrt{1 + c^2 x^2}} dx + \frac{1}{72} \left(5 b^2 c^2 d^3\right) \int \left(x + c^2 x^3\right) dx + \frac{1}{48} \left(5 b^2 c^2 d^3\right) \int x \, dx + \frac{1}{8} \left(b^2 c^2 d^3\right) \int \left(x + c^2 x^3\right) dx + \frac{1}{16} \left(3 b^2 c^2 d^3\right) \int x \, dx + \frac{1}{2} \left(b^2 c^2 d^3\right) \int x \, dx$$

$$= \frac{71}{144} b^2 c^2 d^3 x^2 + \frac{7}{144} b^2 c^4 d^3 x^4 + \frac{1}{108} b^2 d^3 \left(1 + c^2 x^2\right)^3 - \frac{19}{24} b c d^3 x \sqrt{1 + c^2 x^2} (a + b \operatorname{arcsinh}(cx)) - \frac{7}{36} b c d^3 x \left(1 + c^2 x^2\right)^{3/2} (a + b \operatorname{arcsinh}(cx)) - \frac{1}{18} b c d^3 x \left(1 + c^2 x^2\right)^{5/2} (a + b \operatorname{arcsinh}(cx)) - \frac{19}{48} d^3 (a + b \operatorname{arcsinh}(cx))^2 + \frac{1}{2} d^3 \left(1 + c^2 x^2\right) (a + b \operatorname{arcsinh}(cx))^2 + \frac{1}{4} d^3 \left(1 + c^2 x^2\right)^2 (a + b \operatorname{arcsinh}(cx))^2 + \frac{1}{6} d^3 \left(1 + c^2 x^2\right)^3 (a + b \operatorname{arcsinh}(cx))^2 + \frac{d^3 (a + b \operatorname{arcsinh}(cx))^3}{3b} + \frac{\left(2 d^3\right) \operatorname{Subst}\left(\int \frac{e^{2\left(\frac{a}{b} - \frac{x}{b}\right)} x^2}{1 - e^{2\left(\frac{a}{b} - \frac{x}{b}\right)}} dx, x, a + b \operatorname{arcsinh}(cx)\right)}{b}$$

$$= \frac{71}{144} b^2 c^2 d^3 x^2 + \frac{7}{144} b^2 c^4 d^3 x^4 + \frac{1}{108} b^2 d^3 \left(1 + c^2 x^2\right)^3 - \frac{19}{24} b c d^3 x \sqrt{1 + c^2 x^2} (a + b \operatorname{arcsinh}(cx)) - \frac{7}{36} b c d^3 x \left(1 + c^2 x^2\right)^{3/2} (a + b \operatorname{arcsinh}(cx)) - \frac{1}{18} b c d^3 x \left(1 + c^2 x^2\right)^{5/2} (a + b \operatorname{arcsinh}(cx)) - \frac{19}{48} d^3 (a + b \operatorname{arcsinh}(cx))^2 + \frac{1}{2} d^3 \left(1 + c^2 x^2\right) (a + b \operatorname{arcsinh}(cx))^2 + \frac{1}{4} d^3 \left(1 + c^2 x^2\right)^2 (a + b \operatorname{arcsinh}(cx))^2 + \frac{1}{6} d^3 \left(1 + c^2 x^2\right)^3 (a + b \operatorname{arcsinh}(cx))^2 + \frac{d^3 (a + b \operatorname{arcsinh}(cx))^3}{3b} + d^3 (a + b \operatorname{arcsinh}(cx))^2 \log\left(1 - e^{-2 \operatorname{arcsinh}(cx)}\right) - \left(2 d^3\right) \operatorname{Subst}\left(\int x \log\left(1 - e^{2\left(\frac{a}{b} - \frac{x}{b}\right)}\right) dx, x, a + b \operatorname{arcsinh}(cx)\right)$$

$$=\frac{71}{144}b^2c^2d^3x^2+\frac{7}{144}b^2c^4d^3x^4+\frac{1}{108}b^2d^3\left(1+c^2x^2\right)^3-\frac{19}{24}bcd^3x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))-\frac{7}{36}bcd^3x\left(1+c^2x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))-\frac{1}{18}bcd^3x\left(1+c^2x^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))-\frac{19}{48}d^3(a+b\operatorname{arcsinh}(cx))^2+\frac{1}{2}d^3\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))^2+\frac{1}{4}d^3\left(1+c^2x^2\right)^2(a+b\operatorname{arcsinh}(cx))^2+\frac{1}{6}d^3\left(1+c^2x^2\right)^3(a+b\operatorname{arcsinh}(cx))^2+\frac{d^3(a+b\operatorname{arcsinh}(cx))^3}{3b}+d^3(a+b\operatorname{arcsinh}(cx))^2\log\left(1-e^{-2\operatorname{arcsinh}(cx)}\right)-bd^3(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,e^{-2\operatorname{arcsinh}(cx)}\right)+\left(bd^3\right)\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(2,e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)dx,x,a+b\operatorname{arcsinh}(cx)\right)$$

$$=\frac{71}{144}b^2c^2d^3x^2+\frac{7}{144}b^2c^4d^3x^4+\frac{1}{108}b^2d^3\left(1+c^2x^2\right)^3-\frac{19}{24}bcd^3x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))-\frac{7}{36}bcd^3x\left(1+c^2x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))-\frac{1}{18}bcd^3x\left(1+c^2x^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))-\frac{19}{48}d^3(a+b\operatorname{arcsinh}(cx))^2+\frac{1}{2}d^3\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))^2+\frac{1}{4}d^3\left(1+c^2x^2\right)^2(a+b\operatorname{arcsinh}(cx))^2+\frac{1}{6}d^3\left(1+c^2x^2\right)^3(a+b\operatorname{arcsinh}(cx))^2+\frac{d^3(a+b\operatorname{arcsinh}(cx))^3}{3b}+d^3(a+b\operatorname{arcsinh}(cx))^2\log\left(1-e^{-2\operatorname{arcsinh}(cx)}\right)-bd^3(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,e^{-2\operatorname{arcsinh}(cx)}\right)-\frac{1}{2}\left(b^2d^3\right)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,x)}{x}dx,x,e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}\right)$$

$$=\frac{71}{144}b^2c^2d^3x^2+\frac{7}{144}b^2c^4d^3x^4+\frac{1}{108}b^2d^3\left(1+c^2x^2\right)^3-\frac{19}{24}bcd^3x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))-\frac{7}{36}bcd^3x\left(1+c^2x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))-\frac{1}{18}bcd^3x\left(1+c^2x^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))-\frac{19}{48}d^3(a+b\operatorname{arcsinh}(cx))^2+\frac{1}{2}d^3\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))^2+\frac{1}{4}d^3\left(1+c^2x^2\right)^2(a+b\operatorname{arcsinh}(cx))^2+\frac{1}{6}d^3\left(1+c^2x^2\right)^3(a+b\operatorname{arcsinh}(cx))^2+\frac{d^3(a+b\operatorname{arcsinh}(cx))^3}{3b}+d^3(a+b\operatorname{arcsinh}(cx))^2\log\left(1-e^{-2\operatorname{arcsinh}(cx)}\right)-bd^3(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,e^{-2\operatorname{arcsinh}(cx)}\right)-\frac{1}{2}b^2d^3\operatorname{PolyLog}\left(3,e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}\right)$$

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.24

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2}{x} dx$$
$$= \frac{d^3 \left(5184a^2c^2x^2 + 2592a^2c^4x^4 + 576a^2c^6x^6 - 3600abcx\sqrt{1+c^2x^2} - 1056abc^3x^3\sqrt{1+c^2x^2} - 192abc^5x^5\sqrt{1\right.}{}$$

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (d^3*(5184*a^2*c^2*x^2 + 2592*a^2*c^4*x^4 + 576*a^2*c^6*x^6 - 3600*a*b*c*x*
Sqrt[1 + c^2*x^2] - 1056*a*b*c^3*x^3*Sqrt[1 + c^2*x^2] - 192*a*b*c^5*x^5*Sq
rt[1 + c^2*x^2] + 10368*a*b*c^2*x^2*ArcSinh[c*x] + 5184*a*b*c^4*x^4*ArcSinh
[c*x] + 1152*a*b*c^6*x^6*ArcSinh[c*x] - 3456*a*b*ArcSinh[c*x]^2 - 1152*b^2*
ArcSinh[c*x]^3 + 783*b^2*Cosh[2*ArcSinh[c*x]] + 1566*b^2*ArcSinh[c*x]^2*Cos
h[2*ArcSinh[c*x]] + 27*b^2*Cosh[4*ArcSinh[c*x]] + 216*b^2*ArcSinh[c*x]^2*Co
sh[4*ArcSinh[c*x]] + b^2*Cosh[6*ArcSinh[c*x]] + 18*b^2*ArcSinh[c*x]^2*Cosh[
6*ArcSinh[c*x]] + 6912*a*b*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])] + 3456*
b^2*ArcSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + 3456*a^2*Log[c*x] - 3600*a
*b*Log[-(c*x) + Sqrt[1 + c^2*x^2]] + 3456*b*(a + b*ArcSinh[c*x])*PolyLog[2,
 E^(2*ArcSinh[c*x])] - 1728*b^2*PolyLog[3, E^(2*ArcSinh[c*x])] - 1566*b^2*A
rcSinh[c*x]*Sinh[2*ArcSinh[c*x]] - 108*b^2*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]
] - 6*b^2*ArcSinh[c*x]*Sinh[6*ArcSinh[c*x]]))/3456
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 686 vs. 2(334) = 668.

Time = 0.33 (sec) , antiderivative size = 687, normalized size of antiderivative = 2.04

method	result
parts	$-\frac{d^3ab\,c^5x^5\sqrt{c^2x^2+1}}{18}-\frac{11d^3ab\,c^3x^3\sqrt{c^2x^2+1}}{36}-\frac{25d^3abcx\sqrt{c^2x^2+1}}{24}-2d^3b^2\,\text{polylog}\left(3,-cx-\sqrt{c^2x^2}\right.$
derivativedivides	$d^3a^2\left(\frac{c^6x^6}{6}+\frac{3c^4x^4}{4}+\frac{3c^2x^2}{2}+\ln(cx)\right)-\frac{d^3ab\,c^5x^5\sqrt{c^2x^2+1}}{18}-\frac{11d^3ab\,c^3x^3\sqrt{c^2x^2+1}}{36}-\frac{25d^3abcx\sqrt{c^2x^2}}{24}$
default	$d^3a^2\left(\frac{c^6x^6}{6}+\frac{3c^4x^4}{4}+\frac{3c^2x^2}{2}+\ln(cx)\right)-\frac{d^3ab\,c^5x^5\sqrt{c^2x^2+1}}{18}-\frac{11d^3ab\,c^3x^3\sqrt{c^2x^2+1}}{36}-\frac{25d^3abcx\sqrt{c^2x^2}}{24}$

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/18*d^3*a*b*c^5*x^5*(c^2*x^2+1)^(1/2)-11/36*d^3*a*b*c^3*x^3*(c^2*x^2+1)^(
1/2)-25/24*d^3*a*b*c*x*(c^2*x^2+1)^(1/2)-2*d^3*b^2*polylog(3,-c*x-(c^2*x^2+
1)^(1/2))-1/3*d^3*b^2*arcsinh(c*x)^3+25/48*d^3*b^2*arcsinh(c*x)^2-2*d^3*b^2
*polylog(3,c*x+(c^2*x^2+1)^(1/2))+2*d^3*a*b*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+
1)^(1/2))+2*d^3*a*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+1/6*d^3*b^2*ar
```

```
csinh(c*x)^2*c^6*x^6+3/4*d^3*b^2*arcsinh(c*x)^2*c^4*x^4+3/2*d^3*b^2*arcsinh
(c*x)^2*c^2*x^2+1/3*d^3*a*b*arcsinh(c*x)*c^6*x^6+3/2*d^3*a*b*arcsinh(c*x)*c
^4*x^4+3*d^3*a*b*arcsinh(c*x)*c^2*x^2-1/18*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)
^(1/2)*c^5*x^5-11/36*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3*x^3-25/24*d
^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x+811/3456*d^3*b^2+d^3*a^2*(1/6*c^6
*x^6+3/4*c^4*x^4+3/2*c^2*x^2+ln(x))+25/48*b^2*c^2*d^3*x^2+11/144*b^2*c^4*d^
3*x^4+d^3*b^2*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*d^3*b^2*arcsinh(
c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+25/24*d^3*a*b*arcsinh(c*x)-d^3*a*b*a
rcsinh(c*x)^2+2*d^3*a*b*polylog(2,c*x+(c^2*x^2+1)^(1/2))+2*d^3*a*b*polylog(
2,-c*x-(c^2*x^2+1)^(1/2))+1/108*d^3*b^2*c^6*x^6+d^3*b^2*arcsinh(c*x)^2*ln(1
+c*x+(c^2*x^2+1)^(1/2))+2*d^3*b^2*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1
/2))
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)^3\,(a+b\text{arcsinh}(cx))^2}{x}\,dx=\int \frac{(c^2dx^2+d)^3(b\,\text{arsinh}\,(cx)+a)^2}{x}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^6*d^3*x^6 + 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 + a^2*d^3
 + (b^2*c^6*d^3*x^6 + 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 + b^2*d^3)*arcs
inh(c*x)^2 + 2*(a*b*c^6*d^3*x^6 + 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2 + a
*b*d^3)*arcsinh(c*x))/x, x)
```

Sympy [F]

$$\begin{aligned}\int \frac{(d+c^2dx^2)^3\,(a+b\text{arcsinh}(cx))^2}{x}\,dx=d^3\Bigg(&\int \frac{a^2}{x}\,dx+\int 3a^2c^2x\,dx+\int 3a^2c^4x^3\,dx\\&+\int a^2c^6x^5\,dx+\int \frac{b^2\,\text{asinh}^2\,(cx)}{x}\,dx\\&+\int \frac{2ab\,\text{asinh}\,(cx)}{x}\,dx+\int 3b^2c^2x\,\text{asinh}^2\,(cx)\,dx\\&+\int 3b^2c^4x^3\,\text{asinh}^2\,(cx)\,dx\\&+\int b^2c^6x^5\,\text{asinh}^2\,(cx)\,dx+\int 6abc^2x\,\text{asinh}\,(cx)\,dx\\&+\int 6abc^4x^3\,\text{asinh}\,(cx)\,dx\\&+\int 2abc^6x^5\,\text{asinh}\,(cx)\,dx\Bigg)\end{aligned}$$

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2/x,x)
```

```
[Out] d**3*(Integral(a**2/x, x) + Integral(3*a**2*c**2*x, x) + Integral(3*a**2*c*
*4*x**3, x) + Integral(a**2*c**6*x**5, x) + Integral(b**2*asinh(c*x)**2/x,
x) + Integral(2*a*b*asinh(c*x)/x, x) + Integral(3*b**2*c**2*x*asinh(c*x)**2
, x) + Integral(3*b**2*c**4*x**3*asinh(c*x)**2, x) + Integral(b**2*c**6*x**
5*asinh(c*x)**2, x) + Integral(6*a*b*c**2*x*asinh(c*x), x) + Integral(6*a*b
*c**4*x**3*asinh(c*x), x) + Integral(2*a*b*c**6*x**5*asinh(c*x), x))
```

Maxima [F]

$$\int \frac{(d + c^2dx^2)^3 (a + b\text{arcsinh}(cx))^2}{x} dx = \int \frac{(c^2dx^2 + d)^3(b\,\text{arsinh}\,(cx) + a)^2}{x} dx$$

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/6*a^2*c^6*d^3*x^6 + 3/4*a^2*c^4*d^3*x^4 + 3/2*a^2*c^2*d^3*x^2 + a^2*d^3*l
og(x) + integrate(b^2*c^6*d^3*x^5*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*a*b*c^
6*d^3*x^5*log(c*x + sqrt(c^2*x^2 + 1)) + 3*b^2*c^4*d^3*x^3*log(c*x + sqrt(c
^2*x^2 + 1))^2 + 6*a*b*c^4*d^3*x^3*log(c*x + sqrt(c^2*x^2 + 1)) + 3*b^2*c^2
*d^3*x*log(c*x + sqrt(c^2*x^2 + 1))^2 + 6*a*b*c^2*d^3*x*log(c*x + sqrt(c^2*
x^2 + 1)) + b^2*d^3*log(c*x + sqrt(c^2*x^2 + 1))^2/x + 2*a*b*d^3*log(c*x +
sqrt(c^2*x^2 + 1))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + c^2dx^2)^3 (a + b\text{arcsinh}(cx))^2}{x} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2}{x}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)^3}{x}\,dx$$

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x,x)
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x, x)
```

3.222 $\int \frac{(d+c^2dx^2)^3(a+b\,\text{arcsinh}(cx))^2}{x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 307

$$\begin{aligned}\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2}{x^2}\,dx &= \frac{122}{25}b^2c^2d^3x+\frac{14}{75}b^2c^4d^3x^3+\frac{2}{125}b^2c^6d^3x^5 \\ &\quad -\frac{22}{5}bcd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) \\ &\quad -\frac{2}{5}bcd^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx)) \\ &\quad -\frac{2}{25}bcd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx)) \\ &\quad +\frac{16}{5}c^2d^3x(a+b\text{arcsinh}(cx))^2 \\ &\quad +\frac{8}{5}c^2d^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 \\ &\quad +\frac{6}{5}c^2d^3x(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2 \\ &\quad -\frac{d^3(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2}{x} \\ &\quad -4bcd^3(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) \\ &\quad -2b^2cd^3\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right) \\ &\quad +2b^2cd^3\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\end{aligned}$$

```
[Out] 122/25*b^2*c^2*d^3*x+14/75*b^2*c^4*d^3*x^3+2/125*b^2*c^6*d^3*x^5-2/5*b*c*d^
3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))-2/25*b*c*d^3*(c^2*x^2+1)^(5/2)*(a+b*
arcsinh(c*x))+16/5*c^2*d^3*x*(a+b*arcsinh(c*x))^2+8/5*c^2*d^3*x*(c^2*x^2+1)
*(a+b*arcsinh(c*x))^2+6/5*c^2*d^3*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2-d^3*
(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2/x-4*b*c*d^3*(a+b*arcsinh(c*x))*arctanh(c
*x+(c^2*x^2+1)^(1/2))-2*b^2*c*d^3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*b^2*c
```

```
*d^3*polylog(2,c*x+(c^2*x^2+1)^(1/2))-22/5*b*c*d^3*(a+b*arcsinh(c*x))*(c^2*
x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5807, 5786, 5772, 5798, 8, 200, 5808, 5806, 5816, 4267, 2317, 2438}

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2}{x^2} \, dx = -4bcd^3\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)) + \frac{6}{5}c^2d^3x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2 + \frac{8}{5}c^2d^3x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2 - \frac{2}{25}bcd^3\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx)) - \frac{2}{5}bcd^3\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{22}{5}bcd^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx)) - \frac{d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2}{x} + \frac{16}{5}c^2d^3x(a+b\text{arcsinh}(cx))^2 - 2b^2cd^3\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right) + 2b^2cd^3\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right) + \frac{2}{125}b^2c^6d^3x^5 + \frac{14}{75}b^2c^4d^3x^3 + \frac{122}{25}b^2c^2d^3x$$

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (122*b^2*c^2*d^3*x)/25 + (14*b^2*c^4*d^3*x^3)/75 + (2*b^2*c^6*d^3*x^5)/125
- (22*b*c*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/5 - (2*b*c*d^3*(1 + c
^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/5 - (2*b*c*d^3*(1 + c^2*x^2)^(5/2)*(a +
 b*ArcSinh[c*x]))/25 + (16*c^2*d^3*x*(a + b*ArcSinh[c*x])^2)/5 + (8*c^2*d^3
*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/5 + (6*c^2*d^3*x*(1 + c^2*x^2)^2*(
a + b*ArcSinh[c*x])^2)/5 - (d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/x -
 4*b*c*d^3*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - 2*b^2*c*d^3*PolyL
og[2, -E^ArcSinh[c*x]] + 2*b^2*c*d^3*PolyLog[2, E^ArcSinh[c*x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\frac{d^3(1+c^2x^2)^3\,(a+b\text{arcsinh}(cx))^2}{x}+\left(6c^2d\right)\int\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2\,dx$$
$$+\left(2bcd^3\right)\int\frac{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))}{x}\,dx$$

$$= \frac{2}{5}bcd^3\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx)) + \frac{6}{5}c^2d^3x\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2 - \frac{d^3(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2}{x} + \frac{1}{5}\left(24c^2d^2\right)\int\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))^2\,dx + \left(2bcd^3\right)\int\frac{\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}\,dx - \frac{1}{5}\left(2b^2c^2d^3\right)\int\left(1+c^2x^2\right)^2\,dx - \frac{1}{5}\left(12bc^3d^3\right)\int x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx$$

$$= \frac{2}{3}bcd^3\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{2}{25}bcd^3\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx)) + \frac{8}{5}c^2d^3x\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2 + \frac{6}{5}c^2d^3x\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2 - \frac{d^3(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2}{x} + \left(2bcd^3\right)\int\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{x}\,dx + \frac{1}{5}\left(16c^2d^3\right)\int(a+b\text{arcsinh}(cx))^2\,dx - \frac{1}{5}\left(2b^2c^2d^3\right)\int\left(1+2c^2x^2+c^4x^4\right)\,dx + \frac{1}{25}\left(12b^2c^2d^3\right)\int\left(1+c^2x^2\right)^2\,dx - \frac{1}{3}\left(2b^2c^2d^3\right)\int\left(1+c^2x^2\right)\,dx - \frac{1}{5}\left(16bc^3d^3\right)\int x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx$$

$$= -\frac{16}{15}b^2c^2d^3x - \frac{22}{45}b^2c^4d^3x^3 - \frac{2}{25}b^2c^6d^3x^5 + 2bcd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) - \frac{2}{5}bcd^3\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{2}{25}bcd^3\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx)) + \frac{16}{5}c^2d^3x(a+b\text{arcsinh}(cx))^2 + \frac{8}{5}c^2d^3x\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2 + \frac{6}{5}c^2d^3x\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2 - \frac{d^3(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2}{x} + \left(2bcd^3\right)\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{1+c^2x^2}}\,dx + \frac{1}{25}\left(12b^2c^2d^3\right)\int\left(1+2c^2x^2+c^4x^4\right)\,dx + \frac{1}{15}\left(16b^2c^2d^3\right)\int\left(1+c^2x^2\right)\,dx - \left(2b^2c^2d^3\right)\int 1\,dx - \frac{1}{5}\left(32bc^3d^3\right)\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx$$

$$= -\frac{38}{25}b^2c^2d^3x + \frac{14}{75}b^2c^4d^3x^3 + \frac{2}{125}b^2c^6d^3x^5 - \frac{22}{5}bcd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) - \frac{2}{5}bcd^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{2}{25}bcd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx)) + \frac{16}{5}c^2d^3x(a+b\text{arcsinh}(cx))^2 + \frac{8}{5}c^2d^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 + \frac{6}{5}c^2d^3x(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2 - \frac{d^3(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2}{x} + (2bcd^3)\,\text{Subst}\left(\int (a+bx)\text{csch}(x)\,dx, x, \text{arcsinh}(cx)\right) + \frac{1}{5}(32b^2c^2d^3)\int 1\,dx$$

$$= \frac{122}{25}b^2c^2d^3x + \frac{14}{75}b^2c^4d^3x^3 + \frac{2}{125}b^2c^6d^3x^5 - \frac{22}{5}bcd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) - \frac{2}{5}bcd^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{2}{25}bcd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx)) + \frac{16}{5}c^2d^3x(a+b\text{arcsinh}(cx))^2 + \frac{8}{5}c^2d^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 + \frac{6}{5}c^2d^3x(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2 - \frac{d^3(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2}{x} - 4bcd^3(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) - (2b^2cd^3)\,\text{Subst}\left(\int \log\left(1-e^x\right)dx, x, \text{arcsinh}(cx)\right) + (2b^2cd^3)\,\text{Subst}\left(\int \log\left(1+e^x\right)dx, x, \text{arcsinh}(cx)\right)$$

$$= \frac{122}{25}b^2c^2d^3x + \frac{14}{75}b^2c^4d^3x^3 + \frac{2}{125}b^2c^6d^3x^5 - \frac{22}{5}bcd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) - \frac{2}{5}bcd^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{2}{25}bcd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx)) + \frac{16}{5}c^2d^3x(a+b\text{arcsinh}(cx))^2 + \frac{8}{5}c^2d^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 + \frac{6}{5}c^2d^3x(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2 - \frac{d^3(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2}{x} - 4bcd^3(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) - (2b^2cd^3)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}dx, x, e^{\text{arcsinh}(cx)}\right) + (2b^2cd^3)\,\text{Subst}\left(\int \frac{\log(1+x)}{x}dx, x, e^{\text{arcsinh}(cx)}\right)$$

$$
\begin{aligned}
&= \frac{122}{25}b^2c^2d^3x + \frac{14}{75}b^2c^4d^3x^3 + \frac{2}{125}b^2c^6d^3x^5 \\
&\quad - \frac{22}{5}bcd^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx)) - \frac{2}{5}bcd^3\left(1+c^2x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad - \frac{2}{25}bcd^3\left(1+c^2x^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx)) + \frac{16}{5}c^2d^3x(a+b\operatorname{arcsinh}(cx))^2 \\
&\quad + \frac{8}{5}c^2d^3x\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))^2 + \frac{6}{5}c^2d^3x\left(1+c^2x^2\right)^2(a+b\operatorname{arcsinh}(cx))^2 \\
&\quad - \frac{d^3\left(1+c^2x^2\right)^3(a+b\operatorname{arcsinh}(cx))^2}{x} - 4bcd^3(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right) \\
&\quad - 2b^2cd^3\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right) + 2b^2cd^3\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.52

$$
\begin{aligned}
&\int \frac{\left(d+c^2dx^2\right)^3(a+b\operatorname{arcsinh}(cx))^2}{x^2}\,dx = \frac{1}{720}d^3\Bigg(-\frac{720a^2}{x} + 2160a^2c^2x + 3460b^2c^2x \\
&\quad + 720a^2c^4x^3 + 144a^2c^6x^5 - \frac{17568}{5}abc\sqrt{1+c^2x^2} \\
&\quad - \frac{2016}{5}abc^3x^2\sqrt{1+c^2x^2} - \frac{288}{5}abc^5x^4\sqrt{1+c^2x^2} \\
&\quad - \frac{1440ab\operatorname{arcsinh}(cx)}{x} + 4320abc^2x\operatorname{arcsinh}(cx) \\
&\quad + 1440abc^4x^3\operatorname{arcsinh}(cx) + 288abc^6x^5\operatorname{arcsinh}(cx) \\
&\quad - 3420b^2c\sqrt{1+c^2x^2}\operatorname{arcsinh}(cx) \\
&\quad - \frac{720b^2\operatorname{arcsinh}(cx)^2}{x} + 1890b^2c^2x\operatorname{arcsinh}(cx)^2 \\
&\quad - 1440abc\operatorname{arctanh}\left(\sqrt{1+c^2x^2}\right) \\
&\quad + 80b^2c^2x\cosh(2\operatorname{arcsinh}(cx)) \\
&\quad + 360b^2c^2x\operatorname{arcsinh}(cx)^2\cosh(2\operatorname{arcsinh}(cx)) \\
&\quad - 90b^2c\operatorname{arcsinh}(cx)\cosh(3\operatorname{arcsinh}(cx)) \\
&\quad - \frac{18}{5}b^2c\operatorname{arcsinh}(cx)\cosh(5\operatorname{arcsinh}(cx)) \\
&\quad + 1440b^2c\operatorname{arcsinh}(cx)\log\left(1-e^{-\operatorname{arcsinh}(cx)}\right) \\
&\quad - 1440b^2c\operatorname{arcsinh}(cx)\log\left(1+e^{-\operatorname{arcsinh}(cx)}\right) \\
&\quad + 1440b^2c\operatorname{PolyLog}\left(2,-e^{-\operatorname{arcsinh}(cx)}\right) \\
&\quad - 1440b^2c\operatorname{PolyLog}\left(2,e^{-\operatorname{arcsinh}(cx)}\right) \\
&\quad - 10b^2c\sinh(3\operatorname{arcsinh}(cx)) \\
&\quad - 45b^2c\operatorname{arcsinh}(cx)^2\sinh(3\operatorname{arcsinh}(cx)) \\
&\quad + \frac{18}{25}b^2c\sinh(5\operatorname{arcsinh}(cx)) \\
&\quad + 9b^2c\operatorname{arcsinh}(cx)^2\sinh(5\operatorname{arcsinh}(cx))\Bigg)
\end{aligned}
$$

[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^2,x]

[Out] (d^3*((-720*a^2)/x + 2160*a^2*c^2*x + 3460*b^2*c^2*x + 720*a^2*c^4*x^3 + 144*a^2*c^6*x^5 - (17568*a*b*c*Sqrt[1 + c^2*x^2])/5 - (2016*a*b*c^3*x^2*Sqrt[1 + c^2*x^2])/5 - (288*a*b*c^5*x^4*Sqrt[1 + c^2*x^2])/5 - (1440*a*b*ArcSinh[c*x])/x + 4320*a*b*c^2*x*ArcSinh[c*x] + 1440*a*b*c^4*x^3*ArcSinh[c*x] + 288*a*b*c^6*x^5*ArcSinh[c*x] - 3420*b^2*c*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - (720*b^2*ArcSinh[c*x]^2)/x + 1890*b^2*c^2*x*ArcSinh[c*x]^2 - 1440*a*b*c*ArcTanh[Sqrt[1 + c^2*x^2]] + 80*b^2*c^2*x*Cosh[2*ArcSinh[c*x]] + 360*b^2*c^2*x*ArcSinh[c*x]^2*Cosh[2*ArcSinh[c*x]] - 90*b^2*c*ArcSinh[c*x]*Cosh[3*ArcSinh[c*x]] - (18*b^2*c*ArcSinh[c*x]*Cosh[5*ArcSinh[c*x]])/5 + 1440*b^2*c*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 1440*b^2*c*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 1440*b^2*c*PolyLog[2, -E^(-ArcSinh[c*x])] - 1440*b^2*c*PolyLog[2, E^(-ArcSinh[c*x])] - 10*b^2*c*Sinh[3*ArcSinh[c*x]] - 45*b^2*c*ArcSinh[c*x]^2*Sinh[3*ArcSinh[c*x]] + (18*b^2*c*Sinh[5*ArcSinh[c*x]])/25 + 9*b^2*c*ArcSinh[c*x]^2*Sinh[5*ArcSinh[c*x]]))/720

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.51

method	result
derivativedivides	$c\left(d^3a^2\left(\frac{c^5x^5}{5}+c^3x^3+3cx-\frac{1}{cx}\right)+\frac{14d^3b^2c^3x^3}{75}+\frac{122d^3b^2cx}{25}+2d^3b^2\,\text{polylog}\left(2,cx+\sqrt{c^2x^2+\right.\right.$
default	$c\left(d^3a^2\left(\frac{c^5x^5}{5}+c^3x^3+3cx-\frac{1}{cx}\right)+\frac{14d^3b^2c^3x^3}{75}+\frac{122d^3b^2cx}{25}+2d^3b^2\,\text{polylog}\left(2,cx+\sqrt{c^2x^2+\right.\right.$
parts	$d^3a^2\left(\frac{c^6x^5}{5}+c^4x^3+3c^2x-\frac{1}{x}\right)-2d^3b^2c\,\text{arcsinh}\,(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)+2d^3b^2c\,\text{a}$

[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^2,x,method=_RETURNVERBOSE)

[Out] c*(d^3*a^2*(1/5*c^5*x^5+c^3*x^3+3*c*x-1/c/x)+14/75*d^3*b^2*c^3*x^3+122/25*d^3*b^2*c*x+2*d^3*b^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*d^3*b^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-2/25*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^4*x^4-14/25*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^2*x^2+2*d^3*b^2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+2/125*d^3*b^2*c^5*x^5-2*d^3*b^2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-d^3*b^2*arcsinh(c*x)^2/c/x+1/5*d^3*b^2*arcsinh(c*x)^2*c^5*x^5+d^3*b^2*arcsinh(c*x)^2*c^3*x^3+3*d^3*b^2*arcsinh(c*x)^2*c*x-122/25*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*d^3*a*b*(1/5*arcsinh(c*x)*c^5*x^5+arcsinh(c*x)*c^3*x^3+3*arcsinh(c*x)*c*x-arcsinh(c*x)/c/x-1/25*c^4*x^4*(c^2*x^2+1)^(1/2)-7/25*c^2*x^2*(c^2*x^2+1)^(1/2)-61/25*(c^2*x^2+1)^(1/2)-arctanh(1/(c^2*x^2+1)^(1/2))))

Fricas [F]

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2}{x^2}\,dx=\int \frac{(c^2dx^2+d)^3(b\,\text{arsinh}\,(cx)+a)^2}{x^2}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^6*d^3*x^6 + 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 + a^2*d^3
 + (b^2*c^6*d^3*x^6 + 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 + b^2*d^3)*arcs
inh(c*x)^2 + 2*(a*b*c^6*d^3*x^6 + 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2 + a
*b*d^3)*arcsinh(c*x))/x^2, x)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2}{x^2}\,dx=d^3\left(\int 3a^2c^2\,dx+\int \frac{a^2}{x^2}\,dx+\int 3a^2c^4x^2\,dx+\int a^2c^6x^4\,dx+\int 3b^2c^2\operatorname{asinh}^2(cx)\,dx+\int \frac{b^2\operatorname{asinh}^2(cx)}{x^2}\,dx+\int 6abc^2\operatorname{asinh}(cx)\,dx+\int \frac{2ab\operatorname{asinh}(cx)}{x^2}\,dx+\int 3b^2c^4x^2\operatorname{asinh}^2(cx)\,dx+\int b^2c^6x^4\operatorname{asinh}^2(cx)\,dx+\int 6abc^4x^2\operatorname{asinh}(cx)\,dx+\int 2abc^6x^4\operatorname{asinh}(cx)\,dx\right)$$

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2/x**2,x)
```

```
[Out] d**3*(Integral(3*a**2*c**2, x) + Integral(a**2/x**2, x) + Integral(3*a**2*c
**4*x**2, x) + Integral(a**2*c**6*x**4, x) + Integral(3*b**2*c**2*asinh(c*x
)**2, x) + Integral(b**2*asinh(c*x)**2/x**2, x) + Integral(6*a*b*c**2*asinh
(c*x), x) + Integral(2*a*b*asinh(c*x)/x**2, x) + Integral(3*b**2*c**4*x**2*
asinh(c*x)**2, x) + Integral(b**2*c**6*x**4*asinh(c*x)**2, x) + Integral(6*
a*b*c**4*x**2*asinh(c*x), x) + Integral(2*a*b*c**6*x**4*asinh(c*x), x))
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)^3\,(a+b\mathrm{arcsinh}(cx))^2}{x^2}\,dx=\int \frac{(c^2dx^2+d)^3(b\operatorname{arsinh}(cx)+a)^2}{x^2}\,dx$$

[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="maxima")

[Out] 1/5*a^2*c^6*d^3*x^5 + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^6*d^3 + a^2*c^4*d^3*x^3 + 2/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^4*d^3 + 3*b^2*c^2*d^3*x*arcsinh(c*x)^2 + 6*b^2*c^2*d^3*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + 3*a^2*c^2*d^3*x + 6*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*c*d^3 - 2*(c*arcsinh(1/(c*abs(x))) + arcsinh(c*x)/x)*a*b*d^3 - a^2*d^3/x + 1/5*(b^2*c^6*d^3*x^6 + 5*b^2*c^4*d^3*x^4 - 5*b^2*d^3)*log(c*x + sqrt(c^2*x^2 + 1))^2/x - integrate(2/5*(b^2*c^9*d^3*x^8 + 6*b^2*c^7*d^3*x^6 + 5*b^2*c^5*d^3*x^4 - 5*b^2*c^3*d^3*x^2 - 5*b^2*c*d^3 + (b^2*c^8*d^3*x^7 + 5*b^2*c^6*d^3*x^5 - 5*b^2*c^2*d^3*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*x^4 + c*x^2 + (c^2*x^3 + x)*sqrt(c^2*x^2 + 1)), x)

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^3\,(a+b\mathrm{arcsinh}(cx))^2}{x^2}\,dx=\text{Exception raised: TypeError}$$

[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^3\,(a+b\mathrm{arcsinh}(cx))^2}{x^2}\,dx=\int \frac{(a+b\operatorname{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)^3}{x^2}\,dx$$

[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x^2,x)

[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x^2, x)

3.223 $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 354

$$\begin{aligned}\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x^3}\,dx &= \frac{21}{32}b^2c^4d^3x^2+\frac{1}{32}b^2c^6d^3x^4 \\ &\quad -\frac{3}{16}bc^3d^3x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) \\ &\quad +\frac{7}{8}bc^3d^3x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx)) \\ &\quad -\frac{bcd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x} \\ &\quad -\frac{3}{32}c^2d^3(a+b\text{arcsinh}(cx))^2 \\ &\quad +\frac{3}{2}c^2d^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 \\ &\quad +\frac{3}{4}c^2d^3(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2 \\ &\quad -\frac{d^3(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2}{2x^2} \\ &\quad +\frac{c^2d^3(a+b\text{arcsinh}(cx))^3}{b} \\ &\quad +3c^2d^3(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right) \\ &\quad +b^2c^2d^3\log(x) \\ &\quad -3bc^2d^3(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right) \\ &\quad -\frac{3}{2}b^2c^2d^3\,\text{PolyLog}\left(3,e^{-2\text{arcsinh}(cx)}\right)\end{aligned}$$

```
[Out] 21/32*b^2*c^4*d^3*x^2+1/32*b^2*c^6*d^3*x^4+7/8*b*c^3*d^3*x*(c^2*x^2+1)^(3/2
)*(a+b*arcsinh(c*x))-b*c*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))/x-3/32*c^
```

```
2*d^3*(a+b*arcsinh(c*x))^2+3/2*c^2*d^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+3/4
*c^2*d^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2-1/2*d^3*(c^2*x^2+1)^3*(a+b*arcs
inh(c*x))^2/x^2+c^2*d^3*(a+b*arcsinh(c*x))^3/b+3*c^2*d^3*(a+b*arcsinh(c*x))
^2*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)+b^2*c^2*d^3*ln(x)-3*b*c^2*d^3*(a+b*arc
sinh(c*x))*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)-3/2*b^2*c^2*d^3*polylog(3
,1/(c*x+(c^2*x^2+1)^(1/2))^2)-3/16*b*c^3*d^3*x*(a+b*arcsinh(c*x))*(c^2*x^2+
1)^(1/2)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {5807, 5808, 5775, 3797, 2221, 2611, 2320, 6724, 5785, 5783, 30, 5786, 14, 272, 45}

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2}{x^3}\,dx = -3bc^2d^3\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)) + \frac{3}{4}c^2d^3\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2 + \frac{3}{2}c^2d^3\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2 - \frac{bcd^3(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))}{x} - \frac{d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2}{2x^2} + \frac{c^2d^3(a+b\text{arcsinh}(cx))^3}{b} - \frac{3}{32}c^2d^3(a+b\text{arcsinh}(cx))^2 + 3c^2d^3\log\left(1-e^{-2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2 + \frac{7}{8}bc^3d^3x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{3}{16}bc^3d^3x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx)) - \frac{3}{2}b^2c^2d^3\,\text{PolyLog}\left(3,e^{-2\text{arcsinh}(cx)}\right) + \frac{1}{32}b^2c^6d^3x^4 + \frac{21}{32}b^2c^4d^3x^2 + b^2c^2d^3\log(x)$$

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] (21*b^2*c^4*d^3*x^2)/32 + (b^2*c^6*d^3*x^4)/32 - (3*b*c^3*d^3*x*Sqrt[1 + c^
2*x^2]*(a + b*ArcSinh[c*x]))/16 + (7*b*c^3*d^3*x*(1 + c^2*x^2)^(3/2)*(a + b
*ArcSinh[c*x]))/8 - (b*c*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x -
(3*c^2*d^3*(a + b*ArcSinh[c*x])^2)/32 + (3*c^2*d^3*(1 + c^2*x^2)*(a + b*Arc
Sinh[c*x])^2)/2 + (3*c^2*d^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/4 - (d
```

```
^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/(2*x^2) + (c^2*d^3*(a + b*ArcSin
h[c*x])^3)/b + 3*c^2*d^3*(a + b*ArcSinh[c*x])^2*Log[1 - E^(-2*ArcSinh[c*x])
] + b^2*c^2*d^3*Log[x] - 3*b*c^2*d^3*(a + b*ArcSinh[c*x])*PolyLog[2, E^(-2*
ArcSinh[c*x])] - (3*b^2*c^2*d^3*PolyLog[3, E^(-2*ArcSinh[c*x])])/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d^3(1+c^2x^2)^3\,(a+b\operatorname{arcsinh}(cx))^2}{2x^2}+\left(3c^2d\right)\int\frac{(d+c^2dx^2)^2\,(a+b\operatorname{arcsinh}(cx))^2}{x}\,dx \\
&\quad +\left(bcd^3\right)\int\frac{(1+c^2x^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))}{x^2}\,dx \\
&= -\frac{bcd^3(1+c^2x^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))}{x} \\
&\quad +\frac{3}{4}c^2d^3\left(1+c^2x^2\right)^2(a+b\operatorname{arcsinh}(cx))^2-\frac{d^3(1+c^2x^2)^3\,(a+b\operatorname{arcsinh}(cx))^2}{2x^2} \\
&\quad +\left(3c^2d^2\right)\int\frac{(d+c^2dx^2)\,(a+b\operatorname{arcsinh}(cx))^2}{x}\,dx+\left(b^2c^2d^3\right)\int\frac{(1+c^2x^2)^2}{x}\,dx \\
&\quad -\frac{1}{2}\left(3bc^3d^3\right)\int\left(1+c^2x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx \\
&\quad +\left(5bc^3d^3\right)\int\left(1+c^2x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx
\end{aligned}
$$

$$= \frac{7}{8}bc^3d^3x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{bcd^3\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{x} + \frac{3}{2}c^2d^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2 + \frac{3}{4}c^2d^3\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2 - \frac{d^3\left(1+c^2x^2\right)^3(a+b\text{arcsinh}(cx))^2}{2x^2} + \left(3c^2d^3\right)\int \frac{(a+b\text{arcsinh}(cx))^2}{x}\,dx + \frac{1}{2}\left(b^2c^2d^3\right)\text{Subst}\left(\int \frac{\left(1+c^2x\right)^2}{x}\,dx, x, x^2\right) - \frac{1}{8}\left(9bc^3d^3\right)\int \sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx - \left(3bc^3d^3\right)\int \sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx + \frac{1}{4}\left(15bc^3d^3\right)\int \sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx + \frac{1}{8}\left(3b^2c^4d^3\right)\int x\left(1+c^2x^2\right)\,dx - \frac{1}{4}\left(5b^2c^4d^3\right)\int x\left(1+c^2x^2\right)\,dx$$

$$= -\frac{3}{16}bc^3d^3x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) + \frac{7}{8}bc^3d^3x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{bcd^3\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{x} + \frac{3}{2}c^2d^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2 + \frac{3}{4}c^2d^3\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2 - \frac{d^3\left(1+c^2x^2\right)^3(a+b\text{arcsinh}(cx))^2}{2x^2} - \frac{\left(3c^2d^3\right)\text{Subst}\left(\int x^2\coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)}{b} + \frac{1}{2}\left(b^2c^2d^3\right)\text{Subst}\left(\int \left(2c^2+\frac{1}{x}+c^4x\right)\,dx, x, x^2\right) - \frac{1}{16}\left(9bc^3d^3\right)\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx - \frac{1}{2}\left(3bc^3d^3\right)\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx + \frac{1}{8}\left(15bc^3d^3\right)\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx + \frac{1}{8}\left(3b^2c^4d^3\right)\int \left(x+c^2x^3\right)\,dx + \frac{1}{16}\left(9b^2c^4d^3\right)\int x\,dx - \frac{1}{4}\left(5b^2c^4d^3\right)\int \left(x+c^2x^3\right)\,dx + \frac{1}{2}\left(3b^2c^4d^3\right)\int x\,dx - \frac{1}{8}\left(15b^2c^4d^3\right)\int x\,dx$$

$$= \frac{21}{32}b^2c^4d^3x^2 + \frac{1}{32}b^2c^6d^3x^4 - \frac{3}{16}bc^3d^3x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) + \frac{7}{8}bc^3d^3x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{bcd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x} - \frac{3}{32}c^2d^3(a+b\text{arcsinh}(cx))^2 + \frac{3}{2}c^2d^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 + \frac{3}{4}c^2d^3(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2 - \frac{d^3(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2}{2x^2} + \frac{c^2d^3(a+b\text{arcsinh}(cx))^3}{b} + b^2c^2d^3\log(x) + \frac{(6c^2d^3)\,\text{Subst}\left(\int \frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x^2}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b}$$

$$= \frac{21}{32}b^2c^4d^3x^2 + \frac{1}{32}b^2c^6d^3x^4 - \frac{3}{16}bc^3d^3x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) + \frac{7}{8}bc^3d^3x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{bcd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x} - \frac{3}{32}c^2d^3(a+b\text{arcsinh}(cx))^2 + \frac{3}{2}c^2d^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 + \frac{3}{4}c^2d^3(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2 - \frac{d^3(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2}{2x^2} + \frac{c^2d^3(a+b\text{arcsinh}(cx))^3}{b} + 3c^2d^3(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right) + b^2c^2d^3\log(x) - (6c^2d^3)\,\text{Subst}\left(\int x\log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)dx, x, a+b\text{arcsinh}(cx)\right)$$

$$= \frac{21}{32}b^2c^4d^3x^2 + \frac{1}{32}b^2c^6d^3x^4 - \frac{3}{16}bc^3d^3x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) + \frac{7}{8}bc^3d^3x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{bcd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x} - \frac{3}{32}c^2d^3(a+b\text{arcsinh}(cx))^2 + \frac{3}{2}c^2d^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 + \frac{3}{4}c^2d^3(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2 - \frac{d^3(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2}{2x^2} + \frac{c^2d^3(a+b\text{arcsinh}(cx))^3}{b} + 3c^2d^3(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right) + b^2c^2d^3\log(x) - 3bc^2d^3(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2, e^{-2\text{arcsinh}(cx)}\right) + (3bc^2d^3)\,\text{Subst}\left(\int \text{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)dx, x, a+b\text{arcsinh}(cx)\right)$$

$$= \frac{21}{32} b^2 c^4 d^3 x^2 + \frac{1}{32} b^2 c^6 d^3 x^4 - \frac{3}{16} b c^3 d^3 x \sqrt{1+c^2x^2} (a + b\operatorname{arcsinh}(cx)) + \frac{7}{8} b c^3 d^3 x (1+c^2x^2)^{3/2} (a + b\operatorname{arcsinh}(cx)) - \frac{b c d^3 (1+c^2x^2)^{5/2} (a + b\operatorname{arcsinh}(cx))}{x} - \frac{3}{32} c^2 d^3 (a + b\operatorname{arcsinh}(cx))^2 + \frac{3}{2} c^2 d^3 (1+c^2x^2) (a + b\operatorname{arcsinh}(cx))^2 + \frac{3}{4} c^2 d^3 (1+c^2x^2)^2 (a + b\operatorname{arcsinh}(cx))^2 - \frac{d^3 (1+c^2x^2)^3 (a + b\operatorname{arcsinh}(cx))^2}{2x^2} + \frac{c^2 d^3 (a + b\operatorname{arcsinh}(cx))^3}{b} + 3 c^2 d^3 (a + b\operatorname{arcsinh}(cx))^2 \log\left(1 - e^{-2\operatorname{arcsinh}(cx)}\right) + b^2 c^2 d^3 \log(x) - 3 b c^2 d^3 (a + b\operatorname{arcsinh}(cx)) \operatorname{PolyLog}\left(2, e^{-2\operatorname{arcsinh}(cx)}\right) - \frac{1}{2} (3 b^2 c^2 d^3) \operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,x)}{x} dx, x, e^{2\left(\frac{a}{b} - \frac{a + b\operatorname{arcsinh}(cx)}{b}\right)}\right)$$

$$= \frac{21}{32} b^2 c^4 d^3 x^2 + \frac{1}{32} b^2 c^6 d^3 x^4 - \frac{3}{16} b c^3 d^3 x \sqrt{1+c^2x^2} (a + b\operatorname{arcsinh}(cx)) + \frac{7}{8} b c^3 d^3 x (1+c^2x^2)^{3/2} (a + b\operatorname{arcsinh}(cx)) - \frac{b c d^3 (1+c^2x^2)^{5/2} (a + b\operatorname{arcsinh}(cx))}{x} - \frac{3}{32} c^2 d^3 (a + b\operatorname{arcsinh}(cx))^2 + \frac{3}{2} c^2 d^3 (1+c^2x^2) (a + b\operatorname{arcsinh}(cx))^2 + \frac{3}{4} c^2 d^3 (1+c^2x^2)^2 (a + b\operatorname{arcsinh}(cx))^2 - \frac{d^3 (1+c^2x^2)^3 (a + b\operatorname{arcsinh}(cx))^2}{2x^2} + \frac{c^2 d^3 (a + b\operatorname{arcsinh}(cx))^3}{b} + 3 c^2 d^3 (a + b\operatorname{arcsinh}(cx))^2 \log\left(1 - e^{-2\operatorname{arcsinh}(cx)}\right) + b^2 c^2 d^3 \log(x) - 3 b c^2 d^3 (a + b\operatorname{arcsinh}(cx)) \operatorname{PolyLog}\left(2, e^{-2\operatorname{arcsinh}(cx)}\right) - \frac{3}{2} b^2 c^2 d^3 \operatorname{PolyLog}\left(3, e^{2\left(\frac{a}{b} - \frac{a + b\operatorname{arcsinh}(cx)}{b}\right)}\right)$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.40

$$\int \frac{(d + c^2 d x^2)^3 (a + b\operatorname{arcsinh}(cx))^2}{x^3} dx = \frac{d^3 \left(-128a^2 + 384a^2c^4x^4 + 64a^2c^6x^6 - 256abcx\sqrt{1+c^2x^2} - 336abc^3x^3\sqrt{1+c^2x^2} - 32abc^5x^5\sqrt{1+c^2x^2} - 2\right.}{}$$

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] (d^3*(-128*a^2 + 384*a^2*c^4*x^4 + 64*a^2*c^6*x^6 - 256*a*b*c*x*Sqrt[1 + c^
2*x^2] - 336*a*b*c^3*x^3*Sqrt[1 + c^2*x^2] - 32*a*b*c^5*x^5*Sqrt[1 + c^2*x^
2] - 256*a*b*ArcSinh[c*x] + 768*a*b*c^4*x^4*ArcSinh[c*x] + 128*a*b*c^6*x^6*
ArcSinh[c*x] - 256*b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - 128*b^2*ArcSinh
[c*x]^2 - 768*a*b*c^2*x^2*ArcSinh[c*x]^2 - 256*b^2*c^2*x^2*ArcSinh[c*x]^3 +
 80*b^2*c^2*x^2*Cosh[2*ArcSinh[c*x]] + 160*b^2*c^2*x^2*ArcSinh[c*x]^2*Cosh[
```

```
2*ArcSinh[c*x]] + b^2*c^2*x^2*Cosh[4*ArcSinh[c*x]] + 8*b^2*c^2*x^2*ArcSinh[
c*x]^2*Cosh[4*ArcSinh[c*x]] + 1536*a*b*c^2*x^2*ArcSinh[c*x]*Log[1 - E^(2*Ar
cSinh[c*x])] + 768*b^2*c^2*x^2*ArcSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] +
 768*a^2*c^2*x^2*Log[x] + 256*b^2*c^2*x^2*Log[c*x] - 336*a*b*c^2*x^2*Log[-(
c*x) + Sqrt[1 + c^2*x^2]] + 768*b*c^2*x^2*(a + b*ArcSinh[c*x])*PolyLog[2, E
^(2*ArcSinh[c*x])] - 384*b^2*c^2*x^2*PolyLog[3, E^(2*ArcSinh[c*x])] - 160*b
^2*c^2*x^2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]] - 4*b^2*c^2*x^2*ArcSinh[c*x]*S
inh[4*ArcSinh[c*x]]))/(256*x^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 771 vs. $2(357) = 714$.

Time = 0.34 (sec) , antiderivative size = 772, normalized size of antiderivative = 2.18

method	result
derivativedivides	$c^2\left(d^3a^2\left(\frac{c^4x^4}{4}+\frac{3c^2x^2}{2}+3\ln\left(cx\right)-\frac{1}{2c^2x^2}\right)+d^3b^2\operatorname{arcsinh}\left(cx\right)-\frac{d^3ab\,c^3x^3\sqrt{c^2x^2+1}}{8}-\frac{21d^3abc}{}\right.$
default	$c^2\left(d^3a^2\left(\frac{c^4x^4}{4}+\frac{3c^2x^2}{2}+3\ln\left(cx\right)-\frac{1}{2c^2x^2}\right)+d^3b^2\operatorname{arcsinh}\left(cx\right)-\frac{d^3ab\,c^3x^3\sqrt{c^2x^2+1}}{8}-\frac{21d^3abc}{}\right.$
parts	$-\frac{d^3ab\operatorname{arcsinh}(cx)}{x^2}+\frac{21d^3ab\,c^2\operatorname{arcsinh}(cx)}{16}-3d^3ab\,c^2\operatorname{arcsinh}\left(cx\right)^2+6d^3ab\,c^2\operatorname{polylog}\left(2,-cx-\right.$

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(d^3*a^2*(1/4*c^4*x^4+3/2*c^2*x^2+3*ln(c*x)-1/2/c^2/x^2)+d^3*b^2*arcsin
h(c*x)-1/8*d^3*a*b*c^3*x^3*(c^2*x^2+1)^(1/2)-21/16*d^3*a*b*c*x*(c^2*x^2+1)^
(1/2)-6*d^3*b^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))-d^3*b^2*arcsinh(c*x)^3+21
/32*d^3*b^2*arcsinh(c*x)^2-6*d^3*b^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))+6*d^3
*a*b*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+6*d^3*a*b*arcsinh(c*x)*ln(1-c
*x-(c^2*x^2+1)^(1/2))+1/4*d^3*b^2*arcsinh(c*x)^2*c^4*x^4+3/2*d^3*b^2*arcsin
h(c*x)^2*c^2*x^2+1/2*d^3*a*b*arcsinh(c*x)*c^4*x^4+3*d^3*a*b*arcsinh(c*x)*c^
2*x^2-1/8*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3*x^3-21/16*d^3*b^2*arcs
inh(c*x)*(c^2*x^2+1)^(1/2)*c*x+d^3*a*b+81/256*d^3*b^2-d^3*b^2*arcsinh(c*x)/
c/x*(c^2*x^2+1)^(1/2)-d^3*a*b*arcsinh(c*x)/c^2/x^2+d^3*b^2*ln(c*x+(c^2*x^2+
1)^(1/2)-1)+d^3*b^2*ln(1+c*x+(c^2*x^2+1)^(1/2))-2*d^3*b^2*ln(c*x+(c^2*x^2+1
)^(1/2))+21/32*b^2*c^2*d^3*x^2+1/32*b^2*c^4*d^3*x^4+3*d^3*b^2*arcsinh(c*x)^
2*ln(1-c*x-(c^2*x^2+1)^(1/2))+6*d^3*b^2*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^
2+1)^(1/2))+21/16*d^3*a*b*arcsinh(c*x)-3*d^3*a*b*arcsinh(c*x)^2+6*d^3*a*b*p
olylog(2,c*x+(c^2*x^2+1)^(1/2))+6*d^3*a*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))
-d^3*a*b/c/x*(c^2*x^2+1)^(1/2)+3*d^3*b^2*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1
)^(1/2))+6*d^3*b^2*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-1/2*d^3*b^
2*arcsinh(c*x)^2/c^2/x^2)
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)^3\,(a+b\mathrm{arcsinh}(cx))^2}{x^3}\,dx = \int \frac{(c^2dx^2+d)^3(b\operatorname{arsinh}(cx)+a)^2}{x^3}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^6*d^3*x^6 + 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 + a^2*d^3
 + (b^2*c^6*d^3*x^6 + 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 + b^2*d^3)*arcs
inh(c*x)^2 + 2*(a*b*c^6*d^3*x^6 + 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2 + a
*b*d^3)*arcsinh(c*x))/x^3, x)
```

Sympy [F]

$$\begin{aligned}\int \frac{(d+c^2dx^2)^3\,(a+b\mathrm{arcsinh}(cx))^2}{x^3}\,dx = d^3\Bigg(&\int \frac{a^2}{x^3}\,dx + \int \frac{3a^2c^2}{x}\,dx + \int 3a^2c^4x\,dx \\ &+ \int a^2c^6x^3\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{x^3}\,dx \\ &+ \int \frac{2ab\operatorname{asinh}(cx)}{x^3}\,dx + \int \frac{3b^2c^2\operatorname{asinh}^2(cx)}{x}\,dx \\ &+ \int 3b^2c^4x\operatorname{asinh}^2(cx)\,dx \\ &+ \int b^2c^6x^3\operatorname{asinh}^2(cx)\,dx + \int \frac{6abc^2\operatorname{asinh}(cx)}{x}\,dx \\ &+ \int 6abc^4x\operatorname{asinh}(cx)\,dx \\ &+ \int 2abc^6x^3\operatorname{asinh}(cx)\,dx\Bigg)\end{aligned}$$

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2/x**3,x)
```

```
[Out] d**3*(Integral(a**2/x**3, x) + Integral(3*a**2*c**2/x, x) + Integral(3*a**2
*c**4*x, x) + Integral(a**2*c**6*x**3, x) + Integral(b**2*asinh(c*x)**2/x**
3, x) + Integral(2*a*b*asinh(c*x)/x**3, x) + Integral(3*b**2*c**2*asinh(c*x
)**2/x, x) + Integral(3*b**2*c**4*x*asinh(c*x)**2, x) + Integral(b**2*c**6*
x**3*asinh(c*x)**2, x) + Integral(6*a*b*c**2*asinh(c*x)/x, x) + Integral(6*
a*b*c**4*x*asinh(c*x), x) + Integral(2*a*b*c**6*x**3*asinh(c*x), x))
```

Maxima [F]

$$\int \frac{(d + c^2dx^2)^3 (a + b\text{arcsinh}(cx))^2}{x^3}\, dx = \int \frac{(c^2dx^2 + d)^3 (b \operatorname{arsinh}(cx) + a)^2}{x^3}\, dx$$

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*c^6*d^3*x^4 + 3/2*a^2*c^4*d^3*x^2 + 3*a^2*c^2*d^3*log(x) - a*b*d^3*
(sqrt(c^2*x^2 + 1)*c/x + arcsinh(c*x)/x^2) - 1/2*a^2*d^3/x^2 + integrate(b^
2*c^6*d^3*x^3*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*a*b*c^6*d^3*x^3*log(c*x +
sqrt(c^2*x^2 + 1)) + 3*b^2*c^4*d^3*x*log(c*x + sqrt(c^2*x^2 + 1))^2 + 6*a*b
*c^4*d^3*x*log(c*x + sqrt(c^2*x^2 + 1)) + 3*b^2*c^2*d^3*log(c*x + sqrt(c^2*
x^2 + 1))^2/x + 6*a*b*c^2*d^3*log(c*x + sqrt(c^2*x^2 + 1))/x + b^2*d^3*log(
c*x + sqrt(c^2*x^2 + 1))^2/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + c^2dx^2)^3 (a + b\text{arcsinh}(cx))^2}{x^3}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + c^2dx^2)^3 (a + b\text{arcsinh}(cx))^2}{x^3}\, dx = \int \frac{(a + b\,\mathrm{asinh}(c\,x))^2\, (d\,c^2\,x^2 + d)^3}{x^3}\, dx$$

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x^3, x)
```

3.224 $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 326

$$\begin{aligned}\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x^4}\,dx &= -\frac{b^2c^2d^3}{3x}+\frac{50}{9}b^2c^4d^3x+\frac{2}{27}b^2c^6d^3x^3\\ &\quad-5bc^3d^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\\ &\quad+\frac{1}{9}bc^3d^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\\ &\quad-\frac{bcd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{3x^2}\\ &\quad+\frac{16}{3}c^4d^3x(a+b\text{arcsinh}(cx))^2\\ &\quad+\frac{8}{3}c^4d^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2\\ &\quad-\frac{2c^2d^3(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{x}\\ &\quad-\frac{d^3(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2}{3x^3}\\ &\quad-\frac{34}{3}bc^3d^3(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)\\ &\quad-\frac{17}{3}b^2c^3d^3\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)\\ &\quad+\frac{17}{3}b^2c^3d^3\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\end{aligned}$$

```
[Out] -1/3*b^2*c^2*d^3/x+50/9*b^2*c^4*d^3*x+2/27*b^2*c^6*d^3*x^3+1/9*b*c^3*d^3*(c
^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))-1/3*b*c*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsi
nh(c*x))/x^2+16/3*c^4*d^3*x*(a+b*arcsinh(c*x))^2+8/3*c^4*d^3*x*(c^2*x^2+1)*
(a+b*arcsinh(c*x))^2-2*c^2*d^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/x-1/3*d^3
*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2/x^3-34/3*b*c^3*d^3*(a+b*arcsinh(c*x))*a
```

```
rctanh(c*x+(c^2*x^2+1)^(1/2))-17/3*b^2*c^3*d^3*polylog(2,-c*x-(c^2*x^2+1)^(
1/2))+17/3*b^2*c^3*d^3*polylog(2,c*x+(c^2*x^2+1)^(1/2))-5*b*c^3*d^3*(a+b*ar
csinh(c*x))*(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {5807, 5786, 5772, 5798, 8, 5808, 5806, 5816, 4267, 2317, 2438, 276}

$$\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = -\frac{34}{3}bc^3d^3\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)) + \frac{16}{3}c^4d^3x(a+b\text{arcsinh}(cx))^2 - \frac{2c^2d^3(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{x} - \frac{bcd^3(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))}{3x^2} - \frac{d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2}{3x^3} + \frac{8}{3}c^4d^3x(c^2x^2+1)(a+b\text{arcsinh}(cx))^2 + \frac{1}{9}bc^3d^3(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx)) - 5bc^3d^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx)) - \frac{17}{3}b^2c^3d^3\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right) + \frac{17}{3}b^2c^3d^3\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right) + \frac{2}{27}b^2c^6d^3x^3 + \frac{50}{9}b^2c^4d^3x - \frac{b^2c^2d^3}{3x}$$

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] -1/3*(b^2*c^2*d^3)/x + (50*b^2*c^4*d^3*x)/9 + (2*b^2*c^6*d^3*x^3)/27 - 5*b*
c^3*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]) + (b*c^3*d^3*(1 + c^2*x^2)^(
3/2)*(a + b*ArcSinh[c*x]))/9 - (b*c*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[
c*x]))/(3*x^2) + (16*c^4*d^3*x*(a + b*ArcSinh[c*x])^2)/3 + (8*c^4*d^3*x*(1
+ c^2*x^2)*(a + b*ArcSinh[c*x])^2)/3 - (2*c^2*d^3*(1 + c^2*x^2)^2*(a + b*Ar
cSinh[c*x])^2)/x - (d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/(3*x^3) - (
34*b*c^3*d^3*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/3 - (17*b^2*c^3*
d^3*PolyLog[2, -E^ArcSinh[c*x]])/3 + (17*b^2*c^3*d^3*PolyLog[2, E^ArcSinh[c
*x]])/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
```

```
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\frac{d^3(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2}{3x^3} + (2c^2d)\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x^2}\,dx + \frac{1}{3}(2bcd^3)\int \frac{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx$$

$$= -\frac{bcd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{3x^2} - \frac{2c^2d^3(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{x} - \frac{d^3(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2}{3x^3} + (8c^4d^2)\int (d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx + \frac{1}{3}(b^2c^2d^3)\int \frac{(1+c^2x^2)^2}{x^2}\,dx + \frac{1}{3}(5bc^3d^3)\int \frac{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{x}\,dx + (4bc^3d^3)\int \frac{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{x}\,dx$$

$$= \frac{17}{9}bc^3d^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{bcd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{3x^2} + \frac{8}{3}c^4d^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 - \frac{2c^2d^3(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{x} - \frac{d^3(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2}{3x^3} + \frac{1}{3}(b^2c^2d^3)\int \left(2c^2+\frac{1}{x^2}+c^4x^2\right)dx + \frac{1}{3}(5bc^3d^3)\int \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{x}\,dx + (4bc^3d^3)\int \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{x}\,dx + \frac{1}{3}(16c^4d^3)\int (a+b\text{arcsinh}(cx))^2\,dx - \frac{1}{9}(5b^2c^4d^3)\int (1+c^2x^2)\,dx - \frac{1}{3}(4b^2c^4d^3)\int (1+c^2x^2)\,dx - \frac{1}{3}(16bc^5d^3)\int x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx$$

$$= -\frac{b^2c^2d^3}{3x} - \frac{11}{9}b^2c^4d^3x - \frac{14}{27}b^2c^6d^3x^3 + \frac{17}{3}bc^3d^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) + \frac{1}{9}bc^3d^3\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{bcd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{3x^2} + \frac{16}{3}c^4d^3x(a+b\text{arcsinh}(cx))^2 + \frac{8}{3}c^4d^3x\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2 - \frac{2c^2d^3(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{x} - \frac{d^3(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2}{3x^3} + \frac{1}{3}\left(5bc^3d^3\right)\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{1+c^2x^2}}\,dx + \left(4bc^3d^3\right)\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{1+c^2x^2}}\,dx - \frac{1}{3}\left(5b^2c^4d^3\right)\int 1\,dx + \frac{1}{9}\left(16b^2c^4d^3\right)\int\left(1+c^2x^2\right)\,dx - \left(4b^2c^4d^3\right)\int 1\,dx - \frac{1}{3}\left(32bc^5d^3\right)\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx$$

$$= -\frac{b^2c^2d^3}{3x} - \frac{46}{9}b^2c^4d^3x + \frac{2}{27}b^2c^6d^3x^3 - 5bc^3d^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) + \frac{1}{9}bc^3d^3\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{bcd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{3x^2} + \frac{16}{3}c^4d^3x(a+b\text{arcsinh}(cx))^2 + \frac{8}{3}c^4d^3x\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2 - \frac{2c^2d^3(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{x} - \frac{d^3(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2}{3x^3} + \frac{1}{3}\left(5bc^3d^3\right)\text{Subst}\left(\int(a+bx)\text{csch}(x)\,dx, x, \text{arcsinh}(cx)\right) + \left(4bc^3d^3\right)\text{Subst}\left(\int(a+bx)\text{csch}(x)\,dx, x, \text{arcsinh}(cx)\right) + \frac{1}{3}\left(32b^2c^4d^3\right)\int 1\,dx$$

$$
\begin{aligned}
&= -\frac{b^2c^2d^3}{3x}+\frac{50}{9}b^2c^4d^3x+\frac{2}{27}b^2c^6d^3x^3-5bc^3d^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\\
&+\frac{1}{9}bc^3d^3\left(1+c^2x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))-\frac{bcd^3(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{3x^2}\\
&+\frac{16}{3}c^4d^3x(a+b\operatorname{arcsinh}(cx))^2+\frac{8}{3}c^4d^3x\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))^2\\
&-\frac{2c^2d^3(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{x}-\frac{d^3(1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))^2}{3x^3}\\
&-\frac{34}{3}bc^3d^3(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)\\
&-\frac{1}{3}\left(5b^2c^3d^3\right)\operatorname{Subst}\left(\int\log\left(1-e^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)\\
&+\frac{1}{3}\left(5b^2c^3d^3\right)\operatorname{Subst}\left(\int\log\left(1+e^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)\\
&-\left(4b^2c^3d^3\right)\operatorname{Subst}\left(\int\log\left(1-e^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)\\
&+\left(4b^2c^3d^3\right)\operatorname{Subst}\left(\int\log\left(1+e^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b^2c^2d^3}{3x}+\frac{50}{9}b^2c^4d^3x+\frac{2}{27}b^2c^6d^3x^3-5bc^3d^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\\
&+\frac{1}{9}bc^3d^3\left(1+c^2x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))-\frac{bcd^3(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{3x^2}\\
&+\frac{16}{3}c^4d^3x(a+b\operatorname{arcsinh}(cx))^2+\frac{8}{3}c^4d^3x\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))^2\\
&-\frac{2c^2d^3(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{x}-\frac{d^3(1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))^2}{3x^3}\\
&-\frac{34}{3}bc^3d^3(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)\\
&-\frac{1}{3}\left(5b^2c^3d^3\right)\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)\\
&+\frac{1}{3}\left(5b^2c^3d^3\right)\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)\\
&-\left(4b^2c^3d^3\right)\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)\\
&+\left(4b^2c^3d^3\right)\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)
\end{aligned}
$$

$$= -\frac{b^2c^2d^3}{3x} + \frac{50}{9}b^2c^4d^3x + \frac{2}{27}b^2c^6d^3x^3 - 5bc^3d^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) + \frac{1}{9}bc^3d^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{bcd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{3x^2} + \frac{16}{3}c^4d^3x(a+b\text{arcsinh}(cx))^2 + \frac{8}{3}c^4d^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 - \frac{2c^2d^3(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{x} - \frac{d^3(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2}{3x^3} - \frac{34}{3}bc^3d^3(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) - \frac{17}{3}b^2c^3d^3\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right) + \frac{17}{3}b^2c^3d^3\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.41

$$\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = \frac{d^3\left(-9a^2-81a^2c^2x^2-9b^2c^2x^2+81a^2c^4x^4+150b^2c^4x^4+9a^2c^6x^6+2b^2c^6x^6-9abcx\sqrt{1+c^2x^2}-150abc^3\right.}{}$$

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] (d^3*(-9*a^2 - 81*a^2*c^2*x^2 - 9*b^2*c^2*x^2 + 81*a^2*c^4*x^4 + 150*b^2*c^
4*x^4 + 9*a^2*c^6*x^6 + 2*b^2*c^6*x^6 - 9*a*b*c*x*Sqrt[1 + c^2*x^2] - 150*a
*b*c^3*x^3*Sqrt[1 + c^2*x^2] - 6*a*b*c^5*x^5*Sqrt[1 + c^2*x^2] - 18*a*b*Arc
Sinh[c*x] - 162*a*b*c^2*x^2*ArcSinh[c*x] + 162*a*b*c^4*x^4*ArcSinh[c*x] + 1
8*a*b*c^6*x^6*ArcSinh[c*x] - 9*b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - 150
*b^2*c^3*x^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - 6*b^2*c^5*x^5*Sqrt[1 + c^2*x^
2]*ArcSinh[c*x] - 9*b^2*ArcSinh[c*x]^2 - 81*b^2*c^2*x^2*ArcSinh[c*x]^2 + 81
*b^2*c^4*x^4*ArcSinh[c*x]^2 + 9*b^2*c^6*x^6*ArcSinh[c*x]^2 - 153*a*b*c^3*x^
3*ArcTanh[Sqrt[1 + c^2*x^2]] + 153*b^2*c^3*x^3*ArcSinh[c*x]*Log[1 - E^(-Arc
Sinh[c*x])] - 153*b^2*c^3*x^3*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 153
*b^2*c^3*x^3*PolyLog[2, -E^(-ArcSinh[c*x])] - 153*b^2*c^3*x^3*PolyLog[2, E^
(-ArcSinh[c*x])]))/(27*x^3)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.44

method	result
derivativedivides	$c^{3}\left(d^{3}a^{2}\left(\frac{c^{3}x^{3}}{3}+3cx-\frac{1}{3c^{3}x^{3}}-\frac{3}{cx}\right)-\frac{d^{3}b^{2}\operatorname{arcsinh}(cx)\sqrt{c^{2}x^{2}+1}}{3c^{2}x^{2}}-\frac{2d^{3}b^{2}\operatorname{arcsinh}(cx)\sqrt{c^{2}x^{2}+1}\,c^{2}x^{2}}{9}+\frac{2d}{}\right.$
default	$c^{3}\left(d^{3}a^{2}\left(\frac{c^{3}x^{3}}{3}+3cx-\frac{1}{3c^{3}x^{3}}-\frac{3}{cx}\right)-\frac{d^{3}b^{2}\operatorname{arcsinh}(cx)\sqrt{c^{2}x^{2}+1}}{3c^{2}x^{2}}-\frac{2d^{3}b^{2}\operatorname{arcsinh}(cx)\sqrt{c^{2}x^{2}+1}\,c^{2}x^{2}}{9}+\frac{2d}{}\right.$
parts	$d^{3}a^{2}\left(\frac{c^{6}x^{3}}{3}+3c^{4}x-\frac{3c^{2}}{x}-\frac{1}{3x^{3}}\right)-\frac{d^{3}b^{2}c\,\operatorname{arcsinh}(cx)\sqrt{c^{2}x^{2}+1}}{3x^{2}}-\frac{2d^{3}b^{2}c^{5}\operatorname{arcsinh}(cx)\sqrt{c^{2}x^{2}+1}\,x^{2}}{9}-\frac{17b^{2}c}{}$

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(d^3*a^2*(1/3*c^3*x^3+3*c*x-1/3/c^3/x^3-3/c/x)-1/3*d^3*b^2/c^2/x^2*arcs
inh(c*x)*(c^2*x^2+1)^(1/2)-2/9*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^2*x
^2+2/27*d^3*b^2*c^3*x^3-1/3*d^3*b^2/c/x+50/9*d^3*b^2*c*x-3*d^3*b^2*arcsinh(
c*x)^2/c/x-1/3*d^3*b^2/c^3/x^3*arcsinh(c*x)^2+1/3*d^3*b^2*arcsinh(c*x)^2*c^
3*x^3+3*d^3*b^2*arcsinh(c*x)^2*c*x-50/9*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1
/2)+17/3*d^3*b^2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-17/3*d^3*b^2*arcs
inh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+17/3*d^3*b^2*polylog(2,c*x+(c^2*x^2+1)
^(1/2))-17/3*d^3*b^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*d^3*a*b*(1/3*arcsi
nh(c*x)*c^3*x^3+3*arcsinh(c*x)*c*x-1/3*arcsinh(c*x)/c^3/x^3-3*arcsinh(c*x)/
c/x-1/9*c^2*x^2*(c^2*x^2+1)^(1/2)-25/9*(c^2*x^2+1)^(1/2)-1/6/c^2/x^2*(c^2*x
^2+1)^(1/2)-17/6*arctanh(1/(c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)^3\,(a+b\text{arcsinh}(cx))^2}{x^4}\,dx=\int \frac{(c^2dx^2+d)^3(b\,\text{arsinh}\,(cx)+a)^2}{x^4}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^6*d^3*x^6 + 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 + a^2*d^3
 + (b^2*c^6*d^3*x^6 + 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 + b^2*d^3)*arcs
inh(c*x)^2 + 2*(a*b*c^6*d^3*x^6 + 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2 + a
*b*d^3)*arcsinh(c*x))/x^4, x)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^3\,(a+b\mathrm{arcsinh}(cx))^2}{x^4}\,dx = d^3\left(\int 3a^2c^4\,dx+\int \frac{a^2}{x^4}\,dx+\int \frac{3a^2c^2}{x^2}\,dx+\int a^2c^6x^2\,dx+\int 3b^2c^4\operatorname{asinh}^2(cx)\,dx+\int \frac{b^2\operatorname{asinh}^2(cx)}{x^4}\,dx+\int 6abc^4\operatorname{asinh}(cx)\,dx+\int \frac{2ab\operatorname{asinh}(cx)}{x^4}\,dx+\int \frac{3b^2c^2\operatorname{asinh}^2(cx)}{x^2}\,dx+\int b^2c^6x^2\operatorname{asinh}^2(cx)\,dx+\int \frac{6abc^2\operatorname{asinh}(cx)}{x^2}\,dx+\int 2abc^6x^2\operatorname{asinh}(cx)\,dx\right)$$

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2/x**4,x)
```

```
[Out] d**3*(Integral(3*a**2*c**4, x) + Integral(a**2/x**4, x) + Integral(3*a**2*c
**2/x**2, x) + Integral(a**2*c**6*x**2, x) + Integral(3*b**2*c**4*asinh(c*x
)**2, x) + Integral(b**2*asinh(c*x)**2/x**4, x) + Integral(6*a*b*c**4*asinh
(c*x), x) + Integral(2*a*b*asinh(c*x)/x**4, x) + Integral(3*b**2*c**2*asinh
(c*x)**2/x**2, x) + Integral(b**2*c**6*x**2*asinh(c*x)**2, x) + Integral(6*
a*b*c**2*asinh(c*x)/x**2, x) + Integral(2*a*b*c**6*x**2*asinh(c*x), x))
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)^3\,(a+b\mathrm{arcsinh}(cx))^2}{x^4}\,dx = \int \frac{(c^2dx^2+d)^3(b\operatorname{arsinh}(cx)+a)^2}{x^4}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*c^6*d^3*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^
2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^6*d^3 + 3*b^2*c^4*d^3*x*arcsinh(c*x)^2
+ 6*b^2*c^4*d^3*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + 3*a^2*c^4*d^3*x +
6*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*c^3*d^3 - 6*(c*arcsinh(1/(c*ab
s(x))) + arcsinh(c*x)/x)*a*b*c^2*d^3 + 1/3*((c^2*arcsinh(1/(c*abs(x))) - sq
rt(c^2*x^2 + 1)/x^2)*c - 2*arcsinh(c*x)/x^3)*a*b*d^3 - 3*a^2*c^2*d^3/x - 1/
3*a^2*d^3/x^3 + 1/3*(b^2*c^6*d^3*x^6 - 9*b^2*c^2*d^3*x^2 - b^2*d^3)*log(c*x
 + sqrt(c^2*x^2 + 1))^2/x^3 - integrate(2/3*(b^2*c^9*d^3*x^8 + b^2*c^7*d^3*
x^6 - 9*b^2*c^5*d^3*x^4 - 10*b^2*c^3*d^3*x^2 - b^2*c*d^3 + (b^2*c^8*d^3*x^7
 - 9*b^2*c^4*d^3*x^3 - b^2*c^2*d^3*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2
*x^2 + 1))/(c^3*x^6 + c*x^4 + (c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2}{x^4} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2}{x^4} \, dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)^3}{x^4} \, dx$$

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x^4, x)
```

3.225 $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 277

$$\begin{aligned}\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = & -\frac{22b^2x}{9c^4d}+\frac{2b^2x^3}{27c^2d}+\frac{22b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^5d} \\ & -\frac{2bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^3d} \\ & -\frac{x(a+b\text{arcsinh}(cx))^2}{c^4d}+\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d} \\ & +\frac{2(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^5d} \\ & -\frac{2ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^5d} \\ & +\frac{2ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^5d} \\ & +\frac{2ib^2\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{c^5d} \\ & -\frac{2ib^2\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{c^5d}\end{aligned}$$

```
[Out] -22/9*b^2*x/c^4/d+2/27*b^2*x^3/c^2/d-x*(a+b*arcsinh(c*x))^2/c^4/d+1/3*x^3*(
a+b*arcsinh(c*x))^2/c^2/d+2*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/
2))/c^5/d-2*I*b*(a+b*arcsinh(c*x))*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^
5/d+2*I*b*(a+b*arcsinh(c*x))*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d+2*I
*b^2*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d-2*I*b^2*polylog(3,I*(c*x+(
c^2*x^2+1)^(1/2)))/c^5/d+22/9*b*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^5/d-
2/9*b*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3/d
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5812, 5789, 4265, 2611, 2320, 6724, 5798, 8, 30}

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{c^5d} - \frac{2ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c^5d} + \frac{2ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c^5d} - \frac{x(a+b\text{arcsinh}(cx))^2}{c^4d} + \frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d} + \frac{22b\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{9c^5d} - \frac{2bx^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{9c^3d} + \frac{2ib^2\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{c^5d} - \frac{2ib^2\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{c^5d} - \frac{22b^2x}{9c^4d} + \frac{2b^2x^3}{27c^2d}$$

```
[In] Int[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] (-22*b^2*x)/(9*c^4*d) + (2*b^2*x^3)/(27*c^2*d) + (22*b*Sqrt[1 + c^2*x^2]*(a
 + b*ArcSinh[c*x]))/(9*c^5*d) - (2*b*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c
*x]))/(9*c^3*d) - (x*(a + b*ArcSinh[c*x])^2)/(c^4*d) + (x^3*(a + b*ArcSinh[
c*x])^2)/(3*c^2*d) + (2*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/(c^5
*d) - ((2*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^5*d
) + ((2*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/(c^5*d) + (
(2*I)*b^2*PolyLog[3, (-I)*E^ArcSinh[c*x]])/(c^5*d) - ((2*I)*b^2*PolyLog[3,
I*E^ArcSinh[c*x]])/(c^5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d} - \frac{\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx}{c^2} - \frac{(2b)\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{3cd}$$

$$= -\frac{2bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^3d} - \frac{x(a+b\text{arcsinh}(cx))^2}{c^4d} + \frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d} + \frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx}{c^4} + \frac{(4b)\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{9c^3d} + \frac{(2b)\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{c^3d} + \frac{(2b^2)\int x^2\,dx}{9c^2d}$$

$$= \frac{2b^2x^3}{27c^2d} + \frac{22b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^5d} - \frac{2bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^3d} - \frac{x(a+b\text{arcsinh}(cx))^2}{c^4d} + \frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d} + \frac{\text{Subst}\left(\int (a+bx)^2\text{sech}(x)\,dx, x, \text{arcsinh}(cx)\right)}{c^5d} - \frac{(4b^2)\int 1\,dx}{9c^4d} - \frac{(2b^2)\int 1\,dx}{c^4d}$$

$$= -\frac{22b^2x}{9c^4d} + \frac{2b^2x^3}{27c^2d} + \frac{22b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^5d} - \frac{2bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^3d} - \frac{x(a+b\text{arcsinh}(cx))^2}{c^4d} + \frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d} + \frac{2(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^5d} - \frac{(2ib)\text{Subst}\left(\int (a+bx)\log\left(1-ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{c^5d} + \frac{(2ib)\text{Subst}\left(\int (a+bx)\log\left(1+ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{c^5d}$$

$$= -\frac{22b^2x}{9c^4d} + \frac{2b^2x^3}{27c^2d} + \frac{22b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^5d} - \frac{2bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^3d} - \frac{x(a+b\text{arcsinh}(cx))^2}{c^4d} + \frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d} + \frac{2(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^5d} - \frac{2ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^5d} + \frac{2ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^5d} + \frac{(2ib^2)\,\text{Subst}\left(\int \text{PolyLog}(2,-ie^x)\,dx, x, \text{arcsinh}(cx)\right)}{c^5d} - \frac{(2ib^2)\,\text{Subst}\left(\int \text{PolyLog}(2,ie^x)\,dx, x, \text{arcsinh}(cx)\right)}{c^5d}$$

$$= -\frac{22b^2x}{9c^4d} + \frac{2b^2x^3}{27c^2d} + \frac{22b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^5d} - \frac{2bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^3d} - \frac{x(a+b\text{arcsinh}(cx))^2}{c^4d} + \frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d} + \frac{2(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^5d} - \frac{2ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^5d} + \frac{2ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^5d} + \frac{(2ib^2)\,\text{Subst}\left(\int \frac{\text{PolyLog}(2,-ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{c^5d} - \frac{(2ib^2)\,\text{Subst}\left(\int \frac{\text{PolyLog}(2,ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{c^5d}$$

$$= -\frac{22b^2x}{9c^4d} + \frac{2b^2x^3}{27c^2d} + \frac{22b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^5d} - \frac{2bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^3d} - \frac{x(a+b\text{arcsinh}(cx))^2}{c^4d} + \frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d} + \frac{2(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^5d} - \frac{2ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^5d} + \frac{2ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^5d} + \frac{2ib^2\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{c^5d} - \frac{2ib^2\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{c^5d}$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.32

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx$$
$$=\frac{-3a^2cx+a^2c^3x^3+3a^2\arctan(cx)-\frac{2}{3}ab\left(-11\sqrt{1+c^2x^2}+c^2x^2\sqrt{1+c^2x^2}+9cx\text{arcsinh}(cx)-3c^3x^3\text{arcsin}\right.}{}$$

[In] Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]

[Out] (-3*a^2*c*x + a^2*c^3*x^3 + 3*a^2*ArcTan[c*x] - (2*a*b*(-11*Sqrt[1 + c^2*x^2] + c^2*x^2*Sqrt[1 + c^2*x^2] + 9*c*x*ArcSinh[c*x] - 3*c^3*x^3*ArcSinh[c*x] - (9*I)*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + (9*I)*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] + (9*I)*PolyLog[2, (-I)*E^ArcSinh[c*x]] - (9*I)*PolyLog[2, I*E^ArcSinh[c*x]]))/3 + 3*b^2*((5*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/2 - (5*c*x*(2 + ArcSinh[c*x]^2))/4 - (ArcSinh[c*x]*Cosh[3*ArcSinh[c*x]])/18 + I*(-(ArcSinh[c*x]^2*(Log[1 - I/E^ArcSinh[c*x]] - Log[1 + I/E^ArcSinh[c*x]])) - 2*ArcSinh[c*x]*(PolyLog[2, (-I)/E^ArcSinh[c*x]] - PolyLog[2, I/E^ArcSinh[c*x]]) - 2*PolyLog[3, (-I)/E^ArcSinh[c*x]] + 2*PolyLog[3, I/E^ArcSinh[c*x]]) + ((2 + 9*ArcSinh[c*x]^2)*Sinh[3*ArcSinh[c*x]])/108))/(3*c^5*d)

Maple [F]

$$\int \frac{x^4(a+b\,\text{arcsinh}\,(cx))^2}{c^2d\,x^2+d}dx$$

[In] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)

[Out] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)

Fricas [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^4}{c^2dx^2+d}\,dx$$

[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="fricas")

[Out] integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)/(c^2*d*x^2 + d), x)

Sympy [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \frac{\int \frac{a^2x^4}{c^2x^2+1}\,dx + \int \frac{b^2x^4\,\text{asinh}^2\,(cx)}{c^2x^2+1}\,dx + \int \frac{2abx^4\,\text{asinh}\,(cx)}{c^2x^2+1}\,dx}{d}$$

```
[In] integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d),x)
[Out] (Integral(a**2*x**4/(c**2*x**2 + 1), x) + Integral(b**2*x**4*asinh(c*x)**2/
(c**2*x**2 + 1), x) + Integral(2*a*b*x**4*asinh(c*x)/(c**2*x**2 + 1), x))/d
```

Maxima [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^4}{c^2dx^2+d}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="maxima")
[Out] 1/3*a^2*((c^2*x^3 - 3*x)/(c^4*d) + 3*arctan(c*x)/(c^5*d)) + integrate(b^2*x
^4*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^2 + d) + 2*a*b*x^4*log(c*x + sqr
t(c^2*x^2 + 1))/(c^2*d*x^2 + d), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \int \frac{x^4\,(a+b\,\text{asinh}(c\,x))^2}{d\,c^2\,x^2+d}\,dx$$

```
[In] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2),x)
[Out] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2), x)
```

3.226 $\int \frac{x^3(a+b\,\mathbf{arcsinh}(cx))^2}{d+c^2dx^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 199

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \frac{b^2x^2}{4c^2d} - \frac{bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2c^3d} + \frac{(a+b\text{arcsinh}(cx))^2}{4c^4d} + \frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d} + \frac{(a+b\text{arcsinh}(cx))^3}{3bc^4d} - \frac{(a+b\text{arcsinh}(cx))^2\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^4d} - \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c^4d} + \frac{b^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{2c^4d}$$

```
[Out] 1/4*b^2*x^2/c^2/d+1/4*(a+b*arcsinh(c*x))^2/c^4/d+1/2*x^2*(a+b*arcsinh(c*x))
^2/c^2/d+1/3*(a+b*arcsinh(c*x))^3/b/c^4/d-(a+b*arcsinh(c*x))^2*ln(1+(c*x+(c
^2*x^2+1)^(1/2))^2)/c^4/d-b*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^2*x^2+1)^
(1/2))^2)/c^4/d+1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/c^4/d-1/2*b*x
*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3/d
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used

$= \{5812, 5797, 3799, 2221, 2611, 2320, 6724, 5783, 30\}$

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = -\frac{b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c^4d} + \frac{(a+b\text{arcsinh}(cx))^3}{3bc^4d} + \frac{(a+b\text{arcsinh}(cx))^2}{4c^4d} - \frac{\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))^2}{c^4d} + \frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d} - \frac{bx\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^3d} + \frac{b^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{2c^4d} + \frac{b^2x^2}{4c^2d}$$

```
[In] Int[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] (b^2*x^2)/(4*c^2*d) - (b*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*c^3*d
) + (a + b*ArcSinh[c*x])^2/(4*c^4*d) + (x^2*(a + b*ArcSinh[c*x])^2)/(2*c^2*
d) + (a + b*ArcSinh[c*x])^3/(3*b*c^4*d) - ((a + b*ArcSinh[c*x])^2*Log[1 + E
^(2*ArcSinh[c*x])])/(c^4*d) - (b*(a + b*ArcSinh[c*x])*PolyLog[2, -E^(2*ArcS
inh[c*x])])/(c^4*d) + (b^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*c^4*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
```

```
- 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d} - \frac{\int \frac{x(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx}{c^2} - \frac{b\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{cd}$$

$$= -\frac{bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2c^3d} + \frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d} - \frac{\text{Subst}\left(\int (a+bx)^2\tanh(x)\,dx, x, \text{arcsinh}(cx)\right)}{c^4d} + \frac{b\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{2c^3d} + \frac{b^2\int x\,dx}{2c^2d}$$

$$= \frac{b^2x^2}{4c^2d} - \frac{bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2c^3d} + \frac{(a+b\text{arcsinh}(cx))^2}{4c^4d} + \frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d} + \frac{(a+b\text{arcsinh}(cx))^3}{3bc^4d} - \frac{2\text{Subst}\left(\int \frac{e^{2x}(a+bx)^2}{1+e^{2x}}\,dx, x, \text{arcsinh}(cx)\right)}{c^4d}$$

$$= \frac{b^2x^2}{4c^2d} - \frac{bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2c^3d} + \frac{(a+b\text{arcsinh}(cx))^2}{4c^4d} + \frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d} + \frac{(a+b\text{arcsinh}(cx))^3}{3bc^4d} - \frac{(a+b\text{arcsinh}(cx))^2\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^4d} + \frac{(2b)\text{Subst}\left(\int (a+bx)\log\left(1+e^{2x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{c^4d}$$

$$= \frac{b^2x^2}{4c^2d} - \frac{bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2c^3d} + \frac{(a+b\text{arcsinh}(cx))^2}{4c^4d} + \frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d} + \frac{(a+b\text{arcsinh}(cx))^3}{3bc^4d} - \frac{(a+b\text{arcsinh}(cx))^2\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^4d} - \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c^4d} + \frac{b^2\text{Subst}\left(\int \text{PolyLog}\left(2,-e^{2x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{c^4d}$$

$$= \frac{b^2x^2}{4c^2d} - \frac{bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2c^3d} + \frac{(a+b\text{arcsinh}(cx))^2}{4c^4d} + \frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d} + \frac{(a+b\text{arcsinh}(cx))^3}{3bc^4d} - \frac{(a+b\text{arcsinh}(cx))^2\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^4d} - \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c^4d} + \frac{b^2\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx, x, e^{2\text{arcsinh}(cx)}\right)}{2c^4d}$$

$$
\begin{aligned}
&= \frac{b^2x^2}{4c^2d} - \frac{bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2c^3d} + \frac{(a+b\text{arcsinh}(cx))^2}{4c^4d} \\
&\quad + \frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d} + \frac{(a+b\text{arcsinh}(cx))^3}{3bc^4d} \\
&\quad - \frac{(a+b\text{arcsinh}(cx))^2\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^4d} \\
&\quad - \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c^4d} + \frac{b^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{2c^4d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.48

$$
\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx =
-\frac{-12a^2c^2x^2+12abcx\sqrt{1+c^2x^2}-24abc^2x^2\text{arcsinh}(cx)-24ab\text{arcsinh}(cx)^2+8b^2\text{arcsinh}(cx)^3-3b^2\cosh
$$

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] -1/24*(-12*a^2*c^2*x^2 + 12*a*b*c*x*Sqrt[1 + c^2*x^2] - 24*a*b*c^2*x^2*ArcS
inh[c*x] - 24*a*b*ArcSinh[c*x]^2 + 8*b^2*ArcSinh[c*x]^3 - 3*b^2*Cosh[2*ArcS
inh[c*x]] - 6*b^2*ArcSinh[c*x]^2*Cosh[2*ArcSinh[c*x]] + 24*b^2*ArcSinh[c*x]
^2*Log[1 + E^(-2*ArcSinh[c*x])] + 48*a*b*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c
*x]] + 48*a*b*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] + 12*a^2*Log[1 + c^2*x
^2] + 12*a*b*Log[-(c*x) + Sqrt[1 + c^2*x^2]] - 24*b^2*ArcSinh[c*x]*PolyLog[
2, -E^(-2*ArcSinh[c*x])] + 48*a*b*PolyLog[2, (-I)*E^ArcSinh[c*x]] + 48*a*b*
PolyLog[2, I*E^ArcSinh[c*x]] - 12*b^2*PolyLog[3, -E^(-2*ArcSinh[c*x])] + 6*
b^2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]])/(c^4*d)
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.72

method	result
derivativedivides	$\frac{\frac{a^2\left(\frac{c^2x^2}{2}-\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d}+\frac{b^2\,\mathrm{arcsinh}(cx)^3}{3d}+\frac{b^2\,\mathrm{arcsinh}(cx)^2c^2x^2}{2d}-\frac{b^2\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,cx}{2d}+\frac{b^2\,\mathrm{arcsinh}(cx)^2}{4d}+\frac{b^2c^2x^2}{4d}+\frac{b^2}{8d}-}{}$
default	$\frac{\frac{a^2\left(\frac{c^2x^2}{2}-\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d}+\frac{b^2\,\mathrm{arcsinh}(cx)^3}{3d}+\frac{b^2\,\mathrm{arcsinh}(cx)^2c^2x^2}{2d}-\frac{b^2\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,cx}{2d}+\frac{b^2\,\mathrm{arcsinh}(cx)^2}{4d}+\frac{b^2c^2x^2}{4d}+\frac{b^2}{8d}-}{}$
parts	$\frac{a^2\left(\frac{x^2}{2c^2}-\frac{\ln\left(c^2x^2+1\right)}{2c^4}\right)}{d}+\frac{b^2\,\mathrm{arcsinh}(cx)^3}{3d\,c^4}+\frac{b^2\,\mathrm{arcsinh}(cx)^2x^2}{2d\,c^2}-\frac{b^2\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,x}{2d\,c^3}+\frac{b^2x^2}{4c^2d}+\frac{b^2\,\mathrm{arcsinh}}{4d\,c}$

```
[In] int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(a^2/d*(1/2*c^2*x^2-1/2*ln(c^2*x^2+1))+1/3*b^2/d*arcsinh(c*x)^3+1/2*b
^2/d*arcsinh(c*x)^2*c^2*x^2-1/2*b^2/d*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x+1/
4*b^2/d*arcsinh(c*x)^2+1/4*b^2/d*c^2*x^2+1/8*b^2/d-b^2/d*arcsinh(c*x)^2*ln(
1+(c*x+(c^2*x^2+1)^(1/2))^2)-b^2/d*arcsinh(c*x)*polylog(2,-(c*x+(c^2*x^2+1)
^(1/2))^2)+1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d+a*b/d*arcsinh(c*
x)^2+a*b/d*arcsinh(c*x)*c^2*x^2-1/2*a*b/d*c*x*(c^2*x^2+1)^(1/2)+1/2*a*b/d*a
rcsinh(c*x)-2*a*b/d*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-a*b/d*poly
log(2,-(c*x+(c^2*x^2+1)^(1/2))^2))
```

Fricas [F]

$$\int \frac{x^3(a+b\mathrm{arcsinh}(cx))^2}{d+c^2dx^2}\,dx=\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2x^3}{c^2dx^2+d}\,dx$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*x) + a^2*x^3)/(c^2*d
*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x^3(a+b\mathrm{arcsinh}(cx))^2}{d+c^2dx^2}\,dx=\frac{\int \frac{a^2x^3}{c^2x^2+1}\,dx+\int \frac{b^2x^3\,\mathrm{asinh}^2\,(cx)}{c^2x^2+1}\,dx+\int \frac{2abx^3\,\mathrm{asinh}\,(cx)}{c^2x^2+1}\,dx}{d}$$

```
[In] integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a**2*x**3/(c**2*x**2 + 1), x) + Integral(b**2*x**3*asinh(c*x)**2/
(c**2*x**2 + 1), x) + Integral(2*a*b*x**3*asinh(c*x)/(c**2*x**2 + 1), x))/d
```

Maxima [F]

$$\int \frac{x^3(a + b\text{arcsinh}(cx))^2}{d + c^2dx^2} \, dx = \int \frac{(b \operatorname{arsinh}(cx) + a)^2 x^3}{c^2dx^2 + d} \, dx$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(x^2/(c^2*d) - log(c^2*x^2 + 1)/(c^4*d)) + 1/2*(b^2*c^2*x^2 - b^2*l
og(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d) + integrate(-(b^2*c
^2*x^2 - (2*a*b*c^4 - b^2*c^4)*x^4 - (b^2*c^2*x^2 + b^2)*log(c^2*x^2 + 1) -
 (b^2*c*x*log(c^2*x^2 + 1) + (2*a*b*c^3 - b^2*c^3)*x^3)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d*x^3 + c^4*d*x + (c^5*d*x^2 + c^3*d)*sqr
t(c^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\text{arcsinh}(cx))^2}{d + c^2dx^2} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\text{arcsinh}(cx))^2}{d + c^2dx^2} \, dx = \int \frac{x^3 \left(a + b \operatorname{asinh}(c\,x)\right)^2}{d\,c^2\,x^2 + d} \, dx$$

```
[In] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2),x)
```

```
[Out] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2), x)
```

3.227 $\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 198

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \frac{2b^2x}{c^2d} - \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c^3d} + \frac{x(a+b\text{arcsinh}(cx))^2}{c^2d} - \frac{2(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^3d} + \frac{2ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^3d} - \frac{2ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^3d} - \frac{2ib^2\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{c^3d} + \frac{2ib^2\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{c^3d}$$

```
[Out] 2*b^2*x/c^2/d+x*(a+b*arcsinh(c*x))^2/c^2/d-2*(a+b*arcsinh(c*x))^2*arctan(c*
x+(c^2*x^2+1)^(1/2))/c^3/d+2*I*b*(a+b*arcsinh(c*x))*polylog(2,-I*(c*x+(c^2*
x^2+1)^(1/2)))/c^3/d-2*I*b*(a+b*arcsinh(c*x))*polylog(2,I*(c*x+(c^2*x^2+1)^
(1/2)))/c^3/d-2*I*b^2*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d+2*I*b^2*p
olylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d-2*b*(a+b*arcsinh(c*x))*(c^2*x^2+1
)^(1/2)/c^3/d
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5812, 5789, 4265, 2611, 2320, 6724, 5798, 8}

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = -\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{c^3d} + \frac{2ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c^3d} - \frac{2ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c^3d} + \frac{x(a+b\text{arcsinh}(cx))^2}{c^2d} - \frac{2b\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^3d} - \frac{2ib^2\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{c^3d} + \frac{2ib^2\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{c^3d} + \frac{2b^2x}{c^2d}$$

```
[In] Int[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] (2*b^2*x)/(c^2*d) - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(c^3*d) +
(x*(a + b*ArcSinh[c*x])^2)/(c^2*d) - (2*(a + b*ArcSinh[c*x])^2*ArcTan[E^Arc
Sinh[c*x]])/(c^3*d) + ((2*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSi
nh[c*x]])/(c^3*d) - ((2*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*
x]])/(c^3*d) - ((2*I)*b^2*PolyLog[3, (-I)*E^ArcSinh[c*x]])/(c^3*d) + ((2*I)
*b^2*PolyLog[3, I*E^ArcSinh[c*x]])/(c^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{x(a+b\text{arcsinh}(cx))^2}{c^2d} - \frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx}{c^2} - \frac{(2b)\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{cd}$$

$$= -\frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c^3d} + \frac{x(a+b\text{arcsinh}(cx))^2}{c^2d} - \frac{\text{Subst}\left(\int (a+bx)^2\text{sech}(x)\,dx, x, \text{arcsinh}(cx)\right)}{c^3d} + \frac{(2b^2)\int 1\,dx}{c^2d}$$

$$= \frac{2b^2x}{c^2d} - \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c^3d} + \frac{x(a+b\text{arcsinh}(cx))^2}{c^2d} - \frac{2(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^3d} + \frac{(2ib)\text{Subst}\left(\int (a+bx)\log\left(1-ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{c^3d} - \frac{(2ib)\text{Subst}\left(\int (a+bx)\log\left(1+ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{c^3d}$$

$$= \frac{2b^2x}{c^2d} - \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c^3d} + \frac{x(a+b\text{arcsinh}(cx))^2}{c^2d} - \frac{2(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^3d} + \frac{2ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^3d} - \frac{2ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^3d} - \frac{(2ib^2)\,\text{Subst}\left(\int \text{PolyLog}\left(2,-ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{c^3d} + \frac{(2ib^2)\,\text{Subst}\left(\int \text{PolyLog}\left(2,ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{c^3d}$$

$$= \frac{2b^2x}{c^2d} - \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c^3d} + \frac{x(a+b\text{arcsinh}(cx))^2}{c^2d} - \frac{2(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^3d} + \frac{2ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^3d} - \frac{2ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^3d} - \frac{(2ib^2)\,\text{Subst}\left(\int \frac{\text{PolyLog}(2,-ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{c^3d} + \frac{(2ib^2)\,\text{Subst}\left(\int \frac{\text{PolyLog}(2,ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{c^3d}$$

$$
\begin{aligned}
&= \frac{2b^2x}{c^2d} - \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c^3d} + \frac{x(a+b\text{arcsinh}(cx))^2}{c^2d} \\
&\quad - \frac{2(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^3d} \\
&\quad + \frac{2ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^3d} \\
&\quad - \frac{2ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^3d} \\
&\quad - \frac{2ib^2\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{c^3d} + \frac{2ib^2\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{c^3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.60

$$
\begin{aligned}
&\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \frac{a^2x}{c^2d} - \frac{a^2\arctan(cx)}{c^3d} \\
&\quad + \frac{2ab\left(-\sqrt{1+c^2x^2}+cx\text{arcsinh}(cx)+\frac{1}{2}i\left(-\frac{1}{2}\text{arcsinh}(cx)^2+2\text{arcsinh}(cx)\log\left(1+ie^{\text{arcsinh}(cx)}\right)+2\,\text{PolyLo}\right.\right.}{} \\
&\quad + \frac{b^2\left(-2\sqrt{1+c^2x^2}\text{arcsinh}(cx)+cx(2+\text{arcsinh}(cx)^2)-i\left(-\text{arcsinh}(cx)^2\left(\log\left(1-ie^{-\text{arcsinh}(cx)}\right)-\log\left(1\right.\right.\right.\right.}{}
\end{aligned}
$$

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] (a^2*x)/(c^2*d) - (a^2*ArcTan[c*x])/(c^3*d) + (2*a*b*(-Sqrt[1 + c^2*x^2] +
c*x*ArcSinh[c*x] + (I/2)*(-1/2*ArcSinh[c*x]^2 + 2*ArcSinh[c*x]*Log[1 + I*E^
ArcSinh[c*x]] + 2*PolyLog[2, (-I)*E^ArcSinh[c*x]]) - (I/2)*(-1/2*ArcSinh[c*
x]^2 + 2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + 2*PolyLog[2, I*E^ArcSinh[
c*x]])))/(c^3*d) + (b^2*(-2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + c*x*(2 + ArcSi
nh[c*x]^2) - I*(-(ArcSinh[c*x]^2*(Log[1 - I/E^ArcSinh[c*x]] - Log[1 + I/E^A
rcSinh[c*x]])) - 2*ArcSinh[c*x]*(PolyLog[2, (-I)/E^ArcSinh[c*x]] - PolyLog[
2, I/E^ArcSinh[c*x]]) - 2*(PolyLog[3, (-I)/E^ArcSinh[c*x]] - PolyLog[3, I/E
^ArcSinh[c*x]]))))/(c^3*d)
```

Maple [F]

$$
\int \frac{x^2(a+b\,\text{arcsinh}\,(cx))^2}{c^2d\,x^2+d}dx
$$

```
[In] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

```
[Out] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

Fricas [F]

$$\int \frac{x^2(a + b\text{arcsinh}(cx))^2}{d + c^2dx^2} \, dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2 x^2}{c^2dx^2 + d} \, dx$$

[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="fricas")

[Out] integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)/(c^2*d *x^2 + d), x)

Sympy [F]

$$\int \frac{x^2(a + b\text{arcsinh}(cx))^2}{d + c^2dx^2} \, dx = \frac{\int \frac{a^2x^2}{c^2x^2+1} \, dx + \int \frac{b^2x^2\,\text{asinh}^2\,(cx)}{c^2x^2+1} \, dx + \int \frac{2abx^2\,\text{asinh}\,(cx)}{c^2x^2+1} \, dx}{d}$$

[In] integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d),x)

[Out] (Integral(a**2*x**2/(c**2*x**2 + 1), x) + Integral(b**2*x**2*asinh(c*x)**2/(c**2*x**2 + 1), x) + Integral(2*a*b*x**2*asinh(c*x)/(c**2*x**2 + 1), x))/d

Maxima [F]

$$\int \frac{x^2(a + b\text{arcsinh}(cx))^2}{d + c^2dx^2} \, dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2 x^2}{c^2dx^2 + d} \, dx$$

[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="maxima")

[Out] a^2*(x/(c^2*d) - arctan(c*x)/(c^3*d)) + integrate(b^2*x^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^2 + d) + 2*a*b*x^2*log(c*x + sqrt(c^2*x^2 + 1))/(c^2 *d*x^2 + d), x)

Giac [F]

$$\int \frac{x^2(a + b\text{arcsinh}(cx))^2}{d + c^2dx^2} \, dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2 x^2}{c^2dx^2 + d} \, dx$$

[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)^2*x^2/(c^2*d*x^2 + d), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\operatorname{arcsinh}(cx))^2}{d+c^2dx^2}\,dx=\int \frac{x^2\,\left(a+b\,\mathrm{asinh}(c\,x)\right)^2}{d\,c^2\,x^2+d}\,dx$$

```
[In] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2),x)
[Out] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2), x)
```

3.228 $\int \frac{x(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 105

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = -\frac{(a+b\text{arcsinh}(cx))^3}{3bc^2d} + \frac{(a+b\text{arcsinh}(cx))^2\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^2d} + \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c^2d} - \frac{b^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{2c^2d}$$

```
[Out] -1/3*(a+b*arcsinh(c*x))^3/b/c^2/d+(a+b*arcsinh(c*x))^2*ln(1+(c*x+(c^2*x^2+1
)^(1/2))^2)/c^2/d+b*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2
)/c^2/d-1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/c^2/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5797, 3799, 2221, 2611, 2320, 6724}

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \frac{b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c^2d} - \frac{(a+b\text{arcsinh}(cx))^3}{3bc^2d} + \frac{\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))^2}{c^2d} - \frac{b^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{2c^2d}$$

```
[In] Int[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] -1/3*(a + b*ArcSinh[c*x])^3/(b*c^2*d) + ((a + b*ArcSinh[c*x])^2*Log[1 + E^(
2*ArcSinh[c*x])])/(c^2*d) + (b*(a + b*ArcSinh[c*x])*PolyLog[2, -E^(2*ArcSin
h[c*x])])/(c^2*d) - (b^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*c^2*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (a+bx)^2\tanh(x)\,dx, x, \text{arcsinh}(cx)\right)}{c^2d}$$
$$= -\frac{(a+b\text{arcsinh}(cx))^3}{3bc^2d} + \frac{2\text{Subst}\left(\int \frac{e^{2x}(a+bx)^2}{1+e^{2x}}\,dx, x, \text{arcsinh}(cx)\right)}{c^2d}$$
$$= -\frac{(a+b\text{arcsinh}(cx))^3}{3bc^2d} + \frac{(a+b\text{arcsinh}(cx))^2\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^2d} - \frac{(2b)\text{Subst}\left(\int (a+bx)\log\left(1+e^{2x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{c^2d}$$
$$= -\frac{(a+b\text{arcsinh}(cx))^3}{3bc^2d} + \frac{(a+b\text{arcsinh}(cx))^2\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^2d} + \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c^2d} - \frac{b^2\text{Subst}\left(\int \text{PolyLog}\left(2,-e^{2x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{c^2d}$$
$$= -\frac{(a+b\text{arcsinh}(cx))^3}{3bc^2d} + \frac{(a+b\text{arcsinh}(cx))^2\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^2d} + \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c^2d} - \frac{b^2\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx, x, e^{2\text{arcsinh}(cx)}\right)}{2c^2d}$$
$$= -\frac{(a+b\text{arcsinh}(cx))^3}{3bc^2d} + \frac{(a+b\text{arcsinh}(cx))^2\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^2d} + \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c^2d} - \frac{b^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{2c^2d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 281 vs. 2(105) = 210.

Time = 0.21 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.68

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx$$
$$= \frac{-6ab\text{arcsinh}(cx)^2 - 2b^2\text{arcsinh}(cx)^3 + 12ab\text{arcsinh}(cx)\log\left(1+\frac{ce^{\text{arcsinh}(cx)}}{\sqrt{-c^2}}\right) + 6b^2\text{arcsinh}(cx)^2\log\left(1+\frac{ce^{\cdots}}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] (-6*a*b*ArcSinh[c*x]^2 - 2*b^2*ArcSinh[c*x]^3 + 12*a*b*ArcSinh[c*x]*Log[1 +
 (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 6*b^2*ArcSinh[c*x]^2*Log[1 + (c*E^ArcSinh
[c*x])/Sqrt[-c^2]] + 12*a*b*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x]
)/c] + 6*b^2*ArcSinh[c*x]^2*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 3*a^2*
Log[1 + c^2*x^2] + 12*b*(a + b*ArcSinh[c*x])*PolyLog[2, (c*E^ArcSinh[c*x])/
Sqrt[-c^2]] + 12*b*(a + b*ArcSinh[c*x])*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*
x])/c] - 12*b^2*PolyLog[3, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 12*b^2*PolyLog[
3, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(6*c^2*d)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.71

method	result
derivativedivides	$\frac{\frac{a^2\ln\left(c^2x^2+1\right)}{2d}+\frac{b^2\left(-\frac{\operatorname{arcsinh}(cx)^3}{3}+\operatorname{arcsinh}(cx)^2\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)+\operatorname{arcsinh}(cx)\operatorname{polylog}\left(2,-\left(cx+\sqrt{c^2x^2+1}\right)^2\right)-\frac{\operatorname{polylog}}{}\right.}{d}}{c^2}$
default	$\frac{\frac{a^2\ln\left(c^2x^2+1\right)}{2d}+\frac{b^2\left(-\frac{\operatorname{arcsinh}(cx)^3}{3}+\operatorname{arcsinh}(cx)^2\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)+\operatorname{arcsinh}(cx)\operatorname{polylog}\left(2,-\left(cx+\sqrt{c^2x^2+1}\right)^2\right)-\frac{\operatorname{polylog}}{}\right.}{d}}{c^2}$
parts	$\frac{a^2\ln\left(c^2x^2+1\right)}{2d\,c^2}+\frac{b^2\left(-\frac{\operatorname{arcsinh}(cx)^3}{3}+\operatorname{arcsinh}(cx)^2\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)+\operatorname{arcsinh}(cx)\operatorname{polylog}\left(2,-\left(cx+\sqrt{c^2x^2+\right.\right.\right.}{d\,c^2}$

```
[In] int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/2*a^2/d*ln(c^2*x^2+1)+b^2/d*(-1/3*arcsinh(c*x)^3+arcsinh(c*x)^2*ln
(1+(c*x+(c^2*x^2+1)^(1/2))^2)+arcsinh(c*x)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2
))^2)-1/2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2))+2*a*b/d*(-1/2*arcsinh(c*x)
^2+arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1/2*polylog(2,-(c*x+(c^2*x^
2+1)^(1/2))^2)))
```

Fricas [F]

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))^2}{d+c^2dx^2}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2x}{c^2dx^2+d}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x*arcsinh(c*x)^2 + 2*a*b*x*arcsinh(c*x) + a^2*x)/(c^2*d*x^2 +
 d), x)
```

Sympy [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \frac{\int \frac{a^2x}{c^2x^2+1}\,dx + \int \frac{b^2x\,\text{asinh}^2\,(cx)}{c^2x^2+1}\,dx + \int \frac{2abx\,\text{asinh}\,(cx)}{c^2x^2+1}\,dx}{d}$$

```
[In] integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d),x)
[Out] (Integral(a**2*x/(c**2*x**2 + 1), x) + Integral(b**2*x*asinh(c*x)**2/(c**2*
x**2 + 1), x) + Integral(2*a*b*x*asinh(c*x)/(c**2*x**2 + 1), x))/d
```

Maxima [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x}{c^2dx^2+d}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="maxima")
[Out] 1/2*b^2*log(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d) + 1/2*a^2*l
og(c^2*d*x^2 + d)/(c^2*d) - integrate(-(2*a*b*c^2*x^2 - (b^2*c^2*x^2 + b^2)
*log(c^2*x^2 + 1) - (b^2*c*x*log(c^2*x^2 + 1) - 2*a*b*c*x)*sqrt(c^2*x^2 + 1
))*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d*x^3 + c^2*d*x + (c^3*d*x^2 + c*d)*sq
rt(c^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x}{c^2dx^2+d}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2*x/(c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \int \frac{x\,(a+b\,\text{asinh}(c\,x))^2}{d\,c^2\,x^2+d}\,dx$$

```
[In] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2),x)
[Out] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2), x)
```

3.229 $\int \frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 138

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2} dx = \frac{2(a+b\text{arcsinh}(cx))^2 \arctan\left(e^{\text{arcsinh}(cx)}\right)}{cd} - \frac{2ib(a+b\text{arcsinh}(cx)) \text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{cd} + \frac{2ib(a+b\text{arcsinh}(cx)) \text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{cd} + \frac{2ib^2 \text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{cd} - \frac{2ib^2 \text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{cd}$$

```
[Out] 2*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/c/d-2*I*b*(a+b*arcsinh
(c*x))*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d+2*I*b*(a+b*arcsinh(c*x))*p
olylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d+2*I*b^2*polylog(3,-I*(c*x+(c^2*x^2+
1)^(1/2)))/c/d-2*I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used

$= \{5789, 4265, 2611, 2320, 6724\}$

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{cd} - \frac{2ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{cd} + \frac{2ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{cd} + \frac{2ib^2\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{cd} - \frac{2ib^2\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{cd}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2),x]
```

```
[Out] (2*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/(c*d) - ((2*I)*b*(a + b*A
rcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*d) + ((2*I)*b*(a + b*ArcSi
nh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/(c*d) + ((2*I)*b^2*PolyLog[3, (-I)*E
^ArcSinh[c*x]])/(c*d) - ((2*I)*b^2*PolyLog[3, I*E^ArcSinh[c*x]])/(c*d)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
```

```
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (a+bx)^2 \text{sech}(x)\,dx, x, \text{arcsinh}(cx)\right)}{cd} \\
&= \frac{2(a+b\text{arcsinh}(cx))^2 \arctan\left(e^{\text{arcsinh}(cx)}\right)}{cd} \\
&\quad - \frac{(2ib)\text{Subst}\left(\int (a+bx)\log\left(1-ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{cd} \\
&\quad + \frac{(2ib)\text{Subst}\left(\int (a+bx)\log\left(1+ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{cd} \\
&= \frac{2(a+b\text{arcsinh}(cx))^2 \arctan\left(e^{\text{arcsinh}(cx)}\right)}{cd} \\
&\quad - \frac{2ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{cd} \\
&\quad + \frac{2ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{cd} \\
&\quad + \frac{(2ib^2)\,\text{Subst}\left(\int \text{PolyLog}\left(2,-ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{cd} \\
&\quad - \frac{(2ib^2)\,\text{Subst}\left(\int \text{PolyLog}\left(2,ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{cd} \\
&= \frac{2(a+b\text{arcsinh}(cx))^2 \arctan\left(e^{\text{arcsinh}(cx)}\right)}{cd} \\
&\quad - \frac{2ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{cd} \\
&\quad + \frac{2ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{cd} \\
&\quad + \frac{(2ib^2)\,\text{Subst}\left(\int \frac{\text{PolyLog}(2,-ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{cd} \\
&\quad - \frac{(2ib^2)\,\text{Subst}\left(\int \frac{\text{PolyLog}(2,ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{cd}
\end{aligned}
$$

$$= \frac{2(a + b\text{arcsinh}(cx))^2 \arctan\left(e^{\text{arcsinh}(cx)}\right)}{cd} - \frac{2ib(a + b\text{arcsinh}(cx))\,\text{PolyLog}\left(2, -ie^{\text{arcsinh}(cx)}\right)}{cd} + \frac{2ib(a + b\text{arcsinh}(cx))\,\text{PolyLog}\left(2, ie^{\text{arcsinh}(cx)}\right)}{cd} + \frac{2ib^2\,\text{PolyLog}\left(3, -ie^{\text{arcsinh}(cx)}\right)}{cd} - \frac{2ib^2\,\text{PolyLog}\left(3, ie^{\text{arcsinh}(cx)}\right)}{cd}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.99

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{d + c^2dx^2}\,dx = -\frac{c\left(a^2\sqrt{-c^2}\arctan(cx) - 2abc\text{arcsinh}(cx)\log\left(1 + \frac{ce^{\text{arcsinh}(cx)}}{\sqrt{-c^2}}\right) - b^2c\text{arcsinh}(cx)^2\log\left(1 + \frac{ce^{\text{arcsinh}(cx)}}{\sqrt{-c^2}}\right) + \right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2),x]
```

```
[Out] -((c*(a^2*Sqrt[-c^2]*ArcTan[c*x] - 2*a*b*c*ArcSinh[c*x]*Log[1 + (c*E^ArcSin
h[c*x])/Sqrt[-c^2]] - b^2*c*ArcSinh[c*x]^2*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[
-c^2]] + 2*a*b*c*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + b^2*
c*ArcSinh[c*x]^2*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 2*b*c*(a + b*ArcS
inh[c*x])*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 2*b*c*(a + b*ArcSinh[
c*x])*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 2*b^2*c*PolyLog[3, (c*E^A
rcSinh[c*x])/Sqrt[-c^2]] + 2*b^2*c*PolyLog[3, (Sqrt[-c^2]*E^ArcSinh[c*x])/c
]))/((-c^2)^(3/2)*d))
```

Maple [F]

$$\int \frac{(a + b\,\text{arcsinh}\,(cx))^2}{c^2d\,x^2 + d}dx$$

```
[In] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{c^2dx^2+d}\,dx$$

[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="fricas")

[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^2*d*x^2 + d), x
)

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \frac{\int \frac{a^2}{c^2x^2+1}\,dx + \int \frac{b^2\,\text{asinh}^2\,(cx)}{c^2x^2+1}\,dx + \int \frac{2ab\,\text{asinh}\,(cx)}{c^2x^2+1}\,dx}{d}$$

[In] integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d),x)

[Out] (Integral(a**2/(c**2*x**2 + 1), x) + Integral(b**2*asinh(c*x)**2/(c**2*x**2
 + 1), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**2 + 1), x))/d

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{c^2dx^2+d}\,dx$$

[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="maxima")

[Out] a^2*arctan(c*x)/(c*d) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d
*x^2 + d) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d), x)

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{c^2dx^2+d}\,dx$$

[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{d\,c^2\,x^2+d}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2),x)
[Out] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2), x)
```

3.230 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 116

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)}\,dx = -\frac{2(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d} - \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d} + \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d} + \frac{b^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{2d} - \frac{b^2\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(cx)}\right)}{2d}$$

```
[Out] -2*(a+b*arcsinh(c*x))^2*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)/d-b*(a+b*arcsinh
(c*x))*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d+b*(a+b*arcsinh(c*x))*polylog
(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d+1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^
2)/d-1/2*b^2*polylog(3,(c*x+(c^2*x^2+1)^(1/2))^2)/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used

$= \{5799, 5569, 4267, 2611, 2320, 6724\}$

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\,(d+c^2dx^2)}\,dx = -\frac{2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{d} - \frac{b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d} + \frac{b\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d} + \frac{b^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{2d} - \frac{b^2\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(cx)}\right)}{2d}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)),x]
```

```
[Out] (-2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSinh[c*x])])/d - (b*(a + b*ArcSi
nh[c*x])*PolyLog[2, -E^(2*ArcSinh[c*x])])/d + (b*(a + b*ArcSinh[c*x])*PolyL
og[2, E^(2*ArcSinh[c*x])])/d + (b^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*d)
- (b^2*PolyLog[3, E^(2*ArcSinh[c*x])])/(2*d)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
```

```
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (a+bx)^2\text{csch}(x)\text{sech}(x)\,dx, x, \text{arcsinh}(cx)\right)}{d}\\
&= \frac{2\text{Subst}\left(\int (a+bx)^2\text{csch}(2x)\,dx, x, \text{arcsinh}(cx)\right)}{d}\\
&= -\frac{2(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d}\\
&\quad -\frac{(2b)\text{Subst}\left(\int (a+bx)\log\left(1-e^{2x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{d}\\
&\quad +\frac{(2b)\text{Subst}\left(\int (a+bx)\log\left(1+e^{2x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{d}\\
&= -\frac{2(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d}\\
&\quad -\frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d}\\
&\quad +\frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d}\\
&\quad +\frac{b^2\text{Subst}\left(\int \text{PolyLog}\left(2,-e^{2x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{d}\\
&\quad -\frac{b^2\text{Subst}\left(\int \text{PolyLog}\left(2,e^{2x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{d}
\end{aligned}$$

$$= -\frac{2(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d} - \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d} + \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d} + \frac{b^2\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx, x, e^{2\text{arcsinh}(cx)}\right)}{2d} - \frac{b^2\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx, x, e^{2\text{arcsinh}(cx)}\right)}{2d}$$

$$= -\frac{2(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d} - \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d} + \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d} + \frac{b^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{2d} - \frac{b^2\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(cx)}\right)}{2d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 400 vs. $2(116) = 232$.

Time = 0.28 (sec) , antiderivative size = 400, normalized size of antiderivative = 3.45

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)}\,dx = -\frac{2a^3+6a^2b\text{arcsinh}(cx)+12ab^2\text{arcsinh}(cx)\log\left(1+\frac{ce^{\text{arcsinh}(cx)}}{\sqrt{-c^2}}\right)+6b^3\text{arcsinh}(cx)^2\log\left(1+\frac{ce^{\text{arcsinh}(cx)}}{\sqrt{-c^2}}\right) -$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)),x]
```

```
[Out] -1/6*(2*a^3 + 6*a^2*b*ArcSinh[c*x] + 12*a*b^2*ArcSinh[c*x]*Log[1 + (c*E^Arc
Sinh[c*x])/Sqrt[-c^2]] + 6*b^3*ArcSinh[c*x]^2*Log[1 + (c*E^ArcSinh[c*x])/Sq
rt[-c^2]] + 12*a*b^2*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] +
6*b^3*ArcSinh[c*x]^2*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 6*a^2*b*Log[1
 - E^(2*ArcSinh[c*x])] - 12*a*b^2*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])]
- 6*b^3*ArcSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + 3*a^2*b*Log[1 + c^2*x^
2] + 12*b^2*(a + b*ArcSinh[c*x])*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]]
+ 12*b^2*(a + b*ArcSinh[c*x])*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 6
*a*b^2*PolyLog[2, E^(2*ArcSinh[c*x])] - 6*b^3*ArcSinh[c*x]*PolyLog[2, E^(2*
ArcSinh[c*x])] - 12*b^3*PolyLog[3, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 12*b^3*
PolyLog[3, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 3*b^3*PolyLog[3, E^(2*ArcSinh[c
*x])])/(b*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(157) = 314.

Time = 0.24 (sec) , antiderivative size = 392, normalized size of antiderivative = 3.38

method	result
parts	$\frac{a^2\left(-\frac{\ln(c^2x^2+1)}{2}+\ln(x)\right)}{d} + \frac{b^2\left(\operatorname{arcsinh}(cx)^2\ln\left(1+cx+\sqrt{c^2x^2+1}\right)+2\operatorname{arcsinh}(cx)\operatorname{polylog}\left(2,-cx-\sqrt{c^2x^2+1}\right)-2\,\mathrm{po}\right.}{}$
derivativedivides	$\frac{a^2\left(\ln(cx)-\frac{\ln(c^2x^2+1)}{2}\right)}{d} + \frac{b^2\left(\operatorname{arcsinh}(cx)^2\ln\left(1+cx+\sqrt{c^2x^2+1}\right)+2\operatorname{arcsinh}(cx)\operatorname{polylog}\left(2,-cx-\sqrt{c^2x^2+1}\right)-2\,\mathrm{po}\right.}{}$
default	$\frac{a^2\left(\ln(cx)-\frac{\ln(c^2x^2+1)}{2}\right)}{d} + \frac{b^2\left(\operatorname{arcsinh}(cx)^2\ln\left(1+cx+\sqrt{c^2x^2+1}\right)+2\operatorname{arcsinh}(cx)\operatorname{polylog}\left(2,-cx-\sqrt{c^2x^2+1}\right)-2\,\mathrm{po}\right.}{}$

```
[In] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] a^2/d*(-1/2*ln(c^2*x^2+1)+ln(x))+b^2/d*(arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)
^(1/2))+2*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-2*polylog(3,-c*x-(
c^2*x^2+1)^(1/2))-arcsinh(c*x)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-arcsinh(c*
x)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+1/2*polylog(3,-(c*x+(c^2*x^2+1)^(1
/2))^2)+arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*arcsinh(c*x)*polylog(2
,c*x+(c^2*x^2+1)^(1/2))-2*polylog(3,c*x+(c^2*x^2+1)^(1/2)))+2*a*b/d*(arcsin
h(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+polylog(2,-c*x-(c^2*x^2+1)^(1/2))-arcsin
h(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2
))^2)+arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+polylog(2,c*x+(c^2*x^2+1)^(1
/2)))
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\,(d+c^2dx^2)}\,dx = \int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^2*d*x^3 + d*x),
 x)
```

Sympy [F]

$$\int \frac{(a + b\operatorname{arcsinh}(cx))^2}{x(d + c^2dx^2)}\,dx = \frac{\int \frac{a^2}{c^2x^3+x}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{c^2x^3+x}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{c^2x^3+x}\,dx}{d}$$

```
[In] integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a**2/(c**2*x**3 + x), x) + Integral(b**2*asinh(c*x)**2/(c**2*x**3
 + x), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**3 + x), x))/d
```

Maxima [F]

$$\int \frac{(a + b\operatorname{arcsinh}(cx))^2}{x(d + c^2dx^2)}\,dx = \int \frac{(b\operatorname{arsinh}(cx) + a)^2}{(c^2dx^2 + d)x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*(log(c^2*x^2 + 1)/d - 2*log(x)/d) + integrate(b^2*log(c*x + sqrt(c
^2*x^2 + 1))^2/(c^2*d*x^3 + d*x) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*
d*x^3 + d*x), x)
```

Giac [F]

$$\int \frac{(a + b\operatorname{arcsinh}(cx))^2}{x(d + c^2dx^2)}\,dx = \int \frac{(b\operatorname{arsinh}(cx) + a)^2}{(c^2dx^2 + d)x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\operatorname{arcsinh}(cx))^2}{x(d + c^2dx^2)}\,dx = \int \frac{(a + b\operatorname{asinh}(c\,x))^2}{x\,(d\,c^2\,x^2 + d)}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)), x)
```

3.231 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 204

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)}\,dx = -\frac{(a+b\text{arcsinh}(cx))^2}{dx} - \frac{2c(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d} - \frac{4bc(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d} - \frac{2b^2c\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d} + \frac{2ibc(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{d} - \frac{2ibc(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{d} + \frac{2b^2c\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d} - \frac{2ib^2c\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{d} + \frac{2ib^2c\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{d}$$

```
[Out] -(a+b*arcsinh(c*x))^2/d/x-2*c*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(
1/2))/d-4*b*c*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))/d-2*b^2*c*p
olylog(2,-c*x-(c^2*x^2+1)^(1/2))/d+2*I*b*c*(a+b*arcsinh(c*x))*polylog(2,-I*
(c*x+(c^2*x^2+1)^(1/2)))/d-2*I*b*c*(a+b*arcsinh(c*x))*polylog(2,I*(c*x+(c^2
*x^2+1)^(1/2)))/d+2*b^2*c*polylog(2,c*x+(c^2*x^2+1)^(1/2))/d-2*I*b^2*c*poly
log(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/d+2*I*b^2*c*polylog(3,I*(c*x+(c^2*x^2+1)^
(1/2)))/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5809, 5789, 4265, 2611, 2320, 6724, 5816, 4267, 2317, 2438}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)}\,dx = -\frac{2c\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{d} - \frac{4bc\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d} + \frac{2ibc\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d} - \frac{2ibc\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d} - \frac{(a+b\text{arcsinh}(cx))^2}{dx} - \frac{2b^2c\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d} + \frac{2b^2c\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d} - \frac{2ib^2c\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{d} + \frac{2ib^2c\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{d}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)),x]
```

```
[Out] -((a + b*ArcSinh[c*x])^2/(d*x)) - (2*c*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcS
inh[c*x]])/d - (4*b*c*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/d - (2*
b^2*c*PolyLog[2, -E^ArcSinh[c*x]])/d + ((2*I)*b*c*(a + b*ArcSinh[c*x])*Poly
Log[2, (-I)*E^ArcSinh[c*x]])/d - ((2*I)*b*c*(a + b*ArcSinh[c*x])*PolyLog[2,
 I*E^ArcSinh[c*x]])/d + (2*b^2*c*PolyLog[2, E^ArcSinh[c*x]])/d - ((2*I)*b^2
*c*PolyLog[3, (-I)*E^ArcSinh[c*x]])/d + ((2*I)*b^2*c*PolyLog[3, I*E^ArcSinh
[c*x]])/d
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+b\operatorname{arcsinh}(cx))^2}{dx} - c^2\int\frac{(a+b\operatorname{arcsinh}(cx))^2}{d+c^2dx^2}\,dx + \frac{(2bc)\int\frac{a+b\operatorname{arcsinh}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{d}\\
&= -\frac{(a+b\operatorname{arcsinh}(cx))^2}{dx} - \frac{c\operatorname{Subst}\left(\int(a+bx)^2\operatorname{sech}(x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{d}\\
&\quad + \frac{(2bc)\operatorname{Subst}\left(\int(a+bx)\operatorname{csch}(x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{d}\\
&= -\frac{(a+b\operatorname{arcsinh}(cx))^2}{dx} - \frac{2c(a+b\operatorname{arcsinh}(cx))^2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{d}\\
&\quad - \frac{4bc(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{d}\\
&\quad + \frac{(2ibc)\operatorname{Subst}\left(\int(a+bx)\log\left(1-ie^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{d}\\
&\quad - \frac{(2ibc)\operatorname{Subst}\left(\int(a+bx)\log\left(1+ie^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{d}\\
&\quad - \frac{(2b^2c)\operatorname{Subst}\left(\int\log\left(1-e^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{d}\\
&\quad + \frac{(2b^2c)\operatorname{Subst}\left(\int\log\left(1+e^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{d}
\end{aligned}$$

$$= -\frac{(a+b\text{arcsinh}(cx))^2}{dx} - \frac{2c(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d} - \frac{4bc(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d} + \frac{2ibc(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{d} - \frac{2ibc(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{d} - \frac{(2ib^2c)\,\text{Subst}\left(\int \text{PolyLog}(2,-ie^x)\,dx,x,\text{arcsinh}(cx)\right)}{d} + \frac{(2ib^2c)\,\text{Subst}\left(\int \text{PolyLog}(2,ie^x)\,dx,x,\text{arcsinh}(cx)\right)}{d} - \frac{(2b^2c)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{d} + \frac{(2b^2c)\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{d}$$

$$= -\frac{(a+b\text{arcsinh}(cx))^2}{dx} - \frac{2c(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d} - \frac{4bc(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d} - \frac{2b^2c\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d} + \frac{2ibc(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{d} - \frac{2ibc(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{d} + \frac{2b^2c\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d} - \frac{(2ib^2c)\,\text{Subst}\left(\int \frac{\text{PolyLog}(2,-ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{d} + \frac{(2ib^2c)\,\text{Subst}\left(\int \frac{\text{PolyLog}(2,ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{d}$$

$$= -\frac{(a+b\text{arcsinh}(cx))^2}{dx} - \frac{2c(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d} - \frac{4bc(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d} - \frac{2b^2c\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d} + \frac{2ibc(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{d} - \frac{2ibc(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{d} + \frac{2b^2c\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d} - \frac{2ib^2c\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{d} + \frac{2ib^2c\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{d}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.78

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^2 (d + c^2dx^2)} dx =$$
$$-\frac{\frac{a^2}{x} + \frac{2ab\text{arcsinh}(cx)}{x} + a^2c\arctan(cx) + 2abc\text{arctanh}\left(\sqrt{1 + c^2x^2}\right) + \frac{1}{2}iabc\left(\text{arcsinh}(cx)\left(\text{arcsinh}(cx) - 4\,\text{lo}\right.\right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)),x]
```

```
[Out] -((a^2/x + (2*a*b*ArcSinh[c*x])/x + a^2*c*ArcTan[c*x] + 2*a*b*c*ArcTanh[Sqr
t[1 + c^2*x^2]] + (I/2)*a*b*c*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 + I*E^A
rcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]]) - (I/2)*a*b*c*(ArcSinh[c
*x]*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - 4*PolyLog[2, I*E^ArcSinh
[c*x]]) - b^2*c*(-(ArcSinh[c*x]^2/(c*x)) + 2*ArcSinh[c*x]*Log[1 - E^(-ArcSi
nh[c*x])] + I*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] - I*ArcSinh[c*x]^2*L
og[1 + I/E^ArcSinh[c*x]] - 2*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 2*Po
lyLog[2, -E^(-ArcSinh[c*x])] + (2*I)*ArcSinh[c*x]*PolyLog[2, (-I)/E^ArcSinh
[c*x]] - (2*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] - 2*PolyLog[2, E^(
-ArcSinh[c*x])] + (2*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] - (2*I)*PolyLog[3,
I/E^ArcSinh[c*x]]))/d)
```

Maple [F]

$$\int \frac{(a + b\ \text{arcsinh}\,(cx))^2}{x^2\,(c^2d\,x^2 + d)}dx$$

```
[In] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d),x)
```

Fricas [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^2 (d + c^2dx^2)} dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(c^2dx^2 + d)x^2} dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^2*d*x^4 + d*x^2
), x)
```

Sympy [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^2 (d + c^2dx^2)} dx = \frac{\int \frac{a^2}{c^2x^4+x^2} dx + \int \frac{b^2 \operatorname{asinh}^2(cx)}{c^2x^4+x^2} dx + \int \frac{2ab \operatorname{asinh}(cx)}{c^2x^4+x^2} dx}{d}$$

```
[In] integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a**2/(c**2*x**4 + x**2), x) + Integral(b**2*asinh(c*x)**2/(c**2*x
**4 + x**2), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**4 + x**2), x))/d
```

Maxima [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^2 (d + c^2dx^2)} dx = \int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(c^2dx^2 + d)x^2} dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -a^2*(c*arctan(c*x)/d + 1/(d*x)) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1
))^2/(c^2*d*x^4 + d*x^2) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^4 +
d*x^2), x)
```

Giac [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^2 (d + c^2dx^2)} dx = \int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(c^2dx^2 + d)x^2} dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^2 (d + c^2dx^2)} dx = \int \frac{(a + b \operatorname{asinh}(c\,x))^2}{x^2\,(d\,c^2\,x^2 + d)} dx$$

```
[In] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)), x)
```

3.232 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 194

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)}\,dx = -\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{dx}-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2}+\frac{2c^2(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d}+\frac{b^2c^2\log(x)}{d}+\frac{bc^2(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d}-\frac{bc^2(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d}-\frac{b^2c^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{2d}+\frac{b^2c^2\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(cx)}\right)}{2d}$$

```
[Out] -1/2*(a+b*arcsinh(c*x))^2/d/x^2+2*c^2*(a+b*arcsinh(c*x))^2*arctanh((c*x+(c^
2*x^2+1)^(1/2))^2)/d+b^2*c^2*ln(x)/d+b*c^2*(a+b*arcsinh(c*x))*polylog(2,-(c
*x+(c^2*x^2+1)^(1/2))^2)/d-b*c^2*(a+b*arcsinh(c*x))*polylog(2,(c*x+(c^2*x^2
+1)^(1/2))^2)/d-1/2*b^2*c^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d+1/2*b^2
*c^2*polylog(3,(c*x+(c^2*x^2+1)^(1/2))^2)/d-b*c*(a+b*arcsinh(c*x))*(c^2*x^2
+1)^(1/2)/d/x
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5809, 5799, 5569, 4267, 2611, 2320, 6724, 5800, 29}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)}\,dx = \frac{2c^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{d} + \frac{bc^2\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d} - \frac{bc^2\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d} - \frac{bc\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{dx} - \frac{(a+b\text{arcsinh}(cx))^2}{2dx^2} - \frac{b^2c^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{2d} + \frac{b^2c^2\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(cx)}\right)}{2d} + \frac{b^2c^2\log(x)}{d}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)),x]
```

```
[Out] -((b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(d*x)) - (a + b*ArcSinh[c*x]
)^2/(2*d*x^2) + (2*c^2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSinh[c*x])])/
d + (b^2*c^2*Log[x])/d + (b*c^2*(a + b*ArcSinh[c*x])*PolyLog[2, -E^(2*ArcSi
nh[c*x])])/d - (b*c^2*(a + b*ArcSinh[c*x])*PolyLog[2, E^(2*ArcSinh[c*x])])/
d - (b^2*c^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*d) + (b^2*c^2*PolyLog[3, E
^(2*ArcSinh[c*x])])/(2*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = -\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2} - c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)}\,dx + \frac{(bc)\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{1+c^2x^2}}\,dx}{d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{dx} - \frac{(a+b\text{arcsinh}(cx))^2}{2dx^2} - \frac{c^2\text{Subst}\left(\int (a+bx)^2\text{csch}(x)\text{sech}(x)\,dx, x, \text{arcsinh}(cx)\right)}{d} + \frac{(b^2c^2)\int \frac{1}{x}\,dx}{d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{dx} - \frac{(a+b\text{arcsinh}(cx))^2}{2dx^2} + \frac{b^2c^2\log(x)}{d} - \frac{(2c^2)\,\text{Subst}\left(\int (a+bx)^2\text{csch}(2x)\,dx, x, \text{arcsinh}(cx)\right)}{d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{dx} - \frac{(a+b\text{arcsinh}(cx))^2}{2dx^2} + \frac{2c^2(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d} + \frac{b^2c^2\log(x)}{d} + \frac{(2bc^2)\,\text{Subst}\left(\int (a+bx)\log\left(1-e^{2x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{d} - \frac{(2bc^2)\,\text{Subst}\left(\int (a+bx)\log\left(1+e^{2x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{dx} - \frac{(a+b\text{arcsinh}(cx))^2}{2dx^2} + \frac{2c^2(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d} + \frac{b^2c^2\log(x)}{d} + \frac{bc^2(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d} - \frac{bc^2(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d} - \frac{(b^2c^2)\,\text{Subst}\left(\int \text{PolyLog}\left(2,-e^{2x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{d} + \frac{(b^2c^2)\,\text{Subst}\left(\int \text{PolyLog}\left(2,e^{2x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{dx} - \frac{(a+b\text{arcsinh}(cx))^2}{2dx^2} + \frac{2c^2(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d} + \frac{b^2c^2\log(x)}{d} + \frac{bc^2(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d} - \frac{bc^2(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d} - \frac{(b^2c^2)\,\text{Subst}\left(\int\frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{2\text{arcsinh}(cx)}\right)}{2d} + \frac{(b^2c^2)\,\text{Subst}\left(\int\frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{2\text{arcsinh}(cx)}\right)}{2d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{dx} - \frac{(a+b\text{arcsinh}(cx))^2}{2dx^2} + \frac{2c^2(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d} + \frac{b^2c^2\log(x)}{d} + \frac{bc^2(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d} - \frac{bc^2(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d} - \frac{b^2c^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{2d} + \frac{b^2c^2\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(cx)}\right)}{2d}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.12

$$\int\frac{(a+b\text{arcsinh}(cx))^2}{x^3\,(d+c^2dx^2)}\,dx = -\frac{\frac{a^2}{x^2}+\frac{2ab\left(cx\sqrt{1+c^2x^2}+\text{arcsinh}(cx)\right)}{x^2}+2a^2c^2\log(x)-a^2c^2\log(1+c^2x^2)+abc^2\left(\text{arcsinh}(cx)\left(\text{arcsinh}(cx)-4\right.\right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)),x]
```

```
[Out] -1/2*(a^2/x^2 + (2*a*b*(c*x*Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/x^2 + 2*a^2*
c^2*Log[x] - a^2*c^2*Log[1 + c^2*x^2] + a*b*c^2*(ArcSinh[c*x]*(ArcSinh[c*x]
 - 4*Log[1 + I*E^ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + a*b*
c^2*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - 4*PolyLog[
2, I*E^ArcSinh[c*x]]) - 2*a*b*c^2*(ArcSinh[c*x]*(ArcSinh[c*x] - 2*Log[1 - E
^(2*ArcSinh[c*x])]) - PolyLog[2, E^(2*ArcSinh[c*x])]) - 2*b^2*c^2*((-1/24*I
```

```
)*Pi^3 - (Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(c*x) - ArcSinh[c*x]^2/(2*c^2*x^2
) + (2*ArcSinh[c*x]^3)/3 + ArcSinh[c*x]^2*Log[1 + E^(-2*ArcSinh[c*x])] - Ar
cSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + Log[(c*x)/Sqrt[1 + c^2*x^2]] + L
og[1 + c^2*x^2]/2 - ArcSinh[c*x]*PolyLog[2, -E^(-2*ArcSinh[c*x])] - ArcSinh
[c*x]*PolyLog[2, E^(2*ArcSinh[c*x])] - PolyLog[3, -E^(-2*ArcSinh[c*x])]/2 +
 PolyLog[3, E^(2*ArcSinh[c*x])]/2))/d
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(231) = 462$.

Time = 0.27 (sec) , antiderivative size = 540, normalized size of antiderivative = 2.78

method	result
derivativedivides	$c^{2} \left(\frac{a^{2} \left(-\frac{1}{2 c^{2} x^{2}}-\ln(c x)+\frac{\ln(c^{2} x^{2}+1)}{2}\right)}{d}+\frac{b^{2} \left(-\frac{\operatorname{arcsinh}(c x) \left(-2 c^{2} x^{2}+2 c x \sqrt{c^{2} x^{2}+1}+\operatorname{arcsinh}(c x)\right)}{2 c^{2} x^{2}}+\ln\left(1+c x+\sqrt{c^{2} x^{2}+1}\right.\right.}{}\right.$
default	$c^{2} \left(\frac{a^{2} \left(-\frac{1}{2 c^{2} x^{2}}-\ln(c x)+\frac{\ln(c^{2} x^{2}+1)}{2}\right)}{d}+\frac{b^{2} \left(-\frac{\operatorname{arcsinh}(c x) \left(-2 c^{2} x^{2}+2 c x \sqrt{c^{2} x^{2}+1}+\operatorname{arcsinh}(c x)\right)}{2 c^{2} x^{2}}+\ln\left(1+c x+\sqrt{c^{2} x^{2}+1}\right.\right.}{}\right.$
parts	$\frac{a^{2} \left(\frac{c^{2} \ln(c^{2} x^{2}+1)}{2}-\frac{1}{2 x^{2}}-c^{2} \ln(x)\right)}{d}+\frac{b^{2} c^{2} \left(-\frac{\operatorname{arcsinh}(c x) \left(-2 c^{2} x^{2}+2 c x \sqrt{c^{2} x^{2}+1}+\operatorname{arcsinh}(c x)\right)}{2 c^{2} x^{2}}+\ln\left(1+c x+\sqrt{c^{2} x^{2}+1}\right)-\right.}{}$

```
[In] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(a^2/d*(-1/2/c^2/x^2-ln(c*x)+1/2*ln(c^2*x^2+1))+b^2/d*(-1/2*arcsinh(c*x
)*(-2*c^2*x^2+2*c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x))/c^2/x^2+ln(1+c*x+(c^2*x
^2+1)^(1/2))-2*ln(c*x+(c^2*x^2+1)^(1/2))+ln(c*x+(c^2*x^2+1)^(1/2)-1)-arcsin
h(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))-2*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2
+1)^(1/2))+2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))+arcsinh(c*x)^2*ln(1+(c*x+(c^
2*x^2+1)^(1/2))^2)+arcsinh(c*x)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*p
olylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)-arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1
/2))-2*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))+2*polylog(3,c*x+(c^2*x
^2+1)^(1/2)))+2*a*b/d*(-1/2*(c*x*(c^2*x^2+1)^(1/2)-c^2*x^2+arcsinh(c*x))/c^
2/x^2-arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-polylog(2,-c*x-(c^2*x^2+1)^(
1/2))+arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1/2*polylog(2,-(c*x+(c^2
*x^2+1)^(1/2))^2)-arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-polylog(2,c*x+(c
^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\,(d+c^2dx^2)}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)x^3}\,dx$$

[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d),x, algorithm="fricas")

[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^2*d*x^5 + d*x^3), x)

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\,(d+c^2dx^2)}\,dx = \frac{\int \frac{a^2}{c^2x^5+x^3}\,dx + \int \frac{b^2\,\text{asinh}^2\,(cx)}{c^2x^5+x^3}\,dx + \int \frac{2ab\,\text{asinh}\,(cx)}{c^2x^5+x^3}\,dx}{d}$$

[In] integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d),x)

[Out] (Integral(a**2/(c**2*x**5 + x**3), x) + Integral(b**2*asinh(c*x)**2/(c**2*x**5 + x**3), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**5 + x**3), x))/d

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\,(d+c^2dx^2)}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)x^3}\,dx$$

[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d),x, algorithm="maxima")

[Out] 1/2*(c^2*log(c^2*x^2 + 1)/d - 2*c^2*log(x)/d - 1/(d*x^2))*a^2 + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^5 + d*x^3) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^5 + d*x^3), x)

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\,(d+c^2dx^2)}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)x^3}\,dx$$

[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d),x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)*x^3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^3(d+c^2dx^2)}\,dx=\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2}{x^3\,(d\,c^2\,x^2+d)}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)),x)
[Out] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)), x)
```

3.233 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 297

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)}\,dx = -\frac{b^2c^2}{3dx}-\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3dx^2}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3}+\frac{c^2(a+b\text{arcsinh}(cx))^2}{dx}+\frac{2c^3(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d}+\frac{14bc^3(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{3d}+\frac{7b^2c^3\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{3d}-\frac{2ibc^3(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{d}+\frac{2ibc^3(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{d}-\frac{7b^2c^3\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{3d}+\frac{2ib^2c^3\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{d}-\frac{2ib^2c^3\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{d}$$

```
[Out] -1/3*b^2*c^2/d/x-1/3*(a+b*arcsinh(c*x))^2/d/x^3+c^2*(a+b*arcsinh(c*x))^2/d/
x+2*c^3*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/d+14/3*b*c^3*(a+
b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))/d+7/3*b^2*c^3*polylog(2,-c*x
-(c^2*x^2+1)^(1/2))/d-2*I*b*c^3*(a+b*arcsinh(c*x))*polylog(2,-I*(c*x+(c^2*x
^2+1)^(1/2)))/d+2*I*b*c^3*(a+b*arcsinh(c*x))*polylog(2,I*(c*x+(c^2*x^2+1)^(
```

```
1/2)))/d-7/3*b^2*c^3*polylog(2,c*x+(c^2*x^2+1)^(1/2))/d+2*I*b^2*c^3*polylog
(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/d-2*I*b^2*c^3*polylog(3,I*(c*x+(c^2*x^2+1)^(
1/2)))/d-1/3*b*c*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/d/x^2
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {5809, 5789, 4265, 2611, 2320, 6724, 5816, 4267, 2317, 2438, 30}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)}\,dx = \frac{2c^3\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{d} + \frac{14bc^3\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3d} - \frac{2ibc^3\operatorname{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d} + \frac{2ibc^3\operatorname{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d} - \frac{bc\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3dx^2} + \frac{c^2(a+b\text{arcsinh}(cx))^2}{dx} - \frac{(a+b\text{arcsinh}(cx))^2}{3dx^3} + \frac{7b^2c^3\operatorname{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{3d} - \frac{7b^2c^3\operatorname{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{3d} + \frac{2ib^2c^3\operatorname{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{d} - \frac{2ib^2c^3\operatorname{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{d} - \frac{b^2c^2}{3dx}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)),x]
```

```
[Out] -1/3*(b^2*c^2)/(d*x) - (b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*d*x^
2) - (a + b*ArcSinh[c*x])^2/(3*d*x^3) + (c^2*(a + b*ArcSinh[c*x])^2)/(d*x)
+ (2*c^3*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/d + (14*b*c^3*(a +
b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/(3*d) + (7*b^2*c^3*PolyLog[2, -E^A
rcSinh[c*x]])/(3*d) - ((2*I)*b*c^3*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^A
rcSinh[c*x]])/d + ((2*I)*b*c^3*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[
c*x]])/d - (7*b^2*c^3*PolyLog[2, E^ArcSinh[c*x]])/(3*d) + ((2*I)*b^2*c^3*Po
lyLog[3, (-I)*E^ArcSinh[c*x]])/d - ((2*I)*b^2*c^3*PolyLog[3, I*E^ArcSinh[c*
x]])/d
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3} - c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)}\,dx + \frac{(2bc)\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{1+c^2x^2}}\,dx}{3d} \\
&= -\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3dx^2} - \frac{(a+b\text{arcsinh}(cx))^2}{3dx^3} \\
&\quad + \frac{c^2(a+b\text{arcsinh}(cx))^2}{dx} + c^4\int \frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx + \frac{(b^2c^2)\int \frac{1}{x^2}\,dx}{3d} \\
&\quad - \frac{(bc^3)\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{3d} - \frac{(2bc^3)\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{d}
\end{aligned}
$$

$$= -\frac{b^2c^2}{3dx} - \frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3dx^2} - \frac{(a+b\text{arcsinh}(cx))^2}{3dx^3} + \frac{c^2(a+b\text{arcsinh}(cx))^2}{dx} + \frac{c^3\text{Subst}\left(\int (a+bx)^2\text{sech}(x)\,dx, x, \text{arcsinh}(cx)\right)}{d} - \frac{(bc^3)\,\text{Subst}(\int (a+bx)\text{csch}(x)\,dx, x, \text{arcsinh}(cx))}{3d} - \frac{(2bc^3)\,\text{Subst}(\int (a+bx)\text{csch}(x)\,dx, x, \text{arcsinh}(cx))}{d}$$

$$= -\frac{b^2c^2}{3dx} - \frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3dx^2} - \frac{(a+b\text{arcsinh}(cx))^2}{3dx^3} + \frac{c^2(a+b\text{arcsinh}(cx))^2}{dx} + \frac{2c^3(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d} + \frac{14bc^3(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{3d} - \frac{(2ibc^3)\,\text{Subst}\left(\int (a+bx)\log\left(1-ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{d} + \frac{(2ibc^3)\,\text{Subst}\left(\int (a+bx)\log\left(1+ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{d} + \frac{(b^2c^3)\,\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{3d} - \frac{(b^2c^3)\,\text{Subst}\left(\int \log\left(1+e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{3d} + \frac{(2b^2c^3)\,\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{d} - \frac{(2b^2c^3)\,\text{Subst}\left(\int \log\left(1+e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{d}$$

$$= -\frac{b^2c^2}{3dx} - \frac{bc\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3dx^2} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{3dx^3} + \frac{c^2(a+b\operatorname{arcsinh}(cx))^2}{dx} + \frac{2c^3(a+b\operatorname{arcsinh}(cx))^2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{d} + \frac{14bc^3(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{3d} - \frac{2ibc^3(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{d} + \frac{2ibc^3(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{d} + \frac{(2ib^2c^3)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2,-ie^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{d} - \frac{(2ib^2c^3)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2,ie^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{d} + \frac{(b^2c^3)\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{3d} - \frac{(b^2c^3)\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{3d} + \frac{(2b^2c^3)\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{d} - \frac{(2b^2c^3)\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{d}$$

$$= -\frac{b^2c^2}{3dx} - \frac{bc\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3dx^2} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{3dx^3} + \frac{c^2(a+b\operatorname{arcsinh}(cx))^2}{dx} + \frac{2c^3(a+b\operatorname{arcsinh}(cx))^2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{d} + \frac{14bc^3(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{3d} + \frac{7b^2c^3\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)}{3d} - \frac{2ibc^3(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{d} + \frac{2ibc^3(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{d} - \frac{7b^2c^3\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)}{3d} + \frac{(2ib^2c^3)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,-ix)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{d} - \frac{(2ib^2c^3)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,ix)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{d}$$

$$= -\frac{b^2c^2}{3dx} - \frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3dx^2} - \frac{(a+b\text{arcsinh}(cx))^2}{3dx^3} + \frac{c^2(a+b\text{arcsinh}(cx))^2}{dx} + \frac{2c^3(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d} + \frac{14bc^3(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{3d} + \frac{7b^2c^3\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{3d} - \frac{2ibc^3(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{d} + \frac{2ibc^3(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{d} - \frac{7b^2c^3\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{3d} + \frac{2ib^2c^3\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{d} - \frac{2ib^2c^3\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{d}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 602 vs. $2(297) = 594$.

Time = 7.33 (sec) , antiderivative size = 602, normalized size of antiderivative = 2.03

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)}\,dx = -\frac{a^2}{3dx^3} + \frac{a^2c^2}{dx} + \frac{a^2c^3\arctan(cx)}{d} + \frac{2ab\left(-\frac{c\sqrt{1+c^2x^2}}{6x^2} - \frac{\text{arcsinh}(cx)}{3x^3} + \frac{1}{6}c^3\text{arctanh}\left(\sqrt{1+c^2x^2}\right) - c^2\left(-\frac{\text{arcsinh}(cx)}{x} - c\,\text{arctanh}\left(\sqrt{1+c^2x^2}\right)\right) - \cdots\right.}{\cdots} + \frac{b^2c^3\left(-4\coth\left(\frac{1}{2}\text{arcsinh}(cx)\right) + 14\text{arcsinh}(cx)^2\coth\left(\frac{1}{2}\text{arcsinh}(cx)\right) - 2\text{arcsinh}(cx)\text{csch}^2\left(\frac{1}{2}\text{arcsinh}(cx)\right) \cdots\right.}{\cdots}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)),x]
```

```
[Out] -1/3*a^2/(d*x^3) + (a^2*c^2)/(d*x) + (a^2*c^3*ArcTan[c*x])/d + (2*a*b*(-1/6
*(c*Sqrt[1 + c^2*x^2])/x^2 - ArcSinh[c*x]/(3*x^3) + (c^3*ArcTanh[Sqrt[1 + c
^2*x^2]])/6 - c^2*(-(ArcSinh[c*x]/x) - c*ArcTanh[Sqrt[1 + c^2*x^2]]) - (I/2
)*c^4*(-1/2*ArcSinh[c*x]^2/c + (2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]])/c
 + (2*PolyLog[2, (-I)*E^ArcSinh[c*x]])/c) + (I/2)*c^4*(-1/2*ArcSinh[c*x]^2/
c + (2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]])/c + (2*PolyLog[2, I*E^ArcSin
h[c*x]])/c)))/d + (b^2*c^3*(-4*Coth[ArcSinh[c*x]/2] + 14*ArcSinh[c*x]^2*Cot
h[ArcSinh[c*x]/2] - 2*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - (c*x*ArcSinh[c*
x]^2*Csch[ArcSinh[c*x]/2]^4)/2 - 56*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])]
 - (24*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] + (24*I)*ArcSinh[c*x]^2*
Log[1 + I/E^ArcSinh[c*x]] + 56*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 56
*PolyLog[2, -E^(-ArcSinh[c*x])] - (48*I)*ArcSinh[c*x]*PolyLog[2, (-I)/E^Arc
Sinh[c*x]] + (48*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] + 56*PolyLog[
2, E^(-ArcSinh[c*x])] - (48*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] + (48*I)*Pol
```

```
yLog[3, I/E^ArcSinh[c*x]] - 2*ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 - (8*ArcS
inh[c*x]^2*Sinh[ArcSinh[c*x]/2]^4)/(c^3*x^3) + 4*Tanh[ArcSinh[c*x]/2] - 14*
ArcSinh[c*x]^2*Tanh[ArcSinh[c*x]/2]))/(24*d)
```

Maple [F]

$$\int \frac{\left(a+b\,\operatorname{arcsinh}\left(c x\right)\right)^{2}}{x^{4}\left(c^{2} d\,x^{2}+d\right)}d x$$

```
[In] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d),x)
[Out] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d),x)
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\left(d+c^2dx^2\right)}\,dx=\int \frac{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^{2}}{(c^{2}dx^{2}+d)x^{4}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d),x, algorithm="fricas")
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^2*d*x^6 + d*x^4
), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\left(d+c^2dx^2\right)}\,dx=\frac{\int \frac{a^{2}}{c^{2}x^{6}+x^{4}}\,dx+\int \frac{b^{2}\operatorname{asinh}^{2}\left(cx\right)}{c^{2}x^{6}+x^{4}}\,dx+\int \frac{2ab\operatorname{asinh}\left(cx\right)}{c^{2}x^{6}+x^{4}}\,dx}{d}$$

```
[In] integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d),x)
[Out] (Integral(a**2/(c**2*x**6 + x**4), x) + Integral(b**2*asinh(c*x)**2/(c**2*x
**6 + x**4), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**6 + x**4), x))/d
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\left(d+c^2dx^2\right)}\,dx=\int \frac{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^{2}}{(c^{2}dx^{2}+d)x^{4}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d),x, algorithm="maxima")
[Out] 1/3*(3*c^3*arctan(c*x)/d + (3*c^2*x^2 - 1)/(d*x^3))*a^2 + integrate(b^2*log
(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^6 + d*x^4) + 2*a*b*log(c*x + sqrt(c^2*
x^2 + 1))/(c^2*d*x^6 + d*x^4), x)
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\,(d+c^2dx^2)}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)x^4}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\,(d+c^2dx^2)}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2}{x^4\,\left(d\,c^2\,x^2+d\right)}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)),x)
[Out] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)), x)
```

3.234 $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 279

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx = \frac{2b^2x}{c^4d^2}+\frac{b(a+b\text{arcsinh}(cx))}{c^5d^2\sqrt{1+c^2x^2}}-\frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c^5d^2}+\frac{3x(a+b\text{arcsinh}(cx))^2}{2c^4d^2}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{2c^2d^2(1+c^2x^2)}-\frac{3(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^5d^2}-\frac{b^2\arctan(cx)}{c^5d^2}+\frac{3ib(a+b\text{arcsinh}(cx))\operatorname{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^5d^2}-\frac{3ib(a+b\text{arcsinh}(cx))\operatorname{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^5d^2}-\frac{3ib^2\operatorname{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{c^5d^2}+\frac{3ib^2\operatorname{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{c^5d^2}$$

```
[Out] 2*b^2*x/c^4/d^2+3/2*x*(a+b*arcsinh(c*x))^2/c^4/d^2-1/2*x^3*(a+b*arcsinh(c*x
))^2/c^2/d^2/(c^2*x^2+1)-3*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2
))/c^5/d^2-b^2*arctan(c*x)/c^5/d^2+3*I*b*(a+b*arcsinh(c*x))*polylog(2,-I*(c
*x+(c^2*x^2+1)^(1/2)))/c^5/d^2-3*I*b*(a+b*arcsinh(c*x))*polylog(2,I*(c*x+(c
^2*x^2+1)^(1/2)))/c^5/d^2-3*I*b^2*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^5
/d^2+3*I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d^2+b*(a+b*arcsinh(c*
x))/c^5/d^2/(c^2*x^2+1)^(1/2)-2*b*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^5/
d^2
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {5810, 5812, 5789, 4265, 2611, 2320, 6724, 5798, 8, 272, 45, 5804, 396, 211}

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx = -\frac{3\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{c^5d^2} + \frac{3ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c^5d^2} - \frac{3ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c^5d^2} + \frac{3x(a+b\text{arcsinh}(cx))^2}{2c^4d^2} - \frac{x^3(a+b\text{arcsinh}(cx))^2}{2c^2d^2(c^2x^2+1)} - \frac{2b\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^5d^2} + \frac{b(a+b\text{arcsinh}(cx))}{c^5d^2\sqrt{c^2x^2+1}} - \frac{3ib^2\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{c^5d^2} + \frac{3ib^2\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{c^5d^2} - \frac{b^2\arctan(cx)}{c^5d^2} + \frac{2b^2x}{c^4d^2}$$

```
[In] Int[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] (2*b^2*x)/(c^4*d^2) + (b*(a + b*ArcSinh[c*x]))/(c^5*d^2*Sqrt[1 + c^2*x^2])
- (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(c^5*d^2) + (3*x*(a + b*ArcS
inh[c*x])^2)/(2*c^4*d^2) - (x^3*(a + b*ArcSinh[c*x])^2)/(2*c^2*d^2*(1 + c^2
*x^2)) - (3*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/(c^5*d^2) - (b^2
*ArcTan[c*x])/(c^5*d^2) + ((3*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^A
rcSinh[c*x]])/(c^5*d^2) - ((3*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcS
inh[c*x]])/(c^5*d^2) - ((3*I)*b^2*PolyLog[3, (-I)*E^ArcSinh[c*x]])/(c^5*d^2
) + ((3*I)*b^2*PolyLog[3, I*E^ArcSinh[c*x]])/(c^5*d^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^3(a+b\text{arcsinh}(cx))^2}{2c^2d^2(1+c^2x^2)}+\frac{b\int \frac{x^3(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{cd^2}+\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx}{2c^2d}\\
&= \frac{b(a+b\text{arcsinh}(cx))}{c^5d^2\sqrt{1+c^2x^2}}+\frac{b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c^5d^2}\\
&\quad+\frac{3x(a+b\text{arcsinh}(cx))^2}{2c^4d^2}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{2c^2d^2(1+c^2x^2)}-\frac{b^2\int \frac{2+c^2x^2}{c^4+c^6x^2}\,dx}{d^2}\\
&\quad-\frac{(3b)\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{c^3d^2}-\frac{3\int \frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx}{2c^4d}\\
&= -\frac{b^2x}{c^4d^2}+\frac{b(a+b\text{arcsinh}(cx))}{c^5d^2\sqrt{1+c^2x^2}}-\frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c^5d^2}\\
&\quad+\frac{3x(a+b\text{arcsinh}(cx))^2}{2c^4d^2}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{2c^2d^2(1+c^2x^2)}-\frac{b^2\int \frac{1}{c^4+c^6x^2}\,dx}{d^2}\\
&\quad-\frac{3\text{Subst}\left(\int (a+bx)^2\text{sech}(x)\,dx,x,\text{arcsinh}(cx)\right)}{2c^5d^2}+\frac{(3b^2)\int 1\,dx}{c^4d^2}\\
&= \frac{2b^2x}{c^4d^2}+\frac{b(a+b\text{arcsinh}(cx))}{c^5d^2\sqrt{1+c^2x^2}}-\frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c^5d^2}\\
&\quad+\frac{3x(a+b\text{arcsinh}(cx))^2}{2c^4d^2}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{2c^2d^2(1+c^2x^2)}\\
&\quad-\frac{3(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^5d^2}-\frac{b^2\arctan(cx)}{c^5d^2}\\
&\quad+\frac{(3ib)\text{Subst}\left(\int (a+bx)\log\left(1-ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{c^5d^2}\\
&\quad-\frac{(3ib)\text{Subst}\left(\int (a+bx)\log\left(1+ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{c^5d^2}\\
&= \frac{2b^2x}{c^4d^2}+\frac{b(a+b\text{arcsinh}(cx))}{c^5d^2\sqrt{1+c^2x^2}}-\frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c^5d^2}\\
&\quad+\frac{3x(a+b\text{arcsinh}(cx))^2}{2c^4d^2}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{2c^2d^2(1+c^2x^2)}\\
&\quad-\frac{3(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^5d^2}-\frac{b^2\arctan(cx)}{c^5d^2}\\
&\quad+\frac{3ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^5d^2}\\
&\quad-\frac{3ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^5d^2}\\
&\quad-\frac{(3ib^2)\,\text{Subst}\left(\int \text{PolyLog}\left(2,-ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{c^5d^2}\\
&\quad+\frac{(3ib^2)\,\text{Subst}\left(\int \text{PolyLog}\left(2,ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{c^5d^2}
\end{aligned}
$$

$$= \frac{2b^2x}{c^4d^2} + \frac{b(a+b\operatorname{arcsinh}(cx))}{c^5d^2\sqrt{1+c^2x^2}} - \frac{2b\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c^5d^2} + \frac{3x(a+b\operatorname{arcsinh}(cx))^2}{2c^4d^2} - \frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{2c^2d^2(1+c^2x^2)} - \frac{3(a+b\operatorname{arcsinh}(cx))^2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c^5d^2} - \frac{b^2\arctan(cx)}{c^5d^2} + \frac{3ib(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{c^5d^2} - \frac{3ib(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{c^5d^2} - \frac{(3ib^2)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,-ix)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{c^5d^2} + \frac{(3ib^2)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,ix)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{c^5d^2}$$

$$= \frac{2b^2x}{c^4d^2} + \frac{b(a+b\operatorname{arcsinh}(cx))}{c^5d^2\sqrt{1+c^2x^2}} - \frac{2b\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c^5d^2} + \frac{3x(a+b\operatorname{arcsinh}(cx))^2}{2c^4d^2} - \frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{2c^2d^2(1+c^2x^2)} - \frac{3(a+b\operatorname{arcsinh}(cx))^2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c^5d^2} - \frac{b^2\arctan(cx)}{c^5d^2} + \frac{3ib(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{c^5d^2} - \frac{3ib(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{c^5d^2} - \frac{3ib^2\operatorname{PolyLog}\left(3,-ie^{\operatorname{arcsinh}(cx)}\right)}{c^5d^2} + \frac{3ib^2\operatorname{PolyLog}\left(3,ie^{\operatorname{arcsinh}(cx)}\right)}{c^5d^2}$$

Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.73

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx$$

$$= \frac{\frac{2a^2x}{c^4} + \frac{a^2x}{c^4+c^6x^2} - \frac{3a^2\arctan(cx)}{c^5} - \frac{2ab\left(\sqrt{1+c^2x^2}+2c^2x^2\sqrt{1+c^2x^2}-3cx\operatorname{arcsinh}(cx)-2c^3x^3\operatorname{arcsinh}(cx)+3i\operatorname{arcsinh}(cx)\log\left(1-ie^{\operatorname{arc}\ldots}\right.\right.}{}}{}$$

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] ((2*a^2*x)/c^4 + (a^2*x)/(c^4 + c^6*x^2) - (3*a^2*ArcTan[c*x])/c^5 - (2*a*b
*(Sqrt[1 + c^2*x^2] + 2*c^2*x^2*Sqrt[1 + c^2*x^2] - 3*c*x*ArcSinh[c*x] - 2*
c^3*x^3*ArcSinh[c*x] + (3*I)*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + (3*I)
```

```
*c^2*x^2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (3*I)*ArcSinh[c*x]*Log[1
+ I*E^ArcSinh[c*x]] - (3*I)*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]]
- (3*I)*(1 + c^2*x^2)*PolyLog[2, (-I)*E^ArcSinh[c*x]] + (3*I)*(1 + c^2*x^2)
*PolyLog[2, I*E^ArcSinh[c*x]]))/(c^5 + c^7*x^2) + (2*b^2*(ArcSinh[c*x]/Sqrt
[1 + c^2*x^2] - 2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + (c*x*ArcSinh[c*x]^2)/(2
+ 2*c^2*x^2) + c*x*(2 + ArcSinh[c*x]^2) + (I/2)*((4*I)*ArcTan[Tanh[ArcSinh[
c*x]/2]] + 3*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] - 3*ArcSinh[c*x]^2*Lo
g[1 + I/E^ArcSinh[c*x]] + 6*ArcSinh[c*x]*PolyLog[2, (-I)/E^ArcSinh[c*x]] -
6*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] + 6*PolyLog[3, (-I)/E^ArcSinh[c
*x]] - 6*PolyLog[3, I/E^ArcSinh[c*x]])))/c^5)/(2*d^2)
```

Maple [F]

$$\int \frac{x^4(a+b\ \mathrm{arcsinh}\,(cx))^2}{(c^2d\,x^2+d)^2}dx$$

```
[In] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

```
[Out] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

Fricas [F]

$$\int \frac{x^4(a+b\mathrm{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx=\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2x^4}{(c^2dx^2+d)^2}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)/(c^4*d
^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\mathrm{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx=\frac{\int \frac{a^2x^4}{c^4x^4+2c^2x^2+1}\,dx+\int \frac{b^2x^4\,\mathrm{asinh}^2\,(cx)}{c^4x^4+2c^2x^2+1}\,dx+\int \frac{2abx^4\,\mathrm{asinh}\,(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

```
[In] integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2*x**4/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b**2*x**4*
asinh(c*x)**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(2*a*b*x**4*asinh
(c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{x^4(a + b\operatorname{arcsinh}(cx))^2}{(d + c^2dx^2)^2}\,dx = \int \frac{(b\operatorname{arsinh}(cx) + a)^2x^4}{(c^2dx^2 + d)^2}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(x/(c^6*d^2*x^2 + c^4*d^2) + 2*x/(c^4*d^2) - 3*arctan(c*x)/(c^5*d^2
)) + integrate(b^2*x^4*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^4 + 2*c^2*
d^2*x^2 + d^2) + 2*a*b*x^4*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^
2*d^2*x^2 + d^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a + b\operatorname{arcsinh}(cx))^2}{(d + c^2dx^2)^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b\operatorname{arcsinh}(cx))^2}{(d + c^2dx^2)^2}\,dx = \int \frac{x^4\,(a + b\,\operatorname{asinh}(c\,x))^2}{(d\,c^2\,x^2 + d)^2}\,dx$$

```
[In] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2,x)
```

```
[Out] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2, x)
```

3.235 $\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 213

$$\begin{aligned}\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx =& -\frac{bx(a+b\text{arcsinh}(cx))}{c^3d^2\sqrt{1+c^2x^2}}+\frac{(a+b\text{arcsinh}(cx))^2}{2c^4d^2}\\&-\frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d^2(1+c^2x^2)}-\frac{(a+b\text{arcsinh}(cx))^3}{3bc^4d^2}\\&+\frac{(a+b\text{arcsinh}(cx))^2\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^4d^2}+\frac{b^2\log(1+c^2x^2)}{2c^4d^2}\\&+\frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c^4d^2}\\&-\frac{b^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{2c^4d^2}\end{aligned}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))^2/c^4/d^2-1/2*x^2*(a+b*arcsinh(c*x))^2/c^2/d^2/(c^2*
x^2+1)-1/3*(a+b*arcsinh(c*x))^3/b/c^4/d^2+(a+b*arcsinh(c*x))^2*ln(1+(c*x+(c
^2*x^2+1)^(1/2))^2)/c^4/d^2+1/2*b^2*ln(c^2*x^2+1)/c^4/d^2+b*(a+b*arcsinh(c*
x))*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c^4/d^2-1/2*b^2*polylog(3,-(c*x+(
c^2*x^2+1)^(1/2))^2)/c^4/d^2-b*x*(a+b*arcsinh(c*x))/c^3/d^2/(c^2*x^2+1)^(1/
2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5810, 5797, 3799, 2221, 2611, 2320, 6724, 5783, 266}

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx = \frac{b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c^4d^2} - \frac{(a+b\text{arcsinh}(cx))^3}{3bc^4d^2} + \frac{(a+b\text{arcsinh}(cx))^2}{2c^4d^2} + \frac{\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))^2}{c^4d^2} - \frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d^2(c^2x^2+1)} - \frac{bx(a+b\text{arcsinh}(cx))}{c^3d^2\sqrt{c^2x^2+1}} - \frac{b^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{2c^4d^2} + \frac{b^2\log\left(c^2x^2+1\right)}{2c^4d^2}$$

```
[In] Int[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] -((b*x*(a + b*ArcSinh[c*x]))/(c^3*d^2*Sqrt[1 + c^2*x^2])) + (a + b*ArcSinh[
c*x])^2/(2*c^4*d^2) - (x^2*(a + b*ArcSinh[c*x])^2)/(2*c^2*d^2*(1 + c^2*x^2)
) - (a + b*ArcSinh[c*x])^3/(3*b*c^4*d^2) + ((a + b*ArcSinh[c*x])^2*Log[1 +
E^(2*ArcSinh[c*x])])/(c^4*d^2) + (b^2*Log[1 + c^2*x^2])/(2*c^4*d^2) + (b*(a
 + b*ArcSinh[c*x])*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c^4*d^2) - (b^2*PolyLo
g[3, -E^(2*ArcSinh[c*x])])/(2*c^4*d^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = -\frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d^2(1+c^2x^2)} + \frac{b\int\frac{x^2(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{cd^2} + \frac{\int\frac{x(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx}{c^2d}$$

$$= -\frac{bx(a+b\text{arcsinh}(cx))}{c^3d^2\sqrt{1+c^2x^2}} - \frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d^2(1+c^2x^2)} + \frac{\text{Subst}\left(\int(a+bx)^2\tanh(x)\,dx,x,\text{arcsinh}(cx)\right)}{c^4d^2} + \frac{b\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{c^3d^2} + \frac{b^2\int\frac{x}{1+c^2x^2}\,dx}{c^2d^2}$$

$$= -\frac{bx(a+b\text{arcsinh}(cx))}{c^3d^2\sqrt{1+c^2x^2}} + \frac{(a+b\text{arcsinh}(cx))^2}{2c^4d^2} - \frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d^2(1+c^2x^2)} - \frac{(a+b\text{arcsinh}(cx))^3}{3bc^4d^2} + \frac{b^2\log(1+c^2x^2)}{2c^4d^2} + \frac{2\text{Subst}\left(\int\frac{e^{2x}(a+bx)^2}{1+e^{2x}}\,dx,x,\text{arcsinh}(cx)\right)}{c^4d^2}$$

$$= -\frac{bx(a+b\text{arcsinh}(cx))}{c^3d^2\sqrt{1+c^2x^2}} + \frac{(a+b\text{arcsinh}(cx))^2}{2c^4d^2} - \frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d^2(1+c^2x^2)} - \frac{(a+b\text{arcsinh}(cx))^3}{3bc^4d^2} + \frac{(a+b\text{arcsinh}(cx))^2\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^4d^2} + \frac{b^2\log(1+c^2x^2)}{2c^4d^2} - \frac{(2b)\text{Subst}\left(\int(a+bx)\log\left(1+e^{2x}\right)\,dx,x,\text{arcsinh}(cx)\right)}{c^4d^2}$$

$$= -\frac{bx(a+b\text{arcsinh}(cx))}{c^3d^2\sqrt{1+c^2x^2}} + \frac{(a+b\text{arcsinh}(cx))^2}{2c^4d^2} - \frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d^2(1+c^2x^2)} - \frac{(a+b\text{arcsinh}(cx))^3}{3bc^4d^2} + \frac{(a+b\text{arcsinh}(cx))^2\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^4d^2} + \frac{b^2\log(1+c^2x^2)}{2c^4d^2} + \frac{b(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c^4d^2} - \frac{b^2\text{Subst}\left(\int\text{PolyLog}\left(2,-e^{2x}\right)\,dx,x,\text{arcsinh}(cx)\right)}{c^4d^2}$$

$$= -\frac{bx(a+b\text{arcsinh}(cx))}{c^3d^2\sqrt{1+c^2x^2}} + \frac{(a+b\text{arcsinh}(cx))^2}{2c^4d^2} - \frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d^2(1+c^2x^2)} - \frac{(a+b\text{arcsinh}(cx))^3}{3bc^4d^2} + \frac{(a+b\text{arcsinh}(cx))^2\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^4d^2} + \frac{b^2\log(1+c^2x^2)}{2c^4d^2} + \frac{b(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c^4d^2} - \frac{b^2\text{Subst}\left(\int\frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{2\text{arcsinh}(cx)}\right)}{2c^4d^2}$$

$$
\begin{aligned}
&= -\frac{bx(a+b\text{arcsinh}(cx))}{c^3d^2\sqrt{1+c^2x^2}} + \frac{(a+b\text{arcsinh}(cx))^2}{2c^4d^2} \\
&\quad - \frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d^2(1+c^2x^2)} - \frac{(a+b\text{arcsinh}(cx))^3}{3bc^4d^2} \\
&\quad + \frac{(a+b\text{arcsinh}(cx))^2\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^4d^2} + \frac{b^2\log(1+c^2x^2)}{2c^4d^2} \\
&\quad + \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c^4d^2} - \frac{b^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{2c^4d^2}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.50

$$
\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx
$$

$$
= \frac{\frac{a^2}{1+c^2x^2} - \frac{ab\left(\sqrt{1+c^2x^2}-i\text{arcsinh}(cx)\right)}{i+cx} - \frac{ab\left(\sqrt{1+c^2x^2}+i\text{arcsinh}(cx)\right)}{-i+cx} - ab\text{arcsinh}(cx)\left(\text{arcsinh}(cx) - 4\log\left(1-ie^{\text{ar}}\right.\right.}{}
$$

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] (a^2/(1 + c^2*x^2) - (a*b*(Sqrt[1 + c^2*x^2] - I*ArcSinh[c*x]))/(I + c*x) -
 (a*b*(Sqrt[1 + c^2*x^2] + I*ArcSinh[c*x]))/(-I + c*x) - a*b*ArcSinh[c*x]*(
ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - a*b*ArcSinh[c*x]*(ArcSinh[c*x
] - 4*Log[1 + I*E^ArcSinh[c*x]]) + a^2*Log[1 + c^2*x^2] + 4*a*b*PolyLog[2,
(-I)*E^ArcSinh[c*x]] + 4*a*b*PolyLog[2, I*E^ArcSinh[c*x]] + 2*b^2*(-((c*x*A
rcSinh[c*x])/Sqrt[1 + c^2*x^2]) + ArcSinh[c*x]^2/(2 + 2*c^2*x^2) + ArcSinh[
c*x]^3/3 + ArcSinh[c*x]^2*Log[1 + E^(-2*ArcSinh[c*x])] + Log[1 + c^2*x^2]/2
 - ArcSinh[c*x]*PolyLog[2, -E^(-2*ArcSinh[c*x])] - PolyLog[3, -E^(-2*ArcSin
h[c*x])]/2))/(2*c^4*d^2)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.50

method	result
derivativedivides	$\frac{a^2\left(\frac{1}{2c^2x^2+2}+\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d^2}+\frac{b^2\left(-\frac{\operatorname{arcsinh}(cx)^3}{3}+\frac{\left(2c^2x^2-2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)+2\right)\operatorname{arcsinh}(cx)}{2c^2x^2+2}-2\ln\left(cx+\sqrt{c^2x^2+1}\right)+\ln\left(1\right.\right.}{}$
default	$\frac{a^2\left(\frac{1}{2c^2x^2+2}+\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d^2}+\frac{b^2\left(-\frac{\operatorname{arcsinh}(cx)^3}{3}+\frac{\left(2c^2x^2-2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)+2\right)\operatorname{arcsinh}(cx)}{2c^2x^2+2}-2\ln\left(cx+\sqrt{c^2x^2+1}\right)+\ln\left(1\right.\right.}{}$
parts	$\frac{a^2\left(\frac{1}{2c^4\left(c^2x^2+1\right)}+\frac{\ln\left(c^2x^2+1\right)}{2c^4}\right)}{d^2}+\frac{b^2\left(-\frac{\operatorname{arcsinh}(cx)^3}{3}+\frac{\left(2c^2x^2-2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)+2\right)\operatorname{arcsinh}(cx)}{2c^2x^2+2}-2\ln\left(cx+\sqrt{c^2x^2\right.\right.}{}$

```
[In] int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(a^2/d^2*(1/2/(c^2*x^2+1)+1/2*ln(c^2*x^2+1))+b^2/d^2*(-1/3*arcsinh(c*
x)^3+1/2*(2*c^2*x^2-2*c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x)+2)*arcsinh(c*x)/(c
^2*x^2+1)-2*ln(c*x+(c^2*x^2+1)^(1/2))+ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+arcsi
nh(c*x)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+arcsinh(c*x)*polylog(2,-(c*x+(c^2
*x^2+1)^(1/2))^2)-1/2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2))+2*a*b/d^2*(-1/
2*arcsinh(c*x)^2+1/2*(-c*x*(c^2*x^2+1)^(1/2)+c^2*x^2+arcsinh(c*x)+1)/(c^2*x
^2+1)+arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1/2*polylog(2,-(c*x+(c^2
*x^2+1)^(1/2))^2)))
```

Fricas [F]

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx=\int \frac{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2x^3}{\left(c^2dx^2+d\right)^2}\,dx$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*x) + a^2*x^3)/(c^4*d
^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx=\frac{\int \frac{a^2x^3}{c^4x^4+2c^2x^2+1}\,dx+\int \frac{b^2x^3\operatorname{asinh}^2\left(cx\right)}{c^4x^4+2c^2x^2+1}\,dx+\int \frac{2abx^3\operatorname{asinh}\left(cx\right)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

```
[In] integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2*x**3/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b**2*x**3*
asinh(c*x)**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(2*a*b*x**3*asinh
(c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^3}{(c^2dx^2+d)^2}\,dx$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(1/(c^6*d^2*x^2 + c^4*d^2) + log(c^2*x^2 + 1)/(c^4*d^2)) + 1/2*(b^2
 + (b^2*c^2*x^2 + b^2)*log(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^
6*d^2*x^2 + c^4*d^2) - integrate(-(2*a*b*c^4*x^4 - b^2*c^2*x^2 - b^2 - (b^2
*c^4*x^4 + 2*b^2*c^2*x^2 + b^2)*log(c^2*x^2 + 1) + (2*a*b*c^3*x^3 - b^2*c*x
 - (b^2*c^3*x^3 + b^2*c*x)*log(c^2*x^2 + 1))*sqrt(c^2*x^2 + 1))*log(c*x + s
qrt(c^2*x^2 + 1))/(c^8*d^2*x^5 + 2*c^6*d^2*x^3 + c^4*d^2*x + (c^7*d^2*x^4 +
 2*c^5*d^2*x^2 + c^3*d^2)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx = \int \frac{x^3\,(a+b\,\text{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^2}\,dx$$

```
[In] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2,x)
```

```
[Out] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2, x)
```

3.236 $\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 213

$$
\begin{aligned}
\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx = & -\frac{b(a+b\text{arcsinh}(cx))}{c^3d^2\sqrt{1+c^2x^2}} - \frac{x(a+b\text{arcsinh}(cx))^2}{2c^2d^2(1+c^2x^2)} \\
& + \frac{(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^3d^2} + \frac{b^2\arctan(cx)}{c^3d^2} \\
& - \frac{ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^3d^2} \\
& + \frac{ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^3d^2} \\
& + \frac{ib^2\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{c^3d^2} - \frac{ib^2\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{c^3d^2}
\end{aligned}
$$

```
[Out] -1/2*x*(a+b*arcsinh(c*x))^2/c^2/d^2/(c^2*x^2+1)+(a+b*arcsinh(c*x))^2*arctan
(c*x+(c^2*x^2+1)^(1/2))/c^3/d^2+b^2*arctan(c*x)/c^3/d^2-I*b*(a+b*arcsinh(c*
x))*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^2+I*b*(a+b*arcsinh(c*x))*po
lylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^2+I*b^2*polylog(3,-I*(c*x+(c^2*x^2
+1)^(1/2)))/c^3/d^2-I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^2-b*(a
+b*arcsinh(c*x))/c^3/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5810, 5789, 4265, 2611, 2320, 6724, 5798, 209}

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx = \frac{\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{c^3d^2} - \frac{ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c^3d^2} + \frac{ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c^3d^2} - \frac{x(a+b\text{arcsinh}(cx))^2}{2c^2d^2(c^2x^2+1)} - \frac{b(a+b\text{arcsinh}(cx))}{c^3d^2\sqrt{c^2x^2+1}} + \frac{ib^2\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{c^3d^2} - \frac{ib^2\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{c^3d^2} + \frac{b^2\arctan(cx)}{c^3d^2}$$

```
[In] Int[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] -((b*(a + b*ArcSinh[c*x]))/(c^3*d^2*Sqrt[1 + c^2*x^2])) - (x*(a + b*ArcSinh
[c*x])^2)/(2*c^2*d^2*(1 + c^2*x^2)) + ((a + b*ArcSinh[c*x])^2*ArcTan[E^ArcS
inh[c*x]])/(c^3*d^2) + (b^2*ArcTan[c*x])/(c^3*d^2) - (I*b*(a + b*ArcSinh[c*
x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^3*d^2) + (I*b*(a + b*ArcSinh[c*x])*
PolyLog[2, I*E^ArcSinh[c*x]])/(c^3*d^2) + (I*b^2*PolyLog[3, (-I)*E^ArcSinh[
c*x]])/(c^3*d^2) - (I*b^2*PolyLog[3, I*E^ArcSinh[c*x]])/(c^3*d^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = -\frac{x(a+b\text{arcsinh}(cx))^2}{2c^2d^2(1+c^2x^2)} + \frac{b\int\frac{x(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{cd^2} + \frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx}{2c^2d}$$

$$= -\frac{b(a+b\text{arcsinh}(cx))}{c^3d^2\sqrt{1+c^2x^2}} - \frac{x(a+b\text{arcsinh}(cx))^2}{2c^2d^2(1+c^2x^2)} + \frac{\text{Subst}\left(\int(a+bx)^2\text{sech}(x)\,dx,x,\text{arcsinh}(cx)\right)}{2c^3d^2} + \frac{b^2\int\frac{1}{1+c^2x^2}\,dx}{c^2d^2}$$

$$= -\frac{b(a+b\text{arcsinh}(cx))}{c^3d^2\sqrt{1+c^2x^2}} - \frac{x(a+b\text{arcsinh}(cx))^2}{2c^2d^2(1+c^2x^2)} + \frac{(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^3d^2} + \frac{b^2\arctan(cx)}{c^3d^2} - \frac{(ib)\text{Subst}\left(\int(a+bx)\log\left(1-ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{c^3d^2} + \frac{(ib)\text{Subst}\left(\int(a+bx)\log\left(1+ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{c^3d^2}$$

$$= -\frac{b(a+b\text{arcsinh}(cx))}{c^3d^2\sqrt{1+c^2x^2}} - \frac{x(a+b\text{arcsinh}(cx))^2}{2c^2d^2(1+c^2x^2)} + \frac{(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^3d^2} + \frac{b^2\arctan(cx)}{c^3d^2} - \frac{ib(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^3d^2} + \frac{ib(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^3d^2} + \frac{(ib^2)\text{Subst}\left(\int\text{PolyLog}\left(2,-ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{c^3d^2} - \frac{(ib^2)\text{Subst}\left(\int\text{PolyLog}\left(2,ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{c^3d^2}$$

$$= -\frac{b(a+b\text{arcsinh}(cx))}{c^3d^2\sqrt{1+c^2x^2}} - \frac{x(a+b\text{arcsinh}(cx))^2}{2c^2d^2(1+c^2x^2)} + \frac{(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^3d^2} + \frac{b^2\arctan(cx)}{c^3d^2} - \frac{ib(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^3d^2} + \frac{ib(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^3d^2} + \frac{(ib^2)\text{Subst}\left(\int\frac{\text{PolyLog}(2,-ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{c^3d^2} - \frac{(ib^2)\text{Subst}\left(\int\frac{\text{PolyLog}(2,ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{c^3d^2}$$

$$= -\frac{b(a+b\operatorname{arcsinh}(cx))}{c^3d^2\sqrt{1+c^2x^2}} - \frac{x(a+b\operatorname{arcsinh}(cx))^2}{2c^2d^2(1+c^2x^2)} + \frac{(a+b\operatorname{arcsinh}(cx))^2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c^3d^2} + \frac{b^2\arctan(cx)}{c^3d^2} - \frac{ib(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{c^3d^2} + \frac{ib(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{c^3d^2} + \frac{ib^2\operatorname{PolyLog}\left(3,-ie^{\operatorname{arcsinh}(cx)}\right)}{c^3d^2} - \frac{ib^2\operatorname{PolyLog}\left(3,ie^{\operatorname{arcsinh}(cx)}\right)}{c^3d^2}$$

Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.81

$$\int \frac{x^2(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx = -\frac{\frac{a^2cx}{1+c^2x^2} + \frac{2b^2\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}} + \frac{b^2cx\operatorname{arcsinh}(cx)^2}{1+c^2x^2} + \frac{ab\left(-i\sqrt{1+c^2x^2}+\operatorname{arcsinh}(cx)\right)}{-i+cx} + \frac{ab\left(i\sqrt{1+c^2x^2}+\operatorname{arcsinh}(cx)\right)}{i+cx} - a^2\operatorname{arcta}}{}$$

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] -1/2*((a^2*c*x)/(1 + c^2*x^2) + (2*b^2*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + (b
^2*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2) + (a*b*((-I)*Sqrt[1 + c^2*x^2] + ArcSi
nh[c*x]))/(-I + c*x) + (a*b*(I*Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/(I + c*x)
 - a^2*ArcTan[c*x] - (I/2)*a*b*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 + I*E^
ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + (I/2)*a*b*(ArcSinh[c*
x]*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - 4*PolyLog[2, I*E^ArcSinh[
c*x]]) + I*b^2*((4*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + ArcSinh[c*x]^2*Log[1 -
 I/E^ArcSinh[c*x]] - ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] + 2*ArcSinh[c
*x]*PolyLog[2, (-I)/E^ArcSinh[c*x]] - 2*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh
[c*x]] + 2*PolyLog[3, (-I)/E^ArcSinh[c*x]] - 2*PolyLog[3, I/E^ArcSinh[c*x]]
))/(c^3*d^2)
```

Maple [F]

$$\int \frac{x^2(a+b\operatorname{arcsinh}(cx))^2}{(c^2d\,x^2+d)^2}dx$$

```
[In] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

```
[Out] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

Fricas [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^2}{(c^2dx^2+d)^2}\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)/(c^4*d
^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx = \frac{\int \frac{a^2x^2}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{b^2x^2\,\text{asinh}^2\,(cx)}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{2abx^2\,\text{asinh}\,(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

```
[In] integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2*x**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b**2*x**2*
asinh(c*x)**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(2*a*b*x**2*asinh
(c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^2}{(c^2dx^2+d)^2}\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*(x/(c^4*d^2*x^2 + c^2*d^2) - arctan(c*x)/(c^3*d^2)) + integrate(b^
2*x^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2) +
2*a*b*x^2*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2),
 x)
```

Giac [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2} dx = \int \frac{(b \operatorname{arsinh}(cx)+a)^2x^2}{(c^2dx^2+d)^2} dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2*x^2/(c^2*d*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2} dx = \int \frac{x^2\,(a+b\,\mathrm{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^2} dx$$

```
[In] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2,x)
[Out] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2, x)
```

3.237 $\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2} dx = \frac{bx(a+b\text{arcsinh}(cx))}{cd^2\sqrt{1+c^2x^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{2c^2d^2(1+c^2x^2)} - \frac{b^2\log(1+c^2x^2)}{2c^2d^2}$$

```
[Out] -1/2*(a+b*arcsinh(c*x))^2/c^2/d^2/(c^2*x^2+1)-1/2*b^2*ln(c^2*x^2+1)/c^2/d^2
+b*x*(a+b*arcsinh(c*x))/c/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5798, 5787, 266}

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2} dx = \frac{bx(a+b\text{arcsinh}(cx))}{cd^2\sqrt{c^2x^2+1}} - \frac{(a+b\text{arcsinh}(cx))^2}{2c^2d^2(c^2x^2+1)} - \frac{b^2\log(c^2x^2+1)}{2c^2d^2}$$

```
[In] Int[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] (b*x*(a + b*ArcSinh[c*x]))/(c*d^2*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])
^2/(2*c^2*d^2*(1 + c^2*x^2)) - (b^2*Log[1 + c^2*x^2])/(2*c^2*d^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(1+c^2x^2\right)}+\frac{b\int\frac{a+b\text{arcsinh}(cx)}{(1+c^2x^2)^{3/2}}\,dx}{cd^2}\\
&= \frac{bx(a+b\text{arcsinh}(cx))}{cd^2\sqrt{1+c^2x^2}}-\frac{(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(1+c^2x^2\right)}-\frac{b^2\int\frac{x}{1+c^2x^2}\,dx}{d^2}\\
&= \frac{bx(a+b\text{arcsinh}(cx))}{cd^2\sqrt{1+c^2x^2}}-\frac{(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(1+c^2x^2\right)}-\frac{b^2\log\left(1+c^2x^2\right)}{2c^2d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.71

$$\begin{aligned}
\int\frac{x(a+b\text{arcsinh}(cx))^2}{\left(d+c^2dx^2\right)^2}\,dx &= -\frac{a^2}{2c^2d^2\left(1+c^2x^2\right)}+\frac{abx}{cd^2\sqrt{1+c^2x^2}}\\
&\quad+\frac{b\left(-a+bcx\sqrt{1+c^2x^2}\right)\text{arcsinh}(cx)}{c^2d^2\left(1+c^2x^2\right)}\\
&\quad-\frac{b^2\text{arcsinh}(cx)^2}{2c^2d^2\left(1+c^2x^2\right)}-\frac{b^2\log\left(1+c^2x^2\right)}{2c^2d^2}
\end{aligned}$$

```
[In] Integrate[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] -1/2*a^2/(c^2*d^2*(1 + c^2*x^2)) + (a*b*x)/(c*d^2*Sqrt[1 + c^2*x^2]) + (b*(
-a + b*c*x*Sqrt[1 + c^2*x^2])*ArcSinh[c*x])/(c^2*d^2*(1 + c^2*x^2)) - (b^2*
ArcSinh[c*x]^2)/(2*c^2*d^2*(1 + c^2*x^2)) - (b^2*Log[1 + c^2*x^2])/(2*c^2*d
^2)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.73

method	result
derivativedivides	$\frac{-\frac{a^2}{2d^2(c^2x^2+1)}+\frac{b^2\left(2\operatorname{arcsinh}(cx)-\frac{\left(2c^2x^2-2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)+2\right)\operatorname{arcsinh}(cx)}{2(c^2x^2+1)}-\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)\right)}{d^2}+\frac{2ab\left(-\frac{\operatorname{arc}}{2(\cdots}\right.}{\cdots}}{c^2}$
default	$\frac{-\frac{a^2}{2d^2(c^2x^2+1)}+\frac{b^2\left(2\operatorname{arcsinh}(cx)-\frac{\left(2c^2x^2-2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)+2\right)\operatorname{arcsinh}(cx)}{2(c^2x^2+1)}-\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)\right)}{d^2}+\frac{2ab\left(-\frac{\operatorname{arc}}{2(\cdots}\right.}{\cdots}}{c^2}$
parts	$-\frac{a^2}{2d^2c^2(c^2x^2+1)}+\frac{b^2\left(2\operatorname{arcsinh}(cx)-\frac{\left(2c^2x^2-2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)+2\right)\operatorname{arcsinh}(cx)}{2(c^2x^2+1)}-\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)\right)}{d^2c^2}$

```
[In] int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-1/2*a^2/d^2/(c^2*x^2+1)+b^2/d^2*(2*arcsinh(c*x)-1/2*(2*c^2*x^2-2*c*
x*(c^2*x^2+1)^(1/2)+arcsinh(c*x)+2)*arcsinh(c*x)/(c^2*x^2+1)-ln(1+(c*x+(c^2
*x^2+1)^(1/2))^2))+2*a*b/d^2*(-1/2/(c^2*x^2+1)*arcsinh(c*x)+1/2*c*x/(c^2*x^
2+1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(79) = 158$.

Time = 0.27 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.18

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx$$
$$=\frac{2\,abc^2x^2+2\,\sqrt{c^2x^2+1}abcx-b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2-a^2+2\,ab-(b^2c^2x^2+b^2)\log\left(c^2x^2+1\right)+2\,(abc\cdots}{2\,(c^4d^2x^2+c^2d^2)}$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*c^2*x^2 + 2*sqrt(c^2*x^2 + 1)*a*b*c*x - b^2*log(c*x + sqrt(c^2*x
^2 + 1))^2 - a^2 + 2*a*b - (b^2*c^2*x^2 + b^2)*log(c^2*x^2 + 1) + 2*(a*b*c^
2*x^2 + sqrt(c^2*x^2 + 1)*b^2*c*x)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^
2*x^2 + a*b)*log(-c*x + sqrt(c^2*x^2 + 1)))/(c^4*d^2*x^2 + c^2*d^2)
```

Sympy [F]

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))^2}{\left(d+c^2dx^2\right)^2}\,dx = \frac{\int \frac{a^2x}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{b^2x\operatorname{asinh}^2(cx)}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{2abx\operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

```
[In] integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**2,x)
[Out] (Integral(a**2*x/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b**2*x*asinh(
c*x)**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(2*a*b*x*asinh(c*x)/(c*
*4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))^2}{\left(d+c^2dx^2\right)^2}\,dx = \int \frac{\left(b\operatorname{arsinh}(cx)+a\right)^2x}{\left(c^2dx^2+d\right)^2}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
[Out] -1/2*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^2 + c^2*d^2) - 1/2*a^2/(
c^4*d^2*x^2 + c^2*d^2) + integrate(((2*a*b*c^2 + b^2*c^2)*x^2 + sqrt(c^2*x^
2 + 1)*(2*a*b*c + b^2*c)*x + b^2)*log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^2*x^5
 + 2*c^4*d^2*x^3 + c^2*d^2*x + (c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2)*sqrt(c
^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))^2}{\left(d+c^2dx^2\right)^2}\,dx = \int \frac{\left(b\operatorname{arsinh}(cx)+a\right)^2x}{\left(c^2dx^2+d\right)^2}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2*x/(c^2*d*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx=\int \frac{x\,(a+b\,\mathrm{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^2}\,dx$$

```
[In] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2,x)
[Out] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2, x)
```

3.238 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 210

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2} \, dx = {} & \frac{b(a+b\text{arcsinh}(cx))}{cd^2\sqrt{1+c^2x^2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{2d^2(1+c^2x^2)} \\ & +\frac{(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{cd^2}-\frac{b^2\arctan(cx)}{cd^2} \\ & -\frac{ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{cd^2} \\ & +\frac{ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{cd^2} \\ & +\frac{ib^2\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{cd^2}-\frac{ib^2\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{cd^2}\end{aligned}$$

```
[Out] 1/2*x*(a+b*arcsinh(c*x))^2/d^2/(c^2*x^2+1)+(a+b*arcsinh(c*x))^2*arctan(c*x+
(c^2*x^2+1)^(1/2))/c/d^2-b^2*arctan(c*x)/c/d^2-I*b*(a+b*arcsinh(c*x))*polyl
og(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2+I*b*(a+b*arcsinh(c*x))*polylog(2,I*(
c*x+(c^2*x^2+1)^(1/2)))/c/d^2+I*b^2*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/c
/d^2-I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2+b*(a+b*arcsinh(c*x))/
c/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5788, 5789, 4265, 2611, 2320, 6724, 5798, 209}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}dx = \frac{\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{cd^2} + \frac{b(a+b\text{arcsinh}(cx))}{cd^2\sqrt{c^2x^2+1}} + \frac{x(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)} - \frac{ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{cd^2} + \frac{ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{cd^2} + \frac{ib^2\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{cd^2} - \frac{ib^2\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{cd^2} - \frac{b^2\arctan(cx)}{cd^2}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^2,x]
```

```
[Out] (b*(a + b*ArcSinh[c*x]))/(c*d^2*Sqrt[1 + c^2*x^2]) + (x*(a + b*ArcSinh[c*x]
)^2)/(2*d^2*(1 + c^2*x^2)) + ((a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]]
)/(c*d^2) - (b^2*ArcTan[c*x])/(c*d^2) - (I*b*(a + b*ArcSinh[c*x])*PolyLog[2
, (-I)*E^ArcSinh[c*x]])/(c*d^2) + (I*b*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^
ArcSinh[c*x]])/(c*d^2) + (I*b^2*PolyLog[3, (-I)*E^ArcSinh[c*x]])/(c*d^2) -
(I*b^2*PolyLog[3, I*E^ArcSinh[c*x]])/(c*d^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{x(a+b\text{arcsinh}(cx))^2}{2d^2(1+c^2x^2)} - \frac{(bc)\int \frac{x(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{d^2} + \frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx}{2d}$$

$$= \frac{b(a+b\text{arcsinh}(cx))}{cd^2\sqrt{1+c^2x^2}} + \frac{x(a+b\text{arcsinh}(cx))^2}{2d^2(1+c^2x^2)} - \frac{b^2\int \frac{1}{1+c^2x^2}\,dx}{d^2} + \frac{\text{Subst}\left(\int (a+bx)^2\text{sech}(x)\,dx, x, \text{arcsinh}(cx)\right)}{2cd^2}$$

$$= \frac{b(a+b\text{arcsinh}(cx))}{cd^2\sqrt{1+c^2x^2}} + \frac{x(a+b\text{arcsinh}(cx))^2}{2d^2(1+c^2x^2)} + \frac{(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{cd^2} - \frac{b^2\arctan(cx)}{cd^2} - \frac{(ib)\text{Subst}\left(\int (a+bx)\log\left(1-ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{cd^2} + \frac{(ib)\text{Subst}\left(\int (a+bx)\log\left(1+ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{cd^2}$$

$$= \frac{b(a+b\text{arcsinh}(cx))}{cd^2\sqrt{1+c^2x^2}} + \frac{x(a+b\text{arcsinh}(cx))^2}{2d^2(1+c^2x^2)} + \frac{(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{cd^2} - \frac{b^2\arctan(cx)}{cd^2} - \frac{ib(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{cd^2} + \frac{ib(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{cd^2} + \frac{(ib^2)\text{Subst}\left(\int \text{PolyLog}\left(2,-ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{cd^2} - \frac{(ib^2)\text{Subst}\left(\int \text{PolyLog}\left(2,ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{cd^2}$$

$$= \frac{b(a+b\text{arcsinh}(cx))}{cd^2\sqrt{1+c^2x^2}} + \frac{x(a+b\text{arcsinh}(cx))^2}{2d^2(1+c^2x^2)} + \frac{(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{cd^2} - \frac{b^2\arctan(cx)}{cd^2} - \frac{ib(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{cd^2} + \frac{ib(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{cd^2} + \frac{(ib^2)\text{Subst}\left(\int \frac{\text{PolyLog}(2,-ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{cd^2} - \frac{(ib^2)\text{Subst}\left(\int \frac{\text{PolyLog}(2,ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{cd^2}$$

$$
\begin{aligned}
&= \frac{b(a+b\operatorname{arcsinh}(cx))}{cd^2\sqrt{1+c^2x^2}} + \frac{x(a+b\operatorname{arcsinh}(cx))^2}{2d^2(1+c^2x^2)} \\
&+ \frac{(a+b\operatorname{arcsinh}(cx))^2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{cd^2} - \frac{b^2\arctan(cx)}{cd^2} \\
&- \frac{ib(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{cd^2} \\
&+ \frac{ib(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{cd^2} \\
&+ \frac{ib^2\operatorname{PolyLog}\left(3,-ie^{\operatorname{arcsinh}(cx)}\right)}{cd^2} - \frac{ib^2\operatorname{PolyLog}\left(3,ie^{\operatorname{arcsinh}(cx)}\right)}{cd^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.92

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx$$

$$= \frac{\frac{a^2x}{1+c^2x^2} + \frac{a^2\arctan(cx)}{c} + \frac{2ab\left(\sqrt{1+c^2x^2}+cx\operatorname{arcsinh}(cx)+i\operatorname{arcsinh}(cx)\log\left(1-ie^{\operatorname{arcsinh}(cx)}\right)+ic^2x^2\operatorname{arcsinh}(cx)\log\left(1-ie^{\operatorname{arcsinh}(cx)}\right)\right.}{}}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^2,x]
```

```
[Out] ((a^2*x)/(1 + c^2*x^2) + (a^2*ArcTan[c*x])/c + (2*a*b*(Sqrt[1 + c^2*x^2] +
c*x*ArcSinh[c*x] + I*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + I*c^2*x^2*Arc
Sinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - I*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*
x]] - I*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] - I*(1 + c^2*x^2)*Po
lyLog[2, (-I)*E^ArcSinh[c*x]] + I*(1 + c^2*x^2)*PolyLog[2, I*E^ArcSinh[c*x]
]))/(c + c^3*x^2) + (2*b^2*(ArcSinh[c*x]/Sqrt[1 + c^2*x^2] + (c*x*ArcSinh[c
*x]^2)/(2 + 2*c^2*x^2) - (I/2)*((-4*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + ArcSi
nh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] - ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*
x]] + 2*ArcSinh[c*x]*PolyLog[2, (-I)/E^ArcSinh[c*x]] - 2*ArcSinh[c*x]*PolyL
og[2, I/E^ArcSinh[c*x]] + 2*PolyLog[3, (-I)/E^ArcSinh[c*x]] - 2*PolyLog[3,
I/E^ArcSinh[c*x]])))/c)/(2*d^2)
```

Maple [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(c^2d\,x^2+d)^2}dx$$

```
[In] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(c^2dx^2+d)^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^4*d^2*x^4 + 2*c
^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx=\frac{\int \frac{a^2}{c^4x^4+2c^2x^2+1}\,dx+\int \frac{b^2\operatorname{asinh}^2(cx)}{c^4x^4+2c^2x^2+1}\,dx+\int \frac{2ab\operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

```
[In] integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b**2*asinh(c*x)
**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(2*a*b*asinh(c*x)/(c**4*x**
4 + 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(c^2dx^2+d)^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(x/(c^2*d^2*x^2 + d^2) + arctan(c*x)/(c*d^2)) + integrate(b^2*log(c
*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2) + 2*a*b*log(c
*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(c^2dx^2+d)^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\operatorname{arcsinh}(cx))^2}{(d + c^2dx^2)^2}\,dx = \int \frac{(a + b\,\mathrm{asinh}(c\,x))^2}{(d\,c^2\,x^2 + d)^2}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^2,x)
[Out] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^2, x)
```

3.239 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^2} dx$

Optimal result

Integrand size = 26, antiderivative size = 193

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^2} dx = -\frac{bcx(a+b\text{arcsinh}(cx))}{d^2\sqrt{1+c^2x^2}}+\frac{(a+b\text{arcsinh}(cx))^2}{2d^2(1+c^2x^2)} -\frac{2(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^2}+\frac{b^2\log(1+c^2x^2)}{2d^2} -\frac{b(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d^2} +\frac{b(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d^2} +\frac{b^2\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{2d^2}-\frac{b^2\text{PolyLog}\left(3,e^{2\text{arcsinh}(cx)}\right)}{2d^2}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))^2/d^2/(c^2*x^2+1)-2*(a+b*arcsinh(c*x))^2*arctanh((c*
x+(c^2*x^2+1)^(1/2))^2)/d^2+1/2*b^2*ln(c^2*x^2+1)/d^2-b*(a+b*arcsinh(c*x))*
polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^2+b*(a+b*arcsinh(c*x))*polylog(2,(c
*x+(c^2*x^2+1)^(1/2))^2)/d^2+1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/
d^2-1/2*b^2*polylog(3,(c*x+(c^2*x^2+1)^(1/2))^2)/d^2-b*c*x*(a+b*arcsinh(c*x
))/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5811, 5799, 5569, 4267, 2611, 2320, 6724, 5787, 266}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^2}\,dx = -\frac{2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{d^2} - \frac{bcx(a+b\text{arcsinh}(cx))}{d^2\sqrt{c^2x^2+1}} + \frac{(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)} - \frac{b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d^2} + \frac{b\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d^2} + \frac{b^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{2d^2} - \frac{b^2\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(cx)}\right)}{2d^2} + \frac{b^2\log(c^2x^2+1)}{2d^2}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^2),x]
```

```
[Out] -((b*c*x*(a + b*ArcSinh[c*x]))/(d^2*Sqrt[1 + c^2*x^2])) + (a + b*ArcSinh[c*
x])^2/(2*d^2*(1 + c^2*x^2)) - (2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSin
h[c*x])])/d^2 + (b^2*Log[1 + c^2*x^2])/(2*d^2) - (b*(a + b*ArcSinh[c*x])*Po
lyLog[2, -E^(2*ArcSinh[c*x])])/d^2 + (b*(a + b*ArcSinh[c*x])*PolyLog[2, E^(
2*ArcSinh[c*x])])/d^2 + (b^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*d^2) - (b^
2*PolyLog[3, E^(2*ArcSinh[c*x])])/(2*d^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
```

```
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{(a+b\text{arcsinh}(cx))^2}{2d^2(1+c^2x^2)} - \frac{(bc)\int \frac{a+b\text{arcsinh}(cx)}{(1+c^2x^2)^{3/2}}\,dx}{d^2} + \frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)}\,dx}{d}$$

$$= -\frac{bcx(a+b\text{arcsinh}(cx))}{d^2\sqrt{1+c^2x^2}} + \frac{(a+b\text{arcsinh}(cx))^2}{2d^2(1+c^2x^2)} + \frac{\text{Subst}\left(\int (a+bx)^2\text{csch}(x)\text{sech}(x)\,dx, x, \text{arcsinh}(cx)\right)}{d^2} + \frac{(b^2c^2)\int \frac{x}{1+c^2x^2}\,dx}{d^2}$$

$$= -\frac{bcx(a+b\text{arcsinh}(cx))}{d^2\sqrt{1+c^2x^2}} + \frac{(a+b\text{arcsinh}(cx))^2}{2d^2(1+c^2x^2)} + \frac{b^2\log(1+c^2x^2)}{2d^2} + \frac{2\text{Subst}\left(\int (a+bx)^2\text{csch}(2x)\,dx, x, \text{arcsinh}(cx)\right)}{d^2}$$

$$= -\frac{bcx(a+b\text{arcsinh}(cx))}{d^2\sqrt{1+c^2x^2}} + \frac{(a+b\text{arcsinh}(cx))^2}{2d^2(1+c^2x^2)} - \frac{2(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^2} + \frac{b^2\log(1+c^2x^2)}{2d^2} - \frac{(2b)\text{Subst}\left(\int (a+bx)\log(1-e^{2x})\,dx, x, \text{arcsinh}(cx)\right)}{d^2} + \frac{(2b)\text{Subst}\left(\int (a+bx)\log(1+e^{2x})\,dx, x, \text{arcsinh}(cx)\right)}{d^2}$$

$$= -\frac{bcx(a+b\text{arcsinh}(cx))}{d^2\sqrt{1+c^2x^2}} + \frac{(a+b\text{arcsinh}(cx))^2}{2d^2(1+c^2x^2)} - \frac{2(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^2} + \frac{b^2\log(1+c^2x^2)}{2d^2} - \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2, -e^{2\text{arcsinh}(cx)}\right)}{d^2} + \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2, e^{2\text{arcsinh}(cx)}\right)}{d^2} + \frac{b^2\text{Subst}\left(\int \text{PolyLog}(2, -e^{2x})\,dx, x, \text{arcsinh}(cx)\right)}{d^2} - \frac{b^2\text{Subst}\left(\int \text{PolyLog}(2, e^{2x})\,dx, x, \text{arcsinh}(cx)\right)}{d^2}$$

$$= -\frac{bcx(a+b\text{arcsinh}(cx))}{d^2\sqrt{1+c^2x^2}} + \frac{(a+b\text{arcsinh}(cx))^2}{2d^2(1+c^2x^2)} - \frac{2(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^2} + \frac{b^2\log(1+c^2x^2)}{2d^2} - \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d^2} + \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d^2} + \frac{b^2\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx, x, e^{2\text{arcsinh}(cx)}\right)}{2d^2} - \frac{b^2\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx, x, e^{2\text{arcsinh}(cx)}\right)}{2d^2}$$

$$= -\frac{bcx(a+b\text{arcsinh}(cx))}{d^2\sqrt{1+c^2x^2}} + \frac{(a+b\text{arcsinh}(cx))^2}{2d^2(1+c^2x^2)} - \frac{2(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^2} + \frac{b^2\log(1+c^2x^2)}{2d^2} - \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d^2} + \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d^2} + \frac{b^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{2d^2} - \frac{b^2\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(cx)}\right)}{2d^2}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.65 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.22

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^2}\,dx$$

$$= \frac{\frac{a^2}{1+c^2x^2} - \frac{ab\left(\sqrt{1+c^2x^2}-i\text{arcsinh}(cx)\right)}{i+cx} - \frac{ab\left(\sqrt{1+c^2x^2}+i\text{arcsinh}(cx)\right)}{-i+cx} - 2ab\text{arcsinh}(cx)^2 + 4ab\text{arcsinh}(cx)\log\left(1-\right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^2),x]
```

```
[Out] (a^2/(1 + c^2*x^2) - (a*b*(Sqrt[1 + c^2*x^2] - I*ArcSinh[c*x]))/(I + c*x) -
 (a*b*(Sqrt[1 + c^2*x^2] + I*ArcSinh[c*x]))/(-I + c*x) - 2*a*b*ArcSinh[c*x]
^2 + 4*a*b*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])] + 2*a^2*Log[c*x] - a^2*
Log[1 + c^2*x^2] + a*b*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 + I*E^ArcSinh[
c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + a*b*(ArcSinh[c*x]*(ArcSinh[c*
x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - 4*PolyLog[2, I*E^ArcSinh[c*x]]) + 2*a*b
```

```
*PolyLog[2, E^(2*ArcSinh[c*x])] + 2*b^2*((I/24)*Pi^3 - (c*x*ArcSinh[c*x])/S
qrt[1 + c^2*x^2] + ArcSinh[c*x]^2/(2 + 2*c^2*x^2) - (2*ArcSinh[c*x]^3)/3 -
ArcSinh[c*x]^2*Log[1 + E^(-2*ArcSinh[c*x])] + ArcSinh[c*x]^2*Log[1 - E^(2*A
rcSinh[c*x])] + Log[1 + c^2*x^2]/2 + ArcSinh[c*x]*PolyLog[2, -E^(-2*ArcSinh
[c*x])] + ArcSinh[c*x]*PolyLog[2, E^(2*ArcSinh[c*x])] + PolyLog[3, -E^(-2*A
rcSinh[c*x])]/2 - PolyLog[3, E^(2*ArcSinh[c*x])]/2))/(2*d^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(228) = 456$.

Time = 0.26 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.76

method	result
derivativedivides	$\frac{a^2\left(\ln(cx)+\frac{1}{2c^2x^2+2}-\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d^2}+\frac{b^2\left(\frac{\left(2c^2x^2-2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)+2\right)\operatorname{arcsinh}(cx)}{2c^2x^2+2}-2\ln\left(cx+\sqrt{c^2x^2+1}\right)+\ln\left(1-\cdots\right)\cdots\right)}{\cdots}$
default	$\frac{a^2\left(\ln(cx)+\frac{1}{2c^2x^2+2}-\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d^2}+\frac{b^2\left(\frac{\left(2c^2x^2-2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)+2\right)\operatorname{arcsinh}(cx)}{2c^2x^2+2}-2\ln\left(cx+\sqrt{c^2x^2+1}\right)+\ln\left(1-\cdots\right)\cdots\right)}{\cdots}$
parts	$\frac{a^2}{2d^2(c^2x^2+1)}-\frac{a^2\ln(c^2x^2+1)}{2d^2}+\frac{a^2\ln(x)}{d^2}+\frac{b^2\left(\frac{\left(2c^2x^2-2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)+2\right)\operatorname{arcsinh}(cx)}{2c^2x^2+2}-2\ln\left(cx+\sqrt{c^2x^2+1}\cdots\right)\cdots\right)}{\cdots}$

```
[In] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/d^2*(ln(c*x)+1/2/(c^2*x^2+1)-1/2*ln(c^2*x^2+1))+b^2/d^2*(1/2*(2*c^2*x^2
-2*c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x)+2)*arcsinh(c*x)/(c^2*x^2+1)-2*ln(c*x+
(c^2*x^2+1)^(1/2))+ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+arcsinh(c*x)^2*ln(1+c*x+
(c^2*x^2+1)^(1/2))+2*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-2*polyl
og(3,-c*x-(c^2*x^2+1)^(1/2))-arcsinh(c*x)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
-arcsinh(c*x)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+1/2*polylog(3,-(c*x+(c^
2*x^2+1)^(1/2))^2)+arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*arcsinh(c*x
)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*polylog(3,c*x+(c^2*x^2+1)^(1/2)))+2*a*
b/d^2*(1/2*(-c*x*(c^2*x^2+1)^(1/2)+c^2*x^2+arcsinh(c*x)+1)/(c^2*x^2+1)+arcs
inh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+polylog(2,-c*x-(c^2*x^2+1)^(1/2))-arcs
inh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*polylog(2,-(c*x+(c^2*x^2+1)^(1
/2))^2)+arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+polylog(2,c*x+(c^2*x^2+1)^
(1/2)))
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^2}\,dx=\int \frac{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}{\left(c^2dx^2+d\right)^2x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^4*d^2*x^5 + 2*c
^2*d^2*x^3 + d^2*x), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^2}\,dx=\frac{\int \frac{a^2}{c^4x^5+2c^2x^3+x}\,dx+\int \frac{b^2\operatorname{asinh}^2\left(cx\right)}{c^4x^5+2c^2x^3+x}\,dx+\int \frac{2ab\operatorname{asinh}\left(cx\right)}{c^4x^5+2c^2x^3+x}\,dx}{d^2}$$

```
[In] integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2/(c**4*x**5 + 2*c**2*x**3 + x), x) + Integral(b**2*asinh(c*x)
**2/(c**4*x**5 + 2*c**2*x**3 + x), x) + Integral(2*a*b*asinh(c*x)/(c**4*x**
5 + 2*c**2*x**3 + x), x))/d**2
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^2}\,dx=\int \frac{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}{\left(c^2dx^2+d\right)^2x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(1/(c^2*d^2*x^2 + d^2) - log(c^2*x^2 + 1)/d^2 + 2*log(x)/d^2) + int
egrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^
2*x) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^
2*x), x)
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^2}\,dx = \int \frac{\left(b\,\text{arsinh}\,(cx)+a\right)^2}{\left(c^2dx^2+d\right)^2x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^2}\,dx = \int \frac{\left(a+b\,\text{asinh}(c\,x)\right)^2}{x\left(d\,c^2\,x^2+d\right)^2}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^2),x)
[Out] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^2), x)
```

3.240 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 287

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^2}\,dx = & -\frac{bc(a+b\text{arcsinh}(cx))}{d^2\sqrt{1+c^2x^2}}-\frac{(a+b\text{arcsinh}(cx))^2}{d^2x(1+c^2x^2)} \\ & -\frac{3c^2x(a+b\text{arcsinh}(cx))^2}{2d^2(1+c^2x^2)} \\ & -\frac{3c(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d^2} \\ & +\frac{b^2c\arctan(cx)}{d^2}-\frac{4bc(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d^2} \\ & -\frac{2b^2c\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d^2} \\ & +\frac{3ibc(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{d^2} \\ & -\frac{3ibc(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{d^2} \\ & +\frac{2b^2c\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d^2}-\frac{3ib^2c\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{d^2} \\ & +\frac{3ib^2c\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{d^2}\end{aligned}$$

```
[Out] -(a+b*arcsinh(c*x))^2/d^2/x/(c^2*x^2+1)-3/2*c^2*x*(a+b*arcsinh(c*x))^2/d^2/
(c^2*x^2+1)-3*c*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/d^2+b^2*
c*arctan(c*x)/d^2-4*b*c*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))/d
^2-2*b^2*c*polylog(2,-c*x-(c^2*x^2+1)^(1/2))/d^2+3*I*b*c*(a+b*arcsinh(c*x))
*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^2-3*I*b*c*(a+b*arcsinh(c*x))*polyl
og(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d^2+2*b^2*c*polylog(2,c*x+(c^2*x^2+1)^(1/2)
```

```
)/d^2-3*I*b^2*c*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^2+3*I*b^2*c*polylog
(3,I*(c*x+(c^2*x^2+1)^(1/2)))/d^2-b*c*(a+b*arcsinh(c*x))/d^2/(c^2*x^2+1)^(1
/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {5809, 5788, 5789, 4265, 2611, 2320, 6724, 5798, 209, 5811, 5816, 4267, 2317, 2438}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^2}\,dx = -\frac{3c\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{d^2} - \frac{4b\text{carctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d^2} - \frac{bc(a+b\text{arcsinh}(cx))}{d^2\sqrt{c^2x^2+1}} - \frac{3c^2x(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)} - \frac{(a+b\text{arcsinh}(cx))^2}{d^2x(c^2x^2+1)} + \frac{3ibc\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d^2} - \frac{3ibc\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d^2} - \frac{2b^2c\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d^2} + \frac{2b^2c\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d^2} - \frac{3ib^2c\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{d^2} + \frac{3ib^2c\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{d^2} + \frac{b^2c\arctan(cx)}{d^2}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^2),x]
```

```
[Out] -((b*c*(a + b*ArcSinh[c*x]))/(d^2*Sqrt[1 + c^2*x^2])) - (a + b*ArcSinh[c*x]
)^2/(d^2*x*(1 + c^2*x^2)) - (3*c^2*x*(a + b*ArcSinh[c*x])^2)/(2*d^2*(1 + c^
2*x^2)) - (3*c*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/d^2 + (b^2*c*
ArcTan[c*x])/d^2 - (4*b*c*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/d^2
 - (2*b^2*c*PolyLog[2, -E^ArcSinh[c*x]])/d^2 + ((3*I)*b*c*(a + b*ArcSinh[c*
x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^2 - ((3*I)*b*c*(a + b*ArcSinh[c*x])*
PolyLog[2, I*E^ArcSinh[c*x]])/d^2 + (2*b^2*c*PolyLog[2, E^ArcSinh[c*x]])/d^
2 - ((3*I)*b^2*c*PolyLog[3, (-I)*E^ArcSinh[c*x]])/d^2 + ((3*I)*b^2*c*PolyLo
g[3, I*E^ArcSinh[c*x]])/d^2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+b\text{arcsinh}(cx))^2}{d^2x\,(1+c^2x^2)}-\left(3c^2\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx+\frac{(2bc)\int\frac{a+b\text{arcsinh}(cx)}{x(1+c^2x^2)^{3/2}}\,dx}{d^2}\\
&= \frac{2bc(a+b\text{arcsinh}(cx))}{d^2\sqrt{1+c^2x^2}}-\frac{(a+b\text{arcsinh}(cx))^2}{d^2x\,(1+c^2x^2)}-\frac{3c^2x(a+b\text{arcsinh}(cx))^2}{2d^2\,(1+c^2x^2)}\\
&\quad+\frac{(2bc)\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{d^2}-\frac{(2b^2c^2)\int\frac{1}{1+c^2x^2}\,dx}{d^2}\\
&\quad+\frac{(3bc^3)\int\frac{x(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{d^2}-\frac{(3c^2)\int\frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx}{2d}\\
&= -\frac{bc(a+b\text{arcsinh}(cx))}{d^2\sqrt{1+c^2x^2}}-\frac{(a+b\text{arcsinh}(cx))^2}{d^2x\,(1+c^2x^2)}-\frac{3c^2x(a+b\text{arcsinh}(cx))^2}{2d^2\,(1+c^2x^2)}\\
&\quad-\frac{2b^2c\arctan(cx)}{d^2}-\frac{(3c)\text{Subst}\left(\int(a+bx)^2\text{sech}(x)\,dx,x,\text{arcsinh}(cx)\right)}{2d^2}\\
&\quad+\frac{(2bc)\text{Subst}\left(\int(a+bx)\text{csch}(x)\,dx,x,\text{arcsinh}(cx)\right)}{d^2}+\frac{(3b^2c^2)\int\frac{1}{1+c^2x^2}\,dx}{d^2}\\
&= -\frac{bc(a+b\text{arcsinh}(cx))}{d^2\sqrt{1+c^2x^2}}-\frac{(a+b\text{arcsinh}(cx))^2}{d^2x\,(1+c^2x^2)}\\
&\quad-\frac{3c^2x(a+b\text{arcsinh}(cx))^2}{2d^2\,(1+c^2x^2)}-\frac{3c(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d^2}\\
&\quad+\frac{b^2c\arctan(cx)}{d^2}-\frac{4bc(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d^2}\\
&\quad+\frac{(3ibc)\text{Subst}\left(\int(a+bx)\log\left(1-ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{d^2}\\
&\quad-\frac{(3ibc)\text{Subst}\left(\int(a+bx)\log\left(1+ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{d^2}\\
&\quad-\frac{(2b^2c)\,\text{Subst}\left(\int\log\left(1-e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{d^2}\\
&\quad+\frac{(2b^2c)\,\text{Subst}\left(\int\log\left(1+e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{d^2}
\end{aligned}$$

$$= -\frac{bc(a+b\operatorname{arcsinh}(cx))}{d^2\sqrt{1+c^2x^2}} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{d^2x\,(1+c^2x^2)} - \frac{3c^2x(a+b\operatorname{arcsinh}(cx))^2}{2d^2\,(1+c^2x^2)} - \frac{3c(a+b\operatorname{arcsinh}(cx))^2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{d^2} + \frac{b^2c\arctan(cx)}{d^2} - \frac{4bc(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{d^2} + \frac{3ibc(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{d^2} - \frac{3ibc(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{d^2} - \frac{(3ib^2c)\operatorname{Subst}\left(\int\operatorname{PolyLog}(2,-ie^x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{d^2} + \frac{(3ib^2c)\operatorname{Subst}\left(\int\operatorname{PolyLog}(2,ie^x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{d^2} - \frac{(2b^2c)\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{d^2} + \frac{(2b^2c)\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{d^2}$$

$$= -\frac{bc(a+b\operatorname{arcsinh}(cx))}{d^2\sqrt{1+c^2x^2}} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{d^2x\,(1+c^2x^2)} - \frac{3c^2x(a+b\operatorname{arcsinh}(cx))^2}{2d^2\,(1+c^2x^2)} - \frac{3c(a+b\operatorname{arcsinh}(cx))^2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{d^2} + \frac{b^2c\arctan(cx)}{d^2} - \frac{4bc(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{d^2} - \frac{2b^2c\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)}{d^2} + \frac{3ibc(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{d^2} - \frac{3ibc(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{d^2} + \frac{2b^2c\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)}{d^2} - \frac{(3ib^2c)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,-ix)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{d^2} + \frac{(3ib^2c)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,ix)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{d^2}$$

$$
\begin{aligned}
&= -\frac{bc(a+b\operatorname{arcsinh}(cx))}{d^2\sqrt{1+c^2x^2}} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x(a+b\operatorname{arcsinh}(cx))^2}{2d^2\left(1+c^2x^2\right)} \\
&- \frac{3c(a+b\operatorname{arcsinh}(cx))^2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{d^2} + \frac{b^2c\arctan(cx)}{d^2} \\
&- \frac{4bc(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{d^2} - \frac{2b^2c\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)}{d^2} \\
&+ \frac{3ibc(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{d^2} \\
&- \frac{3ibc(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{d^2} + \frac{2b^2c\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)}{d^2} \\
&- \frac{3ib^2c\operatorname{PolyLog}\left(3,-ie^{\operatorname{arcsinh}(cx)}\right)}{d^2} + \frac{3ib^2c\operatorname{PolyLog}\left(3,ie^{\operatorname{arcsinh}(cx)}\right)}{d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 7.06 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.91

$$
\begin{aligned}
&\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^2\left(d+c^2dx^2\right)^2}\,dx = -\frac{a^2}{d^2x} - \frac{a^2c^2x}{2d^2\left(1+c^2x^2\right)} - \frac{3a^2c\arctan(cx)}{2d^2} \\
&+ \frac{2abc\left(\frac{\sqrt{1+c^2x^2}+i\operatorname{arcsinh}(cx)}{4(-1-icx)} - \frac{\operatorname{arcsinh}(cx)}{cx} - \frac{i\sqrt{1+c^2x^2}+\operatorname{arcsinh}(cx)}{4(i+cx)} - \operatorname{arctanh}\left(\sqrt{1+c^2x^2}\right) + \frac{3}{4}i\left(-\frac{1}{2}\operatorname{arcsinh}(\right.\right.}{} \\
&+ \frac{b^2c\left(-\frac{2\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}} - \frac{cx\operatorname{arcsinh}(cx)^2}{1+c^2x^2} + 4\arctan\left(\tanh\left(\frac{1}{2}\operatorname{arcsinh}(cx)\right)\right) - \operatorname{arcsinh}(cx)^2\coth\left(\frac{1}{2}\operatorname{arcsinh}(cx)\right.\right.}{}
\end{aligned}
$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^2),x]
```

```
[Out] -(a^2/(d^2*x)) - (a^2*c^2*x)/(2*d^2*(1 + c^2*x^2)) - (3*a^2*c*ArcTan[c*x])/
(2*d^2) + (2*a*b*c*((Sqrt[1 + c^2*x^2] + I*ArcSinh[c*x])/(4*(-1 - I*c*x)) -
 ArcSinh[c*x]/(c*x) - (I*Sqrt[1 + c^2*x^2] + ArcSinh[c*x])/(4*(I + c*x)) -
ArcTanh[Sqrt[1 + c^2*x^2]] + ((3*I)/4)*(-1/2*ArcSinh[c*x]^2 + 2*ArcSinh[c*x
]*Log[1 + I*E^ArcSinh[c*x]] + 2*PolyLog[2, (-I)*E^ArcSinh[c*x]]) - ((3*I)/4
)*(-1/2*ArcSinh[c*x]^2 + 2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + 2*PolyL
og[2, I*E^ArcSinh[c*x]])))/d^2 + (b^2*c*((-2*ArcSinh[c*x])/Sqrt[1 + c^2*x^2
] - (c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2) + 4*ArcTan[Tanh[ArcSinh[c*x]/2]] - A
rcSinh[c*x]^2*Coth[ArcSinh[c*x]/2] + 4*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x
])] + (3*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] - (3*I)*ArcSinh[c*x]^2
*Log[1 + I/E^ArcSinh[c*x]] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 4*
PolyLog[2, -E^(-ArcSinh[c*x])] + (6*I)*ArcSinh[c*x]*PolyLog[2, (-I)/E^ArcSi
nh[c*x]] - (6*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] - 4*PolyLog[2, E
^(-ArcSinh[c*x])] + (6*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] - (6*I)*PolyLog[3
, I/E^ArcSinh[c*x]] + ArcSinh[c*x]^2*Tanh[ArcSinh[c*x]/2]))/(2*d^2)
```

Maple [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^2\,(c^2d\,x^2+d)^2}\,dx$$

```
[In] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^2,x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^2,x)
```

Fricas [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2}{x^2\,(d+c^2dx^2)^2}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(c^2dx^2+d)^2x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^4*d^2*x^6 + 2*c
^2*d^2*x^4 + d^2*x^2), x)
```

Sympy [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2}{x^2\,(d+c^2dx^2)^2}\,dx = \frac{\int \frac{a^2}{c^4x^6+2c^2x^4+x^2}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{c^4x^6+2c^2x^4+x^2}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{c^4x^6+2c^2x^4+x^2}\,dx}{d^2}$$

```
[In] integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2/(c**4*x**6 + 2*c**2*x**4 + x**2), x) + Integral(b**2*asinh(c
*x)**2/(c**4*x**6 + 2*c**2*x**4 + x**2), x) + Integral(2*a*b*asinh(c*x)/(c*
*4*x**6 + 2*c**2*x**4 + x**2), x))/d**2
```

Maxima [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2}{x^2\,(d+c^2dx^2)^2}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(c^2dx^2+d)^2x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*((3*c^2*x^2 + 2)/(c^2*d^2*x^3 + d^2*x) + 3*c*arctan(c*x)/d^2) + in
tegrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d
^2*x^2) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 +
 d^2*x^2), x)
```

Giac [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^2 (d + c^2 dx^2)^2} \, dx = \int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(c^2 dx^2 + d)^2 x^2} \, dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^2 (d + c^2 dx^2)^2} \, dx = \int \frac{(a + b \operatorname{asinh}(c\,x))^2}{x^2 \, (d\,c^2\,x^2 + d)^2} \, dx$$

```
[In] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^2),x)
[Out] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^2), x)
```

3.241 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 253

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^2}\,dx = -\frac{bc(a+b\text{arcsinh}(cx))}{d^2x\sqrt{1+c^2x^2}} - \frac{c^2(a+b\text{arcsinh}(cx))^2}{d^2(1+c^2x^2)} - \frac{(a+b\text{arcsinh}(cx))^2}{2d^2x^2(1+c^2x^2)} + \frac{4c^2(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^2} + \frac{b^2c^2\log(x)}{d^2} - \frac{b^2c^2\log(1+c^2x^2)}{2d^2} + \frac{2bc^2(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d^2} - \frac{2bc^2(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d^2} - \frac{b^2c^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{d^2} + \frac{b^2c^2\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(cx)}\right)}{d^2}$$

```
[Out] -c^2*(a+b*arcsinh(c*x))^2/d^2/(c^2*x^2+1)-1/2*(a+b*arcsinh(c*x))^2/d^2/x^2/
(c^2*x^2+1)+4*c^2*(a+b*arcsinh(c*x))^2*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)/d
^2+b^2*c^2*ln(x)/d^2-1/2*b^2*c^2*ln(c^2*x^2+1)/d^2+2*b*c^2*(a+b*arcsinh(c*x
))*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^2-2*b*c^2*(a+b*arcsinh(c*x))*pol
ylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d^2-b^2*c^2*polylog(3,-(c*x+(c^2*x^2+1)^(
1/2))^2)/d^2+b^2*c^2*polylog(3,(c*x+(c^2*x^2+1)^(1/2))^2)/d^2-b*c*(a+b*arcs
inh(c*x))/d^2/x/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {5809, 5811, 5799, 5569, 4267, 2611, 2320, 6724, 5787, 266, 277, 197, 5804, 457, 78}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^2}\,dx = \frac{4c^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{d^2} + \frac{2bc^2\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d^2} - \frac{2bc^2\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d^2} - \frac{c^2(a+b\text{arcsinh}(cx))^2}{d^2(c^2x^2+1)} - \frac{bc(a+b\text{arcsinh}(cx))}{d^2x\sqrt{c^2x^2+1}} - \frac{(a+b\text{arcsinh}(cx))^2}{2d^2x^2(c^2x^2+1)} - \frac{b^2c^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{d^2} + \frac{b^2c^2\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(cx)}\right)}{d^2} - \frac{b^2c^2\log(c^2x^2+1)}{2d^2} + \frac{b^2c^2\log(x)}{d^2}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^2),x]
```

```
[Out] -((b*c*(a + b*ArcSinh[c*x]))/(d^2*x*Sqrt[1 + c^2*x^2])) - (c^2*(a + b*ArcSi
nh[c*x])^2)/(d^2*(1 + c^2*x^2)) - (a + b*ArcSinh[c*x])^2/(2*d^2*x^2*(1 + c^
2*x^2)) + (4*c^2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSinh[c*x])])/d^2 +
(b^2*c^2*Log[x])/d^2 - (b^2*c^2*Log[1 + c^2*x^2])/(2*d^2) + (2*b*c^2*(a + b
*ArcSinh[c*x])*PolyLog[2, -E^(2*ArcSinh[c*x])])/d^2 - (2*b*c^2*(a + b*ArcSi
nh[c*x])*PolyLog[2, E^(2*ArcSinh[c*x])])/d^2 - (b^2*c^2*PolyLog[3, -E^(2*Ar
cSinh[c*x])])/d^2 + (b^2*c^2*PolyLog[3, E^(2*ArcSinh[c*x])])/d^2
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
```

```
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+b\text{arcsinh}(cx))^2}{2d^2x^2(1+c^2x^2)} - (2c^2)\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^2}\,dx + \frac{(bc)\int \frac{a+b\text{arcsinh}(cx)}{x^2(1+c^2x^2)^{3/2}}\,dx}{d^2} \\
&= -\frac{bc(a+b\text{arcsinh}(cx))}{d^2x\sqrt{1+c^2x^2}} - \frac{2bc^3x(a+b\text{arcsinh}(cx))}{d^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{c^2(a+b\text{arcsinh}(cx))^2}{d^2(1+c^2x^2)} - \frac{(a+b\text{arcsinh}(cx))^2}{2d^2x^2(1+c^2x^2)} - \frac{(b^2c^2)\int \frac{-1-2c^2x^2}{x(1+c^2x^2)}\,dx}{d^2} \\
&\quad + \frac{(2bc^3)\int \frac{a+b\text{arcsinh}(cx)}{(1+c^2x^2)^{3/2}}\,dx}{d^2} - \frac{(2c^2)\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)}\,dx}{d} \\
&= -\frac{bc(a+b\text{arcsinh}(cx))}{d^2x\sqrt{1+c^2x^2}} - \frac{c^2(a+b\text{arcsinh}(cx))^2}{d^2(1+c^2x^2)} - \frac{(a+b\text{arcsinh}(cx))^2}{2d^2x^2(1+c^2x^2)} \\
&\quad - \frac{(2c^2)\,\text{Subst}\left(\int (a+bx)^2\text{csch}(x)\text{sech}(x)\,dx, x, \text{arcsinh}(cx)\right)}{d^2} \\
&\quad - \frac{(b^2c^2)\,\text{Subst}\left(\int \frac{-1-2c^2x}{x(1+c^2x)}\,dx, x, x^2\right)}{2d^2} - \frac{(2b^2c^4)\int \frac{x}{1+c^2x^2}\,dx}{d^2} \\
&= -\frac{bc(a+b\text{arcsinh}(cx))}{d^2x\sqrt{1+c^2x^2}} - \frac{c^2(a+b\text{arcsinh}(cx))^2}{d^2(1+c^2x^2)} - \frac{(a+b\text{arcsinh}(cx))^2}{2d^2x^2(1+c^2x^2)} \\
&\quad - \frac{b^2c^2\log(1+c^2x^2)}{d^2} - \frac{(4c^2)\,\text{Subst}\left(\int (a+bx)^2\text{csch}(2x)\,dx, x, \text{arcsinh}(cx)\right)}{d^2} \\
&\quad - \frac{(b^2c^2)\,\text{Subst}\left(\int \left(-\frac{1}{x}-\frac{c^2}{1+c^2x}\right)dx, x, x^2\right)}{2d^2} \\
&= -\frac{bc(a+b\text{arcsinh}(cx))}{d^2x\sqrt{1+c^2x^2}} - \frac{c^2(a+b\text{arcsinh}(cx))^2}{d^2(1+c^2x^2)} - \frac{(a+b\text{arcsinh}(cx))^2}{2d^2x^2(1+c^2x^2)} \\
&\quad + \frac{4c^2(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^2} + \frac{b^2c^2\log(x)}{d^2} \\
&\quad - \frac{b^2c^2\log(1+c^2x^2)}{2d^2} + \frac{(4bc^2)\,\text{Subst}\left(\int (a+bx)\log(1-e^{2x})\,dx, x, \text{arcsinh}(cx)\right)}{d^2} \\
&\quad - \frac{(4bc^2)\,\text{Subst}\left(\int (a+bx)\log(1+e^{2x})\,dx, x, \text{arcsinh}(cx)\right)}{d^2}
\end{aligned}$$

$$= -\frac{bc(a+b\operatorname{arcsinh}(cx))}{d^2x\sqrt{1+c^2x^2}} - \frac{c^2(a+b\operatorname{arcsinh}(cx))^2}{d^2(1+c^2x^2)} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{2d^2x^2(1+c^2x^2)} + \frac{4c^2(a+b\operatorname{arcsinh}(cx))^2\operatorname{arctanh}\left(e^{2\operatorname{arcsinh}(cx)}\right)}{d^2} + \frac{b^2c^2\log(x)}{d^2} - \frac{b^2c^2\log(1+c^2x^2)}{2d^2} + \frac{2bc^2(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{d^2} - \frac{2bc^2(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(cx)}\right)}{d^2} - \frac{(2b^2c^2)\operatorname{Subst}\left(\int \operatorname{PolyLog}(2,-e^{2x})\,dx, x, \operatorname{arcsinh}(cx)\right)}{d^2} + \frac{(2b^2c^2)\operatorname{Subst}\left(\int \operatorname{PolyLog}(2,e^{2x})\,dx, x, \operatorname{arcsinh}(cx)\right)}{d^2}$$

$$= -\frac{bc(a+b\operatorname{arcsinh}(cx))}{d^2x\sqrt{1+c^2x^2}} - \frac{c^2(a+b\operatorname{arcsinh}(cx))^2}{d^2(1+c^2x^2)} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{2d^2x^2(1+c^2x^2)} + \frac{4c^2(a+b\operatorname{arcsinh}(cx))^2\operatorname{arctanh}\left(e^{2\operatorname{arcsinh}(cx)}\right)}{d^2} + \frac{b^2c^2\log(x)}{d^2} - \frac{b^2c^2\log(1+c^2x^2)}{2d^2} + \frac{2bc^2(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{d^2} - \frac{2bc^2(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(cx)}\right)}{d^2} - \frac{(b^2c^2)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,-x)}{x}\,dx, x, e^{2\operatorname{arcsinh}(cx)}\right)}{d^2} + \frac{(b^2c^2)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,x)}{x}\,dx, x, e^{2\operatorname{arcsinh}(cx)}\right)}{d^2}$$

$$= -\frac{bc(a+b\operatorname{arcsinh}(cx))}{d^2x\sqrt{1+c^2x^2}} - \frac{c^2(a+b\operatorname{arcsinh}(cx))^2}{d^2(1+c^2x^2)} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{2d^2x^2(1+c^2x^2)} + \frac{4c^2(a+b\operatorname{arcsinh}(cx))^2\operatorname{arctanh}\left(e^{2\operatorname{arcsinh}(cx)}\right)}{d^2} + \frac{b^2c^2\log(x)}{d^2} - \frac{b^2c^2\log(1+c^2x^2)}{2d^2} + \frac{2bc^2(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{d^2} - \frac{2bc^2(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(cx)}\right)}{d^2} - \frac{b^2c^2\operatorname{PolyLog}\left(3,-e^{2\operatorname{arcsinh}(cx)}\right)}{d^2} + \frac{b^2c^2\operatorname{PolyLog}\left(3,e^{2\operatorname{arcsinh}(cx)}\right)}{d^2}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.54 (sec) , antiderivative size = 506, normalized size of antiderivative = 2.00

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^2}\,dx$$
$$= \frac{-\frac{a^2}{x^2}-\frac{a^2c^2}{1+c^2x^2}-4a^2c^2\log(x)+2a^2c^2\log(1+c^2x^2)+ab\Big(-\frac{2c\sqrt{1+c^2x^2}}{x}+\frac{c^2\sqrt{1+c^2x^2}}{-i+cx}+\frac{c^2\sqrt{1+c^2x^2}}{i+cx}-\frac{2\text{arcsinh}(cx)}{x^2}}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^2),x]
```

```
[Out] (-(a^2/x^2) - (a^2*c^2)/(1 + c^2*x^2) - 4*a^2*c^2*Log[x] + 2*a^2*c^2*Log[1
+ c^2*x^2] + a*b*((-2*c*Sqrt[1 + c^2*x^2])/x + (c^2*Sqrt[1 + c^2*x^2])/(-I
+ c*x) + (c^2*Sqrt[1 + c^2*x^2])/(I + c*x) - (2*ArcSinh[c*x])/x^2 + (c^2*Ar
cSinh[c*x])/(-1 - I*c*x) - (I*c^2*ArcSinh[c*x])/(I + c*x) + 8*c^2*ArcSinh[c
*x]*Log[1 - I*E^ArcSinh[c*x]] + 8*c^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]
] - 8*c^2*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])] + 8*c^2*PolyLog[2, (-I)*
E^ArcSinh[c*x]] + 8*c^2*PolyLog[2, I*E^ArcSinh[c*x]] - 4*c^2*PolyLog[2, E^(
2*ArcSinh[c*x])]) + b^2*c^2*((2*c*x*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - (2*Sq
rt[1 + c^2*x^2]*ArcSinh[c*x])/(c*x) - ArcSinh[c*x]^2/(c^2*x^2) - ArcSinh[c*
x]^2/(1 + c^2*x^2) - 4*ArcSinh[c*x]^2*Log[1 - E^(-2*ArcSinh[c*x])] + 4*ArcS
inh[c*x]^2*Log[1 + E^(-2*ArcSinh[c*x])] + 2*Log[(c*x)/Sqrt[1 + c^2*x^2]] -
4*ArcSinh[c*x]*PolyLog[2, -E^(-2*ArcSinh[c*x])] + 4*ArcSinh[c*x]*PolyLog[2,
 E^(-2*ArcSinh[c*x])] - 2*PolyLog[3, -E^(-2*ArcSinh[c*x])] + 2*PolyLog[3, E
^(-2*ArcSinh[c*x])]))/(2*d^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 585 vs. 2(292) = 584.

Time = 0.30 (sec) , antiderivative size = 586, normalized size of antiderivative = 2.32

method	result
derivativedivides	$c^2\left(\frac{a^2\left(-\frac{1}{2c^2x^2}-2\ln(cx)-\frac{1}{2(c^2x^2+1)}+\ln(c^2x^2+1)\right)}{d^2}+\frac{b^2\left(-\frac{\left(2\,\mathrm{arcsinh}(cx)c^2x^2+2cx\sqrt{c^2x^2+1}+\mathrm{arcsinh}(cx)\right)\mathrm{arcsinh}(c}{2c^2x^2(c^2x^2+1)}\right.}{}\right.$
default	$c^2\left(\frac{a^2\left(-\frac{1}{2c^2x^2}-2\ln(cx)-\frac{1}{2(c^2x^2+1)}+\ln(c^2x^2+1)\right)}{d^2}+\frac{b^2\left(-\frac{\left(2\,\mathrm{arcsinh}(cx)c^2x^2+2cx\sqrt{c^2x^2+1}+\mathrm{arcsinh}(cx)\right)\mathrm{arcsinh}(c}{2c^2x^2(c^2x^2+1)}\right.}{}\right.$
parts	$\frac{a^2\left(\frac{c^4\left(-\frac{1}{c^2(c^2x^2+1)}+\frac{2\ln(c^2x^2+1)}{c^2}\right)}{2}-\frac{1}{2x^2}-2c^2\ln(x)\right)}{d^2}+\frac{b^2c^2\left(-\frac{\left(2\,\mathrm{arcsinh}(cx)c^2x^2+2cx\sqrt{c^2x^2+1}+\mathrm{arcsinh}(cx)\right)\mathrm{arcsin}}{2c^2x^2(c^2x^2+1)}\right.}{}$

```
[In] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(a^2/d^2*(-1/2/c^2/x^2-2*ln(c*x)-1/2/(c^2*x^2+1)+ln(c^2*x^2+1))+b^2/d^2
*(-1/2*(2*arcsinh(c*x)*c^2*x^2+2*c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x))*arcsin
h(c*x)/c^2/x^2/(c^2*x^2+1)+ln(1+c*x+(c^2*x^2+1)^(1/2))-ln(1+(c*x+(c^2*x^2+1
)^(1/2))^2)+ln(c*x+(c^2*x^2+1)^(1/2)-1)-2*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+
1)^(1/2))-4*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+4*polylog(3,-c*x
-(c^2*x^2+1)^(1/2))+2*arcsinh(c*x)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+2*arcs
inh(c*x)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)-polylog(3,-(c*x+(c^2*x^2+1)^
(1/2))^2)-2*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))-4*arcsinh(c*x)*polyl
og(2,c*x+(c^2*x^2+1)^(1/2))+4*polylog(3,c*x+(c^2*x^2+1)^(1/2)))+2*a*b/d^2*(
-1/2*(2*arcsinh(c*x)*c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x))/c^2/x^2/(c
^2*x^2+1)-2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-2*polylog(2,-c*x-(c^2*
x^2+1)^(1/2))+2*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+polylog(2,-(c*
x+(c^2*x^2+1)^(1/2))^2)-2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-2*polylo
g(2,c*x+(c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^2}\,dx=\int \frac{\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^2}{\left(c^2dx^2+d\right)^2x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^4*d^2*x^7 + 2*c
^2*d^2*x^5 + d^2*x^3), x)
```

Sympy [F]

$$\int \frac{(a + b\operatorname{arcsinh}(cx))^2}{x^3\left(d + c^2dx^2\right)^2}\,dx = \frac{\int \frac{a^2}{c^4x^7+2c^2x^5+x^3}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{c^4x^7+2c^2x^5+x^3}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{c^4x^7+2c^2x^5+x^3}\,dx}{d^2}$$

```
[In] integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d)**2,x)
[Out] (Integral(a**2/(c**4*x**7 + 2*c**2*x**5 + x**3), x) + Integral(b**2*asinh(c
*x)**2/(c**4*x**7 + 2*c**2*x**5 + x**3), x) + Integral(2*a*b*asinh(c*x)/(c*
*4*x**7 + 2*c**2*x**5 + x**3), x))/d**2
```

Maxima [F]

$$\int \frac{(a + b\operatorname{arcsinh}(cx))^2}{x^3\left(d + c^2dx^2\right)^2}\,dx = \int \frac{\left(b\operatorname{arsinh}(cx) + a\right)^2}{\left(c^2dx^2 + d\right)^2x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^2,x, algorithm="maxima")
[Out] 1/2*a^2*(2*c^2*log(c^2*x^2 + 1)/d^2 - 4*c^2*log(x)/d^2 - (2*c^2*x^2 + 1)/(c
^2*d^2*x^4 + d^2*x^2)) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*
d^2*x^7 + 2*c^2*d^2*x^5 + d^2*x^3) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^
4*d^2*x^7 + 2*c^2*d^2*x^5 + d^2*x^3), x)
```

Giac [F]

$$\int \frac{(a + b\operatorname{arcsinh}(cx))^2}{x^3\left(d + c^2dx^2\right)^2}\,dx = \int \frac{\left(b\operatorname{arsinh}(cx) + a\right)^2}{\left(c^2dx^2 + d\right)^2x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\operatorname{arcsinh}(cx))^2}{x^3\left(d + c^2dx^2\right)^2}\,dx = \int \frac{\left(a + b\operatorname{asinh}(c\,x)\right)^2}{x^3\left(d\,c^2\,x^2 + d\right)^2}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^2),x)
[Out] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^2), x)
```

3.242 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 401

$$
\begin{aligned}
\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^2}\,dx = & -\frac{b^2c^2}{3d^2x}+\frac{2bc^3(a+b\text{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}} \\
& -\frac{bc(a+b\text{arcsinh}(cx))}{3d^2x^2\sqrt{1+c^2x^2}}-\frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3(1+c^2x^2)} \\
& +\frac{5c^2(a+b\text{arcsinh}(cx))^2}{3d^2x(1+c^2x^2)}+\frac{5c^4x(a+b\text{arcsinh}(cx))^2}{2d^2(1+c^2x^2)} \\
& +\frac{5c^3(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d^2}-\frac{b^2c^3\arctan(cx)}{d^2} \\
& +\frac{26bc^3(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{3d^2} \\
& +\frac{13b^2c^3\operatorname{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{3d^2} \\
& -\frac{5ibc^3(a+b\text{arcsinh}(cx))\operatorname{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{d^2} \\
& +\frac{5ibc^3(a+b\text{arcsinh}(cx))\operatorname{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{d^2} \\
& -\frac{13b^2c^3\operatorname{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{3d^2} \\
& +\frac{5ib^2c^3\operatorname{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{d^2} \\
& -\frac{5ib^2c^3\operatorname{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{d^2}
\end{aligned}
$$

```
[Out] -1/3*b^2*c^2/d^2/x-1/3*(a+b*arcsinh(c*x))^2/d^2/x^3/(c^2*x^2+1)+5/3*c^2*(a+
b*arcsinh(c*x))^2/d^2/x/(c^2*x^2+1)+5/2*c^4*x*(a+b*arcsinh(c*x))^2/d^2/(c^2
```

```
*x^2+1)+5*c^3*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/d^2-b^2*c^
3*arctan(c*x)/d^2+26/3*b*c^3*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/
2))/d^2+13/3*b^2*c^3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))/d^2-5*I*b*c^3*(a+b*a
rcsinh(c*x))*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^2+5*I*b*c^3*(a+b*arcsi
nh(c*x))*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d^2-13/3*b^2*c^3*polylog(2,c*
x+(c^2*x^2+1)^(1/2))/d^2+5*I*b^2*c^3*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/
d^2-5*I*b^2*c^3*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/d^2+2/3*b*c^3*(a+b*arc
sinh(c*x))/d^2/(c^2*x^2+1)^(1/2)-1/3*b*c*(a+b*arcsinh(c*x))/d^2/x^2/(c^2*x^
2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {5809, 5788, 5789, 4265, 2611, 2320, 6724, 5798, 209, 5811, 5816, 4267, 2317, 2438, 331}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^2}\,dx = \frac{5c^3\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{d^2} + \frac{26bc^3\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3d^2} - \frac{5ibc^3\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d^2} + \frac{5ibc^3\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d^2} + \frac{5c^2(a+b\text{arcsinh}(cx))^2}{3d^2x(c^2x^2+1)} - \frac{bc(a+b\text{arcsinh}(cx))}{3d^2x^2\sqrt{c^2x^2+1}} - \frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3(c^2x^2+1)} + \frac{5c^4x(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)} + \frac{2bc^3(a+b\text{arcsinh}(cx))}{3d^2\sqrt{c^2x^2+1}} + \frac{13b^2c^3\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{3d^2} - \frac{13b^2c^3\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{3d^2} + \frac{5ib^2c^3\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{d^2} - \frac{5ib^2c^3\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{d^2} - \frac{b^2c^3\arctan(cx)}{d^2} - \frac{b^2c^2}{3d^2x}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^2),x]
```

```
[Out] -1/3*(b^2*c^2)/(d^2*x) + (2*b*c^3*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt[1 + c^2
*x^2]) - (b*c*(a + b*ArcSinh[c*x]))/(3*d^2*x^2*Sqrt[1 + c^2*x^2]) - (a + b*
ArcSinh[c*x])^2/(3*d^2*x^3*(1 + c^2*x^2)) + (5*c^2*(a + b*ArcSinh[c*x])^2)/
```

```
(3*d^2*x*(1 + c^2*x^2)) + (5*c^4*x*(a + b*ArcSinh[c*x])^2)/(2*d^2*(1 + c^2*
x^2)) + (5*c^3*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/d^2 - (b^2*c^
3*ArcTan[c*x])/d^2 + (26*b*c^3*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]]
)/(3*d^2) + (13*b^2*c^3*PolyLog[2, -E^ArcSinh[c*x]])/(3*d^2) - ((5*I)*b*c^3
*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^2 + ((5*I)*b*c^3*(
a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/d^2 - (13*b^2*c^3*PolyLog
[2, E^ArcSinh[c*x]])/(3*d^2) + ((5*I)*b^2*c^3*PolyLog[3, (-I)*E^ArcSinh[c*x
]])/d^2 - ((5*I)*b^2*c^3*PolyLog[3, I*E^ArcSinh[c*x]])/d^2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
```

```
- 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

integral

$$= -\frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3(1+c^2x^2)} - \frac{1}{3}(5c^2)\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^2}dx + \frac{(2bc)\int \frac{a+b\text{arcsinh}(cx)}{x^3(1+c^2x^2)^{3/2}}dx}{3d^2}$$

$$= -\frac{bc(a+b\text{arcsinh}(cx))}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3(1+c^2x^2)} + \frac{5c^2(a+b\text{arcsinh}(cx))^2}{3d^2x(1+c^2x^2)} + (5c^4)\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}dx + \frac{(b^2c^2)\int \frac{1}{x^2(1+c^2x^2)}dx}{3d^2} - \frac{(bc^3)\int \frac{a+b\text{arcsinh}(cx)}{x(1+c^2x^2)^{3/2}}dx}{d^2} - \frac{(10bc^3)\int \frac{a+b\text{arcsinh}(cx)}{x(1+c^2x^2)^{3/2}}dx}{3d^2}$$

$$= -\frac{b^2c^2}{3d^2x} - \frac{13bc^3(a+b\text{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}} - \frac{bc(a+b\text{arcsinh}(cx))}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3(1+c^2x^2)} + \frac{5c^2(a+b\text{arcsinh}(cx))^2}{3d^2x(1+c^2x^2)} + \frac{5c^4x(a+b\text{arcsinh}(cx))^2}{2d^2(1+c^2x^2)} - \frac{(bc^3)\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{d^2} - \frac{(10bc^3)\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{3d^2} - \frac{(b^2c^4)\int \frac{1}{1+c^2x^2}\,dx}{3d^2} + \frac{(b^2c^4)\int \frac{1}{1+c^2x^2}\,dx}{d^2} + \frac{(10b^2c^4)\int \frac{1}{1+c^2x^2}\,dx}{3d^2} - \frac{(5bc^5)\int \frac{x(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{d^2} + \frac{(5c^4)\int \frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx}{2d}$$

$$= -\frac{b^2c^2}{3d^2x} + \frac{2bc^3(a+b\text{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}} - \frac{bc(a+b\text{arcsinh}(cx))}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3(1+c^2x^2)} + \frac{5c^2(a+b\text{arcsinh}(cx))^2}{3d^2x(1+c^2x^2)} + \frac{5c^4x(a+b\text{arcsinh}(cx))^2}{2d^2(1+c^2x^2)} + \frac{4b^2c^3\arctan(cx)}{d^2} + \frac{(5c^3)\,\text{Subst}\left(\int (a+bx)^2\text{sech}(x)\,dx, x, \text{arcsinh}(cx)\right)}{2d^2} - \frac{(bc^3)\,\text{Subst}(\int (a+bx)\text{csch}(x)\,dx, x, \text{arcsinh}(cx))}{d^2} - \frac{(10bc^3)\,\text{Subst}(\int (a+bx)\text{csch}(x)\,dx, x, \text{arcsinh}(cx))}{3d^2} - \frac{(5b^2c^4)\int \frac{1}{1+c^2x^2}\,dx}{d^2}$$

$$= -\frac{b^2c^2}{3d^2x} + \frac{2bc^3(a+b\text{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}} - \frac{bc(a+b\text{arcsinh}(cx))}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3(1+c^2x^2)} + \frac{5c^2(a+b\text{arcsinh}(cx))^2}{3d^2x(1+c^2x^2)} + \frac{5c^4x(a+b\text{arcsinh}(cx))^2}{2d^2(1+c^2x^2)} + \frac{5c^3(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d^2} - \frac{b^2c^3\arctan(cx)}{d^2} + \frac{26bc^3(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{3d^2} - \frac{(5ibc^3)\,\text{Subst}\left(\int (a+bx)\log\left(1-ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{d^2} + \frac{(5ibc^3)\,\text{Subst}\left(\int (a+bx)\log\left(1+ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{d^2} + \frac{(b^2c^3)\,\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{d^2} - \frac{(b^2c^3)\,\text{Subst}\left(\int \log\left(1+e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{d^2} + \frac{(10b^2c^3)\,\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{3d^2} - \frac{(10b^2c^3)\,\text{Subst}\left(\int \log\left(1+e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{3d^2}$$

$$= -\frac{b^2c^2}{3d^2x} + \frac{2bc^3(a+b\operatorname{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}} - \frac{bc(a+b\operatorname{arcsinh}(cx))}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{3d^2x^3(1+c^2x^2)} + \frac{5c^2(a+b\operatorname{arcsinh}(cx))^2}{3d^2x(1+c^2x^2)} + \frac{5c^4x(a+b\operatorname{arcsinh}(cx))^2}{2d^2(1+c^2x^2)} + \frac{5c^3(a+b\operatorname{arcsinh}(cx))^2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{d^2} - \frac{b^2c^3\arctan(cx)}{d^2} + \frac{26bc^3(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{3d^2} - \frac{5ibc^3(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{d^2} + \frac{5ibc^3(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{d^2} + \frac{(5ib^2c^3)\operatorname{Subst}\left(\int \operatorname{PolyLog}(2,-ie^x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{d^2} - \frac{(5ib^2c^3)\operatorname{Subst}\left(\int \operatorname{PolyLog}(2,ie^x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{d^2} + \frac{(b^2c^3)\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{d^2} - \frac{(b^2c^3)\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{d^2} + \frac{(10b^2c^3)\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{3d^2} - \frac{(10b^2c^3)\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{3d^2}$$

$$= -\frac{b^2c^2}{3d^2x} + \frac{2bc^3(a+b\operatorname{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}} - \frac{bc(a+b\operatorname{arcsinh}(cx))}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{3d^2x^3(1+c^2x^2)} + \frac{5c^2(a+b\operatorname{arcsinh}(cx))^2}{3d^2x(1+c^2x^2)} + \frac{5c^4x(a+b\operatorname{arcsinh}(cx))^2}{2d^2(1+c^2x^2)} + \frac{5c^3(a+b\operatorname{arcsinh}(cx))^2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{d^2} - \frac{b^2c^3\arctan(cx)}{d^2} + \frac{26bc^3(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{3d^2} + \frac{13b^2c^3\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)}{3d^2} - \frac{5ibc^3(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{d^2} + \frac{5ibc^3(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{d^2} - \frac{13b^2c^3\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)}{3d^2} + \frac{(5ib^2c^3)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,-ix)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{d^2} - \frac{(5ib^2c^3)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,ix)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{d^2}$$

$$
\begin{aligned}
&= -\frac{b^2c^2}{3d^2x} + \frac{2bc^3(a+b\operatorname{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}} - \frac{bc(a+b\operatorname{arcsinh}(cx))}{3d^2x^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{(a+b\operatorname{arcsinh}(cx))^2}{3d^2x^3(1+c^2x^2)} + \frac{5c^2(a+b\operatorname{arcsinh}(cx))^2}{3d^2x(1+c^2x^2)} + \frac{5c^4x(a+b\operatorname{arcsinh}(cx))^2}{2d^2(1+c^2x^2)} \\
&\quad + \frac{5c^3(a+b\operatorname{arcsinh}(cx))^2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{d^2} - \frac{b^2c^3\arctan(cx)}{d^2} \\
&\quad + \frac{26bc^3(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{3d^2} + \frac{13b^2c^3\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)}{3d^2} \\
&\quad - \frac{5ibc^3(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{d^2} \\
&\quad + \frac{5ibc^3(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{d^2} - \frac{13b^2c^3\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)}{3d^2} \\
&\quad + \frac{5ib^2c^3\operatorname{PolyLog}\left(3,-ie^{\operatorname{arcsinh}(cx)}\right)}{d^2} - \frac{5ib^2c^3\operatorname{PolyLog}\left(3,ie^{\operatorname{arcsinh}(cx)}\right)}{d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 8.04 (sec) , antiderivative size = 764, normalized size of antiderivative = 1.91

$$
\begin{aligned}
\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^2}\,dx &= -\frac{a^2}{3d^2x^3} + \frac{2a^2c^2}{d^2x} + \frac{a^2c^4x}{2d^2(1+c^2x^2)} + \frac{5a^2c^3\arctan(cx)}{2d^2} \\
&\quad + \frac{2ab\left(-\frac{c\sqrt{1+c^2x^2}}{6x^2} - \frac{c^3\left(\sqrt{1+c^2x^2}+i\operatorname{arcsinh}(cx)\right)}{4(-1-icx)} - \frac{\operatorname{arcsinh}(cx)}{3x^3} + \frac{c^4\left(i\sqrt{1+c^2x^2}+\operatorname{arcsinh}(cx)\right)}{4(ic+c^2x)} + \frac{1}{6}c^3\operatorname{arctanh}\left(\sqrt{1+c\right.\right.}{} \\
&\quad + \frac{b^2c^3\left(\frac{24\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}} + \frac{12cx\operatorname{arcsinh}(cx)^2}{1+c^2x^2} - 48\arctan\left(\tanh\left(\frac{1}{2}\operatorname{arcsinh}(cx)\right)\right) - 4\coth\left(\frac{1}{2}\operatorname{arcsinh}(cx)\right) + 26\operatorname{arc}\right.}{}
\end{aligned}
$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^2),x]
```

```
[Out] -1/3*a^2/(d^2*x^3) + (2*a^2*c^2)/(d^2*x) + (a^2*c^4*x)/(2*d^2*(1 + c^2*x^2)
) + (5*a^2*c^3*ArcTan[c*x])/(2*d^2) + (2*a*b*(-1/6*(c*Sqrt[1 + c^2*x^2])/x^
2 - (c^3*(Sqrt[1 + c^2*x^2] + I*ArcSinh[c*x]))/(4*(-1 - I*c*x)) - ArcSinh[c
*x]/(3*x^3) + (c^4*(I*Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/(4*(I*c + c^2*x))
+ (c^3*ArcTanh[Sqrt[1 + c^2*x^2]])/6 - 2*c^2*(-(ArcSinh[c*x]/x) - c*ArcTanh
[Sqrt[1 + c^2*x^2]]) - ((5*I)/4)*c^4*(-1/2*ArcSinh[c*x]^2/c + (2*ArcSinh[c*
x]*Log[1 + I*E^ArcSinh[c*x]])/c + (2*PolyLog[2, (-I)*E^ArcSinh[c*x]])/c) +
((5*I)/4)*c^4*(-1/2*ArcSinh[c*x]^2/c + (2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[
c*x]])/c + (2*PolyLog[2, I*E^ArcSinh[c*x]])/c)))/d^2 + (b^2*c^3*((24*ArcSin
h[c*x])/Sqrt[1 + c^2*x^2] + (12*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2) - 48*ArcT
an[Tanh[ArcSinh[c*x]/2]] - 4*Coth[ArcSinh[c*x]/2] + 26*ArcSinh[c*x]^2*Coth[
ArcSinh[c*x]/2] - 2*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - (c*x*ArcSinh[c*x]
^2*Csch[ArcSinh[c*x]/2]^4)/2 - 104*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])]
- (60*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] + (60*I)*ArcSinh[c*x]^2*L
```

```
og[1 + I/E^ArcSinh[c*x]] + 104*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 10
4*PolyLog[2, -E^(-ArcSinh[c*x])] - (120*I)*ArcSinh[c*x]*PolyLog[2, (-I)/E^A
rcSinh[c*x]] + (120*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] + 104*Poly
Log[2, E^(-ArcSinh[c*x])] - (120*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] + (120*
I)*PolyLog[3, I/E^ArcSinh[c*x]] - 2*ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 - (
8*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2]^4)/(c^3*x^3) + 4*Tanh[ArcSinh[c*x]/2]
 - 26*ArcSinh[c*x]^2*Tanh[ArcSinh[c*x]/2]))/(24*d^2)
```

Maple [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^4\left(c^2d\,x^2+d\right)^2}dx$$

```
[In] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^2,x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^2,x)
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\left(d+c^2dx^2\right)^2}\,dx = \int \frac{\left(b\operatorname{arsinh}(cx)+a\right)^2}{\left(c^2dx^2+d\right)^2x^4}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^4*d^2*x^8 + 2*c
^2*d^2*x^6 + d^2*x^4), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\left(d+c^2dx^2\right)^2}\,dx = \frac{\int \frac{a^2}{c^4x^8+2c^2x^6+x^4}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{c^4x^8+2c^2x^6+x^4}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{c^4x^8+2c^2x^6+x^4}\,dx}{d^2}$$

```
[In] integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2/(c**4*x**8 + 2*c**2*x**6 + x**4), x) + Integral(b**2*asinh(c
*x)**2/(c**4*x**8 + 2*c**2*x**6 + x**4), x) + Integral(2*a*b*asinh(c*x)/(c*
*4*x**8 + 2*c**2*x**6 + x**4), x))/d**2
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^2}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(c^2dx^2+d)^2x^4}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(15*c^3*arctan(c*x)/d^2 + (15*c^4*x^4 + 10*c^2*x^2 - 2)/(c^2*d^2*x^5 +
d^2*x^3))*a^2 + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^8 +
 2*c^2*d^2*x^6 + d^2*x^4) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^8
 + 2*c^2*d^2*x^6 + d^2*x^4), x)
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^2}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(c^2dx^2+d)^2x^4}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^2*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^2}\,dx=\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2}{x^4\,(d\,c^2\,x^2+d)^2}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^2),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^2), x)
```

3.243 $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 320

$$\begin{aligned}\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx &= -\frac{b^2x}{12c^4d^3(1+c^2x^2)}+\frac{b(a+b\text{arcsinh}(cx))}{6c^5d^3(1+c^2x^2)^{3/2}}\\ &\quad-\frac{5b(a+b\text{arcsinh}(cx))}{4c^5d^3\sqrt{1+c^2x^2}}\\ &\quad-\frac{x^3(a+b\text{arcsinh}(cx))^2}{4c^2d^3(1+c^2x^2)^2}-\frac{3x(a+b\text{arcsinh}(cx))^2}{8c^4d^3(1+c^2x^2)}\\ &\quad+\frac{3(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4c^5d^3}+\frac{7b^2\arctan(cx)}{6c^5d^3}\\ &\quad-\frac{3ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{4c^5d^3}\\ &\quad+\frac{3ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{4c^5d^3}\\ &\quad+\frac{3ib^2\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{4c^5d^3}\\ &\quad-\frac{3ib^2\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{4c^5d^3}\end{aligned}$$

```
[Out] -1/12*b^2*x/c^4/d^3/(c^2*x^2+1)+1/6*b*(a+b*arcsinh(c*x))/c^5/d^3/(c^2*x^2+1
)^(3/2)-1/4*x^3*(a+b*arcsinh(c*x))^2/c^2/d^3/(c^2*x^2+1)^2-3/8*x*(a+b*arcsi
nh(c*x))^2/c^4/d^3/(c^2*x^2+1)+3/4*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2
+1)^(1/2))/c^5/d^3+7/6*b^2*arctan(c*x)/c^5/d^3-3/4*I*b*(a+b*arcsinh(c*x))*p
olylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d^3+3/4*I*b*(a+b*arcsinh(c*x))*pol
ylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d^3+3/4*I*b^2*polylog(3,-I*(c*x+(c^2*
x^2+1)^(1/2)))/c^5/d^3-3/4*I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d
^3-5/4*b*(a+b*arcsinh(c*x))/c^5/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5810, 5789, 4265, 2611, 2320, 6724, 5798, 209, 272, 45, 5804, 12, 393}

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \frac{3\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx))^2}{4c^5d^3} - \frac{3ib\,\text{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx))}{4c^5d^3} + \frac{3ib\,\text{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx))}{4c^5d^3} - \frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{4c^2d^3(c^2x^2+1)^2} - \frac{5b(a+b\operatorname{arcsinh}(cx))}{4c^5d^3\sqrt{c^2x^2+1}} + \frac{b(a+b\operatorname{arcsinh}(cx))}{6c^5d^3(c^2x^2+1)^{3/2}} - \frac{3x(a+b\operatorname{arcsinh}(cx))^2}{8c^4d^3(c^2x^2+1)} + \frac{3ib^2\,\text{PolyLog}\left(3,-ie^{\operatorname{arcsinh}(cx)}\right)}{4c^5d^3} - \frac{3ib^2\,\text{PolyLog}\left(3,ie^{\operatorname{arcsinh}(cx)}\right)}{4c^5d^3} + \frac{7b^2\arctan(cx)}{6c^5d^3} - \frac{b^2x}{12c^4d^3(c^2x^2+1)}$$

```
[In] Int[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] -1/12*(b^2*x)/(c^4*d^3*(1 + c^2*x^2)) + (b*(a + b*ArcSinh[c*x]))/(6*c^5*d^3
*(1 + c^2*x^2)^(3/2)) - (5*b*(a + b*ArcSinh[c*x]))/(4*c^5*d^3*Sqrt[1 + c^2*
x^2]) - (x^3*(a + b*ArcSinh[c*x])^2)/(4*c^2*d^3*(1 + c^2*x^2)^2) - (3*x*(a
+ b*ArcSinh[c*x])^2)/(8*c^4*d^3*(1 + c^2*x^2)) + (3*(a + b*ArcSinh[c*x])^2*
ArcTan[E^ArcSinh[c*x]])/(4*c^5*d^3) + (7*b^2*ArcTan[c*x])/(6*c^5*d^3) - (((
3*I)/4)*b*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^5*d^3) +
 (((3*I)/4)*b*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/(c^5*d^3)
+ (((3*I)/4)*b^2*PolyLog[3, (-I)*E^ArcSinh[c*x]])/(c^5*d^3) - (((3*I)/4)*b^
2*PolyLog[3, I*E^ArcSinh[c*x]])/(c^5*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = -\frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{b\int\frac{x^3(a+b\operatorname{arcsinh}(cx))}{(1+c^2x^2)^{5/2}}\,dx}{2cd^3}+\frac{3\int\frac{x^2(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx}{4c^2d}$$

$$= \frac{b(a+b\operatorname{arcsinh}(cx))}{6c^5d^3(1+c^2x^2)^{3/2}} - \frac{b(a+b\operatorname{arcsinh}(cx))}{2c^5d^3\sqrt{1+c^2x^2}} - \frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{4c^2d^3(1+c^2x^2)^2} - \frac{3x(a+b\operatorname{arcsinh}(cx))^2}{8c^4d^3(1+c^2x^2)} - \frac{b^2\int\frac{-2-3c^2x^2}{3c^4(1+c^2x^2)^2}\,dx}{2d^3} + \frac{(3b)\int\frac{x(a+b\operatorname{arcsinh}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{4c^3d^3} + \frac{3\int\frac{(a+b\operatorname{arcsinh}(cx))^2}{d+c^2dx^2}\,dx}{8c^4d^2}$$

$$= \frac{b(a+b\operatorname{arcsinh}(cx))}{6c^5d^3(1+c^2x^2)^{3/2}} - \frac{5b(a+b\operatorname{arcsinh}(cx))}{4c^5d^3\sqrt{1+c^2x^2}} - \frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{4c^2d^3(1+c^2x^2)^2} - \frac{3x(a+b\operatorname{arcsinh}(cx))^2}{8c^4d^3(1+c^2x^2)} + \frac{3\operatorname{Subst}\left(\int(a+bx)^2\operatorname{sech}(x)\,dx, x, \operatorname{arcsinh}(cx)\right)}{8c^5d^3} - \frac{b^2\int\frac{-2-3c^2x^2}{(1+c^2x^2)^2}\,dx}{6c^4d^3} + \frac{(3b^2)\int\frac{1}{1+c^2x^2}\,dx}{4c^4d^3}$$

$$= -\frac{b^2x}{12c^4d^3(1+c^2x^2)} + \frac{b(a+b\operatorname{arcsinh}(cx))}{6c^5d^3(1+c^2x^2)^{3/2}} - \frac{5b(a+b\operatorname{arcsinh}(cx))}{4c^5d^3\sqrt{1+c^2x^2}} - \frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{4c^2d^3(1+c^2x^2)^2} - \frac{3x(a+b\operatorname{arcsinh}(cx))^2}{8c^4d^3(1+c^2x^2)} + \frac{3(a+b\operatorname{arcsinh}(cx))^2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{4c^5d^3} + \frac{3b^2\arctan(cx)}{4c^5d^3} - \frac{(3ib)\operatorname{Subst}\left(\int(a+bx)\log\left(1-ie^x\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{4c^5d^3} + \frac{(3ib)\operatorname{Subst}\left(\int(a+bx)\log\left(1+ie^x\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{4c^5d^3} + \frac{(5b^2)\int\frac{1}{1+c^2x^2}\,dx}{12c^4d^3}$$

$$= -\frac{b^2x}{12c^4d^3(1+c^2x^2)} + \frac{b(a+b\operatorname{arcsinh}(cx))}{6c^5d^3(1+c^2x^2)^{3/2}} - \frac{5b(a+b\operatorname{arcsinh}(cx))}{4c^5d^3\sqrt{1+c^2x^2}} - \frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{4c^2d^3(1+c^2x^2)^2} - \frac{3x(a+b\operatorname{arcsinh}(cx))^2}{8c^4d^3(1+c^2x^2)} + \frac{3(a+b\operatorname{arcsinh}(cx))^2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{4c^5d^3} + \frac{7b^2\arctan(cx)}{6c^5d^3} - \frac{3ib(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{4c^5d^3} + \frac{3ib(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{4c^5d^3} + \frac{(3ib^2)\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(2,-ie^x\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{4c^5d^3} - \frac{(3ib^2)\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(2,ie^x\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{4c^5d^3}$$

$$= -\frac{b^2x}{12c^4d^3(1+c^2x^2)} + \frac{b(a+b\operatorname{arcsinh}(cx))}{6c^5d^3(1+c^2x^2)^{3/2}} - \frac{5b(a+b\operatorname{arcsinh}(cx))}{4c^5d^3\sqrt{1+c^2x^2}} - \frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{4c^2d^3(1+c^2x^2)^2} - \frac{3x(a+b\operatorname{arcsinh}(cx))^2}{8c^4d^3(1+c^2x^2)} + \frac{3(a+b\operatorname{arcsinh}(cx))^2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{4c^5d^3} + \frac{7b^2\arctan(cx)}{6c^5d^3} - \frac{3ib(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{4c^5d^3} + \frac{3ib(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{4c^5d^3} + \frac{(3ib^2)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,-ix)}{x}dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{4c^5d^3} - \frac{(3ib^2)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,ix)}{x}dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{4c^5d^3}$$

$$= -\frac{b^2x}{12c^4d^3(1+c^2x^2)} + \frac{b(a+b\operatorname{arcsinh}(cx))}{6c^5d^3(1+c^2x^2)^{3/2}} - \frac{5b(a+b\operatorname{arcsinh}(cx))}{4c^5d^3\sqrt{1+c^2x^2}} - \frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{4c^2d^3(1+c^2x^2)^2} - \frac{3x(a+b\operatorname{arcsinh}(cx))^2}{8c^4d^3(1+c^2x^2)} + \frac{3(a+b\operatorname{arcsinh}(cx))^2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{4c^5d^3} + \frac{7b^2\arctan(cx)}{6c^5d^3} - \frac{3ib(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{4c^5d^3} + \frac{3ib(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{4c^5d^3} + \frac{3ib^2\operatorname{PolyLog}\left(3,-ie^{\operatorname{arcsinh}(cx)}\right)}{4c^5d^3} - \frac{3ib^2\operatorname{PolyLog}\left(3,ie^{\operatorname{arcsinh}(cx)}\right)}{4c^5d^3}$$

Mathematica [A] (verified)

Time = 2.35 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.72

$$\int\frac{x^4(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \frac{\frac{6a^2cx}{(1+c^2x^2)^2} - \frac{15a^2cx}{1+c^2x^2} + \frac{15ab\left(\sqrt{1+c^2x^2}-i\operatorname{arcsinh}(cx)\right)}{-1+icx} + \frac{15ab\left(\sqrt{1+c^2x^2}+i\operatorname{arcsinh}(cx)\right)}{-1-icx} - \frac{iab\left((-2i+cx)\sqrt{1+c^2x^2}+3\operatorname{arcsinh}(cx)\right.}{(-i+cx)^2}}{}$$

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] ((6*a^2*c*x)/(1 + c^2*x^2)^2 - (15*a^2*c*x)/(1 + c^2*x^2) + (15*a*b*(Sqrt[1
 + c^2*x^2] - I*ArcSinh[c*x]))/(-1 + I*c*x) + (15*a*b*(Sqrt[1 + c^2*x^2] +
I*ArcSinh[c*x]))/(-1 - I*c*x) - (I*a*b*((-2*I + c*x)*Sqrt[1 + c^2*x^2] + 3*
```

```
ArcSinh[c*x]))/(-I + c*x)^2 + (I*a*b*((2*I + c*x)*Sqrt[1 + c^2*x^2] + 3*Arc
Sinh[c*x]))/(I + c*x)^2 + 9*a^2*ArcTan[c*x] + ((9*I)/2)*a*b*(ArcSinh[c*x]*(
ArcSinh[c*x] - 4*Log[1 + I*E^ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c
*x]]) - ((9*I)/2)*a*b*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c
*x]]) - 4*PolyLog[2, I*E^ArcSinh[c*x]]) + b^2*((-2*c*x)/(1 + c^2*x^2) + (4*
ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) - (30*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + (
6*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2)^2 - (15*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^
2) + 56*ArcTan[Tanh[ArcSinh[c*x]/2]] - (9*I)*ArcSinh[c*x]^2*Log[1 - I/E^Arc
Sinh[c*x]] + (9*I)*ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] - (18*I)*ArcSin
h[c*x]*PolyLog[2, (-I)/E^ArcSinh[c*x]] + (18*I)*ArcSinh[c*x]*PolyLog[2, I/E
^ArcSinh[c*x]] - (18*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] + (18*I)*PolyLog[3,
 I/E^ArcSinh[c*x]]))/(24*c^5*d^3)
```

Maple [F]

$$\int \frac{x^{4}(a+b\,\operatorname{arcsinh}(cx))^{2}}{(c^{2}d\,x^{2}+d)^{3}}dx$$

```
[In] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
[Out] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

Fricas [F]

$$\int \frac{x^4(a+b\mathrm{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx=\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2x^4}{(c^2dx^2+d)^3}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
[Out] integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)/(c^6*d
^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\mathrm{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \frac{\int \frac{a^2x^4}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx+\int \frac{b^2x^4\operatorname{asinh}^2(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx+\int \frac{2abx^4\operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

```
[In] integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**3,x)
[Out] (Integral(a**2*x**4/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integ
ral(b**2*x**4*asinh(c*x)**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x)
 + Integral(2*a*b*x**4*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 +
1), x))/d**3
```

Maxima [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2x^4}{(c^2dx^2+d)^3}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a^2*((5*c^2*x^3 + 3*x)/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) - 3*arc
tan(c*x)/(c^5*d^3)) + integrate(b^2*x^4*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^6
*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3) + 2*a*b*x^4*log(c*x + sqrt(
c^2*x^2 + 1))/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Giac [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2x^4}{(c^2dx^2+d)^3}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^4/(c^2*d*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \int \frac{x^4\,(a+b\,\mathrm{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^3}\,dx$$

```
[In] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3,x)
```

```
[Out] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3, x)
```

3.244 $\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 167

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = -\frac{b^2}{12c^4d^3(1+c^2x^2)}+\frac{bx^3(a+b\text{arcsinh}(cx))}{6cd^3(1+c^2x^2)^{3/2}}+\frac{bx(a+b\text{arcsinh}(cx))}{2c^3d^3\sqrt{1+c^2x^2}}-\frac{(a+b\text{arcsinh}(cx))^2}{4c^4d^3}+\frac{x^4(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2}-\frac{b^2\log(1+c^2x^2)}{3c^4d^3}$$

```
[Out] -1/12*b^2/c^4/d^3/(c^2*x^2+1)+1/6*b*x^3*(a+b*arcsinh(c*x))/c/d^3/(c^2*x^2+1
)^(3/2)-1/4*(a+b*arcsinh(c*x))^2/c^4/d^3+1/4*x^4*(a+b*arcsinh(c*x))^2/d^3/(
c^2*x^2+1)^2-1/3*b^2*ln(c^2*x^2+1)/c^4/d^3+1/2*b*x*(a+b*arcsinh(c*x))/c^3/d
^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5800, 5810, 5783, 266, 272, 45}

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = -\frac{(a+b\text{arcsinh}(cx))^2}{4c^4d^3}+\frac{x^4(a+b\text{arcsinh}(cx))^2}{4d^3(c^2x^2+1)^2}+\frac{bx^3(a+b\text{arcsinh}(cx))}{6cd^3(c^2x^2+1)^{3/2}}+\frac{bx(a+b\text{arcsinh}(cx))}{2c^3d^3\sqrt{c^2x^2+1}}-\frac{b^2}{12c^4d^3(c^2x^2+1)}-\frac{b^2\log(c^2x^2+1)}{3c^4d^3}$$

```
[In] Int[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] -1/12*b^2/(c^4*d^3*(1 + c^2*x^2)) + (b*x^3*(a + b*ArcSinh[c*x]))/(6*c*d^3*(
1 + c^2*x^2)^(3/2)) + (b*x*(a + b*ArcSinh[c*x]))/(2*c^3*d^3*Sqrt[1 + c^2*x^
2]) - (a + b*ArcSinh[c*x])^2/(4*c^4*d^3) + (x^4*(a + b*ArcSinh[c*x])^2)/(4*
d^3*(1 + c^2*x^2)^2) - (b^2*Log[1 + c^2*x^2])/(3*c^4*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
```

```
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rubi steps

$$\text{integral} = \frac{x^4(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} - \frac{(bc)\int \frac{x^4(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{5/2}}\,dx}{2d^3}$$

$$= \frac{bx^3(a+b\text{arcsinh}(cx))}{6cd^3(1+c^2x^2)^{3/2}} + \frac{x^4(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} - \frac{b^2\int \frac{x^3}{(1+c^2x^2)^2}\,dx}{6d^3} - \frac{b\int \frac{x^2(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{2cd^3}$$

$$= \frac{bx^3(a+b\text{arcsinh}(cx))}{6cd^3(1+c^2x^2)^{3/2}} + \frac{bx(a+b\text{arcsinh}(cx))}{2c^3d^3\sqrt{1+c^2x^2}} + \frac{x^4(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} - \frac{b^2\text{Subst}\left(\int \frac{x}{(1+c^2x)^2}\,dx, x, x^2\right)}{12d^3} - \frac{b\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{2c^3d^3} - \frac{b^2\int \frac{x}{1+c^2x^2}\,dx}{2c^2d^3}$$

$$= \frac{bx^3(a+b\text{arcsinh}(cx))}{6cd^3(1+c^2x^2)^{3/2}} + \frac{bx(a+b\text{arcsinh}(cx))}{2c^3d^3\sqrt{1+c^2x^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{4c^4d^3} + \frac{x^4(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} - \frac{b^2\log(1+c^2x^2)}{4c^4d^3} - \frac{b^2\text{Subst}\left(\int \left(-\frac{1}{c^2(1+c^2x)^2} + \frac{1}{c^2(1+c^2x)}\right)dx, x, x^2\right)}{12d^3}$$

$$= -\frac{b^2}{12c^4d^3(1+c^2x^2)} + \frac{bx^3(a+b\text{arcsinh}(cx))}{6cd^3(1+c^2x^2)^{3/2}} + \frac{bx(a+b\text{arcsinh}(cx))}{2c^3d^3\sqrt{1+c^2x^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{4c^4d^3} + \frac{x^4(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} - \frac{b^2\log(1+c^2x^2)}{3c^4d^3}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.11

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = -\frac{3a^2+b^2+6a^2c^2x^2+b^2c^2x^2-6abcx\sqrt{1+c^2x^2}-8abc^3x^3\sqrt{1+c^2x^2}+2b\left(-bcx\sqrt{1+c^2x^2}(3+4c^2x^2)-\right.}{12c^4d^3(1+c^2x^2)}$$

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] -1/12*(3*a^2 + b^2 + 6*a^2*c^2*x^2 + b^2*c^2*x^2 - 6*a*b*c*x*Sqrt[1 + c^2*x
^2] - 8*a*b*c^3*x^3*Sqrt[1 + c^2*x^2] + 2*b*(-(b*c*x*Sqrt[1 + c^2*x^2]*(3 +
```

4*c^2*x^2)) + a*(3 + 6*c^2*x^2))*ArcSinh[c*x] + 3*b^2*(1 + 2*c^2*x^2)*ArcSinh[c*x]^2 + 4*(b + b*c^2*x^2)^2*Log[1 + c^2*x^2])/(c^4*d^3*(1 + c^2*x^2)^2)

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.63

method	result
derivativedivides	$\frac{a^2\left(\frac{1}{4(c^2x^2+1)^2}-\frac{1}{2(c^2x^2+1)}\right)}{d^3}+\frac{b^2\left(\frac{4\,\mathrm{arcsinh}(cx)}{3}-\frac{-8\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+8\,\mathrm{arcsinh}(cx)c^4x^4+6\,\mathrm{arcsinh}(cx)^2x^2c^2-6\,\mathrm{arcs}}{12(c^4x^4}\right.}{}$
default	$\frac{a^2\left(\frac{1}{4(c^2x^2+1)^2}-\frac{1}{2(c^2x^2+1)}\right)}{d^3}+\frac{b^2\left(\frac{4\,\mathrm{arcsinh}(cx)}{3}-\frac{-8\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+8\,\mathrm{arcsinh}(cx)c^4x^4+6\,\mathrm{arcsinh}(cx)^2x^2c^2-6\,\mathrm{arcs}}{12(c^4x^4}\right.}{}$
parts	$\frac{a^2\left(-\frac{1}{2c^4(c^2x^2+1)}+\frac{1}{4c^4(c^2x^2+1)^2}\right)}{d^3}+\frac{b^2\left(\frac{4\,\mathrm{arcsinh}(cx)}{3}-\frac{-8\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+8\,\mathrm{arcsinh}(cx)c^4x^4+6\,\mathrm{arcsinh}(cx)^2x^2}{}\right.}{}$

[In] int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)

[Out] 1/c^4*(a^2/d^3*(1/4/(c^2*x^2+1)^2-1/2/(c^2*x^2+1))+b^2/d^3*(4/3*arcsinh(c*x)-1/12*(-8*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3+8*arcsinh(c*x)*c^4*x^4+6*arcsinh(c*x)^2*x^2*c^2-6*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)+16*arcsinh(c*x)*c^2*x^2+3*arcsinh(c*x)^2+c^2*x^2+8*arcsinh(c*x)+1)/(c^4*x^4+2*c^2*x^2+1)-2/3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2))+2*a*b/d^3*(1/4/(c^2*x^2+1)^2*arcsinh(c*x)-1/2/(c^2*x^2+1)*arcsinh(c*x)-1/12/(c^2*x^2+1)^(3/2)*c*x+1/3*c*x/(c^2*x^2+1)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.69

$$\int \frac{x^3(a+b\,\mathrm{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$$

$$=\frac{8\,abc^4x^4-(6\,a^2-16\,ab+b^2)c^2x^2-3\,(2\,b^2c^2x^2+b^2)\log\left(cx+\sqrt{c^2x^2+1}\right)^2-3\,a^2+8\,ab-b^2-4\,(b^2c^4x^4}{}$$

[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")

[Out] 1/12*(8*a*b*c^4*x^4 - (6*a^2 - 16*a*b + b^2)*c^2*x^2 - 3*(2*b^2*c^2*x^2 + b^2)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 3*a^2 + 8*a*b - b^2 - 4*(b^2*c^4*x^4 +

```
 2*b^2*c^2*x^2 + b^2)*log(c^2*x^2 + 1) + 2*(3*a*b*c^4*x^4 + (4*b^2*c^3*x^3
+ 3*b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 6*(a*b*c^4*x
^4 + 2*a*b*c^2*x^2 + a*b)*log(-c*x + sqrt(c^2*x^2 + 1)) + 2*(4*a*b*c^3*x^3
+ 3*a*b*c*x)*sqrt(c^2*x^2 + 1))/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3)
```

Sympy [F]

$$\int \frac{x^3(a + b\text{arcsinh}(cx))^2}{(d + c^2dx^2)^3}\,dx = \frac{\int \frac{a^2x^3}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{b^2x^3\operatorname{asinh}^2(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{2abx^3\operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

```
[In] integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2*x**3/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integ
ral(b**2*x**3*asinh(c*x)**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x)
 + Integral(2*a*b*x**3*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 +
1), x))/d**3
```

Maxima [F]

$$\int \frac{x^3(a + b\text{arcsinh}(cx))^2}{(d + c^2dx^2)^3}\,dx = \int \frac{(b\operatorname{arsinh}(cx) + a)^2x^3}{(c^2dx^2 + d)^3}\,dx$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*c^2*x^2 + 1)*a^2/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) - 1/4*(2*b
^2*c^2*x^2 + b^2)*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^8*d^3*x^4 + 2*c^6*d^3*x
^2 + c^4*d^3) + integrate(1/2*(3*b^2*c^2*x^2 + 2*(2*a*b*c^4 + b^2*c^4)*x^4
+ b^2 + (b^2*c*x + 2*(2*a*b*c^3 + b^2*c^3)*x^3)*sqrt(c^2*x^2 + 1))*log(c*x
+ sqrt(c^2*x^2 + 1))/(c^10*d^3*x^7 + 3*c^8*d^3*x^5 + 3*c^6*d^3*x^3 + c^4*d^
3*x + (c^9*d^3*x^6 + 3*c^7*d^3*x^4 + 3*c^5*d^3*x^2 + c^3*d^3)*sqrt(c^2*x^2
+ 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\text{arcsinh}(cx))^2}{(d + c^2dx^2)^3} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\text{arcsinh}(cx))^2}{(d + c^2dx^2)^3} \, dx = \int \frac{x^3 \left(a + b \operatorname{asinh}(c\,x)\right)^2}{\left(d\,c^2\,x^2 + d\right)^3} \, dx$$

```
[In] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3,x)
```

```
[Out] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3, x)
```

3.245 $\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 318

$$\begin{aligned}\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx &= \frac{b^2x}{12c^2d^3(1+c^2x^2)} - \frac{b(a+b\text{arcsinh}(cx))}{6c^3d^3(1+c^2x^2)^{3/2}} + \frac{b(a+b\text{arcsinh}(cx))}{4c^3d^3\sqrt{1+c^2x^2}} \\ &\quad - \frac{x(a+b\text{arcsinh}(cx))^2}{4c^2d^3(1+c^2x^2)^2} + \frac{x(a+b\text{arcsinh}(cx))^2}{8c^2d^3(1+c^2x^2)} \\ &\quad + \frac{(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4c^3d^3} - \frac{b^2\arctan(cx)}{6c^3d^3} \\ &\quad - \frac{ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{4c^3d^3} \\ &\quad + \frac{ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{4c^3d^3} \\ &\quad + \frac{ib^2\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{4c^3d^3} - \frac{ib^2\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{4c^3d^3}\end{aligned}$$

```
[Out] 1/12*b^2*x/c^2/d^3/(c^2*x^2+1)-1/6*b*(a+b*arcsinh(c*x))/c^3/d^3/(c^2*x^2+1)
^(3/2)-1/4*x*(a+b*arcsinh(c*x))^2/c^2/d^3/(c^2*x^2+1)^2+1/8*x*(a+b*arcsinh(
c*x))^2/c^2/d^3/(c^2*x^2+1)+1/4*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)
^(1/2))/c^3/d^3-1/6*b^2*arctan(c*x)/c^3/d^3-1/4*I*b*(a+b*arcsinh(c*x))*poly
log(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^3+1/4*I*b*(a+b*arcsinh(c*x))*polylo
g(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^3+1/4*I*b^2*polylog(3,-I*(c*x+(c^2*x^2
+1)^(1/2)))/c^3/d^3-1/4*I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^3+
1/4*b*(a+b*arcsinh(c*x))/c^3/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5810, 5788, 5789, 4265, 2611, 2320, 6724, 5798, 209, 205}

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \frac{\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{4c^3d^3} - \frac{ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{4c^3d^3} + \frac{ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{4c^3d^3} + \frac{x(a+b\text{arcsinh}(cx))^2}{8c^2d^3(c^2x^2+1)} - \frac{x(a+b\text{arcsinh}(cx))^2}{4c^2d^3(c^2x^2+1)^2} + \frac{b(a+b\text{arcsinh}(cx))}{4c^3d^3\sqrt{c^2x^2+1}} - \frac{b(a+b\text{arcsinh}(cx))}{6c^3d^3(c^2x^2+1)^{3/2}} + \frac{ib^2\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{4c^3d^3} - \frac{ib^2\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{4c^3d^3} - \frac{b^2\arctan(cx)}{6c^3d^3} + \frac{b^2x}{12c^2d^3(c^2x^2+1)}$$

```
[In] Int[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] (b^2*x)/(12*c^2*d^3*(1 + c^2*x^2)) - (b*(a + b*ArcSinh[c*x]))/(6*c^3*d^3*(1
 + c^2*x^2)^(3/2)) + (b*(a + b*ArcSinh[c*x]))/(4*c^3*d^3*Sqrt[1 + c^2*x^2])
 - (x*(a + b*ArcSinh[c*x])^2)/(4*c^2*d^3*(1 + c^2*x^2)^2) + (x*(a + b*ArcSi
nh[c*x])^2)/(8*c^2*d^3*(1 + c^2*x^2)) + ((a + b*ArcSinh[c*x])^2*ArcTan[E^Ar
cSinh[c*x]])/(4*c^3*d^3) - (b^2*ArcTan[c*x])/(6*c^3*d^3) - ((I/4)*b*(a + b*
ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^3*d^3) + ((I/4)*b*(a + b*
ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/(c^3*d^3) + ((I/4)*b^2*PolyLog[
3, (-I)*E^ArcSinh[c*x]])/(c^3*d^3) - ((I/4)*b^2*PolyLog[3, I*E^ArcSinh[c*x]
])/(c^3*d^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
```

```
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x(a+b\text{arcsinh}(cx))^2}{4c^2d^3(1+c^2x^2)^2}+\frac{b\int\frac{x(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{5/2}}\,dx}{2cd^3}+\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx}{4c^2d}\\
&= -\frac{b(a+b\text{arcsinh}(cx))}{6c^3d^3(1+c^2x^2)^{3/2}}-\frac{x(a+b\text{arcsinh}(cx))^2}{4c^2d^3(1+c^2x^2)^2}+\frac{x(a+b\text{arcsinh}(cx))^2}{8c^2d^3(1+c^2x^2)}\\
&\quad+\frac{b^2\int\frac{1}{(1+c^2x^2)^2}\,dx}{6c^2d^3}-\frac{b\int\frac{x(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{4cd^3}+\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx}{8c^2d^2}\\
&= \frac{b^2x}{12c^2d^3(1+c^2x^2)}-\frac{b(a+b\text{arcsinh}(cx))}{6c^3d^3(1+c^2x^2)^{3/2}}+\frac{b(a+b\text{arcsinh}(cx))}{4c^3d^3\sqrt{1+c^2x^2}}\\
&\quad-\frac{x(a+b\text{arcsinh}(cx))^2}{4c^2d^3(1+c^2x^2)^2}+\frac{x(a+b\text{arcsinh}(cx))^2}{8c^2d^3(1+c^2x^2)}\\
&\quad+\frac{\text{Subst}\left(\int(a+bx)^2\text{sech}(x)\,dx,x,\text{arcsinh}(cx)\right)}{8c^3d^3}+\frac{b^2\int\frac{1}{1+c^2x^2}\,dx}{12c^2d^3}-\frac{b^2\int\frac{1}{1+c^2x^2}\,dx}{4c^2d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b^2 x}{12c^2 d^3 (1+c^2x^2)} - \frac{b(a+b\operatorname{arcsinh}(cx))}{6c^3 d^3 (1+c^2x^2)^{3/2}} + \frac{b(a+b\operatorname{arcsinh}(cx))}{4c^3 d^3 \sqrt{1+c^2x^2}} \\
&\quad - \frac{x(a+b\operatorname{arcsinh}(cx))^2}{4c^2 d^3 (1+c^2x^2)^2} + \frac{x(a+b\operatorname{arcsinh}(cx))^2}{8c^2 d^3 (1+c^2x^2)} \\
&\quad + \frac{(a+b\operatorname{arcsinh}(cx))^2 \arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{4c^3 d^3} - \frac{b^2 \arctan(cx)}{6c^3 d^3} \\
&\quad - \frac{(ib)\text{Subst}\left(\int (a+bx)\log\left(1-ie^x\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{4c^3 d^3} \\
&\quad + \frac{(ib)\text{Subst}\left(\int (a+bx)\log\left(1+ie^x\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{4c^3 d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b^2 x}{12c^2 d^3 (1+c^2x^2)} - \frac{b(a+b\operatorname{arcsinh}(cx))}{6c^3 d^3 (1+c^2x^2)^{3/2}} + \frac{b(a+b\operatorname{arcsinh}(cx))}{4c^3 d^3 \sqrt{1+c^2x^2}} \\
&\quad - \frac{x(a+b\operatorname{arcsinh}(cx))^2}{4c^2 d^3 (1+c^2x^2)^2} + \frac{x(a+b\operatorname{arcsinh}(cx))^2}{8c^2 d^3 (1+c^2x^2)} \\
&\quad + \frac{(a+b\operatorname{arcsinh}(cx))^2 \arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{4c^3 d^3} - \frac{b^2 \arctan(cx)}{6c^3 d^3} \\
&\quad - \frac{ib(a+b\operatorname{arcsinh}(cx))\,\text{PolyLog}\left(2, -ie^{\operatorname{arcsinh}(cx)}\right)}{4c^3 d^3} \\
&\quad + \frac{ib(a+b\operatorname{arcsinh}(cx))\,\text{PolyLog}\left(2, ie^{\operatorname{arcsinh}(cx)}\right)}{4c^3 d^3} \\
&\quad + \frac{(ib^2)\,\text{Subst}\left(\int \text{PolyLog}\left(2, -ie^x\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{4c^3 d^3} \\
&\quad - \frac{(ib^2)\,\text{Subst}\left(\int \text{PolyLog}\left(2, ie^x\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{4c^3 d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b^2 x}{12c^2 d^3 (1+c^2x^2)} - \frac{b(a+b\operatorname{arcsinh}(cx))}{6c^3 d^3 (1+c^2x^2)^{3/2}} + \frac{b(a+b\operatorname{arcsinh}(cx))}{4c^3 d^3 \sqrt{1+c^2x^2}} \\
&\quad - \frac{x(a+b\operatorname{arcsinh}(cx))^2}{4c^2 d^3 (1+c^2x^2)^2} + \frac{x(a+b\operatorname{arcsinh}(cx))^2}{8c^2 d^3 (1+c^2x^2)} \\
&\quad + \frac{(a+b\operatorname{arcsinh}(cx))^2 \arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{4c^3 d^3} - \frac{b^2 \arctan(cx)}{6c^3 d^3} \\
&\quad - \frac{ib(a+b\operatorname{arcsinh}(cx))\,\text{PolyLog}\left(2, -ie^{\operatorname{arcsinh}(cx)}\right)}{4c^3 d^3} \\
&\quad + \frac{ib(a+b\operatorname{arcsinh}(cx))\,\text{PolyLog}\left(2, ie^{\operatorname{arcsinh}(cx)}\right)}{4c^3 d^3} \\
&\quad + \frac{(ib^2)\,\text{Subst}\left(\int \frac{\text{PolyLog}(2,-ix)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{4c^3 d^3} \\
&\quad - \frac{(ib^2)\,\text{Subst}\left(\int \frac{\text{PolyLog}(2,ix)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{4c^3 d^3}
\end{aligned}
$$

$$= \frac{b^2x}{12c^2d^3(1+c^2x^2)} - \frac{b(a+b\text{arcsinh}(cx))}{6c^3d^3(1+c^2x^2)^{3/2}} + \frac{b(a+b\text{arcsinh}(cx))}{4c^3d^3\sqrt{1+c^2x^2}} - \frac{x(a+b\text{arcsinh}(cx))^2}{4c^2d^3(1+c^2x^2)^2} + \frac{x(a+b\text{arcsinh}(cx))^2}{8c^2d^3(1+c^2x^2)} + \frac{(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4c^3d^3} - \frac{b^2\arctan(cx)}{6c^3d^3} - \frac{ib(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{4c^3d^3} + \frac{ib(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{4c^3d^3} + \frac{ib^2\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{4c^3d^3} - \frac{ib^2\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{4c^3d^3}$$

Mathematica [A] (verified)

Time = 2.03 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.73

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \frac{-\frac{6a^2cx}{(1+c^2x^2)^2} + \frac{3a^2cx}{1+c^2x^2} + \frac{ab\left((2+icx)\sqrt{1+c^2x^2}+3i\text{arcsinh}(cx)\right)}{(-i+cx)^2} + \frac{3ab\left(-i\sqrt{1+c^2x^2}+\text{arcsinh}(cx)\right)}{-i+cx} + \frac{3ab\left(i\sqrt{1+c^2x^2}+\text{arcsinh}(cx)\right)}{i+cx}}{}$$

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] ((-6*a^2*c*x)/(1 + c^2*x^2)^2 + (3*a^2*c*x)/(1 + c^2*x^2) + (a*b*((2 + I*c*
x)*Sqrt[1 + c^2*x^2] + (3*I)*ArcSinh[c*x]))/(-I + c*x)^2 + (3*a*b*((-I)*Sqr
t[1 + c^2*x^2] + ArcSinh[c*x]))/(-I + c*x) + (3*a*b*(I*Sqrt[1 + c^2*x^2] +
ArcSinh[c*x]))/(I + c*x) - (I*a*b*((2*I + c*x)*Sqrt[1 + c^2*x^2] + 3*ArcSin
h[c*x]))/(I + c*x)^2 + 3*a^2*ArcTan[c*x] + ((3*I)/2)*a*b*(ArcSinh[c*x]*(Arc
Sinh[c*x] - 4*Log[1 + I*E^ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]
]) - ((3*I)/2)*a*b*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]
]) - 4*PolyLog[2, I*E^ArcSinh[c*x]]) + b^2*((2*c*x)/(1 + c^2*x^2) - (4*ArcS
inh[c*x])/(1 + c^2*x^2)^(3/2) + (6*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - (6*c*x
*ArcSinh[c*x]^2)/(1 + c^2*x^2)^2 + (3*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2) - 8
*ArcTan[Tanh[ArcSinh[c*x]/2]] - (3*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*
x]] + (3*I)*ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] - (6*I)*ArcSinh[c*x]*P
olyLog[2, (-I)/E^ArcSinh[c*x]] + (6*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[
c*x]] - (6*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] + (6*I)*PolyLog[3, I/E^ArcSin
h[c*x]]))/(24*c^3*d^3)
```

Maple [F]

$$\int \frac{x^2(a + b\ \text{arcsinh}\,(cx))^2}{(c^2d\,x^2 + d)^3}dx$$

[In] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)

[Out] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)

Fricas [F]

$$\int \frac{x^2(a + b\text{arcsinh}(cx))^2}{(d + c^2dx^2)^3}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2x^2}{(c^2dx^2 + d)^3}\,dx$$

[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")

[Out] integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)

Sympy [F]

$$\int \frac{x^2(a + b\text{arcsinh}(cx))^2}{(d + c^2dx^2)^3}\,dx \\ = \frac{\int \frac{a^2x^2}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{b^2x^2\,\text{asinh}^2\,(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{2abx^2\,\text{asinh}\,(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

[In] integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**3,x)

[Out] (Integral(a**2*x**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral(b**2*x**2*asinh(c*x)**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral(2*a*b*x**2*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3

Maxima [F]

$$\int \frac{x^2(a + b\text{arcsinh}(cx))^2}{(d + c^2dx^2)^3}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2x^2}{(c^2dx^2 + d)^3}\,dx$$

[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")

[Out] 1/8*a^2*((c^2*x^3 - x)/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3) + arctan(c*x)/(c^3*d^3)) + integrate(b^2*x^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3) + 2*a*b*x^2*log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)

Giac [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^2}{(c^2dx^2+d)^3}\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2*x^2/(c^2*d*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \int \frac{x^2\,(a+b\,\text{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^3}\,dx$$

```
[In] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3,x)
[Out] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3, x)
```

3.246 $\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3} dx = \frac{b^2}{12c^2d^3(1+c^2x^2)} + \frac{bx(a+b\text{arcsinh}(cx))}{6cd^3(1+c^2x^2)^{3/2}} + \frac{bx(a+b\text{arcsinh}(cx))}{3cd^3\sqrt{1+c^2x^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{4c^2d^3(1+c^2x^2)^2} - \frac{b^2\log(1+c^2x^2)}{6c^2d^3}$$

```
[Out] 1/12*b^2/c^2/d^3/(c^2*x^2+1)+1/6*b*x*(a+b*arcsinh(c*x))/c/d^3/(c^2*x^2+1)^(
3/2)-1/4*(a+b*arcsinh(c*x))^2/c^2/d^3/(c^2*x^2+1)^2-1/6*b^2*ln(c^2*x^2+1)/c
^2/d^3+1/3*b*x*(a+b*arcsinh(c*x))/c/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5798, 5788, 5787, 266, 267}

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3} dx = \frac{bx(a+b\text{arcsinh}(cx))}{3cd^3\sqrt{c^2x^2+1}} + \frac{bx(a+b\text{arcsinh}(cx))}{6cd^3(c^2x^2+1)^{3/2}} - \frac{(a+b\text{arcsinh}(cx))^2}{4c^2d^3(c^2x^2+1)^2} + \frac{b^2}{12c^2d^3(c^2x^2+1)} - \frac{b^2\log(c^2x^2+1)}{6c^2d^3}$$

```
[In] Int[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] b^2/(12*c^2*d^3*(1 + c^2*x^2)) + (b*x*(a + b*ArcSinh[c*x]))/(6*c*d^3*(1 + c
^2*x^2)^(3/2)) + (b*x*(a + b*ArcSinh[c*x]))/(3*c*d^3*Sqrt[1 + c^2*x^2]) - (
```

```
a + b*ArcSinh[c*x])^2/(4*c^2*d^3*(1 + c^2*x^2)^2) - (b^2*Log[1 + c^2*x^2])/
(6*c^2*d^3)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{(a+b\text{arcsinh}(cx))^2}{4c^2d^3(1+c^2x^2)^2}+\frac{b\int\frac{a+b\text{arcsinh}(cx)}{(1+c^2x^2)^{5/2}}dx}{2cd^3}$$

$$=\frac{bx(a+b\text{arcsinh}(cx))}{6cd^3(1+c^2x^2)^{3/2}}-\frac{(a+b\text{arcsinh}(cx))^2}{4c^2d^3(1+c^2x^2)^2}-\frac{b^2\int\frac{x}{(1+c^2x^2)^2}dx}{6d^3}+\frac{b\int\frac{a+b\text{arcsinh}(cx)}{(1+c^2x^2)^{3/2}}dx}{3cd^3}$$

$$
\begin{aligned}
&= \frac{b^2}{12c^2d^3\left(1+c^2x^2\right)} + \frac{bx(a+b\operatorname{arcsinh}(cx))}{6cd^3\left(1+c^2x^2\right)^{3/2}} \\
&\quad + \frac{bx(a+b\operatorname{arcsinh}(cx))}{3cd^3\sqrt{1+c^2x^2}} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{4c^2d^3\left(1+c^2x^2\right)^2} - \frac{b^2\int\frac{x}{1+c^2x^2}\,dx}{3d^3} \\
&= \frac{b^2}{12c^2d^3\left(1+c^2x^2\right)} + \frac{bx(a+b\operatorname{arcsinh}(cx))}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{bx(a+b\operatorname{arcsinh}(cx))}{3cd^3\sqrt{1+c^2x^2}} \\
&\quad - \frac{(a+b\operatorname{arcsinh}(cx))^2}{4c^2d^3\left(1+c^2x^2\right)^2} - \frac{b^2\log\left(1+c^2x^2\right)}{6c^2d^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.05

$$
\int \frac{x(a+b\operatorname{arcsinh}(cx))^2}{\left(d+c^2dx^2\right)^3}\,dx
= \frac{-3a^2+b^2+b^2c^2x^2+6abcx\sqrt{1+c^2x^2}+4abc^3x^3\sqrt{1+c^2x^2}+2b\left(-3a+bcx\sqrt{1+c^2x^2}\left(3+2c^2x^2\right)\right)\operatorname{arcsin}}{12d^3\left(c+c^3x^2\right)^2}
$$

```
[In] Integrate[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] (-3*a^2 + b^2 + b^2*c^2*x^2 + 6*a*b*c*x*Sqrt[1 + c^2*x^2] + 4*a*b*c^3*x^3*S
qrt[1 + c^2*x^2] + 2*b*(-3*a + b*c*x*Sqrt[1 + c^2*x^2]*(3 + 2*c^2*x^2))*Arc
Sinh[c*x] - 3*b^2*ArcSinh[c*x]^2 - 2*(b + b*c^2*x^2)^2*Log[1 + c^2*x^2])/(1
2*d^3*(c + c^3*x^2)^2)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.57

method	result
derivativedivides	$\frac{-\frac{a^2}{4d^3\left(c^2x^2+1\right)^2}+\frac{b^2\left(\frac{2\operatorname{arcsinh}(cx)}{3}-\frac{-4\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+4\operatorname{arcsinh}(cx)c^4x^4-6\operatorname{arcsinh}(cx)cx\sqrt{c^2x^2+1}+8\operatorname{arcsinh}(cx)}{12\left(c^4x^4+2c^2x^2+1\right)}\right.}{d^3}}{c^2}$
default	$\frac{-\frac{a^2}{4d^3\left(c^2x^2+1\right)^2}+\frac{b^2\left(\frac{2\operatorname{arcsinh}(cx)}{3}-\frac{-4\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+4\operatorname{arcsinh}(cx)c^4x^4-6\operatorname{arcsinh}(cx)cx\sqrt{c^2x^2+1}+8\operatorname{arcsinh}(cx)}{12\left(c^4x^4+2c^2x^2+1\right)}\right.}{d^3}}{c^2}$
parts	$-\frac{a^2}{4d^3c^2(c^2x^2+1)^2}+\frac{b^2\left(\frac{2\operatorname{arcsinh}(cx)}{3}-\frac{-4\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+4\operatorname{arcsinh}(cx)c^4x^4-6\operatorname{arcsinh}(cx)cx\sqrt{c^2x^2+1}+8\operatorname{arcsi}}{12\left(c^4x^4+2c^2x^2+1\right)}\right.}{d^3c^2}$

```
[In] int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-1/4*a^2/d^3/(c^2*x^2+1)^2+b^2/d^3*(2/3*arcsinh(c*x)-1/12*(-4*arcsin
h(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3+4*arcsinh(c*x)*c^4*x^4-6*arcsinh(c*x)*c*x*
(c^2*x^2+1)^(1/2)+8*arcsinh(c*x)*c^2*x^2+3*arcsinh(c*x)^2-c^2*x^2+4*arcsinh
(c*x)-1)/(c^4*x^4+2*c^2*x^2+1)-1/3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2))+2*a*b/d
^3*(-1/4/(c^2*x^2+1)^2*arcsinh(c*x)+1/12/(c^2*x^2+1)^(3/2)*c*x+1/6*c*x/(c^2
*x^2+1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(131) = 262$.

Time = 0.27 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.88

$$\int \frac{x(a + b\text{arcsinh}(cx))^2}{(d + c^2dx^2)^3}\,dx$$
$$= \frac{4\,abc^4x^4 + (8\,ab + b^2)c^2x^2 - 3\,b^2\log\left(cx + \sqrt{c^2x^2+1}\right)^2 - 3\,a^2 + 4\,ab + b^2 - 2\,(b^2c^4x^4 + 2\,b^2c^2x^2 + b^2)\log\left(c\right.}{}$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*a*b*c^4*x^4 + (8*a*b + b^2)*c^2*x^2 - 3*b^2*log(c*x + sqrt(c^2*x^2
+ 1))^2 - 3*a^2 + 4*a*b + b^2 - 2*(b^2*c^4*x^4 + 2*b^2*c^2*x^2 + b^2)*log(c
^2*x^2 + 1) + 2*(3*a*b*c^4*x^4 + 6*a*b*c^2*x^2 + (2*b^2*c^3*x^3 + 3*b^2*c*x
)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 6*(a*b*c^4*x^4 + 2*a*b*
c^2*x^2 + a*b)*log(-c*x + sqrt(c^2*x^2 + 1)) + 2*(2*a*b*c^3*x^3 + 3*a*b*c*x
)*sqrt(c^2*x^2 + 1))/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3)
```

Sympy [F]

$$\int \frac{x(a + b\text{arcsinh}(cx))^2}{(d + c^2dx^2)^3}\,dx$$
$$= \frac{\int \frac{a^2x}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{b^2x\,\text{asinh}^2\,(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{2abx\,\text{asinh}\,(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

```
[In] integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2*x/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral
(b**2*x*asinh(c*x)**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Int
egral(2*a*b*x*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d
**3
```

Maxima [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x}{(c^2dx^2+d)^3}\,dx$$

[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")

[Out] -1/4*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3) - 1/4*a^2/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3) + integrate(1/2*((4*a*b*c^2 + b^2*c^2)*x^2 + sqrt(c^2*x^2 + 1)*(4*a*b*c + b^2*c)*x + b^2)*log(c*x + sqrt(c^2*x^2 + 1))/(c^8*d^3*x^7 + 3*c^6*d^3*x^5 + 3*c^4*d^3*x^3 + c^2*d^3*x + (c^7*d^3*x^6 + 3*c^5*d^3*x^4 + 3*c^3*d^3*x^2 + c*d^3)*sqrt(c^2*x^2 + 1)), x)

Giac [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x}{(c^2dx^2+d)^3}\,dx$$

[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)^2*x/(c^2*d*x^2 + d)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx=\int \frac{x\,(a+b\,\text{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^3}\,dx$$

[In] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3,x)

[Out] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3, x)

3.247 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 309

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = -\frac{b^2x}{12d^3(1+c^2x^2)}+\frac{b(a+b\text{arcsinh}(cx))}{6cd^3(1+c^2x^2)^{3/2}}+\frac{3b(a+b\text{arcsinh}(cx))}{4cd^3\sqrt{1+c^2x^2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2}+\frac{3x(a+b\text{arcsinh}(cx))^2}{8d^3(1+c^2x^2)}+\frac{3(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4cd^3}-\frac{5b^2\arctan(cx)}{6cd^3}-\frac{3ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{4cd^3}+\frac{3ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{4cd^3}+\frac{3ib^2\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{4cd^3}-\frac{3ib^2\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{4cd^3}$$

```
[Out] -1/12*b^2*x/d^3/(c^2*x^2+1)+1/6*b*(a+b*arcsinh(c*x))/c/d^3/(c^2*x^2+1)^(3/2
)+1/4*x*(a+b*arcsinh(c*x))^2/d^3/(c^2*x^2+1)^2+3/8*x*(a+b*arcsinh(c*x))^2/d
^3/(c^2*x^2+1)+3/4*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/c/d^3
-5/6*b^2*arctan(c*x)/c/d^3-3/4*I*b*(a+b*arcsinh(c*x))*polylog(2,-I*(c*x+(c^
2*x^2+1)^(1/2)))/c/d^3+3/4*I*b*(a+b*arcsinh(c*x))*polylog(2,I*(c*x+(c^2*x^2
+1)^(1/2)))/c/d^3+3/4*I*b^2*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^3-3/4
*I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^3+3/4*b*(a+b*arcsinh(c*x))/
c/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5788, 5789, 4265, 2611, 2320, 6724, 5798, 209, 205}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \frac{3\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{4cd^3} + \frac{3b(a+b\text{arcsinh}(cx))}{4cd^3\sqrt{c^2x^2+1}} + \frac{b(a+b\text{arcsinh}(cx))}{6cd^3(c^2x^2+1)^{3/2}} + \frac{3x(a+b\text{arcsinh}(cx))^2}{8d^3(c^2x^2+1)} + \frac{x(a+b\text{arcsinh}(cx))^2}{4d^3(c^2x^2+1)^2} - \frac{3ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{4cd^3} + \frac{3ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{4cd^3} + \frac{3ib^2\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{4cd^3} - \frac{3ib^2\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{4cd^3} - \frac{5b^2\arctan(cx)}{6cd^3} - \frac{b^2x}{12d^3(c^2x^2+1)}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^3,x]
```

```
[Out] -1/12*(b^2*x)/(d^3*(1 + c^2*x^2)) + (b*(a + b*ArcSinh[c*x]))/(6*c*d^3*(1 +
c^2*x^2)^(3/2)) + (3*b*(a + b*ArcSinh[c*x]))/(4*c*d^3*Sqrt[1 + c^2*x^2]) +
(x*(a + b*ArcSinh[c*x])^2)/(4*d^3*(1 + c^2*x^2)^2) + (3*x*(a + b*ArcSinh[c*
x])^2)/(8*d^3*(1 + c^2*x^2)) + (3*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c
*x]])/(4*c*d^3) - (5*b^2*ArcTan[c*x])/(6*c*d^3) - (((3*I)/4)*b*(a + b*ArcSi
nh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*d^3) + (((3*I)/4)*b*(a + b*Arc
Sinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/(c*d^3) + (((3*I)/4)*b^2*PolyLog[3
, (-I)*E^ArcSinh[c*x]])/(c*d^3) - (((3*I)/4)*b^2*PolyLog[3, I*E^ArcSinh[c*x
]])/(c*d^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
```

```
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(a+b\text{arcsinh}(cx))^2}{4d^3\left(1+c^2x^2\right)^2} - \frac{(bc)\int \frac{x(a+b\text{arcsinh}(cx))}{\left(1+c^2x^2\right)^{5/2}}\,dx}{2d^3} + \frac{3\int \frac{(a+b\text{arcsinh}(cx))^2}{\left(d+c^2dx^2\right)^2}\,dx}{4d} \\
&= \frac{b(a+b\text{arcsinh}(cx))}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{x(a+b\text{arcsinh}(cx))^2}{4d^3\left(1+c^2x^2\right)^2} + \frac{3x(a+b\text{arcsinh}(cx))^2}{8d^3\left(1+c^2x^2\right)} \\
&\quad - \frac{b^2\int \frac{1}{\left(1+c^2x^2\right)^2}\,dx}{6d^3} - \frac{(3bc)\int \frac{x(a+b\text{arcsinh}(cx))}{\left(1+c^2x^2\right)^{3/2}}\,dx}{4d^3} + \frac{3\int \frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx}{8d^2} \\
&= -\frac{b^2x}{12d^3\left(1+c^2x^2\right)} + \frac{b(a+b\text{arcsinh}(cx))}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{3b(a+b\text{arcsinh}(cx))}{4cd^3\sqrt{1+c^2x^2}} \\
&\quad + \frac{x(a+b\text{arcsinh}(cx))^2}{4d^3\left(1+c^2x^2\right)^2} + \frac{3x(a+b\text{arcsinh}(cx))^2}{8d^3\left(1+c^2x^2\right)} - \frac{b^2\int \frac{1}{1+c^2x^2}\,dx}{12d^3} \\
&\quad - \frac{\left(3b^2\right)\int \frac{1}{1+c^2x^2}\,dx}{4d^3} + \frac{3\text{Subst}\left(\int (a+bx)^2\text{sech}(x)\,dx, x, \text{arcsinh}(cx)\right)}{8cd^3} \\
&= -\frac{b^2x}{12d^3\left(1+c^2x^2\right)} + \frac{b(a+b\text{arcsinh}(cx))}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{3b(a+b\text{arcsinh}(cx))}{4cd^3\sqrt{1+c^2x^2}} \\
&\quad + \frac{x(a+b\text{arcsinh}(cx))^2}{4d^3\left(1+c^2x^2\right)^2} + \frac{3x(a+b\text{arcsinh}(cx))^2}{8d^3\left(1+c^2x^2\right)} \\
&\quad + \frac{3(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4cd^3} - \frac{5b^2\arctan(cx)}{6cd^3} \\
&\quad - \frac{(3ib)\text{Subst}\left(\int (a+bx)\log\left(1-ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{4cd^3} \\
&\quad + \frac{(3ib)\text{Subst}\left(\int (a+bx)\log\left(1+ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{4cd^3}
\end{aligned}$$

$$=-\frac{b^2x}{12d^3(1+c^2x^2)}+\frac{b(a+b\text{arcsinh}(cx))}{6cd^3(1+c^2x^2)^{3/2}}+\frac{3b(a+b\text{arcsinh}(cx))}{4cd^3\sqrt{1+c^2x^2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2}+\frac{3x(a+b\text{arcsinh}(cx))^2}{8d^3(1+c^2x^2)}+\frac{3(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4cd^3}-\frac{5b^2\arctan(cx)}{6cd^3}-\frac{3ib(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{4cd^3}+\frac{3ib(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{4cd^3}+\frac{(3ib^2)\text{Subst}\left(\int\text{PolyLog}(2,-ie^x)\,dx,x,\text{arcsinh}(cx)\right)}{4cd^3}-\frac{(3ib^2)\text{Subst}\left(\int\text{PolyLog}(2,ie^x)\,dx,x,\text{arcsinh}(cx)\right)}{4cd^3}$$

$$=-\frac{b^2x}{12d^3(1+c^2x^2)}+\frac{b(a+b\text{arcsinh}(cx))}{6cd^3(1+c^2x^2)^{3/2}}+\frac{3b(a+b\text{arcsinh}(cx))}{4cd^3\sqrt{1+c^2x^2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2}+\frac{3x(a+b\text{arcsinh}(cx))^2}{8d^3(1+c^2x^2)}+\frac{3(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4cd^3}-\frac{5b^2\arctan(cx)}{6cd^3}-\frac{3ib(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{4cd^3}+\frac{3ib(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{4cd^3}+\frac{(3ib^2)\text{Subst}\left(\int\frac{\text{PolyLog}(2,-ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{4cd^3}-\frac{(3ib^2)\text{Subst}\left(\int\frac{\text{PolyLog}(2,ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{4cd^3}$$

$$=-\frac{b^2x}{12d^3(1+c^2x^2)}+\frac{b(a+b\text{arcsinh}(cx))}{6cd^3(1+c^2x^2)^{3/2}}+\frac{3b(a+b\text{arcsinh}(cx))}{4cd^3\sqrt{1+c^2x^2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2}+\frac{3x(a+b\text{arcsinh}(cx))^2}{8d^3(1+c^2x^2)}+\frac{3(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4cd^3}-\frac{5b^2\arctan(cx)}{6cd^3}-\frac{3ib(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{4cd^3}+\frac{3ib(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{4cd^3}+\frac{3ib^2\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{4cd^3}-\frac{3ib^2\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{4cd^3}$$

Mathematica [A] (verified)

Time = 2.42 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.77

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$$
$$= \frac{\frac{6a^2x}{(1+c^2x^2)^2}+\frac{9a^2x}{1+c^2x^2}+\frac{9a^2\arctan(cx)}{c}+\frac{ab\left(\frac{9\left(-i\sqrt{1+c^2x^2}+\text{arcsinh}(cx)\right)}{-i+cx}+\frac{9\left(i\sqrt{1+c^2x^2}+\text{arcsinh}(cx)\right)}{i+cx}-\frac{i\left((-2i+cx)\sqrt{1+c^2x^2}+3\text{arcsi}\right.}{(-i+cx)^2}\right.}{}}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^3,x]
```

```
[Out] ((6*a^2*x)/(1 + c^2*x^2)^2 + (9*a^2*x)/(1 + c^2*x^2) + (9*a^2*ArcTan[c*x])/
c + (a*b*((9*((-I)*Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/(-I + c*x) + (9*(I*Sq
rt[1 + c^2*x^2] + ArcSinh[c*x]))/(I + c*x) - (I*((-2*I + c*x)*Sqrt[1 + c^2*
x^2] + 3*ArcSinh[c*x]))/(-I + c*x)^2 + (I*((2*I + c*x)*Sqrt[1 + c^2*x^2] +
3*ArcSinh[c*x]))/(I + c*x)^2 + ((9*I)/2)*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Lo
g[1 + I*E^ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]]) - ((9*I)/2)*(
ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - 4*PolyLog[2, I*
E^ArcSinh[c*x]])))/c + (b^2*((-2*c*x)/(1 + c^2*x^2) + (4*ArcSinh[c*x])/(1 +
 c^2*x^2)^(3/2) + (18*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + (6*c*x*ArcSinh[c*x]
^2)/(1 + c^2*x^2)^2 + (9*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2) - 40*ArcTan[Tanh
[ArcSinh[c*x]/2]] - (9*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] + (9*I)*
ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] - (18*I)*ArcSinh[c*x]*PolyLog[2, (
-I)/E^ArcSinh[c*x]] + (18*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] - (1
8*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] + (18*I)*PolyLog[3, I/E^ArcSinh[c*x]])
)/c)/(24*d^3)
```

Maple [F]

$$\int \frac{(a+b\,\text{arcsinh}\,(cx))^2}{(c^2d\,x^2+d)^3}dx$$

```
[In] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(c^2dx^2+d)^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^6*d^3*x^6 + 3*c
^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \frac{\int \frac{a^2}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

```
[In] integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral(b
**2*asinh(c*x)**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integra
l(2*a*b*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

Maxima [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(c^2dx^2+d)^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*((3*c^2*x^3 + 5*x)/(c^4*d^3*x^4 + 2*c^2*d^3*x^2 + d^3) + 3*arctan(c
*x)/(c*d^3)) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^6*d^3*x^6 +
3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(
c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Giac [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \int \frac{(a+b\,\mathrm{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^3}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^3,x)
[Out] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^3, x)
```

3.248 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^3} dx$

Optimal result

Integrand size = 26, antiderivative size = 275

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^3} dx = & -\frac{b^2}{12d^3(1+c^2x^2)} - \frac{bcx(a+b\text{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}} \\ & - \frac{4bcx(a+b\text{arcsinh}(cx))}{3d^3\sqrt{1+c^2x^2}} \\ & + \frac{(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} + \frac{(a+b\text{arcsinh}(cx))^2}{2d^3(1+c^2x^2)} \\ & - \frac{2(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^3} + \frac{2b^2\log(1+c^2x^2)}{3d^3} \\ & - \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d^3} \\ & + \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d^3} \\ & + \frac{b^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{2d^3} - \frac{b^2\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(cx)}\right)}{2d^3}\end{aligned}$$

```
[Out] -1/12*b^2/d^3/(c^2*x^2+1)-1/6*b*c*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^(3/2
)+1/4*(a+b*arcsinh(c*x))^2/d^3/(c^2*x^2+1)^2+1/2*(a+b*arcsinh(c*x))^2/d^3/(
c^2*x^2+1)-2*(a+b*arcsinh(c*x))^2*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)/d^3+2/
3*b^2*ln(c^2*x^2+1)/d^3-b*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^2*x^2+1)^(1
/2))^2)/d^3+b*(a+b*arcsinh(c*x))*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d^3+1
/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^3-1/2*b^2*polylog(3,(c*x+(c^
2*x^2+1)^(1/2))^2)/d^3-4/3*b*c*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {5811, 5799, 5569, 4267, 2611, 2320, 6724, 5787, 266, 5788, 267}

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^3}\,dx = -\frac{2\operatorname{arctanh}\left(e^{2\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx))^2}{d^3} - \frac{4bcx(a+b\operatorname{arcsinh}(cx))}{3d^3\sqrt{c^2x^2+1}} - \frac{bcx(a+b\operatorname{arcsinh}(cx))}{6d^3\left(c^2x^2+1\right)^{3/2}} + \frac{(a+b\operatorname{arcsinh}(cx))^2}{2d^3\left(c^2x^2+1\right)} + \frac{(a+b\operatorname{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2} - \frac{b\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx))}{d^3} + \frac{b\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx))}{d^3} + \frac{b^2\operatorname{PolyLog}\left(3,-e^{2\operatorname{arcsinh}(cx)}\right)}{2d^3} - \frac{b^2\operatorname{PolyLog}\left(3,e^{2\operatorname{arcsinh}(cx)}\right)}{2d^3} - \frac{b^2}{12d^3\left(c^2x^2+1\right)} + \frac{2b^2\log\left(c^2x^2+1\right)}{3d^3}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^3),x]
```

```
[Out] -1/12*b^2/(d^3*(1 + c^2*x^2)) - (b*c*x*(a + b*ArcSinh[c*x]))/(6*d^3*(1 + c^
2*x^2)^(3/2)) - (4*b*c*x*(a + b*ArcSinh[c*x]))/(3*d^3*Sqrt[1 + c^2*x^2]) +
(a + b*ArcSinh[c*x])^2/(4*d^3*(1 + c^2*x^2)^2) + (a + b*ArcSinh[c*x])^2/(2*
d^3*(1 + c^2*x^2)) - (2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSinh[c*x])])
/d^3 + (2*b^2*Log[1 + c^2*x^2])/(3*d^3) - (b*(a + b*ArcSinh[c*x])*PolyLog[2
, -E^(2*ArcSinh[c*x])])/d^3 + (b*(a + b*ArcSinh[c*x])*PolyLog[2, E^(2*ArcSi
nh[c*x])])/d^3 + (b^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*d^3) - (b^2*PolyL
og[3, E^(2*ArcSinh[c*x])])/(2*d^3)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} - \frac{(bc)\int \frac{a+b\text{arcsinh}(cx)}{(1+c^2x^2)^{5/2}}\,dx}{2d^3} + \frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^2}\,dx}{d} \\
&= -\frac{bcx(a+b\text{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}} + \frac{(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} + \frac{(a+b\text{arcsinh}(cx))^2}{2d^3(1+c^2x^2)} \\
&\quad - \frac{(bc)\int \frac{a+b\text{arcsinh}(cx)}{(1+c^2x^2)^{3/2}}\,dx}{3d^3} - \frac{(bc)\int \frac{a+b\text{arcsinh}(cx)}{(1+c^2x^2)^{3/2}}\,dx}{d^3} \\
&\quad + \frac{(b^2c^2)\int \frac{x}{(1+c^2x^2)^2}\,dx}{6d^3} + \frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)}\,dx}{d^2} \\
&= -\frac{b^2}{12d^3(1+c^2x^2)} - \frac{bcx(a+b\text{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}} - \frac{4bcx(a+b\text{arcsinh}(cx))}{3d^3\sqrt{1+c^2x^2}} \\
&\quad + \frac{(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} + \frac{(a+b\text{arcsinh}(cx))^2}{2d^3(1+c^2x^2)} \\
&\quad + \frac{\text{Subst}\left(\int (a+bx)^2\text{csch}(x)\text{sech}(x)\,dx, x, \text{arcsinh}(cx)\right)}{d^3} \\
&\quad + \frac{(b^2c^2)\int \frac{x}{1+c^2x^2}\,dx}{3d^3} + \frac{(b^2c^2)\int \frac{x}{1+c^2x^2}\,dx}{d^3}
\end{aligned}$$

$$= -\frac{b^2}{12d^3(1+c^2x^2)} - \frac{bcx(a+b\text{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}} - \frac{4bcx(a+b\text{arcsinh}(cx))}{3d^3\sqrt{1+c^2x^2}} + \frac{(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} + \frac{(a+b\text{arcsinh}(cx))^2}{2d^3(1+c^2x^2)} + \frac{2b^2\log(1+c^2x^2)}{3d^3} + \frac{2\text{Subst}\left(\int (a+bx)^2\text{csch}(2x)\,dx, x, \text{arcsinh}(cx)\right)}{d^3}$$

$$= -\frac{b^2}{12d^3(1+c^2x^2)} - \frac{bcx(a+b\text{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}} - \frac{4bcx(a+b\text{arcsinh}(cx))}{3d^3\sqrt{1+c^2x^2}} + \frac{(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} + \frac{(a+b\text{arcsinh}(cx))^2}{2d^3(1+c^2x^2)} - \frac{2(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^3} + \frac{2b^2\log(1+c^2x^2)}{3d^3} - \frac{(2b)\text{Subst}\left(\int (a+bx)\log(1-e^{2x})\,dx, x, \text{arcsinh}(cx)\right)}{d^3} + \frac{(2b)\text{Subst}\left(\int (a+bx)\log(1+e^{2x})\,dx, x, \text{arcsinh}(cx)\right)}{d^3}$$

$$= -\frac{b^2}{12d^3(1+c^2x^2)} - \frac{bcx(a+b\text{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}} - \frac{4bcx(a+b\text{arcsinh}(cx))}{3d^3\sqrt{1+c^2x^2}} + \frac{(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} + \frac{(a+b\text{arcsinh}(cx))^2}{2d^3(1+c^2x^2)} - \frac{2(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^3} + \frac{2b^2\log(1+c^2x^2)}{3d^3} - \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2, -e^{2\text{arcsinh}(cx)}\right)}{d^3} + \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2, e^{2\text{arcsinh}(cx)}\right)}{d^3} + \frac{b^2\text{Subst}\left(\int \text{PolyLog}(2, -e^{2x})\,dx, x, \text{arcsinh}(cx)\right)}{d^3} - \frac{b^2\text{Subst}\left(\int \text{PolyLog}(2, e^{2x})\,dx, x, \text{arcsinh}(cx)\right)}{d^3}$$

$$
\begin{aligned}
&= -\frac{b^2}{12d^3(1+c^2x^2)} - \frac{bcx(a+b\operatorname{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}} - \frac{4bcx(a+b\operatorname{arcsinh}(cx))}{3d^3\sqrt{1+c^2x^2}} \\
&\quad + \frac{(a+b\operatorname{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} + \frac{(a+b\operatorname{arcsinh}(cx))^2}{2d^3(1+c^2x^2)} \\
&\quad - \frac{2(a+b\operatorname{arcsinh}(cx))^2\operatorname{arctanh}\left(e^{2\operatorname{arcsinh}(cx)}\right)}{d^3} + \frac{2b^2\log(1+c^2x^2)}{3d^3} \\
&\quad - \frac{b(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{d^3} \\
&\quad + \frac{b(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(cx)}\right)}{d^3} \\
&\quad + \frac{b^2\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,-x)}{x}\,dx,x,e^{2\operatorname{arcsinh}(cx)}\right)}{2d^3} \\
&\quad - \frac{b^2\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,x)}{x}\,dx,x,e^{2\operatorname{arcsinh}(cx)}\right)}{2d^3} \\
&= -\frac{b^2}{12d^3(1+c^2x^2)} - \frac{bcx(a+b\operatorname{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}} - \frac{4bcx(a+b\operatorname{arcsinh}(cx))}{3d^3\sqrt{1+c^2x^2}} \\
&\quad + \frac{(a+b\operatorname{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} + \frac{(a+b\operatorname{arcsinh}(cx))^2}{2d^3(1+c^2x^2)} \\
&\quad - \frac{2(a+b\operatorname{arcsinh}(cx))^2\operatorname{arctanh}\left(e^{2\operatorname{arcsinh}(cx)}\right)}{d^3} + \frac{2b^2\log(1+c^2x^2)}{3d^3} \\
&\quad - \frac{b(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{d^3} \\
&\quad + \frac{b(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(cx)}\right)}{d^3} \\
&\quad + \frac{b^2\operatorname{PolyLog}\left(3,-e^{2\operatorname{arcsinh}(cx)}\right)}{2d^3} - \frac{b^2\operatorname{PolyLog}\left(3,e^{2\operatorname{arcsinh}(cx)}\right)}{2d^3}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.16 (sec) , antiderivative size = 560, normalized size of antiderivative = 2.04

$$
\int\frac{(a+b\operatorname{arcsinh}(cx))^2}{x(d+c^2dx^2)^3}\,dx
$$

$$
= \frac{\frac{6a^2}{(1+c^2x^2)^2} + \frac{12a^2}{1+c^2x^2} + 24a^2\log(cx) - 12a^2\log(1+c^2x^2) + ab\left(-\frac{15\left(\sqrt{1+c^2x^2}-i\operatorname{arcsinh}(cx)\right)}{i+cx} - \frac{15\left(\sqrt{1+c^2x^2}+i\operatorname{arcs}\right.}{-i+cx}\right.}{}
$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^3),x]
```

```
[Out] ((6*a^2)/(1 + c^2*x^2)^2 + (12*a^2)/(1 + c^2*x^2) + 24*a^2*Log[c*x] - 12*a^
2*Log[1 + c^2*x^2] + a*b*((-15*(Sqrt[1 + c^2*x^2] - I*ArcSinh[c*x]))/(I + c
*x) - (15*(Sqrt[1 + c^2*x^2] + I*ArcSinh[c*x]))/(-I + c*x) - 24*ArcSinh[c*x
]^2 - ((-2*I + c*x)*Sqrt[1 + c^2*x^2] + 3*ArcSinh[c*x])/(-I + c*x)^2 - ((2*
I + c*x)*Sqrt[1 + c^2*x^2] + 3*ArcSinh[c*x])/(I + c*x)^2 + 48*ArcSinh[c*x]*
Log[1 - E^(2*ArcSinh[c*x])] + 12*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 + I*
E^ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + 12*(ArcSinh[c*x]*(A
rcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - 4*PolyLog[2, I*E^ArcSinh[c*x]]
) + 24*PolyLog[2, E^(2*ArcSinh[c*x])]) + b^2*(I*Pi^3 - 2/(1 + c^2*x^2) - (4
*c*x*ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) - (32*c*x*ArcSinh[c*x])/Sqrt[1 + c^2
*x^2] + (6*ArcSinh[c*x]^2)/(1 + c^2*x^2)^2 + (12*ArcSinh[c*x]^2)/(1 + c^2*x
^2) - 16*ArcSinh[c*x]^3 - 24*ArcSinh[c*x]^2*Log[1 + E^(-2*ArcSinh[c*x])] +
24*ArcSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + 16*Log[1 + c^2*x^2] + 24*Ar
cSinh[c*x]*PolyLog[2, -E^(-2*ArcSinh[c*x])] + 24*ArcSinh[c*x]*PolyLog[2, E^
(2*ArcSinh[c*x])] + 12*PolyLog[3, -E^(-2*ArcSinh[c*x])] - 12*PolyLog[3, E^(
2*ArcSinh[c*x])]))/(24*d^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 675 vs. $2(300) = 600$.

Time = 0.31 (sec) , antiderivative size = 676, normalized size of antiderivative = 2.46

method	result
derivativedivides	$\frac{a^2\left(\ln(cx)+\frac{1}{4\left(c^2x^2+1\right)^2}+\frac{1}{2c^2x^2+2}-\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d^3}+\frac{b^2\left(\frac{-16\ \operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+16\ \operatorname{arcsinh}(cx)c^4x^4+6\operatorname{arcsinh}(cx)^2x}{}\right.}{}$
default	$\frac{a^2\left(\ln(cx)+\frac{1}{4\left(c^2x^2+1\right)^2}+\frac{1}{2c^2x^2+2}-\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d^3}+\frac{b^2\left(\frac{-16\ \operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+16\ \operatorname{arcsinh}(cx)c^4x^4+6\operatorname{arcsinh}(cx)^2x}{}\right.}{}$
parts	$\frac{a^2\left(-\frac{c^2\left(-\frac{1}{c^2\left(c^2x^2+1\right)}-\frac{1}{2c^2\left(c^2x^2+1\right)^2}+\frac{\ln\left(c^2x^2+1\right)}{c^2}\right)}{2}+\ln(x)\right)}{d^3}+\frac{b^2\left(\frac{-16\ \operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+16\ \operatorname{arcsinh}(cx)c^4x^4+}{}\right.}{}$

```
[In] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/d^3*(ln(c*x)+1/4/(c^2*x^2+1)^2+1/2/(c^2*x^2+1)-1/2*ln(c^2*x^2+1))+b^2/d
^3*(1/12*(-16*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3+16*arcsinh(c*x)*c^4*x^
4+6*arcsinh(c*x)^2*x^2*c^2-18*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)+32*arcsinh
(c*x)*c^2*x^2+9*arcsinh(c*x)^2-c^2*x^2+16*arcsinh(c*x)-1)/(c^4*x^4+2*c^2*x^
2+1)-8/3*ln(c*x+(c^2*x^2+1)^(1/2))+4/3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+arcs
inh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*arcsinh(c*x)*polylog(2,-c*x-(c^2*x
^2+1)^(1/2))-2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))-arcsinh(c*x)^2*ln(1+(c*x+(
c^2*x^2+1)^(1/2))^2)-arcsinh(c*x)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+1/2
```

```
*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)+arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^
(1/2))+2*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*polylog(3,c*x+(c^2
*x^2+1)^(1/2)))+2*a*b/d^3*(1/12*(-8*c^3*x^3*(c^2*x^2+1)^(1/2)+8*c^4*x^4+6*a
rcsinh(c*x)*c^2*x^2-9*c*x*(c^2*x^2+1)^(1/2)+16*c^2*x^2+9*arcsinh(c*x)+8)/(c
^4*x^4+2*c^2*x^2+1)+arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+polylog(2,-c*x
-(c^2*x^2+1)^(1/2))-arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*polylo
g(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+po
lylog(2,c*x+(c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x\,(d+c^2dx^2)^3}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(c^2dx^2+d)^3x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^6*d^3*x^7 + 3*c
^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x), x)
```

Sympy [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x\,(d+c^2dx^2)^3}\,dx = \frac{\int \frac{a^2}{c^6x^7+3c^4x^5+3c^2x^3+x}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{c^6x^7+3c^4x^5+3c^2x^3+x}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{c^6x^7+3c^4x^5+3c^2x^3+x}\,dx}{d^3}$$

```
[In] integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2/(c**6*x**7 + 3*c**4*x**5 + 3*c**2*x**3 + x), x) + Integral(b
**2*asinh(c*x)**2/(c**6*x**7 + 3*c**4*x**5 + 3*c**2*x**3 + x), x) + Integra
l(2*a*b*asinh(c*x)/(c**6*x**7 + 3*c**4*x**5 + 3*c**2*x**3 + x), x))/d**3
```

Maxima [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x\,(d+c^2dx^2)^3}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(c^2dx^2+d)^3x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*((2*c^2*x^2 + 3)/(c^4*d^3*x^4 + 2*c^2*d^3*x^2 + d^3) - 2*log(c^2*x^
2 + 1)/d^3 + 4*log(x)/d^3) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(
c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x) + 2*a*b*log(c*x + sqrt
(c^2*x^2 + 1))/(c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x), x)
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\,(d+c^2dx^2)^3}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^3x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^3,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^3*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\,(d+c^2dx^2)^3}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2}{x\,(d\,c^2\,x^2+d)^3}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^3),x)
[Out] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^3), x)
```

3.249 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 389

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^3}\,dx &= \frac{b^2c^2x}{12d^3(1+c^2x^2)} - \frac{bc(a+b\text{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}} \\ &- \frac{7bc(a+b\text{arcsinh}(cx))}{4d^3\sqrt{1+c^2x^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{d^3x(1+c^2x^2)^2} \\ &- \frac{5c^2x(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} - \frac{15c^2x(a+b\text{arcsinh}(cx))^2}{8d^3(1+c^2x^2)} \\ &- \frac{15c(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4d^3} \\ &+ \frac{11b^2c\arctan(cx)}{6d^3} - \frac{4bc(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d^3} \\ &- \frac{2b^2c\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d^3} \\ &+ \frac{15ibc(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{4d^3} \\ &- \frac{15ibc(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{4d^3} \\ &+ \frac{2b^2c\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d^3} \\ &- \frac{15ib^2c\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{4d^3} \\ &+ \frac{15ib^2c\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{4d^3}\end{aligned}$$

```
[Out] 1/12*b^2*c^2*x/d^3/(c^2*x^2+1)-1/6*b*c*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^(
3/2)-(a+b*arcsinh(c*x))^2/d^3/x/(c^2*x^2+1)^2-5/4*c^2*x*(a+b*arcsinh(c*x))^
```

```
2/d^3/(c^2*x^2+1)^2-15/8*c^2*x*(a+b*arcsinh(c*x))^2/d^3/(c^2*x^2+1)-15/4*c*
(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/d^3+11/6*b^2*c*arctan(c*
x)/d^3-4*b*c*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))/d^3-2*b^2*c*
polylog(2,-c*x-(c^2*x^2+1)^(1/2))/d^3+15/4*I*b*c*(a+b*arcsinh(c*x))*polylog
(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^3-15/4*I*b*c*(a+b*arcsinh(c*x))*polylog(2,
I*(c*x+(c^2*x^2+1)^(1/2)))/d^3+2*b^2*c*polylog(2,c*x+(c^2*x^2+1)^(1/2))/d^3
-15/4*I*b^2*c*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^3+15/4*I*b^2*c*polylo
g(3,I*(c*x+(c^2*x^2+1)^(1/2)))/d^3-7/4*b*c*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+
1)^(1/2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {5809, 5788, 5789, 4265, 2611, 2320, 6724, 5798, 209, 205, 5811, 5816, 4267, 2317, 2438}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^3}\,dx = -\frac{15c\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{4d^3} - \frac{4bc\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d^3} - \frac{7bc(a+b\text{arcsinh}(cx))}{4d^3\sqrt{c^2x^2+1}} - \frac{bc(a+b\text{arcsinh}(cx))}{6d^3(c^2x^2+1)^{3/2}} - \frac{15c^2x(a+b\text{arcsinh}(cx))^2}{8d^3(c^2x^2+1)} - \frac{5c^2x(a+b\text{arcsinh}(cx))^2}{4d^3(c^2x^2+1)^2} - \frac{(a+b\text{arcsinh}(cx))^2}{d^3x(c^2x^2+1)^2} + \frac{15ibc\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{4d^3} - \frac{15ibc\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{4d^3} - \frac{2b^2c\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d^3} + \frac{2b^2c\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d^3} - \frac{15ib^2c\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{4d^3} + \frac{15ib^2c\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{4d^3} + \frac{11b^2c\arctan(cx)}{6d^3} + \frac{b^2c^2x}{12d^3(c^2x^2+1)}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^3),x]
```

```
[Out] (b^2*c^2*x)/(12*d^3*(1 + c^2*x^2)) - (b*c*(a + b*ArcSinh[c*x]))/(6*d^3*(1 +
 c^2*x^2)^(3/2)) - (7*b*c*(a + b*ArcSinh[c*x]))/(4*d^3*Sqrt[1 + c^2*x^2]) -
 (a + b*ArcSinh[c*x])^2/(d^3*x*(1 + c^2*x^2)^2) - (5*c^2*x*(a + b*ArcSinh[c
*x])^2)/(4*d^3*(1 + c^2*x^2)^2) - (15*c^2*x*(a + b*ArcSinh[c*x])^2)/(8*d^3*
(1 + c^2*x^2)) - (15*c*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/(4*d^
3) + (11*b^2*c*ArcTan[c*x])/(6*d^3) - (4*b*c*(a + b*ArcSinh[c*x])*ArcTanh[E
^ArcSinh[c*x]])/d^3 - (2*b^2*c*PolyLog[2, -E^ArcSinh[c*x]])/d^3 + (((15*I)/
4)*b*c*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^3 - (((15*I)
/4)*b*c*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/d^3 + (2*b^2*c*P
olyLog[2, E^ArcSinh[c*x]])/d^3 - (((15*I)/4)*b^2*c*PolyLog[3, (-I)*E^ArcSin
h[c*x]])/d^3 + (((15*I)/4)*b^2*c*PolyLog[3, I*E^ArcSinh[c*x]])/d^3
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
```

```
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = -\frac{(a+b\operatorname{arcsinh}(cx))^2}{d^3x\left(1+c^2x^2\right)^2} - \left(5c^2\right)\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\left(d+c^2dx^2\right)^3}\,dx + \frac{(2bc)\int \frac{a+b\operatorname{arcsinh}(cx)}{x\left(1+c^2x^2\right)^{5/2}}\,dx}{d^3}$$

$$= \frac{2bc(a+b\text{arcsinh}(cx))}{3d^3(1+c^2x^2)^{3/2}} - \frac{(a+b\text{arcsinh}(cx))^2}{d^3x(1+c^2x^2)^2} - \frac{5c^2x(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} + \frac{(2bc)\int \frac{a+b\text{arcsinh}(cx)}{x(1+c^2x^2)^{3/2}}\,dx}{d^3} - \frac{(2b^2c^2)\int \frac{1}{(1+c^2x^2)^2}\,dx}{3d^3} + \frac{(5bc^3)\int \frac{x(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{5/2}}\,dx}{2d^3} - \frac{(15c^2)\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx}{4d}$$

$$= -\frac{b^2c^2x}{3d^3(1+c^2x^2)} - \frac{bc(a+b\text{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}} + \frac{2bc(a+b\text{arcsinh}(cx))}{d^3\sqrt{1+c^2x^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{d^3x(1+c^2x^2)^2} - \frac{5c^2x(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} - \frac{15c^2x(a+b\text{arcsinh}(cx))^2}{8d^3(1+c^2x^2)} + \frac{(2bc)\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{d^3} - \frac{(b^2c^2)\int \frac{1}{1+c^2x^2}\,dx}{3d^3} + \frac{(5b^2c^2)\int \frac{1}{(1+c^2x^2)^2}\,dx}{6d^3} - \frac{(2b^2c^2)\int \frac{1}{1+c^2x^2}\,dx}{d^3} + \frac{(15bc^3)\int \frac{x(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{4d^3} - \frac{(15c^2)\int \frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx}{8d^2}$$

$$= \frac{b^2c^2x}{12d^3(1+c^2x^2)} - \frac{bc(a+b\text{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}} - \frac{7bc(a+b\text{arcsinh}(cx))}{4d^3\sqrt{1+c^2x^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{d^3x(1+c^2x^2)^2} - \frac{5c^2x(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} - \frac{15c^2x(a+b\text{arcsinh}(cx))^2}{8d^3(1+c^2x^2)} - \frac{7b^2c\arctan(cx)}{3d^3} - \frac{(15c)\text{Subst}\left(\int (a+bx)^2\text{sech}(x)\,dx, x, \text{arcsinh}(cx)\right)}{8d^3} + \frac{(2bc)\text{Subst}\left(\int (a+bx)\text{csch}(x)\,dx, x, \text{arcsinh}(cx)\right)}{d^3} + \frac{(5b^2c^2)\int \frac{1}{1+c^2x^2}\,dx}{12d^3} + \frac{(15b^2c^2)\int \frac{1}{1+c^2x^2}\,dx}{4d^3}$$

$$= \frac{b^2c^2x}{12d^3(1+c^2x^2)} - \frac{bc(a+b\text{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}} - \frac{7bc(a+b\text{arcsinh}(cx))}{4d^3\sqrt{1+c^2x^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{d^3x(1+c^2x^2)^2} - \frac{5c^2x(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} - \frac{15c^2x(a+b\text{arcsinh}(cx))^2}{8d^3(1+c^2x^2)} - \frac{15c(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4d^3} + \frac{11b^2c\arctan(cx)}{6d^3} - \frac{4bc(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d^3} + \frac{(15ibc)\text{Subst}\left(\int(a+bx)\log(1-ie^x)\,dx,x,\text{arcsinh}(cx)\right)}{4d^3} - \frac{(15ibc)\text{Subst}\left(\int(a+bx)\log(1+ie^x)\,dx,x,\text{arcsinh}(cx)\right)}{4d^3} - \frac{(2b^2c)\,\text{Subst}\left(\int\log(1-e^x)\,dx,x,\text{arcsinh}(cx)\right)}{d^3} + \frac{(2b^2c)\,\text{Subst}\left(\int\log(1+e^x)\,dx,x,\text{arcsinh}(cx)\right)}{d^3}$$

$$= \frac{b^2c^2x}{12d^3(1+c^2x^2)} - \frac{bc(a+b\text{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}} - \frac{7bc(a+b\text{arcsinh}(cx))}{4d^3\sqrt{1+c^2x^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{d^3x(1+c^2x^2)^2} - \frac{5c^2x(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} - \frac{15c^2x(a+b\text{arcsinh}(cx))^2}{8d^3(1+c^2x^2)} - \frac{15c(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4d^3} + \frac{11b^2c\arctan(cx)}{6d^3} - \frac{4bc(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d^3} + \frac{15ibc(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{4d^3} - \frac{15ibc(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{4d^3} - \frac{(15ib^2c)\,\text{Subst}\left(\int\text{PolyLog}(2,-ie^x)\,dx,x,\text{arcsinh}(cx)\right)}{4d^3} + \frac{(15ib^2c)\,\text{Subst}\left(\int\text{PolyLog}(2,ie^x)\,dx,x,\text{arcsinh}(cx)\right)}{4d^3} - \frac{(2b^2c)\,\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{d^3} + \frac{(2b^2c)\,\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{d^3}$$

$$= \frac{b^2c^2x}{12d^3(1+c^2x^2)} - \frac{bc(a+b\text{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}} - \frac{7bc(a+b\text{arcsinh}(cx))}{4d^3\sqrt{1+c^2x^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{d^3x(1+c^2x^2)^2} - \frac{5c^2x(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} - \frac{15c^2x(a+b\text{arcsinh}(cx))^2}{8d^3(1+c^2x^2)} - \frac{15c(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4d^3} + \frac{11b^2c\arctan(cx)}{6d^3} - \frac{4bc(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d^3} - \frac{2b^2c\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d^3} + \frac{15ibc(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{4d^3} - \frac{15ibc(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{4d^3} + \frac{2b^2c\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d^3} - \frac{(15ib^2c)\,\text{Subst}\left(\int\frac{\text{PolyLog}(2,-ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{4d^3} + \frac{(15ib^2c)\,\text{Subst}\left(\int\frac{\text{PolyLog}(2,ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{4d^3}$$

$$= \frac{b^2c^2x}{12d^3(1+c^2x^2)} - \frac{bc(a+b\text{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}} - \frac{7bc(a+b\text{arcsinh}(cx))}{4d^3\sqrt{1+c^2x^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{d^3x(1+c^2x^2)^2} - \frac{5c^2x(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} - \frac{15c^2x(a+b\text{arcsinh}(cx))^2}{8d^3(1+c^2x^2)} - \frac{15c(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4d^3} + \frac{11b^2c\arctan(cx)}{6d^3} - \frac{4bc(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d^3} - \frac{2b^2c\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d^3} + \frac{15ibc(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{4d^3} - \frac{15ibc(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{4d^3} + \frac{2b^2c\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d^3} - \frac{15ib^2c\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{4d^3} + \frac{15ib^2c\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{4d^3}$$

Mathematica [A] (verified)

Time = 7.51 (sec) , antiderivative size = 716, normalized size of antiderivative = 1.84

$$\int\frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^3}\,dx = -\frac{a^2}{d^3x} - \frac{a^2c^2x}{4d^3(1+c^2x^2)^2} - \frac{7a^2c^2x}{8d^3(1+c^2x^2)} - \frac{15a^2c\arctan(cx)}{8d^3} + \frac{2abc\left(\frac{7\left(\sqrt{1+c^2x^2}+i\text{arcsinh}(cx)\right)}{16(-1-icx)} - \frac{\text{arcsinh}(cx)}{cx} - \frac{7\left(i\sqrt{1+c^2x^2}+\text{arcsinh}(cx)\right)}{16(i+cx)} + \frac{i\left((-2i+cx)\sqrt{1+c^2x^2}+3\text{arcsinh}(cx)\right)}{48(-i+cx)^2} - \cdots\right.}{\cdots} + \frac{b^2c\left(\frac{2cx}{1+c^2x^2} - \frac{4\text{arcsinh}(cx)}{(1+c^2x^2)^{3/2}} - \frac{42\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}} - \frac{6cx\text{arcsinh}(cx)^2}{(1+c^2x^2)^2} - \frac{21cx\text{arcsinh}(cx)^2}{1+c^2x^2} + 88\arctan\left(\tanh\left(\frac{1}{2}\text{arcsinh}\cdots\right.\right.\right.}{\cdots}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^3),x]
```

```
[Out] -(a^2/(d^3*x)) - (a^2*c^2*x)/(4*d^3*(1 + c^2*x^2)^2) - (7*a^2*c^2*x)/(8*d^3
*(1 + c^2*x^2)) - (15*a^2*c*ArcTan[c*x])/(8*d^3) + (2*a*b*c*((7*(Sqrt[1 + c
^2*x^2] + I*ArcSinh[c*x]))/(16*(-1 - I*c*x)) - ArcSinh[c*x]/(c*x) - (7*(I*S
qrt[1 + c^2*x^2] + ArcSinh[c*x]))/(16*(I + c*x)) + ((I/48)*((-2*I + c*x)*Sq
rt[1 + c^2*x^2] + 3*ArcSinh[c*x]))/(-I + c*x)^2 - ((I/48)*((2*I + c*x)*Sqrt
[1 + c^2*x^2] + 3*ArcSinh[c*x]))/(I + c*x)^2 - ArcTanh[Sqrt[1 + c^2*x^2]] +
 ((15*I)/16)*(-1/2*ArcSinh[c*x]^2 + 2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]
] + 2*PolyLog[2, (-I)*E^ArcSinh[c*x]]) - ((15*I)/16)*(-1/2*ArcSinh[c*x]^2 +
 2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + 2*PolyLog[2, I*E^ArcSinh[c*x]])
))/d^3 + (b^2*c*((2*c*x)/(1 + c^2*x^2) - (4*ArcSinh[c*x])/(1 + c^2*x^2)^(3/
2) - (42*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - (6*c*x*ArcSinh[c*x]^2)/(1 + c^2*
x^2)^2 - (21*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2) + 88*ArcTan[Tanh[ArcSinh[c*x
]/2]] - 12*ArcSinh[c*x]^2*Coth[ArcSinh[c*x]/2] + 48*ArcSinh[c*x]*Log[1 - E^
(-ArcSinh[c*x])] + (45*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] - (45*I)
*ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] - 48*ArcSinh[c*x]*Log[1 + E^(-Arc
Sinh[c*x])] + 48*PolyLog[2, -E^(-ArcSinh[c*x])] + (90*I)*ArcSinh[c*x]*PolyL
og[2, (-I)/E^ArcSinh[c*x]] - (90*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x
]] - 48*PolyLog[2, E^(-ArcSinh[c*x])] + (90*I)*PolyLog[3, (-I)/E^ArcSinh[c*
x]] - (90*I)*PolyLog[3, I/E^ArcSinh[c*x]] + 12*ArcSinh[c*x]^2*Tanh[ArcSinh[
c*x]/2]))/(24*d^3)
```

Maple [F]

$$\int \frac{(a+b\ \text{arcsinh}\,(cx))^2}{x^2\,(c^2d\,x^2+d)^3}dx$$

```
[In] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^3,x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^3,x)
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\,(d+c^2dx^2)^3}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^3x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^6*d^3*x^8 + 3*c
^4*d^3*x^6 + 3*c^2*d^3*x^4 + d^3*x^2), x)
```

Sympy [F]

$$\int \frac{(a + b\mathrm{arcsinh}(cx))^2}{x^2\left(d + c^2dx^2\right)^3}\,dx$$
$$= \frac{\int \frac{a^2}{c^6x^8+3c^4x^6+3c^2x^4+x^2}\,dx + \int \frac{b^2\,\mathrm{asinh}^2\,(cx)}{c^6x^8+3c^4x^6+3c^2x^4+x^2}\,dx + \int \frac{2ab\,\mathrm{asinh}\,(cx)}{c^6x^8+3c^4x^6+3c^2x^4+x^2}\,dx}{d^3}$$

```
[In] integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2/(c**6*x**8 + 3*c**4*x**6 + 3*c**2*x**4 + x**2), x) + Integra
l(b**2*asinh(c*x)**2/(c**6*x**8 + 3*c**4*x**6 + 3*c**2*x**4 + x**2), x) + I
ntegral(2*a*b*asinh(c*x)/(c**6*x**8 + 3*c**4*x**6 + 3*c**2*x**4 + x**2), x)
)/d**3
```

Maxima [F]

$$\int \frac{(a + b\mathrm{arcsinh}(cx))^2}{x^2\left(d + c^2dx^2\right)^3}\,dx = \int \frac{\left(b\,\mathrm{arsinh}\,(cx) + a\right)^2}{\left(c^2dx^2 + d\right)^3x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a^2*((15*c^4*x^4 + 25*c^2*x^2 + 8)/(c^4*d^3*x^5 + 2*c^2*d^3*x^3 + d^3*
x) + 15*c*arctan(c*x)/d^3) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(
c^6*d^3*x^8 + 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 + d^3*x^2) + 2*a*b*log(c*x + sq
rt(c^2*x^2 + 1))/(c^6*d^3*x^8 + 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 + d^3*x^2), x
)
```

Giac [F]

$$\int \frac{(a + b\mathrm{arcsinh}(cx))^2}{x^2\left(d + c^2dx^2\right)^3}\,dx = \int \frac{\left(b\,\mathrm{arsinh}\,(cx) + a\right)^2}{\left(c^2dx^2 + d\right)^3x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^3*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^2\left(d+c^2dx^2\right)^3}\,dx=\int \frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)^2}{x^2\left(d\,c^2\,x^2+d\right)^3}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^3),x)
[Out] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^3), x)
```

3.250 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 381

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^3}\,dx = {} & \frac{b^2c^2}{12d^3(1+c^2x^2)} - \frac{bc(a+b\text{arcsinh}(cx))}{d^3x(1+c^2x^2)^{3/2}} \\ & - \frac{5bc^3x(a+b\text{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}} \\ & + \frac{4bc^3x(a+b\text{arcsinh}(cx))}{3d^3\sqrt{1+c^2x^2}} - \frac{3c^2(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} \\ & - \frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2(1+c^2x^2)^2} - \frac{3c^2(a+b\text{arcsinh}(cx))^2}{2d^3(1+c^2x^2)} \\ & + \frac{6c^2(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^3} \\ & + \frac{b^2c^2\log(x)}{d^3} - \frac{7b^2c^2\log(1+c^2x^2)}{6d^3} \\ & + \frac{3bc^2(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d^3} \\ & - \frac{3bc^2(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d^3} \\ & - \frac{3b^2c^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{2d^3} \\ & + \frac{3b^2c^2\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(cx)}\right)}{2d^3}\end{aligned}$$

```
[Out] 1/12*b^2*c^2/d^3/(c^2*x^2+1)-b*c*(a+b*arcsinh(c*x))/d^3/x/(c^2*x^2+1)^(3/2)
-5/6*b*c^3*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^(3/2)-3/4*c^2*(a+b*arcsinh(
c*x))^2/d^3/(c^2*x^2+1)^2-1/2*(a+b*arcsinh(c*x))^2/d^3/x^2/(c^2*x^2+1)^2-3/
2*c^2*(a+b*arcsinh(c*x))^2/d^3/(c^2*x^2+1)+6*c^2*(a+b*arcsinh(c*x))^2*arcta
```

```
nh((c*x+(c^2*x^2+1)^(1/2))^2)/d^3+b^2*c^2*ln(x)/d^3-7/6*b^2*c^2*ln(c^2*x^2+
1)/d^3+3*b*c^2*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^3
-3*b*c^2*(a+b*arcsinh(c*x))*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d^3-3/2*b^
2*c^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^3+3/2*b^2*c^2*polylog(3,(c*x+
(c^2*x^2+1)^(1/2))^2)/d^3+4/3*b*c^3*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^(1
/2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.731$, Rules used = {5809, 5811, 5799, 5569, 4267, 2611, 2320, 6724, 5787, 266, 5788, 267, 277, 198, 197, 5804, 12, 1265, 907}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^3}\,dx = \frac{6c^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{d^3} + \frac{3bc^2\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d^3} - \frac{3bc^2\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d^3} - \frac{3c^2(a+b\text{arcsinh}(cx))^2}{2d^3\left(c^2x^2+1\right)} - \frac{3c^2(a+b\text{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2} - \frac{bc(a+b\text{arcsinh}(cx))}{d^3x\left(c^2x^2+1\right)^{3/2}} - \frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2\left(c^2x^2+1\right)^2} + \frac{4bc^3x(a+b\text{arcsinh}(cx))}{3d^3\sqrt{c^2x^2+1}} - \frac{5bc^3x(a+b\text{arcsinh}(cx))}{6d^3\left(c^2x^2+1\right)^{3/2}} - \frac{3b^2c^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{2d^3} + \frac{3b^2c^2\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(cx)}\right)}{2d^3} + \frac{b^2c^2}{12d^3\left(c^2x^2+1\right)} - \frac{7b^2c^2\log\left(c^2x^2+1\right)}{6d^3} + \frac{b^2c^2\log(x)}{d^3}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^3),x]
```

```
[Out] (b^2*c^2)/(12*d^3*(1 + c^2*x^2)) - (b*c*(a + b*ArcSinh[c*x]))/(d^3*x*(1 + c
^2*x^2)^(3/2)) - (5*b*c^3*x*(a + b*ArcSinh[c*x]))/(6*d^3*(1 + c^2*x^2)^(3/2
)) + (4*b*c^3*x*(a + b*ArcSinh[c*x]))/(3*d^3*Sqrt[1 + c^2*x^2]) - (3*c^2*(a
 + b*ArcSinh[c*x])^2)/(4*d^3*(1 + c^2*x^2)^2) - (a + b*ArcSinh[c*x])^2/(2*d
^3*x^2*(1 + c^2*x^2)^2) - (3*c^2*(a + b*ArcSinh[c*x])^2)/(2*d^3*(1 + c^2*x^
2)) + (6*c^2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSinh[c*x])])/d^3 + (b^2
*c^2*Log[x])/d^3 - (7*b^2*c^2*Log[1 + c^2*x^2])/(6*d^3) + (3*b*c^2*(a + b*A
rcSinh[c*x])*PolyLog[2, -E^(2*ArcSinh[c*x])])/d^3 - (3*b*c^2*(a + b*ArcSinh
```

```
[c*x])*PolyLog[2, E^(2*ArcSinh[c*x])])/d^3 - (3*b^2*c^2*PolyLog[3, -E^(2*Ar
cSinh[c*x])])/(2*d^3) + (3*b^2*c^2*PolyLog[3, E^(2*ArcSinh[c*x])])/(2*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2(1+c^2x^2)^2}-\left(3c^2\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^3}\,dx+\frac{(bc)\int\frac{a+b\text{arcsinh}(cx)}{x^2(1+c^2x^2)^{5/2}}\,dx}{d^3}\\
&= -\frac{bc(a+b\text{arcsinh}(cx))}{d^3x(1+c^2x^2)^{3/2}}-\frac{4bc^3x(a+b\text{arcsinh}(cx))}{3d^3(1+c^2x^2)^{3/2}}-\frac{8bc^3x(a+b\text{arcsinh}(cx))}{3d^3\sqrt{1+c^2x^2}}\\
&\quad-\frac{3c^2(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2}-\frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2(1+c^2x^2)^2}-\frac{(b^2c^2)\int\frac{-3-12c^2x^2-8c^4x^4}{3x(1+c^2x^2)^2}\,dx}{d^3}\\
&\quad+\frac{(3bc^3)\int\frac{a+b\text{arcsinh}(cx)}{(1+c^2x^2)^{5/2}}\,dx}{2d^3}-\frac{(3c^2)\int\frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^2}\,dx}{d}\\
&= -\frac{bc(a+b\text{arcsinh}(cx))}{d^3x(1+c^2x^2)^{3/2}}-\frac{5bc^3x(a+b\text{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}}-\frac{8bc^3x(a+b\text{arcsinh}(cx))}{3d^3\sqrt{1+c^2x^2}}\\
&\quad-\frac{3c^2(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2}-\frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2(1+c^2x^2)^2}-\frac{3c^2(a+b\text{arcsinh}(cx))^2}{2d^3(1+c^2x^2)}\\
&\quad-\frac{(b^2c^2)\int\frac{-3-12c^2x^2-8c^4x^4}{x(1+c^2x^2)^2}\,dx}{3d^3}+\frac{(bc^3)\int\frac{a+b\text{arcsinh}(cx)}{(1+c^2x^2)^{3/2}}\,dx}{d^3}\\
&\quad+\frac{(3bc^3)\int\frac{a+b\text{arcsinh}(cx)}{(1+c^2x^2)^{3/2}}\,dx}{d^3}-\frac{(b^2c^4)\int\frac{x}{(1+c^2x^2)^2}\,dx}{2d^3}-\frac{(3c^2)\int\frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)}\,dx}{d^2}\\
&= \frac{b^2c^2}{4d^3(1+c^2x^2)}-\frac{bc(a+b\text{arcsinh}(cx))}{d^3x(1+c^2x^2)^{3/2}}-\frac{5bc^3x(a+b\text{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}}\\
&\quad+\frac{4bc^3x(a+b\text{arcsinh}(cx))}{3d^3\sqrt{1+c^2x^2}}-\frac{3c^2(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2}\\
&\quad-\frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2(1+c^2x^2)^2}-\frac{3c^2(a+b\text{arcsinh}(cx))^2}{2d^3(1+c^2x^2)}\\
&\quad-\frac{(3c^2)\,\text{Subst}\left(\int(a+bx)^2\text{csch}(x)\text{sech}(x)\,dx,x,\text{arcsinh}(cx)\right)}{d^3}\\
&\quad-\frac{(b^2c^2)\,\text{Subst}\left(\int\frac{-3-12c^2x-8c^4x^2}{x(1+c^2x)^2}\,dx,x,x^2\right)}{6d^3}-\frac{(b^2c^4)\int\frac{x}{1+c^2x^2}\,dx}{d^3}-\frac{(3b^2c^4)\int\frac{x}{1+c^2x^2}\,dx}{d^3}
\end{aligned}
$$

$$= \frac{b^2c^2}{4d^3(1+c^2x^2)} - \frac{bc(a+b\operatorname{arcsinh}(cx))}{d^3x(1+c^2x^2)^{3/2}} - \frac{5bc^3x(a+b\operatorname{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}} + \frac{4bc^3x(a+b\operatorname{arcsinh}(cx))}{3d^3\sqrt{1+c^2x^2}} - \frac{3c^2(a+b\operatorname{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{2d^3x^2(1+c^2x^2)^2} - \frac{3c^2(a+b\operatorname{arcsinh}(cx))^2}{2d^3(1+c^2x^2)} - \frac{2b^2c^2\log(1+c^2x^2)}{d^3} - \frac{(6c^2)\operatorname{Subst}\left(\int(a+bx)^2\operatorname{csch}(2x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{d^3} - \frac{(b^2c^2)\operatorname{Subst}\left(\int\left(-\frac{3}{x}-\frac{c^2}{(1+c^2x)^2}-\frac{5c^2}{1+c^2x}\right)dx,x,x^2\right)}{6d^3}$$

$$= \frac{b^2c^2}{12d^3(1+c^2x^2)} - \frac{bc(a+b\operatorname{arcsinh}(cx))}{d^3x(1+c^2x^2)^{3/2}} - \frac{5bc^3x(a+b\operatorname{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}} + \frac{4bc^3x(a+b\operatorname{arcsinh}(cx))}{3d^3\sqrt{1+c^2x^2}} - \frac{3c^2(a+b\operatorname{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{2d^3x^2(1+c^2x^2)^2} - \frac{3c^2(a+b\operatorname{arcsinh}(cx))^2}{2d^3(1+c^2x^2)} + \frac{6c^2(a+b\operatorname{arcsinh}(cx))^2\operatorname{arctanh}\left(e^{2\operatorname{arcsinh}(cx)}\right)}{d^3} + \frac{b^2c^2\log(x)}{d^3} - \frac{7b^2c^2\log(1+c^2x^2)}{6d^3} + \frac{(6bc^2)\operatorname{Subst}\left(\int(a+bx)\log(1-e^{2x})\,dx,x,\operatorname{arcsinh}(cx)\right)}{d^3} - \frac{(6bc^2)\operatorname{Subst}\left(\int(a+bx)\log(1+e^{2x})\,dx,x,\operatorname{arcsinh}(cx)\right)}{d^3}$$

$$= \frac{b^2c^2}{12d^3(1+c^2x^2)} - \frac{bc(a+b\operatorname{arcsinh}(cx))}{d^3x(1+c^2x^2)^{3/2}} - \frac{5bc^3x(a+b\operatorname{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}} + \frac{4bc^3x(a+b\operatorname{arcsinh}(cx))}{3d^3\sqrt{1+c^2x^2}} - \frac{3c^2(a+b\operatorname{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{2d^3x^2(1+c^2x^2)^2} - \frac{3c^2(a+b\operatorname{arcsinh}(cx))^2}{2d^3(1+c^2x^2)} + \frac{6c^2(a+b\operatorname{arcsinh}(cx))^2\operatorname{arctanh}\left(e^{2\operatorname{arcsinh}(cx)}\right)}{d^3} + \frac{b^2c^2\log(x)}{d^3} - \frac{7b^2c^2\log(1+c^2x^2)}{6d^3} + \frac{3bc^2(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{d^3} - \frac{3bc^2(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(cx)}\right)}{d^3} - \frac{(3b^2c^2)\operatorname{Subst}\left(\int\operatorname{PolyLog}(2,-e^{2x})\,dx,x,\operatorname{arcsinh}(cx)\right)}{d^3} + \frac{(3b^2c^2)\operatorname{Subst}\left(\int\operatorname{PolyLog}(2,e^{2x})\,dx,x,\operatorname{arcsinh}(cx)\right)}{d^3}$$

$$= \frac{b^2c^2}{12d^3(1+c^2x^2)} - \frac{bc(a+b\operatorname{arcsinh}(cx))}{d^3x(1+c^2x^2)^{3/2}} - \frac{5bc^3x(a+b\operatorname{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}} + \frac{4bc^3x(a+b\operatorname{arcsinh}(cx))}{3d^3\sqrt{1+c^2x^2}} - \frac{3c^2(a+b\operatorname{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{2d^3x^2(1+c^2x^2)^2} - \frac{3c^2(a+b\operatorname{arcsinh}(cx))^2}{2d^3(1+c^2x^2)} + \frac{6c^2(a+b\operatorname{arcsinh}(cx))^2\operatorname{arctanh}\left(e^{2\operatorname{arcsinh}(cx)}\right)}{d^3} + \frac{b^2c^2\log(x)}{d^3} - \frac{7b^2c^2\log(1+c^2x^2)}{6d^3} + \frac{3bc^2(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{d^3} - \frac{3bc^2(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(cx)}\right)}{d^3} - \frac{(3b^2c^2)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,-x)}{x}\,dx,x,e^{2\operatorname{arcsinh}(cx)}\right)}{2d^3} + \frac{(3b^2c^2)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,x)}{x}\,dx,x,e^{2\operatorname{arcsinh}(cx)}\right)}{2d^3}$$

$$= \frac{b^2c^2}{12d^3(1+c^2x^2)} - \frac{bc(a+b\operatorname{arcsinh}(cx))}{d^3x(1+c^2x^2)^{3/2}} - \frac{5bc^3x(a+b\operatorname{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}} + \frac{4bc^3x(a+b\operatorname{arcsinh}(cx))}{3d^3\sqrt{1+c^2x^2}} - \frac{3c^2(a+b\operatorname{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{2d^3x^2(1+c^2x^2)^2} - \frac{3c^2(a+b\operatorname{arcsinh}(cx))^2}{2d^3(1+c^2x^2)} + \frac{6c^2(a+b\operatorname{arcsinh}(cx))^2\operatorname{arctanh}\left(e^{2\operatorname{arcsinh}(cx)}\right)}{d^3} + \frac{b^2c^2\log(x)}{d^3} - \frac{7b^2c^2\log(1+c^2x^2)}{6d^3} + \frac{3bc^2(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{d^3} - \frac{3bc^2(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(cx)}\right)}{d^3} - \frac{3b^2c^2\operatorname{PolyLog}\left(3,-e^{2\operatorname{arcsinh}(cx)}\right)}{2d^3} + \frac{3b^2c^2\operatorname{PolyLog}\left(3,e^{2\operatorname{arcsinh}(cx)}\right)}{2d^3}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.93 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.84

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^3}\,dx =$$

$$-\frac{\frac{2a^2}{x^2}+\frac{a^2c^2}{(1+c^2x^2)^2}+\frac{4a^2c^2}{1+c^2x^2}+12a^2c^2\log(x)-6a^2c^2\log(1+c^2x^2)-\frac{1}{6}ab\left(\frac{27c^2\left(\sqrt{1+c^2x^2}-i\text{arcsinh}(cx)\right)}{i+cx}+\frac{27c^2\left(\sqrt{\right.}\right.}{}\right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^3),x]
```

```
[Out] -1/4*((2*a^2)/x^2 + (a^2*c^2)/(1 + c^2*x^2)^2 + (4*a^2*c^2)/(1 + c^2*x^2) +
 12*a^2*c^2*Log[x] - 6*a^2*c^2*Log[1 + c^2*x^2] - (a*b*((27*c^2*(Sqrt[1 + c
^2*x^2] - I*ArcSinh[c*x]))/(I + c*x) + (27*c^2*(Sqrt[1 + c^2*x^2] + I*ArcSi
nh[c*x]))/(-I + c*x) - (24*(c*x*Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/x^2 + (c
^2*((-2*I + c*x)*Sqrt[1 + c^2*x^2] + 3*ArcSinh[c*x]))/(-I + c*x)^2 + (c^2*(
(2*I + c*x)*Sqrt[1 + c^2*x^2] + 3*ArcSinh[c*x]))/(I + c*x)^2 - 36*c^2*(ArcS
inh[c*x]*(ArcSinh[c*x] - 4*Log[1 + I*E^ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E
^ArcSinh[c*x]]) - 36*c^2*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSin
h[c*x]]) - 4*PolyLog[2, I*E^ArcSinh[c*x]]) + 72*c^2*(ArcSinh[c*x]*(ArcSinh[
c*x] - 2*Log[1 - E^(2*ArcSinh[c*x])]) - PolyLog[2, E^(2*ArcSinh[c*x])]))))/6
 - 4*b^2*c^2*((-1/8*I)*Pi^3 + (12 + 12*c^2*x^2)^(-1) + (c*x*ArcSinh[c*x])/(
6*(1 + c^2*x^2)^(3/2)) + (7*c*x*ArcSinh[c*x])/(3*Sqrt[1 + c^2*x^2]) - (Sqrt
[1 + c^2*x^2]*ArcSinh[c*x])/(c*x) - ArcSinh[c*x]^2/(2*c^2*x^2) - ArcSinh[c*
x]^2/(4*(1 + c^2*x^2)^2) - ArcSinh[c*x]^2/(1 + c^2*x^2) + 2*ArcSinh[c*x]^3
+ 3*ArcSinh[c*x]^2*Log[1 + E^(-2*ArcSinh[c*x])] - 3*ArcSinh[c*x]^2*Log[1 -
E^(2*ArcSinh[c*x])] + Log[(c*x)/Sqrt[1 + c^2*x^2]] - (2*Log[1 + c^2*x^2])/3
 - 3*ArcSinh[c*x]*PolyLog[2, -E^(-2*ArcSinh[c*x])] - 3*ArcSinh[c*x]*PolyLog
[2, E^(2*ArcSinh[c*x])] - (3*PolyLog[3, -E^(-2*ArcSinh[c*x])])/2 + (3*PolyL
og[3, E^(2*ArcSinh[c*x])])/2))/d^3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 831 vs. 2(402) = 804.

Time = 0.36 (sec) , antiderivative size = 832, normalized size of antiderivative = 2.18

method	result
derivativedivides	$c^2\left(\frac{a^2\left(-\frac{1}{2c^2x^2}-3\ln(cx)-\frac{1}{4(c^2x^2+1)^2}-\frac{1}{c^2x^2+1}+\frac{3\ln(c^2x^2+1)}{2}\right)}{d^3}+\frac{b^2\left(-\frac{-16\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^5c^5+16\,\mathrm{arcsinh}(c\right.}{}\right.\right.$
default	$c^2\left(\frac{a^2\left(-\frac{1}{2c^2x^2}-3\ln(cx)-\frac{1}{4(c^2x^2+1)^2}-\frac{1}{c^2x^2+1}+\frac{3\ln(c^2x^2+1)}{2}\right)}{d^3}+\frac{b^2\left(-\frac{-16\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^5c^5+16\,\mathrm{arcsinh}(c\right.}{}\right.\right.$
parts	$\frac{a^2\left(\frac{c^4\left(-\frac{2}{c^2(c^2x^2+1)}-\frac{1}{2c^2(c^2x^2+1)^2}+\frac{3\ln(c^2x^2+1)}{c^2}\right)}{2}-\frac{1}{2x^2}-3c^2\ln(x)\right)}{d^3}+\frac{b^2c^2\left(-\frac{-16\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^5c^5+16\right.}{}\right.$

```
[In] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(a^2/d^3*(-1/2/c^2/x^2-3*ln(c*x)-1/4/(c^2*x^2+1)^2-1/(c^2*x^2+1)+3/2*ln
(c^2*x^2+1))+b^2/d^3*(-1/12/(c^4*x^4+2*c^2*x^2+1)/c^2/x^2*(-16*arcsinh(c*x)
*(c^2*x^2+1)^(1/2)*x^5*c^5+16*arcsinh(c*x)*c^6*x^6+18*arcsinh(c*x)^2*x^4*c^
4-6*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3+32*arcsinh(c*x)*c^4*x^4+27*arcsi
nh(c*x)^2*x^2*c^2-c^4*x^4+12*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)+16*arcsinh(
c*x)*c^2*x^2+6*arcsinh(c*x)^2-c^2*x^2)+ln(1+c*x+(c^2*x^2+1)^(1/2))+8/3*ln(c
*x+(c^2*x^2+1)^(1/2))-7/3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+ln(c*x+(c^2*x^2+1
)^(1/2)-1)-3*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))-6*arcsinh(c*x)*poly
log(2,-c*x-(c^2*x^2+1)^(1/2))+6*polylog(3,-c*x-(c^2*x^2+1)^(1/2))+3*arcsinh
(c*x)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+3*arcsinh(c*x)*polylog(2,-(c*x+(c^2
*x^2+1)^(1/2))^2)-3/2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)-3*arcsinh(c*x)^
2*ln(1-c*x-(c^2*x^2+1)^(1/2))-6*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2
))+6*polylog(3,c*x+(c^2*x^2+1)^(1/2)))+2*a*b/d^3*(-1/12/(c^4*x^4+2*c^2*x^2+
1)/c^2/x^2*(-8*c^5*x^5*(c^2*x^2+1)^(1/2)+8*c^6*x^6+18*arcsinh(c*x)*c^4*x^4-
3*c^3*x^3*(c^2*x^2+1)^(1/2)+16*c^4*x^4+27*arcsinh(c*x)*c^2*x^2+6*c*x*(c^2*x
^2+1)^(1/2)+8*c^2*x^2+6*arcsinh(c*x))-3*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(
1/2))-3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+3*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2
+1)^(1/2))^2)+3/2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)-3*arcsinh(c*x)*ln(1
-c*x-(c^2*x^2+1)^(1/2))-3*polylog(2,c*x+(c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^3}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^3x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^3,x, algorithm="fricas")
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^6*d^3*x^9 + 3*c
^4*d^3*x^7 + 3*c^2*d^3*x^5 + d^3*x^3), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^3}\,dx = \frac{\int \frac{a^2}{c^6x^9+3c^4x^7+3c^2x^5+x^3}\,dx + \int \frac{b^2\,\text{asinh}^2\,(cx)}{c^6x^9+3c^4x^7+3c^2x^5+x^3}\,dx + \int \frac{2ab\,\text{asinh}\,(cx)}{c^6x^9+3c^4x^7+3c^2x^5+x^3}\,dx}{d^3}$$

```
[In] integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d)**3,x)
[Out] (Integral(a**2/(c**6*x**9 + 3*c**4*x**7 + 3*c**2*x**5 + x**3), x) + Integra
l(b**2*asinh(c*x)**2/(c**6*x**9 + 3*c**4*x**7 + 3*c**2*x**5 + x**3), x) + I
ntegral(2*a*b*asinh(c*x)/(c**6*x**9 + 3*c**4*x**7 + 3*c**2*x**5 + x**3), x)
)/d**3
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^3}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^3x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^3,x, algorithm="maxima")
[Out] -1/4*a^2*((6*c^4*x^4 + 9*c^2*x^2 + 2)/(c^4*d^3*x^6 + 2*c^2*d^3*x^4 + d^3*x^
2) - 6*c^2*log(c^2*x^2 + 1)/d^3 + 12*c^2*log(x)/d^3) + integrate(b^2*log(c*
x + sqrt(c^2*x^2 + 1))^2/(c^6*d^3*x^9 + 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5 + d^3
*x^3) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3*x^9 + 3*c^4*d^3*x^7 + 3
*c^2*d^3*x^5 + d^3*x^3), x)
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^3}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^3x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^3,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^3*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^3}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2}{x^3\,(d\,c^2\,x^2+d)^3}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^3),x)
[Out] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^3), x)
```

3.251 $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 529

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^3}\,dx = -\frac{b^2c^2}{2d^3x}+\frac{b^2c^2}{6d^3x(1+c^2x^2)}+\frac{b^2c^4x}{12d^3(1+c^2x^2)} - \frac{bc^3(a+b\text{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}}-\frac{bc(a+b\text{arcsinh}(cx))}{3d^3x^2(1+c^2x^2)^{3/2}} + \frac{29bc^3(a+b\text{arcsinh}(cx))}{12d^3\sqrt{1+c^2x^2}}-\frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3(1+c^2x^2)^2} + \frac{7c^2(a+b\text{arcsinh}(cx))^2}{3d^3x(1+c^2x^2)^2}+\frac{35c^4x(a+b\text{arcsinh}(cx))^2}{12d^3(1+c^2x^2)^2} + \frac{35c^4x(a+b\text{arcsinh}(cx))^2}{8d^3(1+c^2x^2)} + \frac{35c^3(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4d^3} - \frac{17b^2c^3\arctan(cx)}{6d^3} + \frac{38bc^3(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{3d^3} + \frac{19b^2c^3\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{3d^3} - \frac{35ibc^3(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{4d^3} + \frac{35ibc^3(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{4d^3} - \frac{19b^2c^3\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{3d^3} + \frac{35ib^2c^3\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{4d^3} - \frac{35ib^2c^3\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{4d^3}$$

```
[Out] -1/2*b^2*c^2/d^3/x+1/6*b^2*c^2/d^3/x/(c^2*x^2+1)+1/12*b^2*c^4*x/d^3/(c^2*x^
2+1)-1/6*b*c^3*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^(3/2)-1/3*b*c*(a+b*arcsin
h(c*x))/d^3/x^2/(c^2*x^2+1)^(3/2)-1/3*(a+b*arcsinh(c*x))^2/d^3/x^3/(c^2*x^2
+1)^2+7/3*c^2*(a+b*arcsinh(c*x))^2/d^3/x/(c^2*x^2+1)^2+35/12*c^4*x*(a+b*arc
sinh(c*x))^2/d^3/(c^2*x^2+1)^2+35/8*c^4*x*(a+b*arcsinh(c*x))^2/d^3/(c^2*x^2
+1)+35/4*c^3*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/d^3-17/6*b^
2*c^3*arctan(c*x)/d^3+38/3*b*c^3*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)
^(1/2))/d^3+19/3*b^2*c^3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))/d^3-35/4*I*b*c^3
*(a+b*arcsinh(c*x))*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^3+35/4*I*b*c^3*
(a+b*arcsinh(c*x))*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d^3-19/3*b^2*c^3*po
```

```
lylog(2,c*x+(c^2*x^2+1)^(1/2))/d^3+35/4*I*b^2*c^3*polylog(3,-I*(c*x+(c^2*x^
2+1)^(1/2)))/d^3-35/4*I*b^2*c^3*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/d^3+29
/12*b*c^3*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.00, number of steps used = 43, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.654$, Rules used = {5809, 5788, 5789, 4265, 2611, 2320, 6724, 5798, 209, 205, 5811, 5816, 4267, 2317, 2438, 296, 331}

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^3}\,dx &= \frac{35c^3\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{4d^3} \\ &+ \frac{38bc^3\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3d^3} \\ &- \frac{35ibc^3\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{4d^3} \\ &+ \frac{35ibc^3\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{4d^3} \\ &+ \frac{7c^2(a+b\text{arcsinh}(cx))^2}{3d^3x(c^2x^2+1)^2} - \frac{bc(a+b\text{arcsinh}(cx))}{3d^3x^2(c^2x^2+1)^{3/2}} \\ &- \frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3(c^2x^2+1)^2} + \frac{35c^4x(a+b\text{arcsinh}(cx))^2}{8d^3(c^2x^2+1)} \\ &+ \frac{35c^4x(a+b\text{arcsinh}(cx))^2}{12d^3(c^2x^2+1)^2} + \frac{29bc^3(a+b\text{arcsinh}(cx))}{12d^3\sqrt{c^2x^2+1}} \\ &- \frac{bc^3(a+b\text{arcsinh}(cx))}{6d^3(c^2x^2+1)^{3/2}} + \frac{19b^2c^3\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{3d^3} \\ &- \frac{19b^2c^3\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{3d^3} \\ &+ \frac{35ib^2c^3\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{4d^3} \\ &- \frac{35ib^2c^3\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{4d^3} - \frac{17b^2c^3\arctan(cx)}{6d^3} \\ &+ \frac{b^2c^2}{6d^3x(c^2x^2+1)} - \frac{b^2c^2}{2d^3x} + \frac{b^2c^4x}{12d^3(c^2x^2+1)}\end{aligned}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^3),x]
```

```
[Out] -1/2*(b^2*c^2)/(d^3*x) + (b^2*c^2)/(6*d^3*x*(1 + c^2*x^2)) + (b^2*c^4*x)/(1
2*d^3*(1 + c^2*x^2)) - (b*c^3*(a + b*ArcSinh[c*x]))/(6*d^3*(1 + c^2*x^2)^(3
/2)) - (b*c*(a + b*ArcSinh[c*x]))/(3*d^3*x^2*(1 + c^2*x^2)^(3/2)) + (29*b*c
^3*(a + b*ArcSinh[c*x]))/(12*d^3*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])^
```

```
2/(3*d^3*x^3*(1 + c^2*x^2)^2) + (7*c^2*(a + b*ArcSinh[c*x])^2)/(3*d^3*x*(1
+ c^2*x^2)^2) + (35*c^4*x*(a + b*ArcSinh[c*x])^2)/(12*d^3*(1 + c^2*x^2)^2)
+ (35*c^4*x*(a + b*ArcSinh[c*x])^2)/(8*d^3*(1 + c^2*x^2)) + (35*c^3*(a + b*
ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/(4*d^3) - (17*b^2*c^3*ArcTan[c*x])/
(6*d^3) + (38*b*c^3*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/(3*d^3) +
 (19*b^2*c^3*PolyLog[2, -E^ArcSinh[c*x]])/(3*d^3) - (((35*I)/4)*b*c^3*(a +
b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^3 + (((35*I)/4)*b*c^3*(a
 + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/d^3 - (19*b^2*c^3*PolyLog[
2, E^ArcSinh[c*x]])/(3*d^3) + (((35*I)/4)*b^2*c^3*PolyLog[3, (-I)*E^ArcSinh
[c*x]])/d^3 - (((35*I)/4)*b^2*c^3*PolyLog[3, I*E^ArcSinh[c*x]])/d^3
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
```

```
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

integral

$$= -\frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3(1+c^2x^2)^2} - \frac{1}{3}(7c^2)\int\frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^3}\,dx + \frac{(2bc)\int\frac{a+b\text{arcsinh}(cx)}{x^3(1+c^2x^2)^{5/2}}\,dx}{3d^3}$$

$$= -\frac{bc(a+b\text{arcsinh}(cx))}{3d^3x^2(1+c^2x^2)^{3/2}} - \frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3(1+c^2x^2)^2} + \frac{7c^2(a+b\text{arcsinh}(cx))^2}{3d^3x(1+c^2x^2)^2} + \frac{1}{3}(35c^4)\int\frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx + \frac{(b^2c^2)\int\frac{1}{x^2(1+c^2x^2)^2}\,dx}{3d^3} - \frac{(5bc^3)\int\frac{a+b\text{arcsinh}(cx)}{x(1+c^2x^2)^{5/2}}\,dx}{3d^3} - \frac{(14bc^3)\int\frac{a+b\text{arcsinh}(cx)}{x(1+c^2x^2)^{5/2}}\,dx}{3d^3}$$

$$= \frac{b^2c^2}{6d^3x(1+c^2x^2)} - \frac{19bc^3(a+b\text{arcsinh}(cx))}{9d^3(1+c^2x^2)^{3/2}} - \frac{bc(a+b\text{arcsinh}(cx))}{3d^3x^2(1+c^2x^2)^{3/2}} - \frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3(1+c^2x^2)^2} + \frac{7c^2(a+b\text{arcsinh}(cx))^2}{3d^3x(1+c^2x^2)^2} + \frac{35c^4x(a+b\text{arcsinh}(cx))^2}{12d^3(1+c^2x^2)^2} + \frac{(b^2c^2)\int\frac{1}{x^2(1+c^2x^2)}\,dx}{2d^3} - \frac{(5bc^3)\int\frac{a+b\text{arcsinh}(cx)}{x(1+c^2x^2)^{3/2}}\,dx}{3d^3} - \frac{(14bc^3)\int\frac{a+b\text{arcsinh}(cx)}{x(1+c^2x^2)^{3/2}}\,dx}{3d^3} + \frac{(5b^2c^4)\int\frac{1}{(1+c^2x^2)^2}\,dx}{9d^3} + \frac{(14b^2c^4)\int\frac{1}{(1+c^2x^2)^2}\,dx}{9d^3} - \frac{(35bc^5)\int\frac{x(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{5/2}}\,dx}{6d^3} + \frac{(35c^4)\int\frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx}{4d}$$

$$= -\frac{b^2c^2}{2d^3x} + \frac{b^2c^2}{6d^3x(1+c^2x^2)} + \frac{19b^2c^4x}{18d^3(1+c^2x^2)} - \frac{bc^3(a+b\text{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}} - \frac{bc(a+b\text{arcsinh}(cx))}{3d^3x^2(1+c^2x^2)^{3/2}} - \frac{19bc^3(a+b\text{arcsinh}(cx))}{3d^3\sqrt{1+c^2x^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3(1+c^2x^2)^2} + \frac{7c^2(a+b\text{arcsinh}(cx))^2}{3d^3x(1+c^2x^2)^2} + \frac{35c^4x(a+b\text{arcsinh}(cx))^2}{12d^3(1+c^2x^2)^2} + \frac{35c^4x(a+b\text{arcsinh}(cx))^2}{8d^3(1+c^2x^2)} - \frac{(5bc^3)\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{3d^3} - \frac{(14bc^3)\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{3d^3} + \frac{(5b^2c^4)\int\frac{1}{1+c^2x^2}\,dx}{18d^3} - \frac{(b^2c^4)\int\frac{1}{1+c^2x^2}\,dx}{2d^3} + \frac{(7b^2c^4)\int\frac{1}{1+c^2x^2}\,dx}{9d^3} + \frac{(5b^2c^4)\int\frac{1}{1+c^2x^2}\,dx}{3d^3} - \frac{(35b^2c^4)\int\frac{1}{(1+c^2x^2)^2}\,dx}{18d^3} + \frac{(14b^2c^4)\int\frac{1}{1+c^2x^2}\,dx}{3d^3} - \frac{(35bc^5)\int\frac{x(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{4d^3} + \frac{(35c^4)\int\frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx}{8d^2}$$

$$=-\frac{b^2c^2}{2d^3x}+\frac{b^2c^2}{6d^3x\,(1+c^2x^2)}+\frac{b^2c^4x}{12d^3\,(1+c^2x^2)}-\frac{bc^3(a+b\text{arcsinh}(cx))}{6d^3\,(1+c^2x^2)^{3/2}}$$
$$-\frac{bc(a+b\text{arcsinh}(cx))}{3d^3x^2\,(1+c^2x^2)^{3/2}}+\frac{29bc^3(a+b\text{arcsinh}(cx))}{12d^3\sqrt{1+c^2x^2}}-\frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3\,(1+c^2x^2)^2}$$
$$+\frac{7c^2(a+b\text{arcsinh}(cx))^2}{3d^3x\,(1+c^2x^2)^2}+\frac{35c^4x(a+b\text{arcsinh}(cx))^2}{12d^3\,(1+c^2x^2)^2}+\frac{35c^4x(a+b\text{arcsinh}(cx))^2}{8d^3\,(1+c^2x^2)}$$
$$+\frac{62b^2c^3\arctan(cx)}{9d^3}+\frac{(35c^3)\,\text{Subst}\left(\int(a+bx)^2\text{sech}(x)\,dx,x,\text{arcsinh}(cx)\right)}{8d^3}$$
$$-\frac{(5bc^3)\,\text{Subst}(\int(a+bx)\text{csch}(x)\,dx,x,\text{arcsinh}(cx))}{3d^3}$$
$$-\frac{(14bc^3)\,\text{Subst}(\int(a+bx)\text{csch}(x)\,dx,x,\text{arcsinh}(cx))}{3d^3}$$
$$-\frac{(35b^2c^4)\int\frac{1}{1+c^2x^2}\,dx}{36d^3}-\frac{(35b^2c^4)\int\frac{1}{1+c^2x^2}\,dx}{4d^3}$$

$$=-\frac{b^2c^2}{2d^3x}+\frac{b^2c^2}{6d^3x\,(1+c^2x^2)}+\frac{b^2c^4x}{12d^3\,(1+c^2x^2)}-\frac{bc^3(a+b\text{arcsinh}(cx))}{6d^3\,(1+c^2x^2)^{3/2}}$$
$$-\frac{bc(a+b\text{arcsinh}(cx))}{3d^3x^2\,(1+c^2x^2)^{3/2}}+\frac{29bc^3(a+b\text{arcsinh}(cx))}{12d^3\sqrt{1+c^2x^2}}-\frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3\,(1+c^2x^2)^2}$$
$$+\frac{7c^2(a+b\text{arcsinh}(cx))^2}{3d^3x\,(1+c^2x^2)^2}+\frac{35c^4x(a+b\text{arcsinh}(cx))^2}{12d^3\,(1+c^2x^2)^2}$$
$$+\frac{35c^4x(a+b\text{arcsinh}(cx))^2}{8d^3\,(1+c^2x^2)}+\frac{35c^3(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4d^3}$$
$$-\frac{17b^2c^3\arctan(cx)}{6d^3}+\frac{38bc^3(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{3d^3}$$
$$-\frac{(35ibc^3)\,\text{Subst}\left(\int(a+bx)\log\left(1-ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{4d^3}$$
$$+\frac{(35ibc^3)\,\text{Subst}\left(\int(a+bx)\log\left(1+ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{4d^3}$$
$$+\frac{(5b^2c^3)\,\text{Subst}\left(\int\log\left(1-e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{3d^3}$$
$$-\frac{(5b^2c^3)\,\text{Subst}\left(\int\log\left(1+e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{3d^3}$$
$$+\frac{(14b^2c^3)\,\text{Subst}\left(\int\log\left(1-e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{3d^3}$$
$$-\frac{(14b^2c^3)\,\text{Subst}\left(\int\log\left(1+e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{3d^3}$$

$$= -\frac{b^2c^2}{2d^3x} + \frac{b^2c^2}{6d^3x(1+c^2x^2)} + \frac{b^2c^4x}{12d^3(1+c^2x^2)} - \frac{bc^3(a+b\text{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}}$$
$$- \frac{bc(a+b\text{arcsinh}(cx))}{3d^3x^2(1+c^2x^2)^{3/2}} + \frac{29bc^3(a+b\text{arcsinh}(cx))}{12d^3\sqrt{1+c^2x^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3(1+c^2x^2)^2}$$
$$+ \frac{7c^2(a+b\text{arcsinh}(cx))^2}{3d^3x(1+c^2x^2)^2} + \frac{35c^4x(a+b\text{arcsinh}(cx))^2}{12d^3(1+c^2x^2)^2}$$
$$+ \frac{35c^4x(a+b\text{arcsinh}(cx))^2}{8d^3(1+c^2x^2)} + \frac{35c^3(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4d^3}$$
$$- \frac{17b^2c^3\arctan(cx)}{6d^3} + \frac{38bc^3(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{3d^3}$$
$$- \frac{35ibc^3(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{4d^3}$$
$$+ \frac{35ibc^3(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{4d^3}$$
$$+ \frac{(35ib^2c^3)\,\text{Subst}\left(\int \text{PolyLog}(2,-ie^x)\,dx, x, \text{arcsinh}(cx)\right)}{4d^3}$$
$$- \frac{(35ib^2c^3)\,\text{Subst}\left(\int \text{PolyLog}(2,ie^x)\,dx, x, \text{arcsinh}(cx)\right)}{4d^3}$$
$$+ \frac{(5b^2c^3)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{3d^3} - \frac{(5b^2c^3)\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{3d^3}$$
$$+ \frac{(14b^2c^3)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{3d^3} - \frac{(14b^2c^3)\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{3d^3}$$

$$=-\frac{b^2c^2}{2d^3x}+\frac{b^2c^2}{6d^3x\left(1+c^2x^2\right)}+\frac{b^2c^4x}{12d^3\left(1+c^2x^2\right)}-\frac{bc^3(a+b\operatorname{arcsinh}(cx))}{6d^3\left(1+c^2x^2\right)^{3/2}}$$
$$-\frac{bc(a+b\operatorname{arcsinh}(cx))}{3d^3x^2\left(1+c^2x^2\right)^{3/2}}+\frac{29bc^3(a+b\operatorname{arcsinh}(cx))}{12d^3\sqrt{1+c^2x^2}}-\frac{(a+b\operatorname{arcsinh}(cx))^2}{3d^3x^3\left(1+c^2x^2\right)^2}$$
$$+\frac{7c^2(a+b\operatorname{arcsinh}(cx))^2}{3d^3x\left(1+c^2x^2\right)^2}+\frac{35c^4x(a+b\operatorname{arcsinh}(cx))^2}{12d^3\left(1+c^2x^2\right)^2}$$
$$+\frac{35c^4x(a+b\operatorname{arcsinh}(cx))^2}{8d^3\left(1+c^2x^2\right)}+\frac{35c^3(a+b\operatorname{arcsinh}(cx))^2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{4d^3}$$
$$-\frac{17b^2c^3\arctan(cx)}{6d^3}+\frac{38bc^3(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{3d^3}$$
$$+\frac{19b^2c^3\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)}{3d^3}-\frac{35ibc^3(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{4d^3}$$
$$+\frac{35ibc^3(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{4d^3}-\frac{19b^2c^3\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)}{3d^3}$$
$$+\frac{(35ib^2c^3)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,-ix)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{4d^3}$$
$$-\frac{(35ib^2c^3)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,ix)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{4d^3}$$

$$=-\frac{b^2c^2}{2d^3x}+\frac{b^2c^2}{6d^3x\left(1+c^2x^2\right)}+\frac{b^2c^4x}{12d^3\left(1+c^2x^2\right)}-\frac{bc^3(a+b\operatorname{arcsinh}(cx))}{6d^3\left(1+c^2x^2\right)^{3/2}}$$
$$-\frac{bc(a+b\operatorname{arcsinh}(cx))}{3d^3x^2\left(1+c^2x^2\right)^{3/2}}+\frac{29bc^3(a+b\operatorname{arcsinh}(cx))}{12d^3\sqrt{1+c^2x^2}}-\frac{(a+b\operatorname{arcsinh}(cx))^2}{3d^3x^3\left(1+c^2x^2\right)^2}$$
$$+\frac{7c^2(a+b\operatorname{arcsinh}(cx))^2}{3d^3x\left(1+c^2x^2\right)^2}+\frac{35c^4x(a+b\operatorname{arcsinh}(cx))^2}{12d^3\left(1+c^2x^2\right)^2}$$
$$+\frac{35c^4x(a+b\operatorname{arcsinh}(cx))^2}{8d^3\left(1+c^2x^2\right)}+\frac{35c^3(a+b\operatorname{arcsinh}(cx))^2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{4d^3}$$
$$-\frac{17b^2c^3\arctan(cx)}{6d^3}+\frac{38bc^3(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{3d^3}$$
$$+\frac{19b^2c^3\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)}{3d^3}-\frac{35ibc^3(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{4d^3}$$
$$+\frac{35ibc^3(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{4d^3}-\frac{19b^2c^3\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)}{3d^3}$$
$$+\frac{35ib^2c^3\operatorname{PolyLog}\left(3,-ie^{\operatorname{arcsinh}(cx)}\right)}{4d^3}-\frac{35ib^2c^3\operatorname{PolyLog}\left(3,ie^{\operatorname{arcsinh}(cx)}\right)}{4d^3}$$

Mathematica [A] (verified)

Time = 9.00 (sec) , antiderivative size = 937, normalized size of antiderivative = 1.77

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^3}\,dx$$
$$= -\frac{a^2}{3d^3x^3}+\frac{3a^2c^2}{d^3x}+\frac{a^2c^4x}{4d^3(1+c^2x^2)^2}+\frac{11a^2c^4x}{8d^3(1+c^2x^2)}+\frac{35a^2c^3\arctan(cx)}{8d^3}$$
$$+\frac{2ab\left(-\frac{c\sqrt{1+c^2x^2}}{6x^2}+\frac{ic^3\left((2i-cx)\sqrt{1+c^2x^2}-3\text{arcsinh}(cx)\right)}{48(-i+cx)^2}-\frac{11c^3\left(\sqrt{1+c^2x^2}+i\text{arcsinh}(cx)\right)}{16(-1-icx)}-\frac{\text{arcsinh}(cx)}{3x^3}+\frac{11c^4\left(i\sqrt{1+c^2x^2}\right.}{16(i}\right.}{}$$
$$+\frac{b^2c^3\left(-\frac{2cx}{1+c^2x^2}+\frac{4\text{arcsinh}(cx)}{(1+c^2x^2)^{3/2}}+\frac{66\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}+\frac{6cx\text{arcsinh}(cx)^2}{(1+c^2x^2)^2}+\frac{33cx\text{arcsinh}(cx)^2}{1+c^2x^2}-136\arctan\left(\tanh\left(\frac{1}{2}\text{arc}\right.\right.\right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^3),x]
```

```
[Out] -1/3*a^2/(d^3*x^3) + (3*a^2*c^2)/(d^3*x) + (a^2*c^4*x)/(4*d^3*(1 + c^2*x^2)
^2) + (11*a^2*c^4*x)/(8*d^3*(1 + c^2*x^2)) + (35*a^2*c^3*ArcTan[c*x])/(8*d^
3) + (2*a*b*(-1/6*(c*Sqrt[1 + c^2*x^2])/x^2 + ((I/48)*c^3*((2*I - c*x)*Sqrt
[1 + c^2*x^2] - 3*ArcSinh[c*x]))/(-I + c*x)^2 - (11*c^3*(Sqrt[1 + c^2*x^2]
+ I*ArcSinh[c*x]))/(16*(-1 - I*c*x)) - ArcSinh[c*x]/(3*x^3) + (11*c^4*(I*Sq
rt[1 + c^2*x^2] + ArcSinh[c*x]))/(16*(I*c + c^2*x)) + ((I/48)*c^3*((2*I + c
*x)*Sqrt[1 + c^2*x^2] + 3*ArcSinh[c*x]))/(I + c*x)^2 + (c^3*ArcTanh[Sqrt[1
+ c^2*x^2]])/6 - 3*c^2*(-(ArcSinh[c*x]/x) - c*ArcTanh[Sqrt[1 + c^2*x^2]]) -
 ((35*I)/16)*c^4*(-1/2*ArcSinh[c*x]^2/c + (2*ArcSinh[c*x]*Log[1 + I*E^ArcSi
nh[c*x]])/c + (2*PolyLog[2, (-I)*E^ArcSinh[c*x]])/c) + ((35*I)/16)*c^4*(-1/
2*ArcSinh[c*x]^2/c + (2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]])/c + (2*Poly
Log[2, I*E^ArcSinh[c*x]])/c)))/d^3 + (b^2*c^3*((-2*c*x)/(1 + c^2*x^2) + (4*
ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) + (66*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + (
6*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2)^2 + (33*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^
2) - 136*ArcTan[Tanh[ArcSinh[c*x]/2]] - 4*Coth[ArcSinh[c*x]/2] + 38*ArcSinh
[c*x]^2*Coth[ArcSinh[c*x]/2] - 2*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - (c*x
*ArcSinh[c*x]^2*Csch[ArcSinh[c*x]/2]^4)/2 - 152*ArcSinh[c*x]*Log[1 - E^(-Ar
cSinh[c*x])] - (105*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] + (105*I)*A
rcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] + 152*ArcSinh[c*x]*Log[1 + E^(-ArcS
inh[c*x])] - 152*PolyLog[2, -E^(-ArcSinh[c*x])] - (210*I)*ArcSinh[c*x]*Poly
Log[2, (-I)/E^ArcSinh[c*x]] + (210*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c
*x]] + 152*PolyLog[2, E^(-ArcSinh[c*x])] - (210*I)*PolyLog[3, (-I)/E^ArcSin
h[c*x]] + (210*I)*PolyLog[3, I/E^ArcSinh[c*x]] - 2*ArcSinh[c*x]*Sech[ArcSin
h[c*x]/2]^2 - (8*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2]^4)/(c^3*x^3) + 4*Tanh[
ArcSinh[c*x]/2] - 38*ArcSinh[c*x]^2*Tanh[ArcSinh[c*x]/2]))/(24*d^3)
```

Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{x^4\,(c^2d\,x^2+d)^3}dx$$

[In] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^3,x)

[Out] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^3,x)

Fricas [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2}{x^4\,(d+c^2dx^2)^3}\,dx = \int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^3x^4}\,dx$$

[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^3,x, algorithm="fricas")

[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^6*d^3*x^10 + 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6 + d^3*x^4), x)

Sympy [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2}{x^4\,(d+c^2dx^2)^3}\,dx = \frac{\int \frac{a^2}{c^6x^{10}+3c^4x^8+3c^2x^6+x^4}\,dx+\int \frac{b^2\,\mathrm{asinh}^2\,(cx)}{c^6x^{10}+3c^4x^8+3c^2x^6+x^4}\,dx+\int \frac{2ab\,\mathrm{asinh}\,(cx)}{c^6x^{10}+3c^4x^8+3c^2x^6+x^4}\,dx}{d^3}$$

[In] integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d)**3,x)

[Out] (Integral(a**2/(c**6*x**10 + 3*c**4*x**8 + 3*c**2*x**6 + x**4), x) + Integral(b**2*asinh(c*x)**2/(c**6*x**10 + 3*c**4*x**8 + 3*c**2*x**6 + x**4), x) + Integral(2*a*b*asinh(c*x)/(c**6*x**10 + 3*c**4*x**8 + 3*c**2*x**6 + x**4), x))/d**3

Maxima [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2}{x^4\,(d+c^2dx^2)^3}\,dx = \int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^3x^4}\,dx$$

[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^3,x, algorithm="maxima")

[Out] 1/24*a^2*(105*c^3*arctan(c*x)/d^3 + (105*c^6*x^6 + 175*c^4*x^4 + 56*c^2*x^2 - 8)/(c^4*d^3*x^7 + 2*c^2*d^3*x^5 + d^3*x^3)) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^6*d^3*x^10 + 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6 + d^3*x^4) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3*x^10 + 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6 + d^3*x^4), x)

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4 (d+c^2dx^2)^3} \, dx = \int \frac{(b \operatorname{arsinh}(cx)+a)^2}{(c^2dx^2+d)^3 x^4} \, dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^3,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^3*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4 (d+c^2dx^2)^3} \, dx = \int \frac{(a+b \operatorname{asinh}(c\,x))^2}{x^4 \left(d\,c^2\,x^2+d\right)^3} \, dx$$

```
[In] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^3),x)
[Out] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^3), x)
```

3.252 $\int \left(\pi+c^2\pi x^2\right)^{5/2} (a+b\text{arcsinh}(cx))^2 \, dx$

Optimal result

Integrand size = 25, antiderivative size = 300

$$\int \left(\pi+c^2\pi x^2\right)^{5/2} (a+b\text{arcsinh}(cx))^2 \, dx = \frac{245b^2\pi^{5/2}x\sqrt{1+c^2x^2}}{1152}+\frac{65b^2\pi^{5/2}x(1+c^2x^2)^{3/2}}{1728}$$
$$+\frac{1}{108}b^2\pi^{5/2}x\left(1+c^2x^2\right)^{5/2}-\frac{115b^2\pi^{5/2}\text{arcsinh}(cx)}{1152c}-\frac{5}{16}bc\pi^{5/2}x^2(a+b\text{arcsinh}(cx))-\frac{5b\pi^{5/2}(1+c^2x^2)^2\,(a+ba}{48c}$$

```
[Out] 65/1728*b^2*Pi^(5/2)*x*(c^2*x^2+1)^(3/2)+1/108*b^2*Pi^(5/2)*x*(c^2*x^2+1)^(
5/2)-115/1152*b^2*Pi^(5/2)*arcsinh(c*x)/c-5/16*b*c*Pi^(5/2)*x^2*(a+b*arcsin
h(c*x))-5/48*b*Pi^(5/2)*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c-1/18*b*Pi^(5/2)*
(c^2*x^2+1)^3*(a+b*arcsinh(c*x))/c+5/24*Pi*x*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arc
sinh(c*x))^2+1/6*x*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))^2+5/48*Pi^(5/2)
*(a+b*arcsinh(c*x))^3/b/c+245/1152*b^2*Pi^(5/2)*x*(c^2*x^2+1)^(1/2)+5/16*Pi
^2*x*(a+b*arcsinh(c*x))^2*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {5786, 5785, 5783, 5776, 327, 221, 5798, 201}

$$\int \left(\pi+c^2\pi x^2\right)^{5/2} (a+b\text{arcsinh}(cx))^2 \, dx =$$
$$-\frac{\pi^{5/2}b(c^2x^2+1)^3\,(a+b\text{arcsinh}(cx))}{18c}-\frac{5\pi^{5/2}b(c^2x^2+1)^2\,(a+b\text{arcsinh}(cx))}{48c}$$
$$+\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))^2+\frac{5}{24}\pi x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))^2+\frac{5}{16}\pi^2x\sqrt{\pi c^2x^2+\pi}(a+barc$$

```
[In] Int[(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (245*b^2*Pi^(5/2)*x*Sqrt[1 + c^2*x^2])/1152 + (65*b^2*Pi^(5/2)*x*(1 + c^2*x
^2)^(3/2))/1728 + (b^2*Pi^(5/2)*x*(1 + c^2*x^2)^(5/2))/108 - (115*b^2*Pi^(5
/2)*ArcSinh[c*x])/(1152*c) - (5*b*c*Pi^(5/2)*x^2*(a + b*ArcSinh[c*x]))/16 -
 (5*b*Pi^(5/2)*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(48*c) - (b*Pi^(5/2)*(
1 + c^2*x^2)^3*(a + b*ArcSinh[c*x]))/(18*c) + (5*Pi^2*x*Sqrt[Pi + c^2*Pi*x^
2]*(a + b*ArcSinh[c*x])^2)/16 + (5*Pi*x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcS
inh[c*x])^2)/24 + (x*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/6 + (5
*Pi^(5/2)*(a + b*ArcSinh[c*x])^3)/(48*b*c)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
```

```
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{6}x\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2+\frac{1}{6}(5\pi)\int\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx-\frac{1}{3}\left(bc\pi^{5/2}\right)\int x\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))\,dx$$

$$= -\frac{b\pi^{5/2}(1+c^2x^2)^3(a+b\text{arcsinh}(cx))}{18c}+\frac{5}{24}\pi x\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2+\frac{1}{6}x\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2+\frac{1}{8}\left(5\pi^2\right)\int\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))^2\,dx+\frac{1}{18}\left(b^2\pi^{5/2}\right)$$

$$= \frac{1}{108}b^2\pi^{5/2}x\left(1+c^2x^2\right)^{5/2}-\frac{5b\pi^{5/2}(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{48c}-\frac{b\pi^{5/2}(1+c^2x^2)^3(a+b\text{arcsinh}(cx))}{18c}+\frac{5}{16}\pi^2x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))^2+\frac{5}{24}\pi x\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2+\frac{1}{6}x\left(\pi+c^2\pi x^2\right)^{5}$$

$$= \frac{65b^2\pi^{5/2}x(1+c^2x^2)^{3/2}}{1728}+\frac{1}{108}b^2\pi^{5/2}x\left(1+c^2x^2\right)^{5/2}-\frac{5}{16}bc\pi^{5/2}x^2(a+b\text{arcsinh}(cx))-\frac{5b\pi^{5/2}(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{48c}-$$

$$= \frac{245b^2\pi^{5/2}x\sqrt{1+c^2x^2}}{1152} + \frac{65b^2\pi^{5/2}x(1+c^2x^2)^{3/2}}{1728} + \frac{1}{108}b^2\pi^{5/2}x(1+c^2x^2)^{5/2} - \frac{5}{16}bc\pi^{5/2}x^2(a+b\text{arcsinh}(cx)) - \frac{5b\pi^{5/2}(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{48c} -$$

$$= \frac{245b^2\pi^{5/2}x\sqrt{1+c^2x^2}}{1152} + \frac{65b^2\pi^{5/2}x(1+c^2x^2)^{3/2}}{1728} + \frac{1}{108}b^2\pi^{5/2}x(1+c^2x^2)^{5/2} - \frac{115b^2\pi^{5/2}\text{arcsinh}(cx)}{1152c} - \frac{5}{16}bc\pi^{5/2}x^2(a+b\text{arcsinh}(cx)) - \frac{5b\pi^{5/2}(1+c^2x^2}{}$$

Mathematica [A] (verified)

Time = 1.87 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.95

$$\int (\pi + c^2\pi x^2)^{5/2}(a + b\text{arcsinh}(cx))^2\,dx = \frac{\pi^{5/2}\left(9504a^2cx\sqrt{1+c^2x^2} + 7488a^2c^3x^3\sqrt{1+c^2x^2} + 2304a^2c^5x^5\sqrt{1+c^2x^2} + 1440b^2\text{arcs}\right.}{}$$

```
[In] Integrate[(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (Pi^(5/2)*(9504*a^2*c*x*Sqrt[1 + c^2*x^2] + 7488*a^2*c^3*x^3*Sqrt[1 + c^2*x
^2] + 2304*a^2*c^5*x^5*Sqrt[1 + c^2*x^2] + 1440*b^2*ArcSinh[c*x]^3 - 3240*a
*b*Cosh[2*ArcSinh[c*x]] - 324*a*b*Cosh[4*ArcSinh[c*x]] - 24*a*b*Cosh[6*ArcS
inh[c*x]] + 1620*b^2*Sinh[2*ArcSinh[c*x]] + 81*b^2*Sinh[4*ArcSinh[c*x]] + 4
*b^2*Sinh[6*ArcSinh[c*x]] + 72*b*ArcSinh[c*x]^2*(60*a + 45*b*Sinh[2*ArcSinh
[c*x]] + 9*b*Sinh[4*ArcSinh[c*x]] + b*Sinh[6*ArcSinh[c*x]]) + 12*ArcSinh[c*
x]*(360*a^2 - 270*b^2*Cosh[2*ArcSinh[c*x]] - 27*b^2*Cosh[4*ArcSinh[c*x]] -
2*b^2*Cosh[6*ArcSinh[c*x]] + 540*a*b*Sinh[2*ArcSinh[c*x]] + 108*a*b*Sinh[4*
ArcSinh[c*x]] + 12*a*b*Sinh[6*ArcSinh[c*x]])))/(13824*c)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.32

method	result
default	$\frac{a^2x(\pi c^2x^2+\pi)^{\frac{5}{2}}}{6} + \frac{5a^2\pi x(\pi c^2x^2+\pi)^{\frac{3}{2}}}{24} + \frac{5a^2\pi^2x\sqrt{\pi c^2x^2+\pi}}{16} + \frac{5a^2\pi^3\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{16\sqrt{\pi c^2}} + \frac{b^2\pi^{\frac{5}{2}}\left(576\,\text{arcsinh}(cx)^2\sqrt{c^{}}\right.}{}$
parts	$\frac{a^2x(\pi c^2x^2+\pi)^{\frac{5}{2}}}{6} + \frac{5a^2\pi x(\pi c^2x^2+\pi)^{\frac{3}{2}}}{24} + \frac{5a^2\pi^2x\sqrt{\pi c^2x^2+\pi}}{16} + \frac{5a^2\pi^3\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{16\sqrt{\pi c^2}} + \frac{b^2\pi^{\frac{5}{2}}\left(576\,\text{arcsinh}(cx)^2\sqrt{c^{}}\right.}{}$

```
[In] int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^2*x*(Pi*c^2*x^2+Pi)^(5/2)+5/24*a^2*Pi*x*(Pi*c^2*x^2+Pi)^(3/2)+5/16*a^
2*Pi^2*x*(Pi*c^2*x^2+Pi)^(1/2)+5/16*a^2*Pi^3*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi
*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/3456*b^2*Pi^(5/2)*(576*arcsinh(c*x)^2*
(c^2*x^2+1)^(1/2)*x^5*c^5-192*arcsinh(c*x)*c^6*x^6+32*c^5*x^5*(c^2*x^2+1)^(
1/2)+1872*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*x^3*c^3-936*arcsinh(c*x)*c^4*x^4
+194*c^3*x^3*(c^2*x^2+1)^(1/2)+2376*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*c*x-23
76*arcsinh(c*x)*c^2*x^2+897*c*x*(c^2*x^2+1)^(1/2)+360*arcsinh(c*x)^3-897*ar
csinh(c*x))/c+1/144*a*b*Pi^(5/2)*(48*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^5*c^5
-8*c^6*x^6+156*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3-39*c^4*x^4+198*arcsin
h(c*x)*c*x*(c^2*x^2+1)^(1/2)-99*c^2*x^2+45*arcsinh(c*x)^2-68)/c
```

Fricas [F]

$$\int \left(\pi + c^2\pi x^2\right)^{5/2} (a + b\text{arcsinh}(cx))^2\, dx = \int \left(\pi + \pi c^2 x^2\right)^{\frac{5}{2}} (b \operatorname{arsinh}(cx) + a)^2\, dx$$

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a^2*c^4*x^4 + 2*pi^2*a^2*c^2*x^2 + pi^
2*a^2 + (pi^2*b^2*c^4*x^4 + 2*pi^2*b^2*c^2*x^2 + pi^2*b^2)*arcsinh(c*x)^2 +
 2*(pi^2*a*b*c^4*x^4 + 2*pi^2*a*b*c^2*x^2 + pi^2*a*b)*arcsinh(c*x)), x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(287) = 574$.

Time = 28.50 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.94

$$\int \left(\pi + c^2\pi x^2\right)^{5/2} (a + b\text{arcsinh}(cx))^2\, dx = \begin{cases} \frac{\pi^{\frac{5}{2}} a^2 c^4 x^5 \sqrt{c^2x^2+1}}{6} + \frac{13\pi^{\frac{5}{2}} a^2 c^2 x^3 \sqrt{c^2x^2+1}}{24} + \frac{11\pi^{\frac{5}{2}} a^2 x \sqrt{c^2x^2+1}}{16} + \frac{5\pi^{\frac{5}{2}} a^2 \operatorname{asinh}(cx)}{16c} - \frac{\pi^{\frac{5}{2}} abc^5x^6}{18} + \ldots \\ \pi^{\frac{5}{2}} a^2 x \end{cases}$$

```
[In] integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((pi**(5/2)*a**2*c**4*x**5*sqrt(c**2*x**2 + 1)/6 + 13*pi**(5/2)*a*
*2*c**2*x**3*sqrt(c**2*x**2 + 1)/24 + 11*pi**(5/2)*a**2*x*sqrt(c**2*x**2 +
1)/16 + 5*pi**(5/2)*a**2*asinh(c*x)/(16*c) - pi**(5/2)*a*b*c**5*x**6/18 + p
i**(5/2)*a*b*c**4*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)/3 - 13*pi**(5/2)*a*b*
c**3*x**4/48 + 13*pi**(5/2)*a*b*c**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/12
 - 11*pi**(5/2)*a*b*c*x**2/16 + 11*pi**(5/2)*a*b*x*sqrt(c**2*x**2 + 1)*asin
h(c*x)/8 + 5*pi**(5/2)*a*b*asinh(c*x)**2/(16*c) - pi**(5/2)*b**2*c**5*x**6*
asinh(c*x)/18 + pi**(5/2)*b**2*c**4*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)**2/
```

```
6 + pi**(5/2)*b**2*c**4*x**5*sqrt(c**2*x**2 + 1)/108 - 13*pi**(5/2)*b**2*c*
*3*x**4*asinh(c*x)/48 + 13*pi**(5/2)*b**2*c**2*x**3*sqrt(c**2*x**2 + 1)*asi
nh(c*x)**2/24 + 97*pi**(5/2)*b**2*c**2*x**3*sqrt(c**2*x**2 + 1)/1728 - 11*p
i**(5/2)*b**2*c*x**2*asinh(c*x)/16 + 11*pi**(5/2)*b**2*x*sqrt(c**2*x**2 + 1
)*asinh(c*x)**2/16 + 299*pi**(5/2)*b**2*x*sqrt(c**2*x**2 + 1)/1152 + 5*pi**
(5/2)*b**2*asinh(c*x)**3/(48*c) - 299*pi**(5/2)*b**2*asinh(c*x)/(1152*c), N
e(c, 0)), (pi**(5/2)*a**2*x, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(\pi + c^2\pi x^2\right)^{5/2} (a + b\text{arcsinh}(cx))^2 \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \left(\pi + c^2\pi x^2\right)^{5/2} (a + b\text{arcsinh}(cx))^2 \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(\pi + c^2\pi x^2\right)^{5/2} (a + b\text{arcsinh}(cx))^2 \, dx = \int (a + b\,\text{asinh}(c\,x))^2 \left(\Pi\,c^2\,x^2 + \Pi\right)^{5/2} \, dx$$

```
[In] int((a + b*asinh(c*x))^2*(Pi + Pi*c^2*x^2)^(5/2),x)
[Out] int((a + b*asinh(c*x))^2*(Pi + Pi*c^2*x^2)^(5/2), x)
```

3.253 $\int (\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))^2 \, dx$

Optimal result

Integrand size = 25, antiderivative size = 210

$$\int (\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))^2 \, dx = \frac{15}{64} b^2\pi^{3/2} x\sqrt{1+c^2x^2} + \frac{1}{32} b^2\pi^{3/2} x (1+c^2x^2)^{3/2} - \frac{9b^2\pi^{3/2}\text{arcsinh}(cx)}{64c} - \frac{3}{8} bc\pi^{3/2} x^2 (a+b\text{arcsinh}(cx)) - \frac{b\pi^{3/2}(1+c^2x^2)^2 (a+b\text{arcsinh}}{8c}$$

```
[Out] 1/32*b^2*Pi^(3/2)*x*(c^2*x^2+1)^(3/2)-9/64*b^2*Pi^(3/2)*arcsinh(c*x)/c-3/8*
b*c*Pi^(3/2)*x^2*(a+b*arcsinh(c*x))-1/8*b*Pi^(3/2)*(c^2*x^2+1)^2*(a+b*arcsi
nh(c*x))/c+1/4*x*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))^2+1/8*Pi^(3/2)*(a
+b*arcsinh(c*x))^3/b/c+15/64*b^2*Pi^(3/2)*x*(c^2*x^2+1)^(1/2)+3/8*Pi*x*(a+b
*arcsinh(c*x))^2*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {5786, 5785, 5783, 5776, 327, 221, 5798, 201}

$$\int (\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))^2 \, dx = \frac{1}{4} x (\pi c^2x^2 + \pi)^{3/2} (a + b\text{arcsinh}(cx))^2 + \frac{3}{8}\pi x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))^2 - \frac{\pi^{3/2} b (c^2x^2+1)^2 (a+b\text{arcsinh}(cx))}{8c} - \frac{3}{8}\pi^{3/2} bcx^2 (a+b\text{arcsinh}(cx)) + \frac{\pi^{3/2}(a+b\text{arcsinh}(cx))^3}{8bc} - \frac{9\pi^{3/2}b^2\text{arcsinh}(cx)}{64c} + \frac{1}{32}\pi^{3/2}b^2x(c^2x^2+1)^{3/2} + \frac{15}{64}$$

```
[In] Int[(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (15*b^2*Pi^(3/2)*x*Sqrt[1 + c^2*x^2])/64 + (b^2*Pi^(3/2)*x*(1 + c^2*x^2)^(3
/2))/32 - (9*b^2*Pi^(3/2)*ArcSinh[c*x])/(64*c) - (3*b*c*Pi^(3/2)*x^2*(a + b
```

```
*ArcSinh[c*x]))/8 - (b*Pi^(3/2)*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(8*c)
 + (3*Pi*x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2)/8 + (x*(Pi + c^2*P
i*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + (Pi^(3/2)*(a + b*ArcSinh[c*x])^3)/
(8*b*c)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
```

```
x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{4}x\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2+\frac{1}{4}(3\pi)\int\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))^2\,dx-\frac{1}{2}\left(bc\pi^{3/2}\right)\int x\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))\,dx$$

$$= -\frac{b\pi^{3/2}\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))}{8c}+\frac{3}{8}\pi x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))^2+\frac{1}{4}x\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2+\frac{1}{8}\left(3\pi^{3/2}\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx+\frac{1}{8}\left(b^2\pi^{3/2}\right)\int\left(1+c^2x^2\right)^{3/2}\,dx-\frac{1}{4}\left(3bc\pi^{3/2}\right)\int x(a+b\text{arcsin}$$

$$= \frac{1}{32}b^2\pi^{3/2}x\left(1+c^2x^2\right)^{3/2}-\frac{3}{8}bc\pi^{3/2}x^2(a+b\text{arcsinh}(cx))-\frac{b\pi^{3/2}\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))}{8c}+\frac{3}{8}\pi x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsin}$$

$$= \frac{15}{64}b^2\pi^{3/2}x\sqrt{1+c^2x^2}+\frac{1}{32}b^2\pi^{3/2}x\left(1+c^2x^2\right)^{3/2}-\frac{3}{8}bc\pi^{3/2}x^2(a+b\text{arcsinh}(cx))-\frac{b\pi^{3/2}\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))}{8c}+\frac{3}{8}\pi x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsin}$$

$$= \frac{15}{64}b^2\pi^{3/2}x\sqrt{1+c^2x^2}+\frac{1}{32}b^2\pi^{3/2}x\left(1+c^2x^2\right)^{3/2}-\frac{9b^2\pi^{3/2}\text{arcsinh}(cx)}{64c}-\frac{3}{8}bc\pi^{3/2}x^2(a+b\text{arcsinh}(cx))-\frac{b\pi^{3/2}\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))}{8c}+\frac{3}{8}\pi x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsin}$$

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.96

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))^2 \, dx = \frac{\pi^{3/2}\left(160a^2cx\sqrt{1+c^2x^2} + 64a^2c^3x^3\sqrt{1+c^2x^2} + 32b^2\text{arcsinh}(cx)^3 - 64ab\cosh(2\text{arcsinh}\right.}{}$$

```
[In] Integrate[(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (Pi^(3/2)*(160*a^2*c*x*Sqrt[1 + c^2*x^2] + 64*a^2*c^3*x^3*Sqrt[1 + c^2*x^2]
 + 32*b^2*ArcSinh[c*x]^3 - 64*a*b*Cosh[2*ArcSinh[c*x]] - 4*a*b*Cosh[4*ArcSi
nh[c*x]] + 32*b^2*Sinh[2*ArcSinh[c*x]] + b^2*Sinh[4*ArcSinh[c*x]] + 8*b*Arc
Sinh[c*x]^2*(12*a + 8*b*Sinh[2*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]]) + 4*
ArcSinh[c*x]*(-16*b^2*Cosh[2*ArcSinh[c*x]] - b^2*Cosh[4*ArcSinh[c*x]] + 4*a
*(6*a + 8*b*Sinh[2*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]]))))/(256*c)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.38

method	result
default	$\frac{a^2x(\pi c^2x^2+\pi)^{\frac{3}{2}}}{4} + \frac{3a^2\pi x\sqrt{\pi c^2x^2+\pi}}{8} + \frac{3a^2\pi^2\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{8\sqrt{\pi c^2}} + \frac{b^2\pi^{\frac{3}{2}}\left(16\,\text{arcsinh}(cx)^2\sqrt{c^2x^2+1}\,x^3c^3-8\,\text{arcsinh}(cx)c\right.}{}$
parts	$\frac{a^2x(\pi c^2x^2+\pi)^{\frac{3}{2}}}{4} + \frac{3a^2\pi x\sqrt{\pi c^2x^2+\pi}}{8} + \frac{3a^2\pi^2\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{8\sqrt{\pi c^2}} + \frac{b^2\pi^{\frac{3}{2}}\left(16\,\text{arcsinh}(cx)^2\sqrt{c^2x^2+1}\,x^3c^3-8\,\text{arcsinh}(cx)c\right.}{}$

```
[In] int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^2*x*(Pi*c^2*x^2+Pi)^(3/2)+3/8*a^2*Pi*x*(Pi*c^2*x^2+Pi)^(1/2)+3/8*a^2*
Pi^2*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/64*
b^2*Pi^(3/2)*(16*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*x^3*c^3-8*arcsinh(c*x)*c^
4*x^4+2*c^3*x^3*(c^2*x^2+1)^(1/2)+40*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*c*x-4
0*arcsinh(c*x)*c^2*x^2+17*c*x*(c^2*x^2+1)^(1/2)+8*arcsinh(c*x)^3-17*arcsinh
(c*x))/c+1/8*a*b*Pi^(3/2)*(4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3-c^4*x^4
+10*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)-5*c^2*x^2+3*arcsinh(c*x)^2-4)/c
```

Fricas [F]

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))^2 \, dx = \int \left(\pi + \pi c^2 x^2\right)^{\frac{3}{2}} (b \operatorname{arsinh}(cx) + a)^2 \, dx$$

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi*a^2*c^2*x^2 + pi*a^2 + (pi*b^2*c^2*x^2 +
 pi*b^2)*arcsinh(c*x)^2 + 2*(pi*a*b*c^2*x^2 + pi*a*b)*arcsinh(c*x)), x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(197) = 394$.

Time = 2.91 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.93

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))^2 \, dx = \begin{cases} \frac{\pi^{\frac{3}{2}} a^2 c^2 x^3 \sqrt{c^2x^2+1}}{4} + \frac{5\pi^{\frac{3}{2}} a^2 x \sqrt{c^2x^2+1}}{8} + \frac{3\pi^{\frac{3}{2}} a^2 \operatorname{asinh}(cx)}{8c} - \frac{\pi^{\frac{3}{2}} abc^3x^4}{8} + \frac{\pi^{\frac{3}{2}} abc^2x^3\sqrt{c^2x^2+1} \operatorname{asinh}(cx)}{2} \\ \pi^{\frac{3}{2}} a^2 x \end{cases}$$

```
[In] integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x))**2,x)
[Out] Piecewise((pi**(3/2)*a**2*c**2*x**3*sqrt(c**2*x**2 + 1)/4 + 5*pi**(3/2)*a**
2*x*sqrt(c**2*x**2 + 1)/8 + 3*pi**(3/2)*a**2*asinh(c*x)/(8*c) - pi**(3/2)*a
*b*c**3*x**4/8 + pi**(3/2)*a*b*c**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/2 -
 5*pi**(3/2)*a*b*c*x**2/8 + 5*pi**(3/2)*a*b*x*sqrt(c**2*x**2 + 1)*asinh(c*x
)/4 + 3*pi**(3/2)*a*b*asinh(c*x)**2/(8*c) - pi**(3/2)*b**2*c**3*x**4*asinh(
c*x)/8 + pi**(3/2)*b**2*c**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)**2/4 + pi*
*(3/2)*b**2*c**2*x**3*sqrt(c**2*x**2 + 1)/32 - 5*pi**(3/2)*b**2*c*x**2*asin
h(c*x)/8 + 5*pi**(3/2)*b**2*x*sqrt(c**2*x**2 + 1)*asinh(c*x)**2/8 + 17*pi**
(3/2)*b**2*x*sqrt(c**2*x**2 + 1)/64 + pi**(3/2)*b**2*asinh(c*x)**3/(8*c) -
17*pi**(3/2)*b**2*asinh(c*x)/(64*c), Ne(c, 0)), (pi**(3/2)*a**2*x, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))^2 \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\operatorname{arcsinh}(cx))^2 \, dx = \text{Exception raised: TypeError}$$

[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\operatorname{arcsinh}(cx))^2 \, dx = \int (a + b\operatorname{asinh}(c\,x))^2 \left(\Pi\,c^2\,x^2 + \Pi\right)^{3/2} \, dx$$

[In] int((a + b*asinh(c*x))^2*(Pi + Pi*c^2*x^2)^(3/2),x)

[Out] int((a + b*asinh(c*x))^2*(Pi + Pi*c^2*x^2)^(3/2), x)

3.254 $\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))^2\, dx$

Optimal result

Integrand size = 25, antiderivative size = 122

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))^2\, dx = \frac{1}{4}b^2\sqrt{\pi}x\sqrt{1+c^2x^2} - \frac{b^2\sqrt{\pi}\text{arcsinh}(cx)}{4c} - \frac{1}{2}bc\sqrt{\pi}x^2(a + b\text{arcsinh}(cx)) + \frac{1}{2}x\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))^2 + \frac{\sqrt{\pi}(a + b\text{arcsinh}(cx))^3}{6bc}$$

```
[Out] -1/4*b^2*arcsinh(c*x)*Pi^(1/2)/c-1/2*b*c*x^2*(a+b*arcsinh(c*x))*Pi^(1/2)+1/
6*(a+b*arcsinh(c*x))^3*Pi^(1/2)/b/c+1/4*b^2*x*Pi^(1/2)*(c^2*x^2+1)^(1/2)+1/
2*x*(a+b*arcsinh(c*x))^2*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5785, 5783, 5776, 327, 221}

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))^2\, dx = \frac{1}{2}x\sqrt{\pi c^2x^2 + \pi}(a + b\text{arcsinh}(cx))^2 - \frac{1}{2}\sqrt{\pi}bcx^2(a + b\text{arcsinh}(cx)) + \frac{\sqrt{\pi}(a + b\text{arcsinh}(cx))^3}{6bc} - \frac{\sqrt{\pi}b^2\text{arcsinh}(cx)}{4c} + \frac{1}{4}\sqrt{\pi}b^2x\sqrt{c^2x^2+1}$$

```
[In] Int[Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (b^2*Sqrt[Pi]*x*Sqrt[1 + c^2*x^2])/4 - (b^2*Sqrt[Pi]*ArcSinh[c*x])/(4*c) -
(b*c*Sqrt[Pi]*x^2*(a + b*ArcSinh[c*x]))/2 + (x*Sqrt[Pi + c^2*Pi*x^2]*(a + b
*ArcSinh[c*x])^2)/2 + (Sqrt[Pi]*(a + b*ArcSinh[c*x])^3)/(6*b*c)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))^2+\frac{1}{2}\sqrt{\pi}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx\\&\quad-\left(bc\sqrt{\pi}\right)\int x(a+b\text{arcsinh}(cx))\,dx\end{aligned}$$

$$= -\frac{1}{2}bc\sqrt{\pi}x^2(a+b\text{arcsinh}(cx)) + \frac{1}{2}x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))^2 + \frac{\sqrt{\pi}(a+b\text{arcsinh}(cx))^3}{6bc} + \frac{1}{2}\left(b^2c^2\sqrt{\pi}\right)\int \frac{x^2}{\sqrt{1+c^2x^2}}dx$$

$$= \frac{1}{4}b^2\sqrt{\pi}x\sqrt{1+c^2x^2} - \frac{1}{2}bc\sqrt{\pi}x^2(a+b\text{arcsinh}(cx)) + \frac{1}{2}x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))^2 + \frac{\sqrt{\pi}(a+b\text{arcsinh}(cx))^3}{6bc} - \frac{1}{4}\left(b^2\sqrt{\pi}\right)\int \frac{1}{\sqrt{1+c^2x^2}}dx$$

$$= \frac{1}{4}b^2\sqrt{\pi}x\sqrt{1+c^2x^2} - \frac{b^2\sqrt{\pi}\text{arcsinh}(cx)}{4c} - \frac{1}{2}bc\sqrt{\pi}x^2(a+b\text{arcsinh}(cx)) + \frac{1}{2}x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))^2 + \frac{\sqrt{\pi}(a+b\text{arcsinh}(cx))^3}{6bc}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{\sqrt{\pi}\left(4b^2\text{arcsinh}(cx)^3 + 6b\text{arcsinh}(cx)^2(2a+b\sinh(2\text{arcsinh}(cx))) + 3\left(4a^2cx\sqrt{1+c^2x^2} - 2ab\cosh(2\text{arcsi}\right.\right.}{2}$$

```
[In] Integrate[Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (Sqrt[Pi]*(4*b^2*ArcSinh[c*x]^3 + 6*b*ArcSinh[c*x]^2*(2*a + b*Sinh[2*ArcSin
h[c*x]]) + 3*(4*a^2*c*x*Sqrt[1 + c^2*x^2] - 2*a*b*Cosh[2*ArcSinh[c*x]] + b^
2*Sinh[2*ArcSinh[c*x]]) + 6*ArcSinh[c*x]*(-(b^2*Cosh[2*ArcSinh[c*x]]) + 2*a
*(a + b*Sinh[2*ArcSinh[c*x]]))))/(24*c)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.47

method	result
default	$\frac{a^2x\sqrt{\pi c^2x^2+\pi}}{2} + \frac{a^2\pi\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{2\sqrt{\pi c^2}} + \frac{b^2\sqrt{\pi}\left(6\,\text{arcsinh}(cx)^2\sqrt{c^2x^2+1}\,cx-6\,\text{arcsinh}(cx)c^2x^2+3cx\sqrt{c^2x^2+1}+2\,\text{arcsi}\right.}{12c}$
parts	$\frac{a^2x\sqrt{\pi c^2x^2+\pi}}{2} + \frac{a^2\pi\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{2\sqrt{\pi c^2}} + \frac{b^2\sqrt{\pi}\left(6\,\text{arcsinh}(cx)^2\sqrt{c^2x^2+1}\,cx-6\,\text{arcsinh}(cx)c^2x^2+3cx\sqrt{c^2x^2+1}+2\,\text{arcsi}\right.}{12c}$

```
[In] int((Pi*c^2*x^2+Pi)^(1/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2*x*(Pi*c^2*x^2+Pi)^(1/2)+1/2*a^2*Pi*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c
^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/12*b^2*Pi^(1/2)*(6*arcsinh(c*x)^2*(c^2*x
```

```
^2+1)^(1/2)*c*x-6*arcsinh(c*x)*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+2*arcsinh(c*
x)^3-3*arcsinh(c*x))/c+1/2*a*b*Pi^(1/2)*(2*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/
2)-c^2*x^2+arcsinh(c*x)^2-1)/c
```

Fricas [F]

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))^2\, dx = \int \sqrt{\pi + \pi c^2x^2}(b\,\text{arsinh}\,(cx) + a)^2\, dx$$

```
[In] integrate((pi*c^2*x^2+pi)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a
^2), x)
```

Sympy [F]

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))^2\, dx = \sqrt{\pi}\left(\int a^2\sqrt{c^2x^2 + 1}\, dx + \int b^2\sqrt{c^2x^2 + 1}\,\text{asinh}^2\,(cx)\, dx + \int 2ab\sqrt{c^2x^2 + 1}\,\text{asinh}\,(cx)\, dx\right)$$

```
[In] integrate((pi*c**2*x**2+pi)**(1/2)*(a+b*asinh(c*x))**2,x)
[Out] sqrt(pi)*(Integral(a**2*sqrt(c**2*x**2 + 1), x) + Integral(b**2*sqrt(c**2*x
**2 + 1)*asinh(c*x)**2, x) + Integral(2*a*b*sqrt(c**2*x**2 + 1)*asinh(c*x),
 x))
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))^2\, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((pi*c^2*x^2+pi)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((pi*c^2*x^2+pi)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))^2\,dx = \int (a + b\,\text{asinh}(c\,x))^2\,\sqrt{\Pi\,c^2\,x^2 + \Pi}\,dx$$

```
[In] int((a + b*asinh(c*x))^2*(Pi + Pi*c^2*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(Pi + Pi*c^2*x^2)^(1/2), x)
```

3.255 $\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{\pi}}$$

```
[Out] 1/3*(a+b*arcsinh(c*x))^3/b/c/Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {5783}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{(a+b\text{arcsinh}(cx))^3}{3\sqrt{\pi}bc}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (a + b*ArcSinh[c*x])^3/(3*b*c*Sqrt[Pi])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{\pi}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{\pi+c^2\pi x^2}}\,dx=\frac{(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{\pi}}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/Sqrt[Pi + c^2*Pi*x^2],x]
[Out] (a + b*ArcSinh[c*x])^3/(3*b*c*Sqrt[Pi])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(21) = 42.

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.88

method	result	size
default	$\frac{a^2\ln\left(\frac{\pi\,c^2x}{\sqrt{\pi\,c^2}}+\sqrt{\pi\,c^2x^2+\pi}\right)}{\sqrt{\pi\,c^2}}+\frac{b^2\,\text{arcsinh}(cx)^3}{3c\sqrt{\pi}}+\frac{ab\,\text{arcsinh}(cx)^2}{c\sqrt{\pi}}$	72
parts	$\frac{a^2\ln\left(\frac{\pi\,c^2x}{\sqrt{\pi\,c^2}}+\sqrt{\pi\,c^2x^2+\pi}\right)}{\sqrt{\pi\,c^2}}+\frac{b^2\,\text{arcsinh}(cx)^3}{3c\sqrt{\pi}}+\frac{ab\,\text{arcsinh}(cx)^2}{c\sqrt{\pi}}$	72

```
[In] int((a+b*arcsinh(c*x))^2/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
[Out] a^2*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/3*b^
2/c/Pi^(1/2)*arcsinh(c*x)^3+a*b*arcsinh(c*x)^2/c/Pi^(1/2)
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{\pi+c^2\pi x^2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{\pi+\pi c^2x^2}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas")
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/sqrt(pi + pi*c^2*x
^2), x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(19) = 38$.

Time = 0.90 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.60

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{\sqrt{\pi + c^2\pi x^2}}\, dx = \begin{cases} a^2 \left(\begin{cases} \frac{\log\left(2\pi c^2 x + 2\sqrt{\pi}\sqrt{\pi c^2 x^2 + \pi}\sqrt{c^2}\right)}{\sqrt{\pi}\sqrt{c^2}} & \text{for}\ \pi c^2 \neq 0 \\ \frac{x}{\sqrt{\pi}} & \text{otherwise} \end{cases} \right) & \text{for}\ b = 0 \\ \frac{a^2 x}{\sqrt{\pi}} & \text{for}\ c = 0 \\ \frac{(a + b \operatorname{asinh}(cx))^3}{3\sqrt{\pi} bc} & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*asinh(c*x))**2/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] Piecewise((a**2*Piecewise((log(2*pi*c**2*x + 2*sqrt(pi)*sqrt(pi*c**2*x**2 +
 pi)*sqrt(c**2))/(sqrt(pi)*sqrt(c**2)), Ne(pi*c**2, 0)), (x/sqrt(pi), True)
), Eq(b, 0)), (a**2*x/sqrt(pi), Eq(c, 0)), ((a + b*asinh(c*x))**3/(3*sqrt(p
i)*b*c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(21) = 42$.

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.88

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{\sqrt{\pi + c^2\pi x^2}}\, dx = \frac{b^2 \operatorname{arsinh}(cx)^3}{3\sqrt{\pi} c} + \frac{ab \operatorname{arsinh}(cx)^2}{\sqrt{\pi} c} + \frac{a^2 \operatorname{arsinh}(cx)}{\sqrt{\pi} c}$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*b^2*arcsinh(c*x)^3/(sqrt(pi)*c) + a*b*arcsinh(c*x)^2/(sqrt(pi)*c) + a^2
*arcsinh(c*x)/(sqrt(pi)*c)
```

Giac [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{\sqrt{\pi + c^2\pi x^2}}\, dx = \int \frac{(b \operatorname{arsinh}(cx) + a)^2}{\sqrt{\pi + \pi c^2 x^2}}\, dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/sqrt(pi + pi*c^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{\pi+c^2\pi x^2}}\,dx=\int \frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)^2}{\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(1/2),x)
[Out] int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(1/2), x)
```

3.256 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 104

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{(a+b\text{arcsinh}(cx))^2}{c\pi^{3/2}} + \frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{2b(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c\pi^{3/2}} - \frac{b^2\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c\pi^{3/2}}$$

```
[Out] (a+b*arcsinh(c*x))^2/c/Pi^(3/2)-2*b*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1
)^(1/2))^2)/c/Pi^(3/2)-b^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/Pi^(3/2)
+x*(a+b*arcsinh(c*x))^2/Pi/(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {5787, 5797, 3799, 2221, 2317, 2438}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}} + \frac{(a+b\text{arcsinh}(cx))^2}{\pi^{3/2}c} - \frac{2b\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))}{\pi^{3/2}c} - \frac{b^2\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{\pi^{3/2}c}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (a + b*ArcSinh[c*x])^2/(c*Pi^(3/2)) + (x*(a + b*ArcSinh[c*x])^2)/(Pi*Sqrt[P
i + c^2*Pi*x^2]) - (2*b*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(
c*Pi^(3/2)) - (b^2*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*Pi^(3/2))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{(2bc)\int \frac{x(a+b\text{arcsinh}(cx))}{1+c^2x^2}\,dx}{\pi^{3/2}}$$

$$= \frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{(2b)\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \text{arcsinh}(cx)\right)}{c\pi^{3/2}}$$

$$=\frac{(a+b\text{arcsinh}(cx))^2}{c\pi^{3/2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi+c^2\pi x^2}}-\frac{(4b)\text{Subst}\left(\int\frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx,x,\text{arcsinh}(cx)\right)}{c\pi^{3/2}}$$

$$=\frac{(a+b\text{arcsinh}(cx))^2}{c\pi^{3/2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi+c^2\pi x^2}}-\frac{2b(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c\pi^{3/2}}+\frac{(2b^2)\,\text{Subst}\left(\int\log\left(1+e^{2x}\right)\,dx,x,\text{arcsinh}(cx)\right)}{c\pi^{3/2}}$$

$$=\frac{(a+b\text{arcsinh}(cx))^2}{c\pi^{3/2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi+c^2\pi x^2}}-\frac{2b(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c\pi^{3/2}}+\frac{b^2\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\text{arcsinh}(cx)}\right)}{c\pi^{3/2}}$$

$$=\frac{(a+b\text{arcsinh}(cx))^2}{c\pi^{3/2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi+c^2\pi x^2}}-\frac{2b(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c\pi^{3/2}}-\frac{b^2\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c\pi^{3/2}}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.47

$$\int\frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\frac{-b^2\left(-cx+\sqrt{1+c^2x^2}\right)\text{arcsinh}(cx)^2+2b\text{arcsinh}(cx)\left(acx-b\sqrt{1+c^2x^2}\log\left(1+\right.\right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (-(b^2*(-(c*x) + Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2) + 2*b*ArcSinh[c*x]*(a*c
*x - b*Sqrt[1 + c^2*x^2]*Log[1 + E^(-2*ArcSinh[c*x])]) + a*(a*c*x - b*Sqrt[
1 + c^2*x^2]*Log[1 + c^2*x^2]) + b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-2*Ar
cSinh[c*x])])/(c*Pi^(3/2)*Sqrt[1 + c^2*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(114) = 228$.

Time = 0.26 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.34

method	result
default	$\frac{a^2 x}{\pi\sqrt{\pi c^2x^2+\pi}}+b^2\left(-\frac{\left(c^2x^2-cx\sqrt{c^2x^2+1}+1\right)\operatorname{arcsinh}(cx)^2}{\pi^{\frac{3}{2}}c(c^2x^2+1)}+\frac{2\operatorname{arcsinh}(cx)^2}{c\,\pi^{\frac{3}{2}}}-\frac{2\operatorname{arcsinh}(cx)\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)}{c\,\pi^{\frac{3}{2}}}-\right.$
parts	$\frac{a^2 x}{\pi\sqrt{\pi c^2x^2+\pi}}+b^2\left(-\frac{\left(c^2x^2-cx\sqrt{c^2x^2+1}+1\right)\operatorname{arcsinh}(cx)^2}{\pi^{\frac{3}{2}}c(c^2x^2+1)}+\frac{2\operatorname{arcsinh}(cx)^2}{c\,\pi^{\frac{3}{2}}}-\frac{2\operatorname{arcsinh}(cx)\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)}{c\,\pi^{\frac{3}{2}}}-\right.$

```
[In] int((a+b*arcsinh(c*x))^2/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2/Pi*x/(Pi*c^2*x^2+Pi)^(1/2)+b^2*(-1/Pi^(3/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1
/2)+1)*arcsinh(c*x)^2/c/(c^2*x^2+1)+2/c/Pi^(3/2)*arcsinh(c*x)^2-2/c/Pi^(3/2
)*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-1/c/Pi^(3/2)*polylog(2,-(c*x
+(c^2*x^2+1)^(1/2))^2))+2*a*b*(2/c/Pi^(3/2)*arcsinh(c*x)-1/Pi^(3/2)*(c^2*x^
2-c*x*(c^2*x^2+1)^(1/2)+1)*arcsinh(c*x)/c/(c^2*x^2+1)-1/c/Pi^(3/2)*ln(1+(c*
x+(c^2*x^2+1)^(1/2))^2))
```

Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(\pi+\pi c^2x^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a
^2)/(pi^2*c^4*x^4 + 2*pi^2*c^2*x^2 + pi^2), x)
```

Sympy [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\frac{\int \frac{a^2}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{b^2\operatorname{asinh}^2(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{2ab\operatorname{asinh}(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

```
[In] integrate((a+b*asinh(c*x))**2/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a**2/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x) +
Integral(b**2*asinh(c*x)**2/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2
 + 1)), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqr
t(c**2*x**2 + 1)), x))/pi**(3/2)
```

Maxima [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(\pi + c^2\pi x^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(\pi + \pi c^2x^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima")
[Out] b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/(pi + pi*c^2*x^2)^(3/2), x) +
2*a*b*x*arcsinh(c*x)/(pi*sqrt(pi + pi*c^2*x^2)) + a^2*x/(pi*sqrt(pi + pi*c^
2*x^2)) - a*b*log(x^2 + 1/c^2)/(pi^(3/2)*c)
```

Giac [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(\pi + c^2\pi x^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(\pi + \pi c^2x^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/(pi + pi*c^2*x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(\pi + c^2\pi x^2)^{3/2}}\,dx = \int \frac{(a + b\,\text{asinh}(c\,x))^2}{(\Pi\,c^2\,x^2 + \Pi)^{3/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(3/2),x)
[Out] int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(3/2), x)
```

3.257 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 204

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{5/2}}\,dx = -\frac{b^2x}{3\pi^{5/2}\sqrt{1+c^2x^2}}+\frac{b(a+b\text{arcsinh}(cx))}{3c\pi^{5/2}(1+c^2x^2)} + \frac{2(a+b\text{arcsinh}(cx))^2}{3c\pi^{5/2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi(\pi+c^2\pi x^2)^{3/2}}+\frac{2x(a+b\text{arcsinh}(cx))^2}{3\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{4b(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3c\pi^{5/2}}-\frac{2b^2\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3c\pi^{5/2}}$$

```
[Out] 1/3*b*(a+b*arcsinh(c*x))/c/Pi^(5/2)/(c^2*x^2+1)+2/3*(a+b*arcsinh(c*x))^2/c/
Pi^(5/2)+1/3*x*(a+b*arcsinh(c*x))^2/Pi/(Pi*c^2*x^2+Pi)^(3/2)-4/3*b*(a+b*arc
sinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/Pi^(5/2)-2/3*b^2*polylog(2,-(c
*x+(c^2*x^2+1)^(1/2))^2)/c/Pi^(5/2)-1/3*b^2*x/Pi^(5/2)/(c^2*x^2+1)^(1/2)+2/
3*x*(a+b*arcsinh(c*x))^2/Pi^2/(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5788, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 197}

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{b(a+b\operatorname{arcsinh}(cx))}{3\pi^{5/2}c\,(c^2x^2+1)} + \frac{2x(a+b\operatorname{arcsinh}(cx))^2}{3\pi^2\sqrt{\pi c^2x^2+\pi}} + \frac{x(a+b\operatorname{arcsinh}(cx))^2}{3\pi\,(\pi c^2x^2+\pi)^{3/2}} + \frac{2(a+b\operatorname{arcsinh}(cx))^2}{3\pi^{5/2}c} - \frac{4b\log\left(e^{2\operatorname{arcsinh}(cx)}+1\right)(a+b\operatorname{arcsinh}(cx))}{3\pi^{5/2}c} - \frac{2b^2\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{3\pi^{5/2}c} - \frac{b^2x}{3\pi^{5/2}\sqrt{c^2x^2+1}}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] -1/3*(b^2*x)/(Pi^(5/2)*Sqrt[1 + c^2*x^2]) + (b*(a + b*ArcSinh[c*x]))/(3*c*P
i^(5/2)*(1 + c^2*x^2)) + (2*(a + b*ArcSinh[c*x])^2)/(3*c*Pi^(5/2)) + (x*(a
+ b*ArcSinh[c*x])^2)/(3*Pi*(Pi + c^2*Pi*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c
*x])^2)/(3*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) - (4*b*(a + b*ArcSinh[c*x])*Log[1 +
E^(2*ArcSinh[c*x])])/(3*c*Pi^(5/2)) - (2*b^2*PolyLog[2, -E^(2*ArcSinh[c*x])
])/(3*c*Pi^(5/2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{x(a+b\text{arcsinh}(cx))^2}{3\pi\,(\pi+c^2\pi x^2)^{3/2}} - \frac{(2bc)\int\frac{x(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^2}\,dx}{3\pi^{5/2}} + \frac{2\int\frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{3/2}}\,dx}{3\pi}$$

$$=\frac{b(a+b\text{arcsinh}(cx))}{3c\pi^{5/2}(1+c^2x^2)}+\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi(\pi+c^2\pi x^2)^{3/2}}+\frac{2x(a+b\text{arcsinh}(cx))^2}{3\pi^2\sqrt{\pi+c^2\pi x^2}}-\frac{b^2\int\frac{1}{(1+c^2x^2)^{3/2}}\,dx}{3\pi^{5/2}}-\frac{(4bc)\int\frac{x(a+b\text{arcsinh}(cx))}{1+c^2x^2}\,dx}{3\pi^{5/2}}$$

$$=-\frac{b^2x}{3\pi^{5/2}\sqrt{1+c^2x^2}}+\frac{b(a+b\text{arcsinh}(cx))}{3c\pi^{5/2}(1+c^2x^2)}+\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi(\pi+c^2\pi x^2)^{3/2}}+\frac{2x(a+b\text{arcsinh}(cx))^2}{3\pi^2\sqrt{\pi+c^2\pi x^2}}-\frac{(4b)\text{Subst}\left(\int(a+bx)\tanh(x)\,dx,x,\text{arcsinh}(cx)\right)}{3c\pi^{5/2}}$$

$$=-\frac{b^2x}{3\pi^{5/2}\sqrt{1+c^2x^2}}+\frac{b(a+b\text{arcsinh}(cx))}{3c\pi^{5/2}(1+c^2x^2)}+\frac{2(a+b\text{arcsinh}(cx))^2}{3c\pi^{5/2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi(\pi+c^2\pi x^2)^{3/2}}+\frac{2x(a+b\text{arcsinh}(cx))^2}{3\pi^2\sqrt{\pi+c^2\pi x^2}}-\frac{(8b)\text{Subst}\left(\int\frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx,x,\text{arcsinh}(cx)\right)}{3c\pi^{5/2}}$$

$$=-\frac{b^2x}{3\pi^{5/2}\sqrt{1+c^2x^2}}+\frac{b(a+b\text{arcsinh}(cx))}{3c\pi^{5/2}(1+c^2x^2)}+\frac{2(a+b\text{arcsinh}(cx))^2}{3c\pi^{5/2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi(\pi+c^2\pi x^2)^{3/2}}+\frac{2x(a+b\text{arcsinh}(cx))^2}{3\pi^2\sqrt{\pi+c^2\pi x^2}}-\frac{4b(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3c\pi^{5/2}}+\frac{(4b^2)\,\text{Subst}\left(\int\log\left(1+e^{2x}\right)\,dx,x,\text{arcsinh}(cx)\right)}{3c\pi^{5/2}}$$

$$=-\frac{b^2x}{3\pi^{5/2}\sqrt{1+c^2x^2}}+\frac{b(a+b\text{arcsinh}(cx))}{3c\pi^{5/2}(1+c^2x^2)}+\frac{2(a+b\text{arcsinh}(cx))^2}{3c\pi^{5/2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi(\pi+c^2\pi x^2)^{3/2}}+\frac{2x(a+b\text{arcsinh}(cx))^2}{3\pi^2\sqrt{\pi+c^2\pi x^2}}-\frac{4b(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3c\pi^{5/2}}+\frac{(2b^2)\,\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\text{arcsinh}(cx)}\right)}{3c\pi^{5/2}}$$

$$=-\frac{b^2x}{3\pi^{5/2}\sqrt{1+c^2x^2}}+\frac{b(a+b\text{arcsinh}(cx))}{3c\pi^{5/2}(1+c^2x^2)}+\frac{2(a+b\text{arcsinh}(cx))^2}{3c\pi^{5/2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi(\pi+c^2\pi x^2)^{3/2}}+\frac{2x(a+b\text{arcsinh}(cx))^2}{3\pi^2\sqrt{\pi+c^2\pi x^2}}-\frac{4b(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3c\pi^{5/2}}-\frac{2b^2\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3c\pi^{5/2}}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.44

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{3a^2cx-b^2cx+2a^2c^3x^3-b^2c^3x^3+ab\sqrt{1+c^2x^2}-b^2\left(-3cx-2c^3x^3+2\sqrt{1+c}\right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (3*a^2*c*x - b^2*c*x + 2*a^2*c^3*x^3 - b^2*c^3*x^3 + a*b*Sqrt[1 + c^2*x^2]
- b^2*(-3*c*x - 2*c^3*x^3 + 2*Sqrt[1 + c^2*x^2] + 2*c^2*x^2*Sqrt[1 + c^2*x^
2])*ArcSinh[c*x]^2 - b*ArcSinh[c*x]*(-6*a*c*x - 4*a*c^3*x^3 - b*Sqrt[1 + c^
2*x^2] + 4*b*(1 + c^2*x^2)^(3/2)*Log[1 + E^(-2*ArcSinh[c*x])]) - 2*a*b*Sqrt
[1 + c^2*x^2]*Log[1 + c^2*x^2] - 2*a*b*c^2*x^2*Sqrt[1 + c^2*x^2]*Log[1 + c^
2*x^2] + 2*b^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(-2*ArcSinh[c*x])])/(3*c*P
i^(5/2)*(1 + c^2*x^2)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1728 vs. 2(192) = 384.

Time = 0.26 (sec) , antiderivative size = 1729, normalized size of antiderivative = 8.48

method	result	size
default	Expression too large to display	1729
parts	Expression too large to display	1729

```
[In] int((a+b*arcsinh(c*x))^2/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4*a*b/Pi^(5/2)*c^4/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)*x^5-4*a*b/P
i^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^6+4/3*a*b/Pi^(5/2)/c
/(3*c^2*x^2+4)/(c^2*x^2+1)^2-3*b^2/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)*arc
sinh(c*x)*x^2-4/3*b^2/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)*arcsinh(c*x)*x
^6+2*b^2/Pi^(5/2)*c^4/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)^2*x^5-22
/3*b^2/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)^2*x^2+16/3*b^2/P
i^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^2+4/3*b^2/Pi^(5/2)*c^7
/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^8-2*b^2/Pi^(5/2)*c^5/(3*c^2*x^2
+4)/(c^2*x^2+1)^2*arcsinh(c*x)^2*x^6+16/3*b^2/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c
^2*x^2+1)^2*arcsinh(c*x)*x^6-20/3*b^2/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1
)^2*arcsinh(c*x)^2*x^4+8*b^2/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsi
nh(c*x)*x^4-4*b^2/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)*arcsinh(c*x)*x^4+1
7/3*b^2/Pi^(5/2)*c^2/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)^2*x^3+6*b
^2/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^4+a^2*(1/3/Pi*x/(Pi*c^2*x^2+P
i)^(3/2)+2/3/Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2))+8/3*a*b/c/Pi^(5/2)*arcsinh(c*x)-
4/3*a*b/c/Pi^(5/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-2/3*b^2*polylog(2,-(c*x+
```

```
(c^2*x^2+1)^(1/2))^2)/c/Pi^(5/2)-4/3*b^2/Pi^(5/2)/(3*c^2*x^2+4)/(c^2*x^2+1)
^(3/2)*x-4/3*b^2/c/Pi^(5/2)*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+4/
3*b^2/Pi^(5/2)/c/(3*c^2*x^2+4)/(c^2*x^2+1)^2+4/3*b^2/c/Pi^(5/2)*arcsinh(c*x
)^2+34/3*a*b/Pi^(5/2)*c^2/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)*x^3-
40/3*a*b/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^4-44/3*a*b
/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^2+4/3*a*b/Pi^(5/2)*c
^7/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^8-4/3*a*b/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*
x^2+1)*x^6+16/3*a*b/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^6+14/3*b^2/P
i^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^2-8/3*b^2/Pi^(5/2)/c/(3*c^2*x^2+4)/
(c^2*x^2+1)^2*arcsinh(c*x)^2+4/3*b^2/Pi^(5/2)/c/(3*c^2*x^2+4)/(c^2*x^2+1)^2
*arcsinh(c*x)+2/3*b^2/Pi^(5/2)*c^7/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^8+10/3*b^2
/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^6+4*b^2/Pi^(5/2)/(3*c^2*x^2+4)/
(c^2*x^2+1)^(3/2)*arcsinh(c*x)^2*x-2/3*b^2/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*
x^2+1)*x^6-b^2/Pi^(5/2)*c^4/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*x^5-5/3*b^2/Pi^
(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)*x^4-7/3*b^2/Pi^(5/2)*c^2/(3*c^2*x^2+4)/
(c^2*x^2+1)^(3/2)*x^3-b^2/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)*x^2-4*a*b/Pi
^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)*x^4+8*a*b/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(
c^2*x^2+1)^2*x^4-3*a*b/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)*x^2+8*a*b/Pi^(5
/2)/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)*x+16/3*a*b/Pi^(5/2)*c/(3*c
^2*x^2+4)/(c^2*x^2+1)^2*x^2-16/3*a*b/Pi^(5/2)/c/(3*c^2*x^2+4)/(c^2*x^2+1)^2
*arcsinh(c*x)
```

Fricas [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(\pi + c^2\pi x^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(\pi + \pi c^2x^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a
^2)/(pi^3*c^6*x^6 + 3*pi^3*c^4*x^4 + 3*pi^3*c^2*x^2 + pi^3), x)
```

Sympy [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(\pi + c^2\pi x^2)^{5/2}}\,dx = \frac{\int \frac{a^2}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{b^2\,\text{asinh}^2\,(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int}{\pi^{\frac{5}{2}}}$$

```
[In] integrate((a+b*asinh(c*x))**2/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a**2/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2
+ 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b**2*asinh(c*x)**2/(c**4*x**4*sq
rt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)),
 x) + Integral(2*a*b*asinh(c*x)/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**
2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x))/pi**(5/2)
```

Maxima [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(\pi + c^2\pi x^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(\pi + \pi c^2x^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*b*c*(1/(pi^(5/2)*c^4*x^2 + pi^(5/2)*c^2) - 2*log(c^2*x^2 + 1)/(pi^(5/
2)*c^2)) + 2/3*a*b*(x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 2*x/(pi^2*sqrt(pi + pi
*c^2*x^2)))*arcsinh(c*x) + 1/3*a^2*(x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 2*x/(p
i^2*sqrt(pi + pi*c^2*x^2))) + b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/
(pi + pi*c^2*x^2)^(5/2), x)
```

Giac [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(\pi + c^2\pi x^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(\pi + \pi c^2x^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(pi + pi*c^2*x^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(\pi + c^2\pi x^2)^{5/2}}\,dx = \int \frac{(a + b\,\text{asinh}(c\,x))^2}{(\Pi\,c^2\,x^2 + \Pi)^{5/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(5/2), x)
```

3.258 $\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 358

$$\begin{aligned}\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx &= -\frac{52b^2\sqrt{d+c^2dx^2}}{225c^4}+\frac{4abx\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}} \\ &\quad-\frac{26b^2(1+c^2x^2)\sqrt{d+c^2dx^2}}{675c^4} \\ &\quad+\frac{2b^2(1+c^2x^2)^2\sqrt{d+c^2dx^2}}{125c^4} \\ &\quad+\frac{4b^2x\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{15c^3\sqrt{1+c^2x^2}} \\ &\quad-\frac{2bx^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{45c\sqrt{1+c^2x^2}} \\ &\quad-\frac{2bcx^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{25\sqrt{1+c^2x^2}} \\ &\quad-\frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{15c^4} \\ &\quad+\frac{x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{15c^2} \\ &\quad+\frac{1}{5}x^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\end{aligned}$$

```
[Out] -52/225*b^2*(c^2*d*x^2+d)^(1/2)/c^4-26/675*b^2*(c^2*x^2+1)*(c^2*d*x^2+d)^(1
/2)/c^4+2/125*b^2*(c^2*x^2+1)^2*(c^2*d*x^2+d)^(1/2)/c^4-2/15*(a+b*arcsinh(c
*x))^2*(c^2*d*x^2+d)^(1/2)/c^4+1/15*x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^
(1/2)/c^2+1/5*x^4*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)+4/15*a*b*x*(c^2*
d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)+4/15*b^2*x*arcsinh(c*x)*(c^2*d*x^2+d)^
(1/2)/c^3/(c^2*x^2+1)^(1/2)-2/45*b*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/
2)/c/(c^2*x^2+1)^(1/2)-2/25*b*c*x^5*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/
(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5806, 5812, 5798, 5772, 267, 5776, 272, 45}

$$\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{15c^2} - \frac{2bcx^5\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{25\sqrt{c^2x^2+1}} + \frac{1}{5}x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2 - \frac{2bx^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{45c\sqrt{c^2x^2+1}} - \frac{2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{15c^4} + \frac{4abx\sqrt{c^2dx^2+d}}{15c^3\sqrt{c^2x^2+1}} + \frac{4b^2x\text{arcsinh}(cx)\sqrt{c^2dx^2+d}}{15c^3\sqrt{c^2x^2+1}} + \frac{2b^2(c^2x^2+1)^2\sqrt{c^2dx^2+d}}{125c^4} - \frac{26b^2(c^2x^2+1)\sqrt{c^2dx^2+d}}{675c^4} - \frac{52b^2\sqrt{c^2dx^2+d}}{225c^4}$$

```
[In] Int[x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-52*b^2*Sqrt[d + c^2*d*x^2])/(225*c^4) + (4*a*b*x*Sqrt[d + c^2*d*x^2])/(15
*c^3*Sqrt[1 + c^2*x^2]) - (26*b^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/(675*c
^4) + (2*b^2*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/(125*c^4) + (4*b^2*x*Sqrt
[d + c^2*d*x^2]*ArcSinh[c*x])/(15*c^3*Sqrt[1 + c^2*x^2]) - (2*b*x^3*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(45*c*Sqrt[1 + c^2*x^2]) - (2*b*c*x^5*Sq
rt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(25*Sqrt[1 + c^2*x^2]) - (2*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(15*c^4) + (x^2*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])^2)/(15*c^2) + (x^4*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])^2)/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
```

```
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}x^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2+\frac{\sqrt{d+c^2dx^2}\int\frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{5\sqrt{1+c^2x^2}} \\
&\quad-\frac{\left(2bc\sqrt{d+c^2dx^2}\right)\int x^4(a+b\text{arcsinh}(cx))\,dx}{5\sqrt{1+c^2x^2}} \\
&= -\frac{2bcx^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{25\sqrt{1+c^2x^2}}+\frac{x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{15c^2} \\
&\quad+\frac{1}{5}x^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2-\frac{\left(2\sqrt{d+c^2dx^2}\right)\int\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{15c^2\sqrt{1+c^2x^2}} \\
&\quad-\frac{\left(2b\sqrt{d+c^2dx^2}\right)\int x^2(a+b\text{arcsinh}(cx))\,dx}{15c\sqrt{1+c^2x^2}}+\frac{\left(2b^2c^2\sqrt{d+c^2dx^2}\right)\int\frac{x^5}{\sqrt{1+c^2x^2}}\,dx}{25\sqrt{1+c^2x^2}} \\
&= -\frac{2bx^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{45c\sqrt{1+c^2x^2}}-\frac{2bcx^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{25\sqrt{1+c^2x^2}} \\
&\quad-\frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{15c^4}+\frac{x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{15c^2} \\
&\quad+\frac{1}{5}x^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2+\frac{\left(2b^2\sqrt{d+c^2dx^2}\right)\int\frac{x^3}{\sqrt{1+c^2x^2}}\,dx}{45\sqrt{1+c^2x^2}} \\
&\quad+\frac{\left(4b\sqrt{d+c^2dx^2}\right)\int(a+b\text{arcsinh}(cx))\,dx}{15c^3\sqrt{1+c^2x^2}} \\
&\quad+\frac{\left(b^2c^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{1+c^2x}}\,dx,x,x^2\right)}{25\sqrt{1+c^2x^2}} \\
&= \frac{4abx\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}}-\frac{2bx^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{45c\sqrt{1+c^2x^2}} \\
&\quad-\frac{2bcx^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{25\sqrt{1+c^2x^2}}-\frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{15c^4} \\
&\quad+\frac{x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{15c^2}+\frac{1}{5}x^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 \\
&\quad+\frac{\left(b^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{x}{\sqrt{1+c^2x}}\,dx,x,x^2\right)}{45\sqrt{1+c^2x^2}}+\frac{\left(4b^2\sqrt{d+c^2dx^2}\right)\int\text{arcsinh}(cx)\,dx}{15c^3\sqrt{1+c^2x^2}} \\
&\quad+\frac{\left(b^2c^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{1}{c^4\sqrt{1+c^2x}}-\frac{2\sqrt{1+c^2x}}{c^4}+\frac{(1+c^2x)^{3/2}}{c^4}\right)dx,x,x^2\right)}{25\sqrt{1+c^2x^2}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2b^2\sqrt{d+c^2dx^2}}{25c^4} + \frac{4abx\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}} - \frac{4b^2(1+c^2x^2)\sqrt{d+c^2dx^2}}{75c^4} \\
&\quad + \frac{2b^2(1+c^2x^2)^2\sqrt{d+c^2dx^2}}{125c^4} + \frac{4b^2x\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{15c^3\sqrt{1+c^2x^2}} \\
&\quad - \frac{2bx^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{45c\sqrt{1+c^2x^2}} \\
&\quad - \frac{2bcx^5\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{25\sqrt{1+c^2x^2}} - \frac{2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{15c^4} \\
&\quad + \frac{x^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{15c^2} + \frac{1}{5}x^4\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2 \\
&\quad + \frac{\left(b^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\left(-\frac{1}{c^2\sqrt{1+c^2x}}+\frac{\sqrt{1+c^2x}}{c^2}\right)dx, x, x^2\right)}{45\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(4b^2\sqrt{d+c^2dx^2}\right)\int\frac{x}{\sqrt{1+c^2x^2}}dx}{15c^2\sqrt{1+c^2x^2}} \\
&= -\frac{52b^2\sqrt{d+c^2dx^2}}{225c^4} + \frac{4abx\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}} - \frac{26b^2(1+c^2x^2)\sqrt{d+c^2dx^2}}{675c^4} \\
&\quad + \frac{2b^2(1+c^2x^2)^2\sqrt{d+c^2dx^2}}{125c^4} + \frac{4b^2x\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{15c^3\sqrt{1+c^2x^2}} \\
&\quad - \frac{2bx^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{45c\sqrt{1+c^2x^2}} - \frac{2bcx^5\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{25\sqrt{1+c^2x^2}} \\
&\quad - \frac{2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{15c^4} + \frac{x^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{15c^2} \\
&\quad + \frac{1}{5}x^4\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.62

$$
\int x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2\,dx
$$
$$
= \frac{\sqrt{d+c^2dx^2}\left(225(-2+3c^2x^2)\left(a+ac^2x^2\right)^2 - 30abcx\sqrt{1+c^2x^2}(-30+5c^2x^2+9c^4x^4) + 2b^2(-428-439c^2\right.}{}
$$

```
[In] Integrate[x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(225*(-2 + 3*c^2*x^2)*(a + a*c^2*x^2)^2 - 30*a*b*c*x*S
qrt[1 + c^2*x^2]*(-30 + 5*c^2*x^2 + 9*c^4*x^4) + 2*b^2*(-428 - 439*c^2*x^2
+ 16*c^4*x^4 + 27*c^6*x^6) - 30*b*(-15*a*(1 + c^2*x^2)^2*(-2 + 3*c^2*x^2) +
 b*c*x*Sqrt[1 + c^2*x^2]*(-30 + 5*c^2*x^2 + 9*c^4*x^4))*ArcSinh[c*x] + 225*
(-2 + 3*c^2*x^2)*(b + b*c^2*x^2)^2*ArcSinh[c*x]^2))/(3375*c^4*(1 + c^2*x^2)
)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1161 vs. 2(310) = 620.

Time = 0.38 (sec) , antiderivative size = 1162, normalized size of antiderivative = 3.25

method	result	size
default	Expression too large to display	1162
parts	Expression too large to display	1162

```
[In] int(x^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(1/5*x^2*(c^2*d*x^2+d)^(3/2)/c^2/d-2/15/d/c^4*(c^2*d*x^2+d)^(3/2))+b^2*
(1/4000*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c
^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(
25*arcsinh(c*x)^2-10*arcsinh(c*x)+2)/c^4/(c^2*x^2+1)-1/864*(d*(c^2*x^2+1))^
(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1
/2)+1)*(9*arcsinh(c*x)^2-6*arcsinh(c*x)+2)/c^4/(c^2*x^2+1)-1/16*(d*(c^2*x^2
+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)
+2)/c^4/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/
2)+1)*(arcsinh(c*x)^2+2*arcsinh(c*x)+2)/c^4/(c^2*x^2+1)-1/864*(d*(c^2*x^2+1
))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)
^(1/2)+1)*(9*arcsinh(c*x)^2+6*arcsinh(c*x)+2)/c^4/(c^2*x^2+1)+1/4000*(d*(c^
2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*
x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c*x
)^2+10*arcsinh(c*x)+2)/c^4/(c^2*x^2+1))+2*a*b*(1/800*(d*(c^2*x^2+1))^(1/2)*
(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^
(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))/c^4/(c^2*x^
2+1)-1/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c
^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))/c^4/(c^2*x^2+1)-1/16*
(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))/c
^4/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)
*(arcsinh(c*x)+1)/c^4/(c^2*x^2+1)-1/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*
c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(3*arcsinh(c
*x)+1)/c^4/(c^2*x^2+1)+1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(
c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(
c^2*x^2+1)^(1/2)+1)*(1+5*arcsinh(c*x))/c^4/(c^2*x^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.88

$$\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{225\left(3\,b^2c^6x^6+4\,b^2c^4x^4-b^2c^2x^2-2\,b^2\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)^2+30\left(45\,abc^6x^6+60\,abc^4x^4-1\right.}{}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/3375*(225*(3*b^2*c^6*x^6 + 4*b^2*c^4*x^4 - b^2*c^2*x^2 - 2*b^2)*sqrt(c^2*
d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 30*(45*a*b*c^6*x^6 + 60*a*b*c^4
*x^4 - 15*a*b*c^2*x^2 - 30*a*b - (9*b^2*c^5*x^5 + 5*b^2*c^3*x^3 - 30*b^2*c*
x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (2
7*(25*a^2 + 2*b^2)*c^6*x^6 + 4*(225*a^2 + 8*b^2)*c^4*x^4 - (225*a^2 + 878*b
^2)*c^2*x^2 - 450*a^2 - 856*b^2 - 30*(9*a*b*c^5*x^5 + 5*a*b*c^3*x^3 - 30*a*
b*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^6*x^2 + c^4)
```

Sympy [F]

$$\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\int x^3\sqrt{d\left(c^2x^2+1\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\,dx$$

```
[In] integrate(x**3*(a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.84

$$\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{1}{15}b^2\left(\frac{3\,(c^2dx^2+d)^{\frac{3}{2}}x^2}{c^2d}-\frac{2\,(c^2dx^2+d)^{\frac{3}{2}}}{c^4d}\right)\text{arsinh}\,(cx)^2$$
$$+\frac{2}{15}ab\left(\frac{3\,(c^2dx^2+d)^{\frac{3}{2}}x^2}{c^2d}-\frac{2\,(c^2dx^2+d)^{\frac{3}{2}}}{c^4d}\right)\text{arsinh}\,(cx)$$
$$+\frac{1}{15}a^2\left(\frac{3\,(c^2dx^2+d)^{\frac{3}{2}}x^2}{c^2d}-\frac{2\,(c^2dx^2+d)^{\frac{3}{2}}}{c^4d}\right)$$
$$+\frac{2}{3375}b^2\left(\frac{27\sqrt{c^2x^2+1}c^2\sqrt{d}x^4-11\sqrt{c^2x^2+1}\sqrt{d}x^2-\frac{428\sqrt{c^2x^2+1}\sqrt{d}}{c^2}}{c^2}-\frac{15\left(9\,c^4\sqrt{d}x^5+5\,c^2\sqrt{d}x^3-30\,\sqrt{}\right.}{c^3}\right.$$
$$-\frac{2\left(9\,c^4\sqrt{d}x^5+5\,c^2\sqrt{d}x^3-30\,\sqrt{d}x\right)ab}{225\,c^3}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/15*b^2*(3*(c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(3/2)/(c^
4*d))*arcsinh(c*x)^2 + 2/15*a*b*(3*(c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) - 2*(c
^2*d*x^2 + d)^(3/2)/(c^4*d))*arcsinh(c*x) + 1/15*a^2*(3*(c^2*d*x^2 + d)^(3/
2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(3/2)/(c^4*d)) + 2/3375*b^2*((27*sqrt(c^
2*x^2 + 1)*c^2*sqrt(d)*x^4 - 11*sqrt(c^2*x^2 + 1)*sqrt(d)*x^2 - 428*sqrt(c^
2*x^2 + 1)*sqrt(d)/c^2)/c^2 - 15*(9*c^4*sqrt(d)*x^5 + 5*c^2*sqrt(d)*x^3 - 3
0*sqrt(d)*x)*arcsinh(c*x)/c^3) - 2/225*(9*c^4*sqrt(d)*x^5 + 5*c^2*sqrt(d)*x
^3 - 30*sqrt(d)*x)*a*b/c^3
```

Giac [F(-2)]

Exception generated.

$$\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\int x^3\,\left(a+b\,\mathrm{asinh}(c\,x)\right)^2\,\sqrt{d\,c^2\,x^2+d}\,dx$$

```
[In] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2),x)
[Out] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2), x)
```

3.259 $\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 291

$$\begin{aligned}\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx &= \frac{b^2x\sqrt{d+c^2dx^2}}{64c^2}+\frac{1}{32}b^2x^3\sqrt{d+c^2dx^2}\\ &\quad-\frac{b^2\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{64c^3\sqrt{1+c^2x^2}}\\ &\quad-\frac{bx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8c\sqrt{1+c^2x^2}}\\ &\quad-\frac{bcx^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}}\\ &\quad+\frac{x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{8c^2}\\ &\quad+\frac{1}{4}x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\\ &\quad-\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{24bc^3\sqrt{1+c^2x^2}}\end{aligned}$$

```
[Out] 1/64*b^2*x*(c^2*d*x^2+d)^(1/2)/c^2+1/32*b^2*x^3*(c^2*d*x^2+d)^(1/2)+1/8*x*(
a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^2+1/4*x^3*(a+b*arcsinh(c*x))^2*(c
^2*d*x^2+d)^(1/2)-1/64*b^2*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)
^(1/2)-1/8*b*x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)
-1/8*b*c*x^4*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/24*
(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5806, 5812, 5783, 5776, 327, 221}

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx = -\frac{bx^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{8c\sqrt{c^2x^2+1}} + \frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{8c^2} - \frac{bcx^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{8\sqrt{c^2x^2+1}} + \frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2 - \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{24bc^3\sqrt{c^2x^2+1}} - \frac{b^2\text{arcsinh}(cx)\sqrt{c^2dx^2+d}}{64c^3\sqrt{c^2x^2+1}} + \frac{b^2x\sqrt{c^2dx^2+d}}{64c^2} + \frac{1}{32}b^2x^3\sqrt{c^2dx^2+d}$$

```
[In] Int[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (b^2*x*Sqrt[d + c^2*d*x^2])/(64*c^2) + (b^2*x^3*Sqrt[d + c^2*d*x^2])/32 - (
b^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(64*c^3*Sqrt[1 + c^2*x^2]) - (b*x^2*S
qrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8*c*Sqrt[1 + c^2*x^2]) - (b*c*x^4
*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c^2*x^2]) + (x*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(8*c^2) + (x^3*Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x])^2)/4 - (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(24
*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2+\frac{\sqrt{d+c^2dx^2}\int\frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{4\sqrt{1+c^2x^2}}\\&\quad-\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int x^3(a+b\text{arcsinh}(cx))\,dx}{2\sqrt{1+c^2x^2}}\end{aligned}$$

$$= -\frac{bcx^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} + \frac{x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{8c^2} + \frac{1}{4}x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 - \frac{\sqrt{d+c^2dx^2}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{8c^2\sqrt{1+c^2x^2}} - \frac{\left(b\sqrt{d+c^2dx^2}\right)\int x(a+b\text{arcsinh}(cx))\,dx}{4c\sqrt{1+c^2x^2}} + \frac{\left(b^2c^2\sqrt{d+c^2dx^2}\right)\int\frac{x^4}{\sqrt{1+c^2x^2}}\,dx}{8\sqrt{1+c^2x^2}}$$

$$= \frac{1}{32}b^2x^3\sqrt{d+c^2dx^2} - \frac{bx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8c\sqrt{1+c^2x^2}} - \frac{bcx^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} + \frac{x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{8c^2} + \frac{1}{4}x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 - \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{24bc^3\sqrt{1+c^2x^2}} - \frac{\left(3b^2\sqrt{d+c^2dx^2}\right)\int\frac{x^2}{\sqrt{1+c^2x^2}}\,dx}{32\sqrt{1+c^2x^2}} + \frac{\left(b^2\sqrt{d+c^2dx^2}\right)\int\frac{x^2}{\sqrt{1+c^2x^2}}\,dx}{8\sqrt{1+c^2x^2}}$$

$$= \frac{b^2x\sqrt{d+c^2dx^2}}{64c^2} + \frac{1}{32}b^2x^3\sqrt{d+c^2dx^2} - \frac{bx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8c\sqrt{1+c^2x^2}} - \frac{bcx^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} + \frac{x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{8c^2} + \frac{1}{4}x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 - \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{24bc^3\sqrt{1+c^2x^2}} + \frac{\left(3b^2\sqrt{d+c^2dx^2}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{64c^2\sqrt{1+c^2x^2}} - \frac{\left(b^2\sqrt{d+c^2dx^2}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{16c^2\sqrt{1+c^2x^2}}$$

$$= \frac{b^2x\sqrt{d+c^2dx^2}}{64c^2} + \frac{1}{32}b^2x^3\sqrt{d+c^2dx^2} - \frac{b^2\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{64c^3\sqrt{1+c^2x^2}} - \frac{bx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8c\sqrt{1+c^2x^2}} - \frac{bcx^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} + \frac{x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{8c^2} + \frac{1}{4}x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 - \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{24bc^3\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.71

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx =$$

$$-\frac{-96a^2cx(1+2c^2x^2)\sqrt{d+c^2dx^2}+96a^2\sqrt{d}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)+\frac{12ab\sqrt{d+c^2dx^2}\left(8\text{arcsinh}(cx)^2+\cosh\ldots\right.}{\ldots}}{\ldots}$$

[In] Integrate[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]

[Out] -1/768*(-96*a^2*c*x*(1 + 2*c^2*x^2)*Sqrt[d + c^2*d*x^2] + 96*a^2*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (12*a*b*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/Sqrt[1 + c^2*x^2] + (b^2*Sqrt[d + c^2*d*x^2]*(32*ArcSinh[c*x]^3 + 12*ArcSinh[c*x]*Cosh[4*ArcSinh[c*x]] - 3*(1 + 8*ArcSinh[c*x]^2)*Sinh[4*ArcSinh[c*x]]))/Sqrt[1 + c^2*x^2])/c^3

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. 2(251) = 502.

Time = 0.27 (sec) , antiderivative size = 618, normalized size of antiderivative = 2.12

method	result
default	$\frac{a^2x(c^2dx^2+d)^{\frac{3}{2}}}{4c^2d}-\frac{a^2x\sqrt{c^2dx^2+d}}{8c^2}-\frac{a^2d\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{8c^2\sqrt{c^2d}}+b^2\left(-\frac{\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)^3}{24\sqrt{c^2x^2+1}\,c^3}+\frac{\sqrt{d(c^2x^2+1)}\,(8c^5\ldots}{\ldots}\right.$
parts	$\frac{a^2x(c^2dx^2+d)^{\frac{3}{2}}}{4c^2d}-\frac{a^2x\sqrt{c^2dx^2+d}}{8c^2}-\frac{a^2d\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{8c^2\sqrt{c^2d}}+b^2\left(-\frac{\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)^3}{24\sqrt{c^2x^2+1}\,c^3}+\frac{\sqrt{d(c^2x^2+1)}\,(8c^5\ldots}{\ldots}\right.$

[In] int(x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/4*a^2*x*(c^2*d*x^2+d)^(3/2)/c^2/d-1/8*a^2/c^2*x*(c^2*d*x^2+d)^(1/2)-1/8*a^2/c^2*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b^2*(-1/24*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^3+1/512*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(8*arcsinh(c*x)^2-4*arcsinh(c*x)+1)/c^3/(c^2*x^2+1)+1/512*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(8*arcsinh(c*x)^2+4*arcsinh(c*x)+1)/c^3/(c^2*x^2+1))+2*a*b*(-1/16*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^2+1/256*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(c*x))/c^3/(c^2*x^2+1)+1/256*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3-8*

```
c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(1+4*arcsinh(c*x))/c^3/(
c^2*x^2+1))
```

Fricas [F]

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\int\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^2x^2\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)*sqrt(c
^2*d*x^2 + d), x)
```

Sympy [F]

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\int x^2\sqrt{d\,(c^2x^2+1)}(a+b\,\text{asinh}\,(cx))^2\;dx$$

```
[In] integrate(x**2*(a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\int \sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)^2x^2\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\int x^2\,(a+b\,\text{asinh}(c\,x))^2\,\sqrt{d\,c^2\,x^2+d}\,dx$$

```
[In] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2),x)
[Out] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2), x)
```

3.260 $\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 180

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\frac{4b^2\sqrt{d+c^2dx^2}}{9c^2}+\frac{2b^2(1+c^2x^2)\sqrt{d+c^2dx^2}}{27c^2}-\frac{2bx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3c\sqrt{1+c^2x^2}}-\frac{2bcx^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}}+\frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2d}$$

```
[Out] 1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/c^2/d+4/9*b^2*(c^2*d*x^2+d)^(1
/2)/c^2+2/27*b^2*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/c^2-2/3*b*x*(a+b*arcsinh(c
*x))*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-2/9*b*c*x^3*(a+b*arcsinh(c*x))
*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used

= {5798, 5784, 455, 45}

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx = -\frac{2bx\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{3c\sqrt{c^2x^2+1}} + \frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2d} - \frac{2bcx^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{9\sqrt{c^2x^2+1}} + \frac{2b^2(c^2x^2+1)\sqrt{c^2dx^2+d}}{27c^2} + \frac{4b^2\sqrt{c^2dx^2+d}}{9c^2}$$

```
[In] Int[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (4*b^2*Sqrt[d + c^2*d*x^2])/(9*c^2) + (2*b^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x
^2])/(27*c^2) - (2*b*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*c*Sqrt[
1 + c^2*x^2]) - (2*b*c*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(9*Sqr
t[1 + c^2*x^2]) + ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(3*c^2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2d} \\
&\quad - \frac{\left(2b\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))\,dx}{3c\sqrt{1+c^2x^2}} \\
&= -\frac{2bx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3c\sqrt{1+c^2x^2}} - \frac{2bcx^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2d} + \frac{\left(2b^2\sqrt{d+c^2dx^2}\right)\int\frac{x\left(1+\frac{c^2x^2}{3}\right)}{\sqrt{1+c^2x^2}}\,dx}{3\sqrt{1+c^2x^2}} \\
&= -\frac{2bx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3c\sqrt{1+c^2x^2}} - \frac{2bcx^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2d} + \frac{\left(b^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{1+\frac{c^2x}{3}}{\sqrt{1+c^2x}}\,dx,x,x^2\right)}{3\sqrt{1+c^2x^2}} \\
&= -\frac{2bx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3c\sqrt{1+c^2x^2}} - \frac{2bcx^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2d} \\
&\quad + \frac{\left(b^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{2}{3\sqrt{1+c^2x}}+\frac{1}{3}\sqrt{1+c^2x}\right)\,dx,x,x^2\right)}{3\sqrt{1+c^2x^2}} \\
&= \frac{4b^2\sqrt{d+c^2dx^2}}{9c^2} + \frac{2b^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{27c^2} - \frac{2bx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3c\sqrt{1+c^2x^2}} \\
&\quad - \frac{2bcx^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} + \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.92

$$
\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{\sqrt{d+c^2dx^2}\left(-6abcx\sqrt{1+c^2x^2}(3+c^2x^2)+9(a+ac^2x^2)^2+2b^2(7+8c^2x^2+c^4x^4)+6b\left(3a(1+c^2x^2)^2-b\right.\right.}{27c^2\left(1+c^2x^2\right)}
$$

```
[In] Integrate[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(-6*a*b*c*x*Sqrt[1 + c^2*x^2]*(3 + c^2*x^2) + 9*(a + a
*c^2*x^2)^2 + 2*b^2*(7 + 8*c^2*x^2 + c^4*x^4) + 6*b*(3*a*(1 + c^2*x^2)^2 -
b*c*x*Sqrt[1 + c^2*x^2]*(3 + c^2*x^2))*ArcSinh[c*x] + 9*(b + b*c^2*x^2)^2*A
rcSinh[c*x]^2))/(27*c^2*(1 + c^2*x^2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(156) = 312$.

Time = 0.26 (sec) , antiderivative size = 657, normalized size of antiderivative = 3.65

method	result
default	$\frac{a^2(c^2d\,x^2+d)^{\frac{3}{2}}}{3c^2d}+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\left(4c^4x^4+4c^3x^3\sqrt{c^2x^2+1}+5c^2x^2+3cx\sqrt{c^2x^2+1}+1\right)\left(9\operatorname{arcsinh}(cx)^2-6\operatorname{arcsinh}(cx)+2\right)}{216c^2(c^2x^2+1)}+\right.$
parts	$\frac{a^2(c^2d\,x^2+d)^{\frac{3}{2}}}{3c^2d}+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\left(4c^4x^4+4c^3x^3\sqrt{c^2x^2+1}+5c^2x^2+3cx\sqrt{c^2x^2+1}+1\right)\left(9\operatorname{arcsinh}(cx)^2-6\operatorname{arcsinh}(cx)+2\right)}{216c^2(c^2x^2+1)}+\right.$

```
[In] int(x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^2*(c^2*d*x^2+d)^(3/2)/c^2/d+b^2*(1/216*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x
^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcs
inh(c*x)^2-6*arcsinh(c*x)+2)/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2
*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)/c^2/(c^2*x^
2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c
*x)^2+2*arcsinh(c*x)+2)/c^2/(c^2*x^2+1)+1/216*(d*(c^2*x^2+1))^(1/2)*(4*c^4*
x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arc
sinh(c*x)^2+6*arcsinh(c*x)+2)/c^2/(c^2*x^2+1))+2*a*b*(1/72*(d*(c^2*x^2+1))^
(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1
/2)+1)*(-1+3*arcsinh(c*x))/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x
^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x
^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)+1)/c^2/(c^2*x^
2+1)+1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^
2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(3*arcsinh(c*x)+1)/c^2/(c^2*x^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.38

$$\int x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$=\frac{9\left(b^2c^4x^4+2\,b^2c^2x^2+b^2\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)^2+6\left(3\,abc^4x^4+6\,abc^2x^2+3\,ab-\left(b^2c^3x^3+3\right.\right.}{}$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/27*(9*(b^2*c^4*x^4 + 2*b^2*c^2*x^2 + b^2)*sqrt(c^2*d*x^2 + d)*log(c*x + s
qrt(c^2*x^2 + 1))^2 + 6*(3*a*b*c^4*x^4 + 6*a*b*c^2*x^2 + 3*a*b - (b^2*c^3*x
^3 + 3*b^2*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x
^2 + 1)) + ((9*a^2 + 2*b^2)*c^4*x^4 + 2*(9*a^2 + 8*b^2)*c^2*x^2 + 9*a^2 + 1
```

```
4*b^2 - 6*(a*b*c^3*x^3 + 3*a*b*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))
/(c^4*x^2 + c^2)
```

Sympy [F]

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\int x\sqrt{d\left(c^2x^2+1\right)}(a+b\,\text{asinh}\,(cx))^2\;dx$$

```
[In] integrate(x*(a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2),x)
[Out] Integral(x*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.02

$$\begin{aligned}\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx &= \frac{2}{27}\,b^2\left(\frac{\sqrt{c^2x^2+1}d^{\frac{3}{2}}x^2+\frac{7\sqrt{c^2x^2+1}d^{\frac{3}{2}}}{c^2}}{d}-\frac{3\left(c^2d^{\frac{3}{2}}x^3+3\,d^{\frac{3}{2}}x\right)\text{arsinh}\,(cx)}{cd}\right)\\&\quad+\frac{(c^2dx^2+d)^{\frac{3}{2}}b^2\,\text{arsinh}\,(cx)^2}{3\,c^2d}+\frac{2\,(c^2dx^2+d)^{\frac{3}{2}}ab\,\text{arsinh}\,(cx)}{3\,c^2d}\\&\quad-\frac{2\left(c^2d^{\frac{3}{2}}x^3+3\,d^{\frac{3}{2}}x\right)ab}{9\,cd}+\frac{(c^2dx^2+d)^{\frac{3}{2}}a^2}{3\,c^2d}\end{aligned}$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
[Out] 2/27*b^2*((sqrt(c^2*x^2 + 1)*d^(3/2)*x^2 + 7*sqrt(c^2*x^2 + 1)*d^(3/2)/c^2)
/d - 3*(c^2*d^(3/2)*x^3 + 3*d^(3/2)*x)*arcsinh(c*x)/(c*d)) + 1/3*(c^2*d*x^2
 + d)^(3/2)*b^2*arcsinh(c*x)^2/(c^2*d) + 2/3*(c^2*d*x^2 + d)^(3/2)*a*b*arcs
inh(c*x)/(c^2*d) - 2/9*(c^2*d^(3/2)*x^3 + 3*d^(3/2)*x)*a*b/(c*d) + 1/3*(c^2
*d*x^2 + d)^(3/2)*a^2/(c^2*d)
```

Giac [F(-2)]

Exception generated.

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

[In] integrate(x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx = \int x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2\,\sqrt{d\,c^2\,x^2+d}\,dx$$

[In] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2),x)

[Out] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2), x)

3.261 $\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 184

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{1}{4}b^2x\sqrt{d+c^2dx^2} - \frac{b^2\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{4c\sqrt{1+c^2x^2}} - \frac{bcx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}} + \frac{1}{2}x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{1+c^2x^2}}$$

```
[Out] 1/4*b^2*x*(c^2*d*x^2+d)^(1/2)+1/2*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2
)-1/4*b^2*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/2*b*c*x^2*
(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/6*(a+b*arcsinh(c
*x))^3*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{5785, 5783, 5776, 327, 221\}$

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{c^2x^2+1}} + \frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2 - \frac{bcx^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2\sqrt{c^2x^2+1}} - \frac{b^2\text{arcsinh}(cx)\sqrt{c^2dx^2+d}}{4c\sqrt{c^2x^2+1}} + \frac{1}{4}b^2x\sqrt{c^2dx^2+d}$$

```
[In] Int[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (b^2*x*Sqrt[d + c^2*d*x^2])/4 - (b^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(4*c
*Sqrt[1 + c^2*x^2]) - (b*c*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(2
*Sqrt[1 + c^2*x^2]) + (x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/2 + (S
qrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[1 + c^2*x^2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2+\frac{\sqrt{d+c^2dx^2}\int\frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}} \\
&\quad-\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int x(a+b\operatorname{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}} \\
&= -\frac{bcx^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2\sqrt{1+c^2x^2}}+\frac{1}{2}x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2 \\
&\quad+\frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^3}{6bc\sqrt{1+c^2x^2}}+\frac{\left(b^2c^2\sqrt{d+c^2dx^2}\right)\int\frac{x^2}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}} \\
&= \frac{1}{4}b^2x\sqrt{d+c^2dx^2}-\frac{bcx^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2\sqrt{1+c^2x^2}} \\
&\quad+\frac{1}{2}x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2 \\
&\quad+\frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^3}{6bc\sqrt{1+c^2x^2}}-\frac{\left(b^2\sqrt{d+c^2dx^2}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{4\sqrt{1+c^2x^2}} \\
&= \frac{1}{4}b^2x\sqrt{d+c^2dx^2}-\frac{b^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{4c\sqrt{1+c^2x^2}}-\frac{bcx^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2\sqrt{1+c^2x^2}} \\
&\quad+\frac{1}{2}x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2+\frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^3}{6bc\sqrt{1+c^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.09

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{1}{24}\left(12a^2x\sqrt{d+c^2dx^2}+\frac{12a^2\sqrt{d}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)}{c}\right.$$
$$+\frac{b^2\sqrt{d+c^2dx^2}(4\text{arcsinh}(cx)^3-6\text{arcsinh}(cx)\cosh(2\text{arcsinh}(cx))+(3+6\text{arcsinh}(cx)^2)\sinh(2\text{arcsinh}(cx$$
$$\left.+\frac{6ab\sqrt{d+c^2dx^2}(-\cosh(2\text{arcsinh}(cx))+2\text{arcsinh}(cx)(\text{arcsinh}(cx)+\sinh(2\text{arcsinh}(cx))))}{c\sqrt{1+c^2x^2}}\right)$$

```
[In] Integrate[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (12*a^2*x*Sqrt[d + c^2*d*x^2] + (12*a^2*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d
+ c^2*d*x^2]])/c + (b^2*Sqrt[d + c^2*d*x^2]*(4*ArcSinh[c*x]^3 - 6*ArcSinh[c
*x]*Cosh[2*ArcSinh[c*x]] + (3 + 6*ArcSinh[c*x]^2)*Sinh[2*ArcSinh[c*x]]))/(c
*Sqrt[1 + c^2*x^2]) + (6*a*b*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSinh[c*x]] + 2
*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/(c*Sqrt[1 + c^2*x^2])
)/24
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(158) = 316$.

Time = 0.22 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.61

method	result
default	$\frac{a^2x\sqrt{c^2d\,x^2+d}}{2}+\frac{a^2d\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2\sqrt{c^2d}}+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)^3}{6\sqrt{c^2x^2+1}\,c}+\frac{\sqrt{d(c^2x^2+1)}\left(2c^3x^3+2c^2x^2\sqrt{c^2x^2+1}+2cx\right.}{16c}\right.$
parts	$\frac{a^2x\sqrt{c^2d\,x^2+d}}{2}+\frac{a^2d\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2\sqrt{c^2d}}+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)^3}{6\sqrt{c^2x^2+1}\,c}+\frac{\sqrt{d(c^2x^2+1)}\left(2c^3x^3+2c^2x^2\sqrt{c^2x^2+1}+2cx\right.}{16c}\right.$

```
[In] int((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2*x*(c^2*d*x^2+d)^(1/2)+1/2*a^2*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2
+d)^(1/2))/(c^2*d)^(1/2)+b^2*(1/6*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c
*arcsinh(c*x)^3+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)
^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2-2*arcsinh(c*x)+1)/c/(c^2*
x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*
c*x-(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2+2*arcsinh(c*x)+1)/c/(c^2*x^2+1))+2
```

```
*a*b*(1/4*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(c*x)^2+1/16*(d*
(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)
^(1/2))*(-1+2*arcsinh(c*x))/c/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3
*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(c*x)
)/c/(c^2*x^2+1))
```

Fricas [F]

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\int \sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)^2\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
), x)
```

Sympy [F]

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\int \sqrt{d(c^2x^2+1)}(a+b\operatorname{asinh}(cx))^2\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2),x)
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,\sqrt{d\,c^2\,x^2+d}\,dx$$

```
[In] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2), x)
```

3.262 $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 338

$$\begin{aligned}\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x}\,dx &= 2b^2\sqrt{d+c^2dx^2}-\frac{2abcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}-\frac{2b^2cx\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\\ &\quad+\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2-\frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}\\ &\quad-\frac{2b\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\operatorname{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}\\ &\quad+\frac{2b\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\operatorname{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}\\ &\quad+\frac{2b^2\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}-\frac{2b^2\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}\end{aligned}$$

```
[Out] 2*b^2*(c^2*d*x^2+d)^(1/2)+(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)-2*a*b*c*
x*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*b^2*c*x*arcsinh(c*x)*(c^2*d*x^2+d
)^(1/2)/(c^2*x^2+1)^(1/2)-2*(a+b*arcsinh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1
/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*b*(a+b*arcsinh(c*x))*polylog(2
,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+2*b*(a+b*arc
sinh(c*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)
^(1/2)+2*b^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2
+1)^(1/2)-2*b^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x
^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5806, 5816, 4267, 2611, 2320, 6724, 5772, 267}

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{x}\,dx$$
$$= -\frac{2\sqrt{c^2dx^2+d}\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}} - \frac{2b\sqrt{c^2dx^2+d}\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2x^2+1}} + \frac{2b\sqrt{c^2dx^2+d}\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2x^2+1}} + \sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^2 - \frac{2abcx\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}} + \frac{2b^2\sqrt{c^2dx^2+d}\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{c^2x^2+1}} - \frac{2b^2\sqrt{c^2dx^2+d}\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{c^2x^2+1}} - \frac{2b^2cx\operatorname{arcsinh}(cx)\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}} + 2b^2\sqrt{c^2dx^2+d}$$

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] 2*b^2*Sqrt[d + c^2*d*x^2] - (2*a*b*c*x*Sqrt[d + c^2*d*x^2])/Sqrt[1 + c^2*x^
2] - (2*b^2*c*x*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2 - (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSi
nh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (2*b*Sqrt[d + c^2*d
*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] +
 (2*b*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]])/
Sqrt[1 + c^2*x^2] + (2*b^2*Sqrt[d + c^2*d*x^2]*PolyLog[3, -E^ArcSinh[c*x]])
/Sqrt[1 + c^2*x^2] - (2*b^2*Sqrt[d + c^2*d*x^2]*PolyLog[3, E^ArcSinh[c*x]])
/Sqrt[1 + c^2*x^2]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{\sqrt{d+c^2dx^2}\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{1+c^2x^2}}\,dx}{\sqrt{1+c^2x^2}} - \frac{\left(2bc\sqrt{d+c^2dx^2}\right)\int (a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}}$$

$$= -\frac{2abcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{\sqrt{d+c^2dx^2}\text{Subst}\left(\int (a+bx)^2\text{csch}(x)\,dx, x, \text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}} - \frac{\left(2b^2c\sqrt{d+c^2dx^2}\right)\int \text{arcsinh}(cx)\,dx}{\sqrt{1+c^2x^2}}$$

$$= -\frac{2abcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{2b^2cx\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}} + \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 - \frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{\left(2b\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int (a+bx)\log(1-e^x)\,dx, x, \text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}} + \frac{\left(2b\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int (a+bx)\log(1+e^x)\,dx, x, \text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}} + \frac{\left(2b^2c^2\sqrt{d+c^2dx^2}\right)\int \frac{x}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{1+c^2x^2}}$$

$$= 2b^2\sqrt{d+c^2dx^2} - \frac{2abcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{2b^2cx\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}} + \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 - \frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{2b\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{2b\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{\left(2b^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int \text{PolyLog}(2,-e^x)\,dx, x, \text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}} - \frac{\left(2b^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int \text{PolyLog}(2,e^x)\,dx, x, \text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}}$$

$$= 2b^2\sqrt{d+c^2dx^2} - \frac{2abcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{2b^2cx\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}} + \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 - \frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{2b\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{2b\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{\left(2b^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{\left(2b^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}$$

$$= 2b^2\sqrt{d+c^2dx^2} - \frac{2abcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{2b^2cx\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}} + \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 - \frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{2b\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{2b\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{2b^2\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{2b^2\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x}\,dx$$
$$= a^2\sqrt{d+c^2dx^2}+a^2\sqrt{d}\log(cx)-a^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right)$$
$$+\frac{2ab\sqrt{d+c^2dx^2}\left(-cx+\sqrt{1+c^2x^2}\text{arcsinh}(cx)+\text{arcsinh}(cx)\log\left(1-e^{-\text{arcsinh}(cx)}\right)-\text{arcsinh}(cx)\log\left(1-\right.\right.}{\sqrt{1+c^2x^2}}$$
$$+\frac{b^2\sqrt{d+c^2dx^2}\left(2\sqrt{1+c^2x^2}-2cx\text{arcsinh}(cx)+\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2+\text{arcsinh}(cx)^2\log\left(1-e^{-\text{arcsinh}(\right.\right.}}{}$$

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] a^2*Sqrt[d + c^2*d*x^2] + a^2*Sqrt[d]*Log[c*x] - a^2*Sqrt[d]*Log[d + Sqrt[d
]*Sqrt[d + c^2*d*x^2]] + (2*a*b*Sqrt[d + c^2*d*x^2]*(-(c*x) + Sqrt[1 + c^2*
x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*
Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^
(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2] + (b^2*Sqrt[d + c^2*d*x^2]*(2*Sqrt[1 +
 c^2*x^2] - 2*c*x*ArcSinh[c*x] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 + ArcSinh
[c*x]^2*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x
])] + 2*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - 2*ArcSinh[c*x]*PolyLo
g[2, E^(-ArcSinh[c*x])] + 2*PolyLog[3, -E^(-ArcSinh[c*x])] - 2*PolyLog[3, E
^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 763 vs. 2(353) = 706.

Time = 0.32 (sec) , antiderivative size = 764, normalized size of antiderivative = 2.26

method	result
default	$-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)a^2+a^2\sqrt{c^2d\,x^2+d}+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\,\left(c^2x^2+cx\sqrt{c^2x^2+1}+1\right)\left(\text{arcsinh}(cx)^2-2\,\text{arcs}\right.}{2c^2x^2+2}\right.$
parts	$-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)a^2+a^2\sqrt{c^2d\,x^2+d}+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\,\left(c^2x^2+cx\sqrt{c^2x^2+1}+1\right)\left(\text{arcsinh}(cx)^2-2\,\text{arcs}\right.}{2c^2x^2+2}\right.$

```
[In] int((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)*a^2+a^2*(c^2*d*x^2+d)^(1
/2)+b^2*(1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(arcsi
nh(c*x)^2-2*arcsinh(c*x)+2)/(c^2*x^2+1)+1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-
c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2+2*arcsinh(c*x)+2)/(c^2*x^2+1)-(d*(
c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/
```

```
2))-2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,-c*x-(
c^2*x^2+1)^(1/2))+2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(3,-c*x-
(c^2*x^2+1)^(1/2))+(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*l
n(1-c*x-(c^2*x^2+1)^(1/2))+2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsin
h(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)
^(1/2)*polylog(3,c*x+(c^2*x^2+1)^(1/2)))+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x
^2+1)*arcsinh(c*x)*x^2*c^2-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*c*
x+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(c*x)+2*a*b*(d*(c^2*x^2+1)
)^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*a*b*(d
*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*a*
b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)
^(1/2))-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x
^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x}\,dx=\int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^2}{x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="fricas")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/x, x)
```

Sympy [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x}\,dx=\int \frac{\sqrt{d\,(c^2x^2+1)}(a+b\,\text{asinh}\,(cx))^2}{x}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2)/x,x)
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x}\,dx=\int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^2}{x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="maxima")
[Out] -(sqrt(d)*arcsinh(1/(c*abs(x))) - sqrt(c^2*d*x^2 + d))*a^2 + integrate(sqrt
(c^2*d*x^2 + d)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/x + 2*sqrt(c^2*d*x^2 + d
)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x}\,dx = \int \frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)^2\,\sqrt{d\,c^2\,x^2+d}}{x}\,dx$$

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x, x)
```

3.263 $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^2} dx$

Optimal result

Integrand size = 28, antiderivative size = 209

$$\begin{aligned}
&\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^2} dx \\
&= -\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x}+\frac{c\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}} \\
&\quad+\frac{c\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{3b\sqrt{1+c^2x^2}} \\
&\quad+\frac{2bc\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad-\frac{b^2c\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}
\end{aligned}$$

```
[Out] -(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x+c*(a+b*arcsinh(c*x))^2*(c^2*d*x
^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/3*c*(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^(1/2
)/b/(c^2*x^2+1)^(1/2)+2*b*c*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2
))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b^2*c*polylog(2,1/(c*x+(c^2*x^2
+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5805, 5775, 3797, 2221, 2317, 2438, 5783}

$$\begin{aligned}
&\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx \\
&= \frac{c\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{3b\sqrt{c^2x^2+1}}+\frac{c\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}} \\
&\quad-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x} \\
&\quad+\frac{2bc\sqrt{c^2dx^2+d}\log\left(1-e^{-2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}} \\
&\quad-\frac{b^2c\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)}{\sqrt{c^2x^2+1}}
\end{aligned}$$

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] -((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x) + (c*Sqrt[d + c^2*d*x^2]*
(a + b*ArcSinh[c*x])^2)/Sqrt[1 + c^2*x^2] + (c*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x])^3)/(3*b*Sqrt[1 + c^2*x^2]) + (2*b*c*Sqrt[d + c^2*d*x^2]*(a + b
*ArcSinh[c*x])*Log[1 - E^(-2*ArcSinh[c*x])])/Sqrt[1 + c^2*x^2] - (b^2*c*Sqr
t[d + c^2*d*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/Sqrt[1 + c^2*x^2]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]], Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x]
, x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], In
t[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Free
Q[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{x}+\frac{\left(2bc\sqrt{d+c^2dx^2}\right)\int\frac{a+b\operatorname{arcsinh}(cx)}{x}\,dx}{\sqrt{1+c^2x^2}}\\
&\quad+\frac{\left(c^2\sqrt{d+c^2dx^2}\right)\int\frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{1+c^2x^2}}\\
&= -\frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{x}+\frac{c\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^3}{3b\sqrt{1+c^2x^2}}\\
&\quad-\frac{\left(2c\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int x\coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$= -\frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{x} + \frac{c\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}} + \frac{c\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^3}{3b\sqrt{1+c^2x^2}} + \frac{\left(4c\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}}$$

$$= -\frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{x} + \frac{c\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}} + \frac{c\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^3}{3b\sqrt{1+c^2x^2}} + \frac{2bc\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\log\left(1-e^{-2\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{\left(2bc\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}}$$

$$= -\frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{x} + \frac{c\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}} + \frac{c\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^3}{3b\sqrt{1+c^2x^2}} + \frac{2bc\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\log\left(1-e^{-2\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{\left(b^2c\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}\right)}{\sqrt{1+c^2x^2}}$$

$$= -\frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{x} + \frac{c\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}} + \frac{c\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^3}{3b\sqrt{1+c^2x^2}} + \frac{2bc\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\log\left(1-e^{-2\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{b^2c\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}\right)}{\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = -\frac{a^2\sqrt{d+c^2dx^2}}{x} + \frac{ab\sqrt{d+c^2dx^2}\left(-2\sqrt{1+c^2x^2}\text{arcsinh}(cx)+cx\text{arcsinh}(cx)^2+2cx\log(cx)\right)}{x\sqrt{1+c^2x^2}} + a^2c\sqrt{d}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right) + \frac{b^2c\sqrt{d+c^2dx^2}\left(\text{arcsinh}(cx)\left(\left(3-\frac{3\sqrt{1+c^2x^2}}{cx}\right)\text{arcsinh}(cx)+\text{arcsinh}(cx)^2+6\log\left(1-e^{-2\text{arcsinh}(cx)}\right)\right)-3\right.}{3\sqrt{1+c^2x^2}}$$

[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^2,x]

[Out] -((a^2*Sqrt[d + c^2*d*x^2])/x) + (a*b*Sqrt[d + c^2*d*x^2]*(-2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + c*x*ArcSinh[c*x]^2 + 2*c*x*Log[c*x]))/(x*Sqrt[1 + c^2*x^2]) + a^2*c*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b^2*c*Sqrt[d + c^2*d*x^2]*(ArcSinh[c*x]*((3 - (3*Sqrt[1 + c^2*x^2])/(c*x))*ArcSinh[c*x] + ArcSinh[c*x]^2 + 6*Log[1 - E^(-2*ArcSinh[c*x])]) - 3*PolyLog[2, E^(-2*ArcSinh[c*x])]))/(3*Sqrt[1 + c^2*x^2])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. 2(207) = 414.

Time = 0.29 (sec) , antiderivative size = 579, normalized size of antiderivative = 2.77

method	result
default	$-\frac{a^2(c^2dx^2+d)^{\frac{3}{2}}}{dx}+a^2c^2x\sqrt{c^2dx^2+d}+\frac{a^2c^2d\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{\sqrt{c^2d}}+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)^3c}{3\sqrt{c^2x^2+1}}-\frac{\sqrt{d(c^2x^2+}}{}\right.$
parts	$-\frac{a^2(c^2dx^2+d)^{\frac{3}{2}}}{dx}+a^2c^2x\sqrt{c^2dx^2+d}+\frac{a^2c^2d\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{\sqrt{c^2d}}+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)^3c}{3\sqrt{c^2x^2+1}}-\frac{\sqrt{d(c^2x^2+}}{}\right.$

[In] int((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^2,x,method=_RETURNVERBOSE)

[Out] -a^2/d/x*(c^2*d*x^2+d)^(3/2)+a^2*c^2*x*(c^2*d*x^2+d)^(1/2)+a^2*c^2*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b^2*(1/3*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^3*c-(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*arcsinh(c*x)^2/x/(c^2*x^2+1)-2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c+2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c+2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c+2*(d*(c^2*x^2+1))^(1/2)

```
/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c+2*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c)+2*a*b*(1/2
*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c-2*(d*(c^2*x^2+1))
^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c-(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*
(c^2*x^2+1)^(1/2)+1)*arcsinh(c*x)/x/(c^2*x^2+1)+(d*(c^2*x^2+1))^(1/2)/(c^2*
x^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c)
```

Fricas [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^2}{x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="fricas
")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \int \frac{\sqrt{d\,(c^2x^2+1)}(a+b\,\text{asinh}\,(cx))^2}{x^2}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2)/x**2,x)
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^2}{x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="maxima
")
[Out] (c*sqrt(d)*arcsinh(c*x) - sqrt(c^2*d*x^2 + d)/x)*a^2 + integrate(sqrt(c^2*d
*x^2 + d)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/x^2 + 2*sqrt(c^2*d*x^2 + d)*a*
b*log(c*x + sqrt(c^2*x^2 + 1))/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \text{Exception raised: TypeError}$$

[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,\sqrt{d\,c^2\,x^2+d}}{x^2}\,dx$$

[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x^2,x)

[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x^2, x)

3.264 $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^3} dx$

Optimal result

Integrand size = 28, antiderivative size = 358

$$\begin{aligned}
&\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^3} dx \\
&= -\frac{bc\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x\sqrt{1+c^2x^2}} - \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2x^2} \\
&\quad - \frac{c^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad - \frac{b^2c^2\sqrt{d+c^2dx^2}\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{bc^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{b^2c^2\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{b^2c^2\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}
\end{aligned}$$

```
[Out] -1/2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^2-b*c*(a+b*arcsinh(c*x))*(c
^2*d*x^2+d)^(1/2)/x/(c^2*x^2+1)^(1/2)-c^2*(a+b*arcsinh(c*x))^2*arctanh(c*x+
(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b^2*c^2*arctanh((c
^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b*c^2*(a+b*arcsinh(c
*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2
)+b*c^2*(a+b*arcsinh(c*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(
1/2)/(c^2*x^2+1)^(1/2)+b^2*c^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2
+d)^(1/2)/(c^2*x^2+1)^(1/2)-b^2*c^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*d
*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5805, 5776, 272, 65, 214, 5816, 4267, 2611, 2320, 6724}

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$$
$$= -\frac{c^2\sqrt{c^2dx^2+d}\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}} - \frac{bc^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}} + \frac{bc^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}} - \frac{bc\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{x\sqrt{c^2x^2+1}} - \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2x^2} + \frac{b^2c^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{c^2x^2+1}} - \frac{b^2c^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{\sqrt{c^2x^2+1}} - \frac{b^2c^2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}$$

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] -((b*c*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(x*Sqrt[1 + c^2*x^2])) - (
Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*x^2) - (c^2*Sqrt[d + c^2*d*x
^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b^
2*c^2*Sqrt[d + c^2*d*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/Sqrt[1 + c^2*x^2] - (
b*c^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])
/Sqrt[1 + c^2*x^2] + (b*c^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyLo
g[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (b^2*c^2*Sqrt[d + c^2*d*x^2]*Poly
Log[3, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b^2*c^2*Sqrt[d + c^2*d*x^2]*P
olyLog[3, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]], Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x]
, x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], In
t[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Free
Q[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2x^2}+\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{x^2}\,dx}{\sqrt{1+c^2x^2}}\\
&\quad+\frac{\left(c^2\sqrt{d+c^2dx^2}\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}}\\
&= -\frac{bc\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x\sqrt{1+c^2x^2}}-\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2x^2}\\
&\quad+\frac{\left(c^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^2\text{csch}(x)\,dx,x,\text{arcsinh}(cx)\right)}{2\sqrt{1+c^2x^2}}\\
&\quad+\frac{\left(b^2c^2\sqrt{d+c^2dx^2}\right)\int\frac{1}{x\sqrt{1+c^2x^2}}\,dx}{\sqrt{1+c^2x^2}}\\
&= -\frac{bc\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x\sqrt{1+c^2x^2}}-\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2x^2}\\
&\quad-\frac{c^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}\\
&\quad-\frac{\left(bc^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)\log\left(1-e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}}\\
&\quad+\frac{\left(bc^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)\log\left(1+e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}}\\
&\quad+\frac{\left(b^2c^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+c^2x}}\,dx,x,x^2\right)}{2\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bc\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x\sqrt{1+c^2x^2}} - \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2x^2} \\
&\quad - \frac{c^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{bc^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(b^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx, x, \sqrt{1+c^2x^2}\right)}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(b^2c^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int \text{PolyLog}\left(2,-e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(b^2c^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int \text{PolyLog}\left(2,e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bc\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x\sqrt{1+c^2x^2}} - \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2x^2} \\
&\quad - \frac{c^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad - \frac{b^2c^2\sqrt{d+c^2dx^2}\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{bc^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(b^2c^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(b^2c^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$= -\frac{bc\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x\sqrt{1+c^2x^2}} - \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2x^2} - \frac{c^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{b^2c^2\sqrt{d+c^2dx^2}\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{\sqrt{1+c^2x^2}} - \frac{bc^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{bc^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{b^2c^2\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{b^2c^2\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 3.79 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$$
$$= \frac{1}{8}\Bigg(-\frac{4a^2\sqrt{d+c^2dx^2}}{x^2} + 4a^2c^2\sqrt{d}\log(x) - 4a^2c^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right) + \frac{2abc^2\sqrt{d+c^2dx^2}\left(-2\coth\left(\frac{1}{2}\text{arcsinh}(cx)\right) - \text{arcsinh}(cx)\text{csch}^2\left(\frac{1}{2}\text{arcsinh}(cx)\right) + 4\text{arcsinh}(cx)\log\left(1-e^{-\cdots}\right.\right.}{\cdots} + \frac{b^2c^2\sqrt{d+c^2dx^2}\left(-4\text{arcsinh}(cx)\coth\left(\frac{1}{2}\text{arcsinh}(cx)\right) - \text{arcsinh}(cx)^2\text{csch}^2\left(\frac{1}{2}\text{arcsinh}(cx)\right) + 4\text{arcsinh}(cx)^2\cdots\right.}{\cdots}$$

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] ((-4*a^2*Sqrt[d + c^2*d*x^2])/x^2 + 4*a^2*c^2*Sqrt[d]*Log[x] - 4*a^2*c^2*Sq
rt[d]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (2*a*b*c^2*Sqrt[d + c^2*d*x^2]
*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh
[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])
] + 4*PolyLog[2, -E^(-ArcSinh[c*x])] - 4*PolyLog[2, E^(-ArcSinh[c*x])] - Ar
cSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/Sqrt[1 + c^2*x
^2] + (b^2*c^2*Sqrt[d + c^2*d*x^2]*(-4*ArcSinh[c*x]*Coth[ArcSinh[c*x]/2] -
ArcSinh[c*x]^2*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]^2*Log[1 - E^(-ArcSin
h[c*x])] - 4*ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] + 8*Log[Tanh[ArcSinh
[c*x]/2]] + 8*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - 8*ArcSinh[c*x]*
PolyLog[2, E^(-ArcSinh[c*x])] + 8*PolyLog[3, -E^(-ArcSinh[c*x])] - 8*PolyLo
```

```
g[3, E^(-ArcSinh[c*x])] - ArcSinh[c*x]^2*Sech[ArcSinh[c*x]/2]^2 + 4*ArcSinh
[c*x]*Tanh[ArcSinh[c*x]/2]))/Sqrt[1 + c^2*x^2])/8
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 754 vs. 2(371) = 742.

Time = 0.35 (sec) , antiderivative size = 755, normalized size of antiderivative = 2.11

method	result
default	$a^2\left(-\frac{(c^2dx^2+d)^{\frac{3}{2}}}{2dx^2}+\frac{c^2\left(\sqrt{c^2dx^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2dx^2+d}}{x}\right)\right)}{2}\right)+b^2\left(-\frac{\left(\operatorname{arcsinh}(cx)c^2x^2+2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(c\right.}{2x^2(c^2x^2+1)}\right.$
parts	$a^2\left(-\frac{(c^2dx^2+d)^{\frac{3}{2}}}{2dx^2}+\frac{c^2\left(\sqrt{c^2dx^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2dx^2+d}}{x}\right)\right)}{2}\right)+b^2\left(-\frac{\left(\operatorname{arcsinh}(cx)c^2x^2+2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(c\right.}{2x^2(c^2x^2+1)}\right.$

```
[In] int((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/2/d/x^2*(c^2*d*x^2+d)^(3/2)+1/2*c^2*((c^2*d*x^2+d)^(1/2)-d^(1/2)*ln
((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)))+b^2*(-1/2*(arcsinh(c*x)*c^2*x^2+2
*c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x))*arcsinh(c*x)*(d*(c^2*x^2+1))^(1/2)/x^2
/(c^2*x^2+1)-1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(
1+c*x+(c^2*x^2+1)^(1/2))*c^2-(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsin
h(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^2+(d*(c^2*x^2+1))^(1/2)/(c^2*x^2
+1)^(1/2)*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*c^2+1/2*(d*(c^2*x^2+1))^(1/2)/(
c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^2+(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*
c^2-(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(3,c*x+(c^2*x^2+1)^(1/2)
)*c^2-2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arctanh(c*x+(c^2*x^2+1)^(1/
2))*c^2)+2*a*b*(-1/2*(arcsinh(c*x)*c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*
x))*(d*(c^2*x^2+1))^(1/2)/x^2/(c^2*x^2+1)-1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^
2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2-1/2*(d*(c^2*x^2+1))
^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^2+1/2*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^
2+1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/
2))*c^2)
```

Fricas [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx=\int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^2}{x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/x^3, x)
```

Sympy [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx=\int \frac{\sqrt{d\,(c^2x^2+1)}(a+b\,\text{asinh}\,(cx))^2}{x^3}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx=\int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^2}{x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(c^2*sqrt(d)*arcsinh(1/(c*abs(x))) - sqrt(c^2*d*x^2 + d)*c^2 + (c^2*d*
x^2 + d)^(3/2)/(d*x^2))*a^2 + integrate(sqrt(c^2*d*x^2 + d)*b^2*log(c*x + s
qrt(c^2*x^2 + 1))^2/x^3 + 2*sqrt(c^2*d*x^2 + d)*a*b*log(c*x + sqrt(c^2*x^2
+ 1))/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx=\int \frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)^2\,\sqrt{d\,c^2\,x^2+d}}{x^3}\,dx$$

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x^3,x)
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x^3, x)
```

3.265 $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 294

$$\begin{aligned}
\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx
&= -\frac{b^2c^2\sqrt{d+c^2dx^2}}{3x}+\frac{b^2c^3\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{3\sqrt{1+c^2x^2}} \\
&\quad -\frac{bc\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3x^2} \\
&\quad +\frac{c^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3\sqrt{1+c^2x^2}}-\frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{3dx^3} \\
&\quad +\frac{2bc^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)}{3\sqrt{1+c^2x^2}} \\
&\quad -\frac{b^2c^3\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)}{3\sqrt{1+c^2x^2}}
\end{aligned}$$

```
[Out] -1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/d/x^3-1/3*b^2*c^2*(c^2*d*x^2+
d)^(1/2)/x+1/3*b^2*c^3*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1
/3*c^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+2/3*b*c^3
*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(
c^2*x^2+1)^(1/2)-1/3*b^2*c^3*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*d*
x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/3*b*c*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)
*(c^2*d*x^2+d)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {5800, 5802, 283, 221, 5775, 3797, 2221, 2317, 2438}

$$\begin{aligned}
&\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx \\
&= -\frac{bc\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{3x^2} \\
&\quad -\frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))^2}{3dx^3}+\frac{c^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3\sqrt{c^2x^2+1}} \\
&\quad +\frac{2bc^3\sqrt{c^2dx^2+d}\log\left(1-e^{-2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}} \\
&\quad -\frac{b^2c^3\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)}{3\sqrt{c^2x^2+1}} \\
&\quad +\frac{b^2c^3\text{arcsinh}(cx)\sqrt{c^2dx^2+d}}{3\sqrt{c^2x^2+1}}-\frac{b^2c^2\sqrt{c^2dx^2+d}}{3x}
\end{aligned}$$

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] -1/3*(b^2*c^2*Sqrt[d + c^2*d*x^2])/x + (b^2*c^3*Sqrt[d + c^2*d*x^2]*ArcSinh
[c*x])/(3*Sqrt[1 + c^2*x^2]) - (b*c*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x]))/(3*x^2) + (c^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
)^2)/(3*Sqrt[1 + c^2*x^2]) - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)
/(3*d*x^3) + (2*b*c^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Log[1 - E^(-
2*ArcSinh[c*x])])/(3*Sqrt[1 + c^2*x^2]) - (b^2*c^3*Sqrt[d + c^2*d*x^2]*Poly
Log[2, E^(-2*ArcSinh[c*x])])/(3*Sqrt[1 + c^2*x^2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 5802

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSinh[c
*x])/(f*(m + 1))), x] + (-Dist[b*c*(d^p/(f*(m + 1))), Int[(f*x)^(m + 1)*(1
+ c^2*x^2)^(p - 1/2), x], x] - Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m + 2
)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
&\text{integral}\\
&=-\frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3dx^3}+\frac{\left(2bc\sqrt{d+c^2dx^2}\right)\int\frac{(1+c^2x^2)(a+b\text{arcsinh}(cx))}{x^3}\,dx}{3\sqrt{1+c^2x^2}}\\
&=-\frac{bc\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3x^2}-\frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3dx^3}\\
&\quad+\frac{\left(b^2c^2\sqrt{d+c^2dx^2}\right)\int\frac{\sqrt{1+c^2x^2}}{x^2}\,dx}{3\sqrt{1+c^2x^2}}+\frac{\left(2bc^3\sqrt{d+c^2dx^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{x}\,dx}{3\sqrt{1+c^2x^2}}\\
&=-\frac{b^2c^2\sqrt{d+c^2dx^2}}{3x}-\frac{bc\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3x^2}\\
&\quad-\frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3dx^3}\\
&\quad-\frac{\left(2c^3\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int x\coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{3\sqrt{1+c^2x^2}}\\
&\quad+\frac{\left(b^2c^4\sqrt{d+c^2dx^2}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{3\sqrt{1+c^2x^2}}\\
&=-\frac{b^2c^2\sqrt{d+c^2dx^2}}{3x}+\frac{b^2c^3\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{3\sqrt{1+c^2x^2}}\\
&\quad-\frac{bc\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3x^2}\\
&\quad+\frac{c^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3\sqrt{1+c^2x^2}}-\frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3dx^3}\\
&\quad+\frac{\left(4c^3\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{3\sqrt{1+c^2x^2}}\\
&=-\frac{b^2c^2\sqrt{d+c^2dx^2}}{3x}+\frac{b^2c^3\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{3\sqrt{1+c^2x^2}}\\
&\quad-\frac{bc\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3x^2}\\
&\quad+\frac{c^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3\sqrt{1+c^2x^2}}-\frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3dx^3}\\
&\quad+\frac{2bc^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)}{3\sqrt{1+c^2x^2}}\\
&\quad-\frac{\left(2bc^3\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{3\sqrt{1+c^2x^2}}
\end{aligned}$$

$$= -\frac{b^2c^2\sqrt{d+c^2dx^2}}{3x} + \frac{b^2c^3\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{3\sqrt{1+c^2x^2}} - \frac{bc\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3x^2} + \frac{c^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3\sqrt{1+c^2x^2}} - \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{3dx^3} + \frac{2bc^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)}{3\sqrt{1+c^2x^2}} + \frac{\left(b^2c^3\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right)}{3\sqrt{1+c^2x^2}}$$

$$= -\frac{b^2c^2\sqrt{d+c^2dx^2}}{3x} + \frac{b^2c^3\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{3\sqrt{1+c^2x^2}} - \frac{bc\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3x^2} + \frac{c^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3\sqrt{1+c^2x^2}} - \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{3dx^3} + \frac{2bc^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)}{3\sqrt{1+c^2x^2}} - \frac{b^2c^3\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right)}{3\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = -\frac{\sqrt{d+c^2dx^2}\left(abcx+a^2\sqrt{1+c^2x^2}+a^2c^2x^2\sqrt{1+c^2x^2}+b^2c^2x^2\sqrt{1+c^2x^2}+b^2\left(-c^3x^3+\sqrt{1+c^2x^2}+c^2x\right.\right.}{}$$

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] -1/3*(Sqrt[d + c^2*d*x^2]*(a*b*c*x + a^2*Sqrt[1 + c^2*x^2] + a^2*c^2*x^2*Sq
rt[1 + c^2*x^2] + b^2*c^2*x^2*Sqrt[1 + c^2*x^2] + b^2*(-(c^3*x^3) + Sqrt[1
+ c^2*x^2] + c^2*x^2*Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2 - b*ArcSinh[c*x]*(-(
b*c*x) - 2*a*(1 + c^2*x^2)^(3/2) + 2*b*c^3*x^3*Log[1 - E^(-2*ArcSinh[c*x])]
) - 2*a*b*c^3*x^3*Log[c*x] + b^2*c^3*x^3*PolyLog[2, E^(-2*ArcSinh[c*x])]))/
(x^3*Sqrt[1 + c^2*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1728 vs. 2(274) = 548.

Time = 0.34 (sec) , antiderivative size = 1729, normalized size of antiderivative = 5.88

method	result	size
default	Expression too large to display	1729
parts	Expression too large to display	1729

```
[In] int((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a*b*(d*(c^2*x^2+1))^(1/2)*(2*arcsinh(c*x)*c^3*x^3-2*ln((c*x+(c^2*x^2+1
)^(1/2))^2-1)*x^3*c^3+2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2+2*arcsinh(c*
x)*(c^2*x^2+1)^(1/2)+c*x)/(c^2*x^2+1)^(1/2)/x^3-1/3*a^2/d/x^3*(c^2*d*x^2+d)
^(3/2)+b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1
/2)*arcsinh(c*x)^2*c^5-b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^
2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^5-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x
^4+3*c^2*x^2+1)/x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c+b^2*(d*(c^2*x^2+1))^(1
/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^7-b^2*(d
*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^5/(c^2*x^2+1)*arcsinh(c*x)^2*
c^8-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^5/(c^2*x^2+1)*a
rcsinh(c*x)*c^8-3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^3/(c^
2*x^2+1)*arcsinh(c*x)^2*c^6-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*
x^2+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)*c^6-10/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c
^4*x^4+3*c^2*x^2+1)*x/(c^2*x^2+1)*arcsinh(c*x)^2*c^4-1/3*b^2*(d*(c^2*x^2+1)
)^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x/(c^2*x^2+1)*arcsinh(c*x)*c^4-5/3*b^2*(d*(
c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x/(c^2*x^2+1)*arcsinh(c*x)^2*c^2+
1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*c^3
-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^3*c^6+2/3*b^2*(d*(
c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^3+2
/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1
/2))*c^3-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^3
+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^3*arcsinh(c*x)*c^6
+b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*c^
7+b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*c
^5+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*
arcsinh(c*x)^2*c^3-b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x
^2+1)^(1/2)*arcsinh(c*x)*c^3-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2
*x^2+1)*x^5/(c^2*x^2+1)*c^8-5/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*
x^2+1)*x^3/(c^2*x^2+1)*c^6-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x
^2+1)*x/(c^2*x^2+1)*c^4-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+
1)/x/(c^2*x^2+1)*c^2-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/
x^3/(c^2*x^2+1)*arcsinh(c*x)^2+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1
/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^3+2/3*b^2*(d*(c^2*x^2+1))^(1
/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^3+1/3*b^2*
```

```
(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x*arcsinh(c*x)*c^4
```

Fricas [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx=\int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^2}{x^4}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="fricas
")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/x^4, x)
```

Sympy [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx=\int \frac{\sqrt{d\,(c^2x^2+1)}(a+b\,\text{asinh}\,(cx))^2}{x^4}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2)/x**4,x)
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx=\int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^2}{x^4}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="maxima
")
[Out] -1/3*((-1)^(2*c^2*d*x^2 + 2*d)*c^2*d^(3/2)*log(2*c^2*d + 2*d/x^2) - c^2*d^(
3/2)*log(x^2 + 1/c^2) + sqrt(c^4*d*x^4 + 2*c^2*d*x^2 + d)*d/x^2)*a*b*c/d -
1/3*b^2*((c^2*sqrt(d)*x^2 + sqrt(d))*sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x
^2 + 1))^2/x^3 - 3*integrate(2/3*((c^2*x^2 + 1)*c^2*sqrt(d)*x + (c^3*sqrt(d
)*x^2 + c*sqrt(d))*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/(c*x^4 +
 sqrt(c^2*x^2 + 1)*x^3), x)) - 2/3*(c^2*d*x^2 + d)^(3/2)*a*b*arcsinh(c*x)/(
d*x^3) - 1/3*(c^2*d*x^2 + d)^(3/2)*a^2/(d*x^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = \text{Exception raised: TypeError}$$

[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,\sqrt{d\,c^2\,x^2+d}}{x^4}\,dx$$

[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x^4,x)

[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x^4, x)

3.266 $\int x^3(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 482

$$\begin{aligned}\int x^3(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = &-\frac{304b^2d\sqrt{d+c^2dx^2}}{3675c^4}\\ &+\frac{4abdx\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}}-\frac{152b^2d(1+c^2x^2)\sqrt{d+c^2dx^2}}{11025c^4}\\ &-\frac{38b^2d(1+c^2x^2)^2\sqrt{d+c^2dx^2}}{6125c^4}+\frac{2b^2d(1+c^2x^2)^3\sqrt{d+c^2dx^2}}{343c^4}\\ &+\frac{4b^2dx\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{35c^3\sqrt{1+c^2x^2}}-\frac{2bdx^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{105c\sqrt{1+c^2x^2}}\\ &-\frac{16bcdx^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{175\sqrt{1+c^2x^2}}-\frac{2bc^3dx^7\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{49\sqrt{1+c^2x^2}}\\ &-\frac{2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{35c^4}+\frac{dx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{35c^2}\\ &+\frac{3}{35}dx^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2+\frac{1}{7}x^4(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2\end{aligned}$$

```
[Out] 1/7*x^4*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2-304/3675*b^2*d*(c^2*d*x^2+
d)^(1/2)/c^4-152/11025*b^2*d*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/c^4-38/6125*b^
2*d*(c^2*x^2+1)^2*(c^2*d*x^2+d)^(1/2)/c^4+2/343*b^2*d*(c^2*x^2+1)^3*(c^2*d*
x^2+d)^(1/2)/c^4-2/35*d*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^4+1/35*d
*x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^2+3/35*d*x^4*(a+b*arcsinh(c
*x))^2*(c^2*d*x^2+d)^(1/2)+4/35*a*b*d*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)
^(1/2)+4/35*b^2*d*x*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)-
2/105*b*d*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-16
/175*b*c*d*x^5*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/4
9*b*c^3*d*x^7*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5808, 5806, 5812, 5798, 5772, 267, 5776, 272, 45, 14, 5803, 12, 457, 78}

$$\int x^3(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\frac{dx^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{35c^2}-\frac{16bcdx^5\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{175\sqrt{c^2x^2+1}}+\frac{1}{7}x^4\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2+\frac{3}{35}dx^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bdx^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{105c\sqrt{c^2x^2+1}}-\frac{2d\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{35c^4}-\frac{2bc^3dx^7\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{49\sqrt{c^2x^2+1}}+\frac{4abdx\sqrt{c^2dx^2+d}}{35c^3\sqrt{c^2x^2+1}}+\frac{4b^2dx\text{arcsinh}(cx)\sqrt{c^2dx^2+d}}{35c^3\sqrt{c^2x^2+1}}+\frac{2b^2d(c^2x^2+1)^3\sqrt{c^2dx^2+d}}{343c^4}-\frac{38b^2d(c^2x^2+1)^2\sqrt{c^2dx^2+d}}{6125c^4}-\frac{304b^2d\sqrt{c^2dx^2+d}}{3675c^4}-\frac{152b^2d(c^2x^2+1)\sqrt{c^2dx^2+d}}{11025c^4}$$

```
[In] Int[x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-304*b^2*d*Sqrt[d + c^2*d*x^2])/(3675*c^4) + (4*a*b*d*x*Sqrt[d + c^2*d*x^2
])/(35*c^3*Sqrt[1 + c^2*x^2]) - (152*b^2*d*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2
])/(11025*c^4) - (38*b^2*d*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/(6125*c^4)
+ (2*b^2*d*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2])/(343*c^4) + (4*b^2*d*x*Sqrt
[d + c^2*d*x^2]*ArcSinh[c*x])/(35*c^3*Sqrt[1 + c^2*x^2]) - (2*b*d*x^3*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(105*c*Sqrt[1 + c^2*x^2]) - (16*b*c*d*
x^5*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(175*Sqrt[1 + c^2*x^2]) - (2*
b*c^3*d*x^7*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(49*Sqrt[1 + c^2*x^2]
) - (2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(35*c^4) + (d*x^2*Sqrt
[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(35*c^2) + (3*d*x^4*Sqrt[d + c^2*d*
x^2]*(a + b*ArcSinh[c*x])^2)/35 + (x^4*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh
[c*x])^2)/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
```

```
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
```

```
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{7}x^4\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{7}(3d)\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx - \frac{\left(2bcd\sqrt{d+c^2dx^2}\right)\int x^4(1+c^2x^2)\,(a+b\text{arcsinh}(cx))\,dx}{7\sqrt{1+c^2x^2}}$$

$$= -\frac{2bcdx^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{35\sqrt{1+c^2x^2}} - \frac{2bc^3dx^7\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{49\sqrt{1+c^2x^2}} + \frac{3}{35}dx^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{7}x^4\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{\left(3d\sqrt{d+c^2dx^2}\right)\int\frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{35\sqrt{1+c^2x^2}} - \frac{\left(6bcd\sqrt{d+c^2dx^2}\right)\int x^4(a+b\text{arcsinh}(cx))\,dx}{35\sqrt{1+c^2x^2}} + \frac{\left(2b^2c^2d\sqrt{d+c^2dx^2}\right)\int\frac{x^5(7+5c^2x^2)}{35\sqrt{1+c^2x^2}}\,dx}{7\sqrt{1+c^2x^2}}$$

$$= -\frac{16bcdx^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{175\sqrt{1+c^2x^2}} - \frac{2bc^3dx^7\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{49\sqrt{1+c^2x^2}} + \frac{dx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{35c^2} + \frac{3}{35}dx^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{7}x^4\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2 - \frac{\left(2d\sqrt{d+c^2dx^2}\right)\int\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{35c^2\sqrt{1+c^2x^2}} - \frac{\left(2bd\sqrt{d+c^2dx^2}\right)\int x^2(a+b\text{arcsinh}(cx))\,dx}{35c\sqrt{1+c^2x^2}} + \frac{\left(2b^2c^2d\sqrt{d+c^2dx^2}\right)\int\frac{x^5(7+5c^2x^2)}{\sqrt{1+c^2x^2}}\,dx}{245\sqrt{1+c^2x^2}} + \frac{\left(6b^2c^2d\sqrt{d+c^2dx^2}\right)\int\frac{x^5}{\sqrt{1+c^2x^2}}\,dx}{175\sqrt{1+c^2x^2}}$$

$$= -\frac{2bdx^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{105c\sqrt{1+c^2x^2}} - \frac{16bcdx^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{175\sqrt{1+c^2x^2}} - \frac{2bc^3dx^7\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{49\sqrt{1+c^2x^2}} - \frac{2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{35c^4} + \frac{dx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{35c^2} + \frac{3}{35}dx^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{7}x^4\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{\left(2b^2d\sqrt{d+c^2dx^2}\right)\int\frac{x^3}{\sqrt{1+c^2x^2}}\,dx}{105\sqrt{1+c^2x^2}} + \frac{\left(4bd\sqrt{d+c^2dx^2}\right)\int(a+b\text{arcsinh}(cx))\,dx}{35c^3\sqrt{1+c^2x^2}} + \frac{\left(b^2c^2d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{x^2(7+5c^2x)}{\sqrt{1+c^2x}}\,dx,x,x^2\right)}{245\sqrt{1+c^2x^2}} + \frac{\left(3b^2c^2d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{1+c^2x}}\,dx,x,x^2\right)}{175\sqrt{1+c^2x^2}}$$

$$= \frac{4abdx\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}} - \frac{2bdx^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{105c\sqrt{1+c^2x^2}} - \frac{16bcdx^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{175\sqrt{1+c^2x^2}} - \frac{2bc^3dx^7\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{49\sqrt{1+c^2x^2}} - \frac{2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{35c^4} + \frac{dx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{35c^2} + \frac{3}{35}dx^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{7}x^4\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{\left(b^2d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{x}{\sqrt{1+c^2x}}\,dx,x,x^2\right)}{105\sqrt{1+c^2x^2}} + \frac{\left(4b^2d\sqrt{d+c^2dx^2}\right)\int\text{arcsinh}(cx)\,dx}{35c^3\sqrt{1+c^2x^2}} + \frac{\left(b^2c^2d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{2}{c^4\sqrt{1+c^2x}}+\frac{\sqrt{1+c^2x}}{c^4}-\frac{8(1+c^2x)^{3/2}}{c^4}+\frac{5(1+c^2x)^{5/2}}{c^4}\right)dx,x,x^2\right)}{245\sqrt{1+c^2x^2}} + \frac{\left(3b^2c^2d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{1}{c^4\sqrt{1+c^2x}}-\frac{2\sqrt{1+c^2x}}{c^4}+\frac{(1+c^2x)^{3/2}}{c^4}\right)dx,x,x^2\right)}{175\sqrt{1+c^2x^2}}$$

$$
\begin{aligned}
&= \frac{62b^2d\sqrt{d+c^2dx^2}}{1225c^4} + \frac{4abdx\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}} - \frac{74b^2d(1+c^2x^2)\sqrt{d+c^2dx^2}}{3675c^4} \\
&\quad - \frac{38b^2d(1+c^2x^2)^2\sqrt{d+c^2dx^2}}{6125c^4} + \frac{2b^2d(1+c^2x^2)^3\sqrt{d+c^2dx^2}}{343c^4} \\
&\quad + \frac{4b^2dx\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{35c^3\sqrt{1+c^2x^2}} - \frac{2bdx^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{105c\sqrt{1+c^2x^2}} \\
&\quad - \frac{16bcdx^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{175\sqrt{1+c^2x^2}} - \frac{2bc^3dx^7\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{49\sqrt{1+c^2x^2}} \\
&\quad - \frac{2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{35c^4} + \frac{dx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{35c^2} \\
&\quad + \frac{3}{35}dx^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{7}x^4\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2 \\
&\quad + \frac{\left(b^2d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(-\frac{1}{c^2\sqrt{1+c^2x}}+\frac{\sqrt{1+c^2x}}{c^2}\right)dx, x, x^2\right)}{105\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(4b^2d\sqrt{d+c^2dx^2}\right)\int\frac{x}{\sqrt{1+c^2x^2}}dx}{35c^2\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{304b^2d\sqrt{d+c^2dx^2}}{3675c^4} + \frac{4abdx\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}} - \frac{152b^2d(1+c^2x^2)\sqrt{d+c^2dx^2}}{11025c^4} \\
&\quad - \frac{38b^2d(1+c^2x^2)^2\sqrt{d+c^2dx^2}}{6125c^4} + \frac{2b^2d(1+c^2x^2)^3\sqrt{d+c^2dx^2}}{343c^4} \\
&\quad + \frac{4b^2dx\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{35c^3\sqrt{1+c^2x^2}} - \frac{2bdx^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{105c\sqrt{1+c^2x^2}} \\
&\quad - \frac{16bcdx^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{175\sqrt{1+c^2x^2}} - \frac{2bc^3dx^7\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{49\sqrt{1+c^2x^2}} \\
&\quad - \frac{2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{35c^4} + \frac{dx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{35c^2} \\
&\quad + \frac{3}{35}dx^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{7}x^4\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.52

$$
\int x^3\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{d\sqrt{d+c^2dx^2}\Big(11025a^2(1+c^2x^2)^3(-2+5c^2x^2) - 210abcx\sqrt{1+c^2x^2}(-210+35c^2x^2+}{}
$$

```
[In] Integrate[x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d*Sqrt[d + c^2*d*x^2]*(11025*a^2*(1 + c^2*x^2)^3*(-2 + 5*c^2*x^2) - 210*a*
b*c*x*Sqrt[1 + c^2*x^2]*(-210 + 35*c^2*x^2 + 168*c^4*x^4 + 75*c^6*x^6) + 2*
b^2*(-18692 - 20371*c^2*x^2 + 499*c^4*x^4 + 3303*c^6*x^6 + 1125*c^8*x^8) -
210*b*(-105*a*(1 + c^2*x^2)^3*(-2 + 5*c^2*x^2) + b*c*x*Sqrt[1 + c^2*x^2]*(-
210 + 35*c^2*x^2 + 168*c^4*x^4 + 75*c^6*x^6))*ArcSinh[c*x] + 11025*b^2*(1 +
 c^2*x^2)^3*(-2 + 5*c^2*x^2)*ArcSinh[c*x]^2))/(385875*c^4*(1 + c^2*x^2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1765 vs. $2(420) = 840$.

Time = 0.39 (sec) , antiderivative size = 1766, normalized size of antiderivative = 3.66

method	result	size
default	Expression too large to display	1766
parts	Expression too large to display	1766

```
[In] int(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(1/7*x^2*(c^2*d*x^2+d)^(5/2)/c^2/d-2/35/d/c^4*(c^2*d*x^2+d)^(5/2))+b^2*
(1/43904*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144
*c^6*x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(
1/2)+25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1)*(49*arcsinh(c*x)^2-14*arcsinh(c*
x)+2)*d/c^4/(c^2*x^2+1)+1/16000*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^
5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*
x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c*x)^2-10*arcsinh(c*x)+2)*d/c^4/(c^2*x^2
+1)-1/1152*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c
^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2-6*arcsinh(c*x)+2)*d/c^4
/(c^2*x^2+1)-3/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*
(arcsinh(c*x)^2-2*arcsinh(c*x)+2)*d/c^4/(c^2*x^2+1)-3/128*(d*(c^2*x^2+1))^(
1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2+2*arcsinh(c*x)+2)*d/
c^4/(c^2*x^2+1)-1/1152*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+
1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2+6*arcsinh(c
*x)+2)*d/c^4/(c^2*x^2+1)+1/16000*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x
^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c
*x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c*x)^2+10*arcsinh(c*x)+2)*d/c^4/(c^2*x^
2+1)+1/43904*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8-64*c^7*x^7*(c^2*x^2+1)^(1/2)
+144*c^6*x^6-112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4-56*c^3*x^3*(c^2*x^2+
1)^(1/2)+25*c^2*x^2-7*c*x*(c^2*x^2+1)^(1/2)+1)*(49*arcsinh(c*x)^2+14*arcsin
h(c*x)+2)*d/c^4/(c^2*x^2+1))+2*a*b*(1/6272*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^
8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2)+10
4*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1
)*(-1+7*arcsinh(c*x))*d/c^4/(c^2*x^2+1)+1/3200*(d*(c^2*x^2+1))^(1/2)*(16*c^
6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+
13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))*d/c^4/(c^2*x^2+1)
```

```
-1/384*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x
^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))*d/c^4/(c^2*x^2+1)-3/128*(
d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))*d/
c^4/(c^2*x^2+1)-3/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+
1)*(arcsinh(c*x)+1)*d/c^4/(c^2*x^2+1)-1/384*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^
4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(3*arcsi
nh(c*x)+1)*d/c^4/(c^2*x^2+1)+1/3200*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^
5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-
5*c*x*(c^2*x^2+1)^(1/2)+1)*(1+5*arcsinh(c*x))*d/c^4/(c^2*x^2+1)+1/6272*(d*(
c^2*x^2+1))^(1/2)*(64*c^8*x^8-64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6-112*
c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4-56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x
^2-7*c*x*(c^2*x^2+1)^(1/2)+1)*(1+7*arcsinh(c*x))*d/c^4/(c^2*x^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.83

$$\int x^3\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\frac{11025\,(5\,b^2c^8dx^8+13\,b^2c^6dx^6+9\,b^2c^4dx^4-b^2c^2dx^2-2\,b^2d)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x\right.}$$

```
[In] integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] 1/385875*(11025*(5*b^2*c^8*d*x^8 + 13*b^2*c^6*d*x^6 + 9*b^2*c^4*d*x^4 - b^2
*c^2*d*x^2 - 2*b^2*d)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))^2 +
210*(525*a*b*c^8*d*x^8 + 1365*a*b*c^6*d*x^6 + 945*a*b*c^4*d*x^4 - 105*a*b*c
^2*d*x^2 - 210*a*b*d - (75*b^2*c^7*d*x^7 + 168*b^2*c^5*d*x^5 + 35*b^2*c^3*d
*x^3 - 210*b^2*c*d*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt
(c^2*x^2 + 1)) + (1125*(49*a^2 + 2*b^2)*c^8*d*x^8 + 9*(15925*a^2 + 734*b^2)
*c^6*d*x^6 + (99225*a^2 + 998*b^2)*c^4*d*x^4 - (11025*a^2 + 40742*b^2)*c^2*
d*x^2 - 2*(11025*a^2 + 18692*b^2)*d - 210*(75*a*b*c^7*d*x^7 + 168*a*b*c^5*d
*x^5 + 35*a*b*c^3*d*x^3 - 210*a*b*c*d*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2
+ d))/(c^6*x^2 + c^4)
```

Sympy [F]

$$\int x^3(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \int x^3\left(d(c^2x^2+1)\right)^{\frac{3}{2}}(a+b\,\text{asinh}\,(cx))^2\,dx$$

```
[In] integrate(x**3*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2,x)
[Out] Integral(x**3*(d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.72

$$\int x^3(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{1}{35}\left(\frac{5\,(c^2dx^2+d)^{\frac{5}{2}}x^2}{c^2d}-\frac{2\,(c^2dx^2+d)^{\frac{5}{2}}}{c^4d}\right)b^2\,\text{arsinh}\,(cx)^2 + \frac{2}{35}\left(\frac{5\,(c^2dx^2+d)^{\frac{5}{2}}x^2}{c^2d}-\frac{2\,(c^2dx^2+d)^{\frac{5}{2}}}{c^4d}\right)ab\,\text{arsinh}\,(cx) + \frac{1}{35}\left(\frac{5\,(c^2dx^2+d)^{\frac{5}{2}}x^2}{c^2d}-\frac{2\,(c^2dx^2+d)^{\frac{5}{2}}}{c^4d}\right)a^2 + \frac{2}{385875}b^2\left(\frac{1125\,\sqrt{c^2x^2+1}c^4d^{\frac{3}{2}}x^6+2178\,\sqrt{c^2x^2+1}c^2d^{\frac{3}{2}}x^4-1679\,\sqrt{c^2x^2+1}d^{\frac{3}{2}}x^2-\frac{18692\,\sqrt{c^2x^2+1}d^{\frac{3}{2}}}{c^2}}{c^2}-10\right. - \frac{2\left(75\,c^6d^{\frac{3}{2}}x^7+168\,c^4d^{\frac{3}{2}}x^5+35\,c^2d^{\frac{3}{2}}x^3-210\,d^{\frac{3}{2}}x\right)ab}{3675\,c^3}$$

```
[In] integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
[Out] 1/35*(5*(c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(5/2)/(c^4*d)
)*b^2*arcsinh(c*x)^2 + 2/35*(5*(c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) - 2*(c^2*d
*x^2 + d)^(5/2)/(c^4*d))*a*b*arcsinh(c*x) + 1/35*(5*(c^2*d*x^2 + d)^(5/2)*x
^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(5/2)/(c^4*d))*a^2 + 2/385875*b^2*((1125*sqr
t(c^2*x^2 + 1)*c^4*d^(3/2)*x^6 + 2178*sqrt(c^2*x^2 + 1)*c^2*d^(3/2)*x^4 - 1
679*sqrt(c^2*x^2 + 1)*d^(3/2)*x^2 - 18692*sqrt(c^2*x^2 + 1)*d^(3/2)/c^2)/c^
2 - 105*(75*c^6*d^(3/2)*x^7 + 168*c^4*d^(3/2)*x^5 + 35*c^2*d^(3/2)*x^3 - 21
0*d^(3/2)*x)*arcsinh(c*x)/c^3) - 2/3675*(75*c^6*d^(3/2)*x^7 + 168*c^4*d^(3/
2)*x^5 + 35*c^2*d^(3/2)*x^3 - 210*d^(3/2)*x)*a*b/c^3
```

Giac [F(-2)]

Exception generated.

$$\int x^3\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: TypeError}$$

[In] integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int x^3\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\int x^3\,\left(a+b\,\mathrm{asinh}(c\,x)\right)^2\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

[In] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2),x)

[Out] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2), x)

3.267 $\int x^2(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 405

$$\begin{aligned}\int x^2\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx &= -\frac{7b^2dx\sqrt{d+c^2dx^2}}{1152c^2}\\
&+\frac{43b^2dx^3\sqrt{d+c^2dx^2}}{1728}+\frac{1}{108}b^2c^2dx^5\sqrt{d+c^2dx^2}\\
&+\frac{7b^2d\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{1152c^3\sqrt{1+c^2x^2}}-\frac{bdx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{16c\sqrt{1+c^2x^2}}\\
&-\frac{7bcdx^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{48\sqrt{1+c^2x^2}}-\frac{bc^3dx^6\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{18\sqrt{1+c^2x^2}}\\
&+\frac{dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{16c^2}+\frac{1}{8}dx^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\\
&+\frac{1}{6}x^3\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{48bc^3\sqrt{1+c^2x^2}}\end{aligned}$$

```
[Out] 1/6*x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2-7/1152*b^2*d*x*(c^2*d*x^2+
d)^(1/2)/c^2+43/1728*b^2*d*x^3*(c^2*d*x^2+d)^(1/2)+1/108*b^2*c^2*d*x^5*(c^2
*d*x^2+d)^(1/2)+1/16*d*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^2+1/8*d
*x^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)+7/1152*b^2*d*arcsinh(c*x)*(c^
2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)-1/16*b*d*x^2*(a+b*arcsinh(c*x))*(c^2
*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-7/48*b*c*d*x^4*(a+b*arcsinh(c*x))*(c^2*
d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/18*b*c^3*d*x^6*(a+b*arcsinh(c*x))*(c^2*d
*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/48*d*(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^(
1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {5808, 5806, 5812, 5783, 5776, 327, 221, 14, 5803, 12, 470}

$$\int x^2\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = -\frac{bdx^2\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{16c\sqrt{c^2x^2+1}}+\frac{dx\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^2}{16c^2} -\frac{7bcdx^4\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{48\sqrt{c^2x^2+1}}+\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2 +\frac{1}{8}dx^3\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^2-\frac{d\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^3}{48bc^3\sqrt{c^2x^2+1}} -\frac{bc^3dx^6\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{18\sqrt{c^2x^2+1}}+\frac{7b^2d\operatorname{arcsinh}(cx)\sqrt{c^2dx^2+d}}{1152c^3\sqrt{c^2x^2+1}} -\frac{7b^2dx\sqrt{c^2dx^2+d}}{1152c^2}+\frac{1}{108}b^2c^2dx^5\sqrt{c^2dx^2+d}+\frac{43b^2dx^3\sqrt{c^2dx^2+d}}{1728}$$

```
[In] Int[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-7*b^2*d*x*Sqrt[d + c^2*d*x^2])/(1152*c^2) + (43*b^2*d*x^3*Sqrt[d + c^2*d*
x^2])/1728 + (b^2*c^2*d*x^5*Sqrt[d + c^2*d*x^2])/108 + (7*b^2*d*Sqrt[d + c^
2*d*x^2]*ArcSinh[c*x])/(1152*c^3*Sqrt[1 + c^2*x^2]) - (b*d*x^2*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x]))/(16*c*Sqrt[1 + c^2*x^2]) - (7*b*c*d*x^4*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(48*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^6*
Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(18*Sqrt[1 + c^2*x^2]) + (d*x*Sqr
t[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*c^2) + (d*x^3*Sqrt[d + c^2*d*x
^2]*(a + b*ArcSinh[c*x])^2)/8 + (x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c
*x])^2)/6 - (d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(48*b*c^3*Sqrt[1
 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
```

```
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{6}x^3\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2+\frac{1}{2}d\int x^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2\,dx \\
&\quad -\frac{\left(bcd\sqrt{d+c^2dx^2}\right)\int x^3(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))\,dx}{3\sqrt{1+c^2x^2}} \\
&= -\frac{bcdx^4\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{12\sqrt{1+c^2x^2}}-\frac{bc^3dx^6\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{18\sqrt{1+c^2x^2}} \\
&\quad +\frac{1}{8}dx^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2 \\
&\quad +\frac{1}{6}x^3\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2+\frac{\left(d\sqrt{d+c^2dx^2}\right)\int\frac{x^2(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{8\sqrt{1+c^2x^2}} \\
&\quad -\frac{\left(bcd\sqrt{d+c^2dx^2}\right)\int x^3(a+b\operatorname{arcsinh}(cx))\,dx}{4\sqrt{1+c^2x^2}} \\
&\quad +\frac{\left(b^2c^2d\sqrt{d+c^2dx^2}\right)\int\frac{x^4(3+2c^2x^2)}{12\sqrt{1+c^2x^2}}\,dx}{3\sqrt{1+c^2x^2}}
\end{aligned}$$

$$= -\frac{7bcdx^4\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{48\sqrt{1+c^2x^2}} - \frac{bc^3dx^6\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{18\sqrt{1+c^2x^2}} + \frac{dx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{16c^2} + \frac{1}{8}dx^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{6}x^3\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2 - \frac{\left(d\sqrt{d+c^2dx^2}\right)\int\frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}dx}{16c^2\sqrt{1+c^2x^2}} - \frac{\left(bd\sqrt{d+c^2dx^2}\right)\int x(a+b\operatorname{arcsinh}(cx))\,dx}{8c\sqrt{1+c^2x^2}} + \frac{\left(b^2c^2d\sqrt{d+c^2dx^2}\right)\int\frac{x^4(3+2c^2x^2)}{\sqrt{1+c^2x^2}}dx}{36\sqrt{1+c^2x^2}} + \frac{\left(b^2c^2d\sqrt{d+c^2dx^2}\right)\int\frac{x^4}{\sqrt{1+c^2x^2}}dx}{16\sqrt{1+c^2x^2}}$$

$$= \frac{1}{64}b^2dx^3\sqrt{d+c^2dx^2} + \frac{1}{108}b^2c^2dx^5\sqrt{d+c^2dx^2} - \frac{bdx^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{16c\sqrt{1+c^2x^2}} - \frac{7bcdx^4\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{48\sqrt{1+c^2x^2}} - \frac{bc^3dx^6\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{18\sqrt{1+c^2x^2}} + \frac{dx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{16c^2} + \frac{1}{8}dx^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{6}x^3\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2 - \frac{d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^3}{48bc^3\sqrt{1+c^2x^2}} - \frac{\left(3b^2d\sqrt{d+c^2dx^2}\right)\int\frac{x^2}{\sqrt{1+c^2x^2}}dx}{64\sqrt{1+c^2x^2}} + \frac{\left(b^2d\sqrt{d+c^2dx^2}\right)\int\frac{x^2}{\sqrt{1+c^2x^2}}dx}{16\sqrt{1+c^2x^2}} + \frac{\left(b^2c^2d\sqrt{d+c^2dx^2}\right)\int\frac{x^4}{\sqrt{1+c^2x^2}}dx}{27\sqrt{1+c^2x^2}}$$

$$= \frac{b^2dx\sqrt{d+c^2dx^2}}{128c^2} + \frac{43b^2dx^3\sqrt{d+c^2dx^2}}{1728} + \frac{1}{108}b^2c^2dx^5\sqrt{d+c^2dx^2} - \frac{bdx^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{16c\sqrt{1+c^2x^2}} - \frac{7bcdx^4\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{48\sqrt{1+c^2x^2}} - \frac{bc^3dx^6\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{18\sqrt{1+c^2x^2}} + \frac{dx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{16c^2} + \frac{1}{8}dx^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{6}x^3\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2 - \frac{d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^3}{48bc^3\sqrt{1+c^2x^2}} - \frac{\left(b^2d\sqrt{d+c^2dx^2}\right)\int\frac{x^2}{\sqrt{1+c^2x^2}}dx}{36\sqrt{1+c^2x^2}} + \frac{\left(3b^2d\sqrt{d+c^2dx^2}\right)\int\frac{1}{\sqrt{1+c^2x^2}}dx}{128c^2\sqrt{1+c^2x^2}} - \frac{\left(b^2d\sqrt{d+c^2dx^2}\right)\int\frac{1}{\sqrt{1+c^2x^2}}dx}{32c^2\sqrt{1+c^2x^2}}$$

$$= -\frac{7b^2dx\sqrt{d+c^2dx^2}}{1152c^2} + \frac{43b^2dx^3\sqrt{d+c^2dx^2}}{1728} + \frac{1}{108}b^2c^2dx^5\sqrt{d+c^2dx^2} - \frac{b^2d\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{128c^3\sqrt{1+c^2x^2}} - \frac{bdx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{16c\sqrt{1+c^2x^2}} - \frac{7bcdx^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{48\sqrt{1+c^2x^2}} - \frac{bc^3dx^6\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{18\sqrt{1+c^2x^2}} + \frac{dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{16c^2} + \frac{1}{8}dx^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{6}x^3\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2 - \frac{d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{48bc^3\sqrt{1+c^2x^2}} + \frac{\left(b^2d\sqrt{d+c^2dx^2}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{72c^2\sqrt{1+c^2x^2}}$$

$$= -\frac{7b^2dx\sqrt{d+c^2dx^2}}{1152c^2} + \frac{43b^2dx^3\sqrt{d+c^2dx^2}}{1728} + \frac{1}{108}b^2c^2dx^5\sqrt{d+c^2dx^2} + \frac{7b^2d\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{1152c^3\sqrt{1+c^2x^2}} - \frac{bdx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{16c\sqrt{1+c^2x^2}} - \frac{7bcdx^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{48\sqrt{1+c^2x^2}} - \frac{bc^3dx^6\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{18\sqrt{1+c^2x^2}} + \frac{dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{16c^2} + \frac{1}{8}dx^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{6}x^3\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2 - \frac{d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{48bc^3\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.25

$$\int x^2\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{864a^2cdx\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2} + 4032a^2c^3dx^3\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2} + 2304a^2c^5dx^5\sqrt{1}}{}$$

```
[In] Integrate[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (864*a^2*c*d*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 4032*a^2*c^3*d*x^3*S
qrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 2304*a^2*c^5*d*x^5*Sqrt[1 + c^2*x^2]
*Sqrt[d + c^2*d*x^2] - 288*b^2*d*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 + 216*a
*b*d*Sqrt[d + c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 108*a*b*d*Sqrt[d + c^2*d*x^
2]*Cosh[4*ArcSinh[c*x]] - 24*a*b*d*Sqrt[d + c^2*d*x^2]*Cosh[6*ArcSinh[c*x]]
 - 864*a^2*d^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2
]] - 108*b^2*d*Sqrt[d + c^2*d*x^2]*Sinh[2*ArcSinh[c*x]] + 27*b^2*d*Sqrt[d +
```

```
c^2*d*x^2]*Sinh[4*ArcSinh[c*x]] + 4*b^2*d*Sqrt[d + c^2*d*x^2]*Sinh[6*ArcSi
nh[c*x]] + 12*b*d*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(18*b*Cosh[2*ArcSinh[c*x
]] - 9*b*Cosh[4*ArcSinh[c*x]] - 2*b*Cosh[6*ArcSinh[c*x]] - 36*a*Sinh[2*ArcS
inh[c*x]] + 36*a*Sinh[4*ArcSinh[c*x]] + 12*a*Sinh[6*ArcSinh[c*x]]) + 72*b*d
*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2*(-12*a - 3*b*Sinh[2*ArcSinh[c*x]] + 3*b
*Sinh[4*ArcSinh[c*x]] + b*Sinh[6*ArcSinh[c*x]]))/(13824*c^3*Sqrt[1 + c^2*x^
2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1551 vs. 2(351) = 702.

Time = 0.36 (sec) , antiderivative size = 1552, normalized size of antiderivative = 3.83

method	result	size
default	Expression too large to display	1552
parts	Expression too large to display	1552

```
[In] int(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^2*x*(c^2*d*x^2+d)^(5/2)/c^2/d-1/24*a^2/c^2*x*(c^2*d*x^2+d)^(3/2)-1/16
*a^2/c^2*d*x*(c^2*d*x^2+d)^(1/2)-1/16*a^2/c^2*d^2*ln(c^2*d*x/(c^2*d)^(1/2)+
(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b^2*(-1/48*(d*(c^2*x^2+1))^(1/2)/(c^2*x^
2+1)^(1/2)/c^3*arcsinh(c*x)^3*d+1/6912*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7+32
*c^6*x^6*(c^2*x^2+1)^(1/2)+64*c^5*x^5+48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x
^3+18*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x+(c^2*x^2+1)^(1/2))*(18*arcsinh(c*x)^2
-6*arcsinh(c*x)+1)*d/c^3/(c^2*x^2+1)+1/1024*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^
5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+
(c^2*x^2+1)^(1/2))*(8*arcsinh(c*x)^2-4*arcsinh(c*x)+1)*d/c^3/(c^2*x^2+1)-1/
256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2
*x^2+1)^(1/2))*(2*arcsinh(c*x)^2-2*arcsinh(c*x)+1)*d/c^3/(c^2*x^2+1)-1/256*
(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2
+1)^(1/2))*(2*arcsinh(c*x)^2+2*arcsinh(c*x)+1)*d/c^3/(c^2*x^2+1)+1/1024*(d*
(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3-8*c^2*
x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(8*arcsinh(c*x)^2+4*arcsinh(
c*x)+1)*d/c^3/(c^2*x^2+1)+1/6912*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7-32*c^6*x
^6*(c^2*x^2+1)^(1/2)+64*c^5*x^5-48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3-18*
c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x-(c^2*x^2+1)^(1/2))*(18*arcsinh(c*x)^2+6*arc
sinh(c*x)+1)*d/c^3/(c^2*x^2+1))+2*a*b*(-1/32*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2
+1)^(1/2)/c^3*arcsinh(c*x)^2*d+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7+32*
c^6*x^6*(c^2*x^2+1)^(1/2)+64*c^5*x^5+48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^
3+18*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x+(c^2*x^2+1)^(1/2))*(-1+6*arcsinh(c*x))
*d/c^3/(c^2*x^2+1)+1/512*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^
2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*
(-1+4*arcsinh(c*x))*d/c^3/(c^2*x^2+1)-1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^
```

```
3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*x))*
d/c^3/(c^2*x^2+1)-1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2
+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(c*x))*d/c^3/(c^2*x^2+1)+1/5
12*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3-
8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(1+4*arcsinh(c*x))*d/c
^3/(c^2*x^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7-32*c^6*x^6*(c^2*x^2
+1)^(1/2)+64*c^5*x^5-48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3-18*c^2*x^2*(c^
2*x^2+1)^(1/2)+6*c*x-(c^2*x^2+1)^(1/2))*(1+6*arcsinh(c*x))*d/c^3/(c^2*x^2+1
))
```

Fricas [F]

$$\int x^2\left(d+c^2dx^2\right)^{3/2}(a+b\mathrm{arcsinh}(cx))^2\,dx=\int\left(c^2dx^2+d\right)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)^2x^2\,dx$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^2*d*x^4 + a^2*d*x^2 + (b^2*c^2*d*x^4 + b^2*d*x^2)*arcsinh(c
*x)^2 + 2*(a*b*c^2*d*x^4 + a*b*d*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int x^2\left(d+c^2dx^2\right)^{3/2}(a+b\mathrm{arcsinh}(cx))^2\,dx=\int x^2\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{asinh}(cx)\right)^2\,dx$$

```
[In] integrate(x**2*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**2*(d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^2\left(d+c^2dx^2\right)^{3/2}(a+b\mathrm{arcsinh}(cx))^2\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int x^2 \left(d+c^2 d x^2\right)^{3/2} (a+b\text{arcsinh}(cx))^2\, dx = \int \left(c^2 d x^2+d\right)^{\frac{3}{2}} (b\,\text{arsinh}\,(cx)+a)^2 x^2\, dx$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(d+c^2 d x^2\right)^{3/2} (a+b\text{arcsinh}(cx))^2\, dx = \int x^2\, (a+b\,\text{asinh}(c\,x))^2\, \left(d\,c^2\,x^2+d\right)^{3/2}\, dx$$

```
[In] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2),x)
[Out] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2), x)
```

3.268 $\int x(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 267

$$\int x(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\frac{16b^2d\sqrt{d+c^2dx^2}}{75c^2}+\frac{8b^2d(1+c^2x^2)\sqrt{d+c^2dx^2}}{225c^2}+\frac{2b^2d(1+c^2x^2)^2\sqrt{d+c^2dx^2}}{125c^2}-\frac{2bdx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{5c\sqrt{1+c^2x^2}}-\frac{4bcdx^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{15\sqrt{1+c^2x^2}}-\frac{2bc^3dx^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{25\sqrt{1+c^2x^2}}+\frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{5c^2d}$$

```
[Out] 1/5*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/c^2/d+16/75*b^2*d*(c^2*d*x^2+d
)^(1/2)/c^2+8/225*b^2*d*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/c^2+2/125*b^2*d*(c^
2*x^2+1)^2*(c^2*d*x^2+d)^(1/2)/c^2-2/5*b*d*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+
d)^(1/2)/c/(c^2*x^2+1)^(1/2)-4/15*b*c*d*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d
)^(1/2)/(c^2*x^2+1)^(1/2)-2/25*b*c^3*d*x^5*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)
^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used

= {5798, 200, 5784, 12, 1261, 712}

$$\int x\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = -\frac{2bdx\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{5c\sqrt{c^2x^2+1}} + \frac{(c^2dx^2+d)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{5c^2d} - \frac{4bcdx^3\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{15\sqrt{c^2x^2+1}} - \frac{2bc^3dx^5\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{25\sqrt{c^2x^2+1}} + \frac{2b^2d(c^2x^2+1)^2\sqrt{c^2dx^2+d}}{125c^2} + \frac{16b^2d\sqrt{c^2dx^2+d}}{75c^2} + \frac{8b^2d(c^2x^2+1)\sqrt{c^2dx^2+d}}{225c^2}$$

```
[In] Int[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (16*b^2*d*Sqrt[d + c^2*d*x^2])/(75*c^2) + (8*b^2*d*(1 + c^2*x^2)*Sqrt[d + c
^2*d*x^2])/(225*c^2) + (2*b^2*d*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/(125*c
^2) - (2*b*d*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c*Sqrt[1 + c^2*
x^2]) - (4*b*c*d*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(15*Sqrt[1 +
 c^2*x^2]) - (2*b*c^3*d*x^5*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(25*S
qrt[1 + c^2*x^2]) + ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(5*c^2*d
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{5c^2d} \\
&\quad - \frac{\left(2bd\sqrt{d+c^2dx^2}\right)\int (1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))\,dx}{5c\sqrt{1+c^2x^2}} \\
&= -\frac{2bdx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5c\sqrt{1+c^2x^2}} - \frac{4bcdx^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{15\sqrt{1+c^2x^2}} \\
&\quad - \frac{2bc^3dx^5\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{25\sqrt{1+c^2x^2}} + \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{5c^2d} \\
&\quad + \frac{\left(2b^2d\sqrt{d+c^2dx^2}\right)\int \frac{x(15+10c^2x^2+3c^4x^4)}{15\sqrt{1+c^2x^2}}\,dx}{5\sqrt{1+c^2x^2}} \\
&= -\frac{2bdx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5c\sqrt{1+c^2x^2}} - \frac{4bcdx^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{15\sqrt{1+c^2x^2}} \\
&\quad - \frac{2bc^3dx^5\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{25\sqrt{1+c^2x^2}} + \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{5c^2d} \\
&\quad + \frac{\left(2b^2d\sqrt{d+c^2dx^2}\right)\int \frac{x(15+10c^2x^2+3c^4x^4)}{\sqrt{1+c^2x^2}}\,dx}{75\sqrt{1+c^2x^2}} \\
&= -\frac{2bdx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5c\sqrt{1+c^2x^2}} - \frac{4bcdx^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{15\sqrt{1+c^2x^2}} \\
&\quad - \frac{2bc^3dx^5\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{25\sqrt{1+c^2x^2}} + \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{5c^2d} \\
&\quad + \frac{\left(b^2d\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{15+10c^2x+3c^4x^2}{\sqrt{1+c^2x}}\,dx, x, x^2\right)}{75\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$= -\frac{2bdx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{5c\sqrt{1+c^2x^2}} - \frac{4bcdx^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{15\sqrt{1+c^2x^2}} - \frac{2bc^3dx^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{25\sqrt{1+c^2x^2}} + \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{5c^2d} + \frac{(b^2d\sqrt{d+c^2dx^2})\,\text{Subst}\left(\int\left(\frac{8}{\sqrt{1+c^2x}}+4\sqrt{1+c^2x}+3(1+c^2x)^{3/2}\right)dx,x,x^2\right)}{75\sqrt{1+c^2x^2}}$$

$$= \frac{16b^2d\sqrt{d+c^2dx^2}}{75c^2} + \frac{8b^2d(1+c^2x^2)\sqrt{d+c^2dx^2}}{225c^2} + \frac{2b^2d(1+c^2x^2)^2\sqrt{d+c^2dx^2}}{125c^2} - \frac{2bdx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{5c\sqrt{1+c^2x^2}} - \frac{4bcdx^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{15\sqrt{1+c^2x^2}} - \frac{2bc^3dx^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{25\sqrt{1+c^2x^2}} + \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{5c^2d}$$

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.74

$$\int x(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{d\sqrt{d+c^2dx^2}\Big(225a^2(1+c^2x^2)^3 - 30abcx\sqrt{1+c^2x^2}(15+10c^2x^2+3c^4x^4) + 2b^2(149+$$

```
[In] Integrate[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d*Sqrt[d + c^2*d*x^2]*(225*a^2*(1 + c^2*x^2)^3 - 30*a*b*c*x*Sqrt[1 + c^2*x
^2]*(15 + 10*c^2*x^2 + 3*c^4*x^4) + 2*b^2*(149 + 187*c^2*x^2 + 47*c^4*x^4 +
 9*c^6*x^6) + 30*b*(15*a*(1 + c^2*x^2)^3 - b*c*x*Sqrt[1 + c^2*x^2]*(15 + 10
*c^2*x^2 + 3*c^4*x^4))*ArcSinh[c*x] + 225*b^2*(1 + c^2*x^2)^3*ArcSinh[c*x]^
2))/(1125*c^2*(1 + c^2*x^2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1148 vs. 2(233) = 466.

Time = 0.37 (sec) , antiderivative size = 1149, normalized size of antiderivative = 4.30

method	result	size
default	Expression too large to display	1149
parts	Expression too large to display	1149

```
[In] int(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a^2*(c^2*d*x^2+d)^(5/2)/c^2/d+b^2*(1/4000*(d*(c^2*x^2+1))^(1/2)*(16*c^6
*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+1
3*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c*x)^2-10*arcsinh(c*x)+2)*
d/c^2/(c^2*x^2+1)+1/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2
+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2-6*arcsinh(
c*x)+2)*d/c^2/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+
1)^(1/2)+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)*d/c^2/(c^2*x^2+1)+1/16*(d*(c^
2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2+2*arcsinh
(c*x)+2)*d/c^2/(c^2*x^2+1)+1/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3
*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2+6
*arcsinh(c*x)+2)*d/c^2/(c^2*x^2+1)+1/4000*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6
-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^
2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c*x)^2+10*arcsinh(c*x)+2)*d/c^
2/(c^2*x^2+1))+2*a*b*(1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c
^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c
^2*x^2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/96*(d*(c^2*x^2+1
))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)
^(1/2)+1)*(-1+3*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*
(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/16*
(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)+1)*d/
c^2/(c^2*x^2+1)+1/96*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)
^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(3*arcsinh(c*x)+1)*d/c^2/(c^2*x
^2+1)+1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+
28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+
1)*(1+5*arcsinh(c*x))*d/c^2/(c^2*x^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.24

$$\int x\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{225\left(b^2c^6dx^6+3\,b^2c^4dx^4+3\,b^2c^2dx^2+b^2d\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)^2+30\,(15\,\ldots}{}$$

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/1125*(225*(b^2*c^6*d*x^6 + 3*b^2*c^4*d*x^4 + 3*b^2*c^2*d*x^2 + b^2*d)*sqr
t(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 30*(15*a*b*c^6*d*x^6 + 45
*a*b*c^4*d*x^4 + 45*a*b*c^2*d*x^2 + 15*a*b*d - (3*b^2*c^5*d*x^5 + 10*b^2*c^
3*d*x^3 + 15*b^2*c*d*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x + sq
rt(c^2*x^2 + 1)) + (9*(25*a^2 + 2*b^2)*c^6*d*x^6 + (675*a^2 + 94*b^2)*c^4*d
*x^4 + (675*a^2 + 374*b^2)*c^2*d*x^2 + (225*a^2 + 298*b^2)*d - 30*(3*a*b*c^
```

```
5*d*x^5 + 10*a*b*c^3*d*x^3 + 15*a*b*c*d*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^
2 + d))/(c^4*x^2 + c^2)
```

Sympy [F]

$$\int x\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx=\int x\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}(a+b\operatorname{asinh}(cx))^2\,dx$$

```
[In] integrate(x*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2,x)
[Out] Integral(x*(d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.86

$$\int x\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx=\frac{(c^2dx^2+d)^{\frac{5}{2}}b^2\operatorname{arsinh}(cx)^2}{5\,c^2d}+\frac{2}{1125}\,b^2\left(\frac{9\sqrt{c^2x^2+1}c^2d^{\frac{5}{2}}x^4+38\sqrt{c^2x^2+1}d^{\frac{5}{2}}x^2+\frac{149\sqrt{c^2x^2+1}d^{\frac{5}{2}}}{c^2}}{d}-\frac{15\left(3\,c^4d^{\frac{5}{2}}x^5+10\,c^2d^{\frac{5}{2}}x^3+15\,d^{\frac{5}{2}}x\right)\text{a}}{cd}\right.$$
$$+\frac{2\,(c^2dx^2+d)^{\frac{5}{2}}ab\operatorname{arsinh}(cx)}{5\,c^2d}+\frac{(c^2dx^2+d)^{\frac{5}{2}}a^2}{5\,c^2d}-\frac{2\left(3\,c^4d^{\frac{5}{2}}x^5+10\,c^2d^{\frac{5}{2}}x^3+15\,d^{\frac{5}{2}}x\right)ab}{75\,cd}$$

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] 1/5*(c^2*d*x^2 + d)^(5/2)*b^2*arcsinh(c*x)^2/(c^2*d) + 2/1125*b^2*((9*sqrt(
c^2*x^2 + 1)*c^2*d^(5/2)*x^4 + 38*sqrt(c^2*x^2 + 1)*d^(5/2)*x^2 + 149*sqrt(
c^2*x^2 + 1)*d^(5/2)/c^2)/d - 15*(3*c^4*d^(5/2)*x^5 + 10*c^2*d^(5/2)*x^3 +
15*d^(5/2)*x)*arcsinh(c*x)/(c*d)) + 2/5*(c^2*d*x^2 + d)^(5/2)*a*b*arcsinh(c
*x)/(c^2*d) + 1/5*(c^2*d*x^2 + d)^(5/2)*a^2/(c^2*d) - 2/75*(3*c^4*d^(5/2)*x
^5 + 10*c^2*d^(5/2)*x^3 + 15*d^(5/2)*x)*a*b/(c*d)
```

Giac [F(-2)]

Exception generated.

$$\int x(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \int x\,(a+b\,\text{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

```
[In] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2), x)
```

3.269 $\int \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))^2 \, dx$

Optimal result

Integrand size = 25, antiderivative size = 294

$$\int \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))^2 \, dx = \frac{15}{64}b^2dx\sqrt{d+c^2dx^2} + \frac{1}{32}b^2dx\left(1+c^2x^2\right)\sqrt{d+c^2dx^2} - \frac{9b^2d\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{64c\sqrt{1+c^2x^2}} - \frac{3bcdx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} - \frac{bd(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8c} + \frac{3}{8}dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{4}x\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{8bc\sqrt{1+c^2x^2}}$$

```
[Out] 1/4*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2+15/64*b^2*d*x*(c^2*d*x^2+d)^
(1/2)+1/32*b^2*d*x*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)-1/8*b*d*(c^2*x^2+1)^(3/2
)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c+3/8*d*x*(a+b*arcsinh(c*x))^2*(c^
2*d*x^2+d)^(1/2)-9/64*b^2*d*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^
(1/2)-3/8*b*c*d*x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2
)+1/8*d*(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {5786, 5785, 5783, 5776, 327, 221, 5798, 201}

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\frac{d\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{8bc\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2+\frac{3}{8}dx\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bd(c^2x^2+1)^{3/2}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{8c}-\frac{3bcdx^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{8\sqrt{c^2x^2+1}}-\frac{9b^2d\text{arcsinh}(cx)\sqrt{c^2dx^2+d}}{64c\sqrt{c^2x^2+1}}+\frac{15}{64}b^2dx\sqrt{c^2dx^2+d}+\frac{1}{32}b^2dx\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}$$

```
[In] Int[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (15*b^2*d*x*Sqrt[d + c^2*d*x^2])/64 + (b^2*d*x*(1 + c^2*x^2)*Sqrt[d + c^2*d
*x^2])/32 - (9*b^2*d*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(64*c*Sqrt[1 + c^2*x
^2]) - (3*b*c*d*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c
^2*x^2]) - (b*d*(1 + c^2*x^2)^(3/2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
))/(8*c) + (3*d*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/8 + (x*(d + c
^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + (d*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x])^3)/(8*b*c*Sqrt[1 + c^2*x^2])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{4}x\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2+\frac{1}{4}(3d)\int\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx - \frac{\left(bcd\sqrt{d+c^2dx^2}\right)\int x(1+c^2x^2)\left(a+b\text{arcsinh}(cx)\right)dx}{2\sqrt{1+c^2x^2}}$$

$$= -\frac{bd(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8c}+\frac{3}{8}dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{4}x\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2+\frac{\left(3d\sqrt{d+c^2dx^2}\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{8\sqrt{1+c^2x^2}} + \frac{\left(b^2d\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)^{3/2}\,dx}{8\sqrt{1+c^2x^2}} - \frac{\left(3bcd\sqrt{d+c^2dx^2}\right)\int x(a+b\text{arcsinh}(cx))\,dx}{4\sqrt{1+c^2x^2}}$$

$$= \frac{1}{32}b^2dx\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}-\frac{3bcdx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} - \frac{bd(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8c} + \frac{3}{8}dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{4}x\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2+\frac{d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{8bc\sqrt{1+c^2x^2}} + \frac{\left(3b^2d\sqrt{d+c^2dx^2}\right)\int\sqrt{1+c^2x^2}\,dx}{32\sqrt{1+c^2x^2}}+\frac{\left(3b^2c^2d\sqrt{d+c^2dx^2}\right)\int\frac{x^2}{\sqrt{1+c^2x^2}}\,dx}{8\sqrt{1+c^2x^2}}$$

$$= \frac{15}{64}b^2dx\sqrt{d+c^2dx^2}+\frac{1}{32}b^2dx\left(1+c^2x^2\right)\sqrt{d+c^2dx^2} - \frac{3bcdx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} - \frac{bd(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8c} + \frac{3}{8}dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{4}x\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2+\frac{d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{8bc\sqrt{1+c^2x^2}} + \frac{\left(3b^2d\sqrt{d+c^2dx^2}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{64\sqrt{1+c^2x^2}}-\frac{\left(3b^2d\sqrt{d+c^2dx^2}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{16\sqrt{1+c^2x^2}}$$

$$
\begin{aligned}
&= \frac{15}{64}b^2dx\sqrt{d+c^2dx^2} + \frac{1}{32}b^2dx(1+c^2x^2)\sqrt{d+c^2dx^2} \\
&\quad - \frac{9b^2d\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{64c\sqrt{1+c^2x^2}} - \frac{3bcdx^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} \\
&\quad - \frac{bd(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{8c} \\
&\quad + \frac{3}{8}dx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2 \\
&\qquad + \frac{1}{4}x(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2 + \frac{d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^3}{8bc\sqrt{1+c^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.34 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.12

$$
\int (d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \frac{96a^2cdx\sqrt{1+c^2x^2}(5+2c^2x^2)\sqrt{d+c^2dx^2} + 288a^2d^{3/2}\sqrt{1+c^2x^2}\log\left(cdx+\sqrt{d}\sqrt{d+\right.}{}
$$

```
[In] Integrate[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (96*a^2*c*d*x*Sqrt[1 + c^2*x^2]*(5 + 2*c^2*x^2)*Sqrt[d + c^2*d*x^2] + 288*a
^2*d^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 32*
b^2*d*Sqrt[d + c^2*d*x^2]*(4*ArcSinh[c*x]^3 - 6*ArcSinh[c*x]*Cosh[2*ArcSinh
[c*x]] + (3 + 6*ArcSinh[c*x]^2)*Sinh[2*ArcSinh[c*x]]) - 192*a*b*d*Sqrt[d +
c^2*d*x^2]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*Ar
cSinh[c*x]])) - 12*a*b*d*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*Arc
Sinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]) - b^2*d*Sqrt[d + c^2*d*x^
2]*(32*ArcSinh[c*x]^3 + 12*ArcSinh[c*x]*Cosh[4*ArcSinh[c*x]] - 3*(1 + 8*Arc
Sinh[c*x]^2)*Sinh[4*ArcSinh[c*x]]))/(768*c*Sqrt[1 + c^2*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 958 vs. 2(254) = 508.

Time = 0.26 (sec) , antiderivative size = 959, normalized size of antiderivative = 3.26

method	result
default	$\frac{x(c^2dx^2+d)^{\frac{3}{2}}a^2}{4}+\frac{3a^2dx\sqrt{c^2dx^2+d}}{8}+\frac{3a^2d^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{8\sqrt{c^2d}}+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^3d}{8\sqrt{c^2x^2+1}\,c}+\frac{\sqrt{d(c^2x^2+1)}\,(8c^5$
parts	$\frac{x(c^2dx^2+d)^{\frac{3}{2}}a^2}{4}+\frac{3a^2dx\sqrt{c^2dx^2+d}}{8}+\frac{3a^2d^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{8\sqrt{c^2d}}+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^3d}{8\sqrt{c^2x^2+1}\,c}+\frac{\sqrt{d(c^2x^2+1)}\,(8c^5$

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x*(c^2*d*x^2+d)^(3/2)*a^2+3/8*a^2*d*x*(c^2*d*x^2+d)^(1/2)+3/8*a^2*d^2*l
n(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b^2*(1/8*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(c*x)^3*d+1/512*(d*(c^2*x^2+1))^(
1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1
)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(8*arcsinh(c*x)^2-4*arcsinh(c*x)+1)*d/c/(c
^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)
+2*c*x+(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2-2*arcsinh(c*x)+1)*d/c/(c^2*x^2+
1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-
(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2+2*arcsinh(c*x)+1)*d/c/(c^2*x^2+1)+1/51
2*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3-8
*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(8*arcsinh(c*x)^2+4*arc
sinh(c*x)+1)*d/c/(c^2*x^2+1))+2*a*b*(3/16*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)
^(1/2)/c*arcsinh(c*x)^2*d+1/256*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*
(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^
(1/2))*(-1+4*arcsinh(c*x))*d/c/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^
3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*
x))*d/c/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^
2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(c*x))*d/c/(c^2*x^2+1)+1/25
6*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3-8
*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(1+4*arcsinh(c*x))*d/c/
(c^2*x^2+1))
```

Fricas [F]

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\int \left(c^2dx^2+d\right)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)^2\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx=\int \left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\,dx$$

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2,x)
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx=\int {\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^{3/2} \,d x$$

```
[In] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2),x)
[Out] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2), x)
```

3.270 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 498

$$\begin{aligned}
\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx &= \frac{22}{9}b^2d\sqrt{d+c^2dx^2} \\
&- \frac{2abcdx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} \\
&+ \frac{2}{27}b^2d(1+c^2x^2)\sqrt{d+c^2dx^2} - \frac{2b^2cdx\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}} \\
&- \frac{2bcdx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} - \frac{2bc^3dx^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} \\
&+ d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{3}(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2 \\
&- \frac{2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&- \frac{2bd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&+ \frac{2bd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&+ \frac{2b^2d\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&- \frac{2b^2d\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}
\end{aligned}$$

```
[Out] 1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2+22/9*b^2*d*(c^2*d*x^2+d)^(1/2)
+2/27*b^2*d*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)+d*(a+b*arcsinh(c*x))^2*(c^2*d*x
^2+d)^(1/2)-2*a*b*c*d*x*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*b^2*c*d*x*a
```

```
rcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/3*b*c*d*x*(a+b*arcsinh(
c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/9*b*c^3*d*x^3*(a+b*arcsinh(c*
x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*d*(a+b*arcsinh(c*x))^2*arctanh(
c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*b*d*(a+b*arc
sinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1
)^(1/2)+2*b*d*(a+b*arcsinh(c*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^
2+d)^(1/2)/(c^2*x^2+1)^(1/2)+2*b^2*d*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2
*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*b^2*d*polylog(3,c*x+(c^2*x^2+1)^(1/2))*
(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5808, 5806, 5816, 4267, 2611, 2320, 6724, 5772, 267, 5784, 455, 45}

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx = -\frac{2d\sqrt{c^2dx^2+d}\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}} - \frac{2bd\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}} + \frac{2bd\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}} - \frac{2bcdx\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}} + \frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2 + d\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2 - \frac{2bc^3dx^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{9\sqrt{c^2x^2+1}} - \frac{2abcdx\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}} + \frac{2b^2d\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{c^2x^2+1}} - \frac{2b^2d\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{\sqrt{c^2x^2+1}} - \frac{2b^2cdx\text{arcsinh}(cx)\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}} + \frac{22}{9}b^2d\sqrt{c^2dx^2+d} + \frac{2}{27}b^2d\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}$$

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (22*b^2*d*Sqrt[d + c^2*d*x^2])/9 - (2*a*b*c*d*x*Sqrt[d + c^2*d*x^2])/Sqrt[1
 + c^2*x^2] + (2*b^2*d*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/27 - (2*b^2*c*d*x
*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - (2*b*c*d*x*Sqrt[d +
c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*Sqrt[1 + c^2*x^2]) - (2*b*c^3*d*x^3*Sqr
t[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(9*Sqrt[1 + c^2*x^2]) + d*Sqrt[d + c
```

```
^2*d*x^2]*(a + b*ArcSinh[c*x])^2 + ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*
x])^2)/3 - (2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSin
h[c*x]])/Sqrt[1 + c^2*x^2] - (2*b*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
)*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (2*b*d*Sqrt[d + c^2*d*x^
2]*(a + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (2*
b^2*d*Sqrt[d + c^2*d*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] -
(2*b^2*d*Sqrt[d + c^2*d*x^2]*PolyLog[3, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
```

```
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2+d\int\frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{x}\,dx \\
&\quad-\frac{\left(2bcd\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))\,dx}{3\sqrt{1+c^2x^2}} \\
&= -\frac{2bcdx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} \\
&\quad-\frac{2bc^3dx^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{9\sqrt{1+c^2x^2}}+d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2 \\
&\quad+\frac{1}{3}\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2+\frac{\left(d\sqrt{d+c^2dx^2}\right)\int\frac{(a+b\operatorname{arcsinh}(cx))^2}{x\sqrt{1+c^2x^2}}\,dx}{\sqrt{1+c^2x^2}} \\
&\quad-\frac{\left(2bcd\sqrt{d+c^2dx^2}\right)\int(a+b\operatorname{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}}+\frac{\left(2b^2c^2d\sqrt{d+c^2dx^2}\right)\int\frac{x\left(1+\frac{c^2x^2}{3}\right)}{\sqrt{1+c^2x^2}}\,dx}{3\sqrt{1+c^2x^2}} \\
&= -\frac{2abcdx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}-\frac{2bcdx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} \\
&\quad-\frac{2bc^3dx^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} \\
&\quad+d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2+\frac{1}{3}\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2 \\
&\quad+\frac{\left(d\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int(a+bx)^2\operatorname{csch}(x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}} \\
&\quad-\frac{\left(2b^2cd\sqrt{d+c^2dx^2}\right)\int\operatorname{arcsinh}(cx)\,dx}{\sqrt{1+c^2x^2}} \\
&\quad+\frac{\left(b^2c^2d\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\frac{1+\frac{c^2x}{3}}{\sqrt{1+c^2x}}\,dx,x,x^2\right)}{3\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$= -\frac{2abcdx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{2b^2cdx\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}} - \frac{2bcdx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} - \frac{2bc^3dx^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} + d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{3}\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2 - \frac{2d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{\left(2bd\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int(a+bx)\log\left(1-e^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}} + \frac{\left(2bd\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int(a+bx)\log\left(1+e^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}} + \frac{\left(b^2c^2d\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\left(\frac{2}{3\sqrt{1+c^2x}}+\frac{1}{3}\sqrt{1+c^2x}\right)dx,x,x^2\right)}{3\sqrt{1+c^2x^2}} + \frac{\left(2b^2c^2d\sqrt{d+c^2dx^2}\right)\int\frac{x}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{22}{9}b^2d\sqrt{d+c^2dx^2} - \frac{2abcdx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{2}{27}b^2d\left(1+c^2x^2\right)\sqrt{d+c^2dx^2} - \frac{2b^2cdx\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}} - \frac{2bcdx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} - \frac{2bc^3dx^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} + d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{3}\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2 - \frac{2d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{2bd\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{2bd\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{\left(2b^2d\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(2,-e^x\right)dx,x,\operatorname{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}} - \frac{\left(2b^2d\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(2,e^x\right)dx,x,\operatorname{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}}$$

$$= \frac{22}{9}b^2d\sqrt{d+c^2dx^2} - \frac{2abcdx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{2}{27}b^2d(1+c^2x^2)\sqrt{d+c^2dx^2} - \frac{2b^2cdx\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}} - \frac{2bcdx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} - \frac{2bc^3dx^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} + d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{3}(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2 - \frac{2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{2bd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{2bd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{\left(2b^2d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{\left(2b^2d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}$$

$$= \frac{22}{9}b^2d\sqrt{d+c^2dx^2} - \frac{2abcdx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{2}{27}b^2d(1+c^2x^2)\sqrt{d+c^2dx^2} - \frac{2b^2cdx\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}} - \frac{2bcdx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} - \frac{2bc^3dx^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} + d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{3}(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2 - \frac{2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{2bd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{2bd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{2b^2d\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{2b^2d\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.04

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx = \frac{1}{3}a^2d(4+c^2x^2)\sqrt{d+c^2dx^2} - \frac{2abd\sqrt{d+c^2dx^2}\left(3cx+c^3x^3-3(1+c^2x^2)^{3/2}\text{arcsinh}(cx)\right)}{9\sqrt{1+c^2x^2}} + a^2d^{3/2}\log(cx) - a^2d^{3/2}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right) + \frac{2abd\sqrt{d+c^2dx^2}\left(-cx+\sqrt{1+c^2x^2}\text{arcsinh}(cx)+\text{arcsinh}\right.}{}$$

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (a^2*d*(4 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/3 - (2*a*b*d*Sqrt[d + c^2*d*x^2]*
(3*c*x + c^3*x^3 - 3*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]))/(9*Sqrt[1 + c^2*x^2
]) + a^2*d^(3/2)*Log[c*x] - a^2*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]
] + (2*a*b*d*Sqrt[d + c^2*d*x^2]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] +
 ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh
[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))/
Sqrt[1 + c^2*x^2] + (b^2*d*Sqrt[d + c^2*d*x^2]*(2*Sqrt[1 + c^2*x^2] - 2*c*x
*ArcSinh[c*x] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 + ArcSinh[c*x]^2*(Log[1 -
E^(-ArcSinh[c*x])] - Log[1 + E^(-ArcSinh[c*x])]) + 2*ArcSinh[c*x]*(PolyLog[
2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]) + 2*(PolyLog[3, -E^
(-ArcSinh[c*x])] - PolyLog[3, E^(-ArcSinh[c*x])])))/Sqrt[1 + c^2*x^2] + (b^
2*d*Sqrt[d + c^2*d*x^2]*(27*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c*x]^2) + (2 + 9
*ArcSinh[c*x]^2)*Cosh[3*ArcSinh[c*x]] - 6*ArcSinh[c*x]*(9*c*x + Sinh[3*ArcS
inh[c*x]])))/(108*Sqrt[1 + c^2*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1052 vs. $2(491) = 982$.

Time = 0.35 (sec) , antiderivative size = 1053, normalized size of antiderivative = 2.11

method	result	size
default	Expression too large to display	1053
parts	Expression too large to display	1053

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(c^2*d*x^2+d)^(3/2)*a^2+a^2*d*(c^2*d*x^2+d)^(1/2)-8/3*a*b*(d*(c^2*x^2+1
))^(1/2)*d/(c^2*x^2+1)^(1/2)*c*x-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1
/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*d+2*a*b*(d*(c^2*x^2+1))^(1/2)/
(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*d-2/9*a*b*(d*(c^
```

```
2*x^2+1))^(1/2)*d/(c^2*x^2+1)^(1/2)*c^3*x^3-a^2*d^(3/2)*ln((2*d+2*d^(1/2)*(
c^2*d*x^2+d)^(1/2))/x)-b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(
c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))*d+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))*d-2*b^2*(d*(c^2*x^2+1))^(1
/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*d+2/27
*b^2*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*c^4*x^4+70/27*b^2*(d*(c^2*x^2+1))^
(1/2)*d/(c^2*x^2+1)*x^2*c^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*a
rcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*d-2*b^2*(d*(c^2*x^2+1))^(1/2)/
(c^2*x^2+1)^(1/2)*polylog(3,c*x+(c^2*x^2+1)^(1/2))*d+2/3*a*b*(d*(c^2*x^2+1)
)^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)*x^4*c^4+10/3*a*b*(d*(c^2*x^2+1))^(1/2)*d
/(c^2*x^2+1)*arcsinh(c*x)*x^2*c^2+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(
1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*d-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^
2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*d+8/3*a*b*(d*(c^2*x^2+1))^(1/2
)*d/(c^2*x^2+1)*arcsinh(c*x)+68/27*b^2*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)+
2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(3,-c*x-(c^2*x^2+1)^(1
/2))*d+4/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)^2+1/3*b^2*(
d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)^2*x^4*c^4+5/3*b^2*(d*(c^2*x
^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)^2*x^2*c^2-2/9*b^2*(d*(c^2*x^2+1))^(
1/2)*d/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^3*c^3-8/3*b^2*(d*(c^2*x^2+1))^(1/2)
*d/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x*c
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)^{3/2}\,(a+b\mathrm{arcsinh}(cx))^2}{x}\,dx=\int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)^2}{x}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x, x)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^{3/2}\,(a+b\mathrm{arcsinh}(cx))^2}{x}\,dx=\int \frac{(d(c^2x^2+1))^{\frac{3}{2}}\,(a+b\operatorname{asinh}(cx))^2}{x}\,dx$$

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2/x,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2/x, x)
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx = \int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{x}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] -1/3*(3*d^(3/2)*arcsinh(1/(c*abs(x))) - (c^2*d*x^2 + d)^(3/2) - 3*sqrt(c^2*
d*x^2 + d)*d)*a^2 + integrate((c^2*d*x^2 + d)^(3/2)*b^2*log(c*x + sqrt(c^2*
x^2 + 1))^2/x + 2*(c^2*d*x^2 + d)^(3/2)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/x,
 x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)^{3/2}}{x}\,dx$$

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x, x)
```

3.271 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2} dx$

Optimal result

Integrand size = 28, antiderivative size = 398

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2} dx = \frac{1}{4}b^2c^2dx\sqrt{d+c^2dx^2} - \frac{5b^2cd\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{4\sqrt{1+c^2x^2}} - \frac{3bc^3dx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}} + bcd\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{3}{2}c^2dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{cd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}} - \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x} + \frac{cd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{2b\sqrt{1+c^2x^2}} + \frac{2bcd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{b^2cd\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}$$

```
[Out] -(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x+1/4*b^2*c^2*d*x*(c^2*d*x^2+d)^(
1/2)+3/2*c^2*d*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)-5/4*b^2*c*d*arcsi
nh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3/2*b*c^3*d*x^2*(a+b*arcsinh(
c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+c*d*(a+b*arcsinh(c*x))^2*(c^2*d
*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/2*c*d*(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^
(1/2)/b/(c^2*x^2+1)^(1/2)+2*b*c*d*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1
)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b^2*c*d*polylog(2,1/(c*x+
(c^2*x^2+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+b*c*d*(a+b*arcs
inh(c*x))*(c^2*x^2+1)^(1/2)*(c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {5807, 5785, 5783, 5776, 327, 221, 5801, 5775, 3797, 2221, 2317, 2438, 201}

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}dx = \frac{3}{2}c^2dx\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2 + \frac{cd\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{2b\sqrt{c^2x^2+1}} + \frac{cd\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}} + bcd\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx)) - \frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))^2}{x} + \frac{2bcd\sqrt{c^2dx^2+d}\log\left(1-e^{-2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}} - \frac{3bc^3dx^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2\sqrt{c^2x^2+1}} - \frac{b^2cd\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)}{\sqrt{c^2x^2+1}} - \frac{5b^2cd\text{arcsinh}(cx)\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}} + \frac{1}{4}b^2c^2dx\sqrt{c^2dx^2+d}$$

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (b^2*c^2*d*x*Sqrt[d + c^2*d*x^2])/4 - (5*b^2*c*d*Sqrt[d + c^2*d*x^2]*ArcSin
h[c*x])/(4*Sqrt[1 + c^2*x^2]) - (3*b*c^3*d*x^2*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x]))/(2*Sqrt[1 + c^2*x^2]) + b*c*d*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*
d*x^2]*(a + b*ArcSinh[c*x]) + (3*c^2*d*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x])^2)/2 + (c*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/Sqrt[1 + c^2
*x^2] - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x + (c*d*Sqrt[d + c^
2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(2*b*Sqrt[1 + c^2*x^2]) + (2*b*c*d*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x])*Log[1 - E^(-2*ArcSinh[c*x])])/Sqrt[1 + c^
2*x^2] - (b^2*c*d*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/Sqrt
[1 + c^2*x^2]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5801

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcSinh[c*x])/(2*p)), x] + (Dist[d
, Int[(d + e*x^2)^(p - 1)*((a + b*ArcSinh[c*x])/x), x], x] - Dist[b*c*(d^p/
(2*p)), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{x} + (3c^2d)\int \sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2\,dx + \frac{\left(2bcd\sqrt{d+c^2dx^2}\right)\int \frac{(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))}{x}\,dx}{\sqrt{1+c^2x^2}}$$

$$= bcd\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{3}{2}c^2dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 - \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x} + \frac{\left(2bcd\sqrt{d+c^2dx^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{x}\,dx}{\sqrt{1+c^2x^2}} + \frac{\left(3c^2d\sqrt{d+c^2dx^2}\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}} - \frac{\left(b^2c^2d\sqrt{d+c^2dx^2}\right)\int\sqrt{1+c^2x^2}\,dx}{\sqrt{1+c^2x^2}} - \frac{\left(3bc^3d\sqrt{d+c^2dx^2}\right)\int x(a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}}$$

$$= -\frac{1}{2}b^2c^2dx\sqrt{d+c^2dx^2} - \frac{3bc^3dx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}} + bcd\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{3}{2}c^2dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 - \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x} + \frac{cd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{2b\sqrt{1+c^2x^2}} - \frac{\left(2cd\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int x\coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}} - \frac{\left(b^2c^2d\sqrt{d+c^2dx^2}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}} + \frac{\left(3b^2c^4d\sqrt{d+c^2dx^2}\right)\int\frac{x^2}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}}$$

$$= \frac{1}{4}b^2c^2dx\sqrt{d+c^2dx^2} - \frac{b^2cd\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{2\sqrt{1+c^2x^2}} - \frac{3bc^3dx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}} + bcd\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{3}{2}c^2dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{cd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}} - \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x} + \frac{cd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{2b\sqrt{1+c^2x^2}} + \frac{\left(4cd\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}} - \frac{\left(3b^2c^2d\sqrt{d+c^2dx^2}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{4\sqrt{1+c^2x^2}}$$

$$
\begin{aligned}
&= \frac{1}{4}b^2c^2dx\sqrt{d+c^2dx^2} - \frac{5b^2cd\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{4\sqrt{1+c^2x^2}} \\
&\quad - \frac{3bc^3dx^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2\sqrt{1+c^2x^2}} \\
&\quad + bcd\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{3}{2}c^2dx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2 + \frac{cd\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}} \\
&\quad - \frac{(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{x} + \frac{cd\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^3}{2b\sqrt{1+c^2x^2}} \\
&\quad + \frac{2bcd\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\log\left(1-e^{-2\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(2bcd\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)dx, x, a+b\operatorname{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{4}b^2c^2dx\sqrt{d+c^2dx^2} - \frac{5b^2cd\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{4\sqrt{1+c^2x^2}} \\
&\quad - \frac{3bc^3dx^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2\sqrt{1+c^2x^2}} \\
&\quad + bcd\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{3}{2}c^2dx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2 + \frac{cd\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}} \\
&\quad - \frac{(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{x} + \frac{cd\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^3}{2b\sqrt{1+c^2x^2}} \\
&\quad + \frac{2bcd\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\log\left(1-e^{-2\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(b^2cd\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}\right)}{\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$= \frac{1}{4}b^2c^2dx\sqrt{d+c^2dx^2} - \frac{5b^2cd\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{4\sqrt{1+c^2x^2}} - \frac{3bc^3dx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}} + bcd\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{3}{2}c^2dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{cd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}} - \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x} + \frac{cd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{2b\sqrt{1+c^2x^2}} + \frac{2bcd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{b^2cd\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right)}{\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 3.50 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.93

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \frac{12a^2d(-2+c^2x^2)\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2} + 24abd\sqrt{d+c^2dx^2}(-2\sqrt{}}{}$$

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (12*a^2*d*(-2 + c^2*x^2)*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 24*a*b*d*S
qrt[d + c^2*d*x^2]*(-2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + c*x*ArcSinh[c*x]^2
+ 2*c*x*Log[c*x]) + 36*a^2*c*d^(3/2)*x*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d
]*Sqrt[d + c^2*d*x^2]] - 8*b^2*d*Sqrt[d + c^2*d*x^2]*(ArcSinh[c*x]*(3*Sqrt[
1 + c^2*x^2]*ArcSinh[c*x] - c*x*ArcSinh[c*x]*(3 + ArcSinh[c*x]) - 6*c*x*Log
[1 - E^(-2*ArcSinh[c*x])]) + 3*c*x*PolyLog[2, E^(-2*ArcSinh[c*x])]) + b^2*c
*d*x*Sqrt[d + c^2*d*x^2]*(4*ArcSinh[c*x]^3 - 6*ArcSinh[c*x]*Cosh[2*ArcSinh[
c*x]] + (3 + 6*ArcSinh[c*x]^2)*Sinh[2*ArcSinh[c*x]]) - 6*a*b*c*d*x*Sqrt[d +
 c^2*d*x^2]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*A
rcSinh[c*x]])))/(24*x*Sqrt[1 + c^2*x^2])
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.19

method	result
default	$-\frac{a^2(c^2d\,x^2+d)^{\frac{5}{2}}}{dx}+a^2c^2x(c^2d\,x^2+d)^{\frac{3}{2}}+\frac{3\sqrt{c^2d\,x^2+d}\,a^2c^2dx}{2}+\frac{3a^2c^2d^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2\sqrt{c^2d}}+\frac{b^2\sqrt{d(c^2x^2+1)}\,(}{}$
parts	$-\frac{a^2(c^2d\,x^2+d)^{\frac{5}{2}}}{dx}+a^2c^2x(c^2d\,x^2+d)^{\frac{3}{2}}+\frac{3\sqrt{c^2d\,x^2+d}\,a^2c^2dx}{2}+\frac{3a^2c^2d^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2\sqrt{c^2d}}+\frac{b^2\sqrt{d(c^2x^2+1)}\,(}{}$

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^2/d/x*(c^2*d*x^2+d)^(5/2)+a^2*c^2*x*(c^2*d*x^2+d)^(3/2)+3/2*(c^2*d*x^2+d
)^(1/2)*a^2*c^2*d*x+3/2*a^2*c^2*d^2*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^
(1/2))/(c^2*d)^(1/2)+1/4*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/x*(2*a
rcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*x^2*c^2-2*arcsinh(c*x)*c^3*x^3+2*arcsinh(c*
x)^3*x*c+c^2*x^2*(c^2*x^2+1)^(1/2)-4*arcsinh(c*x)^2*x*c+8*arcsinh(c*x)*ln(1
+c*x+(c^2*x^2+1)^(1/2))*x*c+8*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*x*c-
4*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2-arcsinh(c*x)*c*x+8*polylog(2,-c*x-(c^2*x
^2+1)^(1/2))*x*c+8*polylog(2,c*x+(c^2*x^2+1)^(1/2))*x*c)*d+1/4*a*b*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/x*(4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2
-2*c^3*x^3+6*arcsinh(c*x)^2*x*c-8*arcsinh(c*x)*c*x+8*ln((c*x+(c^2*x^2+1)^(1
/2))^2-1)*x*c-8*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-c*x)*d
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}{x^2}\,dx=\int \frac{(c^2dx^2+d)^{\frac{3}{2}}\,(b\,\text{arsinh}\,(cx)+a)^2}{x^2}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^2, x)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx=\int \frac{(d(c^2x^2+1))^{\frac{3}{2}}(a+b\,\text{asinh}\,(cx))^2}{x^2}\,dx$$

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2/x**2,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)^{3/2}}{x^2}\,dx$$

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x^2, x)
```

3.272 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^3} dx$

Optimal result

Integrand size = 28, antiderivative size = 541

$$\begin{aligned}\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^3}dx &= 2b^2c^2d\sqrt{d+c^2dx^2} \\ &- \frac{3abc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{3b^2c^3dx\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}} \\ &- \frac{bcd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x\sqrt{1+c^2x^2}} + \frac{bc^3dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}} \\ &+ \frac{3}{2}c^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 - \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{2x^2} \\ &- \frac{3c^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\ &- \frac{b^2c^2d\sqrt{d+c^2dx^2}\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{\sqrt{1+c^2x^2}} \\ &- \frac{3bc^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\operatorname{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\ &+ \frac{3bc^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\operatorname{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\ &+ \frac{3b^2c^2d\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\ &- \frac{3b^2c^2d\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}\end{aligned}$$

```
[Out] -1/2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^2+2*b^2*c^2*d*(c^2*d*x^2+d)
^(1/2)+3/2*c^2*d*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)-3*a*b*c^3*d*x*(c^
2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3*b^2*c^3*d*x*arcsinh(c*x)*(c^2*d*x^2+d)
```

```
^(1/2)/(c^2*x^2+1)^(1/2)-b*c*d*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x/(c^
2*x^2+1)^(1/2)+b*c^3*d*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)
^(1/2)-3*c^2*d*(a+b*arcsinh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x
^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b^2*c^2*d*arctanh((c^2*x^2+1)^(1/2))*(c^2*d*x
^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3*b*c^2*d*(a+b*arcsinh(c*x))*polylog(2,-c*x-(
c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+3*b*c^2*d*(a+b*arcs
inh(c*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^
(1/2)+3*b^2*c^2*d*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^
2*x^2+1)^(1/2)-3*b^2*c^2*d*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(
1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.536$, Rules used = {5807, 5806, 5816, 4267, 2611, 2320, 6724, 5772, 267, 14, 5803, 457, 81, 65, 214}

$$\begin{aligned}&\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx = \\
&-\frac{3c^2d\sqrt{c^2dx^2+d}\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}} \\
&-\frac{3bc^2d\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}} \\
&+\frac{3bc^2d\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}} \\
&+\frac{3}{2}c^2d\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bcd\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{x\sqrt{c^2x^2+1}} \\
&-\frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))^2}{2x^2}+\frac{bc^3dx\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}} \\
&-\frac{3abc^3dx\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}+\frac{3b^2c^2d\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{c^2x^2+1}} \\
&-\frac{3b^2c^2d\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{\sqrt{c^2x^2+1}}-\frac{3b^2c^3dx\text{arcsinh}(cx)\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}} \\
&-\frac{b^2c^2d\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}+2b^2c^2d\sqrt{c^2dx^2+d}\end{aligned}$$

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] 2*b^2*c^2*d*Sqrt[d + c^2*d*x^2] - (3*a*b*c^3*d*x*Sqrt[d + c^2*d*x^2])/Sqrt[
1 + c^2*x^2] - (3*b^2*c^3*d*x*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/Sqrt[1 + c^
2*x^2] - (b*c*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(x*Sqrt[1 + c^2*x
^2]) + (b*c^3*d*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^
```

```
2] + (3*c^2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/2 - ((d + c^2*d*x
^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(2*x^2) - (3*c^2*d*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b^2*c^2*
d*Sqrt[d + c^2*d*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/Sqrt[1 + c^2*x^2] - (3*b*
c^2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])
/Sqrt[1 + c^2*x^2] + (3*b*c^2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Po
lyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (3*b^2*c^2*d*Sqrt[d + c^2*d*x
^2]*PolyLog[3, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (3*b^2*c^2*d*Sqrt[d +
c^2*d*x^2]*PolyLog[3, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
```

```
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{2x^2} \\ &\quad+\frac{1}{2}(3c^2d)\int\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x}\,dx \\ &\quad+\frac{\left(bcd\sqrt{d+c^2dx^2}\right)\int\frac{(1+c^2x^2)(a+b\text{arcsinh}(cx))}{x^2}\,dx}{\sqrt{1+c^2x^2}}\end{aligned}$$

$$= -\frac{bcd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x\sqrt{1+c^2x^2}} + \frac{bc^3dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}} + \frac{3}{2}c^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 - \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{2x^2} + \frac{\left(3c^2d\sqrt{d+c^2dx^2}\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}} - \frac{\left(b^2c^2d\sqrt{d+c^2dx^2}\right)\int\frac{-1+c^2x^2}{x\sqrt{1+c^2x^2}}\,dx}{\sqrt{1+c^2x^2}} - \frac{\left(3bc^3d\sqrt{d+c^2dx^2}\right)\int(a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}}$$

$$= -\frac{3abc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{bcd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x\sqrt{1+c^2x^2}} + \frac{bc^3dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}} + \frac{3}{2}c^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 - \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{2x^2} + \frac{\left(3c^2d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^2\text{csch}(x)\,dx,x,\text{arcsinh}(cx)\right)}{2\sqrt{1+c^2x^2}} - \frac{\left(b^2c^2d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{-1+c^2x}{x\sqrt{1+c^2x}}\,dx,x,x^2\right)}{2\sqrt{1+c^2x^2}} - \frac{\left(3b^2c^3d\sqrt{d+c^2dx^2}\right)\int\text{arcsinh}(cx)\,dx}{\sqrt{1+c^2x^2}}$$

$$= -b^2c^2d\sqrt{d+c^2dx^2} - \frac{3abc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{3b^2c^3dx\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}} - \frac{bcd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x\sqrt{1+c^2x^2}} + \frac{bc^3dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}} + \frac{3}{2}c^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 - \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{2x^2} - \frac{3c^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{\left(3bc^2d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)\log\left(1-e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}} + \frac{\left(3bc^2d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)\log\left(1+e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}} + \frac{\left(b^2c^2d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+c^2x}}\,dx,x,x^2\right)}{2\sqrt{1+c^2x^2}} + \frac{\left(3b^2c^4d\sqrt{d+c^2dx^2}\right)\int\frac{x}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{1+c^2x^2}}$$

$$= 2b^2c^2d\sqrt{d+c^2dx^2} - \frac{3abc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{3b^2c^3dx\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}} - \frac{bcd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x\sqrt{1+c^2x^2}} + \frac{bc^3dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}} + \frac{3}{2}c^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 - \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{2x^2} - \frac{3c^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{3bc^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{3bc^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{\left(b^2d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx,x,\sqrt{1+c^2x^2}\right)}{\sqrt{1+c^2x^2}} + \frac{\left(3b^2c^2d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\text{PolyLog}(2,-e^x)\,dx,x,\text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}} - \frac{\left(3b^2c^2d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\text{PolyLog}(2,e^x)\,dx,x,\text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}}$$

$$= 2b^2c^2d\sqrt{d+c^2dx^2} - \frac{3abc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{3b^2c^3dx\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}} - \frac{bcd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x\sqrt{1+c^2x^2}} + \frac{bc^3dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}} + \frac{3}{2}c^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 - \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{2x^2} - \frac{3c^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{b^2c^2d\sqrt{d+c^2dx^2}\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{\sqrt{1+c^2x^2}} - \frac{3bc^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{3bc^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{\left(3b^2c^2d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{\left(3b^2c^2d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}$$

$$
\begin{aligned}
&= 2b^2c^2d\sqrt{d+c^2dx^2} - \frac{3abc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{3b^2c^3dx\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}} \\
&\quad - \frac{bcd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x\sqrt{1+c^2x^2}} + \frac{bc^3dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{3}{2}c^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 - \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{2x^2} \\
&\quad - \frac{3c^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad - \frac{b^2c^2d\sqrt{d+c^2dx^2}\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{\sqrt{1+c^2x^2}} \\
&\quad - \frac{3bc^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{3bc^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{3b^2c^2d\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad - \frac{3b^2c^2d\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 7.79 (sec) , antiderivative size = 771, normalized size of antiderivative = 1.43

$$
\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx = \left(a^2c^2d - \frac{a^2d}{2x^2}\right)\sqrt{d(1+c^2x^2)} + \frac{3}{2}a^2c^2d^{3/2}\log(x) - \frac{3}{2}a^2c^2d^{3/2}\log\left(a\right.
$$

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] (a^2*c^2*d - (a^2*d)/(2*x^2))*Sqrt[d*(1 + c^2*x^2)] + (3*a^2*c^2*d^(3/2)*Lo
g[x])/2 - (3*a^2*c^2*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]])/2 + (2
*a*b*c^2*d*Sqrt[d*(1 + c^2*x^2)]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] +
 ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh
[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))/
Sqrt[1 + c^2*x^2] + b^2*c^2*d*Sqrt[d*(1 + c^2*x^2)]*(2 - (2*c*x*ArcSinh[c*x
])/Sqrt[1 + c^2*x^2] + ArcSinh[c*x]^2 + (ArcSinh[c*x]^2*(Log[1 - E^(-ArcSin
h[c*x])] - Log[1 + E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2] + (2*ArcSinh[c*x]
*(PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))/Sqrt[1 +
 c^2*x^2] + (2*(PolyLog[3, -E^(-ArcSinh[c*x])] - PolyLog[3, E^(-ArcSinh[c*x
])]))/Sqrt[1 + c^2*x^2]) + (a*b*c^2*d*Sqrt[d*(1 + c^2*x^2)]*(-2*Coth[ArcSin
h[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]*Log[1 - E^
(-ArcSinh[c*x])] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 4*PolyLog[2,
 -E^(-ArcSinh[c*x])] - 4*PolyLog[2, E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Sech[
```

```
ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/(4*Sqrt[1 + c^2*x^2]) + (b^2*c
^2*d*Sqrt[d*(1 + c^2*x^2)]*(-4*ArcSinh[c*x]*Coth[ArcSinh[c*x]/2] - ArcSinh[
c*x]^2*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]^2*Log[1 - E^(-ArcSinh[c*x])]
 - 4*ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] + 8*Log[Tanh[ArcSinh[c*x]/2]
] + 8*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - 8*ArcSinh[c*x]*PolyLog[
2, E^(-ArcSinh[c*x])] + 8*PolyLog[3, -E^(-ArcSinh[c*x])] - 8*PolyLog[3, E^(
-ArcSinh[c*x])] - ArcSinh[c*x]^2*Sech[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]*Ta
nh[ArcSinh[c*x]/2]))/(8*Sqrt[1 + c^2*x^2])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 933, normalized size of antiderivative = 1.72

method	result
default	$a^2\left(-\frac{(c^2dx^2+d)^{\frac{5}{2}}}{2dx^2}+\frac{3c^2\left(\frac{(c^2dx^2+d)^{\frac{3}{2}}}{3}+d\left(\sqrt{c^2dx^2+d}-\sqrt{d}\ln\left(\frac{2d+2\sqrt{d}\sqrt{c^2dx^2+d}}{x}\right)\right)\right)}{2}\right)-\frac{3b^2\sqrt{d(c^2x^2+1)}\ \mathrm{polylog}\left(3,cx\right.}{\sqrt{c^2x^2+1}}$
parts	$a^2\left(-\frac{(c^2dx^2+d)^{\frac{5}{2}}}{2dx^2}+\frac{3c^2\left(\frac{(c^2dx^2+d)^{\frac{3}{2}}}{3}+d\left(\sqrt{c^2dx^2+d}-\sqrt{d}\ln\left(\frac{2d+2\sqrt{d}\sqrt{c^2dx^2+d}}{x}\right)\right)\right)}{2}\right)-\frac{3b^2\sqrt{d(c^2x^2+1)}\ \mathrm{polylog}\left(3,cx\right.}{\sqrt{c^2x^2+1}}$

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/2/d/x^2*(c^2*d*x^2+d)^(5/2)+3/2*c^2*(1/3*(c^2*d*x^2+d)^(3/2)+d*((c^
2*d*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x))))-3*b^2
*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(3,c*x+(c^2*x^2+1)^(1/2))*c
^2*d+3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(3,-c*x-(c^2*x^2+
1)^(1/2))*c^2*d+2*b^2*(d*(c^2*x^2+1))^(1/2)*c^4*d/(c^2*x^2+1)*x^2+1/2*b^2*(
d*(c^2*x^2+1))^(1/2)*c^2*d/(c^2*x^2+1)*arcsinh(c*x)^2-1/2*b^2*arcsinh(c*x)^
2*(d*(c^2*x^2+1))^(1/2)*d/x^2/(c^2*x^2+1)-2*b^2*(d*(c^2*x^2+1))^(1/2)*c^3*d
/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x-b^2*arcsinh(c*x)*(d*(c^2*x^2+1))^(1/2)*d/
x/(c^2*x^2+1)^(1/2)*c+2*b^2*(d*(c^2*x^2+1))^(1/2)*c^2*d/(c^2*x^2+1)+b^2*(d*
(c^2*x^2+1))^(1/2)*c^4*d/(c^2*x^2+1)*arcsinh(c*x)^2*x^2-2*b^2*(d*(c^2*x^2+1
))^(1/2)/(c^2*x^2+1)^(1/2)*arctanh(c*x+(c^2*x^2+1)^(1/2))*c^2*d+3/2*b^2*(d*
(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1
/2))*c^2*d-3/2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*l
n(1+c*x+(c^2*x^2+1)^(1/2))*c^2*d-3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1
/2)*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^2*d+3*b^2*(d*(c^2*x^2+
1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c
^2*d+a*b*(d*(c^2*x^2+1))^(1/2)*(2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2+3*
arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*x^2*c^2-3*arcsinh(c*x)*ln(1+c*x+(c
^2*x^2+1)^(1/2))*x^2*c^2-2*c^3*x^3+3*polylog(2,c*x+(c^2*x^2+1)^(1/2))*x^2*c
```

```
^2-3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*x^2*c^2-arcsinh(c*x)*(c^2*x^2+1)^(1/
2)-c*x)*d/(c^2*x^2+1)^(1/2)/x^2
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx=\int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{x^3}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^3, x)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx=\int \frac{(d(c^2x^2+1))^{\frac{3}{2}}(a+b\,\text{asinh}\,(cx))^2}{x^3}\,dx$$

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2/x**3,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2/x**3, x)
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx=\int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{x^3}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(3*c^2*d^(3/2)*arcsinh(1/(c*abs(x))) - (c^2*d*x^2 + d)^(3/2)*c^2 - 3*s
qrt(c^2*d*x^2 + d)*c^2*d + (c^2*d*x^2 + d)^(5/2)/(d*x^2))*a^2 + integrate((
c^2*d*x^2 + d)^(3/2)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/x^3 + 2*(c^2*d*x^2
+ d)^(3/2)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + c^2dx^2)^{3/2} (a + b\text{arcsinh}(cx))^2}{x^3} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + c^2dx^2)^{3/2} (a + b\text{arcsinh}(cx))^2}{x^3} dx = \int \frac{(a + b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2 + d)^{3/2}}{x^3} dx$$

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x^3, x)
```

3.273 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^4} dx$

Optimal result

Integrand size = 28, antiderivative size = 378

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^4} dx = -\frac{b^2c^2d\sqrt{d+c^2dx^2}}{3x} + \frac{b^2c^3d\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{3\sqrt{1+c^2x^2}} - \frac{bcd\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3x^2} - \frac{c^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x} + \frac{4c^3d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3\sqrt{1+c^2x^2}} - \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x^3} + \frac{c^3d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{3b\sqrt{1+c^2x^2}} + \frac{8bc^3d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)}{3\sqrt{1+c^2x^2}} - \frac{4b^2c^3d\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)}{3\sqrt{1+c^2x^2}}$$

```
[Out] -1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^3-1/3*b^2*c^2*d*(c^2*d*x^2+
d)^(1/2)/x-c^2*d*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x+1/3*b^2*c^3*d*a
rcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+4/3*c^3*d*(a+b*arcsinh(c*
x))^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/3*c^3*d*(a+b*arcsinh(c*x))^3*
(c^2*d*x^2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+8/3*b*c^3*d*(a+b*arcsinh(c*x))*ln(1
-1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-4/3*b^2
*c^3*d*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+
1)^(1/2)-1/3*b*c*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)*(c^2*d*x^2+d)^(1/2)
/x^2
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {5807, 5805, 5775, 3797, 2221, 2317, 2438, 5783, 5802, 283, 221}

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{x^4}\,dx = -\frac{c^2d\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^2}{x} - \frac{bcd\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{3x^2} - \frac{(c^2dx^2+d)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{3x^3} + \frac{c^3d\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^3}{3b\sqrt{c^2x^2+1}} + \frac{4c^3d\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^2}{3\sqrt{c^2x^2+1}} + \frac{8bc^3d\sqrt{c^2dx^2+d}\log\left(1-e^{-2\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx))}{3\sqrt{c^2x^2+1}} - \frac{4b^2c^3d\sqrt{c^2dx^2+d}\operatorname{PolyLog}\left(2,e^{-2\operatorname{arcsinh}(cx)}\right)}{3\sqrt{c^2x^2+1}} + \frac{b^2c^3d\operatorname{arcsinh}(cx)\sqrt{c^2dx^2+d}}{3\sqrt{c^2x^2+1}} - \frac{b^2c^2d\sqrt{c^2dx^2+d}}{3x}$$

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] -1/3*(b^2*c^2*d*Sqrt[d + c^2*d*x^2])/x + (b^2*c^3*d*Sqrt[d + c^2*d*x^2]*Arc
Sinh[c*x])/(3*Sqrt[1 + c^2*x^2]) - (b*c*d*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*
x^2]*(a + b*ArcSinh[c*x]))/(3*x^2) - (c^2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcS
inh[c*x])^2)/x + (4*c^3*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*Sq
rt[1 + c^2*x^2]) - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(3*x^3) +
 (c^3*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*Sqrt[1 + c^2*x^2])
 + (8*b*c^3*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Log[1 - E^(-2*ArcSin
h[c*x])])/(3*Sqrt[1 + c^2*x^2]) - (4*b^2*c^3*d*Sqrt[d + c^2*d*x^2]*PolyLog[
2, E^(-2*ArcSinh[c*x])])/(3*Sqrt[1 + c^2*x^2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5802

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSinh[c
*x])/(f*(m + 1))), x] + (-Dist[b*c*(d^p/(f*(m + 1))), Int[(f*x)^(m + 1)*(1
+ c^2*x^2)^(p - 1/2), x], x] - Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m + 2
)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]], Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x]
, x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], In
t[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Free
Q[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{3x^3}+\left(c^2d\right)\int\frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{x^2}\,dx \\
&\quad+\frac{\left(2bcd\sqrt{d+c^2dx^2}\right)\int\frac{(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))}{x^3}\,dx}{3\sqrt{1+c^2x^2}} \\
&= -\frac{bcd\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3x^2} \\
&\quad-\frac{c^2d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{x}-\frac{\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{3x^3} \\
&\quad+\frac{\left(b^2c^2d\sqrt{d+c^2dx^2}\right)\int\frac{\sqrt{1+c^2x^2}}{x^2}\,dx}{3\sqrt{1+c^2x^2}}+\frac{\left(2bc^3d\sqrt{d+c^2dx^2}\right)\int\frac{a+b\operatorname{arcsinh}(cx)}{x}\,dx}{3\sqrt{1+c^2x^2}} \\
&\quad+\frac{\left(2bc^3d\sqrt{d+c^2dx^2}\right)\int\frac{a+b\operatorname{arcsinh}(cx)}{x}\,dx}{\sqrt{1+c^2x^2}}+\frac{\left(c^4d\sqrt{d+c^2dx^2}\right)\int\frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{1+c^2x^2}}
\end{aligned}$$

$$=-\frac{b^2c^2d\sqrt{d+c^2dx^2}}{3x}-\frac{bcd\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3x^2}$$
$$-\frac{c^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x}$$
$$-\frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x^3}+\frac{c^3d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{3b\sqrt{1+c^2x^2}}$$
$$-\frac{\left(2c^3d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int x\coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{3\sqrt{1+c^2x^2}}$$
$$-\frac{\left(2c^3d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int x\coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}}$$
$$+\frac{\left(b^2c^4d\sqrt{d+c^2dx^2}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{3\sqrt{1+c^2x^2}}$$

$$=-\frac{b^2c^2d\sqrt{d+c^2dx^2}}{3x}+\frac{b^2c^3d\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{3\sqrt{1+c^2x^2}}$$
$$-\frac{bcd\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3x^2}$$
$$-\frac{c^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x}+\frac{4c^3d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3\sqrt{1+c^2x^2}}$$
$$-\frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x^3}+\frac{c^3d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{3b\sqrt{1+c^2x^2}}$$
$$+\frac{\left(4c^3d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{3\sqrt{1+c^2x^2}}$$
$$+\frac{\left(4c^3d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}}$$

$$
\begin{aligned}
&= -\frac{b^2c^2d\sqrt{d+c^2dx^2}}{3x} + \frac{b^2c^3d\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{3\sqrt{1+c^2x^2}} \\
&\quad - \frac{bcd\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3x^2} \\
&\quad - \frac{c^2d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{x} + \frac{4c^3d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{3\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{3x^3} + \frac{c^3d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^3}{3b\sqrt{1+c^2x^2}} \\
&\quad + \frac{8bc^3d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\log\left(1-e^{-2\operatorname{arcsinh}(cx)}\right)}{3\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(2bc^3d\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)dx, x, a+b\operatorname{arcsinh}(cx)\right)}{3\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(2bc^3d\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)dx, x, a+b\operatorname{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b^2c^2d\sqrt{d+c^2dx^2}}{3x} + \frac{b^2c^3d\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{3\sqrt{1+c^2x^2}} \\
&\quad - \frac{bcd\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3x^2} \\
&\quad - \frac{c^2d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{x} + \frac{4c^3d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{3\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{3x^3} + \frac{c^3d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^3}{3b\sqrt{1+c^2x^2}} \\
&\quad + \frac{8bc^3d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\log\left(1-e^{-2\operatorname{arcsinh}(cx)}\right)}{3\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(b^2c^3d\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}\right)}{3\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(b^2c^3d\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}\right)}{\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$= -\frac{b^2c^2d\sqrt{d+c^2dx^2}}{3x} + \frac{b^2c^3d\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{3\sqrt{1+c^2x^2}} - \frac{bcd\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3x^2} - \frac{c^2d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{x} + \frac{4c^3d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{3\sqrt{1+c^2x^2}} - \frac{(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{3x^3} + \frac{c^3d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^3}{3b\sqrt{1+c^2x^2}} + \frac{8bc^3d\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\log\left(1-e^{-2\operatorname{arcsinh}(cx)}\right)}{3\sqrt{1+c^2x^2}} - \frac{4b^2c^3d\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}\right)}{3\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.21

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{x^4}\,dx = \frac{-abcdx\sqrt{d+c^2dx^2} - a^2d\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2} - 4a^2c^2dx^2\sqrt{1+c^2x^2}}{}$$

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] (-(a*b*c*d*x*Sqrt[d + c^2*d*x^2]) - a^2*d*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*
x^2] - 4*a^2*c^2*d*x^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] - b^2*c^2*d*x^
2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + b*d*Sqrt[d + c^2*d*x^2]*(3*a*c^3*
x^3 - b*(-4*c^3*x^3 + Sqrt[1 + c^2*x^2] + 4*c^2*x^2*Sqrt[1 + c^2*x^2]))*Arc
Sinh[c*x]^2 + b^2*c^3*d*x^3*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 + b*d*Sqrt[d
 + c^2*d*x^2]*ArcSinh[c*x]*(-(b*c*x) - 2*a*Sqrt[1 + c^2*x^2]*(1 + 4*c^2*x^2
) + 8*b*c^3*x^3*Log[1 - E^(-2*ArcSinh[c*x])]) + 8*a*b*c^3*d*x^3*Sqrt[d + c^
2*d*x^2]*Log[c*x] + 3*a^2*c^3*d^(3/2)*x^3*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqr
t[d]*Sqrt[d + c^2*d*x^2]] - 4*b^2*c^3*d*x^3*Sqrt[d + c^2*d*x^2]*PolyLog[2,
E^(-2*ArcSinh[c*x])])/(3*x^3*Sqrt[1 + c^2*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1928 vs. 2(350) = 700.

Time = 0.37 (sec) , antiderivative size = 1929, normalized size of antiderivative = 5.10

method	result	size
default	Expression too large to display	1929
parts	Expression too large to display	1929

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^4,x,method=_RETURNVERBOSE)
[Out] 1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/x^3*(3*arcsinh(c*x)^2*x^3*c
^3-8*arcsinh(c*x)*c^3*x^3+8*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*x^3*c^3-8*arcsi
nh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-c*x)*d+8
/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1
/2))*c^3*d+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^3*c
^3*d+8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^
2+1)^(1/2))*c^3*d-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c
*x)^2*c^3*d+1/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)/(c^2*x
^2+1)^(1/2)*c^3-4/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^
3*c^6-1/3*a^2/d/x^3*(c^2*d*x^2+d)^(5/2)+2/3*a^2*c^4*x*(c^2*d*x^2+d)^(3/2)+8
/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x
^2+1)^(1/2))*c^3*d-29/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1
)*x^3/(c^2*x^2+1)*c^6-10/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^
2+1)*x/(c^2*x^2+1)*c^4+16/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x
^2+1)*x^3*arcsinh(c*x)*c^6+4/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^
2*x^2+1)*x*arcsinh(c*x)*c^4-1/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c
^2*x^2+1)/x/(c^2*x^2+1)*c^2-1/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c
^2*x^2+1)/x^3/(c^2*x^2+1)*arcsinh(c*x)^2-20/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(
24*c^4*x^4+9*c^2*x^2+1)*x^5/(c^2*x^2+1)*c^8+8*b^2*(d*(c^2*x^2+1))^(1/2)*d/(
24*c^4*x^4+9*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*c^7+3*b^2*(d*(c^2*x^2+1))^(1/
2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*c^5+4/3*b^2*(d*(c^2*x^2
+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^3-
3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*ar
csinh(c*x)*c^3-2/3*a^2*c^2/d/x*(c^2*d*x^2+d)^(5/2)+a^2*c^4*d*x*(c^2*d*x^2+d
)^(1/2)+a^2*c^4*d^2*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(
1/2)+32*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^4/(c^2*x^2+1
)^(1/2)*arcsinh(c*x)^2*c^7+12*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2
*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^5-8*b^2*(d*(c^2*x^2+1))^(1/2
)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^5-1/3*b^2
*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)/x^2/(c^2*x^2+1)^(1/2)*arc
sinh(c*x)*c-32*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^5/(c^
2*x^2+1)*arcsinh(c*x)^2*c^8-16/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*
c^2*x^2+1)*x^5/(c^2*x^2+1)*arcsinh(c*x)*c^8+8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(
c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^3*d-52*b^2*(d*(
```

```
c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)^2
*c^6-20/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^3/(c^2*x^2
+1)*arcsinh(c*x)*c^6-73/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2
+1)*x/(c^2*x^2+1)*arcsinh(c*x)^2*c^4-4/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^
4*x^4+9*c^2*x^2+1)*x/(c^2*x^2+1)*arcsinh(c*x)*c^4-14/3*b^2*(d*(c^2*x^2+1))^
(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)/x/(c^2*x^2+1)*arcsinh(c*x)^2*c^2
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}{x^4}\,dx=\int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{x^4}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^4, x)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}{x^4}\,dx=\int \frac{(d(c^2x^2+1))^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(cx))^2}{x^4}\,dx$$

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2/x**4,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2/x**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}{x^4}\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\text{arcsinh}(cx)\right)^2}{x^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\text{arcsinh}(cx)\right)^2}{x^4}\,dx = \int \frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)^2\left(d\,c^2\,x^2+d\right)^{3/2}}{x^4}\,dx$$

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x^4, x)
```

3.274 $\int x^3(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 625

$$\begin{aligned}\int x^3\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx &= -\frac{160b^2d^2\sqrt{d+c^2dx^2}}{3969c^4} \\ &+\frac{4abd^2x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}}-\frac{80b^2d^2(1+c^2x^2)\sqrt{d+c^2dx^2}}{11907c^4} \\ &-\frac{4b^2d^2(1+c^2x^2)^2\sqrt{d+c^2dx^2}}{1323c^4}-\frac{50b^2d^2(1+c^2x^2)^3\sqrt{d+c^2dx^2}}{27783c^4} \\ &+\frac{2b^2d^2(1+c^2x^2)^4\sqrt{d+c^2dx^2}}{729c^4}+\frac{4b^2d^2x\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{63c^3\sqrt{1+c^2x^2}} \\ &-\frac{2bd^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{189c\sqrt{1+c^2x^2}}-\frac{2bcd^2x^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{21\sqrt{1+c^2x^2}} \\ &-\frac{38bc^3d^2x^7\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{441\sqrt{1+c^2x^2}} \\ &-\frac{2bc^5d^2x^9\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{81\sqrt{1+c^2x^2}}-\frac{2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{63c^4} \\ &+\frac{d^2x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{63c^2}+\frac{1}{21}d^2x^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 \\ &+\frac{5}{63}dx^4\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2+\frac{1}{9}x^4\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\end{aligned}$$

```
[Out] 5/63*d*x^4*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2+1/9*x^4*(c^2*d*x^2+d)^(
5/2)*(a+b*arcsinh(c*x))^2-160/3969*b^2*d^2*(c^2*d*x^2+d)^(1/2)/c^4-80/11907
*b^2*d^2*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/c^4-4/1323*b^2*d^2*(c^2*x^2+1)^2*(
c^2*d*x^2+d)^(1/2)/c^4-50/27783*b^2*d^2*(c^2*x^2+1)^3*(c^2*d*x^2+d)^(1/2)/c
^4+2/729*b^2*d^2*(c^2*x^2+1)^4*(c^2*d*x^2+d)^(1/2)/c^4-2/63*d^2*(a+b*arcsin
h(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^4+1/63*d^2*x^2*(a+b*arcsinh(c*x))^2*(c^2*d*
```

```
x^2+d)^(1/2)/c^2+1/21*d^2*x^4*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)+4/63
*a*b*d^2*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)+4/63*b^2*d^2*x*arcsinh
(c*x)*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)-2/189*b*d^2*x^3*(a+b*arcsin
h(c*x))*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-2/21*b*c*d^2*x^5*(a+b*arcsi
nh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-38/441*b*c^3*d^2*x^7*(a+b*ar
csinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/81*b*c^5*d^2*x^9*(a+b*a
rcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {5808, 5806, 5812, 5798, 5772, 267, 5776, 272, 45, 14, 5803, 12, 457, 78, 276, 1265, 911, 1167}

$$\int x^3\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\frac{d^2x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{63c^2}-\frac{2bcd^2x^5\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{21\sqrt{c^2x^2+1}}+\frac{1}{21}d^2x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bd^2x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{189c\sqrt{c^2x^2+1}}+\frac{1}{9}x^4\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2+\frac{5}{63}dx^4\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{2bc^5d^2x^9\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{81\sqrt{c^2x^2+1}}-\frac{2d^2\sqrt{c^2dx^2+d}(a+bar}{63c^4}$$

```
[In] Int[x^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-160*b^2*d^2*Sqrt[d + c^2*d*x^2])/(3969*c^4) + (4*a*b*d^2*x*Sqrt[d + c^2*d
*x^2])/(63*c^3*Sqrt[1 + c^2*x^2]) - (80*b^2*d^2*(1 + c^2*x^2)*Sqrt[d + c^2*
d*x^2])/(11907*c^4) - (4*b^2*d^2*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/(1323
*c^4) - (50*b^2*d^2*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2])/(27783*c^4) + (2*b
^2*d^2*(1 + c^2*x^2)^4*Sqrt[d + c^2*d*x^2])/(729*c^4) + (4*b^2*d^2*x*Sqrt[d
 + c^2*d*x^2]*ArcSinh[c*x])/(63*c^3*Sqrt[1 + c^2*x^2]) - (2*b*d^2*x^3*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(189*c*Sqrt[1 + c^2*x^2]) - (2*b*c*d^2
*x^5*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(21*Sqrt[1 + c^2*x^2]) - (38
*b*c^3*d^2*x^7*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(441*Sqrt[1 + c^2*
x^2]) - (2*b*c^5*d^2*x^9*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(81*Sqrt
[1 + c^2*x^2]) - (2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(63*c^4
) + (d^2*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(63*c^2) + (d^2*x^
4*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/21 + (5*d*x^4*(d + c^2*d*x^2)
^(3/2)*(a + b*ArcSinh[c*x])^2)/63 + (x^4*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSi
nh[c*x])^2)/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
```

```
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{9}x^4\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{9}(5d)\int x^3\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx - \frac{\left(2bcd^2\sqrt{d+c^2dx^2}\right)\int x^4\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))\,dx}{9\sqrt{1+c^2x^2}}$$

$$= -\frac{2bcd^2x^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{45\sqrt{1+c^2x^2}} - \frac{4bc^3d^2x^7\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{63\sqrt{1+c^2x^2}} - \frac{2bc^5d^2x^9\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{81\sqrt{1+c^2x^2}} + \frac{5}{63}dx^4\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{9}x^4\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{21}\left(5d^2\right)\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx - \frac{\left(10bcd^2\sqrt{d+c^2dx^2}\right)\int x^4\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))\,dx}{63\sqrt{1+c^2x^2}} + \frac{\left(2b^2c^2d^2\sqrt{d+c^2dx^2}\right)\int \frac{x^5\left(63+90c^2x^2+35c^4x^4\right)}{315\sqrt{1+c^2x^2}}\,dx}{9\sqrt{1+c^2x^2}}$$

$$= -\frac{8bcd^2x^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{105\sqrt{1+c^2x^2}} - \frac{38bc^3d^2x^7\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{441\sqrt{1+c^2x^2}} - \frac{2bc^5d^2x^9\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{81\sqrt{1+c^2x^2}} + \frac{1}{21}d^2x^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{5}{63}dx^4\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{9}x^4\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2 + \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{21\sqrt{1+c^2x^2}} - \frac{\left(2bcd^2\sqrt{d+c^2dx^2}\right)\int x^4(a+b\text{arcsinh}(cx))\,dx}{21\sqrt{1+c^2x^2}} + \frac{\left(2b^2c^2d^2\sqrt{d+c^2dx^2}\right)\int \frac{x^5\left(63+90c^2x^2+35c^4x^4\right)}{\sqrt{1+c^2x^2}}\,dx}{2835\sqrt{1+c^2x^2}} + \frac{\left(10b^2c^2d^2\sqrt{d+c^2dx^2}\right)\int \frac{x^5\left(7+5c^2x^2\right)}{35\sqrt{1+c^2x^2}}\,dx}{63\sqrt{1+c^2x^2}}$$

$$= -\frac{2bcd^2x^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{21\sqrt{1+c^2x^2}} - \frac{38bc^3d^2x^7\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{441\sqrt{1+c^2x^2}} - \frac{2bc^5d^2x^9\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{81\sqrt{1+c^2x^2}} + \frac{d^2x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{63c^2} + \frac{1}{21}d^2x^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{5}{63}dx^4\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{9}x^4\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2 - \frac{\left(2d^2\sqrt{d+c^2dx^2}\right)\int\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{63c^2\sqrt{1+c^2x^2}} - \frac{\left(2bd^2\sqrt{d+c^2dx^2}\right)\int x^2(a+b\text{arcsinh}(cx))\,dx}{63c\sqrt{1+c^2x^2}} + \frac{\left(b^2c^2d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{x^2(63+90c^2x+35c^4x^2)}{\sqrt{1+c^2x}}\,dx,x,x^2\right)}{2835\sqrt{1+c^2x^2}} + \frac{\left(2b^2c^2d^2\sqrt{d+c^2dx^2}\right)\int\frac{x^5(7+5c^2x^2)}{\sqrt{1+c^2x^2}}\,dx}{441\sqrt{1+c^2x^2}} + \frac{\left(2b^2c^2d^2\sqrt{d+c^2dx^2}\right)\int\frac{x^5}{\sqrt{1+c^2x^2}}\,dx}{105\sqrt{1+c^2x^2}}$$

$$= -\frac{2bd^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{189c\sqrt{1+c^2x^2}} - \frac{2bcd^2x^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{21\sqrt{1+c^2x^2}} - \frac{38bc^3d^2x^7\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{441\sqrt{1+c^2x^2}} - \frac{2bc^5d^2x^9\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{81\sqrt{1+c^2x^2}} - \frac{2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{63c^4} + \frac{d^2x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{63c^2} + \frac{1}{21}d^2x^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{5}{63}dx^4\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{9}x^4\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2 + \frac{\left(2b^2d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(-\frac{1}{c^2}+\frac{x^2}{c^2}\right)^2(8+20x^2+35x^4)\,dx,x,\sqrt{1+c^2x^2}\right)}{2835\sqrt{1+c^2x^2}} + \frac{\left(2b^2d^2\sqrt{d+c^2dx^2}\right)\int\frac{x^3}{\sqrt{1+c^2x^2}}\,dx}{189\sqrt{1+c^2x^2}} + \frac{\left(4bd^2\sqrt{d+c^2dx^2}\right)\int(a+b\text{arcsinh}(cx))\,dx}{63c^3\sqrt{1+c^2x^2}} + \frac{\left(b^2c^2d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{x^2(7+5c^2x)}{\sqrt{1+c^2x}}\,dx,x,x^2\right)}{441\sqrt{1+c^2x^2}} + \frac{\left(b^2c^2d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{1+c^2x}}\,dx,x,x^2\right)}{105\sqrt{1+c^2x^2}}$$

$$= \frac{4abd^2x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}} - \frac{2bd^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{189c\sqrt{1+c^2x^2}} - \frac{2bcd^2x^5\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{21\sqrt{1+c^2x^2}} - \frac{38bc^3d^2x^7\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{441\sqrt{1+c^2x^2}} - \frac{2bc^5d^2x^9\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{81\sqrt{1+c^2x^2}} - \frac{2d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{63c^4} + \frac{d^2x^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{63c^2} + \frac{1}{21}d^2x^4\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2 + \frac{5}{63}dx^4\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{9}x^4\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))^2 + \frac{\left(2b^2d^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\left(\frac{8}{c^4}+\frac{4x^2}{c^4}+\frac{3x^4}{c^4}-\frac{50x^6}{c^4}+\frac{35x^8}{c^4}\right)dx, x, \sqrt{1+c^2x^2}\right)}{2835\sqrt{1+c^2x^2}} + \frac{\left(b^2d^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\frac{x}{\sqrt{1+c^2x}}dx, x, x^2\right)}{189\sqrt{1+c^2x^2}} + \frac{\left(4b^2d^2\sqrt{d+c^2dx^2}\right)\int\operatorname{arcsinh}(cx)\,dx}{63c^3\sqrt{1+c^2x^2}} + \frac{\left(b^2c^2d^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\left(\frac{2}{c^4\sqrt{1+c^2x}}+\frac{\sqrt{1+c^2x}}{c^4}-\frac{8(1+c^2x)^{3/2}}{c^4}+\frac{5(1+c^2x)^{5/2}}{c^4}\right)dx, x, x^2\right)}{441\sqrt{1+c^2x^2}} + \frac{\left(b^2c^2d^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\left(\frac{1}{c^4\sqrt{1+c^2x}}-\frac{2\sqrt{1+c^2x}}{c^4}+\frac{(1+c^2x)^{3/2}}{c^4}\right)dx, x, x^2\right)}{105\sqrt{1+c^2x^2}}$$

$$= \frac{134b^2d^2\sqrt{d+c^2dx^2}}{3969c^4} + \frac{4abd^2x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}} - \frac{122b^2d^2(1+c^2x^2)\sqrt{d+c^2dx^2}}{11907c^4} - \frac{4b^2d^2(1+c^2x^2)^2\sqrt{d+c^2dx^2}}{1323c^4} - \frac{50b^2d^2(1+c^2x^2)^3\sqrt{d+c^2dx^2}}{27783c^4} + \frac{2b^2d^2(1+c^2x^2)^4\sqrt{d+c^2dx^2}}{729c^4} + \frac{4b^2d^2x\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{63c^3\sqrt{1+c^2x^2}} - \frac{2bd^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{189c\sqrt{1+c^2x^2}} - \frac{2bcd^2x^5\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{21\sqrt{1+c^2x^2}} - \frac{38bc^3d^2x^7\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{441\sqrt{1+c^2x^2}} - \frac{2bc^5d^2x^9\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{81\sqrt{1+c^2x^2}} - \frac{2d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{63c^4} + \frac{d^2x^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{63c^2} + \frac{1}{21}d^2x^4\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2 + \frac{5}{63}dx^4\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{9}x^4\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))^2 + \frac{\left(b^2d^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\left(-\frac{1}{c^2\sqrt{1+c^2x}}+\frac{\sqrt{1+c^2x}}{c^2}\right)dx, x, x^2\right)}{189\sqrt{1+c^2x^2}} - \frac{\left(4b^2d^2\sqrt{d+c^2dx^2}\right)\int\frac{x}{\sqrt{1+c^2x^2}}\,dx}{63c^2\sqrt{1+c^2x^2}}$$

$$= -\frac{160b^2d^2\sqrt{d+c^2dx^2}}{3969c^4} + \frac{4abd^2x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}} - \frac{80b^2d^2(1+c^2x^2)\sqrt{d+c^2dx^2}}{11907c^4} - \frac{4b^2d^2(1+c^2x^2)^2\sqrt{d+c^2dx^2}}{1323c^4} - \frac{50b^2d^2(1+c^2x^2)^3\sqrt{d+c^2dx^2}}{27783c^4} + \frac{2b^2d^2(1+c^2x^2)^4\sqrt{d+c^2dx^2}}{729c^4} + \frac{4b^2d^2x\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{63c^3\sqrt{1+c^2x^2}} - \frac{2bd^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{189c\sqrt{1+c^2x^2}} - \frac{2bcd^2x^5\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{21\sqrt{1+c^2x^2}} - \frac{38bc^3d^2x^7\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{441\sqrt{1+c^2x^2}} - \frac{2bc^5d^2x^9\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{81\sqrt{1+c^2x^2}} - \frac{2d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{63c^4} + \frac{d^2x^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{63c^2} + \frac{1}{21}d^2x^4\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2 + \frac{5}{63}dx^4\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{9}x^4\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))^2$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.44

$$\int x^3(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \frac{d^2\sqrt{d+c^2dx^2}\Big(3969a^2(1+c^2x^2)^4(-2+7c^2x^2)-126abcx\sqrt{1+c^2x^2}(-126+21c^2x^2+\cdots}{}$$

```
[In] Integrate[x^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*Sqrt[d + c^2*d*x^2]*(3969*a^2*(1 + c^2*x^2)^4*(-2 + 7*c^2*x^2) - 126*a
*b*c*x*Sqrt[1 + c^2*x^2]*(-126 + 21*c^2*x^2 + 189*c^4*x^4 + 171*c^6*x^6 + 4
9*c^8*x^8) + 2*b^2*(-6140 - 7039*c^2*x^2 + 106*c^4*x^4 + 2152*c^6*x^6 + 149
0*c^8*x^8 + 343*c^10*x^10) - 126*b*(-63*a*(1 + c^2*x^2)^4*(-2 + 7*c^2*x^2)
+ b*c*x*Sqrt[1 + c^2*x^2]*(-126 + 21*c^2*x^2 + 189*c^4*x^4 + 171*c^6*x^6 +
49*c^8*x^8))*ArcSinh[c*x] + 3969*b^2*(1 + c^2*x^2)^4*(-2 + 7*c^2*x^2)*ArcSi
nh[c*x]^2))/(250047*c^4*(1 + c^2*x^2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2013 vs. 2(549) = 1098.

Time = 0.40 (sec) , antiderivative size = 2014, normalized size of antiderivative = 3.22

method	result	size
default	Expression too large to display	2014
parts	Expression too large to display	2014

```
[In] int(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(1/9*x^2*(c^2*d*x^2+d)^(7/2)/c^2/d-2/63/d/c^4*(c^2*d*x^2+d)^(7/2))+b^2*
(1/373248*(d*(c^2*x^2+1))^(1/2)*(256*c^10*x^10+256*c^9*x^9*(c^2*x^2+1)^(1/2
)+704*c^8*x^8+576*c^7*x^7*(c^2*x^2+1)^(1/2)+688*c^6*x^6+432*c^5*x^5*(c^2*x^
2+1)^(1/2)+280*c^4*x^4+120*c^3*x^3*(c^2*x^2+1)^(1/2)+41*c^2*x^2+9*c*x*(c^2*
x^2+1)^(1/2)+1)*(81*arcsinh(c*x)^2-18*arcsinh(c*x)+2)*d^2/c^4/(c^2*x^2+1)+3
/175616*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*
c^6*x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1
/2)+25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1)*(49*arcsinh(c*x)^2-14*arcsinh(c*x
)+2)*d^2/c^4/(c^2*x^2+1)-1/1728*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*
(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2-6*
arcsinh(c*x)+2)*d^2/c^4/(c^2*x^2+1)-3/256*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*
x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)*d^2/c^4/(c^2*x^2+1
)-3/256*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*
x)^2+2*arcsinh(c*x)+2)*d^2/c^4/(c^2*x^2+1)-1/1728*(d*(c^2*x^2+1))^(1/2)*(4*
```

```
c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(9
*arcsinh(c*x)^2+6*arcsinh(c*x)+2)*d^2/c^4/(c^2*x^2+1)+3/175616*(d*(c^2*x^2+
1))^(1/2)*(64*c^8*x^8-64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6-112*c^5*x^5*
(c^2*x^2+1)^(1/2)+104*c^4*x^4-56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2-7*c*x
*(c^2*x^2+1)^(1/2)+1)*(49*arcsinh(c*x)^2+14*arcsinh(c*x)+2)*d^2/c^4/(c^2*x^
2+1)+1/373248*(d*(c^2*x^2+1))^(1/2)*(256*c^10*x^10-256*c^9*x^9*(c^2*x^2+1)^
(1/2)+704*c^8*x^8-576*c^7*x^7*(c^2*x^2+1)^(1/2)+688*c^6*x^6-432*c^5*x^5*(c^
2*x^2+1)^(1/2)+280*c^4*x^4-120*c^3*x^3*(c^2*x^2+1)^(1/2)+41*c^2*x^2-9*c*x*(
c^2*x^2+1)^(1/2)+1)*(81*arcsinh(c*x)^2+18*arcsinh(c*x)+2)*d^2/c^4/(c^2*x^2+
1))+2*a*b*(1/41472*(d*(c^2*x^2+1))^(1/2)*(256*c^10*x^10+256*c^9*x^9*(c^2*x^
2+1)^(1/2)+704*c^8*x^8+576*c^7*x^7*(c^2*x^2+1)^(1/2)+688*c^6*x^6+432*c^5*x^
5*(c^2*x^2+1)^(1/2)+280*c^4*x^4+120*c^3*x^3*(c^2*x^2+1)^(1/2)+41*c^2*x^2+9*
c*x*(c^2*x^2+1)^(1/2)+1)*(-1+9*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)+3/25088*(d
*(c^2*x^2+1))^(1/2)*(64*c^8*x^8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6+11
2*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2
*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+7*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)-1/5
76*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3
*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)-3/256*(d*
(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))*d^2/
c^4/(c^2*x^2+1)-3/256*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+
1)*(arcsinh(c*x)+1)*d^2/c^4/(c^2*x^2+1)-1/576*(d*(c^2*x^2+1))^(1/2)*(4*c^4*
x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(3*arc
sinh(c*x)+1)*d^2/c^4/(c^2*x^2+1)+3/25088*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8-
64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6-112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*
c^4*x^4-56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2-7*c*x*(c^2*x^2+1)^(1/2)+1)*
(1+7*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)+1/41472*(d*(c^2*x^2+1))^(1/2)*(256*c
^10*x^10-256*c^9*x^9*(c^2*x^2+1)^(1/2)+704*c^8*x^8-576*c^7*x^7*(c^2*x^2+1)^
(1/2)+688*c^6*x^6-432*c^5*x^5*(c^2*x^2+1)^(1/2)+280*c^4*x^4-120*c^3*x^3*(c^
2*x^2+1)^(1/2)+41*c^2*x^2-9*c*x*(c^2*x^2+1)^(1/2)+1)*(1+9*arcsinh(c*x))*d^2
/c^4/(c^2*x^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 525, normalized size of antiderivative = 0.84

$$\int x^3\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \frac{3969\left(7\,b^2c^{10}d^2x^{10}+26\,b^2c^8d^2x^8+34\,b^2c^6d^2x^6+16\,b^2c^4d^2x^4-b^2c^2d^2x^2-2\,b^2d^2\right)\sqrt{c^2dx}}{}$$

```
[In] integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] 1/250047*(3969*(7*b^2*c^10*d^2*x^10 + 26*b^2*c^8*d^2*x^8 + 34*b^2*c^6*d^2*x
^6 + 16*b^2*c^4*d^2*x^4 - b^2*c^2*d^2*x^2 - 2*b^2*d^2)*sqrt(c^2*d*x^2 + d)*
log(c*x + sqrt(c^2*x^2 + 1))^2 + 126*(441*a*b*c^10*d^2*x^10 + 1638*a*b*c^8*
d^2*x^8 + 2142*a*b*c^6*d^2*x^6 + 1008*a*b*c^4*d^2*x^4 - 63*a*b*c^2*d^2*x^2
- 126*a*b*d^2 - (49*b^2*c^9*d^2*x^9 + 171*b^2*c^7*d^2*x^7 + 189*b^2*c^5*d^2
*x^5 + 21*b^2*c^3*d^2*x^3 - 126*b^2*c*d^2*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*
x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (343*(81*a^2 + 2*b^2)*c^10*d^2*x^10
 + 2*(51597*a^2 + 1490*b^2)*c^8*d^2*x^8 + 2*(67473*a^2 + 2152*b^2)*c^6*d^2*
x^6 + 4*(15876*a^2 + 53*b^2)*c^4*d^2*x^4 - (3969*a^2 + 14078*b^2)*c^2*d^2*x
^2 - 2*(3969*a^2 + 6140*b^2)*d^2 - 126*(49*a*b*c^9*d^2*x^9 + 171*a*b*c^7*d^
2*x^7 + 189*a*b*c^5*d^2*x^5 + 21*a*b*c^3*d^2*x^3 - 126*a*b*c*d^2*x)*sqrt(c^
2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^6*x^2 + c^4)
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))^2 \, dx = \text{Timed out}$$

```
[In] integrate(x**3*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.62

$$\int x^3 (d + c^2dx^2)^{5/2} (a+b\text{arcsinh}(cx))^2 \, dx = \frac{1}{63} \left(\frac{7 (c^2dx^2+d)^{\frac{7}{2}} x^2}{c^2 d} - \frac{2 (c^2dx^2+d)^{\frac{7}{2}}}{c^4 d}\right) b^2 \operatorname{arsinh}(cx)^2 + \frac{2}{63} \left(\frac{7 (c^2dx^2+d)^{\frac{7}{2}} x^2}{c^2 d} - \frac{2 (c^2dx^2+d)^{\frac{7}{2}}}{c^4 d}\right) ab \operatorname{arsinh}(cx) + \frac{1}{63} \left(\frac{7 (c^2dx^2+d)^{\frac{7}{2}} x^2}{c^2 d} - \frac{2 (c^2dx^2+d)^{\frac{7}{2}}}{c^4 d}\right) a^2 + \frac{2}{250047} b^2 \left(\frac{343 \sqrt{c^2x^2+1} c^6 d^{\frac{5}{2}} x^8 + 1147 \sqrt{c^2x^2+1} c^4 d^{\frac{5}{2}} x^6 + 1005 \sqrt{c^2x^2+1} c^2 d^{\frac{5}{2}} x^4 - 899 \sqrt{c^2x^2+1} d^{\frac{5}{2}} x^2}{c^2} \right. - \frac{2 \left(49 c^8 d^{\frac{5}{2}} x^9 + 171 c^6 d^{\frac{5}{2}} x^7 + 189 c^4 d^{\frac{5}{2}} x^5 + 21 c^2 d^{\frac{5}{2}} x^3 - 126 d^{\frac{5}{2}} x\right) ab}{3969 c^3}$$

```
[In] integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
```

```
[Out] 1/63*(7*(c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(7/2)/(c^4*d)
)*b^2*arcsinh(c*x)^2 + 2/63*(7*(c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) - 2*(c^2*d
*x^2 + d)^(7/2)/(c^4*d))*a*b*arcsinh(c*x) + 1/63*(7*(c^2*d*x^2 + d)^(7/2)*x
^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(7/2)/(c^4*d))*a^2 + 2/250047*b^2*((343*sqrt
(c^2*x^2 + 1)*c^6*d^(5/2)*x^8 + 1147*sqrt(c^2*x^2 + 1)*c^4*d^(5/2)*x^6 + 10
05*sqrt(c^2*x^2 + 1)*c^2*d^(5/2)*x^4 - 899*sqrt(c^2*x^2 + 1)*d^(5/2)*x^2 -
6140*sqrt(c^2*x^2 + 1)*d^(5/2)/c^2)/c^2 - 63*(49*c^8*d^(5/2)*x^9 + 171*c^6*
d^(5/2)*x^7 + 189*c^4*d^(5/2)*x^5 + 21*c^2*d^(5/2)*x^3 - 126*d^(5/2)*x)*arc
sinh(c*x)/c^3) - 2/3969*(49*c^8*d^(5/2)*x^9 + 171*c^6*d^(5/2)*x^7 + 189*c^4
*d^(5/2)*x^5 + 21*c^2*d^(5/2)*x^3 - 126*d^(5/2)*x)*a*b/c^3
```

Giac [F(-2)]

Exception generated.

$$\int x^3 \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))^2\,dx = \int x^3\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^{5/2}\,dx$$

```
[In] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2), x)
```

3.275 $\int x^2(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 536

$$\begin{aligned}
\int x^2(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx &= -\frac{359b^2d^2x\sqrt{d+c^2dx^2}}{36864c^2} \\
&+\frac{1079b^2d^2x^3\sqrt{d+c^2dx^2}}{55296}+\frac{209b^2c^2d^2x^5\sqrt{d+c^2dx^2}}{13824}+\frac{1}{256}b^2c^4d^2x^7\sqrt{d+c^2dx^2} \\
&+\frac{359b^2d^2\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{36864c^3\sqrt{1+c^2x^2}}-\frac{5bd^2x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{128c\sqrt{1+c^2x^2}} \\
&-\frac{59bcd^2x^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{384\sqrt{1+c^2x^2}} \\
&-\frac{17bc^3d^2x^6\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{144\sqrt{1+c^2x^2}} \\
&-\frac{bc^5d^2x^8\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{32\sqrt{1+c^2x^2}} \\
&+\frac{5d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{128c^2}+\frac{5}{64}d^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 \\
&+\frac{5}{48}dx^3(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2 \\
&+\frac{1}{8}x^3(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2-\frac{5d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{384bc^3\sqrt{1+c^2x^2}}
\end{aligned}$$

```
[Out] 5/48*d*x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2+1/8*x^3*(c^2*d*x^2+d)^(
5/2)*(a+b*arcsinh(c*x))^2-359/36864*b^2*d^2*x*(c^2*d*x^2+d)^(1/2)/c^2+1079/
55296*b^2*d^2*x^3*(c^2*d*x^2+d)^(1/2)+209/13824*b^2*c^2*d^2*x^5*(c^2*d*x^2+
d)^(1/2)+1/256*b^2*c^4*d^2*x^7*(c^2*d*x^2+d)^(1/2)+5/128*d^2*x*(a+b*arcsinh
(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^2+5/64*d^2*x^3*(a+b*arcsinh(c*x))^2*(c^2*d*x
^2+d)^(1/2)+359/36864*b^2*d^2*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2
```

```
+1)^(1/2)-5/128*b*d^2*x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2
+1)^(1/2)-59/384*b*c*d^2*x^4*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^
2+1)^(1/2)-17/144*b*c^3*d^2*x^6*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2
*x^2+1)^(1/2)-1/32*b*c^5*d^2*x^8*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^
2*x^2+1)^(1/2)-5/384*d^2*(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^(1/2)/b/c^3/(c^
2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5808, 5806, 5812, 5783, 5776, 327, 221, 14, 5803, 12, 470, 272, 45, 1281}

$$\int x^2\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=-\frac{5bd^2x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{128c\sqrt{c^2x^2+1}}+\frac{5d^2x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{128c^2}-\frac{59bcd^2x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{384\sqrt{c^2x^2+1}}+\frac{5}{64}d^2x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2+\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2+\frac{5}{48}dx^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{bc^5d^2x^8\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{32\sqrt{c^2x^2+1}}-\frac{5d^2\sqrt{c^2dx^2+d}(a+b\text{arcsi}}{384bc^3\sqrt{c^2x^2+1}}$$

```
[In] Int[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-359*b^2*d^2*x*Sqrt[d + c^2*d*x^2])/(36864*c^2) + (1079*b^2*d^2*x^3*Sqrt[d
 + c^2*d*x^2])/55296 + (209*b^2*c^2*d^2*x^5*Sqrt[d + c^2*d*x^2])/13824 + (b
^2*c^4*d^2*x^7*Sqrt[d + c^2*d*x^2])/256 + (359*b^2*d^2*Sqrt[d + c^2*d*x^2]*
ArcSinh[c*x])/(36864*c^3*Sqrt[1 + c^2*x^2]) - (5*b*d^2*x^2*Sqrt[d + c^2*d*x
^2]*(a + b*ArcSinh[c*x]))/(128*c*Sqrt[1 + c^2*x^2]) - (59*b*c*d^2*x^4*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(384*Sqrt[1 + c^2*x^2]) - (17*b*c^3*d^
2*x^6*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(144*Sqrt[1 + c^2*x^2]) - (
b*c^5*d^2*x^8*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(32*Sqrt[1 + c^2*x^
2]) + (5*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(128*c^2) + (5*d
^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/64 + (5*d*x^3*(d + c^2*d
*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/48 + (x^3*(d + c^2*d*x^2)^(5/2)*(a + b*
ArcSinh[c*x])^2)/8 - (5*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(38
4*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
```

```
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{8}x^3\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2 \\
&\quad+\frac{1}{8}(5d)\int x^2\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx \\
&\quad-\frac{\left(bcd^2\sqrt{d+c^2dx^2}\right)\int x^3\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))\,dx}{4\sqrt{1+c^2x^2}} \\
&= -\frac{bcd^2x^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{16\sqrt{1+c^2x^2}}-\frac{bc^3d^2x^6\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{12\sqrt{1+c^2x^2}} \\
&\quad-\frac{bc^5d^2x^8\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{32\sqrt{1+c^2x^2}} \\
&\quad+\frac{5}{48}dx^3\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2+\frac{1}{8}x^3\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2 \\
&\quad+\frac{1}{16}\left(5d^2\right)\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx \\
&\quad-\frac{\left(5bcd^2\sqrt{d+c^2dx^2}\right)\int x^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))\,dx}{24\sqrt{1+c^2x^2}} \\
&\quad+\frac{\left(b^2c^2d^2\sqrt{d+c^2dx^2}\right)\int\frac{x^4\left(6+8c^2x^2+3c^4x^4\right)}{24\sqrt{1+c^2x^2}}\,dx}{4\sqrt{1+c^2x^2}}
\end{aligned}$$

$$= -\frac{11bcd^2x^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{96\sqrt{1+c^2x^2}} - \frac{17bc^3d^2x^6\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{144\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^8\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{32\sqrt{1+c^2x^2}} + \frac{5}{64}d^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{5}{48}dx^3\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{8}x^3\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2 + \frac{\left(5d^2\sqrt{d+c^2dx^2}\right)\int\frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}dx}{64\sqrt{1+c^2x^2}} - \frac{\left(5bcd^2\sqrt{d+c^2dx^2}\right)\int x^3(a+b\text{arcsinh}(cx))\,dx}{32\sqrt{1+c^2x^2}} + \frac{\left(b^2c^2d^2\sqrt{d+c^2dx^2}\right)\int\frac{x^4(6+8c^2x^2+3c^4x^4)}{\sqrt{1+c^2x^2}}dx}{96\sqrt{1+c^2x^2}} + \frac{\left(5b^2c^2d^2\sqrt{d+c^2dx^2}\right)\int\frac{x^4(3+2c^2x^2)}{12\sqrt{1+c^2x^2}}dx}{24\sqrt{1+c^2x^2}}$$

$$= \frac{1}{256}b^2c^4d^2x^7\sqrt{d+c^2dx^2} - \frac{59bcd^2x^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{384\sqrt{1+c^2x^2}} - \frac{17bc^3d^2x^6\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{144\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^8\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{32\sqrt{1+c^2x^2}} + \frac{5d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{128c^2} + \frac{5}{64}d^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{5}{48}dx^3\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{8}x^3\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2 + \frac{\left(b^2d^2\sqrt{d+c^2dx^2}\right)\int\frac{x^4(48c^2+43c^4x^2)}{\sqrt{1+c^2x^2}}dx}{768\sqrt{1+c^2x^2}} - \frac{\left(5d^2\sqrt{d+c^2dx^2}\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}dx}{128c^2\sqrt{1+c^2x^2}} - \frac{\left(5bd^2\sqrt{d+c^2dx^2}\right)\int x(a+b\text{arcsinh}(cx))\,dx}{64c\sqrt{1+c^2x^2}} + \frac{\left(5b^2c^2d^2\sqrt{d+c^2dx^2}\right)\int\frac{x^4(3+2c^2x^2)}{\sqrt{1+c^2x^2}}dx}{288\sqrt{1+c^2x^2}} + \frac{\left(5b^2c^2d^2\sqrt{d+c^2dx^2}\right)\int\frac{x^4}{\sqrt{1+c^2x^2}}dx}{128\sqrt{1+c^2x^2}}$$

$$= \frac{5}{512} b^2 d^2 x^3 \sqrt{d+c^2 d x^2} + \frac{209 b^2 c^2 d^2 x^5 \sqrt{d+c^2 d x^2}}{13824} + \frac{1}{256} b^2 c^4 d^2 x^7 \sqrt{d+c^2 d x^2} - \frac{5 b d^2 x^2 \sqrt{d+c^2 d x^2} (a+b \operatorname{arcsinh}(cx))}{128 c \sqrt{1+c^2 x^2}} - \frac{59 b c d^2 x^4 \sqrt{d+c^2 d x^2} (a+b \operatorname{arcsinh}(cx))}{384 \sqrt{1+c^2 x^2}} - \frac{17 b c^3 d^2 x^6 \sqrt{d+c^2 d x^2} (a+b \operatorname{arcsinh}(cx))}{144 \sqrt{1+c^2 x^2}} - \frac{b c^5 d^2 x^8 \sqrt{d+c^2 d x^2} (a+b \operatorname{arcsinh}(cx))}{32 \sqrt{1+c^2 x^2}} + \frac{5 d^2 x \sqrt{d+c^2 d x^2} (a+b \operatorname{arcsinh}(cx))^2}{128 c^2} + \frac{5}{64} d^2 x^3 \sqrt{d+c^2 d x^2} (a+b \operatorname{arcsinh}(cx))^2 + \frac{5}{48} d x^3 \left(d+c^2 d x^2\right)^{3/2} (a+b \operatorname{arcsinh}(cx))^2 + \frac{1}{8} x^3 \left(d+c^2 d x^2\right)^{5/2} (a+b \operatorname{arcsinh}(cx))^2 - \frac{5 d^2 \sqrt{d+c^2 d x^2} (a+b \operatorname{arcsinh}(cx))^3}{384 b c^3 \sqrt{1+c^2 x^2}} - \frac{\left(15 b^2 d^2 \sqrt{d+c^2 d x^2}\right) \int \frac{x^2}{\sqrt{1+c^2 x^2}} \, dx}{512 \sqrt{1+c^2 x^2}} + \frac{\left(5 b^2 d^2 \sqrt{d+c^2 d x^2}\right) \int \frac{x^2}{\sqrt{1+c^2 x^2}} \, dx}{128 \sqrt{1+c^2 x^2}} + \frac{\left(73 b^2 c^2 d^2 \sqrt{d+c^2 d x^2}\right) \int \frac{x^4}{\sqrt{1+c^2 x^2}} \, dx}{4608 \sqrt{1+c^2 x^2}} + \frac{\left(5 b^2 c^2 d^2 \sqrt{d+c^2 d x^2}\right) \int \frac{x^4}{\sqrt{1+c^2 x^2}} \, dx}{216 \sqrt{1+c^2 x^2}}$$

$$= \frac{5 b^2 d^2 x \sqrt{d+c^2 d x^2}}{1024 c^2} + \frac{1079 b^2 d^2 x^3 \sqrt{d+c^2 d x^2}}{55296} + \frac{209 b^2 c^2 d^2 x^5 \sqrt{d+c^2 d x^2}}{13824} + \frac{1}{256} b^2 c^4 d^2 x^7 \sqrt{d+c^2 d x^2} - \frac{5 b d^2 x^2 \sqrt{d+c^2 d x^2} (a+b \operatorname{arcsinh}(cx))}{128 c \sqrt{1+c^2 x^2}} - \frac{59 b c d^2 x^4 \sqrt{d+c^2 d x^2} (a+b \operatorname{arcsinh}(cx))}{384 \sqrt{1+c^2 x^2}} - \frac{17 b c^3 d^2 x^6 \sqrt{d+c^2 d x^2} (a+b \operatorname{arcsinh}(cx))}{144 \sqrt{1+c^2 x^2}} - \frac{b c^5 d^2 x^8 \sqrt{d+c^2 d x^2} (a+b \operatorname{arcsinh}(cx))}{32 \sqrt{1+c^2 x^2}} + \frac{5 d^2 x \sqrt{d+c^2 d x^2} (a+b \operatorname{arcsinh}(cx))^2}{128 c^2} + \frac{5}{64} d^2 x^3 \sqrt{d+c^2 d x^2} (a+b \operatorname{arcsinh}(cx))^2 + \frac{5}{48} d x^3 \left(d+c^2 d x^2\right)^{3/2} (a+b \operatorname{arcsinh}(cx))^2 + \frac{1}{8} x^3 \left(d+c^2 d x^2\right)^{5/2} (a+b \operatorname{arcsinh}(cx))^2 - \frac{5 d^2 \sqrt{d+c^2 d x^2} (a+b \operatorname{arcsinh}(cx))^3}{384 b c^3 \sqrt{1+c^2 x^2}} - \frac{\left(73 b^2 d^2 \sqrt{d+c^2 d x^2}\right) \int \frac{x^2}{\sqrt{1+c^2 x^2}} \, dx}{6144 \sqrt{1+c^2 x^2}} - \frac{\left(5 b^2 d^2 \sqrt{d+c^2 d x^2}\right) \int \frac{x^2}{\sqrt{1+c^2 x^2}} \, dx}{288 \sqrt{1+c^2 x^2}} + \frac{\left(15 b^2 d^2 \sqrt{d+c^2 d x^2}\right) \int \frac{1}{\sqrt{1+c^2 x^2}} \, dx}{1024 c^2 \sqrt{1+c^2 x^2}} - \frac{\left(5 b^2 d^2 \sqrt{d+c^2 d x^2}\right) \int \frac{1}{\sqrt{1+c^2 x^2}} \, dx}{256 c^2 \sqrt{1+c^2 x^2}}$$

$$= -\frac{359b^2d^2x\sqrt{d+c^2dx^2}}{36864c^2} + \frac{1079b^2d^2x^3\sqrt{d+c^2dx^2}}{55296} + \frac{209b^2c^2d^2x^5\sqrt{d+c^2dx^2}}{13824} + \frac{1}{256}b^2c^4d^2x^7\sqrt{d+c^2dx^2} - \frac{5b^2d^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{1024c^3\sqrt{1+c^2x^2}} - \frac{5bd^2x^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{128c\sqrt{1+c^2x^2}} - \frac{59bcd^2x^4\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{384\sqrt{1+c^2x^2}} - \frac{17bc^3d^2x^6\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{144\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^8\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{32\sqrt{1+c^2x^2}} + \frac{5d^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{128c^2} + \frac{5}{64}d^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2 + \frac{5}{48}dx^3\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{8}x^3\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))^2 - \frac{5d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^3}{384bc^3\sqrt{1+c^2x^2}} + \frac{\left(73b^2d^2\sqrt{d+c^2dx^2}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{12288c^2\sqrt{1+c^2x^2}} + \frac{\left(5b^2d^2\sqrt{d+c^2dx^2}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{576c^2\sqrt{1+c^2x^2}}$$

$$= -\frac{359b^2d^2x\sqrt{d+c^2dx^2}}{36864c^2} + \frac{1079b^2d^2x^3\sqrt{d+c^2dx^2}}{55296} + \frac{209b^2c^2d^2x^5\sqrt{d+c^2dx^2}}{13824} + \frac{1}{256}b^2c^4d^2x^7\sqrt{d+c^2dx^2} + \frac{359b^2d^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{36864c^3\sqrt{1+c^2x^2}} - \frac{5bd^2x^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{128c\sqrt{1+c^2x^2}} - \frac{59bcd^2x^4\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{384\sqrt{1+c^2x^2}} - \frac{17bc^3d^2x^6\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{144\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^8\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{32\sqrt{1+c^2x^2}} + \frac{5d^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{128c^2} + \frac{5}{64}d^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2 + \frac{5}{48}dx^3\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{8}x^3\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))^2 - \frac{5d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^3}{384bc^3\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 2.84 (sec) , antiderivative size = 619, normalized size of antiderivative = 1.15

$$\int x^2\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \frac{d^2\Big(34560a^2cx\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2} + 271872a^2c^3x^3\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2} + 313344a^2c}{}$$

```
[In] Integrate[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*(34560*a^2*c*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 271872*a^2*c^3*
x^3*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 313344*a^2*c^5*x^5*Sqrt[1 + c^2
*x^2]*Sqrt[d + c^2*d*x^2] + 110592*a^2*c^7*x^7*Sqrt[1 + c^2*x^2]*Sqrt[d + c
^2*d*x^2] - 11520*b^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 + 13824*a*b*Sqrt[d
 + c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 3456*a*b*Sqrt[d + c^2*d*x^2]*Cosh[4*Ar
cSinh[c*x]] - 1536*a*b*Sqrt[d + c^2*d*x^2]*Cosh[6*ArcSinh[c*x]] - 216*a*b*S
qrt[d + c^2*d*x^2]*Cosh[8*ArcSinh[c*x]] - 34560*a^2*Sqrt[d]*Sqrt[1 + c^2*x^
2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 6912*b^2*Sqrt[d + c^2*d*x^2]*
Sinh[2*ArcSinh[c*x]] + 864*b^2*Sqrt[d + c^2*d*x^2]*Sinh[4*ArcSinh[c*x]] + 2
56*b^2*Sqrt[d + c^2*d*x^2]*Sinh[6*ArcSinh[c*x]] + 27*b^2*Sqrt[d + c^2*d*x^2
]*Sinh[8*ArcSinh[c*x]] + 24*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(576*b*Cosh[
2*ArcSinh[c*x]] - 144*b*Cosh[4*ArcSinh[c*x]] - 64*b*Cosh[6*ArcSinh[c*x]] -
9*b*Cosh[8*ArcSinh[c*x]] - 1152*a*Sinh[2*ArcSinh[c*x]] + 576*a*Sinh[4*ArcSi
nh[c*x]] + 384*a*Sinh[6*ArcSinh[c*x]] + 72*a*Sinh[8*ArcSinh[c*x]]) + 288*b*
Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2*(-120*a - 48*b*Sinh[2*ArcSinh[c*x]] + 24
*b*Sinh[4*ArcSinh[c*x]] + 16*b*Sinh[6*ArcSinh[c*x]] + 3*b*Sinh[8*ArcSinh[c*
x]])))/(884736*c^3*Sqrt[1 + c^2*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2279 vs. 2(468) = 936.

Time = 0.40 (sec) , antiderivative size = 2280, normalized size of antiderivative = 4.25

method	result	size
default	Expression too large to display	2280
parts	Expression too large to display	2280

```
[In] int(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*a^2*x*(c^2*d*x^2+d)^(7/2)/c^2/d-1/48*a^2/c^2*x*(c^2*d*x^2+d)^(5/2)-5/19
2*a^2/c^2*d*x*(c^2*d*x^2+d)^(3/2)-5/128*a^2/c^2*d^2*x*(c^2*d*x^2+d)^(1/2)-5
/128*a^2/c^2*d^3*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2
)+b^2*(-5/384*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^3*d^
2+1/65536*(d*(c^2*x^2+1))^(1/2)*(128*c^9*x^9+128*c^8*x^8*(c^2*x^2+1)^(1/2)+
320*c^7*x^7+256*c^6*x^6*(c^2*x^2+1)^(1/2)+272*c^5*x^5+160*c^4*x^4*(c^2*x^2+
1)^(1/2)+88*c^3*x^3+32*c^2*x^2*(c^2*x^2+1)^(1/2)+8*c*x+(c^2*x^2+1)^(1/2))*(
32*arcsinh(c*x)^2-8*arcsinh(c*x)+1)*d^2/c^3/(c^2*x^2+1)+1/6912*(d*(c^2*x^2+
1))^(1/2)*(32*c^7*x^7+32*c^6*x^6*(c^2*x^2+1)^(1/2)+64*c^5*x^5+48*c^4*x^4*(c
^2*x^2+1)^(1/2)+38*c^3*x^3+18*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x+(c^2*x^2+1)^(
1/2))*(18*arcsinh(c*x)^2-6*arcsinh(c*x)+1)*d^2/c^3/(c^2*x^2+1)+1/2048*(d*(c
^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^
2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(8*arcsinh(c*x)^2-4*arcsinh(c*
x)+1)*d^2/c^3/(c^2*x^2+1)-1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*
(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2-2*arcsinh(c*x)
```

```
+1)*d^2/c^3/(c^2*x^2+1)-1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c
^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2+2*arcsinh(c*x)+1
)*d^2/c^3/(c^2*x^2+1)+1/2048*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^
2*x^2+1)^(1/2)+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/
2))*(8*arcsinh(c*x)^2+4*arcsinh(c*x)+1)*d^2/c^3/(c^2*x^2+1)+1/6912*(d*(c^2*
x^2+1))^(1/2)*(32*c^7*x^7-32*c^6*x^6*(c^2*x^2+1)^(1/2)+64*c^5*x^5-48*c^4*x^
4*(c^2*x^2+1)^(1/2)+38*c^3*x^3-18*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x-(c^2*x^2+
1)^(1/2))*(18*arcsinh(c*x)^2+6*arcsinh(c*x)+1)*d^2/c^3/(c^2*x^2+1)+1/65536*
(d*(c^2*x^2+1))^(1/2)*(128*c^9*x^9-128*c^8*x^8*(c^2*x^2+1)^(1/2)+320*c^7*x^
7-256*c^6*x^6*(c^2*x^2+1)^(1/2)+272*c^5*x^5-160*c^4*x^4*(c^2*x^2+1)^(1/2)+8
8*c^3*x^3-32*c^2*x^2*(c^2*x^2+1)^(1/2)+8*c*x-(c^2*x^2+1)^(1/2))*(32*arcsinh
(c*x)^2+8*arcsinh(c*x)+1)*d^2/c^3/(c^2*x^2+1))+2*a*b*(-5/256*(d*(c^2*x^2+1)
)^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^2*d^2+1/16384*(d*(c^2*x^2+1))^(1
/2)*(128*c^9*x^9+128*c^8*x^8*(c^2*x^2+1)^(1/2)+320*c^7*x^7+256*c^6*x^6*(c^2
*x^2+1)^(1/2)+272*c^5*x^5+160*c^4*x^4*(c^2*x^2+1)^(1/2)+88*c^3*x^3+32*c^2*x
^2*(c^2*x^2+1)^(1/2)+8*c*x+(c^2*x^2+1)^(1/2))*(-1+8*arcsinh(c*x))*d^2/c^3/(
c^2*x^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7+32*c^6*x^6*(c^2*x^2+1)^
(1/2)+64*c^5*x^5+48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3+18*c^2*x^2*(c^2*x^
2+1)^(1/2)+6*c*x+(c^2*x^2+1)^(1/2))*(-1+6*arcsinh(c*x))*d^2/c^3/(c^2*x^2+1)
+1/1024*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3
*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(c*x
))*d^2/c^3/(c^2*x^2+1)-1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^
2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*x))*d^2/c^3/(c^2*x^
2+1)-1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c
*x-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(c*x))*d^2/c^3/(c^2*x^2+1)+1/1024*(d*(c^2
*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3-8*c^2*x^2*
(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(1+4*arcsinh(c*x))*d^2/c^3/(c^2*
x^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7-32*c^6*x^6*(c^2*x^2+1)^(1/2
)+64*c^5*x^5-48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3-18*c^2*x^2*(c^2*x^2+1)
^(1/2)+6*c*x-(c^2*x^2+1)^(1/2))*(1+6*arcsinh(c*x))*d^2/c^3/(c^2*x^2+1)+1/16
384*(d*(c^2*x^2+1))^(1/2)*(128*c^9*x^9-128*c^8*x^8*(c^2*x^2+1)^(1/2)+320*c^
7*x^7-256*c^6*x^6*(c^2*x^2+1)^(1/2)+272*c^5*x^5-160*c^4*x^4*(c^2*x^2+1)^(1/
2)+88*c^3*x^3-32*c^2*x^2*(c^2*x^2+1)^(1/2)+8*c*x-(c^2*x^2+1)^(1/2))*(1+8*ar
csinh(c*x))*d^2/c^3/(c^2*x^2+1))
```

Fricas [F]

$$\int x^2 \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))^2\,dx=\int \left(c^2dx^2+d\right)^{\frac{5}{2}}(b\operatorname{arsinh}\left(cx\right)+a)^2x^2\,dx$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^4*d^2*x^6 + 2*a^2*c^2*d^2*x^4 + a^2*d^2*x^2 + (b^2*c^4*d^2*
x^6 + 2*b^2*c^2*d^2*x^4 + b^2*d^2*x^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^6
+ 2*a*b*c^2*d^2*x^4 + a*b*d^2*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 (d + c^2 dx^2)^{5/2} (a + b\text{arcsinh}(cx))^2 \, dx = \text{Timed out}$$

```
[In] integrate(x**2*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^2 (d + c^2 dx^2)^{5/2} (a + b\text{arcsinh}(cx))^2 \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int x^2 (d + c^2 dx^2)^{5/2} (a + b\text{arcsinh}(cx))^2 \, dx = \int (c^2 dx^2 + d)^{\frac{5}{2}} (b \operatorname{arsinh}(cx) + a)^2 x^2 \, dx$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(d+c^2 d x^2\right)^{5/2} (a+b\operatorname{arcsinh}(cx))^2\,dx=\int x^2\,\left(a+b\,\mathrm{asinh}(c\,x)\right)^2\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

```
[In] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2),x)
[Out] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2), x)
```

3.276 $\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 366

$$\int x\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\frac{32b^2d^2\sqrt{d+c^2dx^2}}{245c^2}+\frac{16b^2d^2(1+c^2x^2)\sqrt{d+c^2dx^2}}{735c^2}+\frac{12b^2d^2(1+c^2x^2)^2\sqrt{d+c^2dx^2}}{1225c^2}+\frac{2b^2d^2(1+c^2x^2)^3\sqrt{d+c^2dx^2}}{343c^2}-\frac{2bd^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{7c\sqrt{1+c^2x^2}}-\frac{2bcd^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{7\sqrt{1+c^2x^2}}-\frac{6bc^3d^2x^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{35\sqrt{1+c^2x^2}}-\frac{2bc^5d^2x^7\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{49\sqrt{1+c^2x^2}}+\frac{\left(d+c^2dx^2\right)^{7/2}(a+b\text{arcsinh}(cx))^2}{7c^2d}$$

```
[Out] 1/7*(c^2*d*x^2+d)^(7/2)*(a+b*arcsinh(c*x))^2/c^2/d+32/245*b^2*d^2*(c^2*d*x^2+d)^(1/2)/c^2+16/735*b^2*d^2*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/c^2+12/1225*b^2*d^2*(c^2*x^2+1)^2*(c^2*d*x^2+d)^(1/2)/c^2+2/343*b^2*d^2*(c^2*x^2+1)^3*(c^2*d*x^2+d)^(1/2)/c^2-2/7*b*d^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-2/7*b*c*d^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-6/35*b*c^3*d^2*x^5*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/49*b*c^5*d^2*x^7*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5798, 200, 5784, 12, 1813, 1864}

$$\int x\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=-\frac{2bd^2x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{7c\sqrt{c^2x^2+1}}-\frac{2bcd^2x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{7\sqrt{c^2x^2+1}}+\frac{\left(c^2dx^2+d\right)^{7/2}(a+b\text{arcsinh}(cx))^2}{7c^2d}-\frac{2bc^5d^2x^7\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{49\sqrt{c^2x^2+1}}-\frac{6bc^3d^2x^5\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{35\sqrt{c^2x^2+1}}+\frac{2b^2d^2\left(c^2x^2+1\right)^3\sqrt{c^2dx^2+d}}{343c^2}+\frac{32b^2d^2\sqrt{c^2dx^2+d}}{245c^2}+\frac{12b^2d^2\left(c^2x^2+1\right)^2\sqrt{c^2dx^2+d}}{1225c^2}+\frac{16b^2d^2\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}}{735c^2}$$

```
[In] Int[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (32*b^2*d^2*Sqrt[d + c^2*d*x^2])/(245*c^2) + (16*b^2*d^2*(1 + c^2*x^2)*Sqrt
[d + c^2*d*x^2])/(735*c^2) + (12*b^2*d^2*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2
])/(1225*c^2) + (2*b^2*d^2*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2])/(343*c^2) -
 (2*b*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(7*c*Sqrt[1 + c^2*x^2
]) - (2*b*c*d^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(7*Sqrt[1 + c
^2*x^2]) - (6*b*c^3*d^2*x^5*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(35*S
qrt[1 + c^2*x^2]) - (2*b*c^5*d^2*x^7*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x
]))/(49*Sqrt[1 + c^2*x^2]) + ((d + c^2*d*x^2)^(7/2)*(a + b*ArcSinh[c*x])^2)
/(7*c^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+c^2dx^2)^{7/2}(a+b\operatorname{arcsinh}(cx))^2}{7c^2d} \\
&\quad - \frac{\left(2bd^2\sqrt{d+c^2dx^2}\right)\int (1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))\,dx}{7c\sqrt{1+c^2x^2}} \\
&= -\frac{2bd^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c\sqrt{1+c^2x^2}} - \frac{2bcd^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7\sqrt{1+c^2x^2}} \\
&\quad - \frac{6bc^3d^2x^5\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{35\sqrt{1+c^2x^2}} - \frac{2bc^5d^2x^7\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{49\sqrt{1+c^2x^2}} \\
&\quad + \frac{(d+c^2dx^2)^{7/2}(a+b\operatorname{arcsinh}(cx))^2}{7c^2d} + \frac{\left(2b^2d^2\sqrt{d+c^2dx^2}\right)\int \frac{x(35+35c^2x^2+21c^4x^4+5c^6x^6)}{35\sqrt{1+c^2x^2}}\,dx}{7\sqrt{1+c^2x^2}} \\
&= -\frac{2bd^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c\sqrt{1+c^2x^2}} - \frac{2bcd^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7\sqrt{1+c^2x^2}} \\
&\quad - \frac{6bc^3d^2x^5\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{35\sqrt{1+c^2x^2}} - \frac{2bc^5d^2x^7\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{49\sqrt{1+c^2x^2}} \\
&\quad + \frac{(d+c^2dx^2)^{7/2}(a+b\operatorname{arcsinh}(cx))^2}{7c^2d} + \frac{\left(2b^2d^2\sqrt{d+c^2dx^2}\right)\int \frac{x(35+35c^2x^2+21c^4x^4+5c^6x^6)}{\sqrt{1+c^2x^2}}\,dx}{245\sqrt{1+c^2x^2}}
\end{aligned}$$

$$= -\frac{2bd^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{7c\sqrt{1+c^2x^2}} - \frac{2bcd^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{7\sqrt{1+c^2x^2}} - \frac{6bc^3d^2x^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{35\sqrt{1+c^2x^2}} - \frac{2bc^5d^2x^7\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{49\sqrt{1+c^2x^2}} + \frac{(d+c^2dx^2)^{7/2}(a+b\text{arcsinh}(cx))^2}{7c^2d} + \frac{\left(b^2d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int \frac{35+35c^2x+21c^4x^2+5c^6x^3}{\sqrt{1+c^2x}}\,dx,x,x^2\right)}{245\sqrt{1+c^2x^2}}$$

$$= -\frac{2bd^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{7c\sqrt{1+c^2x^2}} - \frac{2bcd^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{7\sqrt{1+c^2x^2}} - \frac{6bc^3d^2x^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{35\sqrt{1+c^2x^2}} - \frac{2bc^5d^2x^7\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{49\sqrt{1+c^2x^2}} + \frac{(d+c^2dx^2)^{7/2}(a+b\text{arcsinh}(cx))^2}{7c^2d} + \frac{\left(b^2d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int \left(\frac{16}{\sqrt{1+c^2x}}+8\sqrt{1+c^2x}+6(1+c^2x)^{3/2}+5(1+c^2x)^{5/2}\right)dx,x,x^2\right)}{245\sqrt{1+c^2x^2}}$$

$$= \frac{32b^2d^2\sqrt{d+c^2dx^2}}{245c^2} + \frac{16b^2d^2(1+c^2x^2)\sqrt{d+c^2dx^2}}{735c^2} + \frac{12b^2d^2(1+c^2x^2)^2\sqrt{d+c^2dx^2}}{1225c^2} + \frac{2b^2d^2(1+c^2x^2)^3\sqrt{d+c^2dx^2}}{343c^2} - \frac{2bd^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{7c\sqrt{1+c^2x^2}} - \frac{2bcd^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{7\sqrt{1+c^2x^2}} - \frac{6bc^3d^2x^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{35\sqrt{1+c^2x^2}} - \frac{2bc^5d^2x^7\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{49\sqrt{1+c^2x^2}} + \frac{(d+c^2dx^2)^{7/2}(a+b\text{arcsinh}(cx))^2}{7c^2d}$$

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.61

$$\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{d^2\sqrt{d+c^2dx^2}\left(3675a^2(1+c^2x^2)^4 - 210abcx\sqrt{1+c^2x^2}(35+35c^2x^2+21c^4x^4+5c^6x^6)\right.}{}$$

```
[In] Integrate[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*Sqrt[d + c^2*d*x^2]*(3675*a^2*(1 + c^2*x^2)^4 - 210*a*b*c*x*Sqrt[1 + c
^2*x^2]*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) + 2*b^2*(2161 + 2918*c^2
*x^2 + 1108*c^4*x^4 + 426*c^6*x^6 + 75*c^8*x^8) + 210*b*(35*a*(1 + c^2*x^2)
```

```
^4 - b*c*x*Sqrt[1 + c^2*x^2]*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6))*Ar
cSinh[c*x] + 3675*b^2*(1 + c^2*x^2)^4*ArcSinh[c*x]^2))/(25725*c^2*(1 + c^2*
x^2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1772 vs. 2(322) = 644.

Time = 0.37 (sec) , antiderivative size = 1773, normalized size of antiderivative = 4.84

method	result	size
default	Expression too large to display	1773
parts	Expression too large to display	1773

```
[In] int(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*a^2*(c^2*d*x^2+d)^(7/2)/c^2/d+b^2*(1/43904*(d*(c^2*x^2+1))^(1/2)*(64*c^
8*x^8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2
)+104*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/
2)+1)*(49*arcsinh(c*x)^2-14*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+1/3200*(d*(
c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^
3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c
*x)^2-10*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+1/384*(d*(c^2*x^2+1))^(1/2)*(4
*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(
9*arcsinh(c*x)^2-6*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+5/128*(d*(c^2*x^2+1)
)^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)
*d^2/c^2/(c^2*x^2+1)+5/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(
1/2)+1)*(arcsinh(c*x)^2+2*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+1/384*(d*(c^2
*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*
x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2+6*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+1/3
200*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x
^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*(25*a
rcsinh(c*x)^2+10*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+1/43904*(d*(c^2*x^2+1)
)^(1/2)*(64*c^8*x^8-64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6-112*c^5*x^5*(c
^2*x^2+1)^(1/2)+104*c^4*x^4-56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2-7*c*x*(
c^2*x^2+1)^(1/2)+1)*(49*arcsinh(c*x)^2+14*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+
1))+2*a*b*(1/6272*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8+64*c^7*x^7*(c^2*x^2+1)^
(1/2)+144*c^6*x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4+56*c^3*x^3*(c^2
*x^2+1)^(1/2)+25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+7*arcsinh(c*x))*d^2
/c^2/(c^2*x^2+1)+1/640*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^
2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^
2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/128*(d*(c^2*x^2+1))
^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(
1/2)+1)*(-1+3*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+5/128*(d*(c^2*x^2+1))^(1/2)
*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+5/
```

```
128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)+1
)*d^2/c^2/(c^2*x^2+1)+1/128*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2
*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(3*arcsinh(c*x)+1)*d^2/c
^2/(c^2*x^2+1)+1/640*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+
1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+
1)^(1/2)+1)*(1+5*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/6272*(d*(c^2*x^2+1))^(
1/2)*(64*c^8*x^8-64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6-112*c^5*x^5*(c^2*
x^2+1)^(1/2)+104*c^4*x^4-56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2-7*c*x*(c^2
*x^2+1)^(1/2)+1)*(1+7*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.22

$$\int x\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{3675\left(b^2c^8d^2x^8+4\,b^2c^6d^2x^6+6\,b^2c^4d^2x^4+4\,b^2c^2d^2x^2+b^2d^2\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2}\right.}{}$$

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/25725*(3675*(b^2*c^8*d^2*x^8 + 4*b^2*c^6*d^2*x^6 + 6*b^2*c^4*d^2*x^4 + 4*
b^2*c^2*d^2*x^2 + b^2*d^2)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))
^2 + 210*(35*a*b*c^8*d^2*x^8 + 140*a*b*c^6*d^2*x^6 + 210*a*b*c^4*d^2*x^4 +
140*a*b*c^2*d^2*x^2 + 35*a*b*d^2 - (5*b^2*c^7*d^2*x^7 + 21*b^2*c^5*d^2*x^5
+ 35*b^2*c^3*d^2*x^3 + 35*b^2*c*d^2*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 +
d)*log(c*x + sqrt(c^2*x^2 + 1)) + (75*(49*a^2 + 2*b^2)*c^8*d^2*x^8 + 12*(12
25*a^2 + 71*b^2)*c^6*d^2*x^6 + 2*(11025*a^2 + 1108*b^2)*c^4*d^2*x^4 + 4*(36
75*a^2 + 1459*b^2)*c^2*d^2*x^2 + (3675*a^2 + 4322*b^2)*d^2 - 210*(5*a*b*c^7
*d^2*x^7 + 21*a*b*c^5*d^2*x^5 + 35*a*b*c^3*d^2*x^3 + 35*a*b*c*d^2*x)*sqrt(c
^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^4*x^2 + c^2)
```

Sympy [F]

$$\int x\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\int x\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\,dx$$

```
[In] integrate(x*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x*(d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.75

$$\int x(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \frac{(c^2dx^2+d)^{\frac{7}{2}}b^2\operatorname{arsinh}(cx)^2}{7\,c^2d}$$
$$+\frac{2\,(c^2dx^2+d)^{\frac{7}{2}}ab\operatorname{arsinh}(cx)}{7\,c^2d}$$
$$+\frac{2}{25725}b^2\left(\frac{75\sqrt{c^2x^2+1}c^4d^{\frac{7}{2}}x^6+351\sqrt{c^2x^2+1}c^2d^{\frac{7}{2}}x^4+757\sqrt{c^2x^2+1}d^{\frac{7}{2}}x^2+\frac{2161\sqrt{c^2x^2+1}d^{\frac{7}{2}}}{c^2}}{d}-\frac{105\left(5\,c^6\right.}{}\right.$$
$$+\frac{(c^2dx^2+d)^{\frac{7}{2}}a^2}{7\,c^2d}-\frac{2\left(5\,c^6d^{\frac{7}{2}}x^7+21\,c^4d^{\frac{7}{2}}x^5+35\,c^2d^{\frac{7}{2}}x^3+35\,d^{\frac{7}{2}}x\right)ab}{245\,cd}$$

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/7*(c^2*d*x^2 + d)^(7/2)*b^2*arcsinh(c*x)^2/(c^2*d) + 2/7*(c^2*d*x^2 + d)^
(7/2)*a*b*arcsinh(c*x)/(c^2*d) + 2/25725*b^2*((75*sqrt(c^2*x^2 + 1)*c^4*d^(
7/2)*x^6 + 351*sqrt(c^2*x^2 + 1)*c^2*d^(7/2)*x^4 + 757*sqrt(c^2*x^2 + 1)*d^
(7/2)*x^2 + 2161*sqrt(c^2*x^2 + 1)*d^(7/2)/c^2)/d - 105*(5*c^6*d^(7/2)*x^7
+ 21*c^4*d^(7/2)*x^5 + 35*c^2*d^(7/2)*x^3 + 35*d^(7/2)*x)*arcsinh(c*x)/(c*d
)) + 1/7*(c^2*d*x^2 + d)^(7/2)*a^2/(c^2*d) - 2/245*(5*c^6*d^(7/2)*x^7 + 21*
c^4*d^(7/2)*x^5 + 35*c^2*d^(7/2)*x^3 + 35*d^(7/2)*x)*a*b/(c*d)
```

Giac [F(-2)]

Exception generated.

$$\int x(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\int x\,(a+b\,\text{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

```
[In] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2),x)
[Out] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2), x)
```

3.277 $\int (d+c^2dx^2)^{5/2} (a+b\text{arcsinh}(cx))^2 \, dx$

Optimal result

Integrand size = 25, antiderivative size = 420

$$\int (d+c^2dx^2)^{5/2} (a+b\text{arcsinh}(cx))^2 \, dx = \frac{245b^2d^2x\sqrt{d+c^2dx^2}}{1152} + \frac{65b^2d^2x(1+c^2x^2)\sqrt{d+c^2dx^2}}{1728} + \frac{1}{108}b^2d^2x(1+c^2x^2)^2\sqrt{d+c^2dx^2} - \frac{115b^2d^2\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{1152c\sqrt{1+c^2x^2}} - \frac{5bcd^2x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{16\sqrt{1+c^2x^2}} - \frac{5bd^2(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{48c} - \frac{bd^2(1+c^2x^2)^{5/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{18c} + \frac{5}{16}d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{5}{24}dx(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{6}x(d+c^2dx^2)^{5/2}(a+b\text{arcsi}$$

```
[Out] 5/24*d*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2+1/6*x*(c^2*d*x^2+d)^(5/2)
*(a+b*arcsinh(c*x))^2+245/1152*b^2*d^2*x*(c^2*d*x^2+d)^(1/2)+65/1728*b^2*d^
2*x*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)+1/108*b^2*d^2*x*(c^2*x^2+1)^2*(c^2*d*x^
2+d)^(1/2)-5/48*b*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1
/2)/c-1/18*b*d^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c
+5/16*d^2*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)-115/1152*b^2*d^2*arcsi
nh(c*x)*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-5/16*b*c*d^2*x^2*(a+b*arcsi
nh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5/48*d^2*(a+b*arcsinh(c*x))^
3*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {5786, 5785, 5783, 5776, 327, 221, 5798, 201}

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\frac{5d^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{48bc\sqrt{c^2x^2+1}}+\frac{5}{16}d^2x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bd^2(c^2x^2+1)^{5/2}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{18c}-\frac{5bd^2(c^2x^2+1)^{3/2}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{48c}-\frac{5bcd^2x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{16\sqrt{c^2x^2+1}}+\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2+\frac{5}{24}dx\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{115b^2d^2\text{arcsinh}(cx)\sqrt{c^2dx^2+}}{1152c\sqrt{c^2x^2+1}}$$

```
[In] Int[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (245*b^2*d^2*x*Sqrt[d + c^2*d*x^2])/1152 + (65*b^2*d^2*x*(1 + c^2*x^2)*Sqrt
[d + c^2*d*x^2])/1728 + (b^2*d^2*x*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/108
 - (115*b^2*d^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(1152*c*Sqrt[1 + c^2*x^2]
) - (5*b*c*d^2*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(16*Sqrt[1 + c
^2*x^2]) - (5*b*d^2*(1 + c^2*x^2)^(3/2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[
c*x]))/(48*c) - (b*d^2*(1 + c^2*x^2)^(5/2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSi
nh[c*x]))/(18*c) + (5*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/16
+ (5*d*x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/24 + (x*(d + c^2*d*x
^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/6 + (5*d^2*Sqrt[d + c^2*d*x^2]*(a + b*Arc
Sinh[c*x])^3)/(48*b*c*Sqrt[1 + c^2*x^2])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{6}x\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2+\frac{1}{6}(5d)\int\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx-\frac{\left(bcd^2\sqrt{d+c^2dx^2}\right)\int x(1+c^2x^2)^2(a+b\text{arcsinh}(cx))\,dx}{3\sqrt{1+c^2x^2}}$$

$$=-\frac{bd^2(1+c^2x^2)^{5/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{18c}+\frac{5}{24}dx\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2+\frac{1}{6}x\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2+\frac{1}{8}\left(5d^2\right)\int\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx+\frac{\left(b^2d^2\sqrt{d+c^2\right.}{1}$$

$$=\frac{1}{108}b^2d^2x\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}-\frac{5bd^2(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{48c}-\frac{bd^2(1+c^2x^2)^{5/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{18c}+\frac{5}{16}d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2+\frac{5}{24}dx\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2+\frac{1}{6}x\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2+\frac{\left(5d^2\sqrt{d+c^2dx^2}\right)\int}{16\sqrt{1}}$$

$$=\frac{65b^2d^2x(1+c^2x^2)\sqrt{d+c^2dx^2}}{1728}+\frac{1}{108}b^2d^2x\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}-\frac{5bcd^2x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{16\sqrt{1+c^2x^2}}-\frac{5bd^2(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{48c}-\frac{bd^2(1+c^2x^2)^{5/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{18c}+\frac{5}{16}d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2+\frac{5}{24}dx\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2+\frac{1}{6}x\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2+\frac{5d^2\sqrt{d+c^2dx^2}(a-}{48bc\sqrt{1-}}$$

$$=\frac{245b^2d^2x\sqrt{d+c^2dx^2}}{1152}+\frac{65b^2d^2x(1+c^2x^2)\sqrt{d+c^2dx^2}}{1728}+\frac{1}{108}b^2d^2x\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}-\frac{5bcd^2x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{16\sqrt{1+c^2x^2}}-\frac{5bd^2(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{48c}-\frac{bd^2(1+c^2x^2)^{5/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{18c}+\frac{5}{16}d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2+\frac{5}{24}dx\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2+\frac{1}{6}x\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2+\frac{5d^2\sqrt{d+c^2dx^2}(a-}{48bc\sqrt{1-}}$$

$$= \frac{245b^2d^2x\sqrt{d+c^2dx^2}}{1152} + \frac{65b^2d^2x(1+c^2x^2)\sqrt{d+c^2dx^2}}{1728}$$
$$+ \frac{1}{108}b^2d^2x\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2} - \frac{115b^2d^2\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{1152c\sqrt{1+c^2x^2}}$$
$$- \frac{5bcd^2x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{16\sqrt{1+c^2x^2}}$$
$$- \frac{5bd^2(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{48c}$$
$$- \frac{bd^2(1+c^2x^2)^{5/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{18c}$$
$$+ \frac{5}{16}d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2$$
$$+ \frac{5}{24}dx\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{6}x\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2 + \frac{5d^2\sqrt{d+c^2dx^2}(a}{48bc\sqrt{1}}$$

Mathematica [A] (verified)

Time = 2.30 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.19

$$\int \left(d+c^2dx^2\right)^{5/2}(a + b\text{arcsinh}(cx))^2\,dx = \frac{d^2\Big(9504a^2cx\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2} + 7488a^2c^3x^3\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2} + 2304a^2c^5x^5}{}$$

```
[In] Integrate[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*(9504*a^2*c*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 7488*a^2*c^3*x^3
*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 2304*a^2*c^5*x^5*Sqrt[1 + c^2*x^2]
*Sqrt[d + c^2*d*x^2] + 1440*b^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 - 3240*a
*b*Sqrt[d + c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 324*a*b*Sqrt[d + c^2*d*x^2]*C
osh[4*ArcSinh[c*x]] - 24*a*b*Sqrt[d + c^2*d*x^2]*Cosh[6*ArcSinh[c*x]] + 432
0*a^2*Sqrt[d]*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] +
1620*b^2*Sqrt[d + c^2*d*x^2]*Sinh[2*ArcSinh[c*x]] + 81*b^2*Sqrt[d + c^2*d*x
^2]*Sinh[4*ArcSinh[c*x]] + 4*b^2*Sqrt[d + c^2*d*x^2]*Sinh[6*ArcSinh[c*x]] -
 12*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(270*b*Cosh[2*ArcSinh[c*x]] + 27*b*C
osh[4*ArcSinh[c*x]] + 2*b*Cosh[6*ArcSinh[c*x]] - 540*a*Sinh[2*ArcSinh[c*x]]
 - 108*a*Sinh[4*ArcSinh[c*x]] - 12*a*Sinh[6*ArcSinh[c*x]]) + 72*b*Sqrt[d +
c^2*d*x^2]*ArcSinh[c*x]^2*(60*a + 45*b*Sinh[2*ArcSinh[c*x]] + 9*b*Sinh[4*Ar
cSinh[c*x]] + b*Sinh[6*ArcSinh[c*x]])))/(13824*c*Sqrt[1 + c^2*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1567 vs. $2(366) = 732$.

Time = 0.30 (sec) , antiderivative size = 1568, normalized size of antiderivative = 3.73

method	result	size
default	Expression too large to display	1568
parts	Expression too large to display	1568

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x*(c^2*d*x^2+d)^(5/2)*a^2+5/24*a^2*d*x*(c^2*d*x^2+d)^(3/2)+5/16*a^2*d^2
*x*(c^2*d*x^2+d)^(1/2)+5/16*a^2*d^3*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^
(1/2))/(c^2*d)^(1/2)+b^2*(5/48*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*ar
csinh(c*x)^3*d^2+1/6912*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7+32*c^6*x^6*(c^2*x
^2+1)^(1/2)+64*c^5*x^5+48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3+18*c^2*x^2*(
c^2*x^2+1)^(1/2)+6*c*x+(c^2*x^2+1)^(1/2))*(18*arcsinh(c*x)^2-6*arcsinh(c*x)
+1)*d^2/c/(c^2*x^2+1)+3/1024*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^
2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/
2))*(8*arcsinh(c*x)^2-4*arcsinh(c*x)+1)*d^2/c/(c^2*x^2+1)+15/256*(d*(c^2*x^
2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))
*(2*arcsinh(c*x)^2-2*arcsinh(c*x)+1)*d^2/c/(c^2*x^2+1)+15/256*(d*(c^2*x^2+1
))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(2
*arcsinh(c*x)^2+2*arcsinh(c*x)+1)*d^2/c/(c^2*x^2+1)+3/1024*(d*(c^2*x^2+1))^
(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+
1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(8*arcsinh(c*x)^2+4*arcsinh(c*x)+1)*d^2/c
/(c^2*x^2+1)+1/6912*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7-32*c^6*x^6*(c^2*x^2+1
)^(1/2)+64*c^5*x^5-48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3-18*c^2*x^2*(c^2*
x^2+1)^(1/2)+6*c*x-(c^2*x^2+1)^(1/2))*(18*arcsinh(c*x)^2+6*arcsinh(c*x)+1)*
d^2/c/(c^2*x^2+1))+2*a*b*(5/32*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*ar
csinh(c*x)^2*d^2+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7+32*c^6*x^6*(c^2*x
^2+1)^(1/2)+64*c^5*x^5+48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3+18*c^2*x^2*(
c^2*x^2+1)^(1/2)+6*c*x+(c^2*x^2+1)^(1/2))*(-1+6*arcsinh(c*x))*d^2/c/(c^2*x^
2+1)+3/512*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*
c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(
c*x))*d^2/c/(c^2*x^2+1)+15/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(
c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*x))*d^2/c/(c^2*x^
2+1)+15/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*
c*x-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(c*x))*d^2/c/(c^2*x^2+1)+3/512*(d*(c^2*x
^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3-8*c^2*x^2*(c
^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(1+4*arcsinh(c*x))*d^2/c/(c^2*x^2+
1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7-32*c^6*x^6*(c^2*x^2+1)^(1/2)+64
*c^5*x^5-48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3-18*c^2*x^2*(c^2*x^2+1)^(1/
2)+6*c*x-(c^2*x^2+1)^(1/2))*(1+6*arcsinh(c*x))*d^2/c/(c^2*x^2+1))
```

Fricas [F]

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\int \left(c^2dx^2+d\right)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\int \left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}(a+b\,\text{asinh}\,(cx))^2\,dx$$

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\int (a+b\,\text{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

```
[In] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2),x)
[Out] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2), x)
```

3.278 $\int \frac{(d+c^2dx^2)^{5/2}(a+b\textbf{arcsinh}(cx))^2}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 635

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx = \frac{598}{225}b^2d^2\sqrt{d+c^2dx^2} - \frac{2abcd^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}$$
$$+\frac{74}{675}b^2d^2(1+c^2x^2)\sqrt{d+c^2dx^2} + \frac{2}{125}b^2d^2(1+c^2x^2)^2\sqrt{d+c^2dx^2}$$
$$-\frac{2b^2cd^2x\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}} - \frac{16bcd^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{15\sqrt{1+c^2x^2}}$$
$$-\frac{22bc^3d^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{45\sqrt{1+c^2x^2}} - \frac{2bc^5d^2x^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{25\sqrt{1+c^2x^2}}$$
$$+d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{3}d(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2$$
$$+\frac{1}{5}(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2 - \frac{2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{2bd^2\sqrt{d+c^2}}{}$$

```
[Out] 1/3*d*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2+1/5*(c^2*d*x^2+d)^(5/2)*(a+b
*arcsinh(c*x))^2+598/225*b^2*d^2*(c^2*d*x^2+d)^(1/2)+74/675*b^2*d^2*(c^2*x^
2+1)*(c^2*d*x^2+d)^(1/2)+2/125*b^2*d^2*(c^2*x^2+1)^2*(c^2*d*x^2+d)^(1/2)+d^
2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)-2*a*b*c*d^2*x*(c^2*d*x^2+d)^(1/2
)/(c^2*x^2+1)^(1/2)-2*b^2*c*d^2*x*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2
+1)^(1/2)-16/15*b*c*d^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1
)^(1/2)-22/45*b*c^3*d^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2
+1)^(1/2)-2/25*b*c^5*d^2*x^5*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^
2+1)^(1/2)-2*d^2*(a+b*arcsinh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*d
*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*b*d^2*(a+b*arcsinh(c*x))*polylog(2,-c*x-(
c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+2*b*d^2*(a+b*arcsin
h(c*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1
/2)+2*b^2*d^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^
```

```
2+1)^(1/2)-2*b^2*d^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(
c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5808, 5806, 5816, 4267, 2611, 2320, 6724, 5772, 267, 5784, 455, 45, 200, 12, 1261, 712}

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx =$$

$$-\frac{2d^2\sqrt{c^2dx^2+d}\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}$$
$$-\frac{2bd^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}$$
$$+\frac{2bd^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}$$
$$-\frac{16bcd^2x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{15\sqrt{c^2x^2+1}}$$
$$+d^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2+\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$
$$+\frac{1}{3}d\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{2bc^5d^2x^5\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{25\sqrt{c^2x^2+1}}-\frac{22bc^3d^2x^3\sqrt{c^2dx^2+d}(a+b\text{a}}{45\sqrt{c^2x^2+1}}$$

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (598*b^2*d^2*Sqrt[d + c^2*d*x^2])/225 - (2*a*b*c*d^2*x*Sqrt[d + c^2*d*x^2])
/Sqrt[1 + c^2*x^2] + (74*b^2*d^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/675 + (
2*b^2*d^2*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/125 - (2*b^2*c*d^2*x*Sqrt[d
+ c^2*d*x^2]*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - (16*b*c*d^2*x*Sqrt[d + c^2*d
*x^2]*(a + b*ArcSinh[c*x]))/(15*Sqrt[1 + c^2*x^2]) - (22*b*c^3*d^2*x^3*Sqrt
[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(45*Sqrt[1 + c^2*x^2]) - (2*b*c^5*d^2
*x^5*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(25*Sqrt[1 + c^2*x^2]) + d^2
*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2 + (d*(d + c^2*d*x^2)^(3/2)*(a +
 b*ArcSinh[c*x])^2)/3 + ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/5 -
(2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/
Sqrt[1 + c^2*x^2] - (2*b*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyL
og[2, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (2*b*d^2*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (2*b^2*d
^2*Sqrt[d + c^2*d*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (2*
b^2*d^2*Sqrt[d + c^2*d*x^2]*PolyLog[3, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
```

```
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))^2+d\int\frac{\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{x}\,dx \\
&\quad -\frac{\left(2bcd^2\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)^2(a+b\operatorname{arcsinh}(cx))\,dx}{5\sqrt{1+c^2x^2}} \\
&= -\frac{2bcd^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5\sqrt{1+c^2x^2}}-\frac{4bc^3d^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{15\sqrt{1+c^2x^2}} \\
&\quad -\frac{2bc^5d^2x^5\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{25\sqrt{1+c^2x^2}}+\frac{1}{3}d\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2 \\
&\quad +\frac{1}{5}\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))^2+d^2\int\frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{x}\,dx \\
&\quad -\frac{\left(2bcd^2\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))\,dx}{3\sqrt{1+c^2x^2}} \\
&\quad +\frac{\left(2b^2c^2d^2\sqrt{d+c^2dx^2}\right)\int\frac{x\left(15+10c^2x^2+3c^4x^4\right)}{15\sqrt{1+c^2x^2}}\,dx}{5\sqrt{1+c^2x^2}}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{16bcd^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{15\sqrt{1+c^2x^2}} - \frac{22bc^3d^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{45\sqrt{1+c^2x^2}} \\
&\quad - \frac{2bc^5d^2x^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{25\sqrt{1+c^2x^2}} + d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 \\
&\quad + \frac{1}{3}d\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{5}\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2 \\
&\quad + \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{1+c^2x^2}}\,dx}{\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(2bcd^2\sqrt{d+c^2dx^2}\right)\int (a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(2b^2c^2d^2\sqrt{d+c^2dx^2}\right)\int \frac{x(15+10c^2x^2+3c^4x^4)}{\sqrt{1+c^2x^2}}\,dx}{75\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(2b^2c^2d^2\sqrt{d+c^2dx^2}\right)\int \frac{x\left(1+\frac{c^2x^2}{3}\right)}{\sqrt{1+c^2x^2}}\,dx}{3\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2abcd^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{16bcd^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{15\sqrt{1+c^2x^2}} \\
&\quad - \frac{22bc^3d^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{45\sqrt{1+c^2x^2}} \\
&\quad - \frac{2bc^5d^2x^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{25\sqrt{1+c^2x^2}} + d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 \\
&\quad + \frac{1}{3}d\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{5}\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2 \\
&\quad + \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int (a+bx)^2\text{csch}(x)\,dx, x, \text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(2b^2cd^2\sqrt{d+c^2dx^2}\right)\int \text{arcsinh}(cx)\,dx}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(b^2c^2d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int \frac{15+10c^2x+3c^4x^2}{\sqrt{1+c^2x}}\,dx, x, x^2\right)}{75\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(b^2c^2d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int \frac{1+\frac{c^2x}{3}}{\sqrt{1+c^2x}}\,dx, x, x^2\right)}{3\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$= -\frac{2abcd^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{2b^2cd^2x\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}$$
$$- \frac{16bcd^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{15\sqrt{1+c^2x^2}}$$
$$- \frac{22bc^3d^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{45\sqrt{1+c^2x^2}}$$
$$- \frac{2bc^5d^2x^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{25\sqrt{1+c^2x^2}} + d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2$$
$$+ \frac{1}{3}d\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{5}\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$
$$- \frac{2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}$$
$$- \frac{\left(2bd^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)\log\left(1-e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}}$$
$$+ \frac{\left(2bd^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)\log\left(1+e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}}$$
$$+ \frac{\left(b^2c^2d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{8}{\sqrt{1+c^2x}} + 4\sqrt{1+c^2x} + 3(1+c^2x)^{3/2}\right)dx, x, x^2\right)}{75\sqrt{1+c^2x^2}}$$
$$+ \frac{\left(b^2c^2d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{2}{3\sqrt{1+c^2x}} + \frac{1}{3}\sqrt{1+c^2x}\right)dx, x, x^2\right)}{3\sqrt{1+c^2x^2}}$$
$$+ \frac{\left(2b^2c^2d^2\sqrt{d+c^2dx^2}\right)\int\frac{x}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{1+c^2x^2}}$$

$$
\begin{aligned}
&= \frac{598}{225}b^2d^2\sqrt{d+c^2dx^2} - \frac{2abcd^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{74}{675}b^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2} + \frac{2}{125}b^2d^2\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2} \\
&\quad - \frac{2b^2cd^2x\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}} - \frac{16bcd^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{15\sqrt{1+c^2x^2}} \\
&\quad - \frac{22bc^3d^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{45\sqrt{1+c^2x^2}} \\
&\quad - \frac{2bc^5d^2x^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{25\sqrt{1+c^2x^2}} + d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 \\
&\quad + \frac{1}{3}d\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{5}\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2 \\
&\quad - \frac{2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad - \frac{2bd^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{2bd^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(2b^2d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int \text{PolyLog}\left(2,-e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(2b^2d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int \text{PolyLog}\left(2,e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{598}{225} b^2 d^2 \sqrt{d+c^2dx^2} - \frac{2abcd^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} \\
&+ \frac{74}{675} b^2 d^2 \left(1+c^2x^2\right) \sqrt{d+c^2dx^2} + \frac{2}{125} b^2 d^2 \left(1+c^2x^2\right)^2 \sqrt{d+c^2dx^2} \\
&- \frac{2b^2cd^2x\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}} - \frac{16bcd^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{15\sqrt{1+c^2x^2}} \\
&- \frac{22bc^3d^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{45\sqrt{1+c^2x^2}} \\
&- \frac{2bc^5d^2x^5\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{25\sqrt{1+c^2x^2}} + d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2 \\
&+ \frac{1}{3} d\left(d+c^2dx^2\right)^{3/2} (a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{5}\left(d+c^2dx^2\right)^{5/2} (a+b\operatorname{arcsinh}(cx))^2 \\
&- \frac{2d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&- \frac{2bd^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&+ \frac{2bd^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&+ \frac{\left(2b^2d^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,-x)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&- \frac{\left(2b^2d^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,x)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$= \frac{598}{225}b^2d^2\sqrt{d+c^2dx^2} - \frac{2abcd^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{74}{675}b^2d^2(1+c^2x^2)\sqrt{d+c^2dx^2} + \frac{2}{125}b^2d^2(1+c^2x^2)^2\sqrt{d+c^2dx^2} - \frac{2b^2cd^2x\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}} - \frac{16bcd^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{15\sqrt{1+c^2x^2}} - \frac{22bc^3d^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{45\sqrt{1+c^2x^2}} - \frac{2bc^5d^2x^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{25\sqrt{1+c^2x^2}} + d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{3}d(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{5}(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2 - \frac{2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{2bd^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{2bd^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{2b^2d^2\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{2b^2d^2\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 3.56 (sec) , antiderivative size = 757, normalized size of antiderivative = 1.19

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx = \frac{1}{15}a^2d^2\sqrt{d+c^2dx^2}(23+11c^2x^2+3c^4x^4) - \frac{4abd^2\sqrt{d+c^2dx^2}\left(3cx+c^3x^3-3(1+c^2x^2)^{3/2}\text{arcsinh}(cx)\right)}{9\sqrt{1+c^2x^2}} + \frac{2abd^3\sqrt{1+c^2x^2}(30cx-5c^3x^3-9c^5x^5+15\sqrt{1+c^2x^2}(-2+c^2x^2+3c^4x^4)\text{arcsinh}(cx))}{225\sqrt{d+c^2dx^2}} - \frac{b^2d^3\sqrt{1+c^2x^2}(480cx(-30+5c^2x^2+9c^4x^4)\text{arcsinh}(cx)+6750\sqrt{1+c^2x^2}(2+\text{arcsinh}(cx)^2)+125(2+9\text{a}}{54000\sqrt{d+c^2dx^2}} + a^2d^{5/2}\log(cx) - a^2d^{5/2}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right) + \frac{2abd^3\sqrt{1+c^2x^2}(-cx+\sqrt{1+c^2x^2}\text{arcsinh}(cx)+\text{arcsinh}(c}{}$$

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (a^2*d^2*Sqrt[d + c^2*d*x^2]*(23 + 11*c^2*x^2 + 3*c^4*x^4))/15 - (4*a*b*d^2
*Sqrt[d + c^2*d*x^2]*(3*c*x + c^3*x^3 - 3*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x])
)/(9*Sqrt[1 + c^2*x^2]) + (2*a*b*d^3*Sqrt[1 + c^2*x^2]*(30*c*x - 5*c^3*x^3
- 9*c^5*x^5 + 15*Sqrt[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x])
)/(225*Sqrt[d + c^2*d*x^2]) - (b^2*d^3*Sqrt[1 + c^2*x^2]*(480*c*x*(-30 + 5*
c^2*x^2 + 9*c^4*x^4)*ArcSinh[c*x] + 6750*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c*x
]^2) + 125*(2 + 9*ArcSinh[c*x]^2)*Cosh[3*ArcSinh[c*x]] - 27*(2 + 25*ArcSinh
[c*x]^2)*Cosh[5*ArcSinh[c*x]]))/(54000*Sqrt[d + c^2*d*x^2]) + a^2*d^(5/2)*L
og[c*x] - a^2*d^(5/2)*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (2*a*b*d^3*Sqr
t[1 + c^2*x^2]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[
1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[
2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))/Sqrt[d + c^2*d*x^2
] + (b^2*d^3*Sqrt[1 + c^2*x^2]*(2*Sqrt[1 + c^2*x^2] - 2*c*x*ArcSinh[c*x] +
Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 + ArcSinh[c*x]^2*(Log[1 - E^(-ArcSinh[c*x]
)] - Log[1 + E^(-ArcSinh[c*x])]) + 2*ArcSinh[c*x]*(PolyLog[2, -E^(-ArcSinh[
c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]) + 2*(PolyLog[3, -E^(-ArcSinh[c*x])]
 - PolyLog[3, E^(-ArcSinh[c*x])])))/Sqrt[d + c^2*d*x^2] + (b^2*d^3*Sqrt[1 +
 c^2*x^2]*(27*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c*x]^2) + (2 + 9*ArcSinh[c*x]^
2)*Cosh[3*ArcSinh[c*x]] - 6*ArcSinh[c*x]*(9*c*x + Sinh[3*ArcSinh[c*x]])))/(
54*Sqrt[d + c^2*d*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1320 vs. 2(614) = 1228.

Time = 0.32 (sec) , antiderivative size = 1321, normalized size of antiderivative = 2.08

method	result	size
default	Expression too large to display	1321
parts	Expression too large to display	1321

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/125*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*c^6*x^6+532/3375*b^2*(d*(c^
2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*c^4*x^4+9872/3375*b^2*(d*(c^2*x^2+1))^(1/2)
*d^2/(c^2*x^2+1)*x^2*c^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcs
inh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*d^2-2*b^2*(d*(c^2*x^2+1))^(1/2)/(
c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*d^2-b^2*(d*
(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1
/2))*d^2+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1-c*
x-(c^2*x^2+1)^(1/2))*d^2-a^2*d^(5/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))
/x)+1/5*(c^2*d*x^2+d)^(5/2)*a^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/
2)*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*d^2+9394/3375*b^2*(d*(c^2*x^2+1))^(1/2
)*d^2/(c^2*x^2+1)+2/5*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x
)*x^6*c^6+28/15*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^4*
```

c^4+68/15*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^2*c^2+1/5*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)^2*x^6*c^6-22/45*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^3*c^3-46/15*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x*c-2/25*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*c^5*x^5+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*d^2-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*d^2-22/45*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*c^3*x^3-46/15*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*c*x+14/15*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)^2*x^4*c^4-2/25*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^5*c^5+34/15*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)^2*x^2*c^2-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*d^2+46/15*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*d^2-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(3,c*x+(c^2*x^2+1)^(1/2))*d^2+23/15*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)^2+1/3*a^2*d*(c^2*d*x^2+d)^(3/2)+a^2*d^2*(c^2*d*x^2+d)^(1/2)

Fricas [F]

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx=\int \frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}{x}\,dx$$

[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="fricas")

[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4 + 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x, x)

Sympy [F]

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx=\int \frac{\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}{x}\,dx$$

[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2/x,x)

[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))**2/x, x)

Maxima [F]

$$\int \frac{(d + c^2dx^2)^{5/2} (a + b\text{arcsinh}(cx))^2}{x} dx = \int \frac{(c^2dx^2 + d)^{\frac{5}{2}} (b \operatorname{arsinh}(cx) + a)^2}{x} dx$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] -1/15*(15*d^(5/2)*arcsinh(1/(c*abs(x))) - 3*(c^2*d*x^2 + d)^(5/2) - 5*(c^2*
d*x^2 + d)^(3/2)*d - 15*sqrt(c^2*d*x^2 + d)*d^2)*a^2 + integrate((c^2*d*x^2
 + d)^(5/2)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/x + 2*(c^2*d*x^2 + d)^(5/2)*
a*b*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + c^2dx^2)^{5/2} (a + b\text{arcsinh}(cx))^2}{x} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + c^2dx^2)^{5/2} (a + b\text{arcsinh}(cx))^2}{x} dx = \int \frac{(a + b \operatorname{asinh}(c x))^2 (d c^2 x^2 + d)^{5/2}}{x} dx$$

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x, x)
```

3.279 $\int \frac{(d+c^2dx^2)^{5/2}(a+b\,\text{arcsinh}(cx))^2}{x^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 530

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \frac{31}{64}b^2c^2d^2x\sqrt{d+c^2dx^2} + \frac{1}{32}b^2c^2d^2x(1+c^2x^2)\sqrt{d+c^2dx^2} - \frac{89b^2cd^2\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{64\sqrt{1+c^2x^2}} - \frac{15bc^3d^2x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} + bcd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) - \frac{1}{8}bcd^2(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{15}{8}c^2d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{cd^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}} + \frac{5}{4}c^2dx(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2 - \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x} + \frac{5cd^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}}{8b\sqrt{1+c^2x^2}}$$

```
[Out] 5/4*c^2*d*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2-(c^2*d*x^2+d)^(5/2)*(a
+b*arcsinh(c*x))^2/x+31/64*b^2*c^2*d^2*x*(c^2*d*x^2+d)^(1/2)+1/32*b^2*c^2*d
^2*x*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)-1/8*b*c*d^2*(c^2*x^2+1)^(3/2)*(a+b*arc
sinh(c*x))*(c^2*d*x^2+d)^(1/2)+15/8*c^2*d^2*x*(a+b*arcsinh(c*x))^2*(c^2*d*x
^2+d)^(1/2)-89/64*b^2*c*d^2*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1
/2)-15/8*b*c^3*d^2*x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(
1/2)+c*d^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5/8*c
*d^2*(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+2*b*c*d^2
*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(
c^2*x^2+1)^(1/2)-b^2*c*d^2*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*d*x^
2+d)^(1/2)/(c^2*x^2+1)^(1/2)+b*c*d^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)*(
c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.536$, Rules used = {5807, 5786, 5785, 5783, 5776, 327, 221, 5798, 201, 5801, 5775, 3797, 2221, 2317, 2438}

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^2}dx = \frac{15}{8}c^2d^2x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2 + \frac{5cd^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{8b\sqrt{c^2x^2+1}} + \frac{cd^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}} - \frac{1}{8}bcd^2(c^2x^2+1)^{3/2}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx)) + bcd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx)) + \frac{2bcd^2\sqrt{c^2dx^2+d}\log\left(1-e^{-2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}} + \frac{5}{4}c^2dx(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))^2 - \frac{(c^2dx^2+d)^{5/2}(a+b\text{arcsinh}(cx))^2}{x} - \frac{15bc^3d^2x^2\sqrt{c^2dx^2+d}(a+ba}{8\sqrt{c^2x^2+1}}$$

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (31*b^2*c^2*d^2*x*Sqrt[d + c^2*d*x^2])/64 + (b^2*c^2*d^2*x*(1 + c^2*x^2)*Sq
rt[d + c^2*d*x^2])/32 - (89*b^2*c*d^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(64
*Sqrt[1 + c^2*x^2]) - (15*b*c^3*d^2*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[
c*x]))/(8*Sqrt[1 + c^2*x^2]) + b*c*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2
]*(a + b*ArcSinh[c*x]) - (b*c*d^2*(1 + c^2*x^2)^(3/2)*Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x]))/8 + (15*c^2*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
*x])^2)/8 + (c*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/Sqrt[1 + c^2
*x^2] + (5*c^2*d*x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 - ((d +
c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x + (5*c*d^2*Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x])^3)/(8*b*Sqrt[1 + c^2*x^2]) + (2*b*c*d^2*Sqrt[d + c^2*d*
x^2]*(a + b*ArcSinh[c*x])*Log[1 - E^(-2*ArcSinh[c*x])])/Sqrt[1 + c^2*x^2] -
 (b^2*c*d^2*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/Sqrt[1 + c
^2*x^2]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
```

```
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5801

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcSinh[c*x])/(2*p)), x] + (Dist[d
, Int[(d + e*x^2)^(p - 1)*((a + b*ArcSinh[c*x])/x), x], x] - Dist[b*c*(d^p/
(2*p)), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
```

```
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x} \\
&\quad + (5c^2d)\int (d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx \\
&\quad + \frac{\left(2bcd^2\sqrt{d+c^2dx^2}\right)\int \frac{(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{x}\,dx}{\sqrt{1+c^2x^2}} \\
&= \frac{1}{2}bcd^2(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) \\
&\quad + \frac{5}{4}c^2dx(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2 - \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x} \\
&\quad + \frac{1}{4}(15c^2d^2)\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx + \frac{\left(2bcd^2\sqrt{d+c^2dx^2}\right)\int \frac{(1+c^2x^2)(a+b\text{arcsinh}(cx))}{x}\,dx}{\sqrt{1+c^2x^2}} \\
&= -\frac{1}{8}b^2c^2d^2x(1+c^2x^2)\sqrt{d+c^2dx^2} + bcd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) \\
&\quad - \frac{1}{8}bcd^2(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{15}{8}c^2d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 \\
&\quad + \frac{5}{4}c^2dx(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2 - \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x} \\
&\quad + \frac{\left(2bcd^2\sqrt{d+c^2dx^2}\right)\int \frac{a+b\text{arcsinh}(cx)}{x}\,dx}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(15c^2d^2\sqrt{d+c^2dx^2}\right)\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{8\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(3b^2c^2d^2\sqrt{d+c^2dx^2}\right)\int \sqrt{1+c^2x^2}\,dx}{8\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(5b^2c^2d^2\sqrt{d+c^2dx^2}\right)\int (1+c^2x^2)^{3/2}\,dx}{8\sqrt{1+c^2x^2}} - \frac{\left(b^2c^2d^2\sqrt{d+c^2dx^2}\right)\int \sqrt{1+c^2x^2}\,dx}{\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(15bc^3d^2\sqrt{d+c^2dx^2}\right)\int x(a+b\text{arcsinh}(cx))\,dx}{4\sqrt{1+c^2x^2}}
\end{aligned}$$

$$= -\frac{11}{16}b^2c^2d^2x\sqrt{d+c^2dx^2} + \frac{1}{32}b^2c^2d^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2} - \frac{15bc^3d^2x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} + bcd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) - \frac{1}{8}bcd^2\left(1+c^2x^2\right)^{3/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{15}{8}c^2d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{5}{4}c^2dx\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2 - \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x} + \frac{5cd^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{8b\sqrt{1+c^2x^2}} - \frac{\left(2cd^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int x\coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}} - \frac{\left(3b^2c^2d^2\sqrt{d+c^2dx^2}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{16\sqrt{1+c^2x^2}} + \frac{\left(15b^2c^2d^2\sqrt{d+c^2dx^2}\right)\int\sqrt{1+c^2x^2}\,dx}{32\sqrt{1+c^2x^2}} - \frac{\left(b^2c^2d^2\sqrt{d+c^2dx^2}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}} + \frac{\left(15b^2c^4d^2\sqrt{d+c^2dx^2}\right)\int\frac{x^2}{\sqrt{1+c^2x^2}}\,dx}{8\sqrt{1+c^2x^2}}$$

$$= \frac{31}{64}b^2c^2d^2x\sqrt{d+c^2dx^2} + \frac{1}{32}b^2c^2d^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2} - \frac{11b^2cd^2\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{16\sqrt{1+c^2x^2}} - \frac{15bc^3d^2x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} + bcd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) - \frac{1}{8}bcd^2\left(1+c^2x^2\right)^{3/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{15}{8}c^2d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{cd^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}} + \frac{5}{4}c^2dx\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2 - \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x} + \frac{5cd^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{8b\sqrt{1+c^2x^2}} + \frac{\left(4cd^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}} + \frac{\left(15b^2c^2d^2\sqrt{d+c^2dx^2}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{64\sqrt{1+c^2x^2}} - \frac{\left(15b^2c^2d^2\sqrt{d+c^2dx^2}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{16\sqrt{1+c^2x^2}}$$

$$= \frac{31}{64}b^2c^2d^2x\sqrt{d+c^2dx^2} + \frac{1}{32}b^2c^2d^2x(1+c^2x^2)\sqrt{d+c^2dx^2} - \frac{89b^2cd^2\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{64\sqrt{1+c^2x^2}} - \frac{15bc^3d^2x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} + bcd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) - \frac{1}{8}bcd^2(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{15}{8}c^2d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{cd^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}} + \frac{5}{4}c^2dx(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2 - \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x} + \frac{5cd^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{8b\sqrt{1+c^2x^2}} + \frac{2bcd^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{\left(2bcd^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)dx, x, a+b\text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}}$$

$$= \frac{31}{64}b^2c^2d^2x\sqrt{d+c^2dx^2} + \frac{1}{32}b^2c^2d^2x(1+c^2x^2)\sqrt{d+c^2dx^2} - \frac{89b^2cd^2\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{64\sqrt{1+c^2x^2}} - \frac{15bc^3d^2x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} + bcd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) - \frac{1}{8}bcd^2(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{15}{8}c^2d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{cd^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}} + \frac{5}{4}c^2dx(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2 - \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x} + \frac{5cd^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{8b\sqrt{1+c^2x^2}} + \frac{2bcd^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{\left(b^2cd^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right)}{\sqrt{1+c^2x^2}}$$

$$
\begin{aligned}
&= \frac{31}{64}b^2c^2d^2x\sqrt{d+c^2dx^2} + \frac{1}{32}b^2c^2d^2x(1+c^2x^2)\sqrt{d+c^2dx^2} \\
&- \frac{89b^2cd^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{64\sqrt{1+c^2x^2}} - \frac{15bc^3d^2x^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} \\
&+ bcd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) - \frac{1}{8}bcd^2(1 \\
&+ c^2x^2)^{3/2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{15}{8}c^2d^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2 \\
&+ \frac{cd^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}} + \frac{5}{4}c^2dx(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2 \\
&- \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{x} + \frac{5cd^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^3}{8b\sqrt{1+c^2x^2}} \\
&+ \frac{2bcd^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\log\left(1-e^{-2\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&- \frac{b^2cd^2\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}\right)}{\sqrt{1+c^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.79 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.04

$$
\int \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{x^2}\,dx = \frac{d^2\left(-256a^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2} + 288a^2c^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2\right.}{}
$$

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (d^2*(-256*a^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 288*a^2*c^2*x^2*Sqrt
[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 64*a^2*c^4*x^4*Sqrt[1 + c^2*x^2]*Sqrt[d
 + c^2*d*x^2] + 160*b^2*c*x*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 - 128*a*b*c*
x*Sqrt[d + c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 4*a*b*c*x*Sqrt[d + c^2*d*x^2]*
Cosh[4*ArcSinh[c*x]] + 512*a*b*c*x*Sqrt[d + c^2*d*x^2]*Log[c*x] + 480*a^2*c
*Sqrt[d]*x*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 256
*b^2*c*x*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])] + 64*b^2*c*x*S
qrt[d + c^2*d*x^2]*Sinh[2*ArcSinh[c*x]] + b^2*c*x*Sqrt[d + c^2*d*x^2]*Sinh[
4*ArcSinh[c*x]] - 4*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(128*a*Sqrt[1 + c^2*
x^2] + 32*b*c*x*Cosh[2*ArcSinh[c*x]] + b*c*x*Cosh[4*ArcSinh[c*x]] - 128*b*c
*x*Log[1 - E^(-2*ArcSinh[c*x])] - 64*a*c*x*Sinh[2*ArcSinh[c*x]] - 4*a*c*x*S
inh[4*ArcSinh[c*x]]) + 8*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2*(60*a*c*x + 3
2*b*c*x - 32*b*Sqrt[1 + c^2*x^2] + 16*b*c*x*Sinh[2*ArcSinh[c*x]] + b*c*x*Si
nh[4*ArcSinh[c*x]])))/(256*x*Sqrt[1 + c^2*x^2])
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.11

method	result
default	$-\frac{a^2(c^2dx^2+d)^{\frac{7}{2}}}{dx}+a^2c^2x(c^2dx^2+d)^{\frac{5}{2}}+\frac{5(c^2dx^2+d)^{\frac{3}{2}}a^2c^2dx}{4}+\frac{15a^2d^2\sqrt{c^2dx^2+d}\,c^2x}{8}+\frac{15a^2c^2d^3\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx\right.}}{8\sqrt{c^2d}}$
parts	$-\frac{a^2(c^2dx^2+d)^{\frac{7}{2}}}{dx}+a^2c^2x(c^2dx^2+d)^{\frac{5}{2}}+\frac{5(c^2dx^2+d)^{\frac{3}{2}}a^2c^2dx}{4}+\frac{15a^2d^2\sqrt{c^2dx^2+d}\,c^2x}{8}+\frac{15a^2c^2d^3\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx\right.}}{8\sqrt{c^2d}}$

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^2/d/x*(c^2*d*x^2+d)^(7/2)+a^2*c^2*x*(c^2*d*x^2+d)^(5/2)+5/4*(c^2*d*x^2+d
)^(3/2)*a^2*c^2*d*x+15/8*a^2*d^2*(c^2*d*x^2+d)^(1/2)*c^2*x+15/8*a^2*c^2*d^3
*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/64*b^2*(d*(c
^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/x*(16*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*x
^4*c^4-8*arcsinh(c*x)*c^5*x^5+2*c^4*x^4*(c^2*x^2+1)^(1/2)+72*arcsinh(c*x)^2
*(c^2*x^2+1)^(1/2)*x^2*c^2-72*arcsinh(c*x)*c^3*x^3+40*arcsinh(c*x)^3*x*c+33
*c^2*x^2*(c^2*x^2+1)^(1/2)-64*arcsinh(c*x)^2*x*c+128*arcsinh(c*x)*ln(1+c*x+
(c^2*x^2+1)^(1/2))*x*c+128*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*x*c-64*
(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2-33*arcsinh(c*x)*c*x+128*polylog(2,-c*x-(c^
2*x^2+1)^(1/2))*x*c+128*polylog(2,c*x+(c^2*x^2+1)^(1/2))*x*c)*d^2+1/64*a*b*
(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/x*(32*arcsinh(c*x)*(c^2*x^2+1)^(1/2
)*x^4*c^4-8*c^5*x^5+144*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2-72*c^3*x^3+1
20*arcsinh(c*x)^2*x*c-128*arcsinh(c*x)*c*x+128*ln((c*x+(c^2*x^2+1)^(1/2))^2
-1)*x*c-128*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-33*c*x)*d^2
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}{x^2}\,dx=\int \frac{(c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{x^2}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^2, x)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \int \frac{(d(c^2x^2+1))^{\frac{5}{2}}\,(a+b\,\text{asinh}\,(cx))^2}{x^2}\,dx$$

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2/x**2,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))**2/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)^{5/2}}{x^2}\,dx$$

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x^2, x)
```

3.280 $\int \frac{(d+c^2dx^2)^{5/2}(a+b\,\mathbf{arcsinh}(cx))^2}{x^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 687

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{x^3}\,dx = \frac{40}{9}b^2c^2d^2\sqrt{d+c^2dx^2} - \frac{5abc^3d^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{2}{27}b^2c^2d^2(1+c^2x^2)\sqrt{d+c^2dx^2} - \frac{5b^2c^3d^2x\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}} - \frac{bcd^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{x\sqrt{1+c^2x^2}} + \frac{bc^3d^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} - \frac{2bc^5d^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} + \frac{5}{2}c^2d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2 + \frac{5}{6}c^2d(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2 - \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{2x^2} - \frac{5c^2d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{b^2c^2d^2\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\sqrt{1+c^2x^2}\right)}{\sqrt{1+c^2x^2}} - \frac{5bc^2d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{5bc^2d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{5b^2c^2d^2\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{5b^2c^2d^2\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}$$

```
[Out] 5/6*c^2*d*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2-1/2*(c^2*d*x^2+d)^(5/2)*
(a+b*arcsinh(c*x))^2/x^2+40/9*b^2*c^2*d^2*(c^2*d*x^2+d)^(1/2)+2/27*b^2*c^2*
d^2*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)+5/2*c^2*d^2*(a+b*arcsinh(c*x))^2*(c^2*d
*x^2+d)^(1/2)-5*a*b*c^3*d^2*x*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-5*b^2*c
^3*d^2*x*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b*c*d^2*(a+b*ar
csinh(c*x))*(c^2*d*x^2+d)^(1/2)/x/(c^2*x^2+1)^(1/2)+1/3*b*c^3*d^2*x*(a+b*ar
csinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/9*b*c^5*d^2*x^3*(a+b*ar
csinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-5*c^2*d^2*(a+b*arcsinh(c*
x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-
b^2*c^2*d^2*arctanh((c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2
)-5*b*c^2*d^2*(a+b*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x
```

```
^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5*b*c^2*d^2*(a+b*arcsinh(c*x))*polylog(2,c*x+
(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5*b^2*c^2*d^2*poly
log(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-5*b^2*c
^2*d^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/
2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5807, 5808, 5806, 5816, 4267, 2611, 2320, 6724, 5772, 267, 5784, 455, 45, 276, 5803, 12, 1265, 911, 1167, 214}

$$\begin{aligned}
&\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^3}dx= \\
&-\frac{5c^2d^2\sqrt{c^2dx^2+d}\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}} \\
&-\frac{5bc^2d^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}} \\
&+\frac{5bc^2d^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}} \\
&+\frac{5}{2}c^2d^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bcd^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{x\sqrt{c^2x^2+1}} \\
&+\frac{5}{6}c^2d\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2} \\
&-\frac{2bc^5d^2x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{9\sqrt{c^2x^2+1}} \\
&+\frac{bc^3d^2x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{5abc^3d^2x\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}} \\
&+\frac{5b^2c^2d^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{c^2x^2+1}} \\
&-\frac{5b^2c^2d^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{\sqrt{c^2x^2+1}} \\
&-\frac{5b^2c^3d^2x\text{arcsinh}(cx)\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}-\frac{b^2c^2d^2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}} \\
&+\frac{40}{9}b^2c^2d^2\sqrt{c^2dx^2+d}+\frac{2}{27}b^2c^2d^2\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}
\end{aligned}$$

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] (40*b^2*c^2*d^2*Sqrt[d + c^2*d*x^2])/9 - (5*a*b*c^3*d^2*x*Sqrt[d + c^2*d*x^
2])/Sqrt[1 + c^2*x^2] + (2*b^2*c^2*d^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/2
```

```
7 - (5*b^2*c^3*d^2*x*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] -
(b*c*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(x*Sqrt[1 + c^2*x^2]) +
(b*c^3*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*Sqrt[1 + c^2*x^2]
) - (2*b*c^5*d^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(9*Sqrt[1 +
c^2*x^2]) + (5*c^2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/2 + (5*c
^2*d*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/6 - ((d + c^2*d*x^2)^(5/
2)*(a + b*ArcSinh[c*x])^2)/(2*x^2) - (5*c^2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*
ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b^2*c^2*d^2*S
qrt[d + c^2*d*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/Sqrt[1 + c^2*x^2] - (5*b*c^2
*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])/
Sqrt[1 + c^2*x^2] + (5*b*c^2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*P
olyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (5*b^2*c^2*d^2*Sqrt[d + c^2*
d*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (5*b^2*c^2*d^2*Sqrt
[d + c^2*d*x^2]*PolyLog[3, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
```

```
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2} \\
&\quad+\frac{1}{2}\left(5c^2d\right)\int\frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx \\
&\quad+\frac{\left(bcd^2\sqrt{d+c^2dx^2}\right)\int\frac{\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))}{x^2}\,dx}{\sqrt{1+c^2x^2}} \\
&= -\frac{bcd^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x\sqrt{1+c^2x^2}}+\frac{2bc^3d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}} \\
&\quad+\frac{bc^5d^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} \\
&\quad+\frac{5}{6}c^2d\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2} \\
&\quad+\frac{1}{2}\left(5c^2d^2\right)\int\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x}\,dx \\
&\quad-\frac{\left(b^2c^2d^2\sqrt{d+c^2dx^2}\right)\int\frac{-3+6c^2x^2+c^4x^4}{3x\sqrt{1+c^2x^2}}\,dx}{\sqrt{1+c^2x^2}} \\
&\quad-\frac{\left(5bc^3d^2\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))\,dx}{3\sqrt{1+c^2x^2}}
\end{aligned}$$

$$= -\frac{bcd^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x\sqrt{1+c^2x^2}} + \frac{bc^3d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} - \frac{2bc^5d^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} + \frac{5}{2}c^2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{5}{6}c^2d\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2 - \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2} + \frac{\left(5c^2d^2\sqrt{d+c^2dx^2}\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}} - \frac{\left(b^2c^2d^2\sqrt{d+c^2dx^2}\right)\int\frac{-3+6c^2x^2+c^4x^4}{x\sqrt{1+c^2x^2}}\,dx}{3\sqrt{1+c^2x^2}} - \frac{\left(5bc^3d^2\sqrt{d+c^2dx^2}\right)\int(a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}} + \frac{\left(5b^2c^4d^2\sqrt{d+c^2dx^2}\right)\int\frac{x\left(1+\frac{c^2x^2}{3}\right)}{\sqrt{1+c^2x^2}}\,dx}{3\sqrt{1+c^2x^2}}$$

$$= -\frac{5abc^3d^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{bcd^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x\sqrt{1+c^2x^2}} + \frac{bc^3d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} - \frac{2bc^5d^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} + \frac{5}{2}c^2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{5}{6}c^2d\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2 - \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2} + \frac{\left(5c^2d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^2\text{csch}(x)\,dx,x,\text{arcsinh}(cx)\right)}{2\sqrt{1+c^2x^2}} - \frac{\left(b^2c^2d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{-3+6c^2x+c^4x^2}{x\sqrt{1+c^2x}}\,dx,x,x^2\right)}{6\sqrt{1+c^2x^2}} - \frac{\left(5b^2c^3d^2\sqrt{d+c^2dx^2}\right)\int\text{arcsinh}(cx)\,dx}{\sqrt{1+c^2x^2}} + \frac{\left(5b^2c^4d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{1+\frac{c^2x}{3}}{\sqrt{1+c^2x}}\,dx,x,x^2\right)}{6\sqrt{1+c^2x^2}}$$

$$= -\frac{5abc^3d^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{5b^2c^3d^2x\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}} - \frac{bcd^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x\sqrt{1+c^2x^2}} + \frac{bc^3d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} - \frac{2bc^5d^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} + \frac{5}{2}c^2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{5}{6}c^2d\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2 - \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2} - \frac{5c^2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{\left(b^2d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{-8+4x^2+x^4}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx,x,\sqrt{1+c^2x^2}\right)}{3\sqrt{1+c^2x^2}} - \frac{\left(5bc^2d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)\log\left(1-e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}} + \frac{\left(5bc^2d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)\log\left(1+e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}} + \frac{\left(5b^2c^4d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{2}{3\sqrt{1+c^2x}}+\frac{1}{3}\sqrt{1+c^2x}\right)\,dx,x,x^2\right)}{6\sqrt{1+c^2x^2}} + \frac{\left(5b^2c^4d^2\sqrt{d+c^2dx^2}\right)\int\frac{x}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{1+c^2x^2}}$$

$$
\begin{aligned}
&= \frac{55}{9}b^2c^2d^2\sqrt{d+c^2dx^2} - \frac{5abc^3d^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{5}{27}b^2c^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2} - \frac{5b^2c^3d^2x\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}} \\
&\quad - \frac{bcd^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{x\sqrt{1+c^2x^2}} + \frac{bc^3d^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} \\
&\quad - \frac{2bc^5d^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} + \frac{5}{2}c^2d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2 \\
&\quad + \frac{5}{6}c^2d\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2 - \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{2x^2} \\
&\quad - \frac{5c^2d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad - \frac{5bc^2d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{5bc^2d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(b^2d^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\left(5c^2+c^2x^2-\frac{3}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\right)dx,x,\sqrt{1+c^2x^2}\right)}{3\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(5b^2c^2d^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(2,-e^x\right)dx,x,\operatorname{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(5b^2c^2d^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(2,e^x\right)dx,x,\operatorname{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$= \frac{40}{9}b^2c^2d^2\sqrt{d+c^2dx^2} - \frac{5abc^3d^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{2}{27}b^2c^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2} - \frac{5b^2c^3d^2x\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}} - \frac{bcd^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x\sqrt{1+c^2x^2}} + \frac{bc^3d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} - \frac{2bc^5d^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} + \frac{5}{2}c^2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{5}{6}c^2d\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2 - \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2} - \frac{5c^2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{5bc^2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{5bc^2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{\left(b^2d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx,x,\sqrt{1+c^2x^2}\right)}{\sqrt{1+c^2x^2}} + \frac{\left(5b^2c^2d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{\left(5b^2c^2d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}$$

$$= \frac{40}{9}b^2c^2d^2\sqrt{d+c^2dx^2} - \frac{5abc^3d^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{2}{27}b^2c^2d^2(1+c^2x^2)\sqrt{d+c^2dx^2} - \frac{5b^2c^3d^2x\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}} - \frac{bcd^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{x\sqrt{1+c^2x^2}} + \frac{bc^3d^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} - \frac{2bc^5d^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} + \frac{5}{2}c^2d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2 + \frac{5}{6}c^2d(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2 - \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{2x^2} - \frac{5c^2d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{b^2c^2d^2\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\sqrt{1+c^2x^2}\right)}{\sqrt{1+c^2x^2}} - \frac{5bc^2d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{5bc^2d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{5b^2c^2d^2\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{5b^2c^2d^2\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 8.04 (sec) , antiderivative size = 990, normalized size of antiderivative = 1.44

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{x^3}\,dx = \sqrt{d(1+c^2x^2)}\left(\frac{7}{3}a^2c^2d^2 - \frac{a^2d^2}{2x^2} + \frac{1}{3}a^2c^4d^2x^2\right) + 2abc^2d^2\left(-\frac{cx\sqrt{d(1+c^2x^2)}(3+c^2x^2)}{9\sqrt{1+c^2x^2}} + \frac{1}{3}(1+c^2x^2)\sqrt{d(1+c^2x^2)}\operatorname{arcsinh}(cx)\right) + \frac{5}{2}a^2c^2d^{5/2}\log(x) - \frac{5}{2}a^2c^2d^{5/2}\log\left(d+\sqrt{d}\sqrt{d(1+c^2x^2)}\right) + \frac{4abc^2d^2\sqrt{d(1+c^2x^2)}\left(-cx+\sqrt{1+c^2x^2}\operatorname{arcsinh}(cx)+\operatorname{arcsinh}(cx\right.}{}$$

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] Sqrt[d*(1 + c^2*x^2)]*((7*a^2*c^2*d^2)/3 - (a^2*d^2)/(2*x^2) + (a^2*c^4*d^2
*x^2)/3) + 2*a*b*c^2*d^2*(-1/9*(c*x*Sqrt[d*(1 + c^2*x^2)]*(3 + c^2*x^2))/Sq
```

```
rt[1 + c^2*x^2] + ((1 + c^2*x^2)*Sqrt[d*(1 + c^2*x^2)]*ArcSinh[c*x])/3) + (
5*a^2*c^2*d^(5/2)*Log[x])/2 - (5*a^2*c^2*d^(5/2)*Log[d + Sqrt[d]*Sqrt[d*(1
+ c^2*x^2)]])/2 + (4*a*b*c^2*d^2*Sqrt[d*(1 + c^2*x^2)]*(-(c*x) + Sqrt[1 + c
^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*
x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2,
 E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2] + 2*b^2*c^2*d^2*Sqrt[d*(1 + c^2*x^2
)]*(2 - (2*c*x*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + ArcSinh[c*x]^2 + (ArcSinh[
c*x]^2*(Log[1 - E^(-ArcSinh[c*x])] - Log[1 + E^(-ArcSinh[c*x])]))/Sqrt[1 +
c^2*x^2] + (2*ArcSinh[c*x]*(PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(
-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2] + (2*(PolyLog[3, -E^(-ArcSinh[c*x])] -
PolyLog[3, E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2]) + (b^2*c^2*d^2*Sqrt[d*(1
 + c^2*x^2)]*(27*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c*x]^2) + (2 + 9*ArcSinh[c*
x]^2)*Cosh[3*ArcSinh[c*x]] - 6*ArcSinh[c*x]*(9*c*x + Sinh[3*ArcSinh[c*x]]))
)/(108*Sqrt[1 + c^2*x^2]) + (a*b*c^2*d^2*Sqrt[d*(1 + c^2*x^2)]*(-2*Coth[Arc
Sinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]*Log[1 -
 E^(-ArcSinh[c*x])] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 4*PolyLog
[2, -E^(-ArcSinh[c*x])] - 4*PolyLog[2, E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Se
ch[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/(4*Sqrt[1 + c^2*x^2]) + (b^
2*c^2*d^2*Sqrt[d*(1 + c^2*x^2)]*(-4*ArcSinh[c*x]*Coth[ArcSinh[c*x]/2] - Arc
Sinh[c*x]^2*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]^2*Log[1 - E^(-ArcSinh[c
*x])] - 4*ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] + 8*Log[Tanh[ArcSinh[c*
x]/2]] + 8*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - 8*ArcSinh[c*x]*Pol
yLog[2, E^(-ArcSinh[c*x])] + 8*PolyLog[3, -E^(-ArcSinh[c*x])] - 8*PolyLog[3
, E^(-ArcSinh[c*x])] - ArcSinh[c*x]^2*Sech[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*
x]*Tanh[ArcSinh[c*x]/2]))/(8*Sqrt[1 + c^2*x^2])
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 720, normalized size of antiderivative = 1.05

method	result
default	$a^2\left(-\frac{(c^2 d x^2+d)^{\frac{7}{2}}}{2 d x^2}+\frac{5c^2\left(\frac{(c^2 d x^2+d)^{\frac{5}{2}}}{5}+d\left(\frac{(c^2 d x^2+d)^{\frac{3}{2}}}{3}+d\left(\sqrt{c^2 d x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2 d x^2+d}}{x}\right)\right)\right)\right)}{2}\right)+\frac{b^2\sqrt{d(c^2x^2}$
parts	$a^2\left(-\frac{(c^2 d x^2+d)^{\frac{7}{2}}}{2 d x^2}+\frac{5c^2\left(\frac{(c^2 d x^2+d)^{\frac{5}{2}}}{5}+d\left(\frac{(c^2 d x^2+d)^{\frac{3}{2}}}{3}+d\left(\sqrt{c^2 d x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2 d x^2+d}}{x}\right)\right)\right)\right)}{2}\right)+\frac{b^2\sqrt{d(c^2x^2}$

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/2/d/x^2*(c^2*d*x^2+d)^(7/2)+5/2*c^2*(1/5*(c^2*d*x^2+d)^(5/2)+d*(1/3
*(c^2*d*x^2+d)^(3/2)+d*((c^2*d*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*
```

```
d*x^2+d)^(1/2))/x)))))+1/54*b^2*(d*(c^2*x^2+1))^(1/2)*(18*(c^2*x^2+1)^(1/2)
*arcsinh(c*x)^2*x^4*c^4-12*arcsinh(c*x)*c^5*x^5+4*c^4*x^4*(c^2*x^2+1)^(1/2)
+126*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*x^2*c^2+135*arcsinh(c*x)^2*ln(1-c*x-(
c^2*x^2+1)^(1/2))*x^2*c^2-135*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))*x^
2*c^2-252*arcsinh(c*x)*c^3*x^3-270*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^
(1/2))*x^2*c^2+270*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*x^2*c^2+24
4*c^2*x^2*(c^2*x^2+1)^(1/2)-108*arctanh(c*x+(c^2*x^2+1)^(1/2))*x^2*c^2+270*
polylog(3,-c*x-(c^2*x^2+1)^(1/2))*x^2*c^2-270*polylog(3,c*x+(c^2*x^2+1)^(1/
2))*x^2*c^2-27*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2-54*arcsinh(c*x)*c*x)*d^2/(c
^2*x^2+1)^(1/2)/x^2+1/9*a*b*(d*(c^2*x^2+1))^(1/2)*(6*arcsinh(c*x)*(c^2*x^2+
1)^(1/2)*x^4*c^4-2*c^5*x^5+42*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2+45*arc
sinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*x^2*c^2-45*arcsinh(c*x)*ln(1+c*x+(c^2
*x^2+1)^(1/2))*x^2*c^2-42*c^3*x^3+45*polylog(2,c*x+(c^2*x^2+1)^(1/2))*x^2*c
^2-45*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*x^2*c^2-9*arcsinh(c*x)*(c^2*x^2+1)^
(1/2)-9*c*x)*d^2/(c^2*x^2+1)^(1/2)/x^2
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}{x^3}\,dx=\int \frac{(c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{x^3}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^3, x)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}{x^3}\,dx=\int \frac{(d(c^2x^2+1))^{\frac{5}{2}}\,(a+b\,\text{asinh}\,(cx))^2}{x^3}\,dx$$

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2/x**3,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))**2/x**3, x)
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}{x^3}\,dx = \int \frac{(c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{x^3}\,dx$$

[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="maxima
")

[Out] -1/6*(15*c^2*d^(5/2)*arcsinh(1/(c*abs(x))) - 3*(c^2*d*x^2 + d)^(5/2)*c^2 -
5*(c^2*d*x^2 + d)^(3/2)*c^2*d - 15*sqrt(c^2*d*x^2 + d)*c^2*d^2 + 3*(c^2*d*x
^2 + d)^(7/2)/(d*x^2))*a^2 + integrate((c^2*d*x^2 + d)^(5/2)*b^2*log(c*x +
sqrt(c^2*x^2 + 1))^2/x^3 + 2*(c^2*d*x^2 + d)^(5/2)*a*b*log(c*x + sqrt(c^2*x
^2 + 1))/x^3, x)

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}{x^3}\,dx = \text{Exception raised: TypeError}$$

[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}{x^3}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)^{5/2}}{x^3}\,dx$$

[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x^3,x)

[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x^3, x)

3.281 $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 561

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = \frac{7}{12}b^2c^4d^2x\sqrt{d+c^2dx^2} - \frac{b^2c^2d^2(1+c^2x^2)\sqrt{d+c^2dx^2}}{3x} - \frac{23b^2c^3d^2\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{12\sqrt{1+c^2x^2}} - \frac{5bc^5d^2x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}} + \frac{7}{3}bc^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) - \frac{bcd^2(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3x^2} + \frac{5}{2}c^4d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{7c^3d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3\sqrt{1+c^2x^2}} - \frac{5c^2d(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x} - \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3} + \frac{5c^3d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{6b\sqrt{1+c^2x^2}} + \frac{14bc^3d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)}{3\sqrt{1+c^2x^2}} - \frac{7b^2c^3d^2\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)}{3\sqrt{1+c^2x^2}}$$

```
[Out] -5/3*c^2*d*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x-1/3*(c^2*d*x^2+d)^(5/
2)*(a+b*arcsinh(c*x))^2/x^3+7/12*b^2*c^4*d^2*x*(c^2*d*x^2+d)^(1/2)-1/3*b^2*
c^2*d^2*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/x-1/3*b*c*d^2*(c^2*x^2+1)^(3/2)*(a+
```

```
b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^2+5/2*c^4*d^2*x*(a+b*arcsinh(c*x))^2*
(c^2*d*x^2+d)^(1/2)-23/12*b^2*c^3*d^2*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2
*x^2+1)^(1/2)-5/2*b*c^5*d^2*x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2
*x^2+1)^(1/2)+7/3*c^3*d^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2
+1)^(1/2)+5/6*c^3*d^2*(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^(1/2)/b/(c^2*x^2+1
)^(1/2)+14/3*b*c^3*d^2*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)
*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-7/3*b^2*c^3*d^2*polylog(2,1/(c*x+(c^
2*x^2+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+7/3*b*c^3*d^2*(a+b
*arcsinh(c*x))*(c^2*x^2+1)^(1/2)*(c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.536$, Rules used = {5807, 5785, 5783, 5776, 327, 221, 5801, 5775, 3797, 2221, 2317, 2438, 201, 5802, 283}

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx =$$
$$-\frac{bcd^2(c^2x^2+1)^{3/2}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{3x^2}-\frac{5c^2d(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x}$$
$$-\frac{(c^2dx^2+d)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}-\frac{5bc^5d^2x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2\sqrt{c^2x^2+1}}$$
$$+\frac{5}{2}c^4d^2x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2+\frac{5c^3d^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{6b\sqrt{c^2x^2+1}}+\frac{7c^3d^2\sqrt{c^2dx^2+d}(a+b\text{arcsin}}{3\sqrt{c^2x^2+1}}$$

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] (7*b^2*c^4*d^2*x*Sqrt[d + c^2*d*x^2])/12 - (b^2*c^2*d^2*(1 + c^2*x^2)*Sqrt[
d + c^2*d*x^2])/(3*x) - (23*b^2*c^3*d^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(
12*Sqrt[1 + c^2*x^2]) - (5*b*c^5*d^2*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x]))/(2*Sqrt[1 + c^2*x^2]) + (7*b*c^3*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x]))/3 - (b*c*d^2*(1 + c^2*x^2)^(3/2)*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x]))/(3*x^2) + (5*c^4*d^2*x*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])^2)/2 + (7*c^3*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x
])^2)/(3*Sqrt[1 + c^2*x^2]) - (5*c^2*d*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh
[c*x])^2)/(3*x) - ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*x^3) +
(5*c^3*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b*Sqrt[1 + c^2*x^
2]) + (14*b*c^3*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Log[1 - E^(-2*
ArcSinh[c*x])])/(3*Sqrt[1 + c^2*x^2]) - (7*b^2*c^3*d^2*Sqrt[d + c^2*d*x^2]*
PolyLog[2, E^(-2*ArcSinh[c*x])])/(3*Sqrt[1 + c^2*x^2])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5801

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcSinh[c*x])/(2*p)), x] + (Dist[d
, Int[(d + e*x^2)^(p - 1)*((a + b*ArcSinh[c*x])/x), x], x] - Dist[b*c*(d^p/
(2*p)), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5802

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSinh[c
```

```
*x])/(f*(m + 1))), x] + (-Dist[b*c*(d^p/(f*(m + 1))), Int[(f*x)^(m + 1)*(1
+ c^2*x^2)^(p - 1/2), x], x] - Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m + 2
)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Rule 5807

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2
)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3} + \frac{1}{3}(5c^2d)\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx + \frac{\left(2bcd^2\sqrt{d+c^2dx^2}\right)\int \frac{(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{x^3}\,dx}{3\sqrt{1+c^2x^2}}$$

$$= -\frac{bcd^2(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3x^2} - \frac{5c^2d(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x} - \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3} + (5c^4d^2)\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx + \frac{\left(b^2c^2d^2\sqrt{d+c^2dx^2}\right)\int \frac{(1+c^2x^2)^{3/2}}{x^2}\,dx}{3\sqrt{1+c^2x^2}} + \frac{\left(4bc^3d^2\sqrt{d\right.}}{}$$

$$=-\frac{b^2c^2d^2(1+c^2x^2)\sqrt{d+c^2dx^2}}{3x}+\frac{7}{3}bc^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))$$
$$-\frac{bcd^2(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3x^2}$$
$$+\frac{5}{2}c^4d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2-\frac{5c^2d(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x}$$
$$-\frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}+\frac{\left(4bc^3d^2\sqrt{d+c^2dx^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{x}\,dx}{3\sqrt{1+c^2x^2}}$$
$$+\frac{\left(10bc^3d^2\sqrt{d+c^2dx^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{x}\,dx}{3\sqrt{1+c^2x^2}}$$
$$+\frac{\left(5c^4d^2\sqrt{d+c^2dx^2}\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}}$$
$$-\frac{\left(2b^2c^4d^2\sqrt{d+c^2dx^2}\right)\int\sqrt{1+c^2x^2}\,dx}{3\sqrt{1+c^2x^2}}+\frac{\left(b^2c^4d^2\sqrt{d+c^2dx^2}\right)\int\sqrt{1+c^2x^2}\,dx}{\sqrt{1+c^2x^2}}$$
$$-\frac{\left(5b^2c^4d^2\sqrt{d+c^2dx^2}\right)\int\sqrt{1+c^2x^2}\,dx}{3\sqrt{1+c^2x^2}}$$
$$-\frac{\left(5bc^5d^2\sqrt{d+c^2dx^2}\right)\int x(a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}}$$

$$=-\frac{2}{3}b^2c^4d^2x\sqrt{d+c^2dx^2}-\frac{b^2c^2d^2(1+c^2x^2)\sqrt{d+c^2dx^2}}{3x}$$
$$-\frac{5bc^5d^2x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}}$$
$$+\frac{7}{3}bc^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))$$
$$-\frac{bcd^2(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3x^2}$$
$$+\frac{5}{2}c^4d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2-\frac{5c^2d(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x}$$
$$-\frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}+\frac{5c^3d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{6b\sqrt{1+c^2x^2}}$$
$$-\frac{\left(4c^3d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int x\coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{3\sqrt{1+c^2x^2}}$$
$$-\frac{\left(10c^3d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int x\coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{3\sqrt{1+c^2x^2}}$$
$$-\frac{\left(b^2c^4d^2\sqrt{d+c^2dx^2}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{3\sqrt{1+c^2x^2}}+\frac{\left(b^2c^4d^2\sqrt{d+c^2dx^2}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}}$$
$$-\frac{\left(5b^2c^4d^2\sqrt{d+c^2dx^2}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{6\sqrt{1+c^2x^2}}+\frac{\left(5b^2c^6d^2\sqrt{d+c^2dx^2}\right)\int\frac{x^2}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}}$$

$$
\begin{aligned}
&= \frac{7}{12}b^2c^4d^2x\sqrt{d+c^2dx^2} - \frac{b^2c^2d^2(1+c^2x^2)\sqrt{d+c^2dx^2}}{3x} \\
&- \frac{2b^2c^3d^2\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{3\sqrt{1+c^2x^2}} - \frac{5bc^5d^2x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}} \\
&+ \frac{7}{3}bc^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) \\
&- \frac{bcd^2(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3x^2} \\
&+ \frac{5}{2}c^4d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{7c^3d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3\sqrt{1+c^2x^2}} \\
&- \frac{5c^2d(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x} - \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3} \\
&+ \frac{5c^3d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{6b\sqrt{1+c^2x^2}} \\
&+ \frac{\left(8c^3d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int \frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{3\sqrt{1+c^2x^2}} \\
&+ \frac{\left(20c^3d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int \frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{3\sqrt{1+c^2x^2}} \\
&- \frac{\left(5b^2c^4d^2\sqrt{d+c^2dx^2}\right)\int \frac{1}{\sqrt{1+c^2x^2}}\,dx}{4\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{7}{12}b^2c^4d^2x\sqrt{d+c^2dx^2} - \frac{b^2c^2d^2(1+c^2x^2)\sqrt{d+c^2dx^2}}{3x} \\
&- \frac{23b^2c^3d^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{12\sqrt{1+c^2x^2}} - \frac{5bc^5d^2x^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2\sqrt{1+c^2x^2}} \\
&+ \frac{7}{3}bc^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&- \frac{bcd^2(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3x^2} \\
&+ \frac{5}{2}c^4d^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2 + \frac{7c^3d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{3\sqrt{1+c^2x^2}} \\
&- \frac{5c^2d(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{3x} - \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3x^3} \\
&+ \frac{5c^3d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^3}{6b\sqrt{1+c^2x^2}} \\
&+ \frac{14bc^3d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\log\left(1-e^{-2\operatorname{arcsinh}(cx)}\right)}{3\sqrt{1+c^2x^2}} \\
&- \frac{\left(4bc^3d^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)dx, x, a+b\operatorname{arcsinh}(cx)\right)}{3\sqrt{1+c^2x^2}} \\
&- \frac{\left(10bc^3d^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)dx, x, a+b\operatorname{arcsinh}(cx)\right)}{3\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$= \frac{7}{12}b^2c^4d^2x\sqrt{d+c^2dx^2} - \frac{b^2c^2d^2(1+c^2x^2)\sqrt{d+c^2dx^2}}{3x} - \frac{23b^2c^3d^2\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{12\sqrt{1+c^2x^2}} - \frac{5bc^5d^2x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}} + \frac{7}{3}bc^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) - \frac{bcd^2(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3x^2} + \frac{5}{2}c^4d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{7c^3d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3\sqrt{1+c^2x^2}} - \frac{5c^2d(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x} - \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3} + \frac{5c^3d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{6b\sqrt{1+c^2x^2}} + \frac{14bc^3d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)}{3\sqrt{1+c^2x^2}} + \frac{\left(2b^2c^3d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right)}{3\sqrt{1+c^2x^2}} + \frac{\left(5b^2c^3d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right)}{3\sqrt{1+c^2x^2}}$$

$$= \frac{7}{12}b^2c^4d^2x\sqrt{d+c^2dx^2} - \frac{b^2c^2d^2(1+c^2x^2)\sqrt{d+c^2dx^2}}{3x} - \frac{23b^2c^3d^2\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{12\sqrt{1+c^2x^2}} - \frac{5bc^5d^2x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}} + \frac{7}{3}bc^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) - \frac{bcd^2(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3x^2} + \frac{5}{2}c^4d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{7c^3d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3\sqrt{1+c^2x^2}} - \frac{5c^2d(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x} - \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3} + \frac{5c^3d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{6b\sqrt{1+c^2x^2}} + \frac{14bc^3d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)}{3\sqrt{1+c^2x^2}} - \frac{7b^2c^3d^2\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right)}{3\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 2.22 (sec) , antiderivative size = 616, normalized size of antiderivative = 1.10

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{x^4}\,dx = \frac{d^2\left(-8abcx\sqrt{d+c^2dx^2}-8a^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}-56a^2c^2x^2\sqrt{1\right.}{}$$

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] (d^2*(-8*a*b*c*x*Sqrt[d + c^2*d*x^2] - 8*a^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2
*d*x^2] - 56*a^2*c^2*x^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] - 8*b^2*c^2*
x^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 12*a^2*c^4*x^4*Sqrt[1 + c^2*x^2
]*Sqrt[d + c^2*d*x^2] + 20*b^2*c^3*x^3*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 -
 6*a*b*c^3*x^3*Sqrt[d + c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] + 112*a*b*c^3*x^3*S
qrt[d + c^2*d*x^2]*Log[c*x] + 60*a^2*c^3*Sqrt[d]*x^3*Sqrt[1 + c^2*x^2]*Log[
c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 56*b^2*c^3*x^3*Sqrt[d + c^2*d*x^2]*P
olyLog[2, E^(-2*ArcSinh[c*x])] + 3*b^2*c^3*x^3*Sqrt[d + c^2*d*x^2]*Sinh[2*A
rcSinh[c*x]] - 2*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(4*b*c*x + 8*a*Sqrt[1 +
 c^2*x^2] + 56*a*c^2*x^2*Sqrt[1 + c^2*x^2] + 3*b*c^3*x^3*Cosh[2*ArcSinh[c*x
]] - 56*b*c^3*x^3*Log[1 - E^(-2*ArcSinh[c*x])] - 6*a*c^3*x^3*Sinh[2*ArcSinh
[c*x]]) + 2*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2*(30*a*c^3*x^3 - 4*b*(-7*c^
3*x^3 + Sqrt[1 + c^2*x^2] + 7*c^2*x^2*Sqrt[1 + c^2*x^2]) + 3*b*c^3*x^3*Sinh
[2*ArcSinh[c*x]])))/(24*x^3*Sqrt[1 + c^2*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2296 vs. 2(511) = 1022.

Time = 0.38 (sec) , antiderivative size = 2297, normalized size of antiderivative = 4.09

method	result	size
default	Expression too large to display	2297
parts	Expression too large to display	2297

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/x^3*(12*arcsinh(c*x)*(c^2*
x^2+1)^(1/2)*x^4*c^4-6*c^5*x^5+30*arcsinh(c*x)^2*x^3*c^3-56*arcsinh(c*x)*c^
3*x^3+56*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*x^3*c^3-56*arcsinh(c*x)*(c^2*x^2+1
)^(1/2)*x^2*c^2-3*c^3*x^3-8*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-4*c*x)*d^2-1/3*a
^2/d/x^3*(c^2*d*x^2+d)^(7/2)+4/3*a^2*c^4*x*(c^2*d*x^2+d)^(5/2)-14/3*b^2*(d*
(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^3*d^2+14/3*b^2*(d*(c^
2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^3*d^2-
7/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^3*c^6-1/4*b^2
*(d*(c^2*x^2+1))^(1/2)*c^3*d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)+5/3*a^2*c^4*d
```

x(c^2*d*x^2+d)^(3/2)+5/2*a^2*c^4*d^2*x*(c^2*d*x^2+d)^(1/2)+5/2*a^2*c^4*d^3*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-4/3*a^2*c^2/d/x*(c^2*d*x^2+d)^(7/2)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*c^3+1/4*b^2*(d*(c^2*x^2+1))^(1/2)*c^6*d^2/(c^2*x^2+1)*x^3+1/4*b^2*(d*(c^2*x^2+1))^(1/2)*c^4*d^2/(c^2*x^2+1)*x+5/6*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^3*c^3*d^2+14/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^3*d^2+14/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^3*d^2+21*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*c^7+7/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^3-5*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^3+49/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^3*arcsinh(c*x)*c^6+7/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x*arcsinh(c*x)*c^4-16/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x/(c^2*x^2+1)*c^4-1/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/x/(c^2*x^2+1)*c^2-1/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/x^3/(c^2*x^2+1)*arcsinh(c*x)^2+14/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^3*d^2+5*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*c^5-1/2*b^2*(d*(c^2*x^2+1))^(1/2)*c^5*d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^2+1/2*b^2*(d*(c^2*x^2+1))^(1/2)*c^6*d^2/(c^2*x^2+1)*arcsinh(c*x)^2*x^3+1/2*b^2*(d*(c^2*x^2+1))^(1/2)*c^4*d^2/(c^2*x^2+1)*arcsinh(c*x)^2*x-56/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^5/(c^2*x^2+1)*c^8-71/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^3/(c^2*x^2+1)*c^6+147*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^7-1/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c+35*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^5-21*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^5-49/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^5/(c^2*x^2+1)*arcsinh(c*x)*c^8-203*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)^2*c^6-56/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)*c^6-190/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x/(c^2*x^2+1)*arcsinh(c*x)^2*c^4-7/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x/(c^2*x^2+1)*arcsinh(c*x)*c^4-23/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/x/(c^2*x^2+1)*arcsinh(c*x)^2*c^2-147*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^5/(c^2*x^2+1)*arcsinh(c*x)^2*c^8

Fricas [F]

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx=\int \frac{(c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{x^4}\,dx$$

[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="fricas
")

[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^4, x)

Sympy [F]

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx=\int \frac{(d(c^2x^2+1))^{\frac{5}{2}}\,(a+b\,\text{asinh}\,(cx))^2}{x^4}\,dx$$

[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2/x**4,x)

[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))**2/x**4, x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx=\text{Exception raised: RuntimeError}$$

[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="maxima
")

[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)^{5/2}}{x^4}\,dx$$

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x^4, x)
```

3.282 $\int \frac{x^4 \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 153

$$\int \frac{x^4 \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = -\frac{15x\sqrt{1+a^2x^2}}{64a^4}+\frac{x^3\sqrt{1+a^2x^2}}{32a^2}+\frac{15\text{arcsinh}(ax)}{64a^5}+\frac{3x^2\text{arcsinh}(ax)}{8a^3}-\frac{x^4\text{arcsinh}(ax)}{8a}-\frac{3x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{8a^4}+\frac{x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{4a^2}+\frac{\text{arcsinh}(ax)^3}{8a^5}$$

```
[Out] 15/64*arcsinh(a*x)/a^5+3/8*x^2*arcsinh(a*x)/a^3-1/8*x^4*arcsinh(a*x)/a+1/8*
arcsinh(a*x)^3/a^5-15/64*x*(a^2*x^2+1)^(1/2)/a^4+1/32*x^3*(a^2*x^2+1)^(1/2)
/a^2-3/8*x*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a^4+1/4*x^3*arcsinh(a*x)^2*(a^2
*x^2+1)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5812, 5783, 5776, 327, 221}

$$\int \frac{x^4 \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{\text{arcsinh}(ax)^3}{8a^5}+\frac{15\text{arcsinh}(ax)}{64a^5}+\frac{3x^2\text{arcsinh}(ax)}{8a^3}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}}{32a^2}-\frac{3x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{8a^4}-\frac{15x\sqrt{a^2x^2+1}}{64a^4}-\frac{x^4\text{arcsinh}(ax)}{8a}$$

```
[In] Int[(x^4*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (-15*x*Sqrt[1 + a^2*x^2])/(64*a^4) + (x^3*Sqrt[1 + a^2*x^2])/(32*a^2) + (15
*ArcSinh[a*x])/(64*a^5) + (3*x^2*ArcSinh[a*x])/(8*a^3) - (x^4*ArcSinh[a*x])
/(8*a) - (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(8*a^4) + (x^3*Sqrt[1 + a^2
*x^2]*ArcSinh[a*x]^2)/(4*a^2) + ArcSinh[a*x]^3/(8*a^5)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{4a^2} - \frac{3\int \frac{x^2\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{4a^2} - \frac{\int x^3\text{arcsinh}(ax)\,dx}{2a}$$

$$
\begin{aligned}
&= -\frac{x^4\operatorname{arcsinh}(ax)}{8a} - \frac{3x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{4a^2} \\
&\quad + \frac{1}{8}\int \frac{x^4}{\sqrt{1+a^2x^2}}\,dx + \frac{3\int \frac{\operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{8a^4} + \frac{3\int x\operatorname{arcsinh}(ax)\,dx}{4a^3} \\
&= \frac{x^3\sqrt{1+a^2x^2}}{32a^2} + \frac{3x^2\operatorname{arcsinh}(ax)}{8a^3} - \frac{x^4\operatorname{arcsinh}(ax)}{8a} - \frac{3x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{8a^4} \\
&\quad + \frac{x^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{4a^2} + \frac{\operatorname{arcsinh}(ax)^3}{8a^5} - \frac{3\int \frac{x^2}{\sqrt{1+a^2x^2}}\,dx}{32a^2} - \frac{3\int \frac{x^2}{\sqrt{1+a^2x^2}}\,dx}{8a^2} \\
&= -\frac{15x\sqrt{1+a^2x^2}}{64a^4} + \frac{x^3\sqrt{1+a^2x^2}}{32a^2} + \frac{3x^2\operatorname{arcsinh}(ax)}{8a^3} - \frac{x^4\operatorname{arcsinh}(ax)}{8a} \\
&\quad - \frac{3x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{4a^2} \\
&\quad + \frac{\operatorname{arcsinh}(ax)^3}{8a^5} + \frac{3\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{64a^4} + \frac{3\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{16a^4} \\
&= -\frac{15x\sqrt{1+a^2x^2}}{64a^4} + \frac{x^3\sqrt{1+a^2x^2}}{32a^2} + \frac{15\operatorname{arcsinh}(ax)}{64a^5} + \frac{3x^2\operatorname{arcsinh}(ax)}{8a^3} - \frac{x^4\operatorname{arcsinh}(ax)}{8a} \\
&\quad - \frac{3x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{4a^2} + \frac{\operatorname{arcsinh}(ax)^3}{8a^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.64

$$
\int \frac{x^4\operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx
$$

$$
= \frac{ax\sqrt{1+a^2x^2}(-15+2a^2x^2) + (15+24a^2x^2-8a^4x^4)\operatorname{arcsinh}(ax) + 8ax\sqrt{1+a^2x^2}(-3+2a^2x^2)\operatorname{arcsinh}(a}{64a^5}
$$

```
[In] Integrate[(x^4*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (a*x*Sqrt[1 + a^2*x^2]*(-15 + 2*a^2*x^2) + (15 + 24*a^2*x^2 - 8*a^4*x^4)*Ar
cSinh[a*x] + 8*a*x*Sqrt[1 + a^2*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[a*x]^2 + 8*Ar
cSinh[a*x]^3)/(64*a^5)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.82

method	result
default	$\frac{16a^3x^3 \operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}-8a^4x^4 \operatorname{arcsinh}(ax)+2a^3x^3\sqrt{a^2x^2+1}-24 \operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}\, ax+24a^2x^2 \operatorname{arcsinh}(ax)+8 \operatorname{arcsinh}(}{64a^5}$

```
[In] int(x^4*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/64*(16*a^3*x^3*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)-8*a^4*x^4*arcsinh(a*x)+2*
a^3*x^3*(a^2*x^2+1)^(1/2)-24*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)*a*x+24*a^2*x^
2*arcsinh(a*x)+8*arcsinh(a*x)^3-15*a*x*(a^2*x^2+1)^(1/2)+15*arcsinh(a*x))/a
^5
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.86

$$\int \frac{x^4\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$$
$$= \frac{8\,(2\,a^3x^3-3\,ax)\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^2+8\,\log\left(ax+\sqrt{a^2x^2+1}\right)^3-(8\,a^4x^4-24\,a^2x^2-15)\,\mathrm{l}}{64\,a^5}$$

```
[In] integrate(x^4*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/64*(8*(2*a^3*x^3 - 3*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^
2 + 8*log(a*x + sqrt(a^2*x^2 + 1))^3 - (8*a^4*x^4 - 24*a^2*x^2 - 15)*log(a*
x + sqrt(a^2*x^2 + 1)) + (2*a^3*x^3 - 15*a*x)*sqrt(a^2*x^2 + 1))/a^5
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.95

$$\int \frac{x^4\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$$
$$= \begin{cases} -\frac{x^4\operatorname{asinh}(ax)}{8a}+\frac{x^3\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}}{32a^2}+\frac{3x^2\operatorname{asinh}(ax)}{8a^3}-\frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{8a^4}-\frac{15x\sqrt{a^2x^2+1}}{64a^4}+\frac{\operatorname{asinh}^3(a}{8a^5} \\ 0 \end{cases}$$

```
[In] integrate(x**4*asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)
[Out] Piecewise((-x**4*asinh(a*x)/(8*a) + x**3*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/
(4*a**2) + x**3*sqrt(a**2*x**2 + 1)/(32*a**2) + 3*x**2*asinh(a*x)/(8*a**3)
- 3*x*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(8*a**4) - 15*x*sqrt(a**2*x**2 + 1)
/(64*a**4) + asinh(a*x)**3/(8*a**5) + 15*asinh(a*x)/(64*a**5), Ne(a, 0)), (
0, True))
```

Maxima [F]

$$\int \frac{x^4 \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^4 \operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}}\,dx$$

[In] integrate(x^4*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(x^4*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)

Giac [F]

$$\int \frac{x^4 \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^4 \operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}}\,dx$$

[In] integrate(x^4*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")

[Out] integrate(x^4*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^4 \operatorname{asinh}(a\,x)^2}{\sqrt{a^2\,x^2+1}}\,dx$$

[In] int((x^4*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2),x)

[Out] int((x^4*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2), x)

3.283 $\int \frac{x^3 \mathbf{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}} dx$

Optimal result

Integrand size = 23, antiderivative size = 122

$$\int \frac{x^3 \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}} dx = -\frac{14\sqrt{1+a^2x^2}}{9a^4} + \frac{2(1+a^2x^2)^{3/2}}{27a^4} + \frac{4x\text{arcsinh}(ax)}{3a^3} - \frac{2x^3\text{arcsinh}(ax)}{9a} - \frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{3a^4} + \frac{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{3a^2}$$

```
[Out] 2/27*(a^2*x^2+1)^(3/2)/a^4+4/3*x*arcsinh(a*x)/a^3-2/9*x^3*arcsinh(a*x)/a-14
/9*(a^2*x^2+1)^(1/2)/a^4-2/3*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a^4+1/3*x^2*a
rcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5812, 5798, 5772, 267, 5776, 272, 45}

$$\int \frac{x^3 \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}} dx = \frac{4x\text{arcsinh}(ax)}{3a^3} + \frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{3a^2} - \frac{2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{3a^4} + \frac{2(a^2x^2+1)^{3/2}}{27a^4} - \frac{14\sqrt{a^2x^2+1}}{9a^4} - \frac{2x^3\text{arcsinh}(ax)}{9a}$$

```
[In] Int[(x^3*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (-14*Sqrt[1 + a^2*x^2])/(9*a^4) + (2*(1 + a^2*x^2)^(3/2))/(27*a^4) + (4*x*A
rcSinh[a*x])/(3*a^3) - (2*x^3*ArcSinh[a*x])/(9*a) - (2*Sqrt[1 + a^2*x^2]*Ar
cSinh[a*x]^2)/(3*a^4) + (x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(3*a^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
```

```
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{3a^2} - \frac{2\int \frac{x\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{3a^2} - \frac{2\int x^2\text{arcsinh}(ax)\,dx}{3a} \\
&= -\frac{2x^3\text{arcsinh}(ax)}{9a} - \frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{3a^4} \\
&\quad + \frac{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{3a^2} + \frac{2}{9}\int \frac{x^3}{\sqrt{1+a^2x^2}}\,dx + \frac{4\int \text{arcsinh}(ax)\,dx}{3a^3} \\
&= \frac{4x\text{arcsinh}(ax)}{3a^3} - \frac{2x^3\text{arcsinh}(ax)}{9a} - \frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{3a^4} \\
&\quad + \frac{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{3a^2} + \frac{1}{9}\text{Subst}\left(\int \frac{x}{\sqrt{1+a^2x}}\,dx, x, x^2\right) - \frac{4\int \frac{x}{\sqrt{1+a^2x^2}}\,dx}{3a^2} \\
&= -\frac{4\sqrt{1+a^2x^2}}{3a^4} + \frac{4x\text{arcsinh}(ax)}{3a^3} - \frac{2x^3\text{arcsinh}(ax)}{9a} - \frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{3a^4} \\
&\quad + \frac{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{3a^2} + \frac{1}{9}\text{Subst}\left(\int \left(-\frac{1}{a^2\sqrt{1+a^2x}} + \frac{\sqrt{1+a^2x}}{a^2}\right)dx, x, x^2\right) \\
&= -\frac{14\sqrt{1+a^2x^2}}{9a^4} + \frac{2(1+a^2x^2)^{3/2}}{27a^4} + \frac{4x\text{arcsinh}(ax)}{3a^3} - \frac{2x^3\text{arcsinh}(ax)}{9a} \\
&\quad - \frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{3a^4} + \frac{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{3a^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.65

$$
\int \frac{x^3\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{2(-20+a^2x^2)\sqrt{1+a^2x^2} - 6ax(-6+a^2x^2)\text{arcsinh}(ax) + 9(-2+a^2x^2)\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{27a^4}
$$

```
[In] Integrate[(x^3*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
[Out] (2*(-20 + a^2*x^2)*Sqrt[1 + a^2*x^2] - 6*a*x*(-6 + a^2*x^2)*ArcSinh[a*x] +
9*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(27*a^4)
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.93

method	result
default	$\frac{9a^4x^4 \operatorname{arcsinh}(ax)^2-9 \operatorname{arcsinh}(ax)^2 a^2x^2-6a^3x^3 \operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}+2a^4x^4-38a^2x^2-18 \operatorname{arcsinh}(ax)^2+36 \operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}\, ax}{27a^4\sqrt{a^2x^2+1}}$

[In] int(x^3*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/27/a^4/(a^2*x^2+1)^(1/2)*(9*a^4*x^4*arcsinh(a*x)^2-9*arcsinh(a*x)^2*a^2*x^2-6*a^3*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)+2*a^4*x^4-38*a^2*x^2-18*arcsinh(a*x)^2+36*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x-40)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

$$\int \frac{x^3 \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\, dx = \frac{9\sqrt{a^2x^2+1}(a^2x^2-2)\log\left(ax+\sqrt{a^2x^2+1}\right)^2 - 6\left(a^3x^3-6\,ax\right)\log\left(ax+\sqrt{a^2x^2+1}\right) + 2\sqrt{a^2x^2+1}(a^2x^2}{27\,a^4}$$

[In] integrate(x^3*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] 1/27*(9*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 2)*log(a*x + sqrt(a^2*x^2 + 1))^2 - 6*(a^3*x^3 - 6*a*x)*log(a*x + sqrt(a^2*x^2 + 1)) + 2*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 20))/a^4

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.99

$$\int \frac{x^3 \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\, dx = \begin{cases} -\frac{2x^3 \operatorname{asinh}(ax)}{9a} + \frac{x^2\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{3a^2} + \frac{2x^2\sqrt{a^2x^2+1}}{27a^2} + \frac{4x\operatorname{asinh}(ax)}{3a^3} - \frac{2\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{3a^4} - \frac{40\sqrt{a^2x^2+1}}{27a^4} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

[In] integrate(x**3*asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)

[Out] Piecewise((-2*x**3*asinh(a*x)/(9*a) + x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(3*a**2) + 2*x**2*sqrt(a**2*x**2 + 1)/(27*a**2) + 4*x*asinh(a*x)/(3*a**3) - 2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(3*a**4) - 40*sqrt(a**2*x**2 + 1)/(27*a**4), Ne(a, 0)), (0, True))

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

$$\int \frac{x^3\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{1}{3}\left(\frac{\sqrt{a^2x^2+1}x^2}{a^2} - \frac{2\sqrt{a^2x^2+1}}{a^4}\right)\text{arsinh}\,(ax)^2 + \frac{2\left(\sqrt{a^2x^2+1}x^2 - \frac{20\sqrt{a^2x^2+1}}{a^2}\right)}{27\,a^2} - \frac{2\,(a^2x^3-6\,x)\,\text{arsinh}\,(ax)}{9\,a^3}$$

```
[In] integrate(x^3*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2 + 1)/a^4)*arcsinh(a*x)^2 +
2/27*(sqrt(a^2*x^2 + 1)*x^2 - 20*sqrt(a^2*x^2 + 1)/a^2)/a^2 - 2/9*(a^2*x^3
- 6*x)*arcsinh(a*x)/a^3
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^3\,\text{asinh}(a\,x)^2}{\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int((x^3*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((x^3*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2), x)
```

3.284 $\int \frac{x^2\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}} dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{x^2\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}} dx = \frac{x\sqrt{1+a^2x^2}}{4a^2} - \frac{\text{arcsinh}(ax)}{4a^3} - \frac{x^2\text{arcsinh}(ax)}{2a} + \frac{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{2a^2} - \frac{\text{arcsinh}(ax)^3}{6a^3}$$

```
[Out] -1/4*arcsinh(a*x)/a^3-1/2*x^2*arcsinh(a*x)/a-1/6*arcsinh(a*x)^3/a^3+1/4*x*(
a^2*x^2+1)^(1/2)/a^2+1/2*x*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5812, 5783, 5776, 327, 221}

$$\int \frac{x^2\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}} dx = -\frac{\text{arcsinh}(ax)^3}{6a^3} - \frac{\text{arcsinh}(ax)}{4a^3} + \frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2a^2} + \frac{x\sqrt{a^2x^2+1}}{4a^2} - \frac{x^2\text{arcsinh}(ax)}{2a}$$

```
[In] Int[(x^2*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (x*Sqrt[1 + a^2*x^2])/(4*a^2) - ArcSinh[a*x]/(4*a^3) - (x^2*ArcSinh[a*x])/(
2*a) + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(2*a^2) - ArcSinh[a*x]^3/(6*a^3
)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{2a^2} - \frac{\int \frac{\operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{2a^2} - \frac{\int x\operatorname{arcsinh}(ax)\,dx}{a} \\
&= -\frac{x^2\operatorname{arcsinh}(ax)}{2a} + \frac{x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{2a^2} - \frac{\operatorname{arcsinh}(ax)^3}{6a^3} + \frac{1}{2}\int \frac{x^2}{\sqrt{1+a^2x^2}}\,dx \\
&= \frac{x\sqrt{1+a^2x^2}}{4a^2} - \frac{x^2\operatorname{arcsinh}(ax)}{2a} + \frac{x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{2a^2} - \frac{\operatorname{arcsinh}(ax)^3}{6a^3} - \frac{\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{4a^2}
\end{aligned}$$

$$= \frac{x\sqrt{1+a^2x^2}}{4a^2} - \frac{\text{arcsinh}(ax)}{4a^3} - \frac{x^2\text{arcsinh}(ax)}{2a} + \frac{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{2a^2} - \frac{\text{arcsinh}(ax)^3}{6a^3}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

$$\int \frac{x^2\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$$
$$= \frac{3ax\sqrt{1+a^2x^2} - 3(1+2a^2x^2)\,\text{arcsinh}(ax) + 6ax\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2 - 2\text{arcsinh}(ax)^3}{12a^3}$$

```
[In] Integrate[(x^2*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (3*a*x*Sqrt[1 + a^2*x^2] - 3*(1 + 2*a^2*x^2)*ArcSinh[a*x] + 6*a*x*Sqrt[1 +
a^2*x^2]*ArcSinh[a*x]^2 - 2*ArcSinh[a*x]^3)/(12*a^3)
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{-6\,\text{arcsinh}(ax)^2\sqrt{a^2x^2+1}\,ax+6a^2x^2\,\text{arcsinh}(ax)+2\,\text{arcsinh}(ax)^3-3ax\sqrt{a^2x^2+1}+3\,\text{arcsinh}(ax)}{12a^3}$	69

```
[In] int(x^2*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(-6*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)*a*x+6*a^2*x^2*arcsinh(a*x)+2*arc
sinh(a*x)^3-3*a*x*(a^2*x^2+1)^(1/2)+3*arcsinh(a*x))/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.17

$$\int \frac{x^2\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$$
$$= \frac{6\,\sqrt{a^2x^2+1}ax\log\left(ax+\sqrt{a^2x^2+1}\right)^2 - 2\,\log\left(ax+\sqrt{a^2x^2+1}\right)^3 + 3\,\sqrt{a^2x^2+1}ax - 3\,(2\,a^2x^2+1)\log\left(ax\right.}{12\,a^3}$$

```
[In] integrate(x^2*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(6*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1))^2 - 2*log(a*x +
sqrt(a^2*x^2 + 1))^3 + 3*sqrt(a^2*x^2 + 1)*a*x - 3*(2*a^2*x^2 + 1)*log(a*x
+ sqrt(a^2*x^2 + 1)))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.90

$$\int \frac{x^2\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} -\frac{x^2\operatorname{asinh}(ax)}{2a} + \frac{x\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{2a^2} + \frac{x\sqrt{a^2x^2+1}}{4a^2} - \frac{\operatorname{asinh}^3(ax)}{6a^3} - \frac{\operatorname{asinh}(ax)}{4a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**2*asinh(a*x)/(2*a) + x*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(2*
a**2) + x*sqrt(a**2*x**2 + 1)/(4*a**2) - asinh(a*x)**3/(6*a**3) - asinh(a*x
)/(4*a**3), Ne(a, 0)), (0, True))
```

Maxima [F]

$$\int \frac{x^2\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2\operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x^2*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{x^2\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2\operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x^2*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2\operatorname{asinh}(a\,x)^2}{\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int((x^2*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^2*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2), x)
```

3.285 $\int \frac{x \textbf{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}} dx$

Optimal result

Integrand size = 21, antiderivative size = 52

$$\int \frac{x\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{2\sqrt{1+a^2x^2}}{a^2} - \frac{2x\text{arcsinh}(ax)}{a} + \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{a^2}$$

```
[Out] -2*x*arcsinh(a*x)/a+2*(a^2*x^2+1)^(1/2)/a^2+arcsinh(a*x)^2*(a^2*x^2+1)^(1/2
)/a^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5798, 5772, 267}

$$\int \frac{x\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{a^2} + \frac{2\sqrt{a^2x^2+1}}{a^2} - \frac{2x\text{arcsinh}(ax)}{a}$$

```
[In] Int[(x*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2])/a^2 - (2*x*ArcSinh[a*x])/a + (Sqrt[1 + a^2*x^2]*ArcSi
nh[a*x]^2)/a^2
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{a^2} - \frac{2\int \text{arcsinh}(ax)\,dx}{a} \\
&= -\frac{2x\text{arcsinh}(ax)}{a} + \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{a^2} + 2\int \frac{x}{\sqrt{1+a^2x^2}}\,dx \\
&= \frac{2\sqrt{1+a^2x^2}}{a^2} - \frac{2x\text{arcsinh}(ax)}{a} + \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{a^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int \frac{x\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{2\sqrt{1+a^2x^2} - 2ax\text{arcsinh}(ax) + \sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{a^2}$$

```
[In] Integrate[(x*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2] - 2*a*x*ArcSinh[a*x] + Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^
2)/a^2
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.23

method	result	size
default	$\frac{\text{arcsinh}(ax)^2a^2x^2+\text{arcsinh}(ax)^2-2\ \text{arcsinh}(ax)\sqrt{a^2x^2+1}\,ax+2a^2x^2+2}{a^2\sqrt{a^2x^2+1}}$	64

```
[In] int(x*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] 1/a^2/(a^2*x^2+1)^(1/2)*(arcsinh(a*x)^2*a^2*x^2+arcsinh(a*x)^2-2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x+2*a^2*x^2+2)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.35

$$\int \frac{x\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = -\frac{2\,ax\log\left(ax+\sqrt{a^2x^2+1}\right)-\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^2-2\sqrt{a^2x^2+1}}{a^2}$$

[In] integrate(x*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] -(2*a*x*log(a*x + sqrt(a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2 - 2*sqrt(a^2*x^2 + 1))/a^2

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{x\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} -\frac{2x\,\text{asinh}\,(ax)}{a}+\frac{\sqrt{a^2x^2+1}\,\text{asinh}^2\,(ax)}{a^2}+\frac{2\sqrt{a^2x^2+1}}{a^2} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

[In] integrate(x*asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)

[Out] Piecewise((-2*x*asinh(a*x)/a + sqrt(a**2*x**2 + 1)*asinh(a*x)**2/a**2 + 2*sqrt(a**2*x**2 + 1)/a**2, Ne(a, 0)), (0, True))

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int \frac{x\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)^2}{a^2}-\frac{2\left(ax\,\text{arsinh}\,(ax)-\sqrt{a^2x^2+1}\right)}{a^2}$$

[In] integrate(x*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")

[Out] sqrt(a^2*x^2 + 1)*arcsinh(a*x)^2/a^2 - 2*(a*x*arcsinh(a*x) - sqrt(a^2*x^2 + 1))/a^2

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.42

$$\int \frac{x\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{a^2} - \frac{2\left(x\log\left(ax+\sqrt{a^2x^2+1}\right)-\frac{\sqrt{a^2x^2+1}}{a}\right)}{a}$$

```
[In] integrate(x*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2/a^2 - 2*(x*log(a*x + sqrt(
a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1)/a)/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x\,\text{asinh}(a\,x)^2}{\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int((x*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2), x)
```

3.286 $\int \frac{\operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 13

$$\int \frac{\operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx=\frac{\operatorname{arcsinh}(ax)^3}{3a}$$

```
[Out] 1/3*arcsinh(a*x)^3/a
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {5783}

$$\int \frac{\operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx=\frac{\operatorname{arcsinh}(ax)^3}{3a}$$

```
[In] Int[ArcSinh[a*x]^2/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ArcSinh[a*x]^3/(3*a)
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\text{integral}=\frac{\operatorname{arcsinh}(ax)^3}{3a}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{\operatorname{arcsinh}(ax)^3}{3a}$$

```
[In] Integrate[ArcSinh[a*x]^2/Sqrt[1 + a^2*x^2],x]
[Out] ArcSinh[a*x]^3/(3*a)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\operatorname{arcsinh}(ax)^3}{3a}$	12
default	$\frac{\operatorname{arcsinh}(ax)^3}{3a}$	12

```
[In] int(arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*arcsinh(a*x)^3/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{\operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{3\,a}$$

```
[In] integrate(arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/3*log(a*x + sqrt(a^2*x^2 + 1))^3/a
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{\operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} \frac{\operatorname{asinh}^3(ax)}{3a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)
[Out] Piecewise((asinh(a*x)**3/(3*a), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{\operatorname{arsinh}(ax)^3}{3a}$$

```
[In] integrate(arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/3*arcsinh(a*x)^3/a
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 3.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{\operatorname{asinh}(a\,x)^3}{3\,a}$$

```
[In] int(asinh(a*x)^2/(a^2*x^2 + 1)^(1/2),x)
[Out] asinh(a*x)^3/(3*a)
```

3.287 $\int \frac{\operatorname{arcsinh}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 68

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx = &-2\operatorname{arcsinh}(ax)^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right)\\ &-2\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right)\\ &+2\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right)\\ &+2\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(ax)}\right)-2\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(ax)}\right)\end{aligned}$$

```
[Out] -2*arcsinh(a*x)^2*arctanh(a*x+(a^2*x^2+1)^(1/2))-2*arcsinh(a*x)*polylog(2,-
a*x-(a^2*x^2+1)^(1/2))+2*arcsinh(a*x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))+2*po
lylog(3,-a*x-(a^2*x^2+1)^(1/2))-2*polylog(3,a*x+(a^2*x^2+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5816, 4267, 2611, 2320, 6724}

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx = &-2\operatorname{arcsinh}(ax)^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right)\\ &-2\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right)\\ &+2\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right)\\ &+2\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(ax)}\right)-2\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(ax)}\right)\end{aligned}$$

```
[In] Int[ArcSinh[a*x]^2/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -2*ArcSinh[a*x]^2*ArcTanh[E^ArcSinh[a*x]] - 2*ArcSinh[a*x]*PolyLog[2, -E^Ar
cSinh[a*x]] + 2*ArcSinh[a*x]*PolyLog[2, E^ArcSinh[a*x]] + 2*PolyLog[3, -E^A
rcSinh[a*x]] - 2*PolyLog[3, E^ArcSinh[a*x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int x^2 \text{csch}(x)\,dx, x, \text{arcsinh}(ax)\right)$$

$$= -2\text{arcsinh}(ax)^2\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) - 2\text{Subst}\left(\int x\log\left(1-e^x\right)\,dx, x, \text{arcsinh}(ax)\right) + 2\text{Subst}\left(\int x\log\left(1+e^x\right)\,dx, x, \text{arcsinh}(ax)\right)$$

$$= -2\text{arcsinh}(ax)^2\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) - 2\text{arcsinh}(ax)\,\text{PolyLog}\left(2, -e^{\text{arcsinh}(ax)}\right) + 2\text{arcsinh}(ax)\,\text{PolyLog}\left(2, e^{\text{arcsinh}(ax)}\right) + 2\text{Subst}\left(\int \text{PolyLog}\left(2,-e^x\right)\,dx, x, \text{arcsinh}(ax)\right) - 2\text{Subst}\left(\int \text{PolyLog}\left(2,e^x\right)\,dx, x, \text{arcsinh}(ax)\right)$$

$$= -2\text{arcsinh}(ax)^2\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) - 2\text{arcsinh}(ax)\,\text{PolyLog}\left(2, -e^{\text{arcsinh}(ax)}\right) + 2\text{arcsinh}(ax)\,\text{PolyLog}\left(2, e^{\text{arcsinh}(ax)}\right) + 2\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx, x, e^{\text{arcsinh}(ax)}\right) - 2\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx, x, e^{\text{arcsinh}(ax)}\right)$$

$$= -2\text{arcsinh}(ax)^2\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) - 2\text{arcsinh}(ax)\,\text{PolyLog}\left(2, -e^{\text{arcsinh}(ax)}\right) + 2\text{arcsinh}(ax)\,\text{PolyLog}\left(2, e^{\text{arcsinh}(ax)}\right) + 2\,\text{PolyLog}\left(3, -e^{\text{arcsinh}(ax)}\right) - 2\,\text{PolyLog}\left(3, e^{\text{arcsinh}(ax)}\right)$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.47

$$\int \frac{\text{arcsinh}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx = \text{arcsinh}(ax)^2\log\left(1-e^{-\text{arcsinh}(ax)}\right) - \text{arcsinh}(ax)^2\log\left(1+e^{-\text{arcsinh}(ax)}\right) + 2\text{arcsinh}(ax)\,\text{PolyLog}\left(2, -e^{-\text{arcsinh}(ax)}\right) - 2\text{arcsinh}(ax)\,\text{PolyLog}\left(2, e^{-\text{arcsinh}(ax)}\right) + 2\,\text{PolyLog}\left(3, -e^{-\text{arcsinh}(ax)}\right) - 2\,\text{PolyLog}\left(3, e^{-\text{arcsinh}(ax)}\right)$$

```
[In] Integrate[ArcSinh[a*x]^2/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] ArcSinh[a*x]^2*Log[1 - E^(-ArcSinh[a*x])] - ArcSinh[a*x]^2*Log[1 + E^(-ArcS
inh[a*x])] + 2*ArcSinh[a*x]*PolyLog[2, -E^(-ArcSinh[a*x])] - 2*ArcSinh[a*x]
*PolyLog[2, E^(-ArcSinh[a*x])] + 2*PolyLog[3, -E^(-ArcSinh[a*x])] - 2*PolyL
og[3, E^(-ArcSinh[a*x])]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.12

method	result
default	$-\operatorname{arcsinh}(ax)^2 \ln\left(1+ax+\sqrt{a^2x^2+1}\right) - 2 \operatorname{arcsinh}(ax) \operatorname{polylog}\left(2, -ax-\sqrt{a^2x^2+1}\right) + 2 \operatorname{pol}$

[In] int(arcsinh(a*x)^2/x/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] -arcsinh(a*x)^2*ln(1+a*x+(a^2*x^2+1)^(1/2))-2*arcsinh(a*x)*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+2*polylog(3,-a*x-(a^2*x^2+1)^(1/2))+arcsinh(a*x)^2*ln(1-a*x-(a^2*x^2+1)^(1/2))+2*arcsinh(a*x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))-2*polylog(3,a*x+(a^2*x^2+1)^(1/2))

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}x}\,dx$$

[In] integrate(arcsinh(a*x)^2/x/(a^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^2/(a^2*x^3 + x), x)

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}^2(ax)}{x\sqrt{a^2x^2+1}}\,dx$$

[In] integrate(asinh(a*x)**2/x/(a**2*x**2+1)**(1/2),x)

[Out] Integral(asinh(a*x)**2/(x*sqrt(a**2*x**2 + 1)), x)

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}x}\,dx$$

[In] integrate(arcsinh(a*x)^2/x/(a^2*x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(arcsinh(a*x)^2/(sqrt(a^2*x^2 + 1)*x), x)

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^2}{\sqrt{a^2x^2+1}x}\,dx$$

```
[In] integrate(arcsinh(a*x)^2/x/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^2/(sqrt(a^2*x^2 + 1)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)^2}{x\,\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int(asinh(a*x)^2/(x*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(asinh(a*x)^2/(x*(a^2*x^2 + 1)^(1/2)), x)
```

3.288 $\int \frac{\text{arcsinh}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 66

$$\int \frac{\text{arcsinh}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx = -a\text{arcsinh}(ax)^2 - \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{x} + 2a\text{arcsinh}(ax)\log\left(1-e^{2\text{arcsinh}(ax)}\right) + a\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right)$$

```
[Out] -a*arcsinh(a*x)^2+2*a*arcsinh(a*x)*ln(1-(a*x+(a^2*x^2+1)^(1/2))^2)+a*polylo
g(2,(a*x+(a^2*x^2+1)^(1/2))^2)-arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5800, 5775, 3797, 2221, 2317, 2438}

$$\int \frac{\text{arcsinh}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx = -\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{x} + a\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right) - a\text{arcsinh}(ax)^2 + 2a\text{arcsinh}(ax)\log\left(1-e^{2\text{arcsinh}(ax)}\right)$$

```
[In] Int[ArcSinh[a*x]^2/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -(a*ArcSinh[a*x]^2) - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/x + 2*a*ArcSinh[a*
x]*Log[1 - E^(2*ArcSinh[a*x])] + a*PolyLog[2, E^(2*ArcSinh[a*x])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{x} + (2a)\int \frac{\text{arcsinh}(ax)}{x}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{x} + (2a)\text{Subst}(\int x\coth(x)\,dx, x, \text{arcsinh}(ax)) \\
&= -a\text{arcsinh}(ax)^2 - \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{x} - (4a)\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \text{arcsinh}(ax)\right)
\end{aligned}$$

$$= -a\text{arcsinh}(ax)^2 - \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{x} + 2a\text{arcsinh}(ax)\log\left(1-e^{2\text{arcsinh}(ax)}\right) - (2a)\text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \text{arcsinh}(ax)\right)$$

$$= -a\text{arcsinh}(ax)^2 - \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{x} + 2a\text{arcsinh}(ax)\log\left(1-e^{2\text{arcsinh}(ax)}\right) - a\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\text{arcsinh}(ax)}\right)$$

$$= -a\text{arcsinh}(ax)^2 - \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{x} + 2a\text{arcsinh}(ax)\log\left(1-e^{2\text{arcsinh}(ax)}\right) + a\,\text{PolyLog}\left(2, e^{2\text{arcsinh}(ax)}\right)$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int \frac{\text{arcsinh}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx = a\left(\text{arcsinh}(ax)\left(\text{arcsinh}(ax) - \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{ax} + 2\log\left(1-e^{-2\text{arcsinh}(ax)}\right)\right) - \text{PolyLog}\left(2, e^{-2\text{arcsinh}(ax)}\right)\right)$$

```
[In] Integrate[ArcSinh[a*x]^2/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] a*(ArcSinh[a*x]*(ArcSinh[a*x] - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(a*x) + 2*
Log[1 - E^(-2*ArcSinh[a*x])]) - PolyLog[2, E^(-2*ArcSinh[a*x])])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.00

method	result
default	$\frac{\left(ax-\sqrt{a^2x^2+1}\right)\text{arcsinh}(ax)^2}{x} - 2a\,\text{arcsinh}\left(ax\right)^2 + 2a\,\text{arcsinh}\left(ax\right)\ln\left(1+ax+\sqrt{a^2x^2+1}\right) + 2a\,\text{polyl}$

```
[In] int(arcsinh(a*x)^2/x^2/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*x-(a^2*x^2+1)^(1/2))/x*arcsinh(a*x)^2-2*a*arcsinh(a*x)^2+2*a*arcsinh(a*x
)*ln(1+a*x+(a^2*x^2+1)^(1/2))+2*a*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+2*a*arc
sinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))+2*a*polylog(2,a*x+(a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\text{arcsinh}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\text{arsinh}\left(ax\right)^2}{\sqrt{a^2x^2+1}x^2}\,dx$$

```
[In] integrate(arcsinh(a*x)^2/x^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^2/(a^2*x^4 + x^2), x)
```

Sympy [F]

$$\int \frac{\text{arcsinh}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\text{asinh}^2\left(ax\right)}{x^2\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(asinh(a*x)**2/x**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)**2/(x**2*sqrt(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{\text{arcsinh}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\text{arsinh}\left(ax\right)^2}{\sqrt{a^2x^2+1}x^2}\,dx$$

```
[In] integrate(arcsinh(a*x)^2/x^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2/x + integrate(2*(a^3*x^2
+ sqrt(a^2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))/(sqrt(a^2*x^2 +
 1)*a*x^2 + (a^2*x^2 + 1)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\text{arcsinh}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x)^2/x^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)^2}{x^2\,\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int(asinh(a*x)^2/(x^2*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(asinh(a*x)^2/(x^2*(a^2*x^2 + 1)^(1/2)), x)
```

3.289 $\int \frac{\text{arcsinh}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 135

$$\begin{aligned}\int \frac{\text{arcsinh}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx = &-\frac{a\text{arcsinh}(ax)}{x}-\frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{2x^2}\\&+a^2\text{arcsinh}(ax)^2\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right)-a^2\text{arctanh}\left(\sqrt{1+a^2x^2}\right)\\&+a^2\text{arcsinh}(ax)\operatorname{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)\\&-a^2\text{arcsinh}(ax)\operatorname{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right)\\&-a^2\operatorname{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right)+a^2\operatorname{PolyLog}\left(3,e^{\text{arcsinh}(ax)}\right)\end{aligned}$$

```
[Out] -a*arcsinh(a*x)/x+a^2*arcsinh(a*x)^2*arctanh(a*x+(a^2*x^2+1)^(1/2))-a^2*arc
tanh((a^2*x^2+1)^(1/2))+a^2*arcsinh(a*x)*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-
a^2*arcsinh(a*x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))-a^2*polylog(3,-a*x-(a^2*x
^2+1)^(1/2))+a^2*polylog(3,a*x+(a^2*x^2+1)^(1/2))-1/2*arcsinh(a*x)^2*(a^2*x
^2+1)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules

used = {5809, 5816, 4267, 2611, 2320, 6724, 5776, 272, 65, 214}

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx = a^2\operatorname{arcsinh}(ax)^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) + a^2\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right) - a^2\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right) - a^2\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(ax)}\right) + a^2\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(ax)}\right) - \frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^2}{2x^2} - a^2\operatorname{arctanh}\left(\sqrt{a^2x^2+1}\right) - \frac{a\operatorname{arcsinh}(ax)}{x}$$

```
[In] Int[ArcSinh[a*x]^2/(x^3*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -((a*ArcSinh[a*x])/x) - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(2*x^2) + a^2*Ar
cSinh[a*x]^2*ArcTanh[E^ArcSinh[a*x]] - a^2*ArcTanh[Sqrt[1 + a^2*x^2]] + a^2
*ArcSinh[a*x]*PolyLog[2, -E^ArcSinh[a*x]] - a^2*ArcSinh[a*x]*PolyLog[2, E^A
rcSinh[a*x]] - a^2*PolyLog[3, -E^ArcSinh[a*x]] + a^2*PolyLog[3, E^ArcSinh[a
*x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{2x^2} + a\int \frac{\text{arcsinh}(ax)}{x^2}\,dx - \frac{1}{2}a^2\int \frac{\text{arcsinh}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{a\text{arcsinh}(ax)}{x} - \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{2x^2} - \frac{1}{2}a^2\text{Subst}\left(\int x^2\text{csch}(x)\,dx, x, \text{arcsinh}(ax)\right) + a^2\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{a\text{arcsinh}(ax)}{x} - \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{2x^2} + a^2\text{arcsinh}(ax)^2\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) + \frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^2\right) + a^2\text{Subst}\left(\int x\log\left(1-e^x\right)\,dx, x, \text{arcsinh}(ax)\right) - a^2\text{Subst}\left(\int x\log\left(1+e^x\right)\,dx, x, \text{arcsinh}(ax)\right)$$

$$= -\frac{a\text{arcsinh}(ax)}{x} - \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{2x^2} + a^2\text{arcsinh}(ax)^2\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) + a^2\text{arcsinh}(ax)\,\text{PolyLog}\left(2, -e^{\text{arcsinh}(ax)}\right) - a^2\text{arcsinh}(ax)\,\text{PolyLog}\left(2, e^{\text{arcsinh}(ax)}\right) - a^2\text{Subst}\left(\int \text{PolyLog}\left(2, -e^x\right)\,dx, x, \text{arcsinh}(ax)\right) + a^2\text{Subst}\left(\int \text{PolyLog}\left(2, e^x\right)\,dx, x, \text{arcsinh}(ax)\right) + \text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right)$$

$$= -\frac{a\text{arcsinh}(ax)}{x} - \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{2x^2} + a^2\text{arcsinh}(ax)^2\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) - a^2\text{arctanh}\left(\sqrt{1+a^2x^2}\right) + a^2\text{arcsinh}(ax)\,\text{PolyLog}\left(2, -e^{\text{arcsinh}(ax)}\right) - a^2\text{arcsinh}(ax)\,\text{PolyLog}\left(2, e^{\text{arcsinh}(ax)}\right) - a^2\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx, x, e^{\text{arcsinh}(ax)}\right) + a^2\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx, x, e^{\text{arcsinh}(ax)}\right)$$

$$= -\frac{a\text{arcsinh}(ax)}{x} - \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{2x^2} + a^2\text{arcsinh}(ax)^2\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) - a^2\text{arctanh}\left(\sqrt{1+a^2x^2}\right) + a^2\text{arcsinh}(ax)\,\text{PolyLog}\left(2, -e^{\text{arcsinh}(ax)}\right) - a^2\text{arcsinh}(ax)\,\text{PolyLog}\left(2, e^{\text{arcsinh}(ax)}\right) - a^2\,\text{PolyLog}\left(3, -e^{\text{arcsinh}(ax)}\right) + a^2\,\text{PolyLog}\left(3, e^{\text{arcsinh}(ax)}\right)$$

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.39

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx = \frac{1}{8}a^2\left(-4\operatorname{arcsinh}(ax)\coth\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right) - \operatorname{arcsinh}(ax)^2\operatorname{csch}^2\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right) - 4\operatorname{arcsinh}(ax)^2\log\left(1-e^{-\operatorname{arcsinh}(ax)}\right) + 4\operatorname{arcsinh}(ax)^2\log\left(1+e^{-\operatorname{arcsinh}(ax)}\right) + 8\log\left(\tanh\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right)\right) - 8\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,-e^{-\operatorname{arcsinh}(ax)}\right) + 8\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,e^{-\operatorname{arcsinh}(ax)}\right) - 8\operatorname{PolyLog}\left(3,-e^{-\operatorname{arcsinh}(ax)}\right) + 8\operatorname{PolyLog}\left(3,e^{-\operatorname{arcsinh}(ax)}\right) - \operatorname{arcsinh}(ax)^2\operatorname{sech}^2\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right) + 4\operatorname{arcsinh}(ax)\tanh\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right)\right)$$

```
[In] Integrate[ArcSinh[a*x]^2/(x^3*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (a^2*(-4*ArcSinh[a*x]*Coth[ArcSinh[a*x]/2] - ArcSinh[a*x]^2*Csch[ArcSinh[a*
x]/2]^2 - 4*ArcSinh[a*x]^2*Log[1 - E^(-ArcSinh[a*x])] + 4*ArcSinh[a*x]^2*Lo
g[1 + E^(-ArcSinh[a*x])] + 8*Log[Tanh[ArcSinh[a*x]/2]] - 8*ArcSinh[a*x]*Pol
yLog[2, -E^(-ArcSinh[a*x])] + 8*ArcSinh[a*x]*PolyLog[2, E^(-ArcSinh[a*x])]
- 8*PolyLog[3, -E^(-ArcSinh[a*x])] + 8*PolyLog[3, E^(-ArcSinh[a*x])] - ArcS
inh[a*x]^2*Sech[ArcSinh[a*x]/2]^2 + 4*ArcSinh[a*x]*Tanh[ArcSinh[a*x]/2]))/8
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.73

method	result
default	$-\frac{\operatorname{arcsinh}(ax)\left(a^2x^2\operatorname{arcsinh}(ax)+2ax\sqrt{a^2x^2+1}+\operatorname{arcsinh}(ax)\right)}{2\sqrt{a^2x^2+1}\,x^2}+\frac{a^2\operatorname{arcsinh}(ax)^2\ln\left(1+ax+\sqrt{a^2x^2+1}\right)}{2}+a^2\operatorname{arcsinh}(ax)\,\text{po}$

```
[In] int(arcsinh(a*x)^2/x^3/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a^2*x^2+1)^(1/2)/x^2*arcsinh(a*x)*(a^2*x^2*arcsinh(a*x)+2*a*x*(a^2*x^
2+1)^(1/2)+arcsinh(a*x))+1/2*a^2*arcsinh(a*x)^2*ln(1+a*x+(a^2*x^2+1)^(1/2))
+a^2*arcsinh(a*x)*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-a^2*polylog(3,-a*x-(a^2
*x^2+1)^(1/2))-1/2*a^2*arcsinh(a*x)^2*ln(1-a*x-(a^2*x^2+1)^(1/2))-a^2*arcsi
nh(a*x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))+a^2*polylog(3,a*x+(a^2*x^2+1)^(1/2
))-2*a^2*arctanh(a*x+(a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^2}{\sqrt{a^2x^2+1}x^3}\,dx$$

```
[In] integrate(arcsinh(a*x)^2/x^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^2/(a^2*x^5 + x^3), x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}^2\left(ax\right)}{x^3\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(asinh(a*x)**2/x**3/(a**2*x**2+1)**(1/2),x)
[Out] Integral(asinh(a*x)**2/(x**3*sqrt(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^2}{\sqrt{a^2x^2+1}x^3}\,dx$$

```
[In] integrate(arcsinh(a*x)^2/x^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^2/(sqrt(a^2*x^2 + 1)*x^3), x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^2}{\sqrt{a^2x^2+1}x^3}\,dx$$

```
[In] integrate(arcsinh(a*x)^2/x^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^2/(sqrt(a^2*x^2 + 1)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{arcsinh}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\text{asinh}(a\,x)^2}{x^3\,\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int(asinh(a*x)^2/(x^3*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(asinh(a*x)^2/(x^3*(a^2*x^2 + 1)^(1/2)), x)
```

3.290 $\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 383

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = -\frac{16abx\sqrt{1+c^2x^2}}{15c^5\sqrt{d+c^2dx^2}}+\frac{298b^2(1+c^2x^2)}{225c^6\sqrt{d+c^2dx^2}}-\frac{76b^2(1+c^2x^2)^2}{675c^6\sqrt{d+c^2dx^2}}+\frac{2b^2(1+c^2x^2)^3}{125c^6\sqrt{d+c^2dx^2}}-\frac{16b^2x\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{15c^5\sqrt{d+c^2dx^2}}+\frac{8bx^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{45c^3\sqrt{d+c^2dx^2}}-\frac{2bx^5\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{25c\sqrt{d+c^2dx^2}}+\frac{8\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{15c^6d}-\frac{4x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{15c^4d}+\frac{x^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{5c^2d}$$

```
[Out] 298/225*b^2*(c^2*x^2+1)/c^6/(c^2*d*x^2+d)^(1/2)-76/675*b^2*(c^2*x^2+1)^2/c^
6/(c^2*d*x^2+d)^(1/2)+2/125*b^2*(c^2*x^2+1)^3/c^6/(c^2*d*x^2+d)^(1/2)-16/15
*a*b*x*(c^2*x^2+1)^(1/2)/c^5/(c^2*d*x^2+d)^(1/2)-16/15*b^2*x*arcsinh(c*x)*(
c^2*x^2+1)^(1/2)/c^5/(c^2*d*x^2+d)^(1/2)+8/45*b*x^3*(a+b*arcsinh(c*x))*(c^2
*x^2+1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-2/25*b*x^5*(a+b*arcsinh(c*x))*(c^2*x^
2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)+8/15*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1
/2)/c^6/d-4/15*x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^4/d+1/5*x^4*(
a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5812, 5798, 5772, 267, 5776, 272, 45}

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = -\frac{2bx^5\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{25c\sqrt{c^2dx^2+d}} + \frac{x^4\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^2}{5c^2d} + \frac{8\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^2}{15c^6d} - \frac{4x^2\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^2}{15c^4d} + \frac{8bx^3\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{45c^3\sqrt{c^2dx^2+d}} - \frac{16abx\sqrt{c^2x^2+1}}{15c^5\sqrt{c^2dx^2+d}} - \frac{16b^2x\sqrt{c^2x^2+1}\operatorname{arcsinh}(cx)}{15c^5\sqrt{c^2dx^2+d}} + \frac{2b^2(c^2x^2+1)^3}{125c^6\sqrt{c^2dx^2+d}} - \frac{76b^2(c^2x^2+1)^2}{675c^6\sqrt{c^2dx^2+d}} + \frac{298b^2(c^2x^2+1)}{225c^6\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x^5*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (-16*a*b*x*Sqrt[1 + c^2*x^2])/(15*c^5*Sqrt[d + c^2*d*x^2]) + (298*b^2*(1 +
c^2*x^2))/(225*c^6*Sqrt[d + c^2*d*x^2]) - (76*b^2*(1 + c^2*x^2)^2)/(675*c^6
*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*x^2)^3)/(125*c^6*Sqrt[d + c^2*d*x^2
]) - (16*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(15*c^5*Sqrt[d + c^2*d*x^2])
 + (8*b*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(45*c^3*Sqrt[d + c^2*d*
x^2]) - (2*b*x^5*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(25*c*Sqrt[d + c^2
*d*x^2]) + (8*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(15*c^6*d) - (4*x
^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(15*c^4*d) + (x^4*Sqrt[d + c
^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(5*c^2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{x^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{5c^2d} - \frac{4\int\frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx}{5c^2} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\int x^4(a+b\text{arcsinh}(cx))\,dx}{5c\sqrt{d+c^2dx^2}}$$

$$= -\frac{2bx^5\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{25c\sqrt{d+c^2dx^2}} - \frac{4x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{15c^4d} + \frac{x^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{5c^2d} + \frac{8\int\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx}{15c^4} + \frac{\left(2b^2\sqrt{1+c^2x^2}\right)\int\frac{x^5}{\sqrt{1+c^2x^2}}\,dx}{25\sqrt{d+c^2dx^2}} + \frac{\left(8b\sqrt{1+c^2x^2}\right)\int x^2(a+b\text{arcsinh}(cx))\,dx}{15c^3\sqrt{d+c^2dx^2}}$$

$$= \frac{8bx^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{45c^3\sqrt{d+c^2dx^2}} - \frac{2bx^5\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{25c\sqrt{d+c^2dx^2}} + \frac{8\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{15c^6d} - \frac{4x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{15c^4d} + \frac{x^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{5c^2d} + \frac{\left(b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{1+c^2x}}\,dx,x,x^2\right)}{25\sqrt{d+c^2dx^2}} - \frac{\left(16b\sqrt{1+c^2x^2}\right)\int(a+b\text{arcsinh}(cx))\,dx}{15c^5\sqrt{d+c^2dx^2}} - \frac{\left(8b^2\sqrt{1+c^2x^2}\right)\int\frac{x^3}{\sqrt{1+c^2x^2}}\,dx}{45c^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{16abx\sqrt{1+c^2x^2}}{15c^5\sqrt{d+c^2dx^2}} + \frac{8bx^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{45c^3\sqrt{d+c^2dx^2}} - \frac{2bx^5\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{25c\sqrt{d+c^2dx^2}} + \frac{8\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{15c^6d} - \frac{4x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{15c^4d} + \frac{x^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{5c^2d} + \frac{\left(b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\left(\frac{1}{c^4\sqrt{1+c^2x}} - \frac{2\sqrt{1+c^2x}}{c^4} + \frac{(1+c^2x)^{3/2}}{c^4}\right)dx,x,x^2\right)}{25\sqrt{d+c^2dx^2}} - \frac{\left(16b^2\sqrt{1+c^2x^2}\right)\int\text{arcsinh}(cx)\,dx}{15c^5\sqrt{d+c^2dx^2}} - \frac{\left(4b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{x}{\sqrt{1+c^2x}}\,dx,x,x^2\right)}{45c^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{16abx\sqrt{1+c^2x^2}}{15c^5\sqrt{d+c^2dx^2}} + \frac{2b^2(1+c^2x^2)}{25c^6\sqrt{d+c^2dx^2}} - \frac{4b^2(1+c^2x^2)^2}{75c^6\sqrt{d+c^2dx^2}} + \frac{2b^2(1+c^2x^2)^3}{125c^6\sqrt{d+c^2dx^2}} - \frac{16b^2x\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{15c^5\sqrt{d+c^2dx^2}} + \frac{8bx^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{45c^3\sqrt{d+c^2dx^2}} - \frac{2bx^5\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{25c\sqrt{d+c^2dx^2}} + \frac{8\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{15c^6d} - \frac{4x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{15c^4d} + \frac{x^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{5c^2d} + \frac{\left(16b^2\sqrt{1+c^2x^2}\right)\int\frac{x}{\sqrt{1+c^2x^2}}\,dx}{15c^4\sqrt{d+c^2dx^2}} - \frac{\left(4b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\left(-\frac{1}{c^2\sqrt{1+c^2x}} + \frac{\sqrt{1+c^2x}}{c^2}\right)dx,x,x^2\right)}{45c^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{16abx\sqrt{1+c^2x^2}}{15c^5\sqrt{d+c^2dx^2}} + \frac{298b^2(1+c^2x^2)}{225c^6\sqrt{d+c^2dx^2}} - \frac{76b^2(1+c^2x^2)^2}{675c^6\sqrt{d+c^2dx^2}} + \frac{2b^2(1+c^2x^2)^3}{125c^6\sqrt{d+c^2dx^2}} - \frac{16b^2x\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{15c^5\sqrt{d+c^2dx^2}} + \frac{8bx^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{45c^3\sqrt{d+c^2dx^2}} - \frac{2bx^5\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{25c\sqrt{d+c^2dx^2}} + \frac{8\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{15c^6d} - \frac{4x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{15c^4d} + \frac{x^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{5c^2d}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.60

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \frac{-30abcx\sqrt{1+c^2x^2}(120-20c^2x^2+9c^4x^4)+225a^2(8+4c^2x^2-c^4x^4+3c^6x^6)+2b^2(2072+1936c^2x^2-\ldots}{}$$

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (-30*a*b*c*x*Sqrt[1 + c^2*x^2]*(120 - 20*c^2*x^2 + 9*c^4*x^4) + 225*a^2*(8
+ 4*c^2*x^2 - c^4*x^4 + 3*c^6*x^6) + 2*b^2*(2072 + 1936*c^2*x^2 - 109*c^4*x
^4 + 27*c^6*x^6) + 30*b*(b*c*x*Sqrt[1 + c^2*x^2]*(-120 + 20*c^2*x^2 - 9*c^4
*x^4) + 15*a*(8 + 4*c^2*x^2 - c^4*x^4 + 3*c^6*x^6))*ArcSinh[c*x] + 225*b^2*
(8 + 4*c^2*x^2 - c^4*x^4 + 3*c^6*x^6)*ArcSinh[c*x]^2)/(3375*c^6*Sqrt[d + c^
2*d*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1226 vs. 2(335) = 670.

Time = 0.33 (sec) , antiderivative size = 1227, normalized size of antiderivative = 3.20

method	result	size
default	Expression too large to display	1227
parts	Expression too large to display	1227

```
[In] int(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(1/5*x^4/c^2/d*(c^2*d*x^2+d)^(1/2)-4/5/c^2*(1/3*x^2/c^2/d*(c^2*d*x^2+d)
^(1/2)-2/3/d/c^4*(c^2*d*x^2+d)^(1/2)))+b^2*(1/4000*(d*(c^2*x^2+1))^(1/2)*(1
6*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1
/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c*x)^2-10*arcsinh(c*x
)+2)/c^6/d/(c^2*x^2+1)-5/864*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^
```

```
2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2-6*arc
sinh(c*x)+2)/c^6/d/(c^2*x^2+1)+5/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2
*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)/c^6/d/(c^2*x^2+1)+5/16*(
d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2+2*ar
csinh(c*x)+2)/c^6/d/(c^2*x^2+1)-5/864*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^
3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x
)^2+6*arcsinh(c*x)+2)/c^6/d/(c^2*x^2+1)+1/4000*(d*(c^2*x^2+1))^(1/2)*(16*c^
6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+
13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c*x)^2+10*arcsinh(c*x)+2)
/c^6/d/(c^2*x^2+1))+2*a*b*(1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x
^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c
*x*(c^2*x^2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))/c^6/d/(c^2*x^2+1)-5/288*(d*(c^2
*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*
x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))/c^6/d/(c^2*x^2+1)+5/16*(d*(c^2*x^2+1))^
(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))/c^6/d/(c^2*x^2+1)
+5/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)
+1)/c^6/d/(c^2*x^2+1)-5/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2
*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(3*arcsinh(c*x)+1)/c^6/d
/(c^2*x^2+1)+1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)
^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)
^(1/2)+1)*(1+5*arcsinh(c*x))/c^6/d/(c^2*x^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.83

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$$
$$= \frac{225\left(3\,b^2c^6x^6-b^2c^4x^4+4\,b^2c^2x^2+8\,b^2\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)^2+30\left(45\,abc^6x^6-15\,abc^4x^4+6\right.}{}$$

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/3375*(225*(3*b^2*c^6*x^6 - b^2*c^4*x^4 + 4*b^2*c^2*x^2 + 8*b^2)*sqrt(c^2*
d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 30*(45*a*b*c^6*x^6 - 15*a*b*c^4
*x^4 + 60*a*b*c^2*x^2 + 120*a*b - (9*b^2*c^5*x^5 - 20*b^2*c^3*x^3 + 120*b^2
*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) +
 (27*(25*a^2 + 2*b^2)*c^6*x^6 - (225*a^2 + 218*b^2)*c^4*x^4 + 4*(225*a^2 +
968*b^2)*c^2*x^2 + 1800*a^2 + 4144*b^2 - 30*(9*a*b*c^5*x^5 - 20*a*b*c^3*x^3
 + 120*a*b*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^8*d*x^2 + c^6*d)
```

Sympy [F]

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{x^5(a+b\operatorname{asinh}(cx))^2}{\sqrt{d(c^2x^2+1)}}\,dx$$

```
[In] integrate(x**5*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
[Out] Integral(x**5*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.92

$$\begin{aligned}
&\int \frac{x^5(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx \\
&= \frac{1}{15}\left(\frac{3\sqrt{c^2dx^2+d}x^4}{c^2d} - \frac{4\sqrt{c^2dx^2+d}x^2}{c^4d} + \frac{8\sqrt{c^2dx^2+d}}{c^6d}\right)b^2\operatorname{arsinh}(cx)^2 \\
&\quad + \frac{2}{15}\left(\frac{3\sqrt{c^2dx^2+d}x^4}{c^2d} - \frac{4\sqrt{c^2dx^2+d}x^2}{c^4d} + \frac{8\sqrt{c^2dx^2+d}}{c^6d}\right)ab\operatorname{arsinh}(cx) \\
&\quad + \frac{1}{15}\left(\frac{3\sqrt{c^2dx^2+d}x^4}{c^2d} - \frac{4\sqrt{c^2dx^2+d}x^2}{c^4d} + \frac{8\sqrt{c^2dx^2+d}}{c^6d}\right)a^2 \\
&\quad + \frac{2}{3375}b^2\left(\frac{27\sqrt{c^2x^2+1}c^2x^4 - 136\sqrt{c^2x^2+1}x^2 + \frac{2072\sqrt{c^2x^2+1}}{c^2}}{c^4\sqrt{d}} - \frac{15\,(9\,c^4x^5 - 20\,c^2x^3 + 120\,x)\operatorname{arsinh}(cx)}{c^5\sqrt{d}}\right) \\
&\quad - \frac{2\,(9\,c^4x^5 - 20\,c^2x^3 + 120\,x)ab}{225\,c^5\sqrt{d}}
\end{aligned}$$

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
[Out] 1/15*(3*sqrt(c^2*d*x^2 + d)*x^4/(c^2*d) - 4*sqrt(c^2*d*x^2 + d)*x^2/(c^4*d)
 + 8*sqrt(c^2*d*x^2 + d)/(c^6*d))*b^2*arcsinh(c*x)^2 + 2/15*(3*sqrt(c^2*d*x
^2 + d)*x^4/(c^2*d) - 4*sqrt(c^2*d*x^2 + d)*x^2/(c^4*d) + 8*sqrt(c^2*d*x^2
+ d)/(c^6*d))*a*b*arcsinh(c*x) + 1/15*(3*sqrt(c^2*d*x^2 + d)*x^4/(c^2*d) -
4*sqrt(c^2*d*x^2 + d)*x^2/(c^4*d) + 8*sqrt(c^2*d*x^2 + d)/(c^6*d))*a^2 + 2/
3375*b^2*((27*sqrt(c^2*x^2 + 1)*c^2*x^4 - 136*sqrt(c^2*x^2 + 1)*x^2 + 2072*
sqrt(c^2*x^2 + 1)/c^2)/(c^4*sqrt(d)) - 15*(9*c^4*x^5 - 20*c^2*x^3 + 120*x)*
arcsinh(c*x)/(c^5*sqrt(d))) - 2/225*(9*c^4*x^5 - 20*c^2*x^3 + 120*x)*a*b/(c
^5*sqrt(d))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{x^5\,(a+b\,\text{asinh}(c\,x))^2}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

```
[In] int((x^5*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^5*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2), x)
```

3.291 $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 323

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = -\frac{15b^2x(1+c^2x^2)}{64c^4\sqrt{d+c^2dx^2}}+\frac{b^2x^3(1+c^2x^2)}{32c^2\sqrt{d+c^2dx^2}}+\frac{15b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{64c^5\sqrt{d+c^2dx^2}}+\frac{3bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{8c^3\sqrt{d+c^2dx^2}}-\frac{bx^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{8c\sqrt{d+c^2dx^2}}-\frac{3x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{8c^4d}+\frac{x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{4c^2d}+\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{8bc^5\sqrt{d+c^2dx^2}}$$

```
[Out] -15/64*b^2*x*(c^2*x^2+1)/c^4/(c^2*d*x^2+d)^(1/2)+1/32*b^2*x^3*(c^2*x^2+1)/c
^2/(c^2*d*x^2+d)^(1/2)+15/64*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c^5/(c^2*d*
x^2+d)^(1/2)+3/8*b*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3/(c^2*d*x^2+
d)^(1/2)-1/8*b*x^4*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/
2)+1/8*(a+b*arcsinh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c^5/(c^2*d*x^2+d)^(1/2)-3/8
*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^4/d+1/4*x^3*(a+b*arcsinh(c*x)
)^2*(c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {5812, 5783, 5776, 327, 221}

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = -\frac{bx^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{8c\sqrt{c^2dx^2+d}} + \frac{x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{4c^2d} + \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{8bc^5\sqrt{c^2dx^2+d}} - \frac{3x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{8c^4d} + \frac{3bx^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{8c^3\sqrt{c^2dx^2+d}} + \frac{15b^2\sqrt{c^2x^2+1}\text{arcsinh}(cx)}{64c^5\sqrt{c^2dx^2+d}} + \frac{b^2x^3(c^2x^2+1)}{32c^2\sqrt{c^2dx^2+d}} - \frac{15b^2x(c^2x^2+1)}{64c^4\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x^4*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (-15*b^2*x*(1 + c^2*x^2))/(64*c^4*Sqrt[d + c^2*d*x^2]) + (b^2*x^3*(1 + c^2*
x^2))/(32*c^2*Sqrt[d + c^2*d*x^2]) + (15*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
)/(64*c^5*Sqrt[d + c^2*d*x^2]) + (3*b*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[
c*x]))/(8*c^3*Sqrt[d + c^2*d*x^2]) - (b*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSin
h[c*x]))/(8*c*Sqrt[d + c^2*d*x^2]) - (3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
h[c*x])^2)/(8*c^4*d) + (x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*
c^2*d) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(8*b*c^5*Sqrt[d + c^2*d
*x^2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{4c^2d} - \frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx}{4c^2} \\
&\quad - \frac{\left(b\sqrt{1+c^2x^2}\right)\int x^3(a+b\text{arcsinh}(cx))\,dx}{2c\sqrt{d+c^2dx^2}} \\
&= -\frac{bx^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{8c\sqrt{d+c^2dx^2}} - \frac{3x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{8c^4d} \\
&\quad + \frac{x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{4c^2d} + \frac{3\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx}{8c^4} \\
&\quad + \frac{\left(b^2\sqrt{1+c^2x^2}\right)\int \frac{x^4}{\sqrt{1+c^2x^2}}\,dx}{8\sqrt{d+c^2dx^2}} + \frac{\left(3b\sqrt{1+c^2x^2}\right)\int x(a+b\text{arcsinh}(cx))\,dx}{4c^3\sqrt{d+c^2dx^2}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{b^2x^3(1+c^2x^2)}{32c^2\sqrt{d+c^2dx^2}} + \frac{3bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{8c^3\sqrt{d+c^2dx^2}} \\
&\quad - \frac{bx^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{8c\sqrt{d+c^2dx^2}} - \frac{3x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{8c^4d} \\
&\quad + \frac{x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{4c^2d} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{8bc^5\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\left(3b^2\sqrt{1+c^2x^2}\right)\int \frac{x^2}{\sqrt{1+c^2x^2}}\,dx}{32c^2\sqrt{d+c^2dx^2}} - \frac{\left(3b^2\sqrt{1+c^2x^2}\right)\int \frac{x^2}{\sqrt{1+c^2x^2}}\,dx}{8c^2\sqrt{d+c^2dx^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{15b^2x(1+c^2x^2)}{64c^4\sqrt{d+c^2dx^2}} + \frac{b^2x^3(1+c^2x^2)}{32c^2\sqrt{d+c^2dx^2}} + \frac{3bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{8c^3\sqrt{d+c^2dx^2}} \\
&\quad - \frac{bx^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{8c\sqrt{d+c^2dx^2}} - \frac{3x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{8c^4d} \\
&\quad + \frac{x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{4c^2d} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{8bc^5\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(3b^2\sqrt{1+c^2x^2}\right)\int \frac{1}{\sqrt{1+c^2x^2}}\,dx}{64c^4\sqrt{d+c^2dx^2}} + \frac{\left(3b^2\sqrt{1+c^2x^2}\right)\int \frac{1}{\sqrt{1+c^2x^2}}\,dx}{16c^4\sqrt{d+c^2dx^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{15b^2x(1+c^2x^2)}{64c^4\sqrt{d+c^2dx^2}} + \frac{b^2x^3(1+c^2x^2)}{32c^2\sqrt{d+c^2dx^2}} \\
&\quad + \frac{15b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{64c^5\sqrt{d+c^2dx^2}} + \frac{3bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{8c^3\sqrt{d+c^2dx^2}} \\
&\quad - \frac{bx^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{8c\sqrt{d+c^2dx^2}} - \frac{3x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{8c^4d} \\
&\quad + \frac{x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{4c^2d} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{8bc^5\sqrt{d+c^2dx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.83

$$
\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx
= \frac{32a^2c\sqrt{d}x(1+c^2x^2)\left(-3+2c^2x^2\right)+96a^2\sqrt{d+c^2dx^2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)+b^2\sqrt{d}\sqrt{1+c^2x^2}(32\text{arcs}}{}
$$

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (32*a^2*c*Sqrt[d]*x*(1 + c^2*x^2)*(-3 + 2*c^2*x^2) + 96*a^2*Sqrt[d + c^2*d*
x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + b^2*Sqrt[d]*Sqrt[1 + c^2*x^
2]*(32*ArcSinh[c*x]^3 - 4*ArcSinh[c*x]*(-16*Cosh[2*ArcSinh[c*x]] + Cosh[4*A
rcSinh[c*x]]) - 32*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]] + 8*ArcSinh[
```

c*x]^2*(-8*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]])) + 4*a*b*Sqrt[d]*Sq
rt[1 + c^2*x^2]*(16*Cosh[2*ArcSinh[c*x]] - Cosh[4*ArcSinh[c*x]] + 4*ArcSinh
[c*x]*(6*ArcSinh[c*x] - 8*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]])))/(2
56*c^5*Sqrt[d]*Sqrt[d + c^2*d*x^2])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 991 vs. $2(283) = 566$.

Time = 0.26 (sec) , antiderivative size = 992, normalized size of antiderivative = 3.07

method	result
default	$\frac{a^2x^3\sqrt{c^2d\,x^2+d}}{4c^2d}-\frac{3a^2x\sqrt{c^2d\,x^2+d}}{8c^4d}+\frac{3a^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{8c^4\sqrt{c^2d}}+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^3}{8\sqrt{c^2x^2+1}\,c^5d}+\frac{\sqrt{d(c^2x^2+1)}\,\left(8c^5x^5\right.}{}\right.$
parts	$\frac{a^2x^3\sqrt{c^2d\,x^2+d}}{4c^2d}-\frac{3a^2x\sqrt{c^2d\,x^2+d}}{8c^4d}+\frac{3a^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{8c^4\sqrt{c^2d}}+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^3}{8\sqrt{c^2x^2+1}\,c^5d}+\frac{\sqrt{d(c^2x^2+1)}\,\left(8c^5x^5\right.}{}\right.$

[In] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/4*a^2*x^3/c^2/d*(c^2*d*x^2+d)^(1/2)-3/8*a^2/c^4*x/d*(c^2*d*x^2+d)^(1/2)+3
/8*a^2/c^4*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b^2*
(1/8*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d*arcsinh(c*x)^3+1/512*(d*
(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*
x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(8*arcsinh(c*x)^2-4*arcsinh(
c*x)+1)/c^5/d/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(
c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2-2*arcsinh(c*x)+
1)/c^5/d/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x
^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2+2*arcsinh(c*x)+1)/c^
5/d/(c^2*x^2+1)+1/512*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1
)^(1/2)+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(8*
arcsinh(c*x)^2+4*arcsinh(c*x)+1)/c^5/d/(c^2*x^2+1))+2*a*b*(3/16*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d*arcsinh(c*x)^2+1/256*(d*(c^2*x^2+1))^(1/
2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^
(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(c*x))/c^5/d/(c^2*x^2+1)-1/16*(
d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+
1)^(1/2))*(-1+2*arcsinh(c*x))/c^5/d/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*
(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(1+2*arcsin
h(c*x))/c^5/d/(c^2*x^2+1)+1/256*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*
(c^2*x^2+1)^(1/2)+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^
(1/2))*(1+4*arcsinh(c*x))/c^5/d/(c^2*x^2+1))

Fricas [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^4}{\sqrt{c^2dx^2+d}}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
[Out] integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)/sqrt(c
^2*d*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{x^4(a+b\,\text{asinh}\,(cx))^2}{\sqrt{d\,(c^2x^2+1)}}\,dx$$

```
[In] integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
[Out] Integral(x**4*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^4}{\sqrt{c^2dx^2+d}}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2*x^4/sqrt(c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{x^4\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

```
[In] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2),x)
[Out] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2), x)
```

3.292 $\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 265

$$\begin{aligned}\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx &= \frac{4abx\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}}-\frac{14b^2(1+c^2x^2)}{9c^4\sqrt{d+c^2dx^2}}\\ &\quad+\frac{2b^2(1+c^2x^2)^2}{27c^4\sqrt{d+c^2dx^2}}+\frac{4b^2x\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{3c^3\sqrt{d+c^2dx^2}}\\ &\quad-\frac{2bx^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c\sqrt{d+c^2dx^2}}\\ &\quad-\frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^4d}\\ &\quad+\frac{x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^2d}\end{aligned}$$

```
[Out] -14/9*b^2*(c^2*x^2+1)/c^4/(c^2*d*x^2+d)^(1/2)+2/27*b^2*(c^2*x^2+1)^2/c^4/(c
^2*d*x^2+d)^(1/2)+4/3*a*b*x*(c^2*x^2+1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)+4/3*b
^2*x*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-2/9*b*x^3*(a+b*
arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)-2/3*(a+b*arcsinh(c*x)
)^2*(c^2*d*x^2+d)^(1/2)/c^4/d+1/3*x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1
/2)/c^2/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5812, 5798, 5772, 267, 5776, 272, 45}

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}dx = \frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d} - \frac{2bx^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{9c\sqrt{c^2dx^2+d}} - \frac{2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^4d} + \frac{4abx\sqrt{c^2x^2+1}}{3c^3\sqrt{c^2dx^2+d}} + \frac{4b^2x\sqrt{c^2x^2+1}\text{arcsinh}(cx)}{3c^3\sqrt{c^2dx^2+d}} + \frac{2b^2(c^2x^2+1)^2}{27c^4\sqrt{c^2dx^2+d}} - \frac{14b^2(c^2x^2+1)}{9c^4\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x^3*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (4*a*b*x*Sqrt[1 + c^2*x^2])/(3*c^3*Sqrt[d + c^2*d*x^2]) - (14*b^2*(1 + c^2*
x^2))/(9*c^4*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*x^2)^2)/(27*c^4*Sqrt[d
+ c^2*d*x^2]) + (4*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(3*c^3*Sqrt[d + c^
2*d*x^2]) - (2*b*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9*c*Sqrt[d +
c^2*d*x^2]) - (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^4*d) + (x
^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^2d} - \frac{2\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx}{3c^2} \\
&\quad - \frac{\left(2b\sqrt{1+c^2x^2}\right)\int x^2(a+b\text{arcsinh}(cx))\,dx}{3c\sqrt{d+c^2dx^2}} \\
&= -\frac{2bx^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c\sqrt{d+c^2dx^2}} - \frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^4d} \\
&\quad + \frac{x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^2d} + \frac{\left(2b^2\sqrt{1+c^2x^2}\right)\int \frac{x^3}{\sqrt{1+c^2x^2}}\,dx}{9\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(4b\sqrt{1+c^2x^2}\right)\int (a+b\text{arcsinh}(cx))\,dx}{3c^3\sqrt{d+c^2dx^2}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{4abx\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}} - \frac{2bx^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c\sqrt{d+c^2dx^2}} \\
&\quad - \frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^4d} + \frac{x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^2d} \\
&\quad + \frac{\left(b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1+c^2x}}\,dx, x, x^2\right)}{9\sqrt{d+c^2dx^2}} + \frac{\left(4b^2\sqrt{1+c^2x^2}\right)\int \text{arcsinh}(cx)\,dx}{3c^3\sqrt{d+c^2dx^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4abx\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}} + \frac{4b^2x\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{3c^3\sqrt{d+c^2dx^2}} - \frac{2bx^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c\sqrt{d+c^2dx^2}} \\
&\quad - \frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^4d} + \frac{x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^2d} \\
&\quad + \frac{\left(b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \left(-\frac{1}{c^2\sqrt{1+c^2x}} + \frac{\sqrt{1+c^2x}}{c^2}\right)dx, x, x^2\right)}{9\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\left(4b^2\sqrt{1+c^2x^2}\right)\int \frac{x}{\sqrt{1+c^2x^2}}\,dx}{3c^2\sqrt{d+c^2dx^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4abx\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}} - \frac{14b^2(1+c^2x^2)}{9c^4\sqrt{d+c^2dx^2}} + \frac{2b^2(1+c^2x^2)^2}{27c^4\sqrt{d+c^2dx^2}} \\
&\quad + \frac{4b^2x\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{3c^3\sqrt{d+c^2dx^2}} - \frac{2bx^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c\sqrt{d+c^2dx^2}} \\
&\quad - \frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^4d} + \frac{x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.66

$$
\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx
$$

$$
= \frac{-6abcx(-6+c^2x^2)\sqrt{1+c^2x^2} + 2b^2(-20-19c^2x^2+c^4x^4) + 9a^2(-2-c^2x^2+c^4x^4) - 6b\big(bcx(-6+c^2x^2 \cdots}{27c^4\sqrt{d+c^2dx^2}}
$$

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (-6*a*b*c*x*(-6 + c^2*x^2)*Sqrt[1 + c^2*x^2] + 2*b^2*(-20 - 19*c^2*x^2 + c^
4*x^4) + 9*a^2*(-2 - c^2*x^2 + c^4*x^4) - 6*b*(b*c*x*(-6 + c^2*x^2)*Sqrt[1
+ c^2*x^2] + a*(6 + 3*c^2*x^2 - 3*c^4*x^4))*ArcSinh[c*x] + 9*b^2*(-2 - c^2*
x^2 + c^4*x^4)*ArcSinh[c*x]^2)/(27*c^4*Sqrt[d + c^2*d*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 705 vs. 2(231) = 462.

Time = 0.29 (sec) , antiderivative size = 706, normalized size of antiderivative = 2.66

method	result
default	$a^2\left(\frac{x^2\sqrt{c^2d\,x^2+d}}{3c^2d}-\frac{2\sqrt{c^2d\,x^2+d}}{3d\,c^4}\right)+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\left(4c^4x^4+4c^3x^3\sqrt{c^2x^2+1}+5c^2x^2+3cx\sqrt{c^2x^2+1}+1\right)\left(9\operatorname{arcsinh}(cx)^2-6\ \text{ar}\right.}{216c^4d(c^2x^2+1)}\right.$
parts	$a^2\left(\frac{x^2\sqrt{c^2d\,x^2+d}}{3c^2d}-\frac{2\sqrt{c^2d\,x^2+d}}{3d\,c^4}\right)+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\left(4c^4x^4+4c^3x^3\sqrt{c^2x^2+1}+5c^2x^2+3cx\sqrt{c^2x^2+1}+1\right)\left(9\operatorname{arcsinh}(cx)^2-6\ \text{ar}\right.}{216c^4d(c^2x^2+1)}\right.$

```
[In] int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(1/3*x^2/c^2/d*(c^2*d*x^2+d)^(1/2)-2/3/d/c^4*(c^2*d*x^2+d)^(1/2))+b^2*(
1/216*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^
2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2-6*arcsinh(c*x)+2)/c^4/d/(c^2
*x^2+1)-3/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(arcsin
h(c*x)^2-2*arcsinh(c*x)+2)/c^4/d/(c^2*x^2+1)-3/8*(d*(c^2*x^2+1))^(1/2)*(c^2
*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2+2*arcsinh(c*x)+2)/c^4/d/(c^2*
x^2+1)+1/216*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5
*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2+6*arcsinh(c*x)+2)/c^4
/d/(c^2*x^2+1))+2*a*b*(1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2
*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))/c^4/
d/(c^2*x^2+1)-3/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(
-1+arcsinh(c*x))/c^4/d/(c^2*x^2+1)-3/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(
c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)+1)/c^4/d/(c^2*x^2+1)+1/72*(d*(c^2*x^2+1))
^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(
1/2)+1)*(3*arcsinh(c*x)+1)/c^4/d/(c^2*x^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.96

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$$
$$=\frac{9\left(b^2c^4x^4-b^2c^2x^2-2\,b^2\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)^2+6\left(3\,abc^4x^4-3\,abc^2x^2-6\,ab-\left(b^2c^3x^3-6\,b\right.\right.}{}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/27*(9*(b^2*c^4*x^4 - b^2*c^2*x^2 - 2*b^2)*sqrt(c^2*d*x^2 + d)*log(c*x + s
qrt(c^2*x^2 + 1))^2 + 6*(3*a*b*c^4*x^4 - 3*a*b*c^2*x^2 - 6*a*b - (b^2*c^3*x
```

```
^3 - 6*b^2*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x
^2 + 1)) + ((9*a^2 + 2*b^2)*c^4*x^4 - (9*a^2 + 38*b^2)*c^2*x^2 - 18*a^2 - 4
0*b^2 - 6*(a*b*c^3*x^3 - 6*a*b*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))
/(c^6*d*x^2 + c^4*d)
```

Sympy [F]

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{x^3(a+b\operatorname{asinh}(cx))^2}{\sqrt{d\,(c^2x^2+1)}}\,dx$$

```
[In] integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.92

$$\begin{aligned}
&\int \frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx \\
&= \frac{1}{3}\,b^2\left(\frac{\sqrt{c^2dx^2+d}x^2}{c^2d} - \frac{2\,\sqrt{c^2dx^2+d}}{c^4d}\right)\operatorname{arsinh}(cx)^2 \\
&\quad + \frac{2}{3}\,ab\left(\frac{\sqrt{c^2dx^2+d}x^2}{c^2d} - \frac{2\,\sqrt{c^2dx^2+d}}{c^4d}\right)\operatorname{arsinh}(cx) \\
&\quad + \frac{1}{3}\,a^2\left(\frac{\sqrt{c^2dx^2+d}x^2}{c^2d} - \frac{2\,\sqrt{c^2dx^2+d}}{c^4d}\right) \\
&\quad + \frac{2}{27}\,b^2\left(\frac{\sqrt{c^2x^2+1}x^2 - \frac{20\,\sqrt{c^2x^2+1}}{c^2}}{c^2\sqrt{d}} - \frac{3\,(c^2x^3-6\,x)\operatorname{arsinh}(cx)}{c^3\sqrt{d}}\right) - \frac{2\,(c^2x^3-6\,x)ab}{9\,c^3\sqrt{d}}
\end{aligned}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/3*b^2*(sqrt(c^2*d*x^2 + d)*x^2/(c^2*d) - 2*sqrt(c^2*d*x^2 + d)/(c^4*d))*a
rcsinh(c*x)^2 + 2/3*a*b*(sqrt(c^2*d*x^2 + d)*x^2/(c^2*d) - 2*sqrt(c^2*d*x^2
 + d)/(c^4*d))*arcsinh(c*x) + 1/3*a^2*(sqrt(c^2*d*x^2 + d)*x^2/(c^2*d) - 2*
sqrt(c^2*d*x^2 + d)/(c^4*d)) + 2/27*b^2*((sqrt(c^2*x^2 + 1)*x^2 - 20*sqrt(c
^2*x^2 + 1)/c^2)/(c^2*sqrt(d)) - 3*(c^2*x^3 - 6*x)*arcsinh(c*x)/(c^3*sqrt(d
))) - 2/9*(c^2*x^3 - 6*x)*a*b/(c^3*sqrt(d))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\text{arcsinh}(cx))^2}{\sqrt{d + c^2dx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\text{arcsinh}(cx))^2}{\sqrt{d + c^2dx^2}}\,dx = \int \frac{x^3\,(a + b\,\mathrm{asinh}(c\,x))^2}{\sqrt{d\,c^2\,x^2 + d}}\,dx$$

```
[In] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2), x)
```

3.293 $\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 204

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \frac{b^2x(1+c^2x^2)}{4c^2\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{4c^3\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2c\sqrt{d+c^2dx^2}} + \frac{x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2c^2d} - \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{6bc^3\sqrt{d+c^2dx^2}}$$

```
[Out] 1/4*b^2*x*(c^2*x^2+1)/c^2/(c^2*d*x^2+d)^(1/2)-1/4*b^2*arcsinh(c*x)*(c^2*x^2
+1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-1/2*b*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^
(1/2)/c/(c^2*d*x^2+d)^(1/2)-1/6*(a+b*arcsinh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c^
3/(c^2*d*x^2+d)^(1/2)+1/2*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used

$= \{5812, 5783, 5776, 327, 221\}$

$$\int \frac{x^2(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \frac{x\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^2}{2c^2d} - \frac{bx^2\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{2c\sqrt{c^2dx^2+d}} - \frac{\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^3}{6bc^3\sqrt{c^2dx^2+d}} - \frac{b^2\sqrt{c^2x^2+1}\operatorname{arcsinh}(cx)}{4c^3\sqrt{c^2dx^2+d}} + \frac{b^2x(c^2x^2+1)}{4c^2\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x^2*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (b^2*x*(1 + c^2*x^2))/(4*c^2*Sqrt[d + c^2*d*x^2]) - (b^2*Sqrt[1 + c^2*x^2]*
ArcSinh[c*x])/(4*c^3*Sqrt[d + c^2*d*x^2]) - (b*x^2*Sqrt[1 + c^2*x^2]*(a + b
*ArcSinh[c*x]))/(2*c*Sqrt[d + c^2*d*x^2]) + (x*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x])^2)/(2*c^2*d) - (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b
*c^3*Sqrt[d + c^2*d*x^2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2c^2d} - \frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx}{2c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int x(a+b\text{arcsinh}(cx))\,dx}{c\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2c\sqrt{d+c^2dx^2}} + \frac{x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2c^2d} - \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{6bc^3\sqrt{d+c^2dx^2}} + \frac{\left(b^2\sqrt{1+c^2x^2}\right)\int \frac{x^2}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2x(1+c^2x^2)}{4c^2\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2c\sqrt{d+c^2dx^2}} + \frac{x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2c^2d} - \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{6bc^3\sqrt{d+c^2dx^2}} - \frac{\left(b^2\sqrt{1+c^2x^2}\right)\int \frac{1}{\sqrt{1+c^2x^2}}\,dx}{4c^2\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2x(1+c^2x^2)}{4c^2\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{4c^3\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2c\sqrt{d+c^2dx^2}} + \frac{x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2c^2d} - \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{6bc^3\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.97

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$$
$$= \frac{12a^2cx(d+c^2dx^2)-12a^2\sqrt{d}\sqrt{d+c^2dx^2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)-6abd\sqrt{1+c^2x^2}(\cosh(2\text{arcsinh}(cx))}{}$$

[In] Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]

[Out] (12*a^2*c*x*(d + c^2*d*x^2) - 12*a^2*Sqrt[d]*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 6*a*b*d*Sqrt[1 + c^2*x^2]*(Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*x]*(ArcSinh[c*x] - Sinh[2*ArcSinh[c*x]])) - b^2*d*Sqrt[1 + c^2*x^2]*(4*ArcSinh[c*x]^3 + 6*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]] - 3*(1 + 2*ArcSinh[c*x]^2)*Sinh[2*ArcSinh[c*x]]))/(24*c^3*d*Sqrt[d + c^2*d*x^2])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. 2(178) = 356.

Time = 0.23 (sec) , antiderivative size = 506, normalized size of antiderivative = 2.48

method	result
default	$\frac{a^2x\sqrt{c^2d\,x^2+d}}{2c^2d}-\frac{a^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2c^2\sqrt{c^2d}}+b^2\left(-\frac{\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)^3}{6\sqrt{c^2x^2+1}\,c^3d}+\frac{\sqrt{d(c^2x^2+1)}\left(2c^3x^3+2c^2x^2\sqrt{c^2x^2+1}+2cx\right.}{16c^3d}\right.$
parts	$\frac{a^2x\sqrt{c^2d\,x^2+d}}{2c^2d}-\frac{a^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2c^2\sqrt{c^2d}}+b^2\left(-\frac{\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)^3}{6\sqrt{c^2x^2+1}\,c^3d}+\frac{\sqrt{d(c^2x^2+1)}\left(2c^3x^3+2c^2x^2\sqrt{c^2x^2+1}+2cx\right.}{16c^3d}\right.$

[In] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2*a^2*x/c^2/d*(c^2*d*x^2+d)^(1/2)-1/2*a^2/c^2*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b^2*(-1/6*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d*arcsinh(c*x)^3+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2-2*arcsinh(c*x)+1)/c^3/d/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2+2*arcsinh(c*x)+1)/c^3/d/(c^2*x^2+1))+2*a*b*(-1/4*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d*arcsinh(c*x)^2+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*x))/c^3/d/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(c*x))/c^3/d/(c^2*x^2+1))

Fricas [F]

$$\int \frac{x^2(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2x^2}{\sqrt{c^2dx^2+d}}\,dx$$

[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")

[Out] integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)/sqrt(c
^2*d*x^2 + d), x)

Sympy [F]

$$\int \frac{x^2(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{x^2(a+b\operatorname{asinh}(cx))^2}{\sqrt{d(c^2x^2+1)}}\,dx$$

[In] integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)

[Out] Integral(x**2*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\text{Exception raised: RuntimeError}$$

[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")

[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.

Giac [F]

$$\int \frac{x^2(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2x^2}{\sqrt{c^2dx^2+d}}\,dx$$

[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)^2*x^2/sqrt(c^2*d*x^2 + d), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{x^2\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

```
[In] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2),x)
[Out] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2), x)
```

3.294 $\int \frac{x(a+b\,\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 138

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=-\frac{2abx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}}+\frac{2b^2(1+c^2x^2)}{c^2\sqrt{d+c^2dx^2}}-\frac{2b^2x\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{c\sqrt{d+c^2dx^2}}+\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{c^2d}$$

```
[Out] 2*b^2*(c^2*x^2+1)/c^2/(c^2*d*x^2+d)^(1/2)-2*a*b*x*(c^2*x^2+1)^(1/2)/c/(c^2*
d*x^2+d)^(1/2)-2*b^2*x*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)
+(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5798, 5772, 267}

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^2d}-\frac{2abx\sqrt{c^2x^2+1}}{c\sqrt{c^2dx^2+d}}-\frac{2b^2x\sqrt{c^2x^2+1}\text{arcsinh}(cx)}{c\sqrt{c^2dx^2+d}}+\frac{2b^2(c^2x^2+1)}{c^2\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (-2*a*b*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*x^2)
)/(c^2*Sqrt[d + c^2*d*x^2]) - (2*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(c*S
qrt[d + c^2*d*x^2]) + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(c^2*d)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{c^2d} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\int(a+b\text{arcsinh}(cx))\,dx}{c\sqrt{d+c^2dx^2}} \\
&= -\frac{2abx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} + \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{c^2d} - \frac{\left(2b^2\sqrt{1+c^2x^2}\right)\int\text{arcsinh}(cx)\,dx}{c\sqrt{d+c^2dx^2}} \\
&= -\frac{2abx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} - \frac{2b^2x\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{c\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{c^2d} + \frac{\left(2b^2\sqrt{1+c^2x^2}\right)\int\frac{x}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}} \\
&= -\frac{2abx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} + \frac{2b^2(1+c^2x^2)}{c^2\sqrt{d+c^2dx^2}} - \frac{2b^2x\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{c\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{c^2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.92

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$$
$$=\frac{\sqrt{d+c^2dx^2}\left(-2abcx+a^2\sqrt{1+c^2x^2}+2b^2\sqrt{1+c^2x^2}-2b\left(bcx-a\sqrt{1+c^2x^2}\right)\text{arcsinh}(cx)+b^2\sqrt{1+c^2x^2}\right.}{c^2d\sqrt{1+c^2x^2}}$$

```
[In] Integrate[(x*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(-2*a*b*c*x + a^2*Sqrt[1 + c^2*x^2] + 2*b^2*Sqrt[1 + c
^2*x^2] - 2*b*(b*c*x - a*Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + b^2*Sqrt[1 + c^2
*x^2]*ArcSinh[c*x]^2))/(c^2*d*Sqrt[1 + c^2*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(126) = 252.

Time = 0.24 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.14

method	result
default	$\frac{a^2\sqrt{c^2d\,x^2+d}}{c^2d}+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2+cx\sqrt{c^2x^2+1}+1\right)\left(\text{arcsinh}(cx)^2-2\,\text{arcsinh}(cx)+2\right)}{2c^2d(c^2x^2+1)}+\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2-cx\sqrt{c^2x^2+\right.}{2c^2}\right.$
parts	$\frac{a^2\sqrt{c^2d\,x^2+d}}{c^2d}+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2+cx\sqrt{c^2x^2+1}+1\right)\left(\text{arcsinh}(cx)^2-2\,\text{arcsinh}(cx)+2\right)}{2c^2d(c^2x^2+1)}+\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2-cx\sqrt{c^2x^2+\right.}{2c^2}\right.$

```
[In] int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2/c^2/d*(c^2*d*x^2+d)^(1/2)+b^2*(1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(
c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)/c^2/d/(c^2*x^2+1)+1/2
*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2+2*
arcsinh(c*x)+2)/c^2/d/(c^2*x^2+1))+2*a*b*(1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^
2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))/c^2/d/(c^2*x^2+1)+1/2*(d*(c^2*
x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)+1)/c^2/d/(c^2
*x^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.30

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$$
$$= \frac{(b^2c^2x^2+b^2)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\left(abc^2x^2-\sqrt{c^2x^2+1}b^2cx+ab\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{}\right.}{c^4dx^2+c^2d}$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] ((b^2*c^2*x^2 + b^2)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2
*(a*b*c^2*x^2 - sqrt(c^2*x^2 + 1)*b^2*c*x + a*b)*sqrt(c^2*d*x^2 + d)*log(c*
x + sqrt(c^2*x^2 + 1)) + ((a^2 + 2*b^2)*c^2*x^2 - 2*sqrt(c^2*x^2 + 1)*a*b*c
*x + a^2 + 2*b^2)*sqrt(c^2*d*x^2 + d))/(c^4*d*x^2 + c^2*d)
```

Sympy [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{x(a+b\operatorname{asinh}(cx))^2}{\sqrt{d(c^2x^2+1)}}\,dx$$

```
[In] integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = -2b^2\left(\frac{x\operatorname{arsinh}(cx)}{c\sqrt{d}}-\frac{\sqrt{c^2x^2+1}}{c^2\sqrt{d}}\right) - \frac{2abx}{c\sqrt{d}}+\frac{\sqrt{c^2dx^2+d}b^2\operatorname{arsinh}(cx)^2}{c^2d} + \frac{2\sqrt{c^2dx^2+d}ab\operatorname{arsinh}(cx)}{c^2d}+\frac{\sqrt{c^2dx^2+d}a^2}{c^2d}$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*b^2*(x*arcsinh(c*x)/(c*sqrt(d)) - sqrt(c^2*x^2 + 1)/(c^2*sqrt(d))) - 2*a
*b*x/(c*sqrt(d)) + sqrt(c^2*d*x^2 + d)*b^2*arcsinh(c*x)^2/(c^2*d) + 2*sqrt(
c^2*d*x^2 + d)*a*b*arcsinh(c*x)/(c^2*d) + sqrt(c^2*d*x^2 + d)*a^2/(c^2*d)
```

Giac [F]

$$\int \frac{x(a + b\text{arcsinh}(cx))^2}{\sqrt{d + c^2dx^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2 x}{\sqrt{c^2dx^2 + d}}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2*x/sqrt(c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b\text{arcsinh}(cx))^2}{\sqrt{d + c^2dx^2}}\,dx = \int \frac{x\,(a + b\,\text{asinh}(c\,x))^2}{\sqrt{d\,c^2\,x^2 + d}}\,dx$$

```
[In] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2),x)
[Out] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2), x)
```

3.295 $\int \frac{(a+b\mathbf{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 47

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+c^2dx^2}}$$

```
[Out] 1/3*(a+b*arcsinh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {5783}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{c^2dx^2+d}}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c*Sqrt[d + c^2*d*x^2])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\text{integral}=\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+c^2dx^2}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 115 vs. 2(47) = 94.

Time = 0.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.45

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \frac{\frac{3ab\sqrt{1+c^2x^2}\operatorname{arcsinh}(cx)^2}{\sqrt{d+c^2dx^2}}+\frac{b^2\sqrt{1+c^2x^2}\operatorname{arcsinh}(cx)^3}{\sqrt{d+c^2dx^2}}+\frac{3a^2\operatorname{arctanh}\left(\frac{c\sqrt{d}x}{\sqrt{d+c^2dx^2}}\right)}{\sqrt{d}}}{3c}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] ((3*a*b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/Sqrt[d + c^2*d*x^2] + (b^2*Sqrt[1
 + c^2*x^2]*ArcSinh[c*x]^3)/Sqrt[d + c^2*d*x^2] + (3*a^2*ArcTanh[(c*Sqrt[d]
*x)/Sqrt[d + c^2*d*x^2]])/Sqrt[d])/(3*c)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(41) = 82.

Time = 0.21 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.55

method	result	size
default	$\frac{a^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{\sqrt{c^2d}}+\frac{b^2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^3}{3\sqrt{c^2x^2+1}\,cd}+\frac{ab\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2}{\sqrt{c^2x^2+1}\,cd}$	120
parts	$\frac{a^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{\sqrt{c^2d}}+\frac{b^2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^3}{3\sqrt{c^2x^2+1}\,cd}+\frac{ab\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2}{\sqrt{c^2x^2+1}\,cd}$	120

```
[In] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/3*b^2*(d*
(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d*arcsinh(c*x)^3+a*b*(d*(c^2*x^2+1))
^(1/2)/(c^2*x^2+1)^(1/2)/c/d*arcsinh(c*x)^2
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{c^2dx^2+d}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/sqrt(c^2*d*x^2 + d
), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(a+b\,\text{asinh}\,(cx))^2}{\sqrt{d\,(c^2x^2+1)}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
[Out] Integral((a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\frac{b^2\,\text{arsinh}\,(cx)^3}{3\,c\sqrt{d}}+\frac{ab\,\text{arsinh}\,(cx)^2}{c\sqrt{d}}+\frac{a^2\,\text{arsinh}\,(cx)}{c\sqrt{d}}$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
[Out] 1/3*b^2*arcsinh(c*x)^3/(c*sqrt(d)) + a*b*arcsinh(c*x)^2/(c*sqrt(d)) + a^2*a
rcsinh(c*x)/(c*sqrt(d))
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{c^2dx^2+d}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/sqrt(c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)^2}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(1/2),x)
[Out] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(1/2), x)
```

3.296 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 223

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{d+c^2dx^2}}\,dx = -\frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} + \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} + \frac{2b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{2b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}}$$

```
[Out] -2*(a+b*arcsinh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c
^2*d*x^2+d)^(1/2)-2*b*(a+b*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*
(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+2*b*(a+b*arcsinh(c*x))*polylog(2,c*x+
(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+2*b^2*polylog(3,-c
*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-2*b^2*polylog(3
,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {5816, 4267, 2611, 2320, 6724}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{d+c^2dx^2}}\,dx = -\frac{2\sqrt{c^2x^2+1}\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}} - \frac{2b\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}} + \frac{2b\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}} + \frac{2b^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{c^2dx^2+d}} - \frac{2b^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{\sqrt{c^2dx^2+d}}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (-2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[
d + c^2*d*x^2] - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^
ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] + (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[
c*x])*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] + (2*b^2*Sqrt[1 + c^2
*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] - (2*b^2*Sqrt[1 + c^
2*x^2]*PolyLog[3, E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
```

```
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1+c^2x^2}\text{Subst}\left(\int (a+bx)^2\text{csch}(x)\,dx, x, \text{arcsinh}(cx)\right)}{\sqrt{d+c^2dx^2}} \\
&= -\frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\log\left(1-e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\log\left(1+e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{\sqrt{d+c^2dx^2}} \\
&= -\frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} \\
&\quad - \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} \\
&\quad + \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(2b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \text{PolyLog}\left(2,-e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\left(2b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \text{PolyLog}\left(2,e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{\sqrt{d+c^2dx^2}}
\end{aligned}$$

$$= -\frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} + \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} + \frac{\left(2b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{\left(2b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}}$$

$$= -\frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} + \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} + \frac{2b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{2b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.19

$$\int\frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{d+c^2dx^2}}\,dx = \frac{a^2\log(cx)}{\sqrt{d}} - \frac{a^2\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right)}{\sqrt{d}} + \frac{2ab\sqrt{1+c^2x^2}\left(\text{arcsinh}(cx)\left(\log\left(1-e^{-\text{arcsinh}(cx)}\right)-\log\left(1+e^{-\text{arcsinh}(cx)}\right)\right)+\text{PolyLog}\left(2,-e^{-\text{arcsinh}(cx)}\right)\right.}{\sqrt{d+c^2dx^2}} + \frac{b^2\sqrt{1+c^2x^2}\left(\text{arcsinh}(cx)^2\log\left(1-e^{-\text{arcsinh}(cx)}\right)-\text{arcsinh}(cx)^2\log\left(1+e^{-\text{arcsinh}(cx)}\right)+2\text{arcsinh}(cx)\,\text{Pol}\right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (a^2*Log[c*x])/Sqrt[d] - (a^2*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/Sqrt[d]
 + (2*a*b*Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(Log[1 - E^(-ArcSinh[c*x])] - Log
[1 + E^(-ArcSinh[c*x])]) + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-
ArcSinh[c*x])]))/Sqrt[d + c^2*d*x^2] + (b^2*Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]
^2*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] +
 2*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - 2*ArcSinh[c*x]*PolyLog[2,
E^(-ArcSinh[c*x])] + 2*PolyLog[3, -E^(-ArcSinh[c*x])] - 2*PolyLog[3, E^(-Ar
cSinh[c*x])]))/Sqrt[d + c^2*d*x^2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 545 vs. 2(250) = 500.

Time = 0.28 (sec) , antiderivative size = 546, normalized size of antiderivative = 2.45

method	result
default	$-\frac{a^2 \ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2 d\,x^2+d}}{x}\right)}{\sqrt{d}}+b^2\left(-\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2 \ln\left(1+cx+\sqrt{c^2x^2+1}\right)}{\sqrt{c^2x^2+1}\,d}-\frac{2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)\operatorname{polylog}\left(2,-\right.}{\sqrt{c^2x^2+1}\,d}\right.$
parts	$-\frac{a^2 \ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2 d\,x^2+d}}{x}\right)}{\sqrt{d}}+b^2\left(-\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2 \ln\left(1+cx+\sqrt{c^2x^2+1}\right)}{\sqrt{c^2x^2+1}\,d}-\frac{2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)\operatorname{polylog}\left(2,-\right.}{\sqrt{c^2x^2+1}\,d}\right.$

```
[In] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a^2/d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)+b^2*(-(d*(c^2*x^2+1)
)^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))-2*(d
*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^
2+1)^(1/2))+2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(3,-c*x-(c^2
*x^2+1)^(1/2))+(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)^2*ln(
1-c*x-(c^2*x^2+1)^(1/2))+2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsin
h(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)
^(1/2)/d*polylog(3,c*x+(c^2*x^2+1)^(1/2)))+2*a*b*((d*(c^2*x^2+1))^(1/2)/(c^
2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+(d*(c^2*x^2+1))^(
1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,c*x+(c^2*x^2+1)^(1/2))-(d*(c^2*x^2+1))^(
1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-(d*(c^2*x
^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,-c*x-(c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x\sqrt{d+c^2dx^2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{c^2dx^2+d}x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^2*d*x^3 + d*x), x)
```

Sympy [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x\sqrt{d+c^2dx^2}}\,dx = \int \frac{(a+b\operatorname{asinh}(cx))^2}{x\sqrt{d(c^2x^2+1)}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d)**(1/2),x)
[Out] Integral((a + b*asinh(c*x))**2/(x*sqrt(d*(c**2*x**2 + 1))), x)
```

Maxima [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x\sqrt{d+c^2dx^2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{c^2dx^2+d}x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
[Out] -a^2*arcsinh(1/(c*abs(x)))/sqrt(d) + integrate(b^2*log(c*x + sqrt(c^2*x^2 +
 1))^2/(sqrt(c^2*d*x^2 + d)*x) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(c
^2*d*x^2 + d)*x), x)
```

Giac [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x\sqrt{d+c^2dx^2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{c^2dx^2+d}x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/(sqrt(c^2*d*x^2 + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x\sqrt{d+c^2dx^2}}\,dx = \int \frac{(a+b\operatorname{asinh}(c\,x))^2}{x\sqrt{d\,c^2\,x^2+d}}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^(1/2)),x)
[Out] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^(1/2)), x)
```

3.297 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 167

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\sqrt{d+c^2dx^2}}\,dx = \frac{c\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{dx} + \frac{2bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{b^2c\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}}$$

```
[Out] c*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+2*b*c*(a+b*arc
sinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d
)^(1/2)-b^2*c*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/(c^2
*d*x^2+d)^(1/2)-(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/d/x
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used

$= \{5800, 5775, 3797, 2221, 2317, 2438\}$

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\sqrt{d+c^2dx^2}}\,dx = -\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{dx} + \frac{c\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}} + \frac{2bc\sqrt{c^2x^2+1}\log\left(1-e^{-2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}} - \frac{b^2c\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)}{\sqrt{c^2dx^2+d}}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^2*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2] - (Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(d*x) + (2*b*c*Sqrt[1 + c^2*x^2]*(a +
b*ArcSinh[c*x])*Log[1 - E^(-2*ArcSinh[c*x])])/Sqrt[d + c^2*d*x^2] - (b^2*c*
Sqrt[1 + c^2*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/Sqrt[d + c^2*d*x^2]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{dx}+\frac{\left(2bc\sqrt{1+c^2x^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{x}\,dx}{\sqrt{d+c^2dx^2}}\\
&= -\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{dx}\\
&\quad-\frac{\left(2c\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int x\coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{\sqrt{d+c^2dx^2}}\\
&= \frac{c\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{dx}\\
&\quad+\frac{\left(4c\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{\sqrt{d+c^2dx^2}}\\
&= \frac{c\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{dx}\\
&\quad+\frac{2bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}}\\
&\quad-\frac{\left(2bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{\sqrt{d+c^2dx^2}}\\
&= \frac{c\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{dx}\\
&\quad+\frac{2bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}}\\
&\quad+\frac{\left(b^2c\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right)}{\sqrt{d+c^2dx^2}}
\end{aligned}$$

$$= \frac{c\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{dx} + \frac{2bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{b^2c\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}\right)}{\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.01

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\sqrt{d+c^2dx^2}}\,dx = \frac{b^2\left(-1-c^2x^2+cx\sqrt{1+c^2x^2}\right)\text{arcsinh}(cx)^2 - 2b\text{arcsinh}(cx)\left(a+ac^2x^2-bcx\sqrt{1+c^2x^2}\log\left(1-e^{-2\text{arcsinh}\right.\right.}{x\sqrt{d+c^2dx^2}}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^2*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (b^2*(-1 - c^2*x^2 + c*x*Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2 - 2*b*ArcSinh[c*
x]*(a + a*c^2*x^2 - b*c*x*Sqrt[1 + c^2*x^2]*Log[1 - E^(-2*ArcSinh[c*x])]) -
 a*(a + a*c^2*x^2 - 2*b*c*x*Sqrt[1 + c^2*x^2]*Log[c*x]) - b^2*c*x*Sqrt[1 +
c^2*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/(x*Sqrt[d + c^2*d*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. 2(171) = 342.

Time = 0.26 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.86

method	result
default	$-\frac{a^2\sqrt{c^2d\,x^2+d}}{dx}+b^2\left(-\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2-cx\sqrt{c^2x^2+1}+1\right)\text{arcsinh}(cx)^2}{(c^2x^2+1)dx}-\frac{2\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)^2c}{\sqrt{c^2x^2+1}\,d}+\frac{2\sqrt{d(c^2x^2+1)}}{\right.}$
parts	$-\frac{a^2\sqrt{c^2d\,x^2+d}}{dx}+b^2\left(-\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2-cx\sqrt{c^2x^2+1}+1\right)\text{arcsinh}(cx)^2}{(c^2x^2+1)dx}-\frac{2\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)^2c}{\sqrt{c^2x^2+1}\,d}+\frac{2\sqrt{d(c^2x^2+1)}}{\right.}$

```
[In] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a^2/d/x*(c^2*d*x^2+d)^(1/2)+b^2*(-(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*
x^2+1)^(1/2)+1)*arcsinh(c*x)^2/(c^2*x^2+1)/d/x-2*(d*(c^2*x^2+1))^(1/2)/(c^2
*x^2+1)^(1/2)/d*arcsinh(c*x)^2*c+2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/
d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c+2*(d*(c^2*x^2+1))^(1/2)/(c^2*x
```

```
^2+1)^(1/2)/d*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c+2*(d*(c^2*x^2+1))^(1/2)/(
c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c+2*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c)+2*a*b*(-
2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*c-(d*(c^2*x^2+1))^
(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*arcsinh(c*x)/(c^2*x^2+1)/d/x+(d*(c^
2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c)
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\sqrt{d+c^2dx^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{c^2dx^2+d}x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^2*d*x^4 + d*x^2), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\sqrt{d+c^2dx^2}}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))^2}{x^2\sqrt{d\,(c^2x^2+1)}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x**2*sqrt(d*(c**2*x**2 + 1))), x)
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\sqrt{d+c^2dx^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{c^2dx^2+d}x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] -((-1)^(2*c^2*d*x^2 + 2*d)*sqrt(d)*log(2*c^2*d + 2*d/x^2) - sqrt(d)*log(x^2
 + 1/c^2))*a*b*c/d + b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/(sqrt(c^2
*d*x^2 + d)*x^2), x) - 2*sqrt(c^2*d*x^2 + d)*a*b*arcsinh(c*x)/(d*x) - sqrt(
c^2*d*x^2 + d)*a^2/(d*x)
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\sqrt{d+c^2dx^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{c^2dx^2+d}x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/(sqrt(c^2*d*x^2 + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\sqrt{d+c^2dx^2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2}{x^2\,\sqrt{d\,c^2\,x^2+d}}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^(1/2)),x)
[Out] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^(1/2)), x)
```

3.298 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 360

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\sqrt{d+c^2dx^2}}\,dx =& -\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{x\sqrt{d+c^2dx^2}} \\ &- \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2dx^2} \\ &+ \frac{c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} \\ &- \frac{b^2c^2\sqrt{1+c^2x^2}\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{\sqrt{d+c^2dx^2}} \\ &+ \frac{bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} \\ &- \frac{bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} \\ &- \frac{b^2c^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} \\ &+ \frac{b^2c^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}}\end{aligned}$$

```
[Out] -b*c*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/x/(c^2*d*x^2+d)^(1/2)+c^2*(a+b*ar
csinh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d
)^(1/2)-b^2*c^2*arctanh((c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^
(1/2)+b*c^2*(a+b*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1
)^(1/2)/(c^2*d*x^2+d)^(1/2)-b*c^2*(a+b*arcsinh(c*x))*polylog(2,c*x+(c^2*x^2
+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-b^2*c^2*polylog(3,-c*x-(c^
2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+b^2*c^2*polylog(3,c*x
```

```
+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-1/2*(a+b*arcsinh(
c*x))^2*(c^2*d*x^2+d)^(1/2)/d/x^2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5809, 5816, 4267, 2611, 2320, 6724, 5776, 272, 65, 214}

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\sqrt{d+c^2dx^2}}\,dx &= \frac{c^2\sqrt{c^2x^2+1}\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}} \\ &+ \frac{bc^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}} \\ &- \frac{bc^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}} \\ &- \frac{bc\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x\sqrt{c^2dx^2+d}} \\ &- \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2dx^2} \\ &- \frac{b^2c^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{c^2dx^2+d}} \\ &+ \frac{b^2c^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{\sqrt{c^2dx^2+d}} \\ &- \frac{b^2c^2\sqrt{c^2x^2+1}\text{arctanh}\left(\sqrt{c^2x^2+1}\right)}{\sqrt{c^2dx^2+d}}\end{aligned}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^3*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] -((b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(x*Sqrt[d + c^2*d*x^2])) - (
Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*d*x^2) + (c^2*Sqrt[1 + c^2*x
^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] - (
b^2*c^2*Sqrt[1 + c^2*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/Sqrt[d + c^2*d*x^2] +
 (b*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])
/Sqrt[d + c^2*d*x^2] - (b*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLo
g[2, E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] - (b^2*c^2*Sqrt[1 + c^2*x^2]*Poly
Log[3, -E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] + (b^2*c^2*Sqrt[1 + c^2*x^2]*P
olyLog[3, E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2dx^2}-\frac{1}{2}c^2\int\frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{d+c^2dx^2}}\,dx\\
&\quad+\frac{\left(bc\sqrt{1+c^2x^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{x^2}\,dx}{\sqrt{d+c^2dx^2}}\\
&=-\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{x\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2dx^2}\\
&\quad-\frac{\left(c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)^2\text{csch}(x)\,dx,x,\text{arcsinh}(cx)\right)}{2\sqrt{d+c^2dx^2}}\\
&\quad+\frac{\left(b^2c^2\sqrt{1+c^2x^2}\right)\int\frac{1}{x\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}
\end{aligned}$$

$$= -\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2dx^2} + \frac{c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} + \frac{\left(bc^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\log\left(1-e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{\sqrt{d+c^2dx^2}} - \frac{\left(bc^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\log\left(1+e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{\sqrt{d+c^2dx^2}} + \frac{\left(b^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+c^2x}}\,dx,x,x^2\right)}{2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2dx^2} + \frac{c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} + \frac{bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} + \frac{\left(b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx,x,\sqrt{1+c^2x^2}\right)}{\sqrt{d+c^2dx^2}} - \frac{\left(b^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\text{PolyLog}\left(2,-e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{\sqrt{d+c^2dx^2}} + \frac{\left(b^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\text{PolyLog}\left(2,e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2dx^2} + \frac{c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{b^2c^2\sqrt{1+c^2x^2}\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{\sqrt{d+c^2dx^2}} + \frac{bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{\left(b^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} + \frac{\left(b^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2dx^2} + \frac{c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{b^2c^2\sqrt{1+c^2x^2}\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{\sqrt{d+c^2dx^2}} + \frac{bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{b^2c^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} + \frac{b^2c^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 3.81 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.26

$$\int\frac{(a+b\text{arcsinh}(cx))^2}{x^3\sqrt{d+c^2dx^2}}\,dx = \frac{-\frac{4a^2\sqrt{d+c^2dx^2}}{x^2} - 4a^2c^2\sqrt{d}\log(x) + 4a^2c^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right) + \frac{2abc^2d^2(1+c^2x^2)^{3/2}\left(-2\coth\left(\frac{1}{2}\text{arcsinh}(cx\right.\right.}{}}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^3*Sqrt[d + c^2*d*x^2]),x]
```

[Out] ((-4*a^2*Sqrt[d + c^2*d*x^2])/x^2 - 4*a^2*c^2*Sqrt[d]*Log[x] + 4*a^2*c^2*Sq
rt[d]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (2*a*b*c^2*d^2*(1 + c^2*x^2)^(
3/2)*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 4*Arc
Sinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] + 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c
*x])] - 4*PolyLog[2, -E^(-ArcSinh[c*x])] + 4*PolyLog[2, E^(-ArcSinh[c*x])]
- ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/(d + c^2*d
*x^2)^(3/2) + (b^2*c^2*d^2*(1 + c^2*x^2)^(3/2)*(-4*ArcSinh[c*x]*Coth[ArcSin
h[c*x]/2] - ArcSinh[c*x]^2*Csch[ArcSinh[c*x]/2]^2 - 4*ArcSinh[c*x]^2*Log[1
- E^(-ArcSinh[c*x])] + 4*ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] + 8*Log[
Tanh[ArcSinh[c*x]/2]] - 8*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] + 8*A
rcSinh[c*x]*PolyLog[2, E^(-ArcSinh[c*x])] - 8*PolyLog[3, -E^(-ArcSinh[c*x])
] + 8*PolyLog[3, E^(-ArcSinh[c*x])] - ArcSinh[c*x]^2*Sech[ArcSinh[c*x]/2]^2
 + 4*ArcSinh[c*x]*Tanh[ArcSinh[c*x]/2]))/(d + c^2*d*x^2)^(3/2))/(8*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 779 vs. $2(373) = 746$.

Time = 0.31 (sec) , antiderivative size = 780, normalized size of antiderivative = 2.17

method	result
default	$-\frac{a^{2} \sqrt{c^{2} d \,x^{2}+d}}{2 d \,x^{2}}+\frac{a^{2} c^{2} \ln \left(\frac{2 d +2 \sqrt{d}\, \sqrt{c^{2} d \,x^{2}+d}}{x}\right)}{2 \sqrt{d}}+b^{2} \left(-\frac{\left(\operatorname{arcsinh}\left(c x \right) c^{2} x^{2}+2 c x \sqrt{c^{2} x^{2}+1}+\operatorname{arcsinh}\left(c x \right)\right) \operatorname{arcsinh}\left(c x \right) \sqrt{d \left(c^{2} x^{2}+1\right.}}{2 x^{2} d \left(c^{2} x^{2}+1\right)}\right.$
parts	$-\frac{a^{2} \sqrt{c^{2} d \,x^{2}+d}}{2 d \,x^{2}}+\frac{a^{2} c^{2} \ln \left(\frac{2 d +2 \sqrt{d}\, \sqrt{c^{2} d \,x^{2}+d}}{x}\right)}{2 \sqrt{d}}+b^{2} \left(-\frac{\left(\operatorname{arcsinh}\left(c x \right) c^{2} x^{2}+2 c x \sqrt{c^{2} x^{2}+1}+\operatorname{arcsinh}\left(c x \right)\right) \operatorname{arcsinh}\left(c x \right) \sqrt{d \left(c^{2} x^{2}+1\right.}}{2 x^{2} d \left(c^{2} x^{2}+1\right)}\right.$

[In] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/2*a^2/d/x^2*(c^2*d*x^2+d)^(1/2)+1/2*a^2*c^2/d^(1/2)*ln((2*d+2*d^(1/2)*(c
^2*d*x^2+d)^(1/2))/x)+b^2*(-1/2*(arcsinh(c*x)*c^2*x^2+2*c*x*(c^2*x^2+1)^(1/
2)+arcsinh(c*x))*arcsinh(c*x)*(d*(c^2*x^2+1))^(1/2)/x^2/d/(c^2*x^2+1)+1/2*(
d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1
)^(1/2))*c^2+(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*polylog
(2,-c*x-(c^2*x^2+1)^(1/2))*c^2-(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*po
lylog(3,-c*x-(c^2*x^2+1)^(1/2))*c^2-1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(
1/2)/d*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^2-(d*(c^2*x^2+1))^(1/2)
/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^2+(d*(
c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(3,c*x+(c^2*x^2+1)^(1/2))*c^2-
2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arctanh(c*x+(c^2*x^2+1)^(1/2))*
c^2)+2*a*b*(-1/2*(arcsinh(c*x)*c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x))*
(d*(c^2*x^2+1))^(1/2)/x^2/d/(c^2*x^2+1)-1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+
1)^(1/2)/d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^2-1/2*(d*(c^2*x^2+1))
^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^2+1/2*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))

```
*c^2+1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,-c*x-(c^2*x^2+
1)^(1/2))*c^2)
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\sqrt{d+c^2dx^2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{c^2dx^2+dx^3}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^2*d*x^5 + d*x^3), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\sqrt{d+c^2dx^2}}\,dx=\int \frac{(a+b\,\text{asinh}\,(cx))^2}{x^3\sqrt{d\,(c^2x^2+1)}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x**3*sqrt(d*(c**2*x**2 + 1))), x)
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\sqrt{d+c^2dx^2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{c^2dx^2+dx^3}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/2*(c^2*arcsinh(1/(c*abs(x)))/sqrt(d) - sqrt(c^2*d*x^2 + d)/(d*x^2))*a^2 +
 integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(sqrt(c^2*d*x^2 + d)*x^3) + 2
*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(c^2*d*x^2 + d)*x^3), x)
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\sqrt{d+c^2dx^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{c^2dx^2+dx^3}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/(sqrt(c^2*d*x^2 + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\sqrt{d+c^2dx^2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2}{x^3\,\sqrt{d\,c^2\,x^2+d}}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^(1/2)),x)
[Out] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^(1/2)), x)
```

3.299 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 299

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\sqrt{d+c^2dx^2}}\,dx =& -\frac{b^2c^2(1+c^2x^2)}{3x\sqrt{d+c^2dx^2}}-\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3x^2\sqrt{d+c^2dx^2}}\\&-\frac{2c^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3\sqrt{d+c^2dx^2}}\\&-\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3dx^3}\\&+\frac{2c^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3dx}\\&-\frac{4bc^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)}{3\sqrt{d+c^2dx^2}}\\&+\frac{2b^2c^3\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)}{3\sqrt{d+c^2dx^2}}\end{aligned}$$

```
[Out] -1/3*b^2*c^2*(c^2*x^2+1)/x/(c^2*d*x^2+d)^(1/2)-1/3*b*c*(a+b*arcsinh(c*x))*(
c^2*x^2+1)^(1/2)/x^2/(c^2*d*x^2+d)^(1/2)-2/3*c^3*(a+b*arcsinh(c*x))^2*(c^2*
x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-4/3*b*c^3*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(
c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+2/3*b^2*c^3*poly
log(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-1/
3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/d/x^3+2/3*c^2*(a+b*arcsinh(c*x))
^2*(c^2*d*x^2+d)^(1/2)/d/x
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {5809, 5800, 5775, 3797, 2221, 2317, 2438, 5776, 270}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\sqrt{d+c^2dx^2}}\,dx = \frac{2c^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3dx} - \frac{bc\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3x^2\sqrt{c^2dx^2+d}} - \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3dx^3} - \frac{2c^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{3\sqrt{c^2dx^2+d}} - \frac{4bc^3\sqrt{c^2x^2+1}\log\left(1-e^{-2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3\sqrt{c^2dx^2+d}} + \frac{2b^2c^3\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)}{3\sqrt{c^2dx^2+d}} - \frac{b^2c^2(c^2x^2+1)}{3x\sqrt{c^2dx^2+d}}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^4*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] -1/3*(b^2*c^2*(1 + c^2*x^2))/(x*Sqrt[d + c^2*d*x^2]) - (b*c*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x]))/(3*x^2*Sqrt[d + c^2*d*x^2]) - (2*c^3*Sqrt[1 + c^2*
x^2]*(a + b*ArcSinh[c*x])^2)/(3*Sqrt[d + c^2*d*x^2]) - (Sqrt[d + c^2*d*x^2]
*(a + b*ArcSinh[c*x])^2)/(3*d*x^3) + (2*c^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcS
inh[c*x])^2)/(3*d*x) - (4*b*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[
1 - E^(-2*ArcSinh[c*x])])/(3*Sqrt[d + c^2*d*x^2]) + (2*b^2*c^3*Sqrt[1 + c^2
*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/(3*Sqrt[d + c^2*d*x^2])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
```

```
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3dx^3} - \frac{1}{3}(2c^2)\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\sqrt{d+c^2dx^2}}\,dx \\
&\quad + \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{a+b\text{arcsinh}(cx)}{x^3}\,dx}{3\sqrt{d+c^2dx^2}} \\
&= -\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3x^2\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3dx^3} \\
&\quad + \frac{2c^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3dx} + \frac{\left(b^2c^2\sqrt{1+c^2x^2}\right)\int \frac{1}{x^2\sqrt{1+c^2x^2}}\,dx}{3\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\left(4bc^3\sqrt{1+c^2x^2}\right)\int \frac{a+b\text{arcsinh}(cx)}{x}\,dx}{3\sqrt{d+c^2dx^2}} \\
&= -\frac{b^2c^2(1+c^2x^2)}{3x\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3x^2\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3dx^3} + \frac{2c^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3dx} \\
&\quad + \frac{\left(4c^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int x\coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)}{3\sqrt{d+c^2dx^2}} \\
&= -\frac{b^2c^2(1+c^2x^2)}{3x\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3x^2\sqrt{d+c^2dx^2}} \\
&\quad - \frac{2c^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3dx^3} \\
&\quad + \frac{2c^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3dx} \\
&\quad - \frac{\left(8c^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{3\sqrt{d+c^2dx^2}}
\end{aligned}$$

$$= -\frac{b^2c^2(1+c^2x^2)}{3x\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3x^2\sqrt{d+c^2dx^2}} - \frac{2c^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{3\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{3dx^3} + \frac{2c^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{3dx} - \frac{4bc^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\log\left(1-e^{-2\operatorname{arcsinh}(cx)}\right)}{3\sqrt{d+c^2dx^2}} + \frac{\left(4bc^3\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)dx, x, a+b\operatorname{arcsinh}(cx)\right)}{3\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2c^2(1+c^2x^2)}{3x\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3x^2\sqrt{d+c^2dx^2}} - \frac{2c^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{3\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{3dx^3} + \frac{2c^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{3dx} - \frac{4bc^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\log\left(1-e^{-2\operatorname{arcsinh}(cx)}\right)}{3\sqrt{d+c^2dx^2}} - \frac{\left(2b^2c^3\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}\right)}{3\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2c^2(1+c^2x^2)}{3x\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3x^2\sqrt{d+c^2dx^2}} - \frac{2c^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{3\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{3dx^3} + \frac{2c^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{3dx} - \frac{4bc^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\log\left(1-e^{-2\operatorname{arcsinh}(cx)}\right)}{3\sqrt{d+c^2dx^2}} + \frac{2b^2c^3\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}\right)}{3\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\sqrt{d+c^2dx^2}}\,dx$$
$$= \frac{-a^2+a^2c^2x^2-b^2c^2x^2+2a^2c^4x^4-b^2c^4x^4-abcx\sqrt{1+c^2x^2}+b^2\left(-1+c^2x^2+2c^4x^4-2c^3x^3\sqrt{1+c^2x^2}\right)\text{a}}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^4*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (-a^2 + a^2*c^2*x^2 - b^2*c^2*x^2 + 2*a^2*c^4*x^4 - b^2*c^4*x^4 - a*b*c*x*S
qrt[1 + c^2*x^2] + b^2*(-1 + c^2*x^2 + 2*c^4*x^4 - 2*c^3*x^3*Sqrt[1 + c^2*x
^2])*ArcSinh[c*x]^2 - b*ArcSinh[c*x]*(b*c*x*Sqrt[1 + c^2*x^2] - 2*a*(-1 + c
^2*x^2 + 2*c^4*x^4) + 4*b*c^3*x^3*Sqrt[1 + c^2*x^2]*Log[1 - E^(-2*ArcSinh[c
*x])]) - 4*a*b*c^3*x^3*Sqrt[1 + c^2*x^2]*Log[c*x] + 2*b^2*c^3*x^3*Sqrt[1 +
c^2*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/(3*x^3*Sqrt[d + c^2*d*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1505 vs. 2(281) = 562.

Time = 0.31 (sec) , antiderivative size = 1506, normalized size of antiderivative = 5.04

method	result	size
default	Expression too large to display	1506
parts	Expression too large to display	1506

```
[In] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/3/d/x^3*(c^2*d*x^2+d)^(1/2)+2/3*c^2/d/x*(c^2*d*x^2+d)^(1/2))+b^2*(d
*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^2*(c^2*x^2+1)^(1/2)*c^5+1/3
*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x*arcsinh(c*x)^2*c^4-2
/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x*arcsinh(c*x)*c^4-4
/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d/x*arcsinh(c*x)^2*c^2
+4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)^2*c^3-1/3*b
^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*c^3*(c^2*x^2+1)^(1/2)+2/
3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^5*c^8-1/3*b^2*(d*(c
^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^3*c^6-2/3*b^2*(d*(c^2*x^2+1))^
(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x*c^4+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*
x^4+2*c^2*x^2-1)/d/x*c^2+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2
-1)/d/x^3*arcsinh(c*x)^2-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*
polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^3-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2
+1)^(1/2)/d*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^3+2/3*b^2*(d*(c^2*x^2+1))^(
1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^3-b^2*(d*
```

```
(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*(c^2*x^2+1)^(1/2)*arcsinh(c*x)
*c^3+2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^3*arcsinh(c*x)
^2*c^6+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^3*arcsinh(
c*x)*c^6+1/3*a*b*(d*(c^2*x^2+1))^(1/2)*(4*arcsinh(c*x)*c^3*x^3-4*ln((c*x+(c
^2*x^2+1)^(1/2))^2-1)*x^3*c^3+4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2-2*ar
csinh(c*x)*(c^2*x^2+1)^(1/2)-c*x)/(c^2*x^2+1)^(1/2)/d/x^3-4/3*b^2*(d*(c^2*x
^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^3*(c^2*x^2+1)*arcsinh(c*x)*c^6-2*b
^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^2*(c^2*x^2+1)^(1/2)*ar
csinh(c*x)^2*c^5+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x*
(c^2*x^2+1)*arcsinh(c*x)*c^4+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2
*x^2-1)/d/x^2*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c+4/3*b^2*(d*(c^2*x^2+1))^(1/2
)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^5*arcsinh(c*x)*c^8-2/3*b^2*(d*(c^2*x^2+1))^(1
/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^3*(c^2*x^2+1)*c^6-4/3*b^2*(d*(c^2*x^2+1))^(
1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^3-4/3*b
^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2
+1)^(1/2))*c^3
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\sqrt{d+c^2dx^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{c^2dx^2+dx^4}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^2*d*x^6 + d*x^4), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\sqrt{d+c^2dx^2}}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))^2}{x^4\sqrt{d\,(c^2x^2+1)}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x**4*sqrt(d*(c**2*x**2 + 1))), x)
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\sqrt{d+c^2dx^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{c^2dx^2+dx^4}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/3*(4*c^2*log(x)/sqrt(d) + 1/(sqrt(d)*x^2))*a*b*c + 2/3*a*b*(2*sqrt(c^2*d
*x^2 + d)*c^2/(d*x) - sqrt(c^2*d*x^2 + d)/(d*x^3))*arcsinh(c*x) + 1/3*a^2*(
2*sqrt(c^2*d*x^2 + d)*c^2/(d*x) - sqrt(c^2*d*x^2 + d)/(d*x^3)) + b^2*integr
ate(log(c*x + sqrt(c^2*x^2 + 1))^2/(sqrt(c^2*d*x^2 + d)*x^4), x)
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\sqrt{d+c^2dx^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{c^2dx^2+dx^4}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(sqrt(c^2*d*x^2 + d)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\sqrt{d+c^2dx^2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2}{x^4\,\sqrt{d\,c^2\,x^2+d}}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^(1/2)), x)
```

3.300 $\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 515

$$\begin{aligned}
\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx &= \frac{16abx\sqrt{1+c^2x^2}}{3c^5d\sqrt{d+c^2dx^2}} \\
&-\frac{32b^2(1+c^2x^2)}{9c^6d\sqrt{d+c^2dx^2}}+\frac{2b^2(1+c^2x^2)^2}{27c^6d\sqrt{d+c^2dx^2}} \\
&+\frac{16b^2x\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{3c^5d\sqrt{d+c^2dx^2}}-\frac{2bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c^5d\sqrt{d+c^2dx^2}} \\
&-\frac{2bx^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^3d\sqrt{d+c^2dx^2}}-\frac{x^4(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}} \\
&-\frac{8\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^6d^2}+\frac{4x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^4d^2} \\
&+\frac{4b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^6d\sqrt{d+c^2dx^2}} \\
&-\frac{2ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^6d\sqrt{d+c^2dx^2}} \\
&+\frac{2ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^6d\sqrt{d+c^2dx^2}}
\end{aligned}$$

```
[Out] -32/9*b^2*(c^2*x^2+1)/c^6/d/(c^2*d*x^2+d)^(1/2)+2/27*b^2*(c^2*x^2+1)^2/c^6/
d/(c^2*d*x^2+d)^(1/2)-x^4*(a+b*arcsinh(c*x))^2/c^2/d/(c^2*d*x^2+d)^(1/2)+16
/3*a*b*x*(c^2*x^2+1)^(1/2)/c^5/d/(c^2*d*x^2+d)^(1/2)+16/3*b^2*x*arcsinh(c*x
)*(c^2*x^2+1)^(1/2)/c^5/d/(c^2*d*x^2+d)^(1/2)-2*b*x*(a+b*arcsinh(c*x))*(c^2
*x^2+1)^(1/2)/c^5/d/(c^2*d*x^2+d)^(1/2)-2/9*b*x^3*(a+b*arcsinh(c*x))*(c^2*x
^2+1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)+4*b*(a+b*arcsinh(c*x))*arctan(c*x+(c^
2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/c^6/d/(c^2*d*x^2+d)^(1/2)-2*I*b^2*polylog
```

```
(2,-I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^6/d/(c^2*d*x^2+d)^(1/2)+
2*I*b^2*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^6/d/(c^2*d
*x^2+d)^(1/2)-8/3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^6/d^2+4/3*x^2*
(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^4/d^2
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5810, 5812, 5798, 5772, 267, 5776, 272, 45, 5789, 4265, 2317, 2438}

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \frac{4b\sqrt{c^2x^2+1}\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c^6d\sqrt{c^2dx^2+d}} - \frac{x^4(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}} - \frac{8\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^6d^2} - \frac{2bx\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^5d\sqrt{c^2dx^2+d}} + \frac{4x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^4d^2} - \frac{2bx^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{9c^3d\sqrt{c^2dx^2+d}} + \frac{16abx\sqrt{c^2x^2+1}}{3c^5d\sqrt{c^2dx^2+d}} - \frac{2ib^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^6d\sqrt{c^2dx^2+d}} + \frac{2ib^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^6d\sqrt{c^2dx^2+d}} + \frac{16b^2x\sqrt{c^2x^2+1}\text{arcsinh}(cx)}{3c^5d\sqrt{c^2dx^2+d}} + \frac{2b^2(c^2x^2+1)^2}{27c^6d\sqrt{c^2dx^2+d}} - \frac{32b^2(c^2x^2+1)}{9c^6d\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x^5*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (16*a*b*x*Sqrt[1 + c^2*x^2])/(3*c^5*d*Sqrt[d + c^2*d*x^2]) - (32*b^2*(1 + c
^2*x^2))/(9*c^6*d*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*x^2)^2)/(27*c^6*d*
Sqrt[d + c^2*d*x^2]) + (16*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(3*c^5*d*S
qrt[d + c^2*d*x^2]) - (2*b*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(c^5*d
*Sqrt[d + c^2*d*x^2]) - (2*b*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9
*c^3*d*Sqrt[d + c^2*d*x^2]) - (x^4*(a + b*ArcSinh[c*x])^2)/(c^2*d*Sqrt[d +
c^2*d*x^2]) - (8*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^6*d^2) +
(4*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^4*d^2) + (4*b*Sqrt[
1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^6*d*Sqrt[d + c
^2*d*x^2]) - ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/
(c^6*d*Sqrt[d + c^2*d*x^2]) + ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^A
rcSinh[c*x]])/(c^6*d*Sqrt[d + c^2*d*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
```

```
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x^4(a+b\operatorname{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}}+\frac{4\int\frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx}{c^2d}\\&\quad+\frac{\left(2b\sqrt{1+c^2x^2}\right)\int\frac{x^4(a+b\operatorname{arcsinh}(cx))}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}}\end{aligned}$$

$$= \frac{2bx^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3c^3d\sqrt{d+c^2dx^2}} - \frac{x^4(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{4x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^4d^2} - \frac{8\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx}{3c^4d} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{x^2(a+b\text{arcsinh}(cx))}{1+c^2x^2}\,dx}{c^3d\sqrt{d+c^2dx^2}} - \frac{\left(8b\sqrt{1+c^2x^2}\right)\int x^2(a+b\text{arcsinh}(cx))\,dx}{3c^3d\sqrt{d+c^2dx^2}} - \frac{\left(2b^2\sqrt{1+c^2x^2}\right)\int \frac{x^3}{\sqrt{1+c^2x^2}}\,dx}{3c^2d\sqrt{d+c^2dx^2}}$$

$$= -\frac{2bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c^5d\sqrt{d+c^2dx^2}} - \frac{2bx^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^3d\sqrt{d+c^2dx^2}} - \frac{x^4(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}} - \frac{8\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^6d^2} + \frac{4x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^4d^2} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{a+b\text{arcsinh}(cx)}{1+c^2x^2}\,dx}{c^5d\sqrt{d+c^2dx^2}} + \frac{\left(16b\sqrt{1+c^2x^2}\right)\int (a+b\text{arcsinh}(cx))\,dx}{3c^5d\sqrt{d+c^2dx^2}} + \frac{\left(2b^2\sqrt{1+c^2x^2}\right)\int \frac{x}{\sqrt{1+c^2x^2}}\,dx}{c^4d\sqrt{d+c^2dx^2}} - \frac{\left(b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1+c^2x}}\,dx, x, x^2\right)}{3c^2d\sqrt{d+c^2dx^2}} + \frac{\left(8b^2\sqrt{1+c^2x^2}\right)\int \frac{x^3}{\sqrt{1+c^2x^2}}\,dx}{9c^2d\sqrt{d+c^2dx^2}}$$

$$= \frac{16abx\sqrt{1+c^2x^2}}{3c^5d\sqrt{d+c^2dx^2}} + \frac{2b^2(1+c^2x^2)}{c^6d\sqrt{d+c^2dx^2}} - \frac{2bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c^5d\sqrt{d+c^2dx^2}} - \frac{2bx^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^3d\sqrt{d+c^2dx^2}} - \frac{x^4(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}} - \frac{8\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^6d^2} + \frac{4x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^4d^2} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}(\int (a+bx)\text{sech}(x)\,dx, x, \text{arcsinh}(cx))}{c^6d\sqrt{d+c^2dx^2}} + \frac{\left(16b^2\sqrt{1+c^2x^2}\right)\int \text{arcsinh}(cx)\,dx}{3c^5d\sqrt{d+c^2dx^2}} - \frac{\left(b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \left(-\frac{1}{c^2\sqrt{1+c^2x}} + \frac{\sqrt{1+c^2x}}{c^2}\right)\,dx, x, x^2\right)}{3c^2d\sqrt{d+c^2dx^2}} + \frac{\left(4b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1+c^2x}}\,dx, x, x^2\right)}{9c^2d\sqrt{d+c^2dx^2}}$$

$$= \frac{16abx\sqrt{1+c^2x^2}}{3c^5d\sqrt{d+c^2dx^2}} + \frac{8b^2(1+c^2x^2)}{3c^6d\sqrt{d+c^2dx^2}} - \frac{2b^2(1+c^2x^2)^2}{9c^6d\sqrt{d+c^2dx^2}} + \frac{16b^2x\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{3c^5d\sqrt{d+c^2dx^2}} - \frac{2bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c^5d\sqrt{d+c^2dx^2}} - \frac{2bx^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^3d\sqrt{d+c^2dx^2}} - \frac{x^4(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}} - \frac{8\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^6d^2} + \frac{4x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^4d^2} + \frac{4b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^6d\sqrt{d+c^2dx^2}} - \frac{\left(2ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1-ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{c^6d\sqrt{d+c^2dx^2}} + \frac{\left(2ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1+ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{c^6d\sqrt{d+c^2dx^2}} - \frac{\left(16b^2\sqrt{1+c^2x^2}\right)\int\frac{x}{\sqrt{1+c^2x^2}}\,dx}{3c^4d\sqrt{d+c^2dx^2}} + \frac{\left(4b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\left(-\frac{1}{c^2\sqrt{1+c^2x}}+\frac{\sqrt{1+c^2x}}{c^2}\right)\,dx,x,x^2\right)}{9c^2d\sqrt{d+c^2dx^2}}$$

$$= \frac{16abx\sqrt{1+c^2x^2}}{3c^5d\sqrt{d+c^2dx^2}} - \frac{32b^2(1+c^2x^2)}{9c^6d\sqrt{d+c^2dx^2}} + \frac{2b^2(1+c^2x^2)^2}{27c^6d\sqrt{d+c^2dx^2}} + \frac{16b^2x\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{3c^5d\sqrt{d+c^2dx^2}} - \frac{2bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c^5d\sqrt{d+c^2dx^2}} - \frac{2bx^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^3d\sqrt{d+c^2dx^2}} - \frac{x^4(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}} - \frac{8\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^6d^2} + \frac{4x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^4d^2} + \frac{4b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^6d\sqrt{d+c^2dx^2}} - \frac{\left(2ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{c^6d\sqrt{d+c^2dx^2}} + \frac{\left(2ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{c^6d\sqrt{d+c^2dx^2}}$$

$$= \frac{16abx\sqrt{1+c^2x^2}}{3c^5d\sqrt{d+c^2dx^2}} - \frac{32b^2(1+c^2x^2)}{9c^6d\sqrt{d+c^2dx^2}} + \frac{2b^2(1+c^2x^2)^2}{27c^6d\sqrt{d+c^2dx^2}}$$
$$+ \frac{16b^2x\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{3c^5d\sqrt{d+c^2dx^2}} - \frac{2bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c^5d\sqrt{d+c^2dx^2}}$$
$$- \frac{2bx^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^3d\sqrt{d+c^2dx^2}} - \frac{x^4(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}}$$
$$- \frac{8\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^6d^2} + \frac{4x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^4d^2}$$
$$+ \frac{4b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^6d\sqrt{d+c^2dx^2}}$$
$$- \frac{2ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^6d\sqrt{d+c^2dx^2}}$$
$$+ \frac{2ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^6d\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 427, normalized size of antiderivative = 0.83

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \frac{-72a^2-94b^2-36a^2c^2x^2-92b^2c^2x^2+9a^2c^4x^4+2b^2c^4x^4+90abcx\sqrt{1+c^2x^2}}{}$$

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (-72*a^2 - 94*b^2 - 36*a^2*c^2*x^2 - 92*b^2*c^2*x^2 + 9*a^2*c^4*x^4 + 2*b^2
*c^4*x^4 + 90*a*b*c*x*Sqrt[1 + c^2*x^2] - 6*a*b*c^3*x^3*Sqrt[1 + c^2*x^2] -
 144*a*b*ArcSinh[c*x] - 72*a*b*c^2*x^2*ArcSinh[c*x] + 18*a*b*c^4*x^4*ArcSin
h[c*x] + 90*b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - 6*b^2*c^3*x^3*Sqrt[1 +
 c^2*x^2]*ArcSinh[c*x] - 72*b^2*ArcSinh[c*x]^2 - 36*b^2*c^2*x^2*ArcSinh[c*x
]^2 + 9*b^2*c^4*x^4*ArcSinh[c*x]^2 + 108*a*b*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[
ArcSinh[c*x]/2]] - (54*I)*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - I/E^Ar
cSinh[c*x]] + (54*I)*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + I/E^ArcSinh
[c*x]] - (54*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[c*x]] + (54
*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]])/(27*c^6*d*Sqrt[d +
c^2*d*x^2])
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.01

method	result
default	$a^2\left(\frac{x^4}{3c^2d\sqrt{c^2d\,x^2+d}}-\frac{4\left(\frac{x^2}{c^2d\sqrt{c^2d\,x^2+d}}+\frac{2}{d\,c^4\sqrt{c^2d\,x^2+d}}\right)}{3c^2}\right)+\frac{b^2\sqrt{d(c^2x^2+1)}\,\left(9\operatorname{arcsinh}(cx)^2x^4c^4-6\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3\right.}{}$
parts	$a^2\left(\frac{x^4}{3c^2d\sqrt{c^2d\,x^2+d}}-\frac{4\left(\frac{x^2}{c^2d\sqrt{c^2d\,x^2+d}}+\frac{2}{d\,c^4\sqrt{c^2d\,x^2+d}}\right)}{3c^2}\right)+\frac{b^2\sqrt{d(c^2x^2+1)}\,\left(9\operatorname{arcsinh}(cx)^2x^4c^4-6\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3\right.}{}$

```
[In] int(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(1/3*x^4/c^2/d/(c^2*d*x^2+d)^(1/2)-4/3/c^2*(x^2/c^2/d/(c^2*d*x^2+d)^(1/
2)+2/d/c^4/(c^2*d*x^2+d)^(1/2)))+1/27*b^2*(d*(c^2*x^2+1))^(1/2)*(9*arcsinh(
c*x)^2*x^4*c^4-6*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3+2*c^4*x^4-36*arcsin
h(c*x)^2*x^2*c^2-54*I*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+I*(c*x+(c^2*x^2+1
)^(1/2)))+54*I*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-I*(c*x+(c^2*x^2+1)^(1/2)
))+90*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)-92*c^2*x^2-54*I*(c^2*x^2+1)^(1/2)*
dilog(1+I*(c*x+(c^2*x^2+1)^(1/2)))+54*I*(c^2*x^2+1)^(1/2)*dilog(1-I*(c*x+(c
^2*x^2+1)^(1/2)))-72*arcsinh(c*x)^2-94)/c^6/d^2/(c^2*x^2+1)+2/9*a*b*(d*(c^2
*x^2+1))^(1/2)*(3*arcsinh(c*x)*c^4*x^4-c^3*x^3*(c^2*x^2+1)^(1/2)-12*arcsinh
(c*x)*c^2*x^2-9*I*(c^2*x^2+1)^(1/2)*ln(c*x+(c^2*x^2+1)^(1/2)-I)+9*I*(c^2*x^
2+1)^(1/2)*ln(c*x+(c^2*x^2+1)^(1/2)+I)+15*c*x*(c^2*x^2+1)^(1/2)-24*arcsinh(
c*x))/c^6/d^2/(c^2*x^2+1)
```

Fricas [F]

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2x^5}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^5*arcsinh(c*x)^2 + 2*a*b*x^5*arcsinh(c*x) + a^2*x^5)*sqrt(c
^2*d*x^2 + d)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{x^5(a+b\operatorname{asinh}(cx))^2}{(d(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**5*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
[Out] Integral(x**5*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2x^5}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
[Out] 1/3*a^2*(x^4/(sqrt(c^2*d*x^2 + d)*c^2*d) - 4*x^2/(sqrt(c^2*d*x^2 + d)*c^4*d
) - 8/(sqrt(c^2*d*x^2 + d)*c^6*d)) + 1/3*(b^2*c^4*sqrt(d)*x^4 - 4*b^2*c^2*s
qrt(d)*x^2 - 8*b^2*sqrt(d))*sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2 + 1))^
2/(c^8*d^2*x^2 + c^6*d^2) + integrate(2/3*((4*b^2*c^3*x^3 + (3*a*b*c^5 - b^
2*c^5)*x^5 + 8*b^2*c*x)*(c^2*x^2 + 1) + (3*b^2*c^4*x^4 + (3*a*b*c^6 - b^2*c
^6)*x^6 + 12*b^2*c^2*x^2 + 8*b^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
 + 1))/(c^10*d^(3/2)*x^5 + 2*c^8*d^(3/2)*x^3 + c^6*d^(3/2)*x + (c^9*d^(3/2)
*x^4 + 2*c^7*d^(3/2)*x^2 + c^5*d^(3/2))*sqrt(c^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{x^5\,(a+b\,\text{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^{3/2}}\,dx$$

```
[In] int((x^5*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2),x)
[Out] int((x^5*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2), x)
```

3.301 $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 400

$$\begin{aligned}\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx &= \frac{b^2x(1+c^2x^2)}{4c^4d\sqrt{d+c^2dx^2}}\\ &-\frac{b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{4c^5d\sqrt{d+c^2dx^2}}-\frac{bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2c^3d\sqrt{d+c^2dx^2}}\\ &-\frac{x^3(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}}+\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{c^5d\sqrt{d+c^2dx^2}}\\ &+\frac{3x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2c^4d^2}-\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{2bc^5d\sqrt{d+c^2dx^2}}\\ &-\frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^5d\sqrt{d+c^2dx^2}}\\ &-\frac{b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c^5d\sqrt{d+c^2dx^2}}\end{aligned}$$

```
[Out] 1/4*b^2*x*(c^2*x^2+1)/c^4/d/(c^2*d*x^2+d)^(1/2)-x^3*(a+b*arcsinh(c*x))^2/c^
2/d/(c^2*d*x^2+d)^(1/2)-1/4*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c^5/d/(c^2*d
*x^2+d)^(1/2)-1/2*b*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3/d/(c^2*d*x
^2+d)^(1/2)+(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/c^5/d/(c^2*d*x^2+d)^(1/2
)-1/2*(a+b*arcsinh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c^5/d/(c^2*d*x^2+d)^(1/2)-2*
b*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c^5/
d/(c^2*d*x^2+d)^(1/2)-b^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)
^(1/2)/c^5/d/(c^2*d*x^2+d)^(1/2)+3/2*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(
1/2)/c^4/d^2
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {5810, 5812, 5783, 5776, 327, 221, 5797, 3799, 2221, 2317, 2438}

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = -\frac{x^3(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}} - \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{2bc^5d\sqrt{c^2dx^2+d}} + \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{c^5d\sqrt{c^2dx^2+d}} - \frac{2b\sqrt{c^2x^2+1}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))}{c^5d\sqrt{c^2dx^2+d}} + \frac{3x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2c^4d^2} - \frac{bx^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^3d\sqrt{c^2dx^2+d}} - \frac{b^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c^5d\sqrt{c^2dx^2+d}} - \frac{b^2\sqrt{c^2x^2+1}\text{arcsinh}(cx)}{4c^5d\sqrt{c^2dx^2+d}} + \frac{b^2x(c^2x^2+1)}{4c^4d\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (b^2*x*(1 + c^2*x^2))/(4*c^4*d*Sqrt[d + c^2*d*x^2]) - (b^2*Sqrt[1 + c^2*x^2
]*ArcSinh[c*x])/(4*c^5*d*Sqrt[d + c^2*d*x^2]) - (b*x^2*Sqrt[1 + c^2*x^2]*(a
 + b*ArcSinh[c*x]))/(2*c^3*d*Sqrt[d + c^2*d*x^2]) - (x^3*(a + b*ArcSinh[c*x
])^2)/(c^2*d*Sqrt[d + c^2*d*x^2]) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])
^2)/(c^5*d*Sqrt[d + c^2*d*x^2]) + (3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
*x])^2)/(2*c^4*d^2) - (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(2*b*c^5*d
*Sqrt[d + c^2*d*x^2]) - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 +
 E^(2*ArcSinh[c*x])])/(c^5*d*Sqrt[d + c^2*d*x^2]) - (b^2*Sqrt[1 + c^2*x^2]*
PolyLog[2, -E^(2*ArcSinh[c*x])])/(c^5*d*Sqrt[d + c^2*d*x^2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^3(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}}+\frac{3\int\frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx}{c^2d} \\
&\quad+\frac{\left(2b\sqrt{1+c^2x^2}\right)\int\frac{x^3(a+b\text{arcsinh}(cx))}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}} \\
&= \frac{bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c^3d\sqrt{d+c^2dx^2}}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}} \\
&\quad+\frac{3x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2c^4d^2}-\frac{3\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx}{2c^4d} \\
&\quad-\frac{\left(2b\sqrt{1+c^2x^2}\right)\int\frac{x(a+b\text{arcsinh}(cx))}{1+c^2x^2}\,dx}{c^3d\sqrt{d+c^2dx^2}} \\
&\quad-\frac{\left(3b\sqrt{1+c^2x^2}\right)\int x(a+b\text{arcsinh}(cx))\,dx}{c^3d\sqrt{d+c^2dx^2}}-\frac{\left(b^2\sqrt{1+c^2x^2}\right)\int\frac{x^2}{\sqrt{1+c^2x^2}}\,dx}{c^2d\sqrt{d+c^2dx^2}}
\end{aligned}$$

$$= -\frac{b^2x(1+c^2x^2)}{2c^4d\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2c^3d\sqrt{d+c^2dx^2}} - \frac{x^3(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{3x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2c^4d^2} - \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{2bc^5d\sqrt{d+c^2dx^2}} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\tanh(x)\,dx,x,\text{arcsinh}(cx)\right)}{c^5d\sqrt{d+c^2dx^2}} + \frac{\left(b^2\sqrt{1+c^2x^2}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{2c^4d\sqrt{d+c^2dx^2}} + \frac{\left(3b^2\sqrt{1+c^2x^2}\right)\int\frac{x^2}{\sqrt{1+c^2x^2}}\,dx}{2c^2d\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2x(1+c^2x^2)}{4c^4d\sqrt{d+c^2dx^2}} + \frac{b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{2c^5d\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2c^3d\sqrt{d+c^2dx^2}} - \frac{x^3(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{c^5d\sqrt{d+c^2dx^2}} + \frac{3x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2c^4d^2} - \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{2bc^5d\sqrt{d+c^2dx^2}} - \frac{\left(4b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx,x,\text{arcsinh}(cx)\right)}{c^5d\sqrt{d+c^2dx^2}} - \frac{\left(3b^2\sqrt{1+c^2x^2}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{4c^4d\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2x(1+c^2x^2)}{4c^4d\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{4c^5d\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2c^3d\sqrt{d+c^2dx^2}} - \frac{x^3(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{c^5d\sqrt{d+c^2dx^2}} + \frac{3x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2c^4d^2} - \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{2bc^5d\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^5d\sqrt{d+c^2dx^2}} + \frac{\left(2b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1+e^{2x}\right)\,dx,x,\text{arcsinh}(cx)\right)}{c^5d\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2x(1+c^2x^2)}{4c^4d\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{4c^5d\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2c^3d\sqrt{d+c^2dx^2}} - \frac{x^3(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{c^5d\sqrt{d+c^2dx^2}} + \frac{3x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2c^4d^2} - \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{2bc^5d\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^5d\sqrt{d+c^2dx^2}} + \frac{\left(b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\text{arcsinh}(cx)}\right)}{c^5d\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2x(1+c^2x^2)}{4c^4d\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{4c^5d\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2c^3d\sqrt{d+c^2dx^2}} - \frac{x^3(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{c^5d\sqrt{d+c^2dx^2}} + \frac{3x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2c^4d^2} - \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{2bc^5d\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^5d\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c^5d\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.72

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \frac{4a^2c\sqrt{d}x(3+c^2x^2) - 12a^2\sqrt{d+c^2dx^2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right) + b^2\sqrt{d}(8c}{}$$

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (4*a^2*c*Sqrt[d]*x*(3 + c^2*x^2) - 12*a^2*Sqrt[d + c^2*d*x^2]*Log[c*d*x + S
qrt[d]*Sqrt[d + c^2*d*x^2]] + b^2*Sqrt[d]*(8*c*x*ArcSinh[c*x]^2 + 8*Sqrt[1
+ c^2*x^2]*PolyLog[2, -E^(-2*ArcSinh[c*x])] + Sqrt[1 + c^2*x^2]*(-4*ArcSinh
[c*x]^3 - 2*ArcSinh[c*x]*(Cosh[2*ArcSinh[c*x]] + 8*Log[1 + E^(-2*ArcSinh[c*
x])]) + 2*ArcSinh[c*x]^2*(-4 + Sinh[2*ArcSinh[c*x]]) + Sinh[2*ArcSinh[c*x]]
)) + 2*a*b*Sqrt[d]*(8*c*x*ArcSinh[c*x] - Sqrt[1 + c^2*x^2]*(6*ArcSinh[c*x]^
2 + Cosh[2*ArcSinh[c*x]] + 4*Log[1 + c^2*x^2] - 2*ArcSinh[c*x]*Sinh[2*ArcSi
nh[c*x]]))))/(8*c^5*d^(3/2)*Sqrt[d + c^2*d*x^2])
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.45

method	result
default	$\frac{a^2x^3}{2c^2d\sqrt{c^2d\,x^2+d}}+\frac{3a^2x}{2c^4d\sqrt{c^2d\,x^2+d}}-\frac{3a^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2c^4d\sqrt{c^2d}}-\frac{b^2\sqrt{d(c^2x^2+1)}\,\left(-2\,\mathrm{arcsinh}(cx)^2\sqrt{c^2x^2+1}\,x^3c^3+2\,\mathrm{arcsin}\right.}{}$
parts	$\frac{a^2x^3}{2c^2d\sqrt{c^2d\,x^2+d}}+\frac{3a^2x}{2c^4d\sqrt{c^2d\,x^2+d}}-\frac{3a^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2c^4d\sqrt{c^2d}}-\frac{b^2\sqrt{d(c^2x^2+1)}\,\left(-2\,\mathrm{arcsinh}(cx)^2\sqrt{c^2x^2+1}\,x^3c^3+2\,\mathrm{arcsin}\right.}{}$

```
[In] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2*x^3/c^2/d/(c^2*d*x^2+d)^(1/2)+3/2*a^2/c^4*x/d/(c^2*d*x^2+d)^(1/2)-3
/2*a^2/c^4/d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-1/
4*b^2/(c^2*x^2+1)^(3/2)*(d*(c^2*x^2+1))^(1/2)*(-2*arcsinh(c*x)^2*(c^2*x^2+1
)^(1/2)*x^3*c^3+2*arcsinh(c*x)*c^4*x^4+2*arcsinh(c*x)^3*x^2*c^2-c^3*x^3*(c^
2*x^2+1)^(1/2)-4*arcsinh(c*x)^2*x^2*c^2+8*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1
)^(1/2))^2)*x^2*c^2-6*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*c*x+3*arcsinh(c*x)*c
^2*x^2+4*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*x^2*c^2+2*arcsinh(c*x)^3-c*x
*(c^2*x^2+1)^(1/2)-4*arcsinh(c*x)^2+8*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1
/2))^2)+arcsinh(c*x)+4*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2))/c^5/d^2-1/4*a
*b/(c^2*x^2+1)^(3/2)*(d*(c^2*x^2+1))^(1/2)*(-4*arcsinh(c*x)*(c^2*x^2+1)^(1/
2)*x^3*c^3+2*c^4*x^4+6*arcsinh(c*x)^2*x^2*c^2-8*arcsinh(c*x)*c^2*x^2+8*ln(1
+(c*x+(c^2*x^2+1)^(1/2))^2)*x^2*c^2-12*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)+3
*c^2*x^2+6*arcsinh(c*x)^2-8*arcsinh(c*x)+8*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+
1)/c^5/d^2
```

Fricas [F]

$$\int \frac{x^4(a+b\mathrm{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2x^4}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)*sqrt(c
^2*d*x^2 + d)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{x^4(a+b\,\text{asinh}\,(cx))^2}{(d\,(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
[Out] Integral(x**4*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^4}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
[Out] 1/2*a^2*(x^3/(sqrt(c^2*d*x^2 + d)*c^2*d) + 3*x/(sqrt(c^2*d*x^2 + d)*c^4*d)
- 3*arcsinh(c*x)/(c^5*d^(3/2))) + integrate(b^2*x^4*log(c*x + sqrt(c^2*x^2
+ 1))^2/(c^2*d*x^2 + d)^(3/2) + 2*a*b*x^4*log(c*x + sqrt(c^2*x^2 + 1))/(c^2
*d*x^2 + d)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{x^4\,\left(a+b\,\text{asinh}(c\,x)\right)^2}{\left(d\,c^2\,x^2+d\right)^{3/2}}\,dx$$

```
[In] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2),x)
[Out] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2), x)
```

3.302 $\int \frac{x^3(a+b\textbf{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 383

$$\begin{aligned}\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx &= -\frac{4abx\sqrt{1+c^2x^2}}{c^3d\sqrt{d+c^2dx^2}}+\frac{2b^2(1+c^2x^2)}{c^4d\sqrt{d+c^2dx^2}}\\ &-\frac{4b^2x\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{c^3d\sqrt{d+c^2dx^2}}+\frac{2bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c^3d\sqrt{d+c^2dx^2}}\\ &-\frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}}+\frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{c^4d^2}\\ &-\frac{4b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^4d\sqrt{d+c^2dx^2}}\\ &+\frac{2ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^4d\sqrt{d+c^2dx^2}}-\frac{2ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^4d\sqrt{d+c^2dx^2}}\end{aligned}$$

```
[Out] 2*b^2*(c^2*x^2+1)/c^4/d/(c^2*d*x^2+d)^(1/2)-x^2*(a+b*arcsinh(c*x))^2/c^2/d/
(c^2*d*x^2+d)^(1/2)-4*a*b*x*(c^2*x^2+1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)-4*b
^2*x*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)+2*b*x*(a+b*ar
csinh(c*x))*(c^2*x^2+1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)-4*b*(a+b*arcsinh(c*
x))*arctan(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/c^4/d/(c^2*d*x^2+d)^(1/
2)+2*I*b^2*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^4/d/(c
^2*d*x^2+d)^(1/2)-2*I*b^2*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^
(1/2)/c^4/d/(c^2*d*x^2+d)^(1/2)+2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/
c^4/d^2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {5810, 5798, 5772, 267, 5812, 5789, 4265, 2317, 2438}

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = -\frac{4b\sqrt{c^2x^2+1}\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c^4d\sqrt{c^2dx^2+d}} - \frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}} + \frac{2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^4d^2} + \frac{2bx\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^3d\sqrt{c^2dx^2+d}} - \frac{4abx\sqrt{c^2x^2+1}}{c^3d\sqrt{c^2dx^2+d}} + \frac{2ib^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^4d\sqrt{c^2dx^2+d}} - \frac{2ib^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^4d\sqrt{c^2dx^2+d}} - \frac{4b^2x\sqrt{c^2x^2+1}\text{arcsinh}(cx)}{c^3d\sqrt{c^2dx^2+d}} + \frac{2b^2(c^2x^2+1)}{c^4d\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (-4*a*b*x*Sqrt[1 + c^2*x^2])/(c^3*d*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*
x^2))/(c^4*d*Sqrt[d + c^2*d*x^2]) - (4*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
)/(c^3*d*Sqrt[d + c^2*d*x^2]) + (2*b*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x
]))/(c^3*d*Sqrt[d + c^2*d*x^2]) - (x^2*(a + b*ArcSinh[c*x])^2)/(c^2*d*Sqrt[
d + c^2*d*x^2]) + (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(c^4*d^2)
- (4*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^4*
d*Sqrt[d + c^2*d*x^2]) + ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^Arc
Sinh[c*x]])/(c^4*d*Sqrt[d + c^2*d*x^2]) - ((2*I)*b^2*Sqrt[1 + c^2*x^2]*Poly
Log[2, I*E^ArcSinh[c*x]])/(c^4*d*Sqrt[d + c^2*d*x^2])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
```

```
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}}+\frac{2\int\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx}{c^2d} \\
&\quad+\frac{\left(2b\sqrt{1+c^2x^2}\right)\int\frac{x^2(a+b\text{arcsinh}(cx))}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}} \\
&= \frac{2bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c^3d\sqrt{d+c^2dx^2}}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}} \\
&\quad+\frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{c^4d^2}-\frac{\left(2b\sqrt{1+c^2x^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{1+c^2x^2}\,dx}{c^3d\sqrt{d+c^2dx^2}} \\
&\quad-\frac{\left(4b\sqrt{1+c^2x^2}\right)\int(a+b\text{arcsinh}(cx))\,dx}{c^3d\sqrt{d+c^2dx^2}}-\frac{\left(2b^2\sqrt{1+c^2x^2}\right)\int\frac{x}{\sqrt{1+c^2x^2}}\,dx}{c^2d\sqrt{d+c^2dx^2}} \\
&= -\frac{4abx\sqrt{1+c^2x^2}}{c^3d\sqrt{d+c^2dx^2}}-\frac{2b^2(1+c^2x^2)}{c^4d\sqrt{d+c^2dx^2}}+\frac{2bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c^3d\sqrt{d+c^2dx^2}} \\
&\quad-\frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}}+\frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{c^4d^2} \\
&\quad-\frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\text{sech}(x)\,dx,x,\text{arcsinh}(cx)\right)}{c^4d\sqrt{d+c^2dx^2}} \\
&\quad-\frac{\left(4b^2\sqrt{1+c^2x^2}\right)\int\text{arcsinh}(cx)\,dx}{c^3d\sqrt{d+c^2dx^2}}
\end{aligned}$$

$$= -\frac{4abx\sqrt{1+c^2x^2}}{c^3d\sqrt{d+c^2dx^2}} - \frac{2b^2(1+c^2x^2)}{c^4d\sqrt{d+c^2dx^2}} - \frac{4b^2x\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{c^3d\sqrt{d+c^2dx^2}} + \frac{2bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c^3d\sqrt{d+c^2dx^2}} - \frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{c^4d^2} - \frac{4b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^4d\sqrt{d+c^2dx^2}} + \frac{\left(2ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1-ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{c^4d\sqrt{d+c^2dx^2}} - \frac{\left(2ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1+ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{c^4d\sqrt{d+c^2dx^2}} + \frac{\left(4b^2\sqrt{1+c^2x^2}\right)\int\frac{x}{\sqrt{1+c^2x^2}}\,dx}{c^2d\sqrt{d+c^2dx^2}}$$

$$= -\frac{4abx\sqrt{1+c^2x^2}}{c^3d\sqrt{d+c^2dx^2}} + \frac{2b^2(1+c^2x^2)}{c^4d\sqrt{d+c^2dx^2}} - \frac{4b^2x\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{c^3d\sqrt{d+c^2dx^2}} + \frac{2bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c^3d\sqrt{d+c^2dx^2}} - \frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{c^4d^2} - \frac{4b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^4d\sqrt{d+c^2dx^2}} + \frac{\left(2ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{c^4d\sqrt{d+c^2dx^2}} - \frac{\left(2ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{c^4d\sqrt{d+c^2dx^2}}$$

$$= -\frac{4abx\sqrt{1+c^2x^2}}{c^3d\sqrt{d+c^2dx^2}} + \frac{2b^2(1+c^2x^2)}{c^4d\sqrt{d+c^2dx^2}} - \frac{4b^2x\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{c^3d\sqrt{d+c^2dx^2}}$$
$$+ \frac{2bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c^3d\sqrt{d+c^2dx^2}}$$
$$- \frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{c^4d^2}$$
$$- \frac{4b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^4d\sqrt{d+c^2dx^2}}$$
$$+ \frac{2ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^4d\sqrt{d+c^2dx^2}}$$
$$- \frac{2ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^4d\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.83

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \frac{2a^2+2b^2+a^2c^2x^2+2b^2c^2x^2-2abcx\sqrt{1+c^2x^2}+4ab\text{arcsinh}(cx)+2abc^2x^2}{}$$

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (2*a^2 + 2*b^2 + a^2*c^2*x^2 + 2*b^2*c^2*x^2 - 2*a*b*c*x*Sqrt[1 + c^2*x^2]
+ 4*a*b*ArcSinh[c*x] + 2*a*b*c^2*x^2*ArcSinh[c*x] - 2*b^2*c*x*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x] + 2*b^2*ArcSinh[c*x]^2 + b^2*c^2*x^2*ArcSinh[c*x]^2 - 4*a
*b*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] + (2*I)*b^2*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] - (2*I)*b^2*Sqrt[1 + c^2*x^2]*A
rcSinh[c*x]*Log[1 + I/E^ArcSinh[c*x]] + (2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog
[2, (-I)/E^ArcSinh[c*x]] - (2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSi
nh[c*x]])/(c^4*d*Sqrt[d + c^2*d*x^2])
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.08

method	result
default	$a^2\left(\frac{x^2}{c^2d\sqrt{c^2d\,x^2+d}}+\frac{2}{d\,c^4\sqrt{c^2d\,x^2+d}}\right)+\frac{b^2\sqrt{d(c^2x^2+1)}\left(\text{arcsinh}(cx)^2x^2c^2+2i\sqrt{c^2x^2+1}\,\text{arcsinh}(cx)\ln\left(1+i\left(cx+\sqrt{c^2x^2+1}\right)\right)\right.}{}$
parts	$a^2\left(\frac{x^2}{c^2d\sqrt{c^2d\,x^2+d}}+\frac{2}{d\,c^4\sqrt{c^2d\,x^2+d}}\right)+\frac{b^2\sqrt{d(c^2x^2+1)}\left(\text{arcsinh}(cx)^2x^2c^2+2i\sqrt{c^2x^2+1}\,\text{arcsinh}(cx)\ln\left(1+i\left(cx+\sqrt{c^2x^2+1}\right)\right)\right.}{}$

```
[In] int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(x^2/c^2/d/(c^2*d*x^2+d)^(1/2)+2/d/c^4/(c^2*d*x^2+d)^(1/2))+b^2*(d*(c^2
*x^2+1))^(1/2)*(arcsinh(c*x)^2*x^2*c^2+2*I*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*l
n(1+I*(c*x+(c^2*x^2+1)^(1/2)))-2*I*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-I*(c
*x+(c^2*x^2+1)^(1/2)))-2*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)+2*c^2*x^2-2*I*(
c^2*x^2+1)^(1/2)*dilog(1-I*(c*x+(c^2*x^2+1)^(1/2)))+2*I*(c^2*x^2+1)^(1/2)*d
ilog(1+I*(c*x+(c^2*x^2+1)^(1/2)))+2*arcsinh(c*x)^2+2)/c^4/d^2/(c^2*x^2+1)+2
*a*b*(d*(c^2*x^2+1))^(1/2)*(arcsinh(c*x)*c^2*x^2+I*(c^2*x^2+1)^(1/2)*ln(c*x
+(c^2*x^2+1)^(1/2)-I)-I*(c^2*x^2+1)^(1/2)*ln(c*x+(c^2*x^2+1)^(1/2)+I)-c*x*(
c^2*x^2+1)^(1/2)+2*arcsinh(c*x))/c^4/d^2/(c^2*x^2+1)
```

Fricas [F]

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^3}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*x) + a^2*x^3)*sqrt(c
^2*d*x^2 + d)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{x^3(a+b\,\text{asinh}\,(cx))^2}{(d\,(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^3}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

```
[Out] -2*a*b*c*(x/(c^4*d^(3/2)) + arctan(c*x)/(c^5*d^(3/2))) + 2*a*b*(x^2/(sqrt(c
^2*d*x^2 + d)*c^2*d) + 2/(sqrt(c^2*d*x^2 + d)*c^4*d))*arcsinh(c*x) + a^2*(x
```

```
^2/(sqrt(c^2*d*x^2 + d)*c^2*d) + 2/(sqrt(c^2*d*x^2 + d)*c^4*d)) + b^2*((c^2
*x^2 + 2)*log(c*x + sqrt(c^2*x^2 + 1))^2/(sqrt(c^2*x^2 + 1)*c^4*d^(3/2)) -
integrate(2*(c^4*x^4 + 3*c^2*x^2 + (c^3*x^3 + 2*c*x)*sqrt(c^2*x^2 + 1) + 2)
*log(c*x + sqrt(c^2*x^2 + 1))/((c^5*d^(3/2)*x^2 + c^3*d^(3/2))*(c^2*x^2 + 1
) + (c^6*d^(3/2)*x^3 + c^4*d^(3/2)*x)*sqrt(c^2*x^2 + 1)), x))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{x^3\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^{3/2}}\,dx$$

```
[In] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2), x)
```

3.303 $\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}} dx$

Optimal result

Integrand size = 28, antiderivative size = 233

$$\begin{aligned}\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = &-\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}}-\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{c^3d\sqrt{d+c^2dx^2}}\\ &+\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc^3d\sqrt{d+c^2dx^2}}+\frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^3d\sqrt{d+c^2dx^2}}\\ &+\frac{b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c^3d\sqrt{d+c^2dx^2}}\end{aligned}$$

```
[Out] -x*(a+b*arcsinh(c*x))^2/c^2/d/(c^2*d*x^2+d)^(1/2)-(a+b*arcsinh(c*x))^2*(c^2
*x^2+1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)+1/3*(a+b*arcsinh(c*x))^3*(c^2*x^2+1
)^(1/2)/b/c^3/d/(c^2*d*x^2+d)^(1/2)+2*b*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x
^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)+b^2*polylog(2,-
(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

= {5810, 5783, 5797, 3799, 2221, 2317, 2438}

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = -\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}} + \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{3bc^3d\sqrt{c^2dx^2+d}} - \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{c^3d\sqrt{c^2dx^2+d}} + \frac{2b\sqrt{c^2x^2+1}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))}{c^3d\sqrt{c^2dx^2+d}} + \frac{b^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c^3d\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((x*(a + b*ArcSinh[c*x])^2)/(c^2*d*Sqrt[d + c^2*d*x^2])) - (Sqrt[1 + c^2*x
^2]*(a + b*ArcSinh[c*x])^2)/(c^3*d*Sqrt[d + c^2*d*x^2]) + (Sqrt[1 + c^2*x^2
]*(a + b*ArcSinh[c*x])^3)/(3*b*c^3*d*Sqrt[d + c^2*d*x^2]) + (2*b*Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c^3*d*Sqrt[d + c
^2*d*x^2]) + (b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c^3*d
*Sqrt[d + c^2*d*x^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}}+\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx}{c^2d} \\
&\quad+\frac{\left(2b\sqrt{1+c^2x^2}\right)\int\frac{x(a+b\text{arcsinh}(cx))}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}} \\
&= -\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}}+\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc^3d\sqrt{d+c^2dx^2}} \\
&\quad+\frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\tanh(x)\,dx,x,\text{arcsinh}(cx)\right)}{c^3d\sqrt{d+c^2dx^2}} \\
&= -\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}}-\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{c^3d\sqrt{d+c^2dx^2}} \\
&\quad+\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc^3d\sqrt{d+c^2dx^2}} \\
&\quad+\frac{\left(4b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx,x,\text{arcsinh}(cx)\right)}{c^3d\sqrt{d+c^2dx^2}}
\end{aligned}
$$

$$= -\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}} - \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{c^3d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc^3d\sqrt{d+c^2dx^2}} + \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^3d\sqrt{d+c^2dx^2}} - \frac{\left(2b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1+e^{2x}\right)\,dx,x,\text{arcsinh}(cx)\right)}{c^3d\sqrt{d+c^2dx^2}}$$

$$= -\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}} - \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{c^3d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc^3d\sqrt{d+c^2dx^2}} + \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^3d\sqrt{d+c^2dx^2}} - \frac{\left(b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\text{arcsinh}(cx)}\right)}{c^3d\sqrt{d+c^2dx^2}}$$

$$= -\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}} - \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{c^3d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc^3d\sqrt{d+c^2dx^2}} + \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^3d\sqrt{d+c^2dx^2}} + \frac{b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c^3d\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.92

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \frac{-3a^2cdx - 3abd\left(2cx\text{arcsinh}(cx) - \sqrt{1+c^2x^2}\left(\text{arcsinh}(cx)^2 + \log\left(1+c^2x^2\right)\right)\right)}{}$$

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (-3*a^2*c*d*x - 3*a*b*d*(2*c*x*ArcSinh[c*x] - Sqrt[1 + c^2*x^2]*(ArcSinh[c*
x]^2 + Log[1 + c^2*x^2])) + 3*a^2*Sqrt[d]*Sqrt[d + c^2*d*x^2]*Log[c*d*x + S
qrt[d]*Sqrt[d + c^2*d*x^2]] + b^2*d*(ArcSinh[c*x]*(-3*c*x*ArcSinh[c*x] + Sq
rt[1 + c^2*x^2]*(ArcSinh[c*x]*(3 + ArcSinh[c*x]) + 6*Log[1 + E^(-2*ArcSinh[
c*x])])) - 3*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-2*ArcSinh[c*x])]))/(3*c^3*d^
2*Sqrt[d + c^2*d*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(231) = 462$.

Time = 0.25 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.05

method	result
default	$-\frac{a^2 x}{c^2 d\sqrt{c^2 d\,x^2+d}}+\frac{a^2 \ln\left(\frac{c^2 dx}{\sqrt{c^2 d}}+\sqrt{c^2 d\,x^2+d}\right)}{c^2 d\sqrt{c^2 d}}+\frac{b^2 \sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(cx)^3}{3\sqrt{c^2x^2+1}\,c^3 d^2}-\frac{b^2 \sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(cx)^2 x}{c^2 d^2 (c^2x^2+1)}-\frac{b^2\sqrt{d(c^2x^2-}}{c^3 d^2}$
parts	$-\frac{a^2 x}{c^2 d\sqrt{c^2 d\,x^2+d}}+\frac{a^2 \ln\left(\frac{c^2 dx}{\sqrt{c^2 d}}+\sqrt{c^2 d\,x^2+d}\right)}{c^2 d\sqrt{c^2 d}}+\frac{b^2 \sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(cx)^3}{3\sqrt{c^2x^2+1}\,c^3 d^2}-\frac{b^2 \sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(cx)^2 x}{c^2 d^2 (c^2x^2+1)}-\frac{b^2\sqrt{d(c^2x^2-}}{c^3 d^2}$

```
[In] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a^2*x/c^2/d/(c^2*d*x^2+d)^(1/2)+a^2/c^2/d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*
x^2+d)^(1/2))/(c^2*d)^(1/2)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)
/c^3/d^2*arcsinh(c*x)^3-b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/c^2/d^2/(c
^2*x^2+1)*x-b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/c^3/d^2/(c^2*x^2+1)^(1
/2)+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d^2*arcsinh(c*x)*ln(1
+(c*x+(c^2*x^2+1)^(1/2))^2)+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3
/d^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x
^2+1)^(1/2)/c^3/d^2*arcsinh(c*x)^2-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)/c^3/d^2*arcsinh(c*x)-2*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/c^2/d^2
/(c^2*x^2+1)*x+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d^2*ln(1+(
c*x+(c^2*x^2+1)^(1/2))^2)
```

Fricas [F]

$$\int \frac{x^2(a+b\mathrm{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^2 x^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)*sqrt(c
^2*d*x^2 + d)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{x^2(a+b\,\text{asinh}\,(cx))^2}{(d\,(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
[Out] Integral(x**2*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^2}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
[Out] -a^2*(x/(sqrt(c^2*d*x^2 + d)*c^2*d) - arcsinh(c*x)/(c^3*d^(3/2))) + integra
te(b^2*x^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^2 + d)^(3/2) + 2*a*b*x^2
*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^2}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2*x^2/(c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{x^2\,(a+b\,\text{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^{3/2}}\,dx$$

```
[In] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2),x)
[Out] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2), x)
```

3.304 $\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 188

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = -\frac{(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{4b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^2d\sqrt{d+c^2dx^2}} - \frac{2ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^2d\sqrt{d+c^2dx^2}} + \frac{2ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^2d\sqrt{d+c^2dx^2}}$$

```
[Out] -(a+b*arcsinh(c*x))^2/c^2/d/(c^2*d*x^2+d)^(1/2)+4*b*(a+b*arcsinh(c*x))*arct
an(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/c^2/d/(c^2*d*x^2+d)^(1/2)-2*I*b
^2*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^2/d/(c^2*d*x^2
+d)^(1/2)+2*I*b^2*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^
2/d/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5798, 5789, 4265, 2317, 2438}

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \frac{4b\sqrt{c^2x^2+1}\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c^2d\sqrt{c^2dx^2+d}} - \frac{(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}} - \frac{2ib^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^2d\sqrt{c^2dx^2+d}} + \frac{2ib^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^2d\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((a + b*ArcSinh[c*x])^2/(c^2*d*Sqrt[d + c^2*d*x^2])) + (4*b*Sqrt[1 + c^2*x
^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^2*d*Sqrt[d + c^2*d*x^2]
) - ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^2*d*Sq
rt[d + c^2*d*x^2]) + ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^ArcSinh[c*
x]])/(c^2*d*Sqrt[d + c^2*d*x^2])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}}$$

$$= -\frac{(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\text{sech}(x)\,dx, x, \text{arcsinh}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}}$$

$$= -\frac{(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{4b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^2d\sqrt{d+c^2dx^2}} - \frac{\left(2ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \log\left(1-ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}} + \frac{\left(2ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \log\left(1+ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}}$$

$$= -\frac{(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{4b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^2d\sqrt{d+c^2dx^2}} - \frac{\left(2ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{c^2d\sqrt{d+c^2dx^2}} + \frac{\left(2ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{c^2d\sqrt{d+c^2dx^2}}$$

$$= -\frac{(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{4b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^2d\sqrt{d+c^2dx^2}} - \frac{2ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2, -ie^{\text{arcsinh}(cx)}\right)}{c^2d\sqrt{d+c^2dx^2}} + \frac{2ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2, ie^{\text{arcsinh}(cx)}\right)}{c^2d\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.15

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = -\frac{a^2 + 2ab\text{arcsinh}(cx) + b^2\text{arcsinh}(cx)^2 - 4ab\sqrt{1+c^2x^2}\arctan\left(\tanh\left(\frac{1}{2}\text{arcsinh}(cx)\right)\right) + 2ib^2\sqrt{1+c^2x^2}\text{arcsin}}{}$$

```
[In] Integrate[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((a^2 + 2*a*b*ArcSinh[c*x] + b^2*ArcSinh[c*x]^2 - 4*a*b*Sqrt[1 + c^2*x^2]*
ArcTan[Tanh[ArcSinh[c*x]/2]] + (2*I)*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log
[1 - I/E^ArcSinh[c*x]] - (2*I)*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + I
/E^ArcSinh[c*x]] + (2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[c*
x]] - (2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]])/(c^2*d*Sqrt
[d + c^2*d*x^2]))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.69

method	result
default	$-\frac{a^2}{c^2 d\sqrt{c^2 d\,x^2+d}}-\frac{b^2\sqrt{d(c^2x^2+1)}\left(2i\sqrt{c^2x^2+1}\,\text{arcsinh}(cx)\ln\left(1+i\left(cx+\sqrt{c^2x^2+1}\right)\right)-2i\sqrt{c^2x^2+1}\,\text{arcsinh}(cx)\ln\left(1-i\left(cx+\sqrt{c^2}\right.\right.\right.}{c^2d^2(c^2}$
parts	$-\frac{a^2}{c^2 d\sqrt{c^2 d\,x^2+d}}-\frac{b^2\sqrt{d(c^2x^2+1)}\left(2i\sqrt{c^2x^2+1}\,\text{arcsinh}(cx)\ln\left(1+i\left(cx+\sqrt{c^2x^2+1}\right)\right)-2i\sqrt{c^2x^2+1}\,\text{arcsinh}(cx)\ln\left(1-i\left(cx+\sqrt{c^2}\right.\right.\right.}{c^2d^2(c^2}$

```
[In] int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a^2/c^2/d/(c^2*d*x^2+d)^(1/2)-b^2*(d*(c^2*x^2+1))^(1/2)*(2*I*(c^2*x^2+1)^(
1/2)*arcsinh(c*x)*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))-2*I*(c^2*x^2+1)^(1/2)*arc
sinh(c*x)*ln(1-I*(c*x+(c^2*x^2+1)^(1/2)))+2*I*(c^2*x^2+1)^(1/2)*dilog(1+I*(
c*x+(c^2*x^2+1)^(1/2)))-2*I*(c^2*x^2+1)^(1/2)*dilog(1-I*(c*x+(c^2*x^2+1)^(1
/2)))+arcsinh(c*x)^2)/c^2/d^2/(c^2*x^2+1)-2*a*b*(d*(c^2*x^2+1))^(1/2)*(-I*(
c^2*x^2+1)^(1/2)*ln(c*x+(c^2*x^2+1)^(1/2)+I)+I*(c^2*x^2+1)^(1/2)*ln(c*x+(c^
2*x^2+1)^(1/2)-I)+arcsinh(c*x))/c^2/d^2/(c^2*x^2+1)
```

Fricas [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2x}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*x*arcsinh(c*x)^2 + 2*a*b*x*arcsinh(c*x) +
 a^2*x)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{x(a+b\operatorname{asinh}(cx))^2}{(d(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2x}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
[Out] -a^2/(sqrt(c^2*d*x^2 + d)*c^2*d) + integrate(b^2*x*log(c*x + sqrt(c^2*x^2 +
 1))^2/(c^2*d*x^2 + d)^(3/2) + 2*a*b*x*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*
x^2 + d)^(3/2), x)
```

Giac [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2x}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2*x/(c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{x\,(a+b\,\mathrm{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^{3/2}}\,dx$$

```
[In] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2),x)
[Out] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2), x)
```

3.305 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}} dx$

Optimal result

Integrand size = 25, antiderivative size = 179

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}}+\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{cd\sqrt{d+c^2dx^2}}-\frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{cd\sqrt{d+c^2dx^2}}-\frac{b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{cd\sqrt{d+c^2dx^2}}$$

```
[Out] x*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(1/2)+(a+b*arcsinh(c*x))^2*(c^2*x^2+
1)^(1/2)/c/d/(c^2*d*x^2+d)^(1/2)-2*b*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+
1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c/d/(c^2*d*x^2+d)^(1/2)-b^2*polylog(2,-(c*x+
(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c/d/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {5787, 5797, 3799, 2221, 2317, 2438}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{cd\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))}{cd\sqrt{c^2dx^2+d}}-\frac{b^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{cd\sqrt{c^2dx^2+d}}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcSinh[c*x])^2)/(d*Sqrt[d + c^2*d*x^2]) + (Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x])^2)/(c*d*Sqrt[d + c^2*d*x^2]) - (2*b*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c*d*Sqrt[d + c^2*d*x^2]) -
(b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*d*Sqrt[d + c^2*d
*x^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}} - \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{x(a+b\text{arcsinh}(cx))}{1+c^2x^2}\,dx}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \text{arcsinh}(cx)\right)}{cd\sqrt{d+c^2dx^2}}$$

$$= \frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{cd\sqrt{d+c^2dx^2}} - \frac{\left(4b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \text{arcsinh}(cx)\right)}{cd\sqrt{d+c^2dx^2}}$$

$$= \frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{cd\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{cd\sqrt{d+c^2dx^2}} + \frac{\left(2b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{cd\sqrt{d+c^2dx^2}}$$

$$= \frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{cd\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{cd\sqrt{d+c^2dx^2}} + \frac{\left(b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\text{arcsinh}(cx)}\right)}{cd\sqrt{d+c^2dx^2}}$$

$$= \frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{cd\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{cd\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2, -e^{2\text{arcsinh}(cx)}\right)}{cd\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.85

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \frac{-b^2\left(-cx+\sqrt{1+c^2x^2}\right)\text{arcsinh}(cx)^2+2b\text{arcsinh}(cx)\left(acx-b\sqrt{1+c^2x^2}\log\left(1+\right.\right.}{}$$

[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^(3/2),x]

[Out] (-(b^2*(-(c*x) + Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2) + 2*b*ArcSinh[c*x]*(a*c*x - b*Sqrt[1 + c^2*x^2]*Log[1 + E^(-2*ArcSinh[c*x])]) + a*(a*c*x - b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2]) + b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-2*ArcSinh[c*x])])/(c*d*Sqrt[d + c^2*d*x^2])

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.92

method	result
default	$\frac{a^2x}{d\sqrt{c^2d\,x^2+d}}+\frac{b^2\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)^2x}{d^2(c^2x^2+1)}+\frac{b^2\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)^2}{c\,d^2\sqrt{c^2x^2+1}}-\frac{2b^2\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)\ln\left(1+\left(cx+\sqrt{c^2x^2}\right.\right.}{\sqrt{c^2x^2+1}\,c\,d^2}$
parts	$\frac{a^2x}{d\sqrt{c^2d\,x^2+d}}+\frac{b^2\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)^2x}{d^2(c^2x^2+1)}+\frac{b^2\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)^2}{c\,d^2\sqrt{c^2x^2+1}}-\frac{2b^2\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)\ln\left(1+\left(cx+\sqrt{c^2x^2}\right.\right.}{\sqrt{c^2x^2+1}\,c\,d^2}$

[In] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)

[Out] a^2/d*x/(c^2*d*x^2+d)^(1/2)+b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/d^2/(c^2*x^2+1)*x+b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/c/d^2/(c^2*x^2+1)^(1/2)-2*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/c/d^2*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/c/d^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+2*a*b/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/c/d^2*arcsinh(c*x)+2*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d^2/(c^2*x^2+1)*x-2*a*b/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/c/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)

Fricas [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + c^2dx^2)^{3/2}} \, dx = \int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(c^2dx^2 + d)^{\frac{3}{2}}} \, dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + c^2dx^2)^{3/2}} \, dx = \int \frac{(a + b \operatorname{asinh}(cx))^2}{(d (c^2x^2 + 1))^{\frac{3}{2}}} \, dx$$

```
[In] integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
[Out] Integral((a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + c^2dx^2)^{3/2}} \, dx = \int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(c^2dx^2 + d)^{\frac{3}{2}}} \, dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
[Out] b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^2 + d)^(3/2), x) + 2*
a*b*x*arcsinh(c*x)/(sqrt(c^2*d*x^2 + d)*d) + a^2*x/(sqrt(c^2*d*x^2 + d)*d)
- a*b*log(x^2 + 1/c^2)/(c*d^(3/2))
```

Giac [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + c^2dx^2)^{3/2}} \, dx = \int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(c^2dx^2 + d)^{\frac{3}{2}}} \, dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^{3/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(3/2),x)
[Out] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(3/2), x)
```

3.306 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 412

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^{3/2}}\,dx &= \frac{(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}} \\ &- \frac{4b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} \\ &- \frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} \\ &- \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} \\ &+ \frac{2ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} - \frac{2ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} \\ &+ \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} \\ &+ \frac{2b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} - \frac{2b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}\end{aligned}$$

```
[Out] (a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(1/2)-4*b*(a+b*arcsinh(c*x))*arctan(c*
x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-2*(a+b*arcsinh
(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(
1/2)-2*b*(a+b*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(
1/2)/d/(c^2*d*x^2+d)^(1/2)+2*I*b^2*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))*(c
^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-2*I*b^2*polylog(2,I*(c*x+(c^2*x^2+1)^
(1/2)))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+2*b*(a+b*arcsinh(c*x))*poly
log(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+2*b^2*
polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-2
```

```
*b^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/
2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5811, 5816, 4267, 2611, 2320, 6724, 5789, 4265, 2317, 2438}

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x(d+c^2dx^2)^{3/2}}\,dx = -\frac{4b\sqrt{c^2x^2+1}\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}$$
$$-\frac{2\sqrt{c^2x^2+1}\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}$$
$$-\frac{2b\sqrt{c^2x^2+1}\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}$$
$$+\frac{2b\sqrt{c^2x^2+1}\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}+\frac{(a+b\operatorname{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}$$
$$+\frac{2ib^2\sqrt{c^2x^2+1}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{c^2dx^2+d}}-\frac{2ib^2\sqrt{c^2x^2+1}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{c^2dx^2+d}}$$
$$+\frac{2b^2\sqrt{c^2x^2+1}\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{c^2dx^2+d}}-\frac{2b^2\sqrt{c^2x^2+1}\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{c^2dx^2+d}}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] (a + b*ArcSinh[c*x])^2/(d*Sqrt[d + c^2*d*x^2]) - (4*b*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) - (2*Sqrt
[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/(d*Sqrt[d + c
^2*d*x^2]) - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcS
inh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2
, (-I)*E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) - ((2*I)*b^2*Sqrt[1 + c^2*x
^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + (2*b*Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*
x^2]) + (2*b^2*Sqrt[1 + c^2*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/(d*Sqrt[d + c
^2*d*x^2]) - (2*b^2*Sqrt[1 + c^2*x^2]*PolyLog[3, E^ArcSinh[c*x]])/(d*Sqrt[d
 + c^2*d*x^2])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
```

```
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{d+c^2dx^2}}\,dx}{d}-\frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{a+b\text{arcsinh}(cx)}{1+c^2x^2}\,dx}{d\sqrt{d+c^2dx^2}}\\
&= \frac{(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}}+\frac{\sqrt{1+c^2x^2}\text{Subst}\left(\int (a+bx)^2\text{csch}(x)\,dx,x,\text{arcsinh}(cx)\right)}{d\sqrt{d+c^2dx^2}}\\
&\quad-\frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\text{sech}(x)\,dx,x,\text{arcsinh}(cx)\right)}{d\sqrt{d+c^2dx^2}}\\
&= \frac{(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}}-\frac{4b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}\\
&\quad-\frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}\\
&\quad-\frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\log\left(1-e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{d\sqrt{d+c^2dx^2}}\\
&\quad+\frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\log\left(1+e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{d\sqrt{d+c^2dx^2}}\\
&\quad+\frac{\left(2ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \log\left(1-ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{d\sqrt{d+c^2dx^2}}\\
&\quad-\frac{\left(2ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \log\left(1+ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{d\sqrt{d+c^2dx^2}}
\end{aligned}$$

$$= \frac{(a+b\operatorname{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}} - \frac{4b\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$- \frac{2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$- \frac{2b\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$+ \frac{2b\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(2ib^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(2ib^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(2b^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \operatorname{PolyLog}(2,-e^x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{d\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(2b^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \operatorname{PolyLog}(2,e^x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{(a+b\operatorname{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}} - \frac{4b\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$- \frac{2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$- \frac{2b\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$+ \frac{2ib^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$- \frac{2ib^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$+ \frac{2b\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(2b^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,-x)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(2b^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,x)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{(a+b\operatorname{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}} - \frac{4b\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$- \frac{2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$- \frac{2b\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$+ \frac{2ib^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$- \frac{2ib^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$+ \frac{2b\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$+ \frac{2b^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} - \frac{2b^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.38

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^{3/2}}\,dx = \frac{a^2d+a^2\sqrt{d}\sqrt{d+c^2dx^2}\log(cx)-a^2\sqrt{d}\sqrt{d+c^2dx^2}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right)+2}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] (a^2*d + a^2*Sqrt[d]*Sqrt[d + c^2*d*x^2]*Log[c*x] - a^2*Sqrt[d]*Sqrt[d + c^
2*d*x^2]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 2*a*b*d*(ArcSinh[c*x] - 2*S
qrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] + Sqrt[1 + c^2*x^2]*ArcSinh[c
*x]*Log[1 - E^(-ArcSinh[c*x])] - Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E^(
-ArcSinh[c*x])] + Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-ArcSinh[c*x])] - Sqrt[1
 + c^2*x^2]*PolyLog[2, E^(-ArcSinh[c*x])]) + b^2*d*(ArcSinh[c*x]^2 + Sqrt[1
 + c^2*x^2]*ArcSinh[c*x]^2*Log[1 - E^(-ArcSinh[c*x])] + (2*I)*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] - (2*I)*Sqrt[1 + c^2*x^2]*ArcSi
nh[c*x]*Log[1 + I/E^ArcSinh[c*x]] - Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Log[1
+ E^(-ArcSinh[c*x])] + 2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*PolyLog[2, -E^(-Arc
Sinh[c*x])] + (2*I)*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[c*x]] - (2*
I)*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]] - 2*Sqrt[1 + c^2*x^2]*Arc
Sinh[c*x]*PolyLog[2, E^(-ArcSinh[c*x])] + 2*Sqrt[1 + c^2*x^2]*PolyLog[3, -E
^(-ArcSinh[c*x])] - 2*Sqrt[1 + c^2*x^2]*PolyLog[3, E^(-ArcSinh[c*x])]))/(d^
2*Sqrt[d + c^2*d*x^2])
```

Maple [F]

$$\int \frac{(a+b\,\operatorname{arcsinh}(cx))^2}{x\,(c^2d\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\,(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\,(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(a+b\,\text{asinh}\,(cx))^2}{x\,(d\,(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x*(d*(c**2*x**2 + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\,(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -a^2*(arcsinh(1/(c*abs(x)))/d^(3/2) - 1/(sqrt(c^2*d*x^2 + d)*d)) + integrat
e(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/((c^2*d*x^2 + d)^(3/2)*x) + 2*a*b*log(
c*x + sqrt(c^2*x^2 + 1))/((c^2*d*x^2 + d)^(3/2)*x), x)
```

Giac [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x (d + c^2dx^2)^{3/2}} \, dx = \int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(c^2dx^2 + d)^{\frac{3}{2}}x} \, dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x (d + c^2dx^2)^{3/2}} \, dx = \int \frac{(a + b \operatorname{asinh}(c\,x))^2}{x\,(d\,c^2\,x^2 + d)^{3/2}} \, dx$$

```
[In] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^(3/2)),x)
[Out] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^(3/2)), x)
```

3.307 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 305

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{3/2}}\,dx &= -\frac{(a+b\text{arcsinh}(cx))^2}{dx\sqrt{d+c^2dx^2}}\\ &- \frac{2c^2x(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}} - \frac{2c\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}}\\ &- \frac{4bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}\\ &+ \frac{4bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}\\ &+ \frac{b^2c\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} + \frac{b^2c\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}\end{aligned}$$

```
[Out] -(a+b*arcsinh(c*x))^2/d/x/(c^2*d*x^2+d)^(1/2)-2*c^2*x*(a+b*arcsinh(c*x))^2/
d/(c^2*d*x^2+d)^(1/2)-2*c*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/d/(c^2*d*x
^2+d)^(1/2)-4*b*c*(a+b*arcsinh(c*x))*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)*(c^
2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+4*b*c*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^
2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+b^2*c*polylog(2,
-(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+b^2*c*p
olylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5809, 5787, 5797, 3799, 2221, 2317, 2438, 5799, 5569, 4267}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{3/2}}\,dx = -\frac{4bc\sqrt{c^2x^2+1}\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}} - \frac{2c^2x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}} - \frac{2c\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}} - \frac{(a+b\text{arcsinh}(cx))^2}{dx\sqrt{c^2dx^2+d}} + \frac{4bc\sqrt{c^2x^2+1}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}} + \frac{b^2c\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d\sqrt{c^2dx^2+d}} + \frac{b^2c\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d\sqrt{c^2dx^2+d}}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -((a + b*ArcSinh[c*x])^2/(d*x*Sqrt[d + c^2*d*x^2])) - (2*c^2*x*(a + b*ArcSi
nh[c*x])^2)/(d*Sqrt[d + c^2*d*x^2]) - (2*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh
[c*x])^2)/(d*Sqrt[d + c^2*d*x^2]) - (4*b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh
[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/(d*Sqrt[d + c^2*d*x^2]) + (4*b*c*Sqrt[1
 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(d*Sqrt[d + c
^2*d*x^2]) + (b^2*c*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(d*S
qrt[d + c^2*d*x^2]) + (b^2*c*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(2*ArcSinh[c*x]
)])/(d*Sqrt[d + c^2*d*x^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
```

```
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+b\text{arcsinh}(cx))^2}{dx\sqrt{d+c^2dx^2}} - \left(2c^2\right)\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx \\
&\quad + \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{a+b\text{arcsinh}(cx)}{x(1+c^2x^2)}\,dx}{d\sqrt{d+c^2dx^2}} \\
&= -\frac{(a+b\text{arcsinh}(cx))^2}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(2bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\text{csch}(x)\text{sech}(x)\,dx, x, \text{arcsinh}(cx)\right)}{d\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(4bc^3\sqrt{1+c^2x^2}\right)\int \frac{x(a+b\text{arcsinh}(cx))}{1+c^2x^2}\,dx}{d\sqrt{d+c^2dx^2}} \\
&= -\frac{(a+b\text{arcsinh}(cx))^2}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(4bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\text{csch}(2x)\,dx, x, \text{arcsinh}(cx)\right)}{d\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(4bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \text{arcsinh}(cx)\right)}{d\sqrt{d+c^2dx^2}} \\
&= -\frac{(a+b\text{arcsinh}(cx))^2}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}} - \frac{2c\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}} \\
&\quad - \frac{4bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(8bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \text{arcsinh}(cx)\right)}{d\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\left(2b^2c\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{d\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(2b^2c\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{d\sqrt{d+c^2dx^2}}
\end{aligned}$$

$$=-\frac{(a+b\text{arcsinh}(cx))^2}{dx\sqrt{d+c^2dx^2}}-\frac{2c^2x(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}}-\frac{2c\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}}-\frac{4bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}+\frac{4bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}-\frac{\left(b^2c\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}+\frac{\left(b^2c\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}-\frac{\left(4b^2c\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1+e^{2x}\right)\,dx,x,\text{arcsinh}(cx)\right)}{d\sqrt{d+c^2dx^2}}$$

$$=-\frac{(a+b\text{arcsinh}(cx))^2}{dx\sqrt{d+c^2dx^2}}-\frac{2c^2x(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}}-\frac{2c\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}}-\frac{4bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}+\frac{4bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}-\frac{b^2c\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}+\frac{b^2c\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}-\frac{\left(2b^2c\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$

$$=-\frac{(a+b\text{arcsinh}(cx))^2}{dx\sqrt{d+c^2dx^2}}-\frac{2c^2x(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}}-\frac{2c\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}}-\frac{4bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}+\frac{4bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}+\frac{b^2c\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}+\frac{b^2c\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.97

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{3/2}}\,dx =$$
$$-\frac{a^2+2a^2c^2x^2+2ab\text{arcsinh}(cx)+4abc^2x^2\text{arcsinh}(cx)+b^2\text{arcsinh}(cx)^2+2b^2c^2x^2\text{arcsinh}(cx)^2-2b^2cx\sqrt{1+}}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -((a^2 + 2*a^2*c^2*x^2 + 2*a*b*ArcSinh[c*x] + 4*a*b*c^2*x^2*ArcSinh[c*x] +
b^2*ArcSinh[c*x]^2 + 2*b^2*c^2*x^2*ArcSinh[c*x]^2 - 2*b^2*c*x*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x]^2 - 2*b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - E^(-
2*ArcSinh[c*x])] - 2*b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E^(-2*A
rcSinh[c*x])] - 2*a*b*c*x*Sqrt[1 + c^2*x^2]*Log[c*x] - a*b*c*x*Sqrt[1 + c^2
*x^2]*Log[1 + c^2*x^2] + b^2*c*x*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-2*ArcSin
h[c*x])] + b^2*c*x*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/(d*x*
Sqrt[d + c^2*d*x^2]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1116 vs. 2(317) = 634.

Time = 0.30 (sec) , antiderivative size = 1117, normalized size of antiderivative = 3.66

method	result	size
default	Expression too large to display	1117
parts	Expression too large to display	1117

```
[In] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/d/x/(c^2*d*x^2+d)^(1/2)-2*c^2/d*x/(c^2*d*x^2+d)^(1/2))-b^2*(arcsinh
(c*x)^2-4*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*x^3*c^
3-2*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x*c-2*(c
^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*x*c-2*(c^2*x^2+1)^
(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*x*c-4*(c^2*x^2+1)^(1/2)*arcs
inh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^3*c^3+4*arcsinh(c*x)*ln(1+(c*x+(
c^2*x^2+1)^(1/2))^2)*x^2*c^2-4*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2
*x^2+1)^(1/2))*x^3*c^3+4*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*x^4*c^4+4
*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*x^4*c^4-2*(c^2*x^2+1)^(1/2)*polyl
og(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*x^3*c^3-4*(c^2*x^2+1)^(1/2)*polylog(2,-c*x
-(c^2*x^2+1)^(1/2))*x^3*c^3-4*(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(
1/2))*x^3*c^3-(c^2*x^2+1)^(1/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*x*c-2
*(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*x*c-2*(c^2*x^2+1)^(1/2
```

```
)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*x*c+4*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)
^(1/2))^2)*x^4*c^4+2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*x^2*c^2+4*arcsin
h(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*x^2*c^2+4*arcsinh(c*x)*ln(1+c*x+(c^2*x^2
+1)^(1/2))*x^2*c^2+2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*x^4*c^4+4*polylo
g(2,-c*x-(c^2*x^2+1)^(1/2))*x^4*c^4+4*polylog(2,c*x+(c^2*x^2+1)^(1/2))*x^4*
c^4+4*polylog(2,c*x+(c^2*x^2+1)^(1/2))*x^2*c^2+4*polylog(2,-c*x-(c^2*x^2+1)
^(1/2))*x^2*c^2)*(2*c^2*x^2+1+2*c*x*(c^2*x^2+1)^(1/2))*(d*(c^2*x^2+1))^(1/2
)/x/d^2/(c^2*x^2+1)-2*a*b*(2*ln((c*x+(c^2*x^2+1)^(1/2))^4-1)*x^4*c^4-2*(c^2
*x^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^4-1)*x^3*c^3+2*ln((c*x+(c^2*x^2+1)
^(1/2))^4-1)*x^2*c^2-(c^2*x^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^4-1)*x*c+
arcsinh(c*x))*(2*c^2*x^2+1+2*c*x*(c^2*x^2+1)^(1/2))*(d*(c^2*x^2+1))^(1/2)/x
/d^2/(c^2*x^2+1)
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\,(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\,(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))^2}{x^2\,(d\,(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d)**(3/2),x)
[Out] Integral((a + b*asinh(c*x))**2/(x**2*(d*(c**2*x**2 + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\,(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

```
[Out] a*b*c*(log(c^2*x^2 + 1)/d^(3/2) + 2*log(x)/d^(3/2)) - 2*(2*c^2*x/(sqrt(c^2*
d*x^2 + d)*d) + 1/(sqrt(c^2*d*x^2 + d)*d*x))*a*b*arcsinh(c*x) - (2*c^2*x/(s
qrt(c^2*d*x^2 + d)*d) + 1/(sqrt(c^2*d*x^2 + d)*d*x))*a^2 + b^2*integrate(lo
g(c*x + sqrt(c^2*x^2 + 1))^2/((c^2*d*x^2 + d)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{\left(b\,\text{arsinh}\,(cx)+a\right)^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{\left(a+b\,\text{asinh}(c\,x)\right)^2}{x^2\,\left(d\,c^2\,x^2+d\right)^{3/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^(3/2)),x)
[Out] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^(3/2)), x)
```

3.308 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 573

$$\begin{aligned}
\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx &= -\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{dx\sqrt{d+c^2dx^2}} \\
&- \frac{3c^2(a+b\text{arcsinh}(cx))^2}{2d\sqrt{d+c^2dx^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{d+c^2dx^2}} \\
&+ \frac{4bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} \\
&+ \frac{3c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} \\
&- \frac{b^2c^2\sqrt{1+c^2x^2}\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{d\sqrt{d+c^2dx^2}} \\
&+ \frac{3bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} \\
&- \frac{2ib^2c^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} \\
&+ \frac{2ib^2c^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} \\
&- \frac{3bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} \\
&- \frac{3b^2c^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} \\
&+ \frac{3b^2c^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}
\end{aligned}$$

```
[Out] -3/2*c^2*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(1/2)-1/2*(a+b*arcsinh(c*x))^
2/d/x^2/(c^2*d*x^2+d)^(1/2)-b*c*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/d/x/(c
^2*d*x^2+d)^(1/2)+4*b*c^2*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))*
(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+3*c^2*(a+b*arcsinh(c*x))^2*arctanh(
c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-b^2*c^2*arct
anh((c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+3*b*c^2*(a+b
*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d
*x^2+d)^(1/2)-2*I*b^2*c^2*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)
^(1/2)/d/(c^2*d*x^2+d)^(1/2)+2*I*b^2*c^2*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)
))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-3*b*c^2*(a+b*arcsinh(c*x))*polyl
og(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-3*b^2*c
^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2
)+3*b^2*c^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2
+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 573, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5809, 5811, 5816, 4267, 2611, 2320, 6724, 5789, 4265, 2317, 2438, 272, 65, 214}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^{3/2}}\,dx = \frac{4bc^2\sqrt{c^2x^2+1}\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}$$
$$+\frac{3c^2\sqrt{c^2x^2+1}\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}$$
$$+\frac{3bc^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}$$
$$-\frac{3bc^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}$$
$$-\frac{3c^2(a+b\text{arcsinh}(cx))^2}{2d\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{dx\sqrt{c^2dx^2+d}}$$
$$-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{c^2dx^2+d}}-\frac{2ib^2c^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{d\sqrt{c^2dx^2+d}}$$
$$+\frac{2ib^2c^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{d\sqrt{c^2dx^2+d}}-\frac{3b^2c^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{d\sqrt{c^2dx^2+d}}$$
$$+\frac{3b^2c^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{d\sqrt{c^2dx^2+d}}-\frac{b^2c^2\sqrt{c^2x^2+1}\text{arctanh}\left(\sqrt{c^2x^2+1}\right)}{d\sqrt{c^2dx^2+d}}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -((b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(d*x*Sqrt[d + c^2*d*x^2])) -
 (3*c^2*(a + b*ArcSinh[c*x])^2)/(2*d*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[
```

```
c*x])^2/(2*d*x^2*Sqrt[d + c^2*d*x^2]) + (4*b*c^2*Sqrt[1 + c^2*x^2]*(a + b*A
rcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + (3*c^2*Sqrt[
1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/(d*Sqrt[d + c^
2*d*x^2]) - (b^2*c^2*Sqrt[1 + c^2*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/(d*Sqrt[
d + c^2*d*x^2]) + (3*b*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2
, -E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) - ((2*I)*b^2*c^2*Sqrt[1 + c^2*x
^2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + ((2*I)*b^2*c
^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2])
- (3*b*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]
])/(d*Sqrt[d + c^2*d*x^2]) - (3*b^2*c^2*Sqrt[1 + c^2*x^2]*PolyLog[3, -E^Arc
Sinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + (3*b^2*c^2*Sqrt[1 + c^2*x^2]*PolyLog[
3, E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{d+c^2dx^2}} - \frac{1}{2}\left(3c^2\right)\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^{3/2}}\,dx \\
&\quad + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{a+b\text{arcsinh}(cx)}{x^2(1+c^2x^2)}\,dx}{d\sqrt{d+c^2dx^2}} \\
&= -\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{dx\sqrt{d+c^2dx^2}} - \frac{3c^2(a+b\text{arcsinh}(cx))^2}{2d\sqrt{d+c^2dx^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\left(3c^2\right)\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{d+c^2dx^2}}\,dx}{2d} + \frac{\left(b^2c^2\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{1+c^2x^2}}\,dx}{d\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\left(bc^3\sqrt{1+c^2x^2}\right)\int \frac{a+b\text{arcsinh}(cx)}{1+c^2x^2}\,dx}{d\sqrt{d+c^2dx^2}} + \frac{\left(3bc^3\sqrt{1+c^2x^2}\right)\int \frac{a+b\text{arcsinh}(cx)}{1+c^2x^2}\,dx}{d\sqrt{d+c^2dx^2}}
\end{aligned}$$

$$= -\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{dx\sqrt{d+c^2dx^2}} - \frac{3c^2(a+b\text{arcsinh}(cx))^2}{2d\sqrt{d+c^2dx^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(3c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)^2\text{csch}(x)\,dx,x,\text{arcsinh}(cx)\right)}{2d\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(bc^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\text{sech}(x)\,dx,x,\text{arcsinh}(cx)\right)}{d\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(3bc^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\text{sech}(x)\,dx,x,\text{arcsinh}(cx)\right)}{d\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(b^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+c^2x}}\,dx,x,x^2\right)}{2d\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{dx\sqrt{d+c^2dx^2}} - \frac{3c^2(a+b\text{arcsinh}(cx))^2}{2d\sqrt{d+c^2dx^2}}$$
$$- \frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{d+c^2dx^2}} + \frac{4bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$+ \frac{3c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx,x,\sqrt{1+c^2x^2}\right)}{d\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(3bc^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\log\left(1-e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{d\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(3bc^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\log\left(1+e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{d\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(ib^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \log\left(1-ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{d\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(ib^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \log\left(1+ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{d\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(3ib^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \log\left(1-ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{d\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(3ib^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \log\left(1+ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{d\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{dx\sqrt{d+c^2dx^2}} - \frac{3c^2(a+b\text{arcsinh}(cx))^2}{2d\sqrt{d+c^2dx^2}}$$
$$- \frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{d+c^2dx^2}} + \frac{4bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$+ \frac{3c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$- \frac{b^2c^2\sqrt{1+c^2x^2}\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{d\sqrt{d+c^2dx^2}}$$
$$+ \frac{3bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$- \frac{3bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(ib^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(ib^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(3ib^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(3ib^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(3b^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\text{PolyLog}\left(2,-e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{d\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(3b^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\text{PolyLog}\left(2,e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{d\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{dx\sqrt{d+c^2dx^2}} - \frac{3c^2(a+b\text{arcsinh}(cx))^2}{2d\sqrt{d+c^2dx^2}}$$
$$- \frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{d+c^2dx^2}} + \frac{4bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$+ \frac{3c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$- \frac{b^2c^2\sqrt{1+c^2x^2}\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{d\sqrt{d+c^2dx^2}}$$
$$+ \frac{3bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\operatorname{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$- \frac{2ib^2c^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$+ \frac{2ib^2c^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$- \frac{3bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\operatorname{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(3b^2c^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,-x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(3b^2c^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{dx\sqrt{d+c^2dx^2}} - \frac{3c^2(a+b\text{arcsinh}(cx))^2}{2d\sqrt{d+c^2dx^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{d+c^2dx^2}} + \frac{4bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} + \frac{3c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} - \frac{b^2c^2\sqrt{1+c^2x^2}\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{d\sqrt{d+c^2dx^2}} + \frac{3bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} - \frac{2ib^2c^2\sqrt{1+c^2x^2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} + \frac{2ib^2c^2\sqrt{1+c^2x^2}\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} - \frac{3bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} - \frac{3b^2c^2\sqrt{1+c^2x^2}\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} + \frac{3b^2c^2\sqrt{1+c^2x^2}\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 7.42 (sec) , antiderivative size = 884, normalized size of antiderivative = 1.54

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^{3/2}}\,dx = \sqrt{d(1+c^2x^2)}\left(-\frac{a^2}{2d^2x^2} - \frac{a^2c^2}{d^2(1+c^2x^2)}\right) - \frac{3a^2c^2\log(x)}{2d^{3/2}} + \frac{3a^2c^2\log\left(d+\sqrt{d}\sqrt{d(1+c^2x^2)}\right)}{2d^{3/2}} + \frac{abc^2\left(-8\text{arcsinh}(cx)+16\sqrt{1+c^2x^2}\arctan\left(\tanh\left(\frac{1}{2}\text{arcsinh}(cx)\right)\right)-2\sqrt{1+c^2x^2}\coth\left(\frac{1}{2}\text{arcsinh}(cx)\right)-\sqrt{1}\right.}{} + \frac{b^2c^2\left(-8\text{arcsinh}(cx)^2-4\sqrt{1+c^2x^2}\text{arcsinh}(cx)\coth\left(\frac{1}{2}\text{arcsinh}(cx)\right)-\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2\text{csch}^2\left(\frac{1}{2}\text{arcsi}\right.\right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] Sqrt[d*(1 + c^2*x^2)]*(-1/2*a^2/(d^2*x^2) - (a^2*c^2)/(d^2*(1 + c^2*x^2)))
- (3*a^2*c^2*Log[x])/(2*d^(3/2)) + (3*a^2*c^2*Log[d + Sqrt[d]*Sqrt[d*(1 + c
^2*x^2)]])/(2*d^(3/2)) + (a*b*c^2*(-8*ArcSinh[c*x] + 16*Sqrt[1 + c^2*x^2]*A
rcTan[Tanh[ArcSinh[c*x]/2]] - 2*Sqrt[1 + c^2*x^2]*Coth[ArcSinh[c*x]/2] - Sq
```

rt[1 + c^2*x^2]*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] + 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 12*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-ArcSinh[c*x])] + 12*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(-ArcSinh[c*x])] - Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Sqrt[1 + c^2*x^2]*Tanh[ArcSinh[c*x]/2]))/(4*d*Sqrt[d*(1 + c^2*x^2)]) + (b^2*c^2*(-8*ArcSinh[c*x]^2 - 4*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Coth[ArcSinh[c*x]/2] - Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Csch[ArcSinh[c*x]/2]^2 - 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Log[1 - E^(-ArcSinh[c*x])] - (16*I)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] + (16*I)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + I/E^ArcSinh[c*x]] + 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] + 8*Sqrt[1 + c^2*x^2]*Log[Tanh[ArcSinh[c*x]/2]] - 24*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - (16*I)*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[c*x]] + (16*I)*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]] + 24*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*PolyLog[2, E^(-ArcSinh[c*x])] - 24*Sqrt[1 + c^2*x^2]*PolyLog[3, -E^(-ArcSinh[c*x])] + 24*Sqrt[1 + c^2*x^2]*PolyLog[3, E^(-ArcSinh[c*x])] - Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Sech[ArcSinh[c*x]/2]^2 + 4*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Tanh[ArcSinh[c*x]/2]))/(8*d*Sqrt[d*(1 + c^2*x^2)])

Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{x^3\,(c^2d\,x^2+d)^{\frac{3}{2}}}dx$$

[In] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(3/2),x)

[Out] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(3/2),x)

Fricas [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2}{x^3\,(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^{\frac{3}{2}}x^3}\,dx$$

[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^4*d^2*x^7 + 2*c^2*d^2*x^5 + d^2*x^3), x)

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}{x^3\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d)**(3/2),x)
[Out] Integral((a + b*asinh(c*x))**2/(x**3*(d*(c**2*x**2 + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
[Out] 1/2*(3*c^2*arcsinh(1/(c*abs(x)))/d^(3/2) - 3*c^2/(sqrt(c^2*d*x^2 + d)*d) -
1/(sqrt(c^2*d*x^2 + d)*d*x^2))*a^2 + integrate(b^2*log(c*x + sqrt(c^2*x^2 +
 1))^2/((c^2*d*x^2 + d)^(3/2)*x^3) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/((c
^2*d*x^2 + d)^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{x^3\,{\left(d\,c^2\,x^2+d\right)}^{3/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^(3/2)),x)
[Out] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^(3/2)), x)
```

3.309 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 452

$$\begin{aligned}
\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{3/2}}\,dx &= -\frac{b^2c^2(1+c^2x^2)}{3dx\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3dx^2\sqrt{d+c^2dx^2}} \\
&- \frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2(a+b\text{arcsinh}(cx))^2}{3dx\sqrt{d+c^2dx^2}} \\
&+ \frac{8c^4x(a+b\text{arcsinh}(cx))^2}{3d\sqrt{d+c^2dx^2}} + \frac{8c^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3d\sqrt{d+c^2dx^2}} \\
&+ \frac{20bc^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{3d\sqrt{d+c^2dx^2}} \\
&- \frac{16bc^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3d\sqrt{d+c^2dx^2}} \\
&- \frac{b^2c^3\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} \\
&- \frac{5b^2c^3\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{3d\sqrt{d+c^2dx^2}}
\end{aligned}$$

```
[Out] -1/3*b^2*c^2*(c^2*x^2+1)/d/x/(c^2*d*x^2+d)^(1/2)-1/3*(a+b*arcsinh(c*x))^2/d
/x^3/(c^2*d*x^2+d)^(1/2)+4/3*c^2*(a+b*arcsinh(c*x))^2/d/x/(c^2*d*x^2+d)^(1/
2)+8/3*c^4*x*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(1/2)-1/3*b*c*(a+b*arcsin
h(c*x))*(c^2*x^2+1)^(1/2)/d/x^2/(c^2*d*x^2+d)^(1/2)+8/3*c^3*(a+b*arcsinh(c*
x))^2*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+20/3*b*c^3*(a+b*arcsinh(c*x))
*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)
-16/3*b*c^3*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^
(1/2)/d/(c^2*d*x^2+d)^(1/2)-b^2*c^3*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*(
c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-5/3*b^2*c^3*polylog(2,(c*x+(c^2*x^2+
1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {5809, 5787, 5797, 3799, 2221, 2317, 2438, 5799, 5569, 4267, 270}

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{3/2}}\,dx &= \frac{20bc^3\sqrt{c^2x^2+1}\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3d\sqrt{c^2dx^2+d}}\\ &+\frac{4c^2(a+b\text{arcsinh}(cx))^2}{3dx\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3dx^2\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\sqrt{c^2dx^2+d}}\\ &+\frac{8c^4x(a+b\text{arcsinh}(cx))^2}{3d\sqrt{c^2dx^2+d}}+\frac{8c^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{3d\sqrt{c^2dx^2+d}}\\ &-\frac{16bc^3\sqrt{c^2x^2+1}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))}{3d\sqrt{c^2dx^2+d}}\\ &-\frac{b^2c^3\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d\sqrt{c^2dx^2+d}}\\ &-\frac{5b^2c^3\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{3d\sqrt{c^2dx^2+d}}-\frac{b^2c^2(c^2x^2+1)}{3dx\sqrt{c^2dx^2+d}}\end{aligned}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -1/3*(b^2*c^2*(1 + c^2*x^2))/(d*x*Sqrt[d + c^2*d*x^2]) - (b*c*Sqrt[1 + c^2*
x^2]*(a + b*ArcSinh[c*x]))/(3*d*x^2*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c
*x])^2/(3*d*x^3*Sqrt[d + c^2*d*x^2]) + (4*c^2*(a + b*ArcSinh[c*x])^2)/(3*d*
x*Sqrt[d + c^2*d*x^2]) + (8*c^4*x*(a + b*ArcSinh[c*x])^2)/(3*d*Sqrt[d + c^2
*d*x^2]) + (8*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(3*d*Sqrt[d + c
^2*d*x^2]) + (20*b*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*
ArcSinh[c*x])])/(3*d*Sqrt[d + c^2*d*x^2]) - (16*b*c^3*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(3*d*Sqrt[d + c^2*d*x^2]) -
(b^2*c^3*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(d*Sqrt[d + c^2
*d*x^2]) - (5*b^2*c^3*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(2*ArcSinh[c*x])])/(3*
d*Sqrt[d + c^2*d*x^2])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
```

```
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+b\operatorname{arcsinh}(cx))^2}{3dx^3\sqrt{d+c^2dx^2}}-\frac{1}{3}\left(4c^2\right)\int\frac{(a+b\operatorname{arcsinh}(cx))^2}{x^2\left(d+c^2dx^2\right)^{3/2}}\,dx\\
&\quad+\frac{\left(2bc\sqrt{1+c^2x^2}\right)\int\frac{a+b\operatorname{arcsinh}(cx)}{x^3(1+c^2x^2)}\,dx}{3d\sqrt{d+c^2dx^2}}\\
&= -\frac{bc\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3dx^2\sqrt{d+c^2dx^2}}-\frac{(a+b\operatorname{arcsinh}(cx))^2}{3dx^3\sqrt{d+c^2dx^2}}+\frac{4c^2(a+b\operatorname{arcsinh}(cx))^2}{3dx\sqrt{d+c^2dx^2}}\\
&\quad+\frac{1}{3}\left(8c^4\right)\int\frac{(a+b\operatorname{arcsinh}(cx))^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx+\frac{\left(b^2c^2\sqrt{1+c^2x^2}\right)\int\frac{1}{x^2\sqrt{1+c^2x^2}}\,dx}{3d\sqrt{d+c^2dx^2}}\\
&\quad-\frac{\left(2bc^3\sqrt{1+c^2x^2}\right)\int\frac{a+b\operatorname{arcsinh}(cx)}{x(1+c^2x^2)}\,dx}{3d\sqrt{d+c^2dx^2}}-\frac{\left(8bc^3\sqrt{1+c^2x^2}\right)\int\frac{a+b\operatorname{arcsinh}(cx)}{x(1+c^2x^2)}\,dx}{3d\sqrt{d+c^2dx^2}}\\
&= -\frac{b^2c^2(1+c^2x^2)}{3dx\sqrt{d+c^2dx^2}}-\frac{bc\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3dx^2\sqrt{d+c^2dx^2}}-\frac{(a+b\operatorname{arcsinh}(cx))^2}{3dx^3\sqrt{d+c^2dx^2}}\\
&\quad+\frac{4c^2(a+b\operatorname{arcsinh}(cx))^2}{3dx\sqrt{d+c^2dx^2}}+\frac{8c^4x(a+b\operatorname{arcsinh}(cx))^2}{3d\sqrt{d+c^2dx^2}}\\
&\quad-\frac{\left(2bc^3\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int(a+bx)\operatorname{csch}(x)\operatorname{sech}(x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{3d\sqrt{d+c^2dx^2}}\\
&\quad-\frac{\left(8bc^3\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int(a+bx)\operatorname{csch}(x)\operatorname{sech}(x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{3d\sqrt{d+c^2dx^2}}\\
&\quad-\frac{\left(16bc^5\sqrt{1+c^2x^2}\right)\int\frac{x(a+b\operatorname{arcsinh}(cx))}{1+c^2x^2}\,dx}{3d\sqrt{d+c^2dx^2}}
\end{aligned}$$

$$= -\frac{b^2c^2(1+c^2x^2)}{3dx\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3dx^2\sqrt{d+c^2dx^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2(a+b\text{arcsinh}(cx))^2}{3dx\sqrt{d+c^2dx^2}} + \frac{8c^4x(a+b\text{arcsinh}(cx))^2}{3d\sqrt{d+c^2dx^2}} - \frac{\left(4bc^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\text{csch}(2x)\,dx, x, \text{arcsinh}(cx)\right)}{3d\sqrt{d+c^2dx^2}} - \frac{\left(16bc^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\text{csch}(2x)\,dx, x, \text{arcsinh}(cx)\right)}{3d\sqrt{d+c^2dx^2}} - \frac{\left(16bc^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \text{arcsinh}(cx)\right)}{3d\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2c^2(1+c^2x^2)}{3dx\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3dx^2\sqrt{d+c^2dx^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2(a+b\text{arcsinh}(cx))^2}{3dx\sqrt{d+c^2dx^2}} + \frac{8c^4x(a+b\text{arcsinh}(cx))^2}{3d\sqrt{d+c^2dx^2}} + \frac{8c^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3d\sqrt{d+c^2dx^2}} + \frac{20bc^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{3d\sqrt{d+c^2dx^2}} - \frac{\left(32bc^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \text{arcsinh}(cx)\right)}{3d\sqrt{d+c^2dx^2}} + \frac{\left(2b^2c^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{3d\sqrt{d+c^2dx^2}} - \frac{\left(2b^2c^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{3d\sqrt{d+c^2dx^2}} + \frac{\left(8b^2c^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{3d\sqrt{d+c^2dx^2}} - \frac{\left(8b^2c^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{3d\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2c^2(1+c^2x^2)}{3dx\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3dx^2\sqrt{d+c^2dx^2}} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2(a+b\operatorname{arcsinh}(cx))^2}{3dx\sqrt{d+c^2dx^2}} + \frac{8c^4x(a+b\operatorname{arcsinh}(cx))^2}{3d\sqrt{d+c^2dx^2}} + \frac{8c^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{3d\sqrt{d+c^2dx^2}} + \frac{20bc^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{2\operatorname{arcsinh}(cx)}\right)}{3d\sqrt{d+c^2dx^2}} - \frac{16bc^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{3d\sqrt{d+c^2dx^2}} + \frac{\left(b^2c^3\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2\operatorname{arcsinh}(cx)}\right)}{3d\sqrt{d+c^2dx^2}} - \frac{\left(b^2c^3\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\operatorname{arcsinh}(cx)}\right)}{3d\sqrt{d+c^2dx^2}} + \frac{\left(4b^2c^3\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2\operatorname{arcsinh}(cx)}\right)}{3d\sqrt{d+c^2dx^2}} - \frac{\left(4b^2c^3\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\operatorname{arcsinh}(cx)}\right)}{3d\sqrt{d+c^2dx^2}} + \frac{\left(16b^2c^3\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\log\left(1+e^{2x}\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{3d\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2c^2(1+c^2x^2)}{3dx\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3dx^2\sqrt{d+c^2dx^2}} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2(a+b\operatorname{arcsinh}(cx))^2}{3dx\sqrt{d+c^2dx^2}} + \frac{8c^4x(a+b\operatorname{arcsinh}(cx))^2}{3d\sqrt{d+c^2dx^2}} + \frac{8c^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{3d\sqrt{d+c^2dx^2}} + \frac{20bc^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{2\operatorname{arcsinh}(cx)}\right)}{3d\sqrt{d+c^2dx^2}} - \frac{16bc^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{3d\sqrt{d+c^2dx^2}} + \frac{5b^2c^3\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{3d\sqrt{d+c^2dx^2}} - \frac{5b^2c^3\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(cx)}\right)}{3d\sqrt{d+c^2dx^2}} + \frac{\left(8b^2c^3\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\operatorname{arcsinh}(cx)}\right)}{3d\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2c^2(1+c^2x^2)}{3dx\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3dx^2\sqrt{d+c^2dx^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2(a+b\text{arcsinh}(cx))^2}{3dx\sqrt{d+c^2dx^2}} + \frac{8c^4x(a+b\text{arcsinh}(cx))^2}{3d\sqrt{d+c^2dx^2}} + \frac{8c^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3d\sqrt{d+c^2dx^2}} + \frac{20bc^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{3d\sqrt{d+c^2dx^2}} - \frac{16bc^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3d\sqrt{d+c^2dx^2}} - \frac{b^2c^3\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} - \frac{5b^2c^3\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{3d\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 438, normalized size of antiderivative = 0.97

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\left(d+c^2dx^2\right)^{3/2}}\,dx = \frac{-a^2+4a^2c^2x^2-b^2c^2x^2+8a^2c^4x^4-b^2c^4x^4-abcx\sqrt{1+c^2x^2}-2ab\text{arcsinh}(cx)+}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] (-a^2 + 4*a^2*c^2*x^2 - b^2*c^2*x^2 + 8*a^2*c^4*x^4 - b^2*c^4*x^4 - a*b*c*x
*Sqrt[1 + c^2*x^2] - 2*a*b*ArcSinh[c*x] + 8*a*b*c^2*x^2*ArcSinh[c*x] + 16*a
*b*c^4*x^4*ArcSinh[c*x] - b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - b^2*ArcS
inh[c*x]^2 + 4*b^2*c^2*x^2*ArcSinh[c*x]^2 + 8*b^2*c^4*x^4*ArcSinh[c*x]^2 -
8*b^2*c^3*x^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 - 10*b^2*c^3*x^3*Sqrt[1 + c^
2*x^2]*ArcSinh[c*x]*Log[1 - E^(-2*ArcSinh[c*x])] - 6*b^2*c^3*x^3*Sqrt[1 + c
^2*x^2]*ArcSinh[c*x]*Log[1 + E^(-2*ArcSinh[c*x])] - 10*a*b*c^3*x^3*Sqrt[1 +
 c^2*x^2]*Log[c*x] - 3*a*b*c^3*x^3*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] + 3*b
^2*c^3*x^3*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-2*ArcSinh[c*x])] + 5*b^2*c^3*x
^3*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/(3*d*x^3*Sqrt[d + c^2
*d*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2607 vs. 2(438) = 876.

Time = 0.37 (sec) , antiderivative size = 2608, normalized size of antiderivative = 5.77

method	result	size
default	Expression too large to display	2608
parts	Expression too large to display	2608

```
[In] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -128/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^2*(c^2*x^2+1
)^(1/2)*arcsinh(c*x)*c^5+40/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^
2-1)/d^2*x^5*c^8-7/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*
x*c^4+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2/x*c^2+1/3*b
^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2/x^3*arcsinh(c*x)^2-10/
3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*polylog(2,-c*x-(c^2*x^2+1
)^(1/2))*c^3+16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arcsinh(c
*x)^2*c^3-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*c^3*(c^
2*x^2+1)^(1/2)-10/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*polylog
(2,c*x+(c^2*x^2+1)^(1/2))*c^3+32/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c
^2*x^2-1)/d^2*x^7*c^10-b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*poly
log(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*c^3+a^2*(-1/3/d/x^3/(c^2*d*x^2+d)^(1/2)-4
/3*c^2*(-1/d/x/(c^2*d*x^2+d)^(1/2)-2*c^2/d*x/(c^2*d*x^2+d)^(1/2)))-64/3*a*b
*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^5*(c^2*x^2+1)*c^8-32/3
*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*(c^2*x^2+1)*c^6+
128/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*arcsinh(c*x
)*c^6+8/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x*(c^2*x^2+
1)*c^4+16*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x*arcsinh(c
*x)*c^4+16/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*(c^2*x^2
+1)^(1/2)*arcsinh(c*x)*c^3-8*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2
-1)/d^2/x*arcsinh(c*x)*c^2+1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x
^2-1)/d^2/x^2*c*(c^2*x^2+1)^(1/2)-32/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4
+7*c^2*x^2-1)/d^2*x^5*(c^2*x^2+1)*c^8+32*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x
^4+7*c^2*x^2-1)/d^2*x^5*arcsinh(c*x)*c^8-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c
^4*x^4+7*c^2*x^2-1)/d^2*x^3*(c^2*x^2+1)*c^6+64/3*b^2*(d*(c^2*x^2+1))^(1/2)/
(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*arcsinh(c*x)^2*c^6+8*b^2*(d*(c^2*x^2+1))^(1
/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*arcsinh(c*x)*c^6+8/3*b^2*(d*(c^2*x^2+1)
)^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^2*c^5*(c^2*x^2+1)^(1/2)+8*b^2*(d*(c^2
*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x*arcsinh(c*x)^2*c^4-8/3*b^2*(d*
(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x*arcsinh(c*x)*c^4+8/3*b^2*(
d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*(c^2*x^2+1)^(1/2)*arcsinh(
c*x)^2*c^3-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*(c^2*x
^2+1)^(1/2)*arcsinh(c*x)*c^3-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/
d^2*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*c^3-4*b^2*(d*(c^2*x^2+1))^
```

```
(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2/x*arcsinh(c*x)^2*c^2-10/3*b^2*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c
^3-10/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arcsinh(c*x)*ln(1-c
*x-(c^2*x^2+1)^(1/2))*c^3+64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x
^2-1)/d^2*x^7*arcsinh(c*x)*c^10+32/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)/d^2*arcsinh(c*x)*c^3+64/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*
x^2-1)/d^2*x^7*c^10+32*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^
2*x^5*c^8+8*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*c^6-8
/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x*c^4-8/3*a*b*(d*(
c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*c^3*(c^2*x^2+1)^(1/2)+2/3*a*b
*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2/x^3*arcsinh(c*x)-2*a*b*(
d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*
c^3-10/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*ln((c*x+(c^2*x^2+1
)^(1/2))^2-1)*c^3-64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^
2*x^5*(c^2*x^2+1)*arcsinh(c*x)*c^8-32/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^
4+7*c^2*x^2-1)/d^2*x^3*(c^2*x^2+1)*arcsinh(c*x)*c^6-64/3*b^2*(d*(c^2*x^2+1)
)^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^2*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^
5+8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x*(c^2*x^2+1)*a
rcsinh(c*x)*c^4+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2/x
^2*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\,(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^{\frac{3}{2}}x^4}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^4*d^2*x^8 + 2*c^2*d^2*x^6 + d^2*x^4), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\,(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(a+b\,\text{asinh}\,(cx))^2}{x^4\,(d\,(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x**4*(d*(c**2*x**2 + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^4 (d + c^2 dx^2)^{3/2}} dx = \int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(c^2 dx^2 + d)^{\frac{3}{2}} x^4} dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/3*(8*c^4*x/(sqrt(c^2*d*x^2 + d)*d) + 4*c^2/(sqrt(c^2*d*x^2 + d)*d*x) - 1/
(sqrt(c^2*d*x^2 + d)*d*x^3))*a^2 + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1
))^2/((c^2*d*x^2 + d)^(3/2)*x^4) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/((c^2
*d*x^2 + d)^(3/2)*x^4), x)
```

Giac [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^4 (d + c^2 dx^2)^{3/2}} dx = \int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(c^2 dx^2 + d)^{\frac{3}{2}} x^4} dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^4 (d + c^2 dx^2)^{3/2}} dx = \int \frac{(a + b \operatorname{asinh}(c x))^2}{x^4 (d c^2 x^2 + d)^{3/2}} dx$$

```
[In] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^(3/2)), x)
```

3.310 $\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 512

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \frac{b^2}{3c^6d^2\sqrt{d+c^2dx^2}}-\frac{16abx\sqrt{1+c^2x^2}}{3c^5d^2\sqrt{d+c^2dx^2}}+\frac{2b^2(1+c^2x^2)}{c^6d^2\sqrt{d+c^2dx^2}}-\frac{16b^2x\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{3c^5d^2\sqrt{d+c^2dx^2}}-\frac{bx^3(a+b\text{arcsinh}(cx))}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}+\frac{11bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3c^5d^2\sqrt{d+c^2dx^2}}-\frac{x^4(a+b\text{arcsinh}(cx))^2}{3c^2d\,(d+c^2dx^2)^{3/2}}-\frac{4x^2(a+b\text{arcsinh}(cx))^2}{3c^4d^2\sqrt{d+c^2dx^2}}+\frac{8\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^6d^3}-\frac{22b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3c^6d^2\sqrt{d+c^2dx^2}}+\frac{11ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3c^6d^2\sqrt{d+c^2dx^2}}-\frac{11ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3c^6d^2\sqrt{d+c^2dx^2}}$$

```
[Out] -1/3*x^4*(a+b*arcsinh(c*x))^2/c^2/d/(c^2*d*x^2+d)^(3/2)+1/3*b^2/c^6/d^2/(c^
2*d*x^2+d)^(1/2)+2*b^2*(c^2*x^2+1)/c^6/d^2/(c^2*d*x^2+d)^(1/2)-4/3*x^2*(a+b
*arcsinh(c*x))^2/c^4/d^2/(c^2*d*x^2+d)^(1/2)-1/3*b*x^3*(a+b*arcsinh(c*x))/c
^3/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-16/3*a*b*x*(c^2*x^2+1)^(1/2)/c
^5/d^2/(c^2*d*x^2+d)^(1/2)-16/3*b^2*x*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c^5/d^
2/(c^2*d*x^2+d)^(1/2)+11/3*b*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^5/d^2
/(c^2*d*x^2+d)^(1/2)-22/3*b*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2)
)*(c^2*x^2+1)^(1/2)/c^6/d^2/(c^2*d*x^2+d)^(1/2)+11/3*I*b^2*polylog(2,-I*(c*
x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^6/d^2/(c^2*d*x^2+d)^(1/2)-11/3*I*
```

```
b^2*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^6/d^2/(c^2*d*x
^2+d)^(1/2)+8/3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^6/d^3
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {5810, 5798, 5772, 267, 5812, 5789, 4265, 2317, 2438, 272, 45}

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = -\frac{22b\sqrt{c^2x^2+1}\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3c^6d^2\sqrt{c^2dx^2+d}}-\frac{x^4(a+b\text{arcsinh}(cx))^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}+\frac{8\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^6d^3}+\frac{11bx\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^5d^2\sqrt{c^2dx^2+d}}-\frac{4x^2(a+b\text{arcsinh}(cx))^2}{3c^4d^2\sqrt{c^2dx^2+d}}-\frac{bx^3(a+b\text{arcsinh}(cx))}{3c^3d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}-\frac{16abx\sqrt{c^2x^2+1}}{3c^5d^2\sqrt{c^2dx^2+d}}+\frac{11ib^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3c^6d^2\sqrt{c^2dx^2+d}}-\frac{11ib^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3c^6d^2\sqrt{c^2dx^2+d}}-\frac{16b^2x\sqrt{c^2x^2+1}\text{arcsinh}(cx)}{3c^5d^2\sqrt{c^2dx^2+d}}+\frac{2b^2(c^2x^2+1)}{c^6d^2\sqrt{c^2dx^2+d}}+\frac{b^2}{3c^6d^2\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x^5*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] b^2/(3*c^6*d^2*Sqrt[d + c^2*d*x^2]) - (16*a*b*x*Sqrt[1 + c^2*x^2])/(3*c^5*d
^2*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*x^2))/(c^6*d^2*Sqrt[d + c^2*d*x^2
]) - (16*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(3*c^5*d^2*Sqrt[d + c^2*d*x^
2]) - (b*x^3*(a + b*ArcSinh[c*x]))/(3*c^3*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^
2*d*x^2]) + (11*b*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c^5*d^2*Sqrt
[d + c^2*d*x^2]) - (x^4*(a + b*ArcSinh[c*x])^2)/(3*c^2*d*(d + c^2*d*x^2)^(3
/2)) - (4*x^2*(a + b*ArcSinh[c*x])^2)/(3*c^4*d^2*Sqrt[d + c^2*d*x^2]) + (8*
Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^6*d^3) - (22*b*Sqrt[1 + c^
2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(3*c^6*d^2*Sqrt[d + c^2
*d*x^2]) + (((11*I)/3)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^ArcSinh[c*x]
])/(c^6*d^2*Sqrt[d + c^2*d*x^2]) - (((11*I)/3)*b^2*Sqrt[1 + c^2*x^2]*PolyLo
g[2, I*E^ArcSinh[c*x]])/(c^6*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^4(a+b\text{arcsinh}(cx))^2}{3c^2d\,(d+c^2dx^2)^{3/2}} + \frac{4\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx}{3c^2d} \\
&\quad + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{x^4(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^2}\,dx}{3cd^2\sqrt{d+c^2dx^2}} \\
&= -\frac{bx^3(a+b\text{arcsinh}(cx))}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^4(a+b\text{arcsinh}(cx))^2}{3c^2d\,(d+c^2dx^2)^{3/2}} - \frac{4x^2(a+b\text{arcsinh}(cx))^2}{3c^4d^2\sqrt{d+c^2dx^2}} \\
&\quad + \frac{8\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx}{3c^4d^2} + \frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{x^2(a+b\text{arcsinh}(cx))}{1+c^2x^2}\,dx}{c^3d^2\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(8b\sqrt{1+c^2x^2}\right)\int \frac{x^2(a+b\text{arcsinh}(cx))}{1+c^2x^2}\,dx}{3c^3d^2\sqrt{d+c^2dx^2}} + \frac{\left(b^2\sqrt{1+c^2x^2}\right)\int \frac{x^3}{(1+c^2x^2)^{3/2}}\,dx}{3c^2d^2\sqrt{d+c^2dx^2}}
\end{aligned}$$

$$= -\frac{bx^3(a+b\operatorname{arcsinh}(cx))}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}+\frac{11bx\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3c^5d^2\sqrt{d+c^2dx^2}}$$
$$-\frac{x^4(a+b\operatorname{arcsinh}(cx))^2}{3c^2d\,(d+c^2dx^2)^{3/2}}-\frac{4x^2(a+b\operatorname{arcsinh}(cx))^2}{3c^4d^2\sqrt{d+c^2dx^2}}$$
$$+\frac{8\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{3c^6d^3}-\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{a+b\operatorname{arcsinh}(cx)}{1+c^2x^2}\,dx}{c^5d^2\sqrt{d+c^2dx^2}}$$
$$-\frac{\left(8b\sqrt{1+c^2x^2}\right)\int\frac{a+b\operatorname{arcsinh}(cx)}{1+c^2x^2}\,dx}{3c^5d^2\sqrt{d+c^2dx^2}}-\frac{\left(16b\sqrt{1+c^2x^2}\right)\int(a+b\operatorname{arcsinh}(cx))\,dx}{3c^5d^2\sqrt{d+c^2dx^2}}$$
$$-\frac{\left(b^2\sqrt{1+c^2x^2}\right)\int\frac{x}{\sqrt{1+c^2x^2}}\,dx}{c^4d^2\sqrt{d+c^2dx^2}}-\frac{\left(8b^2\sqrt{1+c^2x^2}\right)\int\frac{x}{\sqrt{1+c^2x^2}}\,dx}{3c^4d^2\sqrt{d+c^2dx^2}}$$
$$+\frac{\left(b^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{x}{(1+c^2x)^{3/2}}\,dx,x,x^2\right)}{6c^2d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{16abx\sqrt{1+c^2x^2}}{3c^5d^2\sqrt{d+c^2dx^2}}-\frac{11b^2(1+c^2x^2)}{3c^6d^2\sqrt{d+c^2dx^2}}-\frac{bx^3(a+b\operatorname{arcsinh}(cx))}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}$$
$$+\frac{11bx\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3c^5d^2\sqrt{d+c^2dx^2}}-\frac{x^4(a+b\operatorname{arcsinh}(cx))^2}{3c^2d\,(d+c^2dx^2)^{3/2}}$$
$$-\frac{4x^2(a+b\operatorname{arcsinh}(cx))^2}{3c^4d^2\sqrt{d+c^2dx^2}}+\frac{8\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{3c^6d^3}$$
$$-\frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}(\int(a+bx)\operatorname{sech}(x)\,dx,x,\operatorname{arcsinh}(cx))}{c^6d^2\sqrt{d+c^2dx^2}}$$
$$-\frac{\left(8b\sqrt{1+c^2x^2}\right)\operatorname{Subst}(\int(a+bx)\operatorname{sech}(x)\,dx,x,\operatorname{arcsinh}(cx))}{3c^6d^2\sqrt{d+c^2dx^2}}$$
$$-\frac{\left(16b^2\sqrt{1+c^2x^2}\right)\int\operatorname{arcsinh}(cx)\,dx}{3c^5d^2\sqrt{d+c^2dx^2}}$$
$$+\frac{\left(b^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\left(-\frac{1}{c^2(1+c^2x)^{3/2}}+\frac{1}{c^2\sqrt{1+c^2x}}\right)\,dx,x,x^2\right)}{6c^2d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2}{3c^6d^2\sqrt{d+c^2dx^2}} - \frac{16abx\sqrt{1+c^2x^2}}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{10b^2(1+c^2x^2)}{3c^6d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{16b^2x\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{bx^3(a+b\text{arcsinh}(cx))}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}$$
$$+ \frac{11bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{x^4(a+b\text{arcsinh}(cx))^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}}$$
$$- \frac{4x^2(a+b\text{arcsinh}(cx))^2}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{8\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^6d^3}$$
$$- \frac{22b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3c^6d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1-ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{c^6d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1+ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{c^6d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(8ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1-ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{3c^6d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(8ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1+ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{3c^6d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(16b^2\sqrt{1+c^2x^2}\right)\int\frac{x}{\sqrt{1+c^2x^2}}\,dx}{3c^4d^2\sqrt{d+c^2dx^2}}$$

$$
\begin{aligned}
&= \frac{b^2}{3c^6d^2\sqrt{d+c^2dx^2}} - \frac{16abx\sqrt{1+c^2x^2}}{3c^5d^2\sqrt{d+c^2dx^2}} + \frac{2b^2(1+c^2x^2)}{c^6d^2\sqrt{d+c^2dx^2}} \\
&\quad - \frac{16b^2x\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{bx^3(a+b\text{arcsinh}(cx))}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} \\
&\quad + \frac{11bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{x^4(a+b\text{arcsinh}(cx))^2}{3c^2d\,(d+c^2dx^2)^{3/2}} \\
&\quad - \frac{4x^2(a+b\text{arcsinh}(cx))^2}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{8\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^6d^3} \\
&\quad - \frac{22b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3c^6d^2\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{c^6d^2\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\left(ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{c^6d^2\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(8ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{3c^6d^2\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\left(8ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{3c^6d^2\sqrt{d+c^2dx^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b^2}{3c^6d^2\sqrt{d+c^2dx^2}} - \frac{16abx\sqrt{1+c^2x^2}}{3c^5d^2\sqrt{d+c^2dx^2}} + \frac{2b^2(1+c^2x^2)}{c^6d^2\sqrt{d+c^2dx^2}} \\
&\quad - \frac{16b^2x\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{bx^3(a+b\text{arcsinh}(cx))}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} \\
&\quad + \frac{11bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{x^4(a+b\text{arcsinh}(cx))^2}{3c^2d\,(d+c^2dx^2)^{3/2}} \\
&\quad - \frac{4x^2(a+b\text{arcsinh}(cx))^2}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{8\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^6d^3} \\
&\quad - \frac{22b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3c^6d^2\sqrt{d+c^2dx^2}} \\
&\quad + \frac{11ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3c^6d^2\sqrt{d+c^2dx^2}} \\
&\quad - \frac{11ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3c^6d^2\sqrt{d+c^2dx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.65

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \frac{\sqrt{d+c^2dx^2}\Big(a^2(8+12c^2x^2+3c^4x^4)+ab\big(2(8+12c^2x^2+3c^4x^4)\,\text{arcsinh}(cx)}{}$$

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(a^2*(8 + 12*c^2*x^2 + 3*c^4*x^4) + a*b*(2*(8 + 12*c^2
*x^2 + 3*c^4*x^4)*ArcSinh[c*x] - Sqrt[1 + c^2*x^2]*(c*x*(5 + 6*c^2*x^2) + 2
2*(1 + c^2*x^2)*ArcTan[Tanh[ArcSinh[c*x]/2]])) + b^2*(c*x*Sqrt[1 + c^2*x^2]
*ArcSinh[c*x] - 6*c*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x] - ArcSinh[c*x]^2 + 3
*(1 + c^2*x^2)^2*(2 + ArcSinh[c*x]^2) + (1 + c^2*x^2)*(1 + 6*ArcSinh[c*x]^2
) + (11*I)*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*(Log[1 - I/E^ArcSinh[c*x]] - Lo
g[1 + I/E^ArcSinh[c*x]]) + (11*I)*(1 + c^2*x^2)^(3/2)*(PolyLog[2, (-I)/E^Ar
cSinh[c*x]] - PolyLog[2, I/E^ArcSinh[c*x]]))))/(3*c^6*d^3*(1 + c^2*x^2)^2)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1041 vs. 2(475) = 950.

Time = 0.40 (sec) , antiderivative size = 1042, normalized size of antiderivative = 2.04

method	result	size
default	Expression too large to display	1042
parts	Expression too large to display	1042

```
[In] int(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(x^4/c^2/d/(c^2*d*x^2+d)^(3/2)-4/c^2*(-x^2/c^2/d/(c^2*d*x^2+d)^(3/2)-2/
3/d/c^4/(c^2*d*x^2+d)^(3/2)))-11/3*I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)/c^6/d^3*ln(c*x+(c^2*x^2+1)^(1/2)+I)-11/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/
(c^2*x^2+1)^(1/2)/c^6/d^3*dilog(1-I*(c*x+(c^2*x^2+1)^(1/2)))+2*b^2*(d*(c^2*
x^2+1))^(1/2)/c^6/d^3/(c^2*x^2+1)+b^2*(d*(c^2*x^2+1))^(1/2)/c^4/d^3/(c^2*x^
2+1)*arcsinh(c*x)^2*x^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/c^4/d^3*a
rcsinh(c*x)^2*x^2+11/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^
3*arcsinh(c*x)*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))-2*b^2*(d*(c^2*x^2+1))^(1/2)/
c^5/d^3/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2
*x^2+1)^(3/2)/c^5/d^3*arcsinh(c*x)*x-11/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*
x^2+1)^(1/2)/c^6/d^3*arcsinh(c*x)*ln(1-I*(c*x+(c^2*x^2+1)^(1/2)))+2*b^2*(d*
(c^2*x^2+1))^(1/2)/c^4/d^3/(c^2*x^2+1)*x^2+b^2*(d*(c^2*x^2+1))^(1/2)/c^6/d^
3/(c^2*x^2+1)*arcsinh(c*x)^2+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/c^
6/d^3+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/c^4/d^3*x^2+5/3*b^2*(d*(c
```

```
^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/c^6/d^3*arcsinh(c*x)^2+2*a*b*(d*(c^2*x^2+1))
^(1/2)/c^4/d^3/(c^2*x^2+1)*arcsinh(c*x)*x^2-2*a*b*(d*(c^2*x^2+1))^(1/2)/c^5
/d^3/(c^2*x^2+1)^(1/2)*x+2*a*b*(d*(c^2*x^2+1))^(1/2)/c^6/d^3/(c^2*x^2+1)*ar
csinh(c*x)+4*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/c^4/d^3*arcsinh(c*x)*x
^2+1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(3/2)/c^5/d^3*x+10/3*a*b*(d*(c
^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/c^6/d^3*arcsinh(c*x)+11/3*I*b^2*(d*(c^2*x^2+
1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^3*dilog(1+I*(c*x+(c^2*x^2+1)^(1/2)))+11/3
*I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^3*ln(c*x+(c^2*x^2+1)^(
1/2)-I)
```

Fricas [F]

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^5}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^5*arcsinh(c*x)^2 + 2*a*b*x^5*arcsinh(c*x) + a^2*x^5)*sqrt(c
^2*d*x^2 + d)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F]

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x^5(a+b\,\text{asinh}\,(cx))^2}{(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**5*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**5*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^5}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

```
[Out] 1/3*a^2*(3*x^4/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 12*x^2/((c^2*d*x^2 + d)^(3/2
)*c^4*d) + 8/((c^2*d*x^2 + d)^(3/2)*c^6*d)) + 1/3*(3*b^2*c^4*sqrt(d)*x^4 +
```

```
12*b^2*c^2*sqrt(d)*x^2 + 8*b^2*sqrt(d))*sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^
2*x^2 + 1))^2/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3) + integrate(-2/3*((1
2*b^2*c^3*x^3 - 3*(a*b*c^5 - b^2*c^5)*x^5 + 8*b^2*c*x)*(c^2*x^2 + 1) + (15*
b^2*c^4*x^4 - 3*(a*b*c^6 - b^2*c^6)*x^6 + 20*b^2*c^2*x^2 + 8*b^2)*sqrt(c^2*
x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/(c^12*d^(5/2)*x^7 + 3*c^10*d^(5/2)*x
^5 + 3*c^8*d^(5/2)*x^3 + c^6*d^(5/2)*x + (c^11*d^(5/2)*x^6 + 3*c^9*d^(5/2)*
x^4 + 3*c^7*d^(5/2)*x^2 + c^5*d^(5/2))*sqrt(c^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x^5\,(a+b\,\mathrm{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^{5/2}}\,dx$$

```
[In] int((x^5*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^5*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2), x)
```

3.311 $\int \frac{x^4(a+b\mathbf{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 398

$$\begin{aligned}\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx =& -\frac{b^2x}{3c^4d^2\sqrt{d+c^2dx^2}}+\frac{b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{3c^5d^2\sqrt{d+c^2dx^2}}\\&-\frac{bx^2(a+b\text{arcsinh}(cx))}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d\,(d+c^2dx^2)^{3/2}}-\frac{x(a+b\text{arcsinh}(cx))^2}{c^4d^2\sqrt{d+c^2dx^2}}\\&-\frac{4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3c^5d^2\sqrt{d+c^2dx^2}}+\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc^5d^2\sqrt{d+c^2dx^2}}\\&+\frac{8b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3c^5d^2\sqrt{d+c^2dx^2}}\\&+\frac{4b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3c^5d^2\sqrt{d+c^2dx^2}}\end{aligned}$$

```
[Out] -1/3*x^3*(a+b*arcsinh(c*x))^2/c^2/d/(c^2*d*x^2+d)^(3/2)-1/3*b^2*x/c^4/d^2/(
c^2*d*x^2+d)^(1/2)-x*(a+b*arcsinh(c*x))^2/c^4/d^2/(c^2*d*x^2+d)^(1/2)-1/3*b
*x^2*(a+b*arcsinh(c*x))/c^3/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+1/3*b
^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c^5/d^2/(c^2*d*x^2+d)^(1/2)-4/3*(a+b*arcs
inh(c*x))^2*(c^2*x^2+1)^(1/2)/c^5/d^2/(c^2*d*x^2+d)^(1/2)+1/3*(a+b*arcsinh(
c*x))^3*(c^2*x^2+1)^(1/2)/b/c^5/d^2/(c^2*d*x^2+d)^(1/2)+8/3*b*(a+b*arcsinh(
c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c^5/d^2/(c^2*d*x^2+
d)^(1/2)+4/3*b^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c^
5/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {5810, 5783, 5797, 3799, 2221, 2317, 2438, 294, 221}

$$\begin{aligned}
\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx &= -\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d\,(c^2dx^2+d)^{3/2}} \\
&+\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{3bc^5d^2\sqrt{c^2dx^2+d}}-\frac{4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{3c^5d^2\sqrt{c^2dx^2+d}} \\
&+\frac{8b\sqrt{c^2x^2+1}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))}{3c^5d^2\sqrt{c^2dx^2+d}}-\frac{x(a+b\text{arcsinh}(cx))^2}{c^4d^2\sqrt{c^2dx^2+d}} \\
&-\frac{bx^2(a+b\text{arcsinh}(cx))}{3c^3d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}+\frac{4b^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3c^5d^2\sqrt{c^2dx^2+d}} \\
&+\frac{b^2\sqrt{c^2x^2+1}\text{arcsinh}(cx)}{3c^5d^2\sqrt{c^2dx^2+d}}-\frac{b^2x}{3c^4d^2\sqrt{c^2dx^2+d}}
\end{aligned}$$

```
[In] Int[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -1/3*(b^2*x)/(c^4*d^2*Sqrt[d + c^2*d*x^2]) + (b^2*Sqrt[1 + c^2*x^2]*ArcSinh
[c*x])/(3*c^5*d^2*Sqrt[d + c^2*d*x^2]) - (b*x^2*(a + b*ArcSinh[c*x]))/(3*c^
3*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (x^3*(a + b*ArcSinh[c*x])^2)
/(3*c^2*d*(d + c^2*d*x^2)^(3/2)) - (x*(a + b*ArcSinh[c*x])^2)/(c^4*d^2*Sqrt
[d + c^2*d*x^2]) - (4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^5*d^2*
Sqrt[d + c^2*d*x^2]) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c^5*
d^2*Sqrt[d + c^2*d*x^2]) + (8*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[
1 + E^(2*ArcSinh[c*x])])/(3*c^5*d^2*Sqrt[d + c^2*d*x^2]) + (4*b^2*Sqrt[1 +
c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c^5*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rubi steps

$$\text{integral} = -\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} + \frac{\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx}{c^2d} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{x^3(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^2}\,dx}{3cd^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx^2(a+b\text{arcsinh}(cx))}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{x(a+b\text{arcsinh}(cx))^2}{c^4d^2\sqrt{d+c^2dx^2}} + \frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx}{c^4d^2} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{x(a+b\text{arcsinh}(cx))}{1+c^2x^2}\,dx}{3c^3d^2\sqrt{d+c^2dx^2}} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{x(a+b\text{arcsinh}(cx))}{1+c^2x^2}\,dx}{c^3d^2\sqrt{d+c^2dx^2}} + \frac{\left(b^2\sqrt{1+c^2x^2}\right)\int \frac{x^2}{(1+c^2x^2)^{3/2}}\,dx}{3c^2d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2x}{3c^4d^2\sqrt{d+c^2dx^2}} - \frac{bx^2(a+b\text{arcsinh}(cx))}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{x(a+b\text{arcsinh}(cx))^2}{c^4d^2\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc^5d^2\sqrt{d+c^2dx^2}} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \text{arcsinh}(cx)\right)}{3c^5d^2\sqrt{d+c^2dx^2}} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \text{arcsinh}(cx)\right)}{c^5d^2\sqrt{d+c^2dx^2}} + \frac{\left(b^2\sqrt{1+c^2x^2}\right)\int \frac{1}{\sqrt{1+c^2x^2}}\,dx}{3c^4d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2x}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{b^2\sqrt{1+c^2x^2}\,\text{arcsinh}(cx)}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{bx^2(a+b\text{arcsinh}(cx))}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{x(a+b\text{arcsinh}(cx))^2}{c^4d^2\sqrt{d+c^2dx^2}} - \frac{4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3c^5d^2\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc^5d^2\sqrt{d+c^2dx^2}} + \frac{\left(4b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \text{arcsinh}(cx)\right)}{3c^5d^2\sqrt{d+c^2dx^2}} + \frac{\left(4b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \text{arcsinh}(cx)\right)}{c^5d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2x}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{bx^2(a+b\text{arcsinh}(cx))}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{x(a+b\text{arcsinh}(cx))^2}{c^4d^2\sqrt{d+c^2dx^2}} - \frac{4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3c^5d^2\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc^5d^2\sqrt{d+c^2dx^2}} + \frac{8b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{\left(2b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1+e^{2x}\right)\,dx,x,\text{arcsinh}(cx)\right)}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{\left(2b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1+e^{2x}\right)\,dx,x,\text{arcsinh}(cx)\right)}{c^5d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2x}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{bx^2(a+b\text{arcsinh}(cx))}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{x(a+b\text{arcsinh}(cx))^2}{c^4d^2\sqrt{d+c^2dx^2}} - \frac{4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3c^5d^2\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc^5d^2\sqrt{d+c^2dx^2}} + \frac{8b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{\left(b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\text{arcsinh}(cx)}\right)}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{\left(b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\text{arcsinh}(cx)}\right)}{c^5d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2x}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{bx^2(a+b\text{arcsinh}(cx))}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{x(a+b\text{arcsinh}(cx))^2}{c^4d^2\sqrt{d+c^2dx^2}} - \frac{4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3c^5d^2\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc^5d^2\sqrt{d+c^2dx^2}} + \frac{8b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3c^5d^2\sqrt{d+c^2dx^2}} + \frac{4b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3c^5d^2\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.90

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \frac{-a^2c\sqrt{d}x(3+4c^2x^2)+ab\sqrt{d}\left(\sqrt{1+c^2x^2}+2cx\text{arcsinh}(cx)-8cx(1+c^2x^2)\right.}{}$$

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (-(a^2*c*Sqrt[d]*x*(3 + 4*c^2*x^2)) + a*b*Sqrt[d]*(Sqrt[1 + c^2*x^2] + 2*c*
x*ArcSinh[c*x] - 8*c*x*(1 + c^2*x^2)*ArcSinh[c*x] + (1 + c^2*x^2)^(3/2)*(3*
ArcSinh[c*x]^2 + 4*Log[1 + c^2*x^2])) + 3*a^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*
x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - b^2*Sqrt[d]*(c*x + c^3*x^3
- Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 3*c*x*ArcSinh[c*x]^2 + 4*c^3*x^3*ArcSinh
[c*x]^2 - 4*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]^2 - (1 + c^2*x^2)^(3/2)*ArcSin
h[c*x]^3 - 8*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 + E^(-2*ArcSinh[c*x])]
+ 4*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(-2*ArcSinh[c*x])]))/(3*c^5*d^(5/2)*(
1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 765 vs. 2(370) = 740.

Time = 0.31 (sec) , antiderivative size = 766, normalized size of antiderivative = 1.92

method	result
default	$-\frac{a^2x^3}{3c^2d(c^2d\,x^2+d)^{\frac{3}{2}}}-\frac{a^2x}{c^4d^2\sqrt{c^2d\,x^2+d}}+\frac{a^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{c^4d^2\sqrt{c^2d}}+\frac{b^2\sqrt{d(c^2x^2+1)}\,\sqrt{c^2x^2+1}\,\left(\text{arcsinh}(cx)^3x^4c^4-4\,\text{arcsinh}\right.}{}$
parts	$-\frac{a^2x^3}{3c^2d(c^2d\,x^2+d)^{\frac{3}{2}}}-\frac{a^2x}{c^4d^2\sqrt{c^2d\,x^2+d}}+\frac{a^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{c^4d^2\sqrt{c^2d}}+\frac{b^2\sqrt{d(c^2x^2+1)}\,\sqrt{c^2x^2+1}\,\left(\text{arcsinh}(cx)^3x^4c^4-4\,\text{arcsinh}\right.}{}$

```
[In] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a^2*x^3/c^2/d/(c^2*d*x^2+d)^(3/2)-a^2/c^4/d^2*x/(c^2*d*x^2+d)^(1/2)+a^
2/c^4/d^2*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/3*b
^2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/(c^6*x^6+3*c^4*x^4+3*c^2*x^2+1)/
c^5/d^3*(arcsinh(c*x)^3*x^4*c^4-4*arcsinh(c*x)^2*x^4*c^4+8*arcsinh(c*x)*ln(
1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^4*c^4-4*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*x^3
*c^3+4*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*x^4*c^4+c^4*x^4+2*arcsinh(c*x)
^3*x^2*c^2-c^3*x^3*(c^2*x^2+1)^(1/2)-8*arcsinh(c*x)^2*x^2*c^2+16*arcsinh(c*
x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^2*c^2-3*arcsinh(c*x)^2*(c^2*x^2+1)^(1/
2)*c*x+arcsinh(c*x)*c^2*x^2+8*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*x^2*c^2
+2*c^2*x^2+arcsinh(c*x)^3-c*x*(c^2*x^2+1)^(1/2)-4*arcsinh(c*x)^2+8*arcsinh(
c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+arcsinh(c*x)+4*polylog(2,-(c*x+(c^2*x^
```

```
2+1)^(1/2))^2)+1)+1/3*a*b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/(c^6*x^6+
3*c^4*x^4+3*c^2*x^2+1)/c^5/d^3*(3*arcsinh(c*x)^2*x^4*c^4-8*arcsinh(c*x)*c^4
*x^4+8*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^4*c^4-8*arcsinh(c*x)*(c^2*x^2+1)^(
1/2)*x^3*c^3+6*arcsinh(c*x)^2*x^2*c^2-16*arcsinh(c*x)*c^2*x^2+16*ln(1+(c*x+
(c^2*x^2+1)^(1/2))^2)*x^2*c^2-6*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)+c^2*x^2+
3*arcsinh(c*x)^2-8*arcsinh(c*x)+8*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1)
```

Fricas [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2x^4}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)*sqrt(c
^2*d*x^2 + d)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x^4(a+b\operatorname{asinh}(cx))^2}{(d(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**4*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2x^4}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/3*(x*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2/((c^2*d*x^2 + d)^(3/2)*c^4
*d)) + x/(sqrt(c^2*d*x^2 + d)*c^4*d^2) - 3*arcsinh(c*x)/(c^5*d^(5/2)))*a^2
+ integrate(b^2*x^4*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^2 + d)^(5/2) +
2*a*b*x^4*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d)^(5/2), x)
```

Giac [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^4}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)^2*x^4/(c^2*d*x^2 + d)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x^4\,(a+b\,\text{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^{5/2}}\,dx$$

[In] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2),x)

[Out] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2), x)

3.312 $\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 307

$$\begin{aligned}\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = &-\frac{b^2}{3c^4d^2\sqrt{d+c^2dx^2}}\\ &-\frac{bx(a+b\text{arcsinh}(cx))}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{3c^2d\,(d+c^2dx^2)^{3/2}}\\ &-\frac{2(a+b\text{arcsinh}(cx))^2}{3c^4d^2\sqrt{d+c^2dx^2}}+\frac{10b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3c^4d^2\sqrt{d+c^2dx^2}}\\ &-\frac{5ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3c^4d^2\sqrt{d+c^2dx^2}}+\frac{5ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3c^4d^2\sqrt{d+c^2dx^2}}\end{aligned}$$

```
[Out] -1/3*x^2*(a+b*arcsinh(c*x))^2/c^2/d/(c^2*d*x^2+d)^(3/2)-1/3*b^2/c^4/d^2/(c^
2*d*x^2+d)^(1/2)-2/3*(a+b*arcsinh(c*x))^2/c^4/d^2/(c^2*d*x^2+d)^(1/2)-1/3*b
*x*(a+b*arcsinh(c*x))/c^3/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+10/3*b*
(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/c^4/d^2/
(c^2*d*x^2+d)^(1/2)-5/3*I*b^2*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^
2+1)^(1/2)/c^4/d^2/(c^2*d*x^2+d)^(1/2)+5/3*I*b^2*polylog(2,I*(c*x+(c^2*x^2+
1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^4/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5810, 5798, 5789, 4265, 2317, 2438, 267}

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \frac{10b\sqrt{c^2x^2+1}\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3c^4d^2\sqrt{c^2dx^2+d}} - \frac{x^2(a+b\text{arcsinh}(cx))^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}} - \frac{2(a+b\text{arcsinh}(cx))^2}{3c^4d^2\sqrt{c^2dx^2+d}} - \frac{bx(a+b\text{arcsinh}(cx))}{3c^3d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}} - \frac{5ib^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3c^4d^2\sqrt{c^2dx^2+d}} + \frac{5ib^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3c^4d^2\sqrt{c^2dx^2+d}} - \frac{b^2}{3c^4d^2\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -1/3*b^2/(c^4*d^2*Sqrt[d + c^2*d*x^2]) - (b*x*(a + b*ArcSinh[c*x]))/(3*c^3*
d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (x^2*(a + b*ArcSinh[c*x])^2)/(
3*c^2*d*(d + c^2*d*x^2)^(3/2)) - (2*(a + b*ArcSinh[c*x])^2)/(3*c^4*d^2*Sqrt
[d + c^2*d*x^2]) + (10*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^Ar
cSinh[c*x]])/(3*c^4*d^2*Sqrt[d + c^2*d*x^2]) - (((5*I)/3)*b^2*Sqrt[1 + c^2*
x^2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^4*d^2*Sqrt[d + c^2*d*x^2]) + (((5*
I)/3)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(c^4*d^2*Sqrt[d +
 c^2*d*x^2])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rubi steps

$$\text{integral} = -\frac{x^2(a+b\operatorname{arcsinh}(cx))^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}}+\frac{2\int\frac{x(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx}{3c^2d}+\frac{\left(2b\sqrt{1+c^2x^2}\right)\int\frac{x^2(a+b\operatorname{arcsinh}(cx))}{(1+c^2x^2)^2}\,dx}{3cd^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx(a+b\operatorname{arcsinh}(cx))}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^2(a+b\operatorname{arcsinh}(cx))^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{2(a+b\operatorname{arcsinh}(cx))^2}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{a+b\operatorname{arcsinh}(cx)}{1+c^2x^2}\,dx}{3c^3d^2\sqrt{d+c^2dx^2}} + \frac{\left(4b\sqrt{1+c^2x^2}\right)\int\frac{a+b\operatorname{arcsinh}(cx)}{1+c^2x^2}\,dx}{3c^3d^2\sqrt{d+c^2dx^2}} + \frac{\left(b^2\sqrt{1+c^2x^2}\right)\int\frac{x}{(1+c^2x^2)^{3/2}}\,dx}{3c^2d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2}{3c^4d^2\sqrt{d+c^2dx^2}} - \frac{bx(a+b\operatorname{arcsinh}(cx))}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^2(a+b\operatorname{arcsinh}(cx))^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{2(a+b\operatorname{arcsinh}(cx))^2}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int(a+bx)\operatorname{sech}(x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{\left(4b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int(a+bx)\operatorname{sech}(x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{3c^4d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2}{3c^4d^2\sqrt{d+c^2dx^2}} - \frac{bx(a+b\operatorname{arcsinh}(cx))}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^2(a+b\operatorname{arcsinh}(cx))^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{2(a+b\operatorname{arcsinh}(cx))^2}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{10b\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{3c^4d^2\sqrt{d+c^2dx^2}} - \frac{\left(ib^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\log\left(1-ie^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{\left(ib^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\log\left(1+ie^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{3c^4d^2\sqrt{d+c^2dx^2}} - \frac{\left(4ib^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\log\left(1-ie^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{\left(4ib^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\log\left(1+ie^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{3c^4d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2}{3c^4d^2\sqrt{d+c^2dx^2}} - \frac{bx(a+b\operatorname{arcsinh}(cx))}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^2(a+b\operatorname{arcsinh}(cx))^2}{3c^2d\,(d+c^2dx^2)^{3/2}} - \frac{2(a+b\operatorname{arcsinh}(cx))^2}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{10b\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{3c^4d^2\sqrt{d+c^2dx^2}} - \frac{\left(ib^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{\left(ib^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{3c^4d^2\sqrt{d+c^2dx^2}} - \frac{\left(4ib^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{\left(4ib^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{3c^4d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2}{3c^4d^2\sqrt{d+c^2dx^2}} - \frac{bx(a+b\operatorname{arcsinh}(cx))}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^2(a+b\operatorname{arcsinh}(cx))^2}{3c^2d\,(d+c^2dx^2)^{3/2}} - \frac{2(a+b\operatorname{arcsinh}(cx))^2}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{10b\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{3c^4d^2\sqrt{d+c^2dx^2}} - \frac{5ib^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{5ib^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{3c^4d^2\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.98

$$\int\frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \frac{-a^2(2+3c^2x^2)+ab\left(-2(2+3c^2x^2)\operatorname{arcsinh}(cx)+\sqrt{1+c^2x^2}\left(-cx+10(1+c\right.\right.}{}$$

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (-(a^2*(2 + 3*c^2*x^2)) + a*b*(-2*(2 + 3*c^2*x^2)*ArcSinh[c*x] + Sqrt[1 + c
^2*x^2]*(-(c*x) + 10*(1 + c^2*x^2)*ArcTan[Tanh[ArcSinh[c*x]/2]])) - b^2*(1
+ c^2*x^2 + c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 2*ArcSinh[c*x]^2 + 3*c^2*x
^2*ArcSinh[c*x]^2 + (5*I)*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 - I/E^ArcS
inh[c*x]] - (5*I)*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 + I/E^ArcSinh[c*x]
] + (5*I)*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)/E^ArcSinh[c*x]] - (5*I)*(1 +
c^2*x^2)^(3/2)*PolyLog[2, I/E^ArcSinh[c*x]]))/(3*c^4*d^2*(1 + c^2*x^2)*Sqrt
[d + c^2*d*x^2])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 703 vs. $2(294) = 588$.

Time = 0.20 (sec) , antiderivative size = 704, normalized size of antiderivative = 2.29

method	result
default	$a^2\left(-\frac{x^2}{c^2 d (c^2 d x^2+d)^{\frac{3}{2}}}-\frac{2}{3 d c^4 (c^2 d x^2+d)^{\frac{3}{2}}}\right)-\frac{b^2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2 x^2}{(c^2x^2+1)^2 d^3 c^2}-\frac{b^2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx) x}{3(c^2x^2+1)^{\frac{3}{2}} d^3 c^3}-\frac{b^2\sqrt{d(c}}{3(c^2x^2}$
parts	$a^2\left(-\frac{x^2}{c^2 d (c^2 d x^2+d)^{\frac{3}{2}}}-\frac{2}{3 d c^4 (c^2 d x^2+d)^{\frac{3}{2}}}\right)-\frac{b^2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2 x^2}{(c^2x^2+1)^2 d^3 c^2}-\frac{b^2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx) x}{3(c^2x^2+1)^{\frac{3}{2}} d^3 c^3}-\frac{b^2\sqrt{d(c}}{3(c^2x^2}$

```
[In] int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-x^2/c^2/d/(c^2*d*x^2+d)^(3/2)-2/3/d/c^4/(c^2*d*x^2+d)^(3/2))-b^2*(d*(
c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/d^3/c^2*arcsinh(c*x)^2*x^2-1/3*b^2*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(3/2)/d^3/c^3*arcsinh(c*x)*x-1/3*b^2*(d*(c^2*x^2+
1))^(1/2)/(c^2*x^2+1)^2/d^3/c^2*x^2-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+
1)^2/d^3/c^4*arcsinh(c*x)^2-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/d^3
/c^4-5/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^3*arcsinh(c*x)
*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))+5/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1
)^(1/2)/c^4/d^3*arcsinh(c*x)*ln(1-I*(c*x+(c^2*x^2+1)^(1/2)))-5/3*I*b^2*(d*(
c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^3*dilog(1+I*(c*x+(c^2*x^2+1)^(1/2
)))+5/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^3*dilog(1-I*(c*
x+(c^2*x^2+1)^(1/2)))-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/d^3/c^2*arc
sinh(c*x)*x^2-1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(3/2)/d^3/c^3*x-4/3
*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/d^3/c^4*arcsinh(c*x)+5/3*I*a*b*(d*
(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^3*ln(c*x+(c^2*x^2+1)^(1/2)+I)-5/
3*I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^3*ln(c*x+(c^2*x^2+1)^
(1/2)-I)
```

Fricas [F]

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2x^3}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*x) + a^2*x^3)*sqrt(c
^2*d*x^2 + d)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F]

$$\int \frac{x^3(a + b\text{arcsinh}(cx))^2}{(d + c^2dx^2)^{5/2}}\,dx = \int \frac{x^3(a + b\,\text{asinh}\,(cx))^2}{(d\,(c^2x^2 + 1))^{\frac{5}{2}}}\,dx$$

[In] integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)

[Out] Integral(x**3*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)

Maxima [F]

$$\int \frac{x^3(a + b\text{arcsinh}(cx))^2}{(d + c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2 x^3}{(c^2dx^2 + d)^{\frac{5}{2}}}\,dx$$

[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")

[Out] -1/3*a*b*c*(x/(c^6*d^(5/2)*x^2 + c^4*d^(5/2)) - 5*arctan(c*x)/(c^5*d^(5/2))
) - 2/3*a*b*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2/((c^2*d*x^2 + d)^(3/2)
*c^4*d))*arcsinh(c*x) - 1/3*a^2*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2/((
c^2*d*x^2 + d)^(3/2)*c^4*d)) + b^2*integrate(x^3*log(c*x + sqrt(c^2*x^2 + 1
))^2/(c^2*d*x^2 + d)^(5/2), x)

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\text{arcsinh}(cx))^2}{(d + c^2dx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x^3\,(a+b\,\text{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^{5/2}}\,dx$$

```
[In] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2),x)
[Out] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2), x)
```

3.313 $\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 312

$$\begin{aligned}\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx &= \frac{b^2x}{3c^2d^2\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{3c^3d^2\sqrt{d+c^2dx^2}} \\ &+ \frac{bx^2(a+b\text{arcsinh}(cx))}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x^3(a+b\text{arcsinh}(cx))^2}{3d\,(d+c^2dx^2)^{3/2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3c^3d^2\sqrt{d+c^2dx^2}} \\ &- \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3c^3d^2\sqrt{d+c^2dx^2}} \\ &- \frac{b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3c^3d^2\sqrt{d+c^2dx^2}}\end{aligned}$$

```
[Out] 1/3*x^3*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(3/2)+1/3*b^2*x/c^2/d^2/(c^2*d
*x^2+d)^(1/2)+1/3*b*x^2*(a+b*arcsinh(c*x))/c/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x
^2+d)^(1/2)-1/3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c^3/d^2/(c^2*d*x^2+d)^(1
/2)+1/3*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/c^3/d^2/(c^2*d*x^2+d)^(1/2)-
2/3*b*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/
c^3/d^2/(c^2*d*x^2+d)^(1/2)-1/3*b^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*(
c^2*x^2+1)^(1/2)/c^3/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {5800, 5810, 5797, 3799, 2221, 2317, 2438, 294, 221}

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \frac{bx^2(a+b\text{arcsinh}(cx))}{3cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}+\frac{x^3(a+b\text{arcsinh}(cx))^2}{3d\,(c^2dx^2+d)^{3/2}} +\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{3c^3d^2\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))}{3c^3d^2\sqrt{c^2dx^2+d}} -\frac{b^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3c^3d^2\sqrt{c^2dx^2+d}}-\frac{b^2\sqrt{c^2x^2+1}\text{arcsinh}(cx)}{3c^3d^2\sqrt{c^2dx^2+d}} +\frac{b^2x}{3c^2d^2\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (b^2*x)/(3*c^2*d^2*Sqrt[d + c^2*d*x^2]) - (b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*
x])/(3*c^3*d^2*Sqrt[d + c^2*d*x^2]) + (b*x^2*(a + b*ArcSinh[c*x]))/(3*c*d^2
*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (x^3*(a + b*ArcSinh[c*x])^2)/(3*d
*(d + c^2*d*x^2)^(3/2)) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^3
*d^2*Sqrt[d + c^2*d*x^2]) - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log
[1 + E^(2*ArcSinh[c*x])])/(3*c^3*d^2*Sqrt[d + c^2*d*x^2]) - (b^2*Sqrt[1 + c
^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c^3*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rubi steps

$$\text{integral} = \frac{x^3(a+b\text{arcsinh}(cx))^2}{3d\left(d+c^2dx^2\right)^{3/2}} - \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int\frac{x^3(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^2}\,dx}{3d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{bx^2(a+b\text{arcsinh}(cx))}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x^3(a+b\text{arcsinh}(cx))^2}{3d\left(d+c^2dx^2\right)^{3/2}} - \frac{\left(b^2\sqrt{1+c^2x^2}\right)\int\frac{x^2}{(1+c^2x^2)^{3/2}}\,dx}{3d^2\sqrt{d+c^2dx^2}} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\int\frac{x(a+b\text{arcsinh}(cx))}{1+c^2x^2}\,dx}{3cd^2\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2x}{3c^2d^2\sqrt{d+c^2dx^2}} + \frac{bx^2(a+b\text{arcsinh}(cx))}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x^3(a+b\text{arcsinh}(cx))^2}{3d\left(d+c^2dx^2\right)^{3/2}} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\tanh(x)\,dx,x,\text{arcsinh}(cx)\right)}{3c^3d^2\sqrt{d+c^2dx^2}} - \frac{\left(b^2\sqrt{1+c^2x^2}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{3c^2d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2x}{3c^2d^2\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{3c^3d^2\sqrt{d+c^2dx^2}} + \frac{bx^2(a+b\text{arcsinh}(cx))}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x^3(a+b\text{arcsinh}(cx))^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3c^3d^2\sqrt{d+c^2dx^2}} - \frac{\left(4b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx,x,\text{arcsinh}(cx)\right)}{3c^3d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2x}{3c^2d^2\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{3c^3d^2\sqrt{d+c^2dx^2}} + \frac{bx^2(a+b\text{arcsinh}(cx))}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x^3(a+b\text{arcsinh}(cx))^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3c^3d^2\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3c^3d^2\sqrt{d+c^2dx^2}} + \frac{\left(2b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1+e^{2x}\right)\,dx,x,\text{arcsinh}(cx)\right)}{3c^3d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2x}{3c^2d^2\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{3c^3d^2\sqrt{d+c^2dx^2}} + \frac{bx^2(a+b\text{arcsinh}(cx))}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x^3(a+b\text{arcsinh}(cx))^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3c^3d^2\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3c^3d^2\sqrt{d+c^2dx^2}} + \frac{\left(b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\text{arcsinh}(cx)}\right)}{3c^3d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2x}{3c^2d^2\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{3c^3d^2\sqrt{d+c^2dx^2}} + \frac{bx^2(a+b\text{arcsinh}(cx))}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x^3(a+b\text{arcsinh}(cx))^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3c^3d^2\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3c^3d^2\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3c^3d^2\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.90

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx = \frac{b^2cx+a^2c^3x^3+b^2c^3x^3-ab\sqrt{1+c^2x^2}-b^2\left(-c^3x^3+\sqrt{1+c^2x^2}+c^2x^2\sqrt{1+c\right.}{}$$

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (b^2*c*x + a^2*c^3*x^3 + b^2*c^3*x^3 - a*b*Sqrt[1 + c^2*x^2] - b^2*(-(c^3*x
^3) + Sqrt[1 + c^2*x^2] + c^2*x^2*Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2 - b*Arc
Sinh[c*x]*(-2*a*c^3*x^3 + b*Sqrt[1 + c^2*x^2] + 2*b*(1 + c^2*x^2)^(3/2)*Log
[1 + E^(-2*ArcSinh[c*x])]) - a*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] - a*b*c
^2*x^2*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] + b^2*(1 + c^2*x^2)^(3/2)*PolyLog
[2, -E^(-2*ArcSinh[c*x])])/(3*c^3*d^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2351 vs. 2(292) = 584.

Time = 0.29 (sec) , antiderivative size = 2352, normalized size of antiderivative = 7.54

method	result	size
default	Expression too large to display	2352
parts	Expression too large to display	2352

```
[In] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*
c^4/d^3*arcsinh(c*x)*x^7-b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*
c^4*x^4+5*c^2*x^2+1)*c^3/d^3*(c^2*x^2+1)^(1/2)*x^6-1/3*b^2*(d*(c^2*x^2+1))^
(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*(c^2*x^2+1)*arcsinh(
c*x)*x^3+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^
2*x^2+1)*c^2/d^3*arcsinh(c*x)*x^5+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+
```

9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^4/d^3*x^7+b^2*(d*(c^2*x^2+1))^(1/2)/(3*
c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d^3*x^5-1/3*b^2*(d*(c^2*x^2+1
))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/c^3/d^3*(c^2*x^2+1)^(
1/2)-1/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^3/c^3*polylog(2,-(c*
x+(c^2*x^2+1)^(1/2))^2)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+
10*c^4*x^4+5*c^2*x^2+1)/d^3*arcsinh(c*x)*x^3+2/3*b^2/(c^2*x^2+1)^(1/2)*(d*(
c^2*x^2+1))^(1/2)/d^3/c^3*arcsinh(c*x)^2+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c
^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*(c^2*x^2+1)*x^3+1/3*b^2*(d*(c^
2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*arcsinh(c*
x)^2*x^3-2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*
x^2+1)*c/d^3*(c^2*x^2+1)^(1/2)*x^4-2/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1)
)^(1/2)/d^3/c^3*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-4/3*b^2*(d*(c^
2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/c/d^3*x^2*(c^2
*x^2+1)^(1/2)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4
+5*c^2*x^2+1)/c^2/d^3*(c^2*x^2+1)*x-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^
8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/c^3/d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^
2-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1
)/c^3/d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c
^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d^3*(c^2*x^2+1)*x^5+b^2*(d*(c^
2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d^3*arcsin
h(c*x)^2*x^5+b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^
2*x^2+1)*c^4/d^3*arcsinh(c*x)^2*x^7+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^
8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*x^3+a^2*(-1/2*x/c^2/d/(c^2*d*x^2+d)
^(3/2)+1/2/c^2*(1/3/d*x/(c^2*d*x^2+d)^(3/2)+2/3/d^2*x/(c^2*d*x^2+d)^(1/2)))
-2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c
/d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*x^4-b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*
x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c/d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*
x^4-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2
+1)/c/d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*x^2-1/3*b^2*(d*(c^2*x^2+1))^(1/2
)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d^3*(c^2*x^2+1)*arcsinh(
c*x)*x^5-b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^
2+1)/c/d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^2-b^2*(d*(c^2*x^2+1))^(1/2)/(3*
c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^3/d^3*(c^2*x^2+1)^(1/2)*arcsinh
(c*x)^2*x^6+1/3*a*b*(d*(c^2*x^2+1))^(1/2)*(c^3*x^3+c^2*x^2*(c^2*x^2+1)^(1/2
)+(c^2*x^2+1)^(1/2))*(-2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^6*c^6+2*(c^2*x^2
+1)^(1/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^5*c^5+6*arcsinh(c*x)*c^4*x^4-6*
ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^4*c^4+2*(c^2*x^2+1)^(1/2)*ln(1+(c*x+(c^2*
x^2+1)^(1/2))^2)*x^3*c^3-c^4*x^4+c^3*x^3*(c^2*x^2+1)^(1/2)+6*arcsinh(c*x)*c
^2*x^2-6*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^2*c^2-2*c^2*x^2+2*arcsinh(c*x)-2
*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-1)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x
^2+1)/c^3/d^3

Fricas [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^2}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")

[Out] integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)*sqrt(c
^2*d*x^2 + d)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)

Sympy [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x^2(a+b\,\text{asinh}\,(cx))^2}{(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

[In] integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)

[Out] Integral(x**2*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)

Maxima [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^2}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")

[Out] -1/3*a*b*c*(1/(c^6*d^(5/2)*x^2 + c^4*d^(5/2)) + log(c^2*x^2 + 1)/(c^4*d^(5/
2))) + 2/3*a*b*(x/(sqrt(c^2*d*x^2 + d)*c^2*d^2) - x/((c^2*d*x^2 + d)^(3/2)*
c^2*d))*arcsinh(c*x) + 1/3*a^2*(x/(sqrt(c^2*d*x^2 + d)*c^2*d^2) - x/((c^2*d
*x^2 + d)^(3/2)*c^2*d)) + b^2*integrate(x^2*log(c*x + sqrt(c^2*x^2 + 1))^2/
(c^2*d*x^2 + d)^(5/2), x)

Giac [F]

$$\int \frac{x^2(a+b\mathrm{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2x^2}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2*x^2/(c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\mathrm{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x^2\,(a+b\,\mathrm{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^{5/2}}\,dx$$

```
[In] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2),x)
[Out] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2), x)
```

3.314 $\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 270

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \frac{b^2}{3c^2d^2\sqrt{d+c^2dx^2}}+\frac{bx(a+b\text{arcsinh}(cx))}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{3c^2d\,(d+c^2dx^2)^{3/2}}+\frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3c^2d^2\sqrt{d+c^2dx^2}} - \frac{ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3c^2d^2\sqrt{d+c^2dx^2}}+\frac{ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3c^2d^2\sqrt{d+c^2dx^2}}$$

```
[Out] -1/3*(a+b*arcsinh(c*x))^2/c^2/d/(c^2*d*x^2+d)^(3/2)+1/3*b^2/c^2/d^2/(c^2*d*
x^2+d)^(1/2)+1/3*b*x*(a+b*arcsinh(c*x))/c/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+
d)^(1/2)+2/3*b*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)
^(1/2)/c^2/d^2/(c^2*d*x^2+d)^(1/2)-1/3*I*b^2*polylog(2,-I*(c*x+(c^2*x^2+1)^
(1/2)))*(c^2*x^2+1)^(1/2)/c^2/d^2/(c^2*d*x^2+d)^(1/2)+1/3*I*b^2*polylog(2,I
*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^2/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used

$= \{5798, 5788, 5789, 4265, 2317, 2438, 267\}$

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \frac{2b\sqrt{c^2x^2+1}\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3c^2d^2\sqrt{c^2dx^2+d}} + \frac{bx(a+b\text{arcsinh}(cx))}{3cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}} - \frac{(a+b\text{arcsinh}(cx))^2}{3c^2d\,(c^2dx^2+d)^{3/2}} - \frac{ib^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3c^2d^2\sqrt{c^2dx^2+d}} + \frac{ib^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3c^2d^2\sqrt{c^2dx^2+d}} + \frac{b^2}{3c^2d^2\sqrt{c^2dx^2+d}}$$

```
[In] Int[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] b^2/(3*c^2*d^2*Sqrt[d + c^2*d*x^2]) + (b*x*(a + b*ArcSinh[c*x]))/(3*c*d^2*S
qrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x])^2/(3*c^2*d*(d
+ c^2*d*x^2)^(3/2)) + (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^
ArcSinh[c*x]])/(3*c^2*d^2*Sqrt[d + c^2*d*x^2]) - ((I/3)*b^2*Sqrt[1 + c^2*x^
2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^2*d^2*Sqrt[d + c^2*d*x^2]) + ((I/3)*
b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(c^2*d^2*Sqrt[d + c^2*d
*x^2])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+b\text{arcsinh}(cx))^2}{3c^2d\,(d+c^2dx^2)^{3/2}}+\frac{\left(2b\sqrt{1+c^2x^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{(1+c^2x^2)^2}\,dx}{3cd^2\sqrt{d+c^2dx^2}}\\
&= \frac{bx(a+b\text{arcsinh}(cx))}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{(a+b\text{arcsinh}(cx))^2}{3c^2d\,(d+c^2dx^2)^{3/2}}\\
&\quad-\frac{\left(b^2\sqrt{1+c^2x^2}\right)\int\frac{x}{(1+c^2x^2)^{3/2}}\,dx}{3d^2\sqrt{d+c^2dx^2}}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{1+c^2x^2}\,dx}{3cd^2\sqrt{d+c^2dx^2}}\\
&= \frac{b^2}{3c^2d^2\sqrt{d+c^2dx^2}}+\frac{bx(a+b\text{arcsinh}(cx))}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{(a+b\text{arcsinh}(cx))^2}{3c^2d\,(d+c^2dx^2)^{3/2}}\\
&\quad+\frac{\left(b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\text{sech}(x)\,dx,x,\text{arcsinh}(cx)\right)}{3c^2d^2\sqrt{d+c^2dx^2}}
\end{aligned}$$

$$= \frac{b^2}{3c^2d^2\sqrt{d+c^2dx^2}} + \frac{bx(a+b\text{arcsinh}(cx))}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{3c^2d\,(d+c^2dx^2)^{3/2}} + \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3c^2d^2\sqrt{d+c^2dx^2}} - \frac{\left(ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1-ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{3c^2d^2\sqrt{d+c^2dx^2}} + \frac{\left(ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1+ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{3c^2d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2}{3c^2d^2\sqrt{d+c^2dx^2}} + \frac{bx(a+b\text{arcsinh}(cx))}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{3c^2d\,(d+c^2dx^2)^{3/2}} + \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3c^2d^2\sqrt{d+c^2dx^2}} - \frac{\left(ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{3c^2d^2\sqrt{d+c^2dx^2}} + \frac{\left(ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{3c^2d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2}{3c^2d^2\sqrt{d+c^2dx^2}} + \frac{bx(a+b\text{arcsinh}(cx))}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{3c^2d\,(d+c^2dx^2)^{3/2}} + \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3c^2d^2\sqrt{d+c^2dx^2}} - \frac{ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3c^2d^2\sqrt{d+c^2dx^2}} + \frac{ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3c^2d^2\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.94

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \frac{-a^2+ab\left(-2\text{arcsinh}(cx)+\sqrt{1+c^2x^2}\left(cx+2(1+c^2x^2)\arctan\left(\tanh\left(\frac{1}{2}\text{arcsin}\right.\right.\right.\right.}{}$$

```
[In] Integrate[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (-a^2 + a*b*(-2*ArcSinh[c*x] + Sqrt[1 + c^2*x^2]*(c*x + 2*(1 + c^2*x^2)*Arc
Tan[Tanh[ArcSinh[c*x]/2]])) + b^2*(1 + c^2*x^2 + c*x*Sqrt[1 + c^2*x^2]*ArcS
inh[c*x] - ArcSinh[c*x]^2 - I*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 - I/E^
ArcSinh[c*x]] + I*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 + I/E^ArcSinh[c*x]
] - I*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)/E^ArcSinh[c*x]] + I*(1 + c^2*x^2)
^(3/2)*PolyLog[2, I/E^ArcSinh[c*x]]))/(3*c^2*d*(d + c^2*d*x^2)^(3/2))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(261) = 522$.

Time = 0.29 (sec) , antiderivative size = 591, normalized size of antiderivative = 2.19

method	result
default	$-\frac{a^2}{3c^2d(c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{b^2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)x}{3(c^2x^2+1)^{\frac{3}{2}}d^3c}+\frac{b^2\sqrt{d(c^2x^2+1)}\,x^2}{3(c^2x^2+1)^2d^3}-\frac{b^2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2}{3(c^2x^2+1)^2d^3c^2}+\frac{b^2\sqrt{d(c^2x^2+1)}}{3(c^2x^2+1)^2d^3c^2}-$
parts	$-\frac{a^2}{3c^2d(c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{b^2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)x}{3(c^2x^2+1)^{\frac{3}{2}}d^3c}+\frac{b^2\sqrt{d(c^2x^2+1)}\,x^2}{3(c^2x^2+1)^2d^3}-\frac{b^2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2}{3(c^2x^2+1)^2d^3c^2}+\frac{b^2\sqrt{d(c^2x^2+1)}}{3(c^2x^2+1)^2d^3c^2}-$

```
[In] int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a^2/c^2/d/(c^2*d*x^2+d)^(3/2)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1
)^(3/2)/d^3/c*arcsinh(c*x)*x+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/d^
3*x^2-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/d^3/c^2*arcsinh(c*x)^2+1/
3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/d^3/c^2-1/3*I*b^2*(d*(c^2*x^2+1))
^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*arcsinh(c*x)*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)
))+1/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*arcsinh(c*x)*l
n(1-I*(c*x+(c^2*x^2+1)^(1/2)))-1/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)/c^2/d^3*dilog(1+I*(c*x+(c^2*x^2+1)^(1/2)))+1/3*I*b^2*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*dilog(1-I*(c*x+(c^2*x^2+1)^(1/2)))+1/3*a*b*
(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(3/2)/d^3/c*x-2/3*a*b*(d*(c^2*x^2+1))^(1/
2)/(c^2*x^2+1)^2/d^3/c^2*arcsinh(c*x)+1/3*I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*
x^2+1)^(1/2)/c^2/d^3*ln(c*x+(c^2*x^2+1)^(1/2)+I)-1/3*I*a*b*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*ln(c*x+(c^2*x^2+1)^(1/2)-I)
```

Fricas [F]

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2x}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*x*arcsinh(c*x)^2 + 2*a*b*x*arcsinh(c*x) +
 a^2*x)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F]

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x(a+b\operatorname{asinh}(cx))^2}{(d(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
[Out] Integral(x*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2x}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
[Out] -1/3*a^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + integrate(b^2*x*log(c*x + sqrt(c^2
*x^2 + 1))^2/(c^2*d*x^2 + d)^(5/2) + 2*a*b*x*log(c*x + sqrt(c^2*x^2 + 1))/(
c^2*d*x^2 + d)^(5/2), x)
```

Giac [F]

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2x}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2*x/(c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x\,(a+b\operatorname{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^{5/2}}\,dx$$

```
[In] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2),x)
[Out] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2), x)
```

3.315 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 292

$$\begin{aligned}
\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx &= -\frac{b^2x}{3d^2\sqrt{d+c^2dx^2}}+\frac{b(a+b\text{arcsinh}(cx))}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} \\
&+\frac{x(a+b\text{arcsinh}(cx))^2}{3d\,(d+c^2dx^2)^{3/2}}+\frac{2x(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}}+\frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3cd^2\sqrt{d+c^2dx^2}} \\
&-\frac{4b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3cd^2\sqrt{d+c^2dx^2}} \\
&-\frac{2b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3cd^2\sqrt{d+c^2dx^2}}
\end{aligned}$$

```
[Out] 1/3*x*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(3/2)-1/3*b^2*x/d^2/(c^2*d*x^2+d
)^(1/2)+2/3*x*(a+b*arcsinh(c*x))^2/d^2/(c^2*d*x^2+d)^(1/2)+1/3*b*(a+b*arcsi
nh(c*x))/c/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+2/3*(a+b*arcsinh(c*x))
^2*(c^2*x^2+1)^(1/2)/c/d^2/(c^2*d*x^2+d)^(1/2)-4/3*b*(a+b*arcsinh(c*x))*ln(
1+(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c/d^2/(c^2*d*x^2+d)^(1/2)-2/
3*b^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c/d^2/(c^2*d*
x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {5788, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 197}

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \frac{b(a+b\operatorname{arcsinh}(cx))}{3cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}} + \frac{2x(a+b\operatorname{arcsinh}(cx))^2}{3d^2\sqrt{c^2dx^2+d}} + \frac{2\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^2}{3cd^2\sqrt{c^2dx^2+d}} - \frac{4b\sqrt{c^2x^2+1}\log\left(e^{2\operatorname{arcsinh}(cx)}+1\right)(a+b\operatorname{arcsinh}(cx))}{3cd^2\sqrt{c^2dx^2+d}} + \frac{x(a+b\operatorname{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}} - \frac{2b^2\sqrt{c^2x^2+1}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{3cd^2\sqrt{c^2dx^2+d}} - \frac{b^2x}{3d^2\sqrt{c^2dx^2+d}}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -1/3*(b^2*x)/(d^2*Sqrt[d + c^2*d*x^2]) + (b*(a + b*ArcSinh[c*x]))/(3*c*d^2*
Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (x*(a + b*ArcSinh[c*x])^2)/(3*d*(d
 + c^2*d*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x])^2)/(3*d^2*Sqrt[d + c^2*d*x
^2]) + (2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c*d^2*Sqrt[d + c^2*d
*x^2]) - (4*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c
*x])])/(3*c*d^2*Sqrt[d + c^2*d*x^2]) - (2*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2,
-E^(2*ArcSinh[c*x])])/(3*c*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x(a+b\text{arcsinh}(cx))^2}{3d\left(d+c^2dx^2\right)^{3/2}}+\frac{2\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx}{3d} \\
&\quad -\frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{x(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^2}\,dx}{3d^2\sqrt{d+c^2dx^2}} \\
&= \frac{b(a+b\text{arcsinh}(cx))}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{3d\left(d+c^2dx^2\right)^{3/2}}+\frac{2x(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} \\
&\quad -\frac{\left(b^2\sqrt{1+c^2x^2}\right)\int \frac{1}{(1+c^2x^2)^{3/2}}\,dx}{3d^2\sqrt{d+c^2dx^2}}-\frac{\left(4bc\sqrt{1+c^2x^2}\right)\int \frac{x(a+b\text{arcsinh}(cx))}{1+c^2x^2}\,dx}{3d^2\sqrt{d+c^2dx^2}} \\
&= -\frac{b^2x}{3d^2\sqrt{d+c^2dx^2}}+\frac{b(a+b\text{arcsinh}(cx))}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} \\
&\quad +\frac{x(a+b\text{arcsinh}(cx))^2}{3d\left(d+c^2dx^2\right)^{3/2}}+\frac{2x(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} \\
&\quad -\frac{\left(4b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\tanh(x)\,dx,x,\text{arcsinh}(cx)\right)}{3cd^2\sqrt{d+c^2dx^2}} \\
&= -\frac{b^2x}{3d^2\sqrt{d+c^2dx^2}}+\frac{b(a+b\text{arcsinh}(cx))}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{3d\left(d+c^2dx^2\right)^{3/2}} \\
&\quad +\frac{2x(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}}+\frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3cd^2\sqrt{d+c^2dx^2}} \\
&\quad -\frac{\left(8b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx,x,\text{arcsinh}(cx)\right)}{3cd^2\sqrt{d+c^2dx^2}} \\
&= -\frac{b^2x}{3d^2\sqrt{d+c^2dx^2}}+\frac{b(a+b\text{arcsinh}(cx))}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{3d\left(d+c^2dx^2\right)^{3/2}} \\
&\quad +\frac{2x(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}}+\frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3cd^2\sqrt{d+c^2dx^2}} \\
&\quad -\frac{4b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3cd^2\sqrt{d+c^2dx^2}} \\
&\quad +\frac{\left(4b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx,x,\text{arcsinh}(cx)\right)}{3cd^2\sqrt{d+c^2dx^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b^2x}{3d^2\sqrt{d+c^2dx^2}} + \frac{b(a+b\text{arcsinh}(cx))}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x(a+b\text{arcsinh}(cx))^2}{3d\left(d+c^2dx^2\right)^{3/2}} \\
&\quad + \frac{2x(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} + \frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3cd^2\sqrt{d+c^2dx^2}} \\
&\quad - \frac{4b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3cd^2\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(2b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\text{arcsinh}(cx)}\right)}{3cd^2\sqrt{d+c^2dx^2}} \\
&= -\frac{b^2x}{3d^2\sqrt{d+c^2dx^2}} + \frac{b(a+b\text{arcsinh}(cx))}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x(a+b\text{arcsinh}(cx))^2}{3d\left(d+c^2dx^2\right)^{3/2}} \\
&\quad + \frac{2x(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} + \frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3cd^2\sqrt{d+c^2dx^2}} \\
&\quad - \frac{4b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3cd^2\sqrt{d+c^2dx^2}} \\
&\quad - \frac{2b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3cd^2\sqrt{d+c^2dx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.81

$$
\int \frac{(a+b\text{arcsinh}(cx))^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx = \frac{a^2cx(3+2c^2x^2)+ab\left((6cx+4c^3x^3)\text{arcsinh}(cx)+\sqrt{1+c^2x^2}(1-2(1+c^2x^2)\log\right.}{}
$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (a^2*c*x*(3 + 2*c^2*x^2) + a*b*((6*c*x + 4*c^3*x^3)*ArcSinh[c*x] + Sqrt[1 +
 c^2*x^2]*(1 - 2*(1 + c^2*x^2)*Log[1 + c^2*x^2])) - b^2*(c*x + c^3*x^3 - Sq
rt[1 + c^2*x^2]*ArcSinh[c*x] - c*x*ArcSinh[c*x]^2 - 2*c*x*(1 + c^2*x^2)*Arc
Sinh[c*x]^2 + 2*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*(ArcSinh[c*x] + 2*Log[1 +
E^(-2*ArcSinh[c*x])]) - 2*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(-2*ArcSinh[c*x
])]))/(3*d^2*(c + c^3*x^2)*Sqrt[d + c^2*d*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2130 vs. 2(274) = 548.

Time = 0.28 (sec) , antiderivative size = 2131, normalized size of antiderivative = 7.30

method	result	size
default	Expression too large to display	2131
parts	Expression too large to display	2131

```
[In] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 7/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c/d^3*x^2
*(c^2*x^2+1)^(1/2)-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c
^2*x^2+4)/c/d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2+4/3*b^2*(d*(c^2*x^2+1))^(1
/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/c/d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x
)+17/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^2/d^
3*arcsinh(c*x)^2*x^3+2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c
^2*x^2+4)/d^3*(c^2*x^2+1)*arcsinh(c*x)*x-16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*
c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^2/d^3*arcsinh(c*x)*x^3-4/3*b^2*(d*(c^2*x
^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^6/d^3*arcsinh(c*x)*x^7+2
/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^4/d^3*(c
^2*x^2+1)*x^5+2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+
4)*c^4/d^3*arcsinh(c*x)^2*x^5-14/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*
c^4*x^4+11*c^2*x^2+4)*c^4/d^3*arcsinh(c*x)*x^5+b^2*(d*(c^2*x^2+1))^(1/2)/(3
*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^3/d^3*(c^2*x^2+1)^(1/2)*x^4+4/3*b^2*(d*
(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^2/d^3*(c^2*x^2+1)*
x^3-4/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^3/c*arcsinh(c*x)*ln(1
+(c*x+(c^2*x^2+1)^(1/2))^2)-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4
*x^4+11*c^2*x^2+4)*c^6/d^3*x^7-3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^
4*x^4+11*c^2*x^2+4)*c^4/d^3*x^5-13/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+1
0*c^4*x^4+11*c^2*x^2+4)*c^2/d^3*x^3+4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^
6+10*c^4*x^4+11*c^2*x^2+4)/c/d^3*(c^2*x^2+1)^(1/2)-2/3*b^2/(c^2*x^2+1)^(1/2
)*(d*(c^2*x^2+1))^(1/2)/d^3/c*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+2/3*b^2
*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/d^3*(c^2*x^2+1)*
x+a^2*(1/3/d*x/(c^2*d*x^2+d)^(3/2)+2/3/d^2*x/(c^2*d*x^2+d)^(1/2))+4/3*b^2*(
d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^4/d^3*(c^2*x^2+1
)*arcsinh(c*x)*x^5-2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2
*x^2+4)*c^3/d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*x^4+10/3*b^2*(d*(c^2*x^2+1
))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^2/d^3*(c^2*x^2+1)*arcsinh(c*
x)*x^3-14/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c
/d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*x^2+b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*
x^6+10*c^4*x^4+11*c^2*x^2+4)*c/d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^2+1/3*a
*b*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x+2*(c^
2*x^2+1)^(1/2))*(-8*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^6*c^6+8*(c^2*x^2+1)^(
1/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^5*c^5-24*ln(1+(c*x+(c^2*x^2+1)^(1/2)
```

)^2)*x^4*c^4+20*(c^2*x^2+1)^(1/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^3*c^3+2*c^4*x^4-2*c^3*x^3*(c^2*x^2+1)^(1/2)+6*arcsinh(c*x)*c^2*x^2-24*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^2*c^2+12*(c^2*x^2+1)^(1/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x*c+4*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+8*arcsinh(c*x)-8*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/c/d^3-2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/d^3*x+4*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/d^3*arcsinh(c*x)^2*x+4/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^3/c*arcsinh(c*x)^2-2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/d^3*arcsinh(c*x)*x

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{(a+b\operatorname{asinh}(cx))^2}{(d(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

[In] integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)

[Out] Integral((a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")

[Out] 1/3*a*b*c*(1/(c^4*d^(5/2)*x^2 + c^2*d^(5/2)) - 2*log(c^2*x^2 + 1)/(c^2*d^(5/2))) + 2/3*a*b*(2*x/(sqrt(c^2*d*x^2 + d)*d^2) + x/((c^2*d*x^2 + d)^(3/2)*d))*arcsinh(c*x) + 1/3*a^2*(2*x/(sqrt(c^2*d*x^2 + d)*d^2) + x/((c^2*d*x^2 + d)^(3/2)*d)) + b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^2 + d)^(5/2), x)

Giac [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(c^2dx^2 + d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + c^2dx^2)^{5/2}}\,dx = \int \frac{(a + b\,\text{asinh}(c\,x))^2}{(d\,c^2\,x^2 + d)^{5/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(5/2),x)
[Out] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(5/2), x)
```

3.316 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 518

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^{5/2}}\,dx &= -\frac{b^2}{3d^2\sqrt{d+c^2dx^2}} \\ &- \frac{bcx(a+b\text{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{(a+b\text{arcsinh}(cx))^2}{3d(d+c^2dx^2)^{3/2}} + \frac{(a+b\text{arcsinh}(cx))^2}{d^2\sqrt{d+c^2dx^2}} \\ &- \frac{14b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} \\ &- \frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} \\ &- \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} \\ &+ \frac{7ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} \\ &- \frac{7ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} \\ &+ \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} \\ &+ \frac{2b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} - \frac{2b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}\end{aligned}$$

```
[Out] 1/3*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(3/2)-1/3*b^2/d^2/(c^2*d*x^2+d)^(1
/2)+(a+b*arcsinh(c*x))^2/d^2/(c^2*d*x^2+d)^(1/2)-1/3*b*c*x*(a+b*arcsinh(c*x
))/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-14/3*b*(a+b*arcsinh(c*x))*arct
an(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-2*(a+b*
```

```
arcsinh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d
*x^2+d)^(1/2)-2*b*(a+b*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2
*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+7/3*I*b^2*polylog(2,-I*(c*x+(c^2*x^2+
1)^(1/2)))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-7/3*I*b^2*polylog(2,I*
(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+2*b*(a+b
*arcsinh(c*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*
d*x^2+d)^(1/2)+2*b^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^
2/(c^2*d*x^2+d)^(1/2)-2*b^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1
/2)/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5811, 5816, 4267, 2611, 2320, 6724, 5789, 4265, 2317, 2438, 5788, 267}

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^{5/2}}\,dx = &-\frac{14b\sqrt{c^2x^2+1}\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3d^2\sqrt{c^2dx^2+d}}\\&-\frac{2\sqrt{c^2x^2+1}\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{d^2\sqrt{c^2dx^2+d}}\\&-\frac{2b\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d^2\sqrt{c^2dx^2+d}}\\&+\frac{2b\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d^2\sqrt{c^2dx^2+d}}\\&-\frac{bcx(a+b\text{arcsinh}(cx))}{3d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}+\frac{(a+b\text{arcsinh}(cx))^2}{d^2\sqrt{c^2dx^2+d}}\\&+\frac{(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}+\frac{7ib^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3d^2\sqrt{c^2dx^2+d}}\\&-\frac{7ib^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3d^2\sqrt{c^2dx^2+d}}\\&+\frac{2b^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{c^2dx^2+d}}\\&-\frac{2b^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{c^2dx^2+d}}-\frac{b^2}{3d^2\sqrt{c^2dx^2+d}}\end{aligned}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] -1/3*b^2/(d^2*Sqrt[d + c^2*d*x^2]) - (b*c*x*(a + b*ArcSinh[c*x]))/(3*d^2*Sq
rt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (a + b*ArcSinh[c*x])^2/(3*d*(d + c^2
```

```
*d*x^2)^(3/2)) + (a + b*ArcSinh[c*x])^2/(d^2*Sqrt[d + c^2*d*x^2]) - (14*b*S
qrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(3*d^2*Sqrt[d
 + c^2*d*x^2]) - (2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcS
inh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSin
h[c*x])*PolyLog[2, -E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) + (((7*I)/3)
*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d
*x^2]) - (((7*I)/3)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(d^
2*Sqrt[d + c^2*d*x^2]) + (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLo
g[2, E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) + (2*b^2*Sqrt[1 + c^2*x^2]*
PolyLog[3, -E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) - (2*b^2*Sqrt[1 + c^
2*x^2]*PolyLog[3, E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
```

```
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(a+b\text{arcsinh}(cx))^2}{3d\,(d+c^2dx^2)^{3/2}}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^{3/2}}\,dx}{d}-\frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{a+b\text{arcsinh}(cx)}{(1+c^2x^2)^2}\,dx}{3d^2\sqrt{d+c^2dx^2}}\\
&=-\frac{bcx(a+b\text{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}+\frac{(a+b\text{arcsinh}(cx))^2}{3d\,(d+c^2dx^2)^{3/2}}+\frac{(a+b\text{arcsinh}(cx))^2}{d^2\sqrt{d+c^2dx^2}}\\
&\quad+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{d+c^2dx^2}}\,dx}{d^2}-\frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{a+b\text{arcsinh}(cx)}{1+c^2x^2}\,dx}{3d^2\sqrt{d+c^2dx^2}}\\
&\quad-\frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{a+b\text{arcsinh}(cx)}{1+c^2x^2}\,dx}{d^2\sqrt{d+c^2dx^2}}+\frac{\left(b^2c^2\sqrt{1+c^2x^2}\right)\int \frac{x}{(1+c^2x^2)^{3/2}}\,dx}{3d^2\sqrt{d+c^2dx^2}}\\
&=-\frac{b^2}{3d^2\sqrt{d+c^2dx^2}}-\frac{bcx(a+b\text{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}+\frac{(a+b\text{arcsinh}(cx))^2}{3d\,(d+c^2dx^2)^{3/2}}\\
&\quad+\frac{(a+b\text{arcsinh}(cx))^2}{d^2\sqrt{d+c^2dx^2}}+\frac{\sqrt{1+c^2x^2}\,\text{Subst}\left(\int (a+bx)^2\text{csch}(x)\,dx,x,\text{arcsinh}(cx)\right)}{d^2\sqrt{d+c^2dx^2}}\\
&\quad-\frac{\left(b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\text{sech}(x)\,dx,x,\text{arcsinh}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}}\\
&\quad-\frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\text{sech}(x)\,dx,x,\text{arcsinh}(cx)\right)}{d^2\sqrt{d+c^2dx^2}}
\end{aligned}$$

$$= -\frac{b^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bcx(a+b\text{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{(a+b\text{arcsinh}(cx))^2}{3d\left(d+c^2dx^2\right)^{3/2}}$$
$$+ \frac{(a+b\text{arcsinh}(cx))^2}{d^2\sqrt{d+c^2dx^2}} - \frac{14b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\log\left(1-e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\log\left(1+e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1-ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1+ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(2ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1-ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(2ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1+ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bcx(a+b\text{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{(a+b\text{arcsinh}(cx))^2}{3d(d+c^2dx^2)^{3/2}}$$
$$+ \frac{(a+b\text{arcsinh}(cx))^2}{d^2\sqrt{d+c^2dx^2}} - \frac{14b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(2ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(2ib^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(2b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \text{PolyLog}\left(2,-e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(2b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \text{PolyLog}\left(2,e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bcx(a+b\operatorname{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{(a+b\operatorname{arcsinh}(cx))^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{(a+b\operatorname{arcsinh}(cx))^2}{d^2\sqrt{d+c^2dx^2}} - \frac{14b\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} - \frac{2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} + \frac{7ib^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} - \frac{7ib^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} + \frac{2b\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} + \frac{\left(2b^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,-x)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} - \frac{\left(2b^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,x)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bcx(a+b\operatorname{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{(a+b\operatorname{arcsinh}(cx))^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{(a+b\operatorname{arcsinh}(cx))^2}{d^2\sqrt{d+c^2dx^2}} - \frac{14b\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} - \frac{2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} + \frac{7ib^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} - \frac{7ib^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} + \frac{2b\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} + \frac{2b^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} - \frac{2b^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 3.21 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x(d+c^2dx^2)^{5/2}}\,dx = \frac{\frac{a^2(4+3c^2x^2)\sqrt{d+c^2dx^2}}{(1+c^2x^2)^2}+3a^2\sqrt{d}\log(cx)-3a^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right)+\frac{abd^2(1+\cdots}{\cdots}}{\cdots}$$

[In] Integrate[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^(5/2)),x]

[Out] ((a^2*(4 + 3*c^2*x^2)*Sqrt[d + c^2*d*x^2])/(1 + c^2*x^2)^2 + 3*a^2*Sqrt[d]*Log[c*x] - 3*a^2*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (a*b*d^2*(1 + c^2*x^2)^(3/2)*(-((c*x)/(1 + c^2*x^2)) + (2*ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) + (6*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - 14*ArcTan[Tanh[ArcSinh[c*x]/2]] + 6*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 6*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 6*PolyLog[2, -E^(-ArcSinh[c*x])] - 6*PolyLog[2, E^(-ArcSinh[c*x])]))/(d + c^2*d*x^2)^(3/2) + (b^2*d^2*(1 + c^2*x^2)^(3/2)*(-(1/Sqrt[1 + c^2*x^2]) - (c*x*ArcSinh[c*x])/(1 + c^2*x^2) + ArcSinh[c*x]^2/(1 + c^2*x^2)^(3/2) + (3*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] + 3*ArcSinh[c*x]^2*Log[1 - E^(-ArcSinh[c*x])] + (7*I)*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] - (7*I)*ArcSinh[c*x]*Log[1 + I/E^ArcSinh[c*x]] - 3*ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] + 6*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] + (7*I)*PolyLog[2, (-I)/E^ArcSinh[c*x]] - (7*I)*PolyLog[2, I/E^ArcSinh[c*x]] - 6*ArcSinh[c*x]*PolyLog[2, E^(-ArcSinh[c*x])] + 6*PolyLog[3, -E^(-ArcSinh[c*x])] - 6*PolyLog[3, E^(-ArcSinh[c*x])]))/(d + c^2*d*x^2)^(3/2))/(3*d^3)

Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{x\,(c^2d\,x^2+d)^{\frac{5}{2}}}dx$$

[In] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(5/2),x)

[Out] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(5/2),x)

Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^{\frac{5}{2}}x}\,dx$$

[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x), x)

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^{5/2}}\,dx = \int \frac{\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}{x\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d)**(5/2),x)
[Out] Integral((a + b*asinh(c*x))**2/(x*(d*(c**2*x**2 + 1))**(5/2)), x)
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^{5/2}}\,dx = \int \frac{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
[Out] -1/3*a^2*(3*arcsinh(1/(c*abs(x)))/d^(5/2) - 3/(sqrt(c^2*d*x^2 + d)*d^2) - 1
/((c^2*d*x^2 + d)^(3/2)*d)) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/
((c^2*d*x^2 + d)^(5/2)*x) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/((c^2*d*x^2
+ d)^(5/2)*x), x)
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^{5/2}}\,dx = \int \frac{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(5/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^{5/2}}\,dx = \int \frac{\left(a+b\operatorname{asinh}(c\,x)\right)^2}{x\left(d\,c^2\,x^2+d\right)^{5/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^(5/2)),x)
[Out] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^(5/2)), x)
```

3.317 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 421

$$\begin{aligned}
\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{5/2}}\,dx &= \frac{b^2c^2x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc(a+b\text{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} \\
&- \frac{(a+b\text{arcsinh}(cx))^2}{dx(d+c^2dx^2)^{3/2}} - \frac{4c^2x(a+b\text{arcsinh}(cx))^2}{3d(d+c^2dx^2)^{3/2}} \\
&- \frac{8c^2x(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{8c\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} \\
&- \frac{4bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} \\
&+ \frac{16bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} \\
&+ \frac{5b^2c\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} + \frac{b^2c\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}
\end{aligned}$$

```
[Out] -(a+b*arcsinh(c*x))^2/d/x/(c^2*d*x^2+d)^(3/2)-4/3*c^2*x*(a+b*arcsinh(c*x))^
2/d/(c^2*d*x^2+d)^(3/2)+1/3*b^2*c^2*x/d^2/(c^2*d*x^2+d)^(1/2)-8/3*c^2*x*(a+
b*arcsinh(c*x))^2/d^2/(c^2*d*x^2+d)^(1/2)-1/3*b*c*(a+b*arcsinh(c*x))/d^2/(c
^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-8/3*c*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^
(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-4*b*c*(a+b*arcsinh(c*x))*arctanh((c*x+(c^2*x^
2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+16/3*b*c*(a+b*arcs
inh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+
d)^(1/2)+5/3*b^2*c*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/
d^2/(c^2*d*x^2+d)^(1/2)+b^2*c*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2
+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5809, 5788, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 197, 5811, 5799, 5569, 4267}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{5/2}}\,dx = -\frac{4bc\sqrt{c^2x^2+1}\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d^2\sqrt{c^2dx^2+d}} - \frac{bc(a+b\text{arcsinh}(cx))}{3d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}} - \frac{8c^2x(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{c^2dx^2+d}} - \frac{8c\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{c^2dx^2+d}} + \frac{16bc\sqrt{c^2x^2+1}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))}{3d^2\sqrt{c^2dx^2+d}} - \frac{4c^2x(a+b\text{arcsinh}(cx))^2}{3d(c^2dx^2+d)^{3/2}} - \frac{(a+b\text{arcsinh}(cx))^2}{dx(c^2dx^2+d)^{3/2}} + \frac{5b^2c\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3d^2\sqrt{c^2dx^2+d}} + \frac{b^2c\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d^2\sqrt{c^2dx^2+d}} + \frac{b^2c^2x}{3d^2\sqrt{c^2dx^2+d}}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] (b^2*c^2*x)/(3*d^2*Sqrt[d + c^2*d*x^2]) - (b*c*(a + b*ArcSinh[c*x]))/(3*d^2
*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x])^2/(d*x*(d +
c^2*d*x^2)^(3/2)) - (4*c^2*x*(a + b*ArcSinh[c*x])^2)/(3*d*(d + c^2*d*x^2)^(
3/2)) - (8*c^2*x*(a + b*ArcSinh[c*x])^2)/(3*d^2*Sqrt[d + c^2*d*x^2]) - (8*c
*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(3*d^2*Sqrt[d + c^2*d*x^2]) - (4
*b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/(d
^2*Sqrt[d + c^2*d*x^2]) + (16*b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Lo
g[1 + E^(2*ArcSinh[c*x])])/(3*d^2*Sqrt[d + c^2*d*x^2]) + (5*b^2*c*Sqrt[1 +
c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*d^2*Sqrt[d + c^2*d*x^2]) + (b^
2*c*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(2*ArcSinh[c*x])])/(d^2*Sqrt[d + c^2*d*x
^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
```

```
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(a+b\text{arcsinh}(cx))^2}{dx\left(d+c^2dx^2\right)^{3/2}} \\
&\quad -\left(4c^2\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx+\frac{\left(2bc\sqrt{1+c^2x^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{x\left(1+c^2x^2\right)^2}\,dx}{d^2\sqrt{d+c^2dx^2}} \\
&= \frac{bc(a+b\text{arcsinh}(cx))}{d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{(a+b\text{arcsinh}(cx))^2}{dx\left(d+c^2dx^2\right)^{3/2}}-\frac{4c^2x(a+b\text{arcsinh}(cx))^2}{3d\left(d+c^2dx^2\right)^{3/2}} \\
&\quad -\frac{\left(8c^2\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx}{3d}+\frac{\left(2bc\sqrt{1+c^2x^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{x\left(1+c^2x^2\right)}\,dx}{d^2\sqrt{d+c^2dx^2}} \\
&\quad -\frac{\left(b^2c^2\sqrt{1+c^2x^2}\right)\int\frac{1}{\left(1+c^2x^2\right)^{3/2}}\,dx}{d^2\sqrt{d+c^2dx^2}}+\frac{\left(8bc^3\sqrt{1+c^2x^2}\right)\int\frac{x(a+b\text{arcsinh}(cx))}{\left(1+c^2x^2\right)^2}\,dx}{3d^2\sqrt{d+c^2dx^2}} \\
&= -\frac{b^2c^2x}{d^2\sqrt{d+c^2dx^2}}-\frac{bc(a+b\text{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{(a+b\text{arcsinh}(cx))^2}{dx\left(d+c^2dx^2\right)^{3/2}} \\
&\quad -\frac{4c^2x(a+b\text{arcsinh}(cx))^2}{3d\left(d+c^2dx^2\right)^{3/2}}-\frac{8c^2x(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} \\
&\quad +\frac{\left(2bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\text{csch}(x)\text{sech}(x)\,dx,x,\text{arcsinh}(cx)\right)}{d^2\sqrt{d+c^2dx^2}} \\
&\quad +\frac{\left(4b^2c^2\sqrt{1+c^2x^2}\right)\int\frac{1}{\left(1+c^2x^2\right)^{3/2}}\,dx}{3d^2\sqrt{d+c^2dx^2}}+\frac{\left(16bc^3\sqrt{1+c^2x^2}\right)\int\frac{x(a+b\text{arcsinh}(cx))}{1+c^2x^2}\,dx}{3d^2\sqrt{d+c^2dx^2}} \\
&= \frac{b^2c^2x}{3d^2\sqrt{d+c^2dx^2}}-\frac{bc(a+b\text{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{(a+b\text{arcsinh}(cx))^2}{dx\left(d+c^2dx^2\right)^{3/2}} \\
&\quad -\frac{4c^2x(a+b\text{arcsinh}(cx))^2}{3d\left(d+c^2dx^2\right)^{3/2}}-\frac{8c^2x(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} \\
&\quad +\frac{\left(4bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\text{csch}(2x)\,dx,x,\text{arcsinh}(cx)\right)}{d^2\sqrt{d+c^2dx^2}} \\
&\quad +\frac{\left(16bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\tanh(x)\,dx,x,\text{arcsinh}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}}
\end{aligned}
$$

$$= \frac{b^2c^2x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc(a+b\operatorname{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{dx\left(d+c^2dx^2\right)^{3/2}} - \frac{4c^2x(a+b\operatorname{arcsinh}(cx))^2}{3d\left(d+c^2dx^2\right)^{3/2}} - \frac{8c^2x(a+b\operatorname{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{8c\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{4bc\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{2\operatorname{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} + \frac{\left(32bc\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \operatorname{arcsinh}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}} - \frac{\left(2b^2c\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{d^2\sqrt{d+c^2dx^2}} + \frac{\left(2b^2c\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2c^2x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc(a+b\operatorname{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{dx\left(d+c^2dx^2\right)^{3/2}} - \frac{4c^2x(a+b\operatorname{arcsinh}(cx))^2}{3d\left(d+c^2dx^2\right)^{3/2}} - \frac{8c^2x(a+b\operatorname{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{8c\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{4bc\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{2\operatorname{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} + \frac{16bc\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} - \frac{\left(b^2c\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\operatorname{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} + \frac{\left(b^2c\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\operatorname{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} - \frac{\left(16b^2c\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2c^2x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc(a+b\operatorname{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{dx\,(d+c^2dx^2)^{3/2}} - \frac{4c^2x(a+b\operatorname{arcsinh}(cx))^2}{3d\,(d+c^2dx^2)^{3/2}} - \frac{8c^2x(a+b\operatorname{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{8c\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{4bc\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{2\operatorname{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} + \frac{16bc\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} - \frac{b^2c\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} + \frac{b^2c\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} - \frac{\left(8b^2c\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\operatorname{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2c^2x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc(a+b\operatorname{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{dx\,(d+c^2dx^2)^{3/2}} - \frac{4c^2x(a+b\operatorname{arcsinh}(cx))^2}{3d\,(d+c^2dx^2)^{3/2}} - \frac{8c^2x(a+b\operatorname{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{8c\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{4bc\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{2\operatorname{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} + \frac{16bc\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} + \frac{5b^2c\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} + \frac{b^2c\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 2.14 (sec) , antiderivative size = 408, normalized size of antiderivative = 0.97

$$\int\frac{(a+b\operatorname{arcsinh}(cx))^2}{x^2\,(d+c^2dx^2)^{5/2}}\,dx = -\frac{3a^2+12a^2c^2x^2-b^2c^2x^2+8a^2c^4x^4-b^2c^4x^4+abcx\sqrt{1+c^2x^2}+6ab\operatorname{arcsinh}(cx)+24abc^2x^2\operatorname{arcsinh}(cx)+1}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] -1/3*(3*a^2 + 12*a^2*c^2*x^2 - b^2*c^2*x^2 + 8*a^2*c^4*x^4 - b^2*c^4*x^4 +
a*b*c*x*Sqrt[1 + c^2*x^2] + 6*a*b*ArcSinh[c*x] + 24*a*b*c^2*x^2*ArcSinh[c*x
] + 16*a*b*c^4*x^4*ArcSinh[c*x] + b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] +
3*b^2*ArcSinh[c*x]^2 + 12*b^2*c^2*x^2*ArcSinh[c*x]^2 + 8*b^2*c^4*x^4*ArcSin
h[c*x]^2 - 8*b^2*c*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]^2 - 6*b^2*c*x*(1 + c^
2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 - E^(-2*ArcSinh[c*x])] - 10*b^2*c*x*(1 + c^
2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 + E^(-2*ArcSinh[c*x])] - 6*a*b*c*x*(1 + c^2
*x^2)^(3/2)*Log[c*x] - 5*a*b*c*x*(1 + c^2*x^2)^(3/2)*Log[1 + c^2*x^2] + 5*b
^2*c*x*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(-2*ArcSinh[c*x])] + 3*b^2*c*x*(1
+ c^2*x^2)^(3/2)*PolyLog[2, E^(-2*ArcSinh[c*x])])/(d*x*(d + c^2*d*x^2)^(3/2
))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3512 vs. $2(411) = 822$.

Time = 0.35 (sec) , antiderivative size = 3513, normalized size of antiderivative = 8.34

method	result	size
default	Expression too large to display	3513
parts	Expression too large to display	3513

```
[In] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 128/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^4
*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^5+272/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*
x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^2*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^3-40*
b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^7*c^8-1
60/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^5*
c^6-29*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^
3*c^4-5*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x
*c^2-9*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3/x*
arcsinh(c*x)^2-3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2
+9)/d^3*c*(c^2*x^2+1)^(1/2)+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d
^3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2
+1)^(1/2)/d^3*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c+5/3*b^2*(d*(c^2*x^2+1))^(1
/2)/(c^2*x^2+1)^(1/2)/d^3*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*c+a^2*(-1/d
/x/(c^2*d*x^2+d)^(3/2)-4*c^2*(1/3/d*x/(c^2*d*x^2+d)^(3/2)+2/3/d^2*x/(c^2*d*
x^2+d)^(1/2)))+64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*
x^2+9)/d^3*x^4*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^5+40*b^2*(d*(c^2*x^2+1))^
(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^3*(c^2*x^2+1)*arcsinh(c*x)*
c^4+136/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3
*x^2*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^3-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*
c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^2*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^3
+8*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x*(c^2
```

*x^2+1)*arcsinh(c*x)*c^2+64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x
^4+26*c^2*x^2+9)/d^3*x^7*(c^2*x^2+1)*arcsinh(c*x)*c^8+160/3*b^2*(d*(c^2*x^2
+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^5*(c^2*x^2+1)*arcsinh(
c*x)*c^6-32/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arcsinh(c*x)*
c-64/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^
9*c^10-224/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/
d^3*x^7*c^8-280/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^
2+9)/d^3*x^5*c^6-48*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*
x^2+9)/d^3*x^3*c^4-8*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2
*x^2+9)/d^3*x*c^2-3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*
x^2+9)/d^3*c*(c^2*x^2+1)^(1/2)-18*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c
^4*x^4+26*c^2*x^2+9)/d^3/x*arcsinh(c*x)+10/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2
*x^2+1)^(1/2)/d^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*c+2*a*b*(d*(c^2*x^2+1))^(
1/2)/(c^2*x^2+1)^(1/2)/d^3*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c+64/3*a*b*(d*(c
^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^7*(c^2*x^2+1)*c^
8+160/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x
^5*(c^2*x^2+1)*c^6-128/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26
*c^2*x^2+9)/d^3*x^5*arcsinh(c*x)*c^6+40*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^
6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^3*(c^2*x^2+1)*c^4-112*a*b*(d*(c^2*x^2+1))^
(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^3*arcsinh(c*x)*c^4-8/3*a*b*
(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^2*c^3*(c^2*
x^2+1)^(1/2)+8*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9
)/d^3*x*(c^2*x^2+1)*c^2-88*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+
26*c^2*x^2+9)/d^3*x*arcsinh(c*x)*c^2+48*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^
6+25*c^4*x^4+26*c^2*x^2+9)/d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c-16/3*b^2*(d
*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arcsinh(c*x)^2*c-32/3*b^2*(d*(c^2
*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^9*c^10-224/3*b^2*(
d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^7*arcsinh(c*
x)*c^8+88/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d
^3*x^5*(c^2*x^2+1)*c^6-64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4
+26*c^2*x^2+9)/d^3*x^5*arcsinh(c*x)^2*c^6-280/3*b^2*(d*(c^2*x^2+1))^(1/2)/(
8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^5*arcsinh(c*x)*c^6-8/3*b^2*(d*(c^2
*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^4*c^5*(c^2*x^2+1)^
(1/2)+80/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^
3*x^3*(c^2*x^2+1)*c^4-56*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26
*c^2*x^2+9)/d^3*x^3*arcsinh(c*x)^2*c^4-48*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*
x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^3*arcsinh(c*x)*c^4-17/3*b^2*(d*(c^2*x^2+
1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^2*c^3*(c^2*x^2+1)^(1/2)
+8*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x*(c^2
*x^2+1)*c^2-44*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9
)/d^3*x*arcsinh(c*x)^2*c^2-8*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^
4+26*c^2*x^2+9)/d^3*x*arcsinh(c*x)*c^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2
+1)^(1/2)/d^3*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c+24*b^2*(d*(c^2*x^2
+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*(c^2*x^2+1)^(1/2)*arcsin
h(c*x)^2*c-3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/

```
d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c+10/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^
2+1)^(1/2)/d^3*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*c+2*b^2*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2
))*c-64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3
*x^9*arcsinh(c*x)*c^10+32/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4
+26*c^2*x^2+9)/d^3*x^7*(c^2*x^2+1)*c^8
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^{\frac{5}{2}}x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^6*d^3*x^8 + 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 + d^3*x^2), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{5/2}}\,dx=\int \frac{(a+b\,\text{asinh}\,(cx))^2}{x^2\,(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x**2*(d*(c**2*x**2 + 1))**(5/2)), x)
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^{\frac{5}{2}}x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/3*a^2*(8*c^2*x/(sqrt(c^2*d*x^2 + d)*d^2) + 4*c^2*x/((c^2*d*x^2 + d)^(3/2
)*d) + 3/((c^2*d*x^2 + d)^(3/2)*d*x)) + integrate(b^2*log(c*x + sqrt(c^2*x^
2 + 1))^2/((c^2*d*x^2 + d)^(5/2)*x^2) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/
((c^2*d*x^2 + d)^(5/2)*x^2), x)
```

Giac [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^2 (d + c^2dx^2)^{5/2}}\, dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(c^2dx^2 + d)^{\frac{5}{2}} x^2}\, dx$$

[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(5/2)*x^2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^2 (d + c^2dx^2)^{5/2}}\, dx = \int \frac{(a + b\,\text{asinh}(c\,x))^2}{x^2\,(d\,c^2\,x^2 + d)^{5/2}}\, dx$$

[In] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^(5/2)),x)

[Out] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^(5/2)), x)

3.318 $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 687

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx=\frac{b^2c^2}{3d^2\sqrt{d+c^2dx^2}}-\frac{bc(a+b\operatorname{arcsinh}(cx))}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{2bc^3x(a+b\operatorname{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{5c^2(a+b\operatorname{arcsinh}(cx))^2}{6d\left(d+c^2dx^2\right)^{3/2}}-\frac{(a+b\operatorname{arcsinh}(cx))^2}{2dx^2\left(d+c^2dx^2\right)^{3/2}}-\frac{5c^2(a+b\operatorname{arcsinh}(cx))^2}{2d^2\sqrt{d+c^2dx^2}}+\frac{26bc^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}}+\frac{5c^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}-\frac{b^2c^2\sqrt{1+c^2x^2}\operatorname{arctanh}\left(\sqrt{1+c^2x^2}\right)}{d^2\sqrt{d+c^2dx^2}}+\frac{5bc^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}-\frac{13ib^2c^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}}+\frac{13ib^2c^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}}-\frac{5bc^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}-\frac{5b^2c^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}+\frac{5b^2c^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}$$

```
[Out] -5/6*c^2*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(3/2)-1/2*(a+b*arcsinh(c*x))^
2/d/x^2/(c^2*d*x^2+d)^(3/2)+1/3*b^2*c^2/d^2/(c^2*d*x^2+d)^(1/2)-5/2*c^2*(a+
b*arcsinh(c*x))^2/d^2/(c^2*d*x^2+d)^(1/2)-b*c*(a+b*arcsinh(c*x))/d^2/x/(c^2
*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-2/3*b*c^3*x*(a+b*arcsinh(c*x))/d^2/(c^2*x
^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+26/3*b*c^2*(a+b*arcsinh(c*x))*arctan(c*x+(c
^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+5*c^2*(a+b*arcsi
nh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+
d)^(1/2)-b^2*c^2*arctanh((c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^
2+d)^(1/2)+5*b*c^2*(a+b*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^
2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-13/3*I*b^2*c^2*polylog(2,-I*(c*x+(c^
2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+13/3*I*b^2*c^2*p
olylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/
```

2)-5*b*c^2*(a+b*arcsinh(c*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-5*b^2*c^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+5*b^2*c^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.607, Rules used = {5809, 5811, 5816, 4267, 2611, 2320, 6724, 5789, 4265, 2317, 2438, 5788, 267, 272, 53, 65, 214}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx=\frac{26bc^2\sqrt{c^2x^2+1}\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3d^2\sqrt{c^2dx^2+d}}+\frac{5c^2\sqrt{c^2x^2+1}\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2}{d^2\sqrt{c^2dx^2+d}}+\frac{5bc^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d^2\sqrt{c^2dx^2+d}}-\frac{5bc^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{d^2\sqrt{c^2dx^2+d}}-\frac{5c^2(a+b\text{arcsinh}(cx))^2}{2d^2\sqrt{c^2dx^2+d}}-\frac{bc(a+b\text{arcsinh}(cx))}{d^2x\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}-\frac{5c^2(a+b\text{arcsinh}(cx))^2}{6d\left(c^2dx^2+d\right)^{3/2}}-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(c^2dx^2+d\right)^{3/2}}-\frac{2bc^3x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}-\frac{13ib^2c^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{13ib^2c^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3d^2\sqrt{c^2dx^2+d}}-\frac{5b^2c^2\sqrt{c^2x^2+1}\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{c^2dx^2+d}}+\frac{5b^2c^2\sqrt{c^2x^2+1}\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{c^2dx^2+d}}-\frac{b^2c^2\sqrt{c^2x^2+1}\text{arctanh}\left(\sqrt{c^2x^2+1}\right)}{d^2\sqrt{c^2dx^2+d}}+\frac{b^2c^2}{3d^2\sqrt{c^2dx^2+d}}$$

[In] Int[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^(5/2)),x]

[Out] (b^2*c^2)/(3*d^2*Sqrt[d + c^2*d*x^2]) - (b*c*(a + b*ArcSinh[c*x]))/(d^2*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (2*b*c^3*x*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (5*c^2*(a + b*ArcSinh[c*x])^2)/(6*d*(d + c^2*d*x^2)^(3/2)) - (a + b*ArcSinh[c*x])^2/(2*d*x^2*(d + c^2*d*x^2)^(3/2)) - (5*c^2*(a + b*ArcSinh[c*x])^2)/(2*d^2*Sqrt[d + c^2*d*x^2]) + (26*b*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(3*d^2*Sqrt[d + c^2*d*x^2]) + (5*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) - (b^2*c^2*Sqrt[1 + c^2*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/(d^2*Sqrt[d + c^2*d*x^2]) + (5*b*c^2*Sqrt

```
[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])/(d^2*Sqrt[d
 + c^2*d*x^2]) - (((13*I)/3)*b^2*c^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^Ar
cSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) + (((13*I)/3)*b^2*c^2*Sqrt[1 + c^2*x
^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) - (5*b*c^2*Sqrt
[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]])/(d^2*Sqrt[d
+ c^2*d*x^2]) - (5*b^2*c^2*Sqrt[1 + c^2*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/(
d^2*Sqrt[d + c^2*d*x^2]) + (5*b^2*c^2*Sqrt[1 + c^2*x^2]*PolyLog[3, E^ArcSin
h[c*x]])/(d^2*Sqrt[d + c^2*d*x^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
```

```
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = -\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(d+c^2dx^2\right)^{3/2}} - \frac{1}{2}\left(5c^2\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^{5/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{x^2\left(1+c^2x^2\right)^2}\,dx}{d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc(a+b\text{arcsinh}(cx))}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{5c^2(a+b\text{arcsinh}(cx))^2}{6d\left(d+c^2dx^2\right)^{3/2}} - \frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(d+c^2dx^2\right)^{3/2}} - \frac{\left(5c^2\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^{3/2}}\,dx}{2d} + \frac{\left(b^2c^2\sqrt{1+c^2x^2}\right)\int\frac{1}{x\left(1+c^2x^2\right)^{3/2}}\,dx}{d^2\sqrt{d+c^2dx^2}} + \frac{\left(5bc^3\sqrt{1+c^2x^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{\left(1+c^2x^2\right)^2}\,dx}{3d^2\sqrt{d+c^2dx^2}} - \frac{\left(3bc^3\sqrt{1+c^2x^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{\left(1+c^2x^2\right)^2}\,dx}{d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc(a+b\text{arcsinh}(cx))}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{2bc^3x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{5c^2(a+b\text{arcsinh}(cx))^2}{6d\left(d+c^2dx^2\right)^{3/2}} - \frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(d+c^2dx^2\right)^{3/2}} - \frac{5c^2(a+b\text{arcsinh}(cx))^2}{2d^2\sqrt{d+c^2dx^2}} - \frac{\left(5c^2\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{d+c^2dx^2}}\,dx}{2d^2} + \frac{\left(b^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{1}{x\left(1+c^2x\right)^{3/2}}\,dx,x,x^2\right)}{2d^2\sqrt{d+c^2dx^2}} + \frac{\left(5bc^3\sqrt{1+c^2x^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{1+c^2x^2}\,dx}{6d^2\sqrt{d+c^2dx^2}} - \frac{\left(3bc^3\sqrt{1+c^2x^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{1+c^2x^2}\,dx}{2d^2\sqrt{d+c^2dx^2}} + \frac{\left(5bc^3\sqrt{1+c^2x^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{1+c^2x^2}\,dx}{d^2\sqrt{d+c^2dx^2}} - \frac{\left(5b^2c^4\sqrt{1+c^2x^2}\right)\int\frac{x}{\left(1+c^2x^2\right)^{3/2}}\,dx}{6d^2\sqrt{d+c^2dx^2}} + \frac{\left(3b^2c^4\sqrt{1+c^2x^2}\right)\int\frac{x}{\left(1+c^2x^2\right)^{3/2}}\,dx}{2d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2c^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc(a+b\text{arcsinh}(cx))}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{2bc^3x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}$$
$$- \frac{5c^2(a+b\text{arcsinh}(cx))^2}{6d\left(d+c^2dx^2\right)^{3/2}} - \frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(d+c^2dx^2\right)^{3/2}} - \frac{5c^2(a+b\text{arcsinh}(cx))^2}{2d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(5c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)^2\text{csch}(x)\,dx,x,\text{arcsinh}(cx)\right)}{2d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(5bc^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\text{sech}(x)\,dx,x,\text{arcsinh}(cx)\right)}{6d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(3bc^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\text{sech}(x)\,dx,x,\text{arcsinh}(cx)\right)}{2d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(5bc^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\text{sech}(x)\,dx,x,\text{arcsinh}(cx)\right)}{d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(b^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+c^2x}}\,dx,x,x^2\right)}{2d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2c^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc(a+b\text{arcsinh}(cx))}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{2bc^3x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}$$
$$- \frac{5c^2(a+b\text{arcsinh}(cx))^2}{6d\left(d+c^2dx^2\right)^{3/2}} - \frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(d+c^2dx^2\right)^{3/2}} - \frac{5c^2(a+b\text{arcsinh}(cx))^2}{2d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{26bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{5c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(b^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx, x, \sqrt{1+c^2x^2}\right)}{d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(5bc^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\log\left(1-e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(5bc^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\log\left(1+e^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(5ib^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \log\left(1-ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{6d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(5ib^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \log\left(1+ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{6d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(3ib^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \log\left(1-ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{2d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(3ib^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \log\left(1+ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{2d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(5ib^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \log\left(1-ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(5ib^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \log\left(1+ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2c^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc(a+b\text{arcsinh}(cx))}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{2bc^3x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}$$
$$- \frac{5c^2(a+b\text{arcsinh}(cx))^2}{6d\,(d+c^2dx^2)^{3/2}} - \frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\,(d+c^2dx^2)^{3/2}} - \frac{5c^2(a+b\text{arcsinh}(cx))^2}{2d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{26bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{5c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{b^2c^2\sqrt{1+c^2x^2}\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{5bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{5bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(5ib^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{6d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(5ib^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{6d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(3ib^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{2d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(3ib^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{2d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(5ib^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(5ib^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(5b^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \text{PolyLog}\left(2,-e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(5b^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \text{PolyLog}\left(2,e^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{d^2\sqrt{d+c^2dx^2}}$$

$$
\begin{aligned}
&= \frac{b^2c^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc(a+b\text{arcsinh}(cx))}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{2bc^3x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} \\
&- \frac{5c^2(a+b\text{arcsinh}(cx))^2}{6d\,(d+c^2dx^2)^{3/2}} - \frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\,(d+c^2dx^2)^{3/2}} - \frac{5c^2(a+b\text{arcsinh}(cx))^2}{2d^2\sqrt{d+c^2dx^2}} \\
&+ \frac{26bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} \\
&+ \frac{5c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} \\
&- \frac{b^2c^2\sqrt{1+c^2x^2}\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{d^2\sqrt{d+c^2dx^2}} \\
&+ \frac{5bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} \\
&- \frac{13ib^2c^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} \\
&+ \frac{13ib^2c^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} \\
&- \frac{5bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} \\
&- \frac{\left(5b^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} \\
&+ \frac{\left(5b^2c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}
\end{aligned}
$$

$$= \frac{b^2c^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc(a+b\text{arcsinh}(cx))}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{2bc^3x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}$$
$$- \frac{5c^2(a+b\text{arcsinh}(cx))^2}{6d(d+c^2dx^2)^{3/2}} - \frac{(a+b\text{arcsinh}(cx))^2}{2dx^2(d+c^2dx^2)^{3/2}} - \frac{5c^2(a+b\text{arcsinh}(cx))^2}{2d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{26bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{5c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{b^2c^2\sqrt{1+c^2x^2}\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{5bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{13ib^2c^2\sqrt{1+c^2x^2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{13ib^2c^2\sqrt{1+c^2x^2}\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{5bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{5b^2c^2\sqrt{1+c^2x^2}\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{5b^2c^2\sqrt{1+c^2x^2}\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 7.78 (sec) , antiderivative size = 983, normalized size of antiderivative = 1.43

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^{5/2}}\,dx = \sqrt{d(1+c^2x^2)}\left(-\frac{a^2}{2d^3x^2} - \frac{a^2c^2}{3d^3(1+c^2x^2)^2} - \frac{2a^2c^2}{d^3(1+c^2x^2)}\right)$$
$$- \frac{5a^2c^2\log(x)}{2d^{5/2}} + \frac{5a^2c^2\log\left(d+\sqrt{d}\sqrt{d(1+c^2x^2)}\right)}{2d^{5/2}}$$
$$+ \frac{abc^2\left(\frac{4cx}{\sqrt{1+c^2x^2}} - 48\text{arcsinh}(cx) - \frac{8\text{arcsinh}(cx)}{1+c^2x^2} + 104\sqrt{1+c^2x^2}\arctan\left(\tanh\left(\frac{1}{2}\text{arcsinh}(cx)\right)\right) - 6\sqrt{1+c^2x^2}\,c\right.}{}$$
$$+ \frac{b^2c^2\left(8 + \frac{8cx\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}} - 48\text{arcsinh}(cx)^2 - \frac{8\text{arcsinh}(cx)^2}{1+c^2x^2} - 12\sqrt{1+c^2x^2}\text{arcsinh}(cx)\coth\left(\frac{1}{2}\text{arcsinh}(cx)\right) - 3\right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^(5/2)),x]
```

[Out] Sqrt[d*(1 + c^2*x^2)]*(-1/2*a^2/(d^3*x^2) - (a^2*c^2)/(3*d^3*(1 + c^2*x^2)^2) - (2*a^2*c^2)/(d^3*(1 + c^2*x^2))) - (5*a^2*c^2*Log[x])/(2*d^(5/2)) + (5*a^2*c^2*Log[d + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]])/(2*d^(5/2)) + (a*b*c^2*((4*c*x)/Sqrt[1 + c^2*x^2] - 48*ArcSinh[c*x] - (8*ArcSinh[c*x])/(1 + c^2*x^2) + 104*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] - 6*Sqrt[1 + c^2*x^2]*Coth[ArcSinh[c*x]/2] - 3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 60*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] + 60*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 60*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-ArcSinh[c*x])] + 60*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(-ArcSinh[c*x])] - 3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 6*Sqrt[1 + c^2*x^2]*Tanh[ArcSinh[c*x]/2]))/(12*d^2*Sqrt[d*(1 + c^2*x^2)]) + (b^2*c^2*(8 + (8*c*x*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - 48*ArcSinh[c*x]^2 - (8*ArcSinh[c*x]^2)/(1 + c^2*x^2) - 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Coth[ArcSinh[c*x]/2] - 3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Csch[ArcSinh[c*x]/2]^2 - 60*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Log[1 - E^(-ArcSinh[c*x])] - (104*I)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] + (104*I)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + I/E^ArcSinh[c*x]] + 60*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] + 24*Sqrt[1 + c^2*x^2]*Log[Tanh[ArcSinh[c*x]/2]] - 120*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - (104*I)*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[c*x]] + (104*I)*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]] + 120*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*PolyLog[2, E^(-ArcSinh[c*x])] - 120*Sqrt[1 + c^2*x^2]*PolyLog[3, -E^(-ArcSinh[c*x])] + 120*Sqrt[1 + c^2*x^2]*PolyLog[3, E^(-ArcSinh[c*x])] - 3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Sech[ArcSinh[c*x]/2]^2 + 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Tanh[ArcSinh[c*x]/2]))/(24*d^2*Sqrt[d*(1 + c^2*x^2)])

Maple [F]

$$\int \frac{(a + b \operatorname{arcsinh}(cx))^2}{x^3 (c^2 d x^2 + d)^{\frac{5}{2}}} dx$$

[In] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(5/2),x)

[Out] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(5/2),x)

Fricas [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^3 (d + c^2 dx^2)^{5/2}} dx = \int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(c^2 dx^2 + d)^{\frac{5}{2}} x^3} dx$$

[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^6*d^3*x^9 + 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5 + d^3*x^3), x)

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx = \int \frac{\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}{x^3\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d)**(5/2),x)
[Out] Integral((a + b*asinh(c*x))**2/(x**3*(d*(c**2*x**2 + 1))**(5/2)), x)
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx = \int \frac{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
[Out] 1/6*a^2*(15*c^2*arcsinh(1/(c*abs(x)))/d^(5/2) - 15*c^2/(sqrt(c^2*d*x^2 + d)
*d^2) - 5*c^2/((c^2*d*x^2 + d)^(3/2)*d) - 3/((c^2*d*x^2 + d)^(3/2)*d*x^2))
+ integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/((c^2*d*x^2 + d)^(5/2)*x^3)
+ 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/((c^2*d*x^2 + d)^(5/2)*x^3), x)
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx = \int \frac{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(5/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx = \int \frac{\left(a+b\operatorname{asinh}(c\,x)\right)^2}{x^3\left(d\,c^2\,x^2+d\right)^{5/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^(5/2)),x)
[Out] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^(5/2)), x)
```

3.319 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 506

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{5/2}}\,dx &= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}}-\frac{2b^2c^4x}{3d^2\sqrt{d+c^2dx^2}}\\
&-\frac{bc(a+b\text{arcsinh}(cx))}{3d^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3(d+c^2dx^2)^{3/2}}\\
&+\frac{2c^2(a+b\text{arcsinh}(cx))^2}{dx(d+c^2dx^2)^{3/2}}+\frac{8c^4x(a+b\text{arcsinh}(cx))^2}{3d(d+c^2dx^2)^{3/2}}\\
&+\frac{16c^4x(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}}+\frac{16c^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}}\\
&+\frac{32bc^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}}\\
&-\frac{32bc^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}}\\
&-\frac{8b^2c^3\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}}\\
&-\frac{8b^2c^3\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}}\end{aligned}$$

```
[Out] -1/3*(a+b*arcsinh(c*x))^2/d/x^3/(c^2*d*x^2+d)^(3/2)+2*c^2*(a+b*arcsinh(c*x)
)^2/d/x/(c^2*d*x^2+d)^(3/2)+8/3*c^4*x*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^
(3/2)-1/3*b^2*c^2/d^2/x/(c^2*d*x^2+d)^(1/2)-2/3*b^2*c^4*x/d^2/(c^2*d*x^2+d)
^(1/2)+16/3*c^4*x*(a+b*arcsinh(c*x))^2/d^2/(c^2*d*x^2+d)^(1/2)-1/3*b*c*(a+b
*arcsinh(c*x))/d^2/x^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+16/3*c^3*(a+b*
arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+32/3*b*c^3*(a+b*a
rcsinh(c*x))*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d^2/(c^2*
```

```
d*x^2+d)^(1/2)-32/3*b*c^3*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2
)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-8/3*b^2*c^3*polylog(2,-(c*x+(c^
2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-8/3*b^2*c^3*po
lylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2
)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.536$, Rules used = {5809, 5788, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 197, 5811, 5799, 5569, 4267, 277}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{5/2}}\,dx = \frac{32bc^3\sqrt{c^2x^2+1}\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3d^2\sqrt{c^2dx^2+d}} - \frac{bc(a+b\text{arcsinh}(cx))}{3d^2x^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}} + \frac{2c^2(a+b\text{arcsinh}(cx))^2}{dx\,(c^2dx^2+d)^{3/2}} - \frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\,(c^2dx^2+d)^{3/2}} + \frac{16c^4x(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{c^2dx^2+d}} + \frac{8c^4x(a+b\text{arcsinh}(cx))^2}{3d\,(c^2dx^2+d)^{3/2}} + \frac{16c^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{c^2dx^2+d}} - \frac{32bc^3\sqrt{c^2x^2+1}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))}{3d^2\sqrt{c^2dx^2+d}} - \frac{8b^2c^3\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3d^2\sqrt{c^2dx^2+d}} - \frac{8b^2c^3\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{3d^2\sqrt{c^2dx^2+d}} - \frac{b^2c^2}{3d^2x\sqrt{c^2dx^2+d}} - \frac{2b^2c^4x}{3d^2\sqrt{c^2dx^2+d}}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] -1/3*(b^2*c^2)/(d^2*x*Sqrt[d + c^2*d*x^2]) - (2*b^2*c^4*x)/(3*d^2*Sqrt[d +
c^2*d*x^2]) - (b*c*(a + b*ArcSinh[c*x]))/(3*d^2*x^2*Sqrt[1 + c^2*x^2]*Sqrt[
d + c^2*d*x^2]) - (a + b*ArcSinh[c*x])^2/(3*d*x^3*(d + c^2*d*x^2)^(3/2)) +
(2*c^2*(a + b*ArcSinh[c*x])^2)/(d*x*(d + c^2*d*x^2)^(3/2)) + (8*c^4*x*(a +
b*ArcSinh[c*x])^2)/(3*d*(d + c^2*d*x^2)^(3/2)) + (16*c^4*x*(a + b*ArcSinh[c
*x])^2)/(3*d^2*Sqrt[d + c^2*d*x^2]) + (16*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcS
inh[c*x])^2)/(3*d^2*Sqrt[d + c^2*d*x^2]) + (32*b*c^3*Sqrt[1 + c^2*x^2]*(a +
 b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/(3*d^2*Sqrt[d + c^2*d*x^2]) -
 (32*b*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x]
)])/(3*d^2*Sqrt[d + c^2*d*x^2]) - (8*b^2*c^3*Sqrt[1 + c^2*x^2]*PolyLog[2, -
E^(2*ArcSinh[c*x])])/(3*d^2*Sqrt[d + c^2*d*x^2]) - (8*b^2*c^3*Sqrt[1 + c^2*
x^2]*PolyLog[2, E^(2*ArcSinh[c*x])])/(3*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5811

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3(d+c^2dx^2)^{3/2}} \\
&\quad -(2c^2)\int\frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{5/2}}\,dx+\frac{\left(2bc\sqrt{1+c^2x^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{x^3(1+c^2x^2)^2}\,dx}{3d^2\sqrt{d+c^2dx^2}} \\
&= -\frac{bc(a+b\text{arcsinh}(cx))}{3d^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3(d+c^2dx^2)^{3/2}}+\frac{2c^2(a+b\text{arcsinh}(cx))^2}{dx(d+c^2dx^2)^{3/2}} \\
&\quad +(8c^4)\int\frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx+\frac{\left(b^2c^2\sqrt{1+c^2x^2}\right)\int\frac{1}{x^2(1+c^2x^2)^{3/2}}\,dx}{3d^2\sqrt{d+c^2dx^2}} \\
&\quad -\frac{\left(4bc^3\sqrt{1+c^2x^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{x(1+c^2x^2)^2}\,dx}{3d^2\sqrt{d+c^2dx^2}}-\frac{\left(4bc^3\sqrt{1+c^2x^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{x(1+c^2x^2)^2}\,dx}{d^2\sqrt{d+c^2dx^2}}
\end{aligned}$$

$$= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} - \frac{8bc^3(a+b\text{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{bc(a+b\text{arcsinh}(cx))}{3d^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{3dx^3(d+c^2dx^2)^{3/2}} + \frac{2c^2(a+b\text{arcsinh}(cx))^2}{dx(d+c^2dx^2)^{3/2}} + \frac{8c^4x(a+b\text{arcsinh}(cx))^2}{3d(d+c^2dx^2)^{3/2}} + \frac{(16c^4)\int\frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}dx}{3d} - \frac{\left(4bc^3\sqrt{1+c^2x^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{x(1+c^2x^2)}dx}{3d^2\sqrt{d+c^2dx^2}} - \frac{\left(4bc^3\sqrt{1+c^2x^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{x(1+c^2x^2)}dx}{d^2\sqrt{d+c^2dx^2}} + \frac{\left(2b^2c^4\sqrt{1+c^2x^2}\right)\int\frac{1}{(1+c^2x^2)^{3/2}}dx}{d^2\sqrt{d+c^2dx^2}} - \frac{\left(16bc^5\sqrt{1+c^2x^2}\right)\int\frac{x(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^2}dx}{3d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} + \frac{2b^2c^4x}{d^2\sqrt{d+c^2dx^2}} - \frac{bc(a+b\text{arcsinh}(cx))}{3d^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{3dx^3(d+c^2dx^2)^{3/2}} + \frac{2c^2(a+b\text{arcsinh}(cx))^2}{dx(d+c^2dx^2)^{3/2}} + \frac{8c^4x(a+b\text{arcsinh}(cx))^2}{3d(d+c^2dx^2)^{3/2}} + \frac{16c^4x(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{\left(4bc^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\text{csch}(x)\text{sech}(x)\,dx,x,\text{arcsinh}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}} - \frac{\left(4bc^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\text{csch}(x)\text{sech}(x)\,dx,x,\text{arcsinh}(cx)\right)}{d^2\sqrt{d+c^2dx^2}} - \frac{\left(8b^2c^4\sqrt{1+c^2x^2}\right)\int\frac{1}{(1+c^2x^2)^{3/2}}dx}{3d^2\sqrt{d+c^2dx^2}} - \frac{\left(32bc^5\sqrt{1+c^2x^2}\right)\int\frac{x(a+b\text{arcsinh}(cx))}{1+c^2x^2}dx}{3d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc(a+b\text{arcsinh}(cx))}{3d^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{3dx^3(d+c^2dx^2)^{3/2}} + \frac{2c^2(a+b\text{arcsinh}(cx))^2}{dx(d+c^2dx^2)^{3/2}} + \frac{8c^4x(a+b\text{arcsinh}(cx))^2}{3d(d+c^2dx^2)^{3/2}} + \frac{16c^4x(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{\left(8bc^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\text{csch}(2x)\,dx,x,\text{arcsinh}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}} - \frac{\left(8bc^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\text{csch}(2x)\,dx,x,\text{arcsinh}(cx)\right)}{d^2\sqrt{d+c^2dx^2}} - \frac{\left(32bc^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\tanh(x)\,dx,x,\text{arcsinh}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}}$$

$$
\begin{aligned}
&= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc(a+b\operatorname{arcsinh}(cx))}{3d^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} \\
&\quad - \frac{(a+b\operatorname{arcsinh}(cx))^2}{3dx^3\left(d+c^2dx^2\right)^{3/2}} + \frac{2c^2(a+b\operatorname{arcsinh}(cx))^2}{dx\left(d+c^2dx^2\right)^{3/2}} + \frac{8c^4x(a+b\operatorname{arcsinh}(cx))^2}{3d\left(d+c^2dx^2\right)^{3/2}} \\
&\quad + \frac{16c^4x(a+b\operatorname{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} + \frac{16c^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} \\
&\quad + \frac{32bc^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{2\operatorname{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\left(64bc^3\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \operatorname{arcsinh}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(4b^2c^3\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\left(4b^2c^3\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(4b^2c^3\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{d^2\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\left(4b^2c^3\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{d^2\sqrt{d+c^2dx^2}}
\end{aligned}
$$

$$= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc(a+b\text{arcsinh}(cx))}{3d^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{3dx^3(d+c^2dx^2)^{3/2}} + \frac{2c^2(a+b\text{arcsinh}(cx))^2}{dx(d+c^2dx^2)^{3/2}} + \frac{8c^4x(a+b\text{arcsinh}(cx))^2}{3d(d+c^2dx^2)^{3/2}} + \frac{16c^4x(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} + \frac{16c^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} + \frac{32bc^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} - \frac{32bc^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} + \frac{\left(2b^2c^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} - \frac{\left(2b^2c^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} + \frac{\left(2b^2c^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} - \frac{\left(2b^2c^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} + \frac{\left(32b^2c^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\log\left(1+e^{2x}\right)\,dx,x,\text{arcsinh}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc(a+b\text{arcsinh}(cx))}{3d^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{3dx^3(d+c^2dx^2)^{3/2}} + \frac{2c^2(a+b\text{arcsinh}(cx))^2}{dx(d+c^2dx^2)^{3/2}} + \frac{8c^4x(a+b\text{arcsinh}(cx))^2}{3d(d+c^2dx^2)^{3/2}} + \frac{16c^4x(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} + \frac{16c^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} + \frac{32bc^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} - \frac{32bc^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} + \frac{8b^2c^3\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} - \frac{8b^2c^3\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} + \frac{\left(16b^2c^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc(a+b\text{arcsinh}(cx))}{3d^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}$$
$$- \frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\left(d+c^2dx^2\right)^{3/2}} + \frac{2c^2(a+b\text{arcsinh}(cx))^2}{dx\left(d+c^2dx^2\right)^{3/2}} + \frac{8c^4x(a+b\text{arcsinh}(cx))^2}{3d\left(d+c^2dx^2\right)^{3/2}}$$
$$+ \frac{16c^4x(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} + \frac{16c^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{32bc^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{32bc^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{8b^2c^3\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{8b^2c^3\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}}$$

Mathematica [A] (verified)

Time = 2.60 (sec) , antiderivative size = 417, normalized size of antiderivative = 0.82

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\left(d+c^2dx^2\right)^{5/2}}\,dx = \frac{\frac{a^2(-1+6c^2x^2+24c^4x^4+16c^6x^6)}{x^3} - \frac{ab\left(-2(-1+6c^2x^2+24c^4x^4+16c^6x^6)\text{arcsinh}(cx)+cx\sqrt{1+c^2x^2}(1+1\right.}{x^3}}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] ((a^2*(-1 + 6*c^2*x^2 + 24*c^4*x^4 + 16*c^6*x^6))/x^3 - (a*b*(-2*(-1 + 6*c^
2*x^2 + 24*c^4*x^4 + 16*c^6*x^6)*ArcSinh[c*x] + c*x*Sqrt[1 + c^2*x^2]*(1 +
16*(c^2*x^2 + c^4*x^4)*Log[c*x] + 8*(c^2*x^2 + c^4*x^4)*Log[1 + c^2*x^2])))
/x^3 + b^2*c^3*(1 + c^2*x^2)^(3/2)*(-((c*x)/Sqrt[1 + c^2*x^2]) - Sqrt[1 + c
^2*x^2]/(c*x) - ArcSinh[c*x]/(c^2*x^2) + ArcSinh[c*x]/(1 + c^2*x^2) - 16*Ar
cSinh[c*x]^2 + (c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2)^(3/2) + (8*c*x*ArcSinh[c*
x]^2)/Sqrt[1 + c^2*x^2] - (Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/(c^3*x^3) + (8
*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/(c*x) - 16*ArcSinh[c*x]*Log[1 - E^(-2*Ar
cSinh[c*x])] - 16*ArcSinh[c*x]*Log[1 + E^(-2*ArcSinh[c*x])] + 8*PolyLog[2,
-E^(-2*ArcSinh[c*x])] + 8*PolyLog[2, E^(-2*ArcSinh[c*x])]))/(3*d*(d + c^2*d
*x^2)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3505 vs. 2(484) = 968.

Time = 0.37 (sec) , antiderivative size = 3506, normalized size of antiderivative = 6.93

method	result	size
default	Expression too large to display	3506
parts	Expression too large to display	3506

```
[In] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 344/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^
2-1)/d^3*x^3*arcsinh(c*x)^2*c^6+64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+
36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^3*arcsinh(c*x)*c^6+22/3*b^2*(d*(c
^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^2*c^
5*(c^2*x^2+1)^(1/2)+12*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*
c^4*x^4+10*c^2*x^2-1)/d^3*x*arcsinh(c*x)^2*c^4-16/3*b^2*(d*(c^2*x^2+1))^(1/
2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x*arcsinh(c*x)*c^4-6
*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/
d^3/x*arcsinh(c*x)^2*c^2-16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d
^3*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^3-16/3*b^2*(d*(c^2*x^2+1))^(1
/2)/(c^2*x^2+1)^(1/2)/d^3*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*c^3+
16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2
-1)/d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^3-4*b^2*(d*(c^2*x^2+1))^(1/2)/(1
2*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*(c^2*x^2+1)^(1/2)*arcsinh
(c*x)*c^3-16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arcsinh(c*x)
*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^3+256/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^
8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^11*arcsinh(c*x)*c^14-64/3*b^2*(
d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^
9*(c^2*x^2+1)*c^12+896/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+3
5*c^4*x^4+10*c^2*x^2-1)/d^3*x^9*arcsinh(c*x)*c^12+1/3*b^2*(d*(c^2*x^2+1))^(
1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3/x*c^2-160/3*b^2*(d
*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^7
*(c^2*x^2+1)*c^10+64*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^
4*x^4+10*c^2*x^2-1)/d^3*x^7*arcsinh(c*x)^2*c^10+1120/3*b^2*(d*(c^2*x^2+1))^
(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^7*arcsinh(c*x)*
c^10+8*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x
^2-1)/d^3*x^6*(c^2*x^2+1)^(1/2)*c^9-128/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8
*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^5*(c^2*x^2+1)*c^8+160*b^2*(d
*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^5
*arcsinh(c*x)^2*c^8+560/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+
35*c^4*x^4+10*c^2*x^2-1)/d^3*x^5*arcsinh(c*x)*c^8+16*b^2*(d*(c^2*x^2+1))^(1
/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^4*(c^2*x^2+1)^(1/
2)*c^7-32/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*
c^2*x^2-1)/d^3*x^3*(c^2*x^2+1)*c^6-176/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*
```

x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^2*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^5-64*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^6*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^9-4*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^2*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^5+16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x*(c^2*x^2+1)*arcsinh(c*x)*c^4+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3/x^2*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c-256/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^9*(c^2*x^2+1)*arcsinh(c*x)*c^12-160*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^5*(c^2*x^2+1)*arcsinh(c*x)*c^8-128*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^4*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^7-80/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^3*(c^2*x^2+1)*arcsinh(c*x)*c^6-640/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^7*(c^2*x^2+1)*arcsinh(c*x)*c^10+a^2*(-1/3/d/x^3/(c^2*d*x^2+d)^(3/2)-2*c^2*(-1/d/x/(c^2*d*x^2+d)^(3/2)-4*c^2*(1/3/d*x/(c^2*d*x^2+d)^(3/2)+2/3/d^2*x/(c^2*d*x^2+d)^(1/2))))+1/3*a*b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*(32*arcsinh(c*x)*c^7*x^7-16*ln((c*x+(c^2*x^2+1)^(1/2))^4-1)*x^7*c^7+32*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^6*x^6+64*arcsinh(c*x)*c^5*x^5-32*ln((c*x+(c^2*x^2+1)^(1/2))^4-1)*x^5*c^5+48*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4*c^4+32*arcsinh(c*x)*c^3*x^3-16*ln((c*x+(c^2*x^2+1)^(1/2))^4-1)*x^3*c^3+12*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2-c^3*x^3-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-c*x)/(c^6*x^6+3*c^4*x^4+3*c^2*x^2+1)/d^3/x^3+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3/x^3*arcsinh(c*x)^2+32/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arcsinh(c*x)^2*c^3-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*c^3-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*c^3*(c^2*x^2+1)^(1/2)-16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^3-16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^3+64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^11*c^14+224/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^9*c^12+88*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^7*c^10+100/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^5*c^8-14/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^3*c^6-3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x*c^4

Fricas [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^4 (d + c^2dx^2)^{5/2}} \, dx = \int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(c^2dx^2 + d)^{\frac{5}{2}}x^4} \, dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^6*d^3*x^10 + 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6 + d^3*x^4), x)
```

Sympy [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^4 (d + c^2dx^2)^{5/2}} \, dx = \int \frac{(a + b \operatorname{asinh}(cx))^2}{x^4 (d (c^2x^2 + 1))^{\frac{5}{2}}} \, dx$$

```
[In] integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x**4*(d*(c**2*x**2 + 1))**(5/2)), x)
```

Maxima [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^4 (d + c^2dx^2)^{5/2}} \, dx = \int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(c^2dx^2 + d)^{\frac{5}{2}}x^4} \, dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/3*a*b*c*(8*c^2*log(c^2*x^2 + 1)/d^(5/2) + 16*c^2*log(x)/d^(5/2) + 1/(c^2
*d^(5/2)*x^4 + d^(5/2)*x^2)) + 2/3*(16*c^4*x/(sqrt(c^2*d*x^2 + d)*d^2) + 8*
c^4*x/((c^2*d*x^2 + d)^(3/2)*d) + 6*c^2/((c^2*d*x^2 + d)^(3/2)*d*x) - 1/((c
^2*d*x^2 + d)^(3/2)*d*x^3))*a*b*arcsinh(c*x) + 1/3*(16*c^4*x/(sqrt(c^2*d*x^
2 + d)*d^2) + 8*c^4*x/((c^2*d*x^2 + d)^(3/2)*d) + 6*c^2/((c^2*d*x^2 + d)^(3
/2)*d*x) - 1/((c^2*d*x^2 + d)^(3/2)*d*x^3))*a^2 + b^2*integrate(log(c*x + s
qrt(c^2*x^2 + 1))^2/((c^2*d*x^2 + d)^(5/2)*x^4), x)
```

Giac [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^4 (d + c^2dx^2)^{5/2}} dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(c^2dx^2 + d)^{\frac{5}{2}} x^4} dx$$

[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(5/2)*x^4), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^4 (d + c^2dx^2)^{5/2}} dx = \int \frac{(a + b\,\text{asinh}(c\,x))^2}{x^4\,(d\,c^2\,x^2 + d)^{5/2}} dx$$

[In] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^(5/2)),x)

[Out] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^(5/2)), x)

3.320 $\int \frac{\operatorname{arcsinh}(ax)^2}{(c+a^2cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 366

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^2}{(c+a^2cx^2)^{7/2}}\,dx =& -\frac{x}{3c^3\sqrt{c+a^2cx^2}} - \frac{x}{30c^3(1+a^2x^2)\sqrt{c+a^2cx^2}} \\ &+ \frac{\operatorname{arcsinh}(ax)}{10ac^3(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} + \frac{4\operatorname{arcsinh}(ax)}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} \\ &+ \frac{x\operatorname{arcsinh}(ax)^2}{5c(c+a^2cx^2)^{5/2}} + \frac{4x\operatorname{arcsinh}(ax)^2}{15c^2(c+a^2cx^2)^{3/2}} + \frac{8x\operatorname{arcsinh}(ax)^2}{15c^3\sqrt{c+a^2cx^2}} \\ &+ \frac{8\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{15ac^3\sqrt{c+a^2cx^2}} - \frac{16\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)\log\left(1+e^{2\operatorname{arcsinh}(ax)}\right)}{15ac^3\sqrt{c+a^2cx^2}} \\ &- \frac{8\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(ax)}\right)}{15ac^3\sqrt{c+a^2cx^2}}\end{aligned}$$

```
[Out] 1/5*x*arcsinh(a*x)^2/c/(a^2*c*x^2+c)^(5/2)+4/15*x*arcsinh(a*x)^2/c^2/(a^2*c
*x^2+c)^(3/2)-1/3*x/c^3/(a^2*c*x^2+c)^(1/2)-1/30*x/c^3/(a^2*x^2+1)/(a^2*c*x
^2+c)^(1/2)+1/10*arcsinh(a*x)/a/c^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2)+8
/15*x*arcsinh(a*x)^2/c^3/(a^2*c*x^2+c)^(1/2)+4/15*arcsinh(a*x)/a/c^3/(a^2*x
^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+8/15*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a/c^3
/(a^2*c*x^2+c)^(1/2)-16/15*arcsinh(a*x)*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)*(a^
2*x^2+1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)-8/15*polylog(2,-(a*x+(a^2*x^2+1)^(
1/2))^2)*(a^2*x^2+1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5788, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 197, 198}

$$\int \frac{\operatorname{arcsinh}(ax)^2}{(c+a^2cx^2)^{7/2}}\,dx = -\frac{8\sqrt{a^2x^2+1}\,\text{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(ax)}\right)}{15ac^3\sqrt{a^2cx^2+c}} + \frac{8x\operatorname{arcsinh}(ax)^2}{15c^3\sqrt{a^2cx^2+c}} + \frac{8\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^2}{15ac^3\sqrt{a^2cx^2+c}} + \frac{4\operatorname{arcsinh}(ax)}{15ac^3\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}} + \frac{\operatorname{arcsinh}(ax)}{10ac^3\left(a^2x^2+1\right)^{3/2}\sqrt{a^2cx^2+c}} - \frac{16\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)\log\left(e^{2\operatorname{arcsinh}(ax)}+1\right)}{15ac^3\sqrt{a^2cx^2+c}} + \frac{4x\operatorname{arcsinh}(ax)^2}{15c^2\left(a^2cx^2+c\right)^{3/2}} + \frac{x\operatorname{arcsinh}(ax)^2}{5c\left(a^2cx^2+c\right)^{5/2}} - \frac{x}{3c^3\sqrt{a^2cx^2+c}} - \frac{x}{30c^3\left(a^2x^2+1\right)\sqrt{a^2cx^2+c}}$$

```
[In] Int[ArcSinh[a*x]^2/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/3*x/(c^3*Sqrt[c + a^2*c*x^2]) - x/(30*c^3*(1 + a^2*x^2)*Sqrt[c + a^2*c*x
^2]) + ArcSinh[a*x]/(10*a*c^3*(1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*x^2]) + (4
*ArcSinh[a*x])/(15*a*c^3*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2]) + (x*ArcSin
h[a*x]^2)/(5*c*(c + a^2*c*x^2)^(5/2)) + (4*x*ArcSinh[a*x]^2)/(15*c^2*(c + a
^2*c*x^2)^(3/2)) + (8*x*ArcSinh[a*x]^2)/(15*c^3*Sqrt[c + a^2*c*x^2]) + (8*S
qrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(15*a*c^3*Sqrt[c + a^2*c*x^2]) - (16*Sqrt[
1 + a^2*x^2]*ArcSinh[a*x]*Log[1 + E^(2*ArcSinh[a*x])])/(15*a*c^3*Sqrt[c + a
^2*c*x^2]) - (8*Sqrt[1 + a^2*x^2]*PolyLog[2, -E^(2*ArcSinh[a*x])])/(15*a*c^
3*Sqrt[c + a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x\text{arcsinh}(ax)^2}{5c\left(c+a^2cx^2\right)^{5/2}}+\frac{4\int\frac{\text{arcsinh}(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx}{5c}-\frac{\left(2a\sqrt{1+a^2x^2}\right)\int\frac{x\text{arcsinh}(ax)}{(1+a^2x^2)^3}\,dx}{5c^3\sqrt{c+a^2cx^2}}\\
&= \frac{\text{arcsinh}(ax)}{10ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}}+\frac{x\text{arcsinh}(ax)^2}{5c\left(c+a^2cx^2\right)^{5/2}}\\
&\quad+\frac{4x\text{arcsinh}(ax)^2}{15c^2\left(c+a^2cx^2\right)^{3/2}}+\frac{8\int\frac{\text{arcsinh}(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx}{15c^2}\\
&\quad-\frac{\sqrt{1+a^2x^2}\int\frac{1}{(1+a^2x^2)^{5/2}}\,dx}{10c^3\sqrt{c+a^2cx^2}}-\frac{\left(8a\sqrt{1+a^2x^2}\right)\int\frac{x\text{arcsinh}(ax)}{(1+a^2x^2)^2}\,dx}{15c^3\sqrt{c+a^2cx^2}}\\
&= -\frac{x}{30c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}}+\frac{\text{arcsinh}(ax)}{10ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}}\\
&\quad+\frac{4\text{arcsinh}(ax)}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}+\frac{x\text{arcsinh}(ax)^2}{5c\left(c+a^2cx^2\right)^{5/2}}\\
&\quad+\frac{4x\text{arcsinh}(ax)^2}{15c^2\left(c+a^2cx^2\right)^{3/2}}+\frac{8x\text{arcsinh}(ax)^2}{15c^3\sqrt{c+a^2cx^2}}-\frac{\sqrt{1+a^2x^2}\int\frac{1}{(1+a^2x^2)^{3/2}}\,dx}{15c^3\sqrt{c+a^2cx^2}}\\
&\quad-\frac{\left(4\sqrt{1+a^2x^2}\right)\int\frac{1}{(1+a^2x^2)^{3/2}}\,dx}{15c^3\sqrt{c+a^2cx^2}}-\frac{\left(16a\sqrt{1+a^2x^2}\right)\int\frac{x\text{arcsinh}(ax)}{1+a^2x^2}\,dx}{15c^3\sqrt{c+a^2cx^2}}\\
&= -\frac{x}{3c^3\sqrt{c+a^2cx^2}}-\frac{x}{30c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}}+\frac{\text{arcsinh}(ax)}{10ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}}\\
&\quad+\frac{4\text{arcsinh}(ax)}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}+\frac{x\text{arcsinh}(ax)^2}{5c\left(c+a^2cx^2\right)^{5/2}}+\frac{4x\text{arcsinh}(ax)^2}{15c^2\left(c+a^2cx^2\right)^{3/2}}\\
&\quad+\frac{8x\text{arcsinh}(ax)^2}{15c^3\sqrt{c+a^2cx^2}}-\frac{\left(16\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x\tanh(x)\,dx,x,\text{arcsinh}(ax)\right)}{15ac^3\sqrt{c+a^2cx^2}}
\end{aligned}
$$

$$= -\frac{x}{3c^3\sqrt{c+a^2cx^2}} - \frac{x}{30c^3(1+a^2x^2)\sqrt{c+a^2cx^2}} + \frac{\text{arcsinh}(ax)}{10ac^3(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} + \frac{4\text{arcsinh}(ax)}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\text{arcsinh}(ax)^2}{5c(c+a^2cx^2)^{5/2}} + \frac{4x\text{arcsinh}(ax)^2}{15c^2(c+a^2cx^2)^{3/2}} + \frac{8x\text{arcsinh}(ax)^2}{15c^3\sqrt{c+a^2cx^2}} + \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{15ac^3\sqrt{c+a^2cx^2}} - \frac{\left(32\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \text{arcsinh}(ax)\right)}{15ac^3\sqrt{c+a^2cx^2}}$$

$$= -\frac{x}{3c^3\sqrt{c+a^2cx^2}} - \frac{x}{30c^3(1+a^2x^2)\sqrt{c+a^2cx^2}} + \frac{\text{arcsinh}(ax)}{10ac^3(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} + \frac{4\text{arcsinh}(ax)}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\text{arcsinh}(ax)^2}{5c(c+a^2cx^2)^{5/2}} + \frac{4x\text{arcsinh}(ax)^2}{15c^2(c+a^2cx^2)^{3/2}} + \frac{8x\text{arcsinh}(ax)^2}{15c^3\sqrt{c+a^2cx^2}} + \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{15ac^3\sqrt{c+a^2cx^2}} - \frac{16\sqrt{1+a^2x^2}\text{arcsinh}(ax)\log\left(1+e^{2\text{arcsinh}(ax)}\right)}{15ac^3\sqrt{c+a^2cx^2}} + \frac{\left(16\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \text{arcsinh}(ax)\right)}{15ac^3\sqrt{c+a^2cx^2}}$$

$$= -\frac{x}{3c^3\sqrt{c+a^2cx^2}} - \frac{x}{30c^3(1+a^2x^2)\sqrt{c+a^2cx^2}} + \frac{\text{arcsinh}(ax)}{10ac^3(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} + \frac{4\text{arcsinh}(ax)}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\text{arcsinh}(ax)^2}{5c(c+a^2cx^2)^{5/2}} + \frac{4x\text{arcsinh}(ax)^2}{15c^2(c+a^2cx^2)^{3/2}} + \frac{8x\text{arcsinh}(ax)^2}{15c^3\sqrt{c+a^2cx^2}} + \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{15ac^3\sqrt{c+a^2cx^2}} - \frac{16\sqrt{1+a^2x^2}\text{arcsinh}(ax)\log\left(1+e^{2\text{arcsinh}(ax)}\right)}{15ac^3\sqrt{c+a^2cx^2}} + \frac{\left(8\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\text{arcsinh}(ax)}\right)}{15ac^3\sqrt{c+a^2cx^2}}$$

$$= -\frac{x}{3c^3\sqrt{c+a^2cx^2}} - \frac{x}{30c^3(1+a^2x^2)\sqrt{c+a^2cx^2}} + \frac{\text{arcsinh}(ax)}{10ac^3(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} + \frac{4\text{arcsinh}(ax)}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\text{arcsinh}(ax)^2}{5c(c+a^2cx^2)^{5/2}} + \frac{4x\text{arcsinh}(ax)^2}{15c^2(c+a^2cx^2)^{3/2}} + \frac{8x\text{arcsinh}(ax)^2}{15c^3\sqrt{c+a^2cx^2}} + \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{15ac^3\sqrt{c+a^2cx^2}} - \frac{16\sqrt{1+a^2x^2}\text{arcsinh}(ax)\log\left(1+e^{2\text{arcsinh}(ax)}\right)}{15ac^3\sqrt{c+a^2cx^2}} - \frac{8\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2, -e^{2\text{arcsinh}(ax)}\right)}{15ac^3\sqrt{c+a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.49

$$\int \frac{\operatorname{arcsinh}(ax)^2}{(c+a^2cx^2)^{7/2}}\,dx = \frac{ax\left(-10-\frac{1}{1+a^2x^2}\right)+\left(-16\sqrt{1+a^2x^2}+\frac{2ax(15+20a^2x^2+8a^4x^4)}{(1+a^2x^2)^2}\right)\operatorname{arcsinh}(ax)^2+\frac{\operatorname{arcsinh}}{}}{30ac^3\sqrt{c}}$$

```
[In] Integrate[ArcSinh[a*x]^2/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] (a*x*(-10 - (1 + a^2*x^2)^(-1)) + (-16*Sqrt[1 + a^2*x^2] + (2*a*x*(15 + 20*
a^2*x^2 + 8*a^4*x^4))/(1 + a^2*x^2)^2)*ArcSinh[a*x]^2 + (ArcSinh[a*x]*(11 +
 8*a^2*x^2 - 32*(1 + a^2*x^2)^2*Log[1 + E^(-2*ArcSinh[a*x])]))/(1 + a^2*x^2
)^(3/2) + 16*Sqrt[1 + a^2*x^2]*PolyLog[2, -E^(-2*ArcSinh[a*x])])/(30*a*c^3*
Sqrt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.56

method	result
default	$\frac{\sqrt{c(a^2x^2+1)}\left(8a^5x^5-8a^4x^4\sqrt{a^2x^2+1}+20a^3x^3-16a^2x^2\sqrt{a^2x^2+1}+15ax-8\sqrt{a^2x^2+1}\right)\left(-64\operatorname{arcsinh}(ax)a^8x^8-64\operatorname{arcsinh}(ax)\sqrt{a^2}\right.}{}$

```
[In] int(arcsinh(a*x)^2/(a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30*(c*(a^2*x^2+1))^(1/2)*(8*a^5*x^5-8*a^4*x^4*(a^2*x^2+1)^(1/2)+20*a^3*x^
3-16*a^2*x^2*(a^2*x^2+1)^(1/2)+15*a*x-8*(a^2*x^2+1)^(1/2))*(-64*arcsinh(a*x
)*a^8*x^8-64*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a^7*x^7-32*a^8*x^8-32*(a^2*x^2+
1)^(1/2)*a^7*x^7-280*arcsinh(a*x)*a^6*x^6-248*arcsinh(a*x)*(a^2*x^2+1)^(1/2
)*a^5*x^5-142*a^6*x^6-126*x^5*a^5*(a^2*x^2+1)^(1/2)+80*a^4*x^4*arcsinh(a*x)
^2-456*a^4*x^4*arcsinh(a*x)-340*a^3*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)-265*
a^4*x^4-156*a^3*x^3*(a^2*x^2+1)^(1/2)+190*arcsinh(a*x)^2*a^2*x^2-328*a^2*x^
2*arcsinh(a*x)-165*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x-235*a^2*x^2-62*a*x*(a
^2*x^2+1)^(1/2)+128*arcsinh(a*x)^2-88*arcsinh(a*x)-80)/(40*a^10*x^10+215*a^
8*x^8+469*a^6*x^6+517*a^4*x^4+287*a^2*x^2+64)/a/c^4+16/15/(a^2*x^2+1)^(1/2)
*(c*(a^2*x^2+1))^(1/2)/a/c^4*arcsinh(a*x)^2-16/15/(a^2*x^2+1)^(1/2)*(c*(a^2
*x^2+1))^(1/2)/a/c^4*arcsinh(a*x)*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)-8/15/(a^2
*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^4*polylog(2,-(a*x+(a^2*x^2+1)^(1/2)
)^2)
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^2}{\left(a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

[In] integrate(arcsinh(a*x)^2/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")

[Out] integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^2/(a^8*c^4*x^8 + 4*a^6*c^4*x^6 + 6*a^4*c^4*x^4 + 4*a^2*c^4*x^2 + c^4), x)

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{\operatorname{asinh}^2\left(ax\right)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{7}{2}}}\,dx$$

[In] integrate(asinh(a*x)**2/(a**2*c*x**2+c)**(7/2),x)

[Out] Integral(asinh(a*x)**2/(c*(a**2*x**2 + 1))**(7/2), x)

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^2}{\left(a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

[In] integrate(arcsinh(a*x)^2/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")

[Out] integrate(arcsinh(a*x)^2/(a^2*c*x^2 + c)^(7/2), x)

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^2}{(c+a^2cx^2)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(arcsinh(a*x)^2/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^2}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)^2}{(c\,a^2\,x^2+c)^{7/2}}\,dx$$

```
[In] int(asinh(a*x)^2/(c + a^2*c*x^2)^(7/2),x)
[Out] int(asinh(a*x)^2/(c + a^2*c*x^2)^(7/2), x)
```

3.321 $\int x^m (d+c^2dx^2)^{5/2} (a+b\text{arcsinh}(cx))^2 \, dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$
\begin{aligned}
\int x^m (d+c^2dx^2)^{5/2} (a+b\text{arcsinh}(cx))^2 \, dx &= \frac{10b^2c^2d^2x^{3+m}\sqrt{d+c^2dx^2}}{(4+m)^3(6+m)} \\
&+ \frac{2b^2c^2d^2(52+15m+m^2)\,x^{3+m}\sqrt{d+c^2dx^2}}{(4+m)^2(6+m)^3} + \frac{2b^2c^4d^2x^{5+m}\sqrt{d+c^2dx^2}}{(6+m)^3} \\
&- \frac{30bcd^2x^{2+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(2+m)^2(4+m)(6+m)\sqrt{1+c^2x^2}} \\
&- \frac{10bcd^2x^{2+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(6+m)\,(8+6m+m^2)\,\sqrt{1+c^2x^2}} - \frac{2bcd^2x^{2+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(12+8m+m^2)\,\sqrt{1+c^2x^2}} \\
&- \frac{10bc^3d^2x^{4+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(4+m)^2(6+m)\sqrt{1+c^2x^2}} - \frac{4bc^3d^2x^{4+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(4+m)(6+m)\sqrt{1+c^2x^2}} \\
&- \frac{2bc^5d^2x^{6+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(6+m)^2\sqrt{1+c^2x^2}} + \frac{15d^2x^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{(6+m)\,(8+6m+m^2)} \\
&+ \frac{5dx^{1+m}(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}{(4+m)(6+m)} + \frac{x^{1+m}(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}{6+m} \\
&+ \frac{30b^2c^2d^2x^{3+m}\sqrt{d+c^2dx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-c^2x^2\right)}{(2+m)^2(3+m)(4+m)(6+m)\sqrt{1+c^2x^2}} \\
&+ \frac{10b^2c^2d^2(10+3m)x^{3+m}\sqrt{d+c^2dx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-c^2x^2\right)}{(2+m)(3+m)(4+m)^3(6+m)\sqrt{1+c^2x^2}} \\
&+ \frac{2b^2c^2d^2(264+130m+15m^2)\,x^{3+m}\sqrt{d+c^2dx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-c^2x^2\right)}{(2+m)(3+m)(4+m)^2(6+m)^3\sqrt{1+c^2x^2}} \\
&+ \frac{15d^3\text{Int}\left(\frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}},x\right)}{(6+m)\,(8+6m+m^2)}
\end{aligned}
$$

```
[Out] 5*d*x^(1+m)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/(4+m)/(6+m)+x^(1+m)*(c
^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/(6+m)+10*b^2*c^2*d^2*x^(3+m)*(c^2*d*
x^2+d)^(1/2)/(4+m)^3/(6+m)+2*b^2*c^2*d^2*(m^2+15*m+52)*x^(3+m)*(c^2*d*x^2+d
)^(1/2)/(4+m)^2/(6+m)^3+2*b^2*c^4*d^2*x^(5+m)*(c^2*d*x^2+d)^(1/2)/(6+m)^3+1
5*d^2*x^(1+m)*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/(6+m)/(m^2+6*m+8)-30
*b*c*d^2*x^(2+m)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(2+m)^2/(4+m)/(6+m)
/(c^2*x^2+1)^(1/2)-10*b*c*d^2*x^(2+m)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2
)/(6+m)/(m^2+6*m+8)/(c^2*x^2+1)^(1/2)-2*b*c*d^2*x^(2+m)*(a+b*arcsinh(c*x))*
(c^2*d*x^2+d)^(1/2)/(m^2+8*m+12)/(c^2*x^2+1)^(1/2)-10*b*c^3*d^2*x^(4+m)*(a+
b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(4+m)^2/(6+m)/(c^2*x^2+1)^(1/2)-4*b*c^3
*d^2*x^(4+m)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(4+m)/(6+m)/(c^2*x^2+1)
^(1/2)-2*b*c^5*d^2*x^(6+m)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(6+m)^2/(
c^2*x^2+1)^(1/2)+10*b^2*c^2*d^2*(10+3*m)*x^(3+m)*hypergeom([1/2, 3/2+1/2*m]
,[5/2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(4+m)^3/(6+m)/(m^2+5*m+6)/(c^2*x
^2+1)^(1/2)+30*b^2*c^2*d^2*x^(3+m)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2*m],-
c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(2+m)^2/(6+m)/(m^2+7*m+12)/(c^2*x^2+1)^(1/2)+2
*b^2*c^2*d^2*(15*m^2+130*m+264)*x^(3+m)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2
*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(4+m)^2/(6+m)^3/(m^2+5*m+6)/(c^2*x^2+1)^(
1/2)+15*d^3*Unintegrable(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)/(6
+m)/(m^2+6*m+8)
```

Rubi [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))^2 \, dx = \int x^m \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))^2 \, dx$$

```
[In] Int[x^m*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (10*b^2*c^2*d^2*x^(3 + m)*Sqrt[d + c^2*d*x^2])/((4 + m)^3*(6 + m)) + (2*b^2
*c^2*d^2*(52 + 15*m + m^2)*x^(3 + m)*Sqrt[d + c^2*d*x^2])/((4 + m)^2*(6 + m
)^3) + (2*b^2*c^4*d^2*x^(5 + m)*Sqrt[d + c^2*d*x^2])/(6 + m)^3 - (30*b*c*d^
2*x^(2 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/((2 + m)^2*(4 + m)*(6
 + m)*Sqrt[1 + c^2*x^2]) - (10*b*c*d^2*x^(2 + m)*Sqrt[d + c^2*d*x^2]*(a + b
*ArcSinh[c*x]))/((6 + m)*(8 + 6*m + m^2)*Sqrt[1 + c^2*x^2]) - (2*b*c*d^2*x^
(2 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/((12 + 8*m + m^2)*Sqrt[1
+ c^2*x^2]) - (10*b*c^3*d^2*x^(4 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x]))/((4 + m)^2*(6 + m)*Sqrt[1 + c^2*x^2]) - (4*b*c^3*d^2*x^(4 + m)*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x]))/((4 + m)*(6 + m)*Sqrt[1 + c^2*x^2]) - (2
*b*c^5*d^2*x^(6 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/((6 + m)^2*S
qrt[1 + c^2*x^2]) + (15*d^2*x^(1 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])^2)/((6 + m)*(8 + 6*m + m^2)) + (5*d*x^(1 + m)*(d + c^2*d*x^2)^(3/2)*(a
```

```
+ b*ArcSinh[c*x])^2)/((4 + m)*(6 + m)) + (x^(1 + m)*(d + c^2*d*x^2)^(5/2)*(
a + b*ArcSinh[c*x])^2)/(6 + m) + (30*b^2*c^2*d^2*x^(3 + m)*Sqrt[d + c^2*d*x
^2]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, -(c^2*x^2)])/((2 + m)^2*(3
 + m)*(4 + m)*(6 + m)*Sqrt[1 + c^2*x^2]) + (10*b^2*c^2*d^2*(10 + 3*m)*x^(3
+ m)*Sqrt[d + c^2*d*x^2]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, -(c^2
*x^2)])/((2 + m)*(3 + m)*(4 + m)^3*(6 + m)*Sqrt[1 + c^2*x^2]) + (2*b^2*c^2*
d^2*(264 + 130*m + 15*m^2)*x^(3 + m)*Sqrt[d + c^2*d*x^2]*Hypergeometric2F1[
1/2, (3 + m)/2, (5 + m)/2, -(c^2*x^2)])/((2 + m)*(3 + m)*(4 + m)^2*(6 + m)^
3*Sqrt[1 + c^2*x^2]) + (15*d^3*Defer[Int][(x^m*(a + b*ArcSinh[c*x])^2)/Sqrt
[d + c^2*d*x^2], x])/((6 + m)*(8 + 6*m + m^2))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^{1+m}(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{6+m} \\
&\quad+\frac{(5d)\int x^m(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx}{6+m} \\
&\quad-\frac{\left(2bcd^2\sqrt{d+c^2dx^2}\right)\int x^{1+m}(1+c^2x^2)^2(a+b\text{arcsinh}(cx))\,dx}{(6+m)\sqrt{1+c^2x^2}} \\
&= -\frac{2bcd^2x^{2+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(12+8m+m^2)\sqrt{1+c^2x^2}} \\
&\quad-\frac{4bc^3d^2x^{4+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(4+m)(6+m)\sqrt{1+c^2x^2}} \\
&\quad-\frac{2bc^5d^2x^{6+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(6+m)^2\sqrt{1+c^2x^2}} \\
&\quad+\frac{5dx^{1+m}(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{(4+m)(6+m)} \\
&\quad+\frac{x^{1+m}(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{6+m} \\
&\quad+\frac{(15d^2)\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx}{(4+m)(6+m)} \\
&\quad+\frac{\left(2b^2c^2d^2\sqrt{d+c^2dx^2}\right)\int\frac{x^{2+m}\left(\frac{1}{2+m}+\frac{2c^2x^2}{4+m}+\frac{c^4x^4}{6+m}\right)}{\sqrt{1+c^2x^2}}\,dx}{(6+m)\sqrt{1+c^2x^2}} \\
&\quad-\frac{\left(10bcd^2\sqrt{d+c^2dx^2}\right)\int x^{1+m}(1+c^2x^2)(a+b\text{arcsinh}(cx))\,dx}{(4+m)(6+m)\sqrt{1+c^2x^2}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2b^2c^4d^2x^{5+m}\sqrt{d+c^2dx^2}}{(6+m)^3} - \frac{10bcd^2x^{2+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(2+m)(4+m)(6+m)\sqrt{1+c^2x^2}} \\
&\quad - \frac{2bcd^2x^{2+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(12+8m+m^2)\sqrt{1+c^2x^2}} \\
&\quad - \frac{10bc^3d^2x^{4+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(4+m)^2(6+m)\sqrt{1+c^2x^2}} \\
&\quad - \frac{4bc^3d^2x^{4+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(4+m)(6+m)\sqrt{1+c^2x^2}} \\
&\quad - \frac{2bc^5d^2x^{6+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(6+m)^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{15d^2x^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{(2+m)(4+m)(6+m)} \\
&\quad + \frac{5dx^{1+m}(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{(4+m)(6+m)} \\
&\quad + \frac{x^{1+m}(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{6+m} + \frac{(15d^3)\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx}{(2+m)(4+m)(6+m)} \\
&\quad + \frac{\left(2b^2d^2\sqrt{d+c^2dx^2}\right)\int \frac{x^{2+m}\left(\frac{c^2(6+m)}{2+m}+\frac{c^4(52+15m+m^2)x^2}{(4+m)(6+m)}\right)}{\sqrt{1+c^2x^2}}\,dx}{(6+m)^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(10b^2c^2d^2\sqrt{d+c^2dx^2}\right)\int \frac{x^{2+m}\left(\frac{1}{2+m}+\frac{c^2x^2}{4+m}\right)}{\sqrt{1+c^2x^2}}\,dx}{(4+m)(6+m)\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(30bcd^2\sqrt{d+c^2dx^2}\right)\int x^{1+m}(a+b\text{arcsinh}(cx))\,dx}{(2+m)(4+m)(6+m)\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$= \frac{10b^2c^2d^2x^{3+m}\sqrt{d+c^2dx^2}}{(4+m)^3(6+m)} + \frac{2b^2c^2d^2(52+15m+m^2)\,x^{3+m}\sqrt{d+c^2dx^2}}{(4+m)^2(6+m)^3}$$
$$+ \frac{2b^2c^4d^2x^{5+m}\sqrt{d+c^2dx^2}}{(6+m)^3} - \frac{30bcd^2x^{2+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(2+m)^2(4+m)(6+m)\sqrt{1+c^2x^2}}$$
$$- \frac{10bcd^2x^{2+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(2+m)(4+m)(6+m)\sqrt{1+c^2x^2}}$$
$$- \frac{2bcd^2x^{2+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(12+8m+m^2)\sqrt{1+c^2x^2}}$$
$$- \frac{10bc^3d^2x^{4+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(4+m)^2(6+m)\sqrt{1+c^2x^2}}$$
$$- \frac{4bc^3d^2x^{4+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(4+m)(6+m)\sqrt{1+c^2x^2}}$$
$$- \frac{2bc^5d^2x^{6+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(6+m)^2\sqrt{1+c^2x^2}}$$
$$+ \frac{15d^2x^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{(2+m)(4+m)(6+m)}$$
$$+ \frac{5dx^{1+m}(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}{(4+m)(6+m)}$$
$$+ \frac{x^{1+m}(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}{6+m}$$
$$+ \frac{(15d^3)\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx}{(2+m)(4+m)(6+m)} + \frac{\left(30b^2c^2d^2\sqrt{d+c^2dx^2}\right)\int \frac{x^{2+m}}{\sqrt{1+c^2x^2}}\,dx}{(2+m)^2(4+m)(6+m)\sqrt{1+c^2x^2}}$$
$$+ \frac{\left(10b^2c^2d^2(10+3m)\sqrt{d+c^2dx^2}\right)\int \frac{x^{2+m}}{\sqrt{1+c^2x^2}}\,dx}{(2+m)(4+m)^3(6+m)\sqrt{1+c^2x^2}}$$
$$+ \frac{\left(2b^2c^2d^2(264+130m+15m^2)\sqrt{d+c^2dx^2}\right)\int \frac{x^{2+m}}{\sqrt{1+c^2x^2}}\,dx}{(2+m)(4+m)^2(6+m)^3\sqrt{1+c^2x^2}}$$

$$= \frac{10b^2c^2d^2x^{3+m}\sqrt{d+c^2dx^2}}{(4+m)^3(6+m)} + \frac{2b^2c^2d^2(52+15m+m^2)\,x^{3+m}\sqrt{d+c^2dx^2}}{(4+m)^2(6+m)^3} + \frac{2b^2c^4d^2x^{5+m}\sqrt{d+c^2dx^2}}{(6+m)^3} - \frac{30bcd^2x^{2+m}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{(2+m)^2(4+m)(6+m)\sqrt{1+c^2x^2}} - \frac{10bcd^2x^{2+m}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{(2+m)(4+m)(6+m)\sqrt{1+c^2x^2}} - \frac{2bcd^2x^{2+m}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{(12+8m+m^2)\sqrt{1+c^2x^2}} - \frac{10bc^3d^2x^{4+m}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{(4+m)^2(6+m)\sqrt{1+c^2x^2}} - \frac{4bc^3d^2x^{4+m}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{(4+m)(6+m)\sqrt{1+c^2x^2}} - \frac{2bc^5d^2x^{6+m}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{(6+m)^2\sqrt{1+c^2x^2}} + \frac{15d^2x^{1+m}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{(2+m)(4+m)(6+m)} + \frac{5dx^{1+m}(d+c^2dx^2)^{3/2}\,(a+b\operatorname{arcsinh}(cx))^2}{(4+m)(6+m)} + \frac{x^{1+m}(d+c^2dx^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))^2}{6+m} + \frac{30b^2c^2d^2x^{3+m}\sqrt{d+c^2dx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-c^2x^2\right)}{(2+m)^2(3+m)(4+m)(6+m)\sqrt{1+c^2x^2}} + \frac{10b^2c^2d^2(10+3m)x^{3+m}\sqrt{d+c^2dx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-c^2x^2\right)}{(2+m)(3+m)(4+m)^3(6+m)\sqrt{1+c^2x^2}} + \frac{2b^2c^2d^2(264+130m+15m^2)\,x^{3+m}\sqrt{d+c^2dx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-c^2x^2\right)}{(2+m)(3+m)(4+m)^2(6+m)^3\sqrt{1+c^2x^2}} + \frac{(15d^3)\int\frac{x^m(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx}{(2+m)(4+m)(6+m)}$$

Mathematica [N/A]

Not integrable

Time = 3.62 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int x^m\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \int x^m\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx$$

```
[In] Integrate[x^m*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
[Out] Integrate[x^m*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int x^{m} \left(c^{2} d\, x^{2}+d\right)^{\frac{5}{2}} \left(a+b \,\operatorname{arcsinh}\left(c x\right)\right)^{2}d x$$

[In] `int(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x)`

[Out] `int(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 4.75

$$\int x^m \left(d+c^2 d x^2\right)^{5/2} (a+b \text{arcsinh}(c x))^2 \, dx=\int { {\left(c^{2} d x^{2} + d\right)}^{\frac{5}{2}} {\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{2} x^{m} \,d x }$$

[In] `integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")`

[Out] `integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4 + 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)*x^m, x)`

Sympy [F(-1)]

Timed out.

$$\int x^m \left(d+c^2 d x^2\right)^{5/2} (a+b \text{arcsinh}(c x))^2 \, dx=\text{Timed out}$$

[In] `integrate(x**m*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2,x)`

[Out] `Timed out`

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int x^m \left(d+c^2 d x^2\right)^{5/2} (a+b \text{arcsinh}(c x))^2 \, dx=\int { {\left(c^{2} d x^{2} + d\right)}^{\frac{5}{2}} {\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{2} x^{m} \,d x }$$

[In] `integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")`

[Out] `integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^2*x^m, x)`

Giac [F(-2)]

Exception generated.

$$\int x^m \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int x^m \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))^2\,dx = \int x^m\,(a+b\,\text{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

```
[In] int(x^m*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x^m*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2), x)
```

3.322 $\int x^m(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int x^m\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\frac{2b^2c^2dx^{3+m}\sqrt{d+c^2dx^2}}{(4+m)^3}-\frac{6bcdx^{2+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(2+m)^2(4+m)\sqrt{1+c^2x^2}}-\frac{2bcdx^{2+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(8+6m+m^2)\sqrt{1+c^2x^2}}-\frac{2bc^3dx^{4+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(4+m)^2\sqrt{1+c^2x^2}}+\frac{3dx^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{8+6m+m^2}+\frac{x^{1+m}\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{4+m}+\frac{6b^2c^2dx^{3+m}\sqrt{d+c^2dx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-c^2x^2\right)}{(2+m)^2(3+m)(4+m)\sqrt{1+c^2x^2}}+\frac{2b^2c^2d(10+3m)x^{3+m}\sqrt{d+c^2dx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-c^2x^2\right)}{(2+m)(3+m)(4+m)^3\sqrt{1+c^2x^2}}+\frac{3d^2\text{Int}\left(\frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}},x\right)}{8+6m+m^2}$$

```
[Out] x^(1+m)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/(4+m)+2*b^2*c^2*d*x^(3+m)*
(c^2*d*x^2+d)^(1/2)/(4+m)^3+3*d*x^(1+m)*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^
(1/2)/(m^2+6*m+8)-6*b*c*d*x^(2+m)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(2
+m)^2/(4+m)/(c^2*x^2+1)^(1/2)-2*b*c*d*x^(2+m)*(a+b*arcsinh(c*x))*(c^2*d*x^2
+d)^(1/2)/(m^2+6*m+8)/(c^2*x^2+1)^(1/2)-2*b*c^3*d*x^(4+m)*(a+b*arcsinh(c*x)
)*(c^2*d*x^2+d)^(1/2)/(4+m)^2/(c^2*x^2+1)^(1/2)+2*b^2*c^2*d*(10+3*m)*x^(3+m
)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(4+m
)^3/(m^2+5*m+6)/(c^2*x^2+1)^(1/2)+6*b^2*c^2*d*x^(3+m)*hypergeom([1/2, 3/2+1
/2*m],[5/2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(2+m)^2/(m^2+7*m+12)/(c^2*x
```

```
^2+1)^(1/2)+3*d^2*Unintegrable(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2)
,x)/(m^2+6*m+8)
```

Rubi [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))^2 \, dx = \int x^m \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))^2 \, dx$$

```
[In] Int[x^m*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (2*b^2*c^2*d*x^(3 + m)*Sqrt[d + c^2*d*x^2])/(4 + m)^3 - (6*b*c*d*x^(2 + m)*
Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/((2 + m)^2*(4 + m)*Sqrt[1 + c^2*x
^2]) - (2*b*c*d*x^(2 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/((8 + 6
*m + m^2)*Sqrt[1 + c^2*x^2]) - (2*b*c^3*d*x^(4 + m)*Sqrt[d + c^2*d*x^2]*(a
+ b*ArcSinh[c*x]))/((4 + m)^2*Sqrt[1 + c^2*x^2]) + (3*d*x^(1 + m)*Sqrt[d +
c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(8 + 6*m + m^2) + (x^(1 + m)*(d + c^2*d*
x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(4 + m) + (6*b^2*c^2*d*x^(3 + m)*Sqrt[d
+ c^2*d*x^2]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, -(c^2*x^2)])/((2
+ m)^2*(3 + m)*(4 + m)*Sqrt[1 + c^2*x^2]) + (2*b^2*c^2*d*(10 + 3*m)*x^(3 +
m)*Sqrt[d + c^2*d*x^2]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, -(c^2*x
^2)])/((2 + m)*(3 + m)*(4 + m)^3*Sqrt[1 + c^2*x^2]) + (3*d^2*Defer[Int][(x^
m*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2], x])/(8 + 6*m + m^2)
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^{1+m}\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{4+m} \\
&\quad + \frac{(3d)\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx}{4+m} \\
&\quad - \frac{\left(2bcd\sqrt{d+c^2dx^2}\right)\int x^{1+m}\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))\,dx}{(4+m)\sqrt{1+c^2x^2}}
\end{aligned}$$

$$= -\frac{2bcdx^{2+m}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{(8+6m+m^2)\sqrt{1+c^2x^2}} - \frac{2bc^3dx^{4+m}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{(4+m)^2\sqrt{1+c^2x^2}} + \frac{3dx^{1+m}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{8+6m+m^2} + \frac{x^{1+m}(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{4+m} + \frac{(3d^2)\int \frac{x^m(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx}{8+6m+m^2} + \frac{\left(2b^2c^2d\sqrt{d+c^2dx^2}\right)\int \frac{x^{2+m}\left(\frac{1}{2+m}+\frac{c^2x^2}{4+m}\right)}{\sqrt{1+c^2x^2}}\,dx}{(4+m)\sqrt{1+c^2x^2}} - \frac{\left(6bcd\sqrt{d+c^2dx^2}\right)\int x^{1+m}(a+b\operatorname{arcsinh}(cx))\,dx}{(2+m)(4+m)\sqrt{1+c^2x^2}}$$

$$= \frac{2b^2c^2dx^{3+m}\sqrt{d+c^2dx^2}}{(4+m)^3} - \frac{6bcdx^{2+m}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{(2+m)^2(4+m)\sqrt{1+c^2x^2}} - \frac{2bcdx^{2+m}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{(8+6m+m^2)\sqrt{1+c^2x^2}} - \frac{2bc^3dx^{4+m}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{(4+m)^2\sqrt{1+c^2x^2}} + \frac{3dx^{1+m}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{8+6m+m^2} + \frac{x^{1+m}(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{4+m} + \frac{(3d^2)\int \frac{x^m(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx}{8+6m+m^2} + \frac{\left(6b^2c^2d\sqrt{d+c^2dx^2}\right)\int \frac{x^{2+m}}{\sqrt{1+c^2x^2}}\,dx}{(2+m)^2(4+m)\sqrt{1+c^2x^2}} + \frac{\left(2b^2c^2d(10+3m)\sqrt{d+c^2dx^2}\right)\int \frac{x^{2+m}}{\sqrt{1+c^2x^2}}\,dx}{(2+m)(4+m)^3\sqrt{1+c^2x^2}}$$

$$= \frac{2b^2c^2dx^{3+m}\sqrt{d+c^2dx^2}}{(4+m)^3} - \frac{6bcdx^{2+m}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{(2+m)^2(4+m)\sqrt{1+c^2x^2}} - \frac{2bcdx^{2+m}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{(8+6m+m^2)\sqrt{1+c^2x^2}} - \frac{2bc^3dx^{4+m}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{(4+m)^2\sqrt{1+c^2x^2}} + \frac{3dx^{1+m}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{8+6m+m^2} + \frac{x^{1+m}(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{4+m} + \frac{6b^2c^2dx^{3+m}\sqrt{d+c^2dx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-c^2x^2\right)}{(2+m)^2(3+m)(4+m)\sqrt{1+c^2x^2}} + \frac{2b^2c^2d(10+3m)x^{3+m}\sqrt{d+c^2dx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-c^2x^2\right)}{(2+m)(3+m)(4+m)^3\sqrt{1+c^2x^2}} + \frac{(3d^2)\int \frac{x^m(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx}{8+6m+m^2}$$

Mathematica [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int x^m \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))^2\,dx=\int x^m \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))^2\,dx$$

```
[In] Integrate[x^m*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
[Out] Integrate[x^m*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.85 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int x^{m} \left(c^{2} d\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b \,\operatorname{arcsinh}\left(c x\right)\right)^{2}d x$$

```
[In] int(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x)
[Out] int(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.86

$$\int x^m \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))^2\,dx=\int { {\left(c^{2} d x^{2} + d\right)}^{\frac{3}{2}} {\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{2} x^{m} \,d x }$$

```
[In] integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)*x^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^m (d + c^2 d x^2)^{3/2} (a + b \operatorname{arcsinh}(cx))^2 \, dx = \text{Timed out}$$

```
[In] integrate(x**m*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int x^m (d + c^2 d x^2)^{3/2} (a + b \operatorname{arcsinh}(cx))^2 \, dx = \int (c^2 d x^2 + d)^{\frac{3}{2}} (b \operatorname{arsinh}(cx) + a)^2 x^m \, dx$$

```
[In] integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^2*x^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int x^m (d + c^2 d x^2)^{3/2} (a + b \operatorname{arcsinh}(cx))^2 \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int x^m (d + c^2 d x^2)^{3/2} (a + b \operatorname{arcsinh}(cx))^2 \, dx = \int x^m (a + b \operatorname{asinh}(c x))^2 (d c^2 x^2 + d)^{3/2} \, dx$$

```
[In] int(x^m*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2),x)
[Out] int(x^m*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2), x)
```

3.323 $\int x^m \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\begin{aligned}\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx &= -\frac{2bcx^{2+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(2+m)^2\sqrt{1+c^2x^2}}+\frac{x^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2+m}\\&\quad+\frac{2b^2c^2x^{3+m}\sqrt{d+c^2dx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-c^2x^2\right)}{(2+m)^2(3+m)\sqrt{1+c^2x^2}}\\&\quad+\frac{d\text{Int}\left(\frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}},x\right)}{2+m}\end{aligned}$$

```
[Out] x^(1+m)*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/(2+m)-2*b*c*x^(2+m)*(a+b*a
rcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(2+m)^2/(c^2*x^2+1)^(1/2)+2*b^2*c^2*x^(3+m
)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(2+m
)^2/(3+m)/(c^2*x^2+1)^(1/2)+d*Unintegrable(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*
x^2+d)^(1/2),x)/(2+m)
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$$

```
[In] Int[x^m*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-2*b*c*x^(2 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/((2 + m)^2*Sqrt
[1 + c^2*x^2]) + (x^(1 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2
+ m) + (2*b^2*c^2*x^(3 + m)*Sqrt[d + c^2*d*x^2]*Hypergeometric2F1[1/2, (3 +
 m)/2, (5 + m)/2, -(c^2*x^2)])/((2 + m)^2*(3 + m)*Sqrt[1 + c^2*x^2]) + (d*D
efer[Int][(x^m*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2], x])/(2 + m)
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2+m}+\frac{d\int\frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx}{2+m}\\
&\quad-\frac{\left(2bc\sqrt{d+c^2dx^2}\right)\int x^{1+m}(a+b\text{arcsinh}(cx))\,dx}{(2+m)\sqrt{1+c^2x^2}}\\
&=-\frac{2bcx^{2+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(2+m)^2\sqrt{1+c^2x^2}}+\frac{x^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2+m}\\
&\quad+\frac{d\int\frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx}{2+m}+\frac{\left(2b^2c^2\sqrt{d+c^2dx^2}\right)\int\frac{x^{2+m}}{\sqrt{1+c^2x^2}}\,dx}{(2+m)^2\sqrt{1+c^2x^2}}\\
&=-\frac{2bcx^{2+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(2+m)^2\sqrt{1+c^2x^2}}+\frac{x^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2+m}\\
&\quad+\frac{2b^2c^2x^{3+m}\sqrt{d+c^2dx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-c^2x^2\right)}{(2+m)^2(3+m)\sqrt{1+c^2x^2}}\\
&\quad+\frac{d\int\frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx}{2+m}
\end{aligned}
$$

Mathematica [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$
\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx
$$

```
[In] Integrate[x^m*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Integrate[x^m*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.71 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int x^{m} \sqrt{c^{2} d\, x^{2}+d}\, \left(a +b \,\operatorname{arcsinh}\left(c x \right)\right)^{2}d x$$

```
[In] int(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.43

$$\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\int \sqrt{c^{2} d x^{2} + d} {\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{2} x^{m}\,{d x}$$

```
[In] integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)*x^m, x)
```

Sympy [N/A]

Not integrable

Time = 21.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\int x^{m} \sqrt{d \left(c^{2} x^{2} + 1\right)} \left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{2}\, dx$$

```
[In] integrate(x**m*(c**2*d*x**2+d)**(1/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**m*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\int \sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)^2x^m\,dx$$

```
[In] integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^2*x^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\int x^m\,(a+b\,\mathrm{asinh}(c\,x))^2\,\sqrt{d\,c^2\,x^2+d}\,dx$$

```
[In] int(x^m*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x^m*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2), x)
```

3.324 $\int \frac{x^m(a+b\textbf{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\text{Int}\left(\frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}},x\right)$$

```
[Out] Unintegrable(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$$

```
[In] Int[(x^m*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] Defer[Int][(x^m*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2], x]
```

Rubi steps

$$\text{integral}=\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.42 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$$

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
[Out] Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m\left(a+b\,\operatorname{arcsinh}\left(cx\right)\right)^2}{\sqrt{c^2d\,x^2+d}}dx$$

```
[In] int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)
[Out] int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.43

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{\left(b\,\operatorname{arsinh}\left(cx\right)+a\right)^2x^m}{\sqrt{c^2dx^2+d}}\,dx$$

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*x^m/sqrt(c^2*d*x^2
 + d), x)
```

Sympy [N/A]

Not integrable

Time = 7.80 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{x^m(a+b\operatorname{asinh}(cx))^2}{\sqrt{d(c^2x^2+1)}}\,dx$$

```
[In] integrate(x**m*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
[Out] Integral(x**m*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2x^m}{\sqrt{c^2dx^2+d}}\,dx$$

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
[Out] integrate((b*arcsinh(c*x) + a)^2*x^m/sqrt(c^2*d*x^2 + d), x)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2x^m}{\sqrt{c^2dx^2+d}}\,dx$$

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2*x^m/sqrt(c^2*d*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 3.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{x^m\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

```
[In] int((x^m*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2),x)
[Out] int((x^m*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2), x)
```

3.325 $\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \text{Int}\left(\frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$$

```
[In] Int[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 4.43 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$$

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
[Out] Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m(a+b\,\text{arcsinh}\,(cx))^2}{(c^2d\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x)
[Out] int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.39

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^m}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)*x^m/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [N/A]

Not integrable

Time = 9.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{x^m(a+b\,\text{asinh}\,(cx))^2}{(d\,(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**m*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
[Out] Integral(x**m*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^m}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
[Out] integrate((b*arcsinh(c*x) + a)^2*x^m/(c^2*d*x^2 + d)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^m}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2*x^m/(c^2*d*x^2 + d)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.99 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{x^m\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^{3/2}}\,dx$$

```
[In] int((x^m*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2),x)
[Out] int((x^m*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2), x)
```

3.326 $\int \frac{x^m(a+b\,\mathrm{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m(a+b\,\mathrm{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \mathrm{Int}\left(\frac{x^m(a+b\,\mathrm{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m(a+b\,\mathrm{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x^m(a+b\,\mathrm{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$$

```
[In] Int[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
[Out] Defer[Int][(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^m(a+b\,\mathrm{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 4.60 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$$

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
[Out] Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m(a+b\,\text{arcsinh}\,(cx))^2}{(c^2d\,x^2+d)^{\frac{5}{2}}}dx$$

```
[In] int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x)
[Out] int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.79

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^m}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)*x^m/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [N/A]

Not integrable

Time = 165.87 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{x^m(a+b\operatorname{asinh}{(cx)})^2}{(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**m*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
[Out] Integral(x**m*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^m}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
[Out] integrate((b*arcsinh(c*x) + a)^2*x^m/(c^2*d*x^2 + d)^(5/2), x)
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^m}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2*x^m/(c^2*d*x^2 + d)^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 3.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{x^m\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^{5/2}} \,d x$$

```
[In] int((x^m*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2),x)
[Out] int((x^m*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2), x)
```

3.327 $\int \frac{x^m \textbf{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\, dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^m \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\, dx = \text{Int}\left(\frac{x^m \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}, x\right)$$

```
[Out] Unintegrable(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\, dx = \int \frac{x^m \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\, dx$$

```
[In] Int[(x^m*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] Defer[Int][(x^m*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2], x]
```

Rubi steps

$$\text{integral} = \int \frac{x^m \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\, dx$$

Mathematica [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^m \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$$

```
[In] Integrate[(x^m*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
[Out] Integrate[(x^m*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^m \operatorname{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx$$

```
[In] int(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x)
[Out] int(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^m*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

Sympy [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^m \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{asinh}^2(ax)}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x**m*asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)
[Out] Integral(x**m*asinh(a*x)**2/sqrt(a**2*x**2 + 1), x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^m*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^m*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

Mupad [N/A]

Not integrable

Time = 3.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{asinh}(a\,x)^2}{\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int((x^m*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^m*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2), x)
```

3.328 $\int (c+a^2cx^2)^3 \operatorname{arcsinh}(ax)^3 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 359

$$\int (c+a^2cx^2)^3 \operatorname{arcsinh}(ax)^3 \, dx = -\frac{413312c^3\sqrt{1+a^2x^2}}{128625a} - \frac{30256c^3(1+a^2x^2)^{3/2}}{385875a} - \frac{2664c^3(1+a^2x^2)^{5/2}}{214375a} - \frac{6c^3(1+a^2x^2)^{7/2}}{2401a} + \frac{4322c^3x\operatorname{arcsinh}(ax)}{1225} + \frac{1514a^2c^3x^3\operatorname{arcsinh}(ax)}{3675} + \frac{702a^4c^3x^5\operatorname{arcsinh}(ax)}{6125} + \frac{6}{343}a^6c^3x^7\operatorname{arcsinh}(ax) - \frac{48c^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{35a} - \frac{8c^3(1+a^2x^2)^{3/2}\operatorname{arcsinh}(ax)^2}{35a} - \frac{18c^3(1+a^2x^2)^{5/2}}{175a}$$

```
[Out] -30256/385875*c^3*(a^2*x^2+1)^(3/2)/a-2664/214375*c^3*(a^2*x^2+1)^(5/2)/a-6
/2401*c^3*(a^2*x^2+1)^(7/2)/a+4322/1225*c^3*x*arcsinh(a*x)+1514/3675*a^2*c^
3*x^3*arcsinh(a*x)+702/6125*a^4*c^3*x^5*arcsinh(a*x)+6/343*a^6*c^3*x^7*arcs
inh(a*x)-8/35*c^3*(a^2*x^2+1)^(3/2)*arcsinh(a*x)^2/a-18/175*c^3*(a^2*x^2+1)
^(5/2)*arcsinh(a*x)^2/a-3/49*c^3*(a^2*x^2+1)^(7/2)*arcsinh(a*x)^2/a+16/35*c
^3*x*arcsinh(a*x)^3+8/35*c^3*x*(a^2*x^2+1)*arcsinh(a*x)^3+6/35*c^3*x*(a^2*x
^2+1)^2*arcsinh(a*x)^3+1/7*c^3*x*(a^2*x^2+1)^3*arcsinh(a*x)^3-413312/128625
*c^3*(a^2*x^2+1)^(1/2)/a-48/35*c^3*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {5786, 5772, 5798, 267, 5784, 455, 45, 200, 12, 1261, 712, 1813, 1864}

$$\int (c+a^2cx^2)^3 \operatorname{arcsinh}(ax)^3 \, dx = \frac{6}{343}a^6c^3x^7\operatorname{arcsinh}(ax) + \frac{702a^4c^3x^5\operatorname{arcsinh}(ax)}{6125} + \frac{1514a^2c^3x^3\operatorname{arcsinh}(ax)}{3675} + \frac{1}{7}c^3x(a^2x^2+1)^3\operatorname{arcsinh}(ax)^3 + \frac{6}{35}c^3x(a^2x^2+1)^2\operatorname{arcsinh}(ax)^3 + \frac{8}{35}c^3x(a^2x^2+1)\operatorname{arcsinh}(ax)^3 - \frac{3c^3(a^2x^2+1)^{7/2}\operatorname{arcsinh}(ax)^2}{49a} - \frac{18c^3(a^2x^2+1)^{5/2}\operatorname{arcsinh}(ax)^2}{175a} - \frac{8c^3(a^2x^2+1)^{3/2}\operatorname{arcsinh}(ax)^2}{35a} - \frac{48c^3\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^2}{35a} - \frac{6c^3(a^2x^2+1)^{7/2}}{2401a} - \frac{2664c^3(a^2x^2+1)^{5/2}}{214375a} - \frac{30256c^3(a^2x^2+1)^{3/2}}{385875a} - \frac{413312c^3\sqrt{a^2x^2+1}}{128625a} + \frac{16}{35}c^3x\operatorname{arcsinh}(ax)^3 + \frac{4322c^3x\operatorname{arcsinh}(ax)}{1225}$$

```
[In] Int[(c + a^2*c*x^2)^3*ArcSinh[a*x]^3,x]
```

```
[Out] (-413312*c^3*Sqrt[1 + a^2*x^2])/(128625*a) - (30256*c^3*(1 + a^2*x^2)^(3/2)
)/(385875*a) - (2664*c^3*(1 + a^2*x^2)^(5/2))/(214375*a) - (6*c^3*(1 + a^2*
x^2)^(7/2))/(2401*a) + (4322*c^3*x*ArcSinh[a*x])/1225 + (1514*a^2*c^3*x^3*A
rcSinh[a*x])/3675 + (702*a^4*c^3*x^5*ArcSinh[a*x])/6125 + (6*a^6*c^3*x^7*Ar
cSinh[a*x])/343 - (48*c^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(35*a) - (8*c^3
*(1 + a^2*x^2)^(3/2)*ArcSinh[a*x]^2)/(35*a) - (18*c^3*(1 + a^2*x^2)^(5/2)*A
rcSinh[a*x]^2)/(175*a) - (3*c^3*(1 + a^2*x^2)^(7/2)*ArcSinh[a*x]^2)/(49*a)
+ (16*c^3*x*ArcSinh[a*x]^3)/35 + (8*c^3*x*(1 + a^2*x^2)*ArcSinh[a*x]^3)/35
+ (6*c^3*x*(1 + a^2*x^2)^2*ArcSinh[a*x]^3)/35 + (c^3*x*(1 + a^2*x^2)^3*ArcS
inh[a*x]^3)/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{7}c^3x\left(1+a^2x^2\right)^3\operatorname{arcsinh}(ax)^3+\frac{1}{7}(6c)\int\left(c+a^2cx^2\right)^2\operatorname{arcsinh}(ax)^3\,dx \\ &\quad-\frac{1}{7}\left(3ac^3\right)\int x\left(1+a^2x^2\right)^{5/2}\operatorname{arcsinh}(ax)^2\,dx \end{aligned}$$

$$= -\frac{3c^3(1+a^2x^2)^{7/2}\operatorname{arcsinh}(ax)^2}{49a} + \frac{6}{35}c^3x(1+a^2x^2)^2\operatorname{arcsinh}(ax)^3 + \frac{1}{7}c^3x(1+a^2x^2)^3\operatorname{arcsinh}(ax)^3 + \frac{1}{35}(24c^2)\int (c+a^2cx^2)\operatorname{arcsinh}(ax)^3\,dx + \frac{1}{49}(6c^3)\int (1+a^2x^2)^3\operatorname{arcsinh}(ax)\,dx - \frac{1}{35}(18ac^3)\int x(1+a^2x^2)^{3/2}\operatorname{arcsinh}(ax)^2\,dx$$

$$= \frac{6}{49}c^3x\operatorname{arcsinh}(ax) + \frac{6}{49}a^2c^3x^3\operatorname{arcsinh}(ax) + \frac{18}{245}a^4c^3x^5\operatorname{arcsinh}(ax) + \frac{6}{343}a^6c^3x^7\operatorname{arcsinh}(ax) - \frac{18c^3(1+a^2x^2)^{5/2}\operatorname{arcsinh}(ax)^2}{175a} - \frac{3c^3(1+a^2x^2)^{7/2}\operatorname{arcsinh}(ax)^2}{49a} + \frac{8}{35}c^3x(1+a^2x^2)\operatorname{arcsinh}(ax)^3 + \frac{6}{35}c^3x(1+a^2x^2)^2\operatorname{arcsinh}(ax)^3 + \frac{1}{7}c^3x(1+a^2x^2)^3\operatorname{arcsinh}(ax)^3 + \frac{1}{175}(36c^3)\int (1+a^2x^2)^2\operatorname{arcsinh}(ax)\,dx + \frac{1}{35}(16c^3)\int \operatorname{arcsinh}(ax)^3\,dx - \frac{1}{49}(6ac^3)\int \frac{x(35+35a^2x^2+21a^4x^4+5a^6x^6)}{35\sqrt{1+a^2x^2}}\,dx - \frac{1}{35}(24ac^3)\int x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2\,dx$$

$$= \frac{402c^3x\operatorname{arcsinh}(ax)}{1225} + \frac{318a^2c^3x^3\operatorname{arcsinh}(ax)}{1225} + \frac{702a^4c^3x^5\operatorname{arcsinh}(ax)}{6125} + \frac{6}{343}a^6c^3x^7\operatorname{arcsinh}(ax) - \frac{8c^3(1+a^2x^2)^{3/2}\operatorname{arcsinh}(ax)^2}{35a} - \frac{18c^3(1+a^2x^2)^{5/2}\operatorname{arcsinh}(ax)^2}{175a} - \frac{3c^3(1+a^2x^2)^{7/2}\operatorname{arcsinh}(ax)^2}{49a} + \frac{16}{35}c^3x\operatorname{arcsinh}(ax)^3 + \frac{8}{35}c^3x(1+a^2x^2)\operatorname{arcsinh}(ax)^3 + \frac{6}{35}c^3x(1+a^2x^2)^2\operatorname{arcsinh}(ax)^3 + \frac{1}{7}c^3x(1+a^2x$$

$$= \frac{962c^3x\operatorname{arcsinh}(ax)}{1225} + \frac{1514a^2c^3x^3\operatorname{arcsinh}(ax)}{3675} + \frac{702a^4c^3x^5\operatorname{arcsinh}(ax)}{6125} + \frac{6}{343}a^6c^3x^7\operatorname{arcsinh}(ax) - \frac{48c^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{35a} - \frac{8c^3(1+a^2x^2)^{3/2}\operatorname{arcsinh}(ax)^2}{35a} - \frac{18c^3(1+a^2x^2)^{5/2}\operatorname{arcsinh}(ax)^2}{175a} - \frac{3c^3(1+a^2x^2)^{7/2}\operatorname{arcsinh}(ax)^2}{49a} + \frac{16}{35}c^3x\operatorname{arcsinh}(ax)^3 + \frac{8}{35}c^3x(1+a^2x^2)\operatorname{arcsinh}(ax)^3 + \frac{6}{35}c^3x(1+a^2x^2)^2\operatorname{arcsinh}(ax)^3 + \frac{1}{7}c^3x(1+a^2x$$

$$= \frac{4322c^3x\text{arcsinh}(ax)}{1225} + \frac{1514a^2c^3x^3\text{arcsinh}(ax)}{3675} + \frac{702a^4c^3x^5\text{arcsinh}(ax)}{6125} + \frac{6}{343}a^6c^3x^7\text{arcsinh}(ax) - \frac{48c^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{35a} - \frac{8c^3(1+a^2x^2)^{3/2}\text{arcsinh}(ax)^2}{35a} - \frac{18c^3(1+a^2x^2)^{5/2}\text{arcsinh}(ax)^2}{175a} - \frac{3c^3(1+a^2x^2)^{7/2}\text{arcsinh}(ax)^2}{49a} + \frac{16}{35}c^3x\text{arcsinh}(ax)^3 + \frac{8}{35}c^3x(1+a^2x^2)\text{arcsinh}(ax)^3 + \frac{6}{35}c^3x(1+a^2x^2)^2\text{arcsinh}(ax)^3 + \frac{1}{7}c^3x(1+a$$

$$= -\frac{960c^3\sqrt{1+a^2x^2}}{343a} - \frac{16c^3(1+a^2x^2)^{3/2}}{1715a} - \frac{36c^3(1+a^2x^2)^{5/2}}{8575a} - \frac{6c^3(1+a^2x^2)^{7/2}}{2401a} + \frac{4322c^3x\text{arcsinh}(ax)}{1225} + \frac{1514a^2c^3x^3\text{arcsinh}(ax)}{3675} + \frac{702a^4c^3x^5\text{arcsinh}(ax)}{6125} + \frac{6}{343}a^6c^3x^7\text{arcsinh}(ax) - \frac{48c^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{35a} - \frac{8c^3(1+a^2x^2)^{3/2}\text{arcsinh}(ax)^2}{35a} - \frac{18c^3(1+$$

$$= -\frac{413312c^3\sqrt{1+a^2x^2}}{128625a} - \frac{30256c^3(1+a^2x^2)^{3/2}}{385875a} - \frac{2664c^3(1+a^2x^2)^{5/2}}{214375a} - \frac{6c^3(1+a^2x^2)^{7/2}}{2401a} + \frac{4322c^3x\text{arcsinh}(ax)}{1225} + \frac{1514a^2c^3x^3\text{arcsinh}(ax)}{3675} + \frac{702a^4c^3x^5\text{arcsinh}(ax)}{6125} + \frac{6}{343}a^6c^3x^7\text{arcsinh}(ax) - \frac{48c^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{35a} - \frac{8c^3(1+a^2x^2)^{3/2}\text{arcsinh}(ax)^2}{35a} - \frac{18c^3(1+$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.47

$$\int (c+a^2cx^2)^3 \text{arcsinh}(ax)^3 \, dx = \frac{c^3\left(-2\sqrt{1+a^2x^2}(22329151 + 747937a^2x^2 + 134541a^4x^4 + 16875a^6x^6) + 210ax(226905 + 26495a^2x^2 + 73$$

```
[In] Integrate[(c + a^2*c*x^2)^3*ArcSinh[a*x]^3,x]
```

```
[Out] (c^3*(-2*Sqrt[1 + a^2*x^2]*(22329151 + 747937*a^2*x^2 + 134541*a^4*x^4 + 16
875*a^6*x^6) + 210*a*x*(226905 + 26495*a^2*x^2 + 7371*a^4*x^4 + 1125*a^6*x^
6)*ArcSinh[a*x] - 11025*Sqrt[1 + a^2*x^2]*(2161 + 757*a^2*x^2 + 351*a^4*x^4
 + 75*a^6*x^6)*ArcSinh[a*x]^2 + 385875*a*x*(35 + 35*a^2*x^2 + 21*a^4*x^4 +
5*a^6*x^6)*ArcSinh[a*x]^3))/(13505625*a)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{c^3\left(1929375 \operatorname{arcsinh}(ax)^3 a^7 x^7-826875 \operatorname{arcsinh}(ax)^2 \sqrt{a^2x^2+1}\, a^6 x^6+8103375 a^5 x^5 \operatorname{arcsinh}(ax)^3+236250 \operatorname{arcsinh}(ax) a^7 x^7\right.}{}$
default	$\frac{c^3\left(1929375 \operatorname{arcsinh}(ax)^3 a^7 x^7-826875 \operatorname{arcsinh}(ax)^2 \sqrt{a^2x^2+1}\, a^6 x^6+8103375 a^5 x^5 \operatorname{arcsinh}(ax)^3+236250 \operatorname{arcsinh}(ax) a^7 x^7\right.}{}$

```
[In] int((a^2*c*x^2+c)^3*arcsinh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13505625/a*c^3*(1929375*arcsinh(a*x)^3*a^7*x^7-826875*arcsinh(a*x)^2*(a^2
*x^2+1)^(1/2)*a^6*x^6+8103375*a^5*x^5*arcsinh(a*x)^3+236250*arcsinh(a*x)*a^
7*x^7-3869775*a^4*x^4*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)-33750*x^6*a^6*(a^2*x
^2+1)^(1/2)+13505625*a^3*x^3*arcsinh(a*x)^3+1547910*a^5*x^5*arcsinh(a*x)-83
45925*a^2*x^2*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)-269082*a^4*x^4*(a^2*x^2+1)^(
1/2)+13505625*a*x*arcsinh(a*x)^3+5563950*a^3*x^3*arcsinh(a*x)-23825025*arcs
inh(a*x)^2*(a^2*x^2+1)^(1/2)-1495874*a^2*x^2*(a^2*x^2+1)^(1/2)+47650050*a*x
*arcsinh(a*x)-44658302*(a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.69

$$\int \left(c+a^2cx^2\right)^3 \operatorname{arcsinh}(ax)^3\,dx$$
$$= \frac{385875\left(5\,a^7c^3x^7+21\,a^5c^3x^5+35\,a^3c^3x^3+35\,ac^3x\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3-11025\left(75\,a^6c^3x^6+351\,a^4c^3x\right.}{}$$

```
[In] integrate((a^2*c*x^2+c)^3*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/13505625*(385875*(5*a^7*c^3*x^7 + 21*a^5*c^3*x^5 + 35*a^3*c^3*x^3 + 35*a*
c^3*x)*log(a*x + sqrt(a^2*x^2 + 1))^3 - 11025*(75*a^6*c^3*x^6 + 351*a^4*c^3
*x^4 + 757*a^2*c^3*x^2 + 2161*c^3)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2
 + 1))^2 + 210*(1125*a^7*c^3*x^7 + 7371*a^5*c^3*x^5 + 26495*a^3*c^3*x^3 + 2
26905*a*c^3*x)*log(a*x + sqrt(a^2*x^2 + 1)) - 2*(16875*a^6*c^3*x^6 + 134541
*a^4*c^3*x^4 + 747937*a^2*c^3*x^2 + 22329151*c^3)*sqrt(a^2*x^2 + 1))/a
```

Sympy [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.99

$$\int (c+a^2cx^2)^3 \operatorname{arcsinh}(ax)^3\,dx$$
$$=\begin{cases} \frac{a^6c^3x^7\operatorname{asinh}^3(ax)}{7}+\frac{6a^6c^3x^7\operatorname{asinh}(ax)}{343}-\frac{3a^5c^3x^6\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{49}-\frac{6a^5c^3x^6\sqrt{a^2x^2+1}}{2401}+\frac{3a^4c^3x^5\operatorname{asinh}^3(ax)}{5}+\frac{702a^4c^3x^5\operatorname{as}}{6125} \\ 0 \end{cases}$$

```
[In] integrate((a**2*c*x**2+c)**3*asinh(a*x)**3,x)
```

```
[Out] Piecewise((a**6*c**3*x**7*asinh(a*x)**3/7 + 6*a**6*c**3*x**7*asinh(a*x)/343
 - 3*a**5*c**3*x**6*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/49 - 6*a**5*c**3*x**6
*sqrt(a**2*x**2 + 1)/2401 + 3*a**4*c**3*x**5*asinh(a*x)**3/5 + 702*a**4*c**
3*x**5*asinh(a*x)/6125 - 351*a**3*c**3*x**4*sqrt(a**2*x**2 + 1)*asinh(a*x)*
*2/1225 - 29898*a**3*c**3*x**4*sqrt(a**2*x**2 + 1)/1500625 + a**2*c**3*x**3
*asinh(a*x)**3 + 1514*a**2*c**3*x**3*asinh(a*x)/3675 - 757*a*c**3*x**2*sqrt
(a**2*x**2 + 1)*asinh(a*x)**2/1225 - 1495874*a*c**3*x**2*sqrt(a**2*x**2 + 1
)/13505625 + c**3*x*asinh(a*x)**3 + 4322*c**3*x*asinh(a*x)/1225 - 2161*c**3
*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(1225*a) - 44658302*c**3*sqrt(a**2*x**2
+ 1)/(13505625*a), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.77

$$\int (c+a^2cx^2)^3 \operatorname{arcsinh}(ax)^3\,dx =$$
$$-\frac{1}{1225}\left(75\sqrt{a^2x^2+1}a^4c^3x^6+351\sqrt{a^2x^2+1}a^2c^3x^4+757\sqrt{a^2x^2+1}c^3x^2+\frac{2161\sqrt{a^2x^2+1}c^3}{a^2}\right)a\operatorname{arsin}$$
$$+\frac{1}{35}\left(5a^6c^3x^7+21a^4c^3x^5+35a^2c^3x^3+35c^3x\right)\operatorname{arsinh}(ax)^3$$
$$-\frac{2}{13505625}\left(16875\sqrt{a^2x^2+1}a^4c^3x^6+134541\sqrt{a^2x^2+1}a^2c^3x^4+747937\sqrt{a^2x^2+1}c^3x^2+\frac{22329151\sqrt{}}{a}\right.$$

```
[In] integrate((a^2*c*x^2+c)^3*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/1225*(75*sqrt(a^2*x^2 + 1)*a^4*c^3*x^6 + 351*sqrt(a^2*x^2 + 1)*a^2*c^3*x
^4 + 757*sqrt(a^2*x^2 + 1)*c^3*x^2 + 2161*sqrt(a^2*x^2 + 1)*c^3/a^2)*a*arcs
inh(a*x)^2 + 1/35*(5*a^6*c^3*x^7 + 21*a^4*c^3*x^5 + 35*a^2*c^3*x^3 + 35*c^3
*x)*arcsinh(a*x)^3 - 2/13505625*(16875*sqrt(a^2*x^2 + 1)*a^4*c^3*x^6 + 1345
41*sqrt(a^2*x^2 + 1)*a^2*c^3*x^4 + 747937*sqrt(a^2*x^2 + 1)*c^3*x^2 + 22329
151*sqrt(a^2*x^2 + 1)*c^3/a^2 - 105*(1125*a^6*c^3*x^7 + 7371*a^4*c^3*x^5 +
26495*a^2*c^3*x^3 + 226905*c^3*x)*arcsinh(a*x)/a)*a
```

Giac [F(-2)]

Exception generated.

$$\int (c + a^2cx^2)^3 \operatorname{arcsinh}(ax)^3 \, dx = \text{Exception raised: TypeError}$$

[In] integrate((a^2*c*x^2+c)^3*arcsinh(a*x)^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int (c + a^2cx^2)^3 \operatorname{arcsinh}(ax)^3 \, dx = \int \operatorname{asinh}(a\,x)^3 \left(c\,a^2\,x^2 + c\right)^3 \, dx$$

[In] int(asinh(a*x)^3*(c + a^2*c*x^2)^3,x)

[Out] int(asinh(a*x)^3*(c + a^2*c*x^2)^3, x)

3.329 $\int (c+a^2cx^2)^2 \operatorname{arcsinh}(ax)^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 265

$$\begin{aligned}
&\int (c+a^2cx^2)^2 \operatorname{arcsinh}(ax)^3\,dx \\
&= -\frac{4144c^2\sqrt{1+a^2x^2}}{1125a} - \frac{272c^2(1+a^2x^2)^{3/2}}{3375a} - \frac{6c^2(1+a^2x^2)^{5/2}}{625a} + \frac{298}{75}c^2x\operatorname{arcsinh}(ax) \\
&\quad + \frac{76}{225}a^2c^2x^3\operatorname{arcsinh}(ax) + \frac{6}{125}a^4c^2x^5\operatorname{arcsinh}(ax) - \frac{8c^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{5a} \\
&\quad - \frac{4c^2(1+a^2x^2)^{3/2}\operatorname{arcsinh}(ax)^2}{15a} - \frac{3c^2(1+a^2x^2)^{5/2}\operatorname{arcsinh}(ax)^2}{25a} \\
&\quad + \frac{8}{15}c^2x\operatorname{arcsinh}(ax)^3 + \frac{4}{15}c^2x(1+a^2x^2)\operatorname{arcsinh}(ax)^3 + \frac{1}{5}c^2x(1+a^2x^2)^2\operatorname{arcsinh}(ax)^3
\end{aligned}$$

```
[Out] -272/3375*c^2*(a^2*x^2+1)^(3/2)/a-6/625*c^2*(a^2*x^2+1)^(5/2)/a+298/75*c^2*
x*arcsinh(a*x)+76/225*a^2*c^2*x^3*arcsinh(a*x)+6/125*a^4*c^2*x^5*arcsinh(a*
x)-4/15*c^2*(a^2*x^2+1)^(3/2)*arcsinh(a*x)^2/a-3/25*c^2*(a^2*x^2+1)^(5/2)*a
rcsinh(a*x)^2/a+8/15*c^2*x*arcsinh(a*x)^3+4/15*c^2*x*(a^2*x^2+1)*arcsinh(a*
x)^3+1/5*c^2*x*(a^2*x^2+1)^2*arcsinh(a*x)^3-4144/1125*c^2*(a^2*x^2+1)^(1/2)
/a-8/5*c^2*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {5786, 5772, 5798, 267, 5784, 455, 45, 200, 12, 1261, 712}

$$\int \left(c+a^2cx^2\right)^2 \operatorname{arcsinh}(ax)^3\,dx = \frac{6}{125}a^4c^2x^5\operatorname{arcsinh}(ax)+\frac{76}{225}a^2c^2x^3\operatorname{arcsinh}(ax)+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\operatorname{arcsinh}(ax)^3+\frac{4}{15}c^2x\left(a^2x^2+1\right)\operatorname{arcsinh}(ax)^3-\frac{3c^2(a^2x^2+1)^{5/2}\operatorname{arcsinh}(ax)^2}{25a}-\frac{4c^2(a^2x^2+1)^{3/2}\operatorname{arcsinh}(ax)^2}{15a}-\frac{8c^2\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^2}{5a}-\frac{6c^2(a^2x^2+1)^{5/2}}{625a}-\frac{272c^2(a^2x^2+1)^{3/2}}{3375a}-\frac{4144c^2\sqrt{a^2x^2+1}}{1125a}+\frac{8}{15}c^2x\operatorname{arcsinh}(ax)^3+\frac{298}{75}c^2x\operatorname{arcsinh}(ax)$$

```
[In] Int[(c + a^2*c*x^2)^2*ArcSinh[a*x]^3,x]
```

```
[Out] (-4144*c^2*Sqrt[1 + a^2*x^2])/(1125*a) - (272*c^2*(1 + a^2*x^2)^(3/2))/(337
5*a) - (6*c^2*(1 + a^2*x^2)^(5/2))/(625*a) + (298*c^2*x*ArcSinh[a*x])/75 +
(76*a^2*c^2*x^3*ArcSinh[a*x])/225 + (6*a^4*c^2*x^5*ArcSinh[a*x])/125 - (8*c
^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(5*a) - (4*c^2*(1 + a^2*x^2)^(3/2)*Arc
Sinh[a*x]^2)/(15*a) - (3*c^2*(1 + a^2*x^2)^(5/2)*ArcSinh[a*x]^2)/(25*a) + (
8*c^2*x*ArcSinh[a*x]^3)/15 + (4*c^2*x*(1 + a^2*x^2)*ArcSinh[a*x]^3)/15 + (c
^2*x*(1 + a^2*x^2)^2*ArcSinh[a*x]^3)/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
```

```
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}c^2x(1+a^2x^2)^2\operatorname{arcsinh}(ax)^3+\frac{1}{5}(4c)\int(c+a^2cx^2)\operatorname{arcsinh}(ax)^3\,dx \\
&\quad -\frac{1}{5}(3ac^2)\int x(1+a^2x^2)^{3/2}\operatorname{arcsinh}(ax)^2\,dx \\
&= -\frac{3c^2(1+a^2x^2)^{5/2}\operatorname{arcsinh}(ax)^2}{25a}+\frac{4}{15}c^2x(1+a^2x^2)\operatorname{arcsinh}(ax)^3 \\
&\quad +\frac{1}{5}c^2x(1+a^2x^2)^2\operatorname{arcsinh}(ax)^3+\frac{1}{25}(6c^2)\int(1+a^2x^2)^2\operatorname{arcsinh}(ax)\,dx \\
&\quad +\frac{1}{15}(8c^2)\int\operatorname{arcsinh}(ax)^3\,dx-\frac{1}{5}(4ac^2)\int x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2\,dx \\
&= \frac{6}{25}c^2x\operatorname{arcsinh}(ax)+\frac{4}{25}a^2c^2x^3\operatorname{arcsinh}(ax)+\frac{6}{125}a^4c^2x^5\operatorname{arcsinh}(ax) \\
&\quad -\frac{4c^2(1+a^2x^2)^{3/2}\operatorname{arcsinh}(ax)^2}{15a}-\frac{3c^2(1+a^2x^2)^{5/2}\operatorname{arcsinh}(ax)^2}{25a} \\
&\quad +\frac{8}{15}c^2x\operatorname{arcsinh}(ax)^3+\frac{4}{15}c^2x(1+a^2x^2)\operatorname{arcsinh}(ax)^3 \\
&\quad +\frac{1}{5}c^2x(1+a^2x^2)^2\operatorname{arcsinh}(ax)^3+\frac{1}{15}(8c^2)\int(1+a^2x^2)\operatorname{arcsinh}(ax)\,dx \\
&\quad -\frac{1}{25}(6ac^2)\int\frac{x(15+10a^2x^2+3a^4x^4)}{15\sqrt{1+a^2x^2}}\,dx-\frac{1}{5}(8ac^2)\int\frac{x\operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{58}{75}c^2x\operatorname{arcsinh}(ax) + \frac{76}{225}a^2c^2x^3\operatorname{arcsinh}(ax) \\
&\quad + \frac{6}{125}a^4c^2x^5\operatorname{arcsinh}(ax) - \frac{8c^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{5a} \\
&\quad - \frac{4c^2(1+a^2x^2)^{3/2}\operatorname{arcsinh}(ax)^2}{15a} - \frac{3c^2(1+a^2x^2)^{5/2}\operatorname{arcsinh}(ax)^2}{25a} \\
&\quad + \frac{8}{15}c^2x\operatorname{arcsinh}(ax)^3 + \frac{4}{15}c^2x(1+a^2x^2)\operatorname{arcsinh}(ax)^3 \\
&\quad + \frac{1}{5}c^2x(1+a^2x^2)^2\operatorname{arcsinh}(ax)^3 + \frac{1}{5}(16c^2)\int \operatorname{arcsinh}(ax)\,dx \\
&\quad - \frac{1}{125}(2ac^2)\int \frac{x(15+10a^2x^2+3a^4x^4)}{\sqrt{1+a^2x^2}}\,dx - \frac{1}{15}(8ac^2)\int \frac{x\left(1+\frac{a^2x^2}{3}\right)}{\sqrt{1+a^2x^2}}\,dx \\
&= \frac{298}{75}c^2x\operatorname{arcsinh}(ax) + \frac{76}{225}a^2c^2x^3\operatorname{arcsinh}(ax) + \frac{6}{125}a^4c^2x^5\operatorname{arcsinh}(ax) \\
&\quad - \frac{8c^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{5a} - \frac{4c^2(1+a^2x^2)^{3/2}\operatorname{arcsinh}(ax)^2}{15a} \\
&\quad - \frac{3c^2(1+a^2x^2)^{5/2}\operatorname{arcsinh}(ax)^2}{25a} + \frac{8}{15}c^2x\operatorname{arcsinh}(ax)^3 \\
&\quad + \frac{4}{15}c^2x(1+a^2x^2)\operatorname{arcsinh}(ax)^3 + \frac{1}{5}c^2x(1+a^2x^2)^2\operatorname{arcsinh}(ax)^3 \\
&\quad - \frac{1}{125}(ac^2)\operatorname{Subst}\left(\int \frac{15+10a^2x+3a^4x^2}{\sqrt{1+a^2x}}\,dx, x, x^2\right) \\
&\quad - \frac{1}{15}(4ac^2)\operatorname{Subst}\left(\int \frac{1+\frac{a^2x}{3}}{\sqrt{1+a^2x}}\,dx, x, x^2\right) - \frac{1}{5}(16ac^2)\int \frac{x}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{16c^2\sqrt{1+a^2x^2}}{5a} + \frac{298}{75}c^2x\operatorname{arcsinh}(ax) + \frac{76}{225}a^2c^2x^3\operatorname{arcsinh}(ax) \\
&\quad + \frac{6}{125}a^4c^2x^5\operatorname{arcsinh}(ax) - \frac{8c^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{5a} \\
&\quad - \frac{4c^2(1+a^2x^2)^{3/2}\operatorname{arcsinh}(ax)^2}{15a} - \frac{3c^2(1+a^2x^2)^{5/2}\operatorname{arcsinh}(ax)^2}{25a} \\
&\quad + \frac{8}{15}c^2x\operatorname{arcsinh}(ax)^3 + \frac{4}{15}c^2x(1+a^2x^2)\operatorname{arcsinh}(ax)^3 + \frac{1}{5}c^2x(1+a^2x^2)^2\operatorname{arcsinh}(ax)^3 \\
&\quad - \frac{1}{125}(ac^2)\operatorname{Subst}\left(\int \left(\frac{8}{\sqrt{1+a^2x}} + 4\sqrt{1+a^2x} + 3(1+a^2x)^{3/2}\right)dx, x, x^2\right) - \frac{1}{15}(4ac^2)\operatorname{Subst}\left(\int\right. \\
&= -\frac{4144c^2\sqrt{1+a^2x^2}}{1125a} - \frac{272c^2(1+a^2x^2)^{3/2}}{3375a} - \frac{6c^2(1+a^2x^2)^{5/2}}{625a} + \frac{298}{75}c^2x\operatorname{arcsinh}(ax) \\
&\quad + \frac{76}{225}a^2c^2x^3\operatorname{arcsinh}(ax) + \frac{6}{125}a^4c^2x^5\operatorname{arcsinh}(ax) - \frac{8c^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{5a} \\
&\quad - \frac{4c^2(1+a^2x^2)^{3/2}\operatorname{arcsinh}(ax)^2}{15a} - \frac{3c^2(1+a^2x^2)^{5/2}\operatorname{arcsinh}(ax)^2}{25a} \\
&\quad + \frac{8}{15}c^2x\operatorname{arcsinh}(ax)^3 + \frac{4}{15}c^2x(1+a^2x^2)\operatorname{arcsinh}(ax)^3 + \frac{1}{5}c^2x(1+a^2x^2)^2\operatorname{arcsinh}(ax)^3
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.52

$$\int \left(c+a^2cx^2\right)^2 \operatorname{arcsinh}(ax)^3 \, dx$$
$$= \frac{c^2\left(-2\sqrt{1+a^2x^2}(31841+842a^2x^2+81a^4x^4)+30ax(2235+190a^2x^2+27a^4x^4)\operatorname{arcsinh}(ax)-225\sqrt{1+a^2x^2}\right.}{16875a}$$

[In] Integrate[(c + a^2*c*x^2)^2*ArcSinh[a*x]^3,x]

[Out] (c^2*(-2*Sqrt[1 + a^2*x^2]*(31841 + 842*a^2*x^2 + 81*a^4*x^4) + 30*a*x*(223
5 + 190*a^2*x^2 + 27*a^4*x^4)*ArcSinh[a*x] - 225*Sqrt[1 + a^2*x^2]*(149 + 3
8*a^2*x^2 + 9*a^4*x^4)*ArcSinh[a*x]^2 + 1125*a*x*(15 + 10*a^2*x^2 + 3*a^4*x
^4)*ArcSinh[a*x]^3))/(16875*a)

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{c^2\left(3375a^5x^5\operatorname{arcsinh}(ax)^3-2025a^4x^4\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}+11250a^3x^3\operatorname{arcsinh}(ax)^3+810a^5x^5\operatorname{arcsinh}(ax)-8550a^2x^2\right.}{}$
default	$\frac{c^2\left(3375a^5x^5\operatorname{arcsinh}(ax)^3-2025a^4x^4\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}+11250a^3x^3\operatorname{arcsinh}(ax)^3+810a^5x^5\operatorname{arcsinh}(ax)-8550a^2x^2\right.}{}$

[In] int((a^2*c*x^2+c)^2*arcsinh(a*x)^3,x,method=_RETURNVERBOSE)

[Out] 1/16875/a*c^2*(3375*a^5*x^5*arcsinh(a*x)^3-2025*a^4*x^4*arcsinh(a*x)^2*(a^2
*x^2+1)^(1/2)+11250*a^3*x^3*arcsinh(a*x)^3+810*a^5*x^5*arcsinh(a*x)-8550*a^
2*x^2*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)-162*a^4*x^4*(a^2*x^2+1)^(1/2)+16875*
a*x*arcsinh(a*x)^3+5700*a^3*x^3*arcsinh(a*x)-33525*arcsinh(a*x)^2*(a^2*x^2+
1)^(1/2)-1684*a^2*x^2*(a^2*x^2+1)^(1/2)+67050*a*x*arcsinh(a*x)-63682*(a^2*x
^2+1)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.77

$$\int \left(c+a^2cx^2\right)^2 \operatorname{arcsinh}(ax)^3 \, dx$$
$$= \frac{1125\left(3a^5c^2x^5+10a^3c^2x^3+15ac^2x\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3-225\left(9a^4c^2x^4+38a^2c^2x^2+149c^2\right)\sqrt{a^2x^2+}}{}$$

```
[In] integrate((a^2*c*x^2+c)^2*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/16875*(1125*(3*a^5*c^2*x^5 + 10*a^3*c^2*x^3 + 15*a*c^2*x)*log(a*x + sqrt(
a^2*x^2 + 1))^3 - 225*(9*a^4*c^2*x^4 + 38*a^2*c^2*x^2 + 149*c^2)*sqrt(a^2*x
^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2 + 30*(27*a^5*c^2*x^5 + 190*a^3*c^2*x
^3 + 2235*a*c^2*x)*log(a*x + sqrt(a^2*x^2 + 1)) - 2*(81*a^4*c^2*x^4 + 842*a
^2*c^2*x^2 + 31841*c^2)*sqrt(a^2*x^2 + 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.99

$$\int \left(c+a^2cx^2\right)^2 \operatorname{arcsinh}(ax)^3\,dx = \begin{cases} \frac{a^4c^2x^5\operatorname{asinh}^3(ax)}{5} + \frac{6a^4c^2x^5\operatorname{asinh}(ax)}{125} - \frac{3a^3c^2x^4\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{25} - \frac{6a^3c^2x^4\sqrt{a^2x^2+1}}{625} + \frac{2a^2c^2x^3\operatorname{asinh}^3(ax)}{3} + \frac{76a^2c^2x^3\operatorname{asi}}{225} \\ 0 \end{cases}$$

```
[In] integrate((a**2*c*x**2+c)**2*asinh(a*x)**3,x)
```

```
[Out] Piecewise((a**4*c**2*x**5*asinh(a*x)**3/5 + 6*a**4*c**2*x**5*asinh(a*x)/125
 - 3*a**3*c**2*x**4*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/25 - 6*a**3*c**2*x**4
*sqrt(a**2*x**2 + 1)/625 + 2*a**2*c**2*x**3*asinh(a*x)**3/3 + 76*a**2*c**2*
x**3*asinh(a*x)/225 - 38*a*c**2*x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/75 -
 1684*a*c**2*x**2*sqrt(a**2*x**2 + 1)/16875 + c**2*x*asinh(a*x)**3 + 298*c*
*2*x*asinh(a*x)/75 - 149*c**2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(75*a) - 63
682*c**2*sqrt(a**2*x**2 + 1)/(16875*a), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.79

$$\begin{aligned}\int \left(c+a^2cx^2\right)^2 \operatorname{arcsinh}(ax)^3\,dx &= -\frac{1}{75}\left(9\sqrt{a^2x^2+1}a^2c^2x^4 + 38\sqrt{a^2x^2+1}c^2x^2 + \frac{149\sqrt{a^2x^2+1}c^2}{a^2}\right)a\operatorname{arsinh}(ax)^2 \\ &\quad + \frac{1}{15}\left(3a^4c^2x^5 + 10a^2c^2x^3 + 15c^2x\right)\operatorname{arsinh}(ax)^3 \\ &\quad - \frac{2}{16875}\left(81\sqrt{a^2x^2+1}a^2c^2x^4 + 842\sqrt{a^2x^2+1}c^2x^2 - \frac{15\left(27a^4c^2x^5 + 190a^2c^2x^3 + 2235c^2x\right)\operatorname{arsinh}(ax)}{a}\right.\end{aligned}$$

```
[In] integrate((a^2*c*x^2+c)^2*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/75*(9*sqrt(a^2*x^2 + 1)*a^2*c^2*x^4 + 38*sqrt(a^2*x^2 + 1)*c^2*x^2 + 149
*sqrt(a^2*x^2 + 1)*c^2/a^2)*a*arcsinh(a*x)^2 + 1/15*(3*a^4*c^2*x^5 + 10*a^2
*c^2*x^3 + 15*c^2*x)*arcsinh(a*x)^3 - 2/16875*(81*sqrt(a^2*x^2 + 1)*a^2*c^2
*x^4 + 842*sqrt(a^2*x^2 + 1)*c^2*x^2 - 15*(27*a^4*c^2*x^5 + 190*a^2*c^2*x^3
 + 2235*c^2*x)*arcsinh(a*x)/a + 31841*sqrt(a^2*x^2 + 1)*c^2/a^2)*a
```

Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^2 \operatorname{arcsinh}(ax)^3\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a^2*c*x^2+c)^2*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^2 \operatorname{arcsinh}(ax)^3\,dx = \int \operatorname{asinh}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

```
[In] int(asinh(a*x)^3*(c + a^2*c*x^2)^2,x)
```

```
[Out] int(asinh(a*x)^3*(c + a^2*c*x^2)^2, x)
```

3.330 $\int (c+a^2cx^2) \operatorname{arcsinh}(ax)^3 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 153

$$\int (c+a^2cx^2) \operatorname{arcsinh}(ax)^3 \, dx = -\frac{40c\sqrt{1+a^2x^2}}{9a} - \frac{2c(1+a^2x^2)^{3/2}}{27a} + \frac{14}{3}cx\operatorname{arcsinh}(ax) + \frac{2}{9}a^2cx^3\operatorname{arcsinh}(ax) - \frac{2c\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{a} - \frac{c(1+a^2x^2)^{3/2}\operatorname{arcsinh}(ax)^2}{3a} + \frac{2}{3}cx\operatorname{arcsinh}(ax)^3 + \frac{1}{3}cx(1+a^2x^2)\operatorname{arcsinh}(ax)^3$$

```
[Out] -2/27*c*(a^2*x^2+1)^(3/2)/a+14/3*c*x*arcsinh(a*x)+2/9*a^2*c*x^3*arcsinh(a*x
)-1/3*c*(a^2*x^2+1)^(3/2)*arcsinh(a*x)^2/a+2/3*c*x*arcsinh(a*x)^3+1/3*c*x*(
a^2*x^2+1)*arcsinh(a*x)^3-40/9*c*(a^2*x^2+1)^(1/2)/a-2*c*arcsinh(a*x)^2*(a^
2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {5786, 5772, 5798, 267, 5784, 455, 45}

$$\int (c+a^2cx^2) \operatorname{arcsinh}(ax)^3 \, dx = \frac{2}{9}a^2cx^3\operatorname{arcsinh}(ax) + \frac{1}{3}cx(a^2x^2+1)\operatorname{arcsinh}(ax)^3 - \frac{c(a^2x^2+1)^{3/2}\operatorname{arcsinh}(ax)^2}{3a} - \frac{2c\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^2}{a} - \frac{2c(a^2x^2+1)^{3/2}}{27a} - \frac{40c\sqrt{a^2x^2+1}}{9a} + \frac{2}{3}cx\operatorname{arcsinh}(ax)^3 + \frac{14}{3}cx\operatorname{arcsinh}(ax)$$

```
[In] Int[(c + a^2*c*x^2)*ArcSinh[a*x]^3,x]
```

```
[Out] (-40*c*Sqrt[1 + a^2*x^2])/(9*a) - (2*c*(1 + a^2*x^2)^(3/2))/(27*a) + (14*c*
x*ArcSinh[a*x])/3 + (2*a^2*c*x^3*ArcSinh[a*x])/9 - (2*c*Sqrt[1 + a^2*x^2]*A
rcSinh[a*x]^2)/a - (c*(1 + a^2*x^2)^(3/2)*ArcSinh[a*x]^2)/(3*a) + (2*c*x*Ar
cSinh[a*x]^3)/3 + (c*x*(1 + a^2*x^2)*ArcSinh[a*x]^3)/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
```

```
c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}cx(1+a^2x^2)\operatorname{arcsinh}(ax)^3+\frac{1}{3}(2c)\int \operatorname{arcsinh}(ax)^3\,dx \\
&\quad -(ac)\int x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2\,dx \\
&= -\frac{c(1+a^2x^2)^{3/2}\operatorname{arcsinh}(ax)^2}{3a}+\frac{2}{3}cx\operatorname{arcsinh}(ax)^3+\frac{1}{3}cx(1+a^2x^2)\operatorname{arcsinh}(ax)^3 \\
&\quad +\frac{1}{3}(2c)\int (1+a^2x^2)\operatorname{arcsinh}(ax)\,dx-(2ac)\int \frac{x\operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx \\
&= \frac{2}{3}cx\operatorname{arcsinh}(ax)+\frac{2}{9}a^2cx^3\operatorname{arcsinh}(ax)-\frac{2c\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{a} \\
&\quad -\frac{c(1+a^2x^2)^{3/2}\operatorname{arcsinh}(ax)^2}{3a}+\frac{2}{3}cx\operatorname{arcsinh}(ax)^3+\frac{1}{3}cx(1+a^2x^2)\operatorname{arcsinh}(ax)^3 \\
&\quad +(4c)\int \operatorname{arcsinh}(ax)\,dx-\frac{1}{3}(2ac)\int \frac{x\left(1+\frac{a^2x^2}{3}\right)}{\sqrt{1+a^2x^2}}\,dx \\
&= \frac{14}{3}cx\operatorname{arcsinh}(ax)+\frac{2}{9}a^2cx^3\operatorname{arcsinh}(ax)-\frac{2c\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{a} \\
&\quad -\frac{c(1+a^2x^2)^{3/2}\operatorname{arcsinh}(ax)^2}{3a}+\frac{2}{3}cx\operatorname{arcsinh}(ax)^3+\frac{1}{3}cx(1+a^2x^2)\operatorname{arcsinh}(ax)^3 \\
&\quad -\frac{1}{3}(ac)\operatorname{Subst}\left(\int \frac{1+\frac{a^2x}{3}}{\sqrt{1+a^2x}}\,dx,x,x^2\right)-(4ac)\int \frac{x}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{4c\sqrt{1+a^2x^2}}{a}+\frac{14}{3}cx\operatorname{arcsinh}(ax)+\frac{2}{9}a^2cx^3\operatorname{arcsinh}(ax)-\frac{2c\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{a} \\
&\quad -\frac{c(1+a^2x^2)^{3/2}\operatorname{arcsinh}(ax)^2}{3a}+\frac{2}{3}cx\operatorname{arcsinh}(ax)^3+\frac{1}{3}cx(1+a^2x^2)\operatorname{arcsinh}(ax)^3 \\
&\quad -\frac{1}{3}(ac)\operatorname{Subst}\left(\int \left(\frac{2}{3\sqrt{1+a^2x}}+\frac{1}{3}\sqrt{1+a^2x}\right)dx,x,x^2\right)
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{40c\sqrt{1+a^2x^2}}{9a} - \frac{2c(1+a^2x^2)^{3/2}}{27a} \\
&\quad + \frac{14}{3}cx\text{arcsinh}(ax) + \frac{2}{9}a^2cx^3\text{arcsinh}(ax) - \frac{2c\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{a} \\
&\quad - \frac{c(1+a^2x^2)^{3/2}\,\text{arcsinh}(ax)^2}{3a} + \frac{2}{3}cx\text{arcsinh}(ax)^3 + \frac{1}{3}cx\left(1+a^2x^2\right)\text{arcsinh}(ax)^3
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.65

$$
\int \left(c+a^2cx^2\right)\text{arcsinh}(ax)^3\,dx
= \frac{c\left(-2\sqrt{1+a^2x^2}(61+a^2x^2) + 6ax(21+a^2x^2)\,\text{arcsinh}(ax) - 9\sqrt{1+a^2x^2}(7+a^2x^2)\,\text{arcsinh}(ax)^2 + 9ax(3+\right.}{27a}
$$

```
[In] Integrate[(c + a^2*c*x^2)*ArcSinh[a*x]^3,x]
```

```
[Out] (c*(-2*Sqrt[1 + a^2*x^2]*(61 + a^2*x^2) + 6*a*x*(21 + a^2*x^2)*ArcSinh[a*x]
 - 9*Sqrt[1 + a^2*x^2]*(7 + a^2*x^2)*ArcSinh[a*x]^2 + 9*a*x*(3 + a^2*x^2)*A
rcSinh[a*x]^3))/(27*a)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{c\left(9a^3x^3\,\text{arcsinh}(ax)^3-9a^2x^2\,\text{arcsinh}(ax)^2\sqrt{a^2x^2+1}+27ax\,\text{arcsinh}(ax)^3+6a^3x^3\,\text{arcsinh}(ax)-63\,\text{arcsinh}(ax)^2\sqrt{a^2x^2+1}-\right.}{27a}$
default	$\frac{c\left(9a^3x^3\,\text{arcsinh}(ax)^3-9a^2x^2\,\text{arcsinh}(ax)^2\sqrt{a^2x^2+1}+27ax\,\text{arcsinh}(ax)^3+6a^3x^3\,\text{arcsinh}(ax)-63\,\text{arcsinh}(ax)^2\sqrt{a^2x^2+1}-\right.}{27a}$

```
[In] int((a^2*c*x^2+c)*arcsinh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/27/a*c*(9*a^3*x^3*arcsinh(a*x)^3-9*a^2*x^2*arcsinh(a*x)^2*(a^2*x^2+1)^(1/
2)+27*a*x*arcsinh(a*x)^3+6*a^3*x^3*arcsinh(a*x)-63*arcsinh(a*x)^2*(a^2*x^2+
1)^(1/2)-2*a^2*x^2*(a^2*x^2+1)^(1/2)+126*a*x*arcsinh(a*x)-122*(a^2*x^2+1)^(
1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.92

$$\int (c + a^2cx^2) \operatorname{arcsinh}(ax)^3 \, dx$$
$$= \frac{9\,(a^3cx^3 + 3\,acx)\log\left(ax + \sqrt{a^2x^2 + 1}\right)^3 - 9\,(a^2cx^2 + 7\,c)\sqrt{a^2x^2 + 1}\log\left(ax + \sqrt{a^2x^2 + 1}\right)^2 + 6\,(a^3cx^3 +}{27\,a}$$

```
[In] integrate((a^2*c*x^2+c)*arcsinh(a*x)^3,x, algorithm="fricas")
[Out] 1/27*(9*(a^3*c*x^3 + 3*a*c*x)*log(a*x + sqrt(a^2*x^2 + 1))^3 - 9*(a^2*c*x^2
 + 7*c)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2 + 6*(a^3*c*x^3 + 2
1*a*c*x)*log(a*x + sqrt(a^2*x^2 + 1)) - 2*(a^2*c*x^2 + 61*c)*sqrt(a^2*x^2 +
 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.98

$$\int (c + a^2cx^2) \operatorname{arcsinh}(ax)^3 \, dx$$
$$= \begin{cases} \frac{a^2cx^3 \operatorname{asinh}^3(ax)}{3} + \frac{2a^2cx^3 \operatorname{asinh}(ax)}{9} - \frac{acx^2\sqrt{a^2x^2+1} \operatorname{asinh}^2(ax)}{3} - \frac{2acx^2\sqrt{a^2x^2+1}}{27} + cx \operatorname{asinh}^3(ax) + \frac{14cx \operatorname{asinh}(ax)}{3} - \frac{7c}{} \\ 0 \end{cases}$$

```
[In] integrate((a**2*c*x**2+c)*asinh(a*x)**3,x)
[Out] Piecewise((a**2*c*x**3*asinh(a*x)**3/3 + 2*a**2*c*x**3*asinh(a*x)/9 - a*c*x
**2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/3 - 2*a*c*x**2*sqrt(a**2*x**2 + 1)/27
 + c*x*asinh(a*x)**3 + 14*c*x*asinh(a*x)/3 - 7*c*sqrt(a**2*x**2 + 1)*asinh(
a*x)**2/(3*a) - 122*c*sqrt(a**2*x**2 + 1)/(27*a), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.81

$$\int (c + a^2cx^2) \operatorname{arcsinh}(ax)^3 \, dx$$
$$= -\frac{1}{3}\left(\sqrt{a^2x^2 + 1}cx^2 + \frac{7\sqrt{a^2x^2 + 1}c}{a^2}\right)a \operatorname{arsinh}(ax)^2 + \frac{1}{3}\,(a^2cx^3 + 3\,cx) \operatorname{arsinh}(ax)^3$$
$$- \frac{2}{27}\left(\sqrt{a^2x^2 + 1}cx^2 - \frac{3\,(a^2cx^3 + 21\,cx) \operatorname{arsinh}(ax)}{a} + \frac{61\sqrt{a^2x^2 + 1}c}{a^2}\right)a$$

```
[In] integrate((a^2*c*x^2+c)*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/3*(sqrt(a^2*x^2 + 1)*c*x^2 + 7*sqrt(a^2*x^2 + 1)*c/a^2)*a*arcsinh(a*x)^2
 + 1/3*(a^2*c*x^3 + 3*c*x)*arcsinh(a*x)^3 - 2/27*(sqrt(a^2*x^2 + 1)*c*x^2 -
 3*(a^2*c*x^3 + 21*c*x)*arcsinh(a*x)/a + 61*sqrt(a^2*x^2 + 1)*c/a^2)*a
```

Giac [F(-2)]

Exception generated.

$$\int (c + a^2cx^2) \operatorname{arcsinh}(ax)^3 \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a^2*c*x^2+c)*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (c + a^2cx^2) \operatorname{arcsinh}(ax)^3 \, dx = \int \operatorname{asinh}(a\,x)^3\,\left(c\,a^2\,x^2+c\right) \,d x$$

```
[In] int(asinh(a*x)^3*(c + a^2*c*x^2),x)
```

```
[Out] int(asinh(a*x)^3*(c + a^2*c*x^2), x)
```

3.331 $\int \frac{\operatorname{arcsinh}(ax)^3}{c+a^2cx^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 174

$$\int \frac{\operatorname{arcsinh}(ax)^3}{c+a^2cx^2} dx = \frac{2\operatorname{arcsinh}(ax)^3 \arctan\left(e^{\operatorname{arcsinh}(ax)}\right)}{ac} - \frac{3i\operatorname{arcsinh}(ax)^2 \operatorname{PolyLog}\left(2, -ie^{\operatorname{arcsinh}(ax)}\right)}{ac} + \frac{3i\operatorname{arcsinh}(ax)^2 \operatorname{PolyLog}\left(2, ie^{\operatorname{arcsinh}(ax)}\right)}{ac} + \frac{6i\operatorname{arcsinh}(ax) \operatorname{PolyLog}\left(3, -ie^{\operatorname{arcsinh}(ax)}\right)}{ac} - \frac{6i\operatorname{arcsinh}(ax) \operatorname{PolyLog}\left(3, ie^{\operatorname{arcsinh}(ax)}\right)}{ac} - \frac{6i \operatorname{PolyLog}\left(4, -ie^{\operatorname{arcsinh}(ax)}\right)}{ac} + \frac{6i \operatorname{PolyLog}\left(4, ie^{\operatorname{arcsinh}(ax)}\right)}{ac}$$

```
[Out] 2*arcsinh(a*x)^3*arctan(a*x+(a^2*x^2+1)^(1/2))/a/c-3*I*arcsinh(a*x)^2*polyl
og(2,-I*(a*x+(a^2*x^2+1)^(1/2)))/a/c+3*I*arcsinh(a*x)^2*polylog(2,I*(a*x+(a
^2*x^2+1)^(1/2)))/a/c+6*I*arcsinh(a*x)*polylog(3,-I*(a*x+(a^2*x^2+1)^(1/2))
)/a/c-6*I*arcsinh(a*x)*polylog(3,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c-6*I*polylog
(4,-I*(a*x+(a^2*x^2+1)^(1/2)))/a/c+6*I*polylog(4,I*(a*x+(a^2*x^2+1)^(1/2)))
/a/c
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5789, 4265, 2611, 6744, 2320, 6724}

$$\int \frac{\text{arcsinh}(ax)^3}{c+a^2cx^2}\,dx = \frac{2\text{arcsinh}(ax)^3\arctan\left(e^{\text{arcsinh}(ax)}\right)}{ac} - \frac{3i\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)}{ac} + \frac{3i\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(ax)}\right)}{ac} + \frac{6i\text{arcsinh}(ax)\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(ax)}\right)}{ac} - \frac{6i\text{arcsinh}(ax)\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(ax)}\right)}{ac} - \frac{6i\,\text{PolyLog}\left(4,-ie^{\text{arcsinh}(ax)}\right)}{ac} + \frac{6i\,\text{PolyLog}\left(4,ie^{\text{arcsinh}(ax)}\right)}{ac}$$

```
[In] Int[ArcSinh[a*x]^3/(c + a^2*c*x^2),x]
```

```
[Out] (2*ArcSinh[a*x]^3*ArcTan[E^ArcSinh[a*x]])/(a*c) - ((3*I)*ArcSinh[a*x]^2*Pol
yLog[2, (-I)*E^ArcSinh[a*x]])/(a*c) + ((3*I)*ArcSinh[a*x]^2*PolyLog[2, I*E^
ArcSinh[a*x]])/(a*c) + ((6*I)*ArcSinh[a*x]*PolyLog[3, (-I)*E^ArcSinh[a*x]])
/(a*c) - ((6*I)*ArcSinh[a*x]*PolyLog[3, I*E^ArcSinh[a*x]])/(a*c) - ((6*I)*P
olyLog[4, (-I)*E^ArcSinh[a*x]])/(a*c) + ((6*I)*PolyLog[4, I*E^ArcSinh[a*x]]
)/(a*c)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int x^3 \text{sech}(x)\,dx, x, \text{arcsinh}(ax)\right)}{ac}\\
&= \frac{2\text{arcsinh}(ax)^3\arctan\left(e^{\text{arcsinh}(ax)}\right)}{ac} - \frac{(3i)\text{Subst}\left(\int x^2\log\left(1-ie^x\right)\,dx, x, \text{arcsinh}(ax)\right)}{ac}\\
&\quad + \frac{(3i)\text{Subst}\left(\int x^2\log\left(1+ie^x\right)\,dx, x, \text{arcsinh}(ax)\right)}{ac}\\
&= \frac{2\text{arcsinh}(ax)^3\arctan\left(e^{\text{arcsinh}(ax)}\right)}{ac} - \frac{3i\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2, -ie^{\text{arcsinh}(ax)}\right)}{ac}\\
&\quad + \frac{3i\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2, ie^{\text{arcsinh}(ax)}\right)}{ac}\\
&\quad + \frac{(6i)\text{Subst}\left(\int x\,\text{PolyLog}\left(2, -ie^x\right)\,dx, x, \text{arcsinh}(ax)\right)}{ac}\\
&\quad - \frac{(6i)\text{Subst}\left(\int x\,\text{PolyLog}\left(2, ie^x\right)\,dx, x, \text{arcsinh}(ax)\right)}{ac}
\end{aligned}$$

$$= \frac{2\text{arcsinh}(ax)^3 \arctan\left(e^{\text{arcsinh}(ax)}\right)}{ac} - \frac{3i\text{arcsinh}(ax)^2 \text{PolyLog}\left(2, -ie^{\text{arcsinh}(ax)}\right)}{ac} + \frac{3i\text{arcsinh}(ax)^2 \text{PolyLog}\left(2, ie^{\text{arcsinh}(ax)}\right)}{ac} + \frac{6i\text{arcsinh}(ax) \text{PolyLog}\left(3, -ie^{\text{arcsinh}(ax)}\right)}{ac} - \frac{6i\text{arcsinh}(ax) \text{PolyLog}\left(3, ie^{\text{arcsinh}(ax)}\right)}{ac} - \frac{(6i)\text{Subst}\left(\int \text{PolyLog}\left(3, -ie^x\right) dx, x, \text{arcsinh}(ax)\right)}{ac} + \frac{(6i)\text{Subst}\left(\int \text{PolyLog}\left(3, ie^x\right) dx, x, \text{arcsinh}(ax)\right)}{ac}$$

$$= \frac{2\text{arcsinh}(ax)^3 \arctan\left(e^{\text{arcsinh}(ax)}\right)}{ac} - \frac{3i\text{arcsinh}(ax)^2 \text{PolyLog}\left(2, -ie^{\text{arcsinh}(ax)}\right)}{ac} + \frac{3i\text{arcsinh}(ax)^2 \text{PolyLog}\left(2, ie^{\text{arcsinh}(ax)}\right)}{ac} + \frac{6i\text{arcsinh}(ax) \text{PolyLog}\left(3, -ie^{\text{arcsinh}(ax)}\right)}{ac} - \frac{6i\text{arcsinh}(ax) \text{PolyLog}\left(3, ie^{\text{arcsinh}(ax)}\right)}{ac} - \frac{(6i)\text{Subst}\left(\int \frac{\text{PolyLog}(3,-ix)}{x} dx, x, e^{\text{arcsinh}(ax)}\right)}{ac} + \frac{(6i)\text{Subst}\left(\int \frac{\text{PolyLog}(3,ix)}{x} dx, x, e^{\text{arcsinh}(ax)}\right)}{ac}$$

$$= \frac{2\text{arcsinh}(ax)^3 \arctan\left(e^{\text{arcsinh}(ax)}\right)}{ac} - \frac{3i\text{arcsinh}(ax)^2 \text{PolyLog}\left(2, -ie^{\text{arcsinh}(ax)}\right)}{ac} + \frac{3i\text{arcsinh}(ax)^2 \text{PolyLog}\left(2, ie^{\text{arcsinh}(ax)}\right)}{ac} + \frac{6i\text{arcsinh}(ax) \text{PolyLog}\left(3, -ie^{\text{arcsinh}(ax)}\right)}{ac} - \frac{6i\text{arcsinh}(ax) \text{PolyLog}\left(3, ie^{\text{arcsinh}(ax)}\right)}{ac} - \frac{6i \text{PolyLog}\left(4, -ie^{\text{arcsinh}(ax)}\right)}{ac} + \frac{6i \text{PolyLog}\left(4, ie^{\text{arcsinh}(ax)}\right)}{ac}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.28

$$\int \frac{\text{arcsinh}(ax)^3}{c+a^2cx^2}\,dx = \frac{-\text{arcsinh}(ax)^3\log\left(1+\frac{ae^{\text{arcsinh}(ax)}}{\sqrt{-a^2}}\right)+\text{arcsinh}(ax)^3\log\left(1+\frac{\sqrt{-a^2}e^{\text{arcsinh}(ax)}}{a}\right)+3\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2\right.}{}$$

```
[In] Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2),x]
```

```
[Out] (-(ArcSinh[a*x]^3*Log[1 + (a*E^ArcSinh[a*x])/Sqrt[-a^2]]) + ArcSinh[a*x]^3*
Log[1 + (Sqrt[-a^2]*E^ArcSinh[a*x])/a] + 3*ArcSinh[a*x]^2*PolyLog[2, (a*E^A
rcSinh[a*x])/Sqrt[-a^2]] - 3*ArcSinh[a*x]^2*PolyLog[2, (Sqrt[-a^2]*E^ArcSin
h[a*x])/a] - 6*ArcSinh[a*x]*PolyLog[3, (a*E^ArcSinh[a*x])/Sqrt[-a^2]] + 6*A
rcSinh[a*x]*PolyLog[3, (Sqrt[-a^2]*E^ArcSinh[a*x])/a] + 6*PolyLog[4, (a*E^A
rcSinh[a*x])/Sqrt[-a^2]] - 6*PolyLog[4, (Sqrt[-a^2]*E^ArcSinh[a*x])/a])/(Sq
rt[-a^2]*c)
```

Maple [F]

$$\int \frac{\text{arcsinh}\left(ax\right)^{3}}{a^{2}c\,x^{2}+c}dx$$

```
[In] int(arcsinh(a*x)^3/(a^2*c*x^2+c),x)
```

```
[Out] int(arcsinh(a*x)^3/(a^2*c*x^2+c),x)
```

Fricas [F]

$$\int \frac{\text{arcsinh}(ax)^3}{c+a^2cx^2}\,dx = \int \frac{\text{arsinh}\left(ax\right)^{3}}{a^{2}cx^{2}+c}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^3/(a^2*c*x^2 + c), x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{c+a^2cx^2}\,dx = \frac{\int \frac{\operatorname{asinh}^3(ax)}{a^2x^2+1}\,dx}{c}$$

```
[In] integrate(asinh(a*x)**3/(a**2*c*x**2+c),x)
[Out] Integral(asinh(a*x)**3/(a**2*x**2 + 1), x)/c
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{c+a^2cx^2}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{a^2cx^2+c}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{c+a^2cx^2}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{a^2cx^2+c}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{c+a^2cx^2}\,dx = \int \frac{\operatorname{asinh}(a\,x)^3}{c\,a^2\,x^2+c}\,dx$$

```
[In] int(asinh(a*x)^3/(c + a^2*c*x^2),x)
[Out] int(asinh(a*x)^3/(c + a^2*c*x^2), x)
```

3.332 $\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 294

$$\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^2}\,dx = \frac{3\text{arcsinh}(ax)^2}{2ac^2\sqrt{1+a^2x^2}}+\frac{x\text{arcsinh}(ax)^3}{2c^2(1+a^2x^2)}-\frac{6\text{arcsinh}(ax)\arctan\left(e^{\text{arcsinh}(ax)}\right)}{ac^2}+\frac{\text{arcsinh}(ax)^3\arctan\left(e^{\text{arcsinh}(ax)}\right)}{ac^2}+\frac{3i\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)}{ac^2}-\frac{3i\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)}{2ac^2}-\frac{3i\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(ax)}\right)}{ac^2}+\frac{3i\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(ax)}\right)}{2ac^2}+\frac{3i\text{arcsinh}(ax)\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(ax)}\right)}{ac^2}-\frac{3i\text{arcsinh}(ax)\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(ax)}\right)}{ac^2}-\frac{3i\,\text{PolyLog}\left(4,-ie^{\text{arcsinh}(ax)}\right)}{ac^2}+\frac{3i\,\text{PolyLog}\left(4,ie^{\text{arcsinh}(ax)}\right)}{ac^2}$$

```
[Out] 1/2*x*arcsinh(a*x)^3/c^2/(a^2*x^2+1)-6*arcsinh(a*x)*arctan(a*x+(a^2*x^2+1)^
(1/2))/a/c^2+arcsinh(a*x)^3*arctan(a*x+(a^2*x^2+1)^(1/2))/a/c^2+3*I*polylog
(2,-I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^2-3/2*I*arcsinh(a*x)^2*polylog(2,-I*(a*x
+(a^2*x^2+1)^(1/2)))/a/c^2-3*I*polylog(2,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^2+3
/2*I*arcsinh(a*x)^2*polylog(2,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^2+3*I*arcsinh(
a*x)*polylog(3,-I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^2-3*I*arcsinh(a*x)*polylog(3
,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^2-3*I*polylog(4,-I*(a*x+(a^2*x^2+1)^(1/2)))
/a/c^2+3*I*polylog(4,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^2+3/2*arcsinh(a*x)^2/a/
c^2/(a^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {5788, 5789, 4265, 2611, 6744, 2320, 6724, 5798, 2317, 2438}

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^2}\,dx = \frac{x\operatorname{arcsinh}(ax)^3}{2c^2(a^2x^2+1)} + \frac{3\operatorname{arcsinh}(ax)^2}{2ac^2\sqrt{a^2x^2+1}} + \frac{\operatorname{arcsinh}(ax)^3\arctan\left(e^{\operatorname{arcsinh}(ax)}\right)}{ac^2} - \frac{6\operatorname{arcsinh}(ax)\arctan\left(e^{\operatorname{arcsinh}(ax)}\right)}{ac^2} - \frac{3i\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(ax)}\right)}{2ac^2} + \frac{3i\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(ax)}\right)}{2ac^2} + \frac{3i\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,-ie^{\operatorname{arcsinh}(ax)}\right)}{ac^2} - \frac{3i\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,ie^{\operatorname{arcsinh}(ax)}\right)}{ac^2} + \frac{3i\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(ax)}\right)}{ac^2} - \frac{3i\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(ax)}\right)}{ac^2} - \frac{3i\operatorname{PolyLog}\left(4,-ie^{\operatorname{arcsinh}(ax)}\right)}{ac^2} + \frac{3i\operatorname{PolyLog}\left(4,ie^{\operatorname{arcsinh}(ax)}\right)}{ac^2}$$

```
[In] Int[ArcSinh[a*x]^3/(c + a^2*c*x^2)^2,x]
```

```
[Out] (3*ArcSinh[a*x]^2)/(2*a*c^2*Sqrt[1 + a^2*x^2]) + (x*ArcSinh[a*x]^3)/(2*c^2*
(1 + a^2*x^2)) - (6*ArcSinh[a*x]*ArcTan[E^ArcSinh[a*x]])/(a*c^2) + (ArcSinh
[a*x]^3*ArcTan[E^ArcSinh[a*x]])/(a*c^2) + ((3*I)*PolyLog[2, (-I)*E^ArcSinh[
a*x]])/(a*c^2) - (((3*I)/2)*ArcSinh[a*x]^2*PolyLog[2, (-I)*E^ArcSinh[a*x]])
/(a*c^2) - ((3*I)*PolyLog[2, I*E^ArcSinh[a*x]])/(a*c^2) + (((3*I)/2)*ArcSin
h[a*x]^2*PolyLog[2, I*E^ArcSinh[a*x]])/(a*c^2) + ((3*I)*ArcSinh[a*x]*PolyLo
g[3, (-I)*E^ArcSinh[a*x]])/(a*c^2) - ((3*I)*ArcSinh[a*x]*PolyLog[3, I*E^Arc
Sinh[a*x]])/(a*c^2) - ((3*I)*PolyLog[4, (-I)*E^ArcSinh[a*x]])/(a*c^2) + ((3
*I)*PolyLog[4, I*E^ArcSinh[a*x]])/(a*c^2)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
```

```
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x\text{arcsinh}(ax)^3}{2c^2(1+a^2x^2)} - \frac{(3a)\int \frac{x\text{arcsinh}(ax)^2}{(1+a^2x^2)^{3/2}}\,dx}{2c^2} + \frac{\int \frac{\text{arcsinh}(ax)^3}{c+a^2cx^2}\,dx}{2c} \\
&= \frac{3\text{arcsinh}(ax)^2}{2ac^2\sqrt{1+a^2x^2}} + \frac{x\text{arcsinh}(ax)^3}{2c^2(1+a^2x^2)} - \frac{3\int \frac{\text{arcsinh}(ax)}{1+a^2x^2}\,dx}{c^2} \\
&\quad + \frac{\text{Subst}\left(\int x^3\text{sech}(x)\,dx, x, \text{arcsinh}(ax)\right)}{2ac^2} \\
&= \frac{3\text{arcsinh}(ax)^2}{2ac^2\sqrt{1+a^2x^2}} + \frac{x\text{arcsinh}(ax)^3}{2c^2(1+a^2x^2)} + \frac{\text{arcsinh}(ax)^3\arctan\left(e^{\text{arcsinh}(ax)}\right)}{ac^2} \\
&\quad - \frac{(3i)\text{Subst}\left(\int x^2\log\left(1-ie^x\right)\,dx, x, \text{arcsinh}(ax)\right)}{2ac^2} \\
&\quad + \frac{(3i)\text{Subst}\left(\int x^2\log\left(1+ie^x\right)\,dx, x, \text{arcsinh}(ax)\right)}{2ac^2} \\
&\quad - \frac{3\text{Subst}\left(\int x\text{sech}(x)\,dx, x, \text{arcsinh}(ax)\right)}{ac^2}
\end{aligned}$$

$$= \frac{3\text{arcsinh}(ax)^2}{2ac^2\sqrt{1+a^2x^2}} + \frac{x\text{arcsinh}(ax)^3}{2c^2(1+a^2x^2)} - \frac{6\text{arcsinh}(ax)\arctan\left(e^{\text{arcsinh}(ax)}\right)}{ac^2} + \frac{\text{arcsinh}(ax)^3\arctan\left(e^{\text{arcsinh}(ax)}\right)}{ac^2} - \frac{3i\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)}{2ac^2} + \frac{3i\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(ax)}\right)}{2ac^2} + \frac{(3i)\text{Subst}\left(\int\log(1-ie^x)\,dx,x,\text{arcsinh}(ax)\right)}{ac^2} - \frac{(3i)\text{Subst}\left(\int\log(1+ie^x)\,dx,x,\text{arcsinh}(ax)\right)}{ac^2} + \frac{(3i)\text{Subst}\left(\int x\,\text{PolyLog}(2,-ie^x)\,dx,x,\text{arcsinh}(ax)\right)}{ac^2} - \frac{(3i)\text{Subst}\left(\int x\,\text{PolyLog}(2,ie^x)\,dx,x,\text{arcsinh}(ax)\right)}{ac^2}$$

$$= \frac{3\text{arcsinh}(ax)^2}{2ac^2\sqrt{1+a^2x^2}} + \frac{x\text{arcsinh}(ax)^3}{2c^2(1+a^2x^2)} - \frac{6\text{arcsinh}(ax)\arctan\left(e^{\text{arcsinh}(ax)}\right)}{ac^2} + \frac{\text{arcsinh}(ax)^3\arctan\left(e^{\text{arcsinh}(ax)}\right)}{ac^2} - \frac{3i\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)}{2ac^2} + \frac{3i\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(ax)}\right)}{2ac^2} + \frac{3i\text{arcsinh}(ax)\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(ax)}\right)}{ac^2} - \frac{3i\text{arcsinh}(ax)\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(ax)}\right)}{ac^2} + \frac{(3i)\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{\text{arcsinh}(ax)}\right)}{ac^2} - \frac{(3i)\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{\text{arcsinh}(ax)}\right)}{ac^2} - \frac{(3i)\text{Subst}\left(\int\text{PolyLog}(3,-ie^x)\,dx,x,\text{arcsinh}(ax)\right)}{ac^2} + \frac{(3i)\text{Subst}\left(\int\text{PolyLog}(3,ie^x)\,dx,x,\text{arcsinh}(ax)\right)}{ac^2}$$

$$
\begin{aligned}
&= \frac{3\text{arcsinh}(ax)^2}{2ac^2\sqrt{1+a^2x^2}} + \frac{x\text{arcsinh}(ax)^3}{2c^2(1+a^2x^2)} - \frac{6\text{arcsinh}(ax)\arctan\left(e^{\text{arcsinh}(ax)}\right)}{ac^2} \\
&+ \frac{\text{arcsinh}(ax)^3\arctan\left(e^{\text{arcsinh}(ax)}\right)}{ac^2} + \frac{3i\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)}{ac^2} \\
&- \frac{3i\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)}{2ac^2} - \frac{3i\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(ax)}\right)}{ac^2} \\
&+ \frac{3i\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(ax)}\right)}{2ac^2} \\
&+ \frac{3i\text{arcsinh}(ax)\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(ax)}\right)}{ac^2} \\
&- \frac{3i\text{arcsinh}(ax)\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(ax)}\right)}{ac^2} \\
&- \frac{(3i)\text{Subst}\left(\int \frac{\text{PolyLog}(3,-ix)}{x}\,dx, x, e^{\text{arcsinh}(ax)}\right)}{ac^2} \\
&+ \frac{(3i)\text{Subst}\left(\int \frac{\text{PolyLog}(3,ix)}{x}\,dx, x, e^{\text{arcsinh}(ax)}\right)}{ac^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3\text{arcsinh}(ax)^2}{2ac^2\sqrt{1+a^2x^2}} + \frac{x\text{arcsinh}(ax)^3}{2c^2(1+a^2x^2)} - \frac{6\text{arcsinh}(ax)\arctan\left(e^{\text{arcsinh}(ax)}\right)}{ac^2} \\
&+ \frac{\text{arcsinh}(ax)^3\arctan\left(e^{\text{arcsinh}(ax)}\right)}{ac^2} + \frac{3i\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)}{ac^2} \\
&- \frac{3i\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)}{2ac^2} - \frac{3i\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(ax)}\right)}{ac^2} \\
&+ \frac{3i\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(ax)}\right)}{2ac^2} \\
&+ \frac{3i\text{arcsinh}(ax)\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(ax)}\right)}{ac^2} \\
&- \frac{3i\text{arcsinh}(ax)\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(ax)}\right)}{ac^2} \\
&- \frac{3i\,\text{PolyLog}\left(4,-ie^{\text{arcsinh}(ax)}\right)}{ac^2} + \frac{3i\,\text{PolyLog}\left(4,ie^{\text{arcsinh}(ax)}\right)}{ac^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.93

$$
\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^2}\,dx =
$$

$$
-\frac{i\Big(7\pi^4 + 8i\pi^3\text{arcsinh}(ax) + 24\pi^2\text{arcsinh}(ax)^2 + \frac{192i\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}} - 32i\pi\text{arcsinh}(ax)^3 + \frac{64iax\text{arcsinh}(ax)^3}{1+a^2x^2} -
$$

```
[In] Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2)^2,x]
```

```
[Out] ((-1/128*I)*(7*Pi^4 + (8*I)*Pi^3*ArcSinh[a*x] + 24*Pi^2*ArcSinh[a*x]^2 + ((
192*I)*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2] - (32*I)*Pi*ArcSinh[a*x]^3 + ((64*
I)*a*x*ArcSinh[a*x]^3)/(1 + a^2*x^2) - 16*ArcSinh[a*x]^4 - 384*ArcSinh[a*x]
*Log[1 - I/E^ArcSinh[a*x]] + (8*I)*Pi^3*Log[1 + I/E^ArcSinh[a*x]] + 384*Arc
Sinh[a*x]*Log[1 + I/E^ArcSinh[a*x]] + 48*Pi^2*ArcSinh[a*x]*Log[1 + I/E^ArcS
inh[a*x]] - (96*I)*Pi*ArcSinh[a*x]^2*Log[1 + I/E^ArcSinh[a*x]] - 64*ArcSinh
[a*x]^3*Log[1 + I/E^ArcSinh[a*x]] - 48*Pi^2*ArcSinh[a*x]*Log[1 - I*E^ArcSin
h[a*x]] + (96*I)*Pi*ArcSinh[a*x]^2*Log[1 - I*E^ArcSinh[a*x]] - (8*I)*Pi^3*L
og[1 + I*E^ArcSinh[a*x]] + 64*ArcSinh[a*x]^3*Log[1 + I*E^ArcSinh[a*x]] + (8
*I)*Pi^3*Log[Tan[(Pi + (2*I)*ArcSinh[a*x])/4]] - 48*(8 + Pi^2 - (4*I)*Pi*Ar
cSinh[a*x] - 4*ArcSinh[a*x]^2)*PolyLog[2, (-I)/E^ArcSinh[a*x]] + 384*PolyLo
g[2, I/E^ArcSinh[a*x]] + 192*ArcSinh[a*x]^2*PolyLog[2, (-I)*E^ArcSinh[a*x]]
 - 48*Pi^2*PolyLog[2, I*E^ArcSinh[a*x]] + (192*I)*Pi*ArcSinh[a*x]*PolyLog[2
, I*E^ArcSinh[a*x]] + (192*I)*Pi*PolyLog[3, (-I)/E^ArcSinh[a*x]] + 384*ArcS
inh[a*x]*PolyLog[3, (-I)/E^ArcSinh[a*x]] - 384*ArcSinh[a*x]*PolyLog[3, (-I)
*E^ArcSinh[a*x]] - (192*I)*Pi*PolyLog[3, I*E^ArcSinh[a*x]] + 384*PolyLog[4,
 (-I)/E^ArcSinh[a*x]] + 384*PolyLog[4, (-I)*E^ArcSinh[a*x]]))/(a*c^2)
```

Maple [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(a^2c\,x^2+c)^2}dx$$

```
[In] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^2,x)
```

```
[Out] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^2,x)
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^2}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{(a^2cx^2+c)^2}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{\operatorname{asinh}^3(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

```
[In] integrate(asinh(a*x)**3/(a**2*c*x**2+c)**2,x)
[Out] Integral(asinh(a*x)**3/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^2}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{(a^2cx^2+c)^2}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^2, x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^2}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{(a^2cx^2+c)^2}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^2}\,dx = \int \frac{\operatorname{asinh}(a\,x)^3}{(c\,a^2\,x^2+c)^2}\,dx$$

```
[In] int(asinh(a*x)^3/(c + a^2*c*x^2)^2,x)
[Out] int(asinh(a*x)^3/(c + a^2*c*x^2)^2, x)
```

3.333 $\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 409

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^3}\,dx = -\frac{1}{4ac^3\sqrt{1+a^2x^2}}-\frac{x\operatorname{arcsinh}(ax)}{4c^3(1+a^2x^2)}+\frac{\operatorname{arcsinh}(ax)^2}{4ac^3(1+a^2x^2)^{3/2}}+\frac{9\operatorname{arcsinh}(ax)^2}{8ac^3\sqrt{1+a^2x^2}}+\frac{x\operatorname{arcsinh}(ax)^3}{4c^3(1+a^2x^2)^2}+\frac{3x\operatorname{arcsinh}(ax)^3}{8c^3(1+a^2x^2)}-\frac{5\operatorname{arcsinh}(ax)\arctan\left(e^{\operatorname{arcsinh}(ax)}\right)}{ac^3}+\frac{3\operatorname{arcsinh}(ax)^3\arctan\left(e^{\operatorname{arcsinh}(ax)}\right)}{4ac^3}+\frac{5i\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(ax)}\right)}{2ac^3}-\frac{9i\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(ax)}\right)}{8ac^3}-\frac{5i\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(ax)}\right)}{2ac^3}+\frac{9i\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(ax)}\right)}{8ac^3}+\frac{9i\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,-ie^{\operatorname{arcsinh}(ax)}\right)}{4ac^3}-\frac{9i\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,ie^{\operatorname{arcsinh}(ax)}\right)}{4ac^3}-\frac{9i\operatorname{PolyLog}\left(4,-ie^{\operatorname{arcsinh}(ax)}\right)}{4ac^3}+\frac{9i\operatorname{PolyLog}\left(4,ie^{\operatorname{arcsinh}(ax)}\right)}{4ac^3}$$

```
[Out] -1/4*x*arcsinh(a*x)/c^3/(a^2*x^2+1)+1/4*arcsinh(a*x)^2/a/c^3/(a^2*x^2+1)^(3
/2)+1/4*x*arcsinh(a*x)^3/c^3/(a^2*x^2+1)^2+3/8*x*arcsinh(a*x)^3/c^3/(a^2*x^
2+1)-5*arcsinh(a*x)*arctan(a*x+(a^2*x^2+1)^(1/2))/a/c^3+3/4*arcsinh(a*x)^3*
arctan(a*x+(a^2*x^2+1)^(1/2))/a/c^3-9/8*I*arcsinh(a*x)^2*polylog(2,-I*(a*x+
(a^2*x^2+1)^(1/2)))/a/c^3+9/4*I*polylog(4,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^3+
9/8*I*arcsinh(a*x)^2*polylog(2,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^3+9/4*I*arcsi
```

```
nh(a*x)*polylog(3,-I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^3-9/4*I*arcsinh(a*x)*poly
log(3,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^3+5/2*I*polylog(2,-I*(a*x+(a^2*x^2+1)^
(1/2)))/a/c^3-5/2*I*polylog(2,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^3-9/4*I*polylo
g(4,-I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^3-1/4/a/c^3/(a^2*x^2+1)^(1/2)+9/8*arcsi
nh(a*x)^2/a/c^3/(a^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {5788, 5789, 4265, 2611, 6744, 2320, 6724, 5798, 2317, 2438, 267}

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^3}\,dx &= \frac{3x\operatorname{arcsinh}(ax)^3}{8c^3(a^2x^2+1)}+\frac{x\operatorname{arcsinh}(ax)^3}{4c^3(a^2x^2+1)^2}+\frac{9\operatorname{arcsinh}(ax)^2}{8ac^3\sqrt{a^2x^2+1}}\\&+\frac{\operatorname{arcsinh}(ax)^2}{4ac^3(a^2x^2+1)^{3/2}}-\frac{x\operatorname{arcsinh}(ax)}{4c^3(a^2x^2+1)}-\frac{1}{4ac^3\sqrt{a^2x^2+1}}\\&+\frac{3\operatorname{arcsinh}(ax)^3\arctan\left(e^{\operatorname{arcsinh}(ax)}\right)}{4ac^3}-\frac{5\operatorname{arcsinh}(ax)\arctan\left(e^{\operatorname{arcsinh}(ax)}\right)}{ac^3}\\&-\frac{9i\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(ax)}\right)}{8ac^3}\\&+\frac{9i\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(ax)}\right)}{8ac^3}\\&+\frac{9i\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,-ie^{\operatorname{arcsinh}(ax)}\right)}{4ac^3}\\&-\frac{9i\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,ie^{\operatorname{arcsinh}(ax)}\right)}{4ac^3}\\&+\frac{5i\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(ax)}\right)}{2ac^3}-\frac{5i\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(ax)}\right)}{2ac^3}\\&-\frac{9i\operatorname{PolyLog}\left(4,-ie^{\operatorname{arcsinh}(ax)}\right)}{4ac^3}+\frac{9i\operatorname{PolyLog}\left(4,ie^{\operatorname{arcsinh}(ax)}\right)}{4ac^3}\end{aligned}$$

```
[In] Int[ArcSinh[a*x]^3/(c + a^2*c*x^2)^3,x]
```

```
[Out] -1/4*1/(a*c^3*Sqrt[1 + a^2*x^2]) - (x*ArcSinh[a*x])/(4*c^3*(1 + a^2*x^2)) +
 ArcSinh[a*x]^2/(4*a*c^3*(1 + a^2*x^2)^(3/2)) + (9*ArcSinh[a*x]^2)/(8*a*c^3
*Sqrt[1 + a^2*x^2]) + (x*ArcSinh[a*x]^3)/(4*c^3*(1 + a^2*x^2)^2) + (3*x*Arc
Sinh[a*x]^3)/(8*c^3*(1 + a^2*x^2)) - (5*ArcSinh[a*x]*ArcTan[E^ArcSinh[a*x]]
)/(a*c^3) + (3*ArcSinh[a*x]^3*ArcTan[E^ArcSinh[a*x]])/(4*a*c^3) + (((5*I)/2
)*PolyLog[2, (-I)*E^ArcSinh[a*x]])/(a*c^3) - (((9*I)/8)*ArcSinh[a*x]^2*Poly
Log[2, (-I)*E^ArcSinh[a*x]])/(a*c^3) - (((5*I)/2)*PolyLog[2, I*E^ArcSinh[a*
x]])/(a*c^3) + (((9*I)/8)*ArcSinh[a*x]^2*PolyLog[2, I*E^ArcSinh[a*x]])/(a*c
^3) + (((9*I)/4)*ArcSinh[a*x]*PolyLog[3, (-I)*E^ArcSinh[a*x]])/(a*c^3) - ((
(9*I)/4)*ArcSinh[a*x]*PolyLog[3, I*E^ArcSinh[a*x]])/(a*c^3) - (((9*I)/4)*Po
```

```
lyLog[4, (-I)*E^ArcSinh[a*x]])/(a*c^3) + (((9*I)/4)*PolyLog[4, I*E^ArcSinh[
a*x]])/(a*c^3)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{x\text{arcsinh}(ax)^3}{4c^3(1+a^2x^2)^2} - \frac{(3a)\int \frac{x\text{arcsinh}(ax)^2}{(1+a^2x^2)^{5/2}}\,dx}{4c^3} + \frac{3\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^2}\,dx}{4c}$$

$$= \frac{\text{arcsinh}(ax)^2}{4ac^3(1+a^2x^2)^{3/2}} + \frac{x\text{arcsinh}(ax)^3}{4c^3(1+a^2x^2)^2} + \frac{3x\text{arcsinh}(ax)^3}{8c^3(1+a^2x^2)} - \frac{\int \frac{\text{arcsinh}(ax)}{(1+a^2x^2)^2}\,dx}{2c^3} - \frac{(9a)\int \frac{x\text{arcsinh}(ax)^2}{(1+a^2x^2)^{3/2}}\,dx}{8c^3} + \frac{3\int \frac{\text{arcsinh}(ax)^3}{c+a^2cx^2}\,dx}{8c^2}$$

$$= -\frac{x\operatorname{arcsinh}(ax)}{4c^3(1+a^2x^2)} + \frac{\operatorname{arcsinh}(ax)^2}{4ac^3(1+a^2x^2)^{3/2}} + \frac{9\operatorname{arcsinh}(ax)^2}{8ac^3\sqrt{1+a^2x^2}} + \frac{x\operatorname{arcsinh}(ax)^3}{4c^3(1+a^2x^2)^2} + \frac{3x\operatorname{arcsinh}(ax)^3}{8c^3(1+a^2x^2)} - \frac{\int \frac{\operatorname{arcsinh}(ax)}{1+a^2x^2}\,dx}{4c^3} - \frac{9\int \frac{\operatorname{arcsinh}(ax)}{1+a^2x^2}\,dx}{4c^3} + \frac{3\text{Subst}\left(\int x^3\operatorname{sech}(x)\,dx, x, \operatorname{arcsinh}(ax)\right)}{8ac^3} + \frac{a\int \frac{x}{(1+a^2x^2)^{3/2}}\,dx}{4c^3}$$

$$= -\frac{1}{4ac^3\sqrt{1+a^2x^2}} - \frac{x\operatorname{arcsinh}(ax)}{4c^3(1+a^2x^2)} + \frac{\operatorname{arcsinh}(ax)^2}{4ac^3(1+a^2x^2)^{3/2}} + \frac{9\operatorname{arcsinh}(ax)^2}{8ac^3\sqrt{1+a^2x^2}} + \frac{x\operatorname{arcsinh}(ax)^3}{4c^3(1+a^2x^2)^2} + \frac{3x\operatorname{arcsinh}(ax)^3}{8c^3(1+a^2x^2)} + \frac{3\operatorname{arcsinh}(ax)^3\arctan\left(e^{\operatorname{arcsinh}(ax)}\right)}{4ac^3} - \frac{(9i)\text{Subst}\left(\int x^2\log\left(1-ie^x\right)\,dx, x, \operatorname{arcsinh}(ax)\right)}{8ac^3} + \frac{(9i)\text{Subst}\left(\int x^2\log\left(1+ie^x\right)\,dx, x, \operatorname{arcsinh}(ax)\right)}{8ac^3} - \frac{\text{Subst}(\int x\operatorname{sech}(x)\,dx, x, \operatorname{arcsinh}(ax))}{4ac^3} - \frac{9\text{Subst}(\int x\operatorname{sech}(x)\,dx, x, \operatorname{arcsinh}(ax))}{4ac^3}$$

$$= -\frac{1}{4ac^3\sqrt{1+a^2x^2}} - \frac{x\operatorname{arcsinh}(ax)}{4c^3(1+a^2x^2)} + \frac{\operatorname{arcsinh}(ax)^2}{4ac^3(1+a^2x^2)^{3/2}} + \frac{9\operatorname{arcsinh}(ax)^2}{8ac^3\sqrt{1+a^2x^2}} + \frac{x\operatorname{arcsinh}(ax)^3}{4c^3(1+a^2x^2)^2} + \frac{3x\operatorname{arcsinh}(ax)^3}{8c^3(1+a^2x^2)} - \frac{5\operatorname{arcsinh}(ax)\arctan\left(e^{\operatorname{arcsinh}(ax)}\right)}{ac^3} + \frac{3\operatorname{arcsinh}(ax)^3\arctan\left(e^{\operatorname{arcsinh}(ax)}\right)}{4ac^3} - \frac{9i\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2, -ie^{\operatorname{arcsinh}(ax)}\right)}{8ac^3} + \frac{9i\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2, ie^{\operatorname{arcsinh}(ax)}\right)}{8ac^3} + \frac{i\text{Subst}\left(\int \log\left(1-ie^x\right)\,dx, x, \operatorname{arcsinh}(ax)\right)}{4ac^3} - \frac{i\text{Subst}\left(\int \log\left(1+ie^x\right)\,dx, x, \operatorname{arcsinh}(ax)\right)}{4ac^3} + \frac{(9i)\text{Subst}\left(\int \log\left(1-ie^x\right)\,dx, x, \operatorname{arcsinh}(ax)\right)}{4ac^3} - \frac{(9i)\text{Subst}\left(\int \log\left(1+ie^x\right)\,dx, x, \operatorname{arcsinh}(ax)\right)}{4ac^3} + \frac{(9i)\text{Subst}\left(\int x\operatorname{PolyLog}\left(2, -ie^x\right)\,dx, x, \operatorname{arcsinh}(ax)\right)}{4ac^3} - \frac{(9i)\text{Subst}\left(\int x\operatorname{PolyLog}\left(2, ie^x\right)\,dx, x, \operatorname{arcsinh}(ax)\right)}{4ac^3}$$

$$
\begin{aligned}
&= -\frac{1}{4ac^3\sqrt{1+a^2x^2}} - \frac{x\text{arcsinh}(ax)}{4c^3(1+a^2x^2)} + \frac{\text{arcsinh}(ax)^2}{4ac^3(1+a^2x^2)^{3/2}} + \frac{9\text{arcsinh}(ax)^2}{8ac^3\sqrt{1+a^2x^2}} \\
&+ \frac{x\text{arcsinh}(ax)^3}{4c^3(1+a^2x^2)^2} + \frac{3x\text{arcsinh}(ax)^3}{8c^3(1+a^2x^2)} - \frac{5\text{arcsinh}(ax)\arctan\left(e^{\text{arcsinh}(ax)}\right)}{ac^3} \\
&+ \frac{3\text{arcsinh}(ax)^3\arctan\left(e^{\text{arcsinh}(ax)}\right)}{4ac^3} - \frac{9i\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)}{8ac^3} \\
&+ \frac{9i\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(ax)}\right)}{8ac^3} \\
&+ \frac{9i\text{arcsinh}(ax)\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(ax)}\right)}{4ac^3} \\
&- \frac{9i\text{arcsinh}(ax)\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(ax)}\right)}{4ac^3} + \frac{i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\text{arcsinh}(ax)}\right)}{4ac^3} \\
&- \frac{i\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{\text{arcsinh}(ax)}\right)}{4ac^3} + \frac{(9i)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\text{arcsinh}(ax)}\right)}{4ac^3} \\
&- \frac{(9i)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{\text{arcsinh}(ax)}\right)}{4ac^3} \\
&- \frac{(9i)\text{Subst}\left(\int \text{PolyLog}\left(3,-ie^x\right)\,dx, x, \text{arcsinh}(ax)\right)}{4ac^3} \\
&+ \frac{(9i)\text{Subst}\left(\int \text{PolyLog}\left(3,ie^x\right)\,dx, x, \text{arcsinh}(ax)\right)}{4ac^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1}{4ac^3\sqrt{1+a^2x^2}} - \frac{x\text{arcsinh}(ax)}{4c^3(1+a^2x^2)} + \frac{\text{arcsinh}(ax)^2}{4ac^3(1+a^2x^2)^{3/2}} \\
&+ \frac{9\text{arcsinh}(ax)^2}{8ac^3\sqrt{1+a^2x^2}} + \frac{x\text{arcsinh}(ax)^3}{4c^3(1+a^2x^2)^2} + \frac{3x\text{arcsinh}(ax)^3}{8c^3(1+a^2x^2)} \\
&- \frac{5\text{arcsinh}(ax)\arctan\left(e^{\text{arcsinh}(ax)}\right)}{ac^3} + \frac{3\text{arcsinh}(ax)^3\arctan\left(e^{\text{arcsinh}(ax)}\right)}{4ac^3} \\
&+ \frac{5i\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)}{2ac^3} - \frac{9i\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)}{8ac^3} \\
&- \frac{5i\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(ax)}\right)}{2ac^3} + \frac{9i\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(ax)}\right)}{8ac^3} \\
&+ \frac{9i\text{arcsinh}(ax)\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(ax)}\right)}{4ac^3} \\
&- \frac{9i\text{arcsinh}(ax)\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(ax)}\right)}{4ac^3} \\
&- \frac{(9i)\text{Subst}\left(\int \frac{\text{PolyLog}(3,-ix)}{x}\,dx, x, e^{\text{arcsinh}(ax)}\right)}{4ac^3} \\
&+ \frac{(9i)\text{Subst}\left(\int \frac{\text{PolyLog}(3,ix)}{x}\,dx, x, e^{\text{arcsinh}(ax)}\right)}{4ac^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1}{4ac^3\sqrt{1+a^2x^2}} - \frac{x\mathrm{arcsinh}(ax)}{4c^3(1+a^2x^2)} + \frac{\mathrm{arcsinh}(ax)^2}{4ac^3(1+a^2x^2)^{3/2}} \\
&\quad + \frac{9\mathrm{arcsinh}(ax)^2}{8ac^3\sqrt{1+a^2x^2}} + \frac{x\mathrm{arcsinh}(ax)^3}{4c^3(1+a^2x^2)^2} + \frac{3x\mathrm{arcsinh}(ax)^3}{8c^3(1+a^2x^2)} \\
&\quad - \frac{5\mathrm{arcsinh}(ax)\arctan\left(e^{\mathrm{arcsinh}(ax)}\right)}{ac^3} + \frac{3\mathrm{arcsinh}(ax)^3\arctan\left(e^{\mathrm{arcsinh}(ax)}\right)}{4ac^3} \\
&\quad + \frac{5i\,\mathrm{PolyLog}\left(2,-ie^{\mathrm{arcsinh}(ax)}\right)}{2ac^3} - \frac{9i\mathrm{arcsinh}(ax)^2\,\mathrm{PolyLog}\left(2,-ie^{\mathrm{arcsinh}(ax)}\right)}{8ac^3} \\
&\quad - \frac{5i\,\mathrm{PolyLog}\left(2,ie^{\mathrm{arcsinh}(ax)}\right)}{2ac^3} + \frac{9i\mathrm{arcsinh}(ax)^2\,\mathrm{PolyLog}\left(2,ie^{\mathrm{arcsinh}(ax)}\right)}{8ac^3} \\
&\quad + \frac{9i\mathrm{arcsinh}(ax)\,\mathrm{PolyLog}\left(3,-ie^{\mathrm{arcsinh}(ax)}\right)}{4ac^3} \\
&\quad - \frac{9i\mathrm{arcsinh}(ax)\,\mathrm{PolyLog}\left(3,ie^{\mathrm{arcsinh}(ax)}\right)}{4ac^3} \\
&\quad - \frac{9i\,\mathrm{PolyLog}\left(4,-ie^{\mathrm{arcsinh}(ax)}\right)}{4ac^3} + \frac{9i\,\mathrm{PolyLog}\left(4,ie^{\mathrm{arcsinh}(ax)}\right)}{4ac^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.74 (sec) , antiderivative size = 654, normalized size of antiderivative = 1.60

$$
\int \frac{\mathrm{arcsinh}(ax)^3}{(c+a^2cx^2)^3}\,dx = -\frac{i\left(21\pi^4 - \frac{128i}{\sqrt{1+a^2x^2}} + 24i\pi^3\mathrm{arcsinh}(ax) - \frac{128iax\mathrm{arcsinh}(ax)}{1+a^2x^2} + 72\pi^2\mathrm{arcsinh}(ax)^2 + \frac{128i\mathrm{arcsinh}(ax)^2}{(1+a^2x^2)^{3/2}} + \frac{576i\mathrm{a}}{\sqrt{}}\right.}{}
$$

```
[In] Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2)^3,x]
```

```
[Out] ((-1/512*I)*(21*Pi^4 - (128*I)/Sqrt[1 + a^2*x^2] + (24*I)*Pi^3*ArcSinh[a*x]
 - ((128*I)*a*x*ArcSinh[a*x])/(1 + a^2*x^2) + 72*Pi^2*ArcSinh[a*x]^2 + ((12
8*I)*ArcSinh[a*x]^2)/(1 + a^2*x^2)^(3/2) + ((576*I)*ArcSinh[a*x]^2)/Sqrt[1
+ a^2*x^2] - (96*I)*Pi*ArcSinh[a*x]^3 + ((128*I)*a*x*ArcSinh[a*x]^3)/(1 + a
^2*x^2)^2 + ((192*I)*a*x*ArcSinh[a*x]^3)/(1 + a^2*x^2) - 48*ArcSinh[a*x]^4
- 1280*ArcSinh[a*x]*Log[1 - I/E^ArcSinh[a*x]] + (24*I)*Pi^3*Log[1 + I/E^Arc
Sinh[a*x]] + 1280*ArcSinh[a*x]*Log[1 + I/E^ArcSinh[a*x]] + 144*Pi^2*ArcSinh
[a*x]*Log[1 + I/E^ArcSinh[a*x]] - (288*I)*Pi*ArcSinh[a*x]^2*Log[1 + I/E^Arc
Sinh[a*x]] - 192*ArcSinh[a*x]^3*Log[1 + I/E^ArcSinh[a*x]] - 144*Pi^2*ArcSin
h[a*x]*Log[1 - I*E^ArcSinh[a*x]] + (288*I)*Pi*ArcSinh[a*x]^2*Log[1 - I*E^Ar
cSinh[a*x]] - (24*I)*Pi^3*Log[1 + I*E^ArcSinh[a*x]] + 192*ArcSinh[a*x]^3*Lo
g[1 + I*E^ArcSinh[a*x]] + (24*I)*Pi^3*Log[Tan[(Pi + (2*I)*ArcSinh[a*x])/4]]
 - 16*(80 + 9*Pi^2 - (36*I)*Pi*ArcSinh[a*x] - 36*ArcSinh[a*x]^2)*PolyLog[2,
 (-I)/E^ArcSinh[a*x]] + 1280*PolyLog[2, I/E^ArcSinh[a*x]] + 576*ArcSinh[a*x
]^2*PolyLog[2, (-I)*E^ArcSinh[a*x]] - 144*Pi^2*PolyLog[2, I*E^ArcSinh[a*x]]
```

```
 + (576*I)*Pi*ArcSinh[a*x]*PolyLog[2, I*E^ArcSinh[a*x]] + (576*I)*Pi*PolyLo
g[3, (-I)/E^ArcSinh[a*x]] + 1152*ArcSinh[a*x]*PolyLog[3, (-I)/E^ArcSinh[a*x
]] - 1152*ArcSinh[a*x]*PolyLog[3, (-I)*E^ArcSinh[a*x]] - (576*I)*Pi*PolyLog
[3, I*E^ArcSinh[a*x]] + 1152*PolyLog[4, (-I)/E^ArcSinh[a*x]] + 1152*PolyLog
[4, (-I)*E^ArcSinh[a*x]]))/(a*c^3)
```

Maple [F]

$$\int \frac{\operatorname{arcsinh}\left(a x\right)^{3}}{\left(a^{2} c \,x^{2}+c \right)^{3}}d x$$

```
[In] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^3,x)
[Out] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^3,x)
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^3}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^{3}}{\left(a^{2} c x^{2} + c\right)^{3}}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
[Out] integral(arcsinh(a*x)^3/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)
, x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{\operatorname{asinh}^{3}{\left(a x \right)}}{a^{6} x^{6} + 3 a^{4} x^{4} + 3 a^{2} x^{2} + 1}\, dx}{c^{3}}$$

```
[In] integrate(asinh(a*x)**3/(a**2*c*x**2+c)**3,x)
[Out] Integral(asinh(a*x)**3/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c**3
```

Maxima [F]

$$\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^3}\,dx = \int \frac{\text{arsinh}\,(ax)^3}{(a^2cx^2+c)^3}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^3, x)
```

Giac [F]

$$\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^3}\,dx = \int \frac{\text{arsinh}\,(ax)^3}{(a^2cx^2+c)^3}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^3}\,dx = \int \frac{\text{asinh}(a\,x)^3}{(c\,a^2\,x^2+c)^3}\,dx$$

```
[In] int(asinh(a*x)^3/(c + a^2*c*x^2)^3,x)
[Out] int(asinh(a*x)^3/(c + a^2*c*x^2)^3, x)
```

3.334 $\int (c+a^2cx^2)^{5/2} \operatorname{arcsinh}(ax)^3 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 509

$$\int (c+a^2cx^2)^{5/2} \operatorname{arcsinh}(ax)^3 \, dx = -\frac{865ac^2x^2\sqrt{c+a^2cx^2}}{2304\sqrt{1+a^2x^2}} - \frac{65a^3c^2x^4\sqrt{c+a^2cx^2}}{2304\sqrt{1+a^2x^2}}$$
$$- \frac{c^2(1+a^2x^2)^{5/2}\sqrt{c+a^2cx^2}}{216a} + \frac{245}{384}c^2x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)$$
$$+ \frac{65}{576}c^2x(1+a^2x^2)\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax) + \frac{1}{36}c^2x(1+a^2x^2)^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)$$
$$- \frac{115c^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{768a\sqrt{1+a^2x^2}} - \frac{15ac^2x^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{32\sqrt{1+a^2x^2}}$$
$$- \frac{5c^2(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{32a} - \frac{c^2(1+a^2x^2)^{5/2}\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{12a}$$
$$+ \frac{5}{16}c^2x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^3 + \frac{5}{24}cx(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^3 + \frac{1}{6}x(c+a^2cx^2)^{5/2}\operatorname{arcsinh}(ax)^3 + \frac{5c^2\sqrt{c+\cdots}}{64\cdots}$$

```
[Out] 5/24*c*x*(a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3+1/6*x*(a^2*c*x^2+c)^(5/2)*arcsi
nh(a*x)^3-1/216*c^2*(a^2*x^2+1)^(5/2)*(a^2*c*x^2+c)^(1/2)/a+245/384*c^2*x*a
rcsinh(a*x)*(a^2*c*x^2+c)^(1/2)+65/576*c^2*x*(a^2*x^2+1)*arcsinh(a*x)*(a^2*
c*x^2+c)^(1/2)+1/36*c^2*x*(a^2*x^2+1)^2*arcsinh(a*x)*(a^2*c*x^2+c)^(1/2)-5/
32*c^2*(a^2*x^2+1)^(3/2)*arcsinh(a*x)^2*(a^2*c*x^2+c)^(1/2)/a-1/12*c^2*(a^2
*x^2+1)^(5/2)*arcsinh(a*x)^2*(a^2*c*x^2+c)^(1/2)/a+5/16*c^2*x*arcsinh(a*x)^
3*(a^2*c*x^2+c)^(1/2)-865/2304*a*c^2*x^2*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1
/2)-65/2304*a^3*c^2*x^4*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1/2)-115/768*c^2*a
rcsinh(a*x)^2*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-15/32*a*c^2*x^2*arcsi
nh(a*x)^2*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1/2)+5/64*c^2*arcsinh(a*x)^4*(a^
2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5786, 5785, 5783, 5776, 5812, 30, 5798, 14, 267}

$$\int \left(c+a^2cx^2\right)^{5/2}\text{arcsinh}(ax)^3\,dx=-\frac{15ac^2x^2\text{arcsinh}(ax)^2\sqrt{a^2cx^2+c}}{32\sqrt{a^2x^2+1}}+\frac{5}{16}c^2x\text{arcsinh}(ax)^3\sqrt{a^2cx^2+c}+\frac{245}{384}c^2x\text{arcsinh}(ax)\sqrt{a^2cx^2+c}+\frac{1}{36}c^2x\left(a^2x^2+1\right)^2\text{arcsinh}(ax)\sqrt{a^2cx^2+c}+\frac{65}{576}c^2x\left(a^2x^2+1\right)\text{arcsinh}(ax)\sqrt{a^2cx^2+c}+\frac{5c^2\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{64a\sqrt{a^2x^2+1}}-\frac{c^2\left(a^2x^2+1\right)^{5/2}\text{arcsinh}(ax)^2\sqrt{a^2cx^2+c}}{12a}-\frac{5c^2\left(a^2x^2+1\right)^{3/2}\text{arcsinh}(ax)^2\sqrt{a^2cx^2+c}}{32a}-\frac{115c^2\text{arcsinh}(ax)^2\sqrt{a^2cx^2+c}}{768a\sqrt{a^2x^2+1}}+\frac{1}{6}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{5/2}+\frac{5}{24}cx\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{865ac^2x^2\sqrt{a^2cx^2+c}}{2304\sqrt{a^2x^2+1}}-\frac{c^2\left(a^2x^2+1\right)^{5/2}\sqrt{}}{216a}$$

```
[In] Int[(c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^3,x]
```

```
[Out] (-865*a*c^2*x^2*Sqrt[c + a^2*c*x^2])/(2304*Sqrt[1 + a^2*x^2]) - (65*a^3*c^2
*x^4*Sqrt[c + a^2*c*x^2])/(2304*Sqrt[1 + a^2*x^2]) - (c^2*(1 + a^2*x^2)^(5/
2)*Sqrt[c + a^2*c*x^2])/(216*a) + (245*c^2*x*Sqrt[c + a^2*c*x^2]*ArcSinh[a*
x])/384 + (65*c^2*x*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x])/576 + (
c^2*x*(1 + a^2*x^2)^2*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x])/36 - (115*c^2*Sqrt[
c + a^2*c*x^2]*ArcSinh[a*x]^2)/(768*a*Sqrt[1 + a^2*x^2]) - (15*a*c^2*x^2*Sq
rt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(32*Sqrt[1 + a^2*x^2]) - (5*c^2*(1 + a^2*
x^2)^(3/2)*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(32*a) - (c^2*(1 + a^2*x^2)^
(5/2)*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(12*a) + (5*c^2*x*Sqrt[c + a^2*c*
x^2]*ArcSinh[a*x]^3)/16 + (5*c*x*(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^3)/24 +
 (x*(c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^3)/6 + (5*c^2*Sqrt[c + a^2*c*x^2]*Ar
cSinh[a*x]^4)/(64*a*Sqrt[1 + a^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{6}x\left(c+a^2cx^2\right)^{5/2}\operatorname{arcsinh}(ax)^3 + \frac{1}{6}(5c)\int \left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^3\,dx - \frac{\left(ac^2\sqrt{c+a^2cx^2}\right)\int x(1+a^2x^2)^2\operatorname{arcsinh}(ax)^2\,dx}{2\sqrt{1+a^2x^2}}$$

$$= -\frac{c^2(1+a^2x^2)^{5/2}\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{12a} + \frac{5}{24}cx\left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^3 + \frac{1}{6}x\left(c+a^2cx^2\right)^{5/2}\operatorname{arcsinh}(ax)^3 + \frac{1}{8}\left(5c^2\right)\int\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^3\,dx + \frac{\left(c^2\sqrt{c+a^2cx^2}\right)\int (1+ a}{6\sqrt{1+}}$$

$$= \frac{1}{36}c^2x\left(1+a^2x^2\right)^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax) - \frac{5c^2(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{32a} - \frac{c^2(1+a^2x^2)^{5/2}\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{12a} + \frac{5}{16}c^2x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^3 + \frac{5}{24}cx\left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^3 + \frac{1}{6}x\left(c+a^2cx^2\right)^{5/2}\operatorname{arcsinh}(ax)^3 + \frac{\left(5c^2\sqrt{c+a^2cx^2}\right)\int (1+a^2x^2)}{36\sqrt{1+a^2x}}$$

$$= -\frac{c^2(1+a^2x^2)^{5/2}\sqrt{c+a^2cx^2}}{216a} + \frac{65}{576}c^2x\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax) + \frac{1}{36}c^2x\left(1+a^2x^2\right)^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax) - \frac{15ac^2x^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{32\sqrt{1+a^2x^2}} - \frac{5c^2(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{32a} - \frac{c^2(1+a^2x^2)^{5/2}\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{12a} + \frac{5}{16}c^2x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^3 + \frac{5}{24}cx\left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^3 + \frac{1}{6}x\left(c+a^2cx^2\right)^{5/2}\operatorname{arcsinh}(ax)$$

$$= -\frac{c^2(1+a^2x^2)^{5/2}\sqrt{c+a^2cx^2}}{216a} + \frac{245}{384}c^2x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax) + \frac{65}{576}c^2x(1+a^2x^2)\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax) + \frac{1}{36}c^2x(1+a^2x^2)^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax) - \frac{15ac^2x^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{32\sqrt{1+a^2x^2}} - \frac{5c^2(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{32a} - \frac{c^2(1+a^2x^2)^{5/2}\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{12a} + \frac{5}{16}c^2x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^3 + \frac{5}{24}cx(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^3 + \frac{1}{6}x(c+a^2cx^2)^{5/2}\operatorname{arcsinh}(ax)^3$$

$$= -\frac{865ac^2x^2\sqrt{c+a^2cx^2}}{2304\sqrt{1+a^2x^2}} - \frac{65a^3c^2x^4\sqrt{c+a^2cx^2}}{2304\sqrt{1+a^2x^2}} - \frac{c^2(1+a^2x^2)^{5/2}\sqrt{c+a^2cx^2}}{216a} + \frac{245}{384}c^2x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax) + \frac{65}{576}c^2x(1+a^2x^2)\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax) + \frac{1}{36}c^2x(1+a^2x^2)^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax) - \frac{115c^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{768a\sqrt{1+a^2x^2}} - \frac{15ac^2x^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{32\sqrt{1+a^2x^2}} - \frac{5c^2(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{32a} - \frac{c^2(1+a^2x^2)^{5/2}\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{12a} + \frac{5}{16}c^2x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^3 + \frac{5}{24}cx(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^3 + \frac{1}{6}x(c+a^2cx^2)^{5/2}\operatorname{arcsinh}(ax)^3$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.35

$$\int (c+a^2cx^2)^{5/2}\operatorname{arcsinh}(ax)^3\,dx = \frac{c^2\sqrt{c+a^2cx^2}(4320\operatorname{arcsinh}(ax)^4 - 9720\cosh(2\operatorname{arcsinh}(ax)) - 243\cosh(4\operatorname{arcsinh}$$

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^3,x]
```

```
[Out] (c^2*Sqrt[c + a^2*c*x^2]*(4320*ArcSinh[a*x]^4 - 9720*Cosh[2*ArcSinh[a*x]] -
 243*Cosh[4*ArcSinh[a*x]] - 8*Cosh[6*ArcSinh[a*x]] - 72*ArcSinh[a*x]^2*(270
*Cosh[2*ArcSinh[a*x]] + 27*Cosh[4*ArcSinh[a*x]] + 2*Cosh[6*ArcSinh[a*x]]) +
 288*ArcSinh[a*x]^3*(45*Sinh[2*ArcSinh[a*x]] + 9*Sinh[4*ArcSinh[a*x]] + Sin
h[6*ArcSinh[a*x]]) + 12*ArcSinh[a*x]*(1620*Sinh[2*ArcSinh[a*x]] + 81*Sinh[4
*ArcSinh[a*x]] + 4*Sinh[6*ArcSinh[a*x]])))/(55296*a*Sqrt[1 + a^2*x^2])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 802, normalized size of antiderivative = 1.58

method	result
default	$\frac{5\sqrt{c(a^2x^2+1)}\,\operatorname{arcsinh}(ax)^4c^2}{64\sqrt{a^2x^2+1}\,a}+\frac{\sqrt{c(a^2x^2+1)}\,\left(32a^7x^7+32x^6a^6\sqrt{a^2x^2+1}+64a^5x^5+48a^4x^4\sqrt{a^2x^2+1}+38a^3x^3+18a^2x^2\sqrt{a^2x^2+1}\right.}{13824a(a^2x^2+1)}$

```
[In] int((a^2*c*x^2+c)^(5/2)*arcsinh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 5/64*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a*arcsinh(a*x)^4*c^2+1/13824*(
c*(a^2*x^2+1))^(1/2)*(32*a^7*x^7+32*x^6*a^6*(a^2*x^2+1)^(1/2)+64*a^5*x^5+48
*a^4*x^4*(a^2*x^2+1)^(1/2)+38*a^3*x^3+18*a^2*x^2*(a^2*x^2+1)^(1/2)+6*a*x+(a
^2*x^2+1)^(1/2))*(36*arcsinh(a*x)^3-18*arcsinh(a*x)^2+6*arcsinh(a*x)-1)*c^2
/a/(a^2*x^2+1)+3/4096*(c*(a^2*x^2+1))^(1/2)*(8*a^5*x^5+8*a^4*x^4*(a^2*x^2+1
)^(1/2)+12*a^3*x^3+8*a^2*x^2*(a^2*x^2+1)^(1/2)+4*a*x+(a^2*x^2+1)^(1/2))*(32
*arcsinh(a*x)^3-24*arcsinh(a*x)^2+12*arcsinh(a*x)-3)*c^2/a/(a^2*x^2+1)+15/5
12*(c*(a^2*x^2+1))^(1/2)*(2*a^3*x^3+2*a^2*x^2*(a^2*x^2+1)^(1/2)+2*a*x+(a^2*
x^2+1)^(1/2))*(4*arcsinh(a*x)^3-6*arcsinh(a*x)^2+6*arcsinh(a*x)-3)*c^2/a/(a
^2*x^2+1)+15/512*(c*(a^2*x^2+1))^(1/2)*(2*a^3*x^3-2*a^2*x^2*(a^2*x^2+1)^(1/
2)+2*a*x-(a^2*x^2+1)^(1/2))*(4*arcsinh(a*x)^3+6*arcsinh(a*x)^2+6*arcsinh(a*
x)+3)*c^2/a/(a^2*x^2+1)+3/4096*(c*(a^2*x^2+1))^(1/2)*(8*a^5*x^5-8*a^4*x^4*(
a^2*x^2+1)^(1/2)+12*a^3*x^3-8*a^2*x^2*(a^2*x^2+1)^(1/2)+4*a*x-(a^2*x^2+1)^(
1/2))*(32*arcsinh(a*x)^3+24*arcsinh(a*x)^2+12*arcsinh(a*x)+3)*c^2/a/(a^2*x^
2+1)+1/13824*(c*(a^2*x^2+1))^(1/2)*(32*a^7*x^7-32*x^6*a^6*(a^2*x^2+1)^(1/2)
+64*a^5*x^5-48*a^4*x^4*(a^2*x^2+1)^(1/2)+38*a^3*x^3-18*a^2*x^2*(a^2*x^2+1)^
(1/2)+6*a*x-(a^2*x^2+1)^(1/2))*(36*arcsinh(a*x)^3+18*arcsinh(a*x)^2+6*arcsi
nh(a*x)+1)*c^2/a/(a^2*x^2+1)
```

Fricas [F]

$$\int \left(c+a^2cx^2\right)^{5/2}\operatorname{arcsinh}(ax)^3\,dx=\int \left(a^2cx^2+c\right)^{\frac{5}{2}}\operatorname{arsinh}\left(ax\right)^3\,dx$$

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arcsinh(a*
x)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{5/2}\text{arcsinh}(ax)^3\,dx=\text{Timed out}$$

```
[In] integrate((a**2*c*x**2+c)**(5/2)*asinh(a*x)**3,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{5/2}\text{arcsinh}(ax)^3\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arcsinh(a*x)^3,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{5/2}\text{arcsinh}(ax)^3\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arcsinh(a*x)^3,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{5/2}\text{arcsinh}(ax)^3\,dx=\int \text{asinh}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^{5/2}\,dx$$

```
[In] int(asinh(a*x)^3*(c + a^2*c*x^2)^(5/2),x)
[Out] int(asinh(a*x)^3*(c + a^2*c*x^2)^(5/2), x)
```

3.335 $\int (c+a^2cx^2)^{3/2} \operatorname{arcsinh}(ax)^3 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 348

$$\int (c+a^2cx^2)^{3/2} \operatorname{arcsinh}(ax)^3 \, dx = -\frac{51acx^2\sqrt{c+a^2cx^2}}{128\sqrt{1+a^2x^2}} - \frac{3a^3cx^4\sqrt{c+a^2cx^2}}{128\sqrt{1+a^2x^2}} + \frac{45}{64}cx\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax) + \frac{3}{32}cx(1+a^2x^2)\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax) - \frac{27c\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{128a\sqrt{1+a^2x^2}} - \frac{9acx^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{16\sqrt{1+a^2x^2}} - \frac{3c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{16a} + \frac{3}{8}cx\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^3 + \frac{1}{4}x(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^3 + \frac{3c\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^4}{32a\sqrt{1+a^2x^2}}$$

```
[Out] 1/4*x*(a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3+45/64*c*x*arcsinh(a*x)*(a^2*c*x^2+
c)^(1/2)+3/32*c*x*(a^2*x^2+1)*arcsinh(a*x)*(a^2*c*x^2+c)^(1/2)-3/16*c*(a^2*
x^2+1)^(3/2)*arcsinh(a*x)^2*(a^2*c*x^2+c)^(1/2)/a+3/8*c*x*arcsinh(a*x)^3*(a
^2*c*x^2+c)^(1/2)-51/128*a*c*x^2*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1/2)-3/12
8*a^3*c*x^4*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1/2)-27/128*c*arcsinh(a*x)^2*(
a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-9/16*a*c*x^2*arcsinh(a*x)^2*(a^2*c*x
^2+c)^(1/2)/(a^2*x^2+1)^(1/2)+3/32*c*arcsinh(a*x)^4*(a^2*c*x^2+c)^(1/2)/a/(
a^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {5786, 5785, 5783, 5776, 5812, 30, 5798, 14}

$$\begin{aligned}\int \left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^3\,dx &= -\frac{9acx^2\text{arcsinh}(ax)^2\sqrt{a^2cx^2+c}}{16\sqrt{a^2x^2+1}} \\ &+\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}+\frac{3}{8}cx\text{arcsinh}(ax)^3\sqrt{a^2cx^2+c} \\ &+\frac{45}{64}cx\text{arcsinh}(ax)\sqrt{a^2cx^2+c}+\frac{3}{32}cx\left(a^2x^2+1\right)\text{arcsinh}(ax)\sqrt{a^2cx^2+c} \\ &+\frac{3c\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{32a\sqrt{a^2x^2+1}}-\frac{3c(a^2x^2+1)^{3/2}\text{arcsinh}(ax)^2\sqrt{a^2cx^2+c}}{16a} \\ &-\frac{27c\text{arcsinh}(ax)^2\sqrt{a^2cx^2+c}}{128a\sqrt{a^2x^2+1}}-\frac{51acx^2\sqrt{a^2cx^2+c}}{128\sqrt{a^2x^2+1}}-\frac{3a^3cx^4\sqrt{a^2cx^2+c}}{128\sqrt{a^2x^2+1}}\end{aligned}$$

```
[In] Int[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^3,x]
```

```
[Out] (-51*a*c*x^2*Sqrt[c + a^2*c*x^2])/(128*Sqrt[1 + a^2*x^2]) - (3*a^3*c*x^4*Sq
rt[c + a^2*c*x^2])/(128*Sqrt[1 + a^2*x^2]) + (45*c*x*Sqrt[c + a^2*c*x^2]*Ar
cSinh[a*x])/64 + (3*c*x*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x])/32
- (27*c*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(128*a*Sqrt[1 + a^2*x^2]) - (9*
a*c*x^2*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(16*Sqrt[1 + a^2*x^2]) - (3*c*(
1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(16*a) + (3*c*x*Sqrt
[c + a^2*c*x^2]*ArcSinh[a*x]^3)/8 + (x*(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^3
)/4 + (3*c*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^4)/(32*a*Sqrt[1 + a^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^3+\frac{1}{4}(3c)\int\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^3\,dx \\
&\quad -\frac{\left(3ac\sqrt{c+a^2cx^2}\right)\int x(1+a^2x^2)\operatorname{arcsinh}(ax)^2\,dx}{4\sqrt{1+a^2x^2}} \\
&= -\frac{3c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{16a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^3 \\
&\quad +\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^3+\frac{\left(3c\sqrt{c+a^2cx^2}\right)\int(1+a^2x^2)^{3/2}\operatorname{arcsinh}(ax)\,dx}{8\sqrt{1+a^2x^2}} \\
&\quad +\frac{\left(3c\sqrt{c+a^2cx^2}\right)\int\frac{\operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{8\sqrt{1+a^2x^2}}-\frac{\left(9ac\sqrt{c+a^2cx^2}\right)\int x\operatorname{arcsinh}(ax)^2\,dx}{8\sqrt{1+a^2x^2}} \\
&= \frac{3}{32}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)-\frac{9acx^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{16\sqrt{1+a^2x^2}} \\
&\quad -\frac{3c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{16a} \\
&\quad +\frac{3}{8}cx\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^3+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^3 \\
&\quad +\frac{3c\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^4}{32a\sqrt{1+a^2x^2}}+\frac{\left(9c\sqrt{c+a^2cx^2}\right)\int\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)\,dx}{32\sqrt{1+a^2x^2}} \\
&\quad -\frac{\left(3ac\sqrt{c+a^2cx^2}\right)\int x(1+a^2x^2)\,dx}{32\sqrt{1+a^2x^2}}+\frac{\left(9a^2c\sqrt{c+a^2cx^2}\right)\int\frac{x^2\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{8\sqrt{1+a^2x^2}} \\
&= \frac{45}{64}cx\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)+\frac{3}{32}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax) \\
&\quad -\frac{9acx^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{16\sqrt{1+a^2x^2}}-\frac{3c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{16a} \\
&\quad +\frac{3}{8}cx\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^3+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^3 \\
&\quad +\frac{3c\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^4}{32a\sqrt{1+a^2x^2}}+\frac{\left(9c\sqrt{c+a^2cx^2}\right)\int\frac{\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{64\sqrt{1+a^2x^2}} \\
&\quad -\frac{\left(9c\sqrt{c+a^2cx^2}\right)\int\frac{\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{16\sqrt{1+a^2x^2}}-\frac{\left(3ac\sqrt{c+a^2cx^2}\right)\int\left(x+a^2x^3\right)\,dx}{32\sqrt{1+a^2x^2}} \\
&\quad -\frac{\left(9ac\sqrt{c+a^2cx^2}\right)\int x\,dx}{64\sqrt{1+a^2x^2}}-\frac{\left(9ac\sqrt{c+a^2cx^2}\right)\int x\,dx}{16\sqrt{1+a^2x^2}}
\end{aligned}$$

$$= -\frac{51acx^2\sqrt{c+a^2cx^2}}{128\sqrt{1+a^2x^2}} - \frac{3a^3cx^4\sqrt{c+a^2cx^2}}{128\sqrt{1+a^2x^2}} + \frac{45}{64}cx\sqrt{c+a^2cx^2}\text{arcsinh}(ax) + \frac{3}{32}cx(1+a^2x^2)\sqrt{c+a^2cx^2}\text{arcsinh}(ax) - \frac{27c\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^2}{128a\sqrt{1+a^2x^2}} - \frac{9acx^2\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^2}{16\sqrt{1+a^2x^2}} - \frac{3c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^2}{16a} + \frac{3}{8}cx\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^3 + \frac{1}{4}x(c+a^2cx^2)^{3/2}\text{arcsinh}(ax)^3 + \frac{3c\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^4}{32a\sqrt{1+a^2x^2}}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.39

$$\int (c+a^2cx^2)^{3/2}\text{arcsinh}(ax)^3\,dx = \frac{c\sqrt{c+a^2cx^2}(96\text{arcsinh}(ax)^4 - 24\text{arcsinh}(ax)^2(16\cosh(2\text{arcsinh}(ax)) + \cosh($$

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^3,x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(96*ArcSinh[a*x]^4 - 24*ArcSinh[a*x]^2*(16*Cosh[2*Ar
cSinh[a*x]] + Cosh[4*ArcSinh[a*x]]) - 3*(64*Cosh[2*ArcSinh[a*x]] + Cosh[4*A
rcSinh[a*x]]) + 32*ArcSinh[a*x]^3*(8*Sinh[2*ArcSinh[a*x]] + Sinh[4*ArcSinh[
a*x]]) + 12*ArcSinh[a*x]*(32*Sinh[2*ArcSinh[a*x]] + Sinh[4*ArcSinh[a*x]])))
/(1024*a*Sqrt[1 + a^2*x^2])
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.39

method	result
default	$\frac{3\sqrt{c(a^2x^2+1)}\,\text{arcsinh}(ax)^4c}{32\sqrt{a^2x^2+1}\,a} + \frac{\sqrt{c(a^2x^2+1)}\left(8a^5x^5+8a^4x^4\sqrt{a^2x^2+1}+12a^3x^3+8a^2x^2\sqrt{a^2x^2+1}+4ax+\sqrt{a^2x^2+1}\right)\left(32\,\text{arcsinh}(a\right.}{2048(a^2x^2+1)a}$

```
[In] int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/32*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a*arcsinh(a*x)^4*c+1/2048*(c*(
a^2*x^2+1))^(1/2)*(8*a^5*x^5+8*a^4*x^4*(a^2*x^2+1)^(1/2)+12*a^3*x^3+8*a^2*x
^2*(a^2*x^2+1)^(1/2)+4*a*x+(a^2*x^2+1)^(1/2))*(32*arcsinh(a*x)^3-24*arcsinh
(a*x)^2+12*arcsinh(a*x)-3)*c/(a^2*x^2+1)/a+1/32*(c*(a^2*x^2+1))^(1/2)*(2*a^
3*x^3+2*a^2*x^2*(a^2*x^2+1)^(1/2)+2*a*x+(a^2*x^2+1)^(1/2))*(4*arcsinh(a*x)^
3-6*arcsinh(a*x)^2+6*arcsinh(a*x)-3)*c/(a^2*x^2+1)/a+1/32*(c*(a^2*x^2+1))^(
```

```
1/2)*(2*a^3*x^3-2*a^2*x^2*(a^2*x^2+1)^(1/2)+2*a*x-(a^2*x^2+1)^(1/2))*(4*arc
sinh(a*x)^3+6*arcsinh(a*x)^2+6*arcsinh(a*x)+3)*c/(a^2*x^2+1)/a+1/2048*(c*(a
^2*x^2+1))^(1/2)*(8*a^5*x^5-8*a^4*x^4*(a^2*x^2+1)^(1/2)+12*a^3*x^3-8*a^2*x^
2*(a^2*x^2+1)^(1/2)+4*a*x-(a^2*x^2+1)^(1/2))*(32*arcsinh(a*x)^3+24*arcsinh(
a*x)^2+12*arcsinh(a*x)+3)*c/(a^2*x^2+1)/a
```

Fricas [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^3\,dx=\int \left(a^2cx^2+c\right)^{\frac{3}{2}}\operatorname{arsinh}\left(ax\right)^3\,dx$$

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3,x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^3, x)
```

Sympy [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^3\,dx=\int \left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{asinh}^3\left(ax\right)dx$$

```
[In] integrate((a**2*c*x**2+c)**(3/2)*asinh(a*x)**3,x)
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*asinh(a*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^3\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2} \operatorname{arcsinh}(ax)^3 \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{3/2} \operatorname{arcsinh}(ax)^3 \, dx = \int \operatorname{asinh}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^{3/2} \,dx$$

```
[In] int(asinh(a*x)^3*(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(asinh(a*x)^3*(c + a^2*c*x^2)^(3/2), x)
```

3.336 $\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 205

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^3\,dx = -\frac{3ax^2\sqrt{c+a^2cx^2}}{8\sqrt{1+a^2x^2}}+\frac{3}{4}x\sqrt{c+a^2cx^2}\text{arcsinh}(ax)-\frac{3\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^2}{8a\sqrt{1+a^2x^2}}-\frac{3ax^2\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^2}{4\sqrt{1+a^2x^2}}+\frac{1}{2}x\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^3+\frac{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^4}{8a\sqrt{1+a^2x^2}}$$

```
[Out] 3/4*x*arcsinh(a*x)*(a^2*c*x^2+c)^(1/2)+1/2*x*arcsinh(a*x)^3*(a^2*c*x^2+c)^(
1/2)-3/8*a*x^2*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1/2)-3/8*arcsinh(a*x)^2*(a^
2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-3/4*a*x^2*arcsinh(a*x)^2*(a^2*c*x^2+c)
^(1/2)/(a^2*x^2+1)^(1/2)+1/8*arcsinh(a*x)^4*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+
1)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5785, 5783, 5776, 5812, 30}

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^3\,dx = \frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a^2cx^2+c}-\frac{3ax^2\text{arcsinh}(ax)^2\sqrt{a^2cx^2+c}}{4\sqrt{a^2x^2+1}}-\frac{3\text{arcsinh}(ax)^2\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{3}{4}x\text{arcsinh}(ax)\sqrt{a^2cx^2+c}-\frac{3ax^2\sqrt{a^2cx^2+c}}{8\sqrt{a^2x^2+1}}$$

```
[In] Int[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^3,x]
```

```
[Out] (-3*a*x^2*Sqrt[c + a^2*c*x^2])/(8*Sqrt[1 + a^2*x^2]) + (3*x*Sqrt[c + a^2*c*
x^2]*ArcSinh[a*x])/4 - (3*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(8*a*Sqrt[1 +
 a^2*x^2]) - (3*a*x^2*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(4*Sqrt[1 + a^2*x
^2]) + (x*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^3)/2 + (Sqrt[c + a^2*c*x^2]*ArcS
inh[a*x]^4)/(8*a*Sqrt[1 + a^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^3+\frac{\sqrt{c+a^2cx^2}\int\frac{\operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{2\sqrt{1+a^2x^2}} \\
&\quad-\frac{\left(3a\sqrt{c+a^2cx^2}\right)\int x\operatorname{arcsinh}(ax)^2\,dx}{2\sqrt{1+a^2x^2}} \\
&= -\frac{3ax^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{4\sqrt{1+a^2x^2}}+\frac{1}{2}x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^3 \\
&\quad+\frac{\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^4}{8a\sqrt{1+a^2x^2}}+\frac{\left(3a^2\sqrt{c+a^2cx^2}\right)\int\frac{x^2\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{2\sqrt{1+a^2x^2}} \\
&= \frac{3}{4}x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)-\frac{3ax^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{4\sqrt{1+a^2x^2}}+\frac{1}{2}x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^3 \\
&\quad+\frac{\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^4}{8a\sqrt{1+a^2x^2}}-\frac{\left(3\sqrt{c+a^2cx^2}\right)\int\frac{\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{4\sqrt{1+a^2x^2}}-\frac{\left(3a\sqrt{c+a^2cx^2}\right)\int x\,dx}{4\sqrt{1+a^2x^2}} \\
&= -\frac{3ax^2\sqrt{c+a^2cx^2}}{8\sqrt{1+a^2x^2}}+\frac{3}{4}x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)-\frac{3\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{8a\sqrt{1+a^2x^2}} \\
&\quad-\frac{3ax^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{4\sqrt{1+a^2x^2}}+\frac{1}{2}x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^3 \\
&\quad+\frac{\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^4}{8a\sqrt{1+a^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.42

$$
\int\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^3\,dx
= \frac{\sqrt{c\left(1+a^2x^2\right)}\left(-3\left(1+2\operatorname{arcsinh}(ax)^2\right)\cosh(2\operatorname{arcsinh}(ax))+2\operatorname{arcsinh}(ax)\left(\operatorname{arcsinh}(ax)^3+\left(3+2\operatorname{arcsinh}(a\right.\right.\right.}{16a\sqrt{1+a^2x^2}}
$$

```
[In] Integrate[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^3,x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(-3*(1 + 2*ArcSinh[a*x]^2)*Cosh[2*ArcSinh[a*x]] + 2*
ArcSinh[a*x]*(ArcSinh[a*x]^3 + (3 + 2*ArcSinh[a*x]^2)*Sinh[2*ArcSinh[a*x]])
))/(16*a*Sqrt[1 + a^2*x^2])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.13

method	result
default	$\frac{\sqrt{c(a^2x^2+1)}\,\operatorname{arcsinh}(ax)^4}{8\sqrt{a^2x^2+1}\,a}+\frac{\sqrt{c(a^2x^2+1)}\left(2a^3x^3+2a^2x^2\sqrt{a^2x^2+1}+2ax+\sqrt{a^2x^2+1}\right)\left(4\operatorname{arcsinh}(ax)^3-6\operatorname{arcsinh}(ax)^2+6\operatorname{arcsinh}\right.}{32(a^2x^2+1)a}$

[In] int(arcsinh(a*x)^3*(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/8*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a*arcsinh(a*x)^4+1/32*(c*(a^2*x^2+1))^(1/2)*(2*a^3*x^3+2*a^2*x^2*(a^2*x^2+1)^(1/2)+2*a*x+(a^2*x^2+1)^(1/2))*(4*arcsinh(a*x)^3-6*arcsinh(a*x)^2+6*arcsinh(a*x)-3)/(a^2*x^2+1)/a+1/32*(c*(a^2*x^2+1))^(1/2)*(2*a^3*x^3-2*a^2*x^2*(a^2*x^2+1)^(1/2)+2*a*x-(a^2*x^2+1)^(1/2))*(4*arcsinh(a*x)^3+6*arcsinh(a*x)^2+6*arcsinh(a*x)+3)/(a^2*x^2+1)/a

Fricas [F]

$$\int \sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^3\,dx=\int \sqrt{a^2cx^2+c}\,\operatorname{arsinh}\left(ax\right)^3\,dx$$

[In] integrate(arcsinh(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^3, x)

Sympy [F]

$$\int \sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^3\,dx=\int \sqrt{c\left(a^2x^2+1\right)}\operatorname{asinh}^3\left(ax\right)dx$$

[In] integrate(asinh(a*x)**3*(a**2*c*x**2+c)**(1/2),x)

[Out] Integral(sqrt(c*(a**2*x**2 + 1))*asinh(a*x)**3, x)

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{c + a^2cx^2}\text{arcsinh}(ax)^3\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(arcsinh(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{c + a^2cx^2}\text{arcsinh}(ax)^3\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c + a^2cx^2}\text{arcsinh}(ax)^3\,dx = \int \text{asinh}(a\,x)^3\,\sqrt{c\,a^2\,x^2 + c}\,dx$$

```
[In] int(asinh(a*x)^3*(c + a^2*c*x^2)^(1/2),x)
[Out] int(asinh(a*x)^3*(c + a^2*c*x^2)^(1/2), x)
```

3.337 $\int \frac{\operatorname{arcsinh}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 40

$$\int \frac{\operatorname{arcsinh}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^4}{4a\sqrt{c+a^2cx^2}}$$

```
[Out] 1/4*arcsinh(a*x)^4*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5783}

$$\int \frac{\operatorname{arcsinh}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^4}{4a\sqrt{a^2cx^2+c}}$$

```
[In] Int[ArcSinh[a*x]^3/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^4)/(4*a*Sqrt[c + a^2*c*x^2])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^4}{4a\sqrt{c+a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{\text{arcsinh}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx=\frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^4}{4a\sqrt{c\left(1+a^2x^2\right)}}$$

```
[In] Integrate[ArcSinh[a*x]^3/Sqrt[c + a^2*c*x^2],x]
[Out] (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^4)/(4*a*Sqrt[c*(1 + a^2*x^2)])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{\sqrt{c\left(a^2x^2+1\right)}\,\text{arcsinh}(ax)^4}{4\sqrt{a^2x^2+1}\,ac}$	39

```
[In] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/4*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c*arcsinh(a*x)^4
```

Fricas [F]

$$\int \frac{\text{arcsinh}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{\text{arsinh}\,(ax)^3}{\sqrt{a^2cx^2+c}}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(arcsinh(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]

$$\int \frac{\text{arcsinh}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{\text{asinh}^3\,(ax)}{\sqrt{c\,(a^2x^2+1)}}\,dx$$

```
[In] integrate(asinh(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(asinh(a*x)**3/sqrt(c*(a**2*x**2 + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.35

$$\int \frac{\operatorname{arcsinh}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \frac{\operatorname{arsinh}(ax)^4}{4\,a\sqrt{c}}$$

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] 1/4*arcsinh(a*x)^4/(a*sqrt(c))
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{\sqrt{a^2cx^2+c}}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)^3}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

```
[In] int(asinh(a*x)^3/(c + a^2*c*x^2)^(1/2),x)
[Out] int(asinh(a*x)^3/(c + a^2*c*x^2)^(1/2), x)
```

3.338 $\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 218

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx &= \frac{x\operatorname{arcsinh}(ax)^3}{c\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{ac\sqrt{c+a^2cx^2}} \\ &\quad - \frac{3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2\log\left(1+e^{2\operatorname{arcsinh}(ax)}\right)}{ac\sqrt{c+a^2cx^2}} \\ &\quad - \frac{3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(ax)}\right)}{ac\sqrt{c+a^2cx^2}} \\ &\quad + \frac{3\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{2\operatorname{arcsinh}(ax)}\right)}{2ac\sqrt{c+a^2cx^2}}\end{aligned}$$

```
[Out] x*arcsinh(a*x)^3/c/(a^2*c*x^2+c)^(1/2)+arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a/c
/(a^2*c*x^2+c)^(1/2)-3*arcsinh(a*x)^2*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)*(a^2*
x^2+1)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)-3*arcsinh(a*x)*polylog(2,-(a*x+(a^2*x^
2+1)^(1/2))^2)*(a^2*x^2+1)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)+3/2*polylog(3,-(a*
x+(a^2*x^2+1)^(1/2))^2)*(a^2*x^2+1)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

= {5787, 5797, 3799, 2221, 2611, 2320, 6724}

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{3\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(ax)}\right)}{ac\sqrt{a^2cx^2+c}} + \frac{3\sqrt{a^2x^2+1}\operatorname{PolyLog}\left(3,-e^{2\operatorname{arcsinh}(ax)}\right)}{2ac\sqrt{a^2cx^2+c}} + \frac{x\operatorname{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^3}{ac\sqrt{a^2cx^2+c}} - \frac{3\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^2\log\left(e^{2\operatorname{arcsinh}(ax)}+1\right)}{ac\sqrt{a^2cx^2+c}}$$

```
[In] Int[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*ArcSinh[a*x]^3)/(c*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x
]^3)/(a*c*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2*Log[1
+ E^(2*ArcSinh[a*x])])/(a*c*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[1 + a^2*x^2]*Arc
Sinh[a*x]*PolyLog[2, -E^(2*ArcSinh[a*x])])/(a*c*Sqrt[c + a^2*c*x^2]) + (3*S
qrt[1 + a^2*x^2]*PolyLog[3, -E^(2*ArcSinh[a*x])])/(2*a*c*Sqrt[c + a^2*c*x^2
])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
```

```
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x\text{arcsinh}(ax)^3}{c\sqrt{c+a^2cx^2}} - \frac{\left(3a\sqrt{1+a^2x^2}\right)\int \frac{x\text{arcsinh}(ax)^2}{1+a^2x^2}\,dx}{c\sqrt{c+a^2cx^2}} \\
&= \frac{x\text{arcsinh}(ax)^3}{c\sqrt{c+a^2cx^2}} - \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x^2\tanh(x)\,dx, x, \text{arcsinh}(ax)\right)}{ac\sqrt{c+a^2cx^2}} \\
&= \frac{x\text{arcsinh}(ax)^3}{c\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{ac\sqrt{c+a^2cx^2}} - \frac{\left(6\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{e^{2x}x^2}{1+e^{2x}}\,dx, x, \text{arcsinh}(ax)\right)}{ac\sqrt{c+a^2cx^2}} \\
&= \frac{x\text{arcsinh}(ax)^3}{c\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{ac\sqrt{c+a^2cx^2}} \\
&\quad - \frac{3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2\log\left(1+e^{2\text{arcsinh}(ax)}\right)}{ac\sqrt{c+a^2cx^2}} \\
&\quad + \frac{\left(6\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x\log\left(1+e^{2x}\right)\,dx, x, \text{arcsinh}(ax)\right)}{ac\sqrt{c+a^2cx^2}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{x\text{arcsinh}(ax)^3}{c\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{ac\sqrt{c+a^2cx^2}} \\
&\quad - \frac{3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2 \log\left(1+e^{2\text{arcsinh}(ax)}\right)}{ac\sqrt{c+a^2cx^2}} \\
&\quad - \frac{3\sqrt{1+a^2x^2}\text{arcsinh}(ax) \operatorname{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)}{ac\sqrt{c+a^2cx^2}} \\
&\quad + \frac{\left(3\sqrt{1+a^2x^2}\right) \operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2,-e^{2x}\right) dx, x, \text{arcsinh}(ax)\right)}{ac\sqrt{c+a^2cx^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x\text{arcsinh}(ax)^3}{c\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{ac\sqrt{c+a^2cx^2}} \\
&\quad - \frac{3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2 \log\left(1+e^{2\text{arcsinh}(ax)}\right)}{ac\sqrt{c+a^2cx^2}} \\
&\quad - \frac{3\sqrt{1+a^2x^2}\text{arcsinh}(ax) \operatorname{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)}{ac\sqrt{c+a^2cx^2}} \\
&\quad + \frac{\left(3\sqrt{1+a^2x^2}\right) \operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,-x)}{x} dx, x, e^{2\text{arcsinh}(ax)}\right)}{2ac\sqrt{c+a^2cx^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x\text{arcsinh}(ax)^3}{c\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{ac\sqrt{c+a^2cx^2}} \\
&\quad - \frac{3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2 \log\left(1+e^{2\text{arcsinh}(ax)}\right)}{ac\sqrt{c+a^2cx^2}} \\
&\quad - \frac{3\sqrt{1+a^2x^2}\text{arcsinh}(ax) \operatorname{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)}{ac\sqrt{c+a^2cx^2}} \\
&\quad + \frac{3\sqrt{1+a^2x^2} \operatorname{PolyLog}\left(3,-e^{2\text{arcsinh}(ax)}\right)}{2ac\sqrt{c+a^2cx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.61

$$
\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^{3/2}} dx = \frac{2ax\text{arcsinh}(ax)^3 - 2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2 \left(\text{arcsinh}(ax) + 3\log\left(1+e^{-2\text{arcsinh}(ax)}\right)\right)}{2ac}
$$

```
[In] Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (2*a*x*ArcSinh[a*x]^3 - 2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2*(ArcSinh[a*x] +
3*Log[1 + E^(-2*ArcSinh[a*x])]) + 6*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]*PolyLog[
2, -E^(-2*ArcSinh[a*x])] + 3*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(-2*ArcSinh[a*
x])])/(2*a*c*Sqrt[c*(1 + a^2*x^2)])
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.20

method	result
default	$\frac{\sqrt{c(a^2x^2+1)}\left(ax-\sqrt{a^2x^2+1}\right)\operatorname{arcsinh}(ax)^3}{a\,c^2(a^2x^2+1)}+\frac{2\sqrt{c(a^2x^2+1)}\,\operatorname{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}\,a\,c^2}-\frac{3\sqrt{c(a^2x^2+1)}\,\operatorname{arcsinh}(ax)^2\ln\left(1+\left(ax+\sqrt{a^2x^2+1}\right)^2\right.}{\sqrt{a^2x^2+1}\,a\,c^2}$

```
[In] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (c*(a^2*x^2+1))^(1/2)*(a*x-(a^2*x^2+1)^(1/2))*arcsinh(a*x)^3/a/c^2/(a^2*x^2
+1)+2/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^2*arcsinh(a*x)^3-3/(a^2*x
^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^2*arcsinh(a*x)^2*ln(1+(a*x+(a^2*x^2+1
)^(1/2))^2)-3/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^2*arcsinh(a*x)*po
lylog(2,-(a*x+(a^2*x^2+1)^(1/2))^2)+3/2/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(
1/2)/a/c^2*polylog(3,-(a*x+(a^2*x^2+1)^(1/2))^2)
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx=\int \frac{\operatorname{arsinh}\left(ax\right)^3}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 +
c^2), x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx=\int \frac{\operatorname{asinh}^3\left(ax\right)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(asinh(a*x)**3/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(asinh(a*x)**3/(c*(a**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)^3}{\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

```
[In] int(asinh(a*x)^3/(c + a^2*c*x^2)^(3/2),x)
[Out] int(asinh(a*x)^3/(c + a^2*c*x^2)^(3/2), x)
```

3.339 $\int \frac{\textbf{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 363

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = &-\frac{x\operatorname{arcsinh}(ax)}{c^2\sqrt{c+a^2cx^2}}+\frac{\operatorname{arcsinh}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}\\
&+\frac{x\operatorname{arcsinh}(ax)^3}{3c\,(c+a^2cx^2)^{3/2}}+\frac{2x\operatorname{arcsinh}(ax)^3}{3c^2\sqrt{c+a^2cx^2}}+\frac{2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{3ac^2\sqrt{c+a^2cx^2}}\\
&-\frac{2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2\log\left(1+e^{2\operatorname{arcsinh}(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\log\left(1+a^2x^2\right)}{2ac^2\sqrt{c+a^2cx^2}}\\
&-\frac{2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}}\\
&+\frac{\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{2\operatorname{arcsinh}(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}}\end{aligned}$$

```
[Out] 1/3*x*arcsinh(a*x)^3/c/(a^2*c*x^2+c)^(3/2)-x*arcsinh(a*x)/c^2/(a^2*c*x^2+c)
^(1/2)+2/3*x*arcsinh(a*x)^3/c^2/(a^2*c*x^2+c)^(1/2)+1/2*arcsinh(a*x)^2/a/c^
2/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+2/3*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2
)/a/c^2/(a^2*c*x^2+c)^(1/2)-2*arcsinh(a*x)^2*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2
)*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)+1/2*ln(a^2*x^2+1)*(a^2*x^2+1)
^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)-2*arcsinh(a*x)*polylog(2,-(a*x+(a^2*x^2+1)
^(1/2))^2)*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)+polylog(3,-(a*x+(a^2
*x^2+1)^(1/2))^2)*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5788, 5787, 5797, 3799, 2221, 2611, 2320, 6724, 5798, 266}

$$\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{2\sqrt{a^2x^2+1}\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)}{ac^2\sqrt{a^2cx^2+c}} + \frac{\sqrt{a^2x^2+1}\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(ax)}\right)}{ac^2\sqrt{a^2cx^2+c}} + \frac{2x\text{arcsinh}(ax)^3}{3c^2\sqrt{a^2cx^2+c}} + \frac{2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{3ac^2\sqrt{a^2cx^2+c}} + \frac{\text{arcsinh}(ax)^2}{2ac^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}} - \frac{x\text{arcsinh}(ax)}{c^2\sqrt{a^2cx^2+c}} - \frac{2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2\log\left(e^{2\text{arcsinh}(ax)}+1\right)}{ac^2\sqrt{a^2cx^2+c}} + \frac{x\text{arcsinh}(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}} + \frac{\sqrt{a^2x^2+1}\log\left(a^2x^2+1\right)}{2ac^2\sqrt{a^2cx^2+c}}$$

```
[In] Int[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] -((x*ArcSinh[a*x])/(c^2*Sqrt[c + a^2*c*x^2])) + ArcSinh[a*x]^2/(2*a*c^2*Sqr
t[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2]) + (x*ArcSinh[a*x]^3)/(3*c*(c + a^2*c*x^
2)^(3/2)) + (2*x*ArcSinh[a*x]^3)/(3*c^2*Sqrt[c + a^2*c*x^2]) + (2*Sqrt[1 +
a^2*x^2]*ArcSinh[a*x]^3)/(3*a*c^2*Sqrt[c + a^2*c*x^2]) - (2*Sqrt[1 + a^2*x^
2]*ArcSinh[a*x]^2*Log[1 + E^(2*ArcSinh[a*x])])/(a*c^2*Sqrt[c + a^2*c*x^2])
+ (Sqrt[1 + a^2*x^2]*Log[1 + a^2*x^2])/(2*a*c^2*Sqrt[c + a^2*c*x^2]) - (2*S
qrt[1 + a^2*x^2]*ArcSinh[a*x]*PolyLog[2, -E^(2*ArcSinh[a*x])])/(a*c^2*Sqrt[
c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(2*ArcSinh[a*x])])/(a*c^
2*Sqrt[c + a^2*c*x^2])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
```

```
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x\text{arcsinh}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}}+\frac{2\int \frac{\text{arcsinh}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{3c}-\frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x\text{arcsinh}(ax)^2}{\left(1+a^2x^2\right)^2}\,dx}{c^2\sqrt{c+a^2cx^2}} \\
&= \frac{\text{arcsinh}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}+\frac{x\text{arcsinh}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}}+\frac{2x\text{arcsinh}(ax)^3}{3c^2\sqrt{c+a^2cx^2}} \\
&\quad -\frac{\sqrt{1+a^2x^2}\int \frac{\text{arcsinh}(ax)}{\left(1+a^2x^2\right)^{3/2}}\,dx}{c^2\sqrt{c+a^2cx^2}}-\frac{\left(2a\sqrt{1+a^2x^2}\right)\int \frac{x\text{arcsinh}(ax)^2}{1+a^2x^2}\,dx}{c^2\sqrt{c+a^2cx^2}} \\
&= -\frac{x\text{arcsinh}(ax)}{c^2\sqrt{c+a^2cx^2}}+\frac{\text{arcsinh}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}+\frac{x\text{arcsinh}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}}+\frac{2x\text{arcsinh}(ax)^3}{3c^2\sqrt{c+a^2cx^2}} \\
&\quad -\frac{\left(2\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x^2\tanh(x)\,dx,x,\text{arcsinh}(ax)\right)}{ac^2\sqrt{c+a^2cx^2}}+\frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x}{1+a^2x^2}\,dx}{c^2\sqrt{c+a^2cx^2}} \\
&= -\frac{x\text{arcsinh}(ax)}{c^2\sqrt{c+a^2cx^2}}+\frac{\text{arcsinh}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}+\frac{x\text{arcsinh}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}} \\
&\quad +\frac{2x\text{arcsinh}(ax)^3}{3c^2\sqrt{c+a^2cx^2}}+\frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{3ac^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\log\left(1+a^2x^2\right)}{2ac^2\sqrt{c+a^2cx^2}} \\
&\quad -\frac{\left(4\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{e^{2x}x^2}{1+e^{2x}}\,dx,x,\text{arcsinh}(ax)\right)}{ac^2\sqrt{c+a^2cx^2}} \\
&= -\frac{x\text{arcsinh}(ax)}{c^2\sqrt{c+a^2cx^2}}+\frac{\text{arcsinh}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}+\frac{x\text{arcsinh}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}}+\frac{2x\text{arcsinh}(ax)^3}{3c^2\sqrt{c+a^2cx^2}} \\
&\quad +\frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{3ac^2\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2\log\left(1+e^{2\text{arcsinh}(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}} \\
&\quad +\frac{\sqrt{1+a^2x^2}\log\left(1+a^2x^2\right)}{2ac^2\sqrt{c+a^2cx^2}}+\frac{\left(4\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x\log\left(1+e^{2x}\right)\,dx,x,\text{arcsinh}(ax)\right)}{ac^2\sqrt{c+a^2cx^2}}
\end{aligned}$$

$$=-\frac{x\text{arcsinh}(ax)}{c^2\sqrt{c+a^2cx^2}}+\frac{\text{arcsinh}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}+\frac{x\text{arcsinh}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}}+\frac{2x\text{arcsinh}(ax)^3}{3c^2\sqrt{c+a^2cx^2}}+\frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{3ac^2\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2\log\left(1+e^{2\text{arcsinh}(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\log\left(1+a^2x^2\right)}{2ac^2\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}}+\frac{\left(2\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int\text{PolyLog}\left(2,-e^{2x}\right)\,dx,x,\text{arcsinh}(ax)\right)}{ac^2\sqrt{c+a^2cx^2}}$$

$$=-\frac{x\text{arcsinh}(ax)}{c^2\sqrt{c+a^2cx^2}}+\frac{\text{arcsinh}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}+\frac{x\text{arcsinh}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}}+\frac{2x\text{arcsinh}(ax)^3}{3c^2\sqrt{c+a^2cx^2}}+\frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{3ac^2\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2\log\left(1+e^{2\text{arcsinh}(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\log\left(1+a^2x^2\right)}{2ac^2\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\text{Subst}\left(\int\frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{2\text{arcsinh}(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}}$$

$$=-\frac{x\text{arcsinh}(ax)}{c^2\sqrt{c+a^2cx^2}}+\frac{\text{arcsinh}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}+\frac{x\text{arcsinh}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}}+\frac{2x\text{arcsinh}(ax)^3}{3c^2\sqrt{c+a^2cx^2}}+\frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{3ac^2\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2\log\left(1+e^{2\text{arcsinh}(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\log\left(1+a^2x^2\right)}{2ac^2\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.54

$$\int\frac{\text{arcsinh}(ax)^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx=\frac{\left(1+a^2x^2\right)^{3/2}\left(-\frac{6ax\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}+\frac{3\text{arcsinh}(ax)^2}{1+a^2x^2}-4\text{arcsinh}(ax)^3+\frac{2ax\text{arcsinh}(ax)^3}{\left(1+a^2x^2\right)^{3/2}}+\frac{4ax}{}\right.}{}$$

```
[In] Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(5/2),x]
[Out] ((1 + a^2*x^2)^(3/2)*((-6*a*x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2] + (3*ArcSinh[
a*x]^2)/(1 + a^2*x^2) - 4*ArcSinh[a*x]^3 + (2*a*x*ArcSinh[a*x]^3)/(1 + a^2*
x^2)^(3/2) + (4*a*x*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2] - 12*ArcSinh[a*x]^2*L
og[1 + E^(-2*ArcSinh[a*x])] + 3*Log[1 + a^2*x^2] + 12*ArcSinh[a*x]*PolyLog[
2, -E^(-2*ArcSinh[a*x])] + 6*PolyLog[3, -E^(-2*ArcSinh[a*x])]))/(6*a*c*(c +
 a^2*c*x^2)^(3/2))
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.52

method	result
default	$\frac{\sqrt{c(a^2x^2+1)}\left(2a^3x^3-2a^2x^2\sqrt{a^2x^2+1}+3ax-2\sqrt{a^2x^2+1}\right)\operatorname{arcsinh}(ax)\left(-6a^4x^4\operatorname{arcsinh}(ax)-6a^3x^3\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}-6a^4x^4-\right.}{6(3a^6x^6}$

```
[In] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(c*(a^2*x^2+1))^(1/2)*(2*a^3*x^3-2*a^2*x^2*(a^2*x^2+1)^(1/2)+3*a*x-2*(a
^2*x^2+1)^(1/2))*arcsinh(a*x)*(-6*a^4*x^4*arcsinh(a*x)-6*a^3*x^3*arcsinh(a*
x)*(a^2*x^2+1)^(1/2)-6*a^4*x^4-6*a^3*x^3*(a^2*x^2+1)^(1/2)+6*arcsinh(a*x)^2
*a^2*x^2-12*a^2*x^2*arcsinh(a*x)-9*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x-18*a^
2*x^2-6*a*x*(a^2*x^2+1)^(1/2)+8*arcsinh(a*x)^2-6*arcsinh(a*x)-12)/(3*a^6*x^
6+10*a^4*x^4+11*a^2*x^2+4)/a/c^3-2/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/
a/c^3*ln(a*x+(a^2*x^2+1)^(1/2))+1/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a
/c^3*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)+4/3/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^
(1/2)/a/c^3*arcsinh(a*x)^3-2/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^3*
arcsinh(a*x)^2*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)-2/(a^2*x^2+1)^(1/2)*(c*(a^2*
x^2+1))^(1/2)/a/c^3*arcsinh(a*x)*polylog(2,-(a*x+(a^2*x^2+1)^(1/2))^2)+1/(a
^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^3*polylog(3,-(a*x+(a^2*x^2+1)^(1/
2))^2)
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^3}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^3/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 +
3*a^2*c^3*x^2 + c^3), x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{asinh}^3\left(ax\right)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(asinh(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(asinh(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)^3}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

```
[In] int(asinh(a*x)^3/(c + a^2*c*x^2)^(5/2),x)
[Out] int(asinh(a*x)^3/(c + a^2*c*x^2)^(5/2), x)
```

3.340 $\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 515

$$
\begin{aligned}
\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{7/2}}\,dx &= -\frac{1}{20ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} - \frac{x\operatorname{arcsinh}(ax)}{c^3\sqrt{c+a^2cx^2}} \\
&- \frac{x\operatorname{arcsinh}(ax)}{10c^3(1+a^2x^2)\sqrt{c+a^2cx^2}} + \frac{3\operatorname{arcsinh}(ax)^2}{20ac^3(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} \\
&+ \frac{2\operatorname{arcsinh}(ax)^2}{5ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\operatorname{arcsinh}(ax)^3}{5c(c+a^2cx^2)^{5/2}} \\
&+ \frac{4x\operatorname{arcsinh}(ax)^3}{15c^2(c+a^2cx^2)^{3/2}} + \frac{8x\operatorname{arcsinh}(ax)^3}{15c^3\sqrt{c+a^2cx^2}} + \frac{8\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{15ac^3\sqrt{c+a^2cx^2}} \\
&- \frac{8\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2\log\left(1+e^{2\operatorname{arcsinh}(ax)}\right)}{5ac^3\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\log(1+a^2x^2)}{2ac^3\sqrt{c+a^2cx^2}} \\
&- \frac{8\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(ax)}\right)}{5ac^3\sqrt{c+a^2cx^2}} \\
&+ \frac{4\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{2\operatorname{arcsinh}(ax)}\right)}{5ac^3\sqrt{c+a^2cx^2}}
\end{aligned}
$$

```
[Out] 1/5*x*arcsinh(a*x)^3/c/(a^2*c*x^2+c)^(5/2)+4/15*x*arcsinh(a*x)^3/c^2/(a^2*c
*x^2+c)^(3/2)-x*arcsinh(a*x)/c^3/(a^2*c*x^2+c)^(1/2)-1/10*x*arcsinh(a*x)/c^
3/(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2)+3/20*arcsinh(a*x)^2/a/c^3/(a^2*x^2+1)^(3/
2)/(a^2*c*x^2+c)^(1/2)+8/15*x*arcsinh(a*x)^3/c^3/(a^2*c*x^2+c)^(1/2)-1/20/a
/c^3/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+2/5*arcsinh(a*x)^2/a/c^3/(a^2*x^
2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+8/15*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a/c^3/
(a^2*c*x^2+c)^(1/2)-8/5*arcsinh(a*x)^2*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)*(a^2
*x^2+1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)+1/2*ln(a^2*x^2+1)*(a^2*x^2+1)^(1/2)
/a/c^3/(a^2*c*x^2+c)^(1/2)-8/5*arcsinh(a*x)*polylog(2,-(a*x+(a^2*x^2+1)^(1/
```

```
2))^2)*(a^2*x^2+1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)+4/5*polylog(3,-(a*x+(a^2
*x^2+1)^(1/2))^2)*(a^2*x^2+1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {5788, 5787, 5797, 3799, 2221, 2611, 2320, 6724, 5798, 266, 267}

$$\begin{aligned}\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^{7/2}}\,dx = &-\frac{8\sqrt{a^2x^2+1}\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)}{5ac^3\sqrt{a^2cx^2+c}}\\ &+\frac{4\sqrt{a^2x^2+1}\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(ax)}\right)}{5ac^3\sqrt{a^2cx^2+c}}+\frac{8x\text{arcsinh}(ax)^3}{15c^3\sqrt{a^2cx^2+c}}\\ &+\frac{8\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{15ac^3\sqrt{a^2cx^2+c}}+\frac{2\text{arcsinh}(ax)^2}{5ac^3\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\\ &+\frac{3\text{arcsinh}(ax)^2}{20ac^3\left(a^2x^2+1\right)^{3/2}\sqrt{a^2cx^2+c}}-\frac{x\text{arcsinh}(ax)}{c^3\sqrt{a^2cx^2+c}}-\frac{x\text{arcsinh}(ax)}{10c^3\left(a^2x^2+1\right)\sqrt{a^2cx^2+c}}\\ &-\frac{8\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2\log\left(e^{2\text{arcsinh}(ax)}+1\right)}{5ac^3\sqrt{a^2cx^2+c}}+\frac{4x\text{arcsinh}(ax)^3}{15c^2\left(a^2cx^2+c\right)^{3/2}}\\ &+\frac{x\text{arcsinh}(ax)^3}{5c\left(a^2cx^2+c\right)^{5/2}}-\frac{1}{20ac^3\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\log\left(a^2x^2+1\right)}{2ac^3\sqrt{a^2cx^2+c}}\end{aligned}$$

```
[In] Int[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/20*1/(a*c^3*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2]) - (x*ArcSinh[a*x])/(c
^3*Sqrt[c + a^2*c*x^2]) - (x*ArcSinh[a*x])/(10*c^3*(1 + a^2*x^2)*Sqrt[c + a
^2*c*x^2]) + (3*ArcSinh[a*x]^2)/(20*a*c^3*(1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*
c*x^2]) + (2*ArcSinh[a*x]^2)/(5*a*c^3*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2]
) + (x*ArcSinh[a*x]^3)/(5*c*(c + a^2*c*x^2)^(5/2)) + (4*x*ArcSinh[a*x]^3)/(
15*c^2*(c + a^2*c*x^2)^(3/2)) + (8*x*ArcSinh[a*x]^3)/(15*c^3*Sqrt[c + a^2*c
*x^2]) + (8*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(15*a*c^3*Sqrt[c + a^2*c*x^2]
) - (8*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2*Log[1 + E^(2*ArcSinh[a*x])])/(5*a*c
^3*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2]*Log[1 + a^2*x^2])/(2*a*c^3*Sqr
t[c + a^2*c*x^2]) - (8*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]*PolyLog[2, -E^(2*ArcS
inh[a*x])])/(5*a*c^3*Sqrt[c + a^2*c*x^2]) + (4*Sqrt[1 + a^2*x^2]*PolyLog[3,
 -E^(2*ArcSinh[a*x])])/(5*a*c^3*Sqrt[c + a^2*c*x^2])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
```

```
1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x\text{arcsinh}(ax)^3}{5c\left(c+a^2cx^2\right)^{5/2}}+\frac{4\int\frac{\text{arcsinh}(ax)^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{5c}-\frac{\left(3a\sqrt{1+a^2x^2}\right)\int\frac{x\text{arcsinh}(ax)^2}{\left(1+a^2x^2\right)^3}\,dx}{5c^3\sqrt{c+a^2cx^2}}\\
&= \frac{3\text{arcsinh}(ax)^2}{20ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}}+\frac{x\text{arcsinh}(ax)^3}{5c\left(c+a^2cx^2\right)^{5/2}}\\
&\quad+\frac{4x\text{arcsinh}(ax)^3}{15c^2\left(c+a^2cx^2\right)^{3/2}}+\frac{8\int\frac{\text{arcsinh}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{15c^2}\\
&\quad-\frac{\left(3\sqrt{1+a^2x^2}\right)\int\frac{\text{arcsinh}(ax)}{\left(1+a^2x^2\right)^{5/2}}\,dx}{10c^3\sqrt{c+a^2cx^2}}-\frac{\left(4a\sqrt{1+a^2x^2}\right)\int\frac{x\text{arcsinh}(ax)^2}{\left(1+a^2x^2\right)^2}\,dx}{5c^3\sqrt{c+a^2cx^2}}
\end{aligned}
$$

$$= -\frac{x\text{arcsinh}(ax)}{10c^3(1+a^2x^2)\sqrt{c+a^2cx^2}} + \frac{3\text{arcsinh}(ax)^2}{20ac^3(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} + \frac{2\text{arcsinh}(ax)^2}{5ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\text{arcsinh}(ax)^3}{5c(c+a^2cx^2)^{5/2}} + \frac{4x\text{arcsinh}(ax)^3}{15c^2(c+a^2cx^2)^{3/2}} + \frac{8x\text{arcsinh}(ax)^3}{15c^3\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\int\frac{\text{arcsinh}(ax)}{(1+a^2x^2)^{3/2}}\,dx}{5c^3\sqrt{c+a^2cx^2}} - \frac{\left(4\sqrt{1+a^2x^2}\right)\int\frac{\text{arcsinh}(ax)}{(1+a^2x^2)^{3/2}}\,dx}{5c^3\sqrt{c+a^2cx^2}} + \frac{\left(a\sqrt{1+a^2x^2}\right)\int\frac{x}{(1+a^2x^2)^2}\,dx}{10c^3\sqrt{c+a^2cx^2}} - \frac{\left(8a\sqrt{1+a^2x^2}\right)\int\frac{x\text{arcsinh}(ax)^2}{1+a^2x^2}\,dx}{5c^3\sqrt{c+a^2cx^2}}$$

$$= -\frac{1}{20ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} - \frac{x\text{arcsinh}(ax)}{c^3\sqrt{c+a^2cx^2}} - \frac{x\text{arcsinh}(ax)}{10c^3(1+a^2x^2)\sqrt{c+a^2cx^2}} + \frac{3\text{arcsinh}(ax)^2}{20ac^3(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} + \frac{2\text{arcsinh}(ax)^2}{5ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\text{arcsinh}(ax)^3}{5c(c+a^2cx^2)^{5/2}} + \frac{4x\text{arcsinh}(ax)^3}{15c^2(c+a^2cx^2)^{3/2}} + \frac{8x\text{arcsinh}(ax)^3}{15c^3\sqrt{c+a^2cx^2}} - \frac{\left(8\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x^2\tanh(x)\,dx,x,\text{arcsinh}(ax)\right)}{5ac^3\sqrt{c+a^2cx^2}} + \frac{\left(a\sqrt{1+a^2x^2}\right)\int\frac{x}{1+a^2x^2}\,dx}{5c^3\sqrt{c+a^2cx^2}} + \frac{\left(4a\sqrt{1+a^2x^2}\right)\int\frac{x}{1+a^2x^2}\,dx}{5c^3\sqrt{c+a^2cx^2}}$$

$$= -\frac{1}{20ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} - \frac{x\text{arcsinh}(ax)}{c^3\sqrt{c+a^2cx^2}} - \frac{x\text{arcsinh}(ax)}{10c^3(1+a^2x^2)\sqrt{c+a^2cx^2}} + \frac{3\text{arcsinh}(ax)^2}{20ac^3(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} + \frac{2\text{arcsinh}(ax)^2}{5ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\text{arcsinh}(ax)^3}{5c(c+a^2cx^2)^{5/2}} + \frac{4x\text{arcsinh}(ax)^3}{15c^2(c+a^2cx^2)^{3/2}} + \frac{8x\text{arcsinh}(ax)^3}{15c^3\sqrt{c+a^2cx^2}} + \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{15ac^3\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\log(1+a^2x^2)}{2ac^3\sqrt{c+a^2cx^2}} - \frac{\left(16\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int\frac{e^{2x}x^2}{1+e^{2x}}\,dx,x,\text{arcsinh}(ax)\right)}{5ac^3\sqrt{c+a^2cx^2}}$$

$$= -\frac{1}{20ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} - \frac{x\text{arcsinh}(ax)}{c^3\sqrt{c+a^2cx^2}} - \frac{x\text{arcsinh}(ax)}{10c^3(1+a^2x^2)\sqrt{c+a^2cx^2}} + \frac{3\text{arcsinh}(ax)^2}{20ac^3(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} + \frac{2\text{arcsinh}(ax)^2}{5ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\text{arcsinh}(ax)^3}{5c(c+a^2cx^2)^{5/2}} + \frac{4x\text{arcsinh}(ax)^3}{15c^2(c+a^2cx^2)^{3/2}} + \frac{8x\text{arcsinh}(ax)^3}{15c^3\sqrt{c+a^2cx^2}} + \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{15ac^3\sqrt{c+a^2cx^2}} - \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2\log\left(1+e^{2\text{arcsinh}(ax)}\right)}{5ac^3\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\log(1+a^2x^2)}{2ac^3\sqrt{c+a^2cx^2}} + \frac{\left(16\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x\log\left(1+e^{2x}\right)\,dx,x,\text{arcsinh}(ax)\right)}{5ac^3\sqrt{c+a^2cx^2}}$$

$$= -\frac{1}{20ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} - \frac{x\text{arcsinh}(ax)}{c^3\sqrt{c+a^2cx^2}} - \frac{x\text{arcsinh}(ax)}{10c^3(1+a^2x^2)\sqrt{c+a^2cx^2}} + \frac{3\text{arcsinh}(ax)^2}{20ac^3(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} + \frac{2\text{arcsinh}(ax)^2}{5ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\text{arcsinh}(ax)^3}{5c(c+a^2cx^2)^{5/2}} + \frac{4x\text{arcsinh}(ax)^3}{15c^2(c+a^2cx^2)^{3/2}} + \frac{8x\text{arcsinh}(ax)^3}{15c^3\sqrt{c+a^2cx^2}} + \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{15ac^3\sqrt{c+a^2cx^2}} - \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2\log\left(1+e^{2\text{arcsinh}(ax)}\right)}{5ac^3\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\log(1+a^2x^2)}{2ac^3\sqrt{c+a^2cx^2}} - \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)}{5ac^3\sqrt{c+a^2cx^2}} + \frac{\left(8\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \text{PolyLog}\left(2,-e^{2x}\right)\,dx,x,\text{arcsinh}(ax)\right)}{5ac^3\sqrt{c+a^2cx^2}}$$

$$= -\frac{1}{20ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} - \frac{x\text{arcsinh}(ax)}{c^3\sqrt{c+a^2cx^2}} - \frac{x\text{arcsinh}(ax)}{10c^3(1+a^2x^2)\sqrt{c+a^2cx^2}} + \frac{3\text{arcsinh}(ax)^2}{20ac^3(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} + \frac{2\text{arcsinh}(ax)^2}{5ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\text{arcsinh}(ax)^3}{5c(c+a^2cx^2)^{5/2}} + \frac{4x\text{arcsinh}(ax)^3}{15c^2(c+a^2cx^2)^{3/2}} + \frac{8x\text{arcsinh}(ax)^3}{15c^3\sqrt{c+a^2cx^2}} + \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{15ac^3\sqrt{c+a^2cx^2}} - \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2\log\left(1+e^{2\text{arcsinh}(ax)}\right)}{5ac^3\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\log(1+a^2x^2)}{2ac^3\sqrt{c+a^2cx^2}} - \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)}{5ac^3\sqrt{c+a^2cx^2}} + \frac{\left(4\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{2\text{arcsinh}(ax)}\right)}{5ac^3\sqrt{c+a^2cx^2}}$$

$$= -\frac{1}{20ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} - \frac{x\text{arcsinh}(ax)}{c^3\sqrt{c+a^2cx^2}} - \frac{x\text{arcsinh}(ax)}{10c^3(1+a^2x^2)\sqrt{c+a^2cx^2}} + \frac{3\text{arcsinh}(ax)^2}{20ac^3(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} + \frac{2\text{arcsinh}(ax)^2}{5ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\text{arcsinh}(ax)^3}{5c(c+a^2cx^2)^{5/2}} + \frac{4x\text{arcsinh}(ax)^3}{15c^2(c+a^2cx^2)^{3/2}} + \frac{8x\text{arcsinh}(ax)^3}{15c^3\sqrt{c+a^2cx^2}} + \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{15ac^3\sqrt{c+a^2cx^2}} - \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2\log\left(1+e^{2\text{arcsinh}(ax)}\right)}{5ac^3\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\log(1+a^2x^2)}{2ac^3\sqrt{c+a^2cx^2}} - \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)}{5ac^3\sqrt{c+a^2cx^2}} + \frac{4\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(ax)}\right)}{5ac^3\sqrt{c+a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.58

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{7/2}}\,dx = \frac{-\frac{3}{\sqrt{1+a^2x^2}} - 60ax\operatorname{arcsinh}(ax) - \frac{6ax\operatorname{arcsinh}(ax)}{1+a^2x^2} + \frac{9\operatorname{arcsinh}(ax)^2}{(1+a^2x^2)^{3/2}} + \frac{24\operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}} + 32a\cdots}{\cdots}$$

```
[In] Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] (-3/Sqrt[1 + a^2*x^2] - 60*a*x*ArcSinh[a*x] - (6*a*x*ArcSinh[a*x])/(1 + a^2
*x^2) + (9*ArcSinh[a*x]^2)/(1 + a^2*x^2)^(3/2) + (24*ArcSinh[a*x]^2)/Sqrt[1
 + a^2*x^2] + 32*a*x*ArcSinh[a*x]^3 + (12*a*x*ArcSinh[a*x]^3)/(1 + a^2*x^2)
^2 + (16*a*x*ArcSinh[a*x]^3)/(1 + a^2*x^2) - 32*Sqrt[1 + a^2*x^2]*ArcSinh[a
*x]^3 - 96*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2*Log[1 + E^(-2*ArcSinh[a*x])] +
30*Sqrt[1 + a^2*x^2]*Log[1 + a^2*x^2] + 96*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]*P
olyLog[2, -E^(-2*ArcSinh[a*x])] + 48*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(-2*Ar
cSinh[a*x])])/(60*a*c^3*Sqrt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 888, normalized size of antiderivative = 1.72

method	result
default	$\frac{\sqrt{c(a^2x^2+1)}\left(8a^5x^5-8a^4x^4\sqrt{a^2x^2+1}+20a^3x^3-16a^2x^2\sqrt{a^2x^2+1}+15ax-8\sqrt{a^2x^2+1}\right)\left(24-1590a^4x^4\operatorname{arcsinh}(ax)-1368a^4x^4\operatorname{arcsi}\cdots\right.}{\cdots}$

```
[In] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*(c*(a^2*x^2+1))^(1/2)*(8*a^5*x^5-8*a^4*x^4*(a^2*x^2+1)^(1/2)+20*a^3*x^
3-16*a^2*x^2*(a^2*x^2+1)^(1/2)+15*a*x-8*(a^2*x^2+1)^(1/2))*(24-1590*a^4*x^4
*arcsinh(a*x)-1368*a^4*x^4*arcsinh(a*x)^2-1410*a^2*x^2*arcsinh(a*x)+105*a^3
*x^3*(a^2*x^2+1)^(1/2)+160*a^4*x^4*arcsinh(a*x)^3+45*a*x*(a^2*x^2+1)^(1/2)-
744*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)*a^5*x^5-1020*a^3*x^3*arcsinh(a*x)^2*(a
^2*x^2+1)^(1/2)-495*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)*a*x+256*arcsinh(a*x)^3
-480*arcsinh(a*x)-264*arcsinh(a*x)^2+96*a^2*x^2-192*arcsinh(a*x)^2*(a^2*x^2
+1)^(1/2)*a^7*x^7-192*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a^7*x^7-840*arcsinh(a*
x)^2*a^6*x^6+84*x^5*a^5*(a^2*x^2+1)^(1/2)-984*arcsinh(a*x)^2*a^2*x^2+144*a^
4*x^4-936*a^3*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)-372*arcsinh(a*x)*(a^2*x^2+
1)^(1/2)*a*x-756*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a^5*x^5+96*a^6*x^6+24*(a^2*
x^2+1)^(1/2)*a^7*x^7+24*a^8*x^8-192*arcsinh(a*x)*a^8*x^8-852*arcsinh(a*x)*a
^6*x^6-192*arcsinh(a*x)^2*a^8*x^8+380*arcsinh(a*x)^3*a^2*x^2)/(40*a^10*x^10
+215*a^8*x^8+469*a^6*x^6+517*a^4*x^4+287*a^2*x^2+64)/a/c^4-2/(a^2*x^2+1)^(1
/2)*(c*(a^2*x^2+1))^(1/2)/a/c^4*ln(a*x+(a^2*x^2+1)^(1/2))+1/(a^2*x^2+1)^(1/
2)*(c*(a^2*x^2+1))^(1/2)/a/c^4*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)+16/15/(a^2*x
```

```
^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^4*arcsinh(a*x)^3-8/5/(a^2*x^2+1)^(1/2
)*(c*(a^2*x^2+1))^(1/2)/a/c^4*arcsinh(a*x)^2*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2
)-8/5/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^4*arcsinh(a*x)*polylog(2,
-(a*x+(a^2*x^2+1)^(1/2))^2)+4/5/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c
^4*polylog(3,-(a*x+(a^2*x^2+1)^(1/2))^2)
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{7/2}}\,dx=\int \frac{\operatorname{arsinh}\left(ax\right)^3}{(a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^3/(a^8*c^4*x^8 + 4*a^6*c^4*x^6 +
6*a^4*c^4*x^4 + 4*a^2*c^4*x^2 + c^4), x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{7/2}}\,dx=\int \frac{\operatorname{asinh}^3\left(ax\right)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(asinh(a*x)**3/(a**2*c*x**2+c)**(7/2),x)
[Out] Integral(asinh(a*x)**3/(c*(a**2*x**2 + 1))**(7/2), x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{7/2}}\,dx=\int \frac{\operatorname{arsinh}\left(ax\right)^3}{(a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^(7/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)^3}{(c\,a^2\,x^2+c)^{7/2}}\,dx$$

```
[In] int(asinh(a*x)^3/(c + a^2*c*x^2)^(7/2),x)
```

```
[Out] int(asinh(a*x)^3/(c + a^2*c*x^2)^(7/2), x)
```

3.341 $\int \frac{x^m \textbf{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^m \operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}} \, dx = \operatorname{Int}\left(\frac{x^m \operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}, x\right)$$

```
[Out] Unintegrable(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}} \, dx = \int \frac{x^m \operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}} \, dx$$

```
[In] Int[(x^m*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] Defer[Int][(x^m*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2], x]
```

Rubi steps

$$\text{integral} = \int \frac{x^m \operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^m \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$$

```
[In] Integrate[(x^m*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
[Out] Integrate[(x^m*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^m \operatorname{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx$$

```
[In] int(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
[Out] int(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^m*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

Sympy [N/A]

Not integrable

Time = 2.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^m \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{asinh}^3(ax)}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x**m*asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
[Out] Integral(x**m*asinh(a*x)**3/sqrt(a**2*x**2 + 1), x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^m*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^m*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

Mupad [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{asinh}(a\,x)^3}{\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int((x^m*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^m*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2), x)
```

3.342 $\int \frac{x^4 \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 187

$$\begin{aligned}\int \frac{x^4 \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx &= \frac{45x^2}{128a^3} - \frac{3x^4}{128a} - \frac{45x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{64a^4} \\ &+ \frac{3x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{32a^2} + \frac{45\text{arcsinh}(ax)^2}{128a^5} + \frac{9x^2\text{arcsinh}(ax)^2}{16a^3} \\ &- \frac{3x^4\text{arcsinh}(ax)^2}{16a} - \frac{3x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{8a^4} \\ &+ \frac{x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{4a^2} + \frac{3\text{arcsinh}(ax)^4}{32a^5}\end{aligned}$$

```
[Out] 45/128*x^2/a^3-3/128*x^4/a+45/128*arcsinh(a*x)^2/a^5+9/16*x^2*arcsinh(a*x)^
2/a^3-3/16*x^4*arcsinh(a*x)^2/a+3/32*arcsinh(a*x)^4/a^5-45/64*x*arcsinh(a*x
)*(a^2*x^2+1)^(1/2)/a^4+3/32*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2-3/8*x*a
rcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a^4+1/4*x^3*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2
)/a^2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used

$= \{5812, 5783, 5776, 30\}$

$$\int \frac{x^4 \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{3\text{arcsinh}(ax)^4}{32a^5} + \frac{45\text{arcsinh}(ax)^2}{128a^5} + \frac{9x^2\text{arcsinh}(ax)^2}{16a^3} + \frac{45x^2}{128a^3} + \frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{4a^2} + \frac{3x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{32a^2} - \frac{3x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{8a^4} - \frac{45x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{64a^4} - \frac{3x^4\text{arcsinh}(ax)^2}{16a} - \frac{3x^4}{128a}$$

```
[In] Int[(x^4*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (45*x^2)/(128*a^3) - (3*x^4)/(128*a) - (45*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]
)/(64*a^4) + (3*x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(32*a^2) + (45*ArcSinh[
a*x]^2)/(128*a^5) + (9*x^2*ArcSinh[a*x]^2)/(16*a^3) - (3*x^4*ArcSinh[a*x]^2
)/(16*a) - (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(8*a^4) + (x^3*Sqrt[1 + a
^2*x^2]*ArcSinh[a*x]^3)/(4*a^2) + (3*ArcSinh[a*x]^4)/(32*a^5)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
```

```
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{4a^2} - \frac{3\int \frac{x^2\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{4a^2} - \frac{3\int x^3\text{arcsinh}(ax)^2\,dx}{4a}$$

$$= -\frac{3x^4\text{arcsinh}(ax)^2}{16a} - \frac{3x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{4a^2} + \frac{3}{8}\int \frac{x^4\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx + \frac{3\int \frac{\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{8a^4} + \frac{9\int x\text{arcsinh}(ax)^2\,dx}{8a^3}$$

$$= \frac{3x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{32a^2} + \frac{9x^2\text{arcsinh}(ax)^2}{16a^3} - \frac{3x^4\text{arcsinh}(ax)^2}{16a} - \frac{3x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{4a^2} + \frac{3\text{arcsinh}(ax)^4}{32a^5} - \frac{9\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{32a^2} - \frac{9\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{8a^2} - \frac{3\int x^3\,dx}{32a}$$

$$= -\frac{3x^4}{128a} - \frac{45x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{64a^4} + \frac{3x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{32a^2} + \frac{9x^2\text{arcsinh}(ax)^2}{16a^3} - \frac{3x^4\text{arcsinh}(ax)^2}{16a} - \frac{3x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{4a^2} + \frac{3\text{arcsinh}(ax)^4}{32a^5} + \frac{9\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{64a^4} + \frac{9\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{16a^4} + \frac{9\int x\,dx}{64a^3} + \frac{9\int x\,dx}{16a^3}$$

$$= \frac{45x^2}{128a^3} - \frac{3x^4}{128a} - \frac{45x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{64a^4} + \frac{3x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{32a^2} + \frac{45\text{arcsinh}(ax)^2}{128a^5} + \frac{9x^2\text{arcsinh}(ax)^2}{16a^3} - \frac{3x^4\text{arcsinh}(ax)^2}{16a} - \frac{3x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{4a^2} + \frac{3\text{arcsinh}(ax)^4}{32a^5}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.65

$$\int \frac{x^4\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{45a^2x^2 - 3a^4x^4 + 6ax\sqrt{1+a^2x^2}(-15+2a^2x^2)\,\text{arcsinh}(ax) + (45+72a^2x^2-24a^4x^4)\,\text{arcsinh}(ax)^2 + 16a}{128a^5}$$

```
[In] Integrate[(x^4*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (45*a^2*x^2 - 3*a^4*x^4 + 6*a*x*Sqrt[1 + a^2*x^2]*(-15 + 2*a^2*x^2)*ArcSinh
[a*x] + (45 + 72*a^2*x^2 - 24*a^4*x^4)*ArcSinh[a*x]^2 + 16*a*x*Sqrt[1 + a^2
*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[a*x]^3 + 12*ArcSinh[a*x]^4)/(128*a^5)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.83

method	result
default	$\frac{32a^3x^3 \operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}-24a^4x^4 \operatorname{arcsinh}(ax)^2+12a^3x^3 \operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}-3a^4x^4-48 \operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}\, ax+72 \operatorname{arcsinh}}{128a^5}$

```
[In] int(x^4*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/128*(32*a^3*x^3*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)-24*a^4*x^4*arcsinh(a*x)^
2+12*a^3*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)-3*a^4*x^4-48*arcsinh(a*x)^3*(a^
2*x^2+1)^(1/2)*a*x+72*arcsinh(a*x)^2*a^2*x^2+12*arcsinh(a*x)^4-90*arcsinh(a
*x)*(a^2*x^2+1)^(1/2)*a*x+45*a^2*x^2+45*arcsinh(a*x)^2+45)/a^5
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.89

$$\int \frac{x^4 \operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\, dx = -\frac{3\, a^4x^4 - 16\, (2\, a^3x^3 - 3\, ax)\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^3 - 45\, a^2x^2 - 12\, \log\left(ax+\sqrt{a^2x^2+1}\right)^4 + 3}{128\, a}$$

```
[In] integrate(x^4*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/128*(3*a^4*x^4 - 16*(2*a^3*x^3 - 3*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt
(a^2*x^2 + 1))^3 - 45*a^2*x^2 - 12*log(a*x + sqrt(a^2*x^2 + 1))^4 + 3*(8*a^
4*x^4 - 24*a^2*x^2 - 15)*log(a*x + sqrt(a^2*x^2 + 1))^2 - 6*(2*a^3*x^3 - 15
*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^5
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.99

$$\int \frac{x^4 \operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\, dx = \begin{cases} -\frac{3x^4 \operatorname{asinh}^2(ax)}{16a} - \frac{3x^4}{128a} + \frac{x^3\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{4a^2} + \frac{3x^3\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{32a^2} + \frac{9x^2\operatorname{asinh}^2(ax)}{16a^3} + \frac{45x^2}{128a^3} - \frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{8a^4} \\ 0 \end{cases}$$

```
[In] integrate(x**4*asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
[Out] Piecewise((-3*x**4*asinh(a*x)**2/(16*a) - 3*x**4/(128*a) + x**3*sqrt(a**2*x
**2 + 1)*asinh(a*x)**3/(4*a**2) + 3*x**3*sqrt(a**2*x**2 + 1)*asinh(a*x)/(32
```

```
*a**2) + 9*x**2*asinh(a*x)**2/(16*a**3) + 45*x**2/(128*a**3) - 3*x*sqrt(a**
2*x**2 + 1)*asinh(a*x)**3/(8*a**4) - 45*x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(6
4*a**4) + 3*asinh(a*x)**4/(32*a**5) + 45*asinh(a*x)**2/(128*a**5), Ne(a, 0)
), (0, True))
```

Maxima [F]

$$\int \frac{x^4 \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^4 \operatorname{arsinh}\left(ax\right)^3}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x^4*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{x^4 \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^4 \operatorname{arsinh}\left(ax\right)^3}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x^4*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^4*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^4 \operatorname{asinh}(a\,x)^3}{\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int((x^4*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^4*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2), x)
```

3.343 $\int \frac{x^3 \operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}} dx$

Optimal result

Integrand size = 23, antiderivative size = 153

$$\int \frac{x^3\operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{40x}{9a^3}-\frac{2x^3}{27a}-\frac{40\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{9a^4}+\frac{2x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{9a^2}+\frac{2x\operatorname{arcsinh}(ax)^2}{a^3}-\frac{x^3\operatorname{arcsinh}(ax)^2}{3a}-\frac{2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{3a^4}+\frac{x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{3a^2}$$

```
[Out] 40/9*x/a^3-2/27*x^3/a+2*x*arcsinh(a*x)^2/a^3-1/3*x^3*arcsinh(a*x)^2/a-40/9*
arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^4+2/9*x^2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a
^2-2/3*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a^4+1/3*x^2*arcsinh(a*x)^3*(a^2*x^2
+1)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5812, 5798, 5772, 8, 5776, 30}

$$\int \frac{x^3\operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{2x\operatorname{arcsinh}(ax)^2}{a^3}+\frac{40x}{9a^3}+\frac{x^2\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^3}{3a^2}+\frac{2x^2\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)}{9a^2}-\frac{2\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^3}{3a^4}-\frac{40\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)}{9a^4}-\frac{x^3\operatorname{arcsinh}(ax)^2}{3a}-\frac{2x^3}{27a}$$

```
[In] Int[(x^3*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (40*x)/(9*a^3) - (2*x^3)/(27*a) - (40*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(9*a^
4) + (2*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(9*a^2) + (2*x*ArcSinh[a*x]^2)/
a^3 - (x^3*ArcSinh[a*x]^2)/(3*a) - (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(3*
a^4) + (x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(3*a^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{3a^2} - \frac{2\int \frac{x\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{3a^2} - \frac{\int x^2\text{arcsinh}(ax)^2\,dx}{a} \\
&= -\frac{x^3\text{arcsinh}(ax)^2}{3a} - \frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{3a^4} + \frac{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{3a^2} \\
&\quad + \frac{2}{3}\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx + \frac{2\int \text{arcsinh}(ax)^2\,dx}{a^3} \\
&= \frac{2x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{9a^2} + \frac{2x\text{arcsinh}(ax)^2}{a^3} - \frac{x^3\text{arcsinh}(ax)^2}{3a} - \frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{3a^4} \\
&\quad + \frac{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{3a^2} - \frac{4\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{9a^2} - \frac{4\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{a^2} - \frac{2\int x^2\,dx}{9a} \\
&= -\frac{2x^3}{27a} - \frac{40\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{9a^4} + \frac{2x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{9a^2} \\
&\quad + \frac{2x\text{arcsinh}(ax)^2}{a^3} - \frac{x^3\text{arcsinh}(ax)^2}{3a} - \frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{3a^4} \\
&\quad + \frac{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{3a^2} + \frac{4\int 1\,dx}{9a^3} + \frac{4\int 1\,dx}{a^3} \\
&= \frac{40x}{9a^3} - \frac{2x^3}{27a} - \frac{40\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{9a^4} + \frac{2x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{9a^2} \\
&\quad + \frac{2x\text{arcsinh}(ax)^2}{a^3} - \frac{x^3\text{arcsinh}(ax)^2}{3a} - \frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{3a^4} \\
&\quad + \frac{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{3a^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.64

$$
\int \frac{x^3\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx
= \frac{-2ax(-60+a^2x^2)+6(-20+a^2x^2)\sqrt{1+a^2x^2}\text{arcsinh}(ax)-9ax(-6+a^2x^2)\text{arcsinh}(ax)^2+9(-2+a^2x^2}{27a^4}
$$

```
[In] Integrate[(x^3*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (-2*a*x*(-60 + a^2*x^2) + 6*(-20 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]
- 9*a*x*(-6 + a^2*x^2)*ArcSinh[a*x]^2 + 9*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2]*
ArcSinh[a*x]^3)/(27*a^4)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.07

method	result
default	$\frac{9a^4x^4 \operatorname{arcsinh}(ax)^3-9\operatorname{arcsinh}(ax)^3a^2x^2-9a^3x^3\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}+6a^4x^4\operatorname{arcsinh}(ax)-114a^2x^2\operatorname{arcsinh}(ax)-2a^3x^3\sqrt{a^2x^2+1}}{27a^4\sqrt{a^2x^2+1}}$

[In] int(x^3*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/27/a^4/(a^2*x^2+1)^(1/2)*(9*a^4*x^4*arcsinh(a*x)^3-9*arcsinh(a*x)^3*a^2*x^2-9*a^3*x^3*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)+6*a^4*x^4*arcsinh(a*x)-114*a^2*x^2*arcsinh(a*x)-2*a^3*x^3*(a^2*x^2+1)^(1/2)-18*arcsinh(a*x)^3+54*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)*a*x-120*arcsinh(a*x)+120*a*x*(a^2*x^2+1)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.84

$$\int \frac{x^3\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = -\frac{2\,a^3x^3-9\,\sqrt{a^2x^2+1}(a^2x^2-2)\log\left(ax+\sqrt{a^2x^2+1}\right)^3+9\,(a^3x^3-6\,ax)\log\left(ax+\sqrt{a^2x^2+1}\right)^2-6\,\sqrt{}}{27\,a^4}$$

[In] integrate(x^3*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] -1/27*(2*a^3*x^3 - 9*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 2)*log(a*x + sqrt(a^2*x^2 + 1))^3 + 9*(a^3*x^3 - 6*a*x)*log(a*x + sqrt(a^2*x^2 + 1))^2 - 6*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 20)*log(a*x + sqrt(a^2*x^2 + 1)) - 120*a*x)/a^4

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.97

$$\int \frac{x^3\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} -\frac{x^3\operatorname{asinh}^2(ax)}{3a}-\frac{2x^3}{27a}+\frac{x^2\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{3a^2}+\frac{2x^2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{9a^2}+\frac{2x\operatorname{asinh}^2(ax)}{a^3}+\frac{40x}{9a^3}-\frac{2\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{3a^4}- \\ 0 \end{cases}$$

[In] integrate(x**3*asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)

[Out] Piecewise((-x**3*asinh(a*x)**2/(3*a) - 2*x**3/(27*a) + x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(3*a**2) + 2*x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)/(9*a**2) + 2*x*asinh(a*x)**2/a**3 + 40*x/(9*a**3) - 2*sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(3*a**4) - 40*sqrt(a**2*x**2 + 1)*asinh(a*x)/(9*a**4), Ne(a, 0)), (0, True))

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.83

$$\int \frac{x^3\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{1}{3}\left(\frac{\sqrt{a^2x^2+1}x^2}{a^2} - \frac{2\sqrt{a^2x^2+1}}{a^4}\right)\text{arsinh}\,(ax)^3 + \frac{2}{27}a\left(\frac{3\left(\sqrt{a^2x^2+1}x^2 - \frac{20\sqrt{a^2x^2+1}}{a^2}\right)\text{arsinh}\,(ax)}{a^3} - \frac{a^2x^3-60\,x}{a^4}\right) - \frac{(a^2x^3-6\,x)\,\text{arsinh}\,(ax)^2}{3\,a^3}$$

```
[In] integrate(x^3*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2 + 1)/a^4)*arcsinh(a*x)^3 +
2/27*a*(3*(sqrt(a^2*x^2 + 1)*x^2 - 20*sqrt(a^2*x^2 + 1)/a^2)*arcsinh(a*x)/a
^3 - (a^2*x^3 - 60*x)/a^4) - 1/3*(a^2*x^3 - 6*x)*arcsinh(a*x)^2/a^3
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^3\,\text{asinh}(a\,x)^3}{\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int((x^3*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((x^3*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2), x)
```

3.344 $\int \frac{x^2 \mathbf{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}} dx$

Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \frac{x^2 \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}} dx = -\frac{3x^2}{8a} + \frac{3x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{4a^2} - \frac{3\text{arcsinh}(ax)^2}{8a^3} - \frac{3x^2\text{arcsinh}(ax)^2}{4a} + \frac{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{2a^2} - \frac{\text{arcsinh}(ax)^4}{8a^3}$$

```
[Out] -3/8*x^2/a-3/8*arcsinh(a*x)^2/a^3-3/4*x^2*arcsinh(a*x)^2/a-1/8*arcsinh(a*x)
^4/a^3+3/4*x*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2+1/2*x*arcsinh(a*x)^3*(a^2*x
^2+1)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5812, 5783, 5776, 30}

$$\int \frac{x^2 \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}} dx = -\frac{\text{arcsinh}(ax)^4}{8a^3} - \frac{3\text{arcsinh}(ax)^2}{8a^3} + \frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2a^2} + \frac{3x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{4a^2} - \frac{3x^2\text{arcsinh}(ax)^2}{4a} - \frac{3x^2}{8a}$$

```
[In] Int[(x^2*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (-3*x^2)/(8*a) + (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(4*a^2) - (3*ArcSinh[
a*x]^2)/(8*a^3) - (3*x^2*ArcSinh[a*x]^2)/(4*a) + (x*Sqrt[1 + a^2*x^2]*ArcSi
nh[a*x]^3)/(2*a^2) - ArcSinh[a*x]^4/(8*a^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{2a^2} - \frac{\int \frac{\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{2a^2} - \frac{3\int x\text{arcsinh}(ax)^2\,dx}{2a} \\
&= -\frac{3x^2\text{arcsinh}(ax)^2}{4a} + \frac{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{2a^2} - \frac{\text{arcsinh}(ax)^4}{8a^3} + \frac{3}{2}\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx \\
&= \frac{3x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{4a^2} - \frac{3x^2\text{arcsinh}(ax)^2}{4a} + \frac{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{2a^2} \\
&\quad - \frac{\text{arcsinh}(ax)^4}{8a^3} - \frac{3\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{4a^2} - \frac{3\int x\,dx}{4a} \\
&= -\frac{3x^2}{8a} + \frac{3x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{4a^2} - \frac{3\text{arcsinh}(ax)^2}{8a^3} \\
&\quad - \frac{3x^2\text{arcsinh}(ax)^2}{4a} + \frac{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{2a^2} - \frac{\text{arcsinh}(ax)^4}{8a^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.79

$$\int \frac{x^2\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = -\frac{3a^2x^2-6ax\sqrt{1+a^2x^2}\text{arcsinh}(ax)+(3+6a^2x^2)\,\text{arcsinh}(ax)^2-4ax\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3+\text{arcsinh}}{8a^3}$$

```
[In] Integrate[(x^2*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -1/8*(3*a^2*x^2 - 6*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + (3 + 6*a^2*x^2)*Ar
cSinh[a*x]^2 - 4*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3 + ArcSinh[a*x]^4)/a^3
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{-4\,\text{arcsinh}(ax)^3\sqrt{a^2x^2+1}\,ax+6\,\text{arcsinh}(ax)^2a^2x^2+\text{arcsinh}(ax)^4-6\,\text{arcsinh}(ax)\sqrt{a^2x^2+1}\,ax+3a^2x^2+3\,\text{arcsinh}(ax)^2+3}{8a^3}$	84

```
[In] int(x^2*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-4*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)*a*x+6*arcsinh(a*x)^2*a^2*x^2+arcs
inh(a*x)^4-6*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x+3*a^2*x^2+3*arcsinh(a*x)^2+
3)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.22

$$\int \frac{x^2\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{4\sqrt{a^2x^2+1}ax\log\left(ax+\sqrt{a^2x^2+1}\right)^3-3\,a^2x^2-\log\left(ax+\sqrt{a^2x^2+1}\right)^4+6\sqrt{a^2x^2+1}ax\log\left(ax+\sqrt{a^2}\right.}{8\,a^3}$$

```
[In] integrate(x^2*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1))^3 - 3*a^2*x^2 - l
og(a*x + sqrt(a^2*x^2 + 1))^4 + 6*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*
x^2 + 1)) - 3*(2*a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2)/a^3
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95

$$\int \frac{x^2\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} -\frac{3x^2\operatorname{asinh}^2(ax)}{4a} - \frac{3x^2}{8a} + \frac{x\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{2a^2} + \frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{4a^2} - \frac{\operatorname{asinh}^4(ax)}{8a^3} - \frac{3\operatorname{asinh}^2(ax)}{8a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
[Out] Piecewise((-3*x**2*asinh(a*x)**2/(4*a) - 3*x**2/(8*a) + x*sqrt(a**2*x**2 +
1)*asinh(a*x)**3/(2*a**2) + 3*x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(4*a**2) - a
sinh(a*x)**4/(8*a**3) - 3*asinh(a*x)**2/(8*a**3), Ne(a, 0)), (0, True))
```

Maxima [F]

$$\int \frac{x^2\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2\operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x^2*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{x^2\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2\operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x^2*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2\,\mathrm{asinh}(a\,x)^3}{\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int((x^2*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^2*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2), x)
```

3.345 $\int \frac{x\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 64

$$\int \frac{x\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = -\frac{6x}{a}+\frac{6\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{a^2}-\frac{3x\text{arcsinh}(ax)^2}{a}+\frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{a^2}$$

```
[Out] -6*x/a-3*x*arcsinh(a*x)^2/a+6*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2+arcsinh(a*
x)^3*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5798, 5772, 8}

$$\int \frac{x\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{a^2}+\frac{6\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{3x\text{arcsinh}(ax)^2}{a}-\frac{6x}{a}$$

```
[In] Int[(x*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (-6*x)/a + (6*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2 - (3*x*ArcSinh[a*x]^2)/a
+ (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/a^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{a^2} - \frac{3\int \text{arcsinh}(ax)^2\,dx}{a}\\
&= -\frac{3x\text{arcsinh}(ax)^2}{a} + \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{a^2} + 6\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx\\
&= \frac{6\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{a^2} - \frac{3x\text{arcsinh}(ax)^2}{a} + \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{a^2} - \frac{6\int 1\,dx}{a}\\
&= -\frac{6x}{a} + \frac{6\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{a^2} - \frac{3x\text{arcsinh}(ax)^2}{a} + \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{a^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.91

$$\int \frac{x\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{-6ax + 6\sqrt{1+a^2x^2}\text{arcsinh}(ax) - 3ax\text{arcsinh}(ax)^2 + \sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{a^2}$$

```
[In] Integrate[(x*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
[Out] (-6*a*x + 6*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] - 3*a*x*ArcSinh[a*x]^2 + Sqrt[1
+ a^2*x^2]*ArcSinh[a*x]^3)/a^2
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.41

method	result	size
default	$\frac{\operatorname{arcsinh}(ax)^{3} a^{2} x^{2}+\operatorname{arcsinh}(ax)^{3}-3 \operatorname{arcsinh}(ax)^{2} \sqrt{a^{2} x^{2}+1}\, a x+6 a^{2} x^{2} \operatorname{arcsinh}(ax)+6 \operatorname{arcsinh}(ax)-6 a x \sqrt{a^{2} x^{2}+1}}{a^{2} \sqrt{a^{2} x^{2}+1}}$	90

```
[In] int(x*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2/(a^2*x^2+1)^(1/2)*(arcsinh(a*x)^3*a^2*x^2+arcsinh(a*x)^3-3*arcsinh(a*
x)^2*(a^2*x^2+1)^(1/2)*a*x+6*a^2*x^2*arcsinh(a*x)+6*arcsinh(a*x)-6*a*x*(a^2
*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.44

$$\int \frac{x\operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = -\frac{3\,ax\log\left(ax+\sqrt{a^2x^2+1}\right)^2-\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^3+6\,ax-6\,\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}$$

```
[In] integrate(x*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(3*a*x*log(a*x + sqrt(a^2*x^2 + 1))^2 - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a
^2*x^2 + 1))^3 + 6*a*x - 6*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/
a^2
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{x\operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} -\frac{3x\operatorname{asinh}^{2}(ax)}{a}-\frac{6x}{a}+\frac{\sqrt{a^2x^2+1}\operatorname{asinh}^{3}(ax)}{a^2}+\frac{6\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{a^2} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

```
[In] integrate(x*asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-3*x*asinh(a*x)**2/a - 6*x/a + sqrt(a**2*x**2 + 1)*asinh(a*x)**3
/a**2 + 6*sqrt(a**2*x**2 + 1)*asinh(a*x)/a**2, Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{x\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = -\frac{3\,x\,\text{arsinh}\,(ax)^2}{a} + \frac{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)^3}{a^2} - \frac{6\left(x - \frac{\sqrt{a^2x^2+1}\,\text{arsinh}(ax)}{a}\right)}{a}$$

```
[In] integrate(x*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -3*x*arcsinh(a*x)^2/a + sqrt(a^2*x^2 + 1)*arcsinh(a*x)^3/a^2 - 6*(x - sqrt(
a^2*x^2 + 1)*arcsinh(a*x)/a)/a
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.58

$$\int \frac{x\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{a^2} - \frac{3\left(x\log\left(ax+\sqrt{a^2x^2+1}\right)^2 + 2\,a\left(\frac{x}{a} - \frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}\right)\right)}{a}$$

```
[In] integrate(x*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^3/a^2 - 3*(x*log(a*x + sqrt(
a^2*x^2 + 1))^2 + 2*a*(x/a - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))
/a^2))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x\,\text{asinh}(a\,x)^3}{\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int((x*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2), x)
```

3.346 $\int \frac{\operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}} dx$

Optimal result

Integrand size = 20, antiderivative size = 13

$$\int \frac{\operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}} dx = \frac{\operatorname{arcsinh}(ax)^4}{4a}$$

```
[Out] 1/4*arcsinh(a*x)^4/a
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {5783}

$$\int \frac{\operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}} dx = \frac{\operatorname{arcsinh}(ax)^4}{4a}$$

```
[In] Int[ArcSinh[a*x]^3/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ArcSinh[a*x]^4/(4*a)
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{\operatorname{arcsinh}(ax)^4}{4a}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{\operatorname{arcsinh}(ax)^4}{4a}$$

```
[In] Integrate[ArcSinh[a*x]^3/Sqrt[1 + a^2*x^2],x]
[Out] ArcSinh[a*x]^4/(4*a)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\operatorname{arcsinh}(ax)^4}{4a}$	12
default	$\frac{\operatorname{arcsinh}(ax)^4}{4a}$	12

```
[In] int(arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/4*arcsinh(a*x)^4/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{\operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^4}{4\,a}$$

```
[In] integrate(arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/4*log(a*x + sqrt(a^2*x^2 + 1))^4/a
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{\operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} \frac{\operatorname{asinh}^4(ax)}{4a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
[Out] Piecewise((asinh(a*x)**4/(4*a), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{\operatorname{arsinh}(ax)^4}{4\,a}$$

```
[In] integrate(arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/4*arcsinh(a*x)^4/a
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{\operatorname{asinh}(a\,x)^4}{4\,a}$$

```
[In] int(asinh(a*x)^3/(a^2*x^2 + 1)^(1/2),x)
[Out] asinh(a*x)^4/(4*a)
```

3.347 $\int \frac{\text{arcsinh}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 102

$$\begin{aligned}\int \frac{\text{arcsinh}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx = & -2\text{arcsinh}(ax)^3\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) \\ & - 3\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right) \\ & + 3\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right) \\ & + 6\text{arcsinh}(ax)\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right) \\ & - 6\text{arcsinh}(ax)\,\text{PolyLog}\left(3,e^{\text{arcsinh}(ax)}\right) \\ & - 6\,\text{PolyLog}\left(4,-e^{\text{arcsinh}(ax)}\right) + 6\,\text{PolyLog}\left(4,e^{\text{arcsinh}(ax)}\right)\end{aligned}$$

```
[Out] -2*arcsinh(a*x)^3*arctanh(a*x+(a^2*x^2+1)^(1/2))-3*arcsinh(a*x)^2*polylog(2
,-a*x-(a^2*x^2+1)^(1/2))+3*arcsinh(a*x)^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))+
6*arcsinh(a*x)*polylog(3,-a*x-(a^2*x^2+1)^(1/2))-6*arcsinh(a*x)*polylog(3,a
*x+(a^2*x^2+1)^(1/2))-6*polylog(4,-a*x-(a^2*x^2+1)^(1/2))+6*polylog(4,a*x+(
a^2*x^2+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

$= \{5816, 4267, 2611, 6744, 2320, 6724\}$

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx = -2\operatorname{arcsinh}(ax)^3\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) - 3\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right) + 3\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right) + 6\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(ax)}\right) - 6\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(ax)}\right) - 6\operatorname{PolyLog}\left(4,-e^{\operatorname{arcsinh}(ax)}\right) + 6\operatorname{PolyLog}\left(4,e^{\operatorname{arcsinh}(ax)}\right)$$

```
[In] Int[ArcSinh[a*x]^3/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -2*ArcSinh[a*x]^3*ArcTanh[E^ArcSinh[a*x]] - 3*ArcSinh[a*x]^2*PolyLog[2, -E^
ArcSinh[a*x]] + 3*ArcSinh[a*x]^2*PolyLog[2, E^ArcSinh[a*x]] + 6*ArcSinh[a*x
]*PolyLog[3, -E^ArcSinh[a*x]] - 6*ArcSinh[a*x]*PolyLog[3, E^ArcSinh[a*x]] -
 6*PolyLog[4, -E^ArcSinh[a*x]] + 6*PolyLog[4, E^ArcSinh[a*x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
```

```
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int x^3\text{csch}(x)\,dx, x, \text{arcsinh}(ax)\right)\\
&= -2\text{arcsinh}(ax)^3\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) - 3\text{Subst}\left(\int x^2\log\left(1-e^x\right)\,dx, x, \text{arcsinh}(ax)\right)\\
&\quad + 3\text{Subst}\left(\int x^2\log\left(1+e^x\right)\,dx, x, \text{arcsinh}(ax)\right)\\
&= -2\text{arcsinh}(ax)^3\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) - 3\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2, -e^{\text{arcsinh}(ax)}\right)\\
&\quad + 3\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2, e^{\text{arcsinh}(ax)}\right)\\
&\quad + 6\text{Subst}\left(\int x\,\text{PolyLog}\left(2, -e^x\right)\,dx, x, \text{arcsinh}(ax)\right)\\
&\quad - 6\text{Subst}\left(\int x\,\text{PolyLog}\left(2, e^x\right)\,dx, x, \text{arcsinh}(ax)\right)\\
&= -2\text{arcsinh}(ax)^3\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) - 3\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2, -e^{\text{arcsinh}(ax)}\right)\\
&\quad + 3\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2, e^{\text{arcsinh}(ax)}\right)\\
&\quad + 6\text{arcsinh}(ax)\,\text{PolyLog}\left(3, -e^{\text{arcsinh}(ax)}\right) - 6\text{arcsinh}(ax)\,\text{PolyLog}\left(3, e^{\text{arcsinh}(ax)}\right)\\
&\quad - 6\text{Subst}\left(\int \text{PolyLog}\left(3, -e^x\right)\,dx, x, \text{arcsinh}(ax)\right)\\
&\quad + 6\text{Subst}\left(\int \text{PolyLog}\left(3, e^x\right)\,dx, x, \text{arcsinh}(ax)\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -2\mathrm{arcsinh}(ax)^3\mathrm{arctanh}\left(e^{\mathrm{arcsinh}(ax)}\right) - 3\mathrm{arcsinh}(ax)^2\,\mathrm{PolyLog}\left(2,-e^{\mathrm{arcsinh}(ax)}\right)\\
&\quad + 3\mathrm{arcsinh}(ax)^2\,\mathrm{PolyLog}\left(2,e^{\mathrm{arcsinh}(ax)}\right) + 6\mathrm{arcsinh}(ax)\,\mathrm{PolyLog}\left(3,-e^{\mathrm{arcsinh}(ax)}\right)\\
&\quad - 6\mathrm{arcsinh}(ax)\,\mathrm{PolyLog}\left(3,e^{\mathrm{arcsinh}(ax)}\right)\\
&\quad - 6\mathrm{Subst}\left(\int \frac{\mathrm{PolyLog}(3,-x)}{x}\,dx, x, e^{\mathrm{arcsinh}(ax)}\right)\\
&\quad + 6\mathrm{Subst}\left(\int \frac{\mathrm{PolyLog}(3,x)}{x}\,dx, x, e^{\mathrm{arcsinh}(ax)}\right)\\
&= -2\mathrm{arcsinh}(ax)^3\mathrm{arctanh}\left(e^{\mathrm{arcsinh}(ax)}\right) - 3\mathrm{arcsinh}(ax)^2\,\mathrm{PolyLog}\left(2,-e^{\mathrm{arcsinh}(ax)}\right)\\
&\quad + 3\mathrm{arcsinh}(ax)^2\,\mathrm{PolyLog}\left(2,e^{\mathrm{arcsinh}(ax)}\right)\\
&\quad + 6\mathrm{arcsinh}(ax)\,\mathrm{PolyLog}\left(3,-e^{\mathrm{arcsinh}(ax)}\right) - 6\mathrm{arcsinh}(ax)\,\mathrm{PolyLog}\left(3,e^{\mathrm{arcsinh}(ax)}\right)\\
&\quad - 6\,\mathrm{PolyLog}\left(4,-e^{\mathrm{arcsinh}(ax)}\right) + 6\,\mathrm{PolyLog}\left(4,e^{\mathrm{arcsinh}(ax)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.43

$$
\begin{aligned}
\int \frac{\mathrm{arcsinh}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx = \frac{1}{8}\big(&\pi^4 - 2\mathrm{arcsinh}(ax)^4 - 8\mathrm{arcsinh}(ax)^3\log\left(1+e^{-\mathrm{arcsinh}(ax)}\right)\\
&+ 8\mathrm{arcsinh}(ax)^3\log\left(1-e^{\mathrm{arcsinh}(ax)}\right)\\
&+ 24\mathrm{arcsinh}(ax)^2\,\mathrm{PolyLog}\left(2,-e^{-\mathrm{arcsinh}(ax)}\right)\\
&+ 24\mathrm{arcsinh}(ax)^2\,\mathrm{PolyLog}\left(2,e^{\mathrm{arcsinh}(ax)}\right)\\
&+ 48\mathrm{arcsinh}(ax)\,\mathrm{PolyLog}\left(3,-e^{-\mathrm{arcsinh}(ax)}\right)\\
&- 48\mathrm{arcsinh}(ax)\,\mathrm{PolyLog}\left(3,e^{\mathrm{arcsinh}(ax)}\right)\\
&+ 48\,\mathrm{PolyLog}\left(4,-e^{-\mathrm{arcsinh}(ax)}\right) + 48\,\mathrm{PolyLog}\left(4,e^{\mathrm{arcsinh}(ax)}\right)\big)
\end{aligned}
$$

```
[In] Integrate[ArcSinh[a*x]^3/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (Pi^4 - 2*ArcSinh[a*x]^4 - 8*ArcSinh[a*x]^3*Log[1 + E^(-ArcSinh[a*x])] + 8*
ArcSinh[a*x]^3*Log[1 - E^ArcSinh[a*x]] + 24*ArcSinh[a*x]^2*PolyLog[2, -E^(-
ArcSinh[a*x])] + 24*ArcSinh[a*x]^2*PolyLog[2, E^ArcSinh[a*x]] + 48*ArcSinh[
a*x]*PolyLog[3, -E^(-ArcSinh[a*x])] - 48*ArcSinh[a*x]*PolyLog[3, E^ArcSinh[
a*x]] + 48*PolyLog[4, -E^(-ArcSinh[a*x])] + 48*PolyLog[4, E^ArcSinh[a*x]])/
8
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.93

method	result
default	$-\operatorname{arcsinh}(ax)^3 \ln\left(1+ax+\sqrt{a^2x^2+1}\right)-3\operatorname{arcsinh}(ax)^2 \operatorname{polylog}\left(2,-ax-\sqrt{a^2x^2+1}\right)+6 \text{ ar}$

[In] int(arcsinh(a*x)^3/x/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] -arcsinh(a*x)^3*ln(1+a*x+(a^2*x^2+1)^(1/2))-3*arcsinh(a*x)^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+6*arcsinh(a*x)*polylog(3,-a*x-(a^2*x^2+1)^(1/2))-6*polylog(4,-a*x-(a^2*x^2+1)^(1/2))+arcsinh(a*x)^3*ln(1-a*x-(a^2*x^2+1)^(1/2))+3*arcsinh(a*x)^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))-6*arcsinh(a*x)*polylog(3,a*x+(a^2*x^2+1)^(1/2))+6*polylog(4,a*x+(a^2*x^2+1)^(1/2))

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx=\int \frac{\operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}x}\,dx$$

[In] integrate(arcsinh(a*x)^3/x/(a^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^3/(a^2*x^3 + x), x)

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx=\int \frac{\operatorname{asinh}^3(ax)}{x\sqrt{a^2x^2+1}}\,dx$$

[In] integrate(asinh(a*x)**3/x/(a**2*x**2+1)**(1/2),x)

[Out] Integral(asinh(a*x)**3/(x*sqrt(a**2*x**2 + 1)), x)

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx=\int \frac{\operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}x}\,dx$$

[In] integrate(arcsinh(a*x)^3/x/(a^2*x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(arcsinh(a*x)^3/(sqrt(a^2*x^2 + 1)*x), x)

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}x}\,dx$$

[In] integrate(arcsinh(a*x)^3/x/(a^2*x^2+1)^(1/2),x, algorithm="giac")

[Out] integrate(arcsinh(a*x)^3/(sqrt(a^2*x^2 + 1)*x), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)^3}{x\,\sqrt{a^2\,x^2+1}}\,dx$$

[In] int(asinh(a*x)^3/(x*(a^2*x^2 + 1)^(1/2)),x)

[Out] int(asinh(a*x)^3/(x*(a^2*x^2 + 1)^(1/2)), x)

3.348 $\int \frac{\text{arcsinh}(ax)^3}{x^2\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int \frac{\text{arcsinh}(ax)^3}{x^2\sqrt{1+a^2x^2}}\,dx = -a\text{arcsinh}(ax)^3 - \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{x} + 3a\text{arcsinh}(ax)^2\log\left(1-e^{2\text{arcsinh}(ax)}\right) + 3a\text{arcsinh}(ax)\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right) - \frac{3}{2}a\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(ax)}\right)$$

```
[Out] -a*arcsinh(a*x)^3+3*a*arcsinh(a*x)^2*ln(1-(a*x+(a^2*x^2+1)^(1/2))^2)+3*a*ar
csinh(a*x)*polylog(2,(a*x+(a^2*x^2+1)^(1/2))^2)-3/2*a*polylog(3,(a*x+(a^2*x
^2+1)^(1/2))^2)-arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5800, 5775, 3797, 2221, 2611, 2320, 6724}

$$\int \frac{\text{arcsinh}(ax)^3}{x^2\sqrt{1+a^2x^2}}\,dx = -\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{x} + 3a\text{arcsinh}(ax)\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right) - \frac{3}{2}a\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(ax)}\right) - a\text{arcsinh}(ax)^3 + 3a\text{arcsinh}(ax)^2\log\left(1-e^{2\text{arcsinh}(ax)}\right)$$

```
[In] Int[ArcSinh[a*x]^3/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -(a*ArcSinh[a*x]^3) - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/x + 3*a*ArcSinh[a*
x]^2*Log[1 - E^(2*ArcSinh[a*x])] + 3*a*ArcSinh[a*x]*PolyLog[2, E^(2*ArcSinh
[a*x])] - (3*a*PolyLog[3, E^(2*ArcSinh[a*x])])/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{x} + (3a)\int \frac{\operatorname{arcsinh}(ax)^2}{x}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{x} + (3a)\operatorname{Subst}\left(\int x^2\coth(x)\,dx, x, \operatorname{arcsinh}(ax)\right) \\
&= -a\operatorname{arcsinh}(ax)^3 - \frac{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{x} - (6a)\operatorname{Subst}\left(\int \frac{e^{2x}x^2}{1-e^{2x}}\,dx, x, \operatorname{arcsinh}(ax)\right) \\
&= -a\operatorname{arcsinh}(ax)^3 - \frac{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{x} + 3a\operatorname{arcsinh}(ax)^2\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right) \\
&\quad - (6a)\operatorname{Subst}\left(\int x\log\left(1-e^{2x}\right)\,dx, x, \operatorname{arcsinh}(ax)\right) \\
&= -a\operatorname{arcsinh}(ax)^3 - \frac{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{x} \\
&\quad + 3a\operatorname{arcsinh}(ax)^2\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right) + 3a\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2, e^{2\operatorname{arcsinh}(ax)}\right) \\
&\quad - (3a)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2, e^{2x}\right)\,dx, x, \operatorname{arcsinh}(ax)\right) \\
&= -a\operatorname{arcsinh}(ax)^3 - \frac{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{x} \\
&\quad + 3a\operatorname{arcsinh}(ax)^2\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right) + 3a\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2, e^{2\operatorname{arcsinh}(ax)}\right) \\
&\quad - \frac{1}{2}(3a)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,x)}{x}\,dx, x, e^{2\operatorname{arcsinh}(ax)}\right) \\
&= -a\operatorname{arcsinh}(ax)^3 - \frac{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{x} + 3a\operatorname{arcsinh}(ax)^2\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right) \\
&\quad + 3a\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2, e^{2\operatorname{arcsinh}(ax)}\right) - \frac{3}{2}a\operatorname{PolyLog}\left(3, e^{2\operatorname{arcsinh}(ax)}\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10

$$\int \frac{\text{arcsinh}(ax)^3}{x^2\sqrt{1+a^2x^2}}\,dx = \frac{1}{8}a\left(i\pi^3 - 8\text{arcsinh}(ax)^3 - \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{ax} + 24\text{arcsinh}(ax)^2\log\left(1-e^{2\text{arcsinh}(ax)}\right) + 24\text{arcsinh}(ax)\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right) - 12\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(ax)}\right)\right)$$

```
[In] Integrate[ArcSinh[a*x]^3/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (a*(I*Pi^3 - 8*ArcSinh[a*x]^3 - (8*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(a*x)
+ 24*ArcSinh[a*x]^2*Log[1 - E^(2*ArcSinh[a*x])] + 24*ArcSinh[a*x]*PolyLog[2
, E^(2*ArcSinh[a*x])] - 12*PolyLog[3, E^(2*ArcSinh[a*x])]))/8
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.12

method	result
default	$\frac{\left(ax-\sqrt{a^2x^2+1}\right)\text{arcsinh}(ax)^3}{x} - 2a\,\text{arcsinh}\,(ax)^3 + 3a\,\text{arcsinh}\,(ax)^2\ln\left(1+ax+\sqrt{a^2x^2+1}\right) + 6a\,\text{arcsin}$

```
[In] int(arcsinh(a*x)^3/x^2/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*x-(a^2*x^2+1)^(1/2))/x*arcsinh(a*x)^3-2*a*arcsinh(a*x)^3+3*a*arcsinh(a*x
)^2*ln(1+a*x+(a^2*x^2+1)^(1/2))+6*a*arcsinh(a*x)*polylog(2,-a*x-(a^2*x^2+1)
^(1/2))-6*a*polylog(3,-a*x-(a^2*x^2+1)^(1/2))+3*a*arcsinh(a*x)^2*ln(1-a*x-(
a^2*x^2+1)^(1/2))+6*a*arcsinh(a*x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))-6*a*pol
ylog(3,a*x+(a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\text{arcsinh}(ax)^3}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\text{arsinh}\,(ax)^3}{\sqrt{a^2x^2+1}x^2}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/x^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^3/(a^2*x^4 + x^2), x)
```

Sympy [F]

$$\int \frac{\text{arcsinh}(ax)^3}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\text{asinh}^3\,(ax)}{x^2\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(asinh(a*x)**3/x**2/(a**2*x**2+1)**(1/2),x)
[Out] Integral(asinh(a*x)**3/(x**2*sqrt(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{\text{arcsinh}(ax)^3}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\text{arsinh}\,(ax)^3}{\sqrt{a^2x^2+1}x^2}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/x^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^3/x + integrate(3*(a^3*x^2
+ sqrt(a^2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^2/(sqrt(a^2*x^2
 + 1)*a*x^2 + (a^2*x^2 + 1)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\text{arcsinh}(ax)^3}{x^2\sqrt{1+a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x)^3/x^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)^3}{x^2\,\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int(asinh(a*x)^3/(x^2*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(asinh(a*x)^3/(x^2*(a^2*x^2 + 1)^(1/2)), x)
```

3.349 $\int \frac{\operatorname{arcsinh}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 210

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx =& -\frac{3a\operatorname{arcsinh}(ax)^2}{2x}-\frac{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{2x^2}\\ &-6a^2\operatorname{arcsinh}(ax)\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right)\\ &+a^2\operatorname{arcsinh}(ax)^3\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right)-3a^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right)\\ &+\frac{3}{2}a^2\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right)\\ &+3a^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right)\\ &-\frac{3}{2}a^2\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right)\\ &-3a^2\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(ax)}\right)\\ &+3a^2\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(ax)}\right)\\ &+3a^2\operatorname{PolyLog}\left(4,-e^{\operatorname{arcsinh}(ax)}\right)-3a^2\operatorname{PolyLog}\left(4,e^{\operatorname{arcsinh}(ax)}\right)\end{aligned}$$

```
[Out] -3/2*a*arcsinh(a*x)^2/x-6*a^2*arcsinh(a*x)*arctanh(a*x+(a^2*x^2+1)^(1/2))+a
^2*arcsinh(a*x)^3*arctanh(a*x+(a^2*x^2+1)^(1/2))-3*a^2*polylog(2,-a*x-(a^2*
x^2+1)^(1/2))+3/2*a^2*arcsinh(a*x)^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+3*a^
2*polylog(2,a*x+(a^2*x^2+1)^(1/2))-3/2*a^2*arcsinh(a*x)^2*polylog(2,a*x+(a^
2*x^2+1)^(1/2))-3*a^2*arcsinh(a*x)*polylog(3,-a*x-(a^2*x^2+1)^(1/2))+3*a^2*
arcsinh(a*x)*polylog(3,a*x+(a^2*x^2+1)^(1/2))+3*a^2*polylog(4,-a*x-(a^2*x^2
+1)^(1/2))-3*a^2*polylog(4,a*x+(a^2*x^2+1)^(1/2))-1/2*arcsinh(a*x)^3*(a^2*x
^2+1)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5809, 5816, 4267, 2611, 6744, 2320, 6724, 5776, 2317, 2438}

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx = a^2\operatorname{arcsinh}(ax)^3\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) - 6a^2\operatorname{arcsinh}(ax)\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) + \frac{3}{2}a^2\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right) - \frac{3}{2}a^2\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right) - 3a^2\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(ax)}\right) + 3a^2\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(ax)}\right) - 3a^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right) + 3a^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right) + 3a^2\operatorname{PolyLog}\left(4,-e^{\operatorname{arcsinh}(ax)}\right) - 3a^2\operatorname{PolyLog}\left(4,e^{\operatorname{arcsinh}(ax)}\right) - \frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^3}{2x^2} - \frac{3a\operatorname{arcsinh}(ax)^2}{2x}$$

```
[In] Int[ArcSinh[a*x]^3/(x^3*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (-3*a*ArcSinh[a*x]^2)/(2*x) - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(2*x^2) -
6*a^2*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]] + a^2*ArcSinh[a*x]^3*ArcTanh[E^A
rcSinh[a*x]] - 3*a^2*PolyLog[2, -E^ArcSinh[a*x]] + (3*a^2*ArcSinh[a*x]^2*Po
lyLog[2, -E^ArcSinh[a*x]])/2 + 3*a^2*PolyLog[2, E^ArcSinh[a*x]] - (3*a^2*Ar
cSinh[a*x]^2*PolyLog[2, E^ArcSinh[a*x]])/2 - 3*a^2*ArcSinh[a*x]*PolyLog[3,
-E^ArcSinh[a*x]] + 3*a^2*ArcSinh[a*x]*PolyLog[3, E^ArcSinh[a*x]] + 3*a^2*Po
lyLog[4, -E^ArcSinh[a*x]] - 3*a^2*PolyLog[4, E^ArcSinh[a*x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{2x^2} + \frac{1}{2}(3a)\int \frac{\text{arcsinh}(ax)^2}{x^2}\,dx - \frac{1}{2}a^2\int \frac{\text{arcsinh}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{3a\text{arcsinh}(ax)^2}{2x} - \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{2x^2} \\
&\quad - \frac{1}{2}a^2\text{Subst}\left(\int x^3\text{csch}(x)\,dx, x, \text{arcsinh}(ax)\right) + (3a^2)\int \frac{\text{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{3a\text{arcsinh}(ax)^2}{2x} - \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{2x^2} + a^2\text{arcsinh}(ax)^3\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) \\
&\quad + \frac{1}{2}(3a^2)\,\text{Subst}\left(\int x^2\log\left(1-e^x\right)\,dx, x, \text{arcsinh}(ax)\right) \\
&\quad - \frac{1}{2}(3a^2)\,\text{Subst}\left(\int x^2\log\left(1+e^x\right)\,dx, x, \text{arcsinh}(ax)\right) \\
&\quad + (3a^2)\,\text{Subst}(\int x\text{csch}(x)\,dx, x, \text{arcsinh}(ax)) \\
&= -\frac{3a\text{arcsinh}(ax)^2}{2x} - \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{2x^2} - 6a^2\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) \\
&\quad + a^2\text{arcsinh}(ax)^3\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) + \frac{3}{2}a^2\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2, -e^{\text{arcsinh}(ax)}\right) \\
&\quad - \frac{3}{2}a^2\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2, e^{\text{arcsinh}(ax)}\right) \\
&\quad - (3a^2)\,\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \text{arcsinh}(ax)\right) \\
&\quad + (3a^2)\,\text{Subst}\left(\int \log\left(1+e^x\right)\,dx, x, \text{arcsinh}(ax)\right) \\
&\quad - (3a^2)\,\text{Subst}\left(\int x\,\text{PolyLog}\left(2, -e^x\right)\,dx, x, \text{arcsinh}(ax)\right) \\
&\quad + (3a^2)\,\text{Subst}\left(\int x\,\text{PolyLog}\left(2, e^x\right)\,dx, x, \text{arcsinh}(ax)\right)
\end{aligned}$$

$$= -\frac{3a\text{arcsinh}(ax)^2}{2x} - \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{2x^2} - 6a^2\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right)$$
$$+ a^2\text{arcsinh}(ax)^3\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) + \frac{3}{2}a^2\text{arcsinh}(ax)^2 \text{PolyLog}\left(2, -e^{\text{arcsinh}(ax)}\right)$$
$$- \frac{3}{2}a^2\text{arcsinh}(ax)^2 \text{PolyLog}\left(2, e^{\text{arcsinh}(ax)}\right)$$
$$- 3a^2\text{arcsinh}(ax) \text{PolyLog}\left(3, -e^{\text{arcsinh}(ax)}\right)$$
$$+ 3a^2\text{arcsinh}(ax) \text{PolyLog}\left(3, e^{\text{arcsinh}(ax)}\right)$$
$$- \left(3a^2\right) \text{Subst}\left(\int \frac{\log(1-x)}{x} dx, x, e^{\text{arcsinh}(ax)}\right)$$
$$+ \left(3a^2\right) \text{Subst}\left(\int \frac{\log(1+x)}{x} dx, x, e^{\text{arcsinh}(ax)}\right)$$
$$+ \left(3a^2\right) \text{Subst}\left(\int \text{PolyLog}\left(3, -e^x\right) dx, x, \text{arcsinh}(ax)\right)$$
$$- \left(3a^2\right) \text{Subst}\left(\int \text{PolyLog}\left(3, e^x\right) dx, x, \text{arcsinh}(ax)\right)$$

$$= -\frac{3a\text{arcsinh}(ax)^2}{2x} - \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{2x^2} - 6a^2\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right)$$
$$+ a^2\text{arcsinh}(ax)^3\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) - 3a^2 \text{PolyLog}\left(2, -e^{\text{arcsinh}(ax)}\right)$$
$$+ \frac{3}{2}a^2\text{arcsinh}(ax)^2 \text{PolyLog}\left(2, -e^{\text{arcsinh}(ax)}\right)$$
$$+ 3a^2 \text{PolyLog}\left(2, e^{\text{arcsinh}(ax)}\right) - \frac{3}{2}a^2\text{arcsinh}(ax)^2 \text{PolyLog}\left(2, e^{\text{arcsinh}(ax)}\right)$$
$$- 3a^2\text{arcsinh}(ax) \text{PolyLog}\left(3, -e^{\text{arcsinh}(ax)}\right)$$
$$+ 3a^2\text{arcsinh}(ax) \text{PolyLog}\left(3, e^{\text{arcsinh}(ax)}\right)$$
$$+ \left(3a^2\right) \text{Subst}\left(\int \frac{\text{PolyLog}(3,-x)}{x} dx, x, e^{\text{arcsinh}(ax)}\right)$$
$$- \left(3a^2\right) \text{Subst}\left(\int \frac{\text{PolyLog}(3,x)}{x} dx, x, e^{\text{arcsinh}(ax)}\right)$$

$$= -\frac{3a\text{arcsinh}(ax)^2}{2x} - \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{2x^2} - 6a^2\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right)$$
$$+ a^2\text{arcsinh}(ax)^3\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) - 3a^2 \text{PolyLog}\left(2, -e^{\text{arcsinh}(ax)}\right)$$
$$+ \frac{3}{2}a^2\text{arcsinh}(ax)^2 \text{PolyLog}\left(2, -e^{\text{arcsinh}(ax)}\right)$$
$$+ 3a^2 \text{PolyLog}\left(2, e^{\text{arcsinh}(ax)}\right) - \frac{3}{2}a^2\text{arcsinh}(ax)^2 \text{PolyLog}\left(2, e^{\text{arcsinh}(ax)}\right)$$
$$- 3a^2\text{arcsinh}(ax) \text{PolyLog}\left(3, -e^{\text{arcsinh}(ax)}\right)$$
$$+ 3a^2\text{arcsinh}(ax) \text{PolyLog}\left(3, e^{\text{arcsinh}(ax)}\right)$$
$$+ 3a^2 \text{PolyLog}\left(4, -e^{\text{arcsinh}(ax)}\right) - 3a^2 \text{PolyLog}\left(4, e^{\text{arcsinh}(ax)}\right)$$

Mathematica [A] (verified)

Time = 3.27 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.45

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx$$
$$= \frac{a\left(-a\pi^4x + 2ax\operatorname{arcsinh}(ax)^4 - 12ax\operatorname{arcsinh}(ax)^2\coth\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right) - 2ax\operatorname{arcsinh}(ax)^3\operatorname{csch}^2\left(\frac{1}{2}\operatorname{arcsinh}(a\right.\right.}{}$$

```
[In] Integrate[ArcSinh[a*x]^3/(x^3*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (a*(-(a*Pi^4*x) + 2*a*x*ArcSinh[a*x]^4 - 12*a*x*ArcSinh[a*x]^2*Coth[ArcSinh
[a*x]/2] - 2*a*x*ArcSinh[a*x]^3*Csch[ArcSinh[a*x]/2]^2 + 48*a*x*ArcSinh[a*x
]*Log[1 - E^(-ArcSinh[a*x])] - 48*a*x*ArcSinh[a*x]*Log[1 + E^(-ArcSinh[a*x]
)] + 8*a*x*ArcSinh[a*x]^3*Log[1 + E^(-ArcSinh[a*x])] - 8*a*x*ArcSinh[a*x]^3
*Log[1 - E^ArcSinh[a*x]] - 24*a*x*(-2 + ArcSinh[a*x]^2)*PolyLog[2, -E^(-Arc
Sinh[a*x])] - 48*a*x*PolyLog[2, E^(-ArcSinh[a*x])] - 24*a*x*ArcSinh[a*x]^2*
PolyLog[2, E^ArcSinh[a*x]] - 48*a*x*ArcSinh[a*x]*PolyLog[3, -E^(-ArcSinh[a*
x])] + 48*a*x*ArcSinh[a*x]*PolyLog[3, E^ArcSinh[a*x]] - 48*a*x*PolyLog[4, -
E^(-ArcSinh[a*x])] - 48*a*x*PolyLog[4, E^ArcSinh[a*x]] + 12*a*x*ArcSinh[a*x
]^2*Tanh[ArcSinh[a*x]/2] - 4*ArcSinh[a*x]^3*Tanh[ArcSinh[a*x]/2]))/(16*x)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.80

method	result
default	$-\frac{\operatorname{arcsinh}(ax)^2\left(a^2x^2\operatorname{arcsinh}(ax)+3ax\sqrt{a^2x^2+1}+\operatorname{arcsinh}(ax)\right)}{2\sqrt{a^2x^2+1}\,x^2}+\frac{a^2\operatorname{arcsinh}(ax)^3\ln\left(1+ax+\sqrt{a^2x^2+1}\right)}{2}+\frac{3a^2\operatorname{arcsinh}(ax)^2\operatorname{polyl}}{}$

```
[In] int(arcsinh(a*x)^3/x^3/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a^2*x^2+1)^(1/2)/x^2*arcsinh(a*x)^2*(a^2*x^2*arcsinh(a*x)+3*a*x*(a^2*
x^2+1)^(1/2)+arcsinh(a*x))+1/2*a^2*arcsinh(a*x)^3*ln(1+a*x+(a^2*x^2+1)^(1/2
))+3/2*a^2*arcsinh(a*x)^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-3*a^2*arcsinh(a
*x)*polylog(3,-a*x-(a^2*x^2+1)^(1/2))+3*a^2*polylog(4,-a*x-(a^2*x^2+1)^(1/2
))-1/2*a^2*arcsinh(a*x)^3*ln(1-a*x-(a^2*x^2+1)^(1/2))-3/2*a^2*arcsinh(a*x)^
2*polylog(2,a*x+(a^2*x^2+1)^(1/2))+3*a^2*arcsinh(a*x)*polylog(3,a*x+(a^2*x^
2+1)^(1/2))-3*a^2*polylog(4,a*x+(a^2*x^2+1)^(1/2))-3*a^2*arcsinh(a*x)*ln(1+
a*x+(a^2*x^2+1)^(1/2))-3*a^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+3*a^2*arcsin
h(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))+3*a^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^3}{\sqrt{a^2x^2+1}x^3}\,dx$$

[In] integrate(arcsinh(a*x)^3/x^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^3/(a^2*x^5 + x^3), x)

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}^3\left(ax\right)}{x^3\sqrt{a^2x^2+1}}\,dx$$

[In] integrate(asinh(a*x)**3/x**3/(a**2*x**2+1)**(1/2),x)

[Out] Integral(asinh(a*x)**3/(x**3*sqrt(a**2*x**2 + 1)), x)

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^3}{\sqrt{a^2x^2+1}x^3}\,dx$$

[In] integrate(arcsinh(a*x)^3/x^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(arcsinh(a*x)^3/(sqrt(a^2*x^2 + 1)*x^3), x)

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^3}{\sqrt{a^2x^2+1}x^3}\,dx$$

[In] integrate(arcsinh(a*x)^3/x^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")

[Out] integrate(arcsinh(a*x)^3/(sqrt(a^2*x^2 + 1)*x^3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)^3}{x^3\,\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int(asinh(a*x)^3/(x^3*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(asinh(a*x)^3/(x^3*(a^2*x^2 + 1)^(1/2)), x)
```

3.350 $\int \frac{(c+a^2cx^2)^3}{\text{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \frac{(c+a^2cx^2)^3}{\text{arcsinh}(ax)}\,dx = \frac{35c^3\text{Chi}(\text{arcsinh}(ax))}{64a} + \frac{21c^3\text{Chi}(3\text{arcsinh}(ax))}{64a} + \frac{7c^3\text{Chi}(5\text{arcsinh}(ax))}{64a} + \frac{c^3\text{Chi}(7\text{arcsinh}(ax))}{64a}$$

```
[Out] 35/64*c^3*Chi(arcsinh(a*x))/a+21/64*c^3*Chi(3*arcsinh(a*x))/a+7/64*c^3*Chi(
5*arcsinh(a*x))/a+1/64*c^3*Chi(7*arcsinh(a*x))/a
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {5791, 3393, 3382}

$$\int \frac{(c+a^2cx^2)^3}{\text{arcsinh}(ax)}\,dx = \frac{35c^3\text{Chi}(\text{arcsinh}(ax))}{64a} + \frac{21c^3\text{Chi}(3\text{arcsinh}(ax))}{64a} + \frac{7c^3\text{Chi}(5\text{arcsinh}(ax))}{64a} + \frac{c^3\text{Chi}(7\text{arcsinh}(ax))}{64a}$$

```
[In] Int[(c + a^2*c*x^2)^3/ArcSinh[a*x],x]
```

```
[Out] (35*c^3*CoshIntegral[ArcSinh[a*x]])/(64*a) + (21*c^3*CoshIntegral[3*ArcSinh
[a*x]])/(64*a) + (7*c^3*CoshIntegral[5*ArcSinh[a*x]])/(64*a) + (c^3*CoshInt
egral[7*ArcSinh[a*x]])/(64*a)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{c^3\text{Subst}\left(\int \frac{\cosh^7(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{a} \\
&= \frac{c^3\text{Subst}\left(\int \left(\frac{35\cosh(x)}{64x} + \frac{21\cosh(3x)}{64x} + \frac{7\cosh(5x)}{64x} + \frac{\cosh(7x)}{64x}\right)\,dx, x, \text{arcsinh}(ax)\right)}{a} \\
&= \frac{c^3\text{Subst}\left(\int \frac{\cosh(7x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{64a} + \frac{(7c^3)\,\text{Subst}\left(\int \frac{\cosh(5x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{64a} \\
&\quad + \frac{(21c^3)\,\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{64a} + \frac{(35c^3)\,\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{64a} \\
&= \frac{35c^3\text{Chi}(\text{arcsinh}(ax))}{64a} + \frac{21c^3\text{Chi}(3\text{arcsinh}(ax))}{64a} \\
&\quad + \frac{7c^3\text{Chi}(5\text{arcsinh}(ax))}{64a} + \frac{c^3\text{Chi}(7\text{arcsinh}(ax))}{64a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

$$\int \frac{(c + a^2cx^2)^3}{\text{arcsinh}(ax)}\,dx = \frac{c^3(35\text{Chi}(\text{arcsinh}(ax)) + 21\text{Chi}(3\text{arcsinh}(ax)) + 7\text{Chi}(5\text{arcsinh}(ax)) + \text{Chi}(7\text{arcsinh}(ax)))}{64a}$$

```
[In] Integrate[(c + a^2*c*x^2)^3/ArcSinh[a*x],x]
[Out] (c^3*(35*CoshIntegral[ArcSinh[a*x]] + 21*CoshIntegral[3*ArcSinh[a*x]] + 7*C
oshIntegral[5*ArcSinh[a*x]] + CoshIntegral[7*ArcSinh[a*x]]))/(64*a)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.63

method	result	size
derivativedivides	$\frac{c^{3}\left(35\,\mathrm{Chi}\left(\mathrm{arcsinh}\left(ax\right)\right)+21\,\mathrm{Chi}\left(3\,\mathrm{arcsinh}\left(ax\right)\right)+7\,\mathrm{Chi}\left(5\,\mathrm{arcsinh}\left(ax\right)\right)+\mathrm{Chi}\left(7\,\mathrm{arcsinh}\left(ax\right)\right)\right)}{64a}$	42
default	$\frac{c^{3}\left(35\,\mathrm{Chi}\left(\mathrm{arcsinh}\left(ax\right)\right)+21\,\mathrm{Chi}\left(3\,\mathrm{arcsinh}\left(ax\right)\right)+7\,\mathrm{Chi}\left(5\,\mathrm{arcsinh}\left(ax\right)\right)+\mathrm{Chi}\left(7\,\mathrm{arcsinh}\left(ax\right)\right)\right)}{64a}$	42

```
[In] int((a^2*c*x^2+c)^3/arcsinh(a*x),x,method=_RETURNVERBOSE)
[Out] 1/64/a*c^3*(35*Chi(arcsinh(a*x))+21*Chi(3*arcsinh(a*x))+7*Chi(5*arcsinh(a*x
))+Chi(7*arcsinh(a*x)))
```

Fricas [F]

$$\int \frac{(c+a^2cx^2)^3}{\operatorname{arcsinh}(ax)}\,dx=\int \frac{(a^2cx^2+c)^3}{\operatorname{arsinh}(ax)}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^3/arcsinh(a*x),x, algorithm="fricas")
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/arcsinh(a*x),
x)
```

Sympy [F]

$$\int \frac{(c+a^2cx^2)^3}{\operatorname{arcsinh}(ax)}\,dx=c^3\left(\int \frac{3a^2x^2}{\operatorname{asinh}(ax)}\,dx+\int \frac{3a^4x^4}{\operatorname{asinh}(ax)}\,dx+\int \frac{a^6x^6}{\operatorname{asinh}(ax)}\,dx+\int \frac{1}{\operatorname{asinh}(ax)}\,dx\right)$$

```
[In] integrate((a**2*c*x**2+c)**3/asinh(a*x),x)
[Out] c**3*(Integral(3*a**2*x**2/asinh(a*x), x) + Integral(3*a**4*x**4/asinh(a*x)
, x) + Integral(a**6*x**6/asinh(a*x), x) + Integral(1/asinh(a*x), x))
```

Maxima [F]

$$\int \frac{(c + a^2cx^2)^3}{\text{arcsinh}(ax)} \, dx = \int \frac{(a^2cx^2 + c)^3}{\text{arsinh}\,(ax)} \, dx$$

```
[In] integrate((a^2*c*x^2+c)^3/arcsinh(a*x),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^3/arcsinh(a*x), x)
```

Giac [F]

$$\int \frac{(c + a^2cx^2)^3}{\text{arcsinh}(ax)} \, dx = \int \frac{(a^2cx^2 + c)^3}{\text{arsinh}\,(ax)} \, dx$$

```
[In] integrate((a^2*c*x^2+c)^3/arcsinh(a*x),x, algorithm="giac")
[Out] integrate((a^2*c*x^2 + c)^3/arcsinh(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c + a^2cx^2)^3}{\text{arcsinh}(ax)} \, dx = \int \frac{(c\,a^2\,x^2 + c)^3}{\text{asinh}\,(a\,x)} \, dx$$

```
[In] int((c + a^2*c*x^2)^3/asinh(a*x),x)
[Out] int((c + a^2*c*x^2)^3/asinh(a*x), x)
```

3.351 $\int \frac{(c+a^2cx^2)^2}{\text{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \frac{(c+a^2cx^2)^2}{\text{arcsinh}(ax)}\,dx = \frac{5c^2\text{Chi}(\text{arcsinh}(ax))}{8a} + \frac{5c^2\text{Chi}(3\text{arcsinh}(ax))}{16a} + \frac{c^2\text{Chi}(5\text{arcsinh}(ax))}{16a}$$

```
[Out] 5/8*c^2*Chi(arcsinh(a*x))/a+5/16*c^2*Chi(3*arcsinh(a*x))/a+1/16*c^2*Chi(5*a
rcsinh(a*x))/a
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {5791, 3393, 3382}

$$\int \frac{(c+a^2cx^2)^2}{\text{arcsinh}(ax)}\,dx = \frac{5c^2\text{Chi}(\text{arcsinh}(ax))}{8a} + \frac{5c^2\text{Chi}(3\text{arcsinh}(ax))}{16a} + \frac{c^2\text{Chi}(5\text{arcsinh}(ax))}{16a}$$

```
[In] Int[(c + a^2*c*x^2)^2/ArcSinh[a*x],x]
```

```
[Out] (5*c^2*CoshIntegral[ArcSinh[a*x]])/(8*a) + (5*c^2*CoshIntegral[3*ArcSinh[a*
x]])/(16*a) + (c^2*CoshIntegral[5*ArcSinh[a*x]])/(16*a)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{c^2\text{Subst}\left(\int \frac{\cosh^5(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{a}\\
&= \frac{c^2\text{Subst}\left(\int \left(\frac{5\cosh(x)}{8x}+\frac{5\cosh(3x)}{16x}+\frac{\cosh(5x)}{16x}\right)\,dx, x, \text{arcsinh}(ax)\right)}{a}\\
&= \frac{c^2\text{Subst}\left(\int \frac{\cosh(5x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{16a}+\frac{(5c^2)\,\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{16a}\\
&\quad+\frac{(5c^2)\,\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{8a}\\
&= \frac{5c^2\text{Chi}(\text{arcsinh}(ax))}{8a}+\frac{5c^2\text{Chi}(3\text{arcsinh}(ax))}{16a}+\frac{c^2\text{Chi}(5\text{arcsinh}(ax))}{16a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int \frac{(c+a^2cx^2)^2}{\text{arcsinh}(ax)}\,dx=\frac{c^2(10\text{Chi}(\text{arcsinh}(ax))+5\text{Chi}(3\text{arcsinh}(ax))+\text{Chi}(5\text{arcsinh}(ax)))}{16a}$$

```
[In] Integrate[(c + a^2*c*x^2)^2/ArcSinh[a*x],x]
[Out] (c^2*(10*CoshIntegral[ArcSinh[a*x]] + 5*CoshIntegral[3*ArcSinh[a*x]] + Cosh
Integral[5*ArcSinh[a*x]]))/(16*a)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.66

method	result	size
derivativedivides	$\frac{c^2(10\,\mathrm{Chi}(\mathrm{arcsinh}(ax))+5\,\mathrm{Chi}(3\,\mathrm{arcsinh}(ax))+\mathrm{Chi}(5\,\mathrm{arcsinh}(ax)))}{16a}$	33
default	$\frac{c^2(10\,\mathrm{Chi}(\mathrm{arcsinh}(ax))+5\,\mathrm{Chi}(3\,\mathrm{arcsinh}(ax))+\mathrm{Chi}(5\,\mathrm{arcsinh}(ax)))}{16a}$	33

```
[In] int((a^2*c*x^2+c)^2/arcsinh(a*x),x,method=_RETURNVERBOSE)
[Out] 1/16/a*c^2*(10*Chi(arcsinh(a*x))+5*Chi(3*arcsinh(a*x))+Chi(5*arcsinh(a*x)))
```

Fricas [F]

$$\int \frac{(c+a^2cx^2)^2}{\mathrm{arcsinh}(ax)}\,dx = \int \frac{(a^2cx^2+c)^2}{\mathrm{arsinh}\,(ax)}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="fricas")
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/arcsinh(a*x), x)
```

Sympy [F]

$$\int \frac{(c+a^2cx^2)^2}{\mathrm{arcsinh}(ax)}\,dx = c^2\left(\int \frac{2a^2x^2}{\mathrm{asinh}\,(ax)}\,dx + \int \frac{a^4x^4}{\mathrm{asinh}\,(ax)}\,dx + \int \frac{1}{\mathrm{asinh}\,(ax)}\,dx\right)$$

```
[In] integrate((a**2*c*x**2+c)**2/asinh(a*x),x)
[Out] c**2*(Integral(2*a**2*x**2/asinh(a*x), x) + Integral(a**4*x**4/asinh(a*x),
x) + Integral(1/asinh(a*x), x))
```

Maxima [F]

$$\int \frac{(c+a^2cx^2)^2}{\mathrm{arcsinh}(ax)}\,dx = \int \frac{(a^2cx^2+c)^2}{\mathrm{arsinh}\,(ax)}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^2/arcsinh(a*x), x)
```

Giac [F]

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{(a^2cx^2+c)^2}{\operatorname{arsinh}(ax)}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="giac")
[Out] integrate((a^2*c*x^2 + c)^2/arcsinh(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{(c\,a^2\,x^2+c)^2}{\operatorname{asinh}(a\,x)}\,dx$$

```
[In] int((c + a^2*c*x^2)^2/asinh(a*x),x)
[Out] int((c + a^2*c*x^2)^2/asinh(a*x), x)
```

3.352 $\int \frac{c+a^2cx^2}{\text{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 29

$$\int \frac{c+a^2cx^2}{\text{arcsinh}(ax)}\,dx = \frac{3c\text{Chi}(\text{arcsinh}(ax))}{4a} + \frac{c\text{Chi}(3\text{arcsinh}(ax))}{4a}$$

```
[Out] 3/4*c*Chi(arcsinh(a*x))/a+1/4*c*Chi(3*arcsinh(a*x))/a
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5791, 3393, 3382}

$$\int \frac{c+a^2cx^2}{\text{arcsinh}(ax)}\,dx = \frac{3c\text{Chi}(\text{arcsinh}(ax))}{4a} + \frac{c\text{Chi}(3\text{arcsinh}(ax))}{4a}$$

```
[In] Int[(c + a^2*c*x^2)/ArcSinh[a*x],x]
```

```
[Out] (3*c*CoshIntegral[ArcSinh[a*x]])/(4*a) + (c*CoshIntegral[3*ArcSinh[a*x]])/(
4*a)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{c\text{Subst}\left(\int \frac{\cosh^3(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{a} \\ &= \frac{c\text{Subst}\left(\int \left(\frac{3\cosh(x)}{4x} + \frac{\cosh(3x)}{4x}\right)\,dx, x, \text{arcsinh}(ax)\right)}{a} \\ &= \frac{c\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{4a} + \frac{(3c)\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{4a} \\ &= \frac{3c\text{Chi}(\text{arcsinh}(ax))}{4a} + \frac{c\text{Chi}(3\text{arcsinh}(ax))}{4a} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{c + a^2cx^2}{\text{arcsinh}(ax)}\,dx = \frac{c(3\text{Chi}(\text{arcsinh}(ax)) + \text{Chi}(3\text{arcsinh}(ax)))}{4a}$$

```
[In] Integrate[(c + a^2*c*x^2)/ArcSinh[a*x],x]
[Out] (c*(3*CoshIntegral[ArcSinh[a*x]] + CoshIntegral[3*ArcSinh[a*x]]))/(4*a)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{c(3\,\text{Chi}(\text{arcsinh}(ax))+\text{Chi}(3\,\text{arcsinh}(ax)))}{4a}$	22
default	$\frac{c(3\,\text{Chi}(\text{arcsinh}(ax))+\text{Chi}(3\,\text{arcsinh}(ax)))}{4a}$	22

```
[In] int((a^2*c*x^2+c)/arcsinh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a*c*(3*Chi(arcsinh(a*x))+Chi(3*arcsinh(a*x)))
```

Fricas [F]

$$\int \frac{c + a^2cx^2}{\operatorname{arcsinh}(ax)} dx = \int \frac{a^2cx^2 + c}{\operatorname{arsinh}(ax)} dx$$

```
[In] integrate((a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)/arcsinh(a*x), x)
```

Sympy [F]

$$\int \frac{c + a^2cx^2}{\operatorname{arcsinh}(ax)} dx = c \left(\int \frac{a^2x^2}{\operatorname{asinh}(ax)} dx + \int \frac{1}{\operatorname{asinh}(ax)} dx \right)$$

```
[In] integrate((a**2*c*x**2+c)/asinh(a*x),x)
[Out] c*(Integral(a**2*x**2/asinh(a*x), x) + Integral(1/asinh(a*x), x))
```

Maxima [F]

$$\int \frac{c + a^2cx^2}{\operatorname{arcsinh}(ax)} dx = \int \frac{a^2cx^2 + c}{\operatorname{arsinh}(ax)} dx$$

```
[In] integrate((a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)/arcsinh(a*x), x)
```

Giac [F]

$$\int \frac{c + a^2cx^2}{\operatorname{arcsinh}(ax)} dx = \int \frac{a^2cx^2 + c}{\operatorname{arsinh}(ax)} dx$$

```
[In] integrate((a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="giac")
[Out] integrate((a^2*c*x^2 + c)/arcsinh(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c + a^2 c x^2}{\operatorname{arcsinh}(ax)} \, dx = \int \frac{c\,a^2\,x^2 + c}{\operatorname{asinh}(a\,x)} \, dx$$

```
[In] int((c + a^2*c*x^2)/asinh(a*x),x)
[Out] int((c + a^2*c*x^2)/asinh(a*x), x)
```

3.353 $\int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)}\,dx = \operatorname{Int}\left(\frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)},x\right)$$

```
[Out] Unintegrable(1/(a^2*c*x^2+c)/arcsinh(a*x),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)}\,dx$$

```
[In] Int[1/((c + a^2*c*x^2)*ArcSinh[a*x]),x]
[Out] Defer[Int][1/((c + a^2*c*x^2)*ArcSinh[a*x]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)\,\text{arcsinh}(ax)}\,dx = \int \frac{1}{(c+a^2cx^2)\,\text{arcsinh}(ax)}\,dx$$

```
[In] Integrate[1/((c + a^2*c*x^2)*ArcSinh[a*x]),x]
[Out] Integrate[1/((c + a^2*c*x^2)*ArcSinh[a*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a^2c\,x^2+c)\,\text{arcsinh}\,(ax)}dx$$

```
[In] int(1/(a^2*c*x^2+c)/arcsinh(a*x),x)
[Out] int(1/(a^2*c*x^2+c)/arcsinh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)\,\text{arcsinh}(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)\,\text{arsinh}\,(ax)}\,dx$$

```
[In] integrate(1/(a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="fricas")
[Out] integral(1/((a^2*c*x^2 + c)*arcsinh(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{1}{(c+a^2cx^2)\,\text{arcsinh}(ax)}\,dx = \frac{\int \frac{1}{a^2x^2\,\text{asinh}\,(ax)+\text{asinh}\,(ax)}\,dx}{c}$$

```
[In] integrate(1/(a**2*c*x**2+c)/asinh(a*x),x)
[Out] Integral(1/(a**2*x**2*asinh(a*x) + asinh(a*x)), x)/c
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)\operatorname{arsinh}(ax)}\,dx$$

```
[In] integrate(1/(a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="maxima")
[Out] integrate(1/((a^2*c*x^2 + c)*arcsinh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)\operatorname{arsinh}(ax)}\,dx$$

```
[In] integrate(1/(a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="giac")
[Out] integrate(1/((a^2*c*x^2 + c)*arcsinh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 2.59 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{\operatorname{asinh}(a\,x)\,\left(c\,a^2\,x^2+c\right)}\,dx$$

```
[In] int(1/(asinh(a*x)*(c + a^2*c*x^2)),x)
[Out] int(1/(asinh(a*x)*(c + a^2*c*x^2)), x)
```

3.354 $\int \frac{1}{(c+a^2cx^2)^2\text{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{1}{(c+a^2cx^2)^2\,\text{arcsinh}(ax)}\,dx=\text{Int}\left(\frac{1}{(c+a^2cx^2)^2\,\text{arcsinh}(ax)},x\right)$$

```
[Out] Unintegrable(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(c+a^2cx^2)^2\,\text{arcsinh}(ax)}\,dx=\int \frac{1}{(c+a^2cx^2)^2\,\text{arcsinh}(ax)}\,dx$$

```
[In] Int[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]),x]
```

```
[Out] Defer[Int][1/((c + a^2*c*x^2)^2*ArcSinh[a*x]), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(c+a^2cx^2)^2\,\text{arcsinh}(ax)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)} \, dx = \int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)} \, dx$$

```
[In] Integrate[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]),x]
[Out] Integrate[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a^2c\,x^2+c)^2 \operatorname{arcsinh}(ax)}dx$$

```
[In] int(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x)
[Out] int(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)} \, dx = \int \frac{1}{(a^2cx^2+c)^2 \operatorname{arsinh}(ax)} \, dx$$

```
[In] integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="fricas")
[Out] integral(1/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcsinh(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)} \, dx = \frac{\int \frac{1}{a^4x^4 \operatorname{asinh}(ax)+2a^2x^2 \operatorname{asinh}(ax)+\operatorname{asinh}(ax)} \, dx}{c^2}$$

```
[In] integrate(1/(a**2*c*x**2+c)**2/asinh(a*x),x)
[Out] Integral(1/(a**4*x**4*asinh(a*x) + 2*a**2*x**2*asinh(a*x) + asinh(a*x)), x)
/c**2
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{\left(c+a^2cx^2\right)^2 \operatorname{arcsinh}(ax)} \, dx = \int \frac{1}{\left(a^2cx^2+c\right)^2 \operatorname{arsinh}\left(ax\right)} \, dx$$

```
[In] integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="maxima")
[Out] integrate(1/((a^2*c*x^2 + c)^2*arcsinh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{\left(c+a^2cx^2\right)^2 \operatorname{arcsinh}(ax)} \, dx = \int \frac{1}{\left(a^2cx^2+c\right)^2 \operatorname{arsinh}\left(ax\right)} \, dx$$

```
[In] integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="giac")
[Out] integrate(1/((a^2*c*x^2 + c)^2*arcsinh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 2.54 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{\left(c+a^2cx^2\right)^2 \operatorname{arcsinh}(ax)} \, dx = \int \frac{1}{\operatorname{asinh}\left(a\,x\right)\,{\left(c\,a^2\,x^2+c\right)}^2} \, dx$$

```
[In] int(1/(asinh(a*x)*(c + a^2*c*x^2)^2),x)
[Out] int(1/(asinh(a*x)*(c + a^2*c*x^2)^2), x)
```

3.355 $\int \frac{x^4\sqrt{1+c^2x^2}}{a+b\operatorname{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 206

$$\begin{aligned}\int \frac{x^4\sqrt{1+c^2x^2}}{a+b\operatorname{arcsinh}(cx)}\,dx = &-\frac{\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{32bc^5}\\&-\frac{\cosh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(\frac{4(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5}\\&+\frac{\cosh\left(\frac{6a}{b}\right)\operatorname{Chi}\left(\frac{6(a+b\operatorname{arcsinh}(cx))}{b}\right)}{32bc^5}\\&+\frac{\log(a+b\operatorname{arcsinh}(cx))}{16bc^5}+\frac{\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{32bc^5}\\&+\frac{\sinh\left(\frac{4a}{b}\right)\operatorname{Shi}\left(\frac{4(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5}\\&-\frac{\sinh\left(\frac{6a}{b}\right)\operatorname{Shi}\left(\frac{6(a+b\operatorname{arcsinh}(cx))}{b}\right)}{32bc^5}\end{aligned}$$

```
[Out] -1/32*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c^5-1/16*Chi(4*(a+b*arcsinh
(c*x))/b)*cosh(4*a/b)/b/c^5+1/32*Chi(6*(a+b*arcsinh(c*x))/b)*cosh(6*a/b)/b/
c^5+1/16*ln(a+b*arcsinh(c*x))/b/c^5+1/32*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2
*a/b)/b/c^5+1/16*Shi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b/c^5-1/32*Shi(6*(
a+b*arcsinh(c*x))/b)*sinh(6*a/b)/b/c^5
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5819, 5556, 3384, 3379, 3382}

$$\int \frac{x^4\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = -\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^5} - \frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^5} + \frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^5} + \frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^5} + \frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^5} - \frac{\sinh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^5} + \frac{\log(a+b\text{arcsinh}(cx))}{16bc^5}$$

```
[In] Int[(x^4*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/32*(Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(b*c^5) - (C
osh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(16*b*c^5) + (Cosh[(
6*a)/b]*CoshIntegral[(6*(a + b*ArcSinh[c*x]))/b])/(32*b*c^5) + Log[a + b*Ar
cSinh[c*x]]/(16*b*c^5) + (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]
))/b])/(32*b*c^5) + (Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b]
)/(16*b*c^5) - (Sinh[(6*a)/b]*SinhIntegral[(6*(a + b*ArcSinh[c*x]))/b])/(32
*b*c^5)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\cosh^2\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc^5}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{16x}+\frac{\cosh\left(\frac{6a}{b}-\frac{6x}{b}\right)}{32x}-\frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{16x}-\frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{32x}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc^5}$$

$$= \frac{\log(a+b\text{arcsinh}(cx))}{16bc^5}+\frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{6a}{b}-\frac{6x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{32bc^5}$$
$$-\frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{32bc^5}$$
$$-\frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{16bc^5}$$

$$= \frac{\log(a+b\operatorname{arcsinh}(cx))}{16bc^5} - \frac{\cosh\left(\frac{2a}{b}\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{32bc^5}$$
$$- \frac{\cosh\left(\frac{4a}{b}\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{16bc^5}$$
$$+ \frac{\cosh\left(\frac{6a}{b}\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{6x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{32bc^5}$$
$$+ \frac{\sinh\left(\frac{2a}{b}\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{32bc^5}$$
$$+ \frac{\sinh\left(\frac{4a}{b}\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{16bc^5}$$
$$- \frac{\sinh\left(\frac{6a}{b}\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{6x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{32bc^5}$$
$$= -\frac{\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{32bc^5} - \frac{\cosh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(\frac{4(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5}$$
$$+ \frac{\cosh\left(\frac{6a}{b}\right)\operatorname{Chi}\left(\frac{6(a+b\operatorname{arcsinh}(cx))}{b}\right)}{32bc^5}$$
$$+ \frac{\log(a+b\operatorname{arcsinh}(cx))}{16bc^5} + \frac{\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{32bc^5}$$
$$+ \frac{\sinh\left(\frac{4a}{b}\right)\operatorname{Shi}\left(\frac{4(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5} - \frac{\sinh\left(\frac{6a}{b}\right)\operatorname{Shi}\left(\frac{6(a+b\operatorname{arcsinh}(cx))}{b}\right)}{32bc^5}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.74

$$\int \frac{x^4\sqrt{1+c^2x^2}}{a+b\operatorname{arcsinh}(cx)}\,dx$$
$$= \frac{-\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(2\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\right) - 2\cosh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(4\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\right) + \cosh\left(\frac{6a}{b}\right)\operatorname{Chi}\left(6\left(\frac{a}{b}+\operatorname{arcsin}\right.\right.}{}$$

```
[In] Integrate[(x^4*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])]) - 2*Cosh[(4*a)/b]*Co
shIntegral[4*(a/b + ArcSinh[c*x])] + Cosh[(6*a)/b]*CoshIntegral[6*(a/b + Ar
cSinh[c*x])] + 2*Log[a + b*ArcSinh[c*x]] + Sinh[(2*a)/b]*SinhIntegral[2*(a/
b + ArcSinh[c*x])] + 2*Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] -
 Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])])/(32*b*c^5)
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.79

method	result
default	$-\frac{e^{\frac{6a}{b}} \operatorname{Ei}_1\left(6 \operatorname{arcsinh}(cx)+\frac{6a}{b}\right)-2 e^{\frac{4a}{b}} \operatorname{Ei}_1\left(4 \operatorname{arcsinh}(cx)+\frac{4a}{b}\right)-e^{\frac{2a}{b}} \operatorname{Ei}_1\left(2 \operatorname{arcsinh}(cx)+\frac{2a}{b}\right)-e^{-\frac{2a}{b}} \operatorname{Ei}_1\left(-2 \operatorname{arcsinh}(cx)-\frac{2a}{b}\right)-2 e^{-}}{64c^5b}$

[In] int(x^4*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)

[Out] -1/64*(exp(6*a/b)*Ei(1,6*arcsinh(c*x)+6*a/b)-2*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)-exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)-2*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)+exp(-6*a/b)*Ei(1,-6*arcsinh(c*x)-6*a/b)-4*ln(a+b*arcsinh(c*x)))/c^5/b

Fricas [F]

$$\int \frac{x^4\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{c^2x^2+1}x^4}{b\operatorname{arsinh}(cx)+a}\,dx$$

[In] integrate(x^4*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")

[Out] integral(sqrt(c^2*x^2 + 1)*x^4/(b*arcsinh(c*x) + a), x)

Sympy [F]

$$\int \frac{x^4\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{x^4\sqrt{c^2x^2+1}}{a+b\operatorname{asinh}(cx)}\,dx$$

[In] integrate(x**4*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)

[Out] Integral(x**4*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)

Maxima [F]

$$\int \frac{x^4\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{c^2x^2+1}x^4}{b\operatorname{arsinh}(cx)+a}\,dx$$

[In] integrate(x^4*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")

[Out] integrate(sqrt(c^2*x^2 + 1)*x^4/(b*arcsinh(c*x) + a), x)

Giac [F]

$$\int \frac{x^4\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{\sqrt{c^2x^2+1}x^4}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate(x^4*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(sqrt(c^2*x^2 + 1)*x^4/(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x^4\,\sqrt{c^2\,x^2+1}}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

```
[In] int((x^4*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)),x)
[Out] int((x^4*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)), x)
```

3.356 $\int \frac{x^3\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 183

$$\int \frac{x^3\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8bc^4} + \frac{\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16bc^4} - \frac{\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{16bc^4} - \frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8bc^4} - \frac{\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^4} + \frac{\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^4}$$

```
[Out] -1/8*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^4-1/16*cosh(3*a/b)*Shi(3*(a+b*
arcsinh(c*x))/b)/b/c^4+1/16*cosh(5*a/b)*Shi(5*(a+b*arcsinh(c*x))/b)/b/c^4+1
/8*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^4+1/16*Chi(3*(a+b*arcsinh(c*x))/
b)*sinh(3*a/b)/b/c^4-1/16*Chi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b/c^4
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used

$= \{5819, 5556, 3384, 3379, 3382\}$

$$\int \frac{x^3\sqrt{1+c^2x^2}}{a+b\operatorname{arcsinh}(cx)}\,dx = \frac{\sinh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{8bc^4} + \frac{\sinh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^4} - \frac{\sinh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^4} - \frac{\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{8bc^4} - \frac{\cosh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^4} + \frac{\cosh\left(\frac{5a}{b}\right)\operatorname{Shi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^4}$$

```
[In] Int[(x^3*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(8*b*c^4) + (CoshIntegral[
(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(16*b*c^4) - (CoshIntegral[(5*(a
 + b*ArcSinh[c*x]))/b]*Sinh[(5*a)/b])/(16*b*c^4) - (Cosh[a/b]*SinhIntegral[
(a + b*ArcSinh[c*x])/b])/(8*b*c^4) - (Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*
ArcSinh[c*x]))/b])/(16*b*c^4) + (Cosh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSi
nh[c*x]))/b])/(16*b*c^4)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\cosh^2\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{bc^4}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16x}-\frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16x}-\frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{8x}\right)dx, x, a+b\operatorname{arcsinh}(cx)\right)}{bc^4}\\
&= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{16bc^4}\\
&\quad+\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{16bc^4}\\
&\quad+\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{8bc^4}\\
&= -\frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{8bc^4}\\
&\quad-\frac{\cosh\left(\frac{3a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{16bc^4}\\
&\quad+\frac{\cosh\left(\frac{5a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{5x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{16bc^4}\\
&\quad+\frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{8bc^4}\\
&\quad+\frac{\sinh\left(\frac{3a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{16bc^4}\\
&\quad-\frac{\sinh\left(\frac{5a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{5x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{16bc^4}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\mathrm{Chi}\left(\frac{a+b\mathrm{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8bc^4} + \frac{\mathrm{Chi}\left(\frac{3(a+b\mathrm{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16bc^4} \\
&\quad - \frac{\mathrm{Chi}\left(\frac{5(a+b\mathrm{arcsinh}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{16bc^4} - \frac{\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\mathrm{arcsinh}(cx)}{b}\right)}{8bc^4} \\
&\quad - \frac{\cosh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3(a+b\mathrm{arcsinh}(cx))}{b}\right)}{16bc^4} + \frac{\cosh\left(\frac{5a}{b}\right)\mathrm{Shi}\left(\frac{5(a+b\mathrm{arcsinh}(cx))}{b}\right)}{16bc^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.74

$$
\int \frac{x^3\sqrt{1+c^2x^2}}{a+b\mathrm{arcsinh}(cx)}\,dx
= \frac{2\mathrm{Chi}\left(\frac{a}{b}+\mathrm{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right)+\mathrm{Chi}\left(3\left(\frac{a}{b}+\mathrm{arcsinh}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)-\mathrm{Chi}\left(5\left(\frac{a}{b}+\mathrm{arcsinh}(cx)\right)\right)\sinh\left(\frac{5a}{b}\right)}{1}
$$

```
[In] Integrate[(x^3*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] + CoshIntegral[3*(a/b + ArcSi
nh[c*x])]*Sinh[(3*a)/b] - CoshIntegral[5*(a/b + ArcSinh[c*x])]*Sinh[(5*a)/b
] - 2*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - Cosh[(3*a)/b]*SinhIntegr
al[3*(a/b + ArcSinh[c*x])] + Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*
x])])/(16*b*c^4)
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.81

method	result
default	$\frac{e^{\frac{5a}{b}}\,\mathrm{Ei}_1\left(5\,\mathrm{arcsinh}(cx)+\frac{5a}{b}\right)-e^{\frac{3a}{b}}\,\mathrm{Ei}_1\left(3\,\mathrm{arcsinh}(cx)+\frac{3a}{b}\right)-2\,e^{\frac{a}{b}}\,\mathrm{Ei}_1\left(\mathrm{arcsinh}(cx)+\frac{a}{b}\right)+2\,e^{-\frac{a}{b}}\,\mathrm{Ei}_1\left(-\mathrm{arcsinh}(cx)-\frac{a}{b}\right)+e^{-\frac{3a}{b}}\,\mathrm{Ei}_1\left(-3\,\mathrm{a}\right.}{32c^4b}$

```
[In] int(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32*(exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-exp(3*a/b)*Ei(1,3*arcsinh(c*x)+
3*a/b)-2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)+2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b
)+exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-
5*a/b))/c^4/b
```

Fricas [F]

$$\int \frac{x^3\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{c^2x^2+1}\,x^3}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^3/(b*arcsinh(c*x) + a), x)
```

Sympy [F]

$$\int \frac{x^3\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{x^3\sqrt{c^2x^2+1}}{a+b\,\text{asinh}\,(cx)}\,dx$$

```
[In] integrate(x**3*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
[Out] Integral(x**3*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{x^3\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{c^2x^2+1}\,x^3}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(c^2*x^2 + 1)*x^3/(b*arcsinh(c*x) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{x^3\,\sqrt{c^2\,x^2+1}}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

```
[In] int((x^3*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)),x)
[Out] int((x^3*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)), x)
```

3.357 $\int \frac{x^2\sqrt{1+c^2x^2}}{a+b\,\mathbf{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 82

$$\int \frac{x^2\sqrt{1+c^2x^2}}{a+b\,\text{arcsinh}(cx)}\,dx = \frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8bc^3} - \frac{\log(a+b\,\text{arcsinh}(cx))}{8bc^3} - \frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8bc^3}$$

```
[Out] 1/8*Chi(4*(a+b*arcsinh(c*x))/b)*cosh(4*a/b)/b/c^3-1/8*ln(a+b*arcsinh(c*x))/
b/c^3-1/8*Shi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b/c^3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5819, 5556, 3384, 3379, 3382}

$$\int \frac{x^2\sqrt{1+c^2x^2}}{a+b\,\text{arcsinh}(cx)}\,dx = \frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8bc^3} - \frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8bc^3} - \frac{\log(a+b\,\text{arcsinh}(cx))}{8bc^3}$$

```
[In] Int[(x^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Cosh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(8*b*c^3) - Log[a
+ b*ArcSinh[c*x]]/(8*b*c^3) - (Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh
[c*x]))/b])/(8*b*c^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\cosh^2\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc^3}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{8x}+\frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{8x}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc^3}$$
$$= -\frac{\log(a+b\text{arcsinh}(cx))}{8bc^3}+\frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8bc^3}$$

$$= -\frac{\log(a+b\text{arcsinh}(cx))}{8bc^3} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8bc^3} - \frac{\sinh\left(\frac{4a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8bc^3}$$

$$= \frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8bc^3} - \frac{\log(a+b\text{arcsinh}(cx))}{8bc^3} - \frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8bc^3}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int \frac{x^2\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right) - \log(a+b\text{arcsinh}(cx)) - \sinh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)}{8bc^3}$$

```
[In] Integrate[(x^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
[Out] (Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcSinh[c*x])] - Log[a + b*ArcSinh[c*x
]] - Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])])/(8*b*c^3)
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{e^{\frac{4a}{b}}\text{Ei}_1\left(4\operatorname{arcsinh}(cx)+\frac{4a}{b}\right)+e^{-\frac{4a}{b}}\text{Ei}_1\left(-4\operatorname{arcsinh}(cx)-\frac{4a}{b}\right)+2\ln(a+b\operatorname{arcsinh}(cx))}{16c^3b}$	67

```
[In] int(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
[Out] -1/16*(exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)+exp(-4*a/b)*Ei(1,-4*arcsinh(c*
x)-4*a/b)+2*ln(a+b*arcsinh(c*x)))/c^3/b
```

Fricas [F]

$$\int \frac{x^2\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{\sqrt{c^2x^2+1}x^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^2/(b*arcsinh(c*x) + a), x)
```

Sympy [F]

$$\int \frac{x^2\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x^2\sqrt{c^2x^2+1}}{a+b\,\text{asinh}\,(cx)}\,dx$$

```
[In] integrate(x**2*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
[Out] Integral(x**2*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{\sqrt{c^2x^2+1}x^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(c^2*x^2 + 1)*x^2/(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{\sqrt{c^2x^2+1}x^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(sqrt(c^2*x^2 + 1)*x^2/(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{x^2\,\sqrt{c^2\,x^2+1}}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

```
[In] int((x^2*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)),x)
[Out] int((x^2*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)), x)
```

3.358 $\int \frac{x\sqrt{1+c^2x^2}}{a+b\,\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 121

$$\int \frac{x\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=-\frac{\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4bc^2}-\frac{\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4bc^2}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4bc^2}+\frac{\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4bc^2}$$

```
[Out] 1/4*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^2+1/4*cosh(3*a/b)*Shi(3*(a+b*ar
csinh(c*x))/b)/b/c^2-1/4*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^2-1/4*Chi(
3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5819, 5556, 3384, 3379, 3382}

$$\int \frac{x\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4bc^2}-\frac{\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4bc^2}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4bc^2}+\frac{\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4bc^2}$$

```
[In] Int[(x*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/4*(CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(b*c^2) - (CoshIntegr
al[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(4*b*c^2) + (Cosh[a/b]*SinhIn
```

tegral[(a + b*ArcSinh[c*x])/b])/(4*b*c^2) + (Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(4*b*c^2)

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5556

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\cosh^2\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc^2}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4x}+\frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{4x}\right)dx, x, a+b\text{arcsinh}(cx)\right)}{bc^2}\\
&= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4bc^2}-\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4bc^2}
\end{aligned}$$

$$=\frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{4bc^2}+\frac{\cosh\left(\frac{3a}{b}\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{4bc^2}-\frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{4bc^2}-\frac{\sinh\left(\frac{3a}{b}\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{4bc^2}$$

$$=-\frac{\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4bc^2}-\frac{\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4bc^2}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4bc^2}+\frac{\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4bc^2}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

$$\int\frac{x\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\frac{-\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right)-\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)+\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)+c}{4bc^2}$$

```
[In] Integrate[(x*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-(CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b]) - CoshIntegral[3*(a/b + ArcS
inh[c*x])]*Sinh[(3*a)/b] + Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + Cos
h[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])])/(4*b*c^2)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{e^{\frac{3a}{b}}\,\text{Ei}_1\left(3\,\text{arcsinh}(cx)+\frac{3a}{b}\right)+e^{\frac{a}{b}}\,\text{Ei}_1\left(\text{arcsinh}(cx)+\frac{a}{b}\right)-e^{-\frac{a}{b}}\,\text{Ei}_1\left(-\text{arcsinh}(cx)-\frac{a}{b}\right)-e^{-\frac{3a}{b}}\,\text{Ei}_1\left(-3\,\text{arcsinh}(cx)-\frac{3a}{b}\right)}{8c^2b}$	100

```
[In] int(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-
exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)-exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b))/
c^2/b
```

Fricas [F]

$$\int \frac{x\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{\sqrt{c^2x^2+1}x}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x/(b*arcsinh(c*x) + a), x)
```

Sympy [F]

$$\int \frac{x\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x\sqrt{c^2x^2+1}}{a+b\,\text{asinh}\,(cx)}\,dx$$

```
[In] integrate(x*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
[Out] Integral(x*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{x\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{\sqrt{c^2x^2+1}x}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(c^2*x^2 + 1)*x/(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{x\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{\sqrt{c^2x^2+1}x}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(sqrt(c^2*x^2 + 1)*x/(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{x\,\sqrt{c^2\,x^2+1}}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

```
[In] int((x*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)),x)
[Out] int((x*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)), x)
```

3.359 $\int \frac{\sqrt{1+c^2x^2}}{a+b\textbf{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc}+\frac{\log(a+b\text{arcsinh}(cx))}{2bc}-\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc}$$

```
[Out] 1/2*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c+1/2*ln(a+b*arcsinh(c*x))/b/
c-1/2*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b/c
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5791, 3393, 3384, 3379, 3382}

$$\int \frac{\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc}-\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc}+\frac{\log(a+b\text{arcsinh}(cx))}{2bc}$$

```
[In] Int[Sqrt[1 + c^2*x^2]/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(2*b*c) + Log[a +
b*ArcSinh[c*x]]/(2*b*c) - (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x
]))/b])/(2*b*c)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cosh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{2x}+\frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2x}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc}\\
&= \frac{\log(a+b\text{arcsinh}(cx))}{2bc}+\frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2bc}\\
&= \frac{\log(a+b\text{arcsinh}(cx))}{2bc}+\frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2bc}\\
&\quad-\frac{\sinh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2bc}
\end{aligned}$$

$$= \frac{\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{2bc} + \frac{\log(a+b\operatorname{arcsinh}(cx))}{2bc} - \frac{\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{2bc}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{1+c^2x^2}}{a+b\operatorname{arcsinh}(cx)}\,dx = \frac{\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(2\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\right)+\log(a+b\operatorname{arcsinh}(cx))-\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(2\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\right)}{2bc}$$

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(a + b*ArcSinh[c*x]),x]
[Out] (Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] + Log[a + b*ArcSinh[c*x
]] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])])/(2*b*c)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{e^{\frac{2a}{b}}\operatorname{Ei}_1\left(2\operatorname{arcsinh}(cx)+\frac{2a}{b}\right)+e^{-\frac{2a}{b}}\operatorname{Ei}_1\left(-2\operatorname{arcsinh}(cx)-\frac{2a}{b}\right)-2\ln(a+b\operatorname{arcsinh}(cx))}{4bc}$	67

```
[In] int((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
[Out] -1/4*(exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)+exp(-2*a/b)*Ei(1,-2*arcsinh(c*x
)-2*a/b)-2*ln(a+b*arcsinh(c*x)))/b/c
```

Fricas [F]

$$\int \frac{\sqrt{1+c^2x^2}}{a+b\operatorname{arcsinh}(cx)}\,dx = \int \frac{\sqrt{c^2x^2+1}}{b\operatorname{arsinh}(cx)+a}\,dx$$

```
[In] integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{c^2x^2+1}}{a+b\,\text{asinh}\,(cx)}\,dx$$

```
[In] integrate((c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
[Out] Integral(sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{c^2x^2+1}}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{c^2x^2+1}}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{c^2\,x^2+1}}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

```
[In] int((c^2*x^2 + 1)^(1/2)/(a + b*asinh(c*x)),x)
[Out] int((c^2*x^2 + 1)^(1/2)/(a + b*asinh(c*x)), x)
```

3.360 $\int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))}\,dx = -\frac{\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b} + \text{Int}\left(\frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))},x\right)$$

```
[Out] cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b-Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b
+Unintegrable(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Int[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])),x]
```

```
[Out] -((CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/b) + (Cosh[a/b]*SinhInte
gral[(a + b*ArcSinh[c*x])/b])/b + Defer[Int][1/(x*Sqrt[1 + c^2*x^2]*(a + b*
ArcSinh[c*x])), x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}+\frac{c^2x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\right)dx\\
&= c^2\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx+\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx\\
&= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b}+\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx\\
&= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b}\\
&\quad-\frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b}\\
&\quad+\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx\\
&= -\frac{\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b}\\
&\quad+\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx
\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 2.53 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))}\,dx=\int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c^2x^2+1}}{x\,(a+b\,\operatorname{arcsinh}\,(cx))}dx$$

```
[In] int((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x)),x)
[Out] int((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\operatorname{arcsinh}(cx))}\,dx = \int \frac{\sqrt{c^2x^2+1}}{(b\operatorname{arsinh}(cx)+a)x}\,dx$$

```
[In] integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(b*x*arcsinh(c*x) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\operatorname{arcsinh}(cx))}\,dx = \int \frac{\sqrt{c^2x^2+1}}{x\,(a+b\operatorname{asinh}(cx))}\,dx$$

```
[In] integrate((c**2*x**2+1)**(1/2)/x/(a+b*asinh(c*x)),x)
[Out] Integral(sqrt(c**2*x**2 + 1)/(x*(a + b*asinh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\operatorname{arcsinh}(cx))}\,dx = \int \frac{\sqrt{c^2x^2+1}}{(b\operatorname{arsinh}(cx)+a)x}\,dx$$

```
[In] integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\sqrt{c^2\,x^2+1}}{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

```
[In] int((c^2*x^2 + 1)^(1/2)/(x*(a + b*asinh(c*x))),x)
```

```
[Out] int((c^2*x^2 + 1)^(1/2)/(x*(a + b*asinh(c*x))), x)
```

3.361 $\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \frac{c\log(a+b\text{arcsinh}(cx))}{b}+\text{Int}\left(\frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))},x\right)$$

```
[Out] c*ln(a+b*arcsinh(c*x))/b+Unintegrable(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^
(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Int[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (c*Log[a + b*ArcSinh[c*x]])/b + Defer[Int][1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*
ArcSinh[c*x])), x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{c^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}+\frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\right)dx\\&= c^2\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx+\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx\\&= \frac{c\log(a+b\text{arcsinh}(cx))}{b}+\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 2.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))}\,dx=\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c^2x^2+1}}{x^2\left(a+b\ \text{arcsinh}\left(cx\right)\right)}dx$$

```
[In] int((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x)),x)
[Out] int((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))}\,dx=\int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\left(cx\right)+a)x^2}\,dx$$

```
[In] integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(b*x^2*arcsinh(c*x) + a*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\sqrt{c^2x^2+1}}{x^2\,(a+b\,\text{asinh}\,(cx))}\,dx$$

```
[In] integrate((c**2*x**2+1)**(1/2)/x**2/(a+b*asinh(c*x)),x)
[Out] Integral(sqrt(c**2*x**2 + 1)/(x**2*(a + b*asinh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\,(cx)+a)x^2}\,dx$$

```
[In] integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\,(cx)+a)x^2}\,dx$$

```
[In] integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))}\,dx=\int \frac{\sqrt{c^2\,x^2+1}}{x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

```
[In] int((c^2*x^2 + 1)^(1/2)/(x^2*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(1/2)/(x^2*(a + b*asinh(c*x))), x)
```

3.362 $\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\textbf{arcsinh}(cx))} dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\operatorname{arcsinh}(cx))} dx = \operatorname{Int}\left(\frac{\sqrt{1+c^2x^2}}{x^3(a+b\operatorname{arcsinh}(cx))}, x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\operatorname{arcsinh}(cx))} dx = \int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\operatorname{arcsinh}(cx))} dx$$

```
[In] Int[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\operatorname{arcsinh}(cx))} dx$$

Mathematica [N/A]

Not integrable

Time = 4.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c^2x^2+1}}{x^3\left(a+b\ \text{arcsinh}\left(cx\right)\right)}dx$$

```
[In] int((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x)
[Out] int((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\left(cx\right)+a)x^3}\,dx$$

```
[In] integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(b*x^3*arcsinh(c*x) + a*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\sqrt{c^2x^2+1}}{x^3\left(a+b\,\text{asinh}\left(cx\right)\right)}\,dx$$

```
[In] integrate((c**2*x**2+1)**(1/2)/x**3/(a+b*asinh(c*x)),x)
[Out] Integral(sqrt(c**2*x**2 + 1)/(x**3*(a + b*asinh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\,(cx)+a)x^3}\,dx$$

```
[In] integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\sqrt{c^2\,x^2+1}}{x^3\,\left(a+b\,\text{asinh}\left(c\,x\right)\right)}\,dx$$

```
[In] int((c^2*x^2 + 1)^(1/2)/(x^3*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(1/2)/(x^3*(a + b*asinh(c*x))), x)
```

3.363 $\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\,\mathbf{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\operatorname{arcsinh}(cx))}\,dx=\operatorname{Int}\left(\frac{\sqrt{1+c^2x^2}}{x^4(a+b\operatorname{arcsinh}(cx))},x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\operatorname{arcsinh}(cx))}\,dx=\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\operatorname{arcsinh}(cx))}\,dx$$

```
[In] Int[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\text{integral}=\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\operatorname{arcsinh}(cx))}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))}\,dx=\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c^2x^2+1}}{x^4\left(a+b\,\text{arcsinh}\left(cx\right)\right)}dx$$

```
[In] int((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x)
[Out] int((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))}\,dx=\int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\left(cx\right)+a)x^4}\,dx$$

```
[In] integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(b*x^4*arcsinh(c*x) + a*x^4), x)
```

Sympy [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))}\,dx=\int \frac{\sqrt{c^2x^2+1}}{x^4\left(a+b\,\text{asinh}\left(cx\right)\right)}\,dx$$

```
[In] integrate((c**2*x**2+1)**(1/2)/x**4/(a+b*asinh(c*x)),x)
[Out] Integral(sqrt(c**2*x**2 + 1)/(x**4*(a + b*asinh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))}\,dx=\int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\,(cx)+a)x^4}\,dx$$

```
[In] integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x^4), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))}\,dx=\int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\,(cx)+a)x^4}\,dx$$

```
[In] integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x^4), x)
```

Mupad [N/A]

Not integrable

Time = 2.64 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))}\,dx=\int \frac{\sqrt{c^2\,x^2+1}}{x^4\,\left(a+b\,\text{asinh}\,(c\,x)\right)}\,dx$$

```
[In] int((c^2*x^2 + 1)^(1/2)/(x^4*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(1/2)/(x^4*(a + b*asinh(c*x))), x)
```

3.364 $\int \frac{x^3(1+c^2x^2)^{3/2}}{a+b\mathbf{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 245

$$\begin{aligned}\int \frac{x^3(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx &= \frac{3\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{64bc^4}\\&+\frac{3\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{64bc^4}\\&-\frac{\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{64bc^4}-\frac{\text{Chi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{7a}{b}\right)}{64bc^4}\\&-\frac{3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64bc^4}-\frac{3\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^4}\\&+\frac{\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^4}+\frac{\cosh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^4}\end{aligned}$$

```
[Out] -3/64*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^4-3/64*cosh(3*a/b)*Shi(3*(a+b
*arcsinh(c*x))/b)/b/c^4+1/64*cosh(5*a/b)*Shi(5*(a+b*arcsinh(c*x))/b)/b/c^4+
1/64*cosh(7*a/b)*Shi(7*(a+b*arcsinh(c*x))/b)/b/c^4+3/64*Chi((a+b*arcsinh(c*
x))/b)*sinh(a/b)/b/c^4+3/64*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^4-1
/64*Chi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b/c^4-1/64*Chi(7*(a+b*arcsinh(c
*x))/b)*sinh(7*a/b)/b/c^4
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5819, 5556, 3384, 3379, 3382}

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{3\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64bc^4} + \frac{3\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^4} - \frac{\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^4} - \frac{\sinh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^4} - \frac{3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64bc^4} - \frac{3\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^4} + \frac{\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^4} + \frac{\cosh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^4}$$

```
[In] Int[(x^3*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (3*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(64*b*c^4) + (3*CoshInte
gral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(64*b*c^4) - (CoshIntegral[
(5*(a + b*ArcSinh[c*x]))/b]*Sinh[(5*a)/b])/(64*b*c^4) - (CoshIntegral[(7*(a
 + b*ArcSinh[c*x]))/b]*Sinh[(7*a)/b])/(64*b*c^4) - (3*Cosh[a/b]*SinhIntegra
l[(a + b*ArcSinh[c*x])/b])/(64*b*c^4) - (3*Cosh[(3*a)/b]*SinhIntegral[(3*(a
 + b*ArcSinh[c*x]))/b])/(64*b*c^4) + (Cosh[(5*a)/b]*SinhIntegral[(5*(a + b*
ArcSinh[c*x]))/b])/(64*b*c^4) + (Cosh[(7*a)/b]*SinhIntegral[(7*(a + b*ArcSi
nh[c*x]))/b])/(64*b*c^4)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\cosh^4\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{bc^4} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{\sinh\left(\frac{7a}{b}-\frac{7x}{b}\right)}{64x}+\frac{\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{64x}-\frac{3\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{64x}-\frac{3\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{64x}\right)dx, x, a+b\operatorname{arcsinh}(cx)\right)}{bc^4} \\
&= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{7a}{b}-\frac{7x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{64bc^4} \\
&\quad -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{64bc^4} \\
&\quad +\frac{3\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{64bc^4} \\
&\quad +\frac{3\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{64bc^4}
\end{aligned}$$

$$= -\frac{\left(3\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{64bc^4} - \frac{\left(3\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{64bc^4} + \frac{\cosh\left(\frac{5a}{b}\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{5x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{64bc^4} + \frac{\cosh\left(\frac{7a}{b}\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{7x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{64bc^4} + \frac{\left(3\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{64bc^4} + \frac{\left(3\sinh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{64bc^4} - \frac{\sinh\left(\frac{5a}{b}\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{5x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{64bc^4} - \frac{\sinh\left(\frac{7a}{b}\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{7x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{64bc^4}$$

$$= \frac{3\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{64bc^4} + \frac{3\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{64bc^4} - \frac{\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{64bc^4} - \frac{\text{Chi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{7a}{b}\right)}{64bc^4} - \frac{3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64bc^4} - \frac{3\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^4} + \frac{\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^4} + \frac{\cosh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^4}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.73

$$\int\frac{x^3(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{3\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right)+3\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)-\text{Chi}\left(5\left(\frac{a}{b}+\right.\right.}{}$$

```
[In] Integrate[(x^3*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (3*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] + 3*CoshIntegral[3*(a/b + Arc
Sinh[c*x])]*Sinh[(3*a)/b] - CoshIntegral[5*(a/b + ArcSinh[c*x])]*Sinh[(5*a)
/b] - CoshIntegral[7*(a/b + ArcSinh[c*x])]*Sinh[(7*a)/b] - 3*Cosh[a/b]*Sinh
```

Integral[a/b + ArcSinh[c*x]] - 3*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] + Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])] + Cosh[(7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])])/(64*b*c^4)

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.80

method	result
default	$\frac{e^{\frac{7a}{b}} \operatorname{Ei}_1\left(7 \operatorname{arcsinh}(cx)+\frac{7a}{b}\right)+e^{\frac{5a}{b}} \operatorname{Ei}_1\left(5 \operatorname{arcsinh}(cx)+\frac{5a}{b}\right)-3 e^{\frac{3a}{b}} \operatorname{Ei}_1\left(3 \operatorname{arcsinh}(cx)+\frac{3a}{b}\right)-3 e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)+3 e^{-\frac{a}{b}} \operatorname{Ei}_1(-}{128c^4 b}$

[In] int(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)

[Out] 1/128*(exp(7*a/b)*Ei(1,7*arcsinh(c*x)+7*a/b)+exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-3*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-3*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)+3*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+3*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)-exp(-7*a/b)*Ei(1,-7*arcsinh(c*x)-7*a/b))/c^4/b

Fricas [F]

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}x^3}{b\operatorname{arsinh}(cx)+a}\,dx$$

[In] integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")

[Out] integral((c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)

Sympy [F]

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x^3(c^2x^2+1)^{\frac{3}{2}}}{a+b\operatorname{asinh}(cx)}\,dx$$

[In] integrate(x**3*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)

[Out] Integral(x**3*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x)), x)

Maxima [F]

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}x^3}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*x^2 + 1)^(3/2)*x^3/(b*arcsinh(c*x) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x^3\,(c^2\,x^2+1)^{3/2}}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

```
[In] int((x^3*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)),x)
[Out] int((x^3*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)), x)
```

3.365 $\int \frac{x^2(1+c^2x^2)^{3/2}}{a+b\textbf{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 206

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = -\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^3} + \frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3} - \frac{\log(a+b\text{arcsinh}(cx))}{16bc^3} + \frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3} - \frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^3} - \frac{\sinh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3}$$

```
[Out] -1/32*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c^3+1/16*Chi(4*(a+b*arcsinh
(c*x))/b)*cosh(4*a/b)/b/c^3+1/32*Chi(6*(a+b*arcsinh(c*x))/b)*cosh(6*a/b)/b/
c^3-1/16*ln(a+b*arcsinh(c*x))/b/c^3+1/32*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2
*a/b)/b/c^3-1/16*Shi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b/c^3-1/32*Shi(6*(
a+b*arcsinh(c*x))/b)*sinh(6*a/b)/b/c^3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5819, 5556, 3384, 3379, 3382}

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{a+b\operatorname{arcsinh}(cx)}\,dx = -\frac{\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{32bc^3} + \frac{\cosh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(\frac{4(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^3} + \frac{\cosh\left(\frac{6a}{b}\right)\operatorname{Chi}\left(\frac{6(a+b\operatorname{arcsinh}(cx))}{b}\right)}{32bc^3} + \frac{\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{32bc^3} - \frac{\sinh\left(\frac{4a}{b}\right)\operatorname{Shi}\left(\frac{4(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^3} - \frac{\sinh\left(\frac{6a}{b}\right)\operatorname{Shi}\left(\frac{6(a+b\operatorname{arcsinh}(cx))}{b}\right)}{32bc^3} - \frac{\log(a+b\operatorname{arcsinh}(cx))}{16bc^3}$$

```
[In] Int[(x^2*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/32*(Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(b*c^3) + (C
osh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(16*b*c^3) + (Cosh[(
6*a)/b]*CoshIntegral[(6*(a + b*ArcSinh[c*x]))/b])/(32*b*c^3) - Log[a + b*Ar
cSinh[c*x]]/(16*b*c^3) + (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]
))/b])/(32*b*c^3) - (Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b]
)/(16*b*c^3) - (Sinh[(6*a)/b]*SinhIntegral[(6*(a + b*ArcSinh[c*x]))/b])/(32
*b*c^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cosh^4\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc^3}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{16x}+\frac{\cosh\left(\frac{6a}{b}-\frac{6x}{b}\right)}{32x}+\frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{16x}-\frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{32x}\right)dx, x, a+b\text{arcsinh}(cx)\right)}{bc^3}\\
&= -\frac{\log(a+b\text{arcsinh}(cx))}{16bc^3}+\frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{6a}{b}-\frac{6x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{32bc^3}\\
&\quad -\frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{32bc^3}\\
&\quad +\frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16bc^3}\\
&= -\frac{\log(a+b\text{arcsinh}(cx))}{16bc^3}-\frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{32bc^3}\\
&\quad +\frac{\cosh\left(\frac{4a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16bc^3}\\
&\quad +\frac{\cosh\left(\frac{6a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{6x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{32bc^3}\\
&\quad +\frac{\sinh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{32bc^3}\\
&\quad -\frac{\sinh\left(\frac{4a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16bc^3}\\
&\quad -\frac{\sinh\left(\frac{6a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{6x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{32bc^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^3} \\
&+ \frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3} \\
&- \frac{\log(a+b\text{arcsinh}(cx))}{16bc^3} + \frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3} \\
&- \frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^3} - \frac{\sinh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.74

$$
\int \frac{x^2(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{-\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right) + 2\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right) + \cosh
$$

```
[In] Integrate[(x^2*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])]) + 2*Cosh[(4*a)/b]*Co
shIntegral[4*(a/b + ArcSinh[c*x])] + Cosh[(6*a)/b]*CoshIntegral[6*(a/b + Ar
cSinh[c*x])] - 2*Log[a + b*ArcSinh[c*x]] + Sinh[(2*a)/b]*SinhIntegral[2*(a/
b + ArcSinh[c*x])] - 2*Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] -
 Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])])/(32*b*c^3)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.79

method	result
default	$-\frac{e^{\frac{6a}{b}}\,\text{Ei}_1\left(6\,\text{arcsinh}(cx)+\frac{6a}{b}\right)+2\,e^{\frac{4a}{b}}\,\text{Ei}_1\left(4\,\text{arcsinh}(cx)+\frac{4a}{b}\right)-e^{\frac{2a}{b}}\,\text{Ei}_1\left(2\,\text{arcsinh}(cx)+\frac{2a}{b}\right)-e^{-\frac{2a}{b}}\,\text{Ei}_1\left(-2\,\text{arcsinh}(cx)-\frac{2a}{b}\right)+2\,e^{-\frac{4a}{b}}}{64c^3b}$

```
[In] int(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64*(exp(6*a/b)*Ei(1,6*arcsinh(c*x)+6*a/b)+2*exp(4*a/b)*Ei(1,4*arcsinh(c*
x)+4*a/b)-exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-exp(-2*a/b)*Ei(1,-2*arcsinh
(c*x)-2*a/b)+2*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)+exp(-6*a/b)*Ei(1,-6*
arcsinh(c*x)-6*a/b)+4*ln(a+b*arcsinh(c*x)))/c^3/b
```

Fricas [F]

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2x^2+1)^{\frac{3}{2}}x^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

[In] integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")

[Out] integral((c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)

Sympy [F]

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{x^2(c^2x^2+1)^{\frac{3}{2}}}{a+b\,\text{asinh}\,(cx)}\,dx$$

[In] integrate(x**2*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)

[Out] Integral(x**2*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x)), x)

Maxima [F]

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2x^2+1)^{\frac{3}{2}}x^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

[In] integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")

[Out] integrate((c^2*x^2 + 1)^(3/2)*x^2/(b*arcsinh(c*x) + a), x)

Giac [F]

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2x^2+1)^{\frac{3}{2}}x^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

[In] integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")

[Out] integrate((c^2*x^2 + 1)^(3/2)*x^2/(b*arcsinh(c*x) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x^2\,\left(c^2\,x^2+1\right)^{3/2}}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

```
[In] int((x^2*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)),x)
[Out] int((x^2*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)), x)
```

3.366 $\int \frac{x(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 183

$$\int \frac{x(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = -\frac{\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8bc^2} - \frac{3\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16bc^2} - \frac{\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{16bc^2} + \frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8bc^2} + \frac{3\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^2} + \frac{\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^2}$$

```
[Out] 1/8*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^2+3/16*cosh(3*a/b)*Shi(3*(a+b*a
rcsinh(c*x))/b)/b/c^2+1/16*cosh(5*a/b)*Shi(5*(a+b*arcsinh(c*x))/b)/b/c^2-1/
8*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^2-3/16*Chi(3*(a+b*arcsinh(c*x))/b
)*sinh(3*a/b)/b/c^2-1/16*Chi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b/c^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{5819, 5556, 3384, 3379, 3382\}$

$$\int \frac{x(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = -\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8bc^2} - \frac{3\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^2} - \frac{\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^2} + \frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8bc^2} + \frac{3\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^2} + \frac{\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^2}$$

```
[In] Int[(x*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/8*(CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(b*c^2) - (3*CoshInte
gral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(16*b*c^2) - (CoshIntegral[
(5*(a + b*ArcSinh[c*x]))/b]*Sinh[(5*a)/b])/(16*b*c^2) + (Cosh[a/b]*SinhInte
gral[(a + b*ArcSinh[c*x])/b])/(8*b*c^2) + (3*Cosh[(3*a)/b]*SinhIntegral[(3*
(a + b*ArcSinh[c*x]))/b])/(16*b*c^2) + (Cosh[(5*a)/b]*SinhIntegral[(5*(a +
b*ArcSinh[c*x]))/b])/(16*b*c^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\cosh^4\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc^2}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16x}+\frac{3\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16x}+\frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{8x}\right)dx, x, a+b\text{arcsinh}(cx)\right)}{bc^2}\\
&= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16bc^2}\\
&\quad -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8bc^2}\\
&\quad -\frac{3\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16bc^2}\\
&= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8bc^2}\\
&\quad +\frac{\left(3\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16bc^2}\\
&\quad +\frac{\cosh\left(\frac{5a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{5x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16bc^2}\\
&\quad -\frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8bc^2}\\
&\quad -\frac{\left(3\sinh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16bc^2}\\
&\quad -\frac{\sinh\left(\frac{5a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{5x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16bc^2}
\end{aligned}
$$

$$= -\frac{\mathrm{Chi}\left(\frac{a+b\mathrm{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8bc^2} - \frac{3\mathrm{Chi}\left(\frac{3(a+b\mathrm{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16bc^2} - \frac{\mathrm{Chi}\left(\frac{5(a+b\mathrm{arcsinh}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{16bc^2} + \frac{\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\mathrm{arcsinh}(cx)}{b}\right)}{8bc^2} + \frac{3\cosh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3(a+b\mathrm{arcsinh}(cx))}{b}\right)}{16bc^2} + \frac{\cosh\left(\frac{5a}{b}\right)\mathrm{Shi}\left(\frac{5(a+b\mathrm{arcsinh}(cx))}{b}\right)}{16bc^2}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.74

$$\int \frac{x(1+c^2x^2)^{3/2}}{a+b\mathrm{arcsinh}(cx)}\,dx = \frac{-2\mathrm{Chi}\left(\frac{a}{b}+\mathrm{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right) - 3\mathrm{Chi}\left(3\left(\frac{a}{b}+\mathrm{arcsinh}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right) - \mathrm{Chi}\left(5\left(\frac{a}{b}\right.\right.}{}$$

```
[In] Integrate[(x*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-2*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - 3*CoshIntegral[3*(a/b + Ar
cSinh[c*x])]*Sinh[(3*a)/b] - CoshIntegral[5*(a/b + ArcSinh[c*x])]*Sinh[(5*a
)/b] + 2*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 3*Cosh[(3*a)/b]*SinhI
ntegral[3*(a/b + ArcSinh[c*x])] + Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSi
nh[c*x])])/(16*b*c^2)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.81

method	result
default	$\frac{e^{\frac{5a}{b}}\mathrm{Ei}_1\left(5\,\mathrm{arcsinh}(cx)+\frac{5a}{b}\right)+3\,e^{\frac{3a}{b}}\mathrm{Ei}_1\left(3\,\mathrm{arcsinh}(cx)+\frac{3a}{b}\right)+2\,e^{\frac{a}{b}}\mathrm{Ei}_1\left(\mathrm{arcsinh}(cx)+\frac{a}{b}\right)-2\,e^{-\frac{a}{b}}\mathrm{Ei}_1\left(-\mathrm{arcsinh}(cx)-\frac{a}{b}\right)-3\,e^{-\frac{3a}{b}}\mathrm{Ei}_1\left(-\right.}{32c^2b}$

```
[In] int(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32*(exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)+3*exp(3*a/b)*Ei(1,3*arcsinh(c*x
)+3*a/b)+2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a
/b)-3*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-exp(-5*a/b)*Ei(1,-5*arcsinh(c
*x)-5*a/b))/c^2/b
```

Fricas [F]

$$\int \frac{x(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}x}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((c^2*x^3 + x)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)
```

Sympy [F]

$$\int \frac{x(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x(c^2x^2+1)^{\frac{3}{2}}}{a+b\,\text{asinh}\,(cx)}\,dx$$

```
[In] integrate(x*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
[Out] Integral(x*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{x(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}x}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*x^2 + 1)^(3/2)*x/(b*arcsinh(c*x) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(1+c^2x^2)^{3/2}}{a+b\operatorname{arcsinh}(cx)}\,dx = \int \frac{x\,(c^2\,x^2+1)^{3/2}}{a+b\,\mathrm{asinh}\,(c\,x)}\,dx$$

```
[In] int((x*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)),x)
[Out] int((x*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)), x)
```

3.367 $\int \frac{(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 144

$$\int \frac{(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8bc} + \frac{3\log(a+b\text{arcsinh}(cx))}{8bc} - \frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc} - \frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8bc}$$

```
[Out] 1/2*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c+1/8*Chi(4*(a+b*arcsinh(c*x)
)/b)*cosh(4*a/b)/b/c+3/8*ln(a+b*arcsinh(c*x))/b/c-1/2*Shi(2*(a+b*arcsinh(c*
x))/b)*sinh(2*a/b)/b/c-1/8*Shi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b/c
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5791, 3393, 3384, 3379, 3382}

$$\int \frac{(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8bc} - \frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc} - \frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8bc} + \frac{3\log(a+b\text{arcsinh}(cx))}{8bc}$$

```
[In] Int[(1 + c^2*x^2)^(3/2)/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(2*b*c) + (Cosh[(4
*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(8*b*c) + (3*Log[a + b*Arc
Sinh[c*x]])/(8*b*c) - (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/
b])/(2*b*c) - (Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(8*b
*c)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cosh^4\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc}\\ &= \frac{\text{Subst}\left(\int \left(\frac{3}{8x}+\frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{8x}+\frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2x}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc}\end{aligned}$$

$$= \frac{3\log(a+b\text{arcsinh}(cx))}{8bc} + \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8bc} + \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2bc}$$

$$= \frac{3\log(a+b\text{arcsinh}(cx))}{8bc} + \frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2bc} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8bc} - \frac{\sinh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2bc} - \frac{\sinh\left(\frac{4a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8bc}$$

$$= \frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8bc} + \frac{3\log(a+b\text{arcsinh}(cx))}{8bc} - \frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc} - \frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8bc}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.76

$$\int \frac{(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)+\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)+3\log}{}$$

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] + Cosh[(4*a)/b]*CoshI
ntegral[4*(a/b + ArcSinh[c*x])] + 3*Log[a + b*ArcSinh[c*x]] - 4*Sinh[(2*a)/
b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] - Sinh[(4*a)/b]*SinhIntegral[4*(a/b
 + ArcSinh[c*x])])/(8*b*c)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.80

method	result
default	$-\frac{e^{\frac{4a}{b}} \mathrm{Ei}_1\left(4\,\mathrm{arcsinh}(cx)+\frac{4a}{b}\right)+4\,e^{\frac{2a}{b}} \mathrm{Ei}_1\left(2\,\mathrm{arcsinh}(cx)+\frac{2a}{b}\right)+4\,e^{-\frac{2a}{b}} \mathrm{Ei}_1\left(-2\,\mathrm{arcsinh}(cx)-\frac{2a}{b}\right)+e^{-\frac{4a}{b}} \mathrm{Ei}_1\left(-4\,\mathrm{arcsinh}(cx)-\frac{4a}{b}\right)-6}{16bc}$

[In] int((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)

[Out] -1/16*(exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)+4*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)+4*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)+exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)-6*ln(a+b*arcsinh(c*x)))/b/c

Fricas [F]

$$\int \frac{(1+c^2x^2)^{3/2}}{a+b\mathrm{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{b\,\mathrm{arsinh}\,(cx)+a}\,dx$$

[In] integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")

[Out] integral((c^2*x^2 + 1)^(3/2)/(b*arcsinh(c*x) + a), x)

Sympy [F]

$$\int \frac{(1+c^2x^2)^{3/2}}{a+b\mathrm{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{a+b\,\mathrm{asinh}\,(cx)}\,dx$$

[In] integrate((c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)

[Out] Integral((c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x)), x)

Maxima [F]

$$\int \frac{(1+c^2x^2)^{3/2}}{a+b\mathrm{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{b\,\mathrm{arsinh}\,(cx)+a}\,dx$$

[In] integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")

[Out] integrate((c^2*x^2 + 1)^(3/2)/(b*arcsinh(c*x) + a), x)

Giac [F]

$$\int \frac{(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((c^2*x^2 + 1)^(3/2)/(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2\,x^2+1)^{3/2}}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

```
[In] int((c^2*x^2 + 1)^(3/2)/(a + b*asinh(c*x)),x)
[Out] int((c^2*x^2 + 1)^(3/2)/(a + b*asinh(c*x)), x)
```

3.368 $\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\textbf{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\operatorname{arcsinh}(cx))}\,dx = -\frac{5\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4b} - \frac{\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4b} + \frac{5\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{4b} + \frac{\cosh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{4b} + \operatorname{Int}\left(\frac{1}{x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))},x\right)$$

```
[Out] 5/4*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b+1/4*cosh(3*a/b)*Shi(3*(a+b*arcsin
h(c*x))/b)/b-5/4*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b-1/4*Chi(3*(a+b*arcsi
nh(c*x))/b)*sinh(3*a/b)/b+Unintegrable(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(
1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\operatorname{arcsinh}(cx))}\,dx = \int \frac{(1+c^2x^2)^{3/2}}{x(a+b\operatorname{arcsinh}(cx))}\,dx$$

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (-5*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(4*b) - (CoshIntegral[(
3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(4*b) + (5*Cosh[a/b]*SinhIntegral
[(a + b*ArcSinh[c*x])/b])/(4*b) + (Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*Arc
Sinh[c*x]))/b])/(4*b) + Defer[Int][1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*
x])), x]
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))} + \frac{2c^2x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))} + \frac{c^4x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\right)dx$$

$$= (2c^2)\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}dx + c^4\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}dx + \int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}dx$$

$$= -\frac{\text{Subst}\left(\int \frac{\sinh^3\left(\frac{a}{b}-\frac{x}{b}\right)}{x}dx, x, a+b\text{arcsinh}(cx)\right)}{b} - \frac{2\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}dx, x, a+b\text{arcsinh}(cx)\right)}{b} + \int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}dx$$

$$= -\frac{i\text{Subst}\left(\int \left(-\frac{i\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4x} + \frac{3i\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{4x}\right)dx, x, a+b\text{arcsinh}(cx)\right)}{b} + \frac{\left(2\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}dx, x, a+b\text{arcsinh}(cx)\right)}{b} - \frac{\left(2\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}dx, x, a+b\text{arcsinh}(cx)\right)}{b} + \int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}dx$$

$$= -\frac{2\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b} + \frac{2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b} - \frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}dx, x, a+b\text{arcsinh}(cx)\right)}{4b} + \frac{3\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}dx, x, a+b\text{arcsinh}(cx)\right)}{4b} + \int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}dx$$

$$
\begin{aligned}
&= -\frac{2\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b}+\frac{2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b} \\
&\quad -\frac{\left(3\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{4b} \\
&\quad +\frac{\cosh\left(\frac{3a}{b}\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{4b} \\
&\quad +\frac{\left(3\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{4b} \\
&\quad -\frac{\sinh\left(\frac{3a}{b}\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{4b} \\
&\quad +\int\frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx \\
&= -\frac{5\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4b}-\frac{\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4b} \\
&\quad +\frac{5\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b}+\frac{\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b} \\
&\quad +\int\frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx
\end{aligned}
$$

Mathematica [N/A]

Not integrable

Time = 2.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int\frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))}\,dx=\int\frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\left(c^{2} x^{2}+1\right)^{\frac{3}{2}}}{x \left(a +b \,\operatorname{arcsinh}\left(c x \right)\right)}d x$$

```
[In] int((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x)),x)
[Out] int((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))}\,dx=\int \frac{{\left(c^{2} x^{2} + 1\right)}^{\frac{3}{2}}}{{\left(b \operatorname{arsinh}\left(c x\right) + a\right)} x} \,d x$$

```
[In] integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((c^2*x^2 + 1)^(3/2)/(b*x*arcsinh(c*x) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 2.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))}\,dx=\int \frac{\left(c^{2} x^{2} + 1\right)^{\frac{3}{2}}}{x \left(a + b \operatorname{asinh}{\left(c x \right)}\right)}\, dx$$

```
[In] integrate((c**2*x**2+1)**(3/2)/x/(a+b*asinh(c*x)),x)
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x*(a + b*asinh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))}\,dx=\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)x}\,dx$$

```
[In] integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))}\,dx=\int \frac{(c^2\,x^2+1)^{3/2}}{x\,\left(a+b\,\text{asinh}\,(c\,x)\right)}\,dx$$

```
[In] int((c^2*x^2 + 1)^(3/2)/(x*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(3/2)/(x*(a + b*asinh(c*x))), x)
```

3.369 $\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\textbf{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\operatorname{arcsinh}(cx))}\,dx = \frac{c\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{2b} + \frac{3c\log(a+b\operatorname{arcsinh}(cx))}{2b} - \frac{c\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{2b} + \operatorname{Int}\left(\frac{1}{x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))},x\right)$$

```
[Out] 1/2*c*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b+3/2*c*ln(a+b*arcsinh(c*x))/
b-1/2*c*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b+Unintegrable(1/x^2/(a+b*a
rcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\operatorname{arcsinh}(cx))}\,dx = \int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\operatorname{arcsinh}(cx))}\,dx$$

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (c*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(2*b) + (3*c*Log
[a + b*ArcSinh[c*x]])/(2*b) - (c*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSi
```

```
nh[c*x]))/b])/(2*b) + Defer[Int][1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*
x])), x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{2c^2}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))} + \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))} + \frac{c^4x^2}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\right) dx \\
&= (2c^2)\int \frac{1}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx + c^4\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx + \int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx \\
&= \frac{2c\log(a+b\operatorname{arcsinh}(cx))}{b} + \frac{c\text{Subst}\left(\int \frac{\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b} + \int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx \\
&= \frac{2c\log(a+b\operatorname{arcsinh}(cx))}{b} - \frac{c\text{Subst}\left(\int \left(\frac{1}{2x}-\frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2x}\right)dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b} + \int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx \\
&= \frac{3c\log(a+b\operatorname{arcsinh}(cx))}{2b} + \frac{c\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{2b} + \int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx \\
&= \frac{3c\log(a+b\operatorname{arcsinh}(cx))}{2b} + \frac{\left(c\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{2b} - \frac{\left(c\sinh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{2b} + \int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx \\
&= \frac{c\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{2b} + \frac{3c\log(a+b\operatorname{arcsinh}(cx))}{2b} - \frac{c\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{2b} + \int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx
\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^2\,(a+b\ \text{arcsinh}\,(cx))}dx$$

```
[In] int((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x)),x)
[Out] int((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)x^2}\,dx$$

```
[In] integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((c^2*x^2 + 1)^(3/2)/(b*x^2*arcsinh(c*x) + a*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^2\left(a+b\operatorname{asinh}\left(cx\right)\right)}\,dx$$

```
[In] integrate((c**2*x**2+1)**(3/2)/x**2/(a+b*asinh(c*x)),x)
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x**2*(a + b*asinh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\operatorname{arsinh}\left(cx\right)+a)x^2}\,dx$$

```
[In] integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\operatorname{arsinh}\left(cx\right)+a)x^2}\,dx$$

```
[In] integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\left(c^2\,x^2+1\right)^{3/2}}{x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

```
[In] int((c^2*x^2 + 1)^(3/2)/(x^2*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(3/2)/(x^2*(a + b*asinh(c*x))), x)
```

3.370 $\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\textbf{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx=\text{Int}\left(\frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))},x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx=\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\text{integral}=\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx$$

Mathematica [N/A]

Not integrable

Time = 4.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^3\,(a+b\,\text{arcsinh}\,(cx))}dx$$

```
[In] int((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x)
[Out] int((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)x^3}\,dx$$

```
[In] integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((c^2*x^2 + 1)^(3/2)/(b*x^3*arcsinh(c*x) + a*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 1.94 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))} \, dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^3\,(a+b\operatorname{asinh}{(cx)})} \, dx$$

```
[In] integrate((c**2*x**2+1)**(3/2)/x**3/(a+b*asinh(c*x)),x)
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x**3*(a + b*asinh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))} \, dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\operatorname{arsinh}(cx)+a)x^3} \, dx$$

```
[In] integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx=\int \frac{{\left(c^2\,x^2+1\right)}^{3/2}}{x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

```
[In] int((c^2*x^2 + 1)^(3/2)/(x^3*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(3/2)/(x^3*(a + b*asinh(c*x))), x)
```

3.371 $\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\textbf{arcsinh}(cx))} dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))} dx = \text{Int}\left(\frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))}, x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))} dx = \int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))} dx$$

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))} dx$$

Mathematica [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx=\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^4\,(a+b\ \text{arcsinh}\,(cx))}dx$$

```
[In] int((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x)
[Out] int((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx=\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)x^4}\,dx$$

```
[In] integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((c^2*x^2 + 1)^(3/2)/(b*x^4*arcsinh(c*x) + a*x^4), x)
```

Sympy [N/A]

Not integrable

Time = 2.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^4\,(a+b\,\text{asinh}\,(cx))}\,dx$$

```
[In] integrate((c**2*x**2+1)**(3/2)/x**4/(a+b*asinh(c*x)),x)
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x**4*(a + b*asinh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)x^4}\,dx$$

```
[In] integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x^4), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)x^4}\,dx$$

```
[In] integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x^4), x)
```

Mupad [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\left(c^2\,x^2+1\right)^{3/2}}{x^4\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

```
[In] int((c^2*x^2 + 1)^(3/2)/(x^4*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(3/2)/(x^4*(a + b*asinh(c*x))), x)
```

3.372 $\int \frac{x^3(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 245

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{3\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{128bc^4} + \frac{\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{32bc^4} - \frac{3\text{Chi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{7a}{b}\right)}{256bc^4} - \frac{\text{Chi}\left(\frac{9(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{9a}{b}\right)}{256bc^4} - \frac{3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{128bc^4} - \frac{\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^4} + \frac{3\cosh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{256bc^4} + \frac{\cosh\left(\frac{9a}{b}\right)\text{Shi}\left(\frac{9(a+b\text{arcsinh}(cx))}{b}\right)}{256bc^4}$$

```
[Out] -3/128*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^4-1/32*cosh(3*a/b)*Shi(3*(a+
b*arcsinh(c*x))/b)/b/c^4+3/256*cosh(7*a/b)*Shi(7*(a+b*arcsinh(c*x))/b)/b/c^
4+1/256*cosh(9*a/b)*Shi(9*(a+b*arcsinh(c*x))/b)/b/c^4+3/128*Chi((a+b*arcsin
h(c*x))/b)*sinh(a/b)/b/c^4+1/32*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c
^4-3/256*Chi(7*(a+b*arcsinh(c*x))/b)*sinh(7*a/b)/b/c^4-1/256*Chi(9*(a+b*arc
sinh(c*x))/b)*sinh(9*a/b)/b/c^4
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5819, 5556, 3384, 3379, 3382}

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{3\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{128bc^4} + \frac{\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^4} - \frac{3\sinh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{256bc^4} - \frac{\sinh\left(\frac{9a}{b}\right)\text{Chi}\left(\frac{9(a+b\text{arcsinh}(cx))}{b}\right)}{256bc^4} - \frac{3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{128bc^4} - \frac{\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^4} + \frac{3\cosh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{256bc^4} + \frac{\cosh\left(\frac{9a}{b}\right)\text{Shi}\left(\frac{9(a+b\text{arcsinh}(cx))}{b}\right)}{256bc^4}$$

```
[In] Int[(x^3*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (3*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(128*b*c^4) + (CoshInteg
ral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(32*b*c^4) - (3*CoshIntegral
[(7*(a + b*ArcSinh[c*x]))/b]*Sinh[(7*a)/b])/(256*b*c^4) - (CoshIntegral[(9*
(a + b*ArcSinh[c*x]))/b]*Sinh[(9*a)/b])/(256*b*c^4) - (3*Cosh[a/b]*SinhInte
gral[(a + b*ArcSinh[c*x])/b])/(128*b*c^4) - (Cosh[(3*a)/b]*SinhIntegral[(3*
(a + b*ArcSinh[c*x]))/b])/(32*b*c^4) + (3*Cosh[(7*a)/b]*SinhIntegral[(7*(a
+ b*ArcSinh[c*x]))/b])/(256*b*c^4) + (Cosh[(9*a)/b]*SinhIntegral[(9*(a + b*
ArcSinh[c*x]))/b])/(256*b*c^4)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\cosh^6\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc^4}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{\sinh\left(\frac{9a}{b}-\frac{9x}{b}\right)}{256x}+\frac{3\sinh\left(\frac{7a}{b}-\frac{7x}{b}\right)}{256x}-\frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{32x}-\frac{3\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{128x}\right)dx,x,a+b\text{arcsinh}(cx)\right)}{bc^4}\\
&= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{9a}{b}-\frac{9x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{256bc^4}\\
&\quad -\frac{3\text{Subst}\left(\int \frac{\sinh\left(\frac{7a}{b}-\frac{7x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{256bc^4}\\
&\quad +\frac{3\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{128bc^4}\\
&\quad +\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{32bc^4}
\end{aligned}$$

$$= -\frac{\left(3\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{128bc^4} - \frac{\cosh\left(\frac{3a}{b}\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{32bc^4} + \frac{\left(3\cosh\left(\frac{7a}{b}\right)\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{7x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{256bc^4} + \frac{\cosh\left(\frac{9a}{b}\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{9x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{256bc^4} + \frac{\left(3\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{128bc^4} + \frac{\sinh\left(\frac{3a}{b}\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{32bc^4} - \frac{\left(3\sinh\left(\frac{7a}{b}\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{7x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{256bc^4} - \frac{\sinh\left(\frac{9a}{b}\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{9x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{256bc^4}$$

$$= \frac{3\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{128bc^4} + \frac{\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{32bc^4} - \frac{3\text{Chi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{7a}{b}\right)}{256bc^4} - \frac{\text{Chi}\left(\frac{9(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{9a}{b}\right)}{256bc^4} - \frac{3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{128bc^4} - \frac{\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^4} + \frac{3\cosh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{256bc^4} + \frac{\cosh\left(\frac{9a}{b}\right)\text{Shi}\left(\frac{9(a+b\text{arcsinh}(cx))}{b}\right)}{256bc^4}$$

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.73

$$\int\frac{x^3(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{6\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right)+8\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)-3\text{Chi}\left(7\left(\right.\right.}{}$$

```
[In] Integrate[(x^3*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
[Out] (6*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] + 8*CoshIntegral[3*(a/b + Arc
Sinh[c*x])]*Sinh[(3*a)/b] - 3*CoshIntegral[7*(a/b + ArcSinh[c*x])]*Sinh[(7*
a)/b] - CoshIntegral[9*(a/b + ArcSinh[c*x])]*Sinh[(9*a)/b] - 6*Cosh[a/b]*Si
```

```
nhIntegral[a/b + ArcSinh[c*x]] - 8*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcS
inh[c*x])] + 3*Cosh[(7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])] + Cosh[(9
*a)/b]*SinhIntegral[9*(a/b + ArcSinh[c*x])])/(256*b*c^4)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.80

method	result
default	$\frac{e^{\frac{9a}{b}} \mathrm{Ei}_1\left(9 \operatorname{arcsinh}(cx)+\frac{9a}{b}\right)+3 e^{\frac{7a}{b}} \mathrm{Ei}_1\left(7 \operatorname{arcsinh}(cx)+\frac{7a}{b}\right)-8 e^{\frac{3a}{b}} \mathrm{Ei}_1\left(3 \operatorname{arcsinh}(cx)+\frac{3a}{b}\right)-6 e^{\frac{a}{b}} \mathrm{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)+6 e^{-\frac{a}{b}} \mathrm{Ei}_1(-}{512c^4b}$

```
[In] int(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/512*(exp(9*a/b)*Ei(1,9*arcsinh(c*x)+9*a/b)+3*exp(7*a/b)*Ei(1,7*arcsinh(c*
x)+7*a/b)-8*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-6*exp(a/b)*Ei(1,arcsinh(c
*x)+a/b)+6*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+8*exp(-3*a/b)*Ei(1,-3*arcsinh(
c*x)-3*a/b)-3*exp(-7*a/b)*Ei(1,-7*arcsinh(c*x)-7*a/b)-exp(-9*a/b)*Ei(1,-9*a
rcsinh(c*x)-9*a/b))/c^4/b
```

Fricas [F]

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}x^3}{b\operatorname{arsinh}(cx)+a}\,dx$$

```
[In] integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^7 + 2*c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a)
, x)
```

Sympy [F]

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x^3(c^2x^2+1)^{\frac{5}{2}}}{a+b\operatorname{asinh}(cx)}\,dx$$

```
[In] integrate(x**3*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**3*(c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}x^3}{b\,\text{arsinh}\,(cx)+a}\,dx$$

[In] integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")

[Out] integrate((c^2*x^2 + 1)^(5/2)*x^3/(b*arcsinh(c*x) + a), x)

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \text{Exception raised: RuntimeError}$$

[In] integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")

[Out] Exception raised: RuntimeError >> an error occurred running a Giac command: INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect eur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x^3\,(c^2\,x^2+1)^{5/2}}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

[In] int((x^3*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)),x)

[Out] int((x^3*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)), x)

3.373 $\int \frac{x^2(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 268

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = -\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3} + \frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3} + \frac{\cosh\left(\frac{8a}{b}\right)\text{Chi}\left(\frac{8(a+b\text{arcsinh}(cx))}{b}\right)}{128bc^3} - \frac{5\log(a+b\text{arcsinh}(cx))}{128bc^3} + \frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3} - \frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3} - \frac{\sinh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3} - \frac{\sinh\left(\frac{8a}{b}\right)\text{Shi}\left(\frac{8(a+b\text{arcsinh}(cx))}{b}\right)}{128bc^3}$$

```
[Out] -1/32*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c^3+1/32*Chi(4*(a+b*arcsinh
(c*x))/b)*cosh(4*a/b)/b/c^3+1/32*Chi(6*(a+b*arcsinh(c*x))/b)*cosh(6*a/b)/b/
c^3+1/128*Chi(8*(a+b*arcsinh(c*x))/b)*cosh(8*a/b)/b/c^3-5/128*ln(a+b*arcsin
h(c*x))/b/c^3+1/32*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b/c^3-1/32*Shi(4
*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b/c^3-1/32*Shi(6*(a+b*arcsinh(c*x))/b)*s
inh(6*a/b)/b/c^3-1/128*Shi(8*(a+b*arcsinh(c*x))/b)*sinh(8*a/b)/b/c^3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5819, 5556, 3384, 3379, 3382}

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = -\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3} + \frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3} + \frac{\cosh\left(\frac{8a}{b}\right)\text{Chi}\left(\frac{8(a+b\text{arcsinh}(cx))}{b}\right)}{128bc^3} + \frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3} - \frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3} - \frac{\sinh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3} - \frac{\sinh\left(\frac{8a}{b}\right)\text{Shi}\left(\frac{8(a+b\text{arcsinh}(cx))}{b}\right)}{128bc^3} - \frac{5\log(a+b\text{arcsinh}(cx))}{128bc^3}$$

```
[In] Int[(x^2*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/32*(Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(b*c^3) + (C
osh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(32*b*c^3) + (Cosh[(
6*a)/b]*CoshIntegral[(6*(a + b*ArcSinh[c*x]))/b])/(32*b*c^3) + (Cosh[(8*a)/
b]*CoshIntegral[(8*(a + b*ArcSinh[c*x]))/b])/(128*b*c^3) - (5*Log[a + b*Arc
Sinh[c*x]])/(128*b*c^3) + (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x
]))/b])/(32*b*c^3) - (Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b
])/(32*b*c^3) - (Sinh[(6*a)/b]*SinhIntegral[(6*(a + b*ArcSinh[c*x]))/b])/(3
2*b*c^3) - (Sinh[(8*a)/b]*SinhIntegral[(8*(a + b*ArcSinh[c*x]))/b])/(128*b*
c^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\cosh^6\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc^3}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{5}{128x}+\frac{\cosh\left(\frac{8a}{b}-\frac{8x}{b}\right)}{128x}+\frac{\cosh\left(\frac{6a}{b}-\frac{6x}{b}\right)}{32x}+\frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{32x}-\frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{32x}\right)dx, x, a+b\text{arcsinh}(cx)\right)}{bc^3}$$

$$= -\frac{5\log(a+b\text{arcsinh}(cx))}{128bc^3}+\frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{8a}{b}-\frac{8x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{128bc^3}$$
$$+\frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{6a}{b}-\frac{6x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{32bc^3}$$
$$+\frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{32bc^3}$$
$$-\frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{32bc^3}$$

$$= -\frac{5\log(a+b\text{arcsinh}(cx))}{128bc^3} - \frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{32bc^3}$$
$$+\frac{\cosh\left(\frac{4a}{b}\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{4x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{32bc^3}$$
$$+\frac{\cosh\left(\frac{6a}{b}\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{6x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{32bc^3}$$
$$+\frac{\cosh\left(\frac{8a}{b}\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{8x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{128bc^3}$$
$$+\frac{\sinh\left(\frac{2a}{b}\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{32bc^3}$$
$$-\frac{\sinh\left(\frac{4a}{b}\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{4x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{32bc^3}$$
$$-\frac{\sinh\left(\frac{6a}{b}\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{6x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{32bc^3}$$
$$-\frac{\sinh\left(\frac{8a}{b}\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{8x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{128bc^3}$$

$$= -\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3}$$
$$+\frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3}$$
$$+\frac{\cosh\left(\frac{8a}{b}\right)\text{Chi}\left(\frac{8(a+b\text{arcsinh}(cx))}{b}\right)}{128bc^3} - \frac{5\log(a+b\text{arcsinh}(cx))}{128bc^3}$$
$$+\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3} - \frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3}$$
$$-\frac{\sinh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3} - \frac{\sinh\left(\frac{8a}{b}\right)\text{Shi}\left(\frac{8(a+b\text{arcsinh}(cx))}{b}\right)}{128bc^3}$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.74

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{-4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)+4\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)+4\,c$$

```
[In] Integrate[(x^2*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] + 4*Cosh[(4*a)/b]*Co
shIntegral[4*(a/b + ArcSinh[c*x])] + 4*Cosh[(6*a)/b]*CoshIntegral[6*(a/b +
ArcSinh[c*x])] + Cosh[(8*a)/b]*CoshIntegral[8*(a/b + ArcSinh[c*x])] - 5*Log
[a + b*ArcSinh[c*x]] + 4*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])]
 - 4*Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 4*Sinh[(6*a)/b]*S
inhIntegral[6*(a/b + ArcSinh[c*x])] - Sinh[(8*a)/b]*SinhIntegral[8*(a/b + A
rcSinh[c*x])])/(128*b*c^3)
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.79

method	result
default	$-\frac{e^{\frac{8a}{b}}\,\text{Ei}_1\left(8\,\text{arcsinh}(cx)+\frac{8a}{b}\right)+4\,e^{\frac{6a}{b}}\,\text{Ei}_1\left(6\,\text{arcsinh}(cx)+\frac{6a}{b}\right)+4\,e^{\frac{4a}{b}}\,\text{Ei}_1\left(4\,\text{arcsinh}(cx)+\frac{4a}{b}\right)-4\,e^{\frac{2a}{b}}\,\text{Ei}_1\left(2\,\text{arcsinh}(cx)+\frac{2a}{b}\right)-4\,e^{-\frac{2a}{b}}}{}$

```
[In] int(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/256*(exp(8*a/b)*Ei(1,8*arcsinh(c*x)+8*a/b)+4*exp(6*a/b)*Ei(1,6*arcsinh(c
*x)+6*a/b)+4*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)-4*exp(2*a/b)*Ei(1,2*arcs
inh(c*x)+2*a/b)-4*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)+4*exp(-4*a/b)*Ei(
1,-4*arcsinh(c*x)-4*a/b)+4*exp(-6*a/b)*Ei(1,-6*arcsinh(c*x)-6*a/b)+exp(-8*a
/b)*Ei(1,-8*arcsinh(c*x)-8*a/b)+10*ln(a+b*arcsinh(c*x)))/c^3/b
```

Fricas [F]

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}x^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^6 + 2*c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a)
, x)
```

Sympy [F]

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{x^2(c^2x^2+1)^{\frac{5}{2}}}{a+b\,\text{asinh}\,(cx)}\,dx$$

```
[In] integrate(x**2*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x)),x)
[Out] Integral(x**2*(c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}x^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*x^2 + 1)^(5/2)*x^2/(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}x^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((c^2*x^2 + 1)^(5/2)*x^2/(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{x^2\,(c^2\,x^2+1)^{5/2}}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

```
[In] int((x^2*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)),x)
[Out] int((x^2*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)), x)
```

3.374 $\int \frac{x(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 245

$$\begin{aligned}\int \frac{x(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx &= -\frac{5\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{64bc^2} \\ &\quad -\frac{9\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{64bc^2} \\ &\quad -\frac{5\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{64bc^2}-\frac{\text{Chi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{7a}{b}\right)}{64bc^2} \\ &\quad +\frac{5\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64bc^2}+\frac{9\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^2} \\ &\quad +\frac{5\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^2}+\frac{\cosh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^2}\end{aligned}$$

```
[Out] 5/64*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^2+9/64*cosh(3*a/b)*Shi(3*(a+b*
arcsinh(c*x))/b)/b/c^2+5/64*cosh(5*a/b)*Shi(5*(a+b*arcsinh(c*x))/b)/b/c^2+1
/64*cosh(7*a/b)*Shi(7*(a+b*arcsinh(c*x))/b)/b/c^2-5/64*Chi((a+b*arcsinh(c*x
))/b)*sinh(a/b)/b/c^2-9/64*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^2-5/
64*Chi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b/c^2-1/64*Chi(7*(a+b*arcsinh(c*
x))/b)*sinh(7*a/b)/b/c^2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5819, 5556, 3384, 3379, 3382}

$$\int \frac{x(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = -\frac{5\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64bc^2} - \frac{9\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^2} - \frac{5\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^2} - \frac{\sinh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^2} + \frac{5\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64bc^2} + \frac{9\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^2} + \frac{5\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^2} + \frac{\cosh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^2}$$

```
[In] Int[(x*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-5*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(64*b*c^2) - (9*CoshInt
egral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(64*b*c^2) - (5*CoshIntegr
al[(5*(a + b*ArcSinh[c*x]))/b]*Sinh[(5*a)/b])/(64*b*c^2) - (CoshIntegral[(7
*(a + b*ArcSinh[c*x]))/b]*Sinh[(7*a)/b])/(64*b*c^2) + (5*Cosh[a/b]*SinhInte
gral[(a + b*ArcSinh[c*x])/b])/(64*b*c^2) + (9*Cosh[(3*a)/b]*SinhIntegral[(3
*(a + b*ArcSinh[c*x]))/b])/(64*b*c^2) + (5*Cosh[(5*a)/b]*SinhIntegral[(5*(a
 + b*ArcSinh[c*x]))/b])/(64*b*c^2) + (Cosh[(7*a)/b]*SinhIntegral[(7*(a + b*
ArcSinh[c*x]))/b])/(64*b*c^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\cosh^6\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc^2} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{\sinh\left(\frac{7a}{b}-\frac{7x}{b}\right)}{64x}+\frac{5\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{64x}+\frac{9\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{64x}+\frac{5\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{64x}\right)dx, x, a+b\text{arcsinh}(cx)\right)}{bc^2} \\
&= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{7a}{b}-\frac{7x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64bc^2} \\
&\quad -\frac{5\text{Subst}\left(\int \frac{\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64bc^2} \\
&\quad -\frac{5\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64bc^2} \\
&\quad -\frac{9\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64bc^2}
\end{aligned}$$

$$= \frac{\left(5\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{64bc^2} + \frac{\left(9\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{64bc^2} + \frac{\left(5\cosh\left(\frac{5a}{b}\right)\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{5x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{64bc^2} + \frac{\cosh\left(\frac{7a}{b}\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{7x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{64bc^2} - \frac{\left(5\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{64bc^2} - \frac{\left(9\sinh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{64bc^2} - \frac{\left(5\sinh\left(\frac{5a}{b}\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{5x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{64bc^2} - \frac{\sinh\left(\frac{7a}{b}\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{7x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{64bc^2}$$

$$= -\frac{5\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{64bc^2} - \frac{9\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{64bc^2} - \frac{5\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{64bc^2} - \frac{\text{Chi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{7a}{b}\right)}{64bc^2} + \frac{5\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64bc^2} + \frac{9\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^2} + \frac{5\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^2} + \frac{\cosh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^2}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.73

$$\int\frac{x(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{-5\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right) - 9\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right) - 5\text{Chi}(5$$

```
[In] Integrate[(x*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-5*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - 9*CoshIntegral[3*(a/b + Ar
cSinh[c*x])]*Sinh[(3*a)/b] - 5*CoshIntegral[5*(a/b + ArcSinh[c*x])]*Sinh[(5
*a)/b] - CoshIntegral[7*(a/b + ArcSinh[c*x])]*Sinh[(7*a)/b] + 5*Cosh[a/b]*S
```

```
inhIntegral[a/b + ArcSinh[c*x]] + 9*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + Arc
Sinh[c*x])] + 5*Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])] + Cosh[(
7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])])/(64*b*c^2)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.80

method	result
default	$\frac{e^{\frac{7a}{b}} \operatorname{Ei}_1\left(7 \operatorname{arcsinh}(cx)+\frac{7a}{b}\right)+5 e^{\frac{5a}{b}} \operatorname{Ei}_1\left(5 \operatorname{arcsinh}(cx)+\frac{5a}{b}\right)+9 e^{\frac{3a}{b}} \operatorname{Ei}_1\left(3 \operatorname{arcsinh}(cx)+\frac{3a}{b}\right)+5 e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)-5 e^{-\frac{a}{b}} \operatorname{Ei}_1(-}{128c^2b}$

```
[In] int(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/128*(exp(7*a/b)*Ei(1,7*arcsinh(c*x)+7*a/b)+5*exp(5*a/b)*Ei(1,5*arcsinh(c*
x)+5*a/b)+9*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+5*exp(a/b)*Ei(1,arcsinh(c
*x)+a/b)-5*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)-9*exp(-3*a/b)*Ei(1,-3*arcsinh(
c*x)-3*a/b)-5*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)-exp(-7*a/b)*Ei(1,-7*a
rcsinh(c*x)-7*a/b))/c^2/b
```

Fricas [F]

$$\int \frac{x(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}x}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^5 + 2*c^2*x^3 + x)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a),
x)
```

Sympy [F]

$$\int \frac{x(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x(c^2x^2+1)^{\frac{5}{2}}}{a+b\,\text{asinh}\,(cx)}\,dx$$

```
[In] integrate(x*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x*(c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{x(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}x}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*x^2 + 1)^(5/2)*x/(b*arcsinh(c*x) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x\,(c^2\,x^2+1)^{5/2}}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

```
[In] int((x*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)),x)
[Out] int((x*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)), x)
```

3.375 $\int \frac{\left(1+c^2x^2\right)^{5/2}}{a+b\,\mathbf{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 206

$$\int \frac{(1+c^2x^2)^{5/2}}{a+b\operatorname{arcsinh}(cx)}\,dx = \frac{15\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{32bc} + \frac{3\cosh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(\frac{4(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc} + \frac{\cosh\left(\frac{6a}{b}\right)\operatorname{Chi}\left(\frac{6(a+b\operatorname{arcsinh}(cx))}{b}\right)}{32bc} + \frac{5\log(a+b\operatorname{arcsinh}(cx))}{16bc} - \frac{15\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{32bc} - \frac{3\sinh\left(\frac{4a}{b}\right)\operatorname{Shi}\left(\frac{4(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc} - \frac{\sinh\left(\frac{6a}{b}\right)\operatorname{Shi}\left(\frac{6(a+b\operatorname{arcsinh}(cx))}{b}\right)}{32bc}$$

```
[Out] 15/32*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c+3/16*Chi(4*(a+b*arcsinh(c
*x))/b)*cosh(4*a/b)/b/c+1/32*Chi(6*(a+b*arcsinh(c*x))/b)*cosh(6*a/b)/b/c+5/
16*ln(a+b*arcsinh(c*x))/b/c-15/32*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b
/c-3/16*Shi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b/c-1/32*Shi(6*(a+b*arcsinh
(c*x))/b)*sinh(6*a/b)/b/c
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5791, 3393, 3384, 3379, 3382}

$$\int \frac{(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{15\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32bc} + \frac{3\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{16bc} + \frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32bc} - \frac{15\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32bc} - \frac{3\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{16bc} - \frac{\sinh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32bc} + \frac{5\log(a+b\text{arcsinh}(cx))}{16bc}$$

```
[In] Int[(1 + c^2*x^2)^(5/2)/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (15*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(32*b*c) + (3*C
osh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(16*b*c) + (Cosh[(6*
a)/b]*CoshIntegral[(6*(a + b*ArcSinh[c*x]))/b])/(32*b*c) + (5*Log[a + b*Arc
Sinh[c*x]])/(16*b*c) - (15*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x
]))/b])/(32*b*c) - (3*Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b
])/(16*b*c) - (Sinh[(6*a)/b]*SinhIntegral[(6*(a + b*ArcSinh[c*x]))/b])/(32*
b*c)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cosh^6\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc}\\
&= \frac{\text{Subst}\left(\int \left(\frac{5}{16x}+\frac{\cosh\left(\frac{6a}{b}-\frac{6x}{b}\right)}{32x}+\frac{3\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{16x}+\frac{15\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{32x}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc}\\
&= \frac{5\log(a+b\text{arcsinh}(cx))}{16bc}+\frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{6a}{b}-\frac{6x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{32bc}\\
&\quad+\frac{3\text{Subst}\left(\int \frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16bc}\\
&\quad+\frac{15\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{32bc}\\
&= \frac{5\log(a+b\text{arcsinh}(cx))}{16bc}+\frac{\left(15\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{32bc}\\
&\quad+\frac{\left(3\cosh\left(\frac{4a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16bc}\\
&\quad+\frac{\cosh\left(\frac{6a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{6x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{32bc}\\
&\quad-\frac{\left(15\sinh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{32bc}\\
&\quad-\frac{\left(3\sinh\left(\frac{4a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16bc}\\
&\quad-\frac{\sinh\left(\frac{6a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{6x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{32bc}
\end{aligned}$$

$$= \frac{15\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32bc} + \frac{3\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{16bc}$$
$$+ \frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32bc} + \frac{5\log(a+b\text{arcsinh}(cx))}{16bc}$$
$$- \frac{15\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32bc}$$
$$- \frac{3\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{16bc} - \frac{\sinh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32bc}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.74

$$\int \frac{(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{15\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right) + 6\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right) + \text{co}}{}$$

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (15*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] + 6*Cosh[(4*a)/b]*Co
shIntegral[4*(a/b + ArcSinh[c*x])] + Cosh[(6*a)/b]*CoshIntegral[6*(a/b + Ar
cSinh[c*x])] + 10*Log[a + b*ArcSinh[c*x]] - 15*Sinh[(2*a)/b]*SinhIntegral[2
*(a/b + ArcSinh[c*x])] - 6*Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x]
)] - Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])])/(32*b*c)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.79

method	result
default	$-\frac{e^{\frac{6a}{b}}\text{Ei}_1\left(6\text{ arcsinh}(cx)+\frac{6a}{b}\right)+6e^{\frac{4a}{b}}\text{Ei}_1\left(4\text{ arcsinh}(cx)+\frac{4a}{b}\right)+15e^{\frac{2a}{b}}\text{Ei}_1\left(2\text{ arcsinh}(cx)+\frac{2a}{b}\right)+15e^{-\frac{2a}{b}}\text{Ei}_1\left(-2\text{ arcsinh}(cx)-\frac{2a}{b}\right)+}{64bc}$

```
[In] int((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64*(exp(6*a/b)*Ei(1,6*arcsinh(c*x)+6*a/b)+6*exp(4*a/b)*Ei(1,4*arcsinh(c*
x)+4*a/b)+15*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)+15*exp(-2*a/b)*Ei(1,-2*a
rcsinh(c*x)-2*a/b)+6*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)+exp(-6*a/b)*Ei
(1,-6*arcsinh(c*x)-6*a/b)-20*ln(a+b*arcsinh(c*x)))/b/c
```

Fricas [F]

$$\int \frac{(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a),
x)
```

Sympy [F]

$$\int \frac{(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{a+b\,\text{asinh}\,(cx)}\,dx$$

```
[In] integrate((c**2*x**2+1)**(5/2)/(a+b*asinh(c*x)),x)
[Out] Integral((c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*x^2 + 1)^(5/2)/(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((c^2*x^2 + 1)^(5/2)/(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{{\left(c^2\,x^2+1\right)}^{5/2}}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

```
[In] int((c^2*x^2 + 1)^(5/2)/(a + b*asinh(c*x)),x)
[Out] int((c^2*x^2 + 1)^(5/2)/(a + b*asinh(c*x)), x)
```

3.376 $\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))}\,dx = -\frac{11\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8b} - \frac{7\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16b} - \frac{\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{16b} + \frac{11\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b} + \frac{7\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b} + \frac{\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b} + \text{Int}\left(\frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))},x\right)$$

```
[Out] 11/8*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b+7/16*cosh(3*a/b)*Shi(3*(a+b*arcs
inh(c*x))/b)/b+1/16*cosh(5*a/b)*Shi(5*(a+b*arcsinh(c*x))/b)/b-11/8*Chi((a+b
*arcsinh(c*x))/b)*sinh(a/b)/b-7/16*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/
b-1/16*Chi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b+Unintegrable(1/x/(a+b*arcs
inh(c*x))/(c^2*x^2+1)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))}\,dx=\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (-11*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(8*b) - (7*CoshIntegra
l[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(16*b) - (CoshIntegral[(5*(a +
 b*ArcSinh[c*x]))/b]*Sinh[(5*a)/b])/(16*b) + (11*Cosh[a/b]*SinhIntegral[(a
+ b*ArcSinh[c*x])/b])/(8*b) + (7*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSi
nh[c*x]))/b])/(16*b) + (Cosh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*x]))
/b])/(16*b) + Defer[Int][1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}+\frac{3c^2x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}+\frac{3c^4x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}+\frac{c^6x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\right)dx \\
&= (3c^2)\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx+(3c^4)\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx \\
&\quad +c^6\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx+\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx \\
&= -\frac{\text{Subst}\left(\int \frac{\sinh^5\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b} \\
&\quad -\frac{3\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b} \\
&\quad -\frac{3\text{Subst}\left(\int \frac{\sinh^3\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b} \\
&\quad +\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx
\end{aligned}$$

$$= \frac{i\text{Subst}\left(\int \left(\frac{i\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16x}-\frac{5i\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16x}+\frac{5i\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{8x}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{b}$$
$$-\frac{(3i)\text{Subst}\left(\int \left(-\frac{i\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4x}+\frac{3i\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{4x}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{b}$$
$$+\frac{\left(3\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b}$$
$$-\frac{\left(3\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b}$$
$$+\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$$
$$= -\frac{3\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b}+\frac{3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b}$$
$$-\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{16b}$$
$$+\frac{5\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{16b}$$
$$-\frac{5\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{8b}$$
$$-\frac{3\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{4b}$$
$$+\frac{9\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{4b}$$
$$+\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$$

$$
\begin{aligned}
&= -\frac{3\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b} + \frac{3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b} \\
&\quad + \frac{\left(5\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b} \\
&\quad - \frac{\left(9\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b} \\
&\quad - \frac{\left(5\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b} \\
&\quad + \frac{\left(3\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b} \\
&\quad + \frac{\cosh\left(\frac{5a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{5x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b} \\
&\quad - \frac{\left(5\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b} \\
&\quad + \frac{\left(9\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b} \\
&\quad + \frac{\left(5\sinh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b} \\
&\quad - \frac{\left(3\sinh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b} \\
&\quad - \frac{\sinh\left(\frac{5a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{5x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b} \\
&\quad + \int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx \\
&= -\frac{11\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8b} - \frac{7\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16b} \\
&\quad - \frac{\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{16b} + \frac{11\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b} \\
&\quad + \frac{7\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b} + \frac{\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b} \\
&\quad + \int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx
\end{aligned}
$$

Mathematica [N/A]

Not integrable

Time = 2.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))}\,dx=\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x\,(a+b\,\operatorname{arcsinh}\,(cx))}dx$$

```
[In] int((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x)),x)
[Out] int((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\operatorname{arsinh}\,(cx)+a)x}\,dx$$

```
[In] integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b*x*arcsinh(c*x) + a*
x), x)
```

Sympy [N/A]

Not integrable

Time = 5.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x\,(a+b\,\text{asinh}\,(cx))}\,dx$$

```
[In] integrate((c**2*x**2+1)**(5/2)/x/(a+b*asinh(c*x)),x)
[Out] Integral((c**2*x**2 + 1)**(5/2)/(x*(a + b*asinh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)x}\,dx$$

```
[In] integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))}\,dx=\int \frac{{\left(c^2\,x^2+1\right)}^{5/2}}{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

```
[In] int((c^2*x^2 + 1)^(5/2)/(x*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(5/2)/(x*(a + b*asinh(c*x))), x)
```

3.377 $\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))} dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))} dx = \frac{c\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b} + \frac{c\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8b} + \frac{15c\log(a+b\text{arcsinh}(cx))}{8b} - \frac{c\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b} - \frac{c\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8b} + \text{Int}\left(\frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}, x\right)$$

```
[Out] c*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b+1/8*c*Chi(4*(a+b*arcsinh(c*x))/
b)*cosh(4*a/b)/b+15/8*c*ln(a+b*arcsinh(c*x))/b-c*Shi(2*(a+b*arcsinh(c*x))/b
)*sinh(2*a/b)/b-1/8*c*Shi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b+Unintegrabl
e(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))} dx = \int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))} dx$$

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (c*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/b + (c*Cosh[(4*a
)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(8*b) + (15*c*Log[a + b*ArcS
inh[c*x]])/(8*b) - (c*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/b
])/b - (c*Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(8*b) + D
efer[Int][1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\text{integral} = \int \left(\frac{3c^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))} + \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))} + \frac{3c^4x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))} + \frac{c^6x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\right) dx$$

$$= (3c^2)\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx + (3c^4)\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx + c^6\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx + \int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$$

$$= \frac{3c\log(a+b\text{arcsinh}(cx))}{b} + \frac{c\text{Subst}\left(\int \frac{\sinh^4\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b} + \frac{(3c)\text{Subst}\left(\int \frac{\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b} + \int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$$

$$= \frac{3c\log(a+b\text{arcsinh}(cx))}{b} + \frac{c\text{Subst}\left(\int \left(\frac{3}{8x}+\frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{8x}-\frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2x}\right) dx, x, a+b\text{arcsinh}(cx)\right)}{b} - \frac{(3c)\text{Subst}\left(\int \left(\frac{1}{2x}-\frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2x}\right) dx, x, a+b\text{arcsinh}(cx)\right)}{b} + \int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$$

$$= \frac{15c\log(a+b\text{arcsinh}(cx))}{8b} + \frac{c\text{Subst}\left(\int \frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b} - \frac{c\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b} + \frac{(3c)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b} + \int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$$

$$
\begin{aligned}
&= \frac{15c\log(a+b\operatorname{arcsinh}(cx))}{8b} - \frac{\left(c\cosh\left(\frac{2a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{2b} \\
&\quad + \frac{\left(3c\cosh\left(\frac{2a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{2b} \\
&\quad + \frac{\left(c\cosh\left(\frac{4a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{8b} \\
&\quad + \frac{\left(c\sinh\left(\frac{2a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{2b} \\
&\quad - \frac{\left(3c\sinh\left(\frac{2a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{2b} \\
&\quad - \frac{\left(c\sinh\left(\frac{4a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{8b} \\
&\quad + \int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx \\
&= \frac{c\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{b} + \frac{c\cosh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(\frac{4(a+b\operatorname{arcsinh}(cx))}{b}\right)}{8b} \\
&\quad + \frac{15c\log(a+b\operatorname{arcsinh}(cx))}{8b} - \frac{c\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{b} \\
&\quad - \frac{c\sinh\left(\frac{4a}{b}\right)\operatorname{Shi}\left(\frac{4(a+b\operatorname{arcsinh}(cx))}{b}\right)}{8b} + \int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx
\end{aligned}
$$

Mathematica [N/A]

Not integrable

Time = 2.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$
\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\operatorname{arcsinh}(cx))}\,dx = \int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\operatorname{arcsinh}(cx))}\,dx
$$

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^2\left(a+b\operatorname{arcsinh}\left(cx\right)\right)}dx$$

```
[In] int((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x)),x)
[Out] int((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\operatorname{arcsinh}(cx))}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\operatorname{arsinh}\left(cx\right)+a)x^2}\,dx$$

```
[In] integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b*x^2*arcsinh(c*x) +
a*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 3.88 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\operatorname{arcsinh}(cx))}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^2\left(a+b\operatorname{asinh}\left(cx\right)\right)}\,dx$$

```
[In] integrate((c**2*x**2+1)**(5/2)/x**2/(a+b*asinh(c*x)),x)
[Out] Integral((c**2*x**2 + 1)**(5/2)/(x**2*(a + b*asinh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\operatorname{arsinh}(cx)+a)x^2}\,dx$$

```
[In] integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\operatorname{arsinh}(cx)+a)x^2}\,dx$$

```
[In] integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2\,x^2+1)^{5/2}}{x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

```
[In] int((c^2*x^2 + 1)^(5/2)/(x^2*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(5/2)/(x^2*(a + b*asinh(c*x))), x)
```

3.378 $\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\,\mathbf{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\,\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{(1+c^2x^2)^{5/2}}{x^3(a+b\,\text{arcsinh}(cx))},x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\,\text{arcsinh}(cx))}\,dx = \int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\,\text{arcsinh}(cx))}\,dx$$

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\,\text{arcsinh}(cx))}\,dx$$

Mathematica [N/A]

Not integrable

Time = 4.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx=\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^3\,(a+b\,\text{arcsinh}\,(cx))}dx$$

```
[In] int((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x)
[Out] int((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)x^3}\,dx$$

```
[In] integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b*x^3*arcsinh(c*x) +
a*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 4.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\operatorname{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^3\,(a+b\operatorname{asinh}(cx))}\,dx$$

```
[In] integrate((c**2*x**2+1)**(5/2)/x**3/(a+b*asinh(c*x)),x)
[Out] Integral((c**2*x**2 + 1)**(5/2)/(x**3*(a + b*asinh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\operatorname{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\operatorname{arsinh}(cx)+a)x^3}\,dx$$

```
[In] integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\operatorname{arcsinh}(cx))}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.72 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx=\int \frac{{\left(c^2\,x^2+1\right)}^{5/2}}{x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

```
[In] int((c^2*x^2 + 1)^(5/2)/(x^3*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(5/2)/(x^3*(a + b*asinh(c*x))), x)
```

3.379 $\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\mathbf{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))},x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx=\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^4\,(a+b\,\operatorname{arcsinh}\,(cx))}dx$$

```
[In] int((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x)
[Out] int((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)x^4}\,dx$$

```
[In] integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b*x^4*arcsinh(c*x) +
a*x^4), x)
```

Sympy [N/A]

Not integrable

Time = 4.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^4\,(a+b\,\text{asinh}\,(cx))}\,dx$$

```
[In] integrate((c**2*x**2+1)**(5/2)/x**4/(a+b*asinh(c*x)),x)
[Out] Integral((c**2*x**2 + 1)**(5/2)/(x**4*(a + b*asinh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)x^4}\,dx$$

```
[In] integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x^4), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)x^4}\,dx$$

```
[In] integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x^4), x)
```

Mupad [N/A]

Not integrable

Time = 2.65 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx=\int \frac{{\left(c^2\,x^2+1\right)}^{5/2}}{x^4\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

```
[In] int((c^2*x^2 + 1)^(5/2)/(x^4*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(5/2)/(x^4*(a + b*asinh(c*x))), x)
```

3.380 $\int \frac{x^4}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 41

$$\int \frac{x^4}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}\,dx = -\frac{\operatorname{Chi}(2\operatorname{arcsinh}(ax))}{2a^5}+\frac{\operatorname{Chi}(4\operatorname{arcsinh}(ax))}{8a^5}+\frac{3\log(\operatorname{arcsinh}(ax))}{8a^5}$$

```
[Out] -1/2*Chi(2*arcsinh(a*x))/a^5+1/8*Chi(4*arcsinh(a*x))/a^5+3/8*ln(arcsinh(a*x
))/a^5
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5819, 3393, 3382}

$$\int \frac{x^4}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}\,dx = -\frac{\operatorname{Chi}(2\operatorname{arcsinh}(ax))}{2a^5}+\frac{\operatorname{Chi}(4\operatorname{arcsinh}(ax))}{8a^5}+\frac{3\log(\operatorname{arcsinh}(ax))}{8a^5}$$

```
[In] Int[x^4/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] -1/2*CoshIntegral[2*ArcSinh[a*x]]/a^5 + CoshIntegral[4*ArcSinh[a*x]]/(8*a^5
) + (3*Log[ArcSinh[a*x]])/(8*a^5)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sinh^4(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{a^5} \\
&= \frac{\text{Subst}\left(\int \left(\frac{3}{8x} - \frac{\cosh(2x)}{2x} + \frac{\cosh(4x)}{8x}\right)\,dx, x, \text{arcsinh}(ax)\right)}{a^5} \\
&= \frac{3\log(\text{arcsinh}(ax))}{8a^5} + \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{8a^5} \\
&\quad - \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{2a^5} \\
&= -\frac{\text{Chi}(2\text{arcsinh}(ax))}{2a^5} + \frac{\text{Chi}(4\text{arcsinh}(ax))}{8a^5} + \frac{3\log(\text{arcsinh}(ax))}{8a^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\begin{aligned}
&\int \frac{x^4}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx \\
&= \frac{-4\text{Chi}(2\text{arcsinh}(ax)) + \text{Chi}(4\text{arcsinh}(ax)) + 3\log(\text{arcsinh}(ax))}{8a^5}
\end{aligned}$$

```
[In] Integrate[x^4/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
[Out] (-4*CoshIntegral[2*ArcSinh[a*x]] + CoshIntegral[4*ArcSinh[a*x]] + 3*Log[Arc
Sinh[a*x]])/(8*a^5)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{3\ln(\operatorname{arcsinh}(ax))-4\operatorname{Chi}(2\operatorname{arcsinh}(ax))+\operatorname{Chi}(4\operatorname{arcsinh}(ax))}{8a^5}$	30

```
[In] int(x^4/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/8*(3*ln(arcsinh(a*x))-4*Chi(2*arcsinh(a*x))+Chi(4*arcsinh(a*x)))/a^5
```

Fricas [F]

$$\int \frac{x^4}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^4}{\sqrt{a^2x^2+1}\operatorname{arsinh}(ax)}\,dx$$

```
[In] integrate(x^4/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^4/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^4}{\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}\,dx$$

```
[In] integrate(x**4/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
[Out] Integral(x**4/(sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^4}{\sqrt{a^2x^2+1}\operatorname{arsinh}(ax)}\,dx$$

```
[In] integrate(x^4/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^4}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}\,dx$$

```
[In] integrate(x^4/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^4}{\text{asinh}\,(a\,x)\,\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int(x^4/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(x^4/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)), x)
```

3.381 $\int \frac{x^3}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 27

$$\int \frac{x^3}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}\,dx = -\frac{3\operatorname{Shi}(\operatorname{arcsinh}(ax))}{4a^4}+\frac{\operatorname{Shi}(3\operatorname{arcsinh}(ax))}{4a^4}$$

```
[Out] -3/4*Shi(arcsinh(a*x))/a^4+1/4*Shi(3*arcsinh(a*x))/a^4
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5819, 3393, 3379}

$$\int \frac{x^3}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}\,dx = \frac{\operatorname{Shi}(3\operatorname{arcsinh}(ax))}{4a^4}-\frac{3\operatorname{Shi}(\operatorname{arcsinh}(ax))}{4a^4}$$

```
[In] Int[x^3/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] (-3*SinhIntegral[ArcSinh[a*x]])/(4*a^4) + SinhIntegral[3*ArcSinh[a*x]]/(4*a
^4)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sinh^3(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{a^4} \\
&= \frac{i\text{Subst}\left(\int \left(\frac{3i\sinh(x)}{4x} - \frac{i\sinh(3x)}{4x}\right)dx, x, \text{arcsinh}(ax)\right)}{a^4} \\
&= \frac{\text{Subst}\left(\int \frac{\sinh(3x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{4a^4} - \frac{3\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{4a^4} \\
&= -\frac{3\text{Shi}(\text{arcsinh}(ax))}{4a^4} + \frac{\text{Shi}(3\text{arcsinh}(ax))}{4a^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \frac{-3\text{Shi}(\text{arcsinh}(ax)) + \text{Shi}(3\text{arcsinh}(ax))}{4a^4}$$

```
[In] Integrate[x^3/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
[Out] (-3*SinhIntegral[ArcSinh[a*x]] + SinhIntegral[3*ArcSinh[a*x]])/(4*a^4)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{3\,\text{Shi}(\text{arcsinh}(ax))-\text{Shi}(3\,\text{arcsinh}(ax))}{4a^4}$	23

```
[In] int(x^3/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(3*Shi(arcsinh(a*x))-Shi(3*arcsinh(a*x)))/a^4
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^3}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}\,dx$$

```
[In] integrate(x^3/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^3/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^3}{\sqrt{a^2x^2+1}\,\text{asinh}\,(ax)}\,dx$$

```
[In] integrate(x**3/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
[Out] Integral(x**3/(sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^3}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}\,dx$$

```
[In] integrate(x^3/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^3}{\text{asinh}\left(a\,x\right)\,\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int(x^3/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(x^3/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)), x)
```

3.382 $\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)} dx$

Optimal result

Integrand size = 23, antiderivative size = 27

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)} dx = \frac{\text{Chi}(2\text{arcsinh}(ax))}{2a^3} - \frac{\log(\text{arcsinh}(ax))}{2a^3}$$

```
[Out] 1/2*Chi(2*arcsinh(a*x))/a^3-1/2*ln(arcsinh(a*x))/a^3
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5819, 3393, 3382}

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)} dx = \frac{\text{Chi}(2\text{arcsinh}(ax))}{2a^3} - \frac{\log(\text{arcsinh}(ax))}{2a^3}$$

```
[In] Int[x^2/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] CoshIntegral[2*ArcSinh[a*x]]/(2*a^3) - Log[ArcSinh[a*x]]/(2*a^3)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sinh^2(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{a^3} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{2x} - \frac{\cosh(2x)}{2x}\right)\,dx, x, \text{arcsinh}(ax)\right)}{a^3} \\
&= -\frac{\log(\text{arcsinh}(ax))}{2a^3} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{2a^3} \\
&= \frac{\text{Chi}(2\text{arcsinh}(ax))}{2a^3} - \frac{\log(\text{arcsinh}(ax))}{2a^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \frac{\text{Chi}(2\text{arcsinh}(ax)) - \log(\text{arcsinh}(ax))}{2a^3}$$

```
[In] Integrate[x^2/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
[Out] (CoshIntegral[2*ArcSinh[a*x]] - Log[ArcSinh[a*x]])/(2*a^3)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\ln(\text{arcsinh}(ax))-\text{Chi}(2\,\text{arcsinh}(ax))}{2a^3}$	21

```
[In] int(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(ln(arcsinh(a*x))-Chi(2*arcsinh(a*x)))/a^3
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^2}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}\,dx$$

```
[In] integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^2}{\sqrt{a^2x^2+1}\,\text{asinh}\,(ax)}\,dx$$

```
[In] integrate(x**2/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
[Out] Integral(x**2/(sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^2}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}\,dx$$

```
[In] integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^2}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}\,dx$$

```
[In] integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^2}{\text{asinh}\,(a\,x)\,\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int(x^2/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(x^2/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)), x)
```

3.383 $\int \frac{x^2}{\sqrt{1+a^2x^2}\,\mathbf{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 27

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx=\frac{\text{Chi}(2\text{arcsinh}(ax))}{2a^3}-\frac{\log(\text{arcsinh}(ax))}{2a^3}$$

```
[Out] 1/2*Chi(2*arcsinh(a*x))/a^3-1/2*ln(arcsinh(a*x))/a^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5819, 3393, 3382}

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx=\frac{\text{Chi}(2\text{arcsinh}(ax))}{2a^3}-\frac{\log(\text{arcsinh}(ax))}{2a^3}$$

```
[In] Int[x^2/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] CoshIntegral[2*ArcSinh[a*x]]/(2*a^3) - Log[ArcSinh[a*x]]/(2*a^3)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sinh^2(x)}{x} \, dx, x, \text{arcsinh}(ax)\right)}{a^3} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{2x} - \frac{\cosh(2x)}{2x}\right) \, dx, x, \text{arcsinh}(ax)\right)}{a^3} \\
&= -\frac{\log(\text{arcsinh}(ax))}{2a^3} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x} \, dx, x, \text{arcsinh}(ax)\right)}{2a^3} \\
&= \frac{\text{Chi}(2\text{arcsinh}(ax))}{2a^3} - \frac{\log(\text{arcsinh}(ax))}{2a^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)} \, dx = \frac{\text{Chi}(2\text{arcsinh}(ax)) - \log(\text{arcsinh}(ax))}{2a^3}$$

```
[In] Integrate[x^2/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
[Out] (CoshIntegral[2*ArcSinh[a*x]] - Log[ArcSinh[a*x]])/(2*a^3)
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\ln(\text{arcsinh}(ax)) - \text{Chi}(2 \text{ arcsinh}(ax))}{2a^3}$	21

```
[In] int(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(ln(arcsinh(a*x))-Chi(2*arcsinh(a*x)))/a^3
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^2}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}\,dx$$

```
[In] integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^2}{\sqrt{a^2x^2+1}\,\text{asinh}\,(ax)}\,dx$$

```
[In] integrate(x**2/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
[Out] Integral(x**2/(sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^2}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}\,dx$$

```
[In] integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^2}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}\,dx$$

```
[In] integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^2}{\text{asinh}\,(a\,x)\,\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int(x^2/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(x^2/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)), x)
```

3.384 $\int \frac{x}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 9

$$\int \frac{x}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}\,dx = \frac{\operatorname{Shi}(\operatorname{arcsinh}(ax))}{a^2}$$

```
[Out] Shi(arcsinh(a*x))/a^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5819, 3379}

$$\int \frac{x}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}\,dx = \frac{\operatorname{Shi}(\operatorname{arcsinh}(ax))}{a^2}$$

```
[In] Int[x/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] SinhIntegral[ArcSinh[a*x]]/a^2
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
```

```
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{a^2}$$
$$= \frac{\text{Shi}(\text{arcsinh}(ax))}{a^2}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \frac{\text{Shi}(\text{arcsinh}(ax))}{a^2}$$

```
[In] Integrate[x/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
[Out] SinhIntegral[ArcSinh[a*x]]/a^2
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{\text{Shi}(\text{arcsinh}(ax))}{a^2}$	10

```
[In] int(x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] Shi(arcsinh(a*x))/a^2
```

Fricas [F]

$$\int \frac{x}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}\,dx$$

```
[In] integrate(x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(x/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}\,dx = \int \frac{x}{\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}\,dx$$

```
[In] integrate(x/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
[Out] Integral(x/(sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}\,dx = \int \frac{x}{\sqrt{a^2x^2+1}\,\operatorname{arsinh}(ax)}\,dx$$

```
[In] integrate(x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}\,dx = \int \frac{x}{\sqrt{a^2x^2+1}\,\operatorname{arsinh}(ax)}\,dx$$

```
[In] integrate(x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}\,dx = \int \frac{x}{\operatorname{asinh}(a\,x)\,\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int(x/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(x/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)), x)
```

3.385 $\int \frac{1}{\sqrt{1+a^2x^2}\mathbf{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 9

$$\int \frac{1}{\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)}\,dx = \frac{\log(\text{arcsinh}(ax))}{a}$$

```
[Out] ln(arcsinh(a*x))/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {5782}

$$\int \frac{1}{\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)}\,dx = \frac{\log(\text{arcsinh}(ax))}{a}$$

```
[In] Int[1/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] Log[ArcSinh[a*x]]/a
```

Rule 5782

```
Int[1/(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sy
mbol] :> Simp[(1/(b*c))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*Log[a + b*A
rcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\text{integral} = \frac{\log(\text{arcsinh}(ax))}{a}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx=\frac{\log(\text{arcsinh}(ax))}{a}$$

```
[In] Integrate[1/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
[Out] Log[ArcSinh[a*x]]/a
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{\ln(\text{arcsinh}(ax))}{a}$	10
default	$\frac{\ln(\text{arcsinh}(ax))}{a}$	10

```
[In] int(1/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(arcsinh(a*x))/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(9) = 18.

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.33

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx=\frac{\log\left(\log\left(ax+\sqrt{a^2x^2+1}\right)\right)}{a}$$

```
[In] integrate(1/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] log(log(a*x + sqrt(a^2*x^2 + 1)))/a
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \frac{\log\left(\text{asinh}\left(ax\right)\right)}{a}$$

```
[In] integrate(1/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
[Out] log(asinh(a*x))/a
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \frac{\log\left(\text{arsinh}\left(ax\right)\right)}{a}$$

```
[In] integrate(1/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] log(arcsinh(a*x))/a
```

Giac [F]

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{1}{\sqrt{a^2x^2+1}\,\text{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(1/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Mupad [B] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \frac{\ln\left(\text{asinh}(a\,x)\right)}{a}$$

```
[In] int(1/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
[Out] log(asinh(a*x))/a
```

3.386 $\int \frac{1}{x\sqrt{1+a^2x^2}\textbf{arcsinh}(ax)} dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)} dx = \text{Int}\left(\frac{1}{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}, x\right)$$

```
[Out] Unintegrable(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)} dx = \int \frac{1}{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)} dx$$

```
[In] Int[1/(x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] Defer[Int][1/(x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)} dx$$

Mathematica [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{1}{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx$$

```
[In] Integrate[1/(x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
[Out] Integrate[1/(x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\ \text{arcsinh}\left(ax\right)\sqrt{a^{2}x^{2}+1}}dx$$

```
[In] int(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
[Out] int(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{1}{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{1}{\sqrt{a^{2}x^{2}+1}x\,\text{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*x^2 + 1)/((a^2*x^3 + x)*arcsinh(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{1}{x\sqrt{a^{2}x^{2}+1}\,\text{asinh}\left(ax\right)}\,dx$$

```
[In] integrate(1/x/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
[Out] Integral(1/(x*sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{1}{\sqrt{a^2x^2+1}x\,\text{arsinh}\,(ax)}\,dx$$

```
[In] integrate(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a^2*x^2 + 1)*x*arcsinh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{1}{\sqrt{a^2x^2+1}x\,\text{arsinh}\,(ax)}\,dx$$

```
[In] integrate(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a^2*x^2 + 1)*x*arcsinh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 2.69 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{1}{x\,\text{asinh}\,(a\,x)\,\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int(1/(x*asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(1/(x*asinh(a*x)*(a^2*x^2 + 1)^(1/2)), x)
```

3.387 $\int \frac{1}{x^2\sqrt{1+a^2x^2}\textbf{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \text{Int}\left(\frac{1}{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)},x\right)$$

```
[Out] Unintegrable(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{1}{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx$$

```
[In] Int[1/(x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] Defer[Int][1/(x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{1}{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx$$

```
[In] Integrate[1/(x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
[Out] Integrate[1/(x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2 \operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}}dx$$

```
[In] int(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
[Out] int(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{1}{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{1}{\sqrt{a^2x^2+1}x^2 \operatorname{arsinh}(ax)}\,dx$$

```
[In] integrate(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*x^2 + 1)/((a^2*x^4 + x^2)*arcsinh(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{1}{x^2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}\,dx$$

```
[In] integrate(1/x**2/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{1}{\sqrt{a^2x^2+1}x^2\,\text{arsinh}\,(ax)}\,dx$$

```
[In] integrate(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a^2*x^2 + 1)*x^2*arcsinh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{1}{\sqrt{a^2x^2+1}x^2\,\text{arsinh}\,(ax)}\,dx$$

```
[In] integrate(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a^2*x^2 + 1)*x^2*arcsinh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 2.79 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{1}{x^2\,\text{asinh}\,(a\,x)\,\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int(1/(x^2*asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(1/(x^2*asinh(a*x)*(a^2*x^2 + 1)^(1/2)), x)
```

3.388 $\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 183

$$\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = -\frac{5\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8bc^6} + \frac{5\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16bc^6} - \frac{\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{16bc^6} + \frac{5\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8bc^6} - \frac{5\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^6} + \frac{\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^6}$$

```
[Out] 5/8*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^6-5/16*cosh(3*a/b)*Shi(3*(a+b*a
rcsinh(c*x))/b)/b/c^6+1/16*cosh(5*a/b)*Shi(5*(a+b*arcsinh(c*x))/b)/b/c^6-5/
8*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^6+5/16*Chi(3*(a+b*arcsinh(c*x))/b
)*sinh(3*a/b)/b/c^6-1/16*Chi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b/c^6
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5819, 3393, 3384, 3379, 3382}

$$\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx = -\frac{5\sinh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{8bc^6} + \frac{5\sinh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^6} - \frac{\sinh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^6} + \frac{5\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{8bc^6} - \frac{5\cosh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^6} + \frac{\cosh\left(\frac{5a}{b}\right)\operatorname{Shi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^6}$$

```
[In] Int[x^5/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (-5*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(8*b*c^6) + (5*CoshInte
gral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(16*b*c^6) - (CoshIntegral[
(5*(a + b*ArcSinh[c*x]))/b]*Sinh[(5*a)/b])/(16*b*c^6) + (5*Cosh[a/b]*SinhIn
tegral[(a + b*ArcSinh[c*x])/b])/(8*b*c^6) - (5*Cosh[(3*a)/b]*SinhIntegral[(
3*(a + b*ArcSinh[c*x]))/b])/(16*b*c^6) + (Cosh[(5*a)/b]*SinhIntegral[(5*(a
+ b*ArcSinh[c*x]))/b])/(16*b*c^6)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\sinh^5\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc^6}\\
&= \frac{i\text{Subst}\left(\int \left(\frac{i\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16x}-\frac{5i\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16x}+\frac{5i\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{8x}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc^6}\\
&= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{16bc^6}\\
&\quad+\frac{5\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{16bc^6}\\
&\quad-\frac{5\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{8bc^6}
\end{aligned}$$

$$= \frac{\left(5\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a + b\text{arcsinh}(cx)\right)}{8bc^6} - \frac{\left(5\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a + b\text{arcsinh}(cx)\right)}{16bc^6} + \frac{\cosh\left(\frac{5a}{b}\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{5x}{b}\right)}{x}\,dx, x, a + b\text{arcsinh}(cx)\right)}{16bc^6} - \frac{\left(5\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a + b\text{arcsinh}(cx)\right)}{8bc^6} + \frac{\left(5\sinh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a + b\text{arcsinh}(cx)\right)}{16bc^6} - \frac{\sinh\left(\frac{5a}{b}\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{5x}{b}\right)}{x}\,dx, x, a + b\text{arcsinh}(cx)\right)}{16bc^6}$$

$$= -\frac{5\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8bc^6} + \frac{5\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16bc^6} - \frac{\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{16bc^6} + \frac{5\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8bc^6} - \frac{5\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^6} + \frac{\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^6}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.74

$$\int\frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = -\frac{10\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right) - 5\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right) + \text{Chi}\left(5\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sin}{}$$

```
[In] Integrate[x^5/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] -1/16*(10*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - 5*CoshIntegral[3*(a/
b + ArcSinh[c*x])]*Sinh[(3*a)/b] + CoshIntegral[5*(a/b + ArcSinh[c*x])]*Sin
h[(5*a)/b] - 10*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 5*Cosh[(3*a)/b
]*SinhIntegral[3*(a/b + ArcSinh[c*x])] - Cosh[(5*a)/b]*SinhIntegral[5*(a/b
+ ArcSinh[c*x])])/(b*c^6)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.81

method	result
default	$\frac{e^{\frac{5a}{b}} \operatorname{Ei}_1\left(5 \operatorname{arcsinh}(cx)+\frac{5a}{b}\right)-5 e^{\frac{3a}{b}} \operatorname{Ei}_1\left(3 \operatorname{arcsinh}(cx)+\frac{3a}{b}\right)+10 e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)-10 e^{-\frac{a}{b}} \operatorname{Ei}_1\left(-\operatorname{arcsinh}(cx)-\frac{a}{b}\right)+5 e^{-\frac{3a}{b}} \operatorname{Ei}_1(}{32c^6 b}$

[In] int(x^5/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/32*(exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-5*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+10*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-10*exp(-a/b)*Ei(1,-arcsinh(c*x) -a/b)+5*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-exp(-5*a/b)*Ei(1,-5*arcsinh (c*x)-5*a/b))/c^6/b

Fricas [F]

$$\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^5}{\sqrt{c^2x^2+1}(b\operatorname{arsinh}(cx)+a)}\,dx$$

[In] integrate(x^5/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(c^2*x^2 + 1)*x^5/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) + a), x)

Sympy [F]

$$\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^5}{(a+b\operatorname{asinh}(cx))\sqrt{c^2x^2+1}}\,dx$$

[In] integrate(x**5/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)

[Out] Integral(x**5/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)

Maxima [F]

$$\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^5}{\sqrt{c^2x^2+1}(b\operatorname{arsinh}(cx)+a)}\,dx$$

[In] integrate(x^5/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(x^5/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^5/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^5}{(a+b\,\text{asinh}\,(c\,x))\,\sqrt{c^2\,x^2+1}}\,dx$$

```
[In] int(x^5/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(x^5/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

3.389 $\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\mathbf{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 144

$$\begin{aligned}\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx =& -\frac{\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{2bc^5} \\ &+ \frac{\cosh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(\frac{4(a+b\operatorname{arcsinh}(cx))}{b}\right)}{8bc^5} \\ &+ \frac{3\log(a+b\operatorname{arcsinh}(cx))}{8bc^5} \\ &+ \frac{\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{2bc^5} \\ &- \frac{\sinh\left(\frac{4a}{b}\right)\operatorname{Shi}\left(\frac{4(a+b\operatorname{arcsinh}(cx))}{b}\right)}{8bc^5}\end{aligned}$$

```
[Out] -1/2*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c^5+1/8*Chi(4*(a+b*arcsinh(c
*x))/b)*cosh(4*a/b)/b/c^5+3/8*ln(a+b*arcsinh(c*x))/b/c^5+1/2*Shi(2*(a+b*arc
sinh(c*x))/b)*sinh(2*a/b)/b/c^5-1/8*Shi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)
/b/c^5
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5819, 3393, 3384, 3379, 3382}

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = -\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc^5} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8bc^5} + \frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc^5} - \frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8bc^5} + \frac{3\log(a+b\text{arcsinh}(cx))}{8bc^5}$$

```
[In] Int[x^4/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] -1/2*(Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(b*c^5) + (Co
sh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(8*b*c^5) + (3*Log[a
+ b*ArcSinh[c*x]])/(8*b*c^5) + (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSin
h[c*x]))/b])/(2*b*c^5) - (Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]
))/b])/(8*b*c^5)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sinh^4\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{bc^5}\\
&= \frac{\text{Subst}\left(\int \left(\frac{3}{8x}+\frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{8x}-\frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2x}\right)\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{bc^5}\\
&= \frac{3\log(a+b\operatorname{arcsinh}(cx))}{8bc^5}+\frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{8bc^5}\\
&\quad -\frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{2bc^5}\\
&= \frac{3\log(a+b\operatorname{arcsinh}(cx))}{8bc^5}-\frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{2bc^5}\\
&\quad +\frac{\cosh\left(\frac{4a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{8bc^5}\\
&\quad +\frac{\sinh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{2bc^5}\\
&\quad -\frac{\sinh\left(\frac{4a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{8bc^5}\\
&= -\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{2bc^5}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\operatorname{arcsinh}(cx))}{b}\right)}{8bc^5}\\
&\quad +\frac{3\log(a+b\operatorname{arcsinh}(cx))}{8bc^5}+\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{2bc^5}\\
&\quad -\frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\operatorname{arcsinh}(cx))}{b}\right)}{8bc^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = -\frac{4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)-\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)-3\log(a+b\text{arcsinh}(cx))-}{8bc^5}$$

```
[In] Integrate[x^4/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] -1/8*(4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] - Cosh[(4*a)/b]*
CoshIntegral[4*(a/b + ArcSinh[c*x])] - 3*Log[a + b*ArcSinh[c*x]] - 4*Sinh[(
2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] + Sinh[(4*a)/b]*SinhIntegral[4
*(a/b + ArcSinh[c*x])])/(b*c^5)
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.80

method	result
default	$-\frac{e^{-\frac{4a}{b}}\text{Ei}_1\left(-4\,\text{arcsinh}(cx)-\frac{4a}{b}\right)+e^{\frac{4a}{b}}\text{Ei}_1\left(4\,\text{arcsinh}(cx)+\frac{4a}{b}\right)-4\,e^{\frac{2a}{b}}\text{Ei}_1\left(2\,\text{arcsinh}(cx)+\frac{2a}{b}\right)-4\,e^{-\frac{2a}{b}}\text{Ei}_1\left(-2\,\text{arcsinh}(cx)-\frac{2a}{b}\right)-}{16c^5b}$

```
[In] int(x^4/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)+exp(4*a/b)*Ei(1,4*arcsinh(c*
x)+4*a/b)-4*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-4*exp(-2*a/b)*Ei(1,-2*arc
sinh(c*x)-2*a/b)-6*ln(a+b*arcsinh(c*x)))/c^5/b
```

Fricas [F]

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^4}{\sqrt{c^2x^2+1}\,(b\,\text{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(x^4/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^4/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) +
a), x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^4}{(a+b\,\text{asinh}\,(cx))\,\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate(x**4/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
[Out] Integral(x**4/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^4}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(x^4/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^4}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(x^4/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^4}{(a+b\,\text{asinh}\,(c\,x))\,\sqrt{c^2\,x^2+1}}\,dx$$

```
[In] int(x^4/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(x^4/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

3.390 $\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\,\mathbf{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 121

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\,\text{arcsinh}(cx))}\,dx = \frac{3\text{Chi}\left(\frac{a+b\,\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4bc^4} - \frac{\text{Chi}\left(\frac{3(a+b\,\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4bc^4} - \frac{3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\,\text{arcsinh}(cx)}{b}\right)}{4bc^4} + \frac{\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\,\text{arcsinh}(cx))}{b}\right)}{4bc^4}$$

```
[Out] -3/4*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^4+1/4*cosh(3*a/b)*Shi(3*(a+b*a
rcsinh(c*x))/b)/b/c^4+3/4*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^4-1/4*Chi
(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^4
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used

$= \{5819, 3393, 3384, 3379, 3382\}$

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx = \frac{3\sinh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{4bc^4} - \frac{\sinh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{4bc^4} - \frac{3\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{4bc^4} + \frac{\cosh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{4bc^4}$$

```
[In] Int[x^3/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (3*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(4*b*c^4) - (CoshIntegra
l[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(4*b*c^4) - (3*Cosh[a/b]*SinhI
ntegral[(a + b*ArcSinh[c*x])/b])/(4*b*c^4) + (Cosh[(3*a)/b]*SinhIntegral[(3
*(a + b*ArcSinh[c*x]))/b])/(4*b*c^4)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\sinh^3\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc^4}\\
&= -\frac{i\text{Subst}\left(\int \left(-\frac{i\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4x}+\frac{3i\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{4x}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc^4}\\
&= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4bc^4}+\frac{3\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4bc^4}\\
&= -\frac{\left(3\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4bc^4}\\
&\quad+\frac{\cosh\left(\frac{3a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4bc^4}\\
&\quad+\frac{\left(3\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4bc^4}\\
&\quad-\frac{\sinh\left(\frac{3a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4bc^4}\\
&= \frac{3\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4bc^4}-\frac{\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4bc^4}\\
&\quad-\frac{3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4bc^4}+\frac{\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4bc^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.76

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$$
$$=\frac{3\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right)-\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)-3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)-}{4bc^4}$$

```
[In] Integrate[x^3/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

[Out] (3*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - CoshIntegral[3*(a/b + ArcSinh[c*x])]*Sinh[(3*a)/b] - 3*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])])/(4*b*c^4)

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\mathrm{e}^{\frac{3a}{b}} \mathrm{Ei}_1\left(3 \operatorname{arcsinh}(cx)+\frac{3a}{b}\right)-3\,\mathrm{e}^{\frac{a}{b}} \mathrm{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)+3\,\mathrm{e}^{-\frac{a}{b}} \mathrm{Ei}_1\left(-\operatorname{arcsinh}(cx)-\frac{a}{b}\right)-\mathrm{e}^{-\frac{3a}{b}} \mathrm{Ei}_1\left(-3 \operatorname{arcsinh}(cx)-\frac{3a}{b}\right)}{8c^4b}$	101

[In] int(x^3/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/8*(exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-3*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)+3*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)-exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b))/c^4/b

Fricas [F]

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^3}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

[In] integrate(x^3/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(c^2*x^2 + 1)*x^3/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) + a), x)

Sympy [F]

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^3}{(a+b\,\text{asinh}\,(cx))\,\sqrt{c^2x^2+1}}\,dx$$

[In] integrate(x**3/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)

[Out] Integral(x**3/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)

Maxima [F]

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^3}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(x^3/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^3}{(a+b\,\text{asinh}\,(c\,x))\,\sqrt{c^2\,x^2+1}}\,dx$$

```
[In] int(x^3/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(x^3/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

3.391 $\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 82

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc^3} - \frac{\log(a+b\text{arcsinh}(cx))}{2bc^3} - \frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc^3}$$

```
[Out] 1/2*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c^3-1/2*ln(a+b*arcsinh(c*x))/
b/c^3-1/2*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b/c^3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5819, 3393, 3384, 3379, 3382}

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc^3} - \frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc^3} - \frac{\log(a+b\text{arcsinh}(cx))}{2bc^3}$$

```
[In] Int[x^2/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(2*b*c^3) - Log[a
+ b*ArcSinh[c*x]]/(2*b*c^3) - (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh
[c*x]))/b])/(2*b*c^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc^3}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{2x}-\frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2x}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc^3}\\
&= -\frac{\log(a+b\text{arcsinh}(cx))}{2bc^3}+\frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2bc^3}
\end{aligned}$$

$$= -\frac{\log(a+b\text{arcsinh}(cx))}{2bc^3} + \frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{2bc^3} - \frac{\sinh\left(\frac{2a}{b}\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{2bc^3}$$

$$= \frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc^3} - \frac{\log(a+b\text{arcsinh}(cx))}{2bc^3} - \frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc^3}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int\frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)-\log(a+b\text{arcsinh}(cx))-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)}{2bc^3}$$

```
[In] Integrate[x^2/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] (Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] - Log[a + b*ArcSinh[c*x
]] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])])/(2*b*c^3)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{e^{\frac{2a}{b}}\,\text{Ei}_1\left(2\,\text{arcsinh}(cx)+\frac{2a}{b}\right)+e^{-\frac{2a}{b}}\,\text{Ei}_1\left(-2\,\text{arcsinh}(cx)-\frac{2a}{b}\right)+2\ln(a+b\,\text{arcsinh}(cx))}{4c^3b}$	67

```
[In] int(x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/4*(exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)+exp(-2*a/b)*Ei(1,-2*arcsinh(c*x
)-2*a/b)+2*ln(a+b*arcsinh(c*x)))/c^3/b
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{x^2}{\sqrt{c^2x^2+1}(b\operatorname{arsinh}(cx)+a)}\,dx$$

[In] integrate(x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(c^2*x^2 + 1)*x^2/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) + a), x)

Sympy [F]

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{x^2}{(a+b\operatorname{asinh}(cx))\sqrt{c^2x^2+1}}\,dx$$

[In] integrate(x**2/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)

[Out] Integral(x**2/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)

Maxima [F]

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{x^2}{\sqrt{c^2x^2+1}(b\operatorname{arsinh}(cx)+a)}\,dx$$

[In] integrate(x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(x^2/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)

Giac [F]

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{x^2}{\sqrt{c^2x^2+1}(b\operatorname{arsinh}(cx)+a)}\,dx$$

[In] integrate(x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")

[Out] integrate(x^2/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^2}{(a+b\,\text{asinh}\,(c\,x))\,\sqrt{c^2\,x^2+1}}\,dx$$

```
[In] int(x^2/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(x^2/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

3.392 $\int \frac{x}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 54

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx=-\frac{\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bc^2}+\frac{\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{bc^2}$$

```
[Out] cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^2-Chi((a+b*arcsinh(c*x))/b)*sinh(a/
b)/b/c^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5819, 3384, 3379, 3382}

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx=\frac{\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{bc^2}-\frac{\sinh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{bc^2}$$

```
[In] Int[x/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] -((CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(b*c^2)) + (Cosh[a/b]*Si
nhIntegral[(a + b*ArcSinh[c*x])/b])/(b*c^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc^2} \\
&= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc^2} \\
&\quad - \frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc^2} \\
&= -\frac{\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bc^2} + \frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = -\frac{\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right)-\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{bc^2}$$

```
[In] Integrate[x/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] -((CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - Cosh[a/b]*SinhIntegral[a/b
+ ArcSinh[c*x]])/(b*c^2))
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{e^{\frac{a}{b}}\,\text{Ei}_1\left(\text{arcsinh}(cx)+\frac{a}{b}\right)-e^{-\frac{a}{b}}\,\text{Ei}_1\left(-\,\text{arcsinh}(cx)-\frac{a}{b}\right)}{2c^2b}$	53

```
[In] int(x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b))/c^2
/b
```

Fricas [F]

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) + a)
, x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x}{(a+b\,\text{asinh}\,(cx))\,\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate(x/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
[Out] Integral(x/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x}{(a+b\,\text{asinh}\,(c\,x))\,\sqrt{c^2\,x^2+1}}\,dx$$

```
[In] int(x/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(x/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

3.393 $\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 16

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \frac{\log(a+b\text{arcsinh}(cx))}{bc}$$

```
[Out] ln(a+b*arcsinh(c*x))/b/c
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {5782}

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \frac{\log(a+b\text{arcsinh}(cx))}{bc}$$

```
[In] Int[1/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] Log[a + b*ArcSinh[c*x]]/(b*c)
```

Rule 5782

```
Int[1/(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sy
mbol] :> Simp[(1/(b*c))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*Log[a + b*A
rcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\text{integral} = \frac{\log(a+b\text{arcsinh}(cx))}{bc}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=\frac{\log(a+b\text{arcsinh}(cx))}{bc}$$

```
[In] Integrate[1/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] Log[a + b*ArcSinh[c*x]]/(b*c)
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\ln(a+b\,\text{arcsinh}(cx))}{bc}$	17
default	$\frac{\ln(a+b\,\text{arcsinh}(cx))}{bc}$	17

```
[In] int(1/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(a+b*arcsinh(c*x))/b/c
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=\frac{\log\left(b\log\left(cx+\sqrt{c^2x^2+1}\right)+a\right)}{bc}$$

```
[In] integrate(1/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] log(b*log(c*x + sqrt(c^2*x^2 + 1)) + a)/(b*c)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.92 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \begin{cases} \frac{x}{a} & \text{for}\: b = 0 \wedge c = 0 \\ \frac{\operatorname{asinh}{(cx)}}{ac} & \text{for}\: b = 0 \\ \frac{x}{a} & \text{for}\: c = 0 \\ \frac{\log{\left(\frac{a}{b} + \operatorname{asinh}{(cx)} \right)}}{bc} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
[Out] Piecewise((x/a, Eq(b, 0) & Eq(c, 0)), (asinh(c*x)/(a*c), Eq(b, 0)), (x/a, E
q(c, 0)), (log(a/b + asinh(c*x))/(b*c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \frac{\log{(b\,\text{arsinh}\,(cx) + a)}}{bc}$$

```
[In] integrate(1/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] log(b*arcsinh(c*x) + a)/(b*c)
```

Giac [F]

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Mupad [B] (verification not implemented)

Time = 2.75 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \frac{\ln(a+b\,\text{asinh}(c\,x))}{b\,c}$$

```
[In] int(1/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
[Out] log(a + b*asinh(c*x))/(b*c)
```

3.394 $\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\,\mathrm{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\,\mathrm{arcsinh}(cx))}\,dx = \mathrm{Int}\left(\frac{1}{x\sqrt{1+c^2x^2}(a+b\,\mathrm{arcsinh}(cx))},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\,\mathrm{arcsinh}(cx))}\,dx = \int \frac{1}{x\sqrt{1+c^2x^2}(a+b\,\mathrm{arcsinh}(cx))}\,dx$$

```
[In] Int[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\sqrt{1+c^2x^2}(a+b\,\mathrm{arcsinh}(cx))}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Integrate[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\left(a+b\,\operatorname{arcsinh}\left(cx\right)\right)\sqrt{c^2x^2+1}}dx$$

```
[In] int(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
[Out] int(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{\sqrt{c^2x^2+1}(b\,\operatorname{arsinh}\left(cx\right)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(a*c^2*x^3 + a*x + (b*c^2*x^3 + b*x)*arcsinh(c*x
)), x)
```

Sympy [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{x\left(a+b\,\text{asinh}\,(cx)\right)\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate(1/x/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
[Out] Integral(1/(x*(a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.92 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{c^2\,x^2+1}}\,dx$$

```
[In] int(1/(x*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(1/(x*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

3.395 $\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\,\textbf{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Int[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Integrate[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2\left(a+b\,\text{arcsinh}\left(cx\right)\right)\sqrt{c^2x^2+1}}dx$$

```
[In] int(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
[Out] int(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\left(cx\right)+a)x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(a*c^2*x^4 + a*x^2 + (b*c^2*x^4 + b*x^2)*arcsinh
(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{x^2\left(a+b\,\text{asinh}\,(cx)\right)\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate(1/x**2/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
[Out] Integral(1/(x**2*(a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.80 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{c^2\,x^2+1}}\,dx$$

```
[In] int(1/(x^2*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(1/(x^2*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

3.396 $\int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\mathbf{arcsinh}(cx))} dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))} dx = \text{Int}\left(\frac{x^2}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}, x\right)$$

```
[Out] Unintegrable(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))} dx = \int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))} dx$$

```
[In] Int[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))} dx$$

Mathematica [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx=\int \frac{x^2}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Integrate[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{(c^2x^2+1)^{\frac{3}{2}}\,(a+b\,\text{arcsinh}\,(cx))}dx$$

```
[In] int(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
[Out] int(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.30

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx=\int \frac{x^2}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^2/(a*c^4*x^4 + 2*a*c^2*x^2 + (b*c^4*x^4 + 2*b*
c^2*x^2 + b)*arcsinh(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}\,dx = \int \frac{x^2}{(a+b\operatorname{asinh}(cx))\,(c^2x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
[Out] Integral(x**2/((a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}\,dx = \int \frac{x^2}{(c^2x^2+1)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)}\,dx$$

```
[In] integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(x^2/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}\,dx = \int \frac{x^2}{(c^2x^2+1)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)}\,dx$$

```
[In] integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(x^2/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{x^2}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(c^2\,x^2+1\right)}^{3/2}} \,d x$$

```
[In] int(x^2/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)),x)
[Out] int(x^2/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)), x)
```

3.397 $\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\mathbf{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{x}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))},x\right)$$

```
[Out] Unintegrable(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Int[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{x}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Integrate[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{x}{(c^2x^2+1)^{\frac{3}{2}}\,(a+b\,\text{arcsinh}\,(cx))}dx$$

```
[In] int(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
[Out] int(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.40

$$\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{x}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x/(a*c^4*x^4 + 2*a*c^2*x^2 + (b*c^4*x^4 + 2*b*c^
2*x^2 + b)*arcsinh(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x}{(a+b\,\text{asinh}\,(cx))\,(c^2x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
[Out] Integral(x/((a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(x/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx=\int \frac{x}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(c^2\,x^2+1\right)}^{3/2}} \,d x$$

```
[In] int(x/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)),x)
[Out] int(x/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)), x)
```

3.398 $\int \frac{1}{(1+c^2x^2)^{3/2}(a+b\,\mathbf{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{\left(1+c^2x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))}\,dx=\operatorname{Int}\left(\frac{1}{\left(1+c^2x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))},x\right)$$

```
[Out] Unintegrable(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(1+c^2x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))}\,dx=\int \frac{1}{\left(1+c^2x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))}\,dx$$

```
[In] Int[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{\left(1+c^2x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))} \, dx = \int \frac{1}{(1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))} \, dx$$

```
[In] Integrate[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(c^2x^2+1)^{\frac{3}{2}} (a+b \operatorname{arcsinh}(cx))} dx$$

```
[In] int(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
[Out] int(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.46

$$\int \frac{1}{(1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))} \, dx = \int \frac{1}{(c^2x^2+1)^{\frac{3}{2}} (b \operatorname{arsinh}(cx)+a)} \, dx$$

```
[In] integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(a*c^4*x^4 + 2*a*c^2*x^2 + (b*c^4*x^4 + 2*b*c^2*
x^2 + b)*arcsinh(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))\,(c^2x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
[Out] Integral(1/((a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(c^2\,x^2+1\right)}^{3/2}}\,dx$$

```
[In] int(1/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)),x)
[Out] int(1/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)), x)
```

3.399 $\int \frac{1}{x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{1}{x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{1}{x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))},x\right)$$

```
[Out] Unintegrable(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Int[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{x (1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))} \, dx = \int \frac{1}{x (1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))} \, dx$$

```
[In] Integrate[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{x \left(c^2x^2+1\right)^{\frac{3}{2}} \left(a+b \,\text{arcsinh}\left(cx\right)\right)}dx$$

```
[In] int(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
[Out] int(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.33

$$\int \frac{1}{x (1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))} \, dx = \int \frac{1}{\left(c^2x^2+1\right)^{\frac{3}{2}} (b \,\text{arsinh}\left(cx\right)+a)x} \, dx$$

```
[In] integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(a*c^4*x^5 + 2*a*c^2*x^3 + a*x + (b*c^4*x^5 + 2*
b*c^2*x^3 + b*x)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 2.75 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{x(a+b\,\text{asinh}\,(cx))\,(c^2x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
[Out] Integral(1/(x*(a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)x}\,dx$$

```
[In] integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x (1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))} \, dx=\int \frac{1}{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(c^2\,x^2+1\right)}^{3/2}} \,d x$$

```
[In] int(1/(x*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)),x)
[Out] int(1/(x*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)), x)
```

3.400 $\int \frac{1}{x^2(1+c^2x^2)^{3/2}(a+b\,\text{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))},x\right)$$

```
[Out] Unintegrable(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Int[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.96 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))} dx = \int \frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))} dx$$

```
[In] Integrate[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2 (c^2x^2+1)^{\frac{3}{2}} (a+b\,\text{arcsinh}\,(cx))}dx$$

```
[In] int(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
[Out] int(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.48

$$\int \frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))} dx = \int \frac{1}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)x^2} dx$$

```
[In] integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(a*c^4*x^6 + 2*a*c^2*x^4 + a*x^2 + (b*c^4*x^6 +
2*b*c^2*x^4 + b*x^2)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))} \, dx = \int \frac{1}{x^2 (a+b \operatorname{asinh}(cx)) (c^2x^2+1)^{\frac{3}{2}}} \, dx$$

```
[In] integrate(1/x**2/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
[Out] Integral(1/(x**2*(a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))} \, dx = \int \frac{1}{(c^2x^2+1)^{\frac{3}{2}} (b \operatorname{arsinh}(cx)+a) x^2} \, dx$$

```
[In] integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))} \, dx = \int \frac{1}{(c^2x^2+1)^{\frac{3}{2}} (b \operatorname{arsinh}(cx)+a) x^2} \, dx$$

```
[In] integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))} \, dx = \int \frac{1}{x^2 \left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\left(c^2\,x^2+1\right)^{3/2}} \,dx$$

```
[In] int(1/(x^2*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)),x)
[Out] int(1/(x^2*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)), x)
```

3.401 $\int \frac{x^m \left(1+c^2x^2\right)^{5/2}}{a+b\mathbf{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^m (1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \text{Int}\left(\frac{x^m (1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)},x\right)$$

```
[Out] Unintegrable(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m (1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x^m (1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx$$

```
[In] Int[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
[Out] Defer[Int][(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^m (1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{x^m(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx$$

```
[In] Integrate[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
[Out] Integrate[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.67 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^m(c^2x^2+1)^{\frac{5}{2}}}{a+b\ \text{arcsinh}\left(cx\right)}dx$$

```
[In] int(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x)
[Out] int(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}x^m}{b\,\text{arsinh}\left(cx\right)+a}\,dx$$

```
[In] integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)*x^m/(b*arcsinh(c*x) +
a), x)
```

Sympy [N/A]

Not integrable

Time = 178.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{x^m(c^2x^2+1)^{\frac{5}{2}}}{a+b\operatorname{asinh}(cx)}\,dx$$

```
[In] integrate(x**m*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x)),x)
[Out] Integral(x**m*(c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}x^m}{b\operatorname{arsinh}(cx)+a}\,dx$$

```
[In] integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*x^2 + 1)^(5/2)*x^m/(b*arcsinh(c*x) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.71 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{x^m\,{\left(c^2\,x^2+1\right)}^{5/2}}{a+b\,\mathrm{asinh}\left(c\,x\right)} \,d x$$

```
[In] int((x^m*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)),x)
[Out] int((x^m*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)), x)
```

3.402 $\int \frac{x^m(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx=\text{Int}\left(\frac{x^m(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)},x\right)$$

```
[Out] Unintegrable(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{x^m(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx$$

```
[In] Int[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
[Out] Defer[Int][(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]), x]
```

Rubi steps

$$\text{integral}=\int \frac{x^m(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{x^m(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx$$

```
[In] Integrate[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
[Out] Integrate[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.62 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^m(c^2x^2+1)^{\frac{3}{2}}}{a+b\ \text{arcsinh}\,(cx)}dx$$

```
[In] int(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
[Out] int(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2x^2+1)^{\frac{3}{2}}x^m}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((c^2*x^2 + 1)^(3/2)*x^m/(b*arcsinh(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 14.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{x^m(c^2x^2+1)^{\frac{3}{2}}}{a+b\operatorname{asinh}(cx)}\,dx$$

```
[In] integrate(x**m*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
[Out] Integral(x**m*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2x^2+1)^{\frac{3}{2}}x^m}{b\operatorname{arsinh}(cx)+a}\,dx$$

```
[In] integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*x^2 + 1)^(3/2)*x^m/(b*arcsinh(c*x) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.81 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x^m\,(c^2\,x^2+1)^{3/2}}{a+b\,\mathrm{asinh}\,(c\,x)}\,dx$$

```
[In] int((x^m*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)),x)
[Out] int((x^m*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)), x)
```

3.403 $\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\,\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\,\text{arcsinh}(cx)}\,dx = \text{Int}\left(\frac{x^m\sqrt{1+c^2x^2}}{a+b\,\text{arcsinh}(cx)},x\right)$$

```
[Out] Unintegrable(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\,\text{arcsinh}(cx)}\,dx = \int \frac{x^m\sqrt{1+c^2x^2}}{a+b\,\text{arcsinh}(cx)}\,dx$$

```
[In] Int[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] Defer[Int][(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^m\sqrt{1+c^2x^2}}{a+b\,\text{arcsinh}(cx)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x^m\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx$$

```
[In] Integrate[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
[Out] Integrate[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.51 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^m\sqrt{c^2x^2+1}}{a+b\ \text{arcsinh}\,(cx)}dx$$

```
[In] int(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
[Out] int(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{\sqrt{c^2x^2+1}x^m}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^m/(b*arcsinh(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x^m\sqrt{c^2x^2+1}}{a+b\,\text{asinh}\,(cx)}\,dx$$

```
[In] integrate(x**m*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
[Out] Integral(x**m*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\operatorname{arcsinh}(cx)}\,dx=\int \frac{\sqrt{c^2x^2+1}x^m}{b\operatorname{arsinh}(cx)+a}\,dx$$

```
[In] integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(c^2*x^2 + 1)*x^m/(b*arcsinh(c*x) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\operatorname{arcsinh}(cx)}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.91 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\operatorname{arcsinh}(cx)}\,dx=\int \frac{x^m\,\sqrt{c^2\,x^2+1}}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

```
[In] int((x^m*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)),x)
[Out] int((x^m*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)), x)
```

3.404 $\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\,\mathbf{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))},x\right)$$

```
[Out] Unintegrable(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Int[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Integrate[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^m}{\sqrt{c^2x^2+1}\,\left(a+b\,\operatorname{arcsinh}\left(cx\right)\right)}dx$$

```
[In] int(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
[Out] int(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{x^m}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\left(cx\right)+a)}\,dx$$

```
[In] integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^m/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) +
a), x)
```

Sympy [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^m}{(a+b\operatorname{asinh}(cx))\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate(x**m/(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
[Out] Integral(x**m/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^m}{\sqrt{c^2x^2+1}(b\operatorname{arsinh}(cx)+a)}\,dx$$

```
[In] integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(x^m/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^m}{\sqrt{c^2x^2+1}(b\operatorname{arsinh}(cx)+a)}\,dx$$

```
[In] integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(x^m/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.79 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^m}{(a+b\,\mathrm{asinh}\,(c\,x))\,\sqrt{c^2\,x^2+1}}\,dx$$

```
[In] int(x^m/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(x^m/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

3.405 $\int \frac{x^m}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))},x\right)$$

```
[Out] Unintegrable(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Int[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx=\int \frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Integrate[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^m}{(c^2x^2+1)^{\frac{3}{2}}\,(a+b\,\operatorname{arcsinh}(cx))}dx$$

```
[In] int(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
[Out] int(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.30

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx=\int \frac{x^m}{(c^2x^2+1)^{\frac{3}{2}}(b\,\operatorname{arsinh}(cx)+a)}\,dx$$

```
[In] integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^m/(a*c^4*x^4 + 2*a*c^2*x^2 + (b*c^4*x^4 + 2*b*
c^2*x^2 + b)*arcsinh(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 2.94 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx=\int \frac{x^m}{(a+b\,\text{asinh}\,(cx))\,(c^2x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**m/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
[Out] Integral(x**m/((a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx=\int \frac{x^m}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(x^m/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx=\int \frac{x^m}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(x^m/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.76 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx=\int \frac{x^m}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(c^2\,x^2+1\right)}^{3/2}} \,d x$$

```
[In] int(x^m/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)),x)
[Out] int(x^m/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)), x)
```

3.406 $\int \frac{(c+a^2cx^2)^3}{\operatorname{arcsinh}(ax)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 94

$$\int \frac{(c+a^2cx^2)^3}{\operatorname{arcsinh}(ax)^2}\,dx = -\frac{c^3(1+a^2x^2)^{7/2}}{a\operatorname{arcsinh}(ax)} + \frac{35c^3\operatorname{Shi}(\operatorname{arcsinh}(ax))}{64a} + \frac{63c^3\operatorname{Shi}(3\operatorname{arcsinh}(ax))}{64a} + \frac{35c^3\operatorname{Shi}(5\operatorname{arcsinh}(ax))}{64a} + \frac{7c^3\operatorname{Shi}(7\operatorname{arcsinh}(ax))}{64a}$$

```
[Out] -c^3*(a^2*x^2+1)^(7/2)/a/arcsinh(a*x)+35/64*c^3*Shi(arcsinh(a*x))/a+63/64*c
^3*Shi(3*arcsinh(a*x))/a+35/64*c^3*Shi(5*arcsinh(a*x))/a+7/64*c^3*Shi(7*arc
sinh(a*x))/a
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {5790, 5819, 5556, 3379}

$$\int \frac{(c+a^2cx^2)^3}{\operatorname{arcsinh}(ax)^2}\,dx = -\frac{c^3(a^2x^2+1)^{7/2}}{a\operatorname{arcsinh}(ax)} + \frac{35c^3\operatorname{Shi}(\operatorname{arcsinh}(ax))}{64a} + \frac{63c^3\operatorname{Shi}(3\operatorname{arcsinh}(ax))}{64a} + \frac{35c^3\operatorname{Shi}(5\operatorname{arcsinh}(ax))}{64a} + \frac{7c^3\operatorname{Shi}(7\operatorname{arcsinh}(ax))}{64a}$$

```
[In] Int[(c + a^2*c*x^2)^3/ArcSinh[a*x]^2,x]
```

```
[Out] -((c^3*(1 + a^2*x^2)^(7/2))/(a*ArcSinh[a*x])) + (35*c^3*SinhIntegral[ArcSin
h[a*x]])/(64*a) + (63*c^3*SinhIntegral[3*ArcSinh[a*x]])/(64*a) + (35*c^3*Si
nhIntegral[5*ArcSinh[a*x]])/(64*a) + (7*c^3*SinhIntegral[7*ArcSinh[a*x]])/(
64*a)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{c^3(1+a^2x^2)^{7/2}}{a\text{arcsinh}(ax)} + (7ac^3)\int \frac{x(1+a^2x^2)^{5/2}}{\text{arcsinh}(ax)}\,dx \\
&= -\frac{c^3(1+a^2x^2)^{7/2}}{a\text{arcsinh}(ax)} + \frac{(7c^3)\,\text{Subst}\left(\int \frac{\cosh^6(x)\sinh(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{a} \\
&= -\frac{c^3(1+a^2x^2)^{7/2}}{a\text{arcsinh}(ax)} \\
&\quad + \frac{(7c^3)\,\text{Subst}\left(\int \left(\frac{5\sinh(x)}{64x} + \frac{9\sinh(3x)}{64x} + \frac{5\sinh(5x)}{64x} + \frac{\sinh(7x)}{64x}\right)\,dx, x, \text{arcsinh}(ax)\right)}{a}
\end{aligned}$$

$$= -\frac{c^3(1+a^2x^2)^{7/2}}{a\text{arcsinh}(ax)} + \frac{(7c^3)\,\text{Subst}\left(\int \frac{\sinh(7x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{64a}$$
$$+ \frac{(35c^3)\,\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{64a}$$
$$+ \frac{(35c^3)\,\text{Subst}\left(\int \frac{\sinh(5x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{64a}$$
$$+ \frac{(63c^3)\,\text{Subst}\left(\int \frac{\sinh(3x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{64a}$$
$$= -\frac{c^3(1+a^2x^2)^{7/2}}{a\text{arcsinh}(ax)} + \frac{35c^3\text{Shi}(\text{arcsinh}(ax))}{64a} + \frac{63c^3\text{Shi}(3\text{arcsinh}(ax))}{64a}$$
$$+ \frac{35c^3\text{Shi}(5\text{arcsinh}(ax))}{64a} + \frac{7c^3\text{Shi}(7\text{arcsinh}(ax))}{64a}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.87

$$\int \frac{(c+a^2cx^2)^3}{\text{arcsinh}(ax)^2}\,dx$$
$$= \frac{c^3\left(-64(1+a^2x^2)^{7/2} + 35\text{arcsinh}(ax)\text{Shi}(\text{arcsinh}(ax)) + 63\text{arcsinh}(ax)\text{Shi}(3\text{arcsinh}(ax)) + 35\text{arcsinh}(ax)\right.}{64a\text{arcsinh}(ax)}$$

```
[In] Integrate[(c + a^2*c*x^2)^3/ArcSinh[a*x]^2,x]
```

```
[Out] (c^3*(-64*(1 + a^2*x^2)^(7/2) + 35*ArcSinh[a*x]*SinhIntegral[ArcSinh[a*x]]
+ 63*ArcSinh[a*x]*SinhIntegral[3*ArcSinh[a*x]] + 35*ArcSinh[a*x]*SinhIntegr
al[5*ArcSinh[a*x]] + 7*ArcSinh[a*x]*SinhIntegral[7*ArcSinh[a*x]]))/(64*a*Ar
cSinh[a*x])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{c^3\left(35\ \text{Shi}(\text{arcsinh}(ax))\,\text{arcsinh}(ax)+63\ \text{Shi}(3\ \text{arcsinh}(ax))\,\text{arcsinh}(ax)+35\ \text{Shi}(5\ \text{arcsinh}(ax))\,\text{arcsinh}(ax)+7\ \text{Shi}(7\ \text{arcs}\right.}{64a\ \text{arcsinh}(ax}$
default	$\frac{c^3\left(35\ \text{Shi}(\text{arcsinh}(ax))\,\text{arcsinh}(ax)+63\ \text{Shi}(3\ \text{arcsinh}(ax))\,\text{arcsinh}(ax)+35\ \text{Shi}(5\ \text{arcsinh}(ax))\,\text{arcsinh}(ax)+7\ \text{Shi}(7\ \text{arcs}\right.}{64a\ \text{arcsinh}(ax}$

```
[In] int((a^2*c*x^2+c)^3/arcsinh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/a*c^3*(35*Shi(arcsinh(a*x))*arcsinh(a*x)+63*Shi(3*arcsinh(a*x))*arcsin
h(a*x)+35*Shi(5*arcsinh(a*x))*arcsinh(a*x)+7*Shi(7*arcsinh(a*x))*arcsinh(a*
x)-35*(a^2*x^2+1)^(1/2)-21*cosh(3*arcsinh(a*x))-7*cosh(5*arcsinh(a*x))-cosh
(7*arcsinh(a*x)))/arcsinh(a*x)
```

Fricas [F]

$$\int \frac{(c + a^2cx^2)^3}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{(a^2cx^2 + c)^3}{\operatorname{arsinh}(ax)^2}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^3/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/arcsinh(a*x)^2
, x)
```

Sympy [F]

$$\int \frac{(c + a^2cx^2)^3}{\operatorname{arcsinh}(ax)^2}\,dx = c^3\left(\int \frac{3a^2x^2}{\operatorname{asinh}^2(ax)}\,dx + \int \frac{3a^4x^4}{\operatorname{asinh}^2(ax)}\,dx + \int \frac{a^6x^6}{\operatorname{asinh}^2(ax)}\,dx + \int \frac{1}{\operatorname{asinh}^2(ax)}\,dx\right)$$

```
[In] integrate((a**2*c*x**2+c)**3/asinh(a*x)**2,x)
```

```
[Out] c**3*(Integral(3*a**2*x**2/asinh(a*x)**2, x) + Integral(3*a**4*x**4/asinh(a
*x)**2, x) + Integral(a**6*x**6/asinh(a*x)**2, x) + Integral(asinh(a*x)**(-
2), x))
```

Maxima [F]

$$\int \frac{(c + a^2cx^2)^3}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{(a^2cx^2 + c)^3}{\operatorname{arsinh}(ax)^2}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^3/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^9*c^3*x^9 + 4*a^7*c^3*x^7 + 6*a^5*c^3*x^5 + 4*a^3*c^3*x^3 + a*c^3*x + (
a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 + c^3)*sqrt(a^2
*x^2 + 1))/((a^3*x^2 + sqrt(a^2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2
+ 1))) + integrate((7*a^10*c^3*x^10 + 29*a^8*c^3*x^8 + 46*a^6*c^3*x^6 + 34*
a^4*c^3*x^4 + 11*a^2*c^3*x^2 + c^3 + (7*a^8*c^3*x^8 + 20*a^6*c^3*x^6 + 18*a
^4*c^3*x^4 + 4*a^2*c^3*x^2 - c^3)*(a^2*x^2 + 1) + 7*(2*a^9*c^3*x^9 + 7*a^7*
c^3*x^7 + 9*a^5*c^3*x^5 + 5*a^3*c^3*x^3 + a*c^3*x)*sqrt(a^2*x^2 + 1))/((a^4
*x^4 + (a^2*x^2 + 1)*a^2*x^2 + 2*a^2*x^2 + 2*(a^3*x^3 + a*x)*sqrt(a^2*x^2 +
 1) + 1)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [F]

$$\int \frac{(c+a^2cx^2)^3}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^3}{\operatorname{arsinh}(ax)^2}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^3/arcsinh(a*x)^2,x, algorithm="giac")
[Out] integrate((a^2*c*x^2 + c)^3/arcsinh(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^3}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{(c\,a^2\,x^2+c)^3}{\operatorname{asinh}(a\,x)^2}\,dx$$

```
[In] int((c + a^2*c*x^2)^3/asinh(a*x)^2,x)
[Out] int((c + a^2*c*x^2)^3/asinh(a*x)^2, x)
```

3.407 $\int \frac{(c+a^2cx^2)^2}{\text{arcsinh}(ax)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 77

$$\int \frac{(c+a^2cx^2)^2}{\text{arcsinh}(ax)^2}\,dx = -\frac{c^2(1+a^2x^2)^{5/2}}{a\text{arcsinh}(ax)} + \frac{5c^2\text{Shi}(\text{arcsinh}(ax))}{8a} + \frac{15c^2\text{Shi}(3\text{arcsinh}(ax))}{16a} + \frac{5c^2\text{Shi}(5\text{arcsinh}(ax))}{16a}$$

```
[Out] -c^2*(a^2*x^2+1)^(5/2)/a/arcsinh(a*x)+5/8*c^2*Shi(arcsinh(a*x))/a+15/16*c^2
*Shi(3*arcsinh(a*x))/a+5/16*c^2*Shi(5*arcsinh(a*x))/a
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {5790, 5819, 5556, 3379}

$$\int \frac{(c+a^2cx^2)^2}{\text{arcsinh}(ax)^2}\,dx = -\frac{c^2(a^2x^2+1)^{5/2}}{a\text{arcsinh}(ax)} + \frac{5c^2\text{Shi}(\text{arcsinh}(ax))}{8a} + \frac{15c^2\text{Shi}(3\text{arcsinh}(ax))}{16a} + \frac{5c^2\text{Shi}(5\text{arcsinh}(ax))}{16a}$$

```
[In] Int[(c + a^2*c*x^2)^2/ArcSinh[a*x]^2,x]
```

```
[Out] -((c^2*(1 + a^2*x^2)^(5/2))/(a*ArcSinh[a*x])) + (5*c^2*SinhIntegral[ArcSinh
[a*x]])/(8*a) + (15*c^2*SinhIntegral[3*ArcSinh[a*x]])/(16*a) + (5*c^2*SinhI
ntegral[5*ArcSinh[a*x]])/(16*a)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule steps

$$\text{integral} = -\frac{c^2(1+a^2x^2)^{5/2}}{a\text{arcsinh}(ax)} + (5ac^2)\int \frac{x(1+a^2x^2)^{3/2}}{\text{arcsinh}(ax)}\,dx$$

$$= -\frac{c^2(1+a^2x^2)^{5/2}}{a\text{arcsinh}(ax)} + \frac{(5c^2)\,\text{Subst}\left(\int \frac{\cosh^4(x)\sinh(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{a}$$

$$= -\frac{c^2(1+a^2x^2)^{5/2}}{a\text{arcsinh}(ax)} + \frac{(5c^2)\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{8x} + \frac{3\sinh(3x)}{16x} + \frac{\sinh(5x)}{16x}\right)dx, x, \text{arcsinh}(ax)\right)}{a}$$

$$= -\frac{c^2(1+a^2x^2)^{5/2}}{a\text{arcsinh}(ax)} + \frac{(5c^2)\,\text{Subst}\left(\int \frac{\sinh(5x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{16a} + \frac{(5c^2)\,\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{8a} + \frac{(15c^2)\,\text{Subst}\left(\int \frac{\sinh(3x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{16a}$$

$$= -\frac{c^2(1+a^2x^2)^{5/2}}{a\text{arcsinh}(ax)} + \frac{5c^2\text{Shi}(\text{arcsinh}(ax))}{8a} + \frac{15c^2\text{Shi}(3\text{arcsinh}(ax))}{16a} + \frac{5c^2\text{Shi}(5\text{arcsinh}(ax))}{16a}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

$$\int \frac{(c+a^2cx^2)^2}{\text{arcsinh}(ax)^2}\,dx = \frac{c^2\left(-16(1+a^2x^2)^{5/2} + 10\text{arcsinh}(ax)\text{Shi}(\text{arcsinh}(ax)) + 15\text{arcsinh}(ax)\text{Shi}(3\text{arcsinh}(ax)) + 5\text{arcsinh}(ax\right.}{16a\text{arcsinh}(ax)}$$

```
[In] Integrate[(c + a^2*c*x^2)^2/ArcSinh[a*x]^2,x]
```

```
[Out] (c^2*(-16*(1 + a^2*x^2)^(5/2) + 10*ArcSinh[a*x]*SinhIntegral[ArcSinh[a*x]]
+ 15*ArcSinh[a*x]*SinhIntegral[3*ArcSinh[a*x]] + 5*ArcSinh[a*x]*SinhIntegra
l[5*ArcSinh[a*x]]))/(16*a*ArcSinh[a*x])
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{c^2\left(10\ \text{Shi}(\text{arcsinh}(ax))\,\text{arcsinh}(ax)+15\ \text{Shi}(3\ \text{arcsinh}(ax))\,\text{arcsinh}(ax)+5\ \text{Shi}(5\ \text{arcsinh}(ax))\,\text{arcsinh}(ax)-5\cosh(3\ \text{ar}\right.}{16a\ \text{arcsinh}(ax)}$
default	$\frac{c^2\left(10\ \text{Shi}(\text{arcsinh}(ax))\,\text{arcsinh}(ax)+15\ \text{Shi}(3\ \text{arcsinh}(ax))\,\text{arcsinh}(ax)+5\ \text{Shi}(5\ \text{arcsinh}(ax))\,\text{arcsinh}(ax)-5\cosh(3\ \text{ar}\right.}{16a\ \text{arcsinh}(ax)}$

```
[In] int((a^2*c*x^2+c)^2/arcsinh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/a*c^2*(10*Shi(arcsinh(a*x))*arcsinh(a*x)+15*Shi(3*arcsinh(a*x))*arcsin
h(a*x)+5*Shi(5*arcsinh(a*x))*arcsinh(a*x)-5*cosh(3*arcsinh(a*x))-cosh(5*arc
sinh(a*x))-10*(a^2*x^2+1)^(1/2))/arcsinh(a*x)
```

Fricas [F]

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)^2}\,dx=\int \frac{(a^2cx^2+c)^2}{\operatorname{arsinh}(ax)^2}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/arcsinh(a*x)^2, x)
```

Sympy [F]

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)^2}\,dx=c^2\left(\int \frac{2a^2x^2}{\operatorname{asinh}^2(ax)}\,dx+\int \frac{a^4x^4}{\operatorname{asinh}^2(ax)}\,dx+\int \frac{1}{\operatorname{asinh}^2(ax)}\,dx\right)$$

```
[In] integrate((a**2*c*x**2+c)**2/asinh(a*x)**2,x)
```

```
[Out] c**2*(Integral(2*a**2*x**2/asinh(a*x)**2, x) + Integral(a**4*x**4/asinh(a*x
)**2, x) + Integral(asinh(a*x)**(-2), x))
```

Maxima [F]

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)^2}\,dx=\int \frac{(a^2cx^2+c)^2}{\operatorname{arsinh}(ax)^2}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^7*c^2*x^7 + 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 + a*c^2*x + (a^6*c^2*x^6 + 3*
a^4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^2*
x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((5*a^8*c^2*x^
8 + 16*a^6*c^2*x^6 + 18*a^4*c^2*x^4 + 8*a^2*c^2*x^2 + (5*a^6*c^2*x^6 + 9*a^
4*c^2*x^4 + 3*a^2*c^2*x^2 - c^2)*(a^2*x^2 + 1) + c^2 + 5*(2*a^7*c^2*x^7 + 5
*a^5*c^2*x^5 + 4*a^3*c^2*x^3 + a*c^2*x)*sqrt(a^2*x^2 + 1))/((a^4*x^4 + (a^2
*x^2 + 1)*a^2*x^2 + 2*a^2*x^2 + 2*(a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1) + 1)*lo
g(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [F]

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^2}{\operatorname{arsinh}(ax)^2}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="giac")
[Out] integrate((a^2*c*x^2 + c)^2/arcsinh(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{(c\,a^2\,x^2+c)^2}{\operatorname{asinh}(a\,x)^2}\,dx$$

```
[In] int((c + a^2*c*x^2)^2/asinh(a*x)^2,x)
[Out] int((c + a^2*c*x^2)^2/asinh(a*x)^2, x)
```

3.408 $\int \frac{c+a^2cx^2}{\text{arcsinh}(ax)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 54

$$\int \frac{c+a^2cx^2}{\text{arcsinh}(ax)^2} \, dx = -\frac{c(1+a^2x^2)^{3/2}}{a\text{arcsinh}(ax)} + \frac{3c\text{Shi}(\text{arcsinh}(ax))}{4a} + \frac{3c\text{Shi}(3\text{arcsinh}(ax))}{4a}$$

```
[Out] -c*(a^2*x^2+1)^(3/2)/a/arcsinh(a*x)+3/4*c*Shi(arcsinh(a*x))/a+3/4*c*Shi(3*a
rcsinh(a*x))/a
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {5790, 5819, 5556, 3379}

$$\int \frac{c+a^2cx^2}{\text{arcsinh}(ax)^2} \, dx = -\frac{c(a^2x^2+1)^{3/2}}{a\text{arcsinh}(ax)} + \frac{3c\text{Shi}(\text{arcsinh}(ax))}{4a} + \frac{3c\text{Shi}(3\text{arcsinh}(ax))}{4a}$$

```
[In] Int[(c + a^2*c*x^2)/ArcSinh[a*x]^2,x]
```

```
[Out] -((c*(1 + a^2*x^2)^(3/2))/(a*ArcSinh[a*x])) + (3*c*SinhIntegral[ArcSinh[a*x
]])/(4*a) + (3*c*SinhIntegral[3*ArcSinh[a*x]])/(4*a)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{c(1+a^2x^2)^{3/2}}{a\text{arcsinh}(ax)} + (3ac)\int \frac{x\sqrt{1+a^2x^2}}{\text{arcsinh}(ax)}\,dx \\
&= -\frac{c(1+a^2x^2)^{3/2}}{a\text{arcsinh}(ax)} + \frac{(3c)\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{a} \\
&= -\frac{c(1+a^2x^2)^{3/2}}{a\text{arcsinh}(ax)} + \frac{(3c)\text{Subst}\left(\int \left(\frac{\sinh(x)}{4x} + \frac{\sinh(3x)}{4x}\right)\,dx, x, \text{arcsinh}(ax)\right)}{a} \\
&= -\frac{c(1+a^2x^2)^{3/2}}{a\text{arcsinh}(ax)} + \frac{(3c)\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{4a} \\
&\quad + \frac{(3c)\text{Subst}\left(\int \frac{\sinh(3x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{4a} \\
&= -\frac{c(1+a^2x^2)^{3/2}}{a\text{arcsinh}(ax)} + \frac{3c\text{Shi}(\text{arcsinh}(ax))}{4a} + \frac{3c\text{Shi}(3\text{arcsinh}(ax))}{4a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)^2}\,dx = \frac{c\left(-4(1+a^2x^2)^{3/2}+3\operatorname{arcsinh}(ax)\operatorname{Shi}(\operatorname{arcsinh}(ax))+3\operatorname{arcsinh}(ax)\operatorname{Shi}(3\operatorname{arcsinh}(ax))\right)}{4a\operatorname{arcsinh}(ax)}$$

```
[In] Integrate[(c + a^2*c*x^2)/ArcSinh[a*x]^2,x]
[Out] (c*(-4*(1 + a^2*x^2)^(3/2) + 3*ArcSinh[a*x]*SinhIntegral[ArcSinh[a*x]] + 3*
ArcSinh[a*x]*SinhIntegral[3*ArcSinh[a*x]]))/(4*a*ArcSinh[a*x])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{c\left(3\,\operatorname{Shi}(\operatorname{arcsinh}(ax))\operatorname{arcsinh}(ax)+3\,\operatorname{Shi}(3\,\operatorname{arcsinh}(ax))\operatorname{arcsinh}(ax)-3\sqrt{a^2x^2+1}-\cosh(3\,\operatorname{arcsinh}(ax))\right)}{4a\,\operatorname{arcsinh}(ax)}$	60
default	$\frac{c\left(3\,\operatorname{Shi}(\operatorname{arcsinh}(ax))\operatorname{arcsinh}(ax)+3\,\operatorname{Shi}(3\,\operatorname{arcsinh}(ax))\operatorname{arcsinh}(ax)-3\sqrt{a^2x^2+1}-\cosh(3\,\operatorname{arcsinh}(ax))\right)}{4a\,\operatorname{arcsinh}(ax)}$	60

```
[In] int((a^2*c*x^2+c)/arcsinh(a*x)^2,x,method=_RETURNVERBOSE)
[Out] 1/4/a*c*(3*Shi(arcsinh(a*x))*arcsinh(a*x)+3*Shi(3*arcsinh(a*x))*arcsinh(a*x
)-3*(a^2*x^2+1)^(1/2)-cosh(3*arcsinh(a*x)))/arcsinh(a*x)
```

Fricas [F]

$$\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{a^2cx^2+c}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

```
[In] integrate((a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)/arcsinh(a*x)^2, x)
```

Sympy [F]

$$\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)^2} dx = c\left(\int \frac{a^2x^2}{\operatorname{asinh}^2(ax)} dx + \int \frac{1}{\operatorname{asinh}^2(ax)} dx\right)$$

```
[In] integrate((a**2*c*x**2+c)/asinh(a*x)**2,x)
[Out] c*(Integral(a**2*x**2/asinh(a*x)**2, x) + Integral(asinh(a*x)**(-2), x))
```

Maxima [F]

$$\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)^2} dx = \int \frac{a^2cx^2+c}{\operatorname{arsinh}(ax)^2} dx$$

```
[In] integrate((a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="maxima")
[Out] -(a^5*c*x^5 + 2*a^3*c*x^3 + a*c*x + (a^4*c*x^4 + 2*a^2*c*x^2 + c)*sqrt(a^2*
x^2 + 1))/((a^3*x^2 + sqrt(a^2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 +
 1))) + integrate((3*a^6*c*x^6 + 7*a^4*c*x^4 + 5*a^2*c*x^2 + (3*a^4*c*x^4 +
 2*a^2*c*x^2 - c)*(a^2*x^2 + 1) + 3*(2*a^5*c*x^5 + 3*a^3*c*x^3 + a*c*x)*sqr
t(a^2*x^2 + 1) + c)/((a^4*x^4 + (a^2*x^2 + 1)*a^2*x^2 + 2*a^2*x^2 + 2*(a^3*
x^3 + a*x)*sqrt(a^2*x^2 + 1) + 1)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [F]

$$\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)^2} dx = \int \frac{a^2cx^2+c}{\operatorname{arsinh}(ax)^2} dx$$

```
[In] integrate((a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="giac")
[Out] integrate((a^2*c*x^2 + c)/arcsinh(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)^2} dx = \int \frac{c\,a^2\,x^2+c}{\operatorname{asinh}(a\,x)^2} dx$$

```
[In] int((c + a^2*c*x^2)/asinh(a*x)^2,x)
[Out] int((c + a^2*c*x^2)/asinh(a*x)^2, x)
```

3.409 $\int \frac{1}{(c+a^2cx^2)\mathbf{arcsinh}(ax)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{1}{(c+a^2cx^2)\,\text{arcsinh}(ax)^2}\,dx = -\frac{1}{ac\sqrt{1+a^2x^2}\text{arcsinh}(ax)} - \frac{a\text{Int}\left(\frac{x}{(1+a^2x^2)^{3/2}\text{arcsinh}(ax)},x\right)}{c}$$

```
[Out] -1/a/c/arcsinh(a*x)/(a^2*x^2+1)^(1/2)-a*Unintegrable(x/(a^2*x^2+1)^(3/2)/ar
csinh(a*x),x)/c
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(c+a^2cx^2)\,\text{arcsinh}(ax)^2}\,dx = \int \frac{1}{(c+a^2cx^2)\,\text{arcsinh}(ax)^2}\,dx$$

```
[In] Int[1/((c + a^2*c*x^2)*ArcSinh[a*x]^2),x]
```

```
[Out] -(1/(a*c*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])) - (a*Defer[Int][x/((1 + a^2*x^2)^
(3/2)*ArcSinh[a*x]), x])/c
```

Rubi steps

$$\text{integral} = -\frac{1}{ac\sqrt{1+a^2x^2}\text{arcsinh}(ax)} - \frac{a\int \frac{x}{(1+a^2x^2)^{3/2}\text{arcsinh}(ax)}\,dx}{c}$$

Mathematica [N/A]

Not integrable

Time = 1.66 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)^2}\,dx=\int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)^2}\,dx$$

```
[In] Integrate[1/((c + a^2*c*x^2)*ArcSinh[a*x]^2),x]
[Out] Integrate[1/((c + a^2*c*x^2)*ArcSinh[a*x]^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a^{2} c\,x^{2}+c\right) \operatorname{arcsinh}\left(a x\right)^{2}}d x$$

```
[In] int(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x)
[Out] int(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)^2}\,dx=\int \frac{1}{\left(a^{2} c x^{2}+c\right) \operatorname{arsinh}\left(a x\right)^{2}}\,dx$$

```
[In] integrate(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="fricas")
[Out] integral(1/((a^2*c*x^2 + c)*arcsinh(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)^2}\,dx=\frac{\int \frac{1}{a^{2} x^{2} \operatorname{asinh}^{2}{\left(a x \right)} + \operatorname{asinh}^{2}{\left(a x \right)}}\, dx}{c}$$

```
[In] integrate(1/(a**2*c*x**2+c)/asinh(a*x)**2,x)
[Out] Integral(1/(a**2*x**2*asinh(a*x)**2 + asinh(a*x)**2), x)/c
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 226, normalized size of antiderivative = 11.89

$$\int \frac{1}{(c+a^2cx^2)\,\text{arcsinh}(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)\,\text{arsinh}\,(ax)^2}\,dx$$

```
[In] integrate(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a*x + sqrt(a^2*x^2 + 1))/((a^3*c*x^2 + sqrt(a^2*x^2 + 1)*a^2*c*x + a*c)*l
og(a*x + sqrt(a^2*x^2 + 1))) - integrate((a^4*x^4 + (a^2*x^2 + 1)^2 + (2*a^
3*x^3 + a*x)*sqrt(a^2*x^2 + 1) - 1)/((a^6*c*x^6 + 3*a^4*c*x^4 + 3*a^2*c*x^2
 + (a^4*c*x^4 + a^2*c*x^2)*(a^2*x^2 + 1) + 2*(a^5*c*x^5 + 2*a^3*c*x^3 + a*c
*x)*sqrt(a^2*x^2 + 1) + c)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)\,\text{arcsinh}(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)\,\text{arsinh}\,(ax)^2}\,dx$$

```
[In] integrate(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)*arcsinh(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.84 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)\,\text{arcsinh}(ax)^2}\,dx = \int \frac{1}{\text{asinh}\,(a\,x)^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

```
[In] int(1/(asinh(a*x)^2*(c + a^2*c*x^2)),x)
```

```
[Out] int(1/(asinh(a*x)^2*(c + a^2*c*x^2)), x)
```

3.410 $\int \frac{1}{(c+a^2cx^2)^2\text{arcsinh}(ax)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{1}{(c+a^2cx^2)^2\,\text{arcsinh}(ax)^2}\,dx = -\frac{1}{ac^2\,(1+a^2x^2)^{3/2}\,\text{arcsinh}(ax)} - \frac{3a\text{Int}\left(\frac{x}{(1+a^2x^2)^{5/2}\text{arcsinh}(ax)},x\right)}{c^2}$$

```
[Out] -1/a/c^2/(a^2*x^2+1)^(3/2)/arcsinh(a*x)-3*a*Unintegrable(x/(a^2*x^2+1)^(5/2
)/arcsinh(a*x),x)/c^2
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+a^2cx^2)^2\,\text{arcsinh}(ax)^2}\,dx = \int \frac{1}{(c+a^2cx^2)^2\,\text{arcsinh}(ax)^2}\,dx$$

```
[In] Int[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]^2),x]
```

```
[Out] -(1/(a*c^2*(1 + a^2*x^2)^(3/2)*ArcSinh[a*x])) - (3*a*Defer[Int][x/((1 + a^2
*x^2)^(5/2)*ArcSinh[a*x]), x])/c^2
```

Rubi steps

$$\text{integral} = -\frac{1}{ac^2\,(1+a^2x^2)^{3/2}\,\text{arcsinh}(ax)} - \frac{(3a)\int \frac{x}{(1+a^2x^2)^{5/2}\text{arcsinh}(ax)}\,dx}{c^2}$$

Mathematica [N/A]

Not integrable

Time = 4.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)^2} \, dx = \int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)^2} \, dx$$

```
[In] Integrate[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]^2),x]
[Out] Integrate[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a^2c\,x^2+c)^2 \operatorname{arcsinh}(ax)^2}dx$$

```
[In] int(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x)
[Out] int(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)^2} \, dx = \int \frac{1}{(a^2cx^2+c)^2 \operatorname{arsinh}(ax)^2} \, dx$$

```
[In] integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="fricas")
[Out] integral(1/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcsinh(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.16

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)^2} \, dx = \frac{\int \frac{1}{a^4x^4 \operatorname{asinh}^2(ax)+2a^2x^2 \operatorname{asinh}^2(ax)+\operatorname{asinh}^2(ax)} \, dx}{c^2}$$

```
[In] integrate(1/(a**2*c*x**2+c)**2/asinh(a*x)**2,x)
[Out] Integral(1/(a**4*x**4*asinh(a*x)**2 + 2*a**2*x**2*asinh(a*x)**2 + asinh(a*x
)**2), x)/c**2
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 325, normalized size of antiderivative = 17.11

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)^2} \, dx = \int \frac{1}{(a^2cx^2+c)^2 \operatorname{arsinh}(ax)^2} \, dx$$

```
[In] integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a*x + sqrt(a^2*x^2 + 1))/((a^5*c^2*x^4 + 2*a^3*c^2*x^2 + a*c^2 + (a^4*c^2
*x^3 + a^2*c^2*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))) - integr
ate((3*a^4*x^4 + 2*a^2*x^2 + (3*a^2*x^2 + 1)*(a^2*x^2 + 1) + 3*(2*a^3*x^3 +
 a*x)*sqrt(a^2*x^2 + 1) - 1)/((a^8*c^2*x^8 + 4*a^6*c^2*x^6 + 6*a^4*c^2*x^4
+ 4*a^2*c^2*x^2 + (a^6*c^2*x^6 + 2*a^4*c^2*x^4 + a^2*c^2*x^2)*(a^2*x^2 + 1)
 + c^2 + 2*(a^7*c^2*x^7 + 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 + a*c^2*x)*sqrt(a^2
*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)^2} \, dx = \int \frac{1}{(a^2cx^2+c)^2 \operatorname{arsinh}(ax)^2} \, dx$$

```
[In] integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*arcsinh(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.91 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)^2} \, dx = \int \frac{1}{\operatorname{asinh}(a\,x)^2\,(c\,a^2\,x^2+c)^2} \, dx$$

```
[In] int(1/(asinh(a*x)^2*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(1/(asinh(a*x)^2*(c + a^2*c*x^2)^2), x)
```

3.411 $\int \frac{x^3\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 213

$$\begin{aligned}\int \frac{x^3\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = & -\frac{x^3(1+c^2x^2)}{bc(a+b\text{arcsinh}(cx))} - \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^4} \\ & - \frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^4} \\ & + \frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^4} \\ & + \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^4} \\ & + \frac{3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^4} \\ & - \frac{5\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^4}\end{aligned}$$

```
[Out] -x^3*(c^2*x^2+1)/b/c/(a+b*arcsinh(c*x))-1/8*Chi((a+b*arcsinh(c*x))/b)*cosh(
a/b)/b^2/c^4-3/16*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^4+5/16*Chi(
5*(a+b*arcsinh(c*x))/b)*cosh(5*a/b)/b^2/c^4+1/8*Shi((a+b*arcsinh(c*x))/b)*s
inh(a/b)/b^2/c^4+3/16*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^4-5/16*
Shi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b^2/c^4
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5814, 5780, 5556, 3384, 3379, 3382}

$$\int \frac{x^3\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^4} - \frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^4} + \frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^4} + \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^4} + \frac{3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^4} - \frac{5\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^4} - \frac{x^3(c^2x^2+1)}{bc(a+b\text{arcsinh}(cx))}$$

```
[In] Int[(x^3*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x^3*(1 + c^2*x^2))/(b*c*(a + b*ArcSinh[c*x]))) - (Cosh[a/b]*CoshIntegral
[(a + b*ArcSinh[c*x])/b])/(8*b^2*c^4) - (3*Cosh[(3*a)/b]*CoshIntegral[(3*(a
 + b*ArcSinh[c*x]))/b])/(16*b^2*c^4) + (5*Cosh[(5*a)/b]*CoshIntegral[(5*(a
+ b*ArcSinh[c*x]))/b])/(16*b^2*c^4) + (Sinh[a/b]*SinhIntegral[(a + b*ArcSin
h[c*x])/b])/(8*b^2*c^4) + (3*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c
*x]))/b])/(16*b^2*c^4) - (5*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*
x]))/b])/(16*b^2*c^4)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^3(1+c^2x^2)}{bc(a+b\text{arcsinh}(cx))}+\frac{3\int\frac{x^2}{a+b\text{arcsinh}(cx)}\,dx}{bc}+\frac{(5c)\int\frac{x^4}{a+b\text{arcsinh}(cx)}\,dx}{b}\\
&= -\frac{x^3(1+c^2x^2)}{bc(a+b\text{arcsinh}(cx))}+\frac{3\text{Subst}\left(\int\frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^4}\\
&\quad+\frac{5\text{Subst}\left(\int\frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^4}\\
&= -\frac{x^3(1+c^2x^2)}{bc(a+b\text{arcsinh}(cx))}+\frac{3\text{Subst}\left(\int\left(\frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4x}-\frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{4x}\right)dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^4}\\
&\quad+\frac{5\text{Subst}\left(\int\left(\frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16x}-\frac{3\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16x}+\frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{8x}\right)dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^4}
\end{aligned}$$

$$= -\frac{x^3(1+c^2x^2)}{bc(a+b\operatorname{arcsinh}(cx))} + \frac{5\text{Subst}\left(\int \frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{16b^2c^4} + \frac{5\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{8b^2c^4} + \frac{3\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{4b^2c^4} - \frac{3\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{4b^2c^4} - \frac{15\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{16b^2c^4}$$

$$= -\frac{x^3(1+c^2x^2)}{bc(a+b\operatorname{arcsinh}(cx))} + \frac{\left(5\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{8b^2c^4} - \frac{\left(3\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{4b^2c^4} + \frac{\left(3\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{4b^2c^4} - \frac{\left(15\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{16b^2c^4} + \frac{\left(5\cosh\left(\frac{5a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{5x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{16b^2c^4} - \frac{\left(5\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{8b^2c^4} + \frac{\left(3\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{4b^2c^4} - \frac{\left(3\sinh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{4b^2c^4} + \frac{\left(15\sinh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{16b^2c^4} - \frac{\left(5\sinh\left(\frac{5a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{5x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{16b^2c^4}$$

$$= -\frac{x^3(1+c^2x^2)}{bc(a+b\text{arcsinh}(cx))} - \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^4} - \frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^4} + \frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^4} + \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^4} + \frac{3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^4} - \frac{5\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^4}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.82

$$\int \frac{x^3\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{\frac{16bc^3x^3}{a+b\text{arcsinh}(cx)} + \frac{16bc^5x^5}{a+b\text{arcsinh}(cx)} + 2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right) + 3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)}{}$$

```
[In] Integrate[(x^3*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/16*((16*b*c^3*x^3)/(a + b*ArcSinh[c*x]) + (16*b*c^5*x^5)/(a + b*ArcSinh[
c*x]) + 2*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] + 3*Cosh[(3*a)/b]*Cosh
Integral[3*(a/b + ArcSinh[c*x])] - 5*Cosh[(5*a)/b]*CoshIntegral[5*(a/b + Ar
cSinh[c*x])] - 2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 3*Sinh[(3*a)/
b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] + 5*Sinh[(5*a)/b]*SinhIntegral[5*(a
/b + ArcSinh[c*x])])/(b^2*c^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. 2(201) = 402.

Time = 0.40 (sec) , antiderivative size = 633, normalized size of antiderivative = 2.97

method	result
default	$-\frac{16c^5x^5-16c^4x^4\sqrt{c^2x^2+1}+20c^3x^3-12c^2x^2\sqrt{c^2x^2+1}+5cx-\sqrt{c^2x^2+1}}{32c^4b(a+b\,\text{arcsinh}(cx))} - \frac{5\,e^{\frac{5a}{b}}\,\text{Ei}_1\left(5\,\text{arcsinh}(cx)+\frac{5a}{b}\right)}{32c^4b^2} + \frac{4c^3x^3-4c^2x^2\sqrt{c^2x^2+1}}{32c^4b(a+b\,\text{arc}}$

```
[In] int(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*(16*c^5*x^5-16*c^4*x^4*(c^2*x^2+1)^(1/2)+20*c^3*x^3-12*c^2*x^2*(c^2*x
^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))/c^4/b/(a+b*arcsinh(c*x))-5/32/c^4/b^2*
exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)+1/32*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)
^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^4/b/(a+b*arcsinh(c*x))+3/32/c^4/b^2*exp(3
```

```
*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+1/16*(-(c^2*x^2+1)^(1/2)+c*x)/c^4/b/(a+b*a
rcsinh(c*x))+1/16/c^4/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)+1/16/c^4/b^2*(arc
sinh(c*x)*Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*b+Ei(1,-arcsinh(c*x)-a/b)*exp(-
a/b)*a+b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))+1/32/c^4/b^2*(4*b*c^3*
x^3+4*(c^2*x^2+1)^(1/2)*b*c^2*x^2+3*arcsinh(c*x)*Ei(1,-3*arcsinh(c*x)-3*a/b
)*exp(-3*a/b)*b+3*Ei(1,-3*arcsinh(c*x)-3*a/b)*exp(-3*a/b)*a+3*b*c*x+(c^2*x^
2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/32/c^4/b^2*(16*b*c^5*x^5+16*(c^2*x^2+1)^
(1/2)*b*c^4*x^4+20*b*c^3*x^3+12*(c^2*x^2+1)^(1/2)*b*c^2*x^2+5*arcsinh(c*x)*
Ei(1,-5*arcsinh(c*x)-5*a/b)*exp(-5*a/b)*b+5*Ei(1,-5*arcsinh(c*x)-5*a/b)*exp
(-5*a/b)*a+5*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

Fricas [F]

$$\int \frac{x^3\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}x^3}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

```
[In] integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^3/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a
^2), x)
```

Sympy [F]

$$\int \frac{x^3\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^3\sqrt{c^2x^2+1}}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

```
[In] integrate(x**3*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(x**3*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x))**2, x)
```

Maxima [F]

$$\int \frac{x^3\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}x^3}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

```
[In] integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -((c^2*x^5 + x^3)*(c^2*x^2 + 1) + (c^3*x^6 + c*x^4)*sqrt(c^2*x^2 + 1))/(a*b
*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^
2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((5*c^3
*x^5 + 2*c*x^3)*(c^2*x^2 + 1)^(3/2) + (10*c^4*x^6 + 11*c^2*x^4 + 3*x^2)*(c^
```

```
2*x^2 + 1) + (5*c^5*x^7 + 9*c^3*x^5 + 4*c*x^3)*sqrt(c^2*x^2 + 1))/(a*b*c^5*
x^4 + (c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c
^2*x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*
x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c
^2*x)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^3\,\sqrt{c^2\,x^2+1}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

```
[In] int((x^3*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((x^3*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2, x)
```

3.412 $\int \frac{x^2\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 93

$$\int \frac{x^2\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{x^2(1+c^2x^2)}{bc(a+b\text{arcsinh}(cx))} - \frac{\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{2b^2c^3} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{2b^2c^3}$$

```
[Out] -x^2*(c^2*x^2+1)/b/c/(a+b*arcsinh(c*x))+1/2*cosh(4*a/b)*Shi(4*(a+b*arcsinh(
c*x))/b)/b^2/c^3-1/2*Chi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b^2/c^3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5814, 5780, 5556, 12, 3384, 3379, 3382}

$$\int \frac{x^2\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{2b^2c^3} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{2b^2c^3} - \frac{x^2(c^2x^2+1)}{bc(a+b\text{arcsinh}(cx))}$$

```
[In] Int[(x^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x^2*(1 + c^2*x^2))/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[(4*(a + b
*ArcSinh[c*x]))/b]*Sinh[(4*a)/b])/(2*b^2*c^3) + (Cosh[(4*a)/b]*SinhIntegral
[(4*(a + b*ArcSinh[c*x]))/b])/(2*b^2*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
```

```
0] && IGtQ[m, -3]
```

Rubi steps

$$\text{integral} = -\frac{x^2(1+c^2x^2)}{bc(a+b\text{arcsinh}(cx))} + \frac{2\int \frac{x}{a+b\text{arcsinh}(cx)}\,dx}{bc} + \frac{(4c)\int \frac{x^3}{a+b\text{arcsinh}(cx)}\,dx}{b}$$

$$= -\frac{x^2(1+c^2x^2)}{bc(a+b\text{arcsinh}(cx))} - \frac{2\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^3} - \frac{4\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^3}$$

$$= -\frac{x^2(1+c^2x^2)}{bc(a+b\text{arcsinh}(cx))} - \frac{2\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^3} - \frac{4\text{Subst}\left(\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{8x} - \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{4x}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^3}$$

$$= -\frac{x^2(1+c^2x^2)}{bc(a+b\text{arcsinh}(cx))} - \frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^2c^3}$$

$$= -\frac{x^2(1+c^2x^2)}{bc(a+b\text{arcsinh}(cx))} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^2c^3} - \frac{\sinh\left(\frac{4a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^2c^3}$$

$$= -\frac{x^2(1+c^2x^2)}{bc(a+b\text{arcsinh}(cx))} - \frac{\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{2b^2c^3} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{2b^2c^3}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.88

$$\int \frac{x^2\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \frac{-\frac{2bc^2x^2(1+c^2x^2)}{a+b\text{arcsinh}(cx)} - \text{Chi}\left(4\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{4a}{b}\right) + \cosh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)}{2b^2c^3}$$

```
[In] Integrate[(x^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
[Out] ((-2*b*c^2*x^2*(1 + c^2*x^2))/(a + b*ArcSinh[c*x]) - CoshIntegral[4*(a/b +
ArcSinh[c*x])]*Sinh[(4*a)/b] + Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[
c*x])])/(2*b^2*c^3)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.55

method	result
default	$-\frac{4b\,c^4x^4+4b\,c^2x^2+\mathrm{e}^{-\frac{4a}{b}}\,\mathrm{Ei}_1\left(-4\,\mathrm{arcsinh}(cx)-\frac{4a}{b}\right)b\,\mathrm{arcsinh}(cx)-\mathrm{e}^{\frac{4a}{b}}\,\mathrm{Ei}_1\left(4\,\mathrm{arcsinh}(cx)+\frac{4a}{b}\right)b\,\mathrm{arcsinh}(cx)+\mathrm{e}^{-\frac{4a}{b}}\,\mathrm{Ei}_1\left(-4\,\mathrm{arcsinh}(c\right.}{4c^3b^2(a+b\,\mathrm{arcsinh}(cx))}$

```
[In] int(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
[Out] -1/4*(4*b*c^4*x^4+4*b*c^2*x^2+exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)*b*arc
sinh(c*x)-exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)*b*arcsinh(c*x)+exp(-4*a/b)*
Ei(1,-4*arcsinh(c*x)-4*a/b)*a-exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)*a)/c^3/
b^2/(a+b*arcsinh(c*x))
```

Fricas [F]

$$\int \frac{x^2\sqrt{1+c^2x^2}}{(a+b\mathrm{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}x^2}{(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^2/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a
^2), x)
```

Sympy [F]

$$\int \frac{x^2\sqrt{1+c^2x^2}}{(a+b\mathrm{arcsinh}(cx))^2}\,dx = \int \frac{x^2\sqrt{c^2x^2+1}}{(a+b\,\mathrm{asinh}\,(cx))^2}\,dx$$

```
[In] integrate(x**2*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(x**2*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x))**2, x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}x^2}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

```
[In] integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^4 + x^2)*(c^2*x^2 + 1) + (c^3*x^5 + c*x^3)*sqrt(c^2*x^2 + 1))/(a*b
*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^
2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((4*c^3
*x^4 + c*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(4*c^4*x^5 + 4*c^2*x^3 + x)*(c^2*x^2
+ 1) + (4*c^5*x^6 + 7*c^3*x^4 + 3*c*x^2)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 +
(c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2
 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqr
t(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*
sqrt(c^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}x^2}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

```
[In] integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)*x^2/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2\sqrt{c^2\,x^2+1}}{(a+b\operatorname{asinh}(c\,x))^2}\,dx$$

```
[In] int((x^2*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((x^2*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2, x)
```

3.413 $\int \frac{x\sqrt{1+c^2x^2}}{(a+b\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 149

$$\int \frac{x\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{x(1+c^2x^2)}{bc(a+b\text{arcsinh}(cx))}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^2}+\frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^2}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^2}-\frac{3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^2}$$

```
[Out] -x*(c^2*x^2+1)/b/c/(a+b*arcsinh(c*x))+1/4*Chi((a+b*arcsinh(c*x))/b)*cosh(a/
b)/b^2/c^2+3/4*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^2-1/4*Shi((a+b
*arcsinh(c*x))/b)*sinh(a/b)/b^2/c^2-3/4*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*
a/b)/b^2/c^2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used

$= \{5814, 5774, 3384, 3379, 3382, 5780, 5556\}$

$$\int \frac{x\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^2} + \frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^2} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^2} - \frac{3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^2} - \frac{x(c^2x^2+1)}{bc(a+b\text{arcsinh}(cx))}$$

```
[In] Int[(x*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x*(1 + c^2*x^2))/(b*c*(a + b*ArcSinh[c*x]))) + (Cosh[a/b]*CoshIntegral[(
a + b*ArcSinh[c*x])/b])/(4*b^2*c^2) + (3*Cosh[(3*a)/b]*CoshIntegral[(3*(a +
 b*ArcSinh[c*x]))/b])/(4*b^2*c^2) - (Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[
c*x])/b])/(4*b^2*c^2) - (3*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x
]))/b])/(4*b^2*c^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x(1+c^2x^2)}{bc(a+b\text{arcsinh}(cx))}+\frac{\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx}{bc}+\frac{(3c)\int \frac{x^2}{a+b\text{arcsinh}(cx)}\,dx}{b}\\
&= -\frac{x(1+c^2x^2)}{bc(a+b\text{arcsinh}(cx))}+\frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^2}\\
&\quad+\frac{3\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^2}\\
&= -\frac{x(1+c^2x^2)}{bc(a+b\text{arcsinh}(cx))}\\
&\quad+\frac{3\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4x}-\frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{4x}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^2}\\
&\quad+\frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^2}\\
&\quad-\frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^2}
\end{aligned}$$

$$= -\frac{x(1+c^2x^2)}{bc(a+b\text{arcsinh}(cx))} + \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c^2} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c^2} + \frac{3\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^2} - \frac{3\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^2}$$

$$= -\frac{x(1+c^2x^2)}{bc(a+b\text{arcsinh}(cx))} + \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c^2} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c^2} - \frac{\left(3\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^2} + \frac{\left(3\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^2} + \frac{\left(3\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^2} - \frac{\left(3\sinh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^2}$$

$$= -\frac{x(1+c^2x^2)}{bc(a+b\text{arcsinh}(cx))} + \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^2} + \frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^2} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^2} - \frac{3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^2}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.85

$$\int \frac{x\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{\frac{4bcx}{a+b\text{arcsinh}(cx)} + \frac{4bc^3x^3}{a+b\text{arcsinh}(cx)} - \cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right) - 3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)}{4b^2c^2}$$

```
[In] Integrate[(x*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

[Out] -1/4*((4*b*c*x)/(a + b*ArcSinh[c*x]) + (4*b*c^3*x^3)/(a + b*ArcSinh[c*x]) - Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - 3*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] + Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 3*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])])/(b^2*c^2)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(141) = 282$.

Time = 0.29 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.44

method	result
default	$-\frac{4c^3x^3-4c^2x^2\sqrt{c^2x^2+1}+3cx-\sqrt{c^2x^2+1}}{8c^2b(a+b\,\mathrm{arcsinh}(cx))}-\frac{3\,\mathrm{e}^{\frac{3a}{b}}\,\mathrm{Ei}_1\left(3\,\mathrm{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b^2}-\frac{-\sqrt{c^2x^2+1}+cx}{8c^2b(a+b\,\mathrm{arcsinh}(cx))}-\frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{Ei}_1\left(\mathrm{arcsinh}(cx)+\frac{a}{b}\right)}{8c^2b^2}$

[In] int(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)

[Out] -1/8*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^2/b/(a+b*arcsinh(c*x))-3/8/c^2/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/8*(-(c^2*x^2+1)^(1/2)+c*x)/c^2/b/(a+b*arcsinh(c*x))-1/8/c^2/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/8/c^2/b^2*(arcsinh(c*x)*Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*b+Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*a+b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/8/c^2/b^2*(4*b*c^3*x^3+4*(c^2*x^2+1)^(1/2)*b*c^2*x^2+3*arcsinh(c*x)*Ei(1,-3*arcsinh(c*x)-3*a/b)*exp(-3*a/b)*b+3*Ei(1,-3*arcsinh(c*x)-3*a/b)*exp(-3*a/b)*a+3*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))

Fricas [F]

$$\int \frac{x\sqrt{1+c^2x^2}}{(a+b\mathrm{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{c^2x^2+1}x}{(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

[In] integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")

[Out] integral(sqrt(c^2*x^2 + 1)*x/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)

Sympy [F]

$$\int \frac{x\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x\sqrt{c^2x^2+1}}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

```
[In] integrate(x*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(x*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x))**2, x)
```

Maxima [F]

$$\int \frac{x\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{c^2x^2+1}x}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

```
[In] integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -((c^2*x^3 + x)*(c^2*x^2 + 1) + (c^3*x^4 + c*x^2)*sqrt(c^2*x^2 + 1))/(a*b*c
^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2
+ 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate((3*(c^2*x
^2 + 1)^(3/2)*c^3*x^3 + (6*c^4*x^4 + 5*c^2*x^2 + 1)*(c^2*x^2 + 1) + (3*c^5*
x^5 + 5*c^3*x^3 + 2*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*a*
b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*x^
2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 + 1
)), x)
```

Giac [F]

$$\int \frac{x\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{c^2x^2+1}x}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

```
[In] integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(sqrt(c^2*x^2 + 1)*x/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{1+c^2x^2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx=\int \frac{x\,\sqrt{c^2\,x^2+1}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

```
[In] int((x*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2,x)
[Out] int((x*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2, x)
```

3.414 $\int \frac{\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{1+c^2x^2}{bc(a+b\text{arcsinh}(cx))} - \frac{\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^2c} + \frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c}$$

```
[Out] (-c^2*x^2-1)/b/c/(a+b*arcsinh(c*x))+cosh(2*a/b)*Shi(2*(a+b*arcsinh(c*x))/b)
/b^2/c-Chi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b^2/c
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5790, 5780, 5556, 12, 3384, 3379, 3382}

$$\int \frac{\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c} + \frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c} - \frac{c^2x^2+1}{bc(a+b\text{arcsinh}(cx))}$$

```
[In] Int[Sqrt[1 + c^2*x^2]/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((1 + c^2*x^2)/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[(2*(a + b*ArcSi
nh[c*x]))/b]*Sinh[(2*a)/b])/(b^2*c) + (Cosh[(2*a)/b]*SinhIntegral[(2*(a + b
*ArcSinh[c*x]))/b])/(b^2*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1+c^2x^2}{bc(a+b\text{arcsinh}(cx))}+\frac{(2c)\int \frac{x}{a+b\text{arcsinh}(cx)}\,dx}{b}\\
&= -\frac{1+c^2x^2}{bc(a+b\text{arcsinh}(cx))}-\frac{2\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c}\\
&= -\frac{1+c^2x^2}{bc(a+b\text{arcsinh}(cx))}-\frac{2\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c}\\
&= -\frac{1+c^2x^2}{bc(a+b\text{arcsinh}(cx))}-\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c}\\
&= -\frac{1+c^2x^2}{bc(a+b\text{arcsinh}(cx))}+\frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c}\\
&\quad -\frac{\sinh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c}\\
&= -\frac{1+c^2x^2}{bc(a+b\text{arcsinh}(cx))}-\frac{\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^2c}+\frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$
\int \frac{\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \frac{-\frac{b+bc^2x^2}{a+b\text{arcsinh}(cx)}-\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right)+\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)}{b^2c}
$$

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-((b + b*c^2*x^2)/(a + b*ArcSinh[c*x])) - CoshIntegral[2*(a/b + ArcSinh[c*
x])]*Sinh[(2*a)/b] + Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])])/(b
^2*c)
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.62

method	result
default	$-\frac{2bc^2x^2+\mathrm{e}^{-\frac{2a}{b}}\,\mathrm{Ei}_1\left(-2\,\mathrm{arcsinh}(cx)-\frac{2a}{b}\right)b\,\mathrm{arcsinh}(cx)-\mathrm{e}^{\frac{2a}{b}}\,\mathrm{Ei}_1\left(2\,\mathrm{arcsinh}(cx)+\frac{2a}{b}\right)b\,\mathrm{arcsinh}(cx)+\mathrm{e}^{-\frac{2a}{b}}\,\mathrm{Ei}_1\left(-2\,\mathrm{arcsinh}(cx)-\frac{2a}{b}\right)a}{2c\,b^2(a+b\,\mathrm{arcsinh}(cx))}$

```
[In] int((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(2*b*c^2*x^2+exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*b*arcsinh(c*x)-ex
p(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)*b*arcsinh(c*x)+exp(-2*a/b)*Ei(1,-2*arcs
inh(c*x)-2*a/b)*a-exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)*a+2*b)/c/b^2/(a+b*a
rcsinh(c*x))
```

Fricas [F]

$$\int \frac{\sqrt{1+c^2x^2}}{(a+b\mathrm{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{c^2x^2+1}}{\left(b\,\mathrm{arsinh}\,(cx)+a\right)^2}\,dx$$

```
[In] integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2),
 x)
```

Sympy [F]

$$\int \frac{\sqrt{1+c^2x^2}}{(a+b\mathrm{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{c^2x^2+1}}{\left(a+b\,\mathrm{asinh}\,(cx)\right)^2}\,dx$$

```
[In] integrate((c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x))**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 + 1)^2 + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(
c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x
 + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((2*c^2*x^2 - 1)*(c^2*x
^2 + 1)^(3/2) + 2*(2*c^3*x^3 + c*x)*(c^2*x^2 + 1) + (2*c^4*x^4 + 3*c^2*x^2
+ 1)*sqrt(c^2*x^2 + 1))/(a*b*c^4*x^4 + (c^2*x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^
2*x^2 + a*b + (b^2*c^4*x^4 + (c^2*x^2 + 1)*b^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^
2 + 2*(b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1
)) + 2*(a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{c^2\,x^2+1}}{(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

```
[In] int((c^2*x^2 + 1)^(1/2)/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2 + 1)^(1/2)/(a + b*asinh(c*x))^2, x)
```

3.415 $\int \frac{\sqrt{1+c^2x^2}}{x(a+b\textbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{1+c^2x^2}{bcx(a+b\text{arcsinh}(cx))}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2}-\frac{\text{Int}\left(\frac{1}{x^2(a+b\text{arcsinh}(cx))},x\right)}{bc}$$

```
[Out] (-c^2*x^2-1)/b/c/x/(a+b*arcsinh(c*x))+Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b
^2-Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2-Unintegrable(1/x^2/(a+b*arcsinh(
c*x)),x)/b/c
```

Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Int[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -((1 + c^2*x^2)/(b*c*x*(a + b*ArcSinh[c*x]))) + (Cosh[a/b]*CoshIntegral[(a
+ b*ArcSinh[c*x])/b])/b^2 - (Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b]
)/b^2 - Defer[Int][1/(x^2*(a + b*ArcSinh[c*x])), x]/(b*c)
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1+c^2x^2}{bcx(a+b\text{arcsinh}(cx))}-\frac{\int \frac{1}{x^2(a+b\text{arcsinh}(cx))}\,dx}{bc}+\frac{c\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx}{b}\\
&= -\frac{1+c^2x^2}{bcx(a+b\text{arcsinh}(cx))}+\frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2}\\
&\quad-\frac{\int \frac{1}{x^2(a+b\text{arcsinh}(cx))}\,dx}{bc}\\
&= -\frac{1+c^2x^2}{bcx(a+b\text{arcsinh}(cx))}-\frac{\int \frac{1}{x^2(a+b\text{arcsinh}(cx))}\,dx}{bc}\\
&\quad+\frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2}\\
&\quad-\frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2}\\
&= -\frac{1+c^2x^2}{bcx(a+b\text{arcsinh}(cx))}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2}\\
&\quad-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2}-\frac{\int \frac{1}{x^2(a+b\text{arcsinh}(cx))}\,dx}{bc}
\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 10.99 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c^2x^2+1}}{x\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^2}dx$$

```
[In] int((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\mathrm{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^2x}\,dx$$

```
[In] integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(b^2*x*arcsinh(c*x)^2 + 2*a*b*x*arcsinh(c*x) + a
^2*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\mathrm{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{x\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^2}\,dx$$

```
[In] integrate((c**2*x**2+1)**(1/2)/x/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(sqrt(c**2*x**2 + 1)/(x*(a + b*asinh(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 392, normalized size of antiderivative = 14.52

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{(b\operatorname{arsinh}(cx)+a)^2x}\,dx$$

```
[In] integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 + 1)^2 + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^3 + sqrt(
c^2*x^2 + 1)*a*b*c^2*x^2 + a*b*c*x + (b^2*c^3*x^3 + sqrt(c^2*x^2 + 1)*b^2*c
^2*x^2 + b^2*c*x)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((c^3*x^3 - 2*c
*x)*(c^2*x^2 + 1)^(3/2) + (2*c^4*x^4 - c^2*x^2 - 1)*(c^2*x^2 + 1) + (c^5*x^
5 + c^3*x^3)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^6 + (c^2*x^2 + 1)*a*b*c^3*x^4 +
2*a*b*c^3*x^4 + a*b*c*x^2 + (b^2*c^5*x^6 + (c^2*x^2 + 1)*b^2*c^3*x^4 + 2*b^
2*c^3*x^4 + b^2*c*x^2 + 2*(b^2*c^4*x^5 + b^2*c^2*x^3)*sqrt(c^2*x^2 + 1))*lo
g(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^5 + a*b*c^2*x^3)*sqrt(c^2*x^2 + 1
)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\operatorname{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.82 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2\,x^2+1}}{x\,(a+b\,\mathrm{asinh}\,(c\,x))^2}\,dx$$

```
[In] int((c^2*x^2 + 1)^(1/2)/(x*(a + b*asinh(c*x))^2),x)
```

```
[Out] int((c^2*x^2 + 1)^(1/2)/(x*(a + b*asinh(c*x))^2), x)
```

3.416 $\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{1+c^2x^2}{bcx^2(a+b\text{arcsinh}(cx))}-\frac{2\text{Int}\left(\frac{1}{x^3(a+b\text{arcsinh}(cx))},x\right)}{bc}$$

```
[Out] (-c^2*x^2-1)/b/c/x^2/(a+b*arcsinh(c*x))-2*Unintegrable(1/x^3/(a+b*arcsinh(c
*x)),x)/b/c
```

Rubi [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Int[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -((1 + c^2*x^2)/(b*c*x^2*(a + b*ArcSinh[c*x]))) - (2*Defer[Int][1/(x^3*(a +
 b*ArcSinh[c*x])), x])/(b*c)
```

Rubi steps

$$\text{integral} = -\frac{1+c^2x^2}{bcx^2(a+b\text{arcsinh}(cx))}-\frac{2\int \frac{1}{x^3(a+b\text{arcsinh}(cx))}\,dx}{bc}$$

Mathematica [N/A]

Not integrable

Time = 2.42 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c^2x^2+1}}{x^2\left(a+b\ \text{arcsinh}\left(cx\right)\right)^2}dx$$

```
[In] int((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x))^2,x)
[Out] int((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2x^2}\,dx$$

```
[In] integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x)
 + a^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{x^2\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}\,dx$$

```
[In] integrate((c**2*x**2+1)**(1/2)/x**2/(a+b*asinh(c*x))**2,x)
[Out] Integral(sqrt(c**2*x**2 + 1)/(x**2*(a + b*asinh(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 378, normalized size of antiderivative = 14.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2x^2}\,dx$$

```
[In] integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -((c^2*x^2 + 1)^2 + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^4 + sqrt(
c^2*x^2 + 1)*a*b*c^2*x^3 + a*b*c*x^2 + (b^2*c^3*x^4 + sqrt(c^2*x^2 + 1)*b^2
*c^2*x^3 + b^2*c*x^2)*log(c*x + sqrt(c^2*x^2 + 1))) - integrate((3*(c^2*x^2
 + 1)^(3/2)*c*x + 2*(2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^3*x^3 + c*x)*sqrt(c^
2*x^2 + 1))/(a*b*c^5*x^7 + (c^2*x^2 + 1)*a*b*c^3*x^5 + 2*a*b*c^3*x^5 + a*b*
c*x^3 + (b^2*c^5*x^7 + (c^2*x^2 + 1)*b^2*c^3*x^5 + 2*b^2*c^3*x^5 + b^2*c*x^
3 + 2*(b^2*c^4*x^6 + b^2*c^2*x^4)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
 + 1)) + 2*(a*b*c^4*x^6 + a*b*c^2*x^4)*sqrt(c^2*x^2 + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2x^2}\,dx$$

```
[In] integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.78 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{c^2\,x^2+1}}{x^2\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

```
[In] int((c^2*x^2 + 1)^(1/2)/(x^2*(a + b*asinh(c*x))^2),x)
[Out] int((c^2*x^2 + 1)^(1/2)/(x^2*(a + b*asinh(c*x))^2), x)
```

3.417 $\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\,\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\operatorname{arcsinh}(cx))^2}\,dx=\operatorname{Int}\left(\frac{\sqrt{1+c^2x^2}}{x^3(a+b\operatorname{arcsinh}(cx))^2},x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\operatorname{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\operatorname{arcsinh}(cx))^2}\,dx$$

```
[In] Int[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\operatorname{arcsinh}(cx))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 16.42 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c^2x^2+1}}{x^3\left(a+b\,\operatorname{arcsinh}\left(cx\right)\right)^2}dx$$

```
[In] int((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x)
[Out] int((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{\left(b\,\operatorname{arsinh}\left(cx\right)+a\right)^2x^3}\,dx$$

```
[In] integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*x)
 + a^2*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{x^3\,(a+b\,\text{asinh}\,(cx))^2}\,dx$$

```
[In] integrate((c**2*x**2+1)**(1/2)/x**3/(a+b*asinh(c*x))**2,x)
[Out] Integral(sqrt(c**2*x**2 + 1)/(x**3*(a + b*asinh(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 403, normalized size of antiderivative = 14.93

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\,(cx)+a)^2x^3}\,dx$$

```
[In] integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -((c^2*x^2 + 1)^2 + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^5 + sqrt(
c^2*x^2 + 1)*a*b*c^2*x^4 + a*b*c*x^3 + (b^2*c^3*x^5 + sqrt(c^2*x^2 + 1)*b^2
*c^2*x^4 + b^2*c*x^3)*log(c*x + sqrt(c^2*x^2 + 1))) - integrate(((c^3*x^3 +
 4*c*x)*(c^2*x^2 + 1)^(3/2) + (2*c^4*x^4 + 7*c^2*x^2 + 3)*(c^2*x^2 + 1) + (
c^5*x^5 + 3*c^3*x^3 + 2*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^8 + (c^2*x^2 + 1
)*a*b*c^3*x^6 + 2*a*b*c^3*x^6 + a*b*c*x^4 + (b^2*c^5*x^8 + (c^2*x^2 + 1)*b^
2*c^3*x^6 + 2*b^2*c^3*x^6 + b^2*c*x^4 + 2*(b^2*c^4*x^7 + b^2*c^2*x^5)*sqrt(
c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^7 + a*b*c^2*x^5)*
sqrt(c^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.80 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2\,x^2+1}}{x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}\,dx$$

```
[In] int((c^2*x^2 + 1)^(1/2)/(x^3*(a + b*asinh(c*x))^2),x)
[Out] int((c^2*x^2 + 1)^(1/2)/(x^3*(a + b*asinh(c*x))^2), x)
```

3.418 $\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))^2},x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Int[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 4.54 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c^2x^2+1}}{x^4\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^2}dx$$

```
[In] int((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x)
[Out] int((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^2x^4}\,dx$$

```
[In] integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x)
 + a^2*x^4), x)
```

Sympy [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{x^4\,(a+b\,\text{asinh}\,(cx))^2}\,dx$$

```
[In] integrate((c**2*x**2+1)**(1/2)/x**4/(a+b*asinh(c*x))**2,x)
[Out] Integral(sqrt(c**2*x**2 + 1)/(x**4*(a + b*asinh(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 406, normalized size of antiderivative = 15.04

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\,(cx)+a)^2x^4}\,dx$$

```
[In] integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -((c^2*x^2 + 1)^2 + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^6 + sqrt(
c^2*x^2 + 1)*a*b*c^2*x^5 + a*b*c*x^4 + (b^2*c^3*x^6 + sqrt(c^2*x^2 + 1)*b^2
*c^2*x^5 + b^2*c*x^4)*log(c*x + sqrt(c^2*x^2 + 1))) - integrate(((2*c^3*x^3
 + 5*c*x)*(c^2*x^2 + 1)^(3/2) + 2*(2*c^4*x^4 + 5*c^2*x^2 + 2)*(c^2*x^2 + 1)
 + (2*c^5*x^5 + 5*c^3*x^3 + 3*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^9 + (c^2*x
^2 + 1)*a*b*c^3*x^7 + 2*a*b*c^3*x^7 + a*b*c*x^5 + (b^2*c^5*x^9 + (c^2*x^2 +
 1)*b^2*c^3*x^7 + 2*b^2*c^3*x^7 + b^2*c*x^5 + 2*(b^2*c^4*x^8 + b^2*c^2*x^6)
*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^8 + a*b*c^2
*x^6)*sqrt(c^2*x^2 + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\,(cx)+a)^2x^4}\,dx$$

```
[In] integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)^2*x^4), x)
```

Mupad [N/A]

Not integrable

Time = 2.90 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{c^2\,x^2+1}}{x^4\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

```
[In] int((c^2*x^2 + 1)^(1/2)/(x^4*(a + b*asinh(c*x))^2),x)
[Out] int((c^2*x^2 + 1)^(1/2)/(x^4*(a + b*asinh(c*x))^2), x)
```

3.419 $\int \frac{x^3(1+c^2x^2)^{3/2}}{(a+b\,\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 277

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{x^3(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))} - \frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64b^2c^4} - \frac{9\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^4} + \frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^4} + \frac{7\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^4} + \frac{3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64b^2c^4} + \frac{9\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^4} - \frac{5\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^4} - \frac{7\sinh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^4}$$

```
[Out] -x^3*(c^2*x^2+1)^2/b/c/(a+b*arcsinh(c*x))-3/64*Chi((a+b*arcsinh(c*x))/b)*co
sh(a/b)/b^2/c^4-9/64*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^4+5/64*C
hi(5*(a+b*arcsinh(c*x))/b)*cosh(5*a/b)/b^2/c^4+7/64*Chi(7*(a+b*arcsinh(c*x)
)/b)*cosh(7*a/b)/b^2/c^4+3/64*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c^4+9
/64*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^4-5/64*Shi(5*(a+b*arcsinh
(c*x))/b)*sinh(5*a/b)/b^2/c^4-7/64*Shi(7*(a+b*arcsinh(c*x))/b)*sinh(7*a/b)/
b^2/c^4
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5814, 5819, 5556, 3384, 3379, 3382}

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64b^2c^4} - \frac{9\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^4} + \frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^4} + \frac{7\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^4} + \frac{3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64b^2c^4} + \frac{9\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^4} - \frac{5\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^4} - \frac{7\sinh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^4} - \frac{x^3(c^2x^2+1)^2}{bc(a+b\text{arcsinh}(cx))}$$

```
[In] Int[(x^3*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x^3*(1 + c^2*x^2)^2)/(b*c*(a + b*ArcSinh[c*x]))) - (3*Cosh[a/b]*CoshInte
gral[(a + b*ArcSinh[c*x])/b])/(64*b^2*c^4) - (9*Cosh[(3*a)/b]*CoshIntegral[
(3*(a + b*ArcSinh[c*x]))/b])/(64*b^2*c^4) + (5*Cosh[(5*a)/b]*CoshIntegral[(
5*(a + b*ArcSinh[c*x]))/b])/(64*b^2*c^4) + (7*Cosh[(7*a)/b]*CoshIntegral[(7
*(a + b*ArcSinh[c*x]))/b])/(64*b^2*c^4) + (3*Sinh[a/b]*SinhIntegral[(a + b*
ArcSinh[c*x])/b])/(64*b^2*c^4) + (9*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*Ar
cSinh[c*x]))/b])/(64*b^2*c^4) - (5*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*Arc
Sinh[c*x]))/b])/(64*b^2*c^4) - (7*Sinh[(7*a)/b]*SinhIntegral[(7*(a + b*ArcS
inh[c*x]))/b])/(64*b^2*c^4)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x^3(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))}+\frac{3\int \frac{x^2(1+c^2x^2)}{a+b\text{arcsinh}(cx)}\,dx}{bc}+\frac{(7c)\int \frac{x^4(1+c^2x^2)}{a+b\text{arcsinh}(cx)}\,dx}{b}\\ &= -\frac{x^3(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))}+\frac{3\text{Subst}\left(\int \frac{\cosh^3\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^4}\\ &\quad+\frac{7\text{Subst}\left(\int \frac{\cosh^3\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^4}\end{aligned}$$

$$= -\frac{x^3(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))} + \frac{3\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16x} + \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16x} - \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{8x}\right) dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^4} + \frac{7\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{7a}{b}-\frac{7x}{b}\right)}{64x} - \frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{64x} - \frac{3\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{64x} + \frac{3\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{64x}\right) dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^4}$$

$$= -\frac{x^3(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))} + \frac{7\text{Subst}\left(\int \frac{\cosh\left(\frac{7a}{b}-\frac{7x}{b}\right)}{x} dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^4} - \frac{7\text{Subst}\left(\int \frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{x} dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^4} + \frac{3\text{Subst}\left(\int \frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{x} dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^4} + \frac{3\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x} dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^4} - \frac{21\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x} dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^4} + \frac{21\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x} dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^4} - \frac{3\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x} dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^4}$$

$$= -\frac{x^3(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))} + \frac{\left(21\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^4}$$
$$- \frac{\left(3\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^4}$$
$$+ \frac{\left(3\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^4}$$
$$- \frac{\left(21\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^4}$$
$$- \frac{\left(7\cosh\left(\frac{5a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{5x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^4}$$
$$+ \frac{\left(3\cosh\left(\frac{5a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{5x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^4}$$
$$+ \frac{\left(7\cosh\left(\frac{7a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{7x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^4}$$
$$- \frac{\left(21\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^4}$$
$$+ \frac{\left(3\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^4}$$
$$- \frac{\left(3\sinh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^4}$$
$$+ \frac{\left(21\sinh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^4}$$
$$+ \frac{\left(7\sinh\left(\frac{5a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{5x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^4}$$
$$- \frac{\left(3\sinh\left(\frac{5a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{5x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^4}$$
$$- \frac{\left(7\sinh\left(\frac{7a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{7x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^4}$$

$$= -\frac{x^3(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))} - \frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64b^2c^4} - \frac{9\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^4} + \frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^4} + \frac{7\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^4} + \frac{3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64b^2c^4} + \frac{9\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^4} - \frac{5\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^4} - \frac{7\sinh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^4}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.44

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \frac{-64bc^3x^3 - 128bc^5x^5 - 64bc^7x^7 - 3(a+b\text{arcsinh}(cx))\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsin}\right.}{}$$

```
[In] Integrate[(x^3*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-64*b*c^3*x^3 - 128*b*c^5*x^5 - 64*b*c^7*x^7 - 3*(a + b*ArcSinh[c*x])*Cosh
[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - 9*(a + b*ArcSinh[c*x])*Cosh[(3*a)/
b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] + 5*a*Cosh[(5*a)/b]*CoshIntegral[5*
(a/b + ArcSinh[c*x])] + 5*b*ArcSinh[c*x]*Cosh[(5*a)/b]*CoshIntegral[5*(a/b
+ ArcSinh[c*x])] + 7*a*Cosh[(7*a)/b]*CoshIntegral[7*(a/b + ArcSinh[c*x])] +
 7*b*ArcSinh[c*x]*Cosh[(7*a)/b]*CoshIntegral[7*(a/b + ArcSinh[c*x])] + 3*a*
Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 3*b*ArcSinh[c*x]*Sinh[a/b]*Sin
hIntegral[a/b + ArcSinh[c*x]] + 9*a*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + Arc
Sinh[c*x])] + 9*b*ArcSinh[c*x]*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[
c*x])] - 5*a*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])] - 5*b*ArcSi
nh[c*x]*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])] - 7*a*Sinh[(7*a)
/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])] - 7*b*ArcSinh[c*x]*Sinh[(7*a)/b]*S
inhIntegral[7*(a/b + ArcSinh[c*x])])/(64*b^2*c^4*(a + b*ArcSinh[c*x]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 957 vs. 2(261) = 522.

Time = 0.31 (sec) , antiderivative size = 958, normalized size of antiderivative = 3.46

method	result	size
default	Expression too large to display	958

```
[In] int(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/128*(64*c^7*x^7-64*c^6*x^6*(c^2*x^2+1)^(1/2)+112*c^5*x^5-80*c^4*x^4*(c^2
*x^2+1)^(1/2)+56*c^3*x^3-24*c^2*x^2*(c^2*x^2+1)^(1/2)+7*c*x-(c^2*x^2+1)^(1/
2))/c^4/(a+b*arcsinh(c*x))/b-7/128/c^4/b^2*exp(7*a/b)*Ei(1,7*arcsinh(c*x)+7
*a/b)-1/128*(16*c^5*x^5-16*c^4*x^4*(c^2*x^2+1)^(1/2)+20*c^3*x^3-12*c^2*x^2*
(c^2*x^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))/c^4/b/(a+b*arcsinh(c*x))-5/128/c
^4/b^2*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)+3/128*(4*c^3*x^3-4*c^2*x^2*(c^
2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^4/b/(a+b*arcsinh(c*x))+9/128/c^4/
b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+3/128*(-(c^2*x^2+1)^(1/2)+c*x)/c^
4/b/(a+b*arcsinh(c*x))+3/128/c^4/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)+3/128/
c^4/b^2*(arcsinh(c*x)*Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*b+Ei(1,-arcsinh(c*x
)-a/b)*exp(-a/b)*a+b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))+3/128/c^4/
b^2*(4*b*c^3*x^3+4*(c^2*x^2+1)^(1/2)*b*c^2*x^2+3*arcsinh(c*x)*Ei(1,-3*arcsi
nh(c*x)-3*a/b)*exp(-3*a/b)*b+3*Ei(1,-3*arcsinh(c*x)-3*a/b)*exp(-3*a/b)*a+3*
b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/128/c^4/b^2*(16*b*c^5*x^5+1
6*(c^2*x^2+1)^(1/2)*b*c^4*x^4+20*b*c^3*x^3+12*(c^2*x^2+1)^(1/2)*b*c^2*x^2+5
*arcsinh(c*x)*Ei(1,-5*arcsinh(c*x)-5*a/b)*exp(-5*a/b)*b+5*Ei(1,-5*arcsinh(c
*x)-5*a/b)*exp(-5*a/b)*a+5*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/
128/c^4/b^2*(64*b*c^7*x^7+64*(c^2*x^2+1)^(1/2)*b*c^6*x^6+112*b*c^5*x^5+80*(
c^2*x^2+1)^(1/2)*b*c^4*x^4+56*b*c^3*x^3+24*(c^2*x^2+1)^(1/2)*b*c^2*x^2+7*ar
csinh(c*x)*Ei(1,-7*arcsinh(c*x)-7*a/b)*exp(-7*a/b)*b+7*Ei(1,-7*arcsinh(c*x)
-7*a/b)*exp(-7*a/b)*a+7*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

Fricas [F]

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(c^2x^2+1)^{\frac{3}{2}}x^3}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcs
inh(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^3(c^2x^2+1)^{\frac{3}{2}}}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

```
[In] integrate(x**3*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**3*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x))**2, x)
```

Maxima [F]

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}x^3}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

```
[In] integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^4*x^7 + 2*c^2*x^5 + x^3)*(c^2*x^2 + 1) + (c^5*x^8 + 2*c^3*x^6 + c*x^4)
*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b
^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 +
1))) + integrate(((7*c^5*x^7 + 9*c^3*x^5 + 2*c*x^3)*(c^2*x^2 + 1)^(3/2) + (
14*c^6*x^8 + 27*c^4*x^6 + 16*c^2*x^4 + 3*x^2)*(c^2*x^2 + 1) + (7*c^7*x^9 +
18*c^5*x^7 + 15*c^3*x^5 + 4*c*x^3)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x
^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*
b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*
x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c
^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^3\,(c^2\,x^2+1)^{3/2}}{(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

```
[In] int((x^3*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2,x)
[Out] int((x^3*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2, x)
```

3.420 $\int \frac{x^2(1+c^2x^2)^{3/2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 219

$$\begin{aligned}\int \frac{x^2(1+c^2x^2)^{3/2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = & -\frac{x^2(1+c^2x^2)^2}{bc(a+b\operatorname{arcsinh}(cx))} \\ & +\frac{\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c^3}-\frac{\operatorname{Chi}\left(\frac{4(a+b\operatorname{arcsinh}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{4b^2c^3} \\ & -\frac{3\operatorname{Chi}\left(\frac{6(a+b\operatorname{arcsinh}(cx))}{b}\right)\sinh\left(\frac{6a}{b}\right)}{16b^2c^3}-\frac{\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16b^2c^3} \\ & +\frac{\cosh\left(\frac{4a}{b}\right)\operatorname{Shi}\left(\frac{4(a+b\operatorname{arcsinh}(cx))}{b}\right)}{4b^2c^3}+\frac{3\cosh\left(\frac{6a}{b}\right)\operatorname{Shi}\left(\frac{6(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16b^2c^3}\end{aligned}$$

```
[Out] -x^2*(c^2*x^2+1)^2/b/c/(a+b*arcsinh(c*x))-1/16*cosh(2*a/b)*Shi(2*(a+b*arcsi
nh(c*x))/b)/b^2/c^3+1/4*cosh(4*a/b)*Shi(4*(a+b*arcsinh(c*x))/b)/b^2/c^3+3/1
6*cosh(6*a/b)*Shi(6*(a+b*arcsinh(c*x))/b)/b^2/c^3+1/16*Chi(2*(a+b*arcsinh(c
*x))/b)*sinh(2*a/b)/b^2/c^3-1/4*Chi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b^2
/c^3-3/16*Chi(6*(a+b*arcsinh(c*x))/b)*sinh(6*a/b)/b^2/c^3
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used

$= \{5814, 5819, 5556, 3384, 3379, 3382\}$

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^3} - \frac{\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^3} - \frac{3\sinh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^3} - \frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^3} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^3} + \frac{3\cosh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^3} - \frac{x^2(c^2x^2+1)^2}{bc(a+b\text{arcsinh}(cx))}$$

```
[In] Int[(x^2*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x^2*(1 + c^2*x^2)^2)/(b*c*(a + b*ArcSinh[c*x]))) + (CoshIntegral[(2*(a +
 b*ArcSinh[c*x]))/b]*Sinh[(2*a)/b])/(16*b^2*c^3) - (CoshIntegral[(4*(a + b*
ArcSinh[c*x]))/b]*Sinh[(4*a)/b])/(4*b^2*c^3) - (3*CoshIntegral[(6*(a + b*Ar
cSinh[c*x]))/b]*Sinh[(6*a)/b])/(16*b^2*c^3) - (Cosh[(2*a)/b]*SinhIntegral[(
2*(a + b*ArcSinh[c*x]))/b])/(16*b^2*c^3) + (Cosh[(4*a)/b]*SinhIntegral[(4*(
a + b*ArcSinh[c*x]))/b])/(4*b^2*c^3) + (3*Cosh[(6*a)/b]*SinhIntegral[(6*(a
+ b*ArcSinh[c*x]))/b])/(16*b^2*c^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^2(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))}+\frac{2\int \frac{x(1+c^2x^2)}{a+b\text{arcsinh}(cx)}\,dx}{bc}+\frac{(6c)\int \frac{x^3(1+c^2x^2)}{a+b\text{arcsinh}(cx)}\,dx}{b}\\
&= -\frac{x^2(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))}-\frac{2\text{Subst}\left(\int \frac{\cosh^3\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^3}\\
&\quad-\frac{6\text{Subst}\left(\int \frac{\cosh^3\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^3}\\
&= -\frac{x^2(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))}\\
&\quad-\frac{2\text{Subst}\left(\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{8x}+\frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{4x}\right)dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^3}\\
&\quad-\frac{6\text{Subst}\left(\int \left(\frac{\sinh\left(\frac{6a}{b}-\frac{6x}{b}\right)}{32x}-\frac{3\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{32x}\right)dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^3}
\end{aligned}
$$

$$= -\frac{x^2(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))} - \frac{3\text{Subst}\left(\int \frac{\sinh\left(\frac{6a}{b}-\frac{6x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^3} - \frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^3} - \frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^2c^3} + \frac{9\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^3}$$

$$= -\frac{x^2(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))} + \frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^2c^3} - \frac{\left(9\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^3} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^3} + \frac{\left(3\cosh\left(\frac{6a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{6x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^3} - \frac{\sinh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^2c^3} + \frac{\left(9\sinh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^3} - \frac{\sinh\left(\frac{4a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^3} - \frac{\left(3\sinh\left(\frac{6a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{6x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^3}$$

$$= -\frac{x^2(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))} + \frac{\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c^3} - \frac{\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{4b^2c^3} - \frac{3\text{Chi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{6a}{b}\right)}{16b^2c^3} - \frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^3} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^3} + \frac{3\cosh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^3}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.40

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{16bc^2x^2+32bc^4x^4+16bc^6x^6-(a+b\text{arcsinh}(cx))\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right)+4(a+b\text{arcsinh}(cx$$

[In] Integrate[(x^2*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]

[Out] -1/16*(16*b*c^2*x^2 + 32*b*c^4*x^4 + 16*b*c^6*x^6 - (a + b*ArcSinh[c*x])*CoshIntegral[2*(a/b + ArcSinh[c*x])]*Sinh[(2*a)/b] + 4*(a + b*ArcSinh[c*x])*CoshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/b] + 3*a*CoshIntegral[6*(a/b + ArcSinh[c*x])]*Sinh[(6*a)/b] + 3*b*ArcSinh[c*x]*CoshIntegral[6*(a/b + ArcSinh[c*x])]*Sinh[(6*a)/b] + a*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] + b*ArcSinh[c*x]*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] - 4*a*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 4*b*ArcSinh[c*x]*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 3*a*Cosh[(6*a)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])] - 3*b*ArcSinh[c*x]*Cosh[(6*a)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])])/(b^2*c^3*(a + b*ArcSinh[c*x]))

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.68

method	result
default	$\frac{-32b\,c^6x^6-64b\,c^4x^4-32b\,c^2x^2+e^{-\frac{2a}{b}}\,\text{Ei}_1\left(-2\,\text{arcsinh}(cx)-\frac{2a}{b}\right)b\,\text{arcsinh}(cx)-4\,e^{-\frac{4a}{b}}\,\text{Ei}_1\left(-4\,\text{arcsinh}(cx)-\frac{4a}{b}\right)b\,\text{arcsinh}(cx)+3\,e^{\frac{6a}{b}}}{}$

[In] int(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)

[Out] 1/32*(-32*b*c^6*x^6-64*b*c^4*x^4-32*b*c^2*x^2+exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*b*arcsinh(c*x)-4*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)*b*arcsinh(c*x)+3*exp(6*a/b)*Ei(1,6*arcsinh(c*x)+6*a/b)*b*arcsinh(c*x)+4*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)*b*arcsinh(c*x)-exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)*b*arcsinh(c*x)-3*exp(-6*a/b)*Ei(1,-6*arcsinh(c*x)-6*a/b)*b*arcsinh(c*x)+exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*a-4*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)*a+3*exp(6*a/b)*Ei(1,6*arcsinh(c*x)+6*a/b)*a+4*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)*a-exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)*a-3*exp(-6*a/b)*Ei(1,-6*arcsinh(c*x)-6*a/b)*a)/c^3/b^2/(a+b*arcsinh(c*x))

Fricas [F]

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}x^2}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral((c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcs
inh(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2(c^2x^2+1)^{\frac{3}{2}}}{(a+b\,\text{asinh}\,(cx))^2}\,dx$$

```
[In] integrate(x**2*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(x**2*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x))**2, x)
```

Maxima [F]

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}x^2}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -((c^4*x^6 + 2*c^2*x^4 + x^2)*(c^2*x^2 + 1) + (c^5*x^7 + 2*c^3*x^5 + c*x^3)
*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b
^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 +
1))) + integrate(((6*c^5*x^6 + 7*c^3*x^4 + c*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(
6*c^6*x^7 + 11*c^4*x^5 + 6*c^2*x^3 + x)*(c^2*x^2 + 1) + 3*(2*c^7*x^8 + 5*c^
5*x^6 + 4*c^3*x^4 + c*x^2)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*
a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*
x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1)
)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 +
 1)), x)
```

Giac [F]

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}x^2}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate((c^2*x^2 + 1)^(3/2)*x^2/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2\,(c^2\,x^2+1)^{3/2}}{(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

```
[In] int((x^2*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2,x)
[Out] int((x^2*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2, x)
```

3.421 $\int \frac{x(1+c^2x^2)^{3/2}}{(a+b\,\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 213

$$\int \frac{x(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{x(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))} + \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^2} + \frac{9\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^2} + \frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^2} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^2} - \frac{9\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^2} - \frac{5\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^2}$$

```
[Out] -x*(c^2*x^2+1)^2/b/c/(a+b*arcsinh(c*x))+1/8*Chi((a+b*arcsinh(c*x))/b)*cosh(
a/b)/b^2/c^2+9/16*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^2+5/16*Chi(
5*(a+b*arcsinh(c*x))/b)*cosh(5*a/b)/b^2/c^2-1/8*Shi((a+b*arcsinh(c*x))/b)*s
inh(a/b)/b^2/c^2-9/16*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^2-5/16*
Shi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b^2/c^2
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used

$= \{5814, 5791, 3393, 3384, 3379, 3382, 5819, 5556\}$

$$\int \frac{x(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^2} + \frac{9\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^2} + \frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^2} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^2} - \frac{9\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^2} - \frac{5\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^2} - \frac{x(c^2x^2+1)^2}{bc(a+b\text{arcsinh}(cx))}$$

```
[In] Int[(x*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x*(1 + c^2*x^2)^2)/(b*c*(a + b*ArcSinh[c*x]))) + (Cosh[a/b]*CoshIntegral
[(a + b*ArcSinh[c*x])/b])/(8*b^2*c^2) + (9*Cosh[(3*a)/b]*CoshIntegral[(3*(a
 + b*ArcSinh[c*x]))/b])/(16*b^2*c^2) + (5*Cosh[(5*a)/b]*CoshIntegral[(5*(a
+ b*ArcSinh[c*x]))/b])/(16*b^2*c^2) - (Sinh[a/b]*SinhIntegral[(a + b*ArcSin
h[c*x])/b])/(8*b^2*c^2) - (9*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c
*x]))/b])/(16*b^2*c^2) - (5*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*
x]))/b])/(16*b^2*c^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{x(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))} + \frac{\int \frac{1+c^2x^2}{a+b\text{arcsinh}(cx)}\,dx}{bc} + \frac{(5c)\int \frac{x^2(1+c^2x^2)}{a+b\text{arcsinh}(cx)}\,dx}{b}$$

$$= -\frac{x(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))} + \frac{\text{Subst}\left(\int \frac{\cosh^3\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^2}$$
$$+ \frac{5\text{Subst}\left(\int \frac{\cosh^3\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^2}$$

$$=-\frac{x(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))}+\frac{\text{Subst}\left(\int\left(\frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4x}+\frac{3\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{4x}\right)dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^2}$$
$$+\frac{5\text{Subst}\left(\int\left(\frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16x}+\frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16x}-\frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{8x}\right)dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^2}$$

$$=-\frac{x(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))}+\frac{\text{Subst}\left(\int\frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}dx,x,a+b\text{arcsinh}(cx)\right)}{4b^2c^2}$$
$$+\frac{5\text{Subst}\left(\int\frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{x}dx,x,a+b\text{arcsinh}(cx)\right)}{16b^2c^2}$$
$$+\frac{5\text{Subst}\left(\int\frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}dx,x,a+b\text{arcsinh}(cx)\right)}{16b^2c^2}$$
$$-\frac{5\text{Subst}\left(\int\frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}dx,x,a+b\text{arcsinh}(cx)\right)}{8b^2c^2}$$
$$+\frac{3\text{Subst}\left(\int\frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}dx,x,a+b\text{arcsinh}(cx)\right)}{4b^2c^2}$$

$$=-\frac{x(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))}-\frac{\left(5\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{x}{b}\right)}{x}dx,x,a+b\text{arcsinh}(cx)\right)}{8b^2c^2}$$
$$+\frac{\left(3\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{x}{b}\right)}{x}dx,x,a+b\text{arcsinh}(cx)\right)}{4b^2c^2}$$
$$+\frac{\cosh\left(\frac{3a}{b}\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{3x}{b}\right)}{x}dx,x,a+b\text{arcsinh}(cx)\right)}{4b^2c^2}$$
$$+\frac{\left(5\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{3x}{b}\right)}{x}dx,x,a+b\text{arcsinh}(cx)\right)}{16b^2c^2}$$
$$+\frac{\left(5\cosh\left(\frac{5a}{b}\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{5x}{b}\right)}{x}dx,x,a+b\text{arcsinh}(cx)\right)}{16b^2c^2}$$
$$+\frac{\left(5\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{x}{b}\right)}{x}dx,x,a+b\text{arcsinh}(cx)\right)}{8b^2c^2}$$
$$-\frac{\left(3\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{x}{b}\right)}{x}dx,x,a+b\text{arcsinh}(cx)\right)}{4b^2c^2}$$
$$-\frac{\sinh\left(\frac{3a}{b}\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{3x}{b}\right)}{x}dx,x,a+b\text{arcsinh}(cx)\right)}{4b^2c^2}$$
$$-\frac{\left(5\sinh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{3x}{b}\right)}{x}dx,x,a+b\text{arcsinh}(cx)\right)}{16b^2c^2}$$
$$-\frac{\left(5\sinh\left(\frac{5a}{b}\right)\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{5x}{b}\right)}{x}dx,x,a+b\text{arcsinh}(cx)\right)}{16b^2c^2}$$

$$= -\frac{x(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))} + \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^2} + \frac{9\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^2} + \frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^2} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^2} - \frac{9\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^2} - \frac{5\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^2}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.38

$$\int \frac{x(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{16bcx+32bc^3x^3+16bc^5x^5-2(a+b\text{arcsinh}(cx))\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)-9(a+b\text{arcsinh}(cx))\cos$$

```
[In] Integrate[(x*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/16*(16*b*c*x + 32*b*c^3*x^3 + 16*b*c^5*x^5 - 2*(a + b*ArcSinh[c*x])*Cosh
[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - 9*(a + b*ArcSinh[c*x])*Cosh[(3*a)/
b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] - 5*a*Cosh[(5*a)/b]*CoshIntegral[5*
(a/b + ArcSinh[c*x])] - 5*b*ArcSinh[c*x]*Cosh[(5*a)/b]*CoshIntegral[5*(a/b
+ ArcSinh[c*x])] + 2*a*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 2*b*Arc
Sinh[c*x]*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 9*a*Sinh[(3*a)/b]*Si
nhIntegral[3*(a/b + ArcSinh[c*x])] + 9*b*ArcSinh[c*x]*Sinh[(3*a)/b]*SinhInt
egral[3*(a/b + ArcSinh[c*x])] + 5*a*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + Arc
Sinh[c*x])] + 5*b*ArcSinh[c*x]*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[
c*x])])/(b^2*c^2*(a + b*ArcSinh[c*x]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. 2(201) = 402.

Time = 0.27 (sec) , antiderivative size = 633, normalized size of antiderivative = 2.97

method	result
default	$-\frac{16c^5x^5-16c^4x^4\sqrt{c^2x^2+1}+20c^3x^3-12c^2x^2\sqrt{c^2x^2+1}+5cx-\sqrt{c^2x^2+1}}{32c^2b(a+b\ \text{arcsinh}(cx))} - \frac{5\,\mathrm{e}^{\frac{5a}{b}}\ \text{Ei}_1\left(5\ \text{arcsinh}(cx)+\frac{5a}{b}\right)}{32c^2b^2} - \frac{3\left(4c^3x^3-4c^2x^2\sqrt{c^2x^2}\right.}{32c^2b(a+b\ \text{a}}$

```
[In] int(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*(16*c^5*x^5-16*c^4*x^4*(c^2*x^2+1)^(1/2)+20*c^3*x^3-12*c^2*x^2*(c^2*x
^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))/c^2/b/(a+b*arcsinh(c*x))-5/32/c^2/b^2*
exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-3/32*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)
^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^2/b/(a+b*arcsinh(c*x))-9/32/c^2/b^2*exp(3
*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/16*(-(c^2*x^2+1)^(1/2)+c*x)/c^2/b/(a+b*a
rcsinh(c*x))-1/16/c^2/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/16/c^2/b^2*(arc
sinh(c*x)*Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*b+Ei(1,-arcsinh(c*x)-a/b)*exp(-
a/b)*a+b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-3/32/c^2/b^2*(4*b*c^3*
x^3+4*(c^2*x^2+1)^(1/2)*b*c^2*x^2+3*arcsinh(c*x)*Ei(1,-3*arcsinh(c*x)-3*a/b
)*exp(-3*a/b)*b+3*Ei(1,-3*arcsinh(c*x)-3*a/b)*exp(-3*a/b)*a+3*b*c*x+(c^2*x^
2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/32/c^2/b^2*(16*b*c^5*x^5+16*(c^2*x^2+1)^
(1/2)*b*c^4*x^4+20*b*c^3*x^3+12*(c^2*x^2+1)^(1/2)*b*c^2*x^2+5*arcsinh(c*x)*
Ei(1,-5*arcsinh(c*x)-5*a/b)*exp(-5*a/b)*b+5*Ei(1,-5*arcsinh(c*x)-5*a/b)*exp
(-5*a/b)*a+5*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

Fricas [F]

$$\int \frac{x(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}x}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

```
[In] integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^3 + x)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsin
h(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{x(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x(c^2x^2+1)^{\frac{3}{2}}}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

```
[In] integrate(x*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x))**2, x)
```

Maxima [F]

$$\int \frac{x(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}x}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^4*x^5 + 2*c^2*x^3 + x)*(c^2*x^2 + 1) + (c^5*x^6 + 2*c^3*x^4 + c*x^2)*s
qrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2
*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1)
)) + integrate((5*(c^5*x^5 + c^3*x^3)*(c^2*x^2 + 1)^(3/2) + (10*c^6*x^6 + 1
7*c^4*x^4 + 8*c^2*x^2 + 1)*(c^2*x^2 + 1) + (5*c^7*x^7 + 12*c^5*x^5 + 9*c^3*
x^3 + 2*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*a*b*c^3*x^2 +
2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^
3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1))*log(c*x + sq
rt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x\,(c^2\,x^2+1)^{3/2}}{(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

```
[In] int((x*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((x*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2, x)
```

3.422 $\int \frac{(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 149

$$\int \frac{(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))} - \frac{\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^2c} - \frac{\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{2b^2c} + \frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{2b^2c}$$

```
[Out] -(c^2*x^2+1)^2/b/c/(a+b*arcsinh(c*x))+cosh(2*a/b)*Shi(2*(a+b*arcsinh(c*x))/
b)/b^2/c+1/2*cosh(4*a/b)*Shi(4*(a+b*arcsinh(c*x))/b)/b^2/c-Chi(2*(a+b*arcsi
nh(c*x))/b)*sinh(2*a/b)/b^2/c-1/2*Chi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b
^2/c
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5790, 5819, 5556, 3384, 3379, 3382}

$$\int \frac{(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c} - \frac{\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{2b^2c} + \frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{2b^2c} - \frac{(c^2x^2+1)^2}{bc(a+b\text{arcsinh}(cx))}$$

```
[In] Int[(1 + c^2*x^2)^(3/2)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((1 + c^2*x^2)^2/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[(2*(a + b*Arc
Sinh[c*x]))/b]*Sinh[(2*a)/b])/(b^2*c) - (CoshIntegral[(4*(a + b*ArcSinh[c*x
]))/b]*Sinh[(4*a)/b])/(2*b^2*c) + (Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*Arc
Sinh[c*x]))/b])/(b^2*c) + (Cosh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x
]))/b])/(2*b^2*c)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
```

```
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))} + \frac{(4c)\int \frac{x(1+c^2x^2)}{a+b\text{arcsinh}(cx)}\,dx}{b}$$

$$= -\frac{(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))} - \frac{4\text{Subst}\left(\int \frac{\cosh^3\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c}$$

$$= -\frac{(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))} - \frac{4\text{Subst}\left(\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{8x} + \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{4x}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c}$$

$$= -\frac{(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))} - \frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^2c} - \frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c}$$

$$= -\frac{(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))} + \frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^2c} - \frac{\sinh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c} - \frac{\sinh\left(\frac{4a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^2c}$$

$$= -\frac{(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))} - \frac{\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^2c} - \frac{\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{2b^2c} + \frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{2b^2c}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.82

$$\int \frac{(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \frac{-\frac{2b(1+c^2x^2)^2}{a+b\text{arcsinh}(cx)} - 2\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right) - \text{Chi}\left(4\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right.$$

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((-2*b*(1 + c^2*x^2)^2)/(a + b*ArcSinh[c*x]) - 2*CoshIntegral[2*(a/b + ArcS
inh[c*x])]*Sinh[(2*a)/b] - CoshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/
b] + 2*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] + Cosh[(4*a)/b]*S
inhIntegral[4*(a/b + ArcSinh[c*x])])/(2*b^2*c)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.71

method	result
default	$-\frac{4b\,c^4x^4+8b\,c^2x^2+e^{-\frac{4a}{b}}\,\text{Ei}_1\left(-4\,\text{arcsinh}(cx)-\frac{4a}{b}\right)b\,\text{arcsinh}(cx)+2\,e^{-\frac{2a}{b}}\,\text{Ei}_1\left(-2\,\text{arcsinh}(cx)-\frac{2a}{b}\right)b\,\text{arcsinh}(cx)-e^{\frac{4a}{b}}\,\text{Ei}_1\left(4\,\text{arcsinh}\right.}{}$

```
[In] int((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(4*b*c^4*x^4+8*b*c^2*x^2+exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)*b*arc
sinh(c*x)+2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*b*arcsinh(c*x)-exp(4*a/
b)*Ei(1,4*arcsinh(c*x)+4*a/b)*b*arcsinh(c*x)-2*exp(2*a/b)*Ei(1,2*arcsinh(c*
x)+2*a/b)*b*arcsinh(c*x)+exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)*a+2*exp(-2
*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*a-exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)*a
-2*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)*a+4*b)/c/b^2/(a+b*arcsinh(c*x))
```

Fricas [F]

$$\int \frac{(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^2 + 1)^(3/2)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
), x)
```

Sympy [F]

$$\int \frac{(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(a+b\,\text{asinh}\,(cx))^2}\,dx$$

[In] integrate((c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)

[Out] Integral((c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x))**2, x)

Maxima [F]

$$\int \frac{(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

[In] integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")

[Out] -((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^5*x^5 + 2*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((4*c^4*x^4 + 3*c^2*x^2 - 1)*(c^2*x^2 + 1)^(3/2) + 4*(2*c^5*x^5 + 3*c^3*x^3 + c*x)*(c^2*x^2 + 1) + (4*c^6*x^6 + 9*c^4*x^4 + 6*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1))/(a*b*c^4*x^4 + (c^2*x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^2*x^2 + a*b + (b^2*c^4*x^4 + (c^2*x^2 + 1)*b^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^2 + 2*(b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)), x)

Giac [F]

$$\int \frac{(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

[In] integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")

[Out] integrate((c^2*x^2 + 1)^(3/2)/(b*arcsinh(c*x) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+c^2x^2)^{3/2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{\left(c^2\,x^2+1\right)^{3/2}}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}\,dx$$

```
[In] int((c^2*x^2 + 1)^(3/2)/(a + b*asinh(c*x))^2,x)
[Out] int((c^2*x^2 + 1)^(3/2)/(a + b*asinh(c*x))^2, x)
```

3.423 $\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx=-\frac{(1+c^2x^2)^2}{bcx(a+b\text{arcsinh}(cx))}+\frac{9\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2}+\frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2}-\frac{9\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2}-\frac{3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2}-\frac{\text{Int}\left(\frac{1+c^2x^2}{x^2(a+b\text{arcsinh}(cx))},x\right)}{bc}$$

```
[Out] -(c^2*x^2+1)^2/b/c/x/(a+b*arcsinh(c*x))+9/4*Chi((a+b*arcsinh(c*x))/b)*cosh(
a/b)/b^2+3/4*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2-9/4*Shi((a+b*arcsi
nh(c*x))/b)*sinh(a/b)/b^2-3/4*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2-U
nintegrable((c^2*x^2+1)/x^2/(a+b*arcsinh(c*x)),x)/b/c
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -((1 + c^2*x^2)^2/(b*c*x*(a + b*ArcSinh[c*x]))) + (9*Cosh[a/b]*CoshIntegral
[(a + b*ArcSinh[c*x])/b])/(4*b^2) + (3*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b
*ArcSinh[c*x]))/b])/(4*b^2) - (9*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x]
)/b])/(4*b^2) - (3*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/
(4*b^2) - Defer[Int][(1 + c^2*x^2)/(x^2*(a + b*ArcSinh[c*x])), x]/(b*c)
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(1+c^2x^2)^2}{bcx(a+b\operatorname{arcsinh}(cx))} - \frac{\int \frac{1+c^2x^2}{x^2(a+b\operatorname{arcsinh}(cx))}\,dx}{bc} + \frac{(3c)\int \frac{1+c^2x^2}{a+b\operatorname{arcsinh}(cx)}\,dx}{b} \\
&= -\frac{(1+c^2x^2)^2}{bcx(a+b\operatorname{arcsinh}(cx))} + \frac{3\operatorname{Subst}\left(\int \frac{\cosh^3\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b^2} \\
&\quad - \frac{\int \frac{1+c^2x^2}{x^2(a+b\operatorname{arcsinh}(cx))}\,dx}{bc} \\
&= -\frac{(1+c^2x^2)^2}{bcx(a+b\operatorname{arcsinh}(cx))} \\
&\quad + \frac{3\operatorname{Subst}\left(\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4x} + \frac{3\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{4x}\right)dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b^2} \\
&\quad - \frac{\int \frac{1+c^2x^2}{x^2(a+b\operatorname{arcsinh}(cx))}\,dx}{bc} \\
&= -\frac{(1+c^2x^2)^2}{bcx(a+b\operatorname{arcsinh}(cx))} + \frac{3\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{4b^2} \\
&\quad + \frac{9\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{4b^2} - \frac{\int \frac{1+c^2x^2}{x^2(a+b\operatorname{arcsinh}(cx))}\,dx}{bc} \\
&= -\frac{(1+c^2x^2)^2}{bcx(a+b\operatorname{arcsinh}(cx))} - \frac{\int \frac{1+c^2x^2}{x^2(a+b\operatorname{arcsinh}(cx))}\,dx}{bc} \\
&\quad + \frac{\left(9\cosh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{4b^2} \\
&\quad + \frac{\left(3\cosh\left(\frac{3a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{4b^2} \\
&\quad - \frac{\left(9\sinh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{4b^2} \\
&\quad - \frac{\left(3\sinh\left(\frac{3a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{4b^2}
\end{aligned}
$$

$$= -\frac{(1+c^2x^2)^2}{bcx(a+b\text{arcsinh}(cx))} + \frac{9\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2} + \frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2} - \frac{9\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2} - \frac{3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2} - \frac{\int \frac{1+c^2x^2}{x^2(a+b\text{arcsinh}(cx))}\,dx}{bc}$$

Mathematica [N/A]

Not integrable

Time = 7.86 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x\left(a+b\,\text{arcsinh}\left(cx\right)\right)^2}dx$$

```
[In] int((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x))^2,x)
[Out] int((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2 x}\,dx$$

```
[In] integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral((c^2*x^2 + 1)^(3/2)/(b^2*x*arcsinh(c*x)^2 + 2*a*b*x*arcsinh(c*x) +
 a^2*x), x)
```

Sympy [N/A]

Not integrable

Time = 3.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x\,(a+b\,\text{asinh}\,(cx))^2}\,dx$$

```
[In] integrate((c**2*x**2+1)**(3/2)/x/(a+b*asinh(c*x))**2,x)
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x*(a + b*asinh(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 433, normalized size of antiderivative = 16.04

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2x}\,dx$$

```
[In] integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^5*x^5 + 2*c^3*x^3 + c*x)*sqr
t(c^2*x^2 + 1))/(a*b*c^3*x^3 + sqrt(c^2*x^2 + 1)*a*b*c^2*x^2 + a*b*c*x + (b
^2*c^3*x^3 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^2 + b^2*c*x)*log(c*x + sqrt(c^2*x^
2 + 1))) + integrate(((3*c^5*x^5 + c^3*x^3 - 2*c*x)*(c^2*x^2 + 1)^(3/2) + (
6*c^6*x^6 + 7*c^4*x^4 - 1)*(c^2*x^2 + 1) + 3*(c^7*x^7 + 2*c^5*x^5 + c^3*x^3
)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^6 + (c^2*x^2 + 1)*a*b*c^3*x^4 + 2*a*b*c^3*x
^4 + a*b*c*x^2 + (b^2*c^5*x^6 + (c^2*x^2 + 1)*b^2*c^3*x^4 + 2*b^2*c^3*x^4 +
 b^2*c*x^2 + 2*(b^2*c^4*x^5 + b^2*c^2*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqr
t(c^2*x^2 + 1)) + 2*(a*b*c^4*x^5 + a*b*c^2*x^3)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.72 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\left(c^2\,x^2+1\right)^{3/2}}{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}\,dx$$

```
[In] int((c^2*x^2 + 1)^(3/2)/(x*(a + b*asinh(c*x))^2),x)
[Out] int((c^2*x^2 + 1)^(3/2)/(x*(a + b*asinh(c*x))^2), x)
```

3.424 $\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\textbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{(1+c^2x^2)^2}{bcx^2(a+b\text{arcsinh}(cx))} - \frac{2\text{Int}\left(\frac{1+c^2x^2}{x^3(a+b\text{arcsinh}(cx))},x\right)}{bc} + \frac{2c\text{Int}\left(\frac{1+c^2x^2}{x(a+b\text{arcsinh}(cx))},x\right)}{b}$$

```
[Out] -(c^2*x^2+1)^2/b/c/x^2/(a+b*arcsinh(c*x))-2*Unintegrable((c^2*x^2+1)/x^3/(a
+b*arcsinh(c*x)),x)/b/c+2*c*Unintegrable((c^2*x^2+1)/x/(a+b*arcsinh(c*x)),x
)/b
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -((1 + c^2*x^2)^2/(b*c*x^2*(a + b*ArcSinh[c*x]))) - (2*Defer[Int][(1 + c^2*
x^2)/(x^3*(a + b*ArcSinh[c*x])), x])/(b*c) + (2*c*Defer[Int][(1 + c^2*x^2)/
(x*(a + b*ArcSinh[c*x])), x])/b
```

Rubi steps

$$\text{integral} = -\frac{(1+c^2x^2)^2}{bcx^2(a+b\text{arcsinh}(cx))} - \frac{2\int \frac{1+c^2x^2}{x^3(a+b\text{arcsinh}(cx))}\,dx}{bc} + \frac{(2c)\int \frac{1+c^2x^2}{x(a+b\text{arcsinh}(cx))}\,dx}{b}$$

Mathematica [N/A]

Not integrable

Time = 3.68 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^2\,(a+b\,\text{arcsinh}\,(cx))^2}dx$$

```
[In] int((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x))^2,x)
[Out] int((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2x^2}\,dx$$

```
[In] integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral((c^2*x^2 + 1)^(3/2)/(b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*
x) + a^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 3.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^2\,(a+b\,\text{asinh}\,(cx))^2}\,dx$$

```
[In] integrate((c**2*x**2+1)**(3/2)/x**2/(a+b*asinh(c*x))**2,x)
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x**2*(a + b*asinh(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 443, normalized size of antiderivative = 16.41

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2x^2}\,dx$$

```
[In] integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^5*x^5 + 2*c^3*x^3 + c*x)*sqr
t(c^2*x^2 + 1))/(a*b*c^3*x^4 + sqrt(c^2*x^2 + 1)*a*b*c^2*x^3 + a*b*c*x^2 +
(b^2*c^3*x^4 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^3 + b^2*c*x^2)*log(c*x + sqrt(c^
2*x^2 + 1))) + integrate(((2*c^5*x^5 - c^3*x^3 - 3*c*x)*(c^2*x^2 + 1)^(3/2)
 + 2*(2*c^6*x^6 + c^4*x^4 - 2*c^2*x^2 - 1)*(c^2*x^2 + 1) + (2*c^7*x^7 + 3*c
^5*x^5 - c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^7 + (c^2*x^2 + 1)*a*b*c^3*x^5 +
 2*a*b*c^3*x^5 + a*b*c*x^3 + (b^2*c^5*x^7 + (c^2*x^2 + 1)*b^2*c^3*x^5 + 2*b
^2*c^3*x^5 + b^2*c*x^3 + 2*(b^2*c^4*x^6 + b^2*c^2*x^4)*sqrt(c^2*x^2 + 1))*l
og(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^6 + a*b*c^2*x^4)*sqrt(c^2*x^2 +
1)), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2x^2}\,dx$$

```
[In] integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.74 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{\left(c^2\,x^2+1\right)^{3/2}}{x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}\,dx$$

```
[In] int((c^2*x^2 + 1)^(3/2)/(x^2*(a + b*asinh(c*x))^2),x)
[Out] int((c^2*x^2 + 1)^(3/2)/(x^2*(a + b*asinh(c*x))^2), x)
```

3.425 $\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\,\textbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx=\text{Int}\left(\frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))^2},x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 12.48 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^3\,(a+b\,\operatorname{arcsinh}\,(cx))^2}dx$$

```
[In] int((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x)
[Out] int((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2x^3}\,dx$$

```
[In] integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral((c^2*x^2 + 1)^(3/2)/(b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*
x) + a^2*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 3.94 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^3\,(a+b\,\text{asinh}\,(cx))^2}\,dx$$

```
[In] integrate((c**2*x**2+1)**(3/2)/x**3/(a+b*asinh(c*x))**2,x)
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x**3*(a + b*asinh(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 441, normalized size of antiderivative = 16.33

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2\,x^3}\,dx$$

```
[In] integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^5*x^5 + 2*c^3*x^3 + c*x)*sqr
t(c^2*x^2 + 1))/(a*b*c^3*x^5 + sqrt(c^2*x^2 + 1)*a*b*c^2*x^4 + a*b*c*x^3 +
(b^2*c^3*x^5 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^4 + b^2*c*x^3)*log(c*x + sqrt(c^
2*x^2 + 1))) + integrate(((c^5*x^5 - 3*c^3*x^3 - 4*c*x)*(c^2*x^2 + 1)^(3/2)
 + (2*c^6*x^6 - 3*c^4*x^4 - 8*c^2*x^2 - 3)*(c^2*x^2 + 1) + (c^7*x^7 - 3*c^3
*x^3 - 2*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^8 + (c^2*x^2 + 1)*a*b*c^3*x^6 +
 2*a*b*c^3*x^6 + a*b*c*x^4 + (b^2*c^5*x^8 + (c^2*x^2 + 1)*b^2*c^3*x^6 + 2*b
^2*c^3*x^6 + b^2*c*x^4 + 2*(b^2*c^4*x^7 + b^2*c^2*x^5)*sqrt(c^2*x^2 + 1))*l
og(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^7 + a*b*c^2*x^5)*sqrt(c^2*x^2 +
1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.74 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\left(c^2\,x^2+1\right)^{3/2}}{x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}\,dx$$

```
[In] int((c^2*x^2 + 1)^(3/2)/(x^3*(a + b*asinh(c*x))^2),x)
[Out] int((c^2*x^2 + 1)^(3/2)/(x^3*(a + b*asinh(c*x))^2), x)
```

3.426 $\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx=-\frac{(1+c^2x^2)^2}{bcx^4(a+b\text{arcsinh}(cx))}-\frac{4\text{Int}\left(\frac{1+c^2x^2}{x^5(a+b\text{arcsinh}(cx))},x\right)}{bc}$$

```
[Out] -(c^2*x^2+1)^2/b/c/x^4/(a+b*arcsinh(c*x))-4*Unintegrable((c^2*x^2+1)/x^5/(a
+b*arcsinh(c*x)),x)/b/c
```

Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -((1 + c^2*x^2)^2/(b*c*x^4*(a + b*ArcSinh[c*x]))) - (4*Defer[Int][(1 + c^2*
x^2)/(x^5*(a + b*ArcSinh[c*x])), x])/(b*c)
```

Rubi steps

$$\text{integral}=-\frac{(1+c^2x^2)^2}{bcx^4(a+b\text{arcsinh}(cx))}-\frac{4\int \frac{1+c^2x^2}{x^5(a+b\text{arcsinh}(cx))}\,dx}{bc}$$

Mathematica [N/A]

Not integrable

Time = 3.56 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^4\left(a+b\ \text{arcsinh}\left(cx\right)\right)^2}dx$$

```
[In] int((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x))^2,x)
[Out] int((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2x^4}\,dx$$

```
[In] integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral((c^2*x^2 + 1)^(3/2)/(b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*
x) + a^2*x^4), x)
```

Sympy [N/A]

Not integrable

Time = 5.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^4\,(a+b\,\text{asinh}\,(cx))^2}\,dx$$

```
[In] integrate((c**2*x**2+1)**(3/2)/x**4/(a+b*asinh(c*x))**2,x)
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x**4*(a + b*asinh(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 428, normalized size of antiderivative = 15.85

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2x^4}\,dx$$

```
[In] integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^5*x^5 + 2*c^3*x^3 + c*x)*sqr
t(c^2*x^2 + 1))/(a*b*c^3*x^6 + sqrt(c^2*x^2 + 1)*a*b*c^2*x^5 + a*b*c*x^4 +
(b^2*c^3*x^6 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^5 + b^2*c*x^4)*log(c*x + sqrt(c^
2*x^2 + 1))) - integrate((5*(c^3*x^3 + c*x)*(c^2*x^2 + 1)^(3/2) + 4*(2*c^4*
x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + 3*(c^5*x^5 + 2*c^3*x^3 + c*x)*sqrt(c^2
*x^2 + 1))/(a*b*c^5*x^9 + (c^2*x^2 + 1)*a*b*c^3*x^7 + 2*a*b*c^3*x^7 + a*b*c
*x^5 + (b^2*c^5*x^9 + (c^2*x^2 + 1)*b^2*c^3*x^7 + 2*b^2*c^3*x^7 + b^2*c*x^5
 + 2*(b^2*c^4*x^8 + b^2*c^2*x^6)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
+ 1)) + 2*(a*b*c^4*x^8 + a*b*c^2*x^6)*sqrt(c^2*x^2 + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2x^4}\,dx$$

```
[In] integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)^2*x^4), x)
```

Mupad [N/A]

Not integrable

Time = 2.90 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\left(c^2\,x^2+1\right)^{3/2}}{x^4\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}\,dx$$

```
[In] int((c^2*x^2 + 1)^(3/2)/(x^4*(a + b*asinh(c*x))^2),x)
[Out] int((c^2*x^2 + 1)^(3/2)/(x^4*(a + b*asinh(c*x))^2), x)
```

3.427 $\int \frac{x^3(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 277

$$\begin{aligned}\int \frac{x^3(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx &= -\frac{x^3(1+c^2x^2)^3}{bc(a+b\text{arcsinh}(cx))} \\ &- \frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{128b^2c^4} - \frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32b^2c^4} \\ &+ \frac{21\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{256b^2c^4} + \frac{9\cosh\left(\frac{9a}{b}\right)\text{Chi}\left(\frac{9(a+b\text{arcsinh}(cx))}{b}\right)}{256b^2c^4} \\ &+ \frac{3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{128b^2c^4} + \frac{3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32b^2c^4} \\ &- \frac{21\sinh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{256b^2c^4} - \frac{9\sinh\left(\frac{9a}{b}\right)\text{Shi}\left(\frac{9(a+b\text{arcsinh}(cx))}{b}\right)}{256b^2c^4}\end{aligned}$$

```
[Out] -x^3*(c^2*x^2+1)^3/b/c/(a+b*arcsinh(c*x))-3/128*Chi((a+b*arcsinh(c*x))/b)*c
osh(a/b)/b^2/c^4-3/32*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^4+21/25
6*Chi(7*(a+b*arcsinh(c*x))/b)*cosh(7*a/b)/b^2/c^4+9/256*Chi(9*(a+b*arcsinh(
c*x))/b)*cosh(9*a/b)/b^2/c^4+3/128*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/
c^4+3/32*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^4-21/256*Shi(7*(a+b*
arcsinh(c*x))/b)*sinh(7*a/b)/b^2/c^4-9/256*Shi(9*(a+b*arcsinh(c*x))/b)*sinh
(9*a/b)/b^2/c^4
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5814, 5819, 5556, 3384, 3379, 3382}

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{128b^2c^4} - \frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32b^2c^4} + \frac{21\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{256b^2c^4} + \frac{9\cosh\left(\frac{9a}{b}\right)\text{Chi}\left(\frac{9(a+b\text{arcsinh}(cx))}{b}\right)}{256b^2c^4} + \frac{3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{128b^2c^4} + \frac{3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32b^2c^4} - \frac{21\sinh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{256b^2c^4} - \frac{9\sinh\left(\frac{9a}{b}\right)\text{Shi}\left(\frac{9(a+b\text{arcsinh}(cx))}{b}\right)}{256b^2c^4} - \frac{x^3(c^2x^2+1)^3}{bc(a+b\text{arcsinh}(cx))}$$

```
[In] Int[(x^3*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x^3*(1 + c^2*x^2)^3)/(b*c*(a + b*ArcSinh[c*x]))) - (3*Cosh[a/b]*CoshInte
gral[(a + b*ArcSinh[c*x])/b])/(128*b^2*c^4) - (3*Cosh[(3*a)/b]*CoshIntegral
[(3*(a + b*ArcSinh[c*x]))/b])/(32*b^2*c^4) + (21*Cosh[(7*a)/b]*CoshIntegral
[(7*(a + b*ArcSinh[c*x]))/b])/(256*b^2*c^4) + (9*Cosh[(9*a)/b]*CoshIntegral
[(9*(a + b*ArcSinh[c*x]))/b])/(256*b^2*c^4) + (3*Sinh[a/b]*SinhIntegral[(a
+ b*ArcSinh[c*x])/b])/(128*b^2*c^4) + (3*Sinh[(3*a)/b]*SinhIntegral[(3*(a +
 b*ArcSinh[c*x]))/b])/(32*b^2*c^4) - (21*Sinh[(7*a)/b]*SinhIntegral[(7*(a +
 b*ArcSinh[c*x]))/b])/(256*b^2*c^4) - (9*Sinh[(9*a)/b]*SinhIntegral[(9*(a +
 b*ArcSinh[c*x]))/b])/(256*b^2*c^4)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{x^3(1+c^2x^2)^3}{bc(a+b\text{arcsinh}(cx))} + \frac{3\int \frac{x^2(1+c^2x^2)^2}{a+b\text{arcsinh}(cx)}\,dx}{bc} + \frac{(9c)\int \frac{x^4(1+c^2x^2)^2}{a+b\text{arcsinh}(cx)}\,dx}{b}$$

$$= -\frac{x^3(1+c^2x^2)^3}{bc(a+b\text{arcsinh}(cx))} + \frac{3\text{Subst}\left(\int \frac{\cosh^5\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^4}$$
$$+ \frac{9\text{Subst}\left(\int \frac{\cosh^5\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^4}$$

$$= -\frac{x^3(1+c^2x^2)^3}{bc(a+b\text{arcsinh}(cx))} + \frac{3\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{7a}{b}-\frac{7x}{b}\right)}{64x} + \frac{3\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{64x} + \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{64x} - \frac{5\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{64x}\right) dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^4} + \frac{9\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{9a}{b}-\frac{9x}{b}\right)}{256x} + \frac{\cosh\left(\frac{7a}{b}-\frac{7x}{b}\right)}{256x} - \frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{64x} - \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{64x} + \frac{3\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{128x}\right) dx, x, a+b\text{arcsi}}{b^2c^4}$$

$$= -\frac{x^3(1+c^2x^2)^3}{bc(a+b\text{arcsinh}(cx))} + \frac{9\text{Subst}\left(\int \frac{\cosh\left(\frac{9a}{b}-\frac{9x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{256b^2c^4} + \frac{9\text{Subst}\left(\int \frac{\cosh\left(\frac{7a}{b}-\frac{7x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{256b^2c^4} + \frac{3\text{Subst}\left(\int \frac{\cosh\left(\frac{7a}{b}-\frac{7x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^4} + \frac{3\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^4} - \frac{9\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^4} + \frac{27\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{128b^2c^4} - \frac{15\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^4}$$

$$= -\frac{x^3(1+c^2x^2)^3}{bc(a+b\text{arcsinh}(cx))} + \frac{\left(27\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{128b^2c^4}$$
$$- \frac{\left(15\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^4}$$
$$+ \frac{\left(3\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^4}$$
$$- \frac{\left(9\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^4}$$
$$+ \frac{\left(9\cosh\left(\frac{7a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{7x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{256b^2c^4}$$
$$+ \frac{\left(3\cosh\left(\frac{7a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{7x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^4}$$
$$+ \frac{\left(9\cosh\left(\frac{9a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{9x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{256b^2c^4}$$
$$- \frac{\left(27\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{128b^2c^4}$$
$$+ \frac{\left(15\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^4}$$
$$- \frac{\left(3\sinh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^4}$$
$$+ \frac{\left(9\sinh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^4}$$
$$- \frac{\left(9\sinh\left(\frac{7a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{7x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{256b^2c^4}$$
$$- \frac{\left(3\sinh\left(\frac{7a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{7x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^4}$$
$$- \frac{\left(9\sinh\left(\frac{9a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{9x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{256b^2c^4}$$

$$= -\frac{x^3(1+c^2x^2)^3}{bc(a+b\text{arcsinh}(cx))} - \frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{128b^2c^4} - \frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32b^2c^4} + \frac{21\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{256b^2c^4} + \frac{9\cosh\left(\frac{9a}{b}\right)\text{Chi}\left(\frac{9(a+b\text{arcsinh}(cx))}{b}\right)}{256b^2c^4} + \frac{3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{128b^2c^4} + \frac{3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32b^2c^4} - \frac{21\sinh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{256b^2c^4} - \frac{9\sinh\left(\frac{9a}{b}\right)\text{Shi}\left(\frac{9(a+b\text{arcsinh}(cx))}{b}\right)}{256b^2c^4}$$

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.47

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{256bc^3x^3 + 768bc^5x^5 + 768bc^7x^7 + 256bc^9x^9 + 6(a+b\text{arcsinh}(cx))\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right) + 24(a}{}$$

```
[In] Integrate[(x^3*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/256*(256*b*c^3*x^3 + 768*b*c^5*x^5 + 768*b*c^7*x^7 + 256*b*c^9*x^9 + 6*(
a + b*ArcSinh[c*x])*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] + 24*(a + b*
ArcSinh[c*x])*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] - 21*a*Cos
h[(7*a)/b]*CoshIntegral[7*(a/b + ArcSinh[c*x])] - 21*b*ArcSinh[c*x]*Cosh[(7
*a)/b]*CoshIntegral[7*(a/b + ArcSinh[c*x])] - 9*a*Cosh[(9*a)/b]*CoshIntegra
l[9*(a/b + ArcSinh[c*x])] - 9*b*ArcSinh[c*x]*Cosh[(9*a)/b]*CoshIntegral[9*(
a/b + ArcSinh[c*x])] - 6*a*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 6*b
*ArcSinh[c*x]*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 24*a*Sinh[(3*a)/
b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] - 24*b*ArcSinh[c*x]*Sinh[(3*a)/b]*S
inhIntegral[3*(a/b + ArcSinh[c*x])] + 21*a*Sinh[(7*a)/b]*SinhIntegral[7*(a/
b + ArcSinh[c*x])] + 21*b*ArcSinh[c*x]*Sinh[(7*a)/b]*SinhIntegral[7*(a/b +
ArcSinh[c*x])] + 9*a*Sinh[(9*a)/b]*SinhIntegral[9*(a/b + ArcSinh[c*x])] + 9
*b*ArcSinh[c*x]*Sinh[(9*a)/b]*SinhIntegral[9*(a/b + ArcSinh[c*x])])/(b^2*c^
4*(a + b*ArcSinh[c*x]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1069 vs. $2(261) = 522$.

Time = 0.34 (sec) , antiderivative size = 1070, normalized size of antiderivative = 3.86

method	result	size
default	Expression too large to display	1070

```
[In] int(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/512*(256*c^9*x^9-256*c^8*x^8*(c^2*x^2+1)^(1/2)+576*c^7*x^7-448*c^6*x^6*(
c^2*x^2+1)^(1/2)+432*c^5*x^5-240*c^4*x^4*(c^2*x^2+1)^(1/2)+120*c^3*x^3-40*c
^2*x^2*(c^2*x^2+1)^(1/2)+9*c*x-(c^2*x^2+1)^(1/2))/c^4/(a+b*arcsinh(c*x))/b-
9/512/c^4/b^2*exp(9*a/b)*Ei(1,9*arcsinh(c*x)+9*a/b)-3/512*(64*c^7*x^7-64*c^
6*x^6*(c^2*x^2+1)^(1/2)+112*c^5*x^5-80*c^4*x^4*(c^2*x^2+1)^(1/2)+56*c^3*x^3
-24*c^2*x^2*(c^2*x^2+1)^(1/2)+7*c*x-(c^2*x^2+1)^(1/2))/c^4/(a+b*arcsinh(c*x
))/b-21/512/c^4/b^2*exp(7*a/b)*Ei(1,7*arcsinh(c*x)+7*a/b)+1/64*(4*c^3*x^3-4
*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^4/b/(a+b*arcsinh(c*x)
)+3/64/c^4/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+3/256*(-(c^2*x^2+1)^(1
/2)+c*x)/c^4/b/(a+b*arcsinh(c*x))+3/256/c^4/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+
a/b)+3/256/c^4/b^2*(arcsinh(c*x)*Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*b+Ei(1,-
arcsinh(c*x)-a/b)*exp(-a/b)*a+b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
+1/64/c^4/b^2*(4*b*c^3*x^3+4*(c^2*x^2+1)^(1/2)*b*c^2*x^2+3*arcsinh(c*x)*Ei(
1,-3*arcsinh(c*x)-3*a/b)*exp(-3*a/b)*b+3*Ei(1,-3*arcsinh(c*x)-3*a/b)*exp(-3
*a/b)*a+3*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-3/512/c^4/b^2*(64*b
*c^7*x^7+64*(c^2*x^2+1)^(1/2)*b*c^6*x^6+112*b*c^5*x^5+80*(c^2*x^2+1)^(1/2)*
b*c^4*x^4+56*b*c^3*x^3+24*(c^2*x^2+1)^(1/2)*b*c^2*x^2+7*arcsinh(c*x)*Ei(1,-
7*arcsinh(c*x)-7*a/b)*exp(-7*a/b)*b+7*Ei(1,-7*arcsinh(c*x)-7*a/b)*exp(-7*a/
b)*a+7*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/512/c^4/b^2*(256*b*c
^9*x^9+256*(c^2*x^2+1)^(1/2)*b*c^8*x^8+576*b*c^7*x^7+448*(c^2*x^2+1)^(1/2)*
b*c^6*x^6+432*b*c^5*x^5+240*(c^2*x^2+1)^(1/2)*b*c^4*x^4+120*b*c^3*x^3+40*(c
^2*x^2+1)^(1/2)*b*c^2*x^2+9*arcsinh(c*x)*Ei(1,-9*arcsinh(c*x)-9*a/b)*exp(-9
*a/b)*b+9*Ei(1,-9*arcsinh(c*x)-9*a/b)*exp(-9*a/b)*a+9*b*c*x+(c^2*x^2+1)^(1/
2)*b)/(a+b*arcsinh(c*x))
```

Fricas [F]

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}x^3}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

```
[In] integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^7 + 2*c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2
+ 2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^3(c^2x^2+1)^{\frac{5}{2}}}{(a+b\,\text{asinh}\,(cx))^2}\,dx$$

```
[In] integrate(x**3*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**3*(c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x))**2, x)
```

Maxima [F]

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}x^3}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^9 + 3*c^4*x^7 + 3*c^2*x^5 + x^3)*(c^2*x^2 + 1) + (c^7*x^10 + 3*c^5
*x^8 + 3*c^3*x^6 + c*x^4)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 +
1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*
log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((9*c^7*x^9 + 20*c^5*x^7 + 13*c^3
*x^5 + 2*c*x^3)*(c^2*x^2 + 1)^(3/2) + 3*(6*c^8*x^10 + 17*c^6*x^8 + 17*c^4*x
^6 + 7*c^2*x^4 + x^2)*(c^2*x^2 + 1) + (9*c^9*x^11 + 31*c^7*x^9 + 39*c^5*x^7
 + 21*c^3*x^5 + 4*c*x^3)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*a*
b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*x^
2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 + 1
)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^3\,(c^2\,x^2+1)^{5/2}}{(a+b\,\mathrm{asinh}\,(c\,x))^2}\,dx$$

```
[In] int((x^3*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2,x)
[Out] int((x^3*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2, x)
```

3.428 $\int \frac{x^2(1+c^2x^2)^{5/2}}{(a+b\textbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 281

$$\begin{aligned}\int \frac{x^2(1+c^2x^2)^{5/2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = &-\frac{x^2(1+c^2x^2)^3}{bc(a+b\operatorname{arcsinh}(cx))}\\ &+\frac{\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c^3}-\frac{\operatorname{Chi}\left(\frac{4(a+b\operatorname{arcsinh}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{8b^2c^3}\\ &-\frac{3\operatorname{Chi}\left(\frac{6(a+b\operatorname{arcsinh}(cx))}{b}\right)\sinh\left(\frac{6a}{b}\right)}{16b^2c^3}-\frac{\operatorname{Chi}\left(\frac{8(a+b\operatorname{arcsinh}(cx))}{b}\right)\sinh\left(\frac{8a}{b}\right)}{16b^2c^3}\\ &-\frac{\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16b^2c^3}+\frac{\cosh\left(\frac{4a}{b}\right)\operatorname{Shi}\left(\frac{4(a+b\operatorname{arcsinh}(cx))}{b}\right)}{8b^2c^3}\\ &+\frac{3\cosh\left(\frac{6a}{b}\right)\operatorname{Shi}\left(\frac{6(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16b^2c^3}+\frac{\cosh\left(\frac{8a}{b}\right)\operatorname{Shi}\left(\frac{8(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16b^2c^3}\end{aligned}$$

```
[Out] -x^2*(c^2*x^2+1)^3/b/c/(a+b*arcsinh(c*x))-1/16*cosh(2*a/b)*Shi(2*(a+b*arcsi
nh(c*x))/b)/b^2/c^3+1/8*cosh(4*a/b)*Shi(4*(a+b*arcsinh(c*x))/b)/b^2/c^3+3/1
6*cosh(6*a/b)*Shi(6*(a+b*arcsinh(c*x))/b)/b^2/c^3+1/16*cosh(8*a/b)*Shi(8*(a
+b*arcsinh(c*x))/b)/b^2/c^3+1/16*Chi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b^
2/c^3-1/8*Chi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b^2/c^3-3/16*Chi(6*(a+b*a
rcsinh(c*x))/b)*sinh(6*a/b)/b^2/c^3-1/16*Chi(8*(a+b*arcsinh(c*x))/b)*sinh(8
*a/b)/b^2/c^3
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5814, 5819, 5556, 3384, 3379, 3382}

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^3} - \frac{\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8b^2c^3} - \frac{3\sinh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^3} - \frac{\sinh\left(\frac{8a}{b}\right)\text{Chi}\left(\frac{8(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^3} - \frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^3} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8b^2c^3} + \frac{3\cosh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^3} + \frac{\cosh\left(\frac{8a}{b}\right)\text{Shi}\left(\frac{8(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^3} - \frac{x^2(c^2x^2+1)^3}{bc(a+b\text{arcsinh}(cx))}$$

```
[In] Int[(x^2*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x^2*(1 + c^2*x^2)^3)/(b*c*(a + b*ArcSinh[c*x]))) + (CoshIntegral[(2*(a +
 b*ArcSinh[c*x]))/b]*Sinh[(2*a)/b])/(16*b^2*c^3) - (CoshIntegral[(4*(a + b*
ArcSinh[c*x]))/b]*Sinh[(4*a)/b])/(8*b^2*c^3) - (3*CoshIntegral[(6*(a + b*Ar
cSinh[c*x]))/b]*Sinh[(6*a)/b])/(16*b^2*c^3) - (CoshIntegral[(8*(a + b*ArcSi
nh[c*x]))/b]*Sinh[(8*a)/b])/(16*b^2*c^3) - (Cosh[(2*a)/b]*SinhIntegral[(2*(
a + b*ArcSinh[c*x]))/b])/(16*b^2*c^3) + (Cosh[(4*a)/b]*SinhIntegral[(4*(a +
 b*ArcSinh[c*x]))/b])/(8*b^2*c^3) + (3*Cosh[(6*a)/b]*SinhIntegral[(6*(a + b
*ArcSinh[c*x]))/b])/(16*b^2*c^3) + (Cosh[(8*a)/b]*SinhIntegral[(8*(a + b*Ar
cSinh[c*x]))/b])/(16*b^2*c^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^2(1+c^2x^2)^3}{bc(a+b\text{arcsinh}(cx))}+\frac{2\int\frac{x(1+c^2x^2)^2}{a+b\text{arcsinh}(cx)}\,dx}{bc}+\frac{(8c)\int\frac{x^3(1+c^2x^2)^2}{a+b\text{arcsinh}(cx)}\,dx}{b}\\
&= -\frac{x^2(1+c^2x^2)^3}{bc(a+b\text{arcsinh}(cx))}-\frac{2\text{Subst}\left(\int\frac{\cosh^5\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^3}\\
&\quad-\frac{8\text{Subst}\left(\int\frac{\cosh^5\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^3}
\end{aligned}$$

$$= -\frac{x^2(1+c^2x^2)^3}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{2\text{Subst}\left(\int \left(\frac{\sinh\left(\frac{6a}{b}-\frac{6x}{b}\right)}{32x} + \frac{\sinh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{8x} + \frac{5\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{32x}\right) dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b^2c^3} - \frac{8\text{Subst}\left(\int \left(\frac{\sinh\left(\frac{8a}{b}-\frac{8x}{b}\right)}{128x} + \frac{\sinh\left(\frac{6a}{b}-\frac{6x}{b}\right)}{64x} - \frac{\sinh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{64x} - \frac{3\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{64x}\right) dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b^2c^3}$$

$$= -\frac{x^2(1+c^2x^2)^3}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{8a}{b}-\frac{8x}{b}\right)}{x} dx, x, a+b\operatorname{arcsinh}(cx)\right)}{16b^2c^3} - \frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{6a}{b}-\frac{6x}{b}\right)}{x} dx, x, a+b\operatorname{arcsinh}(cx)\right)}{16b^2c^3} - \frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{6a}{b}-\frac{6x}{b}\right)}{x} dx, x, a+b\operatorname{arcsinh}(cx)\right)}{8b^2c^3} + \frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{x} dx, x, a+b\operatorname{arcsinh}(cx)\right)}{8b^2c^3} - \frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{x} dx, x, a+b\operatorname{arcsinh}(cx)\right)}{4b^2c^3} - \frac{5\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x} dx, x, a+b\operatorname{arcsinh}(cx)\right)}{16b^2c^3} + \frac{3\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x} dx, x, a+b\operatorname{arcsinh}(cx)\right)}{8b^2c^3}$$

$$= -\frac{x^2(1+c^2x^2)^3}{bc(a+b\text{arcsinh}(cx))} + \frac{\left(5\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^3}$$

$$- \frac{\left(3\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^3}$$

$$- \frac{\cosh\left(\frac{4a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^3}$$

$$+ \frac{\cosh\left(\frac{4a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^3}$$

$$+ \frac{\cosh\left(\frac{6a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{6x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^3}$$

$$+ \frac{\cosh\left(\frac{6a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{6x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^3}$$

$$+ \frac{\cosh\left(\frac{8a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{8x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^3}$$

$$- \frac{\left(5\sinh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^3}$$

$$+ \frac{\left(3\sinh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^3}$$

$$+ \frac{\sinh\left(\frac{4a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^3}$$

$$- \frac{\sinh\left(\frac{4a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^3}$$

$$- \frac{\sinh\left(\frac{6a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{6x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^3}$$

$$- \frac{\sinh\left(\frac{6a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{6x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^3}$$

$$- \frac{\sinh\left(\frac{8a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{8x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^3}$$

$$= -\frac{x^2(1+c^2x^2)^3}{bc(a+b\text{arcsinh}(cx))} + \frac{\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c^3} - \frac{\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{8b^2c^3} - \frac{3\text{Chi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{6a}{b}\right)}{16b^2c^3} - \frac{\text{Chi}\left(\frac{8(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{8a}{b}\right)}{16b^2c^3} - \frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^3} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8b^2c^3} + \frac{3\cosh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^3} + \frac{\cosh\left(\frac{8a}{b}\right)\text{Shi}\left(\frac{8(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^3}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.47

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{16bc^2x^2 + 48bc^4x^4 + 48bc^6x^6 + 16bc^8x^8 - (a+b\text{arcsinh}(cx))\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right) + 2(a+ba}{}$$

```
[In] Integrate[(x^2*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/16*(16*b*c^2*x^2 + 48*b*c^4*x^4 + 48*b*c^6*x^6 + 16*b*c^8*x^8 - (a + b*A
rcSinh[c*x])*CoshIntegral[2*(a/b + ArcSinh[c*x])]*Sinh[(2*a)/b] + 2*(a + b*
ArcSinh[c*x])*CoshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/b] + 3*a*Cosh
Integral[6*(a/b + ArcSinh[c*x])]*Sinh[(6*a)/b] + 3*b*ArcSinh[c*x]*CoshInteg
ral[6*(a/b + ArcSinh[c*x])]*Sinh[(6*a)/b] + a*CoshIntegral[8*(a/b + ArcSinh
[c*x])]*Sinh[(8*a)/b] + b*ArcSinh[c*x]*CoshIntegral[8*(a/b + ArcSinh[c*x])]
*Sinh[(8*a)/b] + a*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] + b*A
rcSinh[c*x]*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] - 2*a*Cosh[(
4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 2*b*ArcSinh[c*x]*Cosh[(4*a)/
b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 3*a*Cosh[(6*a)/b]*SinhIntegral[6*
(a/b + ArcSinh[c*x])] - 3*b*ArcSinh[c*x]*Cosh[(6*a)/b]*SinhIntegral[6*(a/b
+ ArcSinh[c*x])] - a*Cosh[(8*a)/b]*SinhIntegral[8*(a/b + ArcSinh[c*x])] - b
*ArcSinh[c*x]*Cosh[(8*a)/b]*SinhIntegral[8*(a/b + ArcSinh[c*x])])/(b^2*c^3*
(a + b*ArcSinh[c*x]))
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.72

method	result
default	$\frac{-96 b c^6 x^6 - 2\,\mathrm{e}^{-\frac{4a}{b}} \mathrm{Ei}_1\left(-4 \operatorname{arcsinh}(cx) - \frac{4a}{b}\right) b \operatorname{arcsinh}(cx) + \mathrm{e}^{-\frac{2a}{b}} \mathrm{Ei}_1\left(-2 \operatorname{arcsinh}(cx) - \frac{2a}{b}\right) b \operatorname{arcsinh}(cx) - \mathrm{e}^{\frac{2a}{b}} \mathrm{Ei}_1\left(2 \operatorname{arcsinh}(cx) + \right.}{}$

[In] int(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)

[Out] 1/32*(-96*b*c^6*x^6-2*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)*b*arcsinh(c*x
)+exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*b*arcsinh(c*x)-exp(2*a/b)*Ei(1,2*
arcsinh(c*x)+2*a/b)*b*arcsinh(c*x)+2*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)*
b*arcsinh(c*x)-exp(-8*a/b)*Ei(1,-8*arcsinh(c*x)-8*a/b)*b*arcsinh(c*x)+exp(8
*a/b)*Ei(1,8*arcsinh(c*x)+8*a/b)*b*arcsinh(c*x)+3*exp(6*a/b)*Ei(1,6*arcsinh
(c*x)+6*a/b)*b*arcsinh(c*x)-3*exp(-6*a/b)*Ei(1,-6*arcsinh(c*x)-6*a/b)*b*arc
sinh(c*x)-96*b*c^4*x^4-32*b*c^2*x^2+3*exp(6*a/b)*Ei(1,6*arcsinh(c*x)+6*a/b)
*a-3*exp(-6*a/b)*Ei(1,-6*arcsinh(c*x)-6*a/b)*a+exp(-2*a/b)*Ei(1,-2*arcsinh(
c*x)-2*a/b)*a-exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)*a-exp(-8*a/b)*Ei(1,-8*a
rcsinh(c*x)-8*a/b)*a+exp(8*a/b)*Ei(1,8*arcsinh(c*x)+8*a/b)*a-2*exp(-4*a/b)*
Ei(1,-4*arcsinh(c*x)-4*a/b)*a+2*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)*a-32*
b*c^8*x^8)/c^3/b^2/(a+b*arcsinh(c*x))

Fricas [F]

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}x^2}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

[In] integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")

[Out] integral((c^4*x^6 + 2*c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2
+ 2*a*b*arcsinh(c*x) + a^2), x)

Sympy [F]

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{x^2(c^2x^2+1)^{\frac{5}{2}}}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

[In] integrate(x**2*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)

[Out] Integral(x**2*(c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x))**2, x)

Maxima [F]

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}x^2}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^8 + 3*c^4*x^6 + 3*c^2*x^4 + x^2)*(c^2*x^2 + 1) + (c^7*x^9 + 3*c^5*
x^7 + 3*c^3*x^5 + c*x^3)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1
)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*l
og(c*x + sqrt(c^2*x^2 + 1))) + integrate(((8*c^7*x^8 + 17*c^5*x^6 + 10*c^3*
x^4 + c*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(8*c^8*x^9 + 22*c^6*x^7 + 21*c^4*x^5 +
 8*c^2*x^3 + x)*(c^2*x^2 + 1) + (8*c^9*x^10 + 27*c^7*x^8 + 33*c^5*x^6 + 17*
c^3*x^4 + 3*c*x^2)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*a*b*c^3*
x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*x^2 + 2*
b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1))*log(c*
x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}x^2}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)*x^2/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2\,(c^2\,x^2+1)^{5/2}}{(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

```
[In] int((x^2*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((x^2*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2, x)
```

3.429 $\int \frac{x(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 275

$$\begin{aligned}\int \frac{x(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx &= -\frac{x(1+c^2x^2)^3}{bc(a+b\text{arcsinh}(cx))}\\
&+\frac{5\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64b^2c^2}+\frac{27\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^2}\\
&+\frac{25\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^2}+\frac{7\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^2}\\
&-\frac{5\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64b^2c^2}-\frac{27\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^2}\\
&-\frac{25\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^2}-\frac{7\sinh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^2}\end{aligned}$$

```
[Out] -x*(c^2*x^2+1)^3/b/c/(a+b*arcsinh(c*x))+5/64*Chi((a+b*arcsinh(c*x))/b)*cosh
(a/b)/b^2/c^2+27/64*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^2+25/64*C
hi(5*(a+b*arcsinh(c*x))/b)*cosh(5*a/b)/b^2/c^2+7/64*Chi(7*(a+b*arcsinh(c*x)
)/b)*cosh(7*a/b)/b^2/c^2-5/64*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c^2-2
7/64*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^2-25/64*Shi(5*(a+b*arcsi
nh(c*x))/b)*sinh(5*a/b)/b^2/c^2-7/64*Shi(7*(a+b*arcsinh(c*x))/b)*sinh(7*a/b
)/b^2/c^2
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {5814, 5791, 3393, 3384, 3379, 3382, 5819, 5556}

$$\begin{aligned}\int \frac{x(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx &= \frac{5\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64b^2c^2}\\ &+\frac{27\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^2}+\frac{25\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^2}\\ &+\frac{7\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^2}-\frac{5\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64b^2c^2}\\ &-\frac{27\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^2}-\frac{25\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^2}\\ &-\frac{7\sinh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^2}-\frac{x(c^2x^2+1)^3}{bc(a+b\text{arcsinh}(cx))}\end{aligned}$$

```
[In] Int[(x*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x*(1 + c^2*x^2)^3)/(b*c*(a + b*ArcSinh[c*x]))) + (5*Cosh[a/b]*CoshIntegr
al[(a + b*ArcSinh[c*x])/b])/(64*b^2*c^2) + (27*Cosh[(3*a)/b]*CoshIntegral[(
3*(a + b*ArcSinh[c*x]))/b])/(64*b^2*c^2) + (25*Cosh[(5*a)/b]*CoshIntegral[(
5*(a + b*ArcSinh[c*x]))/b])/(64*b^2*c^2) + (7*Cosh[(7*a)/b]*CoshIntegral[(7
*(a + b*ArcSinh[c*x]))/b])/(64*b^2*c^2) - (5*Sinh[a/b]*SinhIntegral[(a + b*
ArcSinh[c*x])/b])/(64*b^2*c^2) - (27*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*A
rcSinh[c*x]))/b])/(64*b^2*c^2) - (25*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*A
rcSinh[c*x]))/b])/(64*b^2*c^2) - (7*Sinh[(7*a)/b]*SinhIntegral[(7*(a + b*Ar
cSinh[c*x]))/b])/(64*b^2*c^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{x(1+c^2x^2)^3}{bc(a+b\text{arcsinh}(cx))} + \frac{\int \frac{(1+c^2x^2)^2}{a+b\text{arcsinh}(cx)}\,dx}{bc} + \frac{(7c)\int \frac{x^2(1+c^2x^2)^2}{a+b\text{arcsinh}(cx)}\,dx}{b}$$

$$= -\frac{x(1+c^2x^2)^3}{bc(a+b\text{arcsinh}(cx))} + \frac{\text{Subst}\left(\int \frac{\cosh^5\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^2} + \frac{7\text{Subst}\left(\int \frac{\cosh^5\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^2}$$

$$= -\frac{x(1+c^2x^2)^3}{bc(a+b\text{arcsinh}(cx))} + \frac{\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16x} + \frac{5\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16x} + \frac{5\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{8x}\right)dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^2} + \frac{7\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{7a}{b}-\frac{7x}{b}\right)}{64x} + \frac{3\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{64x} + \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{64x} - \frac{5\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{64x}\right)dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^2}$$

$$= -\frac{x(1+c^2x^2)^3}{bc(a+b\text{arcsinh}(cx))} + \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^2} + \frac{7\text{Subst}\left(\int \frac{\cosh\left(\frac{7a}{b}-\frac{7x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^2} + \frac{7\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^2} + \frac{5\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^2} + \frac{21\text{Subst}\left(\int \frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^2} - \frac{35\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^2} + \frac{5\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^2}$$

$$= -\frac{x(1+c^2x^2)^3}{bc(a+b\text{arcsinh}(cx))} - \frac{\left(35\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{64b^2c^2}$$
$$+\frac{\left(5\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{8b^2c^2}$$
$$+\frac{\left(7\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{64b^2c^2}$$
$$+\frac{\left(5\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{16b^2c^2}$$
$$+\frac{\cosh\left(\frac{5a}{b}\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{5x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{16b^2c^2}$$
$$+\frac{\left(21\cosh\left(\frac{5a}{b}\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{5x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{64b^2c^2}$$
$$+\frac{\left(7\cosh\left(\frac{7a}{b}\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{7x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{64b^2c^2}$$
$$+\frac{\left(35\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{64b^2c^2}$$
$$-\frac{\left(5\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{8b^2c^2}$$
$$-\frac{\left(7\sinh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{64b^2c^2}$$
$$-\frac{\left(5\sinh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{16b^2c^2}$$
$$-\frac{\sinh\left(\frac{5a}{b}\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{5x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{16b^2c^2}$$
$$-\frac{\left(21\sinh\left(\frac{5a}{b}\right)\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{5x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{64b^2c^2}$$
$$-\frac{\left(7\sinh\left(\frac{7a}{b}\right)\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{7x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{64b^2c^2}$$

$$
\begin{aligned}
&= -\frac{x(1+c^2x^2)^3}{bc(a+b\text{arcsinh}(cx))} + \frac{5\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64b^2c^2} \\
&\quad + \frac{27\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^2} \\
&\quad + \frac{25\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^2} + \frac{7\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^2} \\
&\quad - \frac{5\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64b^2c^2} - \frac{27\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^2} \\
&\quad - \frac{25\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^2} - \frac{7\sinh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.47

$$
\int \frac{x(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx =
$$
$$
-\frac{64bcx + 192bc^3x^3 + 192bc^5x^5 + 64bc^7x^7 - 5(a+b\text{arcsinh}(cx))\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right) - 27(a+b\text{ar}}{}
$$

```
[In] Integrate[(x*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/64*(64*b*c*x + 192*b*c^3*x^3 + 192*b*c^5*x^5 + 64*b*c^7*x^7 - 5*(a + b*A
rcSinh[c*x])*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - 27*(a + b*ArcSinh
[c*x])*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] - 25*a*Cosh[(5*a)
/b]*CoshIntegral[5*(a/b + ArcSinh[c*x])] - 25*b*ArcSinh[c*x]*Cosh[(5*a)/b]*
CoshIntegral[5*(a/b + ArcSinh[c*x])] - 7*a*Cosh[(7*a)/b]*CoshIntegral[7*(a/
b + ArcSinh[c*x])] - 7*b*ArcSinh[c*x]*Cosh[(7*a)/b]*CoshIntegral[7*(a/b + A
rcSinh[c*x])] + 5*a*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 5*b*ArcSin
h[c*x]*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 27*a*Sinh[(3*a)/b]*Sinh
Integral[3*(a/b + ArcSinh[c*x])] + 27*b*ArcSinh[c*x]*Sinh[(3*a)/b]*SinhInte
gral[3*(a/b + ArcSinh[c*x])] + 25*a*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + Arc
Sinh[c*x])] + 25*b*ArcSinh[c*x]*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh
[c*x])] + 7*a*Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])] + 7*b*ArcS
inh[c*x]*Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])])/(b^2*c^2*(a +
b*ArcSinh[c*x]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 957 vs. $2(259) = 518$.

Time = 0.28 (sec) , antiderivative size = 958, normalized size of antiderivative = 3.48

method	result	size
default	Expression too large to display	958

```
[In] int(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/128*(64*c^7*x^7-64*c^6*x^6*(c^2*x^2+1)^(1/2)+112*c^5*x^5-80*c^4*x^4*(c^2
*x^2+1)^(1/2)+56*c^3*x^3-24*c^2*x^2*(c^2*x^2+1)^(1/2)+7*c*x-(c^2*x^2+1)^(1/
2))/c^2/(a+b*arcsinh(c*x))/b-7/128/c^2/b^2*exp(7*a/b)*Ei(1,7*arcsinh(c*x)+7
*a/b)-5/128*(16*c^5*x^5-16*c^4*x^4*(c^2*x^2+1)^(1/2)+20*c^3*x^3-12*c^2*x^2*
(c^2*x^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))/c^2/b/(a+b*arcsinh(c*x))-25/128/
c^2/b^2*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-9/128*(4*c^3*x^3-4*c^2*x^2*(c
^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^2/b/(a+b*arcsinh(c*x))-27/128/c^
2/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-5/128*(-(c^2*x^2+1)^(1/2)+c*x)/
c^2/b/(a+b*arcsinh(c*x))-5/128/c^2/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-5/12
8/c^2/b^2*(arcsinh(c*x)*Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*b+Ei(1,-arcsinh(c
*x)-a/b)*exp(-a/b)*a+b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-9/128/c^
2/b^2*(4*b*c^3*x^3+4*(c^2*x^2+1)^(1/2)*b*c^2*x^2+3*arcsinh(c*x)*Ei(1,-3*arc
sinh(c*x)-3*a/b)*exp(-3*a/b)*b+3*Ei(1,-3*arcsinh(c*x)-3*a/b)*exp(-3*a/b)*a+
3*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-5/128/c^2/b^2*(16*b*c^5*x^5
+16*(c^2*x^2+1)^(1/2)*b*c^4*x^4+20*b*c^3*x^3+12*(c^2*x^2+1)^(1/2)*b*c^2*x^2
+5*arcsinh(c*x)*Ei(1,-5*arcsinh(c*x)-5*a/b)*exp(-5*a/b)*b+5*Ei(1,-5*arcsinh
(c*x)-5*a/b)*exp(-5*a/b)*a+5*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-
1/128/c^2/b^2*(64*b*c^7*x^7+64*(c^2*x^2+1)^(1/2)*b*c^6*x^6+112*b*c^5*x^5+80
*(c^2*x^2+1)^(1/2)*b*c^4*x^4+56*b*c^3*x^3+24*(c^2*x^2+1)^(1/2)*b*c^2*x^2+7*
arcsinh(c*x)*Ei(1,-7*arcsinh(c*x)-7*a/b)*exp(-7*a/b)*b+7*Ei(1,-7*arcsinh(c*
x)-7*a/b)*exp(-7*a/b)*a+7*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

Fricas [F]

$$\int \frac{x(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}x}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

```
[In] integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^5 + 2*c^2*x^3 + x)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 +
2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{x(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x(c^2x^2+1)^{\frac{5}{2}}}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

```
[In] integrate(x*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x*(c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x))**2, x)
```

Maxima [F]

$$\int \frac{x(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}x}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

```
[In] integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^7 + 3*c^4*x^5 + 3*c^2*x^3 + x)*(c^2*x^2 + 1) + (c^7*x^8 + 3*c^5*x^
6 + 3*c^3*x^4 + c*x^2)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*
a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log
(c*x + sqrt(c^2*x^2 + 1))) + integrate((7*(c^7*x^7 + 2*c^5*x^5 + c^3*x^3)*(
c^2*x^2 + 1)^(3/2) + (14*c^8*x^8 + 37*c^6*x^6 + 33*c^4*x^4 + 11*c^2*x^2 + 1
)*(c^2*x^2 + 1) + (7*c^9*x^9 + 23*c^7*x^7 + 27*c^5*x^5 + 13*c^3*x^3 + 2*c*x
)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x
^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2
*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
+ 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(1+c^2x^2)^{5/2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{x\,(c^2\,x^2+1)^{5/2}}{(a+b\,\operatorname{asinh}(c\,x))^2}\,dx$$

```
[In] int((x*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2,x)
[Out] int((x*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2, x)
```

3.430 $\int \frac{(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 216

$$\int \frac{(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{(1+c^2x^2)^3}{bc(a+b\text{arcsinh}(cx))} - \frac{15\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c} - \frac{3\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{4b^2c} - \frac{3\text{Chi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{6a}{b}\right)}{16b^2c} + \frac{15\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c} + \frac{3\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c} + \frac{3\cosh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c}$$

```
[Out] -(c^2*x^2+1)^3/b/c/(a+b*arcsinh(c*x))+15/16*cosh(2*a/b)*Shi(2*(a+b*arcsinh(
c*x))/b)/b^2/c+3/4*cosh(4*a/b)*Shi(4*(a+b*arcsinh(c*x))/b)/b^2/c+3/16*cosh(
6*a/b)*Shi(6*(a+b*arcsinh(c*x))/b)/b^2/c-15/16*Chi(2*(a+b*arcsinh(c*x))/b)*
sinh(2*a/b)/b^2/c-3/4*Chi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b^2/c-3/16*Ch
i(6*(a+b*arcsinh(c*x))/b)*sinh(6*a/b)/b^2/c
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

$= \{5790, 5819, 5556, 3384, 3379, 3382\}$

$$\int \frac{(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{15\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c} - \frac{3\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c} - \frac{3\sinh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c} + \frac{15\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c} + \frac{3\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c} + \frac{3\cosh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c} - \frac{(c^2x^2+1)^3}{bc(a+b\text{arcsinh}(cx))}$$

```
[In] Int[(1 + c^2*x^2)^(5/2)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((1 + c^2*x^2)^3/(b*c*(a + b*ArcSinh[c*x]))) - (15*CoshIntegral[(2*(a + b*
ArcSinh[c*x]))/b]*Sinh[(2*a)/b])/(16*b^2*c) - (3*CoshIntegral[(4*(a + b*Arc
Sinh[c*x]))/b]*Sinh[(4*a)/b])/(4*b^2*c) - (3*CoshIntegral[(6*(a + b*ArcSinh
[c*x]))/b]*Sinh[(6*a)/b])/(16*b^2*c) + (15*Cosh[(2*a)/b]*SinhIntegral[(2*(a
 + b*ArcSinh[c*x]))/b])/(16*b^2*c) + (3*Cosh[(4*a)/b]*SinhIntegral[(4*(a +
b*ArcSinh[c*x]))/b])/(4*b^2*c) + (3*Cosh[(6*a)/b]*SinhIntegral[(6*(a + b*Ar
cSinh[c*x]))/b])/(16*b^2*c)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(1+c^2x^2)^3}{bc(a+b\text{arcsinh}(cx))}+\frac{(6c)\int \frac{x(1+c^2x^2)^2}{a+b\text{arcsinh}(cx)}\,dx}{b}\\
&= -\frac{(1+c^2x^2)^3}{bc(a+b\text{arcsinh}(cx))}-\frac{6\text{Subst}\left(\int \frac{\cosh^5\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c}\\
&= -\frac{(1+c^2x^2)^3}{bc(a+b\text{arcsinh}(cx))}\\
&\quad-\frac{6\text{Subst}\left(\int \left(\frac{\sinh\left(\frac{6a}{b}-\frac{6x}{b}\right)}{32x}+\frac{\sinh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{8x}+\frac{5\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{32x}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c}\\
&= -\frac{(1+c^2x^2)^3}{bc(a+b\text{arcsinh}(cx))}-\frac{3\text{Subst}\left(\int \frac{\sinh\left(\frac{6a}{b}-\frac{6x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{16b^2c}\\
&\quad-\frac{3\text{Subst}\left(\int \frac{\sinh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{4b^2c}\\
&\quad-\frac{15\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{16b^2c}
\end{aligned}$$

$$= -\frac{(1+c^2x^2)^3}{bc(a+b\operatorname{arcsinh}(cx))} + \frac{\left(15\cosh\left(\frac{2a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{16b^2c}$$
$$+ \frac{\left(3\cosh\left(\frac{4a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{4b^2c}$$
$$+ \frac{\left(3\cosh\left(\frac{6a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{6x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{16b^2c}$$
$$- \frac{\left(15\sinh\left(\frac{2a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{16b^2c}$$
$$- \frac{\left(3\sinh\left(\frac{4a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{4b^2c}$$
$$- \frac{\left(3\sinh\left(\frac{6a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{6x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{16b^2c}$$

$$= -\frac{(1+c^2x^2)^3}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{15\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c}$$
$$- \frac{3\operatorname{Chi}\left(\frac{4(a+b\operatorname{arcsinh}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{4b^2c} - \frac{3\operatorname{Chi}\left(\frac{6(a+b\operatorname{arcsinh}(cx))}{b}\right)\sinh\left(\frac{6a}{b}\right)}{16b^2c}$$
$$+ \frac{15\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16b^2c}$$
$$+ \frac{3\cosh\left(\frac{4a}{b}\right)\operatorname{Shi}\left(\frac{4(a+b\operatorname{arcsinh}(cx))}{b}\right)}{4b^2c} + \frac{3\cosh\left(\frac{6a}{b}\right)\operatorname{Shi}\left(\frac{6(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16b^2c}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.44

$$\int \frac{(1+c^2x^2)^{5/2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx =$$
$$-\frac{16b + 48bc^2x^2 + 48bc^4x^4 + 16bc^6x^6 + 15(a+b\operatorname{arcsinh}(cx))\operatorname{Chi}\left(2\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right) + 12(a+ba}{}$$

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/16*(16*b + 48*b*c^2*x^2 + 48*b*c^4*x^4 + 16*b*c^6*x^6 + 15*(a + b*ArcSin
h[c*x])*CoshIntegral[2*(a/b + ArcSinh[c*x])]*Sinh[(2*a)/b] + 12*(a + b*ArcS
inh[c*x])*CoshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/b] + 3*a*CoshInte
gral[6*(a/b + ArcSinh[c*x])]*Sinh[(6*a)/b] + 3*b*ArcSinh[c*x]*CoshIntegral[
6*(a/b + ArcSinh[c*x])]*Sinh[(6*a)/b] - 15*a*Cosh[(2*a)/b]*SinhIntegral[2*(
a/b + ArcSinh[c*x])] - 15*b*ArcSinh[c*x]*Cosh[(2*a)/b]*SinhIntegral[2*(a/b
```

```
+ ArcSinh[c*x])] - 12*a*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])]
- 12*b*ArcSinh[c*x]*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 3*
a*Cosh[(6*a)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])] - 3*b*ArcSinh[c*x]*Cos
h[(6*a)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])])/(b^2*c*(a + b*ArcSinh[c*x]
))
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.73

method	result
default	$\frac{-32b\,c^{6}x^{6}-96b\,c^{4}x^{4}-96b\,c^{2}x^{2}+3\,\mathrm{e}^{\frac{6a}{b}}\,\mathrm{Ei}_{1}\left(6\,\mathrm{arcsinh}(cx)+\frac{6a}{b}\right)b\,\mathrm{arcsinh}(cx)+12\,\mathrm{e}^{\frac{4a}{b}}\,\mathrm{Ei}_{1}\left(4\,\mathrm{arcsinh}(cx)+\frac{4a}{b}\right)b\,\mathrm{arcsinh}(cx)+15\,\mathrm{e}^{\frac{2a}{b}}\,\mathrm{Ei}_{1}}{}$

```
[In] int((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32*(-32*b*c^6*x^6-96*b*c^4*x^4-96*b*c^2*x^2+3*exp(6*a/b)*Ei(1,6*arcsinh(c
*x)+6*a/b)*b*arcsinh(c*x)+12*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)*b*arcsin
h(c*x)+15*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)*b*arcsinh(c*x)-3*exp(-6*a/b
)*Ei(1,-6*arcsinh(c*x)-6*a/b)*b*arcsinh(c*x)-15*exp(-2*a/b)*Ei(1,-2*arcsinh
(c*x)-2*a/b)*b*arcsinh(c*x)-12*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)*b*ar
csinh(c*x)+3*exp(6*a/b)*Ei(1,6*arcsinh(c*x)+6*a/b)*a+12*exp(4*a/b)*Ei(1,4*a
rcsinh(c*x)+4*a/b)*a+15*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)*a-3*exp(-6*a/
b)*Ei(1,-6*arcsinh(c*x)-6*a/b)*a-15*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)
*a-12*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)*a-32*b)/c/b^2/(a+b*arcsinh(c*
x))
```

Fricas [F]

$$\int \frac{(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 +
2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

[In] integrate((c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)

[Out] Integral((c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x))**2, x)

Maxima [F]

$$\int \frac{(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

[In] integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")

[Out] -((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^7*x^7 + 3*c^5*x^5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((6*c^6*x^6 + 11*c^4*x^4 + 4*c^2*x^2 - 1)*(c^2*x^2 + 1)^(3/2) + 6*(2*c^7*x^7 + 5*c^5*x^5 + 4*c^3*x^3 + c*x)*(c^2*x^2 + 1) + (6*c^8*x^8 + 19*c^6*x^6 + 21*c^4*x^4 + 9*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1))/(a*b*c^4*x^4 + (c^2*x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^2*x^2 + a*b + (b^2*c^4*x^4 + (c^2*x^2 + 1)*b^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^2 + 2*(b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)), x)

Giac [F]

$$\int \frac{(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

[In] integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")

[Out] integrate((c^2*x^2 + 1)^(5/2)/(b*arcsinh(c*x) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+c^2x^2)^{5/2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{\left(c^2\,x^2+1\right)^{5/2}}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}\,dx$$

```
[In] int((c^2*x^2 + 1)^(5/2)/(a + b*asinh(c*x))^2,x)
[Out] int((c^2*x^2 + 1)^(5/2)/(a + b*asinh(c*x))^2, x)
```

3.431 $\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\,\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\begin{aligned}\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx &= -\frac{(1+c^2x^2)^3}{bcx(a+b\text{arcsinh}(cx))} \\ &+ \frac{25\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2} + \frac{25\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2} \\ &+ \frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2} - \frac{25\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2} \\ &- \frac{25\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2} \\ &- \frac{5\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2} - \frac{\text{Int}\left(\frac{(1+c^2x^2)^2}{x^2(a+b\text{arcsinh}(cx))},x\right)}{bc}\end{aligned}$$

```
[Out] -(c^2*x^2+1)^3/b/c/x/(a+b*arcsinh(c*x))+25/8*Chi((a+b*arcsinh(c*x))/b)*cosh
(a/b)/b^2+25/16*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2+5/16*Chi(5*(a+b
*arcsinh(c*x))/b)*cosh(5*a/b)/b^2-25/8*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/
b^2-25/16*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2-5/16*Shi(5*(a+b*arcsi
nh(c*x))/b)*sinh(5*a/b)/b^2-Unintegrable((c^2*x^2+1)^2/x^2/(a+b*arcsinh(c*x
)),x)/b/c
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -((1 + c^2*x^2)^3/(b*c*x*(a + b*ArcSinh[c*x]))) + (25*Cosh[a/b]*CoshIntegra
l[(a + b*ArcSinh[c*x])/b])/(8*b^2) + (25*Cosh[(3*a)/b]*CoshIntegral[(3*(a +
 b*ArcSinh[c*x]))/b])/(16*b^2) + (5*Cosh[(5*a)/b]*CoshIntegral[(5*(a + b*Ar
cSinh[c*x]))/b])/(16*b^2) - (25*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])
/b])/(8*b^2) - (25*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/
(16*b^2) - (5*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*x]))/b])/(16*b
^2) - Defer[Int][(1 + c^2*x^2)^2/(x^2*(a + b*ArcSinh[c*x])), x]/(b*c)
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(1+c^2x^2)^3}{bcx(a+b\text{arcsinh}(cx))}-\frac{\int \frac{(1+c^2x^2)^2}{x^2(a+b\text{arcsinh}(cx))}\,dx}{bc}+\frac{(5c)\int \frac{(1+c^2x^2)^2}{a+b\text{arcsinh}(cx)}\,dx}{b}\\
&= -\frac{(1+c^2x^2)^3}{bcx(a+b\text{arcsinh}(cx))}+\frac{5\text{Subst}\left(\int \frac{\cosh^5\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2}\\
&\quad -\frac{\int \frac{(1+c^2x^2)^2}{x^2(a+b\text{arcsinh}(cx))}\,dx}{bc}\\
&= -\frac{(1+c^2x^2)^3}{bcx(a+b\text{arcsinh}(cx))}\\
&\quad +\frac{5\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16x}+\frac{5\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16x}+\frac{5\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{8x}\right)dx,x,a+b\text{arcsinh}(cx)\right)}{b^2}\\
&\quad -\frac{\int \frac{(1+c^2x^2)^2}{x^2(a+b\text{arcsinh}(cx))}\,dx}{bc}\\
&= -\frac{(1+c^2x^2)^3}{bcx(a+b\text{arcsinh}(cx))}+\frac{5\text{Subst}\left(\int \frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{16b^2}\\
&\quad +\frac{25\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{16b^2}\\
&\quad +\frac{25\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{8b^2}-\frac{\int \frac{(1+c^2x^2)^2}{x^2(a+b\text{arcsinh}(cx))}\,dx}{bc}
\end{aligned}$$

$$=-\frac{(1+c^2x^2)^3}{bcx(a+b\text{arcsinh}(cx))}-\frac{\int \frac{(1+c^2x^2)^2}{x^2(a+b\text{arcsinh}(cx))}\,dx}{bc}+\frac{\left(25\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{8b^2}+\frac{\left(25\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{16b^2}+\frac{\left(5\cosh\left(\frac{5a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{5x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{16b^2}-\frac{\left(25\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{8b^2}-\frac{\left(25\sinh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{16b^2}-\frac{\left(5\sinh\left(\frac{5a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{5x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{16b^2}$$

$$=-\frac{(1+c^2x^2)^3}{bcx(a+b\text{arcsinh}(cx))}+\frac{25\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2}+\frac{25\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2}+\frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2}-\frac{25\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2}-\frac{25\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2}-\frac{5\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2}-\frac{\int \frac{(1+c^2x^2)^2}{x^2(a+b\text{arcsinh}(cx))}\,dx}{bc}$$

Mathematica [N/A]

Not integrable

Time = 9.48 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\left(c^{2} x^{2}+1\right)^{\frac{5}{2}}}{x \left(a +b \,\operatorname{arcsinh}\left(c x \right)\right)^{2}}d x$$

```
[In] int((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x))^2,x)
[Out] int((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.19

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{{\left(c^{2} x^{2} + 1\right)}^{\frac{5}{2}}}{{\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{2} x} \,d x$$

```
[In] integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b^2*x*arcsinh(c*x)^2
+ 2*a*b*x*arcsinh(c*x) + a^2*x), x)
```

Sympy [N/A]

Not integrable

Time = 7.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{\left(c^{2} x^{2} + 1\right)^{\frac{5}{2}}}{x \left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{2}}\, dx$$

```
[In] integrate((c**2*x**2+1)**(5/2)/x/(a+b*asinh(c*x))**2,x)
[Out] Integral((c**2*x**2 + 1)**(5/2)/(x*(a + b*asinh(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 480, normalized size of antiderivative = 17.78

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2x}\,dx$$

```
[In] integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^7*x^7 + 3*c^5*x^
5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^3 + sqrt(c^2*x^2 + 1)*a*
b*c^2*x^2 + a*b*c*x + (b^2*c^3*x^3 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^2 + b^2*c*
x)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((5*c^7*x^7 + 8*c^5*x^5 + c^3*
x^3 - 2*c*x)*(c^2*x^2 + 1)^(3/2) + (10*c^8*x^8 + 23*c^6*x^6 + 15*c^4*x^4 +
c^2*x^2 - 1)*(c^2*x^2 + 1) + 5*(c^9*x^9 + 3*c^7*x^7 + 3*c^5*x^5 + c^3*x^3)*
sqrt(c^2*x^2 + 1))/(a*b*c^5*x^6 + (c^2*x^2 + 1)*a*b*c^3*x^4 + 2*a*b*c^3*x^4
 + a*b*c*x^2 + (b^2*c^5*x^6 + (c^2*x^2 + 1)*b^2*c^3*x^4 + 2*b^2*c^3*x^4 + b
^2*c*x^2 + 2*(b^2*c^4*x^5 + b^2*c^2*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(
c^2*x^2 + 1)) + 2*(a*b*c^4*x^5 + a*b*c^2*x^3)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2\,x^2+1)^{5/2}}{x\,(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

```
[In] int((c^2*x^2 + 1)^(5/2)/(x*(a + b*asinh(c*x))^2),x)
```

```
[Out] int((c^2*x^2 + 1)^(5/2)/(x*(a + b*asinh(c*x))^2), x)
```

3.432 $\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{(1+c^2x^2)^3}{bcx^2(a+b\text{arcsinh}(cx))} - \frac{2\text{Int}\left(\frac{(1+c^2x^2)^2}{x^3(a+b\text{arcsinh}(cx))},x\right)}{bc} + \frac{4c\text{Int}\left(\frac{(1+c^2x^2)^2}{x(a+b\text{arcsinh}(cx))},x\right)}{b}$$

```
[Out] -(c^2*x^2+1)^3/b/c/x^2/(a+b*arcsinh(c*x))-2*Unintegrable((c^2*x^2+1)^2/x^3/
(a+b*arcsinh(c*x)),x)/b/c+4*c*Unintegrable((c^2*x^2+1)^2/x/(a+b*arcsinh(c*x
)),x)/b
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -((1 + c^2*x^2)^3/(b*c*x^2*(a + b*ArcSinh[c*x]))) - (2*Defer[Int][(1 + c^2*
x^2)^2/(x^3*(a + b*ArcSinh[c*x])), x])/(b*c) + (4*c*Defer[Int][(1 + c^2*x^2
)^2/(x*(a + b*ArcSinh[c*x])), x])/b
```

Rubi steps

$$\text{integral} = -\frac{(1+c^2x^2)^3}{bcx^2(a+b\text{arcsinh}(cx))} - \frac{2\int \frac{(1+c^2x^2)^2}{x^3(a+b\text{arcsinh}(cx))}\,dx}{bc} + \frac{(4c)\int \frac{(1+c^2x^2)^2}{x(a+b\text{arcsinh}(cx))}\,dx}{b}$$

Mathematica [N/A]

Not integrable

Time = 3.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^2\left(a+b\,\text{arcsinh}\left(cx\right)\right)^2}dx$$

```
[In] int((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x))^2,x)
[Out] int((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2x^2}\,dx$$

```
[In] integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b^2*x^2*arcsinh(c*x)^
2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 8.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^2\,(a+b\,\text{asinh}\,(cx))^2}\,dx$$

```
[In] integrate((c**2*x**2+1)**(5/2)/x**2/(a+b*asinh(c*x))**2,x)
[Out] Integral((c**2*x**2 + 1)**(5/2)/(x**2*(a + b*asinh(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 491, normalized size of antiderivative = 18.19

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2x^2}\,dx$$

```
[In] integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^7*x^7 + 3*c^5*x^
5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^4 + sqrt(c^2*x^2 + 1)*a*
b*c^2*x^3 + a*b*c*x^2 + (b^2*c^3*x^4 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^3 + b^2*
c*x^2)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((4*c^7*x^7 + 5*c^5*x^5 -
2*c^3*x^3 - 3*c*x)*(c^2*x^2 + 1)^(3/2) + 2*(4*c^8*x^8 + 8*c^6*x^6 + 3*c^4*x
^4 - 2*c^2*x^2 - 1)*(c^2*x^2 + 1) + (4*c^9*x^9 + 11*c^7*x^7 + 9*c^5*x^5 + c
^3*x^3 - c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^7 + (c^2*x^2 + 1)*a*b*c^3*x^5 +
 2*a*b*c^3*x^5 + a*b*c*x^3 + (b^2*c^5*x^7 + (c^2*x^2 + 1)*b^2*c^3*x^5 + 2*b
^2*c^3*x^5 + b^2*c*x^3 + 2*(b^2*c^4*x^6 + b^2*c^2*x^4)*sqrt(c^2*x^2 + 1))*l
og(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^6 + a*b*c^2*x^4)*sqrt(c^2*x^2 +
1)), x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2x^2}\,dx$$

```
[In] integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.90 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\left(c^2\,x^2+1\right)^{5/2}}{x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}\,dx$$

```
[In] int((c^2*x^2 + 1)^(5/2)/(x^2*(a + b*asinh(c*x))^2),x)
[Out] int((c^2*x^2 + 1)^(5/2)/(x^2*(a + b*asinh(c*x))^2), x)
```

3.433 $\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\operatorname{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\operatorname{arcsinh}(cx))^2}\,dx=\operatorname{Int}\left(\frac{(1+c^2x^2)^{5/2}}{x^3(a+b\operatorname{arcsinh}(cx))^2},x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\operatorname{arcsinh}(cx))^2}\,dx=\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\operatorname{arcsinh}(cx))^2}\,dx$$

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\operatorname{arcsinh}(cx))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 12.58 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^3\,(a+b\,\operatorname{arcsinh}\,(cx))^2}dx$$

```
[In] int((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x)
[Out] int((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2x^3}\,dx$$

```
[In] integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b^2*x^3*arcsinh(c*x)^
2 + 2*a*b*x^3*arcsinh(c*x) + a^2*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 7.79 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^3\,(a+b\,\text{asinh}\,(cx))^2}\,dx$$

```
[In] integrate((c**2*x**2+1)**(5/2)/x**3/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((c**2*x**2 + 1)**(5/2)/(x**3*(a + b*asinh(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 492, normalized size of antiderivative = 18.22

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2x^3}\,dx$$

```
[In] integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^7*x^7 + 3*c^5*x^
5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^5 + sqrt(c^2*x^2 + 1)*a*
b*c^2*x^4 + a*b*c*x^3 + (b^2*c^3*x^5 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^4 + b^2*
c*x^3)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((3*c^7*x^7 + 2*c^5*x^5 -
5*c^3*x^3 - 4*c*x)*(c^2*x^2 + 1)^(3/2) + 3*(2*c^8*x^8 + 3*c^6*x^6 - c^4*x^4
 - 3*c^2*x^2 - 1)*(c^2*x^2 + 1) + (3*c^9*x^9 + 7*c^7*x^7 + 3*c^5*x^5 - 3*c^
3*x^3 - 2*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^8 + (c^2*x^2 + 1)*a*b*c^3*x^6
+ 2*a*b*c^3*x^6 + a*b*c*x^4 + (b^2*c^5*x^8 + (c^2*x^2 + 1)*b^2*c^3*x^6 + 2*
b^2*c^3*x^6 + b^2*c*x^4 + 2*(b^2*c^4*x^7 + b^2*c^2*x^5)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^7 + a*b*c^2*x^5)*sqrt(c^2*x^2 +
 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.74 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\left(c^2\,x^2+1\right)^{5/2}}{x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}\,dx$$

```
[In] int((c^2*x^2 + 1)^(5/2)/(x^3*(a + b*asinh(c*x))^2),x)
[Out] int((c^2*x^2 + 1)^(5/2)/(x^3*(a + b*asinh(c*x))^2), x)
```

3.434 $\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\,\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\operatorname{arcsinh}(cx))^2}\,dx=\operatorname{Int}\left(\frac{(1+c^2x^2)^{5/2}}{x^4(a+b\operatorname{arcsinh}(cx))^2},x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\operatorname{arcsinh}(cx))^2}\,dx=\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\operatorname{arcsinh}(cx))^2}\,dx$$

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\operatorname{arcsinh}(cx))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.83 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.52 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^4\left(a+b\ \text{arcsinh}\left(cx\right)\right)^2}dx$$

```
[In] int((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x)
[Out] int((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2x^4}\,dx$$

```
[In] integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b^2*x^4*arcsinh(c*x)^
2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4), x)
```

Sympy [N/A]

Not integrable

Time = 11.45 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^4\left(a+b\,\text{asinh}\,(cx)\right)^2}\,dx$$

```
[In] integrate((c**2*x**2+1)**(5/2)/x**4/(a+b*asinh(c*x))**2,x)
[Out] Integral((c**2*x**2 + 1)**(5/2)/(x**4*(a + b*asinh(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 491, normalized size of antiderivative = 18.19

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{\left(b\,\text{arsinh}\,(cx)+a\right)^2x^4}\,dx$$

```
[In] integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^7*x^7 + 3*c^5*x^
5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^6 + sqrt(c^2*x^2 + 1)*a*
b*c^2*x^5 + a*b*c*x^4 + (b^2*c^3*x^6 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^5 + b^2*
c*x^4)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((2*c^7*x^7 - c^5*x^5 - 8*
c^3*x^3 - 5*c*x)*(c^2*x^2 + 1)^(3/2) + 2*(2*c^8*x^8 + c^6*x^6 - 6*c^4*x^4 -
 7*c^2*x^2 - 2)*(c^2*x^2 + 1) + (2*c^9*x^9 + 3*c^7*x^7 - 3*c^5*x^5 - 7*c^3*
x^3 - 3*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^9 + (c^2*x^2 + 1)*a*b*c^3*x^7 +
2*a*b*c^3*x^7 + a*b*c*x^5 + (b^2*c^5*x^9 + (c^2*x^2 + 1)*b^2*c^3*x^7 + 2*b^
2*c^3*x^7 + b^2*c*x^5 + 2*(b^2*c^4*x^8 + b^2*c^2*x^6)*sqrt(c^2*x^2 + 1))*lo
g(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^8 + a*b*c^2*x^6)*sqrt(c^2*x^2 + 1
)), x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{\left(b\,\text{arsinh}\,(cx)+a\right)^2x^4}\,dx$$

```
[In] integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)^2*x^4), x)
```

Mupad [N/A]

Not integrable

Time = 2.72 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\left(c^2\,x^2+1\right)^{5/2}}{x^4\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}\,dx$$

```
[In] int((c^2*x^2 + 1)^(5/2)/(x^4*(a + b*asinh(c*x))^2),x)
[Out] int((c^2*x^2 + 1)^(5/2)/(x^4*(a + b*asinh(c*x))^2), x)
```

3.435 $\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 204

$$\begin{aligned}\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx &= -\frac{x^5}{bc(a+b\text{arcsinh}(cx))} \\ &+ \frac{5\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^6} \\ &- \frac{15\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^6} \\ &+ \frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^6} \\ &- \frac{5\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^6} \\ &+ \frac{15\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^6} \\ &- \frac{5\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^6}\end{aligned}$$

```
[Out] -x^5/b/c/(a+b*arcsinh(c*x))+5/8*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b^2/c^6
-15/16*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^6+5/16*Chi(5*(a+b*arcs
inh(c*x))/b)*cosh(5*a/b)/b^2/c^6-5/8*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^
2/c^6+15/16*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^6-5/16*Shi(5*(a+b
*arcsinh(c*x))/b)*sinh(5*a/b)/b^2/c^6
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5818, 5780, 5556, 3384, 3379, 3382}

$$\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}\,dx = \frac{5\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{8b^2c^6} - \frac{15\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16b^2c^6} + \frac{5\cosh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16b^2c^6} - \frac{5\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{8b^2c^6} + \frac{15\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16b^2c^6} - \frac{5\sinh\left(\frac{5a}{b}\right)\operatorname{Shi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16b^2c^6} - \frac{x^5}{bc(a+b\operatorname{arcsinh}(cx))}$$

```
[In] Int[x^5/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^5/(b*c*(a + b*ArcSinh[c*x]))) + (5*Cosh[a/b]*CoshIntegral[(a + b*ArcSin
h[c*x])/b])/(8*b^2*c^6) - (15*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcSinh[
c*x]))/b])/(16*b^2*c^6) + (5*Cosh[(5*a)/b]*CoshIntegral[(5*(a + b*ArcSinh[c
*x]))/b])/(16*b^2*c^6) - (5*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])
/(8*b^2*c^6) + (15*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/
(16*b^2*c^6) - (5*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*x]))/b])/(
16*b^2*c^6)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^5}{bc(a+b\text{arcsinh}(cx))}+\frac{5\int\frac{x^4}{a+b\text{arcsinh}(cx)}\,dx}{bc}\\
&= -\frac{x^5}{bc(a+b\text{arcsinh}(cx))}+\frac{5\text{Subst}\left(\int\frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^6}\\
&= -\frac{x^5}{bc(a+b\text{arcsinh}(cx))}\\
&\quad+\frac{5\text{Subst}\left(\int\left(\frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16x}-\frac{3\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16x}+\frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{8x}\right)dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^6}
\end{aligned}
$$

$$= -\frac{x^5}{bc(a+b\text{arcsinh}(cx))} + \frac{5\text{Subst}\left(\int \frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^6} + \frac{5\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^6} - \frac{15\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^6}$$

$$= -\frac{x^5}{bc(a+b\text{arcsinh}(cx))} + \frac{\left(5\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^6} - \frac{\left(15\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^6} + \frac{\left(5\cosh\left(\frac{5a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{5x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^6} - \frac{\left(5\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^6} + \frac{\left(15\sinh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^6} - \frac{\left(5\sinh\left(\frac{5a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{5x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^6}$$

$$= -\frac{x^5}{bc(a+b\text{arcsinh}(cx))} + \frac{5\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^6} - \frac{15\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^6} + \frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^6} - \frac{5\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^6} + \frac{15\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^6} - \frac{5\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^6}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.77

$$\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}\,dx = -\frac{x^5}{bc(a+b\operatorname{arcsinh}(cx))} + \frac{5\left(2\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)-3\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(3\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\right)+\cosh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(5\left(\frac{a}{b}+\operatorname{arcsin}\right.\right.}{}$$

[In] Integrate[x^5/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]

[Out] -(x^5/(b*c*(a + b*ArcSinh[c*x]))) + (5*(2*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - 3*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] + Cosh[(5*a)/b]*CoshIntegral[5*(a/b + ArcSinh[c*x])] - 2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 3*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] - Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])]))/(16*b^2*c^6)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. 2(192) = 384.

Time = 0.27 (sec) , antiderivative size = 633, normalized size of antiderivative = 3.10

method	result
default	$-\frac{16c^5x^5-16c^4x^4\sqrt{c^2x^2+1}+20c^3x^3-12c^2x^2\sqrt{c^2x^2+1}+5cx-\sqrt{c^2x^2+1}}{32c^6b(a+b\operatorname{arcsinh}(cx))}-\frac{5\,e^{\frac{5a}{b}}\operatorname{Ei}_1\left(5\operatorname{arcsinh}(cx)+\frac{5a}{b}\right)}{32c^6b^2}+\frac{\frac{5c^3x^3}{8}-\frac{5c^2x^2\sqrt{c^2x^2+1}}{8}}{c^6b(a+b\operatorname{arcs}}$

[In] int(x^5/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/32*(16*c^5*x^5-16*c^4*x^4*(c^2*x^2+1)^(1/2)+20*c^3*x^3-12*c^2*x^2*(c^2*x^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))/c^6/b/(a+b*arcsinh(c*x))-5/32/c^6/b^2*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)+5/32*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^6/b/(a+b*arcsinh(c*x))+15/32/c^6/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-5/16*(-(c^2*x^2+1)^(1/2)+c*x)/c^6/b/(a+b*arcsinh(c*x))-5/16/c^6/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-5/16/c^6/b^2*(arcsinh(c*x)*Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*b+Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*a+b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))+5/32/c^6/b^2*(4*b*c^3*x^3+4*(c^2*x^2+1)^(1/2)*b*c^2*x^2+3*arcsinh(c*x)*Ei(1,-3*arcsinh(c*x)-3*a/b)*exp(-3*a/b)*b+3*Ei(1,-3*arcsinh(c*x)-3*a/b)*exp(-3*a/b)*a+3*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/32/c^6/b^2*(16*b*c^5*x^5+16*(c^2*x^2+1)^(1/2)*b*c^4*x^4+20*b*c^3*x^3+12*(c^2*x^2+1)^(1/2)*b*c^2*x^2+5*arcsinh(c*x)*Ei(1,-5*arcsinh(c*x)-5*a/b)*exp(-5*a/b)*b+5*Ei(1,-5*arcsinh(c*x)-5*a/b)*exp(-5*a/b)*a+5*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))

Fricas [F]

$$\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^5}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

[In] integrate(x^5/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(c^2*x^2 + 1)*x^5/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)

Sympy [F]

$$\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^5}{(a+b\,\text{asinh}\,(cx))^2\sqrt{c^2x^2+1}}\,dx$$

[In] integrate(x**5/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)

[Out] Integral(x**5/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)

Maxima [F]

$$\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^5}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

[In] integrate(x^5/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")

[Out] -(c^3*x^8 + c*x^6 + (c^2*x^7 + x^5)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c^2*x + ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + integrate((5*c^5*x^9 + 11*c^3*x^7 + 6*c*x^5 + (5*c^3*x^7 + 4*c*x^5)*(c^2*x^2 + 1) + 5*(2*c^4*x^8 + 3*c^2*x^6 + x^4)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(3/2)*a*b*c^3*x^2 + 2*(a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((c^2*x^2 + 1)^(3/2)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 + 2*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^5/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^5}{(a+b\,\text{asinh}\,(c\,x))^2\,\sqrt{c^2\,x^2+1}}\,dx$$

```
[In] int(x^5/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(x^5/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

3.436 $\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 141

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{x^4}{bc(a+b\text{arcsinh}(cx))} + \frac{\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^2c^5} - \frac{\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{2b^2c^5} - \frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c^5} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{2b^2c^5}$$

```
[Out] -x^4/b/c/(a+b*arcsinh(c*x))-cosh(2*a/b)*Shi(2*(a+b*arcsinh(c*x))/b)/b^2/c^5
+1/2*cosh(4*a/b)*Shi(4*(a+b*arcsinh(c*x))/b)/b^2/c^5+Chi(2*(a+b*arcsinh(c*x
))/b)*sinh(2*a/b)/b^2/c^5-1/2*Chi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b^2/c
^5
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5818, 5780, 5556, 3384, 3379, 3382}

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c^5} - \frac{\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{2b^2c^5} - \frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c^5} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{2b^2c^5} - \frac{x^4}{bc(a+b\text{arcsinh}(cx))}$$

```
[In] Int[x^4/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^4/(b*c*(a + b*ArcSinh[c*x]))) + (CoshIntegral[(2*(a + b*ArcSinh[c*x]))/
b]*Sinh[(2*a)/b])/(b^2*c^5) - (CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b]*Sin
h[(4*a)/b])/(2*b^2*c^5) - (Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x
]))/b])/(b^2*c^5) + (Cosh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b]
)/(2*b^2*c^5)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^4}{bc(a+b\operatorname{arcsinh}(cx))}+\frac{4\int \frac{x^3}{a+b\operatorname{arcsinh}(cx)}\,dx}{bc}\\
&= -\frac{x^4}{bc(a+b\operatorname{arcsinh}(cx))}-\frac{4\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{b^2c^5}\\
&= -\frac{x^4}{bc(a+b\operatorname{arcsinh}(cx))}-\frac{4\text{Subst}\left(\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{8x}-\frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{4x}\right)\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{b^2c^5}\\
&= -\frac{x^4}{bc(a+b\operatorname{arcsinh}(cx))}-\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{x}\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{2b^2c^5}\\
&\quad+\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{b^2c^5}
\end{aligned}
$$

$$= -\frac{x^4}{bc(a+b\text{arcsinh}(cx))} - \frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^5} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^2c^5} + \frac{\sinh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^5} - \frac{\sinh\left(\frac{4a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^2c^5}$$

$$= -\frac{x^4}{bc(a+b\text{arcsinh}(cx))} + \frac{\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^2c^5} - \frac{\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{2b^2c^5} - \frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c^5} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{2b^2c^5}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \frac{-\frac{2bc^4x^4}{a+b\text{arcsinh}(cx)} + 2\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right) - \text{Chi}\left(4\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{4a}{b}\right) - 2\cosh\left(\frac{2a}{b}\right)\text{S}}{2b^2c^5}$$

```
[In] Integrate[x^4/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] ((-2*b*c^4*x^4)/(a + b*ArcSinh[c*x]) + 2*CoshIntegral[2*(a/b + ArcSinh[c*x]
)]*Sinh[(2*a)/b] - CoshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/b] - 2*C
osh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] + Cosh[(4*a)/b]*SinhInteg
ral[4*(a/b + ArcSinh[c*x])])/(2*b^2*c^5)
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.72

method	result
default	$-\frac{4b\,c^4x^4+\mathrm{e}^{-\frac{4a}{b}}\,\mathrm{Ei}_1\left(-4\,\mathrm{arcsinh}(cx)-\frac{4a}{b}\right)b\,\mathrm{arcsinh}(cx)-\mathrm{e}^{\frac{4a}{b}}\,\mathrm{Ei}_1\left(4\,\mathrm{arcsinh}(cx)+\frac{4a}{b}\right)b\,\mathrm{arcsinh}(cx)+2\,\mathrm{e}^{\frac{2a}{b}}\,\mathrm{Ei}_1\left(2\,\mathrm{arcsinh}(cx)+\frac{2a}{b}\right)b}{}$

[In] int(x^4/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/4*(4*b*c^4*x^4+exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)*b*arcsinh(c*x)-exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)*b*arcsinh(c*x)+2*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)*b*arcsinh(c*x)-2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*b*arcsinh(c*x)+exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)*a-exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)*a+2*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)*a-2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*a)/c^5/b^2/(a+b*arcsinh(c*x))

Fricas [F]

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^4}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

[In] integrate(x^4/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(c^2*x^2 + 1)*x^4/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)

Sympy [F]

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^4}{(a+b\,\text{asinh}\,(cx))^2\,\sqrt{c^2x^2+1}}\,dx$$

[In] integrate(x**4/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)

[Out] Integral(x**4/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)

Maxima [F]

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^4}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^4/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^7 + c*x^5 + (c^2*x^6 + x^4)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c
^2*x + ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + i
ntegrate((4*c^5*x^8 + 9*c^3*x^6 + 5*c*x^4 + (4*c^3*x^6 + 3*c*x^4)*(c^2*x^2
+ 1) + 4*(2*c^4*x^7 + 3*c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(3
/2)*a*b*c^3*x^2 + 2*(a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((c^2*x^2 + 1
)^(3/2)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*
x^4 + 2*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)
) + (a*b*c^5*x^4 + 2*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^4}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^4/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^4}{(a+b\,\text{asinh}\,(c\,x))^2\,\sqrt{c^2\,x^2+1}}\,dx$$

```
[In] int(x^4/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(x^4/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

3.437 $\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 142

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{x^3}{bc(a+b\text{arcsinh}(cx))} - \frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^4} + \frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^4} + \frac{3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^4} - \frac{3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^4}$$

```
[Out] -x^3/b/c/(a+b*arcsinh(c*x))-3/4*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b^2/c^4
+3/4*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^4+3/4*Shi((a+b*arcsinh(c
*x))/b)*sinh(a/b)/b^2/c^4-3/4*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c
^4
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5818, 5780, 5556, 3384, 3379, 3382}

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^4} + \frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^4} + \frac{3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^4} - \frac{3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^4} - \frac{x^3}{bc(a+b\text{arcsinh}(cx))}$$

```
[In] Int[x^3/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^3/(b*c*(a + b*ArcSinh[c*x]))) - (3*Cosh[a/b]*CoshIntegral[(a + b*ArcSin
h[c*x])/b])/(4*b^2*c^4) + (3*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcSinh[c
*x]))/b])/(4*b^2*c^4) + (3*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/
(4*b^2*c^4) - (3*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(4
*b^2*c^4)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^3}{bc(a+b\text{arcsinh}(cx))}+\frac{3\int \frac{x^2}{a+b\text{arcsinh}(cx)}\,dx}{bc}\\
&= -\frac{x^3}{bc(a+b\text{arcsinh}(cx))}+\frac{3\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^4}\\
&= -\frac{x^3}{bc(a+b\text{arcsinh}(cx))}+\frac{3\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4x}-\frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{4x}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^4}\\
&= -\frac{x^3}{bc(a+b\text{arcsinh}(cx))}+\frac{3\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{4b^2c^4}\\
&\quad -\frac{3\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{4b^2c^4}
\end{aligned}
$$

$$= -\frac{x^3}{bc(a+b\text{arcsinh}(cx))} - \frac{\left(3\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{x}{b}\right)}{x}dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^4} + \frac{\left(3\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{3x}{b}\right)}{x}dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^4} + \frac{\left(3\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{x}{b}\right)}{x}dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^4} - \frac{\left(3\sinh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{3x}{b}\right)}{x}dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^4}$$

$$= -\frac{x^3}{bc(a+b\text{arcsinh}(cx))} - \frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^4} + \frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^4} + \frac{3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^4} - \frac{3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^4}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.80

$$\int\frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}dx = -\frac{x^3}{bc(a+b\text{arcsinh}(cx))} + \frac{3\left(-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)+\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx\right.\right.}{4b^2c^4}$$

```
[In] Integrate[x^3/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^3/(b*c*(a + b*ArcSinh[c*x]))) + (3*(-(Cosh[a/b]*CoshIntegral[a/b + ArcS
inh[c*x]]) + Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] + Sinh[a/b]
*SinhIntegral[a/b + ArcSinh[c*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + Arc
Sinh[c*x])]))/(4*b^2*c^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. 2(134) = 268.

Time = 0.26 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.56

method	result
default	$-\frac{4c^3x^3-4c^2x^2\sqrt{c^2x^2+1}+3cx-\sqrt{c^2x^2+1}}{8c^4b(a+b\,\mathrm{arcsinh}(cx))}-\frac{3\,\mathrm{e}^{\frac{3a}{b}}\,\mathrm{Ei}_1\left(3\,\mathrm{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^4b^2}+\frac{-\frac{3\sqrt{c^2x^2+1}}{8}+\frac{3cx}{8}}{c^4b(a+b\,\mathrm{arcsinh}(cx))}+\frac{3\,\mathrm{e}^{\frac{a}{b}}\,\mathrm{Ei}_1\left(\mathrm{arcsinh}(cx)+\frac{a}{b}\right.}{8c^4b^2}$

```
[In] int(x^3/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^4/b/
(a+b*arcsinh(c*x))-3/8/c^4/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+3/8*(-
(c^2*x^2+1)^(1/2)+c*x)/c^4/b/(a+b*arcsinh(c*x))+3/8/c^4/b^2*exp(a/b)*Ei(1,a
rcsinh(c*x)+a/b)+3/8/c^4/b^2*(arcsinh(c*x)*Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b
)*b+Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*a+b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arc
sinh(c*x))-1/8/c^4/b^2*(4*b*c^3*x^3+4*(c^2*x^2+1)^(1/2)*b*c^2*x^2+3*arcsinh
(c*x)*Ei(1,-3*arcsinh(c*x)-3*a/b)*exp(-3*a/b)*b+3*Ei(1,-3*arcsinh(c*x)-3*a/
b)*exp(-3*a/b)*a+3*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\mathrm{arcsinh}(cx))^2}\,dx=\int \frac{x^3}{\sqrt{c^2x^2+1}\,(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^3/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^3/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(c
*x)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\mathrm{arcsinh}(cx))^2}\,dx=\int \frac{x^3}{(a+b\,\mathrm{asinh}\,(cx))^2\,\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate(x**3/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^3}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^3/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^6 + c*x^4 + (c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c
^2*x + ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + i
ntegrate((3*c^5*x^7 + 7*c^3*x^5 + 4*c*x^3 + (3*c^3*x^5 + 2*c*x^3)*(c^2*x^2
+ 1) + 3*(2*c^4*x^6 + 3*c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(3
/2)*a*b*c^3*x^2 + 2*(a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((c^2*x^2 + 1
)^(3/2)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*
x^4 + 2*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)
) + (a*b*c^5*x^4 + 2*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^3}{(a+b\,\text{asinh}\,(c\,x))^2\,\sqrt{c^2\,x^2+1}}\,dx$$

```
[In] int(x^3/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(x^3/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

3.438 $\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\,\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 79

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{x^2}{bc(a+b\text{arcsinh}(cx))} - \frac{\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^2c^3} + \frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c^3}$$

```
[Out] -x^2/b/c/(a+b*arcsinh(c*x))+cosh(2*a/b)*Shi(2*(a+b*arcsinh(c*x))/b)/b^2/c^3
-Chi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b^2/c^3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5818, 5780, 5556, 12, 3384, 3379, 3382}

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c^3} + \frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c^3} - \frac{x^2}{bc(a+b\text{arcsinh}(cx))}$$

```
[In] Int[x^2/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^2/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[(2*(a + b*ArcSinh[c*x]))/
b]*Sinh[(2*a)/b])/(b^2*c^3) + (Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh
[c*x]))/b])/(b^2*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
```

```
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^2}{bc(a+b\text{arcsinh}(cx))}+\frac{2\int \frac{x}{a+b\text{arcsinh}(cx)}\,dx}{bc}\\
&= -\frac{x^2}{bc(a+b\text{arcsinh}(cx))}-\frac{2\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^3}\\
&= -\frac{x^2}{bc(a+b\text{arcsinh}(cx))}-\frac{2\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^3}\\
&= -\frac{x^2}{bc(a+b\text{arcsinh}(cx))}-\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^3}\\
&= -\frac{x^2}{bc(a+b\text{arcsinh}(cx))}+\frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^3}\\
&\quad -\frac{\sinh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^3}\\
&= -\frac{x^2}{bc(a+b\text{arcsinh}(cx))}-\frac{\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^2c^3}+\frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \frac{-\frac{bc^2x^2}{a+b\text{arcsinh}(cx)}-\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right)+\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)}{b^2c^3}$$

```
[In] Integrate[x^2/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] (-((b*c^2*x^2)/(a + b*ArcSinh[c*x])) - CoshIntegral[2*(a/b + ArcSinh[c*x])]
*Sinh[(2*a)/b] + Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])])/(b^2*c
^3)
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.71

method	result
default	$-\frac{2b\,c^{2}x^{2}+\mathrm{e}^{-\frac{2a}{b}}\,\mathrm{Ei}_{1}\left(-2\,\mathrm{arcsinh}(cx)-\frac{2a}{b}\right)b\,\mathrm{arcsinh}(cx)-\mathrm{e}^{\frac{2a}{b}}\,\mathrm{Ei}_{1}\left(2\,\mathrm{arcsinh}(cx)+\frac{2a}{b}\right)b\,\mathrm{arcsinh}(cx)+\mathrm{e}^{-\frac{2a}{b}}\,\mathrm{Ei}_{1}\left(-2\,\mathrm{arcsinh}(cx)-\frac{2a}{b}\right)a}{2c^{3}b^{2}(a+b\,\mathrm{arcsinh}(cx))}$

[In] int(x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/2*(2*b*c^2*x^2+exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*b*arcsinh(c*x)-exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)*b*arcsinh(c*x)+exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*a-exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)*a)/c^3/b^2/(a+b*arcsinh(c*x))

Fricas [F]

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\mathrm{arcsinh}(cx))^2}\,dx=\int \frac{x^2}{\sqrt{c^2x^2+1}(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

[In] integrate(x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(c^2*x^2 + 1)*x^2/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)

Sympy [F]

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\mathrm{arcsinh}(cx))^2}\,dx=\int \frac{x^2}{(a+b\,\mathrm{asinh}\,(cx))^2\sqrt{c^2x^2+1}}\,dx$$

[In] integrate(x**2/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)

[Out] Integral(x**2/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)

Maxima [F]

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\mathrm{arcsinh}(cx))^2}\,dx=\int \frac{x^2}{\sqrt{c^2x^2+1}(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

[In] integrate(x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")

```
[Out] -(c^3*x^5 + c*x^3 + (c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c
^2*x + ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + i
ntegrate((2*c^5*x^6 + 5*c^3*x^4 + 3*c*x^2 + (2*c^3*x^4 + c*x^2)*(c^2*x^2 +
1) + 2*(2*c^4*x^5 + 3*c^2*x^3 + x)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(3/2)*
a*b*c^3*x^2 + 2*(a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((c^2*x^2 + 1)^(3
/2)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4
+ 2*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) +
(a*b*c^5*x^4 + 2*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{(a+b\,\text{asinh}\,(c\,x))^2\,\sqrt{c^2\,x^2+1}}\,dx$$

```
[In] int(x^2/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(x^2/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

3.439 $\int \frac{x}{\sqrt{1+c^2x^2}(a+b\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 73

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{x}{bc(a+b\text{arcsinh}(cx))} + \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c^2} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c^2}$$

```
[Out] -x/b/c/(a+b*arcsinh(c*x))+Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b^2/c^2-Shi((
a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5818, 5774, 3384, 3379, 3382}

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c^2} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c^2} - \frac{x}{bc(a+b\text{arcsinh}(cx))}$$

```
[In] Int[x/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x/(b*c*(a + b*ArcSinh[c*x]))) + (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*
x])/b])/(b^2*c^2) - (Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b^2*c
^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x}{bc(a+b\text{arcsinh}(cx))}+\frac{\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx}{bc}\\ &= -\frac{x}{bc(a+b\text{arcsinh}(cx))}+\frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^2}\end{aligned}$$

$$= -\frac{x}{bc(a + b\text{arcsinh}(cx))} + \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a + b\text{arcsinh}(cx)\right)}{b^2c^2} - \frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a + b\text{arcsinh}(cx)\right)}{b^2c^2}$$

$$= -\frac{x}{bc(a + b\text{arcsinh}(cx))} + \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c^2} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c^2}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{x}{\sqrt{1 + c^2x^2}(a + b\text{arcsinh}(cx))^2}\,dx = \frac{-\frac{bcx}{a+b\text{arcsinh}(cx)} + \cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b} + \text{arcsinh}(cx)\right) - \sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b} + \text{arcsinh}(cx)\right)}{b^2c^2}$$

```
[In] Integrate[x/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] (-((b*c*x)/(a + b*ArcSinh[c*x])) + Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x
]] - Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(b^2*c^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(73) = 146.

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.07

method	result
default	$-\frac{-\sqrt{c^2x^2+1}+cx}{2c^2b(a+b\,\text{arcsinh}(cx))} - \frac{e^{\frac{a}{b}}\,\text{Ei}_1\left(\text{arcsinh}(cx)+\frac{a}{b}\right)}{2c^2b^2} - \frac{\text{arcsinh}(cx)\,\text{Ei}_1\left(-\text{arcsinh}(cx)-\frac{a}{b}\right)e^{-\frac{a}{b}}b+\text{Ei}_1\left(-\text{arcsinh}(cx)-\frac{a}{b}\right)e^{-\frac{a}{b}}a+bc}{2c^2b^2(a+b\,\text{arcsinh}(cx))}$

```
[In] int(x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-(c^2*x^2+1)^(1/2)+c*x)/c^2/b/(a+b*arcsinh(c*x))-1/2/c^2/b^2*exp(a/b)
*Ei(1,arcsinh(c*x)+a/b)-1/2/c^2/b^2*(arcsinh(c*x)*Ei(1,-arcsinh(c*x)-a/b)*e
xp(-a/b)*b+Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*a+b*c*x+(c^2*x^2+1)^(1/2)*b)/(
a+b*arcsinh(c*x))
```

Fricas [F]

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x}{\sqrt{c^2x^2+1}\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}\,dx$$

[In] integrate(x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(c^2*x^2 + 1)*x/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)

Sympy [F]

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\sqrt{c^2x^2+1}}\,dx$$

[In] integrate(x/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)

[Out] Integral(x/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)

Maxima [F]

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x}{\sqrt{c^2x^2+1}\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}\,dx$$

[In] integrate(x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")

[Out] -(c^3*x^4 + c*x^2 + (c^2*x^3 + x)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c^2*x + ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + integrate((c^5*x^5 + (c^2*x^2 + 1)*c^3*x^3 + 3*c^3*x^3 + 2*c*x + (2*c^4*x^4 + 3*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(3/2)*a*b*c^3*x^2 + 2*(a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((c^2*x^2 + 1)^(3/2)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 + 2*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)

Giac [F]

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x}{(a+b\,\text{asinh}\,(c\,x))^2\,\sqrt{c^2\,x^2+1}}\,dx$$

```
[In] int(x/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(x/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

3.440 $\int \frac{1}{\sqrt{1+c^2x^2}(a+b\,\mathrm{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 18

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}\,dx = -\frac{1}{bc(a+b\operatorname{arcsinh}(cx))}$$

```
[Out] -1/b/c/(a+b*arcsinh(c*x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {5783}

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}\,dx = -\frac{1}{bc(a+b\operatorname{arcsinh}(cx))}$$

```
[In] Int[1/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
[Out] -(1/(b*c*(a + b*ArcSinh[c*x])))
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = -\frac{1}{bc(a+b\operatorname{arcsinh}(cx))}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx=-\frac{1}{bc(a+b\text{arcsinh}(cx))}$$

```
[In] Integrate[1/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
[Out] -(1/(b*c*(a + b*ArcSinh[c*x])))
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$-\frac{1}{bc(a+b\,\text{arcsinh}(cx))}$	19
default	$-\frac{1}{bc(a+b\,\text{arcsinh}(cx))}$	19

```
[In] int(1/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/b/c/(a+b*arcsinh(c*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx=-\frac{1}{b^2c\log\left(cx+\sqrt{c^2x^2+1}\right)+abc}$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/(b^2*c*log(c*x + sqrt(c^2*x^2 + 1)) + a*b*c)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.

Time = 2.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \begin{cases} \frac{x}{a^2} & \text{for}\ b=0 \wedge c=0 \\ \frac{\operatorname{asinh}(cx)}{a^2c} & \text{for}\ b=0 \\ \frac{x}{a^2} & \text{for}\ c=0 \\ -\frac{1}{abc+b^2c\operatorname{asinh}(cx)} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
[Out] Piecewise((x/a**2, Eq(b, 0) & Eq(c, 0)), (asinh(c*x)/(a**2*c), Eq(b, 0)), (
x/a**2, Eq(c, 0)), (-1/(a*b*c + b**2*c*asinh(c*x)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{1}{(b\operatorname{arsinh}(cx)+a)bc}$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/((b*arcsinh(c*x) + a)*b*c)
```

Giac [F]

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\sqrt{c^2x^2+1}(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [B] (verification not implemented)

Time = 2.65 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{1}{c\,\text{asinh}\left(c\,x\right)\,b^2+a\,c\,b}$$

```
[In] int(1/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
[Out] -1/(b^2*c*asinh(c*x) + a*b*c)
```

3.441 $\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{1}{bcx(a+b\text{arcsinh}(cx))} - \frac{\text{Int}\left(\frac{1}{x^2(a+b\text{arcsinh}(cx))},x\right)}{bc}$$

```
[Out] -1/b/c/x/(a+b*arcsinh(c*x))-Unintegrable(1/x^2/(a+b*arcsinh(c*x)),x)/b/c
```

Rubi [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Int[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(1/(b*c*x*(a + b*ArcSinh[c*x]))) - Defer[Int][1/(x^2*(a + b*ArcSinh[c*x]))
, x]/(b*c)
```

Rubi steps

$$\text{integral} = -\frac{1}{bcx(a+b\text{arcsinh}(cx))} - \frac{\int \frac{1}{x^2(a+b\text{arcsinh}(cx))}\,dx}{bc}$$

Mathematica [N/A]

Not integrable

Time = 6.48 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\left(a+b\,\operatorname{arcsinh}\left(cx\right)\right)^{2}\sqrt{c^{2}x^{2}+1}}dx$$

```
[In] int(1/x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
[Out] int(1/x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.78

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\sqrt{c^2x^2+1}\left(b\,\text{arsinh}\left(cx\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^2*x^3 + a^2*x + (b^2*c^2*x^3 + b^2*x)*arc
sinh(c*x)^2 + 2*(a*b*c^2*x^3 + a*b*x)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.70 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{x\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate(1/x/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
[Out] Integral(1/(x*(a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 416, normalized size of antiderivative = 15.41

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\sqrt{c^2x^2+1}\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/((c^2*x^2 + 1)*a*b*c^2*x^2 + ((c^2*x
^2 + 1)*b^2*c^2*x^2 + (b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x +
sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)) - integrate
((c^5*x^5 + c^3*x^3 + (c^3*x^3 + 2*c*x)*(c^2*x^2 + 1) + (2*c^4*x^4 + 3*c^2*
x^2 + 1)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(3/2)*a*b*c^3*x^4 + 2*(a*b*c^4*x
^5 + a*b*c^2*x^3)*(c^2*x^2 + 1) + ((c^2*x^2 + 1)^(3/2)*b^2*c^3*x^4 + 2*(b^2
*c^4*x^5 + b^2*c^2*x^3)*(c^2*x^2 + 1) + (b^2*c^5*x^6 + 2*b^2*c^3*x^4 + b^2*
c*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^6 + 2*a
*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.81 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\sqrt{c^2\,x^2+1}}\,dx$$

```
[In] int(1/(x*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(1/(x*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

3.442 $\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{1}{bcx^2(a+b\text{arcsinh}(cx))} - \frac{2\text{Int}\left(\frac{1}{x^3(a+b\text{arcsinh}(cx))},x\right)}{bc}$$

```
[Out] -1/b/c/x^2/(a+b*arcsinh(c*x))-2*Unintegrable(1/x^3/(a+b*arcsinh(c*x)),x)/b/
c
```

Rubi [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Int[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(1/(b*c*x^2*(a + b*ArcSinh[c*x]))) - (2*Defer[Int][1/(x^3*(a + b*ArcSinh[c
*x])), x])/(b*c)
```

Rubi steps

$$\text{integral} = -\frac{1}{bcx^2(a+b\text{arcsinh}(cx))} - \frac{2\int \frac{1}{x^3(a+b\text{arcsinh}(cx))}\,dx}{bc}$$

Mathematica [N/A]

Not integrable

Time = 2.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2\left(a+b\,\operatorname{arcsinh}\left(cx\right)\right)^2\sqrt{c^2x^2+1}}dx$$

```
[In] int(1/x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
[Out] int(1/x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.00

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\sqrt{c^2x^2+1}\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas
")
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^2*x^4 + a^2*x^2 + (b^2*c^2*x^4 + b^2*x^2)
*arcsinh(c*x)^2 + 2*(a*b*c^2*x^4 + a*b*x^2)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{x^2\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate(1/x**2/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
[Out] Integral(1/(x**2*(a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 427, normalized size of antiderivative = 15.81

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\sqrt{c^2x^2+1}(b\operatorname{arsinh}\left(cx\right)+a)^2x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima
")
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/((c^2*x^2 + 1)*a*b*c^2*x^3 + ((c^2*x
^2 + 1)*b^2*c^2*x^3 + (b^2*c^3*x^4 + b^2*c*x^2)*sqrt(c^2*x^2 + 1))*log(c*x
+ sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)) - integ
rate((2*c^5*x^5 + 3*c^3*x^3 + (2*c^3*x^3 + 3*c*x)*(c^2*x^2 + 1) + c*x + 2*(
2*c^4*x^4 + 3*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(3/2)*a*b*c^3*
x^5 + 2*(a*b*c^4*x^6 + a*b*c^2*x^4)*(c^2*x^2 + 1) + ((c^2*x^2 + 1)^(3/2)*b^
2*c^3*x^5 + 2*(b^2*c^4*x^6 + b^2*c^2*x^4)*(c^2*x^2 + 1) + (b^2*c^5*x^7 + 2*
b^2*c^3*x^5 + b^2*c*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) +
(a*b*c^5*x^7 + 2*a*b*c^3*x^5 + a*b*c*x^3)*sqrt(c^2*x^2 + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\sqrt{c^2x^2+1}(b\operatorname{arsinh}\left(cx\right)+a)^2x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.72 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{x^2\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\sqrt{c^2\,x^2+1}}\,dx$$

```
[In] int(1/(x^2*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(1/(x^2*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

3.443 $\int \frac{x^3}{(1+c^2x^2)^{3/2}(a+b\textbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^3}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{x^3}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2},x\right)$$

```
[Out] Unintegrable(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^3}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Int[x^3/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][x^3/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^3}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 53.72 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{(1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))^2} \, dx = \int \frac{x^3}{(1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))^2} \, dx$$

```
[In] Integrate[x^3/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[x^3/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{(c^2x^2+1)^{\frac{3}{2}} (a+b \operatorname{arcsinh}(cx))^2} dx$$

```
[In] int(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
[Out] int(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.89

$$\int \frac{x^3}{(1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))^2} \, dx = \int \frac{x^3}{(c^2x^2+1)^{\frac{3}{2}} (b \operatorname{arsinh}(cx)+a)^2} \, dx$$

```
[In] integrate(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^3/(a^2*c^4*x^4 + 2*a^2*c^2*x^2 + (b^2*c^4*x^4
+ 2*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 + 2*a*b*c^2*x^
2 + a*b)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.83 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^3}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^3}{(a+b\,\text{asinh}\,(cx))^2\,(c^2x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**3/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(x**3/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 475, normalized size of antiderivative = 17.59

$$\int \frac{x^3}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^3}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -(c*x^4 + sqrt(c^2*x^2 + 1)*x^3)/((c^2*x^2 + 1)*a*b*c^2*x + ((c^2*x^2 + 1)*
b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + integrate((c^5*x^7 + 5*
c^3*x^5 + 4*c*x^3 + (c^3*x^5 + 2*c*x^3)*(c^2*x^2 + 1) + (2*c^4*x^6 + 7*c^2*
x^4 + 3*x^2)*sqrt(c^2*x^2 + 1))/((a*b*c^5*x^4 + a*b*c^3*x^2)*(c^2*x^2 + 1)^
(3/2) + 2*(a*b*c^6*x^5 + 2*a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((b^2*c
^5*x^4 + b^2*c^3*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^6*x^5 + 2*b^2*c^4*x^3
+ b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^7*x^6 + 3*b^2*c^5*x^4 + 3*b^2*c^3*x^2 +
 b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^7*x^6 + 3*
a*b*c^5*x^4 + 3*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.61 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^3}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{3/2}}\,dx$$

```
[In] int(x^3/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)),x)
[Out] int(x^3/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)), x)
```

3.444 $\int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{x^2}{bc(1+c^2x^2)(a+b\text{arcsinh}(cx))}+\frac{2\text{Int}\left(\frac{x}{(1+c^2x^2)^2(a+b\text{arcsinh}(cx))},x\right)}{bc}$$

```
[Out] -x^2/b/c/(c^2*x^2+1)/(a+b*arcsinh(c*x))+2*Unintegrable(x/(c^2*x^2+1)^2/(a+b
*arcsinh(c*x)),x)/b/c
```

Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Int[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^2/(b*c*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))) + (2*Defer[Int][x/((1 + c^2
*x^2)^2*(a + b*ArcSinh[c*x])), x])/(b*c)
```

Rubi steps

$$\text{integral} = -\frac{x^2}{bc(1+c^2x^2)(a+b\text{arcsinh}(cx))}+\frac{2\int \frac{x}{(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}\,dx}{bc}$$

Mathematica [N/A]

Not integrable

Time = 3.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{(c^2x^2+1)^{\frac{3}{2}}\,(a+b\,\text{arcsinh}\,(cx))^2}dx$$

```
[In] int(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
[Out] int(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.89

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^2/(a^2*c^4*x^4 + 2*a^2*c^2*x^2 + (b^2*c^4*x^4
+ 2*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 + 2*a*b*c^2*x^
2 + a*b)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.80 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{(a+b\,\text{asinh}\,(cx))^2\,(c^2x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(x**2/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 447, normalized size of antiderivative = 16.56

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -(c*x^3 + sqrt(c^2*x^2 + 1)*x^2)/((c^2*x^2 + 1)*a*b*c^2*x + ((c^2*x^2 + 1)*
b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + integrate((3*c^3*x^4 +
(c^2*x^2 + 1)*c*x^2 + 3*c*x^2 + 2*(2*c^2*x^3 + x)*sqrt(c^2*x^2 + 1))/((a*b*
c^5*x^4 + a*b*c^3*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*c^6*x^5 + 2*a*b*c^4*x^3
 + a*b*c^2*x)*(c^2*x^2 + 1) + ((b^2*c^5*x^4 + b^2*c^3*x^2)*(c^2*x^2 + 1)^(3
/2) + 2*(b^2*c^6*x^5 + 2*b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^7*
x^6 + 3*b^2*c^5*x^4 + 3*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + s
qrt(c^2*x^2 + 1)) + (a*b*c^7*x^6 + 3*a*b*c^5*x^4 + 3*a*b*c^3*x^2 + a*b*c)*s
qrt(c^2*x^2 + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(x^2/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))^2} \, dx = \int \frac{x^2}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{3/2}} \,d x$$

```
[In] int(x^2/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)),x)
[Out] int(x^2/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)), x)
```

3.445 $\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2} dx = \operatorname{Int}\left(\frac{x}{(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}, x\right)$$

```
[Out] Unintegrable(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2} dx = \int \frac{x}{(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2} dx$$

```
[In] Int[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{x}{(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2} dx$$

Mathematica [N/A]

Not integrable

Time = 54.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{x}{(c^2x^2+1)^{\frac{3}{2}}(a+b\,\text{arcsinh}\,(cx))^2}dx$$

```
[In] int(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
[Out] int(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 4.12

$$\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x/(a^2*c^4*x^4 + 2*a^2*c^2*x^2 + (b^2*c^4*x^4 +
2*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 + 2*a*b*c^2*x^2
+ a*b)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.84 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{x}{\left(1+c^2x^2\right)^{3/2}\left(a+b\text{arcsinh}(cx)\right)^2}\,dx=\int \frac{x}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\left(c^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(x/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 461, normalized size of antiderivative = 18.44

$$\int \frac{x}{\left(1+c^2x^2\right)^{3/2}\left(a+b\text{arcsinh}(cx)\right)^2}\,dx=\int \frac{x}{\left(c^2x^2+1\right)^{\frac{3}{2}}\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}\,dx$$

```
[In] integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -(c*x^2 + sqrt(c^2*x^2 + 1)*x)/((c^2*x^2 + 1)*a*b*c^2*x + ((c^2*x^2 + 1)*b^
2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 +
 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((c^5*x^5 + (c^2
*x^2 + 1)*c^3*x^3 - c^3*x^3 - 2*c*x + (2*c^4*x^4 - c^2*x^2 - 1)*sqrt(c^2*x^
2 + 1))/((a*b*c^5*x^4 + a*b*c^3*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*c^6*x^5 +
 2*a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((b^2*c^5*x^4 + b^2*c^3*x^2)*(c
^2*x^2 + 1)^(3/2) + 2*(b^2*c^6*x^5 + 2*b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 +
1) + (b^2*c^7*x^6 + 3*b^2*c^5*x^4 + 3*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1
))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^7*x^6 + 3*a*b*c^5*x^4 + 3*a*b*c^3*
x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\left(1+c^2x^2\right)^{3/2}\left(a+b\text{arcsinh}(cx)\right)^2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.64 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{3/2}} \,d x$$

```
[In] int(x/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)),x)
[Out] int(x/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)), x)
```

3.446 $\int \frac{1}{(1+c^2x^2)^{3/2}(a+b\textbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{1}{bc(1+c^2x^2)(a+b\text{arcsinh}(cx))} - \frac{2c\text{Int}\left(\frac{x}{(1+c^2x^2)^2(a+b\text{arcsinh}(cx))},x\right)}{b}$$

```
[Out] -1/b/c/(c^2*x^2+1)/(a+b*arcsinh(c*x))-2*c*Unintegrable(x/(c^2*x^2+1)^2/(a+b
*arcsinh(c*x)),x)/b
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Int[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(1/(b*c*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))) - (2*c*Defer[Int][x/((1 + c^2
*x^2)^2*(a + b*ArcSinh[c*x])), x])/b
```

Rubi steps

$$\text{integral} = -\frac{1}{bc(1+c^2x^2)(a+b\text{arcsinh}(cx))} - \frac{(2c)\int \frac{x}{(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}\,dx}{b}$$

Mathematica [N/A]

Not integrable

Time = 2.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(c^2x^2+1)^{\frac{3}{2}}\,(a+b\,\text{arcsinh}\,(cx))^2}dx$$

```
[In] int(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
[Out] int(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 4.25

$$\int \frac{1}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^4*x^4 + 2*a^2*c^2*x^2 + (b^2*c^4*x^4 + 2*
b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 + 2*a*b*c^2*x^2 +
a*b)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 2.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\operatorname{asinh}(cx))^2\,(c^2x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(1/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 446, normalized size of antiderivative = 18.58

$$\int \frac{1}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(c^2x^2+1)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

```
[In] integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -(c*x + sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c^2*x + ((c^2*x^2 + 1)*b^2*c^
2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))
 + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((2*c^4*x^4 + c^2*x^
2 + (2*c^2*x^2 + 1)*(c^2*x^2 + 1) + 2*(2*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1) -
 1)/((a*b*c^4*x^4 + a*b*c^2*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*c^5*x^5 + 2*a
*b*c^3*x^3 + a*b*c*x)*(c^2*x^2 + 1) + ((b^2*c^4*x^4 + b^2*c^2*x^2)*(c^2*x^2
 + 1)^(3/2) + 2*(b^2*c^5*x^5 + 2*b^2*c^3*x^3 + b^2*c*x)*(c^2*x^2 + 1) + (b^
2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*sqrt(c^2*x^2 + 1))*log(c*x
 + sqrt(c^2*x^2 + 1)) + (a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)
*sqrt(c^2*x^2 + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(c^2x^2+1)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

```
[In] integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{3/2}} \,d x$$

```
[In] int(1/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)),x)
[Out] int(1/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)), x)
```

3.447 $\int \frac{1}{x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx=\text{Int}\left(\frac{1}{x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2},x\right)$$

```
[Out] Unintegrable(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Int[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 35.98 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\left(c^2x^2+1\right)^{\frac{3}{2}}\left(a+b\,\operatorname{arcsinh}\left(cx\right)\right)^2}dx$$

```
[In] int(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
[Out] int(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.96

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{\left(c^2x^2+1\right)^{\frac{3}{2}}\left(b\,\operatorname{arsinh}\left(cx\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^4*x^5 + 2*a^2*c^2*x^3 + a^2*x + (b^2*c^4*
x^5 + 2*b^2*c^2*x^3 + b^2*x)*arcsinh(c*x)^2 + 2*(a*b*c^4*x^5 + 2*a*b*c^2*x^
3 + a*b*x)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 4.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{x\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\left(c^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(1/(x*(a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 477, normalized size of antiderivative = 17.67

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{\left(c^2x^2+1\right)^{\frac{3}{2}}\left(b\,\text{arsinh}\left(cx\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -(c*x + sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c^2*x^2 + ((c^2*x^2 + 1)*b^2*
c^2*x^2 + (b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
 + 1)) + (a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)) - integrate((3*c^5*x^5
+ 3*c^3*x^3 + (3*c^3*x^3 + 2*c*x)*(c^2*x^2 + 1) + (6*c^4*x^4 + 5*c^2*x^2 +
1)*sqrt(c^2*x^2 + 1))/((a*b*c^5*x^6 + a*b*c^3*x^4)*(c^2*x^2 + 1)^(3/2) + 2*
(a*b*c^6*x^7 + 2*a*b*c^4*x^5 + a*b*c^2*x^3)*(c^2*x^2 + 1) + ((b^2*c^5*x^6 +
 b^2*c^3*x^4)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^6*x^7 + 2*b^2*c^4*x^5 + b^2*c^
2*x^3)*(c^2*x^2 + 1) + (b^2*c^7*x^8 + 3*b^2*c^5*x^6 + 3*b^2*c^3*x^4 + b^2*c
*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^7*x^8 + 3*a*
b*c^5*x^6 + 3*a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\text{arcsinh}(cx)\right)^2}\,dx=\int \frac{1}{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{3/2}}\,dx$$

```
[In] int(1/(x*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)),x)
[Out] int(1/(x*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)), x)
```

3.448 $\int \frac{1}{x^2(1+c^2x^2)^{3/2}(a+b\,\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\operatorname{arcsinh}(cx)\right)^2}\,dx=\operatorname{Int}\left(\frac{1}{x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\operatorname{arcsinh}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\operatorname{arcsinh}(cx)\right)^2}\,dx=\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\operatorname{arcsinh}(cx)\right)^2}\,dx$$

```
[In] Int[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\operatorname{arcsinh}(cx)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 15.91 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))^2} dx = \int \frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))^2} dx$$

```
[In] Integrate[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2 (c^2x^2+1)^{\frac{3}{2}} (a+b \,\text{arcsinh}\,(cx))^2} dx$$

```
[In] int(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
[Out] int(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.19

$$\int \frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))^2} dx = \int \frac{1}{(c^2x^2+1)^{\frac{3}{2}} (b\,\text{arsinh}\,(cx)+a)^2 x^2} dx$$

```
[In] integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^4*x^6 + 2*a^2*c^2*x^4 + a^2*x^2 + (b^2*c^
4*x^6 + 2*b^2*c^2*x^4 + b^2*x^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*x^6 + 2*a*b*c^
2*x^4 + a*b*x^2)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 2.95 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))^2} dx = \int \frac{1}{x^2 (a+b\,\text{asinh}\,(cx))^2 (c^2x^2+1)^{\frac{3}{2}}} dx$$

```
[In] integrate(1/x**2/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(1/(x**2*(a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 485, normalized size of antiderivative = 17.96

$$\int \frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))^2} dx = \int \frac{1}{(c^2x^2+1)^{\frac{3}{2}} (b\,\text{arsinh}\,(cx)+a)^2 x^2} dx$$

```
[In] integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
[Out] -(c*x + sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c^2*x^3 + ((c^2*x^2 + 1)*b^2*
c^2*x^3 + (b^2*c^3*x^4 + b^2*c*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x
^2 + 1)) + (a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)) - integrate((4*c^5*
x^5 + 5*c^3*x^3 + (4*c^3*x^3 + 3*c*x)*(c^2*x^2 + 1) + c*x + 2*(4*c^4*x^4 +
4*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1))/((a*b*c^5*x^7 + a*b*c^3*x^5)*(c^2*x^2 + 1
)^(3/2) + 2*(a*b*c^6*x^8 + 2*a*b*c^4*x^6 + a*b*c^2*x^4)*(c^2*x^2 + 1) + ((b
^2*c^5*x^7 + b^2*c^3*x^5)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^6*x^8 + 2*b^2*c^4*
x^6 + b^2*c^2*x^4)*(c^2*x^2 + 1) + (b^2*c^7*x^9 + 3*b^2*c^5*x^7 + 3*b^2*c^3
*x^5 + b^2*c*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^
7*x^9 + 3*a*b*c^5*x^7 + 3*a*b*c^3*x^5 + a*b*c*x^3)*sqrt(c^2*x^2 + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))^2} dx = \int \frac{1}{(c^2x^2+1)^{\frac{3}{2}} (b\,\text{arsinh}\,(cx)+a)^2 x^2} dx$$

```
[In] integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.59 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))^2} \, dx = \int \frac{1}{x^2 \left(a+b\,\text{asinh}\left(c\,x\right)\right)^2 \left(c^2\,x^2+1\right)^{3/2}} \, dx$$

```
[In] int(1/(x^2*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)),x)
[Out] int(1/(x^2*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)), x)
```

3.449 $\int \frac{x^3}{(1+c^2x^2)^{5/2}(a+b\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^3}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx=\text{Int}\left(\frac{x^3}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2},x\right)$$

```
[Out] Unintegrable(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^3}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Int[x^3/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][x^3/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{x^3}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx$$

Mathematica [F(-1)]

Timed out.

$$\int \frac{x^3}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \$\text{Aborted}$$

```
[In] Integrate[x^3/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] $Aborted
```

Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{(c^2x^2+1)^{\frac{5}{2}}\,(a+b\,\text{arcsinh}\,(cx))^2}dx$$

```
[In] int(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 5.07

$$\int \frac{x^3}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^3}{(c^2x^2+1)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^3/(a^2*c^6*x^6 + 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2
 + (b^2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2
 + 2*(a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 3.91 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^3}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^3}{(a+b\,\text{asinh}\,(cx))^2\,(c^2x^2+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**3/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(x**3/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 584, normalized size of antiderivative = 21.63

$$\int \frac{x^3}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^3}{(c^2x^2+1)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -(c*x^4 + sqrt(c^2*x^2 + 1)*x^3)/((a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) +
 ((b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2 +
b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 + 2*a
*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((c^5*x^7 - 3*c^3*x^5 - 4
*c*x^3 + (c^3*x^5 - 2*c*x^3)*(c^2*x^2 + 1) + (2*c^4*x^6 - 5*c^2*x^4 - 3*x^2
)*sqrt(c^2*x^2 + 1))/((a*b*c^7*x^6 + 2*a*b*c^5*x^4 + a*b*c^3*x^2)*(c^2*x^2
+ 1)^(3/2) + 2*(a*b*c^8*x^7 + 3*a*b*c^6*x^5 + 3*a*b*c^4*x^3 + a*b*c^2*x)*(c
^2*x^2 + 1) + ((b^2*c^7*x^6 + 2*b^2*c^5*x^4 + b^2*c^3*x^2)*(c^2*x^2 + 1)^(3
/2) + 2*(b^2*c^8*x^7 + 3*b^2*c^6*x^5 + 3*b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2
+ 1) + (b^2*c^9*x^8 + 4*b^2*c^7*x^6 + 6*b^2*c^5*x^4 + 4*b^2*c^3*x^2 + b^2*c
)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x^8 + 4*a*b*c^
7*x^6 + 6*a*b*c^5*x^4 + 4*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^3}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{5/2}}\,dx$$

```
[In] int(x^3/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)),x)
[Out] int(x^3/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)), x)
```

3.450 $\int \frac{x^2}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^2}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx=\text{Int}\left(\frac{x^2}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2},x\right)$$

```
[Out] Unintegrable(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^2}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Int[x^2/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][x^2/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{x^2}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 10.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[x^2/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[x^2/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{(c^2x^2+1)^{\frac{5}{2}}(a+b\,\text{arcsinh}\,(cx))^2}dx$$

```
[In] int(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
[Out] int(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 137, normalized size of antiderivative = 5.07

$$\int \frac{x^2}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{(c^2x^2+1)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^2/(a^2*c^6*x^6 + 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2
 + (b^2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2
 + 2*(a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 3.74 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{(a+b\operatorname{asinh}{(cx)})^2\,(c^2x^2+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**2/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(x**2/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 585, normalized size of antiderivative = 21.67

$$\int \frac{x^2}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{(c^2x^2+1)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -(c*x^3 + sqrt(c^2*x^2 + 1)*x^2)/((a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) +
 ((b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2 +
b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 + 2*a
*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((2*c^5*x^6 - c^3*x^4 - 3
*c*x^2 + (2*c^3*x^4 - c*x^2)*(c^2*x^2 + 1) + 2*(2*c^4*x^5 - c^2*x^3 - x)*sq
rt(c^2*x^2 + 1))/((a*b*c^7*x^6 + 2*a*b*c^5*x^4 + a*b*c^3*x^2)*(c^2*x^2 + 1)
^(3/2) + 2*(a*b*c^8*x^7 + 3*a*b*c^6*x^5 + 3*a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x
^2 + 1) + ((b^2*c^7*x^6 + 2*b^2*c^5*x^4 + b^2*c^3*x^2)*(c^2*x^2 + 1)^(3/2)
+ 2*(b^2*c^8*x^7 + 3*b^2*c^6*x^5 + 3*b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1)
 + (b^2*c^9*x^8 + 4*b^2*c^7*x^6 + 6*b^2*c^5*x^4 + 4*b^2*c^3*x^2 + b^2*c)*sq
rt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x^8 + 4*a*b*c^7*x^
6 + 6*a*b*c^5*x^4 + 4*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{(c^2x^2+1)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(x^2/((c^2*x^2 + 1)^(5/2)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^2}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{5/2}} \,d x$$

```
[In] int(x^2/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)),x)
[Out] int(x^2/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)), x)
```

3.451 $\int \frac{x}{(1+c^2x^2)^{5/2}(a+b\,\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{x}{(1+c^2x^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))^2}\,dx = \operatorname{Int}\left(\frac{x}{(1+c^2x^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))^2},x\right)$$

```
[Out] Unintegrable(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{(1+c^2x^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{x}{(1+c^2x^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))^2}\,dx$$

```
[In] Int[x/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][x/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{x}{(1+c^2x^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))^2}\,dx$$

Mathematica [F(-1)]

Timed out.

$$\int \frac{x}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \$\text{Aborted}$$

```
[In] Integrate[x/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] $Aborted
```

Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{x}{(c^2x^2+1)^{\frac{5}{2}}\,(a+b\,\text{arcsinh}\,(cx))^2}dx$$

```
[In] int(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
[Out] int(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 5.40

$$\int \frac{x}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x}{(c^2x^2+1)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x/(a^2*c^6*x^6 + 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2 +
 (b^2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 +
 2*(a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 3.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{x}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x}{(a+b\,\text{asinh}\,(cx))^2\,(c^2x^2+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(x/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 568, normalized size of antiderivative = 22.72

$$\int \frac{x}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x}{(c^2x^2+1)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x^2 + sqrt(c^2*x^2 + 1)*x)/((a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + (
(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2 + b^
2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 + 2*a*b
*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((3*c^5*x^5 + 3*(c^2*x^2 +
1)*c^3*x^3 + c^3*x^3 - 2*c*x + (6*c^4*x^4 + c^2*x^2 - 1)*sqrt(c^2*x^2 + 1))
/((a*b*c^7*x^6 + 2*a*b*c^5*x^4 + a*b*c^3*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*
c^8*x^7 + 3*a*b*c^6*x^5 + 3*a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((b^2*
c^7*x^6 + 2*b^2*c^5*x^4 + b^2*c^3*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^8*x^7
 + 3*b^2*c^6*x^5 + 3*b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^9*x^8
+ 4*b^2*c^7*x^6 + 6*b^2*c^5*x^4 + 4*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))
*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x^8 + 4*a*b*c^7*x^6 + 6*a*b*c^5*x^
4 + 4*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.53 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x}{(a+b\,\text{asinh}\,(c\,x))^2\,(c^2\,x^2+1)^{5/2}}\,dx$$

```
[In] int(x/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)),x)
```

```
[Out] int(x/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)), x)
```

3.452 $\int \frac{1}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{1}{bc\,(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}-\frac{4c\text{Int}\left(\frac{x}{(1+c^2x^2)^3(a+b\text{arcsinh}(cx))},x\right)}{b}$$

```
[Out] -1/b/c/(c^2*x^2+1)^2/(a+b*arcsinh(c*x))-4*c*Unintegrable(x/(c^2*x^2+1)^3/(a
+b*arcsinh(c*x)),x)/b
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Int[1/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(1/(b*c*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))) - (4*c*Defer[Int][x/((1 + c
^2*x^2)^3*(a + b*ArcSinh[c*x])), x])/b
```

Rubi steps

$$\text{integral} = -\frac{1}{bc\,(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}-\frac{(4c)\int \frac{x}{(1+c^2x^2)^3(a+b\text{arcsinh}(cx))}\,dx}{b}$$

Mathematica [N/A]

Not integrable

Time = 4.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[1/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[1/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(c^2x^2+1)^{\frac{5}{2}}\,(a+b\,\text{arcsinh}\,(cx))^2}dx$$

```
[In] int(1/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
[Out] int(1/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 5.58

$$\int \frac{1}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{(c^2x^2+1)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(1/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^6*x^6 + 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2 + (
b^2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2
*(a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 3.83 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\operatorname{asinh}(cx))^2\,(c^2x^2+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(1/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 554, normalized size of antiderivative = 23.08

$$\int \frac{1}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(c^2x^2+1)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

```
[In] integrate(1/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -(c*x + sqrt(c^2*x^2 + 1))/((a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((b^2
*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2 + b^2*c)
*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 + 2*a*b*c^3
*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((4*c^4*x^4 + 3*c^2*x^2 + (4*c^
2*x^2 + 1)*(c^2*x^2 + 1) + 4*(2*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1) - 1)/((a*b
*c^6*x^6 + 2*a*b*c^4*x^4 + a*b*c^2*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*c^7*x^
7 + 3*a*b*c^5*x^5 + 3*a*b*c^3*x^3 + a*b*c*x)*(c^2*x^2 + 1) + ((b^2*c^6*x^6
+ 2*b^2*c^4*x^4 + b^2*c^2*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^7*x^7 + 3*b^2
*c^5*x^5 + 3*b^2*c^3*x^3 + b^2*c*x)*(c^2*x^2 + 1) + (b^2*c^8*x^8 + 4*b^2*c^
6*x^6 + 6*b^2*c^4*x^4 + 4*b^2*c^2*x^2 + b^2)*sqrt(c^2*x^2 + 1))*log(c*x + s
qrt(c^2*x^2 + 1)) + (a*b*c^8*x^8 + 4*a*b*c^6*x^6 + 6*a*b*c^4*x^4 + 4*a*b*c^
2*x^2 + a*b)*sqrt(c^2*x^2 + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(c^2x^2+1)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

```
[In] integrate(1/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(1/((c^2*x^2 + 1)^(5/2)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2\,\left(c^2\,x^2+1\right)^{5/2}}\,dx$$

```
[In] int(1/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)),x)
[Out] int(1/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)), x)
```

3.453 $\int \frac{1}{x(1+c^2x^2)^{5/2}(a+b\,\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{1}{x\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx=\text{Int}\left(\frac{1}{x\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2},x\right)$$

```
[Out] Unintegrable(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{x\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Int[1/(x*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][1/(x*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx$$

Mathematica [F(-1)]

Timed out.

$$\int \frac{1}{x\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx=\$\text{Aborted}$$

```
[In] Integrate[1/(x*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] $Aborted
```

Maple [N/A] (verified)

Not integrable

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\left(c^2x^2+1\right)^{\frac{5}{2}}\left(a+b\,\text{arcsinh}\,(cx)\right)^2}dx$$

```
[In] int(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 5.15

$$\int \frac{1}{x\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{\left(c^2x^2+1\right)^{\frac{5}{2}}\left(b\,\text{arsinh}\,(cx)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^6*x^7 + 3*a^2*c^4*x^5 + 3*a^2*c^2*x^3 + a
^2*x + (b^2*c^6*x^7 + 3*b^2*c^4*x^5 + 3*b^2*c^2*x^3 + b^2*x)*arcsinh(c*x)^2
 + 2*(a*b*c^6*x^7 + 3*a*b*c^4*x^5 + 3*a*b*c^2*x^3 + a*b*x)*arcsinh(c*x)), x
)
```

Sympy [N/A]

Not integrable

Time = 8.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{x(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{x(a+b\,\text{asinh}\,(cx))^2\,(c^2x^2+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(1/(x*(a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 584, normalized size of antiderivative = 21.63

$$\int \frac{1}{x(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(c^2x^2+1)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2x}\,dx$$

```
[In] integrate(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -(c*x + sqrt(c^2*x^2 + 1))/((a*b*c^4*x^4 + a*b*c^2*x^2)*(c^2*x^2 + 1) + ((b
^2*c^4*x^4 + b^2*c^2*x^2)*(c^2*x^2 + 1) + (b^2*c^5*x^5 + 2*b^2*c^3*x^3 + b^
2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^5 + 2*a
*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)) - integrate((5*c^5*x^5 + 5*c^3*x^3
 + (5*c^3*x^3 + 2*c*x)*(c^2*x^2 + 1) + (10*c^4*x^4 + 7*c^2*x^2 + 1)*sqrt(c^
2*x^2 + 1))/((a*b*c^7*x^8 + 2*a*b*c^5*x^6 + a*b*c^3*x^4)*(c^2*x^2 + 1)^(3/2
) + 2*(a*b*c^8*x^9 + 3*a*b*c^6*x^7 + 3*a*b*c^4*x^5 + a*b*c^2*x^3)*(c^2*x^2
+ 1) + ((b^2*c^7*x^8 + 2*b^2*c^5*x^6 + b^2*c^3*x^4)*(c^2*x^2 + 1)^(3/2) + 2
*(b^2*c^8*x^9 + 3*b^2*c^6*x^7 + 3*b^2*c^4*x^5 + b^2*c^2*x^3)*(c^2*x^2 + 1)
+ (b^2*c^9*x^10 + 4*b^2*c^7*x^8 + 6*b^2*c^5*x^6 + 4*b^2*c^3*x^4 + b^2*c*x^2
)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x^10 + 4*a*b*c
^7*x^8 + 6*a*b*c^5*x^6 + 4*a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+c^2x^2\right)^{5/2}\left(a+b\text{arcsinh}(cx)\right)^2}\,dx=\int \frac{1}{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{5/2}} \,d x$$

```
[In] int(1/(x*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)),x)
[Out] int(1/(x*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)), x)
```

3.454 $\int \frac{1}{x^2(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{1}{x^2 (1+c^2x^2)^{5/2} (a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{1}{x^2 (1+c^2x^2)^{5/2} (a+b\text{arcsinh}(cx))^2},x\right)$$

```
[Out] Unintegrable(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2 (1+c^2x^2)^{5/2} (a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{x^2 (1+c^2x^2)^{5/2} (a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Int[1/(x^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][1/(x^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2 (1+c^2x^2)^{5/2} (a+b\text{arcsinh}(cx))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 11.46 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2 (1 + c^2x^2)^{5/2} (a + b\text{arcsinh}(cx))^2} dx = \int \frac{1}{x^2 (1 + c^2x^2)^{5/2} (a + b\text{arcsinh}(cx))^2} dx$$

```
[In] Integrate[1/(x^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[1/(x^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.43 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2 (c^2x^2 + 1)^{\frac{5}{2}} (a + b \operatorname{arcsinh}(cx))^2} dx$$

```
[In] int(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
[Out] int(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 5.37

$$\int \frac{1}{x^2 (1 + c^2x^2)^{5/2} (a + b\text{arcsinh}(cx))^2} dx = \int \frac{1}{(c^2x^2 + 1)^{\frac{5}{2}} (b \operatorname{arsinh}(cx) + a)^2 x^2} dx$$

```
[In] integrate(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^6*x^8 + 3*a^2*c^4*x^6 + 3*a^2*c^2*x^4 + a
^2*x^2 + (b^2*c^6*x^8 + 3*b^2*c^4*x^6 + 3*b^2*c^2*x^4 + b^2*x^2)*arcsinh(c*
x)^2 + 2*(a*b*c^6*x^8 + 3*a*b*c^4*x^6 + 3*a*b*c^2*x^4 + a*b*x^2)*arcsinh(c*
x)), x)
```

Sympy [N/A]

Not integrable

Time = 6.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1+c^2x^2)^{5/2} (a+b\text{arcsinh}(cx))^2} \, dx = \int \frac{1}{x^2 (a+b\operatorname{asinh}(cx))^2 (c^2x^2+1)^{\frac{5}{2}}} \, dx$$

```
[In] integrate(1/x**2/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(1/(x**2*(a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 592, normalized size of antiderivative = 21.93

$$\int \frac{1}{x^2 (1+c^2x^2)^{5/2} (a+b\text{arcsinh}(cx))^2} \, dx = \int \frac{1}{(c^2x^2+1)^{\frac{5}{2}} (b\,\text{arsinh}(cx)+a)^2 x^2} \, dx$$

```
[In] integrate(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
[Out] -(c*x + sqrt(c^2*x^2 + 1))/((a*b*c^4*x^5 + a*b*c^2*x^3)*(c^2*x^2 + 1) + ((b
^2*c^4*x^5 + b^2*c^2*x^3)*(c^2*x^2 + 1) + (b^2*c^5*x^6 + 2*b^2*c^3*x^4 + b^
2*c*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^6 + 2
*a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)) - integrate((6*c^5*x^5 + 7*c^3
*x^3 + 3*(2*c^3*x^3 + c*x)*(c^2*x^2 + 1) + c*x + 2*(6*c^4*x^4 + 5*c^2*x^2 +
 1)*sqrt(c^2*x^2 + 1))/((a*b*c^7*x^9 + 2*a*b*c^5*x^7 + a*b*c^3*x^5)*(c^2*x^
2 + 1)^(3/2) + 2*(a*b*c^8*x^10 + 3*a*b*c^6*x^8 + 3*a*b*c^4*x^6 + a*b*c^2*x^
4)*(c^2*x^2 + 1) + ((b^2*c^7*x^9 + 2*b^2*c^5*x^7 + b^2*c^3*x^5)*(c^2*x^2 +
1)^(3/2) + 2*(b^2*c^8*x^10 + 3*b^2*c^6*x^8 + 3*b^2*c^4*x^6 + b^2*c^2*x^4)*(
c^2*x^2 + 1) + (b^2*c^9*x^11 + 4*b^2*c^7*x^9 + 6*b^2*c^5*x^7 + 4*b^2*c^3*x^
5 + b^2*c*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x
^11 + 4*a*b*c^7*x^9 + 6*a*b*c^5*x^7 + 4*a*b*c^3*x^5 + a*b*c*x^3)*sqrt(c^2*x
^2 + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{\left(c^2x^2+1\right)^{\frac{5}{2}}\left(b\,\text{arsinh}\left(cx\right)+a\right)^2x^2}\,dx$$

```
[In] integrate(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(1/((c^2*x^2 + 1)^(5/2)*(b*arcsinh(c*x) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{x^2\left(a+b\,\text{asinh}\left(c\,x\right)\right)^2\left(c^2\,x^2+1\right)^{5/2}}\,dx$$

```
[In] int(1/(x^2*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)),x)
[Out] int(1/(x^2*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)), x)
```

3.455 $\int \frac{x^m (1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^m (1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2} dx = \text{Int}\left(\frac{x^m (1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}, x\right)$$

```
[Out] Unintegrable(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m (1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2} dx = \int \frac{x^m (1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2} dx$$

```
[In] Int[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Defer[Int][(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{x^m (1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2} dx$$

Mathematica [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
[Out] Integrate[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.68 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^m\left(c^2x^2+1\right)^{\frac{5}{2}}}{\left(a+b\ \text{arcsinh}\left(cx\right)\right)^2}dx$$

```
[In] int(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
[Out] int(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}x^m}{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}\,dx$$

```
[In] integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)*x^m/(b^2*arcsinh(c*x)^
2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\text{Timed out}$$

```
[In] integrate(x**m*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 537, normalized size of antiderivative = 19.89

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}x^m}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1)*x^m + (c^7*x^7 + 3*c^
5*x^5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1)*x^m)/(a*b*c^3*x^2 + sqrt(c^2*x^2
 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*
c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate((((c^7*(m + 6)*x^7 + c^5*(3*m +
 11)*x^5 + c^3*(3*m + 4)*x^3 + c*(m - 1)*x)*(c^2*x^2 + 1)^(3/2)*x^m + (2*c^
8*(m + 6)*x^8 + c^6*(7*m + 30)*x^6 + 3*c^4*(3*m + 8)*x^4 + c^2*(5*m + 6)*x^
2 + m)*(c^2*x^2 + 1)*x^m + (c^9*(m + 6)*x^9 + c^7*(4*m + 19)*x^7 + 3*c^5*(2
*m + 7)*x^5 + c^3*(4*m + 9)*x^3 + c*(m + 1)*x)*sqrt(c^2*x^2 + 1)*x^m)/(a*b*
c^5*x^5 + (c^2*x^2 + 1)*a*b*c^3*x^3 + 2*a*b*c^3*x^3 + a*b*c*x + (b^2*c^5*x^
5 + (c^2*x^2 + 1)*b^2*c^3*x^3 + 2*b^2*c^3*x^3 + b^2*c*x + 2*(b^2*c^4*x^4 +
b^2*c^2*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x
^4 + a*b*c^2*x^2)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.74 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^m\,\left(c^2\,x^2+1\right)^{5/2}}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}\,dx$$

```
[In] int((x^m*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2,x)
[Out] int((x^m*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2, x)
```

3.456 $\int \frac{x^m(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\text{Int}\left(\frac{x^m(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2},x\right)$$

```
[Out] Unintegrable(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^m(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Int[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
[Out] Defer[Int][(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{x^m(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^m(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
[Out] Integrate[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.56 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^m(c^2x^2+1)^{\frac{3}{2}}}{(a+b\ \operatorname{arcsinh}(cx))^2}dx$$

```
[In] int(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
[Out] int(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(c^2x^2+1)^{\frac{3}{2}}x^m}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral((c^2*x^2 + 1)^(3/2)*x^m/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) +
 a^2), x)
```

Sympy [N/A]

Not integrable

Time = 34.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m(c^2x^2+1)^{\frac{3}{2}}}{(a+b\,\text{asinh}\,(cx))^2}\,dx$$

```
[In] integrate(x**m*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(x**m*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 480, normalized size of antiderivative = 17.78

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}x^m}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1)*x^m + (c^5*x^5 + 2*c^3*x^3 + c*x)
*sqrt(c^2*x^2 + 1)*x^m)/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c
+ (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^
2 + 1))) + integrate(((c^5*(m + 4)*x^5 + c^3*(2*m + 3)*x^3 + c*(m - 1)*x)*(
c^2*x^2 + 1)^(3/2)*x^m + (2*c^6*(m + 4)*x^6 + c^4*(5*m + 12)*x^4 + 4*c^2*(m
 + 1)*x^2 + m)*(c^2*x^2 + 1)*x^m + (c^7*(m + 4)*x^7 + 3*c^5*(m + 3)*x^5 + 3
*c^3*(m + 2)*x^3 + c*(m + 1)*x)*sqrt(c^2*x^2 + 1)*x^m)/(a*b*c^5*x^5 + (c^2*
x^2 + 1)*a*b*c^3*x^3 + 2*a*b*c^3*x^3 + a*b*c*x + (b^2*c^5*x^5 + (c^2*x^2 +
1)*b^2*c^3*x^3 + 2*b^2*c^3*x^3 + b^2*c*x + 2*(b^2*c^4*x^4 + b^2*c^2*x^2)*sq
rt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^4 + a*b*c^2*x^
2)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.82 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^m\,{\left(c^2\,x^2+1\right)}^{3/2}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

```
[In] int((x^m*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2,x)
[Out] int((x^m*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2, x)
```

3.457 $\int \frac{x^m\sqrt{1+c^2x^2}}{(a+b\textbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^m\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\text{Int}\left(\frac{x^m\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2},x\right)$$

```
[Out] Unintegrable(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^m\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Int[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
[Out] Defer[Int][(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{x^m\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^m\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
[Out] Integrate[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.55 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^m\sqrt{c^2x^2+1}}{(a+b\,\text{arcsinh}\,(cx))^2}dx$$

```
[In] int(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
[Out] int(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{x^m\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{c^2x^2+1}x^m}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^m/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a
^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^m\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^m\sqrt{c^2x^2+1}}{(a+b\operatorname{asinh}{(cx)})^2}\,dx$$

```
[In] integrate(x**m*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(x**m*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 424, normalized size of antiderivative = 15.70

$$\int \frac{x^m\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{c^2x^2+1}x^m}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

```
[In] integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -((c^2*x^2 + 1)^2*x^m + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1)*x^m)/(a*b*c^3*x^2
 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b
^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((c^3*(m + 2)*x
^3 + c*(m - 1)*x)*(c^2*x^2 + 1)^(3/2)*x^m + (2*c^4*(m + 2)*x^4 + c^2*(3*m +
 2)*x^2 + m)*(c^2*x^2 + 1)*x^m + (c^5*(m + 2)*x^5 + c^3*(2*m + 3)*x^3 + c*(
m + 1)*x)*sqrt(c^2*x^2 + 1)*x^m)/(a*b*c^5*x^5 + (c^2*x^2 + 1)*a*b*c^3*x^3 +
 2*a*b*c^3*x^3 + a*b*c*x + (b^2*c^5*x^5 + (c^2*x^2 + 1)*b^2*c^3*x^3 + 2*b^2
*c^3*x^3 + b^2*c*x + 2*(b^2*c^4*x^4 + b^2*c^2*x^2)*sqrt(c^2*x^2 + 1))*log(c
*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^4 + a*b*c^2*x^2)*sqrt(c^2*x^2 + 1)),
 x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^m\,\sqrt{c^2\,x^2+1}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

```
[In] int((x^m*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2,x)
[Out] int((x^m*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2, x)
```

3.458 $\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{x^m}{bc(a+b\text{arcsinh}(cx))}+\frac{m\text{Int}\left(\frac{x^{-1+m}}{a+b\text{arcsinh}(cx)},x\right)}{bc}$$

```
[Out] -x^m/b/c/(a+b*arcsinh(c*x))+m*Unintegrable(x^(-1+m)/(a+b*arcsinh(c*x)),x)/b
/c
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Int[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^m/(b*c*(a + b*ArcSinh[c*x]))) + (m*Defer[Int][x^(-1 + m)/(a + b*ArcSinh
[c*x]), x])/(b*c)
```

Rubi steps

$$\text{integral} = -\frac{x^m}{bc(a+b\text{arcsinh}(cx))}+\frac{m\int \frac{x^{-1+m}}{a+b\text{arcsinh}(cx)}\,dx}{bc}$$

Mathematica [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^{m}}{\sqrt{c^{2} x^{2}+1}\, \left(a +b \,\operatorname{arcsinh}\left(c x \right)\right)^{2}}d x$$

```
[In] int(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
[Out] int(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.70

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^{m}}{\sqrt{c^{2} x^{2}+1}{\left(b \operatorname{arsinh}\left(c x\right)+a\right)}^{2}}\,{d x}$$

```
[In] integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^m/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(c
*x)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 2.84 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m}{\left(a+b\,\text{asinh}\,(cx)\right)^2\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate(x**m/(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(x**m/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

Maxima [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 441, normalized size of antiderivative = 16.33

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m}{\sqrt{c^2x^2+1}\left(b\,\text{arsinh}\,(cx)+a\right)^2}\,dx$$

```
[In] integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -((c^2*x^2 + 1)^(3/2)*x^m + (c^3*x^3 + c*x)*x^m)/((c^2*x^2 + 1)*a*b*c^2*x +
 ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*
x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + integra
te(((c^3*m*x^3 + c*(m - 1)*x)*(c^2*x^2 + 1)*x^m + (2*c^4*m*x^4 + 3*c^2*m*x^
2 + m)*sqrt(c^2*x^2 + 1)*x^m + (c^5*m*x^5 + c^3*(2*m + 1)*x^3 + c*(m + 1)*x
)*x^m)/((c^2*x^2 + 1)^(3/2)*a*b*c^3*x^3 + 2*(a*b*c^4*x^4 + a*b*c^2*x^2)*(c^
2*x^2 + 1) + ((c^2*x^2 + 1)^(3/2)*b^2*c^3*x^3 + 2*(b^2*c^4*x^4 + b^2*c^2*x^
2)*(c^2*x^2 + 1) + (b^2*c^5*x^5 + 2*b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1
))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^5 + 2*a*b*c^3*x^3 + a*b*c*x)*s
qrt(c^2*x^2 + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m}{\sqrt{c^2x^2+1}\left(b\,\text{arsinh}\,(cx)+a\right)^2}\,dx$$

```
[In] integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(x^m/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2\,\sqrt{c^2\,x^2+1}}\,dx$$

```
[In] int(x^m/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(x^m/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

3.459 $\int \frac{x^m}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx=\text{Int}\left(\frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2},x\right)$$

```
[Out] Unintegrable(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Int[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^m}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^m}{(c^2x^2+1)^{\frac{3}{2}}(a+b\,\text{arcsinh}\,(cx))^2}dx$$

```
[In] int(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
[Out] int(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.89

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^m}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^m/(a^2*c^4*x^4 + 2*a^2*c^2*x^2 + (b^2*c^4*x^4
+ 2*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 + 2*a*b*c^2*x^
2 + a*b)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 11.89 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m}{(a+b\,\text{asinh}\,(cx))^2\,(c^2x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**m/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(x**m/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 506, normalized size of antiderivative = 18.74

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -(c*x*x^m + sqrt(c^2*x^2 + 1)*x^m)/((c^2*x^2 + 1)*a*b*c^2*x + ((c^2*x^2 + 1
)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x
^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + integrate(((c^3*(m -
2)*x^3 + c*(m - 1)*x)*(c^2*x^2 + 1)*x^m + (2*c^4*(m - 2)*x^4 + c^2*(3*m - 2
)*x^2 + m)*sqrt(c^2*x^2 + 1)*x^m + (c^5*(m - 2)*x^5 + c^3*(2*m - 1)*x^3 + c
*(m + 1)*x)*x^m)/((a*b*c^5*x^5 + a*b*c^3*x^3)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*
c^6*x^6 + 2*a*b*c^4*x^4 + a*b*c^2*x^2)*(c^2*x^2 + 1) + ((b^2*c^5*x^5 + b^2*
c^3*x^3)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^6*x^6 + 2*b^2*c^4*x^4 + b^2*c^2*x^2
)*(c^2*x^2 + 1) + (b^2*c^7*x^7 + 3*b^2*c^5*x^5 + 3*b^2*c^3*x^3 + b^2*c*x)*s
qrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^7*x^7 + 3*a*b*c^5*x
^5 + 3*a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(x^m/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.71 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^m}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{3/2}} \,d x$$

```
[In] int(x^m/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)),x)
[Out] int(x^m/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)), x)
```

3.460 $\int \frac{x^m}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^m}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{x^m}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2},x\right)$$

```
[Out] Unintegrable(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Int[x^m/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][x^m/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^m}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^m}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[x^m/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[x^m/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^m}{(c^2x^2+1)^{\frac{5}{2}}\,(a+b\,\text{arcsinh}\,(cx))^2}dx$$

```
[In] int(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
[Out] int(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 5.07

$$\int \frac{x^m}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^m}{(c^2x^2+1)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^m/(a^2*c^6*x^6 + 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2
 + (b^2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2
 + 2*(a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 82.64 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^m}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m}{(a+b\,\text{asinh}\,(cx))^2\,(c^2x^2+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**m/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(x**m/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 613, normalized size of antiderivative = 22.70

$$\int \frac{x^m}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m}{(c^2x^2+1)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -(c*x*x^m + sqrt(c^2*x^2 + 1)*x^m)/((a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1)
 + ((b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2
+ b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 + 2
*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + integrate(((c^3*(m - 4)*x^3 + c*
(m - 1)*x)*(c^2*x^2 + 1)*x^m + (2*c^4*(m - 4)*x^4 + c^2*(3*m - 4)*x^2 + m)*
sqrt(c^2*x^2 + 1)*x^m + (c^5*(m - 4)*x^5 + c^3*(2*m - 3)*x^3 + c*(m + 1)*x)
*x^m)/((a*b*c^7*x^7 + 2*a*b*c^5*x^5 + a*b*c^3*x^3)*(c^2*x^2 + 1)^(3/2) + 2*
(a*b*c^8*x^8 + 3*a*b*c^6*x^6 + 3*a*b*c^4*x^4 + a*b*c^2*x^2)*(c^2*x^2 + 1) +
 ((b^2*c^7*x^7 + 2*b^2*c^5*x^5 + b^2*c^3*x^3)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*
c^8*x^8 + 3*b^2*c^6*x^6 + 3*b^2*c^4*x^4 + b^2*c^2*x^2)*(c^2*x^2 + 1) + (b^2
*c^9*x^9 + 4*b^2*c^7*x^7 + 6*b^2*c^5*x^5 + 4*b^2*c^3*x^3 + b^2*c*x)*sqrt(c^
2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x^9 + 4*a*b*c^7*x^7 + 6
*a*b*c^5*x^5 + 4*a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1+c^2x^2)^{5/2} (a+b\text{arcsinh}(cx))^2} \, dx = \int \frac{x^m}{(c^2x^2+1)^{\frac{5}{2}} (b \operatorname{arsinh}(cx)+a)^2} \, dx$$

```
[In] integrate(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(x^m/((c^2*x^2 + 1)^(5/2)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1+c^2x^2)^{5/2} (a+b\text{arcsinh}(cx))^2} \, dx = \int \frac{x^m}{(a+b \operatorname{asinh}(c\,x))^2 (c^2\,x^2+1)^{5/2}} \, dx$$

```
[In] int(x^m/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)),x)
[Out] int(x^m/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)), x)
```

3.461 $\int \frac{1}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 13

$$\int \frac{1}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}\,dx = -\frac{1}{2a\operatorname{arcsinh}(ax)^2}$$

```
[Out] -1/2/a/arcsinh(a*x)^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {5783}

$$\int \frac{1}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}\,dx = -\frac{1}{2a\operatorname{arcsinh}(ax)^2}$$

```
[In] Int[1/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3),x]
```

```
[Out] -1/2*1/(a*ArcSinh[a*x]^2)
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = -\frac{1}{2a\operatorname{arcsinh}(ax)^2}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}\,dx = -\frac{1}{2a\text{arcsinh}(ax)^2}$$

```
[In] Integrate[1/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3),x]
[Out] -1/2*1/(a*ArcSinh[a*x]^2)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\frac{1}{2a\,\text{arcsinh}(ax)^2}$	12
default	$-\frac{1}{2a\,\text{arcsinh}(ax)^2}$	12

```
[In] int(1/arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2/a/arcsinh(a*x)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}\,dx = -\frac{1}{2\,a\log\left(ax+\sqrt{a^2x^2+1}\right)^2}$$

```
[In] integrate(1/arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/2/(a*log(a*x + sqrt(a^2*x^2 + 1))^2)
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}\,dx = -\frac{1}{2a\,\text{asinh}^2\,(ax)}$$

```
[In] integrate(1/asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
[Out] -1/(2*a*asinh(a*x)**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}\,dx = -\frac{1}{2\,a\,\text{arsinh}\,(ax)^2}$$

```
[In] integrate(1/arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/2/(a*arcsinh(a*x)^2)
```

Giac [F]

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}\,dx = \int \frac{1}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)^3}\,dx$$

```
[In] integrate(1/arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^3), x)
```

Mupad [B] (verification not implemented)

Time = 2.68 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}\,dx = -\frac{1}{2\,a\,\text{asinh}\,(a\,x)^2}$$

```
[In] int(1/(asinh(a*x)^3*(a^2*x^2 + 1)^(1/2)),x)
[Out] -1/(2*a*asinh(a*x)^2)
```

3.462 $\int \frac{x^3(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 254

$$\int \frac{x^3(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=-\frac{2dx^3(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} -\frac{3de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}+\frac{de^{\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\text{erf}\left(\frac{\sqrt{6}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4} -\frac{3de^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}+\frac{de^{-\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\text{erfi}\left(\frac{\sqrt{6}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}$$

```
[Out] -3/32*d*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi
^(1/2)/b^(3/2)/c^4-3/32*d*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^
(1/2)*Pi^(1/2)/b^(3/2)/c^4/exp(2*a/b)+1/32*d*exp(6*a/b)*erf(6^(1/2)*(a+b*ar
csinh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^4+1/32*d*erfi(6^(1/2)
*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^4/exp(6*a/b)-
2*d*x^3*(c^2*x^2+1)^(3/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used

$= \{5814, 5819, 5556, 3388, 2211, 2236, 2235\}$

$$\int \frac{x^3(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = -\frac{3\sqrt{\frac{\pi}{2}}de^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4} + \frac{\sqrt{\frac{3\pi}{2}}de^{\frac{6a}{b}}\text{erf}\left(\frac{\sqrt{6}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4} - \frac{3\sqrt{\frac{\pi}{2}}de^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4} + \frac{\sqrt{\frac{3\pi}{2}}de^{-\frac{6a}{b}}\text{erfi}\left(\frac{\sqrt{6}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4} - \frac{2dx^3(c^2x^2+1)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

```
[In] Int[(x^3*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d*x^3*(1 + c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (3*d*E^((2*
a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/
2)*c^4) + (d*E^((6*a)/b)*Sqrt[(3*Pi)/2]*Erf[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x
]])/Sqrt[b]])/(16*b^(3/2)*c^4) - (3*d*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*A
rcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^4*E^((2*a)/b)) + (d*Sqrt[(3*Pi)/2]*Er
fi[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^4*E^((6*a)/b)
)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2dx^3(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\operatorname{arcsinh}(cx)}}+\frac{(6d)\int \frac{x^2\sqrt{1+c^2x^2}}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx}{bc}+\frac{(12cd)\int \frac{x^4\sqrt{1+c^2x^2}}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx}{b}\\
&= -\frac{2dx^3(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\operatorname{arcsinh}(cx)}}+\frac{(6d)\operatorname{Subst}\left(\int \frac{\cosh^2\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{b^2c^4}\\
&\quad+\frac{(12d)\operatorname{Subst}\left(\int \frac{\cosh^2\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{b^2c^4}\\
&= -\frac{2dx^3(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\operatorname{arcsinh}(cx)}}+\frac{(6d)\operatorname{Subst}\left(\int \left(-\frac{1}{8\sqrt{x}}+\frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{8\sqrt{x}}\right)dx,x,a+b\operatorname{arcsinh}(cx)\right)}{b^2c^4}\\
&\quad+\frac{(12d)\operatorname{Subst}\left(\int \left(\frac{1}{16\sqrt{x}}+\frac{\cosh\left(\frac{6a}{b}-\frac{6x}{b}\right)}{32\sqrt{x}}-\frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{16\sqrt{x}}-\frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{32\sqrt{x}}\right)dx,x,a+b\operatorname{arcsinh}(cx)\right)}{b^2c^4}
\end{aligned}
$$

$$= -\frac{2dx^3(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{(3d)\text{Subst}\left(\int \frac{\cosh\left(\frac{6a}{b}-\frac{6x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^4} - \frac{(3d)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^4}$$

$$= -\frac{2dx^3(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{(3d)\text{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^4} - \frac{(3d)\text{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^4} + \frac{(3d)\text{Subst}\left(\int \frac{e^{-i\left(\frac{6ia}{b}-\frac{6ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^4} + \frac{(3d)\text{Subst}\left(\int \frac{e^{i\left(\frac{6ia}{b}-\frac{6ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^4}$$

$$= -\frac{2dx^3(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{(3d)\text{Subst}\left(\int e^{\frac{6a}{b}-\frac{6x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{8b^2c^4} - \frac{(3d)\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{8b^2c^4} - \frac{(3d)\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{8b^2c^4} + \frac{(3d)\text{Subst}\left(\int e^{-\frac{6a}{b}+\frac{6x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{8b^2c^4}$$

$$= -\frac{2dx^3(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{3de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4} + \frac{de^{\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\text{erf}\left(\frac{\sqrt{6}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4} - \frac{3de^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4} + \frac{de^{-\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\text{erfi}\left(\frac{\sqrt{6}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.91

$$\int \frac{x^3(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \frac{de^{-\frac{6a}{b}}\left(\sqrt{6}\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\Gamma\left(\frac{1}{2},-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)-3\sqrt{2}e^{\frac{4a}{b}}\sqrt{-\frac{a+b\text{arcsin}}{b}}\right.}{}$$

```
[In] Integrate[(x^3*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (d*(Sqrt[6]*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-6*(a + b*ArcSinh[c
*x]))/b] - 3*Sqrt[2]*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2,
 (-2*(a + b*ArcSinh[c*x]))/b] + 3*Sqrt[2]*E^((8*a)/b)*Sqrt[a/b + ArcSinh[c*
x]]*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/b] - Sqrt[6]*E^((12*a)/b)*Sqrt[a/b
+ ArcSinh[c*x]]*Gamma[1/2, (6*(a + b*ArcSinh[c*x]))/b] - 8*E^((6*a)/b)*Sinh
[2*ArcSinh[c*x]]^3))/(32*b*c^4*E^((6*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F]

$$\int \frac{x^3(c^2d\,x^2+d)}{(a+b\,\text{arcsinh}\,(cx))^{\frac{3}{2}}}dx$$

```
[In] int(x^3*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x^3*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^3(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = d\left(\int \frac{x^3}{a\sqrt{a+b\,\text{asinh}\,(cx)}+b\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx + \int \frac{c^2x^5}{a\sqrt{a+b\,\text{asinh}\,(cx)}+b\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx\right)$$

```
[In] integrate(x**3*(c**2*d*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
[Out] d*(Integral(x**3/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*asinh
(c*x)), x) + Integral(c**2*x**5/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*as
inh(c*x))*asinh(c*x)), x))
```

Maxima [F]

$$\int \frac{x^3(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{(c^2dx^2+d)x^3}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate((c^2*d*x^2 + d)*x^3/(b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^3*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{x^3\,\left(d\,c^2\,x^2+d\right)}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}}\,dx$$

```
[In] int((x^3*(d + c^2*d*x^2))/(a + b*asinh(c*x))^(3/2),x)
[Out] int((x^3*(d + c^2*d*x^2))/(a + b*asinh(c*x))^(3/2), x)
```

3.463 $\int \frac{x^2(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 335

$$\int \frac{x^2(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = -\frac{2dx^2(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3} - \frac{de^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3} - \frac{de^{\frac{5a}{b}}\sqrt{5\pi}\,\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3} - \frac{de^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3} + \frac{de^{-\frac{3a}{b}}\sqrt{3\pi}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3} + \frac{de^{-\frac{5a}{b}}\sqrt{5\pi}\,\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}$$

```
[Out] 1/8*d*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3-1
/8*d*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3/exp(a/b)-1
/16*d*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(
1/2)/b^(3/2)/c^3+1/16*d*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1
/2)*Pi^(1/2)/b^(3/2)/c^3/exp(3*a/b)-1/16*d*exp(5*a/b)*erf(5^(1/2)*(a+b*arcs
inh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c^3+1/16*d*erfi(5^(1/2)*(
a+b*arcsinh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c^3/exp(5*a/b)-2*
d*x^2*(c^2*x^2+1)^(3/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {5814, 5819, 5556, 3389, 2211, 2236, 2235}

$$\int \frac{x^2(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \frac{\sqrt{\pi}de^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3} - \frac{\sqrt{3\pi}de^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3} - \frac{\sqrt{5\pi}de^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3} - \frac{\sqrt{\pi}de^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3} + \frac{\sqrt{3\pi}de^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3} + \frac{\sqrt{5\pi}de^{-\frac{5a}{b}}\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3} - \frac{2dx^2(c^2x^2+1)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

```
[In] Int[(x^2*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d*x^2*(1 + c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (d*E^(a/b)*
Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*b^(3/2)*c^3) - (d*E^((3*
a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/
2)*c^3) - (d*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/
Sqrt[b]])/(16*b^(3/2)*c^3) - (d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt
[b]])/(8*b^(3/2)*c^3*E^(a/b)) + (d*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcS
inh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^3*E^((3*a)/b)) + (d*Sqrt[5*Pi]*Erfi[(Sqr
t[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^3*E^((5*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2dx^2(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{(4d)\int\frac{x\sqrt{1+c^2x^2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{bc}+\frac{(10cd)\int\frac{x^3\sqrt{1+c^2x^2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{b}\\ &= -\frac{2dx^2(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}-\frac{(4d)\text{Subst}\left(\int\frac{\cosh^2\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^3}\\ &\quad-\frac{(10d)\text{Subst}\left(\int\frac{\cosh^2\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^3}\end{aligned}$$

$$= -\frac{2dx^2(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{(4d)\text{Subst}\left(\int \left(\frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}} + \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right) dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^3}$$
$$- \frac{(10d)\text{Subst}\left(\int \left(\frac{\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16\sqrt{x}} - \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16\sqrt{x}} - \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{8\sqrt{x}}\right) dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^3}$$

$$= -\frac{2dx^2(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{(5d)\text{Subst}\left(\int \frac{\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{\sqrt{x}} dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^3}$$
$$+ \frac{(5d)\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}} dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^3}$$
$$- \frac{d\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}} dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^3}$$
$$- \frac{d\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}} dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^3}$$
$$+ \frac{(5d)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}} dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^3}$$

$$= -\frac{2dx^2(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{(5d)\text{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^3}$$
$$- \frac{(5d)\text{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^3}$$
$$- \frac{(5d)\text{Subst}\left(\int \frac{e^{-i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^3}$$
$$+ \frac{(5d)\text{Subst}\left(\int \frac{e^{i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^3}$$
$$- \frac{d\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^2c^3}$$
$$+ \frac{d\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^2c^3}$$
$$- \frac{d\text{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^2c^3}$$
$$+ \frac{d\text{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^2c^3}$$
$$+ \frac{(5d)\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^3}$$
$$- \frac{(5d)\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^3}$$

$$= -\frac{2dx^2(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{(5d)\text{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{8b^2c^3}$$
$$+ \frac{(5d)\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{8b^2c^3}$$
$$- \frac{(5d)\text{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{8b^2c^3}$$
$$+ \frac{(5d)\text{Subst}\left(\int e^{-\frac{5a}{b}+\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{8b^2c^3}$$
$$- \frac{d\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c^3}$$
$$- \frac{d\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c^3}$$
$$+ \frac{d\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c^3}$$
$$+ \frac{d\text{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c^3}$$
$$+ \frac{(5d)\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{4b^2c^3}$$
$$- \frac{(5d)\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{4b^2c^3}$$

$$= -\frac{2dx^2(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{de^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3}$$
$$- \frac{de^{\frac{3a}{b}}\sqrt{3\pi}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3} - \frac{de^{\frac{5a}{b}}\sqrt{5\pi}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}$$
$$- \frac{de^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3} + \frac{de^{-\frac{3a}{b}}\sqrt{3\pi}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}$$
$$+ \frac{de^{-\frac{5a}{b}}\sqrt{5\pi}\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.30

$$\int \frac{x^2(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \frac{de^{-5\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}\left(-e^{\frac{5a}{b}}-e^{\frac{5a}{b}+2\text{arcsinh}(cx)}+2e^{\frac{5a}{b}+4\text{arcsinh}(cx)}+2e^{\frac{5a}{b}+6\text{arcsinh}(cx)}-\right.}{}$$

```
[In] Integrate[(x^2*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (d*(-E^((5*a)/b) - E^((5*a)/b + 2*ArcSinh[c*x]) + 2*E^((5*a)/b + 4*ArcSinh[
c*x]) + 2*E^((5*a)/b + 6*ArcSinh[c*x]) - E^((5*a)/b + 8*ArcSinh[c*x]) - E^(
(5*a)/b + 10*ArcSinh[c*x]) - 2*E^((6*a)/b + 5*ArcSinh[c*x])*Sqrt[a/b + ArcS
inh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + Sqrt[5]*E^(5*ArcSinh[c*x])*Sqrt[
-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-5*(a + b*ArcSinh[c*x]))/b] + Sqrt[3
]*E^((2*a)/b + 5*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (
-3*(a + b*ArcSinh[c*x]))/b] - 2*E^((4*a)/b + 5*ArcSinh[c*x])*Sqrt[-((a + b*
ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)] + Sqrt[3]*E^((8*a)/
b + 5*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c
*x]))/b] + Sqrt[5]*E^(5*((2*a)/b + ArcSinh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*
Gamma[1/2, (5*(a + b*ArcSinh[c*x]))/b]))/(16*b*c^3*E^(5*(a/b + ArcSinh[c*x]
))*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F]

$$\int \frac{x^2(c^2d\,x^2+d)}{(a+b\,\text{arcsinh}\,(cx))^{\frac{3}{2}}}dx$$

```
[In] int(x^2*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x^2*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = d\left(\int \frac{x^2}{a\sqrt{a+b\,\text{asinh}\,(cx)}+b\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx + \int \frac{c^2x^4}{a\sqrt{a+b\,\text{asinh}\,(cx)}+b\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx\right)$$

```
[In] integrate(x**2*(c**2*d*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
[Out] d*(Integral(x**2/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*asinh
(c*x)), x) + Integral(c**2*x**4/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*as
inh(c*x))*asinh(c*x)), x))
```

Maxima [F]

$$\int \frac{x^2(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{(c^2dx^2+d)x^2}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate((c^2*d*x^2 + d)*x^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{(c^2dx^2+d)x^2}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
[Out] integrate((c^2*d*x^2 + d)*x^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{x^2\,(d\,c^2\,x^2+d)}{(a+b\,\text{asinh}\,(c\,x))^{3/2}}\,dx$$

```
[In] int((x^2*(d + c^2*d*x^2))/(a + b*asinh(c*x))^(3/2),x)
[Out] int((x^2*(d + c^2*d*x^2))/(a + b*asinh(c*x))^(3/2), x)
```

3.464 $\int \frac{x(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 236

$$\int \frac{x(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = -\frac{2dx(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{de^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2} + \frac{de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c^2} + \frac{de^{-\frac{4a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2} + \frac{de^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c^2}$$

```
[Out] 1/4*d*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(
1/2)/b^(3/2)/c^2+1/4*d*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/
2)*Pi^(1/2)/b^(3/2)/c^2/exp(2*a/b)+1/4*d*exp(4*a/b)*erf(2*(a+b*arcsinh(c*x)
)^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^2+1/4*d*erfi(2*(a+b*arcsinh(c*x))^(1/2)
/b^(1/2))*Pi^(1/2)/b^(3/2)/c^2/exp(4*a/b)-2*d*x*(c^2*x^2+1)^(3/2)/b/c/(a+b*
arcsinh(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used

= {5814, 5791, 3393, 3388, 2211, 2236, 2235, 5819, 5556}

$$\int \frac{x(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \frac{\sqrt{\pi}de^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2} + \frac{\sqrt{\frac{\pi}{2}}de^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c^2} + \frac{\sqrt{\pi}de^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2} + \frac{\sqrt{\frac{\pi}{2}}de^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c^2} - \frac{2dx(c^2x^2+1)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

```
[In] Int[(x*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d*x*(1 + c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (d*E^((4*a)/b
)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^2) + (d*
E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(2*
b^(3/2)*c^2) + (d*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b
^(3/2)*c^2*E^((4*a)/b)) + (d*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*
x]])/Sqrt[b]])/(2*b^(3/2)*c^2*E^((2*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{2dx(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{(2d)\int \frac{\sqrt{1+c^2x^2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{bc} + \frac{(8cd)\int \frac{x^2\sqrt{1+c^2x^2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{b}$$

$$= -\frac{2dx(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{(2d)\text{Subst}\left(\int \frac{\cosh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^2} + \frac{(8d)\text{Subst}\left(\int \frac{\cosh^2\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^2}$$

$$= -\frac{2dx(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{(2d)\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}}+\frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^2} + \frac{(8d)\text{Subst}\left(\int \left(-\frac{1}{8\sqrt{x}}+\frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{8\sqrt{x}}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^2}$$

$$= -\frac{2dx(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{d\text{Subst}\left(\int \frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^2} + \frac{d\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^2}$$

$$= -\frac{2dx(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{d\text{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^2c^2} + \frac{d\text{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^2c^2} + \frac{d\text{Subst}\left(\int \frac{e^{-i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^2c^2} + \frac{d\text{Subst}\left(\int \frac{e^{i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^2c^2}$$

$$= -\frac{2dx(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{d\text{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c^2} + \frac{d\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c^2} + \frac{d\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c^2} + \frac{d\text{Subst}\left(\int e^{-\frac{4a}{b}+\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c^2}$$

$$= -\frac{2dx(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\operatorname{arcsinh}(cx)}} + \frac{de^{\frac{4a}{b}}\sqrt{\pi}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2} + \frac{de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c^2} + \frac{de^{-\frac{4a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2} + \frac{de^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c^2}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.96

$$\int \frac{x(d+c^2dx^2)}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \frac{de^{-\frac{4a}{b}}\left(\sqrt{-\frac{a+b\operatorname{arcsinh}(cx)}{b}}\Gamma\left(\frac{1}{2},-\frac{4(a+b\operatorname{arcsinh}(cx))}{b}\right)+\sqrt{2}e^{\frac{2a}{b}}\sqrt{-\frac{a+b\operatorname{arcsinh}(cx)}{b}}\Gamma\right.}{}$$

```
[In] Integrate[(x*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (d*(Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-4*(a + b*ArcSinh[c*x]))/b]
 + Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-2*(a +
b*ArcSinh[c*x]))/b] - E^((4*a)/b)*(Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c
*x]]*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/b] + E^((4*a)/b)*Sqrt[a/b + ArcSin
h[c*x]]*Gamma[1/2, (4*(a + b*ArcSinh[c*x]))/b] + 2*Sinh[2*ArcSinh[c*x]] + S
inh[4*ArcSinh[c*x]])))/(4*b*c^2*E^((4*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F]

$$\int \frac{x(c^2d\,x^2+d)}{(a+b\,\operatorname{arcsinh}(cx))^{\frac{3}{2}}}dx$$

```
[In] int(x*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = d\left(\int \frac{x}{a\sqrt{a+b\,\text{asinh}\,(cx)}+b\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx + \int \frac{c^2x^3}{a\sqrt{a+b\,\text{asinh}\,(cx)}+b\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx\right)$$

```
[In] integrate(x*(c**2*d*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d*(Integral(x/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*asinh(c*
x)), x) + Integral(c**2*x**3/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh
(c*x))*asinh(c*x)), x))
```

Maxima [F]

$$\int \frac{x(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{(c^2dx^2+d)x}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)*x/(b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(d + c^2dx^2)}{(a + b\text{arcsinh}(cx))^{3/2}} \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(d + c^2dx^2)}{(a + b\text{arcsinh}(cx))^{3/2}} \, dx = \int \frac{x\,(d\,c^2\,x^2 + d)}{(a + b\,\text{asinh}\,(c\,x))^{3/2}} \, dx$$

```
[In] int((x*(d + c^2*d*x^2))/(a + b*asinh(c*x))^(3/2),x)
```

```
[Out] int((x*(d + c^2*d*x^2))/(a + b*asinh(c*x))^(3/2), x)
```

3.465 $\int \frac{d+c^2dx^2}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 228

$$\int \frac{d+c^2dx^2}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = -\frac{2d(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\operatorname{arcsinh}(cx)}} - \frac{3de^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c} - \frac{de^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c} + \frac{3de^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c} + \frac{de^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}$$

```
[Out] -3/4*d*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c+3/
4*d*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c/exp(a/b)-1/4*
d*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)
/b^(3/2)/c+1/4*d*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^
(1/2)/b^(3/2)/c/exp(3*a/b)-2*d*(c^2*x^2+1)^(3/2)/b/c/(a+b*arcsinh(c*x))^(1/
2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used

$= \{5790, 5819, 5556, 3389, 2211, 2236, 2235\}$

$$\int \frac{d+c^2dx^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = -\frac{3\sqrt{\pi}de^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c} - \frac{\sqrt{3\pi}de^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c} + \frac{3\sqrt{\pi}de^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c} + \frac{\sqrt{3\pi}de^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c} - \frac{2d(c^2x^2+1)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

```
[In] Int[(d + c^2*d*x^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d*(1 + c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (3*d*E^(a/b)*Sq
rt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c) - (d*E^((3*a)/b
)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c)
 + (3*d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c*E^(a/
b)) + (d*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^
(3/2)*c*E^((3*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2d(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{(6cd)\int\frac{x\sqrt{1+c^2x^2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{b}\\
&= -\frac{2d(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}-\frac{(6d)\text{Subst}\left(\int\frac{\cosh^2\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c}\\
&= -\frac{2d(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}-\frac{(6d)\text{Subst}\left(\int\left(\frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}}+\frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c}\\
&= -\frac{2d(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}-\frac{(3d)\text{Subst}\left(\int\frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{2b^2c}\\
&\quad-\frac{(3d)\text{Subst}\left(\int\frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{2b^2c}
\end{aligned}$$

$$= -\frac{2d(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{(3d)\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c} + \frac{(3d)\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c} - \frac{(3d)\text{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c} + \frac{(3d)\text{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c}$$

$$= -\frac{2d(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{(3d)\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{2b^2c} - \frac{(3d)\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{2b^2c} + \frac{(3d)\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{2b^2c} + \frac{(3d)\text{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{2b^2c}$$

$$= -\frac{2d(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{3de^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c} - \frac{de^{\frac{3a}{b}}\sqrt{3\pi}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c} + \frac{3de^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c} + \frac{de^{-\frac{3a}{b}}\sqrt{3\pi}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.29

$$\int \frac{d+c^2dx^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \frac{de^{-3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}\left(-e^{\frac{3a}{b}} - 3e^{\frac{3a}{b}+2\text{arcsinh}(cx)} - 3e^{\frac{3a}{b}+4\text{arcsinh}(cx)} - e^{\frac{3a}{b}+6\text{arcsinh}(cx)} + \right.}{}$$

```
[In] Integrate[(d + c^2*d*x^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (d*(-E^((3*a)/b) - 3*E^((3*a)/b + 2*ArcSinh[c*x]) - 3*E^((3*a)/b + 4*ArcSin
h[c*x]) - E^((3*a)/b + 6*ArcSinh[c*x]) + 3*E^((4*a)/b + 3*ArcSinh[c*x])*Sqr
t[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + Sqrt[3]*E^(3*ArcSinh
[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))
/b] + 3*E^((2*a)/b + 3*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[
1/2, -((a + b*ArcSinh[c*x])/b)] + Sqrt[3]*E^((6*a)/b + 3*ArcSinh[c*x])*Sqrt
[a/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b]))/(4*b*c*E^(3*(
a/b + ArcSinh[c*x]))*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F]

$$\int \frac{c^2 d\,x^2 + d}{(a + b\,\operatorname{arcsinh}(cx))^{\frac{3}{2}}}dx$$

```
[In] int((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{d + c^2dx^2}{(a + b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{d + c^2dx^2}{(a + b\text{arcsinh}(cx))^{3/2}}\,dx = d\left(\int \frac{c^2x^2}{a\sqrt{a + b\,\text{asinh}\,(cx)} + b\sqrt{a + b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx + \int \frac{1}{a\sqrt{a + b\,\text{asinh}\,(cx)} + b\sqrt{a + b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx\right)$$

```
[In] integrate((c**2*d*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d*(Integral(c**2*x**2/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*
asinh(c*x)), x) + Integral(1/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh
(c*x))*asinh(c*x)), x))
```

Maxima [F]

$$\int \frac{d + c^2dx^2}{(a + b\text{arcsinh}(cx))^{3/2}} \, dx = \int \frac{c^2dx^2 + d}{(b \operatorname{arsinh}(cx) + a)^{\frac{3}{2}}} \, dx$$

```
[In] integrate((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate((c^2*d*x^2 + d)/(b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{d + c^2dx^2}{(a + b\text{arcsinh}(cx))^{3/2}} \, dx = \int \frac{c^2dx^2 + d}{(b \operatorname{arsinh}(cx) + a)^{\frac{3}{2}}} \, dx$$

```
[In] integrate((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
[Out] integrate((c^2*d*x^2 + d)/(b*arcsinh(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d + c^2dx^2}{(a + b\text{arcsinh}(cx))^{3/2}} \, dx = \int \frac{d\,c^2\,x^2 + d}{(a + b \operatorname{asinh}(c\,x))^{3/2}} \, dx$$

```
[In] int((d + c^2*d*x^2)/(a + b*asinh(c*x))^(3/2),x)
[Out] int((d + c^2*d*x^2)/(a + b*asinh(c*x))^(3/2), x)
```

3.466 $\int \frac{d+c^2dx^2}{x(a+b\,\text{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{d+c^2dx^2}{x(a+b\,\text{arcsinh}(cx))^{3/2}}\,dx = -\frac{2d(1+c^2x^2)^{3/2}}{bcx\sqrt{a+b\,\text{arcsinh}(cx)}} + \frac{de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\,\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}} + \frac{de^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\,\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}} - \frac{2d\text{Int}\left(\frac{1}{x^2\sqrt{1+c^2x^2}\sqrt{a+b\,\text{arcsinh}(cx)}},x\right)}{bc}$$

```
[Out] 1/2*d*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(
1/2)/b^(3/2)+1/2*d*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*P
i^(1/2)/b^(3/2)/exp(2*a/b)-2*d*(c^2*x^2+1)^(3/2)/b/c/x/(a+b*arcsinh(c*x))^(
1/2)-2*d*Unintegrable(1/x^2/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^(1/2),x)/b
/c
```

Rubi [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{d+c^2dx^2}{x(a+b\,\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{d+c^2dx^2}{x(a+b\,\text{arcsinh}(cx))^{3/2}}\,dx$$

```
[In] Int[(d + c^2*d*x^2)/(x*(a + b*ArcSinh[c*x])^(3/2)),x]
```

```
[Out] (-2*d*(1 + c^2*x^2)^(3/2))/(b*c*x*Sqrt[a + b*ArcSinh[c*x]]) + (d*E^((2*a)/b
)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/b^(3/2) + (d*
Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(b^(3/2)*E^((2
*a)/b)) - (2*d*Defer[Int][1/(x^2*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]]
), x])/(b*c)
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2d(1+c^2x^2)^{3/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}} - \frac{(2d)\int \frac{\sqrt{1+c^2x^2}}{x^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{bc} + \frac{(4cd)\int \frac{\sqrt{1+c^2x^2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{b} \\
&= -\frac{2d(1+c^2x^2)^{3/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}} + \frac{(4d)\text{Subst}\left(\int \frac{\cosh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2} \\
&\quad - \frac{(2d)\int \left(\frac{c^2}{\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}} + \frac{1}{x^2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}\right)dx}{bc} \\
&= -\frac{2d(1+c^2x^2)^{3/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}} + \frac{(4d)\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}} + \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\right)dx, x, a+b\text{arcsinh}(cx)\right)}{b^2} \\
&\quad - \frac{(2d)\int \frac{1}{x^2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{bc} - \frac{(2cd)\int \frac{1}{\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{b} \\
&= -\frac{2d(1+c^2x^2)^{3/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}} + \frac{(2d)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2} \\
&\quad - \frac{(2d)\int \frac{1}{x^2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{bc} \\
&= -\frac{2d(1+c^2x^2)^{3/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}} + \frac{d\text{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2} \\
&\quad + \frac{d\text{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2} - \frac{(2d)\int \frac{1}{x^2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{bc} \\
&= -\frac{2d(1+c^2x^2)^{3/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}} + \frac{(2d)\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2} \\
&\quad + \frac{(2d)\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2} - \frac{(2d)\int \frac{1}{x^2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{bc}
\end{aligned}$$

$$= -\frac{2d(1+c^2x^2)^{3/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}} + \frac{de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}} + \frac{de^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}} - \frac{(2d)\int \frac{1}{x^2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}dx}{bc}$$

Mathematica [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{d+c^2dx^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{d+c^2dx^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx$$

```
[In] Integrate[(d + c^2*d*x^2)/(x*(a + b*ArcSinh[c*x])^(3/2)),x]
[Out] Integrate[(d + c^2*d*x^2)/(x*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{c^2d\,x^2+d}{x\left(a+b\,\text{arcsinh}\left(cx\right)\right)^{\frac{3}{2}}}dx$$

```
[In] int((c^2*d*x^2+d)/x/(a+b*arcsinh(c*x))^(3/2),x)
[Out] int((c^2*d*x^2+d)/x/(a+b*arcsinh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{d+c^2dx^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 4.46 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.19

$$\int \frac{d+c^2dx^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx = d\left(\int \frac{c^2x^2}{ax\sqrt{a+b\,\text{asinh}\,(cx)}+bx\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx + \int \frac{1}{ax\sqrt{a+b\,\text{asinh}\,(cx)}+bx\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx\right)$$

```
[In] integrate((c**2*d*x**2+d)/x/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d*(Integral(c**2*x**2/(a*x*sqrt(a + b*asinh(c*x)) + b*x*sqrt(a + b*asinh(c*
x))*asinh(c*x)), x) + Integral(1/(a*x*sqrt(a + b*asinh(c*x)) + b*x*sqrt(a +
 b*asinh(c*x))*asinh(c*x)), x))
```

Maxima [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{d+c^2dx^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{c^2dx^2+d}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((c^2*d*x^2+d)/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)/((b*arcsinh(c*x) + a)^(3/2)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{d+c^2dx^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.85 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{d+c^2dx^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{d\,c^2\,x^2+d}{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

```
[In] int((d + c^2*d*x^2)/(x*(a + b*asinh(c*x))^(3/2)),x)
[Out] int((d + c^2*d*x^2)/(x*(a + b*asinh(c*x))^(3/2)), x)
```

3.467 $\int \frac{x^3(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 474

$$\int \frac{x^3(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = -\frac{2d^2x^3(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} - \frac{3d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} + \frac{d^2e^{\frac{8a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{2\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} + \frac{d^2e^{\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\,\text{erf}\left(\frac{\sqrt{6}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} - \frac{d^2e^{-\frac{4a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} - \frac{3d^2e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} + \frac{d^2e^{-\frac{8a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{2\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} + \frac{d^2e^{-\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{6}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}$$

```
[Out] -3/64*d^2*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*
Pi^(1/2)/b^(3/2)/c^4+1/64*d^2*exp(8*a/b)*erf(2*2^(1/2)*(a+b*arcsinh(c*x))^(
1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/c^4-3/64*d^2*erfi(2^(1/2)*(a+b*arcsi
nh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/c^4/exp(2*a/b)+1/64*d^2*er
fi(2*2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/c^4
/exp(8*a/b)-1/32*d^2*exp(4*a/b)*erf(2*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^
(1/2)/b^(3/2)/c^4-1/32*d^2*erfi(2*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2
)/b^(3/2)/c^4/exp(4*a/b)+1/64*d^2*exp(6*a/b)*erf(6^(1/2)*(a+b*arcsinh(c*x))
^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^4+1/64*d^2*erfi(6^(1/2)*(a+b*arc
sinh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^4/exp(6*a/b)-2*d^2*x^3
*(c^2*x^2+1)^(5/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5814, 5819, 5556, 3388, 2211, 2236, 2235}

$$\int \frac{x^3(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = -\frac{\sqrt{\pi}d^2e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} - \frac{3\sqrt{\frac{\pi}{2}}d^2e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} + \frac{\sqrt{\frac{\pi}{2}}d^2e^{\frac{8a}{b}}\text{erf}\left(\frac{2\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} + \frac{\sqrt{\frac{3\pi}{2}}d^2e^{\frac{6a}{b}}\text{erf}\left(\frac{\sqrt{6}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} - \frac{\sqrt{\pi}d^2e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} - \frac{3\sqrt{\frac{\pi}{2}}d^2e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} + \frac{\sqrt{\frac{\pi}{2}}d^2e^{-\frac{8a}{b}}\text{erfi}\left(\frac{2\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} + \frac{\sqrt{\frac{3\pi}{2}}d^2e^{-\frac{6a}{b}}\text{erfi}\left(\frac{\sqrt{6}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} - \frac{2d^2x^3(c^2x^2+1)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

```
[In] Int[(x^3*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d^2*x^3*(1 + c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (d^2*E^((
4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4
) - (3*d^2*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sq
rt[b]])/(32*b^(3/2)*c^4) + (d^2*E^((8*a)/b)*Sqrt[Pi/2]*Erf[(2*Sqrt[2]*Sqrt[
a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4) + (d^2*E^((6*a)/b)*Sqrt[(3*
Pi)/2]*Erf[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4) -
(d^2*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4*E
^((4*a)/b)) - (3*d^2*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqr
t[b]])/(32*b^(3/2)*c^4*E^((2*a)/b)) + (d^2*Sqrt[Pi/2]*Erfi[(2*Sqrt[2]*Sqrt[
a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4*E^((8*a)/b)) + (d^2*Sqrt[(3*
Pi)/2]*Erfi[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4*E^
((6*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{2d^2x^3(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{(6d^2)\int \frac{x^2(1+c^2x^2)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{bc} + \frac{(16cd^2)\int \frac{x^4(1+c^2x^2)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{b}$$

$$= -\frac{2d^2x^3(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{(6d^2)\,\text{Subst}\left(\int \frac{\cosh^4\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^4} + \frac{(16d^2)\,\text{Subst}\left(\int \frac{\cosh^4\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^4}$$

$$= -\frac{2d^2x^3(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{(6d^2)\,\text{Subst}\left(\int \left(-\frac{1}{16\sqrt{x}} + \frac{\cosh\left(\frac{6a}{b}-\frac{6x}{b}\right)}{32\sqrt{x}} + \frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{16\sqrt{x}} - \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{32\sqrt{x}}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^4} + \frac{(16d^2)\,\text{Subst}\left(\int \left(\frac{3}{128\sqrt{x}} + \frac{\cosh\left(\frac{8a}{b}-\frac{8x}{b}\right)}{128\sqrt{x}} - \frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{32\sqrt{x}}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^4}$$

$$= -\frac{2d^2x^3(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{d^2\text{Subst}\left(\int \frac{\cosh\left(\frac{8a}{b}-\frac{8x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^4} + \frac{(3d^2)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{6a}{b}-\frac{6x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^4} - \frac{(3d^2)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^4} + \frac{(3d^2)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^4} - \frac{d^2\text{Subst}\left(\int \frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^2c^4}$$

$$= -\frac{2d^2x^3(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{d^2\text{Subst}\left(\int \frac{e^{-i\left(\frac{8ia}{b}-\frac{8ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^4}$$

$$+ \frac{d^2\text{Subst}\left(\int \frac{e^{i\left(\frac{8ia}{b}-\frac{8ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^4}$$

$$- \frac{(3d^2)\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{32b^2c^4}$$

$$- \frac{(3d^2)\,\text{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{32b^2c^4}$$

$$+ \frac{(3d^2)\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{6ia}{b}-\frac{6ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{32b^2c^4}$$

$$+ \frac{(3d^2)\,\text{Subst}\left(\int \frac{e^{i\left(\frac{6ia}{b}-\frac{6ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{32b^2c^4}$$

$$+ \frac{(3d^2)\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^4}$$

$$+ \frac{(3d^2)\,\text{Subst}\left(\int \frac{e^{i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^4}$$

$$- \frac{d^2\text{Subst}\left(\int \frac{e^{-i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^4}$$

$$- \frac{d^2\text{Subst}\left(\int \frac{e^{i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^4}$$

$$= -\frac{2d^2x^3(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{d^2\text{Subst}\left(\int e^{\frac{8a}{b}-\frac{8x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{8b^2c^4}$$
$$+ \frac{d^2\text{Subst}\left(\int e^{-\frac{8a}{b}+\frac{8x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{8b^2c^4}$$
$$+ \frac{(3d^2)\,\text{Subst}\left(\int e^{\frac{6a}{b}-\frac{6x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{16b^2c^4}$$
$$- \frac{(3d^2)\,\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{16b^2c^4}$$
$$- \frac{(3d^2)\,\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{16b^2c^4}$$
$$+ \frac{(3d^2)\,\text{Subst}\left(\int e^{-\frac{6a}{b}+\frac{6x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{16b^2c^4}$$
$$+ \frac{(3d^2)\,\text{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{8b^2c^4}$$
$$+ \frac{(3d^2)\,\text{Subst}\left(\int e^{-\frac{4a}{b}+\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{8b^2c^4}$$
$$- \frac{d^2\text{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{2b^2c^4}$$
$$- \frac{d^2\text{Subst}\left(\int e^{-\frac{4a}{b}+\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{2b^2c^4}$$

$$= -\frac{2d^2x^3(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}$$
$$- \frac{3d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} + \frac{d^2e^{\frac{8a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{2\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}$$
$$+ \frac{d^2e^{\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\,\text{erf}\left(\frac{\sqrt{6}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} - \frac{d^2e^{-\frac{4a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}$$
$$- \frac{3d^2e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}$$
$$+ \frac{d^2e^{-\frac{8a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{2\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} + \frac{d^2e^{-\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{6}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 462, normalized size of antiderivative = 0.97

$$\int \frac{x^3(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \frac{d^2e^{-\frac{8a}{b}}\left(\sqrt{2}\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\Gamma\left(\frac{1}{2},-\frac{8(a+b\text{arcsinh}(cx))}{b}\right)+\sqrt{6}e^{\frac{2a}{b}}\sqrt{-\frac{a+b\text{arcsinh}}{b}}\right.}{}$$

```
[In] Integrate[(x^3*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (d^2*(Sqrt[2]*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-8*(a + b*ArcSinh
[c*x]))/b] + Sqrt[6]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2,
 (-6*(a + b*ArcSinh[c*x]))/b] - 2*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b
)]*Gamma[1/2, (-4*(a + b*ArcSinh[c*x]))/b] - 3*Sqrt[2]*E^((6*a)/b)*Sqrt[-((
a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-2*(a + b*ArcSinh[c*x]))/b] + 3*Sqrt[2]
*E^((10*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/
b] + 2*E^((12*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (4*(a + b*ArcSinh[c
*x]))/b] - Sqrt[6]*E^((14*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (6*(a +
 b*ArcSinh[c*x]))/b] - Sqrt[2]*E^((16*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[
1/2, (8*(a + b*ArcSinh[c*x]))/b] + 6*E^((8*a)/b)*Sinh[2*ArcSinh[c*x]] + 2*E
^((8*a)/b)*Sinh[4*ArcSinh[c*x]] - 2*E^((8*a)/b)*Sinh[6*ArcSinh[c*x]] - E^((
8*a)/b)*Sinh[8*ArcSinh[c*x]]))/(64*b*c^4*E^((8*a)/b)*Sqrt[a + b*ArcSinh[c*x
]])
```

Maple [F]

$$\int \frac{x^3(c^2d\,x^2+d)^2}{(a+b\,\text{arcsinh}\,(cx))^{\frac{3}{2}}}dx$$

```
[In] int(x^3*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x^3*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas
")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^3(d+c^2dx^2)^2}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = d^2\left(\int \frac{x^3}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,dx + \int \frac{2c^2x^5}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,dx + \int \frac{c^4x^7}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,dx\right)$$

```
[In] integrate(x**3*(c**2*d*x**2+d)**2/(a+b*asinh(c*x))**(3/2),x)
[Out] d**2*(Integral(x**3/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*as
inh(c*x)), x) + Integral(2*c**2*x**5/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a +
 b*asinh(c*x))*asinh(c*x)), x) + Integral(c**4*x**7/(a*sqrt(a + b*asinh(c*x
)) + b*sqrt(a + b*asinh(c*x))*asinh(c*x)), x))
```

Maxima [F]

$$\int \frac{x^3(d+c^2dx^2)^2}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \int \frac{(c^2dx^2+d)^2x^3}{(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima
")
[Out] integrate((c^2*d*x^2 + d)^2*x^3/(b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(d+c^2dx^2)^2}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^3*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{x^3\,{\left(d\,c^2\,x^2+d\right)}^2}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

```
[In] int((x^3*(d + c^2*d*x^2)^2)/(a + b*asinh(c*x))^(3/2),x)
[Out] int((x^3*(d + c^2*d*x^2)^2)/(a + b*asinh(c*x))^(3/2), x)
```

3.468 $\int \frac{x^2(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 457

$$\int \frac{x^2(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = -\frac{2d^2x^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{5d^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} - \frac{d^2e^{\frac{3a}{b}}\sqrt{3\pi}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} - \frac{3d^2e^{\frac{5a}{b}}\sqrt{5\pi}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} - \frac{d^2e^{\frac{7a}{b}}\sqrt{7\pi}\text{erf}\left(\frac{\sqrt{7}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} - \frac{5d^2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} + \frac{d^2e^{-\frac{3a}{b}}\sqrt{3\pi}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} + \frac{3d^2e^{-\frac{5a}{b}}\sqrt{5\pi}\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} + \frac{d^2e^{-\frac{7a}{b}}\sqrt{7\pi}\text{erfi}\left(\frac{\sqrt{7}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}$$

```
[Out] 5/64*d^2*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^
3-5/64*d^2*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3/exp(
a/b)-1/64*d^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1
/2)*Pi^(1/2)/b^(3/2)/c^3+1/64*d^2*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(
1/2))*3^(1/2)*Pi^(1/2)/b^(3/2)/c^3/exp(3*a/b)-3/64*d^2*exp(5*a/b)*erf(5^(1/
2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c^3+3/64*d^2*
erfi(5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c^3
/exp(5*a/b)-1/64*d^2*exp(7*a/b)*erf(7^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2
))*7^(1/2)*Pi^(1/2)/b^(3/2)/c^3+1/64*d^2*erfi(7^(1/2)*(a+b*arcsinh(c*x))^(1
/2)/b^(1/2))*7^(1/2)*Pi^(1/2)/b^(3/2)/c^3/exp(7*a/b)-2*d^2*x^2*(c^2*x^2+1)^
(5/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5814, 5819, 5556, 3389, 2211, 2236, 2235}

$$\int \frac{x^2(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \frac{5\sqrt{\pi}d^2e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} - \frac{\sqrt{3\pi}d^2e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} - \frac{3\sqrt{5\pi}d^2e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} - \frac{\sqrt{7\pi}d^2e^{\frac{7a}{b}}\text{erf}\left(\frac{\sqrt{7}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} - \frac{5\sqrt{\pi}d^2e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} + \frac{\sqrt{3\pi}d^2e^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} + \frac{3\sqrt{5\pi}d^2e^{-\frac{5a}{b}}\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} + \frac{\sqrt{7\pi}d^2e^{-\frac{7a}{b}}\text{erfi}\left(\frac{\sqrt{7}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} - \frac{2d^2x^2(c^2x^2+1)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

```
[In] Int[(x^2*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d^2*x^2*(1 + c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (5*d^2*E^
(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(64*b^(3/2)*c^3) - (d
^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/
(64*b^(3/2)*c^3) - (3*d^2*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a + b*Ar
cSinh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3) - (d^2*E^((7*a)/b)*Sqrt[7*Pi]*Erf[(
Sqrt[7]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3) - (5*d^2*Sqrt[
Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(64*b^(3/2)*c^3*E^(a/b)) + (d^2
*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c
^3*E^((3*a)/b)) + (3*d^2*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])
/Sqrt[b]])/(64*b^(3/2)*c^3*E^((5*a)/b)) + (d^2*Sqrt[7*Pi]*Erfi[(Sqrt[7]*Sqr
t[a + b*ArcSinh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3*E^((7*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{2d^2x^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{(4d^2)\int \frac{x(1+c^2x^2)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{bc} + \frac{(14cd^2)\int \frac{x^3(1+c^2x^2)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{b}$$

$$= -\frac{2d^2x^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{(4d^2)\,\text{Subst}\left(\int \frac{\cosh^4\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^3} - \frac{(14d^2)\,\text{Subst}\left(\int \frac{\cosh^4\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^3}$$

$$= -\frac{2d^2x^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{(4d^2)\,\text{Subst}\left(\int \left(\frac{\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16\sqrt{x}} + \frac{3\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16\sqrt{x}} + \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{8\sqrt{x}}\right)dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^3} - \frac{(14d^2)\,\text{Subst}\left(\int \left(\frac{\sinh\left(\frac{7a}{b}-\frac{7x}{b}\right)}{64\sqrt{x}} + \frac{\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{64\sqrt{x}} - \frac{3\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{64\sqrt{x}} - \frac{3\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{64\sqrt{x}}\right)dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^3}$$

$$= -\frac{2d^2x^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{(7d^2)\,\text{Subst}\left(\int \frac{\sinh\left(\frac{7a}{b}-\frac{7x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{32b^2c^3} - \frac{(7d^2)\,\text{Subst}\left(\int \frac{\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{32b^2c^3} - \frac{d^2\text{Subst}\left(\int \frac{\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^3} - \frac{d^2\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^2c^3} + \frac{(21d^2)\,\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{32b^2c^3} + \frac{(21d^2)\,\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{32b^2c^3} - \frac{(3d^2)\,\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^3}$$

$$= -\frac{2d^2x^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{(7d^2)\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^3}$$

$$+ \frac{(7d^2)\,\text{Subst}\left(\int \frac{e^{i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^3}$$

$$- \frac{(7d^2)\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{7ia}{b}-\frac{7ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^3}$$

$$+ \frac{(7d^2)\,\text{Subst}\left(\int \frac{e^{i\left(\frac{7ia}{b}-\frac{7ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^3}$$

$$- \frac{d^2\text{Subst}\left(\int \frac{e^{-i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^3}$$

$$+ \frac{d^2\text{Subst}\left(\int \frac{e^{i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^3}$$

$$- \frac{d^2\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^3}$$

$$+ \frac{d^2\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^3}$$

$$+ \frac{(21d^2)\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^3}$$

$$- \frac{(21d^2)\,\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^3}$$

$$+ \frac{(21d^2)\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^3}$$

$$- \frac{(21d^2)\,\text{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64b^2c^3}$$

$$- \frac{(3d^2)\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^3}$$

$$+ \frac{(3d^2)\,\text{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^3}$$

$$= -\frac{2d^2x^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\operatorname{arcsinh}(cx)}} - \frac{(7d^2)\operatorname{Subst}\left(\int e^{\frac{7a}{b}-\frac{7x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{32b^2c^3}$$

$$- \frac{(7d^2)\operatorname{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{32b^2c^3}$$

$$+ \frac{(7d^2)\operatorname{Subst}\left(\int e^{-\frac{5a}{b}+\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{32b^2c^3}$$

$$+ \frac{(7d^2)\operatorname{Subst}\left(\int e^{-\frac{7a}{b}+\frac{7x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{32b^2c^3}$$

$$- \frac{d^2\operatorname{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{4b^2c^3}$$

$$+ \frac{d^2\operatorname{Subst}\left(\int e^{-\frac{5a}{b}+\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{4b^2c^3}$$

$$- \frac{d^2\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{2b^2c^3}$$

$$+ \frac{d^2\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{2b^2c^3}$$

$$+ \frac{(21d^2)\operatorname{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{32b^2c^3}$$

$$+ \frac{(21d^2)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{32b^2c^3}$$

$$- \frac{(21d^2)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{32b^2c^3}$$

$$- \frac{(21d^2)\operatorname{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{32b^2c^3}$$

$$- \frac{(3d^2)\operatorname{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{4b^2c^3}$$

$$+ \frac{(3d^2)\operatorname{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{4b^2c^3}$$

$$= -\frac{2d^2x^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{5d^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}$$
$$- \frac{d^2e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} - \frac{3d^2e^{\frac{5a}{b}}\sqrt{5\pi}\,\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}$$
$$- \frac{d^2e^{\frac{7a}{b}}\sqrt{7\pi}\,\text{erf}\left(\frac{\sqrt{7}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} - \frac{5d^2e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}$$
$$+ \frac{d^2e^{-\frac{3a}{b}}\sqrt{3\pi}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} + \frac{3d^2e^{-\frac{5a}{b}}\sqrt{5\pi}\,\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}$$
$$+ \frac{d^2e^{-\frac{7a}{b}}\sqrt{7\pi}\,\text{erfi}\left(\frac{\sqrt{7}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}$$

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.26

$$\int \frac{x^2(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx =$$
$$-\frac{d^2e^{-7\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}\left(e^{\frac{7a}{b}} + 3e^{\frac{7a}{b}+2\text{arcsinh}(cx)} + e^{\frac{7a}{b}+4\text{arcsinh}(cx)} - 5e^{\frac{7a}{b}+6\text{arcsinh}(cx)} - 5e^{\frac{7a}{b}+8\text{arcsinh}(cx)} + e^{\frac{7a}{b}+10\text{arcs}}\right.}{}$$

```
[In] Integrate[(x^2*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] -1/64*(d^2*(E^((7*a)/b) + 3*E^((7*a)/b + 2*ArcSinh[c*x]) + E^((7*a)/b + 4*A
rcSinh[c*x]) - 5*E^((7*a)/b + 6*ArcSinh[c*x]) - 5*E^((7*a)/b + 8*ArcSinh[c*
x]) + E^((7*a)/b + 10*ArcSinh[c*x]) + 3*E^((7*a)/b + 12*ArcSinh[c*x]) + E^(
(7*a)/b + 14*ArcSinh[c*x]) + 5*E^((8*a)/b + 7*ArcSinh[c*x])*Sqrt[a/b + ArcS
inh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] - Sqrt[7]*E^(7*ArcSinh[c*x])*Sqrt[
-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-7*(a + b*ArcSinh[c*x]))/b] - 3*Sqrt
[5]*E^((2*a)/b + 7*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2,
 (-5*(a + b*ArcSinh[c*x]))/b] - Sqrt[3]*E^((4*a)/b + 7*ArcSinh[c*x])*Sqrt[-
((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] + 5*E^((6
*a)/b + 7*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b
*ArcSinh[c*x])/b)] - Sqrt[3]*E^((10*a)/b + 7*ArcSinh[c*x])*Sqrt[a/b + ArcSi
nh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b] - 3*Sqrt[5]*E^((12*a)/b + 7
*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (5*(a + b*ArcSinh[c*x]))
/b] - Sqrt[7]*E^(7*((2*a)/b + ArcSinh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*Gamma
[1/2, (7*(a + b*ArcSinh[c*x]))/b]))/(b*c^3*E^(7*(a/b + ArcSinh[c*x]))*Sqrt[
a + b*ArcSinh[c*x]])
```

Maple [F]

$$\int \frac{x^2 (c^2 d\, x^2 + d)^2}{(a + b \operatorname{arcsinh}(cx))^{\frac{3}{2}}} dx$$

```
[In] int(x^2*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x^2*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 (d + c^2 d x^2)^2}{(a + b\operatorname{arcsinh}(cx))^{3/2}} dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas
")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2 (d + c^2 d x^2)^2}{(a + b\operatorname{arcsinh}(cx))^{3/2}} dx = d^2 \left(\int \frac{x^2}{a\sqrt{a + b\operatorname{asinh}(cx)} + b\sqrt{a + b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)} dx + \int \frac{2c^2 x^4}{a\sqrt{a + b\operatorname{asinh}(cx)} + b\sqrt{a + b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)} dx + \int \frac{c^4 x^6}{a\sqrt{a + b\operatorname{asinh}(cx)} + b\sqrt{a + b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)} dx \right)$$

```
[In] integrate(x**2*(c**2*d*x**2+d)**2/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(x**2/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*as
inh(c*x)), x) + Integral(2*c**2*x**4/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a +
 b*asinh(c*x))*asinh(c*x)), x) + Integral(c**4*x**6/(a*sqrt(a + b*asinh(c*x
)) + b*sqrt(a + b*asinh(c*x))*asinh(c*x)), x))
```

Maxima [F]

$$\int \frac{x^2(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{(c^2dx^2+d)^2x^2}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

[In] integrate(x^2*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima
")

[Out] integrate((c^2*d*x^2 + d)^2*x^2/(b*arcsinh(c*x) + a)^(3/2), x)

Giac [F]

$$\int \frac{x^2(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{(c^2dx^2+d)^2x^2}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

[In] integrate(x^2*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")

[Out] integrate((c^2*d*x^2 + d)^2*x^2/(b*arcsinh(c*x) + a)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{x^2\,(d\,c^2\,x^2+d)^2}{(a+b\,\text{asinh}\,(c\,x))^{3/2}}\,dx$$

[In] int((x^2*(d + c^2*d*x^2)^2)/(a + b*asinh(c*x))^(3/2),x)

[Out] int((x^2*(d + c^2*d*x^2)^2)/(a + b*asinh(c*x))^(3/2), x)

3.469 $\int \frac{x(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 358

$$
\begin{aligned}
\int \frac{x(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}} \, dx &= -\frac{2d^2x(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} \\
&+ \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2} + \frac{5d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2} \\
&+ \frac{d^2e^{\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\,\text{erf}\left(\frac{\sqrt{6}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2} + \frac{d^2e^{-\frac{4a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2} \\
&+ \frac{5d^2e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2} + \frac{d^2e^{-\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{6}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}
\end{aligned}
$$

```
[Out] 5/32*d^2*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*P
i^(1/2)/b^(3/2)/c^2+5/32*d^2*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))
*2^(1/2)*Pi^(1/2)/b^(3/2)/c^2/exp(2*a/b)+1/4*d^2*exp(4*a/b)*erf(2*(a+b*arcs
inh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^2+1/4*d^2*erfi(2*(a+b*arcsinh(c
*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^2/exp(4*a/b)+1/32*d^2*exp(6*a/b)*erf
(6^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^2+1/3
2*d^2*erfi(6^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/
2)/c^2/exp(6*a/b)-2*d^2*x*(c^2*x^2+1)^(5/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5814, 5791, 3393, 3388, 2211, 2236, 2235, 5819, 5556}

$$\int \frac{x(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \frac{\sqrt{\pi}d^2e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2} + \frac{5\sqrt{\frac{\pi}{2}}d^2e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2} + \frac{\sqrt{\frac{3\pi}{2}}d^2e^{\frac{6a}{b}}\text{erf}\left(\frac{\sqrt{6}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2} + \frac{\sqrt{\pi}d^2e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2} + \frac{5\sqrt{\frac{\pi}{2}}d^2e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2} + \frac{\sqrt{\frac{3\pi}{2}}d^2e^{-\frac{6a}{b}}\text{erfi}\left(\frac{\sqrt{6}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2} - \frac{2d^2x(c^2x^2+1)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

```
[In] Int[(x*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d^2*x*(1 + c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (d^2*E^((4*
a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^2) +
 (5*d^2*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[
b]])/(16*b^(3/2)*c^2) + (d^2*E^((6*a)/b)*Sqrt[(3*Pi)/2]*Erf[(Sqrt[6]*Sqrt[a
 + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^2) + (d^2*Sqrt[Pi]*Erfi[(2*Sqrt
[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^2*E^((4*a)/b)) + (5*d^2*Sqrt[P
i/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^2*E^((
2*a)/b)) + (d^2*Sqrt[(3*Pi)/2]*Erfi[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt
[b]])/(16*b^(3/2)*c^2*E^((6*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
```

```
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{2d^2x(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{(2d^2)\int \frac{(1+c^2x^2)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{bc} + \frac{(12cd^2)\int \frac{x^2(1+c^2x^2)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{b}$$

$$= -\frac{2d^2x(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{(2d^2)\,\text{Subst}\left(\int \frac{\cosh^4\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^2} + \frac{(12d^2)\,\text{Subst}\left(\int \frac{\cosh^4\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^2}$$

$$= -\frac{2d^2x(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{(2d^2)\,\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}} + \frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{8\sqrt{x}} + \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\right)dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^2} + \frac{(12d^2)\,\text{Subst}\left(\int \left(-\frac{1}{16\sqrt{x}} + \frac{\cosh\left(\frac{6a}{b}-\frac{6x}{b}\right)}{32\sqrt{x}} + \frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{16\sqrt{x}} - \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{32\sqrt{x}}\right)dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^2}$$

$$= -\frac{2d^2x(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{d^2\text{Subst}\left(\int \frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^2} + \frac{(3d^2)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{6a}{b}-\frac{6x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^2} - \frac{(3d^2)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^2} + \frac{(3d^2)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^2} + \frac{d^2\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^2}$$

$$= -\frac{2d^2x(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{d^2\text{Subst}\left(\int \frac{e^{-i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^2}$$

$$+ \frac{d^2\text{Subst}\left(\int \frac{e^{i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^2}$$

$$- \frac{(3d^2)\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^2}$$

$$- \frac{(3d^2)\,\text{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^2}$$

$$+ \frac{(3d^2)\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{6ia}{b}-\frac{6ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^2}$$

$$+ \frac{(3d^2)\,\text{Subst}\left(\int \frac{e^{i\left(\frac{6ia}{b}-\frac{6ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c^2}$$

$$+ \frac{(3d^2)\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^2}$$

$$+ \frac{(3d^2)\,\text{Subst}\left(\int \frac{e^{i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c^2}$$

$$+ \frac{d^2\text{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^2c^2}$$

$$+ \frac{d^2\text{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^2c^2}$$

$$= -\frac{2d^2x(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{d^2\text{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{4b^2c^2}$$
$$+ \frac{d^2\text{Subst}\left(\int e^{-\frac{4a}{b}+\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{4b^2c^2}$$
$$+ \frac{(3d^2)\,\text{Subst}\left(\int e^{\frac{6a}{b}-\frac{6x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{8b^2c^2}$$
$$- \frac{(3d^2)\,\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{8b^2c^2}$$
$$- \frac{(3d^2)\,\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{8b^2c^2}$$
$$+ \frac{(3d^2)\,\text{Subst}\left(\int e^{-\frac{6a}{b}+\frac{6x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{8b^2c^2}$$
$$+ \frac{(3d^2)\,\text{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{4b^2c^2}$$
$$+ \frac{(3d^2)\,\text{Subst}\left(\int e^{-\frac{4a}{b}+\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{4b^2c^2}$$
$$+ \frac{d^2\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c^2}$$
$$+ \frac{d^2\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c^2}$$

$$= -\frac{2d^2x(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}$$
$$+ \frac{5d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2} + \frac{d^2e^{\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\,\text{erf}\left(\frac{\sqrt{6}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}$$
$$+ \frac{d^2e^{-\frac{4a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2} + \frac{5d^2e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}$$
$$+ \frac{d^2e^{-\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{6}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.98

$$\int \frac{x(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx =$$
$$-\frac{d^2e^{-\frac{6a}{b}}\left(-\sqrt{6}\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\Gamma\left(\frac{1}{2},-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)-8e^{\frac{2a}{b}}\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\Gamma\left(\frac{1}{2},-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)\right.}{}$$

```
[In] Integrate[(x*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] -1/32*(d^2*(-(Sqrt[6]*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-6*(a + b
*ArcSinh[c*x]))/b]) - 8*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1
/2, (-4*(a + b*ArcSinh[c*x]))/b] - 5*Sqrt[2]*E^((4*a)/b)*Sqrt[-((a + b*ArcS
inh[c*x])/b)]*Gamma[1/2, (-2*(a + b*ArcSinh[c*x]))/b] + 5*Sqrt[2]*E^((8*a)/
b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/b] + 8*E^((
10*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (4*(a + b*ArcSinh[c*x]))/b] +
Sqrt[6]*E^((12*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (6*(a + b*ArcSinh[
c*x]))/b] + 10*E^((6*a)/b)*Sinh[2*ArcSinh[c*x]] + 8*E^((6*a)/b)*Sinh[4*ArcS
inh[c*x]] + 2*E^((6*a)/b)*Sinh[6*ArcSinh[c*x]]))/(b*c^2*E^((6*a)/b)*Sqrt[a
+ b*ArcSinh[c*x]])
```

Maple [F]

$$\int \frac{x(c^2d\,x^2+d)^2}{(a+b\,\text{arcsinh}\,(cx))^{\frac{3}{2}}}dx$$

```
[In] int(x*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = d^2\left(\int \frac{x}{a\sqrt{a+b\,\text{asinh}\,(cx)}+b\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx + \int \frac{2c^2x^3}{a\sqrt{a+b\,\text{asinh}\,(cx)}+b\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx + \int \frac{c^4x^5}{a\sqrt{a+b\,\text{asinh}\,(cx)}+b\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx\right)$$

```
[In] integrate(x*(c**2*d*x**2+d)**2/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(x/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*asinh
(c*x)), x) + Integral(2*c**2*x**3/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*
asinh(c*x))*asinh(c*x)), x) + Integral(c**4*x**5/(a*sqrt(a + b*asinh(c*x))
+ b*sqrt(a + b*asinh(c*x))*asinh(c*x)), x))
```

Maxima [F]

$$\int \frac{x(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{(c^2dx^2+d)^2x}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)^2*x/(b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{x\,(d\,c^2\,x^2+d)^2}{(a+b\,\text{asinh}\,(c\,x))^{3/2}}\,dx$$

```
[In] int((x*(d + c^2*d*x^2)^2)/(a + b*asinh(c*x))^(3/2),x)
[Out] int((x*(d + c^2*d*x^2)^2)/(a + b*asinh(c*x))^(3/2), x)
```

3.470 $\int \frac{(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 346

$$\int \frac{(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = -\frac{2d^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{5d^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c} - \frac{5d^2e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c} - \frac{d^2e^{\frac{5a}{b}}\sqrt{5\pi}\,\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c} + \frac{5d^2e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c} + \frac{5d^2e^{-\frac{3a}{b}}\sqrt{3\pi}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c} + \frac{d^2e^{-\frac{5a}{b}}\sqrt{5\pi}\,\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}$$

```
[Out] -5/8*d^2*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c+
5/8*d^2*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c/exp(a/b)-
5/16*d^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*P
i^(1/2)/b^(3/2)/c+5/16*d^2*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3
^(1/2)*Pi^(1/2)/b^(3/2)/c/exp(3*a/b)-1/16*d^2*exp(5*a/b)*erf(5^(1/2)*(a+b*a
rcsinh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c+1/16*d^2*erfi(5^(1/2
)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c/exp(5*a/b)-2
*d^2*(c^2*x^2+1)^(5/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {5790, 5819, 5556, 3389, 2211, 2236, 2235}

$$\int \frac{(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = -\frac{5\sqrt{\pi}d^2e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c} - \frac{5\sqrt{3\pi}d^2e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c} - \frac{\sqrt{5\pi}d^2e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c} + \frac{5\sqrt{\pi}d^2e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c} + \frac{5\sqrt{3\pi}d^2e^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c} + \frac{\sqrt{5\pi}d^2e^{-\frac{5a}{b}}\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c} - \frac{2d^2(c^2x^2+1)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

```
[In] Int[(d + c^2*d*x^2)^2/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d^2*(1 + c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (5*d^2*E^(a/b
)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*b^(3/2)*c) - (5*d^2*E^
((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b
^(3/2)*c) - (d^2*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x
]])/Sqrt[b]])/(16*b^(3/2)*c) + (5*d^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]
]/Sqrt[b]])/(8*b^(3/2)*c*E^(a/b)) + (5*d^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a
+ b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c*E^((3*a)/b)) + (d^2*Sqrt[5*Pi]*E
rfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c*E^((5*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2d^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{(10cd^2)\int \frac{x(1+c^2x^2)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{b} \\
&= -\frac{2d^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{(10d^2)\,\text{Subst}\left(\int \frac{\cosh^4\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c} \\
&= -\frac{2d^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} \\
&\quad - \frac{(10d^2)\,\text{Subst}\left(\int \left(\frac{\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16\sqrt{x}} + \frac{3\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16\sqrt{x}} + \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{8\sqrt{x}}\right)dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c}
\end{aligned}$$

$$= -\frac{2d^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{(5d^2)\,\text{Subst}\left(\int \frac{\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c}$$
$$- \frac{(5d^2)\,\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c}$$
$$- \frac{(15d^2)\,\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c}$$

$$= -\frac{2d^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{(5d^2)\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c}$$
$$+ \frac{(5d^2)\,\text{Subst}\left(\int \frac{e^{i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c}$$
$$- \frac{(5d^2)\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c}$$
$$+ \frac{(5d^2)\,\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8b^2c}$$
$$- \frac{(15d^2)\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c}$$
$$+ \frac{(15d^2)\,\text{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16b^2c}$$

$$= -\frac{2d^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{(5d^2)\,\text{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{8b^2c}$$
$$+ \frac{(5d^2)\,\text{Subst}\left(\int e^{-\frac{5a}{b}+\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{8b^2c}$$
$$- \frac{(5d^2)\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{4b^2c}$$
$$+ \frac{(5d^2)\,\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{4b^2c}$$
$$- \frac{(15d^2)\,\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{8b^2c}$$
$$+ \frac{(15d^2)\,\text{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{8b^2c}$$

$$= -\frac{2d^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{5d^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c}$$
$$- \frac{5d^2e^{\frac{3a}{b}}\sqrt{3\pi}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c} - \frac{d^2e^{\frac{5a}{b}}\sqrt{5\pi}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}$$
$$+ \frac{5d^2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c} + \frac{5d^2e^{-\frac{3a}{b}}\sqrt{3\pi}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}$$
$$+ \frac{d^2e^{-\frac{5a}{b}}\sqrt{5\pi}\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}$$

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.27

$$\int \frac{(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \frac{d^2e^{-5\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}\left(-e^{\frac{5a}{b}} - 5e^{\frac{5a}{b}+2\text{arcsinh}(cx)} - 10e^{\frac{5a}{b}+4\text{arcsinh}(cx)} - 10e^{\frac{5a}{b}+6\text{arcsin}}\right.}{}$$

```
[In] Integrate[(d + c^2*d*x^2)^2/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (d^2*(-E^((5*a)/b) - 5*E^((5*a)/b + 2*ArcSinh[c*x]) - 10*E^((5*a)/b + 4*Arc
Sinh[c*x]) - 10*E^((5*a)/b + 6*ArcSinh[c*x]) - 5*E^((5*a)/b + 8*ArcSinh[c*x
]) - E^((5*a)/b + 10*ArcSinh[c*x]) + 10*E^((6*a)/b + 5*ArcSinh[c*x])*Sqrt[a
/b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + Sqrt[5]*E^(5*ArcSinh[c*
x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-5*(a + b*ArcSinh[c*x]))/b]
 + 5*Sqrt[3]*E^((2*a)/b + 5*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*G
amma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] + 10*E^((4*a)/b + 5*ArcSinh[c*x])*Sq
rt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)] + 5*Sqr
t[3]*E^((8*a)/b + 5*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a
 + b*ArcSinh[c*x]))/b] + Sqrt[5]*E^(5*((2*a)/b + ArcSinh[c*x]))*Sqrt[a/b +
ArcSinh[c*x]]*Gamma[1/2, (5*(a + b*ArcSinh[c*x]))/b]))/(16*b*c*E^(5*(a/b +
ArcSinh[c*x]))*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F]

$$\int \frac{\left(c^{2} d\,x^{2}+d\right)^{2}}{\left(a+b \,\operatorname{arcsinh}\left(c x\right)\right)^{\frac{3}{2}}}d x$$

```
[In] int((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = d^{2}\left(\int \frac{2c^{2}x^{2}}{a\sqrt{a+b\operatorname{asinh}{\left(cx \right)}}+b\sqrt{a+b\operatorname{asinh}{\left(cx \right)}}\operatorname{asinh}{\left(cx \right)}}\,dx + \int \frac{c^{4}x^{4}}{a\sqrt{a+b\operatorname{asinh}{\left(cx \right)}}+b\sqrt{a+b\operatorname{asinh}{\left(cx \right)}}\operatorname{asinh}{\left(cx \right)}}\,dx + \int \frac{1}{a\sqrt{a+b\operatorname{asinh}{\left(cx \right)}}+b\sqrt{a+b\operatorname{asinh}{\left(cx \right)}}\operatorname{asinh}{\left(cx \right)}}\,dx\right)$$

```
[In] integrate((c**2*d*x**2+d)**2/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(2*c**2*x**2/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c
*x))*asinh(c*x)), x) + Integral(c**4*x**4/(a*sqrt(a + b*asinh(c*x)) + b*sqr
t(a + b*asinh(c*x))*asinh(c*x)), x) + Integral(1/(a*sqrt(a + b*asinh(c*x))
+ b*sqrt(a + b*asinh(c*x))*asinh(c*x)), x))
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{(c^2dx^2+d)^2}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate((c^2*d*x^2 + d)^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{(c^2dx^2+d)^2}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
[Out] integrate((c^2*d*x^2 + d)^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{(d\,c^2\,x^2+d)^2}{(a+b\,\text{asinh}\,(c\,x))^{3/2}}\,dx$$

```
[In] int((d + c^2*d*x^2)^2/(a + b*asinh(c*x))^(3/2),x)
[Out] int((d + c^2*d*x^2)^2/(a + b*asinh(c*x))^(3/2), x)
```

3.471 $\int \frac{(d+c^2dx^2)^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\begin{aligned}\int \frac{(d+c^2dx^2)^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx = &-\frac{2d^2(1+c^2x^2)^{5/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}}\\
&+\frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}}-\frac{d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}}\\
&+\frac{d^2e^{\frac{2a}{b}}\sqrt{2\pi}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}}+\frac{d^2e^{-\frac{4a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}}\\
&-\frac{d^2e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{d^2e^{-\frac{2a}{b}}\sqrt{2\pi}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}}\\
&-\frac{2d^2\text{Int}\left(\frac{1}{x^2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}},x\right)}{bc}\end{aligned}$$

```
[Out] 3/4*d^2*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi
^(1/2)/b^(3/2)+3/4*d^2*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/
2)*Pi^(1/2)/b^(3/2)/exp(2*a/b)+1/4*d^2*exp(4*a/b)*erf(2*(a+b*arcsinh(c*x))^
(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)+1/4*d^2*erfi(2*(a+b*arcsinh(c*x))^(1/2)/b^(
1/2))*Pi^(1/2)/b^(3/2)/exp(4*a/b)-2*d^2*(c^2*x^2+1)^(5/2)/b/c/x/(a+b*arcsin
h(c*x))^(1/2)-2*d^2*Unintegrable(1/x^2/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))
^(1/2),x)/b/c
```

Rubi [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+c^2dx^2)^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{(d+c^2dx^2)^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx$$

```
[In] Int[(d + c^2*d*x^2)^2/(x*(a + b*ArcSinh[c*x])^(3/2)),x]
```

```
[Out] (-2*d^2*(1 + c^2*x^2)^(5/2))/(b*c*x*Sqrt[a + b*ArcSinh[c*x]]) + (d^2*E^((4*
a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)) - (d^
2*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(
2*b^(3/2)) + (d^2*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*
x]])/Sqrt[b]])/b^(3/2) + (d^2*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c*x]])/Sq
rt[b]])/(4*b^(3/2)*E^((4*a)/b)) - (d^2*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*
ArcSinh[c*x]])/Sqrt[b]])/(2*b^(3/2)*E^((2*a)/b)) + (d^2*Sqrt[2*Pi]*Erfi[(Sq
rt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(b^(3/2)*E^((2*a)/b)) - (2*d^2*De
fer[Int][1/(x^2*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]]), x])/(b*c)
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2d^2(1+c^2x^2)^{5/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}} - \frac{(2d^2)\int \frac{(1+c^2x^2)^{3/2}}{x^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{bc} \\
&\quad + \frac{(8cd^2)\int \frac{(1+c^2x^2)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{b} \\
&= -\frac{2d^2(1+c^2x^2)^{5/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}} + \frac{(8d^2)\,\text{Subst}\left(\int \frac{\cosh^4\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2} \\
&\quad - \frac{(2d^2)\int \left(\frac{2c^2}{\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}} + \frac{1}{x^2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}} + \frac{c^4x^2}{\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}\right)dx}{bc} \\
&= -\frac{2d^2(1+c^2x^2)^{5/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}} \\
&\quad + \frac{(8d^2)\,\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}} + \frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{8\sqrt{x}} + \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\right)dx, x, a+b\text{arcsinh}(cx)\right)}{b^2} \\
&\quad - \frac{(2d^2)\int \frac{1}{x^2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{bc} - \frac{(4cd^2)\int \frac{1}{\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{b} \\
&\quad - \frac{(2c^3d^2)\int \frac{x^2}{\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{b}
\end{aligned}$$

$$= -\frac{2d^2(1+c^2x^2)^{5/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}} - \frac{2d^2\sqrt{a+b\text{arcsinh}(cx)}}{b^2} + \frac{d^2\text{Subst}\left(\int \frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2} - \frac{(2d^2)\,\text{Subst}\left(\int \frac{\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2} + \frac{(4d^2)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2} - \frac{(2d^2)\int \frac{1}{x^2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{bc}$$

$$= -\frac{2d^2(1+c^2x^2)^{5/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}} - \frac{2d^2\sqrt{a+b\text{arcsinh}(cx)}}{b^2} + \frac{d^2\text{Subst}\left(\int \frac{e^{-i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^2} + \frac{d^2\text{Subst}\left(\int \frac{e^{i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^2} + \frac{(2d^2)\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2} + \frac{(2d^2)\,\text{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2} + \frac{(2d^2)\,\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}} - \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2} - \frac{(2d^2)\int \frac{1}{x^2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{bc}$$

$$= -\frac{2d^2(1+c^2x^2)^{5/2}}{bcx\sqrt{a+b\operatorname{arcsinh}(cx)}} + \frac{d^2\operatorname{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{b^2}$$
$$+ \frac{d^2\operatorname{Subst}\left(\int e^{-\frac{4a}{b}+\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{b^2}$$
$$- \frac{d^2\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b^2}$$
$$+ \frac{(4d^2)\operatorname{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{b^2}$$
$$+ \frac{(4d^2)\operatorname{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{b^2}$$
$$- \frac{(2d^2)\int \frac{1}{x^2\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx}{bc}$$

$$= -\frac{2d^2(1+c^2x^2)^{5/2}}{bcx\sqrt{a+b\operatorname{arcsinh}(cx)}} + \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}}$$
$$+ \frac{d^2e^{\frac{2a}{b}}\sqrt{2\pi}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}} + \frac{d^2e^{-\frac{4a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}}$$
$$+ \frac{d^2e^{-\frac{2a}{b}}\sqrt{2\pi}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}}$$
$$- \frac{d^2\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{2b^2}$$
$$- \frac{d^2\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{2b^2}$$
$$- \frac{(2d^2)\int \frac{1}{x^2\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx}{bc}$$

$$= -\frac{2d^2(1+c^2x^2)^{5/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}} + \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}} + \frac{d^2e^{\frac{2a}{b}}\sqrt{2\pi}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}} + \frac{d^2e^{-\frac{4a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}} + \frac{d^2e^{-\frac{2a}{b}}\sqrt{2\pi}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}} - \frac{d^2\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2} - \frac{d^2\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2} - \frac{(2d^2)\int \frac{1}{x^2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}dx}{bc}$$

$$= -\frac{2d^2(1+c^2x^2)^{5/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}} + \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}} - \frac{d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}} + \frac{d^2e^{\frac{2a}{b}}\sqrt{2\pi}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}} + \frac{d^2e^{-\frac{4a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}} - \frac{d^2e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}} + \frac{d^2e^{-\frac{2a}{b}}\sqrt{2\pi}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}} - \frac{(2d^2)\int \frac{1}{x^2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}dx}{bc}$$

Mathematica [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(d+c^2dx^2)^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{(d+c^2dx^2)^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx$$

```
[In] Integrate[(d + c^2*d*x^2)^2/(x*(a + b*ArcSinh[c*x])^(3/2)),x]
[Out] Integrate[(d + c^2*d*x^2)^2/(x*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\left(c^{2} d \,x^{2}+d \right)^{2}}{x \left(a +b \,\operatorname{arcsinh}\left(c x \right)\right)^{\frac{3}{2}}}d x$$

[In] int((c^2*d*x^2+d)^2/x/(a+b*arcsinh(c*x))^(3/2),x)

[Out] int((c^2*d*x^2+d)^2/x/(a+b*arcsinh(c*x))^(3/2),x)

Fricas [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\text{Exception raised: TypeError}$$

[In] integrate((c^2*d*x^2+d)^2/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)

Sympy [N/A]

Not integrable

Time = 5.97 (sec) , antiderivative size = 131, normalized size of antiderivative = 4.68

$$\int \frac{(d+c^2dx^2)^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx=d^2\left(\int \frac{2c^2x^2}{ax\sqrt{a+b\operatorname{asinh}(cx)}+bx\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,dx + \int \frac{c^4x^4}{ax\sqrt{a+b\operatorname{asinh}(cx)}+bx\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,dx + \int \frac{1}{ax\sqrt{a+b\operatorname{asinh}(cx)}+bx\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,dx\right)$$

[In] integrate((c**2*d*x**2+d)**2/x/(a+b*asinh(c*x))**(3/2),x)

[Out] d**2*(Integral(2*c**2*x**2/(a*x*sqrt(a + b*asinh(c*x)) + b*x*sqrt(a + b*asi
nh(c*x))*asinh(c*x)), x) + Integral(c**4*x**4/(a*x*sqrt(a + b*asinh(c*x)) +
b*x*sqrt(a + b*asinh(c*x))*asinh(c*x)), x) + Integral(1/(a*x*sqrt(a + b*as
inh(c*x)) + b*x*sqrt(a + b*asinh(c*x))*asinh(c*x)), x))

Maxima [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(d+c^2dx^2)^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{(c^2dx^2+d)^2}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^2/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate((c^2*d*x^2 + d)^2/((b*arcsinh(c*x) + a)^(3/2)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^2/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.83 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(d+c^2dx^2)^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{(d\,c^2\,x^2+d)^2}{x\,(a+b\,\text{asinh}\,(c\,x))^{3/2}}\,dx$$

```
[In] int((d + c^2*d*x^2)^2/(x*(a + b*asinh(c*x))^(3/2)),x)
[Out] int((d + c^2*d*x^2)^2/(x*(a + b*asinh(c*x))^(3/2)), x)
```

3.472 $\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\text{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 319

$$\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\text{arcsinh}(ax)}\,dx = \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)} + \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\text{arcsinh}(ax)} + \frac{c\sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^{3/2}}{4a\sqrt{1+a^2x^2}} + \frac{c\sqrt{\pi}\sqrt{c+a^2cx^2}\,\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{256a\sqrt{1+a^2x^2}} + \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{16a\sqrt{1+a^2x^2}} - \frac{c\sqrt{\pi}\sqrt{c+a^2cx^2}\,\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{256a\sqrt{1+a^2x^2}} - \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{16a\sqrt{1+a^2x^2}}$$

```
[Out] 1/4*c*arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/32*c*erf
(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^
2+1)^(1/2)-1/32*c*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*
x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/256*c*erf(2*arcsinh(a*x)^(1/2))*Pi^(1/2)
*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-1/256*c*erfi(2*arcsinh(a*x)^(1/2))
*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/4*x*(a^2*c*x^2+c)^(3/2)
*arcsinh(a*x)^(1/2)+3/8*c*x*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5786, 5785, 5783, 5780, 5556, 12, 3389, 2211, 2235, 2236, 5819}

$$\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\operatorname{arcsinh}(ax)}\,dx = \frac{\sqrt{\pi}c\sqrt{a^2cx^2+c}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{256a\sqrt{a^2x^2+1}} + \frac{\sqrt{\frac{\pi}{2}}c\sqrt{a^2cx^2+c}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a\sqrt{a^2x^2+1}} - \frac{\sqrt{\pi}c\sqrt{a^2cx^2+c}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{256a\sqrt{a^2x^2+1}} - \frac{\sqrt{\frac{\pi}{2}}c\sqrt{a^2cx^2+c}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a\sqrt{a^2x^2+1}} + \frac{1}{4}x\sqrt{\operatorname{arcsinh}(ax)}\left(a^2cx^2+c\right)^{3/2} + \frac{c\operatorname{arcsinh}(ax)^{3/2}\sqrt{a^2cx^2+c}}{4a\sqrt{a^2x^2+1}} + \frac{3}{8}cx\sqrt{\operatorname{arcsinh}(ax)}\sqrt{a^2cx^2+c}$$

```
[In] Int[(c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (3*c*x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/8 + (x*(c + a^2*c*x^2)^(3/2)
*Sqrt[ArcSinh[a*x]])/4 + (c*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))/(4*a*Sq
rt[1 + a^2*x^2]) + (c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]
])/(256*a*Sqrt[1 + a^2*x^2]) + (c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2
]*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*x^2]) - (c*Sqrt[Pi]*Sqrt[c + a^2*
c*x^2]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(256*a*Sqrt[1 + a^2*x^2]) - (c*Sqrt[Pi/2
]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*
x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
```

```
c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\operatorname{arcsinh}(ax)} \\
&\quad+\frac{1}{4}(3c)\int\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}\,dx-\frac{\left(ac\sqrt{c+a^2cx^2}\right)\int\frac{x(1+a^2x^2)}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx}{8\sqrt{1+a^2x^2}} \\
&= \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\operatorname{arcsinh}(ax)} \\
&\quad+\frac{\left(3c\sqrt{c+a^2cx^2}\right)\int\frac{\sqrt{\operatorname{arcsinh}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{8\sqrt{1+a^2x^2}} \\
&\quad-\frac{\left(c\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int\frac{\cosh^3(x)\sinh(x)}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{8a\sqrt{1+a^2x^2}} \\
&\quad-\frac{\left(3ac\sqrt{c+a^2cx^2}\right)\int\frac{x}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx}{16\sqrt{1+a^2x^2}} \\
&= \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\operatorname{arcsinh}(ax)} \\
&\quad+\frac{c\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{4a\sqrt{1+a^2x^2}} \\
&\quad-\frac{\left(c\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int\left(\frac{\sinh(2x)}{4\sqrt{x}}+\frac{\sinh(4x)}{8\sqrt{x}}\right)dx,x,\operatorname{arcsinh}(ax)\right)}{8a\sqrt{1+a^2x^2}} \\
&\quad-\frac{\left(3c\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int\frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{16a\sqrt{1+a^2x^2}}
\end{aligned}$$

$$= \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} + \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\operatorname{arcsinh}(ax)} + \frac{c\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{4a\sqrt{1+a^2x^2}} - \frac{\left(c\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{\sinh(4x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{64a\sqrt{1+a^2x^2}} - \frac{\left(c\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{32a\sqrt{1+a^2x^2}} - \frac{\left(3c\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{16a\sqrt{1+a^2x^2}}$$

$$= \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} + \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\operatorname{arcsinh}(ax)} + \frac{c\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{4a\sqrt{1+a^2x^2}} + \frac{\left(c\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{128a\sqrt{1+a^2x^2}} - \frac{\left(c\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{e^{4x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{128a\sqrt{1+a^2x^2}} + \frac{\left(c\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{64a\sqrt{1+a^2x^2}} - \frac{\left(c\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{64a\sqrt{1+a^2x^2}} - \frac{\left(3c\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{32a\sqrt{1+a^2x^2}}$$

$$= \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} + \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\operatorname{arcsinh}(ax)}$$
$$+ \frac{c\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{4a\sqrt{1+a^2x^2}}$$
$$+ \frac{\left(c\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{64a\sqrt{1+a^2x^2}}$$
$$- \frac{\left(c\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int e^{4x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{64a\sqrt{1+a^2x^2}}$$
$$+ \frac{\left(c\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{32a\sqrt{1+a^2x^2}}$$
$$- \frac{\left(c\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{32a\sqrt{1+a^2x^2}}$$
$$+ \frac{\left(3c\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{64a\sqrt{1+a^2x^2}}$$
$$- \frac{\left(3c\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{64a\sqrt{1+a^2x^2}}$$

$$= \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} + \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\operatorname{arcsinh}(ax)}$$
$$+ \frac{c\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{4a\sqrt{1+a^2x^2}} + \frac{c\sqrt{\pi}\sqrt{c+a^2cx^2}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{256a\sqrt{1+a^2x^2}}$$
$$+ \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a\sqrt{1+a^2x^2}} - \frac{c\sqrt{\pi}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{256a\sqrt{1+a^2x^2}}$$
$$- \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a\sqrt{1+a^2x^2}}$$
$$+ \frac{\left(3c\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{32a\sqrt{1+a^2x^2}}$$
$$- \frac{\left(3c\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{32a\sqrt{1+a^2x^2}}$$

$$= \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} + \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\operatorname{arcsinh}(ax)} + \frac{c\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{4a\sqrt{1+a^2x^2}} + \frac{c\sqrt{\pi}\sqrt{c+a^2cx^2}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{256a\sqrt{1+a^2x^2}} + \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a\sqrt{1+a^2x^2}} - \frac{c\sqrt{\pi}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{256a\sqrt{1+a^2x^2}} - \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a\sqrt{1+a^2x^2}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.45

$$\int (c+a^2cx^2)^{3/2}\sqrt{\operatorname{arcsinh}(ax)}\,dx = \frac{c\sqrt{c+a^2cx^2}\left(-\sqrt{-\operatorname{arcsinh}(ax)}\Gamma\left(\frac{3}{2},-4\operatorname{arcsinh}(ax)\right) - 8\sqrt{2}\sqrt{-\operatorname{arcsinh}(ax)}\right.}{}$$

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(-(Sqrt[-ArcSinh[a*x]]*Gamma[3/2, -4*ArcSinh[a*x]])
- 8*Sqrt[2]*Sqrt[-ArcSinh[a*x]]*Gamma[3/2, -2*ArcSinh[a*x]] + Sqrt[ArcSinh[
a*x]]*(32*ArcSinh[a*x]^(3/2) - 8*Sqrt[2]*Gamma[3/2, 2*ArcSinh[a*x]] - Gamma
[3/2, 4*ArcSinh[a*x]])))/(128*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

Maple [F]

$$\int \left(a^2c\,x^2+c\right)^{\frac{3}{2}}\sqrt{\operatorname{arcsinh}(ax)}dx$$

```
[In] int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2} \sqrt{\operatorname{arcsinh}(ax)}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \left(c+a^2cx^2\right)^{3/2} \sqrt{\operatorname{arcsinh}(ax)}\, dx = \int \left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}} \sqrt{\operatorname{asinh}(ax)}\, dx$$

```
[In] integrate((a**2*c*x**2+c)**(3/2)*asinh(a*x)**(1/2),x)
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*sqrt(asinh(a*x)), x)
```

Maxima [F]

$$\int \left(c+a^2cx^2\right)^{3/2} \sqrt{\operatorname{arcsinh}(ax)}\, dx = \int \left(a^2cx^2+c\right)^{\frac{3}{2}} \sqrt{\operatorname{arsinh}(ax)}\, dx$$

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(3/2)*sqrt(arcsinh(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2} \sqrt{\operatorname{arcsinh}(ax)}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\operatorname{arcsinh}(ax)}\,dx=\int \sqrt{\operatorname{asinh}\left(a\,x\right)}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

```
[In] int(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2),x)
[Out] int(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.473 $\int \sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 186

$$\int \sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}\,dx = \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)} + \frac{\sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^{3/2}}{3a\sqrt{1+a^2x^2}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{16a\sqrt{1+a^2x^2}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{16a\sqrt{1+a^2x^2}}$$

```
[Out] 1/3*arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/32*erf(2^(
1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)
^(1/2)-1/32*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)
^(1/2)/a/(a^2*x^2+1)^(1/2)+1/2*x*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5785, 5783, 5780, 5556, 12, 3389, 2211, 2235, 2236}

$$\int \sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}\,dx = \frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a\sqrt{a^2x^2+1}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a\sqrt{a^2x^2+1}} + \frac{\operatorname{arcsinh}(ax)^{3/2}\sqrt{a^2cx^2+c}}{3a\sqrt{a^2x^2+1}} + \frac{1}{2}x\sqrt{\operatorname{arcsinh}(ax)}\sqrt{a^2cx^2+c}$$

```
[In] Int[Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/2 + (Sqrt[c + a^2*c*x^2]*ArcSinh
[a*x]^(3/2))/(3*a*Sqrt[1 + a^2*x^2]) + (Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[
Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*x^2]) - (Sqrt[Pi/2]*Sqrt[c
+ a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)} + \frac{\sqrt{c+a^2cx^2}\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{2\sqrt{1+a^2x^2}} - \frac{\left(a\sqrt{c+a^2cx^2}\right)\int\frac{x}{\sqrt{\text{arcsinh}(ax)}}\,dx}{4\sqrt{1+a^2x^2}}$$

$$= \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} + \frac{\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{3a\sqrt{1+a^2x^2}} - \frac{\sqrt{c+a^2cx^2}\operatorname{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{4a\sqrt{1+a^2x^2}}$$

$$= \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} + \frac{\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{3a\sqrt{1+a^2x^2}} - \frac{\sqrt{c+a^2cx^2}\operatorname{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{4a\sqrt{1+a^2x^2}}$$

$$= \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} + \frac{\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{3a\sqrt{1+a^2x^2}} - \frac{\sqrt{c+a^2cx^2}\operatorname{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{8a\sqrt{1+a^2x^2}}$$

$$= \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} + \frac{\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{3a\sqrt{1+a^2x^2}} + \frac{\sqrt{c+a^2cx^2}\operatorname{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{16a\sqrt{1+a^2x^2}} - \frac{\sqrt{c+a^2cx^2}\operatorname{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{16a\sqrt{1+a^2x^2}}$$

$$= \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} + \frac{\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{3a\sqrt{1+a^2x^2}} + \frac{\sqrt{c+a^2cx^2}\operatorname{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{8a\sqrt{1+a^2x^2}} - \frac{\sqrt{c+a^2cx^2}\operatorname{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{8a\sqrt{1+a^2x^2}}$$

$$= \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} + \frac{\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{3a\sqrt{1+a^2x^2}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a\sqrt{1+a^2x^2}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a\sqrt{1+a^2x^2}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.56

$$\int \sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}\,dx$$
$$= \frac{\sqrt{c\,(1+a^2x^2)}\Big(16\text{arcsinh}(ax)^2-3\sqrt{2}\sqrt{-\text{arcsinh}(ax)}\Gamma\left(\frac{3}{2},-2\text{arcsinh}(ax)\right)-3\sqrt{2}\sqrt{\text{arcsinh}(ax)}\Gamma\left(\frac{3}{2},2\text{arcs}\right.}{48a\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}$$

```
[In] Integrate[Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(16*ArcSinh[a*x]^2 - 3*Sqrt[2]*Sqrt[-ArcSinh[a*x]]*G
amma[3/2, -2*ArcSinh[a*x]] - 3*Sqrt[2]*Sqrt[ArcSinh[a*x]]*Gamma[3/2, 2*ArcS
inh[a*x]]))/(48*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

Maple [F]

$$\int \sqrt{a^2c\,x^2+c}\,\sqrt{\text{arcsinh}\,(ax)}dx$$

```
[In] int((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}\,dx = \int \sqrt{c\,(a^2x^2+1)}\sqrt{\text{asinh}\,(ax)}\,dx$$

```
[In] integrate((a**2*c*x**2+c)**(1/2)*asinh(a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*sqrt(asinh(a*x)), x)
```

Maxima [F]

$$\int \sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}\,dx = \int \sqrt{a^2cx^2+c}\sqrt{\operatorname{arsinh}(ax)}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a^2*c*x^2 + c)*sqrt(arcsinh(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}\,dx = \int \sqrt{\operatorname{asinh}(a\,x)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

```
[In] int(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2),x)
[Out] int(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.474 $\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{\sqrt{c+a^2cx^2}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 42

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{\sqrt{c+a^2cx^2}} \, dx = \frac{2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{3a\sqrt{c+a^2cx^2}}$$

```
[Out] 2/3*arcsinh(a*x)^(3/2)*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {5783}

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{\sqrt{c+a^2cx^2}} \, dx = \frac{2\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^{3/2}}{3a\sqrt{a^2cx^2+c}}$$

```
[In] Int[Sqrt[ArcSinh[a*x]]/Sqrt[c + a^2*c*x^2],x]
[Out] (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(3*a*Sqrt[c + a^2*c*x^2])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{3a\sqrt{c+a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{3a\sqrt{c\,(1+a^2x^2)}}$$

```
[In] Integrate[Sqrt[ArcSinh[a*x]]/Sqrt[c + a^2*c*x^2],x]
[Out] (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(3*a*Sqrt[c*(1 + a^2*x^2)])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{2\operatorname{arcsinh}(ax)^{\frac{3}{2}}\sqrt{a^2x^2+1}}{3a\sqrt{c(a^2x^2+1)}}$	36

```
[In] int(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*arcsinh(a*x)^(3/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\operatorname{asinh}\left(ax\right)}}{\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

```
[In] integrate(asinh(a*x)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(sqrt(asinh(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\operatorname{arsinh}\left(ax\right)}}{\sqrt{a^2cx^2+c}}\,dx$$

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(arcsinh(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\operatorname{arsinh}\left(ax\right)}}{\sqrt{a^2cx^2+c}}\,dx$$

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(arcsinh(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\operatorname{asinh}\left(a\,x\right)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

```
[In] int(asinh(a*x)^(1/2)/(c + a^2*c*x^2)^(1/2),x)
[Out] int(asinh(a*x)^(1/2)/(c + a^2*c*x^2)^(1/2), x)
```

3.475 $\int \frac{\sqrt{\text{arcsinh}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{\text{arcsinh}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{x\sqrt{\text{arcsinh}(ax)}}{c\sqrt{c+a^2cx^2}} - \frac{a\sqrt{1+a^2x^2}\,\text{Int}\left(\frac{x}{(1+a^2x^2)\sqrt{\text{arcsinh}(ax)}},x\right)}{2c\sqrt{c+a^2cx^2}}$$

```
[Out] x*arcsinh(a*x)^(1/2)/c/(a^2*c*x^2+c)^(1/2)-1/2*a*(a^2*x^2+1)^(1/2)*Unintegr
able(x/(a^2*x^2+1)/arcsinh(a*x)^(1/2),x)/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{\text{arcsinh}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{\text{arcsinh}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$$

```
[In] Int[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[ArcSinh[a*x]])/(c*Sqrt[c + a^2*c*x^2]) - (a*Sqrt[1 + a^2*x^2]*Defer
[Int][x/((1 + a^2*x^2)*Sqrt[ArcSinh[a*x]]), x])/(2*c*Sqrt[c + a^2*c*x^2])
```

Rubi steps

$$\text{integral} = \frac{x\sqrt{\text{arcsinh}(ax)}}{c\sqrt{c+a^2cx^2}} - \frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x}{(1+a^2x^2)\sqrt{\text{arcsinh}(ax)}}\,dx}{2c\sqrt{c+a^2cx^2}}$$

Mathematica [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$$

```
[In] Integrate[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(3/2),x]
[Out] Integrate[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(a^2c\,x^2+c)^{\frac{3}{2}}}dx$$

```
[In] int(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
[Out] int(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{\operatorname{asinh}(ax)}}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(asinh(a*x)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
[Out] Integral(sqrt(asinh(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{\operatorname{arsinh}(ax)}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(arcsinh(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{\operatorname{arsinh}(ax)}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(arcsinh(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{\operatorname{asinh}(a\,x)}}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

```
[In] int(asinh(a*x)^(1/2)/(c + a^2*c*x^2)^(3/2),x)
[Out] int(asinh(a*x)^(1/2)/(c + a^2*c*x^2)^(3/2), x)
```

3.476 $\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{x\sqrt{\operatorname{arcsinh}(ax)}}{3c\,(c+a^2cx^2)^{3/2}} + \frac{2x\sqrt{\operatorname{arcsinh}(ax)}}{3c^2\sqrt{c+a^2cx^2}} - \frac{a\sqrt{1+a^2x^2}\operatorname{Int}\left(\frac{x}{(1+a^2x^2)^2\sqrt{\operatorname{arcsinh}(ax)}},x\right)}{6c^2\sqrt{c+a^2cx^2}} - \frac{a\sqrt{1+a^2x^2}\operatorname{Int}\left(\frac{x}{(1+a^2x^2)\sqrt{\operatorname{arcsinh}(ax)}},x\right)}{3c^2\sqrt{c+a^2cx^2}}$$

```
[Out] 1/3*x*arcsinh(a*x)^(1/2)/c/(a^2*c*x^2+c)^(3/2)+2/3*x*arcsinh(a*x)^(1/2)/c^2
/(a^2*c*x^2+c)^(1/2)-1/6*a*(a^2*x^2+1)^(1/2)*Unintegrable(x/(a^2*x^2+1)^2/a
rcsinh(a*x)^(1/2),x)/c^2/(a^2*c*x^2+c)^(1/2)-1/3*a*(a^2*x^2+1)^(1/2)*Uninte
grable(x/(a^2*x^2+1)/arcsinh(a*x)^(1/2),x)/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$$

```
[In] Int[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[ArcSinh[a*x]])/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*x*Sqrt[ArcSinh[a*x]
])/(3*c^2*Sqrt[c + a^2*c*x^2]) - (a*Sqrt[1 + a^2*x^2]*Defer[Int][x/((1 + a^
```

```
2*x^2)^2*Sqrt[ArcSinh[a*x]]), x])/(6*c^2*Sqrt[c + a^2*c*x^2]) - (a*Sqrt[1 +
 a^2*x^2]*Defer[Int][x/((1 + a^2*x^2)*Sqrt[ArcSinh[a*x]]), x])/(3*c^2*Sqrt[
c + a^2*c*x^2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x\sqrt{\operatorname{arcsinh}(ax)}}{3c\,(c+a^2cx^2)^{3/2}}+\frac{2\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx}{3c}-\frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x}{(1+a^2x^2)^2\sqrt{\operatorname{arcsinh}(ax)}}\,dx}{6c^2\sqrt{c+a^2cx^2}}\\
&= \frac{x\sqrt{\operatorname{arcsinh}(ax)}}{3c\,(c+a^2cx^2)^{3/2}}+\frac{2x\sqrt{\operatorname{arcsinh}(ax)}}{3c^2\sqrt{c+a^2cx^2}}-\frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x}{(1+a^2x^2)^2\sqrt{\operatorname{arcsinh}(ax)}}\,dx}{6c^2\sqrt{c+a^2cx^2}}\\
&\quad-\frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x}{(1+a^2x^2)\sqrt{\operatorname{arcsinh}(ax)}}\,dx}{3c^2\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 2.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx=\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$$

```
[In] Integrate[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(5/2),x]
[Out] Integrate[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(5/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(ax\right)}}{\left(a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}}dx$$

```
[In] int(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
[Out] int(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 23.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\sqrt{\operatorname{asinh}(ax)}}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(asinh(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
[Out] Integral(sqrt(asinh(a*x))/(c*(a**2*x**2 + 1))**(5/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\sqrt{\operatorname{arsinh}(ax)}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate(sqrt(arcsinh(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

Giac [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\sqrt{\operatorname{arsinh}\left(ax\right)}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
[Out] integrate(sqrt(arcsinh(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.80 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\sqrt{\operatorname{asinh}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

```
[In] int(asinh(a*x)^(1/2)/(c + a^2*c*x^2)^(5/2),x)
[Out] int(asinh(a*x)^(1/2)/(c + a^2*c*x^2)^(5/2), x)
```

3.477 $\int \left(c+a^2cx^2\right)^{3/2} \textbf{arcsinh}(ax)^{3/2}\, dx$

Optimal result

Integrand size = 23, antiderivative size = 449

$$\int \left(c+a^2cx^2\right)^{3/2} \operatorname{arcsinh}(ax)^{3/2}\, dx = -\frac{27c\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}{256a\sqrt{1+a^2x^2}} - \frac{9acx^2\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}{32\sqrt{1+a^2x^2}} - \frac{3c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}{32a} + \frac{3}{8}cx\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2} + \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^{3/2} + \frac{3c\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2}}{20a\sqrt{1+a^2x^2}} + \frac{3c\sqrt{\pi}\sqrt{c+\cdots}}{2\cdots}$$

```
[Out] 1/4*x*(a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2)+3/8*c*x*arcsinh(a*x)^(3/2)*(a^
2*c*x^2+c)^(1/2)+3/20*c*arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1
)^(1/2)+3/128*c*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2
+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/128*c*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1
/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/2048*c*erf(2*arcsinh
(a*x)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/2048*c*erfi
(2*arcsinh(a*x)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-3/3
2*c*(a^2*x^2+1)^(3/2)*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)/a-27/256*c*(a^
2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)/a/(a^2*x^2+1)^(1/2)-9/32*a*c*x^2*(a^2*c
*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)/(a^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {5786, 5785, 5783, 5777, 5819, 3393, 3388, 2211, 2235, 2236, 5798, 5791}

$$\int \left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^{3/2}\,dx = \frac{3\sqrt{\pi}c\sqrt{a^2cx^2+c}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{2048a\sqrt{a^2x^2+1}} + \frac{3\sqrt{\frac{\pi}{2}}c\sqrt{a^2cx^2+c}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a\sqrt{a^2x^2+1}} + \frac{3\sqrt{\pi}c\sqrt{a^2cx^2+c}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{2048a\sqrt{a^2x^2+1}} + \frac{3\sqrt{\frac{\pi}{2}}c\sqrt{a^2cx^2+c}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a\sqrt{a^2x^2+1}} + \frac{3c\operatorname{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}}{20a\sqrt{a^2x^2+1}} + \frac{1}{4}x\operatorname{arcsinh}(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2} + \frac{3}{8}cx\operatorname{arcsinh}(ax)^{3/2}\sqrt{a^2cx^2+c} - \frac{3c(a^2x^2+1)^{3/2}\sqrt{\operatorname{arcsinh}(ax)}\sqrt{a^2cx^2+}}{32a}$$

```
[In] Int[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-27*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(256*a*Sqrt[1 + a^2*x^2]) -
(9*a*c*x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(32*Sqrt[1 + a^2*x^2]) -
 (3*c*(1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(32*a) +
(3*c*x*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))/8 + (x*(c + a^2*c*x^2)^(3/2)
*ArcSinh[a*x]^(3/2))/4 + (3*c*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2))/(20*a
*Sqrt[1 + a^2*x^2]) + (3*c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[
a*x]]])/(2048*a*Sqrt[1 + a^2*x^2]) + (3*c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Er
f[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2]) + (3*c*Sqrt[Pi]*Sqr
t[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(2048*a*Sqrt[1 + a^2*x^2]) + (
3*c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(64*a*
Sqrt[1 + a^2*x^2])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{3/2}+\frac{1}{4}(3c)\int\sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^{3/2}\,dx - \frac{\left(3ac\sqrt{c+a^2cx^2}\right)\int x(1+a^2x^2)\sqrt{\text{arcsinh}(ax)}\,dx}{8\sqrt{1+a^2x^2}}$$

$$= -\frac{3c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{32a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^{3/2} + \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{3/2}+\frac{\left(3c\sqrt{c+a^2cx^2}\right)\int\frac{(1+a^2x^2)^{3/2}}{\sqrt{\text{arcsinh}(ax)}}\,dx}{64\sqrt{1+a^2x^2}}+\frac{\left(3c\sqrt{c+a^2cx^2}\right)\int\frac{\text{arcsin}}{\sqrt{1+}}}{8\sqrt{1+a^2x^2}}$$

$$= -\frac{9acx^2\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{32\sqrt{1+a^2x^2}} - \frac{3c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{32a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^{3/2} + \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{3/2}+\frac{3c\sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^{5/2}}{20a\sqrt{1+a^2x^2}}+\frac{\left(3c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int\frac{\cosh^4(}{\sqrt{x}}\right.}{64a\sqrt{1+a^2}}$$

$$
\begin{aligned}
&= -\frac{9acx^2\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{32\sqrt{1+a^2x^2}} \\
&\quad -\frac{3c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{32a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2} \\
&\quad +\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{3/2}+\frac{3c\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}{20a\sqrt{1+a^2x^2}}+\frac{\left(3c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int\left(\frac{3}{8\sqrt{x}}-\cdots\right.\right.}{64a\cdots}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{9c\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{256a\sqrt{1+a^2x^2}}-\frac{9acx^2\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{32\sqrt{1+a^2x^2}} \\
&\quad -\frac{3c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{32a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2} \\
&\quad +\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{3/2}+\frac{3c\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}{20a\sqrt{1+a^2x^2}}+\frac{\left(3c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int\frac{\cosh(4x)}{\sqrt{x}}\cdots\right.}{512a\sqrt{1+a^2\cdots}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{27c\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{256a\sqrt{1+a^2x^2}}-\frac{9acx^2\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{32\sqrt{1+a^2x^2}} \\
&\quad -\frac{3c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{32a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2} \\
&\quad +\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{3/2}+\frac{3c\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}{20a\sqrt{1+a^2x^2}}+\frac{\left(3c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int\frac{e^{-4x}}{\sqrt{x}}dx\cdots\right.}{1024a\sqrt{1+a^2x\cdots}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{27c\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{256a\sqrt{1+a^2x^2}}-\frac{9acx^2\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{32\sqrt{1+a^2x^2}} \\
&\quad -\frac{3c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{32a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2} \\
&\quad +\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{3/2}+\frac{3c\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}{20a\sqrt{1+a^2x^2}}+\frac{\left(3c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int e^{-4x^2}\cdots\right.}{512a\sqrt{1+a^2\cdots}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{27c\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{256a\sqrt{1+a^2x^2}}-\frac{9acx^2\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{32\sqrt{1+a^2x^2}} \\
&\quad -\frac{3c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{32a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2} \\
&\quad +\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{3/2}+\frac{3c\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}{20a\sqrt{1+a^2x^2}}+\frac{3c\sqrt{\pi}\sqrt{c+a^2cx^2}\text{erf}\left(2\sqrt{\text{arcsinh}\cdots}\right.}{2048a\sqrt{1+a^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{27c\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{256a\sqrt{1+a^2x^2}}-\frac{9acx^2\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{32\sqrt{1+a^2x^2}} \\
&\quad -\frac{3c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{32a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2} \\
&\quad +\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{3/2}+\frac{3c\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}{20a\sqrt{1+a^2x^2}}+\frac{3c\sqrt{\pi}\sqrt{c+a^2cx^2}\text{erf}\left(2\sqrt{\text{arcsinh}\cdots}\right.}{2048a\sqrt{1+a^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.41

$$\int (c+a^2cx^2)^{3/2} \operatorname{arcsinh}(ax)^{3/2} \, dx = \frac{c\sqrt{c+a^2cx^2}\Big(384\operatorname{arcsinh}(ax)^3 - 480\operatorname{arcsinh}(ax)\cosh(2\operatorname{arcsinh}(ax)) + 60\sqrt{2}$$

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2),x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(384*ArcSinh[a*x]^3 - 480*ArcSinh[a*x]*Cosh[2*ArcSin
h[a*x]] + 60*Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]]
+ 60*Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + 5*Sqr
t[-ArcSinh[a*x]]*Gamma[5/2, -4*ArcSinh[a*x]] - 5*Sqrt[ArcSinh[a*x]]*Gamma[5
/2, 4*ArcSinh[a*x]] + 640*ArcSinh[a*x]^2*Sinh[2*ArcSinh[a*x]]))/(2560*a*Sqr
t[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

Maple [F]

$$\int \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \operatorname{arcsinh}\left(a x \right)^{\frac{3}{2}}d x$$

```
[In] int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (c+a^2cx^2)^{3/2} \operatorname{arcsinh}(ax)^{3/2} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate((a**2*c*x**2+c)**(3/2)*asinh(a*x)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^{3/2}\,dx=\int \left(a^2cx^2+c\right)^{\frac{3}{2}}\operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^{3/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^{3/2}\,dx=\int \operatorname{asinh}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

```
[In] int(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2),x)
[Out] int(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.478 $\int \sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 271

$$\begin{aligned}
&\int \sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}\,dx = \\
&-\frac{3\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}{16a\sqrt{1+a^2x^2}}-\frac{3ax^2\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}{8\sqrt{1+a^2x^2}} \\
&+\frac{1}{2}x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}+\frac{\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2}}{5a\sqrt{1+a^2x^2}} \\
&+\frac{3\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a\sqrt{1+a^2x^2}}+\frac{3\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a\sqrt{1+a^2x^2}}
\end{aligned}$$

```
[Out] 1/2*x*arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)+1/5*arcsinh(a*x)^(5/2)*(a^2*c*
x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/128*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1
/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/128*erfi(2^(1/2)*arc
sinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-3
/16*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)/a/(a^2*x^2+1)^(1/2)-3/8*a*x^2*(a
^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)/(a^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used

$= \{5785, 5783, 5777, 5819, 3393, 3388, 2211, 2235, 2236\}$

$$\int \sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}\,dx = \frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a\sqrt{a^2x^2+1}} + \frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a\sqrt{a^2x^2+1}} + \frac{\operatorname{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}}{5a\sqrt{a^2x^2+1}} + \frac{1}{2}x\operatorname{arcsinh}(ax)^{3/2}\sqrt{a^2cx^2+c} - \frac{3ax^2\sqrt{\operatorname{arcsinh}(ax)}\sqrt{a^2cx^2+c}}{8\sqrt{a^2x^2+1}} - \frac{3\sqrt{\operatorname{arcsinh}(ax)}\sqrt{a^2cx^2+c}}{16a\sqrt{a^2x^2+1}}$$

```
[In] Int[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-3*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(16*a*Sqrt[1 + a^2*x^2]) - (3*a
*x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(8*Sqrt[1 + a^2*x^2]) + (x*Sqr
t[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))/2 + (Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^
(5/2))/(5*a*Sqrt[1 + a^2*x^2]) + (3*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt
[2]*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2]) + (3*Sqrt[Pi/2]*Sqrt[c +
a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}x\sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^{3/2} + \frac{\sqrt{c+a^2cx^2}\int \frac{\text{arcsinh}(ax)^{3/2}}{\sqrt{1+a^2x^2}}\,dx}{2\sqrt{1+a^2x^2}} - \frac{\left(3a\sqrt{c+a^2cx^2}\right)\int x\sqrt{\text{arcsinh}(ax)}\,dx}{4\sqrt{1+a^2x^2}}$$

$$= -\frac{3ax^2\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2} + \frac{\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2}}{5a\sqrt{1+a^2x^2}} + \frac{\left(3a^2\sqrt{c+a^2cx^2}\right)\int \frac{x^2}{\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx}{16\sqrt{1+a^2x^2}}$$

$$= -\frac{3ax^2\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2} + \frac{\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2}}{5a\sqrt{1+a^2x^2}} + \frac{\left(3\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{\sinh^2(x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{16a\sqrt{1+a^2x^2}}$$

$$= -\frac{3ax^2\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2} + \frac{\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2}}{5a\sqrt{1+a^2x^2}} - \frac{\left(3\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \left(\frac{1}{2\sqrt{x}} - \frac{\cosh(2x)}{2\sqrt{x}}\right)dx, x, \operatorname{arcsinh}(ax)\right)}{16a\sqrt{1+a^2x^2}}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}{16a\sqrt{1+a^2x^2}} - \frac{3ax^2\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2} + \frac{\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2}}{5a\sqrt{1+a^2x^2}} + \frac{\left(3\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{32a\sqrt{1+a^2x^2}}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}{16a\sqrt{1+a^2x^2}} - \frac{3ax^2\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2} + \frac{\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2}}{5a\sqrt{1+a^2x^2}} + \frac{\left(3\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{64a\sqrt{1+a^2x^2}} + \frac{\left(3\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{64a\sqrt{1+a^2x^2}}$$

$$
\begin{aligned}
&= -\frac{3\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}{16a\sqrt{1+a^2x^2}} - \frac{3ax^2\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}{8\sqrt{1+a^2x^2}} \\
&\quad + \frac{1}{2}x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2} + \frac{\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2}}{5a\sqrt{1+a^2x^2}} \\
&\quad + \frac{\left(3\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{32a\sqrt{1+a^2x^2}} \\
&\quad + \frac{\left(3\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{32a\sqrt{1+a^2x^2}} \\
&= -\frac{3\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}{16a\sqrt{1+a^2x^2}} - \frac{3ax^2\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}{8\sqrt{1+a^2x^2}} \\
&\quad + \frac{1}{2}x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2} + \frac{\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2}}{5a\sqrt{1+a^2x^2}} \\
&\quad + \frac{3\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a\sqrt{1+a^2x^2}} \\
&\quad + \frac{3\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a\sqrt{1+a^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.46

$$
\int \sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}\,dx = \frac{\sqrt{c\left(1+a^2x^2\right)}\left(15\sqrt{2\pi}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right) + 15\sqrt{2\pi}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\right.\right.}{}
$$

```
[In] Integrate[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2),x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(15*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + 15*
Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + 8*Sqrt[ArcSinh[a*x]]*(-15*Cos
h[2*ArcSinh[a*x]] + 4*ArcSinh[a*x]*(4*ArcSinh[a*x] + 5*Sinh[2*ArcSinh[a*x]]
))))/(640*a*Sqrt[1 + a^2*x^2])
```

Maple [F]

$$\int \operatorname{arcsinh}(ax)^{\frac{3}{2}} \sqrt{a^2c\,x^2+c}dx$$

```
[In] int(arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
[Out] int(arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}\,dx = \int \sqrt{c\,(a^2x^2+1)}\operatorname{asinh}^{\frac{3}{2}}(ax)\,dx$$

```
[In] integrate(asinh(a*x)**(3/2)*(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*asinh(a*x)**(3/2), x)
```

Maxima [F]

$$\int \sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}\,dx = \int \sqrt{a^2cx^2+c}\operatorname{arsinh}(ax)^{\frac{3}{2}}\,dx$$

```
[In] integrate(arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}\,dx = \int \text{asinh}(a\,x)^{3/2}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

```
[In] int(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.479 $\int \frac{\text{arcsinh}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 42

$$\int \frac{\text{arcsinh}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx=\frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{5/2}}{5a\sqrt{c+a^2cx^2}}$$

```
[Out] 2/5*arcsinh(a*x)^(5/2)*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {5783}

$$\int \frac{\text{arcsinh}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx=\frac{2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{5/2}}{5a\sqrt{a^2cx^2+c}}$$

```
[In] Int[ArcSinh[a*x]^(3/2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(5/2))/(5*a*Sqrt[c + a^2*c*x^2])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\text{integral}=\frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{5/2}}{5a\sqrt{c+a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \frac{2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{5/2}}{5a\sqrt{c\left(1+a^2x^2\right)}}$$

```
[In] Integrate[ArcSinh[a*x]^(3/2)/Sqrt[c + a^2*c*x^2],x]
[Out] (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(5/2))/(5*a*Sqrt[c*(1 + a^2*x^2)])
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{2\operatorname{arcsinh}(ax)^{\frac{5}{2}}\sqrt{a^2x^2+1}}{5a\sqrt{c(a^2x^2+1)}}$	36

```
[In] int(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/5*arcsinh(a*x)^(5/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{asinh}^{\frac{3}{2}}(ax)}{\sqrt{c\,(a^2x^2+1)}}\,dx$$

```
[In] integrate(asinh(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(asinh(a*x)**(3/2)/sqrt(c*(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^{\frac{3}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

```
[In] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^(3/2)/sqrt(a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^{\frac{3}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

```
[In] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^(3/2)/sqrt(a^2*c*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)^{3/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

```
[In] int(asinh(a*x)^(3/2)/(c + a^2*c*x^2)^(1/2),x)
[Out] int(asinh(a*x)^(3/2)/(c + a^2*c*x^2)^(1/2), x)
```

3.480 $\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{x\operatorname{arcsinh}(ax)^{3/2}}{c\sqrt{c+a^2cx^2}} - \frac{3a\sqrt{1+a^2x^2}\operatorname{Int}\left(\frac{x\sqrt{\operatorname{arcsinh}(ax)}}{1+a^2x^2},x\right)}{2c\sqrt{c+a^2cx^2}}$$

```
[Out] x*arcsinh(a*x)^(3/2)/c/(a^2*c*x^2+c)^(1/2)-3/2*a*(a^2*x^2+1)^(1/2)*Unintegr
able(x*arcsinh(a*x)^(1/2)/(a^2*x^2+1),x)/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{arcsinh}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

```
[In] Int[ArcSinh[a*x]^(3/2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*ArcSinh[a*x]^(3/2))/(c*Sqrt[c + a^2*c*x^2]) - (3*a*Sqrt[1 + a^2*x^2]*Def
er[Int][(x*Sqrt[ArcSinh[a*x]])/(1 + a^2*x^2), x])/(2*c*Sqrt[c + a^2*c*x^2])
```

Rubi steps

$$\text{integral} = \frac{x\operatorname{arcsinh}(ax)^{3/2}}{c\sqrt{c+a^2cx^2}} - \frac{\left(3a\sqrt{1+a^2x^2}\right)\int \frac{x\sqrt{\operatorname{arcsinh}(ax)}}{1+a^2x^2}\,dx}{2c\sqrt{c+a^2cx^2}}$$

Mathematica [N/A]

Not integrable

Time = 1.91 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{arcsinh}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

```
[In] Integrate[ArcSinh[a*x]^(3/2)/(c + a^2*c*x^2)^(3/2),x]
[Out] Integrate[ArcSinh[a*x]^(3/2)/(c + a^2*c*x^2)^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{arcsinh}(ax)^{\frac{3}{2}}}{(a^2c\,x^2+c)^{\frac{3}{2}}}dx$$

```
[In] int(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
[Out] int(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 9.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{asinh}^{\frac{3}{2}}(ax)}{(c\,(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(asinh(a*x)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
[Out] Integral(asinh(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2 + c)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2 + c)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.79 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

```
[In] int(asinh(a*x)^(3/2)/(c + a^2*c*x^2)^(3/2),x)
[Out] int(asinh(a*x)^(3/2)/(c + a^2*c*x^2)^(3/2), x)
```

3.481 $\int (c+a^2cx^2)^{3/2} \operatorname{arcsinh}(ax)^{5/2} dx$

Optimal result

Integrand size = 23, antiderivative size = 514

$$\int (c+a^2cx^2)^{3/2} \operatorname{arcsinh}(ax)^{5/2} dx = \frac{225}{512}cx\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} + \frac{15}{256}cx(1+a^2x^2)\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} - \frac{45c\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{256a\sqrt{1+a^2x^2}} - \frac{15acx^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{32\sqrt{1+a^2x^2}} - \frac{5c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{32a} + \frac{3}{8}cx\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2} + \frac{1}{4}x(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^{5/2} + \frac{3c\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{7/2}}{28a\sqrt{1+a^2x^2}} + \frac{15c\sqrt{\pi}\sqrt{c-}}{1}$$

```
[Out] 1/4*x*(a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2)-5/32*c*(a^2*x^2+1)^(3/2)*arcsi
nh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a+3/8*c*x*arcsinh(a*x)^(5/2)*(a^2*c*x^2+c
)^(1/2)-45/256*c*arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)
-15/32*a*c*x^2*arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1/2)+3/2
8*c*arcsinh(a*x)^(7/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+15/512*c*erf
(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^
2+1)^(1/2)-15/512*c*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*
c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+15/16384*c*erf(2*arcsinh(a*x)^(1/2))*Pi^
(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-15/16384*c*erfi(2*arcsinh(a*x
)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+225/512*c*x*(a^2*
c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)+15/256*c*x*(a^2*x^2+1)*(a^2*c*x^2+c)^(1/2
)*arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {5786, 5785, 5783, 5777, 5812, 5780, 5556, 12, 3389, 2211, 2235, 2236, 5798, 5819}

$$\int \left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^{5/2}\,dx=\frac{15\sqrt{\pi}c\sqrt{a^2cx^2+c}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{16384a\sqrt{a^2x^2+1}}+\frac{15\sqrt{\frac{\pi}{2}}c\sqrt{a^2cx^2+c}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{256a\sqrt{a^2x^2+1}}-\frac{15\sqrt{\pi}c\sqrt{a^2cx^2+c}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{16384a\sqrt{a^2x^2+1}}-\frac{15\sqrt{\frac{\pi}{2}}c\sqrt{a^2cx^2+c}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{256a\sqrt{a^2x^2+1}}+\frac{3c\operatorname{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{28a\sqrt{a^2x^2+1}}+\frac{1}{4}x\operatorname{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}+\frac{3}{8}cx\operatorname{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}-\frac{5c(a^2x^2+1)^{3/2}\operatorname{arcsinh}(ax)^{3/2}\sqrt{a^2cx^2+}}{32a}$$

```
[In] Int[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2),x]
```

```
[Out] (225*c*x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/512 + (15*c*x*(1 + a^2*x^2
)*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/256 - (45*c*Sqrt[c + a^2*c*x^2]*A
rcSinh[a*x]^(3/2))/(256*a*Sqrt[1 + a^2*x^2]) - (15*a*c*x^2*Sqrt[c + a^2*c*x
^2]*ArcSinh[a*x]^(3/2))/(32*Sqrt[1 + a^2*x^2]) - (5*c*(1 + a^2*x^2)^(3/2)*S
qrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))/(32*a) + (3*c*x*Sqrt[c + a^2*c*x^2]*
ArcSinh[a*x]^(5/2))/8 + (x*(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2))/4 + (3
*c*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(7/2))/(28*a*Sqrt[1 + a^2*x^2]) + (15*c
*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(16384*a*Sqrt[1 +
a^2*x^2]) + (15*c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a
*x]]])/(256*a*Sqrt[1 + a^2*x^2]) - (15*c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erfi[
2*Sqrt[ArcSinh[a*x]]])/(16384*a*Sqrt[1 + a^2*x^2]) - (15*c*Sqrt[Pi/2]*Sqrt[
c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(256*a*Sqrt[1 + a^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
```

```
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^{5/2} + \frac{1}{4}(3c)\int\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2}\,dx - \frac{\left(5ac\sqrt{c+a^2cx^2}\right)\int x(1+a^2x^2)\operatorname{arcsinh}(ax)^{3/2}\,dx}{8\sqrt{1+a^2x^2}}$$

$$= -\frac{5c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}{32a} + \frac{3}{8}cx\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}$$
$$+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{5/2} + \frac{\left(15c\sqrt{c+a^2cx^2}\right)\int\left(1+a^2x^2\right)^{3/2}\sqrt{\text{arcsinh}(ax)}\,dx}{64\sqrt{1+a^2x^2}} + \frac{\left(3c\sqrt{c+a\right.}{}$$

$$= \frac{15}{256}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)} - \frac{15acx^2\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}{32\sqrt{1+a^2x^2}}$$
$$-\frac{5c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}{32a}$$
$$+\frac{3}{8}cx\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2} + \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{5/2} + \frac{3c\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{7/2}}{28a\sqrt{1+a^2x^2}} + \cdots$$

$$= \frac{225}{512}cx\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)} + \frac{15}{256}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}$$
$$-\frac{15acx^2\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}{32\sqrt{1+a^2x^2}} - \frac{5c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}{32a}$$
$$+\frac{3}{8}cx\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2} + \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{5/2} + \frac{3c\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{7/2}}{28a\sqrt{1+a^2x^2}} + \cdots$$

$$= \frac{225}{512}cx\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)} + \frac{15}{256}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}$$
$$-\frac{45c\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}{256a\sqrt{1+a^2x^2}} - \frac{15acx^2\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}{32\sqrt{1+a^2x^2}}$$
$$-\frac{5c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}{32a}$$
$$+\frac{3}{8}cx\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2} + \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{5/2} + \frac{3c\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{7/2}}{28a\sqrt{1+a^2x^2}} - \cdots$$

$$= \frac{225}{512}cx\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)} + \frac{15}{256}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}$$
$$-\frac{45c\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}{256a\sqrt{1+a^2x^2}} - \frac{15acx^2\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}{32\sqrt{1+a^2x^2}}$$
$$-\frac{5c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}{32a}$$
$$+\frac{3}{8}cx\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2} + \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{5/2} + \frac{3c\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{7/2}}{28a\sqrt{1+a^2x^2}} - \cdots$$

$$
\begin{aligned}
&= \frac{225}{512}cx\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} + \frac{15}{256}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} \\
&\quad - \frac{45c\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{256a\sqrt{1+a^2x^2}} - \frac{15acx^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{32\sqrt{1+a^2x^2}} \\
&\quad - \frac{5c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{32a} \\
&\quad + \frac{3}{8}cx\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2} + \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^{5/2} + \frac{3c\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{7/2}}{28a\sqrt{1+a^2x^2}} +
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{225}{512}cx\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} + \frac{15}{256}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} \\
&\quad - \frac{45c\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{256a\sqrt{1+a^2x^2}} - \frac{15acx^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{32\sqrt{1+a^2x^2}} \\
&\quad - \frac{5c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{32a} \\
&\quad + \frac{3}{8}cx\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2} + \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^{5/2} + \frac{3c\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{7/2}}{28a\sqrt{1+a^2x^2}} +
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{225}{512}cx\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} + \frac{15}{256}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} \\
&\quad - \frac{45c\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{256a\sqrt{1+a^2x^2}} - \frac{15acx^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{32\sqrt{1+a^2x^2}} \\
&\quad - \frac{5c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{32a} \\
&\quad + \frac{3}{8}cx\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2} + \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^{5/2} + \frac{3c\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{7/2}}{28a\sqrt{1+a^2x^2}} +
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{225}{512}cx\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} + \frac{15}{256}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} \\
&\quad - \frac{45c\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{256a\sqrt{1+a^2x^2}} - \frac{15acx^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{32\sqrt{1+a^2x^2}} \\
&\quad - \frac{5c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{32a} \\
&\quad + \frac{3}{8}cx\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2} + \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^{5/2} + \frac{3c\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{7/2}}{28a\sqrt{1+a^2x^2}} +
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.39

$$\int (c+a^2cx^2)^{3/2} \operatorname{arcsinh}(ax)^{5/2}\, dx = \frac{c\sqrt{c+a^2cx^2}\Big(1536\operatorname{arcsinh}(ax)^4 - 4480\operatorname{arcsinh}(ax)^2\cosh(2\operatorname{arcsinh}(ax)) + 420}{}$$

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2),x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(1536*ArcSinh[a*x]^4 - 4480*ArcSinh[a*x]^2*Cosh[2*Ar
cSinh[a*x]] + 420*Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*
x]]] - 420*Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] -
 7*Sqrt[-ArcSinh[a*x]]*Gamma[7/2, -4*ArcSinh[a*x]] - 7*Sqrt[ArcSinh[a*x]]*G
amma[7/2, 4*ArcSinh[a*x]] + 3360*ArcSinh[a*x]*Sinh[2*ArcSinh[a*x]] + 3584*A
rcSinh[a*x]^3*Sinh[2*ArcSinh[a*x]]))/(14336*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSin
h[a*x]])
```

Maple [F]

$$\int \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \operatorname{arcsinh}\left(a x \right)^{\frac{5}{2}}d x$$

```
[In] int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (c+a^2cx^2)^{3/2} \operatorname{arcsinh}(ax)^{5/2}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^{5/2}\,dx=\text{Timed out}$$

```
[In] integrate((a**2*c*x**2+c)**(3/2)*asinh(a*x)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^{5/2}\,dx=\int \left(a^2cx^2+c\right)^{\frac{3}{2}}\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^{5/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^{5/2}\,dx=\int \operatorname{asinh}(a\,x)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}\,dx$$

```
[In] int(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2),x)
[Out] int(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.482 $\int \sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 298

$$\int \sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^{5/2}\,dx = \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}$$
$$-\frac{5\sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^{3/2}}{16a\sqrt{1+a^2x^2}}-\frac{5ax^2\sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^{3/2}}{8\sqrt{1+a^2x^2}}$$
$$+\frac{1}{2}x\sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^{5/2}+\frac{\sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^{7/2}}{7a\sqrt{1+a^2x^2}}+\frac{15\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{256a\sqrt{1+a^2x^2}}-15\sqrt{\ldots}$$

```
[Out] 1/2*x*arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)-5/16*arcsinh(a*x)^(3/2)*(a^2*c
*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-5/8*a*x^2*arcsinh(a*x)^(3/2)*(a^2*c*x^2+c
)^(1/2)/(a^2*x^2+1)^(1/2)+1/7*arcsinh(a*x)^(7/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2
*x^2+1)^(1/2)+15/512*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*
c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-15/512*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*
2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+15/32*x*(a^2*c*x^2
+c)^(1/2)*arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules

used = {5785, 5783, 5777, 5812, 5780, 5556, 12, 3389, 2211, 2235, 2236}

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}\,dx = \frac{15\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{256a\sqrt{a^2x^2+1}} - \frac{15\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{256a\sqrt{a^2x^2+1}} + \frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}} + \frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c} - \frac{5ax^2\text{arcsinh}(ax)^{3/2}\sqrt{a^2cx^2+c}}{8\sqrt{a^2x^2+1}} - \frac{5\text{arcsinh}(ax)^{3/2}\sqrt{a^2cx^2+c}}{16a\sqrt{a^2x^2+1}} + \frac{15}{32}x\sqrt{\text{arcsinh}(ax)}\sqrt{a^2cx^2+c}$$

```
[In] Int[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2),x]
```

```
[Out] (15*x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/32 - (5*Sqrt[c + a^2*c*x^2]*A
rcSinh[a*x]^(3/2))/(16*a*Sqrt[1 + a^2*x^2]) - (5*a*x^2*Sqrt[c + a^2*c*x^2]*
ArcSinh[a*x]^(3/2))/(8*Sqrt[1 + a^2*x^2]) + (x*Sqrt[c + a^2*c*x^2]*ArcSinh[
a*x]^(5/2))/2 + (Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(7/2))/(7*a*Sqrt[1 + a^2*
x^2]) + (15*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])
/(256*a*Sqrt[1 + a^2*x^2]) - (15*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2
]*Sqrt[ArcSinh[a*x]]])/(256*a*Sqrt[1 + a^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
```

```
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}x\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2} + \frac{\sqrt{c+a^2cx^2}\int \frac{\text{arcsinh}(ax)^{5/2}}{\sqrt{1+a^2x^2}}\,dx}{2\sqrt{1+a^2x^2}} - \frac{\left(5a\sqrt{c+a^2cx^2}\right)\int x\text{arcsinh}(ax)^{3/2}\,dx}{4\sqrt{1+a^2x^2}}$$

$$= -\frac{5ax^2\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2} + \frac{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{7/2}}{7a\sqrt{1+a^2x^2}} + \frac{\left(15a^2\sqrt{c+a^2cx^2}\right)\int \frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{16\sqrt{1+a^2x^2}}$$

$$= \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)} - \frac{5ax^2\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2} + \frac{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{7/2}}{7a\sqrt{1+a^2x^2}} - \frac{\left(15\sqrt{c+a^2cx^2}\right)\int \frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{32\sqrt{1+a^2x^2}} - \frac{\left(15a\sqrt{c+a^2cx^2}\right)\int \frac{x}{\sqrt{\text{arcsinh}(ax)}}\,dx}{64\sqrt{1+a^2x^2}}$$

$$= \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)} - \frac{5\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}{16a\sqrt{1+a^2x^2}} - \frac{5ax^2\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2} + \frac{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{7/2}}{7a\sqrt{1+a^2x^2}} - \frac{\left(15\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{64a\sqrt{1+a^2x^2}}$$

$$= \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)} - \frac{5\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}{16a\sqrt{1+a^2x^2}} - \frac{5ax^2\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2} + \frac{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{7/2}}{7a\sqrt{1+a^2x^2}} - \frac{\left(15\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{64a\sqrt{1+a^2x^2}}$$

$$= \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} - \frac{5\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{16a\sqrt{1+a^2x^2}} - \frac{5ax^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2} + \frac{\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{7/2}}{7a\sqrt{1+a^2x^2}} - \frac{\left(15\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{128a\sqrt{1+a^2x^2}}$$

$$= \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} - \frac{5\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{16a\sqrt{1+a^2x^2}} - \frac{5ax^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2} + \frac{\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{7/2}}{7a\sqrt{1+a^2x^2}} + \frac{\left(15\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{256a\sqrt{1+a^2x^2}} - \frac{\left(15\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{256a\sqrt{1+a^2x^2}}$$

$$= \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} - \frac{5\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{16a\sqrt{1+a^2x^2}} - \frac{5ax^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2} + \frac{\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{7/2}}{7a\sqrt{1+a^2x^2}} + \frac{\left(15\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{128a\sqrt{1+a^2x^2}} - \frac{\left(15\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{128a\sqrt{1+a^2x^2}}$$

$$= \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} - \frac{5\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{16a\sqrt{1+a^2x^2}} - \frac{5ax^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2} + \frac{\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{7/2}}{7a\sqrt{1+a^2x^2}} + \frac{15\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{256a\sqrt{1+a^2x^2}} - \frac{15\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{256a\sqrt{1+a^2x^2}}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.45

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}\,dx = \frac{\sqrt{c(1+a^2x^2)}\left(105\sqrt{2\pi}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-105\sqrt{2\pi}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsi}\right.\right.}{}$$

```
[In] Integrate[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2),x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(105*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]] - 10
5*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + 8*Sqrt[ArcSinh[a*x]]*(64*Ar
cSinh[a*x]^3 - 140*ArcSinh[a*x]*Cosh[2*ArcSinh[a*x]] + 7*(15 + 16*ArcSinh[a
*x]^2)*Sinh[2*ArcSinh[a*x]])))/(3584*a*Sqrt[1 + a^2*x^2])
```

Maple [F]

$$\int \operatorname{arcsinh}(ax)^{\frac{5}{2}}\sqrt{a^2c\,x^2+c}dx$$

```
[In] int(arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(asinh(a*x)**(5/2)*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}\,dx = \int \sqrt{a^2cx^2+c}\,\text{arsinh}\,(ax)^{\frac{5}{2}}\,dx$$

```
[In] integrate(arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}\,dx = \int \text{asinh}(a\,x)^{5/2}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

```
[In] int(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2),x)
[Out] int(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.483 $\int \frac{\text{arcsinh}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 42

$$\int \frac{\text{arcsinh}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx=\frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{7/2}}{7a\sqrt{c+a^2cx^2}}$$

```
[Out] 2/7*arcsinh(a*x)^(7/2)*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {5783}

$$\int \frac{\text{arcsinh}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx=\frac{2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{7/2}}{7a\sqrt{a^2cx^2+c}}$$

```
[In] Int[ArcSinh[a*x]^(5/2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(7/2))/(7*a*Sqrt[c + a^2*c*x^2])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\text{integral}=\frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{7/2}}{7a\sqrt{c+a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx=\frac{2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{7/2}}{7a\sqrt{c(1+a^2x^2)}}$$

```
[In] Integrate[ArcSinh[a*x]^(5/2)/Sqrt[c + a^2*c*x^2],x]
[Out] (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(7/2))/(7*a*Sqrt[c*(1 + a^2*x^2)])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{2\operatorname{arcsinh}(ax)^{\frac{7}{2}}\sqrt{a^2x^2+1}}{7a\sqrt{c(a^2x^2+1)}}$	36

```
[In] int(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/7*arcsinh(a*x)^(7/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(asinh(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

```
[In] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^(5/2)/sqrt(a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

```
[In] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^(5/2)/sqrt(a^2*c*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)^{5/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

```
[In] int(asinh(a*x)^(5/2)/(c + a^2*c*x^2)^(1/2),x)
[Out] int(asinh(a*x)^(5/2)/(c + a^2*c*x^2)^(1/2), x)
```

3.484 $\int \frac{\text{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\text{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx=\frac{x\text{arcsinh}(ax)^{5/2}}{c\sqrt{c+a^2cx^2}}-\frac{5a\sqrt{1+a^2x^2}\text{Int}\left(\frac{x\text{arcsinh}(ax)^{3/2}}{1+a^2x^2},x\right)}{2c\sqrt{c+a^2cx^2}}$$

```
[Out] x*arcsinh(a*x)^(5/2)/c/(a^2*c*x^2+c)^(1/2)-5/2*a*(a^2*x^2+1)^(1/2)*Unintegr
able(x*arcsinh(a*x)^(3/2)/(a^2*x^2+1),x)/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx=\int \frac{\text{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

```
[In] Int[ArcSinh[a*x]^(5/2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*ArcSinh[a*x]^(5/2))/(c*Sqrt[c + a^2*c*x^2]) - (5*a*Sqrt[1 + a^2*x^2]*Def
er[Int][(x*ArcSinh[a*x]^(3/2))/(1 + a^2*x^2), x])/(2*c*Sqrt[c + a^2*c*x^2])
```

Rubi steps

$$\text{integral}=\frac{x\text{arcsinh}(ax)^{5/2}}{c\sqrt{c+a^2cx^2}}-\frac{\left(5a\sqrt{1+a^2x^2}\right)\int \frac{x\text{arcsinh}(ax)^{3/2}}{1+a^2x^2}\,dx}{2c\sqrt{c+a^2cx^2}}$$

Mathematica [N/A]

Not integrable

Time = 1.88 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

```
[In] Integrate[ArcSinh[a*x]^(5/2)/(c + a^2*c*x^2)^(3/2),x]
[Out] Integrate[ArcSinh[a*x]^(5/2)/(c + a^2*c*x^2)^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{arcsinh}\left(ax\right)^{\frac{5}{2}}}{\left(a^{2} c\,x^{2}+c\right)^{\frac{3}{2}}}dx$$

```
[In] int(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
[Out] int(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(asinh(a*x)**(5/2)/(a**2*c*x**2+c)**(3/2),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2 + c)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2 + c)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

```
[In] int(asinh(a*x)^(5/2)/(c + a^2*c*x^2)^(3/2),x)
[Out] int(asinh(a*x)^(5/2)/(c + a^2*c*x^2)^(3/2), x)
```

3.485 $\int (a^2+x^2)^{3/2} \sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 309

$$\int (a^2+x^2)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx = \frac{3}{8}a^2x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)} + \frac{1}{4}x(a^2+x^2)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)} + \frac{a^3\sqrt{a^2+x^2}\,\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{4\sqrt{1+\frac{x^2}{a^2}}} + \frac{a^3\sqrt{\pi}\sqrt{a^2+x^2}\,\text{erf}\left(2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{256\sqrt{1+\frac{x^2}{a^2}}} + \frac{a^3\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\,\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{16\sqrt{1+\frac{x^2}{a^2}}} - \frac{a^3\sqrt{\pi}\sqrt{a^2+x^2}\,\text{erfi}\left(2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{256\sqrt{1+\frac{x^2}{a^2}}} - \frac{a^3\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\,\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{16\sqrt{1+\frac{x^2}{a^2}}}$$

```
[Out] 1/4*a^3*arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+1/32*a^3*erf(2
^(1/2)*arcsinh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/
2)-1/32*a^3*erfi(2^(1/2)*arcsinh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/
2)/(1+x^2/a^2)^(1/2)+1/256*a^3*erf(2*arcsinh(x/a)^(1/2))*Pi^(1/2)*(a^2+x^2)
^(1/2)/(1+x^2/a^2)^(1/2)-1/256*a^3*erfi(2*arcsinh(x/a)^(1/2))*Pi^(1/2)*(a^2
+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+1/4*x*(a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2)+3/8*
a^2*x*(a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5786, 5785, 5783, 5780, 5556, 12, 3389, 2211, 2235, 2236, 5819}

$$\int \left(a^2+x^2\right)^{3/2} \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)} \, dx = \frac{3}{8}a^2x\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)} + \frac{\sqrt{\pi}a^3\sqrt{a^2+x^2}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{256\sqrt{\frac{x^2}{a^2}+1}} + \frac{\sqrt{\frac{\pi}{2}}a^3\sqrt{a^2+x^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x^2}{a^2}+1}} - \frac{\sqrt{\pi}a^3\sqrt{a^2+x^2}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{256\sqrt{\frac{x^2}{a^2}+1}} - \frac{\sqrt{\frac{\pi}{2}}a^3\sqrt{a^2+x^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x^2}{a^2}+1}} + \frac{a^3\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{4\sqrt{\frac{x^2}{a^2}+1}}$$

```
[In] Int[(a^2 + x^2)^(3/2)*Sqrt[ArcSinh[x/a]],x]
```

```
[Out] (3*a^2*x*Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/8 + (x*(a^2 + x^2)^(3/2)*Sqrt[
ArcSinh[x/a]])/4 + (a^3*Sqrt[a^2 + x^2]*ArcSinh[x/a]^(3/2))/(4*Sqrt[1 + x^2
/a^2]) + (a^3*Sqrt[Pi]*Sqrt[a^2 + x^2]*Erf[2*Sqrt[ArcSinh[x/a]]])/(256*Sqrt
[1 + x^2/a^2]) + (a^3*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[x
/a]]])/(16*Sqrt[1 + x^2/a^2]) - (a^3*Sqrt[Pi]*Sqrt[a^2 + x^2]*Erfi[2*Sqrt[A
rcSinh[x/a]]])/(256*Sqrt[1 + x^2/a^2]) - (a^3*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Er
fi[Sqrt[2]*Sqrt[ArcSinh[x/a]]])/(16*Sqrt[1 + x^2/a^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
```

```
+ c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)} \\
&\quad+\frac{1}{4}\left(3a^2\right)\int\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx-\frac{\left(a\sqrt{a^2+x^2}\right)\int\frac{x\left(1+\frac{x^2}{a^2}\right)}{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}\,dx}{8\sqrt{1+\frac{x^2}{a^2}}} \\
&= \frac{3}{8}a^2x\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}+\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)} \\
&\quad-\frac{\left(3a\sqrt{a^2+x^2}\right)\int\frac{x}{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}\,dx}{16\sqrt{1+\frac{x^2}{a^2}}}+\frac{\left(3a^2\sqrt{a^2+x^2}\right)\int\frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{1+\frac{x^2}{a^2}}}\,dx}{8\sqrt{1+\frac{x^2}{a^2}}} \\
&\quad-\frac{\left(a^3\sqrt{a^2+x^2}\right)\operatorname{Subst}\left(\int\frac{\cosh^3(x)\sinh(x)}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{8\sqrt{1+\frac{x^2}{a^2}}} \\
&= \frac{3}{8}a^2x\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)} \\
&\quad+\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}+\frac{a^3\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{4\sqrt{1+\frac{x^2}{a^2}}} \\
&\quad-\frac{\left(a^3\sqrt{a^2+x^2}\right)\operatorname{Subst}\left(\int\left(\frac{\sinh(2x)}{4\sqrt{x}}+\frac{\sinh(4x)}{8\sqrt{x}}\right)\,dx,x,\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{8\sqrt{1+\frac{x^2}{a^2}}} \\
&\quad-\frac{\left(3a^3\sqrt{a^2+x^2}\right)\operatorname{Subst}\left(\int\frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{16\sqrt{1+\frac{x^2}{a^2}}}
\end{aligned}$$

$$= \frac{3}{8}a^2x\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}$$
$$+ \frac{a^3\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{4\sqrt{1+\frac{x^2}{a^2}}} - \frac{\left(a^3\sqrt{a^2+x^2}\right)\operatorname{Subst}\left(\int \frac{\sinh(4x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{64\sqrt{1+\frac{x^2}{a^2}}}$$
$$- \frac{\left(a^3\sqrt{a^2+x^2}\right)\operatorname{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{32\sqrt{1+\frac{x^2}{a^2}}}$$
$$- \frac{\left(3a^3\sqrt{a^2+x^2}\right)\operatorname{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,dx, x, \operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{16\sqrt{1+\frac{x^2}{a^2}}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}$$
$$+ \frac{a^3\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{4\sqrt{1+\frac{x^2}{a^2}}} + \frac{\left(a^3\sqrt{a^2+x^2}\right)\operatorname{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{128\sqrt{1+\frac{x^2}{a^2}}}$$
$$- \frac{\left(a^3\sqrt{a^2+x^2}\right)\operatorname{Subst}\left(\int \frac{e^{4x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{128\sqrt{1+\frac{x^2}{a^2}}}$$
$$+ \frac{\left(a^3\sqrt{a^2+x^2}\right)\operatorname{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{64\sqrt{1+\frac{x^2}{a^2}}}$$
$$- \frac{\left(a^3\sqrt{a^2+x^2}\right)\operatorname{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{64\sqrt{1+\frac{x^2}{a^2}}}$$
$$- \frac{\left(3a^3\sqrt{a^2+x^2}\right)\operatorname{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{32\sqrt{1+\frac{x^2}{a^2}}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}$$
$$+ \frac{a^3\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{4\sqrt{1+\frac{x^2}{a^2}}} + \frac{\left(a^3\sqrt{a^2+x^2}\right)\operatorname{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{64\sqrt{1+\frac{x^2}{a^2}}}$$
$$- \frac{\left(a^3\sqrt{a^2+x^2}\right)\operatorname{Subst}\left(\int e^{4x^2}\,dx, x, \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{64\sqrt{1+\frac{x^2}{a^2}}}$$
$$+ \frac{\left(a^3\sqrt{a^2+x^2}\right)\operatorname{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{32\sqrt{1+\frac{x^2}{a^2}}}$$
$$- \frac{\left(a^3\sqrt{a^2+x^2}\right)\operatorname{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{32\sqrt{1+\frac{x^2}{a^2}}}$$
$$+ \frac{\left(3a^3\sqrt{a^2+x^2}\right)\operatorname{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{64\sqrt{1+\frac{x^2}{a^2}}}$$
$$- \frac{\left(3a^3\sqrt{a^2+x^2}\right)\operatorname{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{64\sqrt{1+\frac{x^2}{a^2}}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}$$
$$+ \frac{a^3\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{4\sqrt{1+\frac{x^2}{a^2}}} + \frac{a^3\sqrt{\pi}\sqrt{a^2+x^2}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{256\sqrt{1+\frac{x^2}{a^2}}}$$
$$+ \frac{a^3\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{64\sqrt{1+\frac{x^2}{a^2}}} - \frac{a^3\sqrt{\pi}\sqrt{a^2+x^2}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{256\sqrt{1+\frac{x^2}{a^2}}}$$
$$- \frac{a^3\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{64\sqrt{1+\frac{x^2}{a^2}}}$$
$$+ \frac{\left(3a^3\sqrt{a^2+x^2}\right)\operatorname{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{32\sqrt{1+\frac{x^2}{a^2}}}$$
$$- \frac{\left(3a^3\sqrt{a^2+x^2}\right)\operatorname{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{32\sqrt{1+\frac{x^2}{a^2}}}$$

$$=\frac{3}{8}a^2x\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}+\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}+\frac{a^3\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{4\sqrt{1+\frac{x^2}{a^2}}}$$
$$+\frac{a^3\sqrt{\pi}\sqrt{a^2+x^2}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{256\sqrt{1+\frac{x^2}{a^2}}}+\frac{a^3\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{16\sqrt{1+\frac{x^2}{a^2}}}$$
$$-\frac{a^3\sqrt{\pi}\sqrt{a^2+x^2}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{256\sqrt{1+\frac{x^2}{a^2}}}-\frac{a^3\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{16\sqrt{1+\frac{x^2}{a^2}}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.50

$$\int \left(a^2+x^2\right)^{3/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx=\frac{a^3\sqrt{a^2+x^2}\left(-\sqrt{-\operatorname{arcsinh}\left(\frac{x}{a}\right)}\Gamma\left(\frac{3}{2},-4\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)-8\sqrt{2}\sqrt{-\operatorname{arcsinh}\left(\frac{x}{a}\right)}\Gamma\left(\frac{3}{2},\right.\right.}{128}$$

```
[In] Integrate[(a^2 + x^2)^(3/2)*Sqrt[ArcSinh[x/a]],x]
```

```
[Out] (a^3*Sqrt[a^2 + x^2]*(-(Sqrt[-ArcSinh[x/a]]*Gamma[3/2, -4*ArcSinh[x/a]]) -
8*Sqrt[2]*Sqrt[-ArcSinh[x/a]]*Gamma[3/2, -2*ArcSinh[x/a]] + Sqrt[ArcSinh[x/
a]]*(32*ArcSinh[x/a]^(3/2) - 8*Sqrt[2]*Gamma[3/2, 2*ArcSinh[x/a]] - Gamma[3
/2, 4*ArcSinh[x/a]])))/(128*Sqrt[1 + x^2/a^2]*Sqrt[ArcSinh[x/a]])
```

Maple [F]

$$\int \left(a^2+x^2\right)^{\frac{3}{2}}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}dx$$

```
[In] int((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x)
```

```
[Out] int((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (a^2 + x^2)^{3/2} \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (a^2 + x^2)^{3/2} \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\, dx = \int (a^2 + x^2)^{\frac{3}{2}} \sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}\, dx$$

```
[In] integrate((a**2+x**2)**(3/2)*asinh(x/a)**(1/2),x)
[Out] Integral((a**2 + x**2)**(3/2)*sqrt(asinh(x/a)), x)
```

Maxima [F]

$$\int (a^2 + x^2)^{3/2} \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\, dx = \int (a^2 + x^2)^{\frac{3}{2}} \sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}\, dx$$

```
[In] integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x, algorithm="maxima")
[Out] integrate((a^2 + x^2)^(3/2)*sqrt(arcsinh(x/a)), x)
```

Giac [F]

$$\int (a^2 + x^2)^{3/2} \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\, dx = \int (a^2 + x^2)^{\frac{3}{2}} \sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}\, dx$$

```
[In] integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x, algorithm="giac")
[Out] integrate((a^2 + x^2)^(3/2)*sqrt(arcsinh(x/a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+x^2\right)^{3/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx=\int \sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}\,\left(a^2+x^2\right)^{3/2}\,dx$$

```
[In] int(asinh(x/a)^(1/2)*(a^2 + x^2)^(3/2),x)
[Out] int(asinh(x/a)^(1/2)*(a^2 + x^2)^(3/2), x)
```

3.486 $\int \sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 176

$$\int \sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx=\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}+\frac{a\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1+\frac{x^2}{a^2}}}+\frac{a\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{16\sqrt{1+\frac{x^2}{a^2}}}-\frac{a\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{16\sqrt{1+\frac{x^2}{a^2}}}$$

```
[Out] 1/3*a*arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+1/32*a*erf(2^(1/
2)*arcsinh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)-1
/32*a*erfi(2^(1/2)*arcsinh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/2)/(1+
x^2/a^2)^(1/2)+1/2*x*(a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used

$= \{5785, 5783, 5780, 5556, 12, 3389, 2211, 2235, 2236\}$

$$\int \sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx = \frac{\sqrt{\frac{\pi}{2}}a\sqrt{a^2+x^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x^2}{a^2}+1}} - \frac{\sqrt{\frac{\pi}{2}}a\sqrt{a^2+x^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x^2}{a^2}+1}} + \frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{\frac{x^2}{a^2}+1}} + \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}$$

```
[In] Int[Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]],x]
```

```
[Out] (x*Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/2 + (a*Sqrt[a^2 + x^2]*ArcSinh[x/a]^
(3/2))/(3*Sqrt[1 + x^2/a^2]) + (a*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erf[Sqrt[2]*Sq
rt[ArcSinh[x/a]]])/(16*Sqrt[1 + x^2/a^2]) - (a*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*E
rfi[Sqrt[2]*Sqrt[ArcSinh[x/a]]])/(16*Sqrt[1 + x^2/a^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)} + \frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{1+\frac{x^2}{a^2}}}\,dx}{2\sqrt{1+\frac{x^2}{a^2}}} - \frac{\sqrt{a^2+x^2}\int \frac{x}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}\,dx}{4a\sqrt{1+\frac{x^2}{a^2}}}$$

$$= \frac{1}{2} x \sqrt{a^2+x^2} \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)} + \frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1+\frac{x^2}{a^2}}} - \frac{\left(a\sqrt{a^2+x^2}\right) \operatorname{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{4\sqrt{1+\frac{x^2}{a^2}}}$$

$$= \frac{1}{2} x \sqrt{a^2+x^2} \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)} + \frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1+\frac{x^2}{a^2}}} - \frac{\left(a\sqrt{a^2+x^2}\right) \operatorname{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,dx, x, \operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{4\sqrt{1+\frac{x^2}{a^2}}}$$

$$= \frac{1}{2} x \sqrt{a^2+x^2} \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)} + \frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1+\frac{x^2}{a^2}}} - \frac{\left(a\sqrt{a^2+x^2}\right) \operatorname{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{8\sqrt{1+\frac{x^2}{a^2}}}$$

$$= \frac{1}{2} x \sqrt{a^2+x^2} \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)} + \frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1+\frac{x^2}{a^2}}} + \frac{\left(a\sqrt{a^2+x^2}\right) \operatorname{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{16\sqrt{1+\frac{x^2}{a^2}}} - \frac{\left(a\sqrt{a^2+x^2}\right) \operatorname{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{16\sqrt{1+\frac{x^2}{a^2}}}$$

$$= \frac{1}{2} x \sqrt{a^2+x^2} \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)} + \frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1+\frac{x^2}{a^2}}} + \frac{\left(a\sqrt{a^2+x^2}\right) \operatorname{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{8\sqrt{1+\frac{x^2}{a^2}}} - \frac{\left(a\sqrt{a^2+x^2}\right) \operatorname{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{8\sqrt{1+\frac{x^2}{a^2}}}$$

$$
= \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)} + \frac{a\sqrt{a^2+x^2}\,\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1+\frac{x^2}{a^2}}} + \frac{a\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\,\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{16\sqrt{1+\frac{x^2}{a^2}}} - \frac{a\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\,\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{16\sqrt{1+\frac{x^2}{a^2}}}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.62

$$
\int \sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx = \frac{a\sqrt{a^2+x^2}\left(16\text{arcsinh}\left(\frac{x}{a}\right)^2 - 3\sqrt{2}\sqrt{-\text{arcsinh}\left(\frac{x}{a}\right)}\Gamma\left(\frac{3}{2}, -2\text{arcsinh}\left(\frac{x}{a}\right)\right) - 3\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\Gamma\left(\frac{3}{2}, 2\text{arcsinh}\left(\frac{x}{a}\right.\right.\right.}{48\sqrt{1+\frac{x^2}{a^2}}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}
$$

```
[In] Integrate[Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]],x]
[Out] (a*Sqrt[a^2 + x^2]*(16*ArcSinh[x/a]^2 - 3*Sqrt[2]*Sqrt[-ArcSinh[x/a]]*Gamma
[3/2, -2*ArcSinh[x/a]] - 3*Sqrt[2]*Sqrt[ArcSinh[x/a]]*Gamma[3/2, 2*ArcSinh[
x/a]]))/(48*Sqrt[1 + x^2/a^2]*Sqrt[ArcSinh[x/a]])
```

Maple [F]

$$
\int \sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}dx
$$

```
[In] int((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x)
[Out] int((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$
\int \sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx = \text{Exception raised: TypeError}
$$

```
[In] integrate((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx = \int \sqrt{a^2+x^2}\sqrt{\text{asinh}\left(\frac{x}{a}\right)}\,dx$$

```
[In] integrate((a**2+x**2)**(1/2)*asinh(x/a)**(1/2),x)
[Out] Integral(sqrt(a**2 + x**2)*sqrt(asinh(x/a)), x)
```

Maxima [F]

$$\int \sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx = \int \sqrt{a^2+x^2}\sqrt{\text{arsinh}\left(\frac{x}{a}\right)}\,dx$$

```
[In] integrate((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a^2 + x^2)*sqrt(arcsinh(x/a)), x)
```

Giac [F]

$$\int \sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx = \int \sqrt{a^2+x^2}\sqrt{\text{arsinh}\left(\frac{x}{a}\right)}\,dx$$

```
[In] integrate((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a^2 + x^2)*sqrt(arcsinh(x/a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx = \int \sqrt{\text{asinh}\left(\frac{x}{a}\right)}\,\sqrt{a^2+x^2}\,dx$$

```
[In] int(asinh(x/a)^(1/2)*(a^2 + x^2)^(1/2),x)
[Out] int(asinh(x/a)^(1/2)*(a^2 + x^2)^(1/2), x)
```

3.487 $\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 39

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx = \frac{2a\sqrt{1+\frac{x^2}{a^2}}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2+x^2}}$$

```
[Out] 2/3*a*arcsinh(x/a)^(3/2)*(1+x^2/a^2)^(1/2)/(a^2+x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {5783}

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx = \frac{2a\sqrt{\frac{x^2}{a^2}+1}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2+x^2}}$$

```
[In] Int[Sqrt[ArcSinh[x/a]]/Sqrt[a^2 + x^2],x]
[Out] (2*a*Sqrt[1 + x^2/a^2]*ArcSinh[x/a]^(3/2))/(3*Sqrt[a^2 + x^2])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{2a\sqrt{1+\frac{x^2}{a^2}}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2+x^2}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx=\frac{2a\sqrt{1+\frac{x^2}{a^2}}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2+x^2}}$$

```
[In] Integrate[Sqrt[ArcSinh[x/a]]/Sqrt[a^2 + x^2],x]
[Out] (2*a*Sqrt[1 + x^2/a^2]*ArcSinh[x/a]^(3/2))/(3*Sqrt[a^2 + x^2])
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{2\operatorname{arcsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}a\sqrt{\frac{a^2+x^2}{a^2}}}{3\sqrt{a^2+x^2}}$	34

```
[In] int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*arcsinh(x/a)^(3/2)*a/(a^2+x^2)^(1/2)*((a^2+x^2)/a^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx=\frac{2}{3}\log\left(\frac{x+\sqrt{a^2+x^2}}{a}\right)^{\frac{3}{2}}$$

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(1/2),x, algorithm="fricas")
[Out] 2/3*log((x + sqrt(a^2 + x^2))/a)^(3/2)
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx=\int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx$$

```
[In] integrate(asinh(x/a)**(1/2)/(a**2+x**2)**(1/2),x)
[Out] Integral(sqrt(asinh(x/a))/sqrt(a**2 + x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx=\int \frac{\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx$$

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(arcsinh(x/a))/sqrt(a^2 + x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx=\int \frac{\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx$$

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(arcsinh(x/a))/sqrt(a^2 + x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx=\int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx$$

```
[In] int(asinh(x/a)^(1/2)/(a^2 + x^2)^(1/2),x)
[Out] int(asinh(x/a)^(1/2)/(a^2 + x^2)^(1/2), x)
```

3.488 $\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx=\frac{x\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2+x^2}}-\frac{\sqrt{1+\frac{x^2}{a^2}}\operatorname{Int}\left(\frac{x}{\left(1+\frac{x^2}{a^2}\right)\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}},x\right)}{2a^3\sqrt{a^2+x^2}}$$

```
[Out] x*arcsinh(x/a)^(1/2)/a^2/(a^2+x^2)^(1/2)-1/2*(1+x^2/a^2)^(1/2)*Unintegrable
(x/(1+x^2/a^2)/arcsinh(x/a)^(1/2),x)/a^3/(a^2+x^2)^(1/2)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx=\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx$$

```
[In] Int[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[ArcSinh[x/a]])/(a^2*Sqrt[a^2 + x^2]) - (Sqrt[1 + x^2/a^2]*Defer[Int
][x/((1 + x^2/a^2)*Sqrt[ArcSinh[x/a]]), x])/(2*a^3*Sqrt[a^2 + x^2])
```

Rubi steps

$$\text{integral}=\frac{x\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2+x^2}}-\frac{\sqrt{1+\frac{x^2}{a^2}}\int \frac{x}{\left(1+\frac{x^2}{a^2}\right)\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}\,dx}{2a^3\sqrt{a^2+x^2}}$$

Mathematica [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx=\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx$$

```
[In] Integrate[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(3/2),x]
[Out] Integrate[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{\frac{3}{2}}}dx$$

```
[In] int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x)
[Out] int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{3/2}}\,dx = \int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(asinh(x/a)**(1/2)/(a**2+x**2)**(3/2),x)
[Out] Integral(sqrt(asinh(x/a))/(a**2 + x**2)**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{3/2}}\,dx = \int \frac{\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(arcsinh(x/a))/(a^2 + x^2)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{3/2}}\,dx = \int \frac{\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(arcsinh(x/a))/(a^2 + x^2)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.89 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{3/2}}\,dx = \int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{3/2}}\,dx$$

```
[In] int(asinh(x/a)^(1/2)/(a^2 + x^2)^(3/2),x)
[Out] int(asinh(x/a)^(1/2)/(a^2 + x^2)^(3/2), x)
```

3.489 $\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx = \frac{x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{3a^2\left(a^2+x^2\right)^{3/2}}+\frac{2x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{3a^4\sqrt{a^2+x^2}} - \frac{\sqrt{1+\frac{x^2}{a^2}}\text{Int}\left(\frac{x}{\left(1+\frac{x^2}{a^2}\right)^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}},x\right)}{6a^5\sqrt{a^2+x^2}} - \frac{\sqrt{1+\frac{x^2}{a^2}}\text{Int}\left(\frac{x}{\left(1+\frac{x^2}{a^2}\right)\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}},x\right)}{3a^5\sqrt{a^2+x^2}}$$

```
[Out] 1/3*x*arcsinh(x/a)^(1/2)/a^2/(a^2+x^2)^(3/2)+2/3*x*arcsinh(x/a)^(1/2)/a^4/(
a^2+x^2)^(1/2)-1/6*(1+x^2/a^2)^(1/2)*Unintegrable(x/(1+x^2/a^2)^2/arcsinh(x
/a)^(1/2),x)/a^5/(a^2+x^2)^(1/2)-1/3*(1+x^2/a^2)^(1/2)*Unintegrable(x/(1+x^
2/a^2)/arcsinh(x/a)^(1/2),x)/a^5/(a^2+x^2)^(1/2)
```

Rubi [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx = \int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx$$

```
[In] Int[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[ArcSinh[x/a]])/(3*a^2*(a^2 + x^2)^(3/2)) + (2*x*Sqrt[ArcSinh[x/a]])
/(3*a^4*Sqrt[a^2 + x^2]) - (Sqrt[1 + x^2/a^2]*Defer[Int][x/((1 + x^2/a^2)^2
```

```
*Sqrt[ArcSinh[x/a]]), x])/(6*a^5*Sqrt[a^2 + x^2]) - (Sqrt[1 + x^2/a^2]*Defe
r[Int][x/((1 + x^2/a^2)*Sqrt[ArcSinh[x/a]]), x])/(3*a^5*Sqrt[a^2 + x^2])
```

Rubi steps

$$\text{integral} = \frac{x\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{3a^2(a^2+x^2)^{3/2}} + \frac{2\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{3/2}}\,dx}{3a^2} - \frac{\sqrt{1+\frac{x^2}{a^2}}\int \frac{x}{\left(1+\frac{x^2}{a^2}\right)^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}\,dx}{6a^5\sqrt{a^2+x^2}}$$

$$= \frac{x\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{3a^2(a^2+x^2)^{3/2}} + \frac{2x\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{3a^4\sqrt{a^2+x^2}} - \frac{\sqrt{1+\frac{x^2}{a^2}}\int \frac{x}{\left(1+\frac{x^2}{a^2}\right)^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}\,dx}{6a^5\sqrt{a^2+x^2}} - \frac{\sqrt{1+\frac{x^2}{a^2}}\int \frac{x}{\left(1+\frac{x^2}{a^2}\right)\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}\,dx}{3a^5\sqrt{a^2+x^2}}$$

Mathematica [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{5/2}}\,dx = \int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{5/2}}\,dx$$

```
[In] Integrate[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(5/2),x]
[Out] Integrate[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(5/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{\frac{5}{2}}}dx$$

```
[In] int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x)
[Out] int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 23.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{5/2}}\,dx = \int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(asinh(x/a)**(1/2)/(a**2+x**2)**(5/2),x)
```

```
[Out] Integral(sqrt(asinh(x/a))/(a**2 + x**2)**(5/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{5/2}}\,dx = \int \frac{\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arcsinh(x/a))/(a^2 + x^2)^(5/2), x)
```

Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx=\int \frac{\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x, algorithm="giac")
[Out] integrate(sqrt(arcsinh(x/a))/(a^2 + x^2)^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.76 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx=\int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx$$

```
[In] int(asinh(x/a)^(1/2)/(a^2 + x^2)^(5/2),x)
[Out] int(asinh(x/a)^(1/2)/(a^2 + x^2)^(5/2), x)
```

3.490 $\int (a^2+x^2)^{3/2} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx$

Optimal result

Integrand size = 22, antiderivative size = 433

$$\int (a^2+x^2)^{3/2} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx = -\frac{27a^3\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{256\sqrt{1+\frac{x^2}{a^2}}} - \frac{9ax^2\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}} - \frac{3(a^2+x^2)^{5/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{32a\sqrt{1+\frac{x^2}{a^2}}} + \frac{3}{8}a^2x\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} + \frac{1}{4}x(a^2+x^2)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} + \frac{3a^3\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{20\sqrt{1+\frac{x^2}{a^2}}} + \frac{3a^3\sqrt{\pi}\sqrt{a^2+ \cdots}}{20 \cdots}$$

```
[Out] 1/4*x*(a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2)+3/8*a^2*x*arcsinh(x/a)^(3/2)*(a^2+
x^2)^(1/2)+3/20*a^3*arcsinh(x/a)^(5/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+3/
128*a^3*erf(2^(1/2)*arcsinh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/2)/(1
+x^2/a^2)^(1/2)+3/128*a^3*erfi(2^(1/2)*arcsinh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)
*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+3/2048*a^3*erf(2*arcsinh(x/a)^(1/2))*Pi^
(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+3/2048*a^3*erfi(2*arcsinh(x/a)^(1/2
))*Pi^(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)-3/32*(a^2+x^2)^(5/2)*arcsinh(
x/a)^(1/2)/a/(1+x^2/a^2)^(1/2)-27/256*a^3*(a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2
)/(1+x^2/a^2)^(1/2)-9/32*a*x^2*(a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2)/(1+x^2/a^
2)^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {5786, 5785, 5783, 5777, 5819, 3393, 3388, 2211, 2235, 2236, 5798, 5791}

$$\int (a^2+x^2)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx=\frac{3}{8}a^2x\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}-\frac{9ax^2\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{32\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{4}x(a^2+x^2)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}-\frac{3(a^2+x^2)^{5/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{32a\sqrt{\frac{x^2}{a^2}+1}}+\frac{3\sqrt{\pi}a^3\sqrt{a^2+x^2}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{2048\sqrt{\frac{x^2}{a^2}+1}}+\frac{3\sqrt{\frac{\pi}{2}}a}{}$$

```
[In] Int[(a^2 + x^2)^(3/2)*ArcSinh[x/a]^(3/2),x]
```

```
[Out] (-27*a^3*Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/(256*Sqrt[1 + x^2/a^2]) - (9*a
*x^2*Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/(32*Sqrt[1 + x^2/a^2]) - (3*(a^2 +
 x^2)^(5/2)*Sqrt[ArcSinh[x/a]])/(32*a*Sqrt[1 + x^2/a^2]) + (3*a^2*x*Sqrt[a^
2 + x^2]*ArcSinh[x/a]^(3/2))/8 + (x*(a^2 + x^2)^(3/2)*ArcSinh[x/a]^(3/2))/4
 + (3*a^3*Sqrt[a^2 + x^2]*ArcSinh[x/a]^(5/2))/(20*Sqrt[1 + x^2/a^2]) + (3*a
^3*Sqrt[Pi]*Sqrt[a^2 + x^2]*Erf[2*Sqrt[ArcSinh[x/a]]])/(2048*Sqrt[1 + x^2/a
^2]) + (3*a^3*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[x/a]]])/(
64*Sqrt[1 + x^2/a^2]) + (3*a^3*Sqrt[Pi]*Sqrt[a^2 + x^2]*Erfi[2*Sqrt[ArcSinh
[x/a]]])/(2048*Sqrt[1 + x^2/a^2]) + (3*a^3*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erfi[
Sqrt[2]*Sqrt[ArcSinh[x/a]]])/(64*Sqrt[1 + x^2/a^2])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} + \frac{1}{4}\left(3a^2\right)\int\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx - \frac{\left(3a\sqrt{a^2+x^2}\right)\int x\left(1+\frac{x^2}{a^2}\right)\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx}{8\sqrt{1+\frac{x^2}{a^2}}}$$

$$= -\frac{3\left(a^2+x^2\right)^{5/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{32a\sqrt{1+\frac{x^2}{a^2}}} + \frac{3}{8}a^2x\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} - \frac{\left(9a\sqrt{a^2+x^2}\right)\int x\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx}{16\sqrt{1+\frac{x^2}{a^2}}} + \frac{\left(3a^2\sqrt{a^2+x^2}\right)\int\frac{\left(1+\frac{x^2}{a^2}\right)^{3/2}}{\sqrt{\operatorname{arcsinh}(}}}{64\sqrt{1+\frac{x^2}{a^2}}}$$

$$= -\frac{9ax^2\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}} - \frac{3\left(a^2+x^2\right)^{5/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{32a\sqrt{1+\frac{x^2}{a^2}}} + \frac{3}{8}a^2x\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} + \frac{3a^3\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{20\sqrt{1+\frac{x^2}{a^2}}} + \frac{\left(9\sqrt{a^2+x^2}\right)\int\frac{x^2}{\sqrt{1+\frac{x^2}{a^2}}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}\,d}{64\sqrt{1+\frac{x^2}{a^2}}}$$

$$= -\frac{9ax^2\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}} - \frac{3(a^2+x^2)^{5/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{32a\sqrt{1+\frac{x^2}{a^2}}} + \frac{3}{8}a^2x\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$
$$+ \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2} + \frac{3a^3\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{20\sqrt{1+\frac{x^2}{a^2}}} + \frac{\left(3a^3\sqrt{a^2+x^2}\right)\text{Subst}\left(\int\left(\frac{3}{8\sqrt{x}} + \text{co}\right.\right.}{64\sqrt{}}$$

$$= \frac{9a^3\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{256\sqrt{1+\frac{x^2}{a^2}}} - \frac{9ax^2\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}}$$
$$- \frac{3(a^2+x^2)^{5/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{32a\sqrt{1+\frac{x^2}{a^2}}} + \frac{3}{8}a^2x\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$
$$+ \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2} + \frac{3a^3\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{20\sqrt{1+\frac{x^2}{a^2}}} + \frac{\left(3a^3\sqrt{a^2+x^2}\right)\text{Subst}\left(\int\frac{\cosh(4x)}{\sqrt{x}}\,dx\right.}{512\sqrt{1+\frac{x^2}{a^2}}}$$

$$= -\frac{27a^3\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{256\sqrt{1+\frac{x^2}{a^2}}} - \frac{9ax^2\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}}$$
$$- \frac{3(a^2+x^2)^{5/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{32a\sqrt{1+\frac{x^2}{a^2}}} + \frac{3}{8}a^2x\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$
$$+ \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2} + \frac{3a^3\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{20\sqrt{1+\frac{x^2}{a^2}}} + \frac{\left(3a^3\sqrt{a^2+x^2}\right)\text{Subst}\left(\int\frac{e^{-4x}}{\sqrt{x}}\,dx, x\right.}{1024\sqrt{1+\frac{x^2}{a^2}}}$$

$$= -\frac{27a^3\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{256\sqrt{1+\frac{x^2}{a^2}}} - \frac{9ax^2\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}}$$
$$- \frac{3(a^2+x^2)^{5/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{32a\sqrt{1+\frac{x^2}{a^2}}} + \frac{3}{8}a^2x\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$
$$+ \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2} + \frac{3a^3\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{20\sqrt{1+\frac{x^2}{a^2}}} + \frac{\left(3a^3\sqrt{a^2+x^2}\right)\text{Subst}\left(\int e^{-4x^2}\,dx,\right.}{512\sqrt{1+\frac{x^2}{a^2}}}$$

$$= -\frac{27a^3\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{256\sqrt{1+\frac{x^2}{a^2}}} - \frac{9ax^2\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}} - \frac{3(a^2+x^2)^{5/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{32a\sqrt{1+\frac{x^2}{a^2}}} + \frac{3}{8}a^2x\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} + \frac{3a^3\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{20\sqrt{1+\frac{x^2}{a^2}}} + \frac{3a^3\sqrt{\pi}\sqrt{a^2+x^2}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right.}{2048\sqrt{1+\frac{x^2}{a^2}}}$$

$$= -\frac{27a^3\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{256\sqrt{1+\frac{x^2}{a^2}}} - \frac{9ax^2\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}} - \frac{3(a^2+x^2)^{5/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{32a\sqrt{1+\frac{x^2}{a^2}}} + \frac{3}{8}a^2x\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} + \frac{3a^3\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{20\sqrt{1+\frac{x^2}{a^2}}} + \frac{3a^3\sqrt{\pi}\sqrt{a^2+x^2}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right.}{2048\sqrt{1+\frac{x^2}{a^2}}}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.48

$$\int \left(a^2+x^2\right)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx = \frac{a^3\sqrt{a^2+x^2}\left(384\operatorname{arcsinh}\left(\frac{x}{a}\right)^3 - 480\operatorname{arcsinh}\left(\frac{x}{a}\right)\cosh\left(2\operatorname{arcsinh}\left(\frac{x}{a}\right)\right) + 60\sqrt{2\pi}\sqrt{\operatorname{ar}\right.}{}$$

```
[In] Integrate[(a^2 + x^2)^(3/2)*ArcSinh[x/a]^(3/2),x]
```

```
[Out] (a^3*Sqrt[a^2 + x^2]*(384*ArcSinh[x/a]^3 - 480*ArcSinh[x/a]*Cosh[2*ArcSinh[
x/a]] + 60*Sqrt[2*Pi]*Sqrt[ArcSinh[x/a]]*Erf[Sqrt[2]*Sqrt[ArcSinh[x/a]]] +
60*Sqrt[2*Pi]*Sqrt[ArcSinh[x/a]]*Erfi[Sqrt[2]*Sqrt[ArcSinh[x/a]]] + 5*Sqrt[
-ArcSinh[x/a]]*Gamma[5/2, -4*ArcSinh[x/a]] - 5*Sqrt[ArcSinh[x/a]]*Gamma[5/2
, 4*ArcSinh[x/a]] + 640*ArcSinh[x/a]^2*Sinh[2*ArcSinh[x/a]]))/(2560*Sqrt[1
+ x^2/a^2]*Sqrt[ArcSinh[x/a]])
```

Maple [F]

$$\int \left(a^2+x^2\right)^{\frac{3}{2}} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}} dx$$

```
[In] int((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x)
```

```
[Out] int((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(a^2+x^2\right)^{3/2} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a^2+x^2\right)^{3/2} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx = \text{Timed out}$$

```
[In] integrate((a**2+x**2)**(3/2)*asinh(x/a)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \left(a^2+x^2\right)^{3/2} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx = \int \left(a^2+x^2\right)^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}} dx$$

```
[In] integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2 + x^2)^(3/2)*arcsinh(x/a)^(3/2), x)
```

Giac [F]

$$\int \left(a^2+x^2\right)^{3/2} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx = \int \left(a^2+x^2\right)^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}} dx$$

```
[In] integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x, algorithm="giac")
[Out] integrate((a^2 + x^2)^(3/2)*arcsinh(x/a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+x^2\right)^{3/2} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx = \int \operatorname{asinh}\left(\frac{x}{a}\right)^{3/2} \left(a^2+x^2\right)^{3/2} dx$$

```
[In] int(asinh(x/a)^(3/2)*(a^2 + x^2)^(3/2),x)
[Out] int(asinh(x/a)^(3/2)*(a^2 + x^2)^(3/2), x)
```

3.491 $\int \sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 259

$$\int \sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx=-\frac{3a\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{16\sqrt{1+\frac{x^2}{a^2}}}-\frac{3x^2\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}+\frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{1+\frac{x^2}{a^2}}}+\frac{3a\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{64\sqrt{1+\frac{x^2}{a^2}}}+\frac{3a\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{64\sqrt{1+\frac{x^2}{a^2}}}$$

```
[Out] 1/2*x*arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2)+1/5*a*arcsinh(x/a)^(5/2)*(a^2+x^2)
^(1/2)/(1+x^2/a^2)^(1/2)+3/128*a*erf(2^(1/2)*arcsinh(x/a)^(1/2))*2^(1/2)*Pi
^(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+3/128*a*erfi(2^(1/2)*arcsinh(x/a)^
(1/2))*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)-3/16*a*(a^2+x^2)^
(1/2)*arcsinh(x/a)^(1/2)/(1+x^2/a^2)^(1/2)-3/8*x^2*(a^2+x^2)^(1/2)*arcsinh(
x/a)^(1/2)/a/(1+x^2/a^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {5785, 5783, 5777, 5819, 3393, 3388, 2211, 2235, 2236}

$$\int \sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx = \frac{3\sqrt{\frac{\pi}{2}}a\sqrt{a^2+x^2}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x^2}{a^2}+1}} + \frac{3\sqrt{\frac{\pi}{2}}a\sqrt{a^2+x^2}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x^2}{a^2}+1}} + \frac{a\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}} + \frac{1}{2}x\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2} - \frac{3x^2\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{8a\sqrt{\frac{x^2}{a^2}+1}} - \frac{3a\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{16\sqrt{\frac{x^2}{a^2}+1}}$$

```
[In] Int[Sqrt[a^2 + x^2]*ArcSinh[x/a]^(3/2),x]
```

```
[Out] (-3*a*Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/(16*Sqrt[1 + x^2/a^2]) - (3*x^2*S
qrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/(8*a*Sqrt[1 + x^2/a^2]) + (x*Sqrt[a^2 +
x^2]*ArcSinh[x/a]^(3/2))/2 + (a*Sqrt[a^2 + x^2]*ArcSinh[x/a]^(5/2))/(5*Sqrt
[1 + x^2/a^2]) + (3*a*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[x
/a]]])/(64*Sqrt[1 + x^2/a^2]) + (3*a*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erfi[Sqrt[2
]*Sqrt[ArcSinh[x/a]]])/(64*Sqrt[1 + x^2/a^2])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}x\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2} + \frac{\sqrt{a^2+x^2}\int \frac{\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{1+\frac{x^2}{a^2}}}\,dx}{2\sqrt{1+\frac{x^2}{a^2}}} - \frac{\left(3\sqrt{a^2+x^2}\right)\int x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx}{4a\sqrt{1+\frac{x^2}{a^2}}}$$

$$= -\frac{3x^2\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}} + \frac{1}{2}x\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2} + \frac{a\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{1+\frac{x^2}{a^2}}} + \frac{\left(3\sqrt{a^2+x^2}\right)\int \frac{x^2}{\sqrt{1+\frac{x^2}{a^2}}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}\,dx}{16a^2\sqrt{1+\frac{x^2}{a^2}}}$$

$$= -\frac{3x^2\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}} + \frac{1}{2}x\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2} + \frac{a\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{1+\frac{x^2}{a^2}}} + \frac{\left(3a\sqrt{a^2+x^2}\right)\text{Subst}\left(\int \frac{\sinh^2(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}\left(\frac{x}{a}\right)\right)}{16\sqrt{1+\frac{x^2}{a^2}}}$$

$$= -\frac{3x^2\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}} + \frac{1}{2}x\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2} + \frac{a\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{1+\frac{x^2}{a^2}}} - \frac{\left(3a\sqrt{a^2+x^2}\right)\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}} - \frac{\cosh(2x)}{2\sqrt{x}}\right)dx, x, \text{arcsinh}\left(\frac{x}{a}\right)\right)}{16\sqrt{1+\frac{x^2}{a^2}}}$$

$$= -\frac{3a\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{16\sqrt{1+\frac{x^2}{a^2}}} - \frac{3x^2\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}} + \frac{1}{2}x\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2} + \frac{a\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{1+\frac{x^2}{a^2}}} + \frac{\left(3a\sqrt{a^2+x^2}\right)\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx, x, \text{arcsinh}\left(\frac{x}{a}\right)\right)}{32\sqrt{1+\frac{x^2}{a^2}}}$$

$$= -\frac{3a\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{16\sqrt{1+\frac{x^2}{a^2}}} - \frac{3x^2\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}} + \frac{1}{2}x\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} + \frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{1+\frac{x^2}{a^2}}} + \frac{\left(3a\sqrt{a^2+x^2}\right)\operatorname{Subst}\left(\int\frac{e^{-2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{64\sqrt{1+\frac{x^2}{a^2}}} + \frac{\left(3a\sqrt{a^2+x^2}\right)\operatorname{Subst}\left(\int\frac{e^{2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{64\sqrt{1+\frac{x^2}{a^2}}}$$

$$= -\frac{3a\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{16\sqrt{1+\frac{x^2}{a^2}}} - \frac{3x^2\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}} + \frac{1}{2}x\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} + \frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{1+\frac{x^2}{a^2}}} + \frac{\left(3a\sqrt{a^2+x^2}\right)\operatorname{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{32\sqrt{1+\frac{x^2}{a^2}}} + \frac{\left(3a\sqrt{a^2+x^2}\right)\operatorname{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{32\sqrt{1+\frac{x^2}{a^2}}}$$

$$= -\frac{3a\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{16\sqrt{1+\frac{x^2}{a^2}}} - \frac{3x^2\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}} + \frac{1}{2}x\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} + \frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{1+\frac{x^2}{a^2}}} + \frac{3a\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{64\sqrt{1+\frac{x^2}{a^2}}} + \frac{3a\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{64\sqrt{1+\frac{x^2}{a^2}}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.51

$$\int \sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx = \frac{a\sqrt{a^2+x^2}\left(15\sqrt{2\pi}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)+15\sqrt{2\pi}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)\right.}{}$$

```
[In] Integrate[Sqrt[a^2 + x^2]*ArcSinh[x/a]^(3/2),x]
```

```
[Out] (a*Sqrt[a^2 + x^2]*(15*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcSinh[x/a]]] + 15*Sqrt
[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[x/a]]] + 8*Sqrt[ArcSinh[x/a]]*(16*ArcSinh[
x/a]^2 - 15*Cosh[2*ArcSinh[x/a]] + 20*ArcSinh[x/a]*Sinh[2*ArcSinh[x/a]])))/
(640*Sqrt[1 + x^2/a^2])
```

Maple [F]

$$\int \text{arcsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}\sqrt{a^2+x^2}\,dx$$

```
[In] int(arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2),x)
```

```
[Out] int(arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx = \int \sqrt{a^2+x^2}\,\text{asinh}^{\frac{3}{2}}\left(\frac{x}{a}\right)dx$$

```
[In] integrate(asinh(x/a)**(3/2)*(a**2+x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + x**2)*asinh(x/a)**(3/2), x)
```

Maxima [F]

$$\int \sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx=\int \sqrt{a^2+x^2}\,\text{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a^2 + x^2)*arcsinh(x/a)^(3/2), x)
```

Giac [F]

$$\int \sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx=\int \sqrt{a^2+x^2}\,\text{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a^2 + x^2)*arcsinh(x/a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx=\int \text{asinh}\left(\frac{x}{a}\right)^{3/2}\,\sqrt{a^2+x^2}\,dx$$

```
[In] int(asinh(x/a)^(3/2)*(a^2 + x^2)^(1/2),x)
[Out] int(asinh(x/a)^(3/2)*(a^2 + x^2)^(1/2), x)
```

3.492 $\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 39

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx=\frac{2a\sqrt{1+\frac{x^2}{a^2}}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2+x^2}}$$

```
[Out] 2/5*a*arcsinh(x/a)^(5/2)*(1+x^2/a^2)^(1/2)/(a^2+x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {5783}

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx=\frac{2a\sqrt{\frac{x^2}{a^2}+1}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2+x^2}}$$

```
[In] Int[ArcSinh[x/a]^(3/2)/Sqrt[a^2 + x^2],x]
[Out] (2*a*Sqrt[1 + x^2/a^2]*ArcSinh[x/a]^(5/2))/(5*Sqrt[a^2 + x^2])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\text{integral}=\frac{2a\sqrt{1+\frac{x^2}{a^2}}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2+x^2}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx=\frac{2a\sqrt{1+\frac{x^2}{a^2}}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2+x^2}}$$

```
[In] Integrate[ArcSinh[x/a]^(3/2)/Sqrt[a^2 + x^2],x]
[Out] (2*a*Sqrt[1 + x^2/a^2]*ArcSinh[x/a]^(5/2))/(5*Sqrt[a^2 + x^2])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{2\operatorname{arcsinh}\left(\frac{x}{a}\right)^{\frac{5}{2}} a\sqrt{\frac{a^2+x^2}{a^2}}}{5\sqrt{a^2+x^2}}$	34

```
[In] int(arcsinh(x/a)^(3/2)/(a^2+x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/5*arcsinh(x/a)^(5/2)*a/(a^2+x^2)^(1/2)*((a^2+x^2)/a^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.51

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx=\frac{2}{5}\log\left(\frac{x+\sqrt{a^2+x^2}}{a}\right)^{\frac{5}{2}}$$

```
[In] integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(1/2),x, algorithm="fricas")
[Out] 2/5*log((x + sqrt(a^2 + x^2))/a)^(5/2)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx=\int \frac{\operatorname{asinh}^{\frac{3}{2}}\left(\frac{x}{a}\right)}{\sqrt{a^2+x^2}}\,dx$$

```
[In] integrate(asinh(x/a)**(3/2)/(a**2+x**2)**(1/2),x)
[Out] Integral(asinh(x/a)**(3/2)/sqrt(a**2 + x**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx=\int \frac{\operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\sqrt{a^2+x^2}}\,dx$$

```
[In] integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(arcsinh(x/a)^(3/2)/sqrt(a^2 + x^2), x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx=\int \frac{\operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\sqrt{a^2+x^2}}\,dx$$

```
[In] integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(x/a)^(3/2)/sqrt(a^2 + x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx=\int \frac{\operatorname{asinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx$$

```
[In] int(asinh(x/a)^(3/2)/(a^2 + x^2)^(1/2),x)
[Out] int(asinh(x/a)^(3/2)/(a^2 + x^2)^(1/2), x)
```

3.493 $\int \frac{\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{(a^2+x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{(a^2+x^2)^{3/2}}\,dx = \frac{x\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2+x^2}} - \frac{3\sqrt{1+\frac{x^2}{a^2}}\,\text{Int}\left(\frac{x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{1+\frac{x^2}{a^2}},x\right)}{2a^3\sqrt{a^2+x^2}}$$

```
[Out] x*arcsinh(x/a)^(3/2)/a^2/(a^2+x^2)^(1/2)-3/2*(1+x^2/a^2)^(1/2)*Unintegrable
(x*arcsinh(x/a)^(1/2)/(1+x^2/a^2),x)/a^3/(a^2+x^2)^(1/2)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{(a^2+x^2)^{3/2}}\,dx = \int \frac{\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{(a^2+x^2)^{3/2}}\,dx$$

```
[In] Int[ArcSinh[x/a]^(3/2)/(a^2 + x^2)^(3/2),x]
```

```
[Out] (x*ArcSinh[x/a]^(3/2))/(a^2*Sqrt[a^2 + x^2]) - (3*Sqrt[1 + x^2/a^2]*Defer[I
nt][(x*Sqrt[ArcSinh[x/a]])/(1 + x^2/a^2), x])/(2*a^3*Sqrt[a^2 + x^2])
```

Rubi steps

$$\text{integral} = \frac{x\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2+x^2}} - \frac{\left(3\sqrt{1+\frac{x^2}{a^2}}\right)\int \frac{x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{1+\frac{x^2}{a^2}}\,dx}{2a^3\sqrt{a^2+x^2}}$$

Mathematica [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{(a^2+x^2)^{3/2}}\,dx=\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{(a^2+x^2)^{3/2}}\,dx$$

```
[In] Integrate[ArcSinh[x/a]^(3/2)/(a^2 + x^2)^(3/2),x]
[Out] Integrate[ArcSinh[x/a]^(3/2)/(a^2 + x^2)^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{(a^2+x^2)^{\frac{3}{2}}}dx$$

```
[In] int(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x)
[Out] int(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{(a^2+x^2)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 9.84 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{(a^2+x^2)^{3/2}}\,dx = \int \frac{\operatorname{asinh}^{\frac{3}{2}}\left(\frac{x}{a}\right)}{(a^2+x^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(asinh(x/a)**(3/2)/(a**2+x**2)**(3/2),x)
[Out] Integral(asinh(x/a)**(3/2)/(a**2 + x**2)**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{(a^2+x^2)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{(a^2+x^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x, algorithm="maxima")
[Out] integrate(arcsinh(x/a)^(3/2)/(a^2 + x^2)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{(a^2+x^2)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{(a^2+x^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x, algorithm="giac")
[Out] integrate(arcsinh(x/a)^(3/2)/(a^2 + x^2)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}}\,dx=\int \frac{\operatorname{asinh}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}}\,dx$$

```
[In] int(asinh(x/a)^(3/2)/(a^2 + x^2)^(3/2),x)
[Out] int(asinh(x/a)^(3/2)/(a^2 + x^2)^(3/2), x)
```

3.494 $\int \frac{x}{\sqrt{1+x^2}\sqrt{\text{arcsinh}(x)}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 33

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{\text{arcsinh}(x)}} \, dx = -\frac{1}{2}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(x)}\right) + \frac{1}{2}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(x)}\right)$$

```
[Out] -1/2*erf(arcsinh(x)^(1/2))*Pi^(1/2)+1/2*erfi(arcsinh(x)^(1/2))*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5819, 3389, 2211, 2235, 2236}

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{\text{arcsinh}(x)}} \, dx = \frac{1}{2}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(x)}\right) - \frac{1}{2}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(x)}\right)$$

```
[In] Int[x/(Sqrt[1 + x^2]*Sqrt[ArcSinh[x]]),x]
```

```
[Out] -1/2*(Sqrt[Pi]*Erf[Sqrt[ArcSinh[x]]]) + (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[x]]])/2
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(x)\right) \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(x)\right)\right) + \frac{1}{2}\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(x)\right) \\
&= -\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(x)}\right) + \text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(x)}\right) \\
&= -\frac{1}{2}\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(x)}\right) + \frac{1}{2}\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(x)}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{\operatorname{arcsinh}(x)}}\,dx = \frac{1}{2}\left(\frac{\sqrt{-\operatorname{arcsinh}(x)}\,\Gamma\left(\frac{1}{2}, -\operatorname{arcsinh}(x)\right)}{\sqrt{\operatorname{arcsinh}(x)}} + \Gamma\left(\frac{1}{2}, \operatorname{arcsinh}(x)\right)\right)$$

```
[In] Integrate[x/(Sqrt[1 + x^2]*Sqrt[ArcSinh[x]]),x]
[Out] ((Sqrt[-ArcSinh[x]]*Gamma[1/2, -ArcSinh[x]])/Sqrt[ArcSinh[x]] + Gamma[1/2,
ArcSinh[x]])/2
```

Maple [F]

$$\int \frac{x}{\sqrt{x^2+1}\,\sqrt{\operatorname{arcsinh}\left(x\right)}}dx$$

```
[In] int(x/(x^2+1)^(1/2)/arcsinh(x)^(1/2),x)
```

```
[Out] int(x/(x^2+1)^(1/2)/arcsinh(x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{\operatorname{arcsinh}(x)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/(x^2+1)^(1/2)/arcsinh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{\operatorname{arcsinh}(x)}}\,dx = \int \frac{x}{\sqrt{x^2+1}\sqrt{\operatorname{asinh}\left(x\right)}}\,dx$$

```
[In] integrate(x/(x**2+1)**(1/2)/asinh(x)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(x**2 + 1)*sqrt(asinh(x))), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{\operatorname{arcsinh}(x)}}\,dx = \int \frac{x}{\sqrt{x^2+1}\sqrt{\operatorname{arsinh}\left(x\right)}}\,dx$$

```
[In] integrate(x/(x^2+1)^(1/2)/arcsinh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^2 + 1)*sqrt(arcsinh(x))), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{\text{arcsinh}(x)}}\,dx = \int \frac{x}{\sqrt{x^2+1}\sqrt{\text{arsinh}\,(x)}}\,dx$$

[In] integrate(x/(x^2+1)^(1/2)/arcsinh(x)^(1/2),x, algorithm="giac")

[Out] integrate(x/(sqrt(x^2 + 1)*sqrt(arcsinh(x))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{\text{arcsinh}(x)}}\,dx = \int \frac{x}{\sqrt{\text{asinh}\,(x)}\,\sqrt{x^2+1}}\,dx$$

[In] int(x/(asinh(x)^(1/2)*(x^2 + 1)^(1/2)),x)

[Out] int(x/(asinh(x)^(1/2)*(x^2 + 1)^(1/2)), x)

3.495 $\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\textbf{arcsinh}(ax)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 396

$$\begin{aligned}\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx &= \frac{5c^2\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}{8a\sqrt{1+a^2x^2}} \\ &+ \frac{3c^2\sqrt{\pi}\sqrt{c+a^2cx^2}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a\sqrt{1+a^2x^2}} \\ &+ \frac{15c^2\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a\sqrt{1+a^2x^2}} \\ &+ \frac{c^2\sqrt{\frac{\pi}{6}}\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{6}\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a\sqrt{1+a^2x^2}} + \frac{3c^2\sqrt{\pi}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a\sqrt{1+a^2x^2}} \\ &+ \frac{15c^2\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a\sqrt{1+a^2x^2}} \\ &+ \frac{c^2\sqrt{\frac{\pi}{6}}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(\sqrt{6}\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a\sqrt{1+a^2x^2}}\end{aligned}$$

```
[Out] 1/384*c^2*erf(6^(1/2)*arcsinh(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1
/2)/a/(a^2*x^2+1)^(1/2)+1/384*c^2*erfi(6^(1/2)*arcsinh(a*x)^(1/2))*6^(1/2)*
Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+15/128*c^2*erf(2^(1/2)*arc
sinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1
5/128*c^2*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(
1/2)/a/(a^2*x^2+1)^(1/2)+3/64*c^2*erf(2*arcsinh(a*x)^(1/2))*Pi^(1/2)*(a^2*c
*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/64*c^2*erfi(2*arcsinh(a*x)^(1/2))*Pi^(1
/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+5/8*c^2*(a^2*c*x^2+c)^(1/2)*arc
sinh(a*x)^(1/2)/a/(a^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5791, 3393, 3388, 2211, 2235, 2236}

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\text{arcsinh}(ax)}}\,dx = \frac{3\sqrt{\pi}c^2\sqrt{a^2cx^2+c}\,\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{64a\sqrt{a^2x^2+1}} + \frac{15\sqrt{\frac{\pi}{2}}c^2\sqrt{a^2cx^2+c}\,\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{64a\sqrt{a^2x^2+1}} + \frac{\sqrt{\frac{\pi}{6}}c^2\sqrt{a^2cx^2+c}\,\text{erf}\left(\sqrt{6}\sqrt{\text{arcsinh}(ax)}\right)}{64a\sqrt{a^2x^2+1}} + \frac{3\sqrt{\pi}c^2\sqrt{a^2cx^2+c}\,\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{64a\sqrt{a^2x^2+1}} + \frac{15\sqrt{\frac{\pi}{2}}c^2\sqrt{a^2cx^2+c}\,\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{64a\sqrt{a^2x^2+1}} + \frac{\sqrt{\frac{\pi}{6}}c^2\sqrt{a^2cx^2+c}\,\text{erfi}\left(\sqrt{6}\sqrt{\text{arcsinh}(ax)}\right)}{64a\sqrt{a^2x^2+1}} + \frac{5c^2\sqrt{\text{arcsinh}(ax)}\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}$$

```
[In] Int[(c + a^2*c*x^2)^(5/2)/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (5*c^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(8*a*Sqrt[1 + a^2*x^2]) + (3
*c^2*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 +
 a^2*x^2]) + (15*c^2*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSin
h[a*x]]])/(64*a*Sqrt[1 + a^2*x^2]) + (c^2*Sqrt[Pi/6]*Sqrt[c + a^2*c*x^2]*Er
f[Sqrt[6]*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2]) + (3*c^2*Sqrt[Pi]*S
qrt[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2]) + (
15*c^2*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(64
*a*Sqrt[1 + a^2*x^2]) + (c^2*Sqrt[Pi/6]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[6]*Sq
rt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh^6(x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{a\sqrt{1+a^2x^2}}\\
&= \frac{\left(c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \left(\frac{5}{16\sqrt{x}}+\frac{15\cosh(2x)}{32\sqrt{x}}+\frac{3\cosh(4x)}{16\sqrt{x}}+\frac{\cosh(6x)}{32\sqrt{x}}\right)\,dx, x, \operatorname{arcsinh}(ax)\right)}{a\sqrt{1+a^2x^2}}\\
&= \frac{5c^2\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}{8a\sqrt{1+a^2x^2}}+\frac{\left(c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(6x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{32a\sqrt{1+a^2x^2}}\\
&\quad+\frac{\left(3c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{16a\sqrt{1+a^2x^2}}\\
&\quad+\frac{\left(15c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{32a\sqrt{1+a^2x^2}}
\end{aligned}
$$

$$=\frac{5c^2\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}{8a\sqrt{1+a^2x^2}}+\frac{\left(c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int\frac{e^{-6x}}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{64a\sqrt{1+a^2x^2}}$$
$$+\frac{\left(c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int\frac{e^{6x}}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{64a\sqrt{1+a^2x^2}}$$
$$+\frac{\left(3c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int\frac{e^{-4x}}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{32a\sqrt{1+a^2x^2}}$$
$$+\frac{\left(3c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int\frac{e^{4x}}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{32a\sqrt{1+a^2x^2}}$$
$$+\frac{\left(15c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int\frac{e^{-2x}}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{64a\sqrt{1+a^2x^2}}$$
$$+\frac{\left(15c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int\frac{e^{2x}}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{64a\sqrt{1+a^2x^2}}$$

$$=\frac{5c^2\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}{8a\sqrt{1+a^2x^2}}$$
$$+\frac{\left(c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int e^{-6x^2}\,dx,x,\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a\sqrt{1+a^2x^2}}$$
$$+\frac{\left(c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int e^{6x^2}\,dx,x,\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a\sqrt{1+a^2x^2}}$$
$$+\frac{\left(3c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int e^{-4x^2}\,dx,x,\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a\sqrt{1+a^2x^2}}$$
$$+\frac{\left(3c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int e^{4x^2}\,dx,x,\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a\sqrt{1+a^2x^2}}$$
$$+\frac{\left(15c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int e^{-2x^2}\,dx,x,\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a\sqrt{1+a^2x^2}}$$
$$+\frac{\left(15c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int e^{2x^2}\,dx,x,\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a\sqrt{1+a^2x^2}}$$

$$= \frac{5c^2\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}{8a\sqrt{1+a^2x^2}} + \frac{3c^2\sqrt{\pi}\sqrt{c+a^2cx^2}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a\sqrt{1+a^2x^2}}$$
$$+ \frac{15c^2\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a\sqrt{1+a^2x^2}}$$
$$+ \frac{c^2\sqrt{\frac{\pi}{6}}\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{6}\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a\sqrt{1+a^2x^2}}$$
$$+ \frac{3c^2\sqrt{\pi}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a\sqrt{1+a^2x^2}}$$
$$+ \frac{15c^2\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a\sqrt{1+a^2x^2}}$$
$$+ \frac{c^2\sqrt{\frac{\pi}{6}}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(\sqrt{6}\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a\sqrt{1+a^2x^2}}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.50

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \frac{c^2\sqrt{c+a^2cx^2}\Big(240\operatorname{arcsinh}(ax)+\sqrt{6}\sqrt{-\operatorname{arcsinh}(ax)}\,\Gamma\left(\frac{1}{2},-6\operatorname{arcsinh}(ax)\right)+18\sqrt{-\mathrm{a}$$

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (c^2*Sqrt[c + a^2*c*x^2]*(240*ArcSinh[a*x] + Sqrt[6]*Sqrt[-ArcSinh[a*x]]*Ga
mma[1/2, -6*ArcSinh[a*x]] + 18*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*
x]] + 45*Sqrt[2]*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]] - 45*Sqrt[
2]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 2*ArcSinh[a*x]] - 18*Sqrt[ArcSinh[a*x]]*Ga
mma[1/2, 4*ArcSinh[a*x]] - Sqrt[6]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 6*ArcSinh[
a*x]]))/(384*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

Maple [F]

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{\sqrt{\operatorname{arcsinh}(ax)}}dx$$

```
[In] int((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Timed out}$$

```
[In] integrate((a**2*c*x**2+c)**(5/2)/asinh(a*x)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(5/2)/sqrt(arcsinh(a*x)), x)
```

Giac [F]

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
[Out] integrate((a^2*c*x^2 + c)^(5/2)/sqrt(arcsinh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx=\int \frac{{\left(c\,a^2\,x^2+c\right)}^{5/2}}{\sqrt{\mathrm{asinh}\left(a\,x\right)}}\,dx$$

```
[In] int((c + a^2*c*x^2)^(5/2)/asinh(a*x)^(1/2),x)
[Out] int((c + a^2*c*x^2)^(5/2)/asinh(a*x)^(1/2), x)
```

3.496 $\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\mathbf{arcsinh}(ax)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 264

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \frac{3c\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}{4a\sqrt{1+a^2x^2}} + \frac{c\sqrt{\pi}\sqrt{c+a^2cx^2}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a\sqrt{1+a^2x^2}} + \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a\sqrt{1+a^2x^2}} + \frac{c\sqrt{\pi}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a\sqrt{1+a^2x^2}} + \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a\sqrt{1+a^2x^2}}$$

```
[Out] 1/8*c*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/
a/(a^2*x^2+1)^(1/2)+1/8*c*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)
*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/32*c*erf(2*arcsinh(a*x)^(1/2))*P
i^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/32*c*erfi(2*arcsinh(a*x)^
(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/4*c*(a^2*c*x^2+c)
^(1/2)*arcsinh(a*x)^(1/2)/a/(a^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

$= \{5791, 3393, 3388, 2211, 2235, 2236\}$

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \frac{\sqrt{\pi}c\sqrt{a^2cx^2+c}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a\sqrt{a^2x^2+1}} + \frac{\sqrt{\frac{\pi}{2}}c\sqrt{a^2cx^2+c}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a\sqrt{a^2x^2+1}} + \frac{\sqrt{\pi}c\sqrt{a^2cx^2+c}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a\sqrt{a^2x^2+1}} + \frac{\sqrt{\frac{\pi}{2}}c\sqrt{a^2cx^2+c}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a\sqrt{a^2x^2+1}} + \frac{3c\sqrt{\operatorname{arcsinh}(ax)}\sqrt{a^2cx^2+c}}{4a\sqrt{a^2x^2+1}}$$

```
[In] Int[(c + a^2*c*x^2)^(3/2)/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (3*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(4*a*Sqrt[1 + a^2*x^2]) + (c*S
qrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(32*a*Sqrt[1 + a^2*x
^2]) + (c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(
4*a*Sqrt[1 + a^2*x^2]) + (c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSin
h[a*x]]])/(32*a*Sqrt[1 + a^2*x^2]) + (c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi
[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(4*a*Sqrt[1 + a^2*x^2])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh^4(x)}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{a\sqrt{1+a^2x^2}}\\
&= \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}}+\frac{\cosh(2x)}{2\sqrt{x}}+\frac{\cosh(4x)}{8\sqrt{x}}\right)dx,x,\text{arcsinh}(ax)\right)}{a\sqrt{1+a^2x^2}}\\
&= \frac{3c\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{4a\sqrt{1+a^2x^2}}+\frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{8a\sqrt{1+a^2x^2}}\\
&\quad+\frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{2a\sqrt{1+a^2x^2}}\\
&= \frac{3c\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{4a\sqrt{1+a^2x^2}}+\frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{16a\sqrt{1+a^2x^2}}\\
&\quad+\frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{4x}}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{16a\sqrt{1+a^2x^2}}\\
&\quad+\frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{4a\sqrt{1+a^2x^2}}\\
&\quad+\frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{4a\sqrt{1+a^2x^2}}
\end{aligned}$$

$$= \frac{3c\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}{4a\sqrt{1+a^2x^2}} + \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{8a\sqrt{1+a^2x^2}}$$
$$+ \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int e^{4x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{8a\sqrt{1+a^2x^2}}$$
$$+ \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{2a\sqrt{1+a^2x^2}}$$
$$+ \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{2a\sqrt{1+a^2x^2}}$$

$$= \frac{3c\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}{4a\sqrt{1+a^2x^2}} + \frac{c\sqrt{\pi}\sqrt{c+a^2cx^2}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a\sqrt{1+a^2x^2}}$$
$$+ \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a\sqrt{1+a^2x^2}} + \frac{c\sqrt{\pi}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a\sqrt{1+a^2x^2}}$$
$$+ \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a\sqrt{1+a^2x^2}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.53

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \frac{c\sqrt{c+a^2cx^2}\left(\sqrt{-\operatorname{arcsinh}(ax)}\Gamma\left(\frac{1}{2},-4\operatorname{arcsinh}(ax)\right)+4\sqrt{2}\sqrt{-\operatorname{arcsinh}(ax)}\Gamma\left(\frac{1}{2},-2\operatorname{ar}\right.\right.}{32a\sqrt{1+}}$$

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*x]] + 4
*Sqrt[2]*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]] + Sqrt[ArcSinh[a*x
]]*(24*Sqrt[ArcSinh[a*x]] - 4*Sqrt[2]*Gamma[1/2, 2*ArcSinh[a*x]] - Gamma[1/
2, 4*ArcSinh[a*x]])))/(32*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

Maple [F]

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{\sqrt{\operatorname{arcsinh}(ax)}}dx$$

```
[In] int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{\sqrt{\operatorname{asinh}(ax)}}\,dx$$

```
[In] integrate((a**2*c*x**2+c)**(3/2)/asinh(a*x)**(1/2),x)
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)/sqrt(asinh(a*x)), x)
```

Maxima [F]

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(3/2)/sqrt(arcsinh(a*x)), x)
```

Giac [F]

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
[Out] integrate((a^2*c*x^2 + c)^(3/2)/sqrt(arcsinh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx=\int \frac{{\left(c\,a^2\,x^2+c\right)}^{3/2}}{\sqrt{\mathrm{asinh}\left(a\,x\right)}} \,d x$$

```
[In] int((c + a^2*c*x^2)^(3/2)/asinh(a*x)^(1/2),x)
[Out] int((c + a^2*c*x^2)^(3/2)/asinh(a*x)^(1/2), x)
```

3.497 $\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\text{arcsinh}(ax)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 156

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\text{arcsinh}(ax)}}\,dx = \frac{\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{a\sqrt{1+a^2x^2}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{4a\sqrt{1+a^2x^2}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{4a\sqrt{1+a^2x^2}}$$

```
[Out] 1/8*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/
(a^2*x^2+1)^(1/2)+1/8*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^
2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)
/a/(a^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5791, 3393, 3388, 2211, 2235, 2236}

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\text{arcsinh}(ax)}}\,dx = \frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{4a\sqrt{a^2x^2+1}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{4a\sqrt{a^2x^2+1}} + \frac{\sqrt{\text{arcsinh}(ax)}\sqrt{a^2cx^2+c}}{a\sqrt{a^2x^2+1}}$$

```
[In] Int[Sqrt[c + a^2*c*x^2]/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(a*Sqrt[1 + a^2*x^2]) + (Sqrt[Pi/2
]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(4*a*Sqrt[1 + a^2*x^
2]) + (Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(4*
a*Sqrt[1 + a^2*x^2])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int \frac{\cosh^2(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{a\sqrt{1+a^2x^2}} \\
&= \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}}+\frac{\cosh(2x)}{2\sqrt{x}}\right)\,dx, x, \text{arcsinh}(ax)\right)}{a\sqrt{1+a^2x^2}} \\
&= \frac{\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{a\sqrt{1+a^2x^2}} + \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{2a\sqrt{1+a^2x^2}} \\
&= \frac{\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{a\sqrt{1+a^2x^2}} + \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{4a\sqrt{1+a^2x^2}} \\
&\quad + \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{4a\sqrt{1+a^2x^2}} \\
&= \frac{\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{a\sqrt{1+a^2x^2}} + \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{2a\sqrt{1+a^2x^2}} \\
&\quad + \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{2a\sqrt{1+a^2x^2}} \\
&= \frac{\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{a\sqrt{1+a^2x^2}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{4a\sqrt{1+a^2x^2}} \\
&\quad + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{4a\sqrt{1+a^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.65

$$
\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\text{arcsinh}(ax)}}\,dx = \frac{\sqrt{c(1+a^2x^2)}\left(8\text{arcsinh}(ax)+\sqrt{2}\sqrt{-\text{arcsinh}(ax)}\,\Gamma\left(\frac{1}{2},-2\text{arcsinh}(ax)\right)-\sqrt{2}\sqrt{\text{arcsinh}(ax)}\,\Gamma\left(\frac{1}{2},2\text{arcsinh}(\right.\right.}{8a\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}
$$

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(8*ArcSinh[a*x] + Sqrt[2]*Sqrt[-ArcSinh[a*x]]*Gamma[
1/2, -2*ArcSinh[a*x]] - Sqrt[2]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 2*ArcSinh[a*x
]]))/(8*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

Maple [F]

$$\int \frac{\sqrt{a^2 c\,x^2 + c}}{\sqrt{\operatorname{arcsinh}\left(a x\right)}}dx$$

```
[In] int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c + a^2 c x^2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\sqrt{c + a^2 c x^2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{\sqrt{c\left(a^{2} x^{2} + 1\right)}}{\sqrt{\operatorname{asinh}\left(a x\right)}}\,dx$$

```
[In] integrate((a**2*c*x**2+c)**(1/2)/asinh(a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/sqrt(asinh(a*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c + a^2 c x^2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{\sqrt{a^{2} c x^{2} + c}}{\sqrt{\operatorname{arsinh}\left(a x\right)}}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/sqrt(arcsinh(a*x)), x)
```

Giac [F]

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a^2*c*x^2 + c)/sqrt(arcsinh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{\sqrt{c\,a^2\,x^2+c}}{\sqrt{\operatorname{asinh}(a\,x)}}\,dx$$

```
[In] int((c + a^2*c*x^2)^(1/2)/asinh(a*x)^(1/2),x)
[Out] int((c + a^2*c*x^2)^(1/2)/asinh(a*x)^(1/2), x)
```

3.498 $\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 40

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}\,dx = \frac{2\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{a\sqrt{c+a^2cx^2}}$$

```
[Out] 2*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {5783}

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}\,dx = \frac{2\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{a\sqrt{a^2cx^2+c}}$$

```
[In] Int[1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]]),x]
[Out] (2*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{a\sqrt{c+a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx=\frac{2\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{a\sqrt{c(1+a^2x^2)}}$$

```
[In] Integrate[1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]]),x]
[Out] (2*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(a*Sqrt[c*(1 + a^2*x^2)])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{2\sqrt{\operatorname{arcsinh}(ax)}\,\sqrt{a^2x^2+1}}{a\sqrt{c(a^2x^2+1)}}$	36

```
[In] int(1/(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*arcsinh(a*x)^(1/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx=\int \frac{1}{\sqrt{c\left(a^2x^2+1\right)}\sqrt{\operatorname{asinh}\left(ax\right)}}\,dx$$

```
[In] integrate(1/(a**2*c*x**2+c)**(1/2)/asinh(a*x)**(1/2),x)
[Out] Integral(1/(sqrt(c*(a**2*x**2 + 1))*sqrt(asinh(a*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{c + a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{\sqrt{a^2cx^2 + c}\sqrt{\operatorname{arsinh}\left(ax\right)}}\,dx$$

```
[In] integrate(1/(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*sqrt(arcsinh(a*x))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{c + a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{\sqrt{a^2cx^2 + c}\sqrt{\operatorname{arsinh}\left(ax\right)}}\,dx$$

```
[In] integrate(1/(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*sqrt(arcsinh(a*x))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{c + a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{\sqrt{\mathrm{asinh}\left(a\,x\right)}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

```
[In] int(1/(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(1/(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.499 $\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\text{arcsinh}(ax)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\text{arcsinh}(ax)}}\,dx = \text{Int}\left(\frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\text{arcsinh}(ax)}},x\right)$$

```
[Out] Unintegrable(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\text{arcsinh}(ax)}}\,dx = \int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\text{arcsinh}(ax)}}\,dx$$

```
[In] Int[1/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]]),x]
[Out] Defer[Int][1/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\text{arcsinh}(ax)}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.83 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$

```
[In] Integrate[1/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]]),x]
[Out] Integrate[1/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(a^{2} c\,x^{2}+c\right)^{\frac{3}{2}} \sqrt{\operatorname{arcsinh}\left(a x\right)}}d x$$

```
[In] int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x)
[Out] int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 3.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\sqrt{\operatorname{asinh}\left(ax\right)}}\,dx$$

```
[In] integrate(1/(a**2*c*x**2+c)**(3/2)/asinh(a*x)**(1/2),x)
[Out] Integral(1/((c*(a**2*x**2 + 1))**(3/2)*sqrt(asinh(a*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*sqrt(arcsinh(a*x))), x)
```

Giac [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*sqrt(arcsinh(a*x))), x)
```

Mupad [N/A]

Not integrable

Time = 2.74 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{\sqrt{\operatorname{asinh}(a\,x)}\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

```
[In] int(1/(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(1/(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.500 $\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \operatorname{Int}\left(\frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\operatorname{arcsinh}(ax)}},x\right)$$

```
[Out] Unintegrable(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$

```
[In] Int[1/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcSinh[a*x]]),x]
[Out] Defer[Int][1/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcSinh[a*x]]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$

```
[In] Integrate[1/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcSinh[a*x]]),x]
[Out] Integrate[1/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcSinh[a*x]]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{(a^2c\,x^2+c)^{\frac{5}{2}}\sqrt{\operatorname{arcsinh}(ax)}}dx$$

```
[In] int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x)
[Out] int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 58.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{(c\,(a^2x^2+1))^{\frac{5}{2}}\sqrt{\operatorname{asinh}(ax)}}\,dx$$

```
[In] integrate(1/(a**2*c*x**2+c)**(5/2)/asinh(a*x)**(1/2),x)
[Out] Integral(1/((c*(a**2*x**2 + 1))**(5/2)*sqrt(asinh(a*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\text{arcsinh}(ax)}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\sqrt{\text{arsinh}\,(ax)}}\,dx$$

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*sqrt(arcsinh(a*x))), x)
```

Giac [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\text{arcsinh}(ax)}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\sqrt{\text{arsinh}\,(ax)}}\,dx$$

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*sqrt(arcsinh(a*x))), x)
```

Mupad [N/A]

Not integrable

Time = 2.72 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\text{arcsinh}(ax)}}\,dx = \int \frac{1}{\sqrt{\text{asinh}\,(a\,x)}\,(c\,a^2\,x^2+c)^{5/2}}\,dx$$

```
[In] int(1/(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(1/(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.501 $\int \frac{(c+a^2cx^2)^{5/2}}{\text{arcsinh}(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 391

$$\begin{aligned}\int \frac{(c+a^2cx^2)^{5/2}}{\text{arcsinh}(ax)^{3/2}}\,dx &= -\frac{2\sqrt{1+a^2x^2}(c+a^2cx^2)^{5/2}}{a\sqrt{\text{arcsinh}(ax)}} \\ &- \frac{3c^2\sqrt{\pi}\sqrt{c+a^2cx^2}\,\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{8a\sqrt{1+a^2x^2}} \\ &- \frac{15c^2\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{16a\sqrt{1+a^2x^2}} \\ &- \frac{c^2\sqrt{\frac{3\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erf}\left(\sqrt{6}\sqrt{\text{arcsinh}(ax)}\right)}{16a\sqrt{1+a^2x^2}} \\ &+ \frac{3c^2\sqrt{\pi}\sqrt{c+a^2cx^2}\,\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{8a\sqrt{1+a^2x^2}} \\ &+ \frac{15c^2\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{16a\sqrt{1+a^2x^2}} \\ &+ \frac{c^2\sqrt{\frac{3\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erfi}\left(\sqrt{6}\sqrt{\text{arcsinh}(ax)}\right)}{16a\sqrt{1+a^2x^2}}\end{aligned}$$

```
[Out] -15/32*c^2*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(
1/2)/a/(a^2*x^2+1)^(1/2)+15/32*c^2*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)
*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-3/8*c^2*erf(2*arcsinh(a*x
)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/8*c^2*erfi(2*ar
csinh(a*x)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-1/32*c^2
*erf(6^(1/2)*arcsinh(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^
```

```
2*x^2+1)^(1/2)+1/32*c^2*erfi(6^(1/2)*arcsinh(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(
a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-2*(a^2*c*x^2+c)^(5/2)*(a^2*x^2+1)^(1
/2)/a/arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5790, 5819, 5556, 3389, 2211, 2235, 2236}

$$\int \frac{(c+a^2cx^2)^{5/2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = -\frac{3\sqrt{\pi}c^2\sqrt{a^2cx^2+c}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a\sqrt{a^2x^2+1}} - \frac{15\sqrt{\frac{\pi}{2}}c^2\sqrt{a^2cx^2+c}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a\sqrt{a^2x^2+1}} - \frac{\sqrt{\frac{3\pi}{2}}c^2\sqrt{a^2cx^2+c}\operatorname{erf}\left(\sqrt{6}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a\sqrt{a^2x^2+1}} + \frac{3\sqrt{\pi}c^2\sqrt{a^2cx^2+c}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a\sqrt{a^2x^2+1}} + \frac{15\sqrt{\frac{\pi}{2}}c^2\sqrt{a^2cx^2+c}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a\sqrt{a^2x^2+1}} + \frac{\sqrt{\frac{3\pi}{2}}c^2\sqrt{a^2cx^2+c}\operatorname{erfi}\left(\sqrt{6}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a\sqrt{a^2x^2+1}} - \frac{2\sqrt{a^2x^2+1}(a^2cx^2+c)^{5/2}}{a\sqrt{\operatorname{arcsinh}(ax)}}$$

```
[In] Int[(c + a^2*c*x^2)^(5/2)/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2]*(c + a^2*c*x^2)^(5/2))/(a*Sqrt[ArcSinh[a*x]]) - (3*c^
2*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(8*a*Sqrt[1 + a^2
*x^2]) - (15*c^2*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*
x]]])/(16*a*Sqrt[1 + a^2*x^2]) - (c^2*Sqrt[(3*Pi)/2]*Sqrt[c + a^2*c*x^2]*Er
f[Sqrt[6]*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*x^2]) + (3*c^2*Sqrt[Pi]*S
qrt[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(8*a*Sqrt[1 + a^2*x^2]) + (1
5*c^2*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(16*
a*Sqrt[1 + a^2*x^2]) + (c^2*Sqrt[(3*Pi)/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[6]
*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*x^2])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{1+a^2x^2}(c+a^2cx^2)^{5/2}}{a\sqrt{\operatorname{arcsinh}(ax)}} + \frac{\left(12ac^2\sqrt{c+a^2cx^2}\right)\int \frac{x(1+a^2x^2)^2}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx}{\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}(c+a^2cx^2)^{5/2}}{a\sqrt{\operatorname{arcsinh}(ax)}} + \frac{\left(12c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{\cosh^5(x)\sinh(x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}(c+a^2cx^2)^{5/2}}{a\sqrt{\operatorname{arcsinh}(ax)}} + \frac{\left(12c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \left(\frac{5\sinh(2x)}{32\sqrt{x}} + \frac{\sinh(4x)}{8\sqrt{x}} + \frac{\sinh(6x)}{32\sqrt{x}}\right)\,dx, x, \operatorname{arcsinh}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}(c+a^2cx^2)^{5/2}}{a\sqrt{\operatorname{arcsinh}(ax)}} + \frac{\left(3c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{\sinh(6x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{8a\sqrt{1+a^2x^2}} + \frac{\left(3c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{\sinh(4x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{2a\sqrt{1+a^2x^2}} + \frac{\left(15c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{8a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}(c+a^2cx^2)^{5/2}}{a\sqrt{\operatorname{arcsinh}(ax)}} - \frac{\left(3c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{e^{-6x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{16a\sqrt{1+a^2x^2}} + \frac{\left(3c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{e^{6x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{16a\sqrt{1+a^2x^2}} - \frac{\left(3c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{4a\sqrt{1+a^2x^2}} + \frac{\left(3c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{e^{4x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{4a\sqrt{1+a^2x^2}} - \frac{\left(15c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{16a\sqrt{1+a^2x^2}} + \frac{\left(15c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{16a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}(c+a^2cx^2)^{5/2}}{a\sqrt{\operatorname{arcsinh}(ax)}} - \frac{\left(3c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int e^{-6x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{8a\sqrt{1+a^2x^2}} + \frac{\left(3c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int e^{6x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{8a\sqrt{1+a^2x^2}} - \frac{\left(3c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{2a\sqrt{1+a^2x^2}} + \frac{\left(3c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int e^{4x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{2a\sqrt{1+a^2x^2}} - \frac{\left(15c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{8a\sqrt{1+a^2x^2}} + \frac{\left(15c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{8a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}(c+a^2cx^2)^{5/2}}{a\sqrt{\operatorname{arcsinh}(ax)}} - \frac{3c^2\sqrt{\pi}\sqrt{c+a^2cx^2}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a\sqrt{1+a^2x^2}} - \frac{15c^2\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a\sqrt{1+a^2x^2}} - \frac{c^2\sqrt{\frac{3\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{6}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a\sqrt{1+a^2x^2}} + \frac{3c^2\sqrt{\pi}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a\sqrt{1+a^2x^2}} + \frac{15c^2\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a\sqrt{1+a^2x^2}} + \frac{c^2\sqrt{\frac{3\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(\sqrt{6}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a\sqrt{1+a^2x^2}}$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.02

$$\int \frac{(c+a^2cx^2)^{5/2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \frac{c^2e^{-6\operatorname{arcsinh}(ax)}\sqrt{c+a^2cx^2}\left(-1-6e^{2\operatorname{arcsinh}(ax)}+e^{4\operatorname{arcsinh}(ax)}-52e^{6\operatorname{arcsinh}(ax)}+e^{8\operatorname{arcsinh}}\right.}{}$$

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (c^2*Sqrt[c + a^2*c*x^2]*(-1 - 6*E^(2*ArcSinh[a*x]) + E^(4*ArcSinh[a*x]) -
52*E^(6*ArcSinh[a*x]) + E^(8*ArcSinh[a*x]) - 6*E^(10*ArcSinh[a*x]) - E^(12*
ArcSinh[a*x]) - 64*a^2*E^(6*ArcSinh[a*x])*x^2 - 16*E^(6*ArcSinh[a*x])*Sqrt[
2*Pi]*Sqrt[ArcSinh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + 16*E^(6*ArcSinh[
a*x])*Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + Sqrt
[6]*E^(6*ArcSinh[a*x])*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -6*ArcSinh[a*x]] + 12
*E^(6*ArcSinh[a*x])*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*x]] - Sqrt[
2]*E^(6*ArcSinh[a*x])*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]] - Sqr
t[2]*E^(6*ArcSinh[a*x])*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 2*ArcSinh[a*x]] + 12*
E^(6*ArcSinh[a*x])*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 4*ArcSinh[a*x]] + Sqrt[6]*
E^(6*ArcSinh[a*x])*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 6*ArcSinh[a*x]]))/(32*a*E^
(6*ArcSinh[a*x])*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

Maple [F]

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{\operatorname{arcsinh}(ax)^{\frac{3}{2}}}dx$$

```
[In] int((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a**2*c*x**2+c)**(5/2)/asinh(a*x)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(c+a^2cx^2)^{5/2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(5/2)/arcsinh(a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{(c+a^2cx^2)^{5/2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="giac")
[Out] integrate((a^2*c*x^2 + c)^(5/2)/arcsinh(a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{(c\,a^2\,x^2+c)^{5/2}}{\operatorname{asinh}(a\,x)^{3/2}}\,dx$$

```
[In] int((c + a^2*c*x^2)^(5/2)/asinh(a*x)^(3/2),x)
[Out] int((c + a^2*c*x^2)^(5/2)/asinh(a*x)^(3/2), x)
```

3.502 $\int \frac{(c+a^2cx^2)^{3/2}}{\text{arcsinh}(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 256

$$\int \frac{(c+a^2cx^2)^{3/2}}{\text{arcsinh}(ax)^{3/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}(c+a^2cx^2)^{3/2}}{a\sqrt{\text{arcsinh}(ax)}} - \frac{c\sqrt{\pi}\sqrt{c+a^2cx^2}\,\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{4a\sqrt{1+a^2x^2}} - \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{a\sqrt{1+a^2x^2}} + \frac{c\sqrt{\pi}\sqrt{c+a^2cx^2}\,\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{4a\sqrt{1+a^2x^2}} + \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{a\sqrt{1+a^2x^2}}$$

```
[Out] -1/2*c*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)
/a/(a^2*x^2+1)^(1/2)+1/2*c*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2
)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-1/4*c*erf(2*arcsinh(a*x)^(1/2))*P
i^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/4*c*erfi(2*arcsinh(a*x)^(
1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-2*(a^2*c*x^2+c)^(3/2
)*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used

$= \{5790, 5819, 5556, 3389, 2211, 2235, 2236\}$

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = -\frac{\sqrt{\pi}c\sqrt{a^2cx^2+c}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a\sqrt{a^2x^2+1}} - \frac{\sqrt{\frac{\pi}{2}}c\sqrt{a^2cx^2+c}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{a\sqrt{a^2x^2+1}} + \frac{\sqrt{\pi}c\sqrt{a^2cx^2+c}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a\sqrt{a^2x^2+1}} + \frac{\sqrt{\frac{\pi}{2}}c\sqrt{a^2cx^2+c}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{a\sqrt{a^2x^2+1}} - \frac{2\sqrt{a^2x^2+1}(a^2cx^2+c)^{3/2}}{a\sqrt{\operatorname{arcsinh}(ax)}}$$

```
[In] Int[(c + a^2*c*x^2)^(3/2)/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2]*(c + a^2*c*x^2)^(3/2))/(a*Sqrt[ArcSinh[a*x]]) - (c*Sq
rt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(4*a*Sqrt[1 + a^2*x^2
]) - (c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(a*
Sqrt[1 + a^2*x^2]) + (c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSinh[a*
x]]])/(4*a*Sqrt[1 + a^2*x^2]) + (c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt
[2]*Sqrt[ArcSinh[a*x]]])/(a*Sqrt[1 + a^2*x^2])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{1+a^2x^2}(c+a^2cx^2)^{3/2}}{a\sqrt{\operatorname{arcsinh}(ax)}} + \frac{\left(8ac\sqrt{c+a^2cx^2}\right)\int \frac{x(1+a^2x^2)}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx}{\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}(c+a^2cx^2)^{3/2}}{a\sqrt{\operatorname{arcsinh}(ax)}} + \frac{\left(8c\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{\cosh^3(x)\sinh(x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}(c+a^2cx^2)^{3/2}}{a\sqrt{\operatorname{arcsinh}(ax)}} + \frac{\left(8c\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \left(\frac{\sinh(2x)}{4\sqrt{x}} + \frac{\sinh(4x)}{8\sqrt{x}}\right)\,dx, x, \operatorname{arcsinh}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}(c+a^2cx^2)^{3/2}}{a\sqrt{\operatorname{arcsinh}(ax)}} + \frac{\left(c\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{\sinh(4x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{a\sqrt{1+a^2x^2}} + \frac{\left(2c\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}(c+a^2cx^2)^{3/2}}{a\sqrt{\operatorname{arcsinh}(ax)}} - \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{2a\sqrt{1+a^2x^2}}$$
$$+ \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{4x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{2a\sqrt{1+a^2x^2}}$$
$$- \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{a\sqrt{1+a^2x^2}}$$
$$+ \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}(c+a^2cx^2)^{3/2}}{a\sqrt{\operatorname{arcsinh}(ax)}} - \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{a\sqrt{1+a^2x^2}}$$
$$+ \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int e^{4x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{a\sqrt{1+a^2x^2}}$$
$$- \frac{\left(2c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{a\sqrt{1+a^2x^2}}$$
$$+ \frac{\left(2c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}(c+a^2cx^2)^{3/2}}{a\sqrt{\operatorname{arcsinh}(ax)}} - \frac{c\sqrt{\pi}\sqrt{c+a^2cx^2}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a\sqrt{1+a^2x^2}}$$
$$- \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{a\sqrt{1+a^2x^2}} + \frac{c\sqrt{\pi}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a\sqrt{1+a^2x^2}}$$
$$+ \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{a\sqrt{1+a^2x^2}}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.88

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx =$$
$$-\frac{ce^{-4\operatorname{arcsinh}(ax)}\sqrt{c+a^2cx^2}\left(1+14e^{4\operatorname{arcsinh}(ax)}+e^{8\operatorname{arcsinh}(ax)}+16a^2e^{4\operatorname{arcsinh}(ax)}x^2+4e^{4\operatorname{arcsinh}(ax)}\sqrt{2\pi}\sqrt{\operatorname{arcsinh}(\right.}{}$$

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/ArcSinh[a*x]^(3/2),x]
```

```
[Out] -1/8*(c*Sqrt[c + a^2*c*x^2]*(1 + 14*E^(4*ArcSinh[a*x]) + E^(8*ArcSinh[a*x])
 + 16*a^2*E^(4*ArcSinh[a*x])*x^2 + 4*E^(4*ArcSinh[a*x])*Sqrt[2*Pi]*Sqrt[Arc
Sinh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]] - 4*E^(4*ArcSinh[a*x])*Sqrt[2*Pi
]*Sqrt[ArcSinh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] - 2*E^(4*ArcSinh[a*x]
)*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*x]] - 2*E^(4*ArcSinh[a*x])*Sq
rt[ArcSinh[a*x]]*Gamma[1/2, 4*ArcSinh[a*x]]))/(a*E^(4*ArcSinh[a*x])*Sqrt[1
+ a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

Maple [F]

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{\operatorname{arcsinh}\left(ax\right)^{\frac{3}{2}}}dx$$

```
[In] int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

```
[In] integrate((a**2*c*x**2+c)**(3/2)/asinh(a*x)**(3/2),x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)/asinh(a*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="maxima")

[Out] integrate((a^2*c*x^2 + c)^(3/2)/arcsinh(a*x)^(3/2), x)

Giac [F]

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="giac")

[Out] integrate((a^2*c*x^2 + c)^(3/2)/arcsinh(a*x)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{(c\,a^2\,x^2+c)^{3/2}}{\operatorname{asinh}(a\,x)^{3/2}}\,dx$$

[In] int((c + a^2*c*x^2)^(3/2)/asinh(a*x)^(3/2),x)

[Out] int((c + a^2*c*x^2)^(3/2)/asinh(a*x)^(3/2), x)

3.503 $\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 152

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{a\sqrt{\operatorname{arcsinh}(ax)}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{a\sqrt{1+a^2x^2}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{a\sqrt{1+a^2x^2}}$$

```
[Out] -1/2*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a
/(a^2*x^2+1)^(1/2)+1/2*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a
^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-2*(a^2*x^2+1)^(1/2)*(a^2*c*x^2+c)^(1/
2)/a/arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5790, 5780, 5556, 12, 3389, 2211, 2235, 2236}

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{a\sqrt{a^2x^2+1}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{a\sqrt{a^2x^2+1}} - \frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{a\sqrt{\operatorname{arcsinh}(ax)}}$$

```
[In] Int[Sqrt[c + a^2*c*x^2]/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2])/(a*Sqrt[ArcSinh[a*x]]) - (Sqrt[P
i/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(a*Sqrt[1 + a^2*x
```

```
^2]) + (Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(a
*Sqrt[1 + a^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{a\sqrt{\operatorname{arcsinh}(ax)}}+\frac{\left(4a\sqrt{c+a^2cx^2}\right)\int\frac{x}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx}{\sqrt{1+a^2x^2}}$$

$$=-\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{a\sqrt{\operatorname{arcsinh}(ax)}}+\frac{\left(4\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int\frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$=-\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{a\sqrt{\operatorname{arcsinh}(ax)}}+\frac{\left(4\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int\frac{\sinh(2x)}{2\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$=-\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{a\sqrt{\operatorname{arcsinh}(ax)}}+\frac{\left(2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int\frac{\sinh(2x)}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$=-\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{a\sqrt{\operatorname{arcsinh}(ax)}}-\frac{\sqrt{c+a^2cx^2}\operatorname{Subst}\left(\int\frac{e^{-2x}}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{a\sqrt{1+a^2x^2}}+\frac{\sqrt{c+a^2cx^2}\operatorname{Subst}\left(\int\frac{e^{2x}}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$=-\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{a\sqrt{\operatorname{arcsinh}(ax)}}-\frac{\left(2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int e^{-2x^2}\,dx,x,\sqrt{\operatorname{arcsinh}(ax)}\right)}{a\sqrt{1+a^2x^2}}+\frac{\left(2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int e^{2x^2}\,dx,x,\sqrt{\operatorname{arcsinh}(ax)}\right)}{a\sqrt{1+a^2x^2}}$$

$$=-\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{a\sqrt{\operatorname{arcsinh}(ax)}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{a\sqrt{1+a^2x^2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{a\sqrt{1+a^2x^2}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\left(4+4a^2x^2+\sqrt{2\pi}\sqrt{\operatorname{arcsinh}(ax)}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)-\sqrt{2\pi}\sqrt{\operatorname{arcsinh}(ax)}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)\right)}{2a\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}$$

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/ArcSinh[a*x]^(3/2),x]
[Out] -1/2*(Sqrt[c + a^2*c*x^2]*(4 + 4*a^2*x^2 + Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Er
f[Sqrt[2]*Sqrt[ArcSinh[a*x]]] - Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erfi[Sqrt[2]*
Sqrt[ArcSinh[a*x]]]))/(a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

Maple [F]

$$\int \frac{\sqrt{a^2c\,x^2+c}}{\operatorname{arcsinh}\left(ax\right)^{\frac{3}{2}}}dx$$

```
[In] int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(3/2),x)
[Out] int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx=\int \frac{\sqrt{c\left(a^2x^2+1\right)}}{\operatorname{asinh}^{\frac{3}{2}}\left(ax\right)}\,dx$$

```
[In] integrate((a**2*c*x**2+c)**(1/2)/asinh(a*x)**(3/2),x)
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/asinh(a*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx=\int \frac{\sqrt{a^2cx^2+c}}{\operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(a^2*c*x^2 + c)/arcsinh(a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx=\int \frac{\sqrt{a^2cx^2+c}}{\operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(a^2*c*x^2 + c)/arcsinh(a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx=\int \frac{\sqrt{c\,a^2\,x^2+c}}{{\operatorname{asinh}\left(a\,x\right)}^{3/2}}\,dx$$

```
[In] int((c + a^2*c*x^2)^(1/2)/asinh(a*x)^(3/2),x)
[Out] int((c + a^2*c*x^2)^(1/2)/asinh(a*x)^(3/2), x)
```

3.504 $\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 40

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}\,dx=-\frac{2\sqrt{1+a^2x^2}}{a\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}$$

```
[Out] -2*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {5783}

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}\,dx=-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}\sqrt{a^2cx^2+c}}$$

```
[In] Int[1/(Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2)),x]
[Out] (-2*Sqrt[1 + a^2*x^2])/(a*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\text{integral}=-\frac{2\sqrt{1+a^2x^2}}{a\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}}{a\sqrt{c\left(1+a^2x^2\right)}\sqrt{\text{arcsinh}(ax)}}$$

```
[In] Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2)),x]
[Out] (-2*Sqrt[1 + a^2*x^2])/(a*Sqrt[c*(1 + a^2*x^2)]*Sqrt[ArcSinh[a*x]])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{2\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}\,a\sqrt{c(a^2x^2+1)}}$	36

```
[In] int(1/arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/arcsinh(a*x)^(1/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.42

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}\,dx = -\frac{2\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}}{\left(a^3cx^2+ac\right)\sqrt{\log\left(ax+\sqrt{a^2x^2+1}\right)}}$$

```
[In] integrate(1/arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)/((a^3*c*x^2 + a*c)*sqrt(log(a*x +
sqrt(a^2*x^2 + 1))))
```

Sympy [F]

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{\sqrt{c\,(a^2x^2+1)}\,\text{asinh}^{\frac{3}{2}}\,(ax)}\,dx$$

```
[In] integrate(1/asinh(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(1/(sqrt(c*(a**2*x**2 + 1))*asinh(a*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}\,\text{arsinh}\,(ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}\,\text{arsinh}\,(ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{\text{asinh}\,(a\,x)^{3/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

```
[In] int(1/(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(1/(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.505 $\int \frac{1}{(c+a^2cx^2)^{3/2}\mathbf{arcsinh}(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^{3/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}}{a(c+a^2cx^2)^{3/2}\sqrt{\operatorname{arcsinh}(ax)}} - \frac{4a\sqrt{1+a^2x^2}\text{Int}\left(\frac{x}{(1+a^2x^2)^2\sqrt{\operatorname{arcsinh}(ax)}},x\right)}{c\sqrt{c+a^2cx^2}}$$

```
[Out] -2*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2)-4*a*(a^2*x^2+
1)^(1/2)*Unintegrable(x/(a^2*x^2+1)^2/arcsinh(a*x)^(1/2),x)/c/(a^2*c*x^2+c)
^(1/2)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^{3/2}}\,dx$$

```
[In] Int[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(a*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]]) - (4*a*
Sqrt[1 + a^2*x^2]*Defer[Int][x/((1 + a^2*x^2)^2*Sqrt[ArcSinh[a*x]]), x])/(c
*Sqrt[c + a^2*c*x^2])
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{1+a^2x^2}}{a\left(c+a^2cx^2\right)^{3/2}\sqrt{\text{arcsinh}(ax)}} - \frac{\left(4a\sqrt{1+a^2x^2}\right)\int\frac{x}{(1+a^2x^2)^2\sqrt{\text{arcsinh}(ax)}}\,dx}{c\sqrt{c+a^2cx^2}}$$

Mathematica [N/A]

Not integrable

Time = 1.91 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{3/2}}\,dx$$

```
[In] Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2)),x]
[Out] Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\text{arcsinh}\left(ax\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x)
[Out] int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 25.93 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^{3/2}}\,dx=\int \frac{1}{(c\,(a^2x^2+1))^{\frac{3}{2}}\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

```
[In] integrate(1/(a**2*c*x**2+c)**(3/2)/asinh(a*x)**(3/2),x)
[Out] Integral(1/((c*(a**2*x**2 + 1))**(3/2)*asinh(a*x)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^{3/2}}\,dx=\int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(3/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^{3/2}}\,dx=\int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 2.69 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\,\text{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{\text{asinh}\,(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

```
[In] int(1/(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(1/(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.506 $\int \frac{1}{(c+a^2cx^2)^{5/2}\textbf{arcsinh}(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\operatorname{arcsinh}(ax)^{3/2}}\,dx=-\frac{2\sqrt{1+a^2x^2}}{a\,(c+a^2cx^2)^{5/2}\sqrt{\operatorname{arcsinh}(ax)}}-\frac{8a\sqrt{1+a^2x^2}\operatorname{Int}\left(\frac{x}{(1+a^2x^2)^3\sqrt{\operatorname{arcsinh}(ax)}},x\right)}{c^2\sqrt{c+a^2cx^2}}$$

```
[Out] -2*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2)-8*a*(a^2*x^2+
1)^(1/2)*Unintegrable(x/(a^2*x^2+1)^3/arcsinh(a*x)^(1/2),x)/c^2/(a^2*c*x^2+
c)^(1/2)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\operatorname{arcsinh}(ax)^{3/2}}\,dx=\int \frac{1}{(c+a^2cx^2)^{5/2}\operatorname{arcsinh}(ax)^{3/2}}\,dx$$

```
[In] Int[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(a*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcSinh[a*x]]) - (8*a*
Sqrt[1 + a^2*x^2]*Defer[Int][x/((1 + a^2*x^2)^3*Sqrt[ArcSinh[a*x]]), x])/(c
^2*Sqrt[c + a^2*c*x^2])
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{1+a^2x^2}}{a\left(c+a^2cx^2\right)^{5/2}\sqrt{\operatorname{arcsinh}(ax)}} - \frac{\left(8a\sqrt{1+a^2x^2}\right)\int \frac{x}{(1+a^2x^2)^3\sqrt{\operatorname{arcsinh}(ax)}}\,dx}{c^2\sqrt{c+a^2cx^2}}$$

Mathematica [N/A]

Not integrable

Time = 1.99 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\operatorname{arcsinh}(ax)^{3/2}}\,dx$$

```
[In] Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(3/2)),x]
[Out] Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(3/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.33 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\operatorname{arcsinh}\left(ax\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x)
[Out] int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\operatorname{arcsinh}(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\,\text{arcsinh}(ax)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a**2*c*x**2+c)**(5/2)/asinh(a*x)**(3/2),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\,\text{arcsinh}(ax)^{3/2}}\,dx=\int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\,\text{arsinh}\,(ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*arcsinh(a*x)^(3/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\,\text{arcsinh}(ax)^{3/2}}\,dx=\int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\,\text{arsinh}\,(ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*arcsinh(a*x)^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\,\text{arcsinh}(ax)^{3/2}}\,dx=\int \frac{1}{\text{asinh}\,(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^{5/2}}\,dx$$

```
[In] int(1/(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(1/(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.507 $\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 296

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}(c+a^2cx^2)^{3/2}}{3a\operatorname{arcsinh}(ax)^{3/2}} - \frac{16cx(1+a^2x^2)\sqrt{c+a^2cx^2}}{3\sqrt{\operatorname{arcsinh}(ax)}} + \frac{2c\sqrt{\pi}\sqrt{c+a^2cx^2}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}} + \frac{2c\sqrt{2\pi}\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}} + \frac{2c\sqrt{\pi}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}} + \frac{2c\sqrt{2\pi}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}}$$

```
[Out] -2/3*(a^2*c*x^2+c)^(3/2)*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(3/2)+2/3*c*erf(2
*arcsinh(a*x)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+2/3*c
*erfi(2*arcsinh(a*x)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2
)+2/3*c*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2
)/a/(a^2*x^2+1)^(1/2)+2/3*c*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/
2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-16/3*c*x*(a^2*x^2+1)*(a^2*c*x^2+
c)^(1/2)/arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules

used = {5790, 5814, 5791, 3393, 3388, 2211, 2235, 2236, 5819, 5556}

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \frac{2\sqrt{\pi}c\sqrt{a^2cx^2+c}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a\sqrt{a^2x^2+1}} + \frac{2\sqrt{2\pi}c\sqrt{a^2cx^2+c}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a\sqrt{a^2x^2+1}} + \frac{2\sqrt{\pi}c\sqrt{a^2cx^2+c}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a\sqrt{a^2x^2+1}} + \frac{2\sqrt{2\pi}c\sqrt{a^2cx^2+c}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a\sqrt{a^2x^2+1}} - \frac{2\sqrt{a^2x^2+1}(a^2cx^2+c)^{3/2}}{3a\operatorname{arcsinh}(ax)^{3/2}} - \frac{16cx(a^2x^2+1)\sqrt{a^2cx^2+c}}{3\sqrt{\operatorname{arcsinh}(ax)}}$$

```
[In] Int[(c + a^2*c*x^2)^(3/2)/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2]*(c + a^2*c*x^2)^(3/2))/(3*a*ArcSinh[a*x]^(3/2)) - (16
*c*x*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2])/(3*Sqrt[ArcSinh[a*x]]) + (2*c*Sqrt[
Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(3*a*Sqrt[1 + a^2*x^2])
+ (2*c*Sqrt[2*Pi]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(3*a
*Sqrt[1 + a^2*x^2]) + (2*c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSinh
[a*x]]])/(3*a*Sqrt[1 + a^2*x^2]) + (2*c*Sqrt[2*Pi]*Sqrt[c + a^2*c*x^2]*Erfi
[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(3*a*Sqrt[1 + a^2*x^2])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
```

```
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
```

```
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sqrt{1+a^2x^2}(c+a^2cx^2)^{3/2}}{3a\text{arcsinh}(ax)^{3/2}} + \frac{\left(8ac\sqrt{c+a^2cx^2}\right)\int \frac{x(1+a^2x^2)}{\text{arcsinh}(ax)^{3/2}}\,dx}{3\sqrt{1+a^2x^2}} \\
&= -\frac{2\sqrt{1+a^2x^2}(c+a^2cx^2)^{3/2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{16cx(1+a^2x^2)\sqrt{c+a^2cx^2}}{3\sqrt{\text{arcsinh}(ax)}} \\
&\quad + \frac{\left(16c\sqrt{c+a^2cx^2}\right)\int \frac{\sqrt{1+a^2x^2}}{\sqrt{\text{arcsinh}(ax)}}\,dx}{3\sqrt{1+a^2x^2}} + \frac{\left(64a^2c\sqrt{c+a^2cx^2}\right)\int \frac{x^2\sqrt{1+a^2x^2}}{\sqrt{\text{arcsinh}(ax)}}\,dx}{3\sqrt{1+a^2x^2}} \\
&= -\frac{2\sqrt{1+a^2x^2}(c+a^2cx^2)^{3/2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{16cx(1+a^2x^2)\sqrt{c+a^2cx^2}}{3\sqrt{\text{arcsinh}(ax)}} \\
&\quad + \frac{\left(16c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh^2(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{3a\sqrt{1+a^2x^2}} \\
&\quad + \frac{\left(64c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh^2(x)\sinh^2(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{3a\sqrt{1+a^2x^2}} \\
&= -\frac{2\sqrt{1+a^2x^2}(c+a^2cx^2)^{3/2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{16cx(1+a^2x^2)\sqrt{c+a^2cx^2}}{3\sqrt{\text{arcsinh}(ax)}} \\
&\quad + \frac{\left(16c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}} + \frac{\cosh(2x)}{2\sqrt{x}}\right)\,dx, x, \text{arcsinh}(ax)\right)}{3a\sqrt{1+a^2x^2}} \\
&\quad + \frac{\left(64c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \left(-\frac{1}{8\sqrt{x}} + \frac{\cosh(4x)}{8\sqrt{x}}\right)\,dx, x, \text{arcsinh}(ax)\right)}{3a\sqrt{1+a^2x^2}} \\
&= -\frac{2\sqrt{1+a^2x^2}(c+a^2cx^2)^{3/2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{16cx(1+a^2x^2)\sqrt{c+a^2cx^2}}{3\sqrt{\text{arcsinh}(ax)}} \\
&\quad + \frac{\left(8c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{3a\sqrt{1+a^2x^2}} \\
&\quad + \frac{\left(8c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{3a\sqrt{1+a^2x^2}}
\end{aligned}$$

$$= -\frac{2\sqrt{1+a^2x^2}(c+a^2cx^2)^{3/2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{16cx(1+a^2x^2)\sqrt{c+a^2cx^2}}{3\sqrt{\text{arcsinh}(ax)}} + \frac{(4c\sqrt{c+a^2cx^2})\,\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{3a\sqrt{1+a^2x^2}} + \frac{(4c\sqrt{c+a^2cx^2})\,\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{3a\sqrt{1+a^2x^2}} + \frac{(4c\sqrt{c+a^2cx^2})\,\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{3a\sqrt{1+a^2x^2}} + \frac{(4c\sqrt{c+a^2cx^2})\,\text{Subst}\left(\int \frac{e^{4x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{3a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}(c+a^2cx^2)^{3/2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{16cx(1+a^2x^2)\sqrt{c+a^2cx^2}}{3\sqrt{\text{arcsinh}(ax)}} + \frac{(8c\sqrt{c+a^2cx^2})\,\text{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}} + \frac{(8c\sqrt{c+a^2cx^2})\,\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}} + \frac{(8c\sqrt{c+a^2cx^2})\,\text{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}} + \frac{(8c\sqrt{c+a^2cx^2})\,\text{Subst}\left(\int e^{4x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}(c+a^2cx^2)^{3/2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{16cx(1+a^2x^2)\sqrt{c+a^2cx^2}}{3\sqrt{\text{arcsinh}(ax)}} + \frac{2c\sqrt{\pi}\sqrt{c+a^2cx^2}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}} + \frac{2c\sqrt{2\pi}\sqrt{c+a^2cx^2}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}} + \frac{2c\sqrt{\pi}\sqrt{c+a^2cx^2}\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}} + \frac{2c\sqrt{2\pi}\sqrt{c+a^2cx^2}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.89

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx =$$
$$-\frac{ce^{-4\operatorname{arcsinh}(ax)}\sqrt{c+a^2cx^2}\left(1+14e^{4\operatorname{arcsinh}(ax)}+e^{8\operatorname{arcsinh}(ax)}+16a^2e^{4\operatorname{arcsinh}(ax)}x^2-8\operatorname{arcsinh}(ax)+8e^{8\operatorname{arcsinh}(a}\right.}{}$$

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/ArcSinh[a*x]^(5/2),x]
```

```
[Out] -1/24*(c*Sqrt[c + a^2*c*x^2]*(1 + 14*E^(4*ArcSinh[a*x]) + E^(8*ArcSinh[a*x]
) + 16*a^2*E^(4*ArcSinh[a*x])*x^2 - 8*ArcSinh[a*x] + 8*E^(8*ArcSinh[a*x])*A
rcSinh[a*x] + 64*a*E^(4*ArcSinh[a*x])*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + 16
*E^(4*ArcSinh[a*x])*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -4*ArcSinh[a*x]] + 16*
Sqrt[2]*E^(4*ArcSinh[a*x])*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -2*ArcSinh[a*x]
] + 16*Sqrt[2]*E^(4*ArcSinh[a*x])*ArcSinh[a*x]^(3/2)*Gamma[1/2, 2*ArcSinh[a
*x]] + 16*E^(4*ArcSinh[a*x])*ArcSinh[a*x]^(3/2)*Gamma[1/2, 4*ArcSinh[a*x]])
)/(a*E^(4*ArcSinh[a*x])*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))
```

Maple [F]

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{\operatorname{arcsinh}(ax)^{\frac{5}{2}}}dx$$

```
[In] int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{\operatorname{asinh}^{\frac{5}{2}}(ax)}\,dx$$

```
[In] integrate((a**2*c*x**2+c)**(3/2)/asinh(a*x)**(5/2),x)
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)/asinh(a*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(3/2)/arcsinh(a*x)^(5/2), x)
```

Giac [F]

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="giac")
[Out] integrate((a^2*c*x^2 + c)^(3/2)/arcsinh(a*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{(c\,a^2\,x^2+c)^{3/2}}{\operatorname{asinh}(a\,x)^{5/2}}\,dx$$

```
[In] int((c + a^2*c*x^2)^(3/2)/asinh(a*x)^(5/2),x)
[Out] int((c + a^2*c*x^2)^(3/2)/asinh(a*x)^(5/2), x)
```

3.508 $\int \frac{\sqrt{c+a^2cx^2}}{\mathbf{arcsinh}(ax)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 182

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{3a\operatorname{arcsinh}(ax)^{3/2}}-\frac{8x\sqrt{c+a^2cx^2}}{3\sqrt{\operatorname{arcsinh}(ax)}} +\frac{2\sqrt{2\pi}\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}}+\frac{2\sqrt{2\pi}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}}$$

```
[Out] 2/3*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/
(a^2*x^2+1)^(1/2)+2/3*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^
2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-2/3*(a^2*x^2+1)^(1/2)*(a^2*c*x^2+c)^(1
/2)/a/arcsinh(a*x)^(3/2)-8/3*x*(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5790, 5778, 3388, 2211, 2235, 2236}

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \frac{2\sqrt{2\pi}\sqrt{a^2cx^2+c}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a\sqrt{a^2x^2+1}} +\frac{2\sqrt{2\pi}\sqrt{a^2cx^2+c}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a\sqrt{a^2x^2+1}}-\frac{8x\sqrt{a^2cx^2+c}}{3\sqrt{\operatorname{arcsinh}(ax)}}-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{3a\operatorname{arcsinh}(ax)^{3/2}}$$

```
[In] Int[Sqrt[c + a^2*c*x^2]/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2])/(3*a*ArcSinh[a*x]^(3/2)) - (8*x*
Sqrt[c + a^2*c*x^2])/(3*Sqrt[ArcSinh[a*x]]) + (2*Sqrt[2*Pi]*Sqrt[c + a^2*c*
```

```
x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(3*a*Sqrt[1 + a^2*x^2]) + (2*Sqrt[2*P
i]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(3*a*Sqrt[1 + a^2*
x^2])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{3a\text{arcsinh}(ax)^{3/2}} + \frac{\left(4a\sqrt{c+a^2cx^2}\right)\int \frac{x}{\text{arcsinh}(ax)^{3/2}}\,dx}{3\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{8x\sqrt{c+a^2cx^2}}{3\sqrt{\text{arcsinh}(ax)}} + \frac{\left(8\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{3a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{8x\sqrt{c+a^2cx^2}}{3\sqrt{\text{arcsinh}(ax)}} + \frac{\left(4\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{3a\sqrt{1+a^2x^2}} + \frac{\left(4\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{3a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{8x\sqrt{c+a^2cx^2}}{3\sqrt{\text{arcsinh}(ax)}} + \frac{\left(8\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}} + \frac{\left(8\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{8x\sqrt{c+a^2cx^2}}{3\sqrt{\text{arcsinh}(ax)}} + \frac{2\sqrt{2\pi}\sqrt{c+a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}} + \frac{2\sqrt{2\pi}\sqrt{c+a^2cx^2}\,\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx =$$
$$-\frac{2\sqrt{c+a^2cx^2}\left(1+a^2x^2+4ax\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)+\sqrt{2}(-\operatorname{arcsinh}(ax))^{3/2}\Gamma\left(\frac{1}{2},-2\operatorname{arcsinh}(ax)\right)+\sqrt{2}\text{arcs}\right.}{3a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}$$

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-2*Sqrt[c + a^2*c*x^2]*(1 + a^2*x^2 + 4*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]
 + Sqrt[2]*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -2*ArcSinh[a*x]] + Sqrt[2]*ArcS
inh[a*x]^(3/2)*Gamma[1/2, 2*ArcSinh[a*x]]))/(3*a*Sqrt[1 + a^2*x^2]*ArcSinh[
a*x]^(3/2))
```

Maple [F]

$$\int \frac{\sqrt{a^2c\,x^2+c}}{\operatorname{arcsinh}\left(ax\right)^{\frac{5}{2}}}dx$$

```
[In] int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{\sqrt{c\,(a^2x^2+1)}}{\operatorname{asinh}^{\frac{5}{2}}(ax)}\,dx$$

```
[In] integrate((a**2*c*x**2+c)**(1/2)/asinh(a*x)**(5/2),x)
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/asinh(a*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x, algorithm="maxima")
[Out] integrate(sqrt(a^2*c*x^2 + c)/arcsinh(a*x)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x, algorithm="giac")
[Out] integrate(sqrt(a^2*c*x^2 + c)/arcsinh(a*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{\sqrt{c\,a^2\,x^2+c}}{\operatorname{asinh}(a\,x)^{5/2}}\,dx$$

```
[In] int((c + a^2*c*x^2)^(1/2)/asinh(a*x)^(5/2),x)
[Out] int((c + a^2*c*x^2)^(1/2)/asinh(a*x)^(5/2), x)
```

3.509 $\int \frac{1}{\sqrt{c+a^2cx^2}\textbf{arcsinh}(ax)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 42

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}}{3a\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}$$

```
[Out] -2/3*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {5783}

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}\,dx = -\frac{2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

```
[In] Int[1/(Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2)),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(3*a*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{1+a^2x^2}}{3a\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}}{3a\sqrt{c\,(1+a^2x^2)}\text{arcsinh}(ax)^{3/2}}$$

```
[In] Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2)),x]
[Out] (-2*Sqrt[1 + a^2*x^2])/(3*a*Sqrt[c*(1 + a^2*x^2)]*ArcSinh[a*x]^(3/2))
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{2\sqrt{a^2x^2+1}}{3\,\text{arcsinh}(ax)^{\frac{3}{2}}\,a\sqrt{c(a^2x^2+1)}}$	36

```
[In] int(1/arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3/arcsinh(a*x)^(3/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.36

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}\,dx = -\frac{2\,\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}}{3\,(a^3cx^2+ac)\log\left(ax+\sqrt{a^2x^2+1}\right)^{\frac{3}{2}}}$$

```
[In] integrate(1/arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] -2/3*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)/((a^3*c*x^2 + a*c)*log(a*x + sqr
t(a^2*x^2 + 1))^(3/2))
```

Sympy [F]

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{\sqrt{c\,(a^2x^2+1)}\,\text{asinh}^{\frac{5}{2}}\,(ax)}\,dx$$

```
[In] integrate(1/asinh(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(1/(sqrt(c*(a**2*x**2 + 1))*asinh(a*x)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}\,\text{arsinh}\,(ax)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}\,\text{arsinh}\,(ax)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{\text{asinh}\,(a\,x)^{5/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

```
[In] int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.510 $\int \frac{1}{(c+a^2cx^2)^{3/2}\text{arcsinh}(ax)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\,\text{arcsinh}(ax)^{5/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}}{3a\,(c+a^2cx^2)^{3/2}\,\text{arcsinh}(ax)^{3/2}} - \frac{4a\sqrt{1+a^2x^2}\,\text{Int}\left(\frac{x}{(1+a^2x^2)^2\text{arcsinh}(ax)^{3/2}},x\right)}{3c\sqrt{c+a^2cx^2}}$$

```
[Out] -2/3*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2)-4/3*a*(a^2*
x^2+1)^(1/2)*Unintegrable(x/(a^2*x^2+1)^2/arcsinh(a*x)^(3/2),x)/c/(a^2*c*x^
2+c)^(1/2)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\,\text{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{(c+a^2cx^2)^{3/2}\,\text{arcsinh}(ax)^{5/2}}\,dx$$

```
[In] Int[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2)),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(3*a*(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2)) - (4*
a*Sqrt[1 + a^2*x^2]*Defer[Int][x/((1 + a^2*x^2)^2*ArcSinh[a*x]^(3/2)), x])/
(3*c*Sqrt[c + a^2*c*x^2])
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{1+a^2x^2}}{3a\,(c+a^2cx^2)^{3/2}\,\text{arcsinh}(ax)^{3/2}} - \frac{\left(4a\sqrt{1+a^2x^2}\right)\int \frac{x}{(1+a^2x^2)^2\text{arcsinh}(ax)^{3/2}}\,dx}{3c\sqrt{c+a^2cx^2}}$$

Mathematica [N/A]

Not integrable

Time = 1.85 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^{5/2}}\,dx=\int \frac{1}{(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^{5/2}}\,dx$$

```
[In] Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2)),x]
[Out] Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \operatorname{arcsinh}\left(a x \right)^{\frac{5}{2}}}d x$$

```
[In] int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x)
[Out] int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a**2*c*x**2+c)**(3/2)/asinh(a*x)**(5/2),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\,\mathrm{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\,\mathrm{arsinh}\,(ax)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="maxima")
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(5/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\,\mathrm{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\,\mathrm{arsinh}\,(ax)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="giac")
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(5/2)), x)
```

Mupad [N/A]

Not integrable

Time = 2.54 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\,\mathrm{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{\mathrm{asinh}\,(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

```
[In] int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.511 $\int \frac{1}{(c+a^2cx^2)^{5/2}\text{arcsinh}(ax)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\,\text{arcsinh}(ax)^{5/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}}{3a\,(c+a^2cx^2)^{5/2}\,\text{arcsinh}(ax)^{3/2}} - \frac{8a\sqrt{1+a^2x^2}\text{Int}\left(\frac{x}{(1+a^2x^2)^3\text{arcsinh}(ax)^{3/2}},x\right)}{3c^2\sqrt{c+a^2cx^2}}$$

```
[Out] -2/3*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2)-8/3*a*(a^2*
x^2+1)^(1/2)*Unintegrable(x/(a^2*x^2+1)^3/arcsinh(a*x)^(3/2),x)/c^2/(a^2*c*
x^2+c)^(1/2)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\,\text{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{(c+a^2cx^2)^{5/2}\,\text{arcsinh}(ax)^{5/2}}\,dx$$

```
[In] Int[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(5/2)),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(3*a*(c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(3/2)) - (8*
a*Sqrt[1 + a^2*x^2]*Defer[Int][x/((1 + a^2*x^2)^3*ArcSinh[a*x]^(3/2)), x])/
(3*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{1+a^2x^2}}{3a\,(c+a^2cx^2)^{5/2}\,\text{arcsinh}(ax)^{3/2}} - \frac{\left(8a\sqrt{1+a^2x^2}\right)\int \frac{x}{(1+a^2x^2)^3\text{arcsinh}(ax)^{3/2}}\,dx}{3c^2\sqrt{c+a^2cx^2}}$$

Mathematica [N/A]

Not integrable

Time = 2.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\operatorname{arcsinh}(ax)^{5/2}}\,dx=\int \frac{1}{(c+a^2cx^2)^{5/2}\operatorname{arcsinh}(ax)^{5/2}}\,dx$$

```
[In] Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(5/2)),x]
[Out] Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(5/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\operatorname{arcsinh}\left(ax\right)^{\frac{5}{2}}}dx$$

```
[In] int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x)
[Out] int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\operatorname{arcsinh}(ax)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\operatorname{arcsinh}(ax)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a**2*c*x**2+c)**(5/2)/asinh(a*x)**(5/2),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x, algorithm="maxima")
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*arcsinh(a*x)^(5/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x, algorithm="giac")
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*arcsinh(a*x)^(5/2)), x)
```

Mupad [N/A]

Not integrable

Time = 2.65 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{\operatorname{asinh}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^{5/2}}\,dx$$

```
[In] int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.512 $\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx$

Optimal result

Integrand size = 28, antiderivative size = 235

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx=-\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{1+n}}{8bc^3(1+n)\sqrt{1+c^2x^2}}+\frac{2^{-2(3+n)}e^{-\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}}-\frac{2^{-2(3+n)}e^{\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}}$$

```
[Out] -1/8*(a+b*arcsinh(c*x))^(1+n)*(c^2*d*x^2+d)^(1/2)/b/c^3/(1+n)/(c^2*x^2+1)^(
1/2)+(a+b*arcsinh(c*x))^n*GAMMA(1+n,-4*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^
(1/2)/(2^(6+2*n))/c^3/exp(4*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1
/2)-exp(4*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,4*(a+b*arcsinh(c*x))/b)*(c^2*
d*x^2+d)^(1/2)/(2^(6+2*n))/c^3/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used

$= \{5819, 5556, 3388, 2212\}$

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx = -\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^{n+1}}{8bc^3(n+1)\sqrt{c^2x^2+1}} + \frac{2^{-2(n+3)}e^{-\frac{4a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{c^2x^2+1}} - \frac{2^{-2(n+3)}e^{\frac{4a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{c^2x^2+1}}$$

```
[In] Int[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] -1/8*(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(b*c^3*(1 + n)*Sqrt
[1 + c^2*x^2]) + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (
-4*(a + b*ArcSinh[c*x]))/b])/(2^(2*(3 + n))*c^3*E^((4*a)/b)*Sqrt[1 + c^2*x^
2]*(-((a + b*ArcSinh[c*x])/b))^n) - (E^((4*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b
*ArcSinh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]))/b])/(2^(2*(3 + n))*c
^3*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
```

```
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{d+c^2dx^2}\text{Subst}\left(\int x^n\cosh^2\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc^3\sqrt{1+c^2x^2}}\\
&= \frac{\sqrt{d+c^2dx^2}\text{Subst}\left(\int\left(-\frac{x^n}{8}+\frac{1}{8}x^n\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc^3\sqrt{1+c^2x^2}}\\
&= -\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{1+n}}{8bc^3(1+n)\sqrt{1+c^2x^2}}\\
&\quad+\frac{\sqrt{d+c^2dx^2}\text{Subst}\left(\int x^n\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{8bc^3\sqrt{1+c^2x^2}}\\
&= -\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{1+n}}{8bc^3(1+n)\sqrt{1+c^2x^2}}\\
&\quad+\frac{\sqrt{d+c^2dx^2}\text{Subst}\left(\int e^{-i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{16bc^3\sqrt{1+c^2x^2}}\\
&\quad+\frac{\sqrt{d+c^2dx^2}\text{Subst}\left(\int e^{i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{16bc^3\sqrt{1+c^2x^2}}\\
&= -\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{1+n}}{8bc^3(1+n)\sqrt{1+c^2x^2}}\\
&\quad+\frac{4^{-3-n}e^{-\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}}\\
&\quad-\frac{4^{-3-n}e^{\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.73

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx$$
$$= \frac{d\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{8(a+b\text{arcsinh}(cx))}{b(1+n)}+4^{-n}e^{-\frac{4a}{b}}\left(-\frac{(a+b\text{arcsinh}(cx))^2}{b^2}\right)^{-n}\left(\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right.\right.}{64c^3\sqrt{d\left(1+c^2x^2\right)}}$$

```
[In] Integrate[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*((-8*(a + b*ArcSinh[c*x]))/(b*(
1 + n)) + ((a/b + ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/b]
 - E^((8*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (4*(a + b*ArcSinh
[c*x]))/b])/(4^n*E^((4*a)/b)*(-((a + b*ArcSinh[c*x])^2/b^2))^n)))/(64*c^3*S
qrt[d*(1 + c^2*x^2)])
```

Maple [F]

$$\int x^{2}(a+b\ \mathrm{arcsinh}\left(cx\right))^{n}\sqrt{c^{2}d\,x^{2}+d}dx$$

```
[In] int(x^2*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int(x^2*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int x^{2}\sqrt{d+c^{2}dx^{2}}(a+b\mathrm{arcsinh}(cx))^{n}\,dx=\int\sqrt{c^{2}dx^{2}+d}(b\,\mathrm{arsinh}\left(cx\right)+a)^{n}x^{2}\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

Sympy [F]

$$\int x^{2}\sqrt{d+c^{2}dx^{2}}(a+b\mathrm{arcsinh}(cx))^{n}\,dx=\int x^{2}\sqrt{d\left(c^{2}x^{2}+1\right)}(a+b\,\mathrm{asinh}\left(cx\right))^{n}\,dx$$

```
[In] integrate(x**2*(a+b*asinh(c*x))**n*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**n, x)
```

Maxima [F]

$$\int x^{2}\sqrt{d+c^{2}dx^{2}}(a+b\mathrm{arcsinh}(cx))^{n}\,dx=\int\sqrt{c^{2}dx^{2}+d}(b\,\mathrm{arsinh}\left(cx\right)+a)^{n}x^{2}\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx=\int\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^nx^2\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx=\int x^2\,(a+b\,\text{asinh}(c\,x))^n\,\sqrt{d\,c^2\,x^2+d}\,dx$$

```
[In] int(x^2*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2),x)
[Out] int(x^2*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2), x)
```

3.513 $\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx$

Optimal result

Integrand size = 26, antiderivative size = 355

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx = \frac{3^{-1-n}e^{-\frac{3a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{8c^2\sqrt{1+c^2x^2}} + \frac{e^{-\frac{a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8c^2\sqrt{1+c^2x^2}} + \frac{e^{a/b}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8c^2\sqrt{1+c^2x^2}} + \frac{3^{-1-n}e^{\frac{3a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{8c^2\sqrt{1+c^2x^2}}$$

```
[Out] 1/8*3^(-1-n)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-3*(a+b*arcsinh(c*x))/b)*(c^2*d
*x^2+d)^(1/2)/c^2/exp(3*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)+
1/8*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(-a-b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/
2)/c^2/exp(a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)+1/8*exp(a/b)*
(a+b*arcsinh(c*x))^n*GAMMA(1+n,(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^
2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)+1/8*3^(-1-n)*exp(3*a/b)*(a+b
*arcsinh(c*x))^n*GAMMA(1+n,3*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^2/
(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5819, 5556, 3389, 2212}

$$\int x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^n\,dx$$
$$=\frac{3^{-n-1}e^{-\frac{3a}{b}}\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^n\left(-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{8c^2\sqrt{c^2x^2+1}}$$
$$+\frac{e^{-\frac{a}{b}}\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^n\left(-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{8c^2\sqrt{c^2x^2+1}}$$
$$+\frac{e^{a/b}\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^n\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{8c^2\sqrt{c^2x^2+1}}$$
$$+\frac{3^{-n-1}e^{\frac{3a}{b}}\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^n\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{8c^2\sqrt{c^2x^2+1}}$$

```
[In] Int[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (3^(-1 - n)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-3*(a
+ b*ArcSinh[c*x]))/b])/(8*c^2*E^((3*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSi
nh[c*x])/b))^n) + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n,
-((a + b*ArcSinh[c*x])/b)])/(8*c^2*E^(a/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcS
inh[c*x])/b))^n) + (E^(a/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamm
a[1 + n, (a + b*ArcSinh[c*x])/b])/(8*c^2*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[
c*x])/b)^n) + (3^(-1 - n)*E^((3*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])^n*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b])/(8*c^2*Sqrt[1 + c^2*x^2]*((
a + b*ArcSinh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int x^n\cosh^2\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc^2\sqrt{1+c^2x^2}}\\
&= -\frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int\left(\frac{1}{4}x^n\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)+\frac{1}{4}x^n\sinh\left(\frac{a}{b}-\frac{x}{b}\right)\right)dx,x,a+b\text{arcsinh}(cx)\right)}{bc^2\sqrt{1+c^2x^2}}\\
&= -\frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int x^n\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{4bc^2\sqrt{1+c^2x^2}}\\
&\quad-\frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int x^n\sinh\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{4bc^2\sqrt{1+c^2x^2}}\\
&= -\frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{8bc^2\sqrt{1+c^2x^2}}\\
&\quad+\frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{8bc^2\sqrt{1+c^2x^2}}\\
&\quad-\frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{8bc^2\sqrt{1+c^2x^2}}\\
&\quad+\frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{8bc^2\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$= \frac{3^{-1-n}e^{-\frac{3a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{8c^2\sqrt{1+c^2x^2}}$$
$$+\frac{e^{-\frac{a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8c^2\sqrt{1+c^2x^2}}$$
$$+\frac{e^{a/b}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8c^2\sqrt{1+c^2x^2}}$$
$$+\frac{3^{-1-n}e^{\frac{3a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{8c^2\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.65

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx$$
$$=\frac{de^{-\frac{3a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(3e^{\frac{4a}{b}}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)^{-n}\Gamma\left(1+n,\frac{a}{b}+\text{arcsinh}(cx)\right)+\left(-\frac{a+b\text{arcsin}}{b}\right.\right.}{}$$

```
[In] Integrate[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*((3*E^((4*a)/b)*Gamma[1 + n, a/
b + ArcSinh[c*x]])/(a/b + ArcSinh[c*x])^n + (Gamma[1 + n, (-3*(a + b*ArcSin
h[c*x]))/b]/3^n + 3*E^((2*a)/b)*Gamma[1 + n, -((a + b*ArcSinh[c*x])/b)] + (
E^((6*a)/b)*(-((a + b*ArcSinh[c*x])/b))^(2*n)*Gamma[1 + n, (3*(a + b*ArcSin
h[c*x]))/b])/(3^n*(-((a + b*ArcSinh[c*x])^2/b^2))^n))/(-((a + b*ArcSinh[c*x
])/b))^n))/(24*c^2*E^((3*a)/b)*Sqrt[d*(1 + c^2*x^2)])
```

Maple [F]

$$\int x(a+b\,\text{arcsinh}\,(cx))^n\sqrt{c^2d\,x^2+d}dx$$

```
[In] int(x*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx=\int\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^n x\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n*x, x)
```

Sympy [F]

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx=\int x\sqrt{d\,(c^2x^2+1)}(a+b\,\text{asinh}\,(cx))^n\,\,dx$$

```
[In] integrate(x*(a+b*asinh(c*x))**n*(c**2*d*x**2+d)**(1/2),x)
[Out] Integral(x*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**n, x)
```

Maxima [F]

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx=\int\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^n x\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n*x, x)
```

Giac [F(-2)]

Exception generated.

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx=\int x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^n\,\sqrt{d\,c^2\,x^2+d}\,dx$$

```
[In] int(x*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2),x)
[Out] int(x*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2), x)
```

3.514 $\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx$

Optimal result

Integrand size = 25, antiderivative size = 235

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx = \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{1+n}}{2bc(1+n)\sqrt{1+c^2x^2}} + \frac{2^{-3-n}e^{-\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}} - \frac{2^{-3-n}e^{\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))^(1+n)*(c^2*d*x^2+d)^(1/2)/b/c/(1+n)/(c^2*x^2+1)^(1/2
)+2^(-3-n)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-2*(a+b*arcsinh(c*x))/b)*(c^2*d*x
^2+d)^(1/2)/c/exp(2*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)-2^(-
3-n)*exp(2*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,2*(a+b*arcsinh(c*x))/b)*(c^2
*d*x^2+d)^(1/2)/c/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used

$= \{5791, 3393, 3388, 2212\}$

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx = \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^{n+1}}{2bc(n+1)\sqrt{c^2x^2+1}} + \frac{2^{-n-3}e^{-\frac{2a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{c^2x^2+1}} - \frac{2^{-n-3}e^{\frac{2a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{c^2x^2+1}}$$

```
[In] Int[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(2*b*c*(1 + n)*Sqrt[1 +
c^2*x^2]) + (2^(-3 - n)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1
+ n, (-2*(a + b*ArcSinh[c*x]))/b])/(c*E^((2*a)/b)*Sqrt[1 + c^2*x^2]*(-((a +
 b*ArcSinh[c*x])/b))^n) - (2^(-3 - n)*E^((2*a)/b)*Sqrt[d + c^2*d*x^2]*(a +
b*ArcSinh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b])/(c*Sqrt[1 + c^2
*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{d+c^2dx^2}\text{Subst}\left(\int x^n\cosh^2\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc\sqrt{1+c^2x^2}}$$
$$= \frac{\sqrt{d+c^2dx^2}\text{Subst}\left(\int\left(\frac{x^n}{2}+\frac{1}{2}x^n\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc\sqrt{1+c^2x^2}}$$
$$= \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{1+n}}{2bc(1+n)\sqrt{1+c^2x^2}} + \frac{\sqrt{d+c^2dx^2}\text{Subst}\left(\int x^n\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{2bc\sqrt{1+c^2x^2}}$$
$$= \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{1+n}}{2bc(1+n)\sqrt{1+c^2x^2}} + \frac{\sqrt{d+c^2dx^2}\text{Subst}\left(\int e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{4bc\sqrt{1+c^2x^2}} + \frac{\sqrt{d+c^2dx^2}\text{Subst}\left(\int e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{4bc\sqrt{1+c^2x^2}}$$
$$= \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{1+n}}{2bc(1+n)\sqrt{1+c^2x^2}} + \frac{2^{-3-n}e^{-\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}} - \frac{2^{-3-n}e^{\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.68

$$\int\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx = \frac{d\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{4a+4b\text{arcsinh}(cx)}{b+bn}+2^{-n}e^{-\frac{2a}{b}}\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\text{arcsinh}(c}{b}\right.\right.}{8c\sqrt{d\left(1+c^2x^2\right)}}$$

```
[In] Integrate[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*((4*a + 4*b*ArcSinh[c*x])/(b +
b*n) + Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b]/(2^n*E^((2*a)/b)*(-((a + b
*ArcSinh[c*x])/b))^n) - (E^((2*a)/b)*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/
b])/(2^n*(a/b + ArcSinh[c*x])^n)))/(8*c*Sqrt[d*(1 + c^2*x^2)])
```

Maple [F]

$$\int (a+b\operatorname{arcsinh}(cx))^n\sqrt{c^2d\,x^2+d}dx$$

```
[In] int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)
[Out] int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx=\int \sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)^n\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n, x)
```

Sympy [F]

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx=\int \sqrt{d\,(c^2x^2+1)}(a+b\operatorname{asinh}(cx))^n\,dx$$

```
[In] integrate((a+b*asinh(c*x))**n*(c**2*d*x**2+d)**(1/2),x)
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**n, x)
```

Maxima [F]

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx=\int \sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)^n\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n, x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d + c^2 d x^2} (a + b \text{arcsinh}(cx))^n \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d + c^2 d x^2} (a + b \text{arcsinh}(cx))^n \, dx = \int (a + b \operatorname{asinh}(c x))^n \sqrt{d c^2 x^2 + d} \, dx$$

```
[In] int((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2), x)
```

3.515 $\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^n}{x} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\begin{aligned}
&\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^n}{x} \, dx \\
&= \frac{de^{-\frac{a}{b}}\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^n \left(-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)^{-n} \Gamma\left(1+n,-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{2\sqrt{d+c^2dx^2}} \\
&\quad + \frac{de^{a/b}\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^n \left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)^{-n} \Gamma\left(1+n,\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{2\sqrt{d+c^2dx^2}} \\
&\quad + d\text{Int}\left(\frac{(a+b\operatorname{arcsinh}(cx))^n}{x\sqrt{d+c^2dx^2}},x\right)
\end{aligned}$$

```
[Out] 1/2*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(-a-b*arcsinh(c*x))/b)*(c^2*x^2+1)^(1/
2)/exp(a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+1/2*d*exp(a/b)*
(a+b*arcsinh(c*x))^n*GAMMA(1+n,(a+b*arcsinh(c*x))/b)*(c^2*x^2+1)^(1/2)/(((a
+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+d*Unintegrable((a+b*arcsinh(c*x)
)^n/x/(c^2*d*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^n}{x} \, dx = \int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^n}{x} \, dx$$

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x,x]
```

```
[Out] (d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c
*x])/b)])/(2*E^(a/b)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (
d*E^(a/b)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (a + b*ArcS
inh[c*x])/b])/(2*Sqrt[d + c^2*d*x^2]*((a + b*ArcSinh[c*x])/b)^n) + d*Defer[
Int][(a + b*ArcSinh[c*x])^n/(x*Sqrt[d + c^2*d*x^2]), x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{d(a+b\text{arcsinh}(cx))^n}{x\sqrt{d+c^2dx^2}}+\frac{c^2dx(a+b\text{arcsinh}(cx))^n}{\sqrt{d+c^2dx^2}}\right)dx\\
&= d\int \frac{(a+b\text{arcsinh}(cx))^n}{x\sqrt{d+c^2dx^2}}\,dx+\left(c^2d\right)\int \frac{x(a+b\text{arcsinh}(cx))^n}{\sqrt{d+c^2dx^2}}\,dx\\
&= d\int \frac{(a+b\text{arcsinh}(cx))^n}{x\sqrt{d+c^2dx^2}}\,dx-\frac{\left(d\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int x^n\sinh\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{b\sqrt{d+c^2dx^2}}\\
&= d\int \frac{(a+b\text{arcsinh}(cx))^n}{x\sqrt{d+c^2dx^2}}\,dx\\
&\quad-\frac{\left(d\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{2b\sqrt{d+c^2dx^2}}\\
&\quad+\frac{\left(d\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{2b\sqrt{d+c^2dx^2}}\\
&= \frac{de^{-\frac{a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{2\sqrt{d+c^2dx^2}}\\
&\quad+\frac{de^{a/b}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arcsinh}(cx)}{b}\right)}{2\sqrt{d+c^2dx^2}}\\
&\quad+d\int \frac{(a+b\text{arcsinh}(cx))^n}{x\sqrt{d+c^2dx^2}}\,dx
\end{aligned}
$$

Mathematica [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x}\,dx=\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x}\,dx$$

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x,x]
```

```
[Out] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^{n}\sqrt{c^{2}d\,x^{2}+d}}{x}dx$$

[In] int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x,x)

[Out] int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x,x)

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\mathrm{arcsinh}(cx))^n}{x}\,dx = \int \frac{\sqrt{c^2dx^2+d}(b\,\mathrm{arsinh}\,(cx)+a)^n}{x}\,dx$$

[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="fricas")

[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n/x, x)

Sympy [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\mathrm{arcsinh}(cx))^n}{x}\,dx = \int \frac{\sqrt{d\,(c^2x^2+1)}(a+b\,\mathrm{asinh}\,(cx))^n}{x}\,dx$$

[In] integrate((a+b*asinh(c*x))**n*(c**2*d*x**2+d)**(1/2)/x,x)

[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**n/x, x)

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\mathrm{arcsinh}(cx))^n}{x}\,dx = \int \frac{\sqrt{c^2dx^2+d}(b\,\mathrm{arsinh}\,(cx)+a)^n}{x}\,dx$$

[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n/x, x)

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x}\,dx = \text{Exception raised: TypeError}$$

[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^n\,\sqrt{d\,c^2\,x^2+d}}{x}\,dx$$

[In] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2))/x,x)

[Out] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2))/x, x)

3.516 $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx = \frac{cd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{1+n}}{b(1+n)\sqrt{d+c^2dx^2}} + d\text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^n}{x^2\sqrt{d+c^2dx^2}},x\right)$$

```
[Out] c*d*(a+b*arcsinh(c*x))^(1+n)*(c^2*x^2+1)^(1/2)/b/(1+n)/(c^2*d*x^2+d)^(1/2)+
d*Unintegrable((a+b*arcsinh(c*x))^n/x^2/(c^2*d*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx = \int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx$$

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x^2,x]
```

```
[Out] (c*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(b*(1 + n)*Sqrt[d + c^
2*d*x^2]) + d*Defer[Int][(a + b*ArcSinh[c*x])^n/(x^2*Sqrt[d + c^2*d*x^2]),
x]
```

Rubi steps

$$\text{integral} = \int \left(\frac{c^2 d (a + b\text{arcsinh}(cx))^n}{\sqrt{d + c^2 d x^2}} + \frac{d(a + b\text{arcsinh}(cx))^n}{x^2 \sqrt{d + c^2 d x^2}} \right) dx$$
$$= d \int \frac{(a + b\text{arcsinh}(cx))^n}{x^2 \sqrt{d + c^2 d x^2}} dx + \left(c^2 d\right) \int \frac{(a + b\text{arcsinh}(cx))^n}{\sqrt{d + c^2 d x^2}} dx$$
$$= \frac{cd\sqrt{1 + c^2 x^2}(a + b\text{arcsinh}(cx))^{1+n}}{b(1+n)\sqrt{d + c^2 d x^2}} + d \int \frac{(a + b\text{arcsinh}(cx))^n}{x^2 \sqrt{d + c^2 d x^2}} dx$$

Mathematica [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{d + c^2 d x^2}(a + b\text{arcsinh}(cx))^n}{x^2} dx = \int \frac{\sqrt{d + c^2 d x^2}(a + b\text{arcsinh}(cx))^n}{x^2} dx$$

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x^2,x]
[Out] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(a + b \operatorname{arcsinh}(cx))^n \sqrt{c^2 d\, x^2 + d}}{x^2} dx$$

```
[In] int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x^2,x)
[Out] int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d + c^2 d x^2}(a + b\text{arcsinh}(cx))^n}{x^2} dx = \int \frac{\sqrt{c^2 d x^2 + d}(b \operatorname{arsinh}(cx) + a)^n}{x^2} dx$$

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="fricas
")
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 4.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx = \int \frac{\sqrt{d\,(c^2x^2+1)}(a+b\,\text{asinh}\,(cx))^n}{x^2}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**n*(c**2*d*x**2+d)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**n/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx = \int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^n}{x^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx = \int \frac{(a+b\,\mathrm{asinh}(c\,x))^n\,\sqrt{d\,c^2\,x^2+d}}{x^2}\,dx$$

```
[In] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2))/x^2,x)
[Out] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2))/x^2, x)
```

3.517 $\int x^2(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n\,dx$

Optimal result

Integrand size = 28, antiderivative size = 616

$$\begin{aligned}\int x^2\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^n\,dx &= -\frac{d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{1+n}}{16bc^3(1+n)\sqrt{1+c^2x^2}}\\
&+\frac{2^{-7-n}3^{-1-n}de^{-\frac{6a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}}\\
&+\frac{2^{-7-2n}de^{-\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}}\\
&-\frac{2^{-7-n}de^{-\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}}\\
&+\frac{2^{-7-n}de^{\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}}\\
&-\frac{2^{-7-2n}de^{\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}}\\
&-\frac{2^{-7-n}3^{-1-n}de^{\frac{6a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}}\end{aligned}$$

```
[Out] -1/16*d*(a+b*arcsinh(c*x))^(1+n)*(c^2*d*x^2+d)^(1/2)/b/c^3/(1+n)/(c^2*x^2+1
)^(1/2)+2^(-7-n)*3^(-1-n)*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-6*(a+b*arcsinh(
c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/exp(6*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^
2*x^2+1)^(1/2)+2^(-7-2*n)*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-4*(a+b*arcsinh(
c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/exp(4*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^
2*x^2+1)^(1/2)-2^(-7-n)*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-2*(a+b*arcsinh(c*
```

```
x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/exp(2*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*
x^2+1)^(1/2)+2^(-7-n)*d*exp(2*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,2*(a+b*ar
csinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+
1)^(1/2)-2^(-7-2*n)*d*exp(4*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,4*(a+b*arcs
inh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)
^(1/2)-2^(-7-n)*3^(-1-n)*d*exp(6*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,6*(a+b
*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x
^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 616, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5819, 5556, 3388, 2212}

$$\int x^2\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^n\,dx=-\frac{d\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^{n+1}}{16bc^3(n+1)\sqrt{c^2x^2+1}}$$
$$+\frac{d2^{-n-7}3^{-n-1}e^{-\frac{6a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{c^2x^2+1}}$$
$$+\frac{d2^{-2n-7}e^{-\frac{4a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{c^2x^2+1}}$$
$$-\frac{d2^{-n-7}e^{-\frac{2a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{c^2x^2+1}}$$
$$+\frac{d2^{-n-7}e^{\frac{2a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{c^2x^2+1}}$$
$$-\frac{d2^{-2n-7}e^{\frac{4a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{c^2x^2+1}}$$
$$-\frac{d2^{-n-7}3^{-n-1}e^{\frac{6a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{c^2x^2+1}}$$

```
[In] Int[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] -1/16*(d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(b*c^3*(1 + n)*S
qrt[1 + c^2*x^2]) + (2^(-7 - n)*3^(-1 - n)*d*Sqrt[d + c^2*d*x^2]*(a + b*Arc
Sinh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcSinh[c*x]))/b])/(c^3*E^((6*a)/b)*Sq
rt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (2^(-7 - 2*n)*d*Sqrt[d + c
^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/b])
/(c^3*E^((4*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) - (2^(-7
 - n)*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-2*(a + b*
```

```
ArcSinh[c*x]))/b])/(c^3*E^((2*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x
])/b))^n) + (2^(-7 - n)*d*E^((2*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b])/(c^3*Sqrt[1 + c^2*x^2]*((a
+ b*ArcSinh[c*x])/b)^n) - (2^(-7 - 2*n)*d*E^((4*a)/b)*Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]))/b])/(c^3*Sqrt[1
 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) - (2^(-7 - n)*3^(-1 - n)*d*E^((6*a)
/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (6*(a + b*ArcSi
nh[c*x]))/b])/(c^3*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int x^n\cosh^4\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc^3\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(-\frac{x^n}{16}+\frac{1}{32}x^n\cosh\left(\frac{6a}{b}-\frac{6x}{b}\right)+\frac{1}{16}x^n\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)-\frac{1}{32}x^n\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)\right.}{bc^3\sqrt{1+c^2x^2}}$$

$$= -\frac{d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{1+n}}{16bc^3(1+n)\sqrt{1+c^2x^2}} + \frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int x^n\cosh\left(\frac{6a}{b}-\frac{6x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{32bc^3\sqrt{1+c^2x^2}} - \frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int x^n\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{32bc^3\sqrt{1+c^2x^2}} + \frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int x^n\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{16bc^3\sqrt{1+c^2x^2}}$$

$$= -\frac{d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{1+n}}{16bc^3(1+n)\sqrt{1+c^2x^2}} - \frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{64bc^3\sqrt{1+c^2x^2}} - \frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{64bc^3\sqrt{1+c^2x^2}} + \frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{-i\left(\frac{6ia}{b}-\frac{6ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{64bc^3\sqrt{1+c^2x^2}} + \frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{i\left(\frac{6ia}{b}-\frac{6ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{64bc^3\sqrt{1+c^2x^2}} + \frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{-i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{32bc^3\sqrt{1+c^2x^2}} + \frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{32bc^3\sqrt{1+c^2x^2}}$$

$$= -\frac{d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{1+n}}{16bc^3(1+n)\sqrt{1+c^2x^2}} + \frac{2^{-7-n}3^{-1-n}de^{-\frac{6a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{6(a+b\text{arcsinh}(}{b}\right.}{c^3\sqrt{1+c^2x^2}} + \frac{2^{-7-2n}de^{-\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}} - \frac{2^{-7-n}de^{-\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}} + \frac{2^{-7-n}de^{\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}} - \frac{2^{-7-2n}de^{\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}} - \frac{2^{-7-n}3^{-1-n}de^{\frac{6a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 2.34 (sec) , antiderivative size = 429, normalized size of antiderivative = 0.70

$$\int x^2\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^n\,dx = -\frac{2^{-7-2n}3^{-1-n}d^2e^{-\frac{6a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{(a+b\text{arcsinh}(cx))^2}{b^2}\right)^{-n}\left(-2^nb(1+n)\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right.}{}$$

```
[In] Integrate[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] -((2^(-7 - 2*n)*3^(-1 - n)*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*(-(
2^n*b*(1 + n)*(a/b + ArcSinh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcSinh[c*x]))
/b]) - 3^(1 + n)*b*E^((2*a)/b)*(1 + n)*(a/b + ArcSinh[c*x])^n*Gamma[1 + n,
(-4*(a + b*ArcSinh[c*x]))/b] + 2^n*3^(1 + n)*b*E^((4*a)/b)*(1 + n)*(a/b + A
rcSinh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b] - 2^n*3^(1 + n)*b*
E^((8*a)/b)*(1 + n)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (2*(a + b*Ar
cSinh[c*x]))/b] + 3^(1 + n)*b*E^((10*a)/b)*(1 + n)*(-((a + b*ArcSinh[c*x])/
b))^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]))/b] + 2^n*E^((6*a)/b)*(2^(3 + n)
*3^(1 + n)*(a + b*ArcSinh[c*x])*(-((a + b*ArcSinh[c*x])^2/b^2))^n + b*E^((6
*a)/b)*(1 + n)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (6*(a + b*ArcSinh
[c*x]))/b])))/(b*c^3*E^((6*a)/b)*(1 + n)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcS
inh[c*x])^2/b^2))^n))
```

Maple [F]

$$\int x^{2} \left(c^{2} d\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arcsinh}\left(c x\right)\right)^{n}d x$$

```
[In] int(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
[Out] int(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
```

Fricas [F]

$$\int x^{2}\left(d+c^{2}dx^{2}\right)^{3/2}(a+b\mathrm{arcsinh}(cx))^{n}\,dx=\int\left(c^{2}dx^{2}+d\right)^{\frac{3}{2}}(b\,\mathrm{arsinh}\left(cx\right)+a)^{n}x^{2}\,dx$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="fricas
")
[Out] integral((c^2*d*x^4 + d*x^2)*sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n, x)
```

Sympy [F(-2)]

Exception generated.

$$\int x^{2}\left(d+c^{2}dx^{2}\right)^{3/2}(a+b\mathrm{arcsinh}(cx))^{n}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(x**2*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**n,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int x^{2}\left(d+c^{2}dx^{2}\right)^{3/2}(a+b\mathrm{arcsinh}(cx))^{n}\,dx=\int\left(c^{2}dx^{2}+d\right)^{\frac{3}{2}}(b\,\mathrm{arsinh}\left(cx\right)+a)^{n}x^{2}\,dx$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxima
")
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

Giac [F]

$$\int x^2 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))^n\, dx = \int \left(c^2dx^2+d\right)^{\frac{3}{2}} (b\,\text{arsinh}\,(cx)+a)^n x^2\, dx$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac")
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))^n\, dx = \int x^2\, (a+b\,\text{asinh}(c\,x))^n\, \left(d\,c^2\,x^2+d\right)^{3/2}\, dx$$

```
[In] int(x^2*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2),x)
[Out] int(x^2*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2), x)
```

3.518 $\int x(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n\,dx$

Optimal result

Integrand size = 26, antiderivative size = 542

$$\int x(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n\,dx = \frac{5^{-1-n}de^{-\frac{5a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{32c^2\sqrt{1+c^2x^2}}$$
$$+\frac{3^{-n}de^{-\frac{3a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32c^2\sqrt{1+c^2x^2}}$$
$$+\frac{de^{-\frac{a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{16c^2\sqrt{1+c^2x^2}}$$
$$+\frac{de^{a/b}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arcsinh}(cx)}{b}\right)}{16c^2\sqrt{1+c^2x^2}}$$
$$+\frac{3^{-n}de^{\frac{3a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32c^2\sqrt{1+c^2x^2}}$$
$$+\frac{5^{-1-n}de^{\frac{5a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{32c^2\sqrt{1+c^2x^2}}$$

```
[Out] 1/32*5^(-1-n)*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-5*(a+b*arcsinh(c*x))/b)*(c^
2*d*x^2+d)^(1/2)/c^2/exp(5*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/
2)+1/32*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-3*(a+b*arcsinh(c*x))/b)*(c^2*d*x^
2+d)^(1/2)/(3^n)/c^2/exp(3*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/
2)+1/16*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(-a-b*arcsinh(c*x))/b)*(c^2*d*x^2+
d)^(1/2)/c^2/exp(a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)+1/16*d*
exp(a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)
```

```
^(1/2)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)+1/32*d*exp(3*a/b)*(
a+b*arcsinh(c*x))^n*GAMMA(1+n,3*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/(
3^n)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)+1/32*5^(-1-n)*d*exp(5
*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,5*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^
(1/2)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5819, 5556, 3389, 2212}

$$\int x\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^n\,dx=\frac{d5^{-n-1}e^{-\frac{5a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{5(a+b\text{arcsi}}{b}\right.}{32c^2\sqrt{c^2x^2+1}}+\frac{d3^{-n}e^{-\frac{3a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32c^2\sqrt{c^2x^2+1}}+\frac{de^{-\frac{a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{16c^2\sqrt{c^2x^2+1}}+\frac{de^{a/b}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{a+b\text{arcsinh}(cx)}{b}\right)}{16c^2\sqrt{c^2x^2+1}}+\frac{d3^{-n}e^{\frac{3a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32c^2\sqrt{c^2x^2+1}}+\frac{d5^{-n-1}e^{\frac{5a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{32c^2\sqrt{c^2x^2+1}}$$

```
[In] Int[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (5^(-1 - n)*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-5*(
a + b*ArcSinh[c*x]))/b])/(32*c^2*E^((5*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*Ar
cSinh[c*x])/b))^n) + (d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1
+ n, (-3*(a + b*ArcSinh[c*x]))/b])/(32*3^n*c^2*E^((3*a)/b)*Sqrt[1 + c^2*x^2
]*(-((a + b*ArcSinh[c*x])/b))^n) + (d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])^n*Gamma[1 + n, -((a + b*ArcSinh[c*x])/b)])/(16*c^2*E^(a/b)*Sqrt[1 + c^2
*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (d*E^(a/b)*Sqrt[d + c^2*d*x^2]*(a +
b*ArcSinh[c*x])^n*Gamma[1 + n, (a + b*ArcSinh[c*x])/b])/(16*c^2*Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])/b)^n) + (d*E^((3*a)/b)*Sqrt[d + c^2*d*x^2]*(a
+ b*ArcSinh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b])/(32*3^n*c^2*S
```

```
qrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) + (5^(-1 - n)*d*E^((5*a)/b)*Sq
rt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (5*(a + b*ArcSinh[c*x
]))/b])/(32*c^2*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int x^n\cosh^4\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc^2\sqrt{1+c^2x^2}}$$

$$= -\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{1}{16}x^n\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)+\frac{3}{16}x^n\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)+\frac{1}{8}x^n\sinh\left(\frac{a}{b}-\frac{x}{b}\right)\right)\,dx,x,a\right.}{bc^2\sqrt{1+c^2x^2}}$$

$$= -\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int x^n\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{16bc^2\sqrt{1+c^2x^2}} - \frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int x^n\sinh\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{8bc^2\sqrt{1+c^2x^2}} - \frac{\left(3d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int x^n\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{16bc^2\sqrt{1+c^2x^2}}$$

$$=-\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{-i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{32bc^2\sqrt{1+c^2x^2}}+\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{32bc^2\sqrt{1+c^2x^2}}-\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{16bc^2\sqrt{1+c^2x^2}}+\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{16bc^2\sqrt{1+c^2x^2}}-\frac{\left(3d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{32bc^2\sqrt{1+c^2x^2}}+\frac{\left(3d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{32bc^2\sqrt{1+c^2x^2}}$$

$$=\frac{5^{-1-n}de^{-\frac{5a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{32c^2\sqrt{1+c^2x^2}}+\frac{3^{-n}de^{-\frac{3a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32c^2\sqrt{1+c^2x^2}}+\frac{de^{-\frac{a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{16c^2\sqrt{1+c^2x^2}}+\frac{de^{a/b}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arcsinh}(cx)}{b}\right)}{16c^2\sqrt{1+c^2x^2}}+\frac{3^{-n}de^{\frac{3a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32c^2\sqrt{1+c^2x^2}}+\frac{5^{-1-n}de^{\frac{5a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{32c^2\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.72

$$\int x(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n\,dx = \frac{15^{-1-n}d^2e^{-\frac{5a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{(a+b\text{arcsinh}(cx))^2}{b^2}\right)^{-2n}\left(2\ 15^{1+n}e^{\frac{6a}{b}}\left(-\frac{a+}{}\right.\right.}{}$$

```
[In] Integrate[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (15^(-1 - n)*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*(2*15^(1 + n)*E^(
(6*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n*(-((a + b*ArcSinh[c*x])^2/b^2))^n*Ga
mma[1 + n, a/b + ArcSinh[c*x]] + 3*(a/b + ArcSinh[c*x])^n*(3^n*(-((a + b*Ar
cSinh[c*x])^2/b^2))^n*Gamma[1 + n, (-5*(a + b*ArcSinh[c*x]))/b] + 5^(1 + n)
*E^((2*a)/b)*(-((a + b*ArcSinh[c*x])^2/b^2))^n*Gamma[1 + n, (-3*(a + b*ArcS
inh[c*x]))/b] + 2*3^n*5^(1 + n)*E^((4*a)/b)*(-((a + b*ArcSinh[c*x])^2/b^2))
^n*Gamma[1 + n, -((a + b*ArcSinh[c*x])/b)] + 5^(1 + n)*E^((8*a)/b)*(-((a +
b*ArcSinh[c*x])/b))^(2*n)*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b] + 3^n*E^
((10*a)/b)*(-((a + b*ArcSinh[c*x])/b))^(2*n)*Gamma[1 + n, (5*(a + b*ArcSinh
[c*x]))/b])))/(32*c^2*E^((5*a)/b)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x
])^2/b^2))^(2*n))
```

Maple [F]

$$\int x\left(c^2d\,x^2+d\right)^{\frac{3}{2}}(a+b\ \text{arcsinh}\,(cx))^n\,dx$$

```
[In] int(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
```

```
[Out] int(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
```

Fricas [F]

$$\int x(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n\,dx = \int\left(c^2dx^2+d\right)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^n x\,dx$$

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="fricas")
```

```
[Out] integral((c^2*d*x^3 + d*x)*sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int x(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^n\,dx=\text{Timed out}$$

```
[In] integrate(x*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**n,x)
[Out] Timed out
```

Maxima [F]

$$\int x(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^n\,dx=\int (c^2dx^2+d)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)^n x\,dx$$

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxima")
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n*x, x)
```

Giac [F(-2)]

Exception generated.

$$\int x(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^n\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^n\,dx=\int x\,(a+b\,\operatorname{asinh}(c\,x))^n\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

```
[In] int(x*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2),x)
[Out] int(x*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2), x)
```

3.519 $\int (d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n\,dx$

Optimal result

Integrand size = 25, antiderivative size = 420

$$\int (d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n\,dx = \frac{3d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{1+n}}{8bc(1+n)\sqrt{1+c^2x^2}} + \frac{2^{-2(3+n)}de^{-\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}} + \frac{2^{-3-n}de^{-\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}} - \frac{2^{-3-n}de^{\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}} - \frac{2^{-2(3+n)}de^{\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}}$$

```
[Out] 3/8*d*(a+b*arcsinh(c*x))^(1+n)*(c^2*d*x^2+d)^(1/2)/b/c/(1+n)/(c^2*x^2+1)^(1
/2)+d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-4*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)
^(1/2)/(2^(6+2*n))/c/exp(4*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/
2)+2^(-3-n)*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-2*(a+b*arcsinh(c*x))/b)*(c^2*
d*x^2+d)^(1/2)/c/exp(2*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)-2
^(-3-n)*d*exp(2*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,2*(a+b*arcsinh(c*x))/b)
*(c^2*d*x^2+d)^(1/2)/c/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)-d*exp(4
*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,4*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^
(1/2)/(2^(6+2*n))/c/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5791, 3393, 3388, 2212}

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^n\,dx=\frac{3d\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^{n+1}}{8bc(n+1)\sqrt{c^2x^2+1}}$$
$$+\frac{d2^{-2(n+3)}e^{-\frac{4a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{c^2x^2+1}}$$
$$+\frac{d2^{-n-3}e^{-\frac{2a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{c^2x^2+1}}$$
$$-\frac{d2^{-n-3}e^{\frac{2a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{c^2x^2+1}}$$
$$-\frac{d2^{-2(n+3)}e^{\frac{4a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{c^2x^2+1}}$$

```
[In] Int[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (3*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(8*b*c*(1 + n)*Sqrt[
1 + c^2*x^2]) + (d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n,
(-4*(a + b*ArcSinh[c*x]))/b])/(2^(2*(3 + n))*c*E^((4*a)/b)*Sqrt[1 + c^2*x^2
]*(-((a + b*ArcSinh[c*x])/b))^n) + (2^(-3 - n)*d*Sqrt[d + c^2*d*x^2]*(a + b
*ArcSinh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b])/(c*E^((2*a)/b)*
Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) - (2^(-3 - n)*d*E^((2*a)/b
)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh
[c*x]))/b])/(c*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) - (d*E^((4*a)/
b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSin
h[c*x]))/b])/(2^(2*(3 + n))*c*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
```

```
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int x^n\cosh^4\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{3x^n}{8}+\frac{1}{8}x^n\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)+\frac{1}{2}x^n\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc\sqrt{1+c^2x^2}}$$

$$=\frac{3d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{1+n}}{8bc(1+n)\sqrt{1+c^2x^2}}+\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int x^n\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{8bc\sqrt{1+c^2x^2}}+\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int x^n\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{2bc\sqrt{1+c^2x^2}}$$

$$=\frac{3d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{1+n}}{8bc(1+n)\sqrt{1+c^2x^2}}+\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{-i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{16bc\sqrt{1+c^2x^2}}+\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{16bc\sqrt{1+c^2x^2}}+\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{4bc\sqrt{1+c^2x^2}}+\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{4bc\sqrt{1+c^2x^2}}$$

$$= \frac{3d\sqrt{d+c^2dx^2}(a+b\mathrm{arcsinh}(cx))^{1+n}}{8bc(1+n)\sqrt{1+c^2x^2}} + \frac{4^{-3-n}de^{-\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\mathrm{arcsinh}(cx))^n\left(-\frac{a+b\mathrm{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4(a+b\mathrm{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}} + \frac{2^{-3-n}de^{-\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\mathrm{arcsinh}(cx))^n\left(-\frac{a+b\mathrm{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\mathrm{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}} - \frac{2^{-3-n}de^{\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\mathrm{arcsinh}(cx))^n\left(\frac{a+b\mathrm{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\mathrm{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}} - \frac{4^{-3-n}de^{\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\mathrm{arcsinh}(cx))^n\left(\frac{a+b\mathrm{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{4(a+b\mathrm{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.69

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\mathrm{arcsinh}(cx))^n\,dx = \frac{d^2\sqrt{1+c^2x^2}(a+b\mathrm{arcsinh}(cx))^n\left(-\frac{8(a+b\mathrm{arcsinh}(cx))}{b(1+n)}+8\left(\frac{4a+4b\mathrm{arcsinh}(cx)}{b+bn}+2^{-n}e^{-\frac{2a}{b}}\left(\right.\right.\right.}{}$$

```
[In] Integrate[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*((-8*(a + b*ArcSinh[c*x]))/(b
*(1 + n)) + 8*((4*a + 4*b*ArcSinh[c*x])/(b + b*n) + Gamma[1 + n, (-2*(a + b
*ArcSinh[c*x]))/b]/(2^n*E^((2*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n) - (E^((2
*a)/b)*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b])/(2^n*(a/b + ArcSinh[c*x])^
n)) + ((a/b + ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/b] - E
^((8*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x
]))/b])/(4^n*E^((4*a)/b)*(-((a + b*ArcSinh[c*x])^2/b^2))^n)))/(64*c*Sqrt[d
+ c^2*d*x^2])
```

Maple [F]

$$\int \left(c^2d\,x^2+d\right)^{\frac{3}{2}}(a+b\,\mathrm{arcsinh}\,(cx))^n\,dx$$

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
```

```
[Out] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
```

Fricas [F]

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^n\,dx=\int \left(c^2dx^2+d\right)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^n\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="fricas")
[Out] integral((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^n\,dx=\text{Timed out}$$

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**n,x)
[Out] Timed out
```

Maxima [F]

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^n\,dx=\int \left(c^2dx^2+d\right)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^n\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxima")
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n, x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^n\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d + c^2 d x^2\right)^{3/2} (a + b\text{arcsinh}(cx))^n \, dx = \int (a + b\,\text{asinh}(c\,x))^n \left(d\,c^2\,x^2 + d\right)^{3/2} dx$$

```
[In] int((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2),x)
[Out] int((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2), x)
```

3.520 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\mathbf{arcsinh}(cx))^n}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n}{x}\,dx = \frac{3^{-1-n}d^2e^{-\frac{3a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma}{8\sqrt{d+c^2dx^2}}$$
$$+\frac{5d^2e^{-\frac{a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8\sqrt{d+c^2dx^2}}$$
$$+\frac{5d^2e^{a/b}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8\sqrt{d+c^2dx^2}}$$
$$+\frac{3^{-1-n}d^2e^{\frac{3a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{8\sqrt{d+c^2dx^2}}$$
$$+d^2\text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^n}{x\sqrt{d+c^2dx^2}},x\right)$$

```
[Out] 1/8*3^(-1-n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-3*(a+b*arcsinh(c*x))/b)*(c
^2*x^2+1)^(1/2)/exp(3*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+
5/8*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(-a-b*arcsinh(c*x))/b)*(c^2*x^2+1)^(
1/2)/exp(a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+5/8*d^2*exp(a
/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(a+b*arcsinh(c*x))/b)*(c^2*x^2+1)^(1/2)/
(((a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+1/8*3^(-1-n)*d^2*exp(3*a/b)*
(a+b*arcsinh(c*x))^n*GAMMA(1+n,3*(a+b*arcsinh(c*x))/b)*(c^2*x^2+1)^(1/2)/((
(a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+d^2*Unintegrable((a+b*arcsinh(
c*x))^n/x/(c^2*d*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^n}{x}\,dx=\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^n}{x}\,dx$$

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x,x]
```

```
[Out] (3^(-1 - n)*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-3*(
a + b*ArcSinh[c*x]))/b])/(8*E^((3*a)/b)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSi
nh[c*x])/b))^n) + (5*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 +
 n, -((a + b*ArcSinh[c*x])/b)])/(8*E^(a/b)*Sqrt[d + c^2*d*x^2]*(-((a + b*Ar
cSinh[c*x])/b))^n) + (5*d^2*E^(a/b)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^
n*Gamma[1 + n, (a + b*ArcSinh[c*x])/b])/(8*Sqrt[d + c^2*d*x^2]*((a + b*ArcS
inh[c*x])/b)^n) + (3^(-1 - n)*d^2*E^((3*a)/b)*Sqrt[1 + c^2*x^2]*(a + b*ArcS
inh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b])/(8*Sqrt[d + c^2*d*x^2
]*((a + b*ArcSinh[c*x])/b)^n) + d^2*Defer[Int][(a + b*ArcSinh[c*x])^n/(x*Sq
rt[d + c^2*d*x^2]), x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d^2(a+b\operatorname{arcsinh}(cx))^n}{x\sqrt{d+c^2dx^2}}+\frac{2c^2d^2x(a+b\operatorname{arcsinh}(cx))^n}{\sqrt{d+c^2dx^2}}+\frac{c^4d^2x^3(a+b\operatorname{arcsinh}(cx))^n}{\sqrt{d+c^2dx^2}}\right)dx\\
&= d^2\int \frac{(a+b\operatorname{arcsinh}(cx))^n}{x\sqrt{d+c^2dx^2}}\,dx+\left(2c^2d^2\right)\int \frac{x(a+b\operatorname{arcsinh}(cx))^n}{\sqrt{d+c^2dx^2}}\,dx\\
&\quad+\left(c^4d^2\right)\int \frac{x^3(a+b\operatorname{arcsinh}(cx))^n}{\sqrt{d+c^2dx^2}}\,dx\\
&= d^2\int \frac{(a+b\operatorname{arcsinh}(cx))^n}{x\sqrt{d+c^2dx^2}}\,dx\\
&\quad-\frac{\left(d^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int x^n\sinh^3\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{b\sqrt{d+c^2dx^2}}\\
&\quad-\frac{\left(2d^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int x^n\sinh\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{b\sqrt{d+c^2dx^2}}
\end{aligned}$$

$$
\begin{aligned}
&= d^2 \int \frac{(a+b\operatorname{arcsinh}(cx))^n}{x\sqrt{d+c^2dx^2}}\,dx \\
&\quad - \frac{\left(id^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \left(-\frac{1}{4}ix^n\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)+\frac{3}{4}ix^n\sinh\left(\frac{a}{b}-\frac{x}{b}\right)\right)dx,x,a+b\operatorname{arcsinh}(cx)\right)}{b\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\left(d^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}x^n\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{b\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(d^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}x^n\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{b\sqrt{d+c^2dx^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{d^2e^{-\frac{a}{b}}\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^n\left(-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{\sqrt{d+c^2dx^2}} \\
&\quad + \frac{d^2e^{a/b}\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^n\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{\sqrt{d+c^2dx^2}} \\
&\quad + d^2\int \frac{(a+b\operatorname{arcsinh}(cx))^n}{x\sqrt{d+c^2dx^2}}\,dx \\
&\quad - \frac{\left(d^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int x^n\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)dx,x,a+b\operatorname{arcsinh}(cx)\right)}{4b\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(3d^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int x^n\sinh\left(\frac{a}{b}-\frac{x}{b}\right)dx,x,a+b\operatorname{arcsinh}(cx)\right)}{4b\sqrt{d+c^2dx^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{d^2e^{-\frac{a}{b}}\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^n\left(-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{\sqrt{d+c^2dx^2}} \\
&\quad + \frac{d^2e^{a/b}\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^n\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{\sqrt{d+c^2dx^2}} \\
&\quad + d^2\int \frac{(a+b\operatorname{arcsinh}(cx))^n}{x\sqrt{d+c^2dx^2}}\,dx \\
&\quad - \frac{\left(d^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}x^n\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{8b\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(d^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}x^n\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{8b\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(3d^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}x^n\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{8b\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\left(3d^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}x^n\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{8b\sqrt{d+c^2dx^2}}
\end{aligned}
$$

$$= \frac{3^{-1-n}d^2e^{-\frac{3a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{8\sqrt{d+c^2dx^2}}$$
$$+\frac{5d^2e^{-\frac{a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8\sqrt{d+c^2dx^2}}$$
$$+\frac{5d^2e^{a/b}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8\sqrt{d+c^2dx^2}}$$
$$+\frac{3^{-1-n}d^2e^{\frac{3a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{8\sqrt{d+c^2dx^2}}$$
$$+d^2\int\frac{(a+b\text{arcsinh}(cx))^n}{x\sqrt{d+c^2dx^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int\frac{(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))^n}{x}\,dx=\int\frac{(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))^n}{x}\,dx$$

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x,x]
[Out] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int\frac{(c^2d\,x^2+d)^{\frac{3}{2}}\,(a+b\ \text{arcsinh}\,(cx))^n}{x}dx$$

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x,x)
[Out] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n}{x}\,dx=\int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^n}{x}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="fricas")
[Out] integral((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n/x, x)
```

Sympy [N/A]

Not integrable

Time = 125.89 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n}{x}\,dx=\int \frac{(d(c^2x^2+1))^{\frac{3}{2}}(a+b\,\text{asinh}\,(cx))^n}{x}\,dx$$

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**n/x,x)
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**n/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n}{x}\,dx=\int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^n}{x}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="maxima")
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n}{x}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^n\,(d\,c^2\,x^2+d)^{3/2}}{x}\,dx$$

```
[In] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2))/x, x)
```

3.521 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx = \frac{3cd^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{1+n}}{2b(1+n)\sqrt{d+c^2dx^2}}$$
$$+\frac{2^{-3-n}cd^2e^{-\frac{2a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{d+c^2dx^2}}$$
$$-\frac{2^{-3-n}cd^2e^{\frac{2a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{d+c^2dx^2}}$$
$$+d^2\text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^n}{x^2\sqrt{d+c^2dx^2}},x\right)$$

```
[Out] 3/2*c*d^2*(a+b*arcsinh(c*x))^(1+n)*(c^2*x^2+1)^(1/2)/b/(1+n)/(c^2*d*x^2+d)^
(1/2)+2^(-3-n)*c*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-2*(a+b*arcsinh(c*x))/b
)*(c^2*x^2+1)^(1/2)/exp(2*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1
/2)-2^(-3-n)*c*d^2*exp(2*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,2*(a+b*arcsinh
(c*x))/b)*(c^2*x^2+1)^(1/2)/(((a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+
d^2*Unintegrable((a+b*arcsinh(c*x))^n/x^2/(c^2*d*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^n}{x^2}\,dx = \int \frac{(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^n}{x^2}\,dx$$

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x^2,x]
```

```
[Out] (3*c*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(2*b*(1 + n)*Sqrt[
d + c^2*d*x^2]) + (2^(-3 - n)*c*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^
n*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b])/(E^((2*a)/b)*Sqrt[d + c^2*d*x^
2]*(-((a + b*ArcSinh[c*x])/b))^n) - (2^(-3 - n)*c*d^2*E^((2*a)/b)*Sqrt[1 +
c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b])/(
Sqrt[d + c^2*d*x^2]*((a + b*ArcSinh[c*x])/b)^n) + d^2*Defer[Int][(a + b*Arc
Sinh[c*x])^n/(x^2*Sqrt[d + c^2*d*x^2]), x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{2c^2d^2(a+b\operatorname{arcsinh}(cx))^n}{\sqrt{d+c^2dx^2}}+\frac{d^2(a+b\operatorname{arcsinh}(cx))^n}{x^2\sqrt{d+c^2dx^2}}+\frac{c^4d^2x^2(a+b\operatorname{arcsinh}(cx))^n}{\sqrt{d+c^2dx^2}}\right)dx\\
&= d^2\int \frac{(a+b\operatorname{arcsinh}(cx))^n}{x^2\sqrt{d+c^2dx^2}}\,dx+\left(2c^2d^2\right)\int \frac{(a+b\operatorname{arcsinh}(cx))^n}{\sqrt{d+c^2dx^2}}\,dx\\
&\quad+\left(c^4d^2\right)\int \frac{x^2(a+b\operatorname{arcsinh}(cx))^n}{\sqrt{d+c^2dx^2}}\,dx\\
&= \frac{2cd^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^{1+n}}{b(1+n)\sqrt{d+c^2dx^2}}+d^2\int \frac{(a+b\operatorname{arcsinh}(cx))^n}{x^2\sqrt{d+c^2dx^2}}\,dx\\
&\quad+\frac{\left(cd^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int x^n\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{b\sqrt{d+c^2dx^2}}\\
&= \frac{2cd^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^{1+n}}{b(1+n)\sqrt{d+c^2dx^2}}+d^2\int \frac{(a+b\operatorname{arcsinh}(cx))^n}{x^2\sqrt{d+c^2dx^2}}\,dx\\
&\quad-\frac{\left(cd^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\left(\frac{x^n}{2}-\frac{1}{2}x^n\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)\right)dx,x,a+b\operatorname{arcsinh}(cx)\right)}{b\sqrt{d+c^2dx^2}}\\
&= \frac{3cd^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^{1+n}}{2b(1+n)\sqrt{d+c^2dx^2}}+d^2\int \frac{(a+b\operatorname{arcsinh}(cx))^n}{x^2\sqrt{d+c^2dx^2}}\,dx\\
&\quad+\frac{\left(cd^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int x^n\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{2b\sqrt{d+c^2dx^2}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{3cd^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^{1+n}}{2b(1+n)\sqrt{d+c^2dx^2}} + d^2\int \frac{(a+b\operatorname{arcsinh}(cx))^n}{x^2\sqrt{d+c^2dx^2}}\,dx \\
&\quad + \frac{\left(cd^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}x^n\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{4b\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(cd^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}x^n\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{4b\sqrt{d+c^2dx^2}} \\
&= \frac{3cd^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^{1+n}}{2b(1+n)\sqrt{d+c^2dx^2}} \\
&\quad + \frac{2^{-3-n}cd^2e^{-\frac{2a}{b}}\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^n\left(-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{\sqrt{d+c^2dx^2}} \\
&\quad - \frac{2^{-3-n}cd^2e^{\frac{2a}{b}}\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^n\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{\sqrt{d+c^2dx^2}} \\
&\quad + d^2\int \frac{(a+b\operatorname{arcsinh}(cx))^n}{x^2\sqrt{d+c^2dx^2}}\,dx
\end{aligned}
$$

Mathematica [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^n}{x^2}\,dx = \int \frac{(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^n}{x^2}\,dx$$

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x^2,x]
[Out] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(c^2d\,x^2+d)^{\frac{3}{2}}(a+b\operatorname{arcsinh}(cx))^n}{x^2}dx$$

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x^2,x)
[Out] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx=\int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^n}{x^2}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="fricas
")
[Out] integral((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx=\text{Timed out}$$

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**n/x**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx=\int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^n}{x^2}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="maxima
")
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{3/2} (a+b\text{arcsinh}(cx))^n}{x^2} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(d+c^2dx^2)^{3/2} (a+b\text{arcsinh}(cx))^n}{x^2} \, dx = \int \frac{(a+b\,\text{asinh}(c\,x))^n\,(d\,c^2\,x^2+d)^{3/2}}{x^2} \, dx$$

```
[In] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2))/x^2, x)
```

3.522 $\int x^2(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^n\,dx$

Optimal result

Integrand size = 28, antiderivative size = 816

$$
\begin{aligned}
\int x^2\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx &= -\frac{5d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{1+n}}{128bc^3(1+n)\sqrt{1+c^2x^2}} \\
&+\frac{2^{-11-3n}d^2e^{-\frac{8a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{8(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}} \\
&+\frac{2^{-7-n}3^{-1-n}d^2e^{-\frac{6a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}} \\
&+\frac{2^{-2(4+n)}d^2e^{-\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}} \\
&-\frac{2^{-7-n}d^2e^{-\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}} \\
&+\frac{2^{-7-n}d^2e^{\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}} \\
&-\frac{2^{-2(4+n)}d^2e^{\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}} \\
&-\frac{2^{-7-n}3^{-1-n}d^2e^{\frac{6a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}} \\
&-\frac{2^{-11-3n}d^2e^{\frac{8a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{8(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}}
\end{aligned}
$$

```
[Out] -5/128*d^2*(a+b*arcsinh(c*x))^(1+n)*(c^2*d*x^2+d)^(1/2)/b/c^3/(1+n)/(c^2*x^
2+1)^(1/2)+2^(-11-3*n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-8*(a+b*arcsinh(c
*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/exp(8*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2
*x^2+1)^(1/2)+2^(-7-n)*3^(-1-n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-6*(a+b*
arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/exp(6*a/b)/(((-a-b*arcsinh(c*x))/b
)^n)/(c^2*x^2+1)^(1/2)+d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-4*(a+b*arcsinh(c
*x))/b)*(c^2*d*x^2+d)^(1/2)/(2^(8+2*n))/c^3/exp(4*a/b)/(((-a-b*arcsinh(c*x)
)/b)^n)/(c^2*x^2+1)^(1/2)-2^(-7-n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-2*(a
+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/exp(2*a/b)/(((-a-b*arcsinh(c*x)
)/b)^n)/(c^2*x^2+1)^(1/2)+2^(-7-n)*d^2*exp(2*a/b)*(a+b*arcsinh(c*x))^n*GAMM
A(1+n,2*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/(((a+b*arcsinh(c*x))/
b)^n)/(c^2*x^2+1)^(1/2)-d^2*exp(4*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,4*(a+
b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/(2^(8+2*n))/c^3/(((a+b*arcsinh(c*x))
/b)^n)/(c^2*x^2+1)^(1/2)-2^(-7-n)*3^(-1-n)*d^2*exp(6*a/b)*(a+b*arcsinh(c*x)
)^n*GAMMA(1+n,6*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/(((a+b*arcsin
h(c*x))/b)^n)/(c^2*x^2+1)^(1/2)-2^(-11-3*n)*d^2*exp(8*a/b)*(a+b*arcsinh(c*x
))^n*GAMMA(1+n,8*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/(((a+b*arcsi
nh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 816, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used

$= \{5819, 5556, 3388, 2212\}$

$$\int x^2\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))^n\,dx = \frac{2^{-3n-11}d^2e^{-\frac{8a}{b}}\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^n\Gamma\left(n+1,-\frac{8(a+b\operatorname{arcsinh}(cx))}{b}\right)\left(-\frac{a+b\operatorname{arcsi}}{b}\right.}{c^3\sqrt{c^2x^2+1}}$$
$$+\frac{2^{-n-7}3^{-n-1}d^2e^{-\frac{6a}{b}}\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^n\Gamma\left(n+1,-\frac{6(a+b\operatorname{arcsinh}(cx))}{b}\right)\left(-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)^{-n}}{c^3\sqrt{c^2x^2+1}}$$
$$+\frac{2^{-2(n+4)}d^2e^{-\frac{4a}{b}}\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^n\Gamma\left(n+1,-\frac{4(a+b\operatorname{arcsinh}(cx))}{b}\right)\left(-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)^{-n}}{c^3\sqrt{c^2x^2+1}}$$
$$-\frac{2^{-n-7}d^2e^{-\frac{2a}{b}}\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^n\Gamma\left(n+1,-\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)\left(-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)^{-n}}{c^3\sqrt{c^2x^2+1}}$$
$$-\frac{5d^2\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^{n+1}}{128bc^3(n+1)\sqrt{c^2x^2+1}}$$
$$+\frac{2^{-n-7}d^2e^{\frac{2a}{b}}\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^n\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{c^3\sqrt{c^2x^2+1}}$$
$$-\frac{2^{-2(n+4)}d^2e^{\frac{4a}{b}}\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^n\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{4(a+b\operatorname{arcsinh}(cx))}{b}\right)}{c^3\sqrt{c^2x^2+1}}$$
$$-\frac{2^{-n-7}3^{-n-1}d^2e^{\frac{6a}{b}}\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^n\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{6(a+b\operatorname{arcsinh}(cx))}{b}\right)}{c^3\sqrt{c^2x^2+1}}$$
$$-\frac{2^{-3n-11}d^2e^{\frac{8a}{b}}\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^n\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{8(a+b\operatorname{arcsinh}(cx))}{b}\right)}{c^3\sqrt{c^2x^2+1}}$$

```
[In] Int[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (-5*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(128*b*c^3*(1 + n
)*Sqrt[1 + c^2*x^2]) + (2^(-11 - 3*n)*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
h[c*x])^n*Gamma[1 + n, (-8*(a + b*ArcSinh[c*x]))/b])/(c^3*E^((8*a)/b)*Sqrt[
1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (2^(-7 - n)*3^(-1 - n)*d^2*Sq
rt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcSinh[c*
x]))/b])/(c^3*E^((6*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n)
+ (d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*A
rcSinh[c*x]))/b])/(2^(2*(4 + n))*c^3*E^((4*a)/b)*Sqrt[1 + c^2*x^2]*(-((a +
b*ArcSinh[c*x])/b))^n) - (2^(-7 - n)*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b])/(c^3*E^((2*a)/b)*Sqrt[1
 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (2^(-7 - n)*d^2*E^((2*a)/b)*Sq
rt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x
]))/b])/(c^3*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) - (d^2*E^((4*a)/
```

```
b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSin
h[c*x]))/b])/(2^(2*(4 + n))*c^3*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^
n) - (2^(-7 - n)*3^(-1 - n)*d^2*E^((6*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcS
inh[c*x])^n*Gamma[1 + n, (6*(a + b*ArcSinh[c*x]))/b])/(c^3*Sqrt[1 + c^2*x^2
]*((a + b*ArcSinh[c*x])/b)^n) - (2^(-11 - 3*n)*d^2*E^((8*a)/b)*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (8*(a + b*ArcSinh[c*x]))/b])/(c
^3*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int x^n\cosh^6\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc^3\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(-\frac{5x^n}{128}+\frac{1}{128}x^n\cosh\left(\frac{8a}{b}-\frac{8x}{b}\right)+\frac{1}{32}x^n\cosh\left(\frac{6a}{b}-\frac{6x}{b}\right)+\frac{1}{32}x^n\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right.\right.\right.}{bc^3\sqrt{1+c^2x^2}}$$

$$= -\frac{5d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^{1+n}}{128bc^3(1+n)\sqrt{1+c^2x^2}} + \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int x^n\cosh\left(\frac{8a}{b}-\frac{8x}{b}\right)\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{128bc^3\sqrt{1+c^2x^2}} + \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int x^n\cosh\left(\frac{6a}{b}-\frac{6x}{b}\right)\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{32bc^3\sqrt{1+c^2x^2}} + \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int x^n\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{32bc^3\sqrt{1+c^2x^2}} - \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int x^n\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{32bc^3\sqrt{1+c^2x^2}}$$

$$= -\frac{5d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^{1+n}}{128bc^3(1+n)\sqrt{1+c^2x^2}} + \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int e^{-i\left(\frac{8ia}{b}-\frac{8ix}{b}\right)}x^n\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{256bc^3\sqrt{1+c^2x^2}} + \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int e^{i\left(\frac{8ia}{b}-\frac{8ix}{b}\right)}x^n\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{256bc^3\sqrt{1+c^2x^2}} - \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}x^n\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{64bc^3\sqrt{1+c^2x^2}} - \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}x^n\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{64bc^3\sqrt{1+c^2x^2}} + \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int e^{-i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}x^n\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{64bc^3\sqrt{1+c^2x^2}} + \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int e^{i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}x^n\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{64bc^3\sqrt{1+c^2x^2}} + \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int e^{-i\left(\frac{6ia}{b}-\frac{6ix}{b}\right)}x^n\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{64bc^3\sqrt{1+c^2x^2}} + \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int e^{i\left(\frac{6ia}{b}-\frac{6ix}{b}\right)}x^n\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{64bc^3\sqrt{1+c^2x^2}}$$

$$= -\frac{5d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{1+n}}{128bc^3(1+n)\sqrt{1+c^2x^2}}$$
$$+\frac{2^{-11-3n}d^2e^{-\frac{8a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{8(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}}$$
$$+\frac{2^{-7-n}3^{-1-n}d^2e^{-\frac{6a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{6(a+b\text{arcsinh}(c}{b}\right.}{c^3\sqrt{1+c^2x^2}}$$
$$+\frac{4^{-4-n}d^2e^{-\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}}$$
$$-\frac{2^{-7-n}d^2e^{-\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}}$$
$$+\frac{2^{-7-n}d^2e^{\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}}$$
$$-\frac{4^{-4-n}d^2e^{\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}}$$
$$-\frac{2^{-7-n}3^{-1-n}d^2e^{\frac{6a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}}$$
$$-\frac{2^{-11-3n}d^2e^{\frac{8a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{8(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 5.62 (sec) , antiderivative size = 667, normalized size of antiderivative = 0.82

$$\int x^2\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx =$$
$$-\frac{2^{-11-3n}3^{-1-n}d^3e^{-\frac{8a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{(a+b\text{arcsinh}(cx))^2}{b^2}\right)^{-n}\left(-3^{1+n}b(1+n)\left(\frac{a}{b}+\text{arcsinh}(cx\right.\right.}{}$$

```
[In] Integrate[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] -((2^(-11 - 3*n)*3^(-1 - n)*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*(-
(3^(1 + n)*b*(1 + n)*(a/b + ArcSinh[c*x])^n*Gamma[1 + n, (-8*(a + b*ArcSinh
[c*x]))/b]) - 4^(2 + n)*b*E^((2*a)/b)*(1 + n)*(a/b + ArcSinh[c*x])^n*Gamma[
1 + n, (-6*(a + b*ArcSinh[c*x]))/b] - 2^(3 + n)*3^(1 + n)*b*E^((4*a)/b)*(1
+ n)*(a/b + ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/b] + 3^(
1 + n)*4^(2 + n)*b*E^((6*a)/b)*(1 + n)*(a/b + ArcSinh[c*x])^n*Gamma[1 + n,
```

```
(-2*(a + b*ArcSinh[c*x]))/b] + E^((8*a)/b)*(5*2^(4 + 3*n)*3^(1 + n)*a*(-((a
 + b*ArcSinh[c*x])^2/b^2))^n + 5*2^(4 + 3*n)*3^(1 + n)*b*ArcSinh[c*x]*(-((a
 + b*ArcSinh[c*x])^2/b^2))^n - 3^(1 + n)*4^(2 + n)*b*E^((2*a)/b)*(1 + n)*(-
((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b] + 2^(3
 + n)*3^(1 + n)*b*E^((4*a)/b)*(1 + n)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1
 + n, (4*(a + b*ArcSinh[c*x]))/b] + 4^(2 + n)*b*E^((6*a)/b)*(-((a + b*ArcSi
nh[c*x])/b))^n*Gamma[1 + n, (6*(a + b*ArcSinh[c*x]))/b] + 4^(2 + n)*b*E^((6
*a)/b)*n*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (6*(a + b*ArcSinh[c*x])
)/b] + 3^(1 + n)*b*E^((8*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (
8*(a + b*ArcSinh[c*x]))/b] + 3^(1 + n)*b*E^((8*a)/b)*n*(-((a + b*ArcSinh[c*
x])/b))^n*Gamma[1 + n, (8*(a + b*ArcSinh[c*x]))/b])))/(b*c^3*E^((8*a)/b)*(1
 + n)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x])^2/b^2))^n))
```

Maple [F]

$$\int x^{2} \left(c^{2} d \,x^{2}+d \right)^{\frac{5}{2}} \left(a +b \,\operatorname{arcsinh}\left(c x \right)\right)^{n}d x$$

```
[In] int(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
[Out] int(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
```

Fricas [F]

$$\int x^2 \left(d+c^2 d x^2\right)^{5/2} (a+b \text{arcsinh}(c x))^n \, dx=\int { {\left(c^{2} d x^{2} + d\right)}^{\frac{5}{2}} {\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{n} x^{2} \,d x }$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="fricas
")
[Out] integral((c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2)*sqrt(c^2*d*x^2 + d)*(b*arc
sinh(c*x) + a)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \left(d+c^2 d x^2\right)^{5/2} (a+b \text{arcsinh}(c x))^n \, dx=\text{Timed out}$$

```
[In] integrate(x**2*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**n,x)
[Out] Timed out
```

Maxima [F]

$$\int x^2 (d + c^2dx^2)^{5/2} (a + b\text{arcsinh}(cx))^n \, dx = \int \left(c^2dx^2 + d\right)^{\frac{5}{2}} (b \operatorname{arsinh}(cx) + a)^n x^2 \, dx$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxima
")
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

Giac [F]

$$\int x^2 (d + c^2dx^2)^{5/2} (a + b\text{arcsinh}(cx))^n \, dx = \int \left(c^2dx^2 + d\right)^{\frac{5}{2}} (b \operatorname{arsinh}(cx) + a)^n x^2 \, dx$$

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac")
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (d + c^2dx^2)^{5/2} (a + b\text{arcsinh}(cx))^n \, dx = \int x^2 \, (a + b \operatorname{asinh}(c\,x))^n \left(d\,c^2\,x^2 + d\right)^{5/2} dx$$

```
[In] int(x^2*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2),x)
[Out] int(x^2*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2), x)
```

3.523 $\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx$

Optimal result

Integrand size = 26, antiderivative size = 745

$$\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx = \frac{7^{-1-n}d^2e^{-\frac{7a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{128c^2\sqrt{1+c^2x^2}}$$
$$+\frac{5^{-n}d^2e^{-\frac{5a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{128c^2\sqrt{1+c^2x^2}}$$
$$+\frac{3^{1-n}d^2e^{-\frac{3a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{128c^2\sqrt{1+c^2x^2}}$$
$$+\frac{5d^2e^{-\frac{a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{128c^2\sqrt{1+c^2x^2}}$$
$$+\frac{5d^2e^{a/b}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arcsinh}(cx)}{b}\right)}{128c^2\sqrt{1+c^2x^2}}$$
$$+\frac{3^{1-n}d^2e^{\frac{3a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{128c^2\sqrt{1+c^2x^2}}$$
$$+\frac{5^{-n}d^2e^{\frac{5a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{128c^2\sqrt{1+c^2x^2}}$$
$$+\frac{7^{-1-n}d^2e^{\frac{7a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{128c^2\sqrt{1+c^2x^2}}$$

```
[Out] 1/128*7^(-1-n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-7*(a+b*arcsinh(c*x))/b)*
(c^2*d*x^2+d)^(1/2)/c^2/exp(7*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^
```

```
(1/2)+1/128*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-5*(a+b*arcsinh(c*x))/b)*(c^
2*d*x^2+d)^(1/2)/(5^n)/c^2/exp(5*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+
1)^(1/2)+1/128*3^(1-n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-3*(a+b*arcsinh(c
*x))/b)*(c^2*d*x^2+d)^(1/2)/c^2/exp(3*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2
*x^2+1)^(1/2)+5/128*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(-a-b*arcsinh(c*x))/
b)*(c^2*d*x^2+d)^(1/2)/c^2/exp(a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)
^(1/2)+5/128*d^2*exp(a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(a+b*arcsinh(c*x))
/b)*(c^2*d*x^2+d)^(1/2)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)+1/
128*3^(1-n)*d^2*exp(3*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,3*(a+b*arcsinh(c*
x))/b)*(c^2*d*x^2+d)^(1/2)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)
+1/128*d^2*exp(5*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,5*(a+b*arcsinh(c*x))/b
)*(c^2*d*x^2+d)^(1/2)/(5^n)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2
)+1/128*7^(-1-n)*d^2*exp(7*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,7*(a+b*arcsi
nh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^
(1/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 745, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used

$= \{5819, 5556, 3389, 2212\}$

$$\int x\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx=\frac{d^2 7^{-n-1}e^{-\frac{7a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{128c^2\sqrt{c^2x^2+1}}$$
$$+\frac{d^2 5^{-n}e^{-\frac{5a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{128c^2\sqrt{c^2x^2+1}}$$
$$+\frac{d^2 3^{1-n}e^{-\frac{3a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{128c^2\sqrt{c^2x^2+1}}$$
$$+\frac{5d^2e^{-\frac{a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{128c^2\sqrt{c^2x^2+1}}$$
$$+\frac{5d^2e^{a/b}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{a+b\text{arcsinh}(cx)}{b}\right)}{128c^2\sqrt{c^2x^2+1}}$$
$$+\frac{d^2 3^{1-n}e^{\frac{3a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{128c^2\sqrt{c^2x^2+1}}$$
$$+\frac{d^2 5^{-n}e^{\frac{5a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{128c^2\sqrt{c^2x^2+1}}$$
$$+\frac{d^2 7^{-n-1}e^{\frac{7a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{128c^2\sqrt{c^2x^2+1}}$$

```
[In] Int[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (7^(-1 - n)*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-7
*(a + b*ArcSinh[c*x]))/b])/(128*c^2*E^((7*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b
*ArcSinh[c*x])/b))^n) + (d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gam
ma[1 + n, (-5*(a + b*ArcSinh[c*x]))/b])/(128*5^n*c^2*E^((5*a)/b)*Sqrt[1 + c
^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (3^(1 - n)*d^2*Sqrt[d + c^2*d*x^2]
*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-3*(a + b*ArcSinh[c*x]))/b])/(128*c^2
*E^((3*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (5*d^2*Sqrt
[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c*x])/
b)])/(128*c^2*E^(a/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (5
*d^2*E^(a/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (a + b
*ArcSinh[c*x])/b])/(128*c^2*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) +
 (3^(1 - n)*d^2*E^((3*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamm
a[1 + n, (3*(a + b*ArcSinh[c*x]))/b])/(128*c^2*Sqrt[1 + c^2*x^2]*((a + b*Ar
cSinh[c*x])/b)^n) + (d^2*E^((5*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x
])^n*Gamma[1 + n, (5*(a + b*ArcSinh[c*x]))/b])/(128*5^n*c^2*Sqrt[1 + c^2*x^
```

```
2]*((a + b*ArcSinh[c*x])/b)^n) + (7^(-1 - n)*d^2*E^((7*a)/b)*Sqrt[d + c^2*d
*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (7*(a + b*ArcSinh[c*x]))/b])/(128
*c^2*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int x^n\cosh^6\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc^2\sqrt{1+c^2x^2}}$$

$$= -\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{1}{64}x^n\sinh\left(\frac{7a}{b}-\frac{7x}{b}\right)+\frac{5}{64}x^n\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)+\frac{9}{64}x^n\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)+\frac{5}{64}x\right.\right.}{bc^2\sqrt{1+c^2x^2}}$$

$$= -\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int x^n\sinh\left(\frac{7a}{b}-\frac{7x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{64bc^2\sqrt{1+c^2x^2}} - \frac{\left(5d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int x^n\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{64bc^2\sqrt{1+c^2x^2}} - \frac{\left(5d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int x^n\sinh\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{64bc^2\sqrt{1+c^2x^2}} - \frac{\left(9d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int x^n\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{64bc^2\sqrt{1+c^2x^2}}$$

$$= -\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{-i\left(\frac{7ia}{b}-\frac{7ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{128bc^2\sqrt{1+c^2x^2}} + \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{i\left(\frac{7ia}{b}-\frac{7ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{128bc^2\sqrt{1+c^2x^2}} - \frac{\left(5d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{128bc^2\sqrt{1+c^2x^2}} + \frac{\left(5d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{128bc^2\sqrt{1+c^2x^2}} - \frac{\left(5d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{-i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{128bc^2\sqrt{1+c^2x^2}} + \frac{\left(5d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{128bc^2\sqrt{1+c^2x^2}} - \frac{\left(9d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{128bc^2\sqrt{1+c^2x^2}} + \frac{\left(9d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{128bc^2\sqrt{1+c^2x^2}}$$

$$= \frac{7^{-1-n}d^2e^{-\frac{7a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{128c^2\sqrt{1+c^2x^2}}$$
$$+\frac{5^{-n}d^2e^{-\frac{5a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{128c^2\sqrt{1+c^2x^2}}$$
$$+\frac{3^{1-n}d^2e^{-\frac{3a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{128c^2\sqrt{1+c^2x^2}}$$
$$+\frac{5d^2e^{-\frac{a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{128c^2\sqrt{1+c^2x^2}}$$
$$+\frac{5d^2e^{a/b}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arcsinh}(cx)}{b}\right)}{128c^2\sqrt{1+c^2x^2}}$$
$$+\frac{3^{1-n}d^2e^{\frac{3a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{128c^2\sqrt{1+c^2x^2}}$$
$$+\frac{5^{-n}d^2e^{\frac{5a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{128c^2\sqrt{1+c^2x^2}}$$
$$+\frac{7^{-1-n}d^2e^{\frac{7a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{128c^2\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 685, normalized size of antiderivative = 0.92

$$\int x\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx = \frac{105^{-1-n}d^3e^{-\frac{7a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\left(-\frac{(a+b\text{arcsinh}(cx))^2}{b^2}\right)}{}$$

```
[In] Integrate[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (105^(-1 - n)*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*(5^(2 + n)*21^(1
 + n)*E^((8*a)/b)*(-((a + b*ArcSinh[c*x])/b))^(2*n)*(-((a + b*ArcSinh[c*x])
^2/b^2))^n*Gamma[1 + n, a/b + ArcSinh[c*x]] + 15^(1 + n)*(-((a + b*ArcSinh[
c*x])^2/b^2))^(2*n)*Gamma[1 + n, (-7*(a + b*ArcSinh[c*x]))/b] + E^((2*a)/b)
*(5*21^(1 + n)*(-((a + b*ArcSinh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, (-5*(a +
b*ArcSinh[c*x]))/b] + 9*35^(1 + n)*E^((2*a)/b)*(-((a + b*ArcSinh[c*x])^2/b^
2))^(2*n)*Gamma[1 + n, (-3*(a + b*ArcSinh[c*x]))/b] + 5^(2 + n)*21^(1 + n)*
E^((4*a)/b)*(-((a + b*ArcSinh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, -((a + b*Arc
```

```
Sinh[c*x])/b)] + 35^(1 + n)*E^((8*a)/b)*(a/b + ArcSinh[c*x])^n*(-((a + b*Ar
cSinh[c*x])/b))^(3*n)*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b] + 8*35^(1 +
n)*E^((8*a)/b)*(-((a + b*ArcSinh[c*x])/b))^(2*n)*(-((a + b*ArcSinh[c*x])^2/
b^2))^n*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b] - 3^(2 + n)*7^(1 + n)*E^((
10*a)/b)*(a/b + ArcSinh[c*x])^n*(-((a + b*ArcSinh[c*x])/b))^(3*n)*Gamma[1 +
 n, (5*(a + b*ArcSinh[c*x]))/b] + 8*21^(1 + n)*E^((10*a)/b)*(-((a + b*ArcSi
nh[c*x])/b))^(2*n)*(-((a + b*ArcSinh[c*x])^2/b^2))^n*Gamma[1 + n, (5*(a + b
*ArcSinh[c*x]))/b] + 15^(1 + n)*E^((12*a)/b)*(a/b + ArcSinh[c*x])^n*(-((a +
 b*ArcSinh[c*x])/b))^(3*n)*Gamma[1 + n, (7*(a + b*ArcSinh[c*x]))/b])))/(128
*c^2*E^((7*a)/b)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x])/b))^n*(-((a +
b*ArcSinh[c*x])^2/b^2))^(2*n))
```

Maple [F]

$$\int x\left(c^2 d\,x^2+d\right)^{\frac{5}{2}} \left(a+b\,\operatorname{arcsinh}\left(c x\right)\right)^{n} dx$$

```
[In] int(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
```

```
[Out] int(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
```

Fricas [F]

$$\int x\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx=\int \left(c^2dx^2+d\right)^{\frac{5}{2}}(b\operatorname{arsinh}\left(cx\right)+a)^n x\,dx$$

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="fricas")
```

```
[Out] integral((c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x)*sqrt(c^2*d*x^2 + d)*(b*arcsi
nh(c*x) + a)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx=\text{Timed out}$$

```
[In] integrate(x*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**n,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx=\int (c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^n x\,dx$$

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxima")
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n*x, x)
```

Giac [F(-2)]

Exception generated.

$$\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx=\int x\,(a+b\,\text{asinh}(c\,x))^n\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

```
[In] int(x*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2),x)
[Out] int(x*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2), x)
```

3.524 $\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx$

Optimal result

Integrand size = 25, antiderivative size = 632

$$\begin{aligned}\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx &= \frac{5d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{1+n}}{16bc(1+n)\sqrt{1+c^2x^2}} \\
&+ \frac{2^{-7-n}3^{-1-n}d^2e^{-\frac{6a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}} \\
&+ \frac{3\ 2^{-7-2n}d^2e^{-\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}} \\
&+ \frac{15\ 2^{-7-n}d^2e^{-\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}} \\
&- \frac{15\ 2^{-7-n}d^2e^{\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}} \\
&- \frac{3\ 2^{-7-2n}d^2e^{\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}} \\
&- \frac{2^{-7-n}3^{-1-n}d^2e^{\frac{6a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}}\end{aligned}$$

```
[Out] 5/16*d^2*(a+b*arcsinh(c*x))^(1+n)*(c^2*d*x^2+d)^(1/2)/b/c/(1+n)/(c^2*x^2+1)
^(1/2)+2^(-7-n)*3^(-1-n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-6*(a+b*arcsinh
(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c/exp(6*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2
*x^2+1)^(1/2)+3*2^(-7-2*n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-4*(a+b*arcsi
nh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c/exp(4*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c
^2*x^2+1)^(1/2)+15*2^(-7-n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-2*(a+b*arcs
```

```
inh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c/exp(2*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(
c^2*x^2+1)^(1/2)-15*2^(-7-n)*d^2*exp(2*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,
2*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c/(((a+b*arcsinh(c*x))/b)^n)/(c
^2*x^2+1)^(1/2)-3*2^(-7-2*n)*d^2*exp(4*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,
4*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c/(((a+b*arcsinh(c*x))/b)^n)/(c
^2*x^2+1)^(1/2)-2^(-7-n)*3^(-1-n)*d^2*exp(6*a/b)*(a+b*arcsinh(c*x))^n*GAMMA
(1+n,6*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c/(((a+b*arcsinh(c*x))/b)^
n)/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 632, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5791, 3393, 3388, 2212}

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx=\frac{5d^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^{n+1}}{16bc(n+1)\sqrt{c^2x^2+1}}$$
$$+\frac{d^22^{-n-7}3^{-n-1}e^{-\frac{6a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{c^2x^2+1}}$$
$$+\frac{3d^22^{-2n-7}e^{-\frac{4a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{c^2x^2+1}}$$
$$+\frac{15d^22^{-n-7}e^{-\frac{2a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{c^2x^2+1}}$$
$$-\frac{15d^22^{-n-7}e^{\frac{2a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{c^2x^2+1}}$$
$$-\frac{3d^22^{-2n-7}e^{\frac{4a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{c^2x^2+1}}$$
$$-\frac{d^22^{-n-7}3^{-n-1}e^{\frac{6a}{b}}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{c^2x^2+1}}$$

```
[In] Int[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (5*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(16*b*c*(1 + n)*Sq
rt[1 + c^2*x^2]) + (2^(-7 - n)*3^(-1 - n)*d^2*Sqrt[d + c^2*d*x^2]*(a + b*Ar
cSinh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcSinh[c*x]))/b])/(c*E^((6*a)/b)*Sqr
t[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (3*2^(-7 - 2*n)*d^2*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/
b])/(c*E^((4*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (15*2
^(-7 - n)*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-2*(
```

```
a + b*ArcSinh[c*x]))/b])/(c*E^((2*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh
[c*x])/b))^n) - (15*2^(-7 - n)*d^2*E^((2*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b])/(c*Sqrt[1 + c^2*x^
2]*((a + b*ArcSinh[c*x])/b)^n) - (3*2^(-7 - 2*n)*d^2*E^((4*a)/b)*Sqrt[d + c
^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]))/b])/
(c*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) - (2^(-7 - n)*3^(-1 - n)*d
^2*E^((6*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (6*(
a + b*ArcSinh[c*x]))/b])/(c*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int x^n\cosh^6\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{5x^n}{16}+\frac{1}{32}x^n\cosh\left(\frac{6a}{b}-\frac{6x}{b}\right)+\frac{3}{16}x^n\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)+\frac{15}{32}x^n\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)\right.}{bc\sqrt{1+c^2x^2}}$$

$$= \frac{5d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{1+n}}{16bc(1+n)\sqrt{1+c^2x^2}} + \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int x^n\cosh\left(\frac{6a}{b}-\frac{6x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{32bc\sqrt{1+c^2x^2}} + \frac{\left(3d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int x^n\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{16bc\sqrt{1+c^2x^2}} + \frac{\left(15d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int x^n\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{32bc\sqrt{1+c^2x^2}}$$

$$= \frac{5d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{1+n}}{16bc(1+n)\sqrt{1+c^2x^2}} + \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{-i\left(\frac{6ia}{b}-\frac{6ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{64bc\sqrt{1+c^2x^2}} + \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{i\left(\frac{6ia}{b}-\frac{6ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{64bc\sqrt{1+c^2x^2}} + \frac{\left(3d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{-i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{32bc\sqrt{1+c^2x^2}} + \frac{\left(3d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{32bc\sqrt{1+c^2x^2}} + \frac{\left(15d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{64bc\sqrt{1+c^2x^2}} + \frac{\left(15d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{64bc\sqrt{1+c^2x^2}}$$

$$= \frac{5d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{1+n}}{16bc(1+n)\sqrt{1+c^2x^2}}$$
$$+ \frac{2^{-7-n}3^{-1-n}d^2e^{-\frac{6a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{6(a+b\text{arcsinh}}{b}\right.}{c\sqrt{1+c^2x^2}}$$
$$+ \frac{3\ 2^{-7-2n}d^2e^{-\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4(a+b\text{arcsinh}(cx)}{b}\right.}{c\sqrt{1+c^2x^2}}$$
$$+ \frac{15\ 2^{-7-n}d^2e^{-\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\text{arcsinh}(cx)}{b}\right.}{c\sqrt{1+c^2x^2}}$$
$$- \frac{15\ 2^{-7-n}d^2e^{\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}}$$
$$- \frac{3\ 2^{-7-2n}d^2e^{\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}}$$
$$- \frac{2^{-7-n}3^{-1-n}d^2e^{\frac{6a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 4.00 (sec) , antiderivative size = 529, normalized size of antiderivative = 0.84

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx = \frac{2^{-7-2n}3^{-1-n}d^3e^{-\frac{6a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{(a+b\text{arcsinh}(cx))^2}{b^2}\right)^{-2n}\left(2^nb(1+n\right.}{}$$

```
[In] Integrate[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (2^(-7 - 2*n)*3^(-1 - n)*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*(2^n*
b*(1 + n)*(a/b + ArcSinh[c*x])^(2*n)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1
+ n, (-6*(a + b*ArcSinh[c*x]))/b] + 3^(2 + n)*b*E^((2*a)/b)*(1 + n)*(a/b +
ArcSinh[c*x])^(2*n)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (-4*(a + b*A
rcSinh[c*x]))/b] + 5*2^n*3^(2 + n)*b*E^((4*a)/b)*(1 + n)*(a/b + ArcSinh[c*x
])^n*(-((a + b*ArcSinh[c*x])^2/b^2))^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]
))/b] - E^((6*a)/b)*(5*2^n*3^(2 + n)*b*E^((2*a)/b)*(1 + n)*(-((a + b*ArcSin
h[c*x])/b))^n*(-((a + b*ArcSinh[c*x])^2/b^2))^n*Gamma[1 + n, (2*(a + b*ArcS
inh[c*x]))/b] + 3^(2 + n)*b*E^((4*a)/b)*(1 + n)*(a/b + ArcSinh[c*x])^n*(-((
a + b*ArcSinh[c*x])/b))^(2*n)*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]))/b] + 2^
n*(-5*2^(3 + n)*3^(1 + n)*(a + b*ArcSinh[c*x])*(-((a + b*ArcSinh[c*x])^2/b^
2))^(2*n) + b*E^((6*a)/b)*(1 + n)*(a/b + ArcSinh[c*x])^n*(-((a + b*ArcSinh[
```

```
c*x])/b))^(2*n)*Gamma[1 + n, (6*(a + b*ArcSinh[c*x]))/b]))))/(b*c*E^((6*a)/
b)*(1 + n)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x])^2/b^2))^(2*n))
```

Maple [F]

$$\int \left(c^2 d\,x^2+d\right)^{\frac{5}{2}} \left(a+b\,\operatorname{arcsinh}\left(cx\right)\right)^n dx$$

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
[Out] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
```

Fricas [F]

$$\int \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))^n\,dx = \int \left(c^2dx^2+d\right)^{\frac{5}{2}} (b\,\text{arsinh}\,(cx)+a)^n\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="fricas")
[Out] integral((c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2)*sqrt(c^2*d*x^2 + d)*(b*arcsinh
(c*x) + a)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))^n\,dx = \text{Timed out}$$

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**n,x)
[Out] Timed out
```

Maxima [F]

$$\int \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))^n\,dx = \int \left(c^2dx^2+d\right)^{\frac{5}{2}} (b\,\text{arsinh}\,(cx)+a)^n\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxima")
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n, x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx=\int (a+b\,\text{asinh}(c\,x))^n\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

```
[In] int((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2), x)
```

3.525 $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n}{x}\,dx = \frac{5^{-1-n}d^3e^{-\frac{5a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma}{32\sqrt{d+c^2dx^2}}$$
$$-\frac{5\ 3^{-1-n}d^3e^{-\frac{3a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32\sqrt{d+c^2dx^2}}$$
$$+\frac{3^{-n}d^3e^{-\frac{3a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{8\sqrt{d+c^2dx^2}}$$
$$+\frac{11d^3e^{-\frac{a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{16\sqrt{d+c^2dx^2}}$$
$$+\frac{11d^3e^{a/b}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arcsinh}(cx)}{b}\right)}{16\sqrt{d+c^2dx^2}}$$
$$-\frac{5\ 3^{-1-n}d^3e^{\frac{3a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32\sqrt{d+c^2dx^2}}$$
$$+\frac{3^{-n}d^3e^{\frac{3a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{8\sqrt{d+c^2dx^2}}$$
$$+\frac{5^{-1-n}d^3e^{\frac{5a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{32\sqrt{d+c^2dx^2}}$$
$$+d^3\text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^n}{x\sqrt{d+c^2dx^2}},x\right)$$

```
[Out] 1/32*5^(-1-n)*d^3*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-5*(a+b*arcsinh(c*x))/b)*(
c^2*x^2+1)^(1/2)/exp(5*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)
-5/32*3^(-1-n)*d^3*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-3*(a+b*arcsinh(c*x))/b)*
(c^2*x^2+1)^(1/2)/exp(3*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2
)+1/8*d^3*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-3*(a+b*arcsinh(c*x))/b)*(c^2*x^2+
1)^(1/2)/(3^n)/exp(3*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+1
1/16*d^3*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(-a-b*arcsinh(c*x))/b)*(c^2*x^2+1)^
(1/2)/exp(a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+11/16*d^3*ex
p(a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(a+b*arcsinh(c*x))/b)*(c^2*x^2+1)^(1/
2)/(((a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)-5/32*3^(-1-n)*d^3*exp(3*a
/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,3*(a+b*arcsinh(c*x))/b)*(c^2*x^2+1)^(1/2
)/(((a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+1/8*d^3*exp(3*a/b)*(a+b*ar
csinh(c*x))^n*GAMMA(1+n,3*(a+b*arcsinh(c*x))/b)*(c^2*x^2+1)^(1/2)/(3^n)/(((
a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+1/32*5^(-1-n)*d^3*exp(5*a/b)*(a
+b*arcsinh(c*x))^n*GAMMA(1+n,5*(a+b*arcsinh(c*x))/b)*(c^2*x^2+1)^(1/2)/(((a
+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+d^3*Unintegrable((a+b*arcsinh(c*
x))^n/x/(c^2*d*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n}{x}\,dx = \int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n}{x}\,dx$$

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x,x]
```

```
[Out] (5^(-1 - n)*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-5*(
a + b*ArcSinh[c*x]))/b])/(32*E^((5*a)/b)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcS
inh[c*x])/b))^n) - (5*3^(-1 - n)*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])
^n*Gamma[1 + n, (-3*(a + b*ArcSinh[c*x]))/b])/(32*E^((3*a)/b)*Sqrt[d + c^2*
d*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSi
nh[c*x])^n*Gamma[1 + n, (-3*(a + b*ArcSinh[c*x]))/b])/(8*3^n*E^((3*a)/b)*Sq
rt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (11*d^3*Sqrt[1 + c^2*x^2
]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c*x])/b)])/(16*E^(a/
b)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (11*d^3*E^(a/b)*Sqr
t[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (a + b*ArcSinh[c*x])/b])
/(16*Sqrt[d + c^2*d*x^2]*((a + b*ArcSinh[c*x])/b)^n) - (5*3^(-1 - n)*d^3*E^
((3*a)/b)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (3*(a + b*A
rcSinh[c*x]))/b])/(32*Sqrt[d + c^2*d*x^2]*((a + b*ArcSinh[c*x])/b)^n) + (d^
3*E^((3*a)/b)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (3*(a +
 b*ArcSinh[c*x]))/b])/(8*3^n*Sqrt[d + c^2*d*x^2]*((a + b*ArcSinh[c*x])/b)^n
```

```
) + (5^(-1 - n)*d^3*E^((5*a)/b)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Ga
mma[1 + n, (5*(a + b*ArcSinh[c*x]))/b])/(32*Sqrt[d + c^2*d*x^2]*((a + b*Arc
Sinh[c*x])/b)^n) + d^3*Defer[Int][(a + b*ArcSinh[c*x])^n/(x*Sqrt[d + c^2*d*
x^2]), x]
```

Rubi steps

$$\text{integral} = \int \left(\frac{d^3(a+b\text{arcsinh}(cx))^n}{x\sqrt{d+c^2dx^2}} + \frac{3c^2d^3x(a+b\text{arcsinh}(cx))^n}{\sqrt{d+c^2dx^2}} + \frac{3c^4d^3x^3(a+b\text{arcsinh}(cx))^n}{\sqrt{d+c^2dx^2}} + \frac{c^6d^3x^5(a+b\text{arcsinh}(cx))^n}{\sqrt{d+c^2dx^2}}\right) dx$$

$$= d^3\int \frac{(a+b\text{arcsinh}(cx))^n}{x\sqrt{d+c^2dx^2}}\,dx + \left(3c^2d^3\right)\int \frac{x(a+b\text{arcsinh}(cx))^n}{\sqrt{d+c^2dx^2}}\,dx + \left(3c^4d^3\right)\int \frac{x^3(a+b\text{arcsinh}(cx))^n}{\sqrt{d+c^2dx^2}}\,dx + \left(c^6d^3\right)\int \frac{x^5(a+b\text{arcsinh}(cx))^n}{\sqrt{d+c^2dx^2}}\,dx$$

$$= d^3\int \frac{(a+b\text{arcsinh}(cx))^n}{x\sqrt{d+c^2dx^2}}\,dx - \frac{\left(d^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int x^n\sinh^5\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)}{b\sqrt{d+c^2dx^2}} - \frac{\left(3d^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int x^n\sinh\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)}{b\sqrt{d+c^2dx^2}} - \frac{\left(3d^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int x^n\sinh^3\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)}{b\sqrt{d+c^2dx^2}}$$

$$= d^3\int \frac{(a+b\text{arcsinh}(cx))^n}{x\sqrt{d+c^2dx^2}}\,dx + \frac{\left(id^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \left(\frac{1}{16}ix^n\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right) - \frac{5}{16}ix^n\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right) + \frac{5}{8}ix^n\sinh\left(\frac{a}{b}-\frac{x}{b}\right)\right)\,dx, x\right.}{b\sqrt{d+c^2dx^2}} - \frac{\left(3id^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \left(-\frac{1}{4}ix^n\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right) + \frac{3}{4}ix^n\sinh\left(\frac{a}{b}-\frac{x}{b}\right)\right)\,dx, x, a+b\text{arcsinh}(cx)\right)}{b\sqrt{d+c^2dx^2}} - \frac{\left(3d^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}x^n\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b\sqrt{d+c^2dx^2}} + \frac{\left(3d^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}x^n\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b\sqrt{d+c^2dx^2}}$$

$$
\begin{aligned}
&= \frac{3d^3 e^{-\frac{a}{b}}\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^n \left(-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{2\sqrt{d+c^2dx^2}} \\
&+ \frac{3d^3 e^{a/b}\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^n \left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{2\sqrt{d+c^2dx^2}} \\
&+ d^3\int \frac{(a+b\operatorname{arcsinh}(cx))^n}{x\sqrt{d+c^2dx^2}}\,dx \\
&- \frac{\left(d^3\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int x^n \sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{16b\sqrt{d+c^2dx^2}} \\
&+ \frac{\left(5d^3\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int x^n \sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{16b\sqrt{d+c^2dx^2}} \\
&- \frac{\left(5d^3\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int x^n \sinh\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{8b\sqrt{d+c^2dx^2}} \\
&- \frac{\left(3d^3\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int x^n \sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{4b\sqrt{d+c^2dx^2}} \\
&+ \frac{\left(9d^3\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int x^n \sinh\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{4b\sqrt{d+c^2dx^2}}
\end{aligned}
$$

$$= \frac{3d^3e^{-\frac{a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{2\sqrt{d+c^2dx^2}}$$
$$+\frac{3d^3e^{a/b}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arcsinh}(cx)}{b}\right)}{2\sqrt{d+c^2dx^2}}$$
$$+d^3\int\frac{(a+b\text{arcsinh}(cx))^n}{x\sqrt{d+c^2dx^2}}\,dx$$
$$-\frac{\left(d^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int e^{-i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{32b\sqrt{d+c^2dx^2}}$$
$$+\frac{\left(d^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int e^{i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{32b\sqrt{d+c^2dx^2}}$$
$$+\frac{\left(5d^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{32b\sqrt{d+c^2dx^2}}$$
$$-\frac{\left(5d^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{32b\sqrt{d+c^2dx^2}}$$
$$-\frac{\left(5d^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{16b\sqrt{d+c^2dx^2}}$$
$$+\frac{\left(5d^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{16b\sqrt{d+c^2dx^2}}$$
$$-\frac{\left(3d^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{8b\sqrt{d+c^2dx^2}}$$
$$+\frac{\left(3d^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{8b\sqrt{d+c^2dx^2}}$$
$$+\frac{\left(9d^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{8b\sqrt{d+c^2dx^2}}$$
$$-\frac{\left(9d^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{8b\sqrt{d+c^2dx^2}}$$

$$
\begin{aligned}
&= \frac{5^{-1-n}d^3e^{-\frac{5a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{32\sqrt{d+c^2dx^2}} \\
&- \frac{5\ 3^{-1-n}d^3e^{-\frac{3a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32\sqrt{d+c^2dx^2}} \\
&+ \frac{3^{-n}d^3e^{-\frac{3a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{8\sqrt{d+c^2dx^2}} \\
&+ \frac{11d^3e^{-\frac{a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{16\sqrt{d+c^2dx^2}} \\
&+ \frac{11d^3e^{a/b}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arcsinh}(cx)}{b}\right)}{16\sqrt{d+c^2dx^2}} \\
&- \frac{5\ 3^{-1-n}d^3e^{\frac{3a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32\sqrt{d+c^2dx^2}} \\
&+ \frac{3^{-n}d^3e^{\frac{3a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{8\sqrt{d+c^2dx^2}} \\
&+ \frac{5^{-1-n}d^3e^{\frac{5a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{32\sqrt{d+c^2dx^2}} \\
&+ d^3\int\frac{(a+b\text{arcsinh}(cx))^n}{x\sqrt{d+c^2dx^2}}\,dx
\end{aligned}
$$

Mathematica [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$
\int\frac{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^n}{x}\,dx=\int\frac{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^n}{x}\,dx
$$

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x,x]
[Out] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\left(c^{2} d \,x^{2}+d \right)^{\frac{5}{2}} \left(a +b \,\operatorname{arcsinh}\left(c x \right)\right)^{n}}{x}d x$$

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x,x)
[Out] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^n}{x}\,dx=\int \frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^n}{x}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="fricas")
[Out] integral((c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2)*sqrt(c^2*d*x^2 + d)*(b*arcsinh
(c*x) + a)^n/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^n}{x}\,dx=\text{Timed out}$$

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**n/x,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^n}{x}\,dx=\int \frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^n}{x}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="maxima")
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n}{x}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^n\,(d\,c^2\,x^2+d)^{5/2}}{x}\,dx$$

```
[In] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2))/x, x)
```

3.526 $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx = \frac{15cd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{1+n}}{8b(1+n)\sqrt{d+c^2dx^2}}$$
$$+\frac{2^{-2(3+n)}cd^3e^{-\frac{4a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{d+c^2dx^2}}$$
$$+\frac{2^{-2-n}cd^3e^{-\frac{2a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{d+c^2dx^2}}$$
$$-\frac{2^{-2-n}cd^3e^{\frac{2a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{d+c^2dx^2}}$$
$$-\frac{2^{-2(3+n)}cd^3e^{\frac{4a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{d+c^2dx^2}}$$
$$+d^3\text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^n}{x^2\sqrt{d+c^2dx^2}},x\right)$$

```
[Out] 15/8*c*d^3*(a+b*arcsinh(c*x))^(1+n)*(c^2*x^2+1)^(1/2)/b/(1+n)/(c^2*d*x^2+d)
^(1/2)+c*d^3*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-4*(a+b*arcsinh(c*x))/b)*(c^2*x
^2+1)^(1/2)/(2^(6+2*n))/exp(4*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d
)^(1/2)+2^(-2-n)*c*d^3*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-2*(a+b*arcsinh(c*x))
/b)*(c^2*x^2+1)^(1/2)/exp(2*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^
(1/2)-2^(-2-n)*c*d^3*exp(2*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,2*(a+b*arcsi
nh(c*x))/b)*(c^2*x^2+1)^(1/2)/(((a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2
)-c*d^3*exp(4*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,4*(a+b*arcsinh(c*x))/b)*(
c^2*x^2+1)^(1/2)/(2^(6+2*n))/(((a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)
+d^3*Unintegrable((a+b*arcsinh(c*x))^n/x^2/(c^2*d*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^n}{x^2}\,dx=\int \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^n}{x^2}\,dx$$

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x^2,x]
```

```
[Out] (15*c*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(8*b*(1 + n)*Sqrt
[d + c^2*d*x^2]) + (c*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1
+ n, (-4*(a + b*ArcSinh[c*x]))/b])/(2^(2*(3 + n))*E^((4*a)/b)*Sqrt[d + c^2*
d*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (2^(-2 - n)*c*d^3*Sqrt[1 + c^2*x^2]
*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b])/(E^((2*a
)/b)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) - (2^(-2 - n)*c*d^3
*E^((2*a)/b)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (2*(a +
b*ArcSinh[c*x]))/b])/(Sqrt[d + c^2*d*x^2]*((a + b*ArcSinh[c*x])/b)^n) - (c*
d^3*E^((4*a)/b)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (4*(a
 + b*ArcSinh[c*x]))/b])/(2^(2*(3 + n))*Sqrt[d + c^2*d*x^2]*((a + b*ArcSinh[
c*x])/b)^n) + d^3*Defer[Int][(a + b*ArcSinh[c*x])^n/(x^2*Sqrt[d + c^2*d*x^2
]), x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3c^2d^3(a+b\operatorname{arcsinh}(cx))^n}{\sqrt{d+c^2dx^2}}+\frac{d^3(a+b\operatorname{arcsinh}(cx))^n}{x^2\sqrt{d+c^2dx^2}}+\frac{3c^4d^3x^2(a+b\operatorname{arcsinh}(cx))^n}{\sqrt{d+c^2dx^2}}+\frac{c^6d^3x^4(a+b\operatorname{arcsinh}(cx))^n}{\sqrt{d+c^2dx^2}}\right)dx\\
&= d^3\int \frac{(a+b\operatorname{arcsinh}(cx))^n}{x^2\sqrt{d+c^2dx^2}}\,dx+\left(3c^2d^3\right)\int \frac{(a+b\operatorname{arcsinh}(cx))^n}{\sqrt{d+c^2dx^2}}\,dx\\
&\quad+\left(3c^4d^3\right)\int \frac{x^2(a+b\operatorname{arcsinh}(cx))^n}{\sqrt{d+c^2dx^2}}\,dx+\left(c^6d^3\right)\int \frac{x^4(a+b\operatorname{arcsinh}(cx))^n}{\sqrt{d+c^2dx^2}}\,dx\\
&= \frac{3cd^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^{1+n}}{b(1+n)\sqrt{d+c^2dx^2}}+d^3\int \frac{(a+b\operatorname{arcsinh}(cx))^n}{x^2\sqrt{d+c^2dx^2}}\,dx\\
&\quad+\frac{\left(cd^3\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int x^n\sinh^4\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{b\sqrt{d+c^2dx^2}}\\
&\quad+\frac{\left(3cd^3\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int x^n\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{b\sqrt{d+c^2dx^2}}
\end{aligned}$$

$$= \frac{3cd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{1+n}}{b(1+n)\sqrt{d+c^2dx^2}} + d^3\int \frac{(a+b\text{arcsinh}(cx))^n}{x^2\sqrt{d+c^2dx^2}}\,dx + \frac{\left(cd^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \left(\frac{3x^n}{8}+\frac{1}{8}x^n\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)-\frac{1}{2}x^n\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)\right)\,dx,x,a+b\text{arcsinh}(cx\right)}{b\sqrt{d+c^2dx^2}} - \frac{\left(3cd^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \left(\frac{x^n}{2}-\frac{1}{2}x^n\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{b\sqrt{d+c^2dx^2}}$$

$$= \frac{15cd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{1+n}}{8b(1+n)\sqrt{d+c^2dx^2}} + d^3\int \frac{(a+b\text{arcsinh}(cx))^n}{x^2\sqrt{d+c^2dx^2}}\,dx + \frac{\left(cd^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int x^n\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{8b\sqrt{d+c^2dx^2}} - \frac{\left(cd^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int x^n\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{2b\sqrt{d+c^2dx^2}} + \frac{\left(3cd^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int x^n\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{2b\sqrt{d+c^2dx^2}}$$

$$= \frac{15cd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{1+n}}{8b(1+n)\sqrt{d+c^2dx^2}} + d^3\int \frac{(a+b\text{arcsinh}(cx))^n}{x^2\sqrt{d+c^2dx^2}}\,dx + \frac{\left(cd^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int e^{-i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{16b\sqrt{d+c^2dx^2}} + \frac{\left(cd^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int e^{i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{16b\sqrt{d+c^2dx^2}} - \frac{\left(cd^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{4b\sqrt{d+c^2dx^2}} - \frac{\left(cd^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{4b\sqrt{d+c^2dx^2}} + \frac{\left(3cd^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{4b\sqrt{d+c^2dx^2}} + \frac{\left(3cd^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}x^n\,dx,x,a+b\text{arcsinh}(cx)\right)}{4b\sqrt{d+c^2dx^2}}$$

$$= \frac{15cd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{1+n}}{8b(1+n)\sqrt{d+c^2dx^2}} + \frac{4^{-3-n}cd^3e^{-\frac{4a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{d+c^2dx^2}} + \frac{2^{-2-n}cd^3e^{-\frac{2a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{d+c^2dx^2}} - \frac{2^{-2-n}cd^3e^{\frac{2a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{d+c^2dx^2}} - \frac{4^{-3-n}cd^3e^{\frac{4a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{d+c^2dx^2}} + d^3\int\frac{(a+b\text{arcsinh}(cx))^n}{x^2\sqrt{d+c^2dx^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int\frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx = \int\frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx$$

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x^2,x]
[Out] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int\frac{(c^2d\,x^2+d)^{\frac{5}{2}}\left(a+b\ \text{arcsinh}\left(cx\right)\right)^n}{x^2}dx$$

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x^2,x)
[Out] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\mathrm{arcsinh}(cx))^n}{x^2}\,dx=\int \frac{(c^2dx^2+d)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^n}{x^2}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="fricas
")
[Out] integral((c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2)*sqrt(c^2*d*x^2 + d)*(b*arcsinh
(c*x) + a)^n/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\mathrm{arcsinh}(cx))^n}{x^2}\,dx=\text{Timed out}$$

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**n/x**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\mathrm{arcsinh}(cx))^n}{x^2}\,dx=\int \frac{(c^2dx^2+d)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^n}{x^2}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="maxima
")
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^n}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^n}{x^2}\,dx = \int \frac{(a+b\operatorname{asinh}(c\,x))^n\,(d\,c^2\,x^2+d)^{5/2}}{x^2}\,dx$$

```
[In] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2))/x^2, x)
```

3.527 $\int \frac{x^m \operatorname{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^m \operatorname{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \operatorname{Int}\left(\frac{x^m \operatorname{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}},x\right)$$

```
[Out] Unintegrable(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m \operatorname{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx$$

```
[In] Int[(x^m*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] Defer[Int][(x^m*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2], x]
```

Rubi steps

$$\text{integral} = \int \frac{x^m \operatorname{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^m \text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx$$

```
[In] Integrate[(x^m*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
[Out] Integrate[(x^m*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^m \operatorname{arcsinh}(ax)^n}{\sqrt{a^2x^2+1}}dx$$

```
[In] int(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
[Out] int(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m \text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^m*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

Sympy [N/A]

Not integrable

Time = 3.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^m \text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{asinh}^n(ax)}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x**m*asinh(a*x)**n/(a**2*x**2+1)**(1/2),x)
[Out] Integral(x**m*asinh(a*x)**n/sqrt(a**2*x**2 + 1), x)
```

Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx=\int \frac{x^m\,\text{arsinh}\left(ax\right)^n}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^m*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx=\int \frac{x^m\,\text{arsinh}\left(ax\right)^n}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^m*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

Mupad [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx=\int \frac{x^m\,\text{asinh}(a\,x)^n}{\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int((x^m*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^m*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2), x)
```

3.528 $\int \frac{x^3 \text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}} dx$

Optimal result

Integrand size = 23, antiderivative size = 113

$$\int \frac{x^3 \text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}} dx = \frac{3^{-1-n}(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,-3\text{arcsinh}(ax))}{8a^4} - \frac{3(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,-\text{arcsinh}(ax))}{8a^4} - \frac{3\Gamma(1+n,\text{arcsinh}(ax))}{8a^4} + \frac{3^{-1-n}\Gamma(1+n,3\text{arcsinh}(ax))}{8a^4}$$

```
[Out] 1/8*3^(-1-n)*arcsinh(a*x)^n*GAMMA(1+n,-3*arcsinh(a*x))/a^4/((-arcsinh(a*x))
^n)-3/8*arcsinh(a*x)^n*GAMMA(1+n,-arcsinh(a*x))/a^4/((-arcsinh(a*x))^n)-3/8
*GAMMA(1+n,arcsinh(a*x))/a^4+1/8*3^(-1-n)*GAMMA(1+n,3*arcsinh(a*x))/a^4
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5819, 3393, 3389, 2212}

$$\int \frac{x^3 \text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}} dx = \frac{3^{-n-1}\text{arcsinh}(ax)^n(-\text{arcsinh}(ax))^{-n}\Gamma(n+1,-3\text{arcsinh}(ax))}{8a^4} - \frac{3\text{arcsinh}(ax)^n(-\text{arcsinh}(ax))^{-n}\Gamma(n+1,-\text{arcsinh}(ax))}{8a^4} - \frac{3\Gamma(n+1,\text{arcsinh}(ax))}{8a^4} + \frac{3^{-n-1}\Gamma(n+1,3\text{arcsinh}(ax))}{8a^4}$$

```
[In] Int[(x^3*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (3^(-1 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -3*ArcSinh[a*x]])/(8*a^4*(-ArcSinh[
a*x])^n) - (3*ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(8*a^4*(-ArcSinh[
a*x])^n) - (3*Gamma[1 + n, ArcSinh[a*x]])/(8*a^4) + (3^(-1 - n)*Gamma[1 + n
, 3*ArcSinh[a*x]])/(8*a^4)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int x^n \sinh^3(x)\,dx, x, \operatorname{arcsinh}(ax)\right)}{a^4}\\
&= \frac{i\text{Subst}\left(\int \left(\frac{3}{4}ix^n\sinh(x) - \frac{1}{4}ix^n\sinh(3x)\right)\,dx, x, \operatorname{arcsinh}(ax)\right)}{a^4}\\
&= \frac{\text{Subst}\left(\int x^n\sinh(3x)\,dx, x, \operatorname{arcsinh}(ax)\right)}{4a^4} - \frac{3\text{Subst}\left(\int x^n\sinh(x)\,dx, x, \operatorname{arcsinh}(ax)\right)}{4a^4}\\
&= -\frac{\text{Subst}\left(\int e^{-3x}x^n\,dx, x, \operatorname{arcsinh}(ax)\right)}{8a^4} + \frac{\text{Subst}\left(\int e^{3x}x^n\,dx, x, \operatorname{arcsinh}(ax)\right)}{8a^4}\\
&\quad + \frac{3\text{Subst}\left(\int e^{-x}x^n\,dx, x, \operatorname{arcsinh}(ax)\right)}{8a^4} - \frac{3\text{Subst}\left(\int e^{x}x^n\,dx, x, \operatorname{arcsinh}(ax)\right)}{8a^4}
\end{aligned}$$

$$= \frac{3^{-1-n}(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,-3\text{arcsinh}(ax))}{8a^4} - \frac{3(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,-\text{arcsinh}(ax))}{8a^4} - \frac{3\Gamma(1+n,\text{arcsinh}(ax))}{8a^4} + \frac{3^{-1-n}\Gamma(1+n,3\text{arcsinh}(ax))}{8a^4}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{x^3\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \frac{3^{-1-n}(-\text{arcsinh}(ax))^{-n}\left(\text{arcsinh}(ax)^n\Gamma(1+n,-3\text{arcsinh}(ax)) - 3^{2+n}\text{arcsinh}(ax)^n\Gamma(1+n,-\text{arcsinh}(ax\right.}{8a^4}$$

```
[In] Integrate[(x^3*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
[Out] (3^(-1 - n)*(ArcSinh[a*x]^n*Gamma[1 + n, -3*ArcSinh[a*x]] - 3^(2 + n)*ArcSi
nh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]] + (-ArcSinh[a*x])^n*(-(3^(2 + n)*Gamm
a[1 + n, ArcSinh[a*x]]) + Gamma[1 + n, 3*ArcSinh[a*x]])))/(8*a^4*(-ArcSinh[
a*x])^n)
```

Maple [F]

$$\int \frac{x^3 \operatorname{arcsinh}(ax)^n}{\sqrt{a^2x^2+1}}dx$$

```
[In] int(x^3*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
[Out] int(x^3*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^3\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^3 \operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x^3*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^3*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

Sympy [F]

$$\int \frac{x^3 \text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^3 \operatorname{asinh}^n(ax)}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x**3*asinh(a*x)**n/(a**2*x**2+1)**(1/2),x)
[Out] Integral(x**3*asinh(a*x)**n/sqrt(a**2*x**2 + 1), x)
```

Maxima [F]

$$\int \frac{x^3 \text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^3 \operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x^3*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^3 \operatorname{asinh}(a\,x)^n}{\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int((x^3*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^3*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2), x)
```

3.529 $\int \frac{x^2\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 80

$$\begin{aligned}\int \frac{x^2\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx &= -\frac{\text{arcsinh}(ax)^{1+n}}{2a^3(1+n)} \\ &\quad+\frac{2^{-3-n}(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,-2\text{arcsinh}(ax))}{a^3} \\ &\quad-\frac{2^{-3-n}\Gamma(1+n,2\text{arcsinh}(ax))}{a^3}\end{aligned}$$

```
[Out] -1/2*arcsinh(a*x)^(1+n)/a^3/(1+n)+2^(-3-n)*arcsinh(a*x)^n*GAMMA(1+n,-2*arcs
inh(a*x))/a^3/((-arcsinh(a*x))^n)-2^(-3-n)*GAMMA(1+n,2*arcsinh(a*x))/a^3
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5819, 3393, 3388, 2212}

$$\begin{aligned}\int \frac{x^2\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx &= -\frac{\text{arcsinh}(ax)^{n+1}}{2a^3(n+1)} \\ &\quad+\frac{2^{-n-3}\text{arcsinh}(ax)^n(-\text{arcsinh}(ax))^{-n}\Gamma(n+1,-2\text{arcsinh}(ax))}{a^3} \\ &\quad-\frac{2^{-n-3}\Gamma(n+1,2\text{arcsinh}(ax))}{a^3}\end{aligned}$$

```
[In] Int[(x^2*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -1/2*ArcSinh[a*x]^(1 + n)/(a^3*(1 + n)) + (2^(-3 - n)*ArcSinh[a*x]^n*Gamma[
1 + n, -2*ArcSinh[a*x]])/(a^3*(-ArcSinh[a*x])^n) - (2^(-3 - n)*Gamma[1 + n,
 2*ArcSinh[a*x]])/a^3
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int x^n \sinh^2(x)\,dx, x, \text{arcsinh}(ax)\right)}{a^3} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{x^n}{2} - \frac{1}{2}x^n\cosh(2x)\right)\,dx, x, \text{arcsinh}(ax)\right)}{a^3} \\
&= -\frac{\text{arcsinh}(ax)^{1+n}}{2a^3(1+n)} + \frac{\text{Subst}\left(\int x^n\cosh(2x)\,dx, x, \text{arcsinh}(ax)\right)}{2a^3} \\
&= -\frac{\text{arcsinh}(ax)^{1+n}}{2a^3(1+n)} + \frac{\text{Subst}\left(\int e^{-2x}x^n\,dx, x, \text{arcsinh}(ax)\right)}{4a^3} + \frac{\text{Subst}\left(\int e^{2x}x^n\,dx, x, \text{arcsinh}(ax)\right)}{4a^3} \\
&= -\frac{\text{arcsinh}(ax)^{1+n}}{2a^3(1+n)} + \frac{2^{-3-n}(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,-2\text{arcsinh}(ax))}{a^3} \\
&\quad - \frac{2^{-3-n}\Gamma(1+n,2\text{arcsinh}(ax))}{a^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08

$$\int \frac{x^2\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \frac{2^{-3-n}(-\text{arcsinh}(ax))^{-n}\left((1+n)\text{arcsinh}(ax)^n\Gamma(1+n,-2\text{arcsinh}(ax))-(-\text{arcsinh}(ax))^n\left(2^{2+n}\text{arcsinh}(a\right.\right.}{a^3(1+n)}$$

```
[In] Integrate[(x^2*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (2^(-3 - n)*((1 + n)*ArcSinh[a*x]^n*Gamma[1 + n, -2*ArcSinh[a*x]] - (-ArcSi
nh[a*x])^n*(2^(2 + n)*ArcSinh[a*x]^(1 + n) + (1 + n)*Gamma[1 + n, 2*ArcSinh
[a*x]])))/(a^3*(1 + n)*(-ArcSinh[a*x])^n)
```

Maple [F]

$$\int \frac{x^2 \operatorname{arcsinh}(ax)^n}{\sqrt{a^2x^2+1}}dx$$

```
[In] int(x^2*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^2*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2 \operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x^2*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

Sympy [F]

$$\int \frac{x^2\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2 \operatorname{asinh}^n(ax)}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x**2*asinh(a*x)**n/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*asinh(a*x)**n/sqrt(a**2*x**2 + 1), x)
```

Maxima [F]

$$\int \frac{x^2 \text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2\,\text{arsinh}\,(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x^2*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{x^2 \text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2\,\text{arsinh}\,(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x^2*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2\,\text{asinh}(a\,x)^n}{\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int((x^2*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^2*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2), x)
```

3.530 $\int \frac{x\textbf{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{x\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \frac{(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,-\text{arcsinh}(ax))}{2a^2} + \frac{\Gamma(1+n,\text{arcsinh}(ax))}{2a^2}$$

```
[Out] 1/2*arcsinh(a*x)^n*GAMMA(1+n,-arcsinh(a*x))/a^2/((-arcsinh(a*x))^n)+1/2*GAM
MA(1+n,arcsinh(a*x))/a^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5819, 3389, 2212}

$$\int \frac{x\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \frac{\text{arcsinh}(ax)^n(-\text{arcsinh}(ax))^{-n}\Gamma(n+1,-\text{arcsinh}(ax))}{2a^2} + \frac{\Gamma(n+1,\text{arcsinh}(ax))}{2a^2}$$

```
[In] Int[(x*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(2*a^2*(-ArcSinh[a*x])^n) + Ga
mma[1 + n, ArcSinh[a*x]]/(2*a^2)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
```

```
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^n \sinh(x)\,dx, x, \text{arcsinh}(ax)\right)}{a^2}\\ &= -\frac{\text{Subst}\left(\int e^{-x}x^n\,dx, x, \text{arcsinh}(ax)\right)}{2a^2}+\frac{\text{Subst}\left(\int e^{x}x^n\,dx, x, \text{arcsinh}(ax)\right)}{2a^2}\\ &= \frac{(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,-\text{arcsinh}(ax))}{2a^2}+\frac{\Gamma(1+n,\text{arcsinh}(ax))}{2a^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{x\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \frac{(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,-\text{arcsinh}(ax))+\Gamma(1+n,\text{arcsinh}(ax))}{2a^2}$$

```
[In] Integrate[(x*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
[Out] ((ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(-ArcSinh[a*x])^n + Gamma[1 +
 n, ArcSinh[a*x]])/(2*a^2)
```

Maple [F]

$$\int \frac{x \operatorname{arcsinh}(ax)^n}{\sqrt{a^2x^2+1}}dx$$

```
[In] int(x*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
[Out] int(x*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{x\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x \operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(x*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

Sympy [F]

$$\int \frac{x\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x \operatorname{asinh}^n(ax)}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x*asinh(a*x)**n/(a**2*x**2+1)**(1/2),x)
[Out] Integral(x*asinh(a*x)**n/sqrt(a**2*x**2 + 1), x)
```

Maxima [F]

$$\int \frac{x\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x \operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{x\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x\,\text{arsinh}\,(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(x*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x\,\text{asinh}(a\,x)^n}{\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int((x*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2), x)
```

3.531 $\int \frac{\mathbf{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 17

$$\int \frac{\operatorname{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \frac{\operatorname{arcsinh}(ax)^{1+n}}{a(1+n)}$$

```
[Out] arcsinh(a*x)^(1+n)/a/(1+n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {5783}

$$\int \frac{\operatorname{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \frac{\operatorname{arcsinh}(ax)^{n+1}}{a(n+1)}$$

```
[In] Int[ArcSinh[a*x]^n/Sqrt[1 + a^2*x^2],x]
[Out] ArcSinh[a*x]^(1 + n)/(a*(1 + n))
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{\operatorname{arcsinh}(ax)^{1+n}}{a(1+n)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \frac{\text{arcsinh}(ax)^{1+n}}{a(1+n)}$$

```
[In] Integrate[ArcSinh[a*x]^n/Sqrt[1 + a^2*x^2],x]
[Out] ArcSinh[a*x]^(1 + n)/(a*(1 + n))
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\text{arcsinh}(ax)^{1+n}}{a(1+n)}$	18
default	$\frac{\text{arcsinh}(ax)^{1+n}}{a(1+n)}$	18

```
[In] int(arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsinh(a*x)^(1+n)/a/(1+n)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(17) = 34.

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 4.88

$$\int \frac{\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \frac{\cosh\left(n\log\left(\log\left(ax+\sqrt{a^2x^2+1}\right)\right)\right)\log\left(ax+\sqrt{a^2x^2+1}\right)+\log\left(ax+\sqrt{a^2x^2+1}\right)\sinh\left(n\log\left(\log\left(ax+\sqrt{a^2x^2+1}\right)\right)\right)}{an+a}$$

```
[In] integrate(arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] (cosh(n*log(log(a*x + sqrt(a^2*x^2 + 1))))*log(a*x + sqrt(a^2*x^2 + 1)) + l
og(a*x + sqrt(a^2*x^2 + 1))*sinh(n*log(log(a*x + sqrt(a^2*x^2 + 1)))))/(a*n
 + a)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(12) = 24$.

Time = 0.36 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.00

$$\int \frac{\operatorname{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} \tilde{\infty}x & \text{for } a=0 \wedge n=-1 \\ 0^n x & \text{for } a=0 \\ \frac{\log(\operatorname{asinh}(ax))}{a} & \text{for } n=-1 \\ \frac{\operatorname{asinh}(ax)\operatorname{asinh}^n(ax)}{an+a} & \text{otherwise} \end{cases}$$

```
[In] integrate(asinh(a*x)**n/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(n, -1)), (0**n*x, Eq(a, 0)), (log(asinh(a*x
))/a, Eq(n, -1)), (asinh(a*x)*asinh(a*x)**n/(a*n + a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \frac{\operatorname{arsinh}(ax)^{n+1}}{a(n+1)}$$

```
[In] integrate(arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(a*x)^(n + 1)/(a*(n + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \frac{\operatorname{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^{n+1}}{a(n+1)}$$

```
[In] integrate(arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] log(a*x + sqrt(a^2*x^2 + 1))^(n + 1)/(a*(n + 1))
```

Mupad [B] (verification not implemented)

Time = 2.93 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

$$\int \frac{\operatorname{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} \frac{\ln(\operatorname{asinh}(a\,x))}{a} & \text{if } n=-1 \\ \frac{\operatorname{asinh}(a\,x)^{n+1}}{a\,(n+1)} & \text{if } n\neq -1 \end{cases}$$

```
[In] int(asinh(a*x)^n/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] piecewise(n == -1, log(asinh(a*x))/a, n ~= -1, asinh(a*x)^(n + 1)/(a*(n + 1
)))
```

3.532 $\int \frac{\text{arcsinh}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\text{arcsinh}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx = \text{Int}\left(\frac{\text{arcsinh}(ax)^n}{x\sqrt{1+a^2x^2}},x\right)$$

```
[Out] Unintegrable(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{arcsinh}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\text{arcsinh}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx$$

```
[In] Int[ArcSinh[a*x]^n/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] Defer[Int][ArcSinh[a*x]^n/(x*Sqrt[1 + a^2*x^2]), x]
```

Rubi steps

$$\text{integral} = \int \frac{\text{arcsinh}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 5.71 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\text{arcsinh}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\text{arcsinh}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx$$

```
[In] Integrate[ArcSinh[a*x]^n/(x*Sqrt[1 + a^2*x^2]),x]
[Out] Integrate[ArcSinh[a*x]^n/(x*Sqrt[1 + a^2*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\text{arcsinh}\,(ax)^n}{x\sqrt{a^2x^2+1}}dx$$

```
[In] int(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x)
[Out] int(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{\text{arcsinh}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\text{arsinh}\,(ax)^n}{\sqrt{a^2x^2+1}x}\,dx$$

```
[In] integrate(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^n/(a^2*x^3 + x), x)
```

Sympy [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\text{arcsinh}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\text{asinh}^n\,(ax)}{x\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(asinh(a*x)**n/x/(a**2*x**2+1)**(1/2),x)
[Out] Integral(asinh(a*x)**n/(x*sqrt(a**2*x**2 + 1)), x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^n}{\sqrt{a^2x^2+1}x}\,dx$$

```
[In] integrate(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^n/(sqrt(a^2*x^2 + 1)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^n}{\sqrt{a^2x^2+1}x}\,dx$$

```
[In] integrate(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^n/(sqrt(a^2*x^2 + 1)*x), x)
```

Mupad [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)^n}{x\,\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int(asinh(a*x)^n/(x*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(asinh(a*x)^n/(x*(a^2*x^2 + 1)^(1/2)), x)
```

3.533 $\int \frac{\operatorname{arcsinh}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx=\operatorname{Int}\left(\frac{\operatorname{arcsinh}(ax)^n}{x^2\sqrt{1+a^2x^2}},x\right)$$

```
[Out] Unintegrable(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx=\int \frac{\operatorname{arcsinh}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx$$

```
[In] Int[ArcSinh[a*x]^n/(x^2*Sqrt[1 + a^2*x^2]),x]
[Out] Defer[Int][ArcSinh[a*x]^n/(x^2*Sqrt[1 + a^2*x^2]), x]
```

Rubi steps

$$\text{integral}=\int \frac{\operatorname{arcsinh}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.86 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arcsinh}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx$$

```
[In] Integrate[ArcSinh[a*x]^n/(x^2*Sqrt[1 + a^2*x^2]),x]
[Out] Integrate[ArcSinh[a*x]^n/(x^2*Sqrt[1 + a^2*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2\sqrt{a^2x^2+1}}dx$$

```
[In] int(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x)
[Out] int(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}x^2}\,dx$$

```
[In] integrate(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^n/(a^2*x^4 + x^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}^n(ax)}{x^2\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(asinh(a*x)**n/x**2/(a**2*x**2+1)**(1/2),x)
[Out] Integral(asinh(a*x)**n/(x**2*sqrt(a**2*x**2 + 1)), x)
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\text{arcsinh}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\text{arsinh}\,(ax)^n}{\sqrt{a^2x^2+1}x^2}\,dx$$

```
[In] integrate(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^n/(sqrt(a^2*x^2 + 1)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\text{arcsinh}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\text{arsinh}\,(ax)^n}{\sqrt{a^2x^2+1}x^2}\,dx$$

```
[In] integrate(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^n/(sqrt(a^2*x^2 + 1)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.73 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\text{arcsinh}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\text{asinh}(a\,x)^n}{x^2\,\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int(asinh(a*x)^n/(x^2*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(asinh(a*x)^n/(x^2*(a^2*x^2 + 1)^(1/2)), x)
```

3.534 $\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 416

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx = -\frac{2ibd^2x\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}} - \frac{3bcd^2x^2\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{1+c^2x^2}} - \frac{2ibc^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}}{9\sqrt{1+c^2x^2}} + \frac{bc^3d^2x^4\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{1+c^2x^2}} + \frac{3}{8}d^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx)) - \frac{1}{4}c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx)) + \frac{2id^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c} + \frac{5d^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{16bc\sqrt{1+c^2x^2}}$$

```
[Out] 3/8*d^2*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/4*c^2*d^
2*x^3*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)+2/3*I*d^2*(c^2
*x^2+1)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c-2/3*I*b*d^
2*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-3/16*b*c*d^2*x^2*
(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-2/9*I*b*c^2*d^2*x^3*(
d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/16*b*c^3*d^2*x^4*(d+
I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+5/16*d^2*(a+b*arcsinh(c*
x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5796, 5838, 5785, 5783, 30, 5798, 5806, 5812}

$$\begin{aligned}
&\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx = \\
&-\frac{1}{4}c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx)) \\
&+\frac{5d^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{16bc\sqrt{c^2x^2+1}} \\
&+\frac{2id^2(c^2x^2+1)\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{3c} \\
&+\frac{3}{8}d^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx)) \\
&-\frac{3bcd^2x^2\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{c^2x^2+1}}-\frac{2ibd^2x\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{c^2x^2+1}} \\
&-\frac{2ibc^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}}{9\sqrt{c^2x^2+1}}+\frac{bc^3d^2x^4\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{c^2x^2+1}}
\end{aligned}$$

```
[In] Int[(d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (((-2*I)/3)*b*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2]
- (3*b*c*d^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(16*Sqrt[1 + c^2*x^2]
) - (((2*I)/9)*b*c^2*d^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 +
c^2*x^2] + (b*c^3*d^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(16*Sqrt[1 +
 c^2*x^2]) + (3*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*
x]))/8 - (c^2*d^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*
x]))/4 + (((2*I)/3)*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(
a + b*ArcSinh[c*x]))/c + (5*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*
ArcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int (d+icdx)^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \left(d^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))+2icd^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))-\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}} + \frac{\left(2icd^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}} - \frac{\left(c^2d^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{2}d^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx)) - \frac{1}{4}c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx)) + \frac{2id^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{3c} + \frac{\left(d^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}} - \frac{\left(2ibd^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int (1+c^2x^2)\,dx}{3\sqrt{1+c^2x^2}} - \frac{\left(bcd^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x\,dx}{2\sqrt{1+c^2x^2}} - \frac{\left(c^2d^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{4\sqrt{1+c^2x^2}} + \frac{\left(bc^3d^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x^3\,dx}{4\sqrt{1+c^2x^2}}$$

$$= -\frac{2ibd^2x\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}} - \frac{bcd^2x^2\sqrt{d+icdx}\sqrt{f-icfx}}{4\sqrt{1+c^2x^2}} - \frac{2ibc^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}}{9\sqrt{1+c^2x^2}} + \frac{bc^3d^2x^4\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{1+c^2x^2}} + \frac{3}{8}d^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx)) - \frac{1}{4}c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx)) + \frac{2id^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))}{3c} + \frac{d^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}} + \frac{\left(d^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{8\sqrt{1+c^2x^2}} + \frac{\left(bcd^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x\,dx}{8\sqrt{1+c^2x^2}}$$

$$= -\frac{2ibd^2x\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}} - \frac{3bcd^2x^2\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{1+c^2x^2}} - \frac{2ibc^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}}{9\sqrt{1+c^2x^2}} + \frac{bc^3d^2x^4\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{1+c^2x^2}} + \frac{3}{8}d^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx)) - \frac{1}{4}c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx)) + \frac{2id^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))}{3c} + \frac{5d^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2}{16bc\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 2.79 (sec) , antiderivative size = 361, normalized size of antiderivative = 0.87

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))\,dx = \frac{48ad^2\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}(16i+9cx+16ic^2x^2-6c^3x^3)+720ad^{5/2}\sqrt{f}\sqrt{1}$$

```
[In] Integrate[(d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (48*a*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*(16*I + 9*c
*x + (16*I)*c^2*x^2 - 6*c^3*x^3) + 720*a*d^(5/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*
```

```
Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 144*b*
d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(2*ArcSinh[c*x]^2 - Cosh[2*ArcSinh[
c*x]] + 2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]]) - (64*I)*b*d^2*Sqrt[d + I*c*d*
x]*Sqrt[f - I*c*f*x]*(9*c*x - 3*ArcSinh[c*x]*(3*Sqrt[1 + c^2*x^2] + Cosh[3*
ArcSinh[c*x]]) + Sinh[3*ArcSinh[c*x]]) + 9*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*
ArcSinh[c*x]]))/(1152*c*Sqrt[1 + c^2*x^2])
```

Maple [F]

$$\int (icdx + d)^{\frac{5}{2}} (a + b \operatorname{arcsinh}(cx)) \sqrt{-icfx + f}dx$$

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x)
```

Fricas [F]

$$\int (d + icdx)^{5/2}\sqrt{f - icfx}(a + b\text{arcsinh}(cx)) dx = \int (i\,cdx + d)^{\frac{5}{2}} \sqrt{-i\,cfx + f}(b\,\text{arsinh}(cx) + a)\,dx$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-(b*c^2*d^2*x^2 - 2*I*b*c*d^2*x - b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I
*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) - (a*c^2*d^2*x^2 - 2*I*a*c*d^2*x -
 a*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d + icdx)^{5/2}\sqrt{f - icfx}(a + b\text{arcsinh}(cx)) dx = \text{Timed out}$$

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))*(f-I*c*f*x)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))\,dx = \int (a+b\,\mathrm{asinh}(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}\,\sqrt{f-c\,f\,x\,1\mathrm{i}}\,dx$$

```
[In] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2), x)
```

3.535 $\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\,\mathrm{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 304

$$\begin{aligned}\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))\,dx = &-\frac{ibdx\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}}-\frac{bcdx^2\sqrt{d+icdx}\sqrt{f-icfx}}{4\sqrt{1+c^2x^2}}\\&-\frac{ibc^2dx^3\sqrt{d+icdx}\sqrt{f-icfx}}{9\sqrt{1+c^2x^2}}+\frac{1}{2}dx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))\\&+\frac{id\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{3c}\\&+\frac{d\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}}\end{aligned}$$

```
[Out] 1/2*d*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)+1/3*I*d*(c^2
*x^2+1)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c-1/3*I*b*d*
x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-1/4*b*c*d*x^2*(d+I*
c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-1/9*I*b*c^2*d*x^3*(d+I*c*d
*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/4*d*(a+b*arcsinh(c*x))^2*(d
+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {5796, 5838, 5785, 5783, 30, 5798}

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx=\frac{d\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{c^2x^2+1}}+\frac{id(c^2x^2+1)\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{3c}+\frac{1}{2}dx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))-\frac{bcdx^2\sqrt{d+icdx}\sqrt{f-icfx}}{4\sqrt{c^2x^2+1}}-\frac{ibdx\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{c^2x^2+1}}-\frac{ibc^2dx^3\sqrt{d+icdx}\sqrt{f-icfx}}{9\sqrt{c^2x^2+1}}$$

```
[In] Int[(d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((-1/3*I)*b*d*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2] - (b
*c*d*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(4*Sqrt[1 + c^2*x^2]) - ((I/9
)*b*c^2*d*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2] + (d*x
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/2 + ((I/3)*d*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/c + (d
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1
 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
```

```
x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int(d+icdx)\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\left(d\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))+icdx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\right)\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}} + \frac{\left(icd\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}}$$

$$
\begin{aligned}
&= \frac{1}{2}dx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{id\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{3c} \\
&\quad + \frac{\left(d\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(ibd\sqrt{d+icdx}\sqrt{f-icfx}\right)\int (1+c^2x^2)\,dx}{3\sqrt{1+c^2x^2}} - \frac{\left(bcd\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x\,dx}{2\sqrt{1+c^2x^2}} \\
&= -\frac{ibdx\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}} - \frac{bcdx^2\sqrt{d+icdx}\sqrt{f-icfx}}{4\sqrt{1+c^2x^2}} \\
&\quad - \frac{ibc^2dx^3\sqrt{d+icdx}\sqrt{f-icfx}}{9\sqrt{1+c^2x^2}} + \frac{1}{2}dx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{id\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{3c} \\
&\quad + \frac{d\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.83 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.90

$$
\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))\,dx = \frac{12ad\sqrt{d+icdx}\sqrt{f-icfx}(2i+3cx+2ic^2x^2)+36ad^{3/2}\sqrt{f}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+ic}\right.}{}
$$

```
[In] Integrate[(d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (12*a*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(2*I + 3*c*x + (2*I)*c^2*x^2) +
 36*a*d^(3/2)*Sqrt[f]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]] + (9*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(2*ArcSinh[c*x]^
2 - Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]]))/Sqrt[1 + c
^2*x^2] - ((2*I)*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(9*c*x - 3*ArcSinh
[c*x]*(3*Sqrt[1 + c^2*x^2] + Cosh[3*ArcSinh[c*x]]) + Sinh[3*ArcSinh[c*x]]))
/Sqrt[1 + c^2*x^2])/(72*c)
```

Maple [F]

$$\int (icdx + d)^{\frac{3}{2}} (a + b \operatorname{arcsinh}(cx)) \sqrt{-icfx + f}dx$$

[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x)

[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x)

Fricas [F]

$$\int (d + icdx)^{3/2}\sqrt{f - icfx}(a + b\text{arcsinh}(cx))\,dx = \int (i\,cdx + d)^{\frac{3}{2}}\sqrt{-i\,cfx + f}(b\,\text{arsinh}\,(cx) + a)\,dx$$

[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algorit hm="fricas")

[Out] integral((I*b*c*d*x + b*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + s qrt(c^2*x^2 + 1)) + (I*a*c*d*x + a*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)

Sympy [F]

$$\int (d + icdx)^{3/2}\sqrt{f - icfx}(a + b\text{arcsinh}(cx))\,dx = \int (id(cx - i))^{\frac{3}{2}}\sqrt{-if(cx + i)}(a + b\,\text{asinh}\,(cx))\,dx$$

[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))*(f-I*c*f*x)**(1/2),x)

[Out] Integral((I*d*(c*x - I))**(3/2)*sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x)), x)

Maxima [F(-2)]

Exception generated.

$$\int (d + icdx)^{3/2}\sqrt{f - icfx}(a + b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algorit hm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ e exponent.

Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad Ar
gument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx = \int (a + b\,\text{asinh}(c\,x))\ (d+c\,d\,x\,1\text{i})^{3/2}\ \sqrt{f-c\,f\,x\,1\text{i}}\,dx$$

```
[In] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2), x)
```

3.536 $\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 147

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx = -\frac{bcx^2\sqrt{d+icdx}\sqrt{f-icfx}}{4\sqrt{1+c^2x^2}}+\frac{1}{2}x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx)) + \frac{\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}}$$

```
[Out] 1/2*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/4*b*c*x^2*(d
+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/4*(a+b*arcsinh(c*x))^
2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {5796, 5785, 5783, 30}

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx = \frac{\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{c^2x^2+1}} + \frac{1}{2}x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))-\frac{bcx^2\sqrt{d+icdx}\sqrt{f-icfx}}{4\sqrt{c^2x^2+1}}$$

```
[In] Int[Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/4*(b*c*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2] + (x*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/2 + (Sqrt[d + I*c*
d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}}\\
&= \frac{1}{2}x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\\
&\quad+\frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}}-\frac{\left(bc\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x\,dx}{2\sqrt{1+c^2x^2}}\\
&= -\frac{bcx^2\sqrt{d+icdx}\sqrt{f-icfx}}{4\sqrt{1+c^2x^2}}+\frac{1}{2}x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\\
&\quad+\frac{\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.59

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx=\frac{1}{2}ax\sqrt{id(-i+cx)}\sqrt{-if(i+cx)}+\frac{a\sqrt{d}\sqrt{f}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{id(-i+cx)}\sqrt{-if(i+cx)}\right)}{2c}-\frac{b\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\sqrt{-df\left(1+c^2x^2\right)}(\cosh(2\text{arcsinh}(cx))-2\text{arcsinh}(cx)(\text{arcsinh}(cx)+\sinh}{8c\sqrt{-((-id+cdx)(if+cfx))}\sqrt{1+c^2x^2}}$$

```
[In] Integrate[Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (a*x*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)])/2 + (a*Sqrt[d]*Sqrt[f]*Lo
g[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(
2*c) - (b*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 +
 c^2*x^2))]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*A
rcSinh[c*x]])))/(8*c*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x
^2])
```

Maple [F]

$$\int (a+b\,\text{arcsinh}\,(cx))\,\sqrt{icdx+d}\,\sqrt{-icfx+f}dx$$

```
[In] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x)
```

Fricas [F]

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx=\int \sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}(b\,\text{arsinh}\,(cx)+a)\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 + 1)
) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a, x)
```

Sympy [F]

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\int \sqrt{id\,(cx-i)}\sqrt{-if\,(cx+i)}(a+b\operatorname{asinh}{(cx)})\;dx$$

```
[In] integrate((a+b*asinh(c*x))*(d+I*c*d*x)**(1/2)*(f-I*c*f*x)**(1/2),x)
[Out] Integral(sqrt(I*d*(c*x - I))*sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad Ar
gument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\int (a+b\,\mathrm{asinh}(c\,x))\,\sqrt{d+c\,d\,x\,1\mathrm{i}}\,\sqrt{f-c\,f\,x\,1\mathrm{i}}\,dx$$

```
[In] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2),x)
[Out] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2), x)
```

3.537 $\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 158

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \frac{ibfx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{if(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{f\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[Out] -I*f*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+I
*b*f*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/2*f*(a+b*arc
sinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5796, 5838, 5783, 5798, 8}

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \frac{f\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{if(c^2x^2+1)\,(a+b\text{arcsinh}(cx))}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{ibfx\sqrt{c^2x^2+1}}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[In] Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

```
[Out] (I*b*f*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (I*f*(1
 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) +
 (f*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+c^2x^2}\int\frac{(f-icfx)(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\int\left(\frac{f(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}-\frac{icfx(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\left(f\sqrt{1+c^2x^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{\left(icf\sqrt{1+c^2x^2}\right)\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= -\frac{if(1+c^2x^2)\left(a+b\text{arcsinh}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{f\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{\left(ibf\sqrt{1+c^2x^2}\right)\int 1\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{ibfx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{if(1+c^2x^2)\left(a+b\text{arcsinh}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{f\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.44

$$\int\frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx$$
$$= \frac{2i\sqrt{d+icdx}\sqrt{f-icfx}\left(bcx-a\sqrt{1+c^2x^2}\right)-2ib\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}\text{arcsinh}(cx)+b\sqrt{d+ic}}{2cd\sqrt{1+c^2x^2}}$$

```
[In] Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((2*I)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(b*c*x - a*Sqrt[1 + c^2*x^2]) -
(2*I)*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
+ b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 + 2*a*Sqrt[d]*Sqrt[f
]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]])/(2*c*d*Sqrt[1 + c^2*x^2])
```

Maple [F]

$$\int\frac{(a+b\,\text{arcsinh}\,(cx))\,\sqrt{-icfx+f}}{\sqrt{icdx+d}}dx$$

```
[In] int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx=\int \frac{\sqrt{-i\,cfx+f}(b\,\text{arsinh}\,(cx)+a)}{\sqrt{i\,cdx+d}}\,dx$$

[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algorit hm="fricas")

[Out] integral((-I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 + 1)) - I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c*d*x - I*d), x)

Sympy [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx=\int \frac{\sqrt{-if\,(cx+i)}(a+b\,\text{asinh}\,(cx))}{\sqrt{id\,(cx-i)}}\,dx$$

[In] integrate((a+b*asinh(c*x))*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(1/2),x)

[Out] Integral(sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))/sqrt(I*d*(c*x - I)), x)

Maxima [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx=\int \frac{\sqrt{-i\,cfx+f}(b\,\text{arsinh}\,(cx)+a)}{\sqrt{i\,cdx+d}}\,dx$$

[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algorit hm="maxima")

[Out] a*(f*arcsinh(c*x)/(c*d*sqrt(f/d)) - I*sqrt(c^2*d*f*x^2 + d*f)/(c*d)) + b*in tegrate(sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))/sqrt(I*c*d*x + d), x)

Giac [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx=\int \frac{\sqrt{-i\,cfx+f}(b\,\text{arsinh}\,(cx)+a)}{\sqrt{i\,cdx+d}}\,dx$$

[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algorit hm="giac")

[Out] integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)/sqrt(I*c*d*x + d), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\sqrt{f-c\,f\,x\,1\text{i}}}{\sqrt{d+c\,d\,x\,1\text{i}}}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(1/2),x)
[Out] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(1/2), x)
```

3.538 $\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 181

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx = \frac{2if^2(1-icx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2bf^2\left(1+c^2x^2\right)^{3/2}\log(i-cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] 2*I*f^2*(1-I*c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c
*f*x)^(3/2)-1/2*f^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(d+I*c*d*x)^
(3/2)/(f-I*c*f*x)^(3/2)-2*b*f^2*(c^2*x^2+1)^(3/2)*ln(I-c*x)/c/(d+I*c*d*x)^(
3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {5796, 5844, 651, 5837, 12, 641, 31, 5783}

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx = -\frac{f^2\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2if^2(1-icx)\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2bf^2\left(c^2x^2+1\right)^{3/2}\log(-cx+i)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[In] Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((2*I)*f^2*(1 - I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2)) - (f^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])
```

```
^2)/(2*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (2*b*f^2*(1 + c^2*x^2
)^(3/2)*Log[I - c*x])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
```

```
3])
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{(1+c^2x^2)^{3/2}\int\frac{(f-icfx)^2(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{(1+c^2x^2)^{3/2}\int\left(-\frac{2i(if^2+cf^2x)(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{3/2}}-\frac{f^2(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(2i(1+c^2x^2)^{3/2}\right)\int\frac{(if^2+cf^2x)(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(f^2(1+c^2x^2)^{3/2}\right)\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{2if^2(1-icx)(1+c^2x^2)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2ibc(1+c^2x^2)^{3/2}\right)\int\frac{f^2(1-icx)}{c(1+c^2x^2)}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{2if^2(1-icx)(1+c^2x^2)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2ibf^2(1+c^2x^2)^{3/2}\right)\int\frac{1-icx}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{2if^2(1-icx)(1+c^2x^2)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2ibf^2(1+c^2x^2)^{3/2}\right)\int\frac{1}{1+icx}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{2if^2(1-icx)(1+c^2x^2)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2bf^2(1+c^2x^2)^{3/2}\log(i-cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Mathematica [A] (verified)

Time = 2.55 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx =$$
$$-\frac{-\frac{4a\sqrt{d+icdx}\sqrt{f-icfx}}{-i+cx}+2a\sqrt{d}\sqrt{f}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)+\frac{b\sqrt{d+icdx}\sqrt{f-icfx}\left(\text{arcsinh}(cx)\left(-4i\right.\right.}{}}{}$$

```
[In] Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

```
[Out] -1/2*((-4*a*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(-I + c*x) + 2*a*Sqrt[d]*S
qrt[f]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] +
 (b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(ArcSinh[c*x]*((-4*I)*Cosh[ArcSinh[
c*x]/2] - 4*Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] +
I*Sinh[ArcSinh[c*x]/2]) + 2*((4*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + Log[1 + c
^2*x^2])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 + c^2*x^
2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(c*d^2)
```

Maple [F]

$$\int \frac{(a+b\,\text{arcsinh}\,(cx))\sqrt{-icfx+f}}{(icdx+d)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx = \int \frac{\sqrt{-i\,cfx+f}(b\,\text{arsinh}\,(cx)+a)}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(-(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 +
1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c^2*d^2*x^2 - 2*I*c*d^2*x -
d^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\int \frac{\sqrt{-if\,(cx+i)}(a+b\,\text{asinh}\,(cx))}{(id\,(cx-i))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(3/2),x)
[Out] Integral(sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))/(I*d*(c*x - I))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\int \frac{\sqrt{-i\,cfx+f}(b\,\text{arsinh}\,(cx)+a)}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algorit
hm="maxima")
[Out] a*(2*I*sqrt(c^2*d*f*x^2 + d*f)/(I*c^2*d^2*x + c*d^2) - f*arcsinh(c*x)/(c*d^
2*sqrt(f/d))) + b*integrate(sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))
/(I*c*d*x + d)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\int \frac{\sqrt{-i\,cfx+f}(b\,\text{arsinh}\,(cx)+a)}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algorit
hm="giac")
[Out] integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)/(I*c*d*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))\,\sqrt{f-c\,f\,x\,1\text{i}}}{(d+c\,d\,x\,1\text{i})^{3/2}}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(3/2),x)
[Out] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(3/2), x)
```

3.539 $\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 187

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \frac{2ibf^3(1+c^2x^2)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{if^3(1-icx)^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf^3(1+c^2x^2)^{5/2}\log(i-cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[Out] 2/3*I*b*f^3*(c^2*x^2+1)^(5/2)/c/(I-c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
+1/3*I*f^3*(1-I*c*x)^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(
f-I*c*f*x)^(5/2)+1/3*b*f^3*(c^2*x^2+1)^(5/2)*ln(I-c*x)/c/(d+I*c*d*x)^(5/2)/
(f-I*c*f*x)^(5/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {5796, 665, 5837, 12, 641, 45}

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \frac{if^3(1-icx)^3\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ibf^3(c^2x^2+1)^{5/2}}{3c(-cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf^3(c^2x^2+1)^{5/2}\log(-cx+i)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[In] Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

```
[Out] (((2*I)/3)*b*f^3*(1 + c^2*x^2)^(5/2))/(c*(I - c*x)*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) + ((I/3)*f^3*(1 - I*c*x)^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*
```

```
x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (b*f^3*(1 + c^2*x^2)^(5/
2)*Log[I - c*x])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\text{integral} = \frac{\left(1+c^2x^2\right)^{5/2}\int\frac{(f-icfx)^3(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{if^3(1-icx)^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(bc\left(1+c^2x^2\right)^{5/2}\right)\int\frac{if^3(1-icx)^3}{3c(1+c^2x^2)^2}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{if^3(1-icx)^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(ibf^3\left(1+c^2x^2\right)^{5/2}\right)\int\frac{(1-icx)^3}{(1+c^2x^2)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{if^3(1-icx)^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(ibf^3\left(1+c^2x^2\right)^{5/2}\right)\int\frac{1-icx}{(1+icx)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{if^3(1-icx)^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(ibf^3\left(1+c^2x^2\right)^{5/2}\right)\int\left(-\frac{2}{(-i+cx)^2}+\frac{i}{-i+cx}\right)dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2ibf^3\left(1+c^2x^2\right)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{if^3(1-icx)^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf^3\left(1+c^2x^2\right)^{5/2}\log(i-cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.75

$$\int\frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(-\left((i+cx)\left(-ib+bcx+a\sqrt{1+c^2x^2}\right)\right)-b(i+\cdots\right.}{3cd^3(-i+cx)^2\sqrt{1+\cdots}}$$

```
[In] Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

```
[Out] (Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-((I + c*x)*((-I)*b + b*c*x + a*Sqrt[
1 + c^2*x^2])) - b*(I + c*x)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + b*(-I + c*x)^
2*Log[d + I*c*d*x]))/(3*c*d^3*(-I + c*x)^2*Sqrt[1 + c^2*x^2])
```

Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))\,\sqrt{-icfx+f}}{(icdx+d)^{\frac{5}{2}}}dx$$

[In] int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x)

[Out] int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x)

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(142) = 284$.

Time = 0.32 (sec) , antiderivative size = 548, normalized size of antiderivative = 2.93

$$\int \frac{\sqrt{f-icfx}(a+b\mathrm{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx =$$

$$-\frac{4\sqrt{c^2x^2+1}\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}bcx+2\left(bc^2x^2+2i\,bcx-b\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2+\right.}{}$$

[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algorit hm="fricas")

[Out] -1/6*(4*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x + 2*(b *c^2*x^2 + 2*I*b*c*x - b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sq rt(c^2*x^2 + 1)) - (c^4*d^3*x^3 - I*c^3*d^3*x^2 + c^2*d^3*x - I*c*d^3)*sqrt (b^2*f/(c^2*d^5))*log(-1/8*((I*b*c^6*x^2 + 2*b*c^5*x - 2*I*b*c^4)*sqrt(c^2* x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + (I*c^9*d^3*x^4 + 2*c^8*d^3* x^3 + I*c^7*d^3*x^2 + 2*c^6*d^3*x)*sqrt(b^2*f/(c^2*d^5)))/(b*c^3*x^3 - I*b* c^2*x^2 + b*c*x - I*b)) + (c^4*d^3*x^3 - I*c^3*d^3*x^2 + c^2*d^3*x - I*c*d^ 3)*sqrt(b^2*f/(c^2*d^5))*log(-1/8*((I*b*c^6*x^2 + 2*b*c^5*x - 2*I*b*c^4)*sq rt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + (-I*c^9*d^3*x^4 - 2* c^8*d^3*x^3 - I*c^7*d^3*x^2 - 2*c^6*d^3*x)*sqrt(b^2*f/(c^2*d^5)))/(b*c^3*x^ 3 - I*b*c^2*x^2 + b*c*x - I*b)) + 2*(a*c^2*x^2 + 2*I*a*c*x - a)*sqrt(I*c*d* x + d)*sqrt(-I*c*f*x + f))/(c^4*d^3*x^3 - I*c^3*d^3*x^2 + c^2*d^3*x - I*c*d ^3)

Sympy [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \int \frac{\sqrt{-if\,(cx+i)}(a+b\,\text{asinh}\,(cx))}{(id\,(cx-i))^{\frac{5}{2}}}\,dx$$

[In] integrate((a+b*asinh(c*x))*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(5/2),x)

[Out] Integral(sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))/(I*d*(c*x - I))**(5/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.17

$$\begin{aligned}\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx &= \frac{1}{3}\,bc\left(\frac{6\,\sqrt{f}}{3i\,c^3d^{\frac{5}{2}}x+3\,c^2d^{\frac{5}{2}}}+\frac{\sqrt{f}\log\left(cx-i\right)}{c^2d^{\frac{5}{2}}}\right)\\ &-\frac{1}{3}\,b\left(\frac{2i\,\sqrt{c^2dfx^2+df}}{c^3d^3x^2-2i\,c^2d^3x-cd^3}+\frac{3i\,\sqrt{c^2dfx^2+df}}{3i\,c^2d^3x+3\,cd^3}\right)\text{arsinh}\,(cx)\\ &-\frac{1}{3}\,a\left(\frac{2i\,\sqrt{c^2dfx^2+df}}{c^3d^3x^2-2i\,c^2d^3x-cd^3}+\frac{3i\,\sqrt{c^2dfx^2+df}}{3i\,c^2d^3x+3\,cd^3}\right)\end{aligned}$$

[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algorithm="maxima")

[Out] 1/3*b*c*(6*sqrt(f)/(3*I*c^3*d^(5/2)*x + 3*c^2*d^(5/2)) + sqrt(f)*log(c*x - I)/(c^2*d^(5/2))) - 1/3*b*(2*I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3) + 3*I*sqrt(c^2*d*f*x^2 + d*f)/(3*I*c^2*d^3*x + 3*c*d^3))*arcsinh(c*x) - 1/3*a*(2*I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3) + 3*I*sqrt(c^2*d*f*x^2 + d*f)/(3*I*c^2*d^3*x + 3*c*d^3))

Giac [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \int \frac{\sqrt{-i\,cfx+f}(b\,\text{arsinh}\,(cx)+a)}{(i\,cdx+d)^{\frac{5}{2}}}\,dx$$

[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algorithm="giac")

[Out] integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)/(I*c*d*x + d)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f - icfx}(a + b\text{arcsinh}(cx))}{(d + icdx)^{5/2}}\,dx = \int \frac{(a + b\,\text{asinh}(c\,x))\,\sqrt{f - c\,f\,x\,1\text{i}}}{(d + c\,d\,x\,1\text{i})^{5/2}}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(5/2),x)
[Out] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(5/2), x)
```

3.540 $\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\,\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 459

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = -\frac{ibdx(d+icdx)^{3/2}(f-icfx)^{3/2}}{5(1+c^2x^2)^{3/2}} - \frac{5bcdx^2(d+icdx)^{3/2}(f-icfx)^{3/2}}{16(1+c^2x^2)^{3/2}} - \frac{2ibc^2dx^3(d+icdx)^{3/2}(f-icfx)^{3/2}}{15(1+c^2x^2)^{3/2}} - \frac{bc^3dx^4(d+icdx)^{3/2}(f-icfx)^{3/2}}{16(1+c^2x^2)^{3/2}} - \frac{ibc^4dx^5(d+icdx)^{3/2}(f-icfx)^{3/2}}{25(1+c^2x^2)^{3/2}} + \frac{1}{4}dx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{3dx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{8(1+c^2x^2)} + \frac{id(d+$$

```
[Out] -1/5*I*b*d*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)-5/16*b*c
*d*x^2*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)-2/15*I*b*c^2*d
*x^3*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)-1/16*b*c^3*d*x^4
*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)-1/25*I*b*c^4*d*x^5*(
d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)+1/4*d*x*(d+I*c*d*x)^(3
/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))+3/8*d*x*(d+I*c*d*x)^(3/2)*(f-I*c*f
*x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)+1/5*I*d*(d+I*c*d*x)^(3/2)*(f-I*c*f
*x)^(3/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c+3/16*d*(d+I*c*d*x)^(3/2)*(f-I*c*
f*x)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*x^2+1)^(3/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {5796, 5838, 5786, 5785, 5783, 30, 14, 5798, 200}

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \frac{3dx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{8(c^2x^2+1)} + \frac{3d(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{16bc(c^2x^2+1)^{3/2}} + \frac{id(c^2x^2+1)(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{5c} + \frac{1}{4}dx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{5bcdx^2(d+icdx)^{3/2}(f-icfx)^{3/2}}{16(c^2x^2+1)^{3/2}} - \frac{ibdx(d+icdx)^{3/2}(f-}{5(c^2x^2+1)^{3/2}}$$

```
[In] Int[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((-1/5*I)*b*d*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(1 + c^2*x^2)^(3/2
) - (5*b*c*d*x^2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(16*(1 + c^2*x^2)
^(3/2)) - (((2*I)/15)*b*c^2*d*x^3*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/
(1 + c^2*x^2)^(3/2) - (b*c^3*d*x^4*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
/(16*(1 + c^2*x^2)^(3/2)) - ((I/25)*b*c^4*d*x^5*(d + I*c*d*x)^(3/2)*(f - I*
c*f*x)^(3/2))/(1 + c^2*x^2)^(3/2) + (d*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^
(3/2)*(a + b*ArcSinh[c*x]))/4 + (3*d*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3
/2)*(a + b*ArcSinh[c*x]))/(8*(1 + c^2*x^2)) + ((I/5)*d*(d + I*c*d*x)^(3/2)*
(f - I*c*f*x)^(3/2)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/c + (3*d*(d + I*c*d
*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(16*b*c*(1 + c^2*x^2)
^(3/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
```

```
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\text{integral} = \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int (d+icdx)\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx}{(1+c^2x^2)^{3/2}}$$

$$= \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(d(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))+icdx(1+c^2x^2)^{3/2}(a+b\text{arcsinh}$$

$$= \frac{\left(d(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int (1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\,dx}{(1+c^2x^2)^{3/2}} + \frac{\left(icd(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\,dx}{(1+c^2x^2)^{3/2}}$$

$$= \frac{1}{4}dx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{id(d+icdx)^{3/2}(f-icfx)^{3/2}\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{5c} + \frac{\left(3d(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx}{4\left(1+c^2x^2\right)^{3/2}} - \frac{\left(ibd(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(1+c^2x^2\right)^2\,dx}{5\left(1+c^2x^2\right)^{3/2}} - \frac{\left(bcd(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int x(1+c^2x^2)\,dx}{4\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{1}{4}dx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx)) + \frac{3dx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{8(1+c^2x^2)} + \frac{id(d+icdx)^{3/2}(f-icfx)^{3/2}(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))}{5c} + \frac{\left(3d(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int\frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}dx}{8(1+c^2x^2)^{3/2}} - \frac{\left(ibd(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int(1+2c^2x^2+c^4x^4)\,dx}{5(1+c^2x^2)^{3/2}} - \frac{\left(bcd(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int(x+c^2x^3)\,dx}{4(1+c^2x^2)^{3/2}} - \frac{\left(3bcd(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int x\,dx}{8(1+c^2x^2)^{3/2}}$$

$$= -\frac{ibdx(d+icdx)^{3/2}(f-icfx)^{3/2}}{5(1+c^2x^2)^{3/2}} - \frac{5bcdx^2(d+icdx)^{3/2}(f-icfx)^{3/2}}{16(1+c^2x^2)^{3/2}} - \frac{2ibc^2dx^3(d+icdx)^{3/2}(f-icfx)^{3/2}}{15(1+c^2x^2)^{3/2}} - \frac{bc^3dx^4(d+icdx)^{3/2}(f-icfx)^{3/2}}{16(1+c^2x^2)^{3/2}} - \frac{ibc^4dx^5(d+icdx)^{3/2}(f-icfx)^{3/2}}{25(1+c^2x^2)^{3/2}} + \frac{1}{4}dx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx)) + \frac{3dx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(c$$

Mathematica [A] (verified)

Time = 3.17 (sec) , antiderivative size = 683, normalized size of antiderivative = 1.49

$$\int(d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \frac{-1200ibcd^2fx\sqrt{d+icdx}\sqrt{f-icfx}+1920iad^2f\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1}$$

```
[In] Integrate[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((-1200*I)*b*c*d^2*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (1920*I)*a*d^2
*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 6000*a*c*d^2*f*x
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (3840*I)*a*c^2*d^2
*f*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 2400*a*c^3*d
^2*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (1920*I)*a
*c^4*d^2*f*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 1800
```

```
*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 - 1200*b*d^2*f*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - 75*b*d^2*f*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 3600*a*d^(5/2)*f^(3/2
)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]] - (200*I)*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*Ar
cSinh[c*x]] + 60*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(
(10*I)*Cosh[3*ArcSinh[c*x]] + (2*I)*Cosh[5*ArcSinh[c*x]] + 5*((4*I)*Sqrt[1
+ c^2*x^2] + 8*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]])) - (24*I)*b*d^2
*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[5*ArcSinh[c*x]])/(9600*c*Sqrt[1
 + c^2*x^2])
```

Maple [F]

$$\int (icdx + d)^{\frac{5}{2}} (-icfx + f)^{\frac{3}{2}} (a + b \operatorname{arcsinh}(cx))\, dx$$

```
[In] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x)
```

Fricas [F]

$$\int (d+icdx)^{5/2}(f - icfx)^{3/2}(a+b\text{arcsinh}(cx))\, dx = \int (i\,cdx + d)^{\frac{5}{2}}(-i\,cfx + f)^{\frac{3}{2}}(b \operatorname{arsinh}(cx) + a)\, dx$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algorit
hm="fricas")
```

```
[Out] integral((I*b*c^3*d^2*f*x^3 + b*c^2*d^2*f*x^2 + I*b*c*d^2*f*x + b*d^2*f)*sq
rt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a*c^3*
d^2*f*x^3 + a*c^2*d^2*f*x^2 + I*a*c*d^2*f*x + a*d^2*f)*sqrt(I*c*d*x + d)*sq
rt(-I*c*f*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\, dx = \text{Timed out}$$

```
[In] integrate((d+I*c*d*x)**(5/2)*(f-I*c*f*x)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad Ar
gument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \int (a+b\,\mathrm{asinh}(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}\,dx$$

```
[In] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2), x)
```

3.541 $\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 247

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = -\frac{5bcx^2(d+icdx)^{3/2}(f-icfx)^{3/2}}{16(1+c^2x^2)^{3/2}} - \frac{bc^3x^4(d+icdx)^{3/2}(f-icfx)^{3/2}}{16(1+c^2x^2)^{3/2}} + \frac{1}{4}x(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{3x(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{8(1+c^2x^2)} + \frac{3(d+icdx}{}$$

```
[Out] -5/16*b*c*x^2*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)-1/16*b*
c^3*x^4*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)+1/4*x*(d+I*c*
d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))+3/8*x*(d+I*c*d*x)^(3/2)*(f-
I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)+3/16*(d+I*c*d*x)^(3/2)*(f-I*c
*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*x^2+1)^(3/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {5796, 5786, 5785, 5783, 30, 14}

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \frac{3x(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{8(c^2x^2+1)} + \frac{3(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{16bc(c^2x^2+1)^{3/2}} + \frac{1}{4}x(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{5bcx^2(d+icdx)^{3/2}(f-icfx)^{3/2}}{16(c^2x^2+1)^{3/2}} - \frac{bc^3x^4(d+icdx)^{3/2}(f-ic}{16(c^2x^2+1)^{3/2}}$$

```
[In] Int[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-5*b*c*x^2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(16*(1 + c^2*x^2)^(3/2
)) - (b*c^3*x^4*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(16*(1 + c^2*x^2)^
(3/2)) + (x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/4
 + (3*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(8*(1
 + c^2*x^2)) + (3*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*
x])^2)/(16*b*c*(1 + c^2*x^2)^(3/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rubi steps

$$\text{integral} = \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{1}{4}x(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{\left(3(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx}{4\left(1+c^2x^2\right)^{3/2}} - \frac{\left(bc(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int x\left(1+c^2x^2\right)\,dx}{4\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{1}{4}x(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{3x(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{8\left(1+c^2x^2\right)} + \frac{\left(3(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{8\left(1+c^2x^2\right)^{3/2}} - \frac{\left(bc(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int\left(x+c^2x^3\right)\,dx}{4\left(1+c^2x^2\right)^{3/2}} - \frac{\left(3bc(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int x\,dx}{8\left(1+c^2x^2\right)^{3/2}}$$

$$= -\frac{5bcx^2(d+icdx)^{3/2}(f-icfx)^{3/2}}{16\left(1+c^2x^2\right)^{3/2}} - \frac{bc^3x^4(d+icdx)^{3/2}(f-icfx)^{3/2}}{16\left(1+c^2x^2\right)^{3/2}} + \frac{1}{4}x(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{3x(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{8\left(1+c^2x^2\right)} +$$

Mathematica [A] (verified)

Time = 2.38 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.43

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \frac{80acdfx\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+32ac^3dfx^3\sqrt{d+icdx}\sqrt{f-icfx}}{}$$

```
[In] Integrate[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (80*a*c*d*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 32*a*
c^3*d*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 24*b*d*
f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 - 16*b*d*f*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - b*d*f*Sqrt[d + I*c*d*x]*Sqr
t[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 48*a*d^(3/2)*f^(3/2)*Sqrt[1 + c^2*x^2
]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 4*b*
d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(8*Sinh[2*ArcSinh[c*x]
] + Sinh[4*ArcSinh[c*x]]))/(128*c*Sqrt[1 + c^2*x^2])
```

Maple [F]

$$\int (icdx+d)^{\frac{3}{2}}(-icfx+f)^{\frac{3}{2}}(a+b\,\text{arcsinh}(cx))\,dx$$

```
[In] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x)
```

Fricas [F]

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \int (i\,cdx+d)^{\frac{3}{2}}(-i\,cfx+f)^{\frac{3}{2}}(b\,\text{arsinh}(cx)+a)\,dx$$

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algorit
hm="fricas")
```

```
[Out] integral((b*c^2*d*f*x^2 + b*d*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c
*x + sqrt(c^2*x^2 + 1)) + (a*c^2*d*f*x^2 + a*d*f)*sqrt(I*c*d*x + d)*sqrt(-I
*c*f*x + f), x)
```

Sympy [F]

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \int (id(cx-i))^{\frac{3}{2}}\,(-if(cx+i))^{\frac{3}{2}}\,(a+b\operatorname{asinh}(cx))\,dx$$

```
[In] integrate((d+I*c*d*x)**(3/2)*(f-I*c*f*x)**(3/2)*(a+b*asinh(c*x)),x)
[Out] Integral((I*d*(c*x - I))**(3/2)*(-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x)),
 x)
```

Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algorit
hm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algorit
hm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx=\int (a+b\,\mathrm{asinh}(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{3/2}\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}\,dx$$

```
[In] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2),x)
[Out] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2), x)
```

3.542 $\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 304

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \frac{ibfx\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}} - \frac{bcfx^2\sqrt{d+icdx}\sqrt{f-icfx}}{4\sqrt{1+c^2x^2}} + \frac{ibc^2fx^3\sqrt{d+icdx}\sqrt{f-icfx}}{9\sqrt{1+c^2x^2}} + \frac{1}{2}fx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx)) - \frac{if\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{3c} + \frac{f\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}}$$

```
[Out] 1/2*f*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/3*I*f*(c^2
*x^2+1)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+1/3*I*b*f*
x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-1/4*b*c*f*x^2*(d+I*
c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/9*I*b*c^2*f*x^3*(d+I*c*d
*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/4*f*(a+b*arcsinh(c*x))^2*(d
+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {5796, 5838, 5785, 5783, 30, 5798}

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\frac{f\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{c^2x^2+1}}-\frac{if(c^2x^2+1)\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{3c}+\frac{1}{2}fx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))-\frac{bcfx^2\sqrt{d+icdx}\sqrt{f-icfx}}{4\sqrt{c^2x^2+1}}+\frac{ibfx\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{c^2x^2+1}}+\frac{ibc^2fx^3\sqrt{d+icdx}\sqrt{f-icfx}}{9\sqrt{c^2x^2+1}}$$

```
[In] Int[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((I/3)*b*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2] - (b*c*
f*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(4*Sqrt[1 + c^2*x^2]) + ((I/9)*b
*c^2*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2] + (f*x*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/2 - ((I/3)*f*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/c + (f*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 + c
^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
```

```
x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int(f-icfx)\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\left(f\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))-icfx\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\right)\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(f\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}} - \frac{\left(icf\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{2}fx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx)) - \frac{if\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c} + \frac{\left(f\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}dx}{2\sqrt{1+c^2x^2}} + \frac{\left(ibf\sqrt{d+icdx}\sqrt{f-icfx}\right)\int(1+c^2x^2)\,dx}{3\sqrt{1+c^2x^2}} - \frac{\left(bcf\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x\,dx}{2\sqrt{1+c^2x^2}}$$

$$= \frac{ibfx\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}} - \frac{bcfx^2\sqrt{d+icdx}\sqrt{f-icfx}}{4\sqrt{1+c^2x^2}} + \frac{ibc^2fx^3\sqrt{d+icdx}\sqrt{f-icfx}}{9\sqrt{1+c^2x^2}} + \frac{1}{2}fx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx)) - \frac{if\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c} + \frac{f\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 2.94 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.90

$$\int\sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \frac{12af\sqrt{d+icdx}\sqrt{f-icfx}(-2i+3cx-2ic^2x^2)+36a\sqrt{d}f^{3/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+\right.}{}$$

```
[In] Integrate[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (12*a*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-2*I + 3*c*x - (2*I)*c^2*x^2)
+ 36*a*Sqrt[d]*f^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]] + (9*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(2*ArcSinh[c*x]
^2 - Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]]))/Sqrt[1 +
c^2*x^2] + ((2*I)*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(9*c*x - 3*ArcSin
h[c*x]*(3*Sqrt[1 + c^2*x^2] + Cosh[3*ArcSinh[c*x]]) + Sinh[3*ArcSinh[c*x]])
)/Sqrt[1 + c^2*x^2])/(72*c)
```

Maple [F]

$$\int (-icfx+f)^{\frac{3}{2}} (a+b \operatorname{arcsinh}(cx)) \sqrt{icdx+d}dx$$

[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x)

[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x)

Fricas [F]

$$\int \sqrt{d+icdx}(f - icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \int \sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)\,dx$$

[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algorit
hm="fricas")

[Out] integral((-I*b*c*f*x + b*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) + (-I*a*c*f*x + a*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f
), x)

Sympy [F]

$$\int \sqrt{d+icdx}(f - icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \int \sqrt{id\,(cx-i)}(-if(cx+i))^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(cx))\,\,dx$$

[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))*(d+I*c*d*x)**(1/2),x)

[Out] Integral(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x)), x)

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algorit
hm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad Ar
gument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \int (a+b\,\text{asinh}(c\,x))\,\sqrt{d+c\,d\,x\,1\text{i}}\,(f-c\,f\,x\,1\text{i})^{3/2}\,dx$$

```
[In] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2), x)
```

3.543 $\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 266

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \frac{2ibf^2x\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{bcf^2x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2if^2(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{f^2x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{3f^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[Out] -2*I*f^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/
2)-1/2*f^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(
1/2)+2*I*b*f^2*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/4*
b*c*f^2*x^2*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+3/4*f^2*(
a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/
2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5796, 5838, 5783, 5798, 8, 5812, 30}

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \frac{3f^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{f^2x(c^2x^2+1)\,(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2if^2(c^2x^2+1)\,(a+b\text{arcsinh}(cx))}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{bcf^2x^2\sqrt{c^2x^2+1}}{4\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ibf^2x\sqrt{c^2x^2+1}}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[In] Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((2*I)*b*f^2*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (
b*c*f^2*x^2*Sqrt[1 + c^2*x^2])/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - ((
2*I)*f^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d + I*c*d*x]*Sqrt[f -
I*c*f*x]) - (f^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(2*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]) + (3*f^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b
*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
```

```
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1+c^2x^2}\int\frac{(f-icfx)^2(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}\\
&= \frac{\sqrt{1+c^2x^2}\int\left(\frac{f^2(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}-\frac{2icf^2x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}-\frac{c^2f^2x^2(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}\\
&= \frac{\left(f^2\sqrt{1+c^2x^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{\left(2icf^2\sqrt{1+c^2x^2}\right)\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}\\
&\quad-\frac{\left(c^2f^2\sqrt{1+c^2x^2}\right)\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}\\
&= -\frac{2if^2(1+c^2x^2)\left(a+b\text{arcsinh}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{f^2x(1+c^2x^2)\left(a+b\text{arcsinh}(cx)\right)}{2\sqrt{d+icdx}\sqrt{f-icfx}}\\
&\quad+\frac{f^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{\left(f^2\sqrt{1+c^2x^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{d+icdx}\sqrt{f-icfx}}\\
&\quad+\frac{\left(2ibf^2\sqrt{1+c^2x^2}\right)\int 1\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{\left(bcf^2\sqrt{1+c^2x^2}\right)\int x\,dx}{2\sqrt{d+icdx}\sqrt{f-icfx}}\\
&= \frac{2ibf^2x\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{bcf^2x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{2if^2(1+c^2x^2)\left(a+b\text{arcsinh}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}\\
&\quad-\frac{f^2x(1+c^2x^2)\left(a+b\text{arcsinh}(cx)\right)}{2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{3f^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 5.47 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.29

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \frac{16ibcfx\sqrt{d+icdx}\sqrt{f-icfx}-16iaf\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c}$$

[In] Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]

[Out] ((16*I)*b*c*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (16*I)*a*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*a*c*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*b*f*(4*I + c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 6*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 + b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] + 12*a*Sqrt[d]*f^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/(8*c*d*Sqrt[1 + c^2*x^2])

Maple [F]

$$\int \frac{(-icfx+f)^{\frac{3}{2}}\left(a+b\,\text{arcsinh}\left(cx\right)\right)}{\sqrt{icdx+d}}dx$$

[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x)

[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x)

Fricas [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\,\text{arsinh}\left(cx\right)+a)}{\sqrt{i\,cdx+d}}\,dx$$

[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algorithm="fricas")

[Out] integral(-((b*c*f*x + I*b*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*c*f*x + I*a*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*d*x - I*d), x)

Sympy [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx=\int \frac{(-if(cx+i))^{\frac{3}{2}}\,(a+b\operatorname{asinh}{(cx)})}{\sqrt{id\,(cx-i)}}\,dx$$

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(1/2),x)
[Out] Integral((-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))/sqrt(I*d*(c*x - I)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algorit
hm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx=\int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{\sqrt{i\,cdx+d}}\,dx$$

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algorit
hm="giac")
[Out] integrate((-I*c*f*x + f)^(3/2)*(b*arcsinh(c*x) + a)/sqrt(I*c*d*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))\,\left(f-c\,f\,x\,1\text{i}\right)^{3/2}}{\sqrt{d+c\,d\,x\,1\text{i}}}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(1/2),x)
[Out] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(1/2), x)
```

3.544 $\int \frac{(f-icfx)^{3/2}(a+b\,\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 284

$$\int \frac{(f-icfx)^{3/2}(a+b\,\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx = -\frac{ibf^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^3(1-icx)\left(1+c^2x^2\right)(a+b\,\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{if^3(1+c^2x^2)^2(a+b\,\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{3f^3(1+c^2x^2)^{3/2}(a+b\,\text{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4bf^3(1+c^2x^2)^{3/2}\log(i-cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] -I*b*f^3*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*I*f^3*(1
-I*c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2
)+I*f^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3
/2)-3/2*f^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(d+I*c*d*x)^(3/2)/(f
-I*c*f*x)^(3/2)-4*b*f^3*(c^2*x^2+1)^(3/2)*ln(I-c*x)/c/(d+I*c*d*x)^(3/2)/(f-
I*c*f*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5796, 5844, 651, 5837, 12, 641, 31, 5783, 5798, 8}

$$\int \frac{(f-icfx)^{3/2}(a+b\,\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx = -\frac{3f^3(c^2x^2+1)^{3/2}(a+b\,\text{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{if^3(c^2x^2+1)^2(a+b\,\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^3(1-icx)\left(c^2x^2+1\right)(a+b\,\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{ibf^3x(c^2x^2+1)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4bf^3(c^2x^2+1)^{3/2}\log(-cx+i)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[In] Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((-I)*b*f^3*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)
) + ((4*I)*f^3*(1 - I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*
d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (I*f^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x
]))/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (3*f^3*(1 + c^2*x^2)^(3/2
)*(a + b*ArcSinh[c*x])^2)/(2*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) -
 (4*b*f^3*(1 + c^2*x^2)^(3/2)*Log[I - c*x])/(c*(d + I*c*d*x)^(3/2)*(f - I*c
*f*x)^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
```

```
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{(1+c^2x^2)^{3/2}\int\frac{(f-icfx)^3(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{(1+c^2x^2)^{3/2}\int\left(-\frac{4i(if^3+cf^3x)(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{3/2}}-\frac{3f^3(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}+\frac{icf^3x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(4i(1+c^2x^2)^{3/2}\right)\int\frac{(if^3+cf^3x)(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(3f^3(1+c^2x^2)^{3/2}\right)\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(icf^3(1+c^2x^2)^{3/2}\right)\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{4if^3(1-icx)\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{if^3\left(1+c^2x^2\right)^2(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{3f^3\left(1+c^2x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(4ibc\left(1+c^2x^2\right)^{3/2}\right)\int\frac{f^3(1-icx)}{c(1+c^2x^2)}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(ibf^3\left(1+c^2x^2\right)^{3/2}\right)\int 1\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{ibf^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^3(1-icx)\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{if^3\left(1+c^2x^2\right)^2(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{3f^3\left(1+c^2x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(4ibf^3\left(1+c^2x^2\right)^{3/2}\right)\int\frac{1-icx}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{ibf^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^3(1-icx)\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{if^3\left(1+c^2x^2\right)^2(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{3f^3\left(1+c^2x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(4ibf^3\left(1+c^2x^2\right)^{3/2}\right)\int\frac{1}{1+icx}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{ibf^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^3(1-icx)\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{if^3\left(1+c^2x^2\right)^2(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{3f^3\left(1+c^2x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4bf^3\left(1+c^2x^2\right)^{3/2}\log(i-cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Mathematica [A] (verified)

Time = 7.01 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.81

$$\int\frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx = \frac{\frac{2af(5+icx)\sqrt{d+icdx}\sqrt{f-icfx}}{d^2(-i+cx)} - \frac{6af^{3/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{d^{3/2}} - \frac{bf\sqrt{d+}}{}}{}$$

```
[In] Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((2*a*f*(5 + I*c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(d^2*(-I + c*x)) -
 (6*a*f^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*
f*x]])/d^(3/2) - (b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(ArcSinh[c*x]*((-
```

```
4*I)*Cosh[ArcSinh[c*x]/2] - 4*Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]^2*(Cosh[
ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 2*((4*I)*ArcTan[Tanh[ArcSinh[c*
x]/2]] + Log[1 + c^2*x^2])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]))
)/(d^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) +
 (2*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh
[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (c*x - 4*ArcTan[Coth[ArcSinh[c*x]/2]]
 - I*Log[1 + c^2*x^2])*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2]) +
 ArcSinh[c*x]*(I*(2 + Sqrt[1 + c^2*x^2])*Cosh[ArcSinh[c*x]/2] - (-2 + Sqrt[
1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2])))/(d^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[
c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(2*c)
```

Maple [F]

$$\int \frac{(-icfx+f)^{\frac{3}{2}} (a+b \operatorname{arcsinh}(cx))}{(icdx+d)^{\frac{3}{2}}} dx$$

[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x)

[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x)

Fricas [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}} dx = \int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)}{(i\,cdx+d)^{\frac{3}{2}}} dx$$

[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algorit
hm="fricas")

[Out] integral(((I*b*c*f*x - b*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) + (I*a*c*f*x - a*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)
)/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)

Sympy [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}} dx = \int \frac{(-if(cx+i))^{\frac{3}{2}} (a+b\operatorname{asinh}(cx))}{(id(cx-i))^{\frac{3}{2}}} dx$$

[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(3/2),x)

[Out] Integral((-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))/(I*d*(c*x - I))**(3/2),
 x)

Maxima [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algorit
hm="maxima")
```

```
[Out] a*(I*(c^2*d*f*x^2 + d*f)^(3/2)/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3) + 6*I*
sqrt(c^2*d*f*x^2 + d*f)*f/(I*c^2*d^2*x + c*d^2) - 3*f^2*arcsinh(c*x)/(c*d^2
*sqrt(f/d))) + b*integrate((-I*c*f*x + f)^(3/2)*log(c*x + sqrt(c^2*x^2 + 1)
)/(I*c*d*x + d)^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))\,\left(f-c\,f\,x\,1\text{i}\right)^{3/2}}{\left(d+c\,d\,x\,1\text{i}\right)^{3/2}}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(3/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(3/2), x)
```

3.545 $\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}} dx$

Optimal result

Integrand size = 35, antiderivative size = 364

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}} dx = \frac{4ibf^4(1+c^2x^2)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bf^4(1+c^2x^2)^{5/2}\,\text{arcsinh}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^4(1-icx)^3\,(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2if^4(1-icx)\,(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^4(1+c^2x^2)^{5/2}\,\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{8bf^4(1+c^2x^2)^{5/2}\log(i-cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[Out] 4/3*I*b*f^4*(c^2*x^2+1)^(5/2)/c/(I-c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
-1/2*b*f^4*(c^2*x^2+1)^(5/2)*arcsinh(c*x)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)
^(5/2)+2/3*I*f^4*(1-I*c*x)^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(
5/2)/(f-I*c*f*x)^(5/2)-2*I*f^4*(1-I*c*x)*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c
/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+f^4*(c^2*x^2+1)^(5/2)*arcsinh(c*x)*(a+
b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+8/3*b*f^4*(c^2*x^2+1)
^(5/2)*ln(I-c*x)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {5796, 683, 667, 221, 5837, 641, 45, 31, 5783}

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = -\frac{2if^4(1-icx)\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^4(1-icx)^3\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^4\left(c^2x^2+1\right)^{5/2}\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bf^4\left(c^2x^2+1\right)^{5/2}\text{arcsinh}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{4ibf^4\left(c^2x^2+1\right)^{5/2}}{3c(-cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{8bf^4\left(c^2x^2+1\right)^{5/2}\log(-cx+i)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[In] Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

```
[Out] (((4*I)/3)*b*f^4*(1 + c^2*x^2)^(5/2))/(c*(I - c*x)*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) - (b*f^4*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]^2)/(2*c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*f^4*(1 - I*c*x)^3*(1 + c^2*x^
2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((2*
I)*f^4*(1 - I*c*x)*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(
5/2)*(f - I*c*f*x)^(5/2)) + (f^4*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]*(a + b*Ar
cSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (8*b*f^4*(1 + c^
2*x^2)^(5/2)*Log[I - c*x])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 683

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((m +
p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(1+c^2x^2)^{5/2}\int\frac{(f-icfx)^4(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&= \frac{2if^4(1-icx)^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&\quad-\frac{2if^4(1-icx)\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&\quad+\frac{f^4(1+c^2x^2)^{5/2}\,\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&\quad-\frac{\left(bc(1+c^2x^2)^{5/2}\right)\int\left(\frac{2if^4(1-icx)^3}{3c(1+c^2x^2)^2}-\frac{2if^4(1-icx)}{c(1+c^2x^2)}+\frac{f^4\text{arcsinh}(cx)}{c\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&= \frac{2if^4(1-icx)^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2if^4(1-icx)\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&\quad+\frac{f^4(1+c^2x^2)^{5/2}\,\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&\quad-\frac{\left(2ibf^4(1+c^2x^2)^{5/2}\right)\int\frac{(1-icx)^3}{(1+c^2x^2)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{\left(2ibf^4(1+c^2x^2)^{5/2}\right)\int\frac{1-icx}{1+c^2x^2}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&\quad-\frac{\left(bf^4(1+c^2x^2)^{5/2}\right)\int\frac{\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&= -\frac{bf^4(1+c^2x^2)^{5/2}\,\text{arcsinh}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2if^4(1-icx)^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&\quad-\frac{2if^4(1-icx)\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&\quad+\frac{f^4(1+c^2x^2)^{5/2}\,\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&\quad-\frac{\left(2ibf^4(1+c^2x^2)^{5/2}\right)\int\frac{1-icx}{(1+icx)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{\left(2ibf^4(1+c^2x^2)^{5/2}\right)\int\frac{1}{1+icx}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}$$

$$= -\frac{bf^4(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^4(1-icx)^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2if^4(1-icx)(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^4(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2bf^4(1+c^2x^2)^{5/2}\log(i-cx)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2ibf^4(1+c^2x^2)^{5/2}\right)\int\left(-\frac{2}{(-i+cx)^2}+\frac{i}{-i+cx}\right)dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{4ibf^4(1+c^2x^2)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bf^4(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^4(1-icx)^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2if^4(1-icx)(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^4(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{8bf^4(1+c^2x^2)^{5/2}\log(i-cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Mathematica [A] (verified)

Time = 9.81 (sec) , antiderivative size = 706, normalized size of antiderivative = 1.94

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \frac{-\frac{16af(-i+2cx)\sqrt{d+icdx}\sqrt{f-icfx}}{d^3(-i+cx)^2} + \frac{12af^{3/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{d^{5/2}} -}{}$$

```
[In] Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

```
[Out] ((-16*a*f*(-I + 2*c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(d^3*(-I + c*x)
^2) + (12*a*f^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]])/d^(5/2) - (b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[ArcSi
nh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*ArcSinh[c*x])/2]*((-14 + (3*I
)*ArcSinh[c*x])*ArcSinh[c*x] - 28*ArcTan[Tanh[ArcSinh[c*x]/2]] + (7*I)*Log[
1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(84*ArcTan[Tanh[ArcSinh[c*x]/2]] - I*(
8 - (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 + 21*Log[1 + c^2*x^2])) + 2*(4 -
(4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 + (56*I)*ArcTan[Tanh[ArcSinh[c*x]/2]]
 + 14*Log[1 + c^2*x^2] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(-14*I + 3*ArcSinh
[c*x]) + (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 7*Log[1 + c^2*x^2]))*Sinh[Ar
cSinh[c*x]/2]))/(d^3*(I + c*x)*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/
2])^4) + ((2*I)*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]/
2] - I*Sinh[ArcSinh[c*x]/2])*((-I)*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] -
 2*ArcTan[Coth[ArcSinh[c*x]/2]] - (I/2)*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*
```

```
x]/2]*(4 + (3*I)*ArcSinh[c*x] - (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*Log
[1 + c^2*x^2])/2) + 2*((2 + Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 2*(2 + Sqrt[1
 + c^2*x^2])*ArcTan[Coth[ArcSinh[c*x]/2]] + (I/2)*(4 + (2 + Sqrt[1 + c^2*x^
2])*Log[1 + c^2*x^2]))*Sinh[ArcSinh[c*x]/2]))/(d^3*(I + c*x)*(Cosh[ArcSinh[
c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^4))/(12*c)
```

Maple [F]

$$\int \frac{(-icfx+f)^{\frac{3}{2}} (a+b \operatorname{arcsinh}(cx))}{(icdx+d)^{\frac{5}{2}}}dx$$

```
[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x)
```

```
[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x)
```

Fricas [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)}{(i\,cdx+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral(((b*c*f*x + I*b*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) + (a*c*f*x + I*a*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)
)/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c*d^3*x + I*d^3), x)
```

Sympy [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \int \frac{(-if(cx+i))^{\frac{3}{2}}\,(a+b\operatorname{asinh}(cx))}{(id\,(cx-i))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(5/2),x)
```

```
[Out] Integral((-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))/(I*d*(c*x - I))**(5/2),
 x)
```

Maxima [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{(i\,cdx+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algorit
hm="maxima")
```

```
[Out] -1/3*a*(-3*I*(c^2*d*f*x^2 + d*f)^(3/2)/(-3*I*c^4*d^4*x^3 - 9*c^3*d^4*x^2 +
9*I*c^2*d^4*x + 3*c*d^4) + 2*I*sqrt(c^2*d*f*x^2 + d*f)*f/(c^3*d^3*x^2 - 2*I
*c^2*d^3*x - c*d^3) + 21*I*sqrt(c^2*d*f*x^2 + d*f)*f/(3*I*c^2*d^3*x + 3*c*d
^3) - 3*f^2*arcsinh(c*x)/(c*d^3*sqrt(f/d))) + b*integrate((-I*c*f*x + f)^(3
/2)*log(c*x + sqrt(c^2*x^2 + 1))/(I*c*d*x + d)^(5/2), x)
```

Giac [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{(i\,cdx+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((-I*c*f*x + f)^(3/2)*(b*arcsinh(c*x) + a)/(I*c*d*x + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\left(f-c\,f\,x\,1\text{i}\right)^{3/2}}{\left(d+c\,d\,x\,1\text{i}\right)^{5/2}}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(5/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(5/2), x)
```

3.546 $\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 344

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx = -\frac{25bcx^2(d+icdx)^{5/2}(f-icfx)^{5/2}}{96(1+c^2x^2)^{5/2}}$$
$$-\frac{5bc^3x^4(d+icdx)^{5/2}(f-icfx)^{5/2}}{96(1+c^2x^2)^{5/2}}-\frac{b(d+icdx)^{5/2}(f-icfx)^{5/2}\sqrt{1+c^2x^2}}{36c}$$
$$+\frac{1}{6}x(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5x(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{16(1+c^2x^2)^2}+\frac{5x(d+ic}{}$$

```
[Out] -25/96*b*c*x^2*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)/(c^2*x^2+1)^(5/2)-5/96*b
*c^3*x^4*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)/(c^2*x^2+1)^(5/2)+1/6*x*(d+I*c
*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))+5/16*x*(d+I*c*d*x)^(5/2)*(
f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^2+5/24*x*(d+I*c*d*x)^(5/2)*
(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)+5/32*(d+I*c*d*x)^(5/2)*(f-
I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*x^2+1)^(5/2)-1/36*b*(d+I*c*d*x
)^(5/2)*(f-I*c*f*x)^(5/2)*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{5796, 5786, 5785, 5783, 30, 14, 267\}$

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \frac{5x(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{24\left(c^2x^2+1\right)} + \frac{5x(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{16\left(c^2x^2+1\right)^2} + \frac{5(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{32bc\left(c^2x^2+1\right)^{5/2}} + \frac{1}{6}x(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx)) - \frac{25bcx^2(d+icdx)^{5/2}(f-icfx)^{5/2}}{96\left(c^2x^2+1\right)^{5/2}} - \frac{b\sqrt{c^2x^2+1}(d+icdx)^5}{36c}$$

```
[In] Int[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-25*b*c*x^2*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))/(96*(1 + c^2*x^2)^(5/
2)) - (5*b*c^3*x^4*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))/(96*(1 + c^2*x^
2)^(5/2)) - (b*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*Sqrt[1 + c^2*x^2])/(
36*c) + (x*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/6
+ (5*x*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(16*(1
 + c^2*x^2)^2) + (5*x*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSin
h[c*x]))/(24*(1 + c^2*x^2)) + (5*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a
 + b*ArcSinh[c*x])^2)/(32*b*c*(1 + c^2*x^2)^(5/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
```

```
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left((d+icdx)^{5/2}(f-icfx)^{5/2}\right)\int\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx}{\left(1+c^2x^2\right)^{5/2}} \\
&= \frac{1}{6}x(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx)) \\
&\quad+\frac{\left(5(d+icdx)^{5/2}(f-icfx)^{5/2}\right)\int\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx}{6\left(1+c^2x^2\right)^{5/2}} \\
&\quad-\frac{\left(bc(d+icdx)^{5/2}(f-icfx)^{5/2}\right)\int x\left(1+c^2x^2\right)^2\,dx}{6\left(1+c^2x^2\right)^{5/2}} \\
&= -\frac{b(d+icdx)^{5/2}(f-icfx)^{5/2}\sqrt{1+c^2x^2}}{36c} \\
&\quad+\frac{1}{6}x(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5x(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{24\left(1+c^2x^2\right)}+
\end{aligned}$$

$$= -\frac{b(d+icdx)^{5/2}(f-icfx)^{5/2}\sqrt{1+c^2x^2}}{36c} + \frac{1}{6}x(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx)) + \frac{5x(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{16(1+c^2x^2)^2}$$

$$= -\frac{25bcx^2(d+icdx)^{5/2}(f-icfx)^{5/2}}{96(1+c^2x^2)^{5/2}} - \frac{5bc^3x^4(d+icdx)^{5/2}(f-icfx)^{5/2}}{96(1+c^2x^2)^{5/2}} - \frac{b(d+icdx)^{5/2}(f-icfx)^{5/2}\sqrt{1+c^2x^2}}{36c} + \frac{1}{6}x(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx)) + \frac{5x(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{16(1+c^2x^2)^2}$$

Mathematica [A] (verified)

Time = 2.76 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.40

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx = \frac{1584acd^2f^2x\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2} + 1248ac^3d^2f^2x^3\sqrt{d+icdx}\sqrt{}}{}$$

```
[In] Integrate[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (1584*a*c*d^2*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] +
 1248*a*c^3*d^2*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^
2] + 384*a*c^5*d^2*f^2*x^5*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2
*x^2] + 360*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 -
270*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - 27
*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] - 2*b*d
^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[6*ArcSinh[c*x]] + 720*a*d^(
5/2)*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]] + 12*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*A
rcSinh[c*x]*(45*Sinh[2*ArcSinh[c*x]] + 9*Sinh[4*ArcSinh[c*x]] + Sinh[6*ArcS
inh[c*x]]))/(2304*c*Sqrt[1 + c^2*x^2])
```

Maple [F]

$$\int (icdx + d)^{\frac{5}{2}} (-icfx + f)^{\frac{5}{2}} (a + b \operatorname{arcsinh}(cx)) dx$$

[In] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x)

[Out] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x)

Fricas [F]

$$\int (d + icdx)^{5/2}(f - icfx)^{5/2}(a + b\operatorname{arcsinh}(cx)) dx = \int (i\,cdx + d)^{\frac{5}{2}}(-i\,cfx + f)^{\frac{5}{2}}(b \operatorname{arsinh}(cx) + a) dx$$

[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")

[Out] integral((b*c^4*d^2*f^2*x^4 + 2*b*c^2*d^2*f^2*x^2 + b*d^2*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*c^4*d^2*f^2*x^4 + 2*a*c^2*d^2*f^2*x^2 + a*d^2*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)

Sympy [F(-1)]

Timed out.

$$\int (d + icdx)^{5/2}(f - icfx)^{5/2}(a + b\operatorname{arcsinh}(cx)) dx = \text{Timed out}$$

[In] integrate((d+I*c*d*x)**(5/2)*(f-I*c*f*x)**(5/2)*(a+b*asinh(c*x)),x)

[Out] Timed out

Maxima [F(-2)]

Exception generated.

$$\int (d + icdx)^{5/2}(f - icfx)^{5/2}(a + b\operatorname{arcsinh}(cx)) dx = \text{Exception raised: RuntimeError}$$

[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.

Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx = \int (a+b\operatorname{asinh}(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}\,dx$$

```
[In] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2), x)
```

3.547 $\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 459

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \frac{ibfx(d+icdx)^{3/2}(f-icfx)^{3/2}}{5(1+c^2x^2)^{3/2}}$$
$$-\frac{5bcfx^2(d+icdx)^{3/2}(f-icfx)^{3/2}}{16(1+c^2x^2)^{3/2}}+\frac{2ibc^2fx^3(d+icdx)^{3/2}(f-icfx)^{3/2}}{15(1+c^2x^2)^{3/2}}$$
$$-\frac{bc^3fx^4(d+icdx)^{3/2}(f-icfx)^{3/2}}{16(1+c^2x^2)^{3/2}}+\frac{ibc^4fx^5(d+icdx)^{3/2}(f-icfx)^{3/2}}{25(1+c^2x^2)^{3/2}}$$
$$+\frac{1}{4}fx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3fx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{8(1+c^2x^2)}-\frac{if(d+$$

```
[Out] 1/5*I*b*f*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)-5/16*b*c*
f*x^2*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)+2/15*I*b*c^2*f*
x^3*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)-1/16*b*c^3*f*x^4*
(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)+1/25*I*b*c^4*f*x^5*(d
+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)+1/4*f*x*(d+I*c*d*x)^(3/
2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))+3/8*f*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*
x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)-1/5*I*f*(d+I*c*d*x)^(3/2)*(f-I*c*f*
x)^(3/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c+3/16*f*(d+I*c*d*x)^(3/2)*(f-I*c*f
*x)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*x^2+1)^(3/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {5796, 5838, 5786, 5785, 5783, 30, 14, 5798, 200}

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \frac{3fx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{8\,(c^2x^2+1)} + \frac{3f(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{16bc\,(c^2x^2+1)^{3/2}} - \frac{if(c^2x^2+1)\,(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{5c} + \frac{1}{4}fx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{5bcfx^2(d+icdx)^{3/2}(f-icfx)^{3/2}}{16\,(c^2x^2+1)^{3/2}} + \frac{ibfx(d+icdx)^{3/2}(f}{5\,(c^2x^2+1)^{3}}$$

```
[In] Int[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((I/5)*b*f*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(1 + c^2*x^2)^(3/2) -
 (5*b*c*f*x^2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(16*(1 + c^2*x^2)^(3
/2)) + (((2*I)/15)*b*c^2*f*x^3*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(1
+ c^2*x^2)^(3/2) - (b*c^3*f*x^4*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(1
6*(1 + c^2*x^2)^(3/2)) + ((I/25)*b*c^4*f*x^5*(d + I*c*d*x)^(3/2)*(f - I*c*f
*x)^(3/2))/(1 + c^2*x^2)^(3/2) + (f*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/
2)*(a + b*ArcSinh[c*x]))/4 + (3*f*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)
*(a + b*ArcSinh[c*x]))/(8*(1 + c^2*x^2)) - ((I/5)*f*(d + I*c*d*x)^(3/2)*(f
- I*c*f*x)^(3/2)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/c + (3*f*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(16*b*c*(1 + c^2*x^2)^(3
/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
```

```
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\text{integral} = \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int(f-icfx)\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int\left(f\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))-icfx\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsi}\right.}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{\left(f(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx}{\left(1+c^2x^2\right)^{3/2}} - \frac{\left(icf(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{1}{4}fx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{if(d+icdx)^{3/2}(f-icfx)^{3/2}\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{5c} + \frac{\left(3f(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx}{4\left(1+c^2x^2\right)^{3/2}} + \frac{\left(ibf(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int\left(1+c^2x^2\right)^2\,dx}{5\left(1+c^2x^2\right)^{3/2}} - \frac{\left(bcf(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int x\left(1+c^2x^2\right)\,dx}{4\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{1}{4}fx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{3fx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{8(1+c^2x^2)} - \frac{if(d+icdx)^{3/2}(f-icfx)^{3/2}(1+c^2x^2)(a+b\text{arcsinh}(cx))}{5c} + \frac{\left(3f(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}dx}{8(1+c^2x^2)^{3/2}} + \frac{\left(ibf(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int (1+2c^2x^2+c^4x^4)\,dx}{5(1+c^2x^2)^{3/2}} - \frac{\left(bcf(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int (x+c^2x^3)\,dx}{4(1+c^2x^2)^{3/2}} - \frac{\left(3bcf(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int x\,dx}{8(1+c^2x^2)^{3/2}}$$

$$= \frac{ibfx(d+icdx)^{3/2}(f-icfx)^{3/2}}{5(1+c^2x^2)^{3/2}} - \frac{5bcfx^2(d+icdx)^{3/2}(f-icfx)^{3/2}}{16(1+c^2x^2)^{3/2}} + \frac{2ibc^2fx^3(d+icdx)^{3/2}(f-icfx)^{3/2}}{15(1+c^2x^2)^{3/2}} - \frac{bc^3fx^4(d+icdx)^{3/2}(f-icfx)^{3/2}}{16(1+c^2x^2)^{3/2}} + \frac{ibc^4fx^5(d+icdx)^{3/2}(f-icfx)^{3/2}}{25(1+c^2x^2)^{3/2}} + \frac{1}{4}fx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{3fx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx}{8(1+c^2x^2)}$$

Mathematica [A] (verified)

Time = 3.16 (sec) , antiderivative size = 683, normalized size of antiderivative = 1.49

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \frac{1200ibcdf^2x\sqrt{d+icdx}\sqrt{f-icfx} - 1920iadf^2\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c}}{}$$

```
[In] Integrate[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((1200*I)*b*c*d*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (1920*I)*a*d*f^
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 6000*a*c*d*f^2*x*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (3840*I)*a*c^2*d*f^
2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 2400*a*c^3*d*
f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (1920*I)*a*
c^4*d*f^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 1800*
```

```
b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 - 1200*b*d*f^2*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - 75*b*d*f^2*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 3600*a*d^(3/2)*f^(5/2)
*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]] + (200*I)*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*Arc
Sinh[c*x]] + 60*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((
-10*I)*Cosh[3*ArcSinh[c*x]] - (2*I)*Cosh[5*ArcSinh[c*x]] + 5*((-4*I)*Sqrt[1
 + c^2*x^2] + 8*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]])) + (24*I)*b*d*
f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[5*ArcSinh[c*x]])/(9600*c*Sqrt[
1 + c^2*x^2])
```

Maple [F]

$$\int (icdx+d)^{\frac{3}{2}} (-icfx+f)^{\frac{5}{2}} (a+b\,\mathrm{arcsinh}\,(cx))\,dx$$

```
[In] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x)
[Out] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x)
```

Fricas [F]

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\mathrm{arcsinh}(cx))\,dx = \int (i\,cdx+d)^{\frac{3}{2}}(-i\,cfx+f)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)\,dx$$

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algorit
hm="fricas")
[Out] integral((-I*b*c^3*d*f^2*x^3 + b*c^2*d*f^2*x^2 - I*b*c*d*f^2*x + b*d*f^2)*s
qrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a*c^
3*d*f^2*x^3 + a*c^2*d*f^2*x^2 - I*a*c*d*f^2*x + a*d*f^2)*sqrt(I*c*d*x + d)*
sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\mathrm{arcsinh}(cx))\,dx = \text{Timed out}$$

```
[In] integrate((d+I*c*d*x)**(3/2)*(f-I*c*f*x)**(5/2)*(a+b*asinh(c*x)),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad Ar
gument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx = \int (a+b\,\mathrm{asinh}(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{3/2}\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}\,dx$$

```
[In] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2), x)
```

3.548 $\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\,\mathbf{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 416

$$\begin{aligned}\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx &= \frac{2ibf^2x\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}} \\ &- \frac{3bcf^2x^2\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{1+c^2x^2}} + \frac{2ibc^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}}{9\sqrt{1+c^2x^2}} \\ &+ \frac{bc^3f^2x^4\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{1+c^2x^2}} + \frac{3}{8}f^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx)) \\ &- \frac{1}{4}c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx)) \\ &- \frac{2if^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))}{3c} \\ &+ \frac{5f^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2}{16bc\sqrt{1+c^2x^2}}\end{aligned}$$

```
[Out] 3/8*f^2*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/4*c^2*f^
2*x^3*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-2/3*I*f^2*(c^2
*x^2+1)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+2/3*I*b*f^
2*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-3/16*b*c*f^2*x^2*
(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+2/9*I*b*c^2*f^2*x^3*(
d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/16*b*c^3*f^2*x^4*(d+
I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+5/16*f^2*(a+b*arcsinh(c*
x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5796, 5838, 5785, 5783, 30, 5798, 5806, 5812}

$$\begin{aligned}&\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\\&-\frac{1}{4}c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\\&+\frac{5f^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{16bc\sqrt{c^2x^2+1}}\\&-\frac{2if^2(c^2x^2+1)\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{3c}\\&+\frac{3}{8}f^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\\&-\frac{3bcf^2x^2\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{c^2x^2+1}}+\frac{2ibf^2x\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{c^2x^2+1}}\\&+\frac{2ibc^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}}{9\sqrt{c^2x^2+1}}+\frac{bc^3f^2x^4\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{c^2x^2+1}}\end{aligned}$$

```
[In] Int[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (((2*I)/3)*b*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2] -
 (3*b*c*f^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(16*Sqrt[1 + c^2*x^2])
 + (((2*I)/9)*b*c^2*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c
^2*x^2] + (b*c^3*f^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(16*Sqrt[1 +
c^2*x^2]) + (3*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x
]))/8 - (c^2*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x
]))/4 - (((2*I)/3)*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(a
 + b*ArcSinh[c*x]))/c + (5*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*A
rcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int (f-icfx)^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \left(f^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))-2icf^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))-\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(f^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}} - \frac{\left(2icf^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}} - \frac{\left(c^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{2}f^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx)) - \frac{1}{4}c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx)) - \frac{2if^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{3c} + \frac{\left(f^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}} + \frac{\left(2ibf^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \left(1+c^2x^2\right)\,dx}{3\sqrt{1+c^2x^2}} - \frac{\left(bcf^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x\,dx}{2\sqrt{1+c^2x^2}} - \frac{\left(c^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{4\sqrt{1+c^2x^2}} + \frac{\left(bc^3f^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x^3\,dx}{4\sqrt{1+c^2x^2}}$$

$$= \frac{2ibf^2x\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}} - \frac{bcf^2x^2\sqrt{d+icdx}\sqrt{f-icfx}}{4\sqrt{1+c^2x^2}} + \frac{2ibc^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}}{9\sqrt{1+c^2x^2}} + \frac{bc^3f^2x^4\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{1+c^2x^2}} + \frac{3}{8}f^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx)) - \frac{1}{4}c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx)) - \frac{2if^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))}{3c} + \frac{f^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}} + \frac{\left(f^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{8\sqrt{1+c^2x^2}} + \frac{\left(bcf^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x\,dx}{8\sqrt{1+c^2x^2}}$$

$$= \frac{2ibf^2x\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}} - \frac{3bcf^2x^2\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{1+c^2x^2}} + \frac{2ibc^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}}{9\sqrt{1+c^2x^2}} + \frac{bc^3f^2x^4\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{1+c^2x^2}} + \frac{3}{8}f^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx)) - \frac{1}{4}c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx)) - \frac{2if^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))}{3c} + \frac{5f^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2}{16bc\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 2.57 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.36

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx = \frac{576ibcf^2x\sqrt{d+icdx}\sqrt{f-icfx} - 768iaf^2\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2} + 432acf^2x}{}$$

```
[In] Integrate[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((576*I)*b*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (768*I)*a*f^2*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 432*a*c*f^2*x*Sqrt[d +
```

```
I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (768*I)*a*c^2*f^2*x^2*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 288*a*c^3*f^2*x^3*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 360*b*f^2*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 - 144*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x]*Cosh[2*ArcSinh[c*x]] + 9*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*
Cosh[4*ArcSinh[c*x]] + 720*a*Sqrt[d]*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x
+ Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + (64*I)*b*f^2*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 12*b*f^2*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((-48*I)*Sqrt[1 + c^2*x^2] - (16*I)*Co
sh[3*ArcSinh[c*x]] + 24*Sinh[2*ArcSinh[c*x]] - 3*Sinh[4*ArcSinh[c*x]]))/(11
52*c*Sqrt[1 + c^2*x^2])
```

Maple [F]

$$\int (-icfx+f)^{\frac{5}{2}} (a+b\,\mathrm{arcsinh}\,(cx))\,\sqrt{icdx+d}dx$$

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x)
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x)
```

Fricas [F]

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\mathrm{arcsinh}(cx))\,dx=\int \sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)\,dx$$

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algorit
hm="fricas")
[Out] integral(-(b*c^2*f^2*x^2 + 2*I*b*c*f^2*x - b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I
*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) - (a*c^2*f^2*x^2 + 2*I*a*c*f^2*x -
 a*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\mathrm{arcsinh}(cx))\,dx=\text{Timed out}$$

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))*(d+I*c*d*x)**(1/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d + icdx}(f - icfx)^{5/2}(a + b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d + icdx}(f - icfx)^{5/2}(a + b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d + icdx}(f - icfx)^{5/2}(a + b\text{arcsinh}(cx))\,dx = \int (a + b\,\text{asinh}(c\,x))\,\sqrt{d + c\,d\,x\,1\text{i}}\,(f - c\,f\,x\,1\text{i})^{5/2}\,dx$$

```
[In] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2), x)
```

3.549 $\int \frac{(f-icfx)^{5/2}(a+b\textbf{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 381

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx=\frac{11ibf^3x\sqrt{1+c^2x^2}}{3\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{3bcf^3x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{ibc^2f^3x^3\sqrt{1+c^2x^2}}{9\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{11if^3(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{3c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{3f^3x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{icf^3x^2(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{3\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{5f^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[Out] -11/3*I*f^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^
(1/2)-3/2*f^3*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x
)^(1/2)+1/3*I*c*f^3*x^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f
-I*c*f*x)^(1/2)+11/3*I*b*f^3*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f
*x)^(1/2)+3/4*b*c*f^3*x^2*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(
1/2)-1/9*I*b*c^2*f^3*x^3*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1
/2)+5/4*f^3*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f
-I*c*f*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5796, 5838, 5783, 5798, 8, 5812, 30}

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx=\frac{5f^3\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{icf^3x^2(c^2x^2+1)\,(a+b\operatorname{arcsinh}(cx))}{3\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{3f^3x(c^2x^2+1)\,(a+b\operatorname{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{11if^3(c^2x^2+1)\,(a+b\operatorname{arcsinh}(cx))}{3c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{3bcf^3x^2\sqrt{c^2x^2+1}}{4\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{11ibf^3x\sqrt{c^2x^2+1}}{3\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{ibc^2f^3x^3\sqrt{c^2x^2+1}}{9\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[In] Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((((11*I)/3)*b*f^3*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
) + (3*b*c*f^3*x^2*Sqrt[1 + c^2*x^2])/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]) - ((I/9)*b*c^2*f^3*x^3*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x]) - (((11*I)/3)*f^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]) - (3*f^3*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + ((I/3)*c*f^3*x^2*(1 + c^2*x^2)*(a
+ b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (5*f^3*Sqrt[1 +
c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
```

```
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+c^2x^2}\int \frac{(f-icfx)^3(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{f^3(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}} - \frac{3icf^3x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}} - \frac{3c^2f^3x^2(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}} + \frac{ic^3f^3x^3(a+b\text{arcsinh}}{\sqrt{1+c^2x^2}}\right.}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\left(f^3\sqrt{1+c^2x^2}\right)\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(3icf^3\sqrt{1+c^2x^2}\right)\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$
$$- \frac{\left(3c^2f^3\sqrt{1+c^2x^2}\right)\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(ic^3f^3\sqrt{1+c^2x^2}\right)\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= -\frac{3if^3(1+c^2x^2)(a+b\text{arcsinh}(cx))}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3f^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{icf^3x^2(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{f^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(3f^3\sqrt{1+c^2x^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}dx}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(3ibf^3\sqrt{1+c^2x^2}\right)\int 1\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(2icf^3\sqrt{1+c^2x^2}\right)\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}dx}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(3bcf^3\sqrt{1+c^2x^2}\right)\int x\,dx}{2\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(ibc^2f^3\sqrt{1+c^2x^2}\right)\int x^2\,dx}{3\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{3ibf^3x\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{3bcf^3x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{ibc^2f^3x^3\sqrt{1+c^2x^2}}{9\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{11if^3(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3f^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{icf^3x^2(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{5f^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(2ibf^3\sqrt{1+c^2x^2}\right)\int 1\,dx}{3\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{11ibf^3x\sqrt{1+c^2x^2}}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{3bcf^3x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{ibc^2f^3x^3\sqrt{1+c^2x^2}}{9\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{11if^3(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3f^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{icf^3x^2(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{5f^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

Mathematica [A] (verified)

Time = 7.17 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.22

$$\int\frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \frac{264ibcf^2x\sqrt{d+icdx}\sqrt{f-icfx} - 8ibc^3f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}}{}$$

```
[In] Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((264*I)*b*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (8*I)*b*c^3*f^2*x^
3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (264*I)*a*f^2*Sqrt[d + I*c*d*x]*Sqr
t[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 108*a*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]*Sqrt[1 + c^2*x^2] + (24*I)*a*c^2*f^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]*Sqrt[1 + c^2*x^2] + 90*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
```

```
]*ArcSinh[c*x]^2 + 27*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcS
inh[c*x]] - 6*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(9*(5*
I + 2*c*x)*Sqrt[1 + c^2*x^2] - I*Cosh[3*ArcSinh[c*x]]) + 180*a*Sqrt[d]*f^(5
/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]])/(72*c*d*Sqrt[1 + c^2*x^2])
```

Maple [F]

$$\int \frac{(-icfx+f)^{\frac{5}{2}} (a+b \operatorname{arcsinh}(cx))}{\sqrt{icdx+d}} dx$$

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}} dx = \int \frac{(-i\,cfx+f)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}{\sqrt{i\,cdx+d}} dx$$

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(((I*b*c^2*f^2*x^2 - 2*b*c*f^2*x - I*b*f^2)*sqrt(I*c*d*x + d)*sqrt(
-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a*c^2*f^2*x^2 - 2*a*c*f^2*x
 - I*a*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*d*x - I*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}} dx = \text{Timed out}$$

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f - icfx)^{5/2}(a + b\text{arcsinh}(cx))}{\sqrt{d + icdx}}\, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f - icfx)^{5/2}(a + b\text{arcsinh}(cx))}{\sqrt{d + icdx}}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f - icfx)^{5/2}(a + b\text{arcsinh}(cx))}{\sqrt{d + icdx}}\, dx = \int \frac{(a + b\,\text{asinh}(c\,x))\,\left(f - c\,f\,x\,1\text{i}\right)^{5/2}}{\sqrt{d + c\,d\,x\,1\text{i}}}\, dx$$

```
[In] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(1/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(1/2), x)
```

3.550 $\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 518

$$\begin{aligned}\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx &= -\frac{3ibf^4x(1+c^2x^2)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} \\ &+ \frac{bcf^4x^2(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bf^4(1-icx)^2\,(1+c^2x^2)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\ &+ \frac{15bf^4(1+c^2x^2)^{3/2}\,\text{arcsinh}(cx)^2}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2if^4(1-icx)^3\,(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\ &+ \frac{15if^4(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5if^4(1-icx)\,(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\ &- \frac{15f^4(1+c^2x^2)^{3/2}\,\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8bf^4(1+c^2x^2)^{3/2}\log(i-cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}\end{aligned}$$

```
[Out] -3/2*I*b*f^4*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+b*c*f^
4*x^2*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+5/4*b*f^4*(1-I*
c*x)^2*(c^2*x^2+1)^(3/2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+15/4*b*f^4*(
c^2*x^2+1)^(3/2)*arcsinh(c*x)^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+2*I*f
^4*(1-I*c*x)^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*
x)^(3/2)+15/2*I*f^4*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f
-I*c*f*x)^(3/2)+5/2*I*f^4*(1-I*c*x)*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/(d+I
*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-15/2*f^4*(c^2*x^2+1)^(3/2)*arcsinh(c*x)*(a+
b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*b*f^4*(c^2*x^2+1)^(
3/2)*ln(I-c*x)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {5796, 683, 685, 655, 221, 5837, 641, 45, 5783}

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\frac{5if^4(1-icx)\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{15if^4\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2if^4(1-icx)^3\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{15f^4\left(c^2x^2+1\right)^{3/2}\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{15bf^4\left(c^2x^2+1\right)^{3/2}\text{arcsinh}(cx)^2}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{bcf^4x^2\left(c^2x^2+1\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{5bf^4(1-icx)^2\left(c^2x^2+1\right)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{3ibf^4x\left(c^2x^2+1\right)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{8bf^4\left(c^2x^2+1\right)^{3/2}\log(-cx+i)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[In] Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

```
[Out] (((-3*I)/2)*b*f^4*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)) + (b*c*f^4*x^2*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f
*x)^(3/2)) + (5*b*f^4*(1 - I*c*x)^2*(1 + c^2*x^2)^(3/2))/(4*c*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2)) + (15*b*f^4*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]^2)
/(4*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((2*I)*f^4*(1 - I*c*x)^3*(
1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/
2)) + (((15*I)/2)*f^4*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) + (((5*I)/2)*f^4*(1 - I*c*x)*(1 + c^2*x^2)^2*(
a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (15*f^4*
(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*(a + b*ArcSinh[c*x]))/(2*c*(d + I*c*d*x)^(
3/2)*(f - I*c*f*x)^(3/2)) - (8*b*f^4*(1 + c^2*x^2)^(3/2)*Log[I - c*x])/(c*(
d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 683

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((m +
p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
```

```
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(1+c^2x^2)^{3/2}\int \frac{(f-icfx)^4(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= \frac{2if^4(1-icx)^3(1+c^2x^2)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{15if^4(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&\quad+\frac{5if^4(1-icx)(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&\quad-\frac{15f^4(1+c^2x^2)^{3/2}\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&\quad-\frac{\left(bc(1+c^2x^2)^{3/2}\right)\int\left(\frac{15if^4}{2c}+\frac{5if^4(1-icx)}{2c}+\frac{2if^4(1-icx)^3}{c(1+c^2x^2)}-\frac{15f^4\text{arcsinh}(cx)}{2c\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= -\frac{15ibf^4x(1+c^2x^2)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{5bf^4(1-icx)^2(1+c^2x^2)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&\quad+\frac{2if^4(1-icx)^3(1+c^2x^2)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&\quad+\frac{15if^4(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&\quad+\frac{5if^4(1-icx)(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&\quad-\frac{15f^4(1+c^2x^2)^{3/2}\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&\quad-\frac{\left(2ibf^4(1+c^2x^2)^{3/2}\right)\int\frac{(1-icx)^3}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{\left(15bf^4(1+c^2x^2)^{3/2}\right)\int\frac{\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}$$

$$= -\frac{15ibf^4x(1+c^2x^2)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bf^4(1-icx)^2(1+c^2x^2)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{15bf^4(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)^2}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2if^4(1-icx)^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{15if^4(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5if^4(1-icx)(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{15f^4(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)(a+b\operatorname{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2ibf^4(1+c^2x^2)^{3/2}\right)\int\frac{(1-icx)^2}{1+icx}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{15ibf^4x(1+c^2x^2)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bf^4(1-icx)^2(1+c^2x^2)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{15bf^4(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)^2}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2if^4(1-icx)^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{15if^4(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5if^4(1-icx)(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{15f^4(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)(a+b\operatorname{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2ibf^4(1+c^2x^2)^{3/2}\right)\int\left(-3+icx+\frac{4}{1+icx}\right)\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{3ibf^4x(1+c^2x^2)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{bcf^4x^2(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bf^4(1-icx)^2(1+c^2x^2)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{15bf^4(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)^2}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2if^4(1-icx)^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{15if^4(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5if^4(1-icx)(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{15f^4(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)(a+b\operatorname{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8bf^4(1+c^2x^2)^{3/2}\log(i-cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Mathematica [A] (verified)

Time = 9.86 (sec) , antiderivative size = 779, normalized size of antiderivative = 1.50

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx = \frac{\frac{4af^2\sqrt{d+icdx}\sqrt{f-icfx}(24+7icx+c^2x^2)}{d^2(-i+cx)} - \frac{60af^{5/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right.}{d^{3/2}}}{}$$

```
[In] Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((4*a*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(24 + (7*I)*c*x + c^2*x^2))/(
d^2*(-I + c*x)) - (60*a*f^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*
d*x]*Sqrt[f - I*c*f*x]])/d^(3/2) - (4*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*
f*x]*(ArcSinh[c*x]*((-4*I)*Cosh[ArcSinh[c*x]/2] - 4*Sinh[ArcSinh[c*x]/2]) +
 ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 2*((4*I)*
ArcTan[Tanh[ArcSinh[c*x]/2]] + Log[1 + c^2*x^2])*(Cosh[ArcSinh[c*x]/2] + I*
Sinh[ArcSinh[c*x]/2])))/(d^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Si
nh[ArcSinh[c*x]/2])) + (16*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(Arc
Sinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (c*x - 4*Arc
Tan[Coth[ArcSinh[c*x]/2]] - I*Log[1 + c^2*x^2])*((-I)*Cosh[ArcSinh[c*x]/2]
+ Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]*(I*(2 + Sqrt[1 + c^2*x^2])*Cosh[ArcS
inh[c*x]/2] - (-2 + Sqrt[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2])))/(d^2*Sqrt[1
+ c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (b*f^2*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*(-10*ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] + I
*Sinh[ArcSinh[c*x]/2]) - (Cosh[2*ArcSinh[c*x]] + 8*((2*I)*c*x + (4*I)*ArcTa
n[Tanh[ArcSinh[c*x]/2]] + Log[1 + c^2*x^2]))*(Cosh[ArcSinh[c*x]/2] + I*Sinh
[ArcSinh[c*x]/2]) + 2*ArcSinh[c*x]*(Sinh[ArcSinh[c*x]/2]*(8 - 8*Sqrt[1 + c^
2*x^2] + I*Sinh[2*ArcSinh[c*x]]) + Cosh[ArcSinh[c*x]/2]*((8*I)*(1 + Sqrt[1
+ c^2*x^2]) + Sinh[2*ArcSinh[c*x]]))))/(d^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh
[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(8*c)
```

Maple [F]

$$\int \frac{(-icfx+f)^{\frac{5}{2}}\,(a+b\,\text{arcsinh}\,(cx))}{(icdx+d)^{\frac{3}{2}}}dx$$

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x)
```

```
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x)
```

Fricas [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\int \frac{(-i\,cfx+f)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(((b*c^2*f^2*x^2 + 2*I*b*c*f^2*x - b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I
*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*c^2*f^2*x^2 + 2*I*a*c*f^2*x -
 a*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^2*d^2*x^2 - 2*I*c*d^2*x -
d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\int \frac{(-i\,cfx+f)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*(c^2*f^3*x^3/(sqrt(c^2*d*f*x^2 + d*f)*d) + 8*I*c*f^3*x^2/(sqrt(c^2*d*f*
x^2 + d*f)*d) + 17*f^3*x/(sqrt(c^2*d*f*x^2 + d*f)*d) - 15*f^3*arcsinh(c*x)/
(sqrt(d*f)*c*d) + 24*I*f^3/(sqrt(c^2*d*f*x^2 + d*f)*c*d))*a + b*integrate((
-I*c*f*x + f)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1))/(I*c*d*x + d)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f - icfx)^{5/2}(a + b\text{arcsinh}(cx))}{(d + icdx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f - icfx)^{5/2}(a + b\text{arcsinh}(cx))}{(d + icdx)^{3/2}}\,dx = \int \frac{(a + b\,\text{asinh}(c\,x))\,\left(f - c\,f\,x\,1\text{i}\right)^{5/2}}{\left(d + c\,d\,x\,1\text{i}\right)^{3/2}}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(3/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(3/2), x)
```

3.551 $\int \frac{(f-icfx)^{5/2}(a+b\,\mathbf{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 472

$$\begin{aligned}\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx &= \frac{ibf^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} \\ &+ \frac{8ibf^5(1+c^2x^2)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{5bf^5(1+c^2x^2)^{5/2}\,\text{arcsinh}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\ &+ \frac{2if^5(1-icx)^4(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\ &- \frac{10if^5(1-icx)^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\ &- \frac{5if^5(1+c^2x^2)^3(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\ &+ \frac{5f^5(1+c^2x^2)^{5/2}\,\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{28bf^5(1+c^2x^2)^{5/2}\log(i-cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}\end{aligned}$$

```
[Out] I*b*f^5*x*(c^2*x^2+1)^(5/2)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+8/3*I*b*f^5
*(c^2*x^2+1)^(5/2)/c/(I-c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-5/2*b*f^5*
(c^2*x^2+1)^(5/2)*arcsinh(c*x)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3*
I*f^5*(1-I*c*x)^4*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c
*f*x)^(5/2)-10/3*I*f^5*(1-I*c*x)^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/(d+I*
c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-5*I*f^5*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))/c/(
d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+5*f^5*(c^2*x^2+1)^(5/2)*arcsinh(c*x)*(a+
b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+28/3*b*f^5*(c^2*x^2+1
)^(5/2)*ln(I-c*x)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5796, 683, 655, 221, 5837, 641, 45, 5783}

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = -\frac{5if^5(c^2x^2+1)^3(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{10if^5(1-icx)^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{2if^5(1-icx)^4(c^2x^2+1)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{5f^5(c^2x^2+1)^{5/2}\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{5bf^5(c^2x^2+1)^{5/2}\text{arcsinh}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibf^5x(c^2x^2+1)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{8ibf^5(c^2x^2+1)^{5/2}}{3c(-cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{28bf^5(c^2x^2+1)^{5/2}\log(-cx+i)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[In] Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

```
[Out] (I*b*f^5*x*(1 + c^2*x^2)^(5/2))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) +
 (((8*I)/3)*b*f^5*(1 + c^2*x^2)^(5/2))/(c*(I - c*x)*(d + I*c*d*x)^(5/2)*(f
- I*c*f*x)^(5/2)) - (5*b*f^5*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]^2)/(2*c*(d +
I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*f^5*(1 - I*c*x)^4*(1 + c^2
*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (
((10*I)/3)*f^5*(1 - I*c*x)^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d +
I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((5*I)*f^5*(1 + c^2*x^2)^3*(a + b*Arc
Sinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (5*f^5*(1 + c^2*x
^2)^(5/2)*ArcSinh[c*x]*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*
c*f*x)^(5/2)) + (28*b*f^5*(1 + c^2*x^2)^(5/2)*Log[I - c*x])/(3*c*(d + I*c*d
*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 683

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((m +
p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(1+c^2x^2)^{5/2}\int \frac{(f-icfx)^5(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&= \frac{2if^5(1-icx)^4(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{10if^5(1-icx)^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{5if^5(1+c^2x^2)^3(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{5f^5(1+c^2x^2)^{5/2}\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{\left(bc(1+c^2x^2)^{5/2}\right)\int\left(-\frac{5if^5}{c}+\frac{2if^5(1-icx)^4}{3c(1+c^2x^2)^2}-\frac{10if^5(1-icx)^2}{3c(1+c^2x^2)}+\frac{5f^5\text{arcsinh}(cx)}{c\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&= \frac{5ibf^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^5(1-icx)^4(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{10if^5(1-icx)^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{5if^5(1+c^2x^2)^3(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5f^5(1+c^2x^2)^{5/2}\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{\left(2ibf^5(1+c^2x^2)^{5/2}\right)\int\frac{(1-icx)^4}{(1+c^2x^2)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(10ibf^5(1+c^2x^2)^{5/2}\right)\int\frac{(1-icx)^2}{1+c^2x^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{\left(5bf^5(1+c^2x^2)^{5/2}\right)\int\frac{\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&= \frac{5ibf^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{5bf^5(1+c^2x^2)^{5/2}\text{arcsinh}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{2if^5(1-icx)^4(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{10if^5(1-icx)^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{5if^5(1+c^2x^2)^3(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{5f^5(1+c^2x^2)^{5/2}\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{\left(2ibf^5(1+c^2x^2)^{5/2}\right)\int\frac{(1-icx)^2}{(1+icx)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(10ibf^5(1+c^2x^2)^{5/2}\right)\int\frac{1-icx}{1+icx}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{5ibf^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{5bf^5(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{2if^5(1-icx)^4(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{10if^5(1-icx)^2(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{5if^5(1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{5f^5(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{\left(2ibf^5(1+c^2x^2)^{5/2}\right)\int\left(1-\frac{4}{(-i+cx)^2}+\frac{4i}{-i+cx}\right)dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{\left(10ibf^5(1+c^2x^2)^{5/2}\right)\int\left(-1-\frac{2i}{-i+cx}\right)dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&= \frac{ibf^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{8ibf^5(1+c^2x^2)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{5bf^5(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^5(1-icx)^4(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{10if^5(1-icx)^2(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{5if^5(1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{5f^5(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{28bf^5(1+c^2x^2)^{5/2}\log(i-cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1005 vs. 2(472) = 944.

Time = 12.47 (sec) , antiderivative size = 1005, normalized size of antiderivative = 2.13

$$
\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \frac{-\frac{4iaf^2\sqrt{d+icdx}\sqrt{f-icfx}(-23-34icx+3c^2x^2)}{d^3(-i+cx)^2} + \frac{60af^{5/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-\right.}}{d^{5/2}}}{}
$$

```
[In] Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

```
[Out] (((-4*I)*a*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-23 - (34*I)*c*x + 3*c^
2*x^2))/(d^3*(-I + c*x)^2) + (60*a*f^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sq
```

```
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/d^(5/2) - (2*b*f^2*Sqrt[d + I*c*d*x]*Sq
rt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*Ar
cSinh[c*x])/2]*((-14 + (3*I)*ArcSinh[c*x])*ArcSinh[c*x] - 28*ArcTan[Tanh[Ar
cSinh[c*x]/2]] + (7*I)*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(84*ArcTan[
Tanh[ArcSinh[c*x]/2]] - I*(8 - (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 + 21*L
og[1 + c^2*x^2])) + 2*(4 - (4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 + (56*I)*A
rcTan[Tanh[ArcSinh[c*x]/2]] + 14*Log[1 + c^2*x^2] + Sqrt[1 + c^2*x^2]*(ArcS
inh[c*x]*(-14*I + 3*ArcSinh[c*x]) + (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 7
*Log[1 + c^2*x^2]))*Sinh[ArcSinh[c*x]/2]))/(d^3*(I + c*x)*(Cosh[ArcSinh[c*x
]/2] + I*Sinh[ArcSinh[c*x]/2])^4) + ((2*I)*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*((-I)*Cosh[(3*Arc
Sinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] - (I/2)*Log[1
+ c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(4 + (3*I)*ArcSinh[c*x] - (6*I)*ArcTan[C
oth[ArcSinh[c*x]/2]] + (3*Log[1 + c^2*x^2])/2) + 2*((2 + Sqrt[1 + c^2*x^2])
*ArcSinh[c*x] + 2*(2 + Sqrt[1 + c^2*x^2])*ArcTan[Coth[ArcSinh[c*x]/2]] + (I
/2)*(4 + (2 + Sqrt[1 + c^2*x^2])*Log[1 + c^2*x^2]))*Sinh[ArcSinh[c*x]/2]))/
(d^3*(I + c*x)*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^4) + (b*f^2*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[
c*x]/2])*(2*(4 + (6*I)*c*x - 6*c^2*x^2 + 52*(-I + c*x)*ArcTan[Coth[ArcSinh[
c*x]/2]] + 13*(1 + I*c*x)*Log[1 + c^2*x^2])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[
ArcSinh[c*x]/2]) + 18*ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh
[c*x]/2])^3 + ArcSinh[c*x]*((-24*I)*Cosh[ArcSinh[c*x]/2] - (35*I)*Cosh[(3*A
rcSinh[c*x])/2] + (3*I)*Cosh[(5*ArcSinh[c*x])/2] - 24*Sinh[ArcSinh[c*x]/2]
+ 35*Sinh[(3*ArcSinh[c*x])/2] + 3*Sinh[(5*ArcSinh[c*x])/2])))/(d^3*(I + c*x
)*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^4))/(12*c)
```

Maple [F]

$$\int \frac{(-icfx+f)^{\frac{5}{2}} (a+b \operatorname{arcsinh}(cx))}{(icdx+d)^{\frac{5}{2}}}dx$$

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x)
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x)
```

Fricas [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)}{(i\,cdx+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral(((-I*b*c^2*f^2*x^2 + 2*b*c*f^2*x + I*b*f^2)*sqrt(I*c*d*x + d)*sqrt
(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a*c^2*f^2*x^2 + 2*a*c*f^2
*x + I*a*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*d^3*x^3 - 3*I*c^2*
d^3*x^2 - 3*c*d^3*x + I*d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx=\int \frac{(-i\,cfx+f)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}{(i\,cdx+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algorit
hm="maxima")
```

```
[Out] -1/3*(3*I*(c^2*d*f*x^2 + d*f)^(5/2)/(c^5*d^5*x^4 - 4*I*c^4*d^5*x^3 - 6*c^3*
d^5*x^2 + 4*I*c^2*d^5*x + c*d^5) - 15*I*(c^2*d*f*x^2 + d*f)^(3/2)*f/(-3*I*c
^4*d^4*x^3 - 9*c^3*d^4*x^2 + 9*I*c^2*d^4*x + 3*c*d^4) + 10*I*sqrt(c^2*d*f*x
^2 + d*f)*f^2/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3) + 105*I*sqrt(c^2*d*f*x^
2 + d*f)*f^2/(3*I*c^2*d^3*x + 3*c*d^3) - 15*f^3*arcsinh(c*x)/(c*d^3*sqrt(f/
d)))*a + b*integrate((-I*c*f*x + f)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1))/(I*c
*d*x + d)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx=\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}}{\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(5/2),x)
[Out] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(5/2), x)
```

3.552 $\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 381

$$
\begin{aligned}
\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx &= -\frac{11ibd^3x\sqrt{1+c^2x^2}}{3\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{3bcd^3x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{ibc^2d^3x^3\sqrt{1+c^2x^2}}{9\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{11id^3(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{3c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3d^3x(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{icd^3x^2(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{5d^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}
\end{aligned}
$$

```
[Out] 11/3*I*d^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(
1/2)-3/2*d^3*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)
^(1/2)-1/3*I*c*d^3*x^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-
I*c*f*x)^(1/2)-11/3*I*b*d^3*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*
x)^(1/2)+3/4*b*c*d^3*x^2*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1
/2)+1/9*I*b*c^2*d^3*x^3*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/
2)+5/4*d^3*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-
I*c*f*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5796, 5838, 5783, 5798, 8, 5812, 30}

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = \frac{5d^3\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{icd^3x^2(c^2x^2+1)\,(a+b\operatorname{arcsinh}(cx))}{3\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3d^3x(c^2x^2+1)\,(a+b\operatorname{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{11id^3(c^2x^2+1)\,(a+b\operatorname{arcsinh}(cx))}{3c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{3bcd^3x^2\sqrt{c^2x^2+1}}{4\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{11ibd^3x\sqrt{c^2x^2+1}}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{ibc^2d^3x^3\sqrt{c^2x^2+1}}{9\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[In] Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

```
[Out] (((-11*I)/3)*b*d^3*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]) + (3*b*c*d^3*x^2*Sqrt[1 + c^2*x^2])/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]) + ((I/9)*b*c^2*d^3*x^3*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x]) + (((11*I)/3)*d^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]) - (3*d^3*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/
(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - ((I/3)*c*d^3*x^2*(1 + c^2*x^2)*(a
 + b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (5*d^3*Sqrt[1 +
 c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
```

```
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+c^2x^2}\int \frac{(d+icdx)^3(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{d^3(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}+\frac{3icd^3x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}-\frac{3c^2d^3x^2(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}-\frac{ic^3d^3x^3(a+b\text{arcsinh}(}{\sqrt{1+c^2x^2}}\right.}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\left(d^3\sqrt{1+c^2x^2}\right)\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{\left(3icd^3\sqrt{1+c^2x^2}\right)\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$-\frac{\left(3c^2d^3\sqrt{1+c^2x^2}\right)\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{\left(ic^3d^3\sqrt{1+c^2x^2}\right)\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$
\begin{aligned}
&= \frac{3id^3(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3d^3x(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad - \frac{icd^3x^2(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{d^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad + \frac{\left(3d^3\sqrt{1+c^2x^2}\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(3ibd^3\sqrt{1+c^2x^2}\right)\int 1\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad + \frac{\left(2icd^3\sqrt{1+c^2x^2}\right)\int \frac{x(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{3\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad + \frac{\left(3bcd^3\sqrt{1+c^2x^2}\right)\int x\,dx}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(ibc^2d^3\sqrt{1+c^2x^2}\right)\int x^2\,dx}{3\sqrt{d+icdx}\sqrt{f-icfx}} \\
&= -\frac{3ibd^3x\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{3bcd^3x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad + \frac{ibc^2d^3x^3\sqrt{1+c^2x^2}}{9\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{11id^3(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{3c\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad - \frac{3d^3x(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{icd^3x^2(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{3\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad + \frac{5d^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(2ibd^3\sqrt{1+c^2x^2}\right)\int 1\,dx}{3\sqrt{d+icdx}\sqrt{f-icfx}} \\
&= -\frac{11ibd^3x\sqrt{1+c^2x^2}}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{3bcd^3x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{ibc^2d^3x^3\sqrt{1+c^2x^2}}{9\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad + \frac{11id^3(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{3c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3d^3x(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad - \frac{icd^3x^2(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{5d^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 7.34 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.22

$$
\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = \frac{-264ibcd^2x\sqrt{d+icdx}\sqrt{f-icfx}+8ibc^3d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}
$$

```
[In] Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-264*I)*b*c*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (8*I)*b*c^3*d^2*x
^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (264*I)*a*d^2*Sqrt[d + I*c*d*x]*Sq
rt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 108*a*c*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]*Sqrt[1 + c^2*x^2] - (24*I)*a*c^2*d^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 90*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
```

```
x]*ArcSinh[c*x]^2 + 27*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*Arc
Sinh[c*x]] - 6*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(9*(-
5*I + 2*c*x)*Sqrt[1 + c^2*x^2] + I*Cosh[3*ArcSinh[c*x]]) + 180*a*d^(5/2)*Sq
rt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqr
t[f - I*c*f*x]])/(72*c*f*Sqrt[1 + c^2*x^2])
```

Maple [F]

$$\int \frac{(icdx+d)^{\frac{5}{2}} (a+b\,\mathrm{arcsinh}\,(cx))}{\sqrt{-icfx+f}}dx$$

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\mathrm{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = \int \frac{(i\,cdx+d)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)}{\sqrt{-i\,cfx+f}}\,dx$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(((-I*b*c^2*d^2*x^2 - 2*b*c*d^2*x + I*b*d^2)*sqrt(I*c*d*x + d)*sqrt
(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a*c^2*d^2*x^2 - 2*a*c*d^2
*x + I*a*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*f*x + I*f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\mathrm{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = \text{Timed out}$$

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = \int \frac{(a+b\operatorname{asinh}(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}}{\sqrt{f-c\,f\,x\,1\mathrm{i}}}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(1/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(1/2), x)
```

3.553 $\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 266

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=-\frac{2ibd^2x\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{bcd^2x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{2id^2(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{d^2x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{3d^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[Out] 2*I*d^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2
)-1/2*d^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1
/2)-2*I*b*d^2*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/4*b
*c*d^2*x^2*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+3/4*d^2*(a
+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2
)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5796, 5838, 5783, 5798, 8, 5812, 30}

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\frac{3d^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{d^2x(c^2x^2+1)\,(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{2id^2(c^2x^2+1)\,(a+b\text{arcsinh}(cx))}{c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{bcd^2x^2\sqrt{c^2x^2+1}}{4\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{2ibd^2x\sqrt{c^2x^2+1}}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[In] Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-2*I)*b*d^2*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) +
(b*c*d^2*x^2*Sqrt[1 + c^2*x^2])/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (
(2*I)*d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]) - (d^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(2*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]) + (3*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*
b*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
```

```
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1+c^2x^2}\int \frac{(d+icdx)^2(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}} \\
&= \frac{\sqrt{1+c^2x^2}\int \left(\frac{d^2(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}+\frac{2icd^2x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}-\frac{c^2d^2x^2(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}} \\
&= \frac{\left(d^2\sqrt{1+c^2x^2}\right)\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{\left(2icd^2\sqrt{1+c^2x^2}\right)\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad -\frac{\left(c^2d^2\sqrt{1+c^2x^2}\right)\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}} \\
&= \frac{2id^2(1+c^2x^2)(a+b\text{arcsinh}(cx))}{c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{d^2x(1+c^2x^2)(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad +\frac{d^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{\left(d^2\sqrt{1+c^2x^2}\right)\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad -\frac{\left(2ibd^2\sqrt{1+c^2x^2}\right)\int 1\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{\left(bcd^2\sqrt{1+c^2x^2}\right)\int x\,dx}{2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&= -\frac{2ibd^2x\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{bcd^2x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad +\frac{2id^2(1+c^2x^2)(a+b\text{arcsinh}(cx))}{c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{d^2x(1+c^2x^2)(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad +\frac{3d^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 5.55 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.29

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = \frac{-16ibcdx\sqrt{d+icdx}\sqrt{f-icfx}+16iad\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+}}{}$$

[In] Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]

[Out] ((-16*I)*b*c*d*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (16*I)*a*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*a*c*d*x*Sqrt[d + I*c*d*x]* Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*b*d*(-4*I + c*x)*Sqrt[d + I*c*d*x]* Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 6*b*d*Sqrt[d + I*c*d*x]* Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 + b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]* Cosh[2*ArcSinh[c*x]] + 12*a*d^(3/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/(8*c*f*Sqrt[1 + c^2*x^2])

Maple [F]

$$\int \frac{(icdx+d)^{\frac{3}{2}}\,(a+b\,\text{arcsinh}\,(cx))}{\sqrt{-icfx+f}}dx$$

[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x)

[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x)

Fricas [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = \int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{\sqrt{-i\,cfx+f}}\,dx$$

[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algorithm="fricas")

[Out] integral(-((b*c*d*x - I*b*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*c*d*x - I*a*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*f*x + I*f), x)

Sympy [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\int \frac{(id(cx-i))^{\frac{3}{2}}\,(a+b\operatorname{asinh}{(cx)})}{\sqrt{-if\,(cx+i)}}\,dx$$

[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(1/2),x)

[Out] Integral((I*d*(c*x - I))**(3/2)*(a + b*asinh(c*x))/sqrt(-I*f*(c*x + I)), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\text{Exception raised: RuntimeError}$$

[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.

Giac [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\operatorname{arsinh}{(cx)}+a)}{\sqrt{-i\,cfx+f}}\,dx$$

[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algorit
hm="giac")

[Out] integrate((I*c*d*x + d)^(3/2)*(b*arcsinh(c*x) + a)/sqrt(-I*c*f*x + f), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\int \frac{(a+b\operatorname{asinh}(c\,x))\,{(d+c\,d\,x\,1\mathrm{i})}^{3/2}}{\sqrt{f-c\,f\,x\,1\mathrm{i}}}\,dx$$

[In] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(1/2),x)

[Out] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(1/2), x)

3.554 $\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 158

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = -\frac{ibdx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{id(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{d\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[Out] I*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-I*
b*d*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/2*d*(a+b*arcs
inh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5796, 5838, 5783, 5798, 8}

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = \frac{d\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{id(c^2x^2+1)\,(a+b\text{arcsinh}(cx))}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{ibdx\sqrt{c^2x^2+1}}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[In] Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-I)*b*d*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (I*d
*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
) + (d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+c^2x^2}\int \frac{(d+icdx)(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\int\left(\frac{d(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}+\frac{icdx(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\left(d\sqrt{1+c^2x^2}\right)\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{\left(icd\sqrt{1+c^2x^2}\right)\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{id(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{d\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{\left(ibd\sqrt{1+c^2x^2}\right)\int 1\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= -\frac{ibdx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{id(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{d\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.44

$$\int\frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}dx$$

$$= \frac{-2i\sqrt{d+icdx}\sqrt{f-icfx}\left(bcx-a\sqrt{1+c^2x^2}\right)+2ib\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}\text{arcsinh}(cx)+b\sqrt{d+\cdots}}{2cf\sqrt{1+c^2x^2}\cdots}$$

```
[In] Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-2*I)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(b*c*x - a*Sqrt[1 + c^2*x^2]) +
 (2*I)*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
 + b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 + 2*a*Sqrt[d]*Sqrt[
f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]])/(2*c*f*Sqrt[1 + c^2*x^2])
```

Maple [F]

$$\int\frac{(a+b\,\text{arcsinh}\,(cx))\,\sqrt{icdx+d}}{\sqrt{-icfx+f}}dx$$

```
[In] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\int \frac{\sqrt{i\,cdx+d}(b\,\text{arsinh}\,(cx)+a)}{\sqrt{-i\,cfx+f}}\,dx$$

[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algorithm="fricas")

[Out] integral((I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 + 1)) + I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c*f*x + I*f), x)

Sympy [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\int \frac{\sqrt{id\,(cx-i)}(a+b\,\text{asinh}\,(cx))}{\sqrt{-if\,(cx+i)}}\,dx$$

[In] integrate((a+b*asinh(c*x))*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(1/2),x)

[Out] Integral(sqrt(I*d*(c*x - I))*(a + b*asinh(c*x))/sqrt(-I*f*(c*x + I)), x)

Maxima [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\int \frac{\sqrt{i\,cdx+d}(b\,\text{arsinh}\,(cx)+a)}{\sqrt{-i\,cfx+f}}\,dx$$

[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algorithm="maxima")

[Out] a*(d*arcsinh(c*x)/(c*f*sqrt(d/f)) + I*sqrt(c^2*d*f*x^2 + d*f)/(c*f)) + b*integrate(sqrt(I*c*d*x + d)*log(c*x + sqrt(c^2*x^2 + 1))/sqrt(-I*c*f*x + f), x)

Giac [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\int \frac{\sqrt{i\,cdx+d}(b\,\text{arsinh}\,(cx)+a)}{\sqrt{-i\,cfx+f}}\,dx$$

[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)/sqrt(-I*c*f*x + f), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))\,\sqrt{d+c\,d\,x\,1\text{i}}}{\sqrt{f-c\,f\,x\,1\text{i}}}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(1/2),x)
[Out] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(1/2), x)
```

3.555 $\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}} dx$

Optimal result

Integrand size = 35, antiderivative size = 59

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}} dx = \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x
)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {5796, 5783}

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}} dx = \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+c^2x^2}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$
$$= \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.92

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx = \frac{b\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2}{2c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{a\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{c\sqrt{d}\sqrt{f}}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]),x]
[Out] (b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/(2*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]) + (a*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]]
)/(c*Sqrt[d]*Sqrt[f])
```

Maple [F]

$$\int \frac{a+b\,\text{arcsinh}\,(cx)}{\sqrt{icdx+d}\,\sqrt{-icfx+f}}dx$$

```
[In] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
[Out] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{d + icdx}\sqrt{f - icfx}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{\sqrt{i\,cdx + d}\sqrt{-i\,cfx + f}}\,dx$$

[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algorithm="fricas")

[Out] integral((sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c^2*d*f*x^2 + d*f), x)

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{d + icdx}\sqrt{f - icfx}}\,dx = \int \frac{a + b\,\text{asinh}\,(cx)}{\sqrt{id\,(cx - i)}\sqrt{-if\,(cx + i)}}\,dx$$

[In] integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(1/2),x)

[Out] Integral((a + b*asinh(c*x))/(sqrt(I*d*(c*x - I))*sqrt(-I*f*(c*x + I))), x)

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.54

$$\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{d + icdx}\sqrt{f - icfx}}\,dx = \frac{b\,\text{arsinh}\,(cx)^2}{2\,\sqrt{df}c} + \frac{a\,\text{arsinh}\,(cx)}{\sqrt{df}c}$$

[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algorithm="maxima")

[Out] 1/2*b*arcsinh(c*x)^2/(sqrt(d*f)*c) + a*arcsinh(c*x)/(sqrt(d*f)*c)

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{d + icdx}\sqrt{f - icfx}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{\sqrt{i\,cdx + d}\sqrt{-i\,cfx + f}}\,dx$$

[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx=\int \frac{a+b\,\mathrm{asinh}(c\,x)}{\sqrt{d+c\,d\,x\,1\mathrm{i}}\,\sqrt{f-c\,f\,x\,1\mathrm{i}}}\,dx$$

```
[In] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2)),x)
[Out] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2)), x)
```

3.556 $\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 111

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx=\frac{f(i+cx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{bf\left(1+c^2x^2\right)^{3/2}\log(i-cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] f*(I+c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3
/2)-b*f*(c^2*x^2+1)^(3/2)*ln(I-c*x)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {5796, 651, 5837, 12, 641, 31}

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx=\frac{f(cx+i)\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{bf\left(c^2x^2+1\right)^{3/2}\log(-cx+i)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (f*(I + c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(f
- I*c*f*x)^(3/2)) - (b*f*(1 + c^2*x^2)^(3/2)*Log[I - c*x])/(c*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\text{integral} = \frac{(1+c^2x^2)^{3/2}\int\frac{(f-icfx)(a+b\operatorname{arcsinh}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{f(i+cx)\,(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(bc(1+c^2x^2)^{3/2}\right)\int\frac{f(i+cx)}{c(1+c^2x^2)}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{f(i+cx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(bf\left(1+c^2x^2\right)^{3/2}\right)\int \frac{i+cx}{1+c^2x^2}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{f(i+cx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(bf\left(1+c^2x^2\right)^{3/2}\right)\int \frac{1}{-i+cx}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{f(i+cx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{bf\left(1+c^2x^2\right)^{3/2}\log(i-cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}\sqrt{f-icfx}}dx = \frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(a\sqrt{1+c^2x^2}+b\sqrt{1+c^2x^2}\text{arcsinh}(cx)+b(i-cx)\log(\right.}{cd^2f(-i+cx)\sqrt{1+c^2x^2}}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a*Sqrt[1 + c^2*x^2] + b*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x] + b*(I - c*x)*Log[d + I*c*d*x]))/(c*d^2*f*(-I + c*x)*Sqrt
[1 + c^2*x^2])
```

Maple [F]

$$\int \frac{a+b\,\text{arcsinh}\,(cx)}{(icdx+d)^{\frac{3}{2}}\sqrt{-icfx+f}}dx$$

```
[In] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 443 vs. 2(89) = 178.

Time = 0.31 (sec) , antiderivative size = 443, normalized size of antiderivative = 3.99

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}\sqrt{f-icfx}}dx = \frac{2\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}b\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(c^2d^2fx-i\,cd^2f\right)\sqrt{\frac{b^2}{c^2d^3f}}}{}$$

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] 1/2*(2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 + 1))
+ (c^2*d^2*f*x - I*c*d^2*f)*sqrt(b^2/(c^2*d^3*f))*log(-1/8*((I*b*c^6*x^2 +
2*b*c^5*x - 2*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
f) - (I*c^9*d^2*f*x^4 + 2*c^8*d^2*f*x^3 + I*c^7*d^2*f*x^2 + 2*c^6*d^2*f*x)*
sqrt(b^2/(c^2*d^3*f)))/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) - (c^2*d^2*
f*x - I*c*d^2*f)*sqrt(b^2/(c^2*d^3*f))*log(-1/8*((I*b*c^6*x^2 + 2*b*c^5*x -
 2*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) - (-I*c^
9*d^2*f*x^4 - 2*c^8*d^2*f*x^3 - I*c^7*d^2*f*x^2 - 2*c^6*d^2*f*x)*sqrt(b^2/(
c^2*d^3*f)))/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) + 2*sqrt(I*c*d*x + d)
*sqrt(-I*c*f*x + f)*a)/(c^2*d^2*f*x - I*c*d^2*f)
```

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + icdx)^{3/2}\sqrt{f - icfx}}\,dx = \int \frac{a + b\operatorname{asinh}(cx)}{(id(cx - i))^{\frac{3}{2}}\sqrt{-if(cx + i)}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/((I*d*(c*x - I))**(3/2)*sqrt(-I*f*(c*x + I))),
x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.88

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + icdx)^{3/2}\sqrt{f - icfx}}\,dx = \frac{i\sqrt{c^2dfx^2 + df}b\operatorname{arsinh}(cx)}{i c^2d^2fx + cd^2f} + \frac{i\sqrt{c^2dfx^2 + df}a}{i c^2d^2fx + cd^2f} - \frac{b\log(icx + 1)}{cd^{\frac{3}{2}}\sqrt{f}}$$

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] I*sqrt(c^2*d*f*x^2 + d*f)*b*arcsinh(c*x)/(I*c^2*d^2*f*x + c*d^2*f) + I*sqrt
(c^2*d*f*x^2 + d*f)*a/(I*c^2*d^2*f*x + c*d^2*f) - b*log(I*c*x + 1)/(c*d^(3/
2)*sqrt(f))
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + icdx)^{3/2}\sqrt{f - icfx}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{(i\,cdx + d)^{\frac{3}{2}}\sqrt{-i\,cfx + f}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((I*c*d*x + d)^(3/2)*sqrt(-I*c*f*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + icdx)^{3/2}\sqrt{f - icfx}}\,dx = \int \frac{a + b\,\text{asinh}(c\,x)}{(d + c\,d\,x\,1\text{i})^{3/2}\,\sqrt{f - c\,f\,x\,1\text{i}}}\,dx$$

```
[In] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2)), x)
```

3.557 $\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 295

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \frac{ibf^2(1+c^2x^2)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^2(1-icx)\,(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2x(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{ibf^2(1+c^2x^2)^{5/2}\arctan(cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bf^2(1+c^2x^2)^{5/2}\log(1+c^2x^2)}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[Out] 1/3*I*b*f^2*(c^2*x^2+1)^(5/2)/c/(I-c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
+2/3*I*f^2*(1-I*c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-
I*c*f*x)^(5/2)+1/3*f^2*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)
/(f-I*c*f*x)^(5/2)-1/3*I*b*f^2*(c^2*x^2+1)^(5/2)*arctan(c*x)/c/(d+I*c*d*x)^
(5/2)/(f-I*c*f*x)^(5/2)-1/6*b*f^2*(c^2*x^2+1)^(5/2)*ln(c^2*x^2+1)/c/(d+I*c*
d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5796, 667, 197, 5837, 641, 46, 209, 266}

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \frac{f^2x(c^2x^2+1)^2\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^2(1-icx)\,(c^2x^2+1)\,(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{ibf^2(c^2x^2+1)^{5/2}\arctan(cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{ibf^2(c^2x^2+1)^{5/2}}{3c(-cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bf^2(c^2x^2+1)^{5/2}\log(c^2x^2+1)}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] ((I/3)*b*f^2*(1 + c^2*x^2)^(5/2))/(c*(I - c*x)*(d + I*c*d*x)^(5/2)*(f - I*c
*f*x)^(5/2)) + (((2*I)/3)*f^2*(1 - I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (f^2*x*(1 + c^2*x^2)^2*(a
+ b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((I/3)*b*f
^2*(1 + c^2*x^2)^(5/2)*ArcTan[c*x])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) - (b*f^2*(1 + c^2*x^2)^(5/2)*Log[1 + c^2*x^2])/(6*c*(d + I*c*d*x)^(5/2
)*(f - I*c*f*x)^(5/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(f-icfx)^2(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&= \frac{2if^2(1-icx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{f^2x\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad -\frac{\left(bc\left(1+c^2x^2\right)^{5/2}\right)\int\left(\frac{2if^2(1-icx)}{3c(1+c^2x^2)^2}+\frac{f^2x}{3(1+c^2x^2)}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&= \frac{2if^2(1-icx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{f^2x\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad -\frac{\left(2ibf^2\left(1+c^2x^2\right)^{5/2}\right)\int\frac{1-icx}{(1+c^2x^2)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{\left(bcf^2\left(1+c^2x^2\right)^{5/2}\right)\int\frac{x}{1+c^2x^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&= \frac{2if^2(1-icx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{f^2x\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad -\frac{bf^2\left(1+c^2x^2\right)^{5/2}\log\left(1+c^2x^2\right)}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{\left(2ibf^2\left(1+c^2x^2\right)^{5/2}\right)\int\frac{1}{(1-icx)(1+icx)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&= \frac{2if^2(1-icx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{f^2x\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad -\frac{bf^2\left(1+c^2x^2\right)^{5/2}\log\left(1+c^2x^2\right)}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{\left(2ibf^2\left(1+c^2x^2\right)^{5/2}\right)\int\left(-\frac{1}{2(-i+cx)^2}+\frac{1}{2(1+c^2x^2)}\right)dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

$$= \frac{ibf^2(1+c^2x^2)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^2(1-icx)\,(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2x(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bf^2(1+c^2x^2)^{5/2}\log(1+c^2x^2)}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(ibf^2(1+c^2x^2)^{5/2}\right)\int\frac{1}{1+c^2x^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{ibf^2(1+c^2x^2)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^2(1-icx)\,(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2x(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{ibf^2(1+c^2x^2)^{5/2}\arctan(cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bf^2(1+c^2x^2)^{5/2}\log(1+c^2x^2)}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.48

$$\int\frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \frac{\sqrt{d+icdx}\sqrt{f-icfx}\left((-2i+cx)\left(-ib+bcx+a\sqrt{1+c^2x^2}\right)+b(-2i+cx)\right.}{3cd^3f(-i+cx)^2\sqrt{1+c^2x}}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((-2*I + c*x)*((-I)*b + b*c*x + a*Sqrt
[1 + c^2*x^2]) + b*(-2*I + c*x)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - b*(-I + c*
x)^2*Log[d + I*c*d*x]))/(3*c*d^3*f*(-I + c*x)^2*Sqrt[1 + c^2*x^2])
```

Maple [F]

$$\int\frac{a+b\,\text{arcsinh}\,(cx)}{(icdx+d)^{\frac{5}{2}}\sqrt{-icfx+f}}dx$$

```
[In] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 576 vs. 2(228) = 456.

Time = 0.34 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.95

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx =$$

$$-\frac{2\sqrt{c^2x^2+1}\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}bcx-2\left(bc^2x^2-i\,bcx+2\,b\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2}\right.}{}$$

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] -1/6*(2*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x - 2*(b
*c^2*x^2 - I*b*c*x + 2*b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sq
rt(c^2*x^2 + 1)) + (c^4*d^3*f*x^3 - I*c^3*d^3*f*x^2 + c^2*d^3*f*x - I*c*d^3
*f)*sqrt(b^2/(c^2*d^5*f))*log(-1/8*((I*b*c^6*x^2 + 2*b*c^5*x - 2*I*b*c^4)*s
qrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + (I*c^9*d^3*f*x^4 +
2*c^8*d^3*f*x^3 + I*c^7*d^3*f*x^2 + 2*c^6*d^3*f*x)*sqrt(b^2/(c^2*d^5*f)))/(
b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) - (c^4*d^3*f*x^3 - I*c^3*d^3*f*x^2
+ c^2*d^3*f*x - I*c*d^3*f)*sqrt(b^2/(c^2*d^5*f))*log(-1/8*((I*b*c^6*x^2 + 2
*b*c^5*x - 2*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f
) + (-I*c^9*d^3*f*x^4 - 2*c^8*d^3*f*x^3 - I*c^7*d^3*f*x^2 - 2*c^6*d^3*f*x)*
sqrt(b^2/(c^2*d^5*f)))/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) - 2*(a*c^2*
x^2 - I*a*c*x + 2*a)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^4*d^3*f*x^3 -
 I*c^3*d^3*f*x^2 + c^2*d^3*f*x - I*c*d^3*f)
```

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \int \frac{a+b\,\text{asinh}\,(cx)}{(id\,(cx-i))^{\frac{5}{2}}\sqrt{-if\,(cx+i)}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/((I*d*(c*x - I))**(5/2)*sqrt(-I*f*(c*x + I))),
x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.79

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \frac{1}{3}\,bc\left(\frac{3}{3i\,c^3d^{\frac{5}{2}}\sqrt{f}x+3\,c^2d^{\frac{5}{2}}\sqrt{f}}-\frac{\log\left(cx-i\right)}{c^2d^{\frac{5}{2}}\sqrt{f}}\right) - \frac{1}{3}\,b\left(\frac{i\,\sqrt{c^2dfx^2+df}}{c^3d^3\,fx^2-2i\,c^2d^3\,fx-cd^3\,f}-\frac{3i\,\sqrt{c^2dfx^2+df}}{3i\,c^2d^3\,fx+3\,cd^3\,f}\right)\text{arsinh}\left(cx\right) - \frac{1}{3}\,a\left(\frac{i\,\sqrt{c^2dfx^2+df}}{c^3d^3\,fx^2-2i\,c^2d^3\,fx-cd^3\,f}-\frac{3i\,\sqrt{c^2dfx^2+df}}{3i\,c^2d^3\,fx+3\,cd^3\,f}\right)$$

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/3*b*c*(3/(3*I*c^3*d^(5/2)*sqrt(f)*x + 3*c^2*d^(5/2)*sqrt(f)) - log(c*x -
I)/(c^2*d^(5/2)*sqrt(f))) - 1/3*b*(I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*d^3*f*x^2
 - 2*I*c^2*d^3*f*x - c*d^3*f) - 3*I*sqrt(c^2*d*f*x^2 + d*f)/(3*I*c^2*d^3*f*
x + 3*c*d^3*f))*arcsinh(c*x) - 1/3*a*(I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*d^3*f*
x^2 - 2*I*c^2*d^3*f*x - c*d^3*f) - 3*I*sqrt(c^2*d*f*x^2 + d*f)/(3*I*c^2*d^3
*f*x + 3*c*d^3*f))
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \int \frac{b\,\text{arsinh}\left(cx\right)+a}{\left(i\,cdx+d\right)^{\frac{5}{2}}\sqrt{-i\,cfx+f}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((I*c*d*x + d)^(5/2)*sqrt(-I*c*f*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \int \frac{a+b\,\mathrm{asinh}(c\,x)}{{\left(d+c\,d\,x\,1\mathrm{i}\right)}^{5/2}\,\sqrt{f-c\,f\,x\,1\mathrm{i}}}\,dx$$

```
[In] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2)), x)
```

3.558 $\int \frac{(d+icdx)^{5/2}(a+b\,\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 517

$$\int \frac{(d+icdx)^{5/2}(a+b\,\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \frac{3ibd^4x(1+c^2x^2)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{bcd^4x^2(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bd^4(1+icx)^2\,(1+c^2x^2)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{15bd^4(1+c^2x^2)^{3/2}\,\text{arcsinh}(cx)^2}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2id^4(1+icx)^3\,(1+c^2x^2)\,(a+b\,\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{15id^4(1+c^2x^2)^2\,(a+b\,\text{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{5id^4(1+icx)\,(1+c^2x^2)^2\,(a+b\,\text{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{15d^4(1+c^2x^2)^{3/2}\,\text{arcsinh}(cx)(a+b\,\text{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8bd^4(1+c^2x^2)^{3/2}\log(i+cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] 3/2*I*b*d^4*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+b*c*d^4
*x^2*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+5/4*b*d^4*(1+I*c
*x)^2*(c^2*x^2+1)^(3/2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+15/4*b*d^4*(c
^2*x^2+1)^(3/2)*arcsinh(c*x)^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*I*d^
4*(1+I*c*x)^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x
)^(3/2)-15/2*I*d^4*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-
I*c*f*x)^(3/2)-5/2*I*d^4*(1+I*c*x)*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/(d+I*
c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-15/2*d^4*(c^2*x^2+1)^(3/2)*arcsinh(c*x)*(a+b
*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*b*d^4*(c^2*x^2+1)^(3
/2)*ln(I+c*x)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {5796, 683, 685, 655, 221, 5837, 641, 45, 5783}

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = -\frac{5id^4(1+icx)\left(c^2x^2+1\right)^2(a+b\operatorname{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{15id^4\left(c^2x^2+1\right)^2(a+b\operatorname{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2id^4(1+icx)^3\left(c^2x^2+1\right)(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{15d^4\left(c^2x^2+1\right)^{3/2}\operatorname{arcsinh}(cx)(a+b\operatorname{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{15bd^4\left(c^2x^2+1\right)^{3/2}\operatorname{arcsinh}(cx)^2}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{bcd^4x^2\left(c^2x^2+1\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bd^4(1+icx)^2\left(c^2x^2+1\right)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{3ibd^4x\left(c^2x^2+1\right)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8bd^4\left(c^2x^2+1\right)^{3/2}\log(cx+i)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[In] Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

```
[Out] (((3*I)/2)*b*d^4*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^
(3/2)) + (b*c*d^4*x^2*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*
x)^(3/2)) + (5*b*d^4*(1 + I*c*x)^2*(1 + c^2*x^2)^(3/2))/(4*c*(d + I*c*d*x)^
(3/2)*(f - I*c*f*x)^(3/2)) + (15*b*d^4*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]^2)/
(4*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((2*I)*d^4*(1 + I*c*x)^3*(1
 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2
)) - (((15*I)/2)*d^4*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2)) - (((5*I)/2)*d^4*(1 + I*c*x)*(1 + c^2*x^2)^2*(a
 + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (15*d^4*(
1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*(a + b*ArcSinh[c*x]))/(2*c*(d + I*c*d*x)^(3
/2)*(f - I*c*f*x)^(3/2)) - (8*b*d^4*(1 + c^2*x^2)^(3/2)*Log[I + c*x])/(c*(d
 + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 683

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((m +
p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
```

```
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(1+c^2x^2)^{3/2}\int \frac{(d+icdx)^4(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&= -\frac{2id^4(1+icx)^3\,(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad -\frac{15id^4(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad -\frac{5id^4(1+icx)\,(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad -\frac{15d^4(1+c^2x^2)^{3/2}\,\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad -\frac{\left(bc(1+c^2x^2)^{3/2}\right)\int\left(-\frac{15id^4}{2c}-\frac{5id^4(1+icx)}{2c}-\frac{2id^4(1+icx)^3}{c(1+c^2x^2)}-\frac{15d^4\text{arcsinh}(cx)}{2c\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&= \frac{15ibd^4x(1+c^2x^2)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{5bd^4(1+icx)^2\,(1+c^2x^2)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad -\frac{2id^4(1+icx)^3\,(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad -\frac{15id^4(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad -\frac{5id^4(1+icx)\,(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad -\frac{15d^4(1+c^2x^2)^{3/2}\,\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad +\frac{\left(2ibd^4(1+c^2x^2)^{3/2}\right)\int\frac{(1+icx)^3}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{\left(15bd^4(1+c^2x^2)^{3/2}\right)\int\frac{\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}$$

$$= \frac{15ibd^4x(1+c^2x^2)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bd^4(1+icx)^2(1+c^2x^2)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{15bd^4(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)^2}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2id^4(1+icx)^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{15id^4(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{5id^4(1+icx)(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{15d^4(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)(a+b\operatorname{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(2ibd^4(1+c^2x^2)^{3/2}\right)\int \frac{(1+icx)^2}{1-icx}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{15ibd^4x(1+c^2x^2)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bd^4(1+icx)^2(1+c^2x^2)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{15bd^4(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)^2}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2id^4(1+icx)^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{15id^4(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{5id^4(1+icx)(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{15d^4(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)(a+b\operatorname{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(2ibd^4(1+c^2x^2)^{3/2}\right)\int \left(-3-icx+\frac{4}{1-icx}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{3ibd^4x(1+c^2x^2)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{bcd^4x^2(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bd^4(1+icx)^2(1+c^2x^2)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{15bd^4(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)^2}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2id^4(1+icx)^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{15id^4(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{5id^4(1+icx)(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{15d^4(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)(a+b\operatorname{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8bd^4(1+c^2x^2)^{3/2}\log(i+cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Mathematica [A] (verified)

Time = 10.37 (sec) , antiderivative size = 781, normalized size of antiderivative = 1.51

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \frac{\frac{4ad^2\sqrt{d+icdx}\sqrt{f-icfx}(24-7icx+c^2x^2)}{f^2(i+cx)} - \frac{60ad^{5/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{f^{3/2}}}{}$$

[In] Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]

[Out] ((4*a*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(24 - (7*I)*c*x + c^2*x^2))/(f^2*(I + c*x)) - (60*a*d^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/f^(3/2) + (4*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + 4*ArcSinh[c*x]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2]) + 2*(4*ArcTan[Tanh[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])))/(f^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + (16*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + (c*x - 4*ArcTan[Coth[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]*((-I)*(2 + Sqrt[1 + c^2*x^2])*Cosh[ArcSinh[c*x]/2] - (-2 + Sqrt[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2])))/(f^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + (b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-10*ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]) + (16*c*x + 32*ArcTan[Tanh[ArcSinh[c*x]/2]] + I*Cosh[2*ArcSinh[c*x]] + (8*I)*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2]) + 2*ArcSinh[c*x]*(Sinh[ArcSinh[c*x]/2]*(8 - 8*Sqrt[1 + c^2*x^2] - I*Sinh[2*ArcSinh[c*x]]) + Cosh[ArcSinh[c*x]/2]*((-8*I)*(1 + Sqrt[1 + c^2*x^2]) + Sinh[2*ArcSinh[c*x]]))))/(f^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(8*c)

Maple [F]

$$\int \frac{(icdx+d)^{\frac{5}{2}}\,(a+b\,\text{arcsinh}\,(cx))}{(-icfx+f)^{\frac{3}{2}}}dx$$

[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x)

[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x)

Fricas [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \int \frac{(i\,cdx+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algorithm="fricas")

[Out] integral(((b*c^2*d^2*x^2 - 2*I*b*c*d^2*x - b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*c^2*d^2*x^2 - 2*I*a*c*d^2*x - a*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^2*f^2*x^2 + 2*I*c*f^2*x - f^2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \text{Timed out}$$

[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(3/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \int \frac{(i\,cdx+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algorithm="maxima")

[Out] 1/2*(c^2*d^3*x^3/(sqrt(c^2*d*f*x^2 + d*f)*f) - 8*I*c*d^3*x^2/(sqrt(c^2*d*f*x^2 + d*f)*f) + 17*d^3*x/(sqrt(c^2*d*f*x^2 + d*f)*f) - 15*d^3*arcsinh(c*x)/(sqrt(d*f)*c*f) - 24*I*d^3/(sqrt(c^2*d*f*x^2 + d*f)*c*f))*a + b*integrate((I*c*d*x + d)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1))/(-I*c*f*x + f)^(3/2), x)

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\left(d+c\,d\,x\,1\text{i}\right)^{5/2}}{\left(f-c\,f\,x\,1\text{i}\right)^{3/2}}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(3/2), x)
```

3.559 $\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 283

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \frac{ibd^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4id^3(1+icx)\,(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{id^3(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{3d^3(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4bd^3(1+c^2x^2)^{3/2}\log(i+cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] I*b*d^3*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*I*d^3*(1+
I*c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
-I*d^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/
2)-3/2*d^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(d+I*c*d*x)^(3/2)/(f-
I*c*f*x)^(3/2)-4*b*d^3*(c^2*x^2+1)^(3/2)*ln(I+c*x)/c/(d+I*c*d*x)^(3/2)/(f-I
*c*f*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5796, 5844, 651, 5837, 12, 641, 31, 5783, 5798, 8}

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = -\frac{3d^3(c^2x^2+1)^{3/2}\,(a+b\text{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{id^3(c^2x^2+1)^2\,(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4id^3(1+icx)\,(c^2x^2+1)\,(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{ibd^3x(c^2x^2+1)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4bd^3(c^2x^2+1)^{3/2}\log(cx+i)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[In] Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

```
[Out] (I*b*d^3*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) -
 ((4*I)*d^3*(1 + I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) - (I*d^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))
/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (3*d^3*(1 + c^2*x^2)^(3/2)*(
a + b*ArcSinh[c*x])^2)/(2*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4
*b*d^3*(1 + c^2*x^2)^(3/2)*Log[I + c*x])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*
x)^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
```

```
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(1+c^2x^2)^{3/2}\int\frac{(d+icdx)^3(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= \frac{(1+c^2x^2)^{3/2}\int\left(-\frac{4i(id^3-cd^3x)(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{3/2}}-\frac{3d^3(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}-\frac{icd^3x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= -\frac{\left(4i(1+c^2x^2)^{3/2}\right)\int\frac{(id^3-cd^3x)(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&\quad-\frac{\left(3d^3(1+c^2x^2)^{3/2}\right)\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&\quad-\frac{\left(icd^3(1+c^2x^2)^{3/2}\right)\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}$$

$$= -\frac{4id^3(1+icx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{id^3\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{3d^3\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(4ibc\left(1+c^2x^2\right)^{3/2}\right)\int\frac{d^3(1+icx)}{c(1+c^2x^2)}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(ibd^3\left(1+c^2x^2\right)^{3/2}\right)\int 1\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{ibd^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4id^3(1+icx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{id^3\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{3d^3\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(4ibd^3\left(1+c^2x^2\right)^{3/2}\right)\int\frac{1+icx}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{ibd^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4id^3(1+icx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{id^3\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{3d^3\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(4ibd^3\left(1+c^2x^2\right)^{3/2}\right)\int\frac{1}{1-icx}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{ibd^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4id^3(1+icx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{id^3\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{3d^3\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4bd^3\left(1+c^2x^2\right)^{3/2}\log(i+cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Mathematica [A] (verified)

Time = 7.29 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.82

$$\int\frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \frac{\frac{2ad(5-icx)\sqrt{d+icdx}\sqrt{f-icfx}}{f^2(i+cx)} - \frac{6ad^{3/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{f^{3/2}} + \frac{bd\sqrt{d+ic}}{}}{}$$

```
[In] Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

```
[Out] ((2*a*d*(5 - I*c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(f^2*(I + c*x)) -
(6*a*d^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f
*x]])/f^(3/2) + (b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]^2*
```

```
(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + 4*ArcSinh[c*x]*((-I)*Cos
h[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2]) + 2*(4*ArcTan[Tanh[ArcSinh[c*x]/2
]] + I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])))/
(f^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + (
2*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c
*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + (c*x - 4*ArcTan[Coth[ArcSinh[c*x]/2]] +
 I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2]) + ArcS
inh[c*x]*((-I)*(2 + Sqrt[1 + c^2*x^2])*Cosh[ArcSinh[c*x]/2] - (-2 + Sqrt[1
+ c^2*x^2])*Sinh[ArcSinh[c*x]/2])))/(f^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*
x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(2*c)
```

Maple [F]

$$\int \frac{(icdx + d)^{\frac{3}{2}} (a + b \operatorname{arcsinh}(cx))}{(-icfx + f)^{\frac{3}{2}}} dx$$

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x)
```

Fricas [F]

$$\int \frac{(d + icdx)^{3/2}(a + b\text{arcsinh}(cx))}{(f - icfx)^{3/2}} dx = \int \frac{(i\,cdx + d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx) + a)}{(-i\,cfx + f)^{\frac{3}{2}}} dx$$

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(((-I*b*c*d*x - b*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) + (-I*a*c*d*x - a*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
f))/(c^2*f^2*x^2 + 2*I*c*f^2*x - f^2), x)
```

Sympy [F]

$$\int \frac{(d + icdx)^{3/2}(a + b\text{arcsinh}(cx))}{(f - icfx)^{3/2}} dx = \int \frac{(id(cx - i))^{\frac{3}{2}} (a + b \operatorname{asinh}(cx))}{(-if(cx + i))^{\frac{3}{2}}} dx$$

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Integral((I*d*(c*x - I))**(3/2)*(a + b*asinh(c*x))/(-I*f*(c*x + I))**(3/2),
 x)
```

Maxima [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algorithm="maxima")

[Out] a*(-I*(c^2*d*f*x^2 + d*f)^(3/2)/(c^3*f^3*x^2 + 2*I*c^2*f^3*x - c*f^3) - 6*I*sqrt(c^2*d*f*x^2 + d*f)*d/(-I*c^2*f^2*x + c*f^2) - 3*d^2*arcsinh(c*x)/(c*f^2*sqrt(d/f))) + b*integrate((I*c*d*x + d)^(3/2)*log(c*x + sqrt(c^2*x^2 + 1))/(-I*c*f*x + f)^(3/2), x)

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to transpose Error: Bad Argument Valuesym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error:

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \int \frac{(a+b\operatorname{asinh}(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{3/2}}{\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}}\,dx$$

[In] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(3/2),x)

[Out] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(3/2), x)

3.560 $\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 180

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = -\frac{2id^2(1+icx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^2\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2bd^2\left(1+c^2x^2\right)^{3/2}\log(i+cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] -2*I*d^2*(1+I*c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*
c*f*x)^(3/2)-1/2*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(d+I*c*d*x)
^(3/2)/(f-I*c*f*x)^(3/2)-2*b*d^2*(c^2*x^2+1)^(3/2)*ln(I+c*x)/c/(d+I*c*d*x)^
(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5796, 5844, 651, 5837, 12, 641, 31, 5783}

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = -\frac{d^2\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2id^2(1+icx)\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2bd^2\left(c^2x^2+1\right)^{3/2}\log(cx+i)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[In] Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

```
[Out] ((-2*I)*d^2*(1 + I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) - (d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]
```

```
)^2)/(2*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (2*b*d^2*(1 + c^2*x^
2)^(3/2)*Log[I + c*x])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
```

```
3])
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{(1+c^2x^2)^{3/2}\int \frac{(d+icdx)^2(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{(1+c^2x^2)^{3/2}\int \left(-\frac{2i(id^2-cd^2x)(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{3/2}}-\frac{d^2(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(2i(1+c^2x^2)^{3/2}\right)\int \frac{(id^2-cd^2x)(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{\left(d^2(1+c^2x^2)^{3/2}\right)\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2id^2(1+icx)(1+c^2x^2)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{\left(2ibc(1+c^2x^2)^{3/2}\right)\int \frac{d^2(1+icx)}{c(1+c^2x^2)}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2id^2(1+icx)(1+c^2x^2)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{\left(2ibd^2(1+c^2x^2)^{3/2}\right)\int \frac{1+icx}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2id^2(1+icx)(1+c^2x^2)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{\left(2ibd^2(1+c^2x^2)^{3/2}\right)\int \frac{1}{1-icx}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2id^2(1+icx)(1+c^2x^2)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2bd^2(1+c^2x^2)^{3/2}\log(i+cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Mathematica [A] (verified)

Time = 2.71 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \frac{\frac{4a\sqrt{d+icdx}\sqrt{f-icfx}}{i+cx} - 2a\sqrt{d}\sqrt{f}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{}$$

[In] Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]

[Out] ((4*a*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(I + c*x) - 2*a*Sqrt[d]*Sqrt[f]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + (b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + 4*ArcSinh[c*x]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2]) + 2*(4*ArcTan[Tanh[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(2*c*f^2)

Maple [F]

$$\int \frac{(a+b\,\text{arcsinh}\,(cx))\,\sqrt{icdx+d}}{(-icfx+f)^{\frac{3}{2}}}dx$$

[In] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x)

[Out] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x)

Fricas [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \int \frac{\sqrt{i\,cdx+d}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algorithm="fricas")

[Out] integral(-(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c^2*f^2*x^2 + 2*I*c*f^2*x - f^2), x)

Sympy [F]

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \int \frac{\sqrt{id\,(cx-i)}(a+b\operatorname{asinh}(cx))}{(-if\,(cx+i))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(3/2),x)
[Out] Integral(sqrt(I*d*(c*x - I))*(a + b*asinh(c*x))/(-I*f*(c*x + I))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \int \frac{\sqrt{i\,cdx+d}(b\operatorname{arsinh}(cx)+a)}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="maxima")
[Out] a*(-2*I*sqrt(c^2*d*f*x^2 + d*f)/(-I*c^2*f^2*x + c*f^2) - d*arcsinh(c*x)/(c*
f^2*sqrt(d/f))) + b*integrate(sqrt(I*c*d*x + d)*log(c*x + sqrt(c^2*x^2 + 1)
)/(-I*c*f*x + f)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \int \frac{\sqrt{i\,cdx+d}(b\operatorname{arsinh}(cx)+a)}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="giac")
[Out] integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)/(-I*c*f*x + f)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \int \frac{(a+b\operatorname{asinh}(c\,x))\,\sqrt{d+c\,d\,x\,1\mathrm{i}}}{(f-c\,f\,x\,1\mathrm{i})^{3/2}}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(3/2),x)
[Out] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(3/2), x)
```

3.561 $\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 112

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx = -\frac{d(i-cx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{bd(1+c^2x^2)^{3/2}\log(i+cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] -d*(I-c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(
3/2)-b*d*(c^2*x^2+1)^(3/2)*ln(I+c*x)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {5796, 651, 5837, 12, 641, 31}

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx = -\frac{d(-cx+i)\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{bd(c^2x^2+1)^{3/2}\log(cx+i)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] -((d*(I - c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(
f - I*c*f*x)^(3/2))) - (b*d*(1 + c^2*x^2)^(3/2)*Log[I + c*x])/(c*(d + I*c*d
*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(1+c^2x^2)^{3/2}\int \frac{(d+icdx)(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&= -\frac{d(i-cx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{\left(bc(1+c^2x^2)^{3/2}\right)\int \frac{d(i-cx)}{c(1+c^2x^2)}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}
$$

$$= -\frac{d(i-cx)\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(bd(1+c^2x^2)^{3/2}\right)\int\frac{i-cx}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{d(i-cx)\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(bd(1+c^2x^2)^{3/2}\right)\int\frac{1}{-i-cx}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{d(i-cx)\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{bd(1+c^2x^2)^{3/2}\log(i+cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.84

$$\int\frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx = \frac{\sqrt{f-icfx}\left(a+iacx+(b+ibcx)\operatorname{arcsinh}(cx)-ib\sqrt{1+c^2x^2}\log(d(-1+icx\right.}{cf^2(i+cx)\sqrt{d+icdx}}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)),x]
[Out] (Sqrt[f - I*c*f*x]*(a + I*a*c*x + (b + I*b*c*x)*ArcSinh[c*x] - I*b*Sqrt[1 +
 c^2*x^2]*Log[d*(-1 + I*c*x)]))/(c*f^2*(I + c*x)*Sqrt[d + I*c*d*x])
```

Maple [F]

$$\int\frac{a+b\operatorname{arcsinh}(cx)}{(-icfx+f)^{\frac{3}{2}}\sqrt{icdx+d}}dx$$

```
[In] int((a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x)
[Out] int((a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 443 vs. 2(88) = 176.

Time = 0.30 (sec) , antiderivative size = 443, normalized size of antiderivative = 3.96

$$\int\frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx = \frac{2\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\,b\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(c^2df^2x+i\,cdf^2\right)\sqrt{\frac{b^2}{c^2df^3}}\,\mathrm{l}}{}$$

```
[In] integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] 1/2*(2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 + 1))
- (c^2*d*f^2*x + I*c*d*f^2)*sqrt(b^2/(c^2*d*f^3))*log(-1/8*((-I*b*c^6*x^2 +
 2*b*c^5*x + 2*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
 f) - (I*c^9*d*f^2*x^4 - 2*c^8*d*f^2*x^3 + I*c^7*d*f^2*x^2 - 2*c^6*d*f^2*x)
*sqrt(b^2/(c^2*d*f^3)))/(b*c^3*x^3 + I*b*c^2*x^2 + b*c*x + I*b)) + (c^2*d*f
^2*x + I*c*d*f^2)*sqrt(b^2/(c^2*d*f^3))*log(-1/8*((-I*b*c^6*x^2 + 2*b*c^5*x
 + 2*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) - (-I*
c^9*d*f^2*x^4 + 2*c^8*d*f^2*x^3 - I*c^7*d*f^2*x^2 + 2*c^6*d*f^2*x)*sqrt(b^2
/(c^2*d*f^3)))/(b*c^3*x^3 + I*b*c^2*x^2 + b*c*x + I*b)) + 2*sqrt(I*c*d*x +
d)*sqrt(-I*c*f*x + f)*a)/(c^2*d*f^2*x + I*c*d*f^2)
```

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{d + icdx}(f - icfx)^{3/2}}\, dx = \int \frac{a + b\,\text{asinh}\,(cx)}{\sqrt{id\,(cx - i)}\,(-if\,(cx + i))^{\frac{3}{2}}}\, dx$$

```
[In] integrate((a+b*asinh(c*x))/(f-I*c*f*x)**(3/2)/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(3/2)),
x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.88

$$\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{d + icdx}(f - icfx)^{3/2}}\, dx = -\frac{i\,\sqrt{c^2dfx^2 + df}b\,\text{arsinh}\,(cx)}{-i\,c^2df^2x + cdf^2} - \frac{i\,\sqrt{c^2dfx^2 + df}a}{-i\,c^2df^2x + cdf^2} - \frac{b\log\,(i\,cx - 1)}{c\sqrt{d}f^{\frac{3}{2}}}$$

```
[In] integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] -I*sqrt(c^2*d*f*x^2 + d*f)*b*arcsinh(c*x)/(-I*c^2*d*f^2*x + c*d*f^2) - I*sq
rt(c^2*d*f*x^2 + d*f)*a/(-I*c^2*d*f^2*x + c*d*f^2) - b*log(I*c*x - 1)/(c*sq
rt(d)*f^(3/2))
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{d + icdx}(f - icfx)^{3/2}} \, dx = \int \frac{b \text{arsinh}\,(cx) + a}{\sqrt{i\,cdx + d}(-i\,cfx + f)^{\frac{3}{2}}} \, dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(I*c*d*x + d)*(-I*c*f*x + f)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{d + icdx}(f - icfx)^{3/2}} \, dx = \int \frac{a + b \,\text{asinh}(c\,x)}{\sqrt{d + c\,d\,x\,1\text{i}}\,(f - c\,f\,x\,1\text{i})^{3/2}} \, dx$$

```
[In] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2)), x)
```

3.562 $\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 103

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx = \frac{x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{b(1+c^2x^2)^{3/2}\log\left(1+c^2x^2\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] x*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-1/2*b*
(c^2*x^2+1)^(3/2)*ln(c^2*x^2+1)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {5796, 5787, 266}

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx = \frac{x(c^2x^2+1)\,(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{b(c^2x^2+1)^{3/2}\log\left(c^2x^2+1\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] (x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(
3/2)) - (b*(1 + c^2*x^2)^(3/2)*Log[1 + c^2*x^2])/(2*c*(d + I*c*d*x)^(3/2)*(
f - I*c*f*x)^(3/2))
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(1+c^2x^2)^{3/2}\int \frac{a+b\text{arcsinh}(cx)}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= \frac{x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(bc(1+c^2x^2)^{3/2}\right)\int \frac{x}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= \frac{x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{b(1+c^2x^2)^{3/2}\log\left(1+c^2x^2\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx = \frac{i\sqrt{f-icfx}\left(2acx+2bcx\text{arcsinh}(cx)-b\sqrt{1+c^2x^2}\log(d(-1+icx))-b\sqrt{\ldots}\right.}{2cdf^2(i+cx)\sqrt{d+icdx}}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] ((I/2)*Sqrt[f - I*c*f*x]*(2*a*c*x + 2*b*c*x*ArcSinh[c*x] - b*Sqrt[1 + c^2*x
^2]*Log[d*(-1 + I*c*x)] - b*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*x]))/(c*d*f^2*(
I + c*x)*Sqrt[d + I*c*d*x])
```

Maple [F]

$$\int \frac{a + b \operatorname{arcsinh}(cx)}{(icdx + d)^{\frac{3}{2}} (-icfx + f)^{\frac{3}{2}}} dx$$

[In] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x)

[Out] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x)

Fricas [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + icdx)^{3/2}(f - icfx)^{3/2}} dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(i\,cdx + d)^{\frac{3}{2}}(-i\,cfx + f)^{\frac{3}{2}}} dx$$

[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algorithm="fricas")

[Out] 1/4*(4*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*x*log(c*x + sqrt(c^2*x^2 + 1)) + 4*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*x + (c^2*d^2*f^2*x^2 + d^2*f^2)*sqrt(b^2/(c^2*d^3*f^3))*log((b*c^2*x^4 + sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f*x^2*sqrt(b^2/(c^2*d^3*f^3)) + b*x^2)/(b*c^4*x^4 + 2*b*c^2*x^2 + b)) - (c^2*d^2*f^2*x^2 + d^2*f^2)*sqrt(b^2/(c^2*d^3*f^3))*log((b*c^2*x^4 - sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f*x^2*sqrt(b^2/(c^2*d^3*f^3)) + b*x^2)/(b*c^4*x^4 + 2*b*c^2*x^2 + b)) - 2*(c^2*d^2*f^2*x^2 + d^2*f^2)*sqrt(b^2/(c^2*d^3*f^3))*log((b*c^2*x^3 + sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f*x*sqrt(b^2/(c^2*d^3*f^3)) + b*x)/(b*c^2*x^2 + b)) + 2*(c^2*d^2*f^2*x^2 + d^2*f^2)*sqrt(b^2/(c^2*d^3*f^3))*log((b*c^2*x^3 - sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f*x*sqrt(b^2/(c^2*d^3*f^3)) + b*x)/(b*c^2*x^2 + b)) + 4*(c^2*d^2*f^2*x^2 + d^2*f^2)*integral(-sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x/(c^4*d^2*f^2*x^4 + 2*c^2*d^2*f^2*x^2 + d^2*f^2), x))/(c^2*d^2*f^2*x^2 + d^2*f^2)

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + icdx)^{3/2}(f - icfx)^{3/2}} dx = \int \frac{a + b \operatorname{asinh}(cx)}{(id (cx - i))^{\frac{3}{2}} (-if (cx + i))^{\frac{3}{2}}} dx$$

[In] integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(3/2),x)

[Out] Integral((a + b*asinh(c*x))/((I*d*(c*x - I))**(3/2)*(-I*f*(c*x + I))**(3/2)), x)

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.80

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx = \frac{bx\,\text{arsinh}\left(cx\right)}{\sqrt{c^2dfx^2+df}\,df} + \frac{ax}{\sqrt{c^2dfx^2+df}\,df} - \frac{b\sqrt{\frac{1}{df}}\log\left(x^2+\frac{1}{c^2}\right)}{2\,cdf}$$

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="maxima")
```

```
[Out] b*x*arcsinh(c*x)/(sqrt(c^2*d*f*x^2 + d*f)*d*f) + a*x/(sqrt(c^2*d*f*x^2 + d*
f)*d*f) - 1/2*b*sqrt(1/(d*f))*log(x^2 + 1/c^2)/(c*d*f)
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx = \int \frac{b\,\text{arsinh}\left(cx\right)+a}{\left(i\,cdx+d\right)^{\frac{3}{2}}\left(-i\,cfx+f\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((I*c*d*x + d)^(3/2)*(-I*c*f*x + f)^(3/2)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx = \int \frac{a+b\,\mathrm{asinh}(c\,x)}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^{3/2}\,{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{3/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2)), x)
```

3.563 $\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}} dx$

Optimal result

Integrand size = 35, antiderivative size = 282

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx = \frac{ibf(1+c^2x^2)^{5/2}}{6c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f(i+cx)(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2fx(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{ibf(1+c^2x^2)^{5/2}\arctan(cx)}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bf(1+c^2x^2)^{5/2}\log(1+c^2x^2)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[Out] 1/6*I*b*f*(c^2*x^2+1)^(5/2)/c/(I-c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1
/3*f*(I+c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)
^(5/2)+2/3*f*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*
x)^(5/2)-1/6*I*b*f*(c^2*x^2+1)^(5/2)*arctan(c*x)/c/(d+I*c*d*x)^(5/2)/(f-I*c
*f*x)^(5/2)-1/3*b*f*(c^2*x^2+1)^(5/2)*ln(c^2*x^2+1)/c/(d+I*c*d*x)^(5/2)/(f-
I*c*f*x)^(5/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5796, 653, 197, 5837, 641, 46, 209, 266}

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx = \frac{2fx(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f(cx+i)(c^2x^2+1)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{ibf(c^2x^2+1)^{5/2}\arctan(cx)}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{ibf(c^2x^2+1)^{5/2}}{6c(-cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bf(c^2x^2+1)^{5/2}\log(c^2x^2+1)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] ((I/6)*b*f*(1 + c^2*x^2)^(5/2))/(c*(I - c*x)*(d + I*c*d*x)^(5/2)*(f - I*c*f
*x)^(5/2)) + (f*(I + c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(3*c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*f*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*
x]))/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((I/6)*b*f*(1 + c^2*x^2)
^(5/2)*ArcTan[c*x])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (b*f*(1 +
 c^2*x^2)^(5/2)*Log[1 + c^2*x^2])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(1+c^2x^2)^{5/2}\int \frac{(f-icfx)(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&= \frac{f(i+cx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2fx(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&\quad-\frac{\left(bc(1+c^2x^2)^{5/2}\right)\int\left(\frac{f(i+cx)}{3c(1+c^2x^2)^2}+\frac{2fx}{3(1+c^2x^2)}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&= \frac{f(i+cx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2fx(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&\quad-\frac{\left(bf(1+c^2x^2)^{5/2}\right)\int\frac{i+cx}{(1+c^2x^2)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{\left(2bcf(1+c^2x^2)^{5/2}\right)\int\frac{x}{1+c^2x^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&= \frac{f(i+cx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2fx(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&\quad-\frac{bf(1+c^2x^2)^{5/2}\log\left(1+c^2x^2\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{\left(bf(1+c^2x^2)^{5/2}\right)\int\frac{1}{(-i+cx)^2(i+cx)}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&= \frac{f(i+cx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2fx(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&\quad-\frac{bf(1+c^2x^2)^{5/2}\log\left(1+c^2x^2\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{\left(bf(1+c^2x^2)^{5/2}\right)\int\left(-\frac{i}{2(-i+cx)^2}+\frac{i}{2(1+c^2x^2)}\right)dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}$$

$$= \frac{ibf(1+c^2x^2)^{5/2}}{6c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f(i+cx)(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2fx(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bf(1+c^2x^2)^{5/2}\log(1+c^2x^2)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(ibf(1+c^2x^2)^{5/2}\right)\int\frac{1}{1+c^2x^2}\,dx}{6(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{ibf(1+c^2x^2)^{5/2}}{6c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f(i+cx)(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2fx(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{ibf(1+c^2x^2)^{5/2}\arctan(cx)}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bf(1+c^2x^2)^{5/2}\log(1+c^2x^2)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.71

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx = \frac{\sqrt{f-icfx}\left(4ia+8acx+8iac^2x^2+2b\sqrt{1+c^2x^2}+4b(i+2cx+2ic^2x^2)\,a\right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] (Sqrt[f - I*c*f*x]*((4*I)*a + 8*a*c*x + (8*I)*a*c^2*x^2 + 2*b*Sqrt[1 + c^2*
x^2] + 4*b*(I + 2*c*x + (2*I)*c^2*x^2)*ArcSinh[c*x] + 3*b*(-1 - I*c*x)*Sqrt
[1 + c^2*x^2]*Log[d*(-1 + I*c*x)] - 5*b*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*x]
- (5*I)*b*c*x*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*x]))/(12*d^2*f^2*Sqrt[d + I*c
*d*x]*(c + c^3*x^2))
```

Maple [F]

$$\int \frac{a+b\,\text{arcsinh}\,(cx)}{(icdx+d)^{\frac{5}{2}}\,(-icfx+f)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x)
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{(i\,cdx+d)^{\frac{5}{2}}(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="fricas")
```

```
[Out] -1/24*(4*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x - 8*(
2*b*c^2*x^2 - 2*I*b*c*x + b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) - 5*(c^4*d^3*f^2*x^3 - I*c^3*d^3*f^2*x^2 + c^2*d^3*f^2*
x - I*c*d^3*f^2)*sqrt(b^2/(c^2*d^5*f^3))*log(-(I*sqrt(c^2*x^2 + 1)*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2*f*x*sqrt(b^2/(c^2*d^5*f^3)) + I*b*c^2*x^
3 + I*b*x)/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) + 3*(c^4*d^3*f^2*x^3 -
I*c^3*d^3*f^2*x^2 + c^2*d^3*f^2*x - I*c*d^3*f^2)*sqrt(b^2/(c^2*d^5*f^3))*lo
g(-(I*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2*f*x*sqrt
(b^2/(c^2*d^5*f^3)) - I*b*c^2*x^3 - I*b*x)/(b*c^3*x^3 + I*b*c^2*x^2 + b*c*x
 + I*b)) + 5*(c^4*d^3*f^2*x^3 - I*c^3*d^3*f^2*x^2 + c^2*d^3*f^2*x - I*c*d^3
*f^2)*sqrt(b^2/(c^2*d^5*f^3))*log(-(-I*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*
sqrt(-I*c*f*x + f)*c*d^2*f*x*sqrt(b^2/(c^2*d^5*f^3)) + I*b*c^2*x^3 + I*b*x)
/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) - 3*(c^4*d^3*f^2*x^3 - I*c^3*d^3*
f^2*x^2 + c^2*d^3*f^2*x - I*c*d^3*f^2)*sqrt(b^2/(c^2*d^5*f^3))*log(-(-I*sqr
t(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2*f*x*sqrt(b^2/(c^2
*d^5*f^3)) - I*b*c^2*x^3 - I*b*x)/(b*c^3*x^3 + I*b*c^2*x^2 + b*c*x + I*b))
+ 8*(c^4*d^3*f^2*x^3 - I*c^3*d^3*f^2*x^2 + c^2*d^3*f^2*x - I*c*d^3*f^2)*sqr
t(b^2/(c^2*d^5*f^3))*log((sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x
 + f)*c*d^2*f*x*sqrt(b^2/(c^2*d^5*f^3)) + b*c^2*x^3 + b*x)/(b*c^2*x^2 + b))
 - 8*(c^4*d^3*f^2*x^3 - I*c^3*d^3*f^2*x^2 + c^2*d^3*f^2*x - I*c*d^3*f^2)*sq
rt(b^2/(c^2*d^5*f^3))*log(-(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f
*x + f)*c*d^2*f*x*sqrt(b^2/(c^2*d^5*f^3)) - b*c^2*x^3 - b*x)/(b*c^2*x^2 + b
)) - 8*(2*a*c^2*x^2 - 2*I*a*c*x + a)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) -
 24*(c^4*d^3*f^2*x^3 - I*c^3*d^3*f^2*x^2 + c^2*d^3*f^2*x - I*c*d^3*f^2)*int
egral(-1/6*sqrt(c^2*x^2 + 1)*(4*b*c*x + I*b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*
x + f)/(c^4*d^3*f^2*x^4 + 2*c^2*d^3*f^2*x^2 + d^3*f^2), x))/(c^4*d^3*f^2*x^
3 - I*c^3*d^3*f^2*x^2 + c^2*d^3*f^2*x - I*c*d^3*f^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.84

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx=\frac{1}{12}bc\left(-\frac{2i\sqrt{d}\sqrt{f}}{c^3d^3f^2x-i\,c^2d^3f^2}-\frac{3\,\log\left(cx+i\right)}{c^2d^{\frac{5}{2}}f^{\frac{3}{2}}}-\frac{5\,\log\left(cx-i\right)}{c^2d^{\frac{5}{2}}f^{\frac{3}{2}}}\right)-\frac{1}{3}b\left(-\frac{3i}{3i\sqrt{c^2dfx^2+df}c^2d^2fx+3\sqrt{c^2dfx^2+df}cd^2f}-\frac{2x}{\sqrt{c^2dfx^2+df}d^2f}\right)\text{arsinh}\left(cx\right)-\frac{1}{3}a\left(-\frac{3i}{3i\sqrt{c^2dfx^2+df}c^2d^2fx+3\sqrt{c^2dfx^2+df}cd^2f}-\frac{2x}{\sqrt{c^2dfx^2+df}d^2f}\right)$$

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="maxima")
[Out] 1/12*b*c*(-2*I*sqrt(d)*sqrt(f)/(c^3*d^3*f^2*x - I*c^2*d^3*f^2) - 3*log(c*x
+ I)/(c^2*d^(5/2)*f^(3/2)) - 5*log(c*x - I)/(c^2*d^(5/2)*f^(3/2))) - 1/3*b*
(-3*I/(3*I*sqrt(c^2*d*f*x^2 + d*f)*c^2*d^2*f*x + 3*sqrt(c^2*d*f*x^2 + d*f)*
c*d^2*f) - 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f))*arcsinh(c*x) - 1/3*a*(-3*I/
(3*I*sqrt(c^2*d*f*x^2 + d*f)*c^2*d^2*f*x + 3*sqrt(c^2*d*f*x^2 + d*f)*c*d^2*
f) - 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0,0
,0]ext_reduce Error: Bad Argument TypeThe choice was done assuming 0=[0,0,0
,0]ext
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{asinh}(c\,x)}{\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2)),x)
[Out] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2)), x)
```

3.564 $\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 470

$$\begin{aligned}
\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx &= -\frac{ibd^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{8ibd^5(1+c^2x^2)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{5bd^5(1+c^2x^2)^{5/2}\,\text{arcsinh}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2id^5(1+icx)^4(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{10id^5(1+icx)^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{5id^5(1+c^2x^2)^3(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{5d^5(1+c^2x^2)^{5/2}\,\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{28bd^5(1+c^2x^2)^{5/2}\log(i+cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}$$

```
[Out] -I*b*d^5*x*(c^2*x^2+1)^(5/2)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+8/3*I*b*d^
5*(c^2*x^2+1)^(5/2)/c/(I+c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-5/2*b*d^5
*(c^2*x^2+1)^(5/2)*arcsinh(c*x)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3
*I*d^5*(1+I*c*x)^4*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*
c*f*x)^(5/2)+10/3*I*d^5*(1+I*c*x)^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/(d+I
*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+5*I*d^5*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))/c/
(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+5*d^5*(c^2*x^2+1)^(5/2)*arcsinh(c*x)*(a
+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+28/3*b*d^5*(c^2*x^2+
1)^(5/2)*ln(I+c*x)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5796, 683, 655, 221, 5837, 641, 45, 5783}

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \frac{5id^5(c^2x^2+1)^3(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{10id^5(1+icx)^2(c^2x^2+1)^2(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^5(1+icx)^4(c^2x^2+1)(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5d^5(c^2x^2+1)^{5/2}\operatorname{arcsinh}(cx)(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{5bd^5(c^2x^2+1)^{5/2}\operatorname{arcsinh}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{ibd^5x(c^2x^2+1)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{8ibd^5(c^2x^2+1)^{5/2}}{3c(cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{28bd^5(c^2x^2+1)^{5/2}\log(cx+i)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[In] Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

```
[Out] ((-I)*b*d^5*x*(1 + c^2*x^2)^(5/2))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)
) + (((8*I)/3)*b*d^5*(1 + c^2*x^2)^(5/2))/(c*(I + c*x)*(d + I*c*d*x)^(5/2)*
(f - I*c*f*x)^(5/2)) - (5*b*d^5*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]^2)/(2*c*(d
 + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((2*I)/3)*d^5*(1 + I*c*x)^4*(1 +
c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
+ (((10*I)/3)*d^5*(1 + I*c*x)^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d
 + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((5*I)*d^5*(1 + c^2*x^2)^3*(a + b*
ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (5*d^5*(1 + c^
2*x^2)^(5/2)*ArcSinh[c*x]*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) + (28*b*d^5*(1 + c^2*x^2)^(5/2)*Log[I + c*x])/(3*c*(d + I*
c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 683

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((m +
p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\text{integral} = \frac{(1+c^2x^2)^{5/2}\int\frac{(d+icdx)^5(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{2id^5(1+icx)^4(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{10id^5(1+icx)^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5id^5(1+c^2x^2)^3(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5d^5(1+c^2x^2)^{5/2}\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(bc(1+c^2x^2)^{5/2}\right)\int\left(\frac{5id^5}{c}-\frac{2id^5(1+icx)^4}{3c(1+c^2x^2)^2}+\frac{10id^5(1+icx)^2}{3c(1+c^2x^2)}+\frac{5d^5\text{arcsinh}(cx)}{c\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{5ibd^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^5(1+icx)^4(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{10id^5(1+icx)^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5id^5(1+c^2x^2)^3(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5d^5(1+c^2x^2)^{5/2}\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2ibd^5(1+c^2x^2)^{5/2}\right)\int\frac{(1+icx)^4}{(1+c^2x^2)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(10ibd^5(1+c^2x^2)^{5/2}\right)\int\frac{(1+icx)^2}{1+c^2x^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(5bd^5(1+c^2x^2)^{5/2}\right)\int\frac{\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{5ibd^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{5bd^5(1+c^2x^2)^{5/2}\text{arcsinh}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^5(1+icx)^4(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{10id^5(1+icx)^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5id^5(1+c^2x^2)^3(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5d^5(1+c^2x^2)^{5/2}\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2ibd^5(1+c^2x^2)^{5/2}\right)\int\frac{(1+icx)^2}{(1-icx)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(10ibd^5(1+c^2x^2)^{5/2}\right)\int\frac{1+icx}{1-icx}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{5ibd^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{5bd^5(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{2id^5(1+icx)^4(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{10id^5(1+icx)^2(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{5id^5(1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{5d^5(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{\left(2ibd^5(1+c^2x^2)^{5/2}\right)\int\left(1-\frac{4}{(i+cx)^2}-\frac{4i}{i+cx}\right)dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{\left(10ibd^5(1+c^2x^2)^{5/2}\right)\int\left(-1+\frac{2i}{i+cx}\right)dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{ibd^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{8ibd^5(1+c^2x^2)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{5bd^5(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^5(1+icx)^4(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{10id^5(1+icx)^2(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{5id^5(1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{5d^5(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{28bd^5(1+c^2x^2)^{5/2}\log(i+cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1083 vs. 2(470) = 940.

Time = 15.42 (sec) , antiderivative size = 1083, normalized size of antiderivative = 2.30

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \frac{\frac{4iad^2\sqrt{d+icdx}\sqrt{f-icfx}(-23+34icx+3c^2x^2)}{f^3(i+cx)^2} + \frac{60ad^{5/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icf}\right.}{f^{5/2}}}{}$$

```
[In] Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

```
[Out] (((4*I)*a*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-23 + (34*I)*c*x + 3*c^2
*x^2))/(f^3*(I + c*x)^2) + (60*a*d^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt
```

```
[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/f^(5/2) - ((2*I)*b*d^2*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(-(Cosh[(
3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] + (I/2)*L
og[1 + c^2*x^2])) + Cosh[ArcSinh[c*x]/2]*(4*I + 3*ArcSinh[c*x] - 6*ArcTan[C
oth[ArcSinh[c*x]/2]] + ((3*I)/2)*Log[1 + c^2*x^2]) + 2*(2 + (2*I)*ArcSinh[c
*x] + (4*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + Log[1 + c^2*x^2] + (Sqrt[1 + c^2
*x^2]*((2*I)*ArcSinh[c*x] + (4*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + Log[1 + c^
2*x^2]))/2)*Sinh[ArcSinh[c*x]/2]))/(f^3*(1 + I*c*x)*(Cosh[ArcSinh[c*x]/2] -
 I*Sinh[ArcSinh[c*x]/2])^4) + (2*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*
(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*ArcSinh[c*x])/2]*(
(14*I - 3*ArcSinh[c*x])*ArcSinh[c*x] + (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]]
- 7*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(8 + (6*I)*ArcSinh[c*x] + 9*Ar
cSinh[c*x]^2 - (84*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 21*Log[1 + c^2*x^2]) -
 (2*I)*(4 + (4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 - (56*I)*ArcTan[Tanh[ArcS
inh[c*x]/2]] + 14*Log[1 + c^2*x^2] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(14*I
+ 3*ArcSinh[c*x]) - (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 7*Log[1 + c^2*x^2
]))*Sinh[ArcSinh[c*x]/2]))/(f^3*(1 + I*c*x)*(Cosh[ArcSinh[c*x]/2] - I*Sinh[
ArcSinh[c*x]/2])^4) - (I*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[Ar
cSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(-(Cosh[(3*ArcSinh[c*x])/2]*(9 - (3
5*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 + (52*I)*ArcTan[Coth[ArcSinh[c*x]/2]]
+ 13*Log[1 + c^2*x^2])) + Cosh[ArcSinh[c*x]/2]*(20 + (24*I)*ArcSinh[c*x] +
27*ArcSinh[c*x]^2 + (156*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 39*Log[1 + c^2*x
^2]) - I*(3*(-I + ArcSinh[c*x])*Cosh[(5*ArcSinh[c*x])/2] + 2*(13 + (7*I)*Ar
cSinh[c*x] + 18*ArcSinh[c*x]^2 + (104*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*
I)*(I + ArcSinh[c*x])*Cosh[2*ArcSinh[c*x]] + 26*Log[1 + c^2*x^2] + Sqrt[1 +
 c^2*x^2]*(6 + (38*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 + (52*I)*ArcTan[Coth[
ArcSinh[c*x]/2]] + 13*Log[1 + c^2*x^2]))*Sinh[ArcSinh[c*x]/2])))/(f^3*(-I +
 c*x)*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^4))/(12*c)
```

Maple [F]

$$\int \frac{(icdx+d)^{\frac{5}{2}}(a+b\,\mathrm{arcsinh}\,(cx))}{(-icfx+f)^{\frac{5}{2}}}dx$$

[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x)

[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x)

Fricas [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \int \frac{(i\,cdx+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral(((I*b*c^2*d^2*x^2 + 2*b*c*d^2*x - I*b*d^2)*sqrt(I*c*d*x + d)*sqrt(
-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a*c^2*d^2*x^2 + 2*a*c*d^2*x
 - I*a*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*f^3*x^3 + 3*I*c^2*f^
3*x^2 - 3*c*f^3*x - I*f^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \int \frac{(i\,cdx+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algorit
hm="maxima")
```

```
[Out] -1/3*(-3*I*(c^2*d*f*x^2 + d*f)^(5/2)/(c^5*f^5*x^4 + 4*I*c^4*f^5*x^3 - 6*c^3
*f^5*x^2 - 4*I*c^2*f^5*x + c*f^5) + 15*I*(c^2*d*f*x^2 + d*f)^(3/2)*d/(3*I*c
^4*f^4*x^3 - 9*c^3*f^4*x^2 - 9*I*c^2*f^4*x + 3*c*f^4) - 10*I*sqrt(c^2*d*f*x
^2 + d*f)*d^2/(c^3*f^3*x^2 + 2*I*c^2*f^3*x - c*f^3) - 105*I*sqrt(c^2*d*f*x^
2 + d*f)*d^2/(-3*I*c^2*f^3*x + 3*c*f^3) - 15*d^3*arcsinh(c*x)/(c*f^3*sqrt(d
/f)))*a + b*integrate((I*c*d*x + d)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1))/(-I*
c*f*x + f)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\left(d+c\,d\,x\,1\text{i}\right)^{5/2}}{\left(f-c\,f\,x\,1\text{i}\right)^{5/2}}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(5/2), x)
```

3.565 $\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}} dx$

Optimal result

Integrand size = 35, antiderivative size = 362

$$\begin{aligned}\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx &= \frac{4ibd^4(1+c^2x^2)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} \\ &- \frac{bd^4(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^4(1+icx)^3\,(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\ &+ \frac{2id^4(1+icx)\,(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\ &+ \frac{d^4(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{8bd^4(1+c^2x^2)^{5/2}\log(i+cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}\end{aligned}$$

```
[Out] 4/3*I*b*d^4*(c^2*x^2+1)^(5/2)/c/(I+c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
-1/2*b*d^4*(c^2*x^2+1)^(5/2)*arcsinh(c*x)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)
^(5/2)-2/3*I*d^4*(1+I*c*x)^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(
5/2)/(f-I*c*f*x)^(5/2)+2*I*d^4*(1+I*c*x)*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c
/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+d^4*(c^2*x^2+1)^(5/2)*arcsinh(c*x)*(a+
b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+8/3*b*d^4*(c^2*x^2+1)
^(5/2)*ln(I+c*x)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {5796, 683, 667, 221, 5837, 641, 45, 31, 5783}

$$\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx=\frac{2id^4(1+icx)\left(c^2x^2+1\right)^2(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2id^4(1+icx)^3\left(c^2x^2+1\right)(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{d^4\left(c^2x^2+1\right)^{5/2}\operatorname{arcsinh}(cx)(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{bd^4\left(c^2x^2+1\right)^{5/2}\operatorname{arcsinh}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{4ibd^4\left(c^2x^2+1\right)^{5/2}}{3c(cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{8bd^4\left(c^2x^2+1\right)^{5/2}\log(cx+i)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[In] Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

```
[Out] (((4*I)/3)*b*d^4*(1 + c^2*x^2)^(5/2))/(c*(I + c*x)*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) - (b*d^4*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]^2)/(2*c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((2*I)/3)*d^4*(1 + I*c*x)^3*(1 + c^2*x^
2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((2*
I)*d^4*(1 + I*c*x)*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(
5/2)*(f - I*c*f*x)^(5/2)) + (d^4*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]*(a + b*Ar
cSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (8*b*d^4*(1 + c^
2*x^2)^(5/2)*Log[I + c*x])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
```

```
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 683

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((m +
p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\text{integral} = \frac{(1+c^2x^2)^{5/2}\int\frac{(d+icdx)^4(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$=-\frac{2id^4(1+icx)^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2id^4(1+icx)\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{d^4\left(1+c^2x^2\right)^{5/2}\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{\left(bc\left(1+c^2x^2\right)^{5/2}\right)\int\left(-\frac{2id^4(1+icx)^3}{3c(1+c^2x^2)^2}+\frac{2id^4(1+icx)}{c(1+c^2x^2)}+\frac{d^4\text{arcsinh}(cx)}{c\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$=-\frac{2id^4(1+icx)^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2id^4(1+icx)\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{d^4\left(1+c^2x^2\right)^{5/2}\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{\left(2ibd^4\left(1+c^2x^2\right)^{5/2}\right)\int\frac{(1+icx)^3}{(1+c^2x^2)^2}dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{\left(2ibd^4\left(1+c^2x^2\right)^{5/2}\right)\int\frac{1+icx}{1+c^2x^2}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{\left(bd^4\left(1+c^2x^2\right)^{5/2}\right)\int\frac{\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$=-\frac{bd^4\left(1+c^2x^2\right)^{5/2}\text{arcsinh}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2id^4(1+icx)^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2id^4(1+icx)\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{d^4\left(1+c^2x^2\right)^{5/2}\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{\left(2ibd^4\left(1+c^2x^2\right)^{5/2}\right)\int\frac{1+icx}{(1-icx)^2}dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{\left(2ibd^4\left(1+c^2x^2\right)^{5/2}\right)\int\frac{1}{1-icx}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$=-\frac{bd^4\left(1+c^2x^2\right)^{5/2}\text{arcsinh}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2id^4(1+icx)^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2id^4(1+icx)\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{d^4\left(1+c^2x^2\right)^{5/2}\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2bd^4\left(1+c^2x^2\right)^{5/2}\log(i+cx)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{\left(2ibd^4\left(1+c^2x^2\right)^{5/2}\right)\int\left(-\frac{2}{(i+cx)^2}-\frac{i}{i+cx}\right)dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$
\begin{aligned}
&= \frac{4ibd^4(1+c^2x^2)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bd^4(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{2id^4(1+icx)^3\left(1+c^2x^2\right)\left(a+b\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{2id^4(1+icx)\left(1+c^2x^2\right)^2\left(a+b\operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{d^4(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{8bd^4(1+c^2x^2)^{5/2}\log(i+cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 10.02 (sec) , antiderivative size = 706, normalized size of antiderivative = 1.95

$$
\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \frac{-\frac{16ad(i+2cx)\sqrt{d+icdx}\sqrt{f-icfx}}{f^3(i+cx)^2} + \frac{12ad^{3/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{f^{5/2}} - \frac{2ibd}{}}{}
$$

```
[In] Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

```
[Out] ((-16*a*d*(I + 2*c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(f^3*(I + c*x)^2
) + (12*a*d^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f -
I*c*f*x]])/f^(5/2) - ((2*I)*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[A
rcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(-(Cosh[(3*ArcSinh[c*x])/2]*(ArcSi
nh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] + (I/2)*Log[1 + c^2*x^2])) + Cosh[
ArcSinh[c*x]/2]*(4*I + 3*ArcSinh[c*x] - 6*ArcTan[Coth[ArcSinh[c*x]/2]] + ((
3*I)/2)*Log[1 + c^2*x^2]) + 2*(2 + (2*I)*ArcSinh[c*x] + (4*I)*ArcTan[Coth[A
rcSinh[c*x]/2]] + Log[1 + c^2*x^2] + (Sqrt[1 + c^2*x^2]*((2*I)*ArcSinh[c*x]
 + (4*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + Log[1 + c^2*x^2]))/2)*Sinh[ArcSinh[
c*x]/2]))/(f^3*(1 + I*c*x)*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^
4) + (b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]/2] + I*Sin
h[ArcSinh[c*x]/2])*(Cosh[(3*ArcSinh[c*x])/2]*((14*I - 3*ArcSinh[c*x])*ArcSi
nh[c*x] + (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] - 7*Log[1 + c^2*x^2]) + Cosh[
ArcSinh[c*x]/2]*(8 + (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 - (84*I)*ArcTan[
Tanh[ArcSinh[c*x]/2]] + 21*Log[1 + c^2*x^2]) - (2*I)*(4 + (4*I)*ArcSinh[c*x
] + 6*ArcSinh[c*x]^2 - (56*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 14*Log[1 + c^2
*x^2] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(14*I + 3*ArcSinh[c*x]) - (28*I)*Ar
cTan[Tanh[ArcSinh[c*x]/2]] + 7*Log[1 + c^2*x^2]))*Sinh[ArcSinh[c*x]/2]))/(f
^3*(1 + I*c*x)*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^4))/(12*c)
```

Maple [F]

$$\int \frac{(icdx+d)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)}{\left(-icfx+f\right)^{\frac{5}{2}}}dx$$

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x)
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x)
```

Fricas [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\mathrm{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\mathrm{arsinh}\left(cx\right)+a)}{\left(-i\,cfx+f\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algorit
hm="fricas")
[Out] integral(((b*c*d*x - I*b*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) + (a*c*d*x - I*a*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)
)/(c^3*f^3*x^3 + 3*I*c^2*f^3*x^2 - 3*c*f^3*x - I*f^3), x)
```

Sympy [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\mathrm{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \int \frac{\left(id(cx-i)\right)^{\frac{3}{2}}\left(a+b\,\mathrm{asinh}\left(cx\right)\right)}{\left(-if\left(cx+i\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(5/2),x)
[Out] Integral((I*d*(c*x - I))**(3/2)*(a + b*asinh(c*x))/(-I*f*(c*x + I))**(5/2),
 x)
```

Maxima [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\mathrm{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\mathrm{arsinh}\left(cx\right)+a)}{\left(-i\,cfx+f\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algorit
hm="maxima")
[Out] -1/3*a*(3*I*(c^2*d*f*x^2 + d*f)^(3/2)/(3*I*c^4*f^4*x^3 - 9*c^3*f^4*x^2 - 9*
I*c^2*f^4*x + 3*c*f^4) - 2*I*sqrt(c^2*d*f*x^2 + d*f)*d/(c^3*f^3*x^2 + 2*I*c
^2*f^3*x - c*f^3) - 21*I*sqrt(c^2*d*f*x^2 + d*f)*d/(-3*I*c^2*f^3*x + 3*c*f^
3) - 3*d^2*arcsinh(c*x)/(c*f^3*sqrt(d/f))) + b*integrate((I*c*d*x + d)^(3/2
)*log(c*x + sqrt(c^2*x^2 + 1))/(-I*c*f*x + f)^(5/2), x)
```

Giac [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algorit
hm="giac")

[Out] integrate((I*c*d*x + d)^(3/2)*(b*arcsinh(c*x) + a)/(-I*c*f*x + f)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\left(d+c\,d\,x\,1\text{i}\right)^{3/2}}{\left(f-c\,f\,x\,1\text{i}\right)^{5/2}}\,dx$$

[In] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(5/2),x)

[Out] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(5/2), x)

3.566 $\int \frac{\sqrt{d+icdx}(a+b\textbf{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 185

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \frac{2ibd^3(1+c^2x^2)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{id^3(1+icx)^3\,(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd^3(1+c^2x^2)^{5/2}\log(i+cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[Out] 2/3*I*b*d^3*(c^2*x^2+1)^(5/2)/c/(I+c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
-1/3*I*d^3*(1+I*c*x)^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(
f-I*c*f*x)^(5/2)+1/3*b*d^3*(c^2*x^2+1)^(5/2)*ln(I+c*x)/c/(d+I*c*d*x)^(5/2)/
(f-I*c*f*x)^(5/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {5796, 665, 5837, 12, 641, 45}

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = -\frac{id^3(1+icx)^3\,(c^2x^2+1)\,(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ibd^3(c^2x^2+1)^{5/2}}{3c(cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd^3(c^2x^2+1)^{5/2}\log(cx+i)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[In] Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

```
[Out] (((2*I)/3)*b*d^3*(1 + c^2*x^2)^(5/2))/(c*(I + c*x)*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) - ((I/3)*d^3*(1 + I*c*x)^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*
```

```
x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (b*d^3*(1 + c^2*x^2)^(5/
2)*Log[I + c*x])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\text{integral} = \frac{(1+c^2x^2)^{5/2}\int \frac{(d+icdx)^3(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{id^3(1+icx)^3(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(bc(1+c^2x^2)^{5/2}\right)\int -\frac{id^3(1+icx)^3}{3c(1+c^2x^2)^2}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{id^3(1+icx)^3(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(ibd^3(1+c^2x^2)^{5/2}\right)\int \frac{(1+icx)^3}{(1+c^2x^2)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{id^3(1+icx)^3(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(ibd^3(1+c^2x^2)^{5/2}\right)\int \frac{1+icx}{(1-icx)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{id^3(1+icx)^3(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(ibd^3(1+c^2x^2)^{5/2}\right)\int \left(-\frac{2}{(i+cx)^2}-\frac{i}{i+cx}\right)dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2ibd^3(1+c^2x^2)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{id^3(1+icx)^3(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd^3(1+c^2x^2)^{5/2}\log(i+cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Mathematica [A] (verified)

Time = 1.75 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = -\frac{id\sqrt{f-icfx}\left((-i+cx)\left(-ia+acx+b\sqrt{1+c^2x^2}\right)+b(-i+cx)^2\text{arcsinh}(cx)-b(i+cx)\sqrt{1+c^2x^2}\log(d\right.}{3cf^3(i+cx)^2\sqrt{d+icdx}}$$

```
[In] Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

```
[Out] ((-1/3*I)*d*Sqrt[f - I*c*f*x]*((-I + c*x)*((-I)*a + a*c*x + b*Sqrt[1 + c^2*
x^2]) + b*(-I + c*x)^2*ArcSinh[c*x] - b*(I + c*x)*Sqrt[1 + c^2*x^2]*Log[d*(
-1 + I*c*x)]))/(c*f^3*(I + c*x)^2*Sqrt[d + I*c*d*x])
```

Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))\,\sqrt{icdx+d}}{(-icfx+f)^{\frac{5}{2}}}dx$$

[In] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x)

[Out] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x)

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(141) = 282$.

Time = 0.33 (sec) , antiderivative size = 548, normalized size of antiderivative = 2.96

$$\int \frac{\sqrt{d+icdx}(a+b\mathrm{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx =$$

$$-\frac{4\,\sqrt{c^2x^2+1}\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}bcx+2\,(bc^2x^2-2i\,bcx-b)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2+\right.}{}$$

[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="fricas")

[Out] -1/6*(4*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x + 2*(b
*c^2*x^2 - 2*I*b*c*x - b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sq
rt(c^2*x^2 + 1)) + (c^4*f^3*x^3 + I*c^3*f^3*x^2 + c^2*f^3*x + I*c*f^3)*sqrt
(b^2*d/(c^2*f^5))*log(-1/8*((-I*b*c^6*x^2 + 2*b*c^5*x + 2*I*b*c^4)*sqrt(c^2
*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + (I*c^9*f^3*x^4 - 2*c^8*f^3
*x^3 + I*c^7*f^3*x^2 - 2*c^6*f^3*x)*sqrt(b^2*d/(c^2*f^5)))/(b*c^3*x^3 + I*b
*c^2*x^2 + b*c*x + I*b)) - (c^4*f^3*x^3 + I*c^3*f^3*x^2 + c^2*f^3*x + I*c*f
^3)*sqrt(b^2*d/(c^2*f^5))*log(-1/8*((-I*b*c^6*x^2 + 2*b*c^5*x + 2*I*b*c^4)*
sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + (-I*c^9*f^3*x^4 +
2*c^8*f^3*x^3 - I*c^7*f^3*x^2 + 2*c^6*f^3*x)*sqrt(b^2*d/(c^2*f^5)))/(b*c^3*
x^3 + I*b*c^2*x^2 + b*c*x + I*b)) + 2*(a*c^2*x^2 - 2*I*a*c*x - a)*sqrt(I*c*
d*x + d)*sqrt(-I*c*f*x + f))/(c^4*f^3*x^3 + I*c^3*f^3*x^2 + c^2*f^3*x + I*c
*f^3)

Sympy [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx=\int \frac{\sqrt{id\,(cx-i)}(a+b\,\text{asinh}\,(cx))}{(-if\,(cx+i))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(5/2),x)
[Out] Integral(sqrt(I*d*(c*x - I))*(a + b*asinh(c*x))/(-I*f*(c*x + I))**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx=-\frac{1}{3}\,bc\left(\frac{6\sqrt{d}}{3i\,c^3f^{\frac{5}{2}}x-3\,c^2f^{\frac{5}{2}}}-\frac{\sqrt{d}\log\left(cx+i\right)}{c^2f^{\frac{5}{2}}}\right)-\frac{1}{3}\,b\left(-\frac{2i\,\sqrt{c^2dfx^2+df}}{c^3f^3x^2+2i\,c^2f^3x-cf^3}-\frac{3i\,\sqrt{c^2dfx^2+df}}{-3i\,c^2f^3x+3\,cf^3}\right)\text{arsinh}\left(cx\right)-\frac{1}{3}\,a\left(-\frac{2i\,\sqrt{c^2dfx^2+df}}{c^3f^3x^2+2i\,c^2f^3x-cf^3}-\frac{3i\,\sqrt{c^2dfx^2+df}}{-3i\,c^2f^3x+3\,cf^3}\right)$$

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="maxima")
[Out] -1/3*b*c*(6*sqrt(d)/(3*I*c^3*f^(5/2)*x - 3*c^2*f^(5/2)) - sqrt(d)*log(c*x +
 I)/(c^2*f^(5/2))) - 1/3*b*(-2*I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*f^3*x^2 + 2*I
*c^2*f^3*x - c*f^3) - 3*I*sqrt(c^2*d*f*x^2 + d*f)/(-3*I*c^2*f^3*x + 3*c*f^3
))*arcsinh(c*x) - 1/3*a*(-2*I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*f^3*x^2 + 2*I*c^
2*f^3*x - c*f^3) - 3*I*sqrt(c^2*d*f*x^2 + d*f)/(-3*I*c^2*f^3*x + 3*c*f^3))
```

Giac [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx=\int \frac{\sqrt{i\,cdx+d}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="giac")
[Out] integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)/(-I*c*f*x + f)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))\,\sqrt{d+c\,d\,x\,1\text{i}}}{(f-c\,f\,x\,1\text{i})^{5/2}}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(5/2),x)
[Out] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(5/2), x)
```

3.567 $\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 294

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx=\frac{ibd^2(1+c^2x^2)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2id^2(1+icx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{d^2x(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibd^2(1+c^2x^2)^{5/2}\arctan(cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{bd^2(1+c^2x^2)^{5/2}\log\left(1+c^2x^2\right)}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[Out] 1/3*I*b*d^2*(c^2*x^2+1)^(5/2)/c/(I+c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
-2/3*I*d^2*(1+I*c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-
I*c*f*x)^(5/2)+1/3*d^2*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)
/(f-I*c*f*x)^(5/2)+1/3*I*b*d^2*(c^2*x^2+1)^(5/2)*arctan(c*x)/c/(d+I*c*d*x)^
(5/2)/(f-I*c*f*x)^(5/2)-1/6*b*d^2*(c^2*x^2+1)^(5/2)*ln(c^2*x^2+1)/c/(d+I*c*
d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5796, 667, 197, 5837, 641, 46, 209, 266}

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx=\frac{d^2x(c^2x^2+1)^2\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2id^2(1+icx)\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibd^2(c^2x^2+1)^{5/2}\arctan(cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibd^2(c^2x^2+1)^{5/2}}{3c(cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{bd^2(c^2x^2+1)^{5/2}\log\left(c^2x^2+1\right)}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] ((I/3)*b*d^2*(1 + c^2*x^2)^(5/2))/(c*(I + c*x)*(d + I*c*d*x)^(5/2)*(f - I*c
*f*x)^(5/2)) - (((2*I)/3)*d^2*(1 + I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (d^2*x*(1 + c^2*x^2)^2*(a
+ b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((I/3)*b*d
^2*(1 + c^2*x^2)^(5/2)*ArcTan[c*x])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) - (b*d^2*(1 + c^2*x^2)^(5/2)*Log[1 + c^2*x^2])/(6*c*(d + I*c*d*x)^(5/2
)*(f - I*c*f*x)^(5/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\text{integral} = \frac{(1+c^2x^2)^{5/2}\int \frac{(d+icdx)^2(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{2id^2(1+icx)(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2x(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(bc(1+c^2x^2)^{5/2}\right)\int\left(-\frac{2id^2(1+icx)}{3c(1+c^2x^2)^2}+\frac{d^2x}{3(1+c^2x^2)}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{2id^2(1+icx)(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2x(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2ibd^2(1+c^2x^2)^{5/2}\right)\int\frac{1+icx}{(1+c^2x^2)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(bcd^2(1+c^2x^2)^{5/2}\right)\int\frac{x}{1+c^2x^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{2id^2(1+icx)(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2x(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bd^2(1+c^2x^2)^{5/2}\log(1+c^2x^2)}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2ibd^2(1+c^2x^2)^{5/2}\right)\int\frac{1}{(1-icx)^2(1+icx)}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{2id^2(1+icx)(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2x(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bd^2(1+c^2x^2)^{5/2}\log(1+c^2x^2)}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2ibd^2(1+c^2x^2)^{5/2}\right)\int\left(-\frac{1}{2(i+cx)^2}+\frac{1}{2(1+c^2x^2)}\right)dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$
\begin{aligned}
&= \frac{ibd^2(1+c^2x^2)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^2(1+icx)\left(1+c^2x^2\right)\left(a+b\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{d^2x\left(1+c^2x^2\right)^2\left(a+b\operatorname{arcsinh}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{bd^2(1+c^2x^2)^{5/2}\log\left(1+c^2x^2\right)}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(ibd^2(1+c^2x^2)^{5/2}\right)\int \frac{1}{1+c^2x^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&= \frac{ibd^2(1+c^2x^2)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^2(1+icx)\left(1+c^2x^2\right)\left(a+b\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{d^2x\left(1+c^2x^2\right)^2\left(a+b\operatorname{arcsinh}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{ibd^2(1+c^2x^2)^{5/2}\arctan(cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bd^2(1+c^2x^2)^{5/2}\log\left(1+c^2x^2\right)}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.47

$$
\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx = \frac{\sqrt{f-icfx}\left((2i+cx)\left(a+iacx+ib\sqrt{1+c^2x^2}\right)+ib(2+icx+c^2x^2)\operatorname{arcsinh}\right.}{3cf^3(i+cx)^2\sqrt{d+icdx}}
$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] (Sqrt[f - I*c*f*x]*((2*I + c*x)*(a + I*a*c*x + I*b*Sqrt[1 + c^2*x^2]) + I*b
*(2 + I*c*x + c^2*x^2)*ArcSinh[c*x] + b*(1 - I*c*x)*Sqrt[1 + c^2*x^2]*Log[d
*(-1 + I*c*x)]))/(3*c*f^3*(I + c*x)^2*Sqrt[d + I*c*d*x])
```

Maple [F]

$$
\int \frac{a+b\ \operatorname{arcsinh}(cx)}{(-icfx+f)^{\frac{5}{2}}\sqrt{icdx+d}}dx
$$

```
[In] int((a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 576 vs. 2(228) = 456.

Time = 0.33 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.96

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx =$$

$$-\frac{2\sqrt{c^2x^2+1}\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\,bcx-2\left(bc^2x^2+i\,bcx+2\,b\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2\right.}}{}$$

```
[In] integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] -1/6*(2*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x - 2*(b
*c^2*x^2 + I*b*c*x + 2*b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sq
rt(c^2*x^2 + 1)) - (c^4*d*f^3*x^3 + I*c^3*d*f^3*x^2 + c^2*d*f^3*x + I*c*d*f
^3)*sqrt(b^2/(c^2*d*f^5))*log(-1/8*((-I*b*c^6*x^2 + 2*b*c^5*x + 2*I*b*c^4)*
sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + (I*c^9*d*f^3*x^4 -
 2*c^8*d*f^3*x^3 + I*c^7*d*f^3*x^2 - 2*c^6*d*f^3*x)*sqrt(b^2/(c^2*d*f^5)))/
(b*c^3*x^3 + I*b*c^2*x^2 + b*c*x + I*b)) + (c^4*d*f^3*x^3 + I*c^3*d*f^3*x^2
 + c^2*d*f^3*x + I*c*d*f^3)*sqrt(b^2/(c^2*d*f^5))*log(-1/8*((-I*b*c^6*x^2 +
 2*b*c^5*x + 2*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
 f) + (-I*c^9*d*f^3*x^4 + 2*c^8*d*f^3*x^3 - I*c^7*d*f^3*x^2 + 2*c^6*d*f^3*x
)*sqrt(b^2/(c^2*d*f^5)))/(b*c^3*x^3 + I*b*c^2*x^2 + b*c*x + I*b)) - 2*(a*c^
2*x^2 + I*a*c*x + 2*a)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^4*d*f^3*x^3
 + I*c^3*d*f^3*x^2 + c^2*d*f^3*x + I*c*d*f^3)
```

Sympy [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx = \int \frac{a+b\operatorname{asinh}(cx)}{\sqrt{id\,(cx-i)}\,(-if\,(cx+i))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))/(f-I*c*f*x)**(5/2)/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(5/2)),
x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.79

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx = -\frac{1}{3}\,bc\left(\frac{3}{3i\,c^3\sqrt{d}f^{\frac{5}{2}}x-3\,c^2\sqrt{d}f^{\frac{5}{2}}}+\frac{\log\left(cx+i\right)}{c^2\sqrt{d}f^{\frac{5}{2}}}\right) - \frac{1}{3}\,b\left(-\frac{i\sqrt{c^2dfx^2+df}}{c^3df^3x^2+2i\,c^2df^3x-cdf^3}+\frac{3i\sqrt{c^2dfx^2+df}}{-3i\,c^2df^3x+3\,cdf^3}\right)\text{arsinh}\left(cx\right) - \frac{1}{3}\,a\left(-\frac{i\sqrt{c^2dfx^2+df}}{c^3df^3x^2+2i\,c^2df^3x-cdf^3}+\frac{3i\sqrt{c^2dfx^2+df}}{-3i\,c^2df^3x+3\,cdf^3}\right)$$

```
[In] integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] -1/3*b*c*(3/(3*I*c^3*sqrt(d)*f^(5/2)*x - 3*c^2*sqrt(d)*f^(5/2)) + log(c*x +
 I)/(c^2*sqrt(d)*f^(5/2))) - 1/3*b*(-I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*d*f^3*x
^2 + 2*I*c^2*d*f^3*x - c*d*f^3) + 3*I*sqrt(c^2*d*f*x^2 + d*f)/(-3*I*c^2*d*f
^3*x + 3*c*d*f^3))*arcsinh(c*x) - 1/3*a*(-I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*d*
f^3*x^2 + 2*I*c^2*d*f^3*x - c*d*f^3) + 3*I*sqrt(c^2*d*f*x^2 + d*f)/(-3*I*c^
2*d*f^3*x + 3*c*d*f^3))
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx = \int \frac{b\,\text{arsinh}\left(cx\right)+a}{\sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(I*c*d*x + d)*(-I*c*f*x + f)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx = \int \frac{a+b\,\mathrm{asinh}(c\,x)}{\sqrt{d+c\,d\,x\,1\mathrm{i}}\,{(f-c\,f\,x\,1\mathrm{i})}^{5/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2)), x)
```

3.568 $\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 282

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx = \frac{ibd(1+c^2x^2)^{5/2}}{6c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{d(i-cx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2dx(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{ibd(1+c^2x^2)^{5/2}\arctan(cx)}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bd(1+c^2x^2)^{5/2}\log\left(1+c^2x^2\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[Out] 1/6*I*b*d*(c^2*x^2+1)^(5/2)/c/(I+c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1
/3*d*(I-c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)
^(5/2)+2/3*d*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*
x)^(5/2)+1/6*I*b*d*(c^2*x^2+1)^(5/2)*arctan(c*x)/c/(d+I*c*d*x)^(5/2)/(f-I*c
*f*x)^(5/2)-1/3*b*d*(c^2*x^2+1)^(5/2)*ln(c^2*x^2+1)/c/(d+I*c*d*x)^(5/2)/(f-
I*c*f*x)^(5/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5796, 653, 197, 5837, 641, 46, 209, 266}

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx = \frac{2dx(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{d(-cx+i)\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{ibd(c^2x^2+1)^{5/2}\arctan(cx)}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{ibd(c^2x^2+1)^{5/2}}{6c(cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bd(c^2x^2+1)^{5/2}\log\left(c^2x^2+1\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] ((I/6)*b*d*(1 + c^2*x^2)^(5/2))/(c*(I + c*x)*(d + I*c*d*x)^(5/2)*(f - I*c*f
*x)^(5/2)) - (d*(I - c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(3*c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*d*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*
x]))/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((I/6)*b*d*(1 + c^2*x^2)
^(5/2)*ArcTan[c*x])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (b*d*(1 +
 c^2*x^2)^(5/2)*Log[1 + c^2*x^2])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5837

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\text{integral} = \frac{(1+c^2x^2)^{5/2}\int \frac{(d+icdx)(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{d(i-cx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2dx(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(bc(1+c^2x^2)^{5/2}\right)\int\left(-\frac{d(i-cx)}{3c(1+c^2x^2)^2}+\frac{2dx}{3(1+c^2x^2)}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{d(i-cx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2dx(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(bd(1+c^2x^2)^{5/2}\right)\int\frac{i-cx}{(1+c^2x^2)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2bcd(1+c^2x^2)^{5/2}\right)\int\frac{x}{1+c^2x^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{d(i-cx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2dx(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bd(1+c^2x^2)^{5/2}\log\left(1+c^2x^2\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(bd(1+c^2x^2)^{5/2}\right)\int\frac{1}{(-i-cx)^2(i-cx)}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{d(i-cx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2dx(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bd(1+c^2x^2)^{5/2}\log\left(1+c^2x^2\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(bd(1+c^2x^2)^{5/2}\right)\int\left(-\frac{i}{2(i+cx)^2}+\frac{i}{2(1+c^2x^2)}\right)dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$
\begin{aligned}
&= \frac{ibd(1+c^2x^2)^{5/2}}{6c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{d(i-cx)\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{2dx(1+c^2x^2)^2\,(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{bd(1+c^2x^2)^{5/2}\log\left(1+c^2x^2\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(ibd(1+c^2x^2)^{5/2}\right)\int\frac{1}{1+c^2x^2}\,dx}{6(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&= \frac{ibd(1+c^2x^2)^{5/2}}{6c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{d(i-cx)\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{2dx(1+c^2x^2)^2\,(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{ibd(1+c^2x^2)^{5/2}\arctan(cx)}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bd(1+c^2x^2)^{5/2}\log\left(1+c^2x^2\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.72

$$
\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx = \frac{\sqrt{f-icfx}\left(4ia-8acx+8iac^2x^2-2b\sqrt{1+c^2x^2}+4ib(1+2icx+2c^2x^2)\right.}{}
$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] (Sqrt[f - I*c*f*x]*((4*I)*a - 8*a*c*x + (8*I)*a*c^2*x^2 - 2*b*Sqrt[1 + c^2*
x^2] + (4*I)*b*(1 + (2*I)*c*x + 2*c^2*x^2)*ArcSinh[c*x] + 5*b*(1 - I*c*x)*S
qrt[1 + c^2*x^2]*Log[d*(-1 + I*c*x)] + 3*b*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*
x] - (3*I)*b*c*x*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*x]))/(12*c*d*f^3*(I + c*x)
^2*Sqrt[d + I*c*d*x])
```

Maple [F]

$$
\int \frac{a+b\,\operatorname{arcsinh}(cx)}{(icdx+d)^{\frac{3}{2}}\,(-icfx+f)^{\frac{5}{2}}}dx
$$

```
[In] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x)
```

Fricas [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + icdx)^{3/2}(f - icfx)^{5/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{(i\,cdx + d)^{\frac{3}{2}}(-i\,cfx + f)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] -1/24*(4*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x - 8*(
2*b*c^2*x^2 + 2*I*b*c*x + b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) - 3*(c^4*d^2*f^3*x^3 + I*c^3*d^2*f^3*x^2 + c^2*d^2*f^3*
x + I*c*d^2*f^3)*sqrt(b^2/(c^2*d^3*f^5))*log(-(I*sqrt(c^2*x^2 + 1)*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f^2*x*sqrt(b^2/(c^2*d^3*f^5)) + I*b*c^2*x^
3 + I*b*x)/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) + 5*(c^4*d^2*f^3*x^3 +
I*c^3*d^2*f^3*x^2 + c^2*d^2*f^3*x + I*c*d^2*f^3)*sqrt(b^2/(c^2*d^3*f^5))*lo
g(-(I*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f^2*x*sqrt
(b^2/(c^2*d^3*f^5)) - I*b*c^2*x^3 - I*b*x)/(b*c^3*x^3 + I*b*c^2*x^2 + b*c*x
 + I*b)) + 3*(c^4*d^2*f^3*x^3 + I*c^3*d^2*f^3*x^2 + c^2*d^2*f^3*x + I*c*d^2
*f^3)*sqrt(b^2/(c^2*d^3*f^5))*log(-(-I*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*
sqrt(-I*c*f*x + f)*c*d*f^2*x*sqrt(b^2/(c^2*d^3*f^5)) + I*b*c^2*x^3 + I*b*x)
/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) - 5*(c^4*d^2*f^3*x^3 + I*c^3*d^2*
f^3*x^2 + c^2*d^2*f^3*x + I*c*d^2*f^3)*sqrt(b^2/(c^2*d^3*f^5))*log(-(-I*sqr
t(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f^2*x*sqrt(b^2/(c^2
*d^3*f^5)) - I*b*c^2*x^3 - I*b*x)/(b*c^3*x^3 + I*b*c^2*x^2 + b*c*x + I*b))
+ 8*(c^4*d^2*f^3*x^3 + I*c^3*d^2*f^3*x^2 + c^2*d^2*f^3*x + I*c*d^2*f^3)*sqr
t(b^2/(c^2*d^3*f^5))*log((sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x
 + f)*c*d*f^2*x*sqrt(b^2/(c^2*d^3*f^5)) + b*c^2*x^3 + b*x)/(b*c^2*x^2 + b))
 - 8*(c^4*d^2*f^3*x^3 + I*c^3*d^2*f^3*x^2 + c^2*d^2*f^3*x + I*c*d^2*f^3)*sq
rt(b^2/(c^2*d^3*f^5))*log(-(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f
*x + f)*c*d*f^2*x*sqrt(b^2/(c^2*d^3*f^5)) - b*c^2*x^3 - b*x)/(b*c^2*x^2 + b
)) - 8*(2*a*c^2*x^2 + 2*I*a*c*x + a)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) -
 24*(c^4*d^2*f^3*x^3 + I*c^3*d^2*f^3*x^2 + c^2*d^2*f^3*x + I*c*d^2*f^3)*int
egral(-1/6*sqrt(c^2*x^2 + 1)*(4*b*c*x - I*b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*
x + f)/(c^4*d^2*f^3*x^4 + 2*c^2*d^2*f^3*x^2 + d^2*f^3), x))/(c^4*d^2*f^3*x^
3 + I*c^3*d^2*f^3*x^2 + c^2*d^2*f^3*x + I*c*d^2*f^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.84

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx=\frac{1}{12}\,bc\left(\frac{2i\,\sqrt{d}\sqrt{f}}{c^3d^2f^3x+i\,c^2d^2f^3}-\frac{5\,\log\left(cx+i\right)}{c^2d^{\frac{3}{2}}f^{\frac{5}{2}}}-\frac{3\,\log\left(cx-i\right)}{c^2d^{\frac{3}{2}}f^{\frac{5}{2}}}\right)$$
$$-\frac{1}{3}\,b\left(\frac{3i}{-3i\,\sqrt{c^2dfx^2+df}c^2df^2x+3\,\sqrt{c^2dfx^2+df}cdf^2}-\frac{2\,x}{\sqrt{c^2dfx^2+df}df^2}\right)\operatorname{arsinh}\left(cx\right)$$
$$-\frac{1}{3}\,a\left(\frac{3i}{-3i\,\sqrt{c^2dfx^2+df}c^2df^2x+3\,\sqrt{c^2dfx^2+df}cdf^2}-\frac{2\,x}{\sqrt{c^2dfx^2+df}df^2}\right)$$

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="maxima")
[Out] 1/12*b*c*(2*I*sqrt(d)*sqrt(f)/(c^3*d^2*f^3*x + I*c^2*d^2*f^3) - 5*log(c*x +
 I)/(c^2*d^(3/2)*f^(5/2)) - 3*log(c*x - I)/(c^2*d^(3/2)*f^(5/2))) - 1/3*b*(
3*I/(-3*I*sqrt(c^2*d*f*x^2 + d*f)*c^2*d*f^2*x + 3*sqrt(c^2*d*f*x^2 + d*f)*c
*d*f^2) - 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d*f^2))*arcsinh(c*x) - 1/3*a*(3*I/(-
3*I*sqrt(c^2*d*f*x^2 + d*f)*c^2*d*f^2*x + 3*sqrt(c^2*d*f*x^2 + d*f)*c*d*f^2
) - 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d*f^2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0,0
,0]ext_reduce Error: Bad Argument TypeThe choice was done assuming 0=[0,0,0
,0]ext
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{{\left(d+c\,d\,x\,1\text{i}\right)}^{3/2}\,{\left(f-c\,f\,x\,1\text{i}\right)}^{5/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2)),x)
[Out] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2)), x)
```

3.569 $\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 203

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx = \frac{b(1+c^2x^2)^{3/2}}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2x(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b(1+c^2x^2)^{5/2}\log(1+c^2x^2)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[Out] 1/6*b*(c^2*x^2+1)^(3/2)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*x*(c^2*x^
2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3*x*(c^2*x^2+
1)^2*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b*(c^2*x^2+
1)^(5/2)*ln(c^2*x^2+1)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5796, 5788, 5787, 266, 267}

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx = \frac{2x(c^2x^2+1)^2\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x(c^2x^2+1)\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b(c^2x^2+1)^{3/2}}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b(c^2x^2+1)^{5/2}\log(c^2x^2+1)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] (b*(1 + c^2*x^2)^(3/2))/(6*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (x*
(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) + (2*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(5/2)*(f
 - I*c*f*x)^(5/2)) - (b*(1 + c^2*x^2)^(5/2)*Log[1 + c^2*x^2])/(3*c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(1+c^2x^2)^{5/2}\int \frac{a+b\text{arcsinh}(cx)}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&= \frac{x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2(1+c^2x^2)^{5/2}\right)\int \frac{a+b\text{arcsinh}(cx)}{(1+c^2x^2)^{3/2}}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{\left(bc(1+c^2x^2)^{5/2}\right)\int \frac{x}{(1+c^2x^2)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&= \frac{b(1+c^2x^2)^{3/2}}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{2x(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2bc(1+c^2x^2)^{5/2}\right)\int \frac{x}{1+c^2x^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&= \frac{b(1+c^2x^2)^{3/2}}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{2x(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b(1+c^2x^2)^{5/2}\log(1+c^2x^2)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.95

$$
\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx = \frac{i\sqrt{f-icfx}\Big(6acx+4ac^3x^3+b\sqrt{1+c^2x^2}+2bcx(3+2c^2x^2)\,\text{arcsinh}(cx)}{6cd^2 j}
$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] ((I/6)*Sqrt[f - I*c*f*x]*(6*a*c*x + 4*a*c^3*x^3 + b*Sqrt[1 + c^2*x^2] + 2*b
*c*x*(3 + 2*c^2*x^2)*ArcSinh[c*x] - 2*b*(1 + c^2*x^2)^(3/2)*Log[d*(-1 + I*c
*x)] - 2*b*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*x] - 2*b*c^2*x^2*Sqrt[1 + c^2*x^
2]*Log[d + I*c*d*x]))/(c*d^2*f^3*(-I + c*x)*(I + c*x)^2*Sqrt[d + I*c*d*x])
```

Maple [F]

$$\int \frac{a+b\,\mathrm{arcsinh}\,(cx)}{(icdx+d)^{\frac{5}{2}}\,(-icfx+f)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x)
```

Fricas [F]

$$\int \frac{a+b\mathrm{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx = \int \frac{b\,\mathrm{arsinh}\,(cx)+a}{(i\,cdx+d)^{\frac{5}{2}}(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] -1/6*(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x^2 - 2*(2
*b*c^2*x^3 + 3*b*x)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2
*x^2 + 1)) - (c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)*sqrt(b^2/(c^2*
d^5*f^5))*log((sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2
*f^2*x^2*sqrt(b^2/(c^2*d^5*f^5)) + b*c^2*x^4 + b*x^2)/(b*c^4*x^4 + 2*b*c^2*
x^2 + b)) + (c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)*sqrt(b^2/(c^2*d
^5*f^5))*log(-(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2
*f^2*x^2*sqrt(b^2/(c^2*d^5*f^5)) - b*c^2*x^4 - b*x^2)/(b*c^4*x^4 + 2*b*c^2*
x^2 + b)) + 2*(c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)*sqrt(b^2/(c^2
*d^5*f^5))*log((sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^
2*f^2*x*sqrt(b^2/(c^2*d^5*f^5)) + b*c^2*x^3 + b*x)/(b*c^2*x^2 + b)) - 2*(c^
4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)*sqrt(b^2/(c^2*d^5*f^5))*log(-(
sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2*f^2*x*sqrt(b^2
/(c^2*d^5*f^5)) - b*c^2*x^3 - b*x)/(b*c^2*x^2 + b)) - 2*(2*a*c^2*x^3 + 3*a*
x)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) - 6*(c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^
3*x^2 + d^3*f^3)*integral(-2/3*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*
c*f*x + f)*b*c*x/(c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3), x))/(c^4*
d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + icdx)^{5/2}(f - icfx)^{5/2}}\, dx = \text{Timed out}$$

```
[In] integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.78

$$\begin{aligned}\int \frac{a + b\text{arcsinh}(cx)}{(d + icdx)^{5/2}(f - icfx)^{5/2}}\, dx &= \frac{1}{6}\, bc\left(\frac{1}{c^4 d^{\frac{5}{2}} f^{\frac{5}{2}} x^2 + c^2 d^{\frac{5}{2}} f^{\frac{5}{2}}} - \frac{2\, \log\left(c^2x^2 + 1\right)}{c^2 d^{\frac{5}{2}} f^{\frac{5}{2}}}\right) \\ &+ \frac{1}{3}\, b\left(\frac{x}{\left(c^2dfx^2 + df\right)^{\frac{3}{2}} df} + \frac{2\, x}{\sqrt{c^2dfx^2 + df}\, d^2 f^2}\right) \text{arsinh}\left(cx\right) \\ &+ \frac{1}{3}\, a\left(\frac{x}{\left(c^2dfx^2 + df\right)^{\frac{3}{2}} df} + \frac{2\, x}{\sqrt{c^2dfx^2 + df}\, d^2 f^2}\right)\end{aligned}$$

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="maxima")
[Out] 1/6*b*c*(1/(c^4*d^(5/2)*f^(5/2)*x^2 + c^2*d^(5/2)*f^(5/2)) - 2*log(c^2*x^2
+ 1)/(c^2*d^(5/2)*f^(5/2))) + 1/3*b*(x/((c^2*d*f*x^2 + d*f)^(3/2)*d*f) + 2*
x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f^2))*arcsinh(c*x) + 1/3*a*(x/((c^2*d*f*x^2
+ d*f)^(3/2)*d*f) + 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f^2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + icdx)^{5/2}(f - icfx)^{5/2}}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0,0
,0]ext_reduce Error: Bad Argument TypeThe choice was done assuming 0=[0,0,0
,0]ext
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx = \int \frac{a+b\,\text{asinh}(c\,x)}{\left(d+c\,d\,x\,1\text{i}\right)^{5/2}\left(f-c\,f\,x\,1\text{i}\right)^{5/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2)),x)
[Out] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2)), x)
```

3.570 $\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 37, antiderivative size = 680

$$\begin{aligned}
\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx &= \frac{8ib^2d^2\sqrt{d+icdx}\sqrt{f-icfx}}{9c} \\
&+ \frac{15}{64}b^2d^2x\sqrt{d+icdx}\sqrt{f-icfx} - \frac{1}{32}b^2c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx} \\
&+ \frac{4ib^2d^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)}{27c} \\
&- \frac{15b^2d^2\sqrt{d+icdx}\sqrt{f-icfx}\text{arcsinh}(cx)}{64c\sqrt{1+c^2x^2}} \\
&- \frac{4ibd^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} \\
&- \frac{3bcd^2x^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} \\
&- \frac{4ibc^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} \\
&+ \frac{bc^3d^2x^4\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} \\
&+ \frac{3}{8}d^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 \\
&- \frac{1}{4}c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 \\
&+ \frac{2id^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3c} \\
&+ \frac{5d^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{24bc\sqrt{1+c^2x^2}}
\end{aligned}$$

```
[Out] 8/9*I*b^2*d^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+15/64*b^2*d^2*x*(d+I*c*
d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/32*b^2*c^2*d^2*x^3*(d+I*c*d*x)^(1/2)*(f-I*c*
```

```
f*x)^(1/2)+4/27*I*b^2*d^2*(c^2*x^2+1)*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c
+3/8*d^2*x*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/4*c^2
*d^2*x^3*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)+2/3*I*d^2
*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c-15/
64*b^2*d^2*arcsinh(c*x)*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c/(c^2*x^2+1)^(
1/2)-4/3*I*b*d^2*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(
c^2*x^2+1)^(1/2)-3/8*b*c*d^2*x^2*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*
c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-4/9*I*b*c^2*d^2*x^3*(a+b*arcsinh(c*x))*(d+I*
c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/8*b*c^3*d^2*x^4*(a+b*arc
sinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+5/24*d^2*(
a+b*arcsinh(c*x))^3*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/
2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {5796, 5838, 5785, 5783, 5776, 327, 221, 5798, 5784, 455, 45, 5806, 5812}

$$\begin{aligned}
&\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx= \\
&-\frac{1}{4}c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 \\
&-\frac{3bcd^2x^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{8\sqrt{c^2x^2+1}} \\
&-\frac{4ibd^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}} \\
&+\frac{5d^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{24bc\sqrt{c^2x^2+1}} \\
&+\frac{2id^2(c^2x^2+1)\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{3c} \\
&-\frac{4ibc^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{9\sqrt{c^2x^2+1}} \\
&+\frac{bc^3d^2x^4\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{8\sqrt{c^2x^2+1}} \\
&+\frac{3}{8}d^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 \\
&-\frac{15b^2d^2\text{arcsinh}(cx)\sqrt{d+icdx}\sqrt{f-icfx}}{64c\sqrt{c^2x^2+1}} \\
&-\frac{1}{32}b^2c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}+\frac{4ib^2d^2(c^2x^2+1)\sqrt{d+icdx}\sqrt{f-icfx}}{27c} \\
&+\frac{15}{64}b^2d^2x\sqrt{d+icdx}\sqrt{f-icfx}+\frac{8ib^2d^2\sqrt{d+icdx}\sqrt{f-icfx}}{9c}
\end{aligned}$$

```
[In] Int[(d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (((8*I)/9)*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/c + (15*b^2*d^2*x*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/64 - (b^2*c^2*d^2*x^3*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x])/32 + (((4*I)/27)*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c
*f*x]*(1 + c^2*x^2))/c - (15*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Ar
cSinh[c*x])/(64*c*Sqrt[1 + c^2*x^2]) - (((4*I)/3)*b*d^2*x*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] - (3*b*c*d^2*x^2
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c^2*
x^2]) - (((4*I)/9)*b*c^2*d^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b
*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] + (b*c^3*d^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c^2*x^2]) + (3*d^2*x*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/8 - (c^2*d^2*x^3*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/4 + (((2*I)/3)*d^2*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/c +
 (5*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^3)/(24*b*c
*Sqrt[1 + c^2*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int(d+icdx)^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\left(d^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2+2icd^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\,dx}{\sqrt{1+c^2x^2}} + \frac{\left(2icd^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\,dx}{\sqrt{1+c^2x^2}} - \frac{\left(c^2d^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{2}d^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 - \frac{1}{4}c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 + \frac{2id^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c} + \frac{\left(d^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}} - \frac{\left(4ibd^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int(1+c^2x^2)(a+b\text{arcsinh}(cx))\,dx}{3\sqrt{1+c^2x^2}} - \frac{\left(bcd^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x(a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}} - \frac{\left(c^2d^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{4\sqrt{1+c^2x^2}} + \frac{\left(bc^3d^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x^3(a+b\text{arcsinh}(cx))\,dx}{2\sqrt{1+c^2x^2}}$$

$$= -\frac{4ibd^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{3\sqrt{1+c^2x^2}}$$
$$-\frac{bcd^2x^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{2\sqrt{1+c^2x^2}}$$
$$-\frac{4ibc^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{9\sqrt{1+c^2x^2}}$$
$$+\frac{bc^3d^2x^4\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{8\sqrt{1+c^2x^2}}$$
$$+\frac{3}{8}d^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2$$
$$-\frac{1}{4}c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2$$
$$+\frac{2id^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)\left(a+b\operatorname{arcsinh}(cx)\right)^2}{3c}$$
$$+\frac{d^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^3}{6bc\sqrt{1+c^2x^2}}$$
$$+\frac{\left(d^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{8\sqrt{1+c^2x^2}}$$
$$+\frac{\left(bcd^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x(a+b\operatorname{arcsinh}(cx))\,dx}{4\sqrt{1+c^2x^2}}$$
$$+\frac{\left(4ib^2cd^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\frac{x\left(1+\frac{c^2x^2}{3}\right)}{\sqrt{1+c^2x^2}}\,dx}{3\sqrt{1+c^2x^2}}$$
$$+\frac{\left(b^2c^2d^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\frac{x^2}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}}$$
$$-\frac{\left(b^2c^4d^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\frac{x^4}{\sqrt{1+c^2x^2}}\,dx}{8\sqrt{1+c^2x^2}}$$

$$= \frac{1}{4}b^2d^2x\sqrt{d+icdx}\sqrt{f-icfx} - \frac{1}{32}b^2c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx} - \frac{4ibd^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} - \frac{3bcd^2x^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} - \frac{4ibc^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} + \frac{bc^3d^2x^4\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} + \frac{3}{8}d^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 - \frac{1}{4}c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 + \frac{2id^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c} + \frac{5d^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{24bc\sqrt{1+c^2x^2}} - \frac{\left(b^2d^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{4\sqrt{1+c^2x^2}} + \frac{\left(2ib^2cd^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\text{Subst}\left(\int\frac{1+\frac{c^2x}{3}}{\sqrt{1+c^2x}}\,dx,x,x^2\right)}{3\sqrt{1+c^2x^2}} + \frac{\left(3b^2c^2d^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\frac{x^2}{\sqrt{1+c^2x^2}}\,dx}{32\sqrt{1+c^2x^2}} - \frac{\left(b^2c^2d^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\frac{x^2}{\sqrt{1+c^2x^2}}\,dx}{8\sqrt{1+c^2x^2}}$$

$$
\begin{aligned}
&= \frac{15}{64}b^2d^2x\sqrt{d+icdx}\sqrt{f-icfx} - \frac{1}{32}b^2c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx} \\
&\quad - \frac{b^2d^2\sqrt{d+icdx}\sqrt{f-icfx}\text{arcsinh}(cx)}{4c\sqrt{1+c^2x^2}} \\
&\quad - \frac{4ibd^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} \\
&\quad - \frac{3bcd^2x^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} \\
&\quad - \frac{4ibc^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} \\
&\quad + \frac{bc^3d^2x^4\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} \\
&\quad + \frac{3}{8}d^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 \\
&\quad - \frac{1}{4}c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 \\
&\quad + \frac{2id^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3c} \\
&\quad + \frac{5d^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{24bc\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(3b^2d^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{64\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(b^2d^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{16\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(2ib^2cd^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\text{Subst}\left(\int\left(\frac{2}{3\sqrt{1+c^2x}}+\frac{1}{3}\sqrt{1+c^2x}\right)\,dx, x, x^2\right)}{3\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{8ib^2d^2\sqrt{d+icdx}\sqrt{f-icfx}}{9c} + \frac{15}{64}b^2d^2x\sqrt{d+icdx}\sqrt{f-icfx} \\
&- \frac{1}{32}b^2c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx} + \frac{4ib^2d^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)}{27c} \\
&- \frac{15b^2d^2\sqrt{d+icdx}\sqrt{f-icfx}\text{arcsinh}(cx)}{64c\sqrt{1+c^2x^2}} \\
&- \frac{4ibd^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} \\
&- \frac{3bcd^2x^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} \\
&- \frac{4ibc^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} \\
&+ \frac{bc^3d^2x^4\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} \\
&+ \frac{3}{8}d^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 \\
&- \frac{1}{4}c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 \\
&+ \frac{2id^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c} \\
&+ \frac{5d^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{24bc\sqrt{1+c^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.60 (sec) , antiderivative size = 890, normalized size of antiderivative = 1.31

$$
\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx = \frac{-6912iabcd^2x\sqrt{d+icdx}\sqrt{f-icfx}+4608ia^2d^2\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+69}{}
$$

```
[In] Integrate[(d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((-6912*I)*a*b*c*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (4608*I)*a^2*d
^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (6912*I)*b^2*d^2
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 2592*a^2*c*d^2*x*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (4608*I)*a^2*c^2*d^2
*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 1728*a^2*c^3*d
^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 1440*b^2*d^2
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 - 1728*a*b*d^2*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] + (256*I)*b^2*d^2*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[3*ArcSinh[c*x]] + 108*a*b*d^2*Sqrt[d + I*c
```

```
*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 4320*a^2*d^(5/2)*Sqrt[f]*Sqr
t[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c
*f*x]] + 864*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x
]] - (768*I)*a*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x
]] - 27*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] +
12*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((-576*I)*b*c*x +
 (576*I)*a*Sqrt[1 + c^2*x^2] - 144*b*Cosh[2*ArcSinh[c*x]] + (192*I)*a*Cosh[
3*ArcSinh[c*x]] + 9*b*Cosh[4*ArcSinh[c*x]] + 288*a*Sinh[2*ArcSinh[c*x]] - (
64*I)*b*Sinh[3*ArcSinh[c*x]] - 36*a*Sinh[4*ArcSinh[c*x]]) + 72*b*d^2*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a + (48*I)*b*Sqrt[1 + c^2*
x^2] + (16*I)*b*Cosh[3*ArcSinh[c*x]] + 24*b*Sinh[2*ArcSinh[c*x]] - 3*b*Sinh
[4*ArcSinh[c*x]]))/(6912*c*Sqrt[1 + c^2*x^2])
```

Maple [F]

$$\int (icdx+d)^{\frac{5}{2}} (a+b\,\mathrm{arcsinh}\,(cx))^2 \sqrt{-icfx+f}dx$$

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x)
```

Fricas [F]

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\mathrm{arcsinh}(cx))^2\,dx = \int (i\,cdx+d)^{\frac{5}{2}}\sqrt{-i\,cfx+f}(b\,\mathrm{arsinh}\,(cx)+a)^2\,dx$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(b^2*c^2*d^2*x^2 - 2*I*b^2*c*d^2*x - b^2*d^2)*sqrt(I*c*d*x + d)*s
qrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 2*(a*b*c^2*d^2*x^2 - 2*I
*a*b*c*d^2*x - a*b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt
(c^2*x^2 + 1)) - (a^2*c^2*d^2*x^2 - 2*I*a^2*c*d^2*x - a^2*d^2)*sqrt(I*c*d*x
 + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx=\text{Timed out}$$

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))**2*(f-I*c*f*x)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx=\int (a+b\,\text{asinh}(c\,x))^2\,(d+c\,d\,x\,1\text{i})^{5/2}\,\sqrt{f-c\,f\,x\,1\text{i}}\,dx$$

```
[In] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2),x)
[Out] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2), x)
```

3.571 $\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\,\mathrm{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 37, antiderivative size = 508

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\,\mathrm{arcsinh}(cx))^2\,dx = \frac{4ib^2d\sqrt{d+icdx}\sqrt{f-icfx}}{9c} + \frac{1}{4}b^2dx\sqrt{d+icdx}\sqrt{f-icfx} + \frac{2ib^2d\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)}{27c} - \frac{b^2d\sqrt{d+icdx}\sqrt{f-icfx}\,\mathrm{arcsinh}(cx)}{4c\sqrt{1+c^2x^2}} - \frac{2ibdx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\,\mathrm{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} - \frac{bcdx^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\,\mathrm{arcsinh}(cx))}{2\sqrt{1+c^2x^2}} - \frac{2ibc^2dx^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\,\mathrm{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} + \frac{1}{2}dx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\,\mathrm{arcsinh}(cx))^2 + \frac{id\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)\,(a+b\,\mathrm{arcsinh}(cx))^2}{3c} + \frac{d\sqrt{d+icdx}\sqrt{f-icfx}(a+b\,\mathrm{arcsinh}(cx))^3}{6bc\sqrt{1+c^2x^2}}$$

```
[Out] 4/9*I*b^2*d*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+1/4*b^2*d*x*(d+I*c*d*x)^(
1/2)*(f-I*c*f*x)^(1/2)+2/27*I*b^2*d*(c^2*x^2+1)*(d+I*c*d*x)^(1/2)*(f-I*c*f*
x)^(1/2)/c+1/2*d*x*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)
+1/3*I*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/
2)/c-1/4*b^2*d*arcsinh(c*x)*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c/(c^2*x^2+
1)^(1/2)-2/3*I*b*d*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)
/(c^2*x^2+1)^(1/2)-1/2*b*c*d*x^2*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*
```

```
c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-2/9*I*b*c^2*d*x^3*(a+b*arcsinh(c*x))*(d+I*c*
d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/6*d*(a+b*arcsinh(c*x))^3*(
d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.297, Rules used = {5796, 5838, 5785, 5783, 5776, 327, 221, 5798, 5784, 455, 45}

$$\begin{aligned}
&\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx= \\
&-\frac{bcdx^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{2\sqrt{c^2x^2+1}} \\
&-\frac{2ibdx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}} \\
&+\frac{d\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{c^2x^2+1}} \\
&+\frac{id(c^2x^2+1)\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{3c} \\
&-\frac{2ibc^2dx^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{9\sqrt{c^2x^2+1}} \\
&+\frac{1}{2}dx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 \\
&-\frac{b^2d\text{arcsinh}(cx)\sqrt{d+icdx}\sqrt{f-icfx}}{4c\sqrt{c^2x^2+1}}+\frac{2ib^2d(c^2x^2+1)\sqrt{d+icdx}\sqrt{f-icfx}}{27c} \\
&+\frac{1}{4}b^2dx\sqrt{d+icdx}\sqrt{f-icfx}+\frac{4ib^2d\sqrt{d+icdx}\sqrt{f-icfx}}{9c}
\end{aligned}$$

```
[In] Int[(d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (((4*I)/9)*b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/c + (b^2*d*x*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x])/4 + (((2*I)/27)*b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]*(1 + c^2*x^2))/c - (b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*A
rcSinh[c*x])/(4*c*Sqrt[1 + c^2*x^2]) - (((2*I)/3)*b*d*x*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] - (b*c*d*x^2*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(2*Sqrt[1 + c^2*x^2])
- (((2*I)/9)*b*c^2*d*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh
[c*x]))/Sqrt[1 + c^2*x^2] + (d*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b
*ArcSinh[c*x])^2)/2 + ((I/3)*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2
*x^2)*(a + b*ArcSinh[c*x])^2)/c + (d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a
 + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[1 + c^2*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int(d+icdx)\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\left(d\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2+icdx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\right)\,d}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\,dx}{\sqrt{1+c^2x^2}} + \frac{\left(icd\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\,dx}{\sqrt{1+c^2x^2}}$$

$$
\begin{aligned}
&= \frac{1}{2}dx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 \\
&\quad + \frac{id\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c} \\
&\quad + \frac{\left(d\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(2ibd\sqrt{d+icdx}\sqrt{f-icfx}\right)\int (1+c^2x^2)(a+b\text{arcsinh}(cx))\,dx}{3\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(bcd\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x(a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2ibdx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} \\
&\quad - \frac{bcdx^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}} \\
&\quad - \frac{2ibc^2dx^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} \\
&\quad + \frac{1}{2}dx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 \\
&\quad + \frac{id\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c} \\
&\quad + \frac{d\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(2ib^2cd\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \frac{x\left(1+\frac{c^2x^2}{3}\right)}{\sqrt{1+c^2x^2}}\,dx}{3\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(b^2c^2d\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \frac{x^2}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{4}b^2dx\sqrt{d+icdx}\sqrt{f-icfx} - \frac{2ibdx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} \\
&\quad - \frac{bcdx^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}} \\
&\quad - \frac{2ibc^2dx^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} \\
&\quad + \frac{1}{2}dx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 \\
&\quad + \frac{id\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3c} \\
&\quad + \frac{d\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(b^2d\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \frac{1}{\sqrt{1+c^2x^2}}\,dx}{4\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(ib^2cd\sqrt{d+icdx}\sqrt{f-icfx}\right)\text{Subst}\left(\int \frac{1+\frac{c^2x}{3}}{\sqrt{1+c^2x}}\,dx,x,x^2\right)}{3\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{4}b^2dx\sqrt{d+icdx}\sqrt{f-icfx} - \frac{b^2d\sqrt{d+icdx}\sqrt{f-icfx}\text{arcsinh}(cx)}{4c\sqrt{1+c^2x^2}} \\
&\quad - \frac{2ibdx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} \\
&\quad - \frac{bcdx^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}} \\
&\quad - \frac{2ibc^2dx^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} \\
&\quad + \frac{1}{2}dx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 \\
&\quad + \frac{id\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3c} \\
&\quad + \frac{d\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(ib^2cd\sqrt{d+icdx}\sqrt{f-icfx}\right)\text{Subst}\left(\int \left(\frac{2}{3\sqrt{1+c^2x}}+\frac{1}{3}\sqrt{1+c^2x}\right)\,dx,x,x^2\right)}{3\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4ib^2d\sqrt{d+icdx}\sqrt{f-icfx}}{9c} + \frac{1}{4}b^2dx\sqrt{d+icdx}\sqrt{f-icfx} \\
&+ \frac{2ib^2d\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)}{27c} - \frac{b^2d\sqrt{d+icdx}\sqrt{f-icfx}\text{arcsinh}(cx)}{4c\sqrt{1+c^2x^2}} \\
&- \frac{2ibdx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} \\
&- \frac{bcdx^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}} \\
&- \frac{2ibc^2dx^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} \\
&+ \frac{1}{2}dx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 \\
&+ \frac{id\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c} \\
&+ \frac{d\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{1+c^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.23 (sec) , antiderivative size = 705, normalized size of antiderivative = 1.39

$$
\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx = \frac{-108iabcdx\sqrt{d+icdx}\sqrt{f-icfx} + 72ia^2d\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2} + 108ib^2d\cdots}{\quad}
$$

```
[In] Integrate[(d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((-108*I)*a*b*c*d*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (72*I)*a^2*d*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (108*I)*b^2*d*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 108*a^2*c*d*x*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (72*I)*a^2*c^2*d*x^2*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 36*b^2*d*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]*ArcSinh[c*x]^3 - 54*a*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
*Cosh[2*ArcSinh[c*x]] + (4*I)*b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cos
h[3*ArcSinh[c*x]] + 108*a^2*d^(3/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x +
 Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 27*b^2*d*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 18*b*d*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]*ArcSinh[c*x]^2*(6*a + (3*I)*b*Sqrt[1 + c^2*x^2] + I*b*Cosh
[3*ArcSinh[c*x]] + 3*b*Sinh[2*ArcSinh[c*x]]) - (12*I)*a*b*d*Sqrt[d + I*c*d*
x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 6*b*d*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]*ArcSinh[c*x]*(-9*b*Cosh[2*ArcSinh[c*x]] + 2*((-9*I)*b*c*x + (9*I
)*a*Sqrt[1 + c^2*x^2] + (3*I)*a*Cosh[3*ArcSinh[c*x]] + 9*a*Sinh[2*ArcSinh[c
*x]] - I*b*Sinh[3*ArcSinh[c*x]])))/(216*c*Sqrt[1 + c^2*x^2])
```

Maple [F]

$$\int \left(icdx + d\right)^{\frac{3}{2}} \left(a + b\,\mathrm{arcsinh}\left(cx\right)\right)^{2} \sqrt{-icfx + f}dx$$

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x)
```

Fricas [F]

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a + b\mathrm{arcsinh}(cx))^2\,dx = \int \left(i\,cdx + d\right)^{\frac{3}{2}} \sqrt{-i\,cfx + f}\left(b\,\mathrm{arsinh}\left(cx\right) + a\right)^{2}dx$$

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((I*b^2*c*d*x + b^2*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x
 + sqrt(c^2*x^2 + 1))^2 - 2*(-I*a*b*c*d*x - a*b*d)*sqrt(I*c*d*x + d)*sqrt(-
I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a^2*c*d*x + a^2*d)*sqrt(I*c*
d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F]

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a + b\mathrm{arcsinh}(cx))^2\,dx = \int \left(id\left(cx - i\right)\right)^{\frac{3}{2}} \sqrt{- if\left(cx + i\right)}\left(a + b\,\mathrm{asinh}\left(cx\right)\right)^{2}\, dx$$

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))**2*(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral((I*d*(c*x - I))**(3/2)*sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))**2,
 x)
```

Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad Ar
gument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,(d+c\,d\,x\,1\text{i})^{3/2}\,\sqrt{f-c\,f\,x\,1\text{i}}\,dx$$

```
[In] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2), x)
```

3.572 $\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 37, antiderivative size = 244

$$\begin{aligned}\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx &= \frac{1}{4}b^2x\sqrt{d+icdx}\sqrt{f-icfx}-\frac{b^2\sqrt{d+icdx}\sqrt{f-icfx}\text{arcsinh}(cx)}{4c\sqrt{1+c^2x^2}}\\ &\quad-\frac{bcx^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}}\\ &\quad+\frac{1}{2}x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\\ &\quad+\frac{\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{1+c^2x^2}}\end{aligned}$$

```
[Out] 1/4*b^2*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)+1/2*x*(a+b*arcsinh(c*x))^2*(d
+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/4*b^2*arcsinh(c*x)*(d+I*c*d*x)^(1/2)*(f
-I*c*f*x)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/2*b*c*x^2*(a+b*arcsinh(c*x))*(d+I*c*d
*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/6*(a+b*arcsinh(c*x))^3*(d+I
*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used

$= \{5796, 5785, 5783, 5776, 327, 221\}$

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{c^2x^2+1}} - \frac{bcx^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{2\sqrt{c^2x^2+1}} + \frac{1}{2}x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 - \frac{b^2\text{arcsinh}(cx)\sqrt{d+icdx}\sqrt{f-icfx}}{4c\sqrt{c^2x^2+1}} + \frac{1}{4}b^2x\sqrt{d+icdx}\sqrt{f-icfx}$$

```
[In] Int[Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (b^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/4 - (b^2*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]*ArcSinh[c*x])/(4*c*Sqrt[1 + c^2*x^2]) - (b*c*x^2*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(2*Sqrt[1 + c^2*x^2]) + (x*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/2 + (Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[1 + c^2*x^2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\,dx}{\sqrt{1+c^2x^2}}\\
&= \frac{1}{2}x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\\
&\quad+\frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}}\\
&\quad-\frac{\left(bc\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x(a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}}\\
&= -\frac{bcx^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}}\\
&\quad+\frac{1}{2}x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\\
&\quad+\frac{\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{1+c^2x^2}}\\
&\quad+\frac{\left(b^2c^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\frac{x^2}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}}\\
&= \frac{1}{4}b^2x\sqrt{d+icdx}\sqrt{f-icfx}-\frac{bcx^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}}\\
&\quad+\frac{1}{2}x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\\
&\quad+\frac{\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{1+c^2x^2}}\\
&\quad-\frac{\left(b^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{4\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{4}b^2x\sqrt{d+icdx}\sqrt{f-icfx} - \frac{b^2\sqrt{d+icdx}\sqrt{f-icfx}\operatorname{arcsinh}(cx)}{4c\sqrt{1+c^2x^2}} \\
&\quad - \frac{bcx^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{2\sqrt{1+c^2x^2}} \\
&\quad + \frac{1}{2}x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2 \\
&\quad + \frac{\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^3}{6bc\sqrt{1+c^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.88 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.44

$$
\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2\,dx
$$

$$
= \frac{12a^2cx\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2} + 4b^2\sqrt{d+icdx}\sqrt{f-icfx}\operatorname{arcsinh}(cx)^3 - 6ab\sqrt{d+icdx}\sqrt{f-icf}
$$

```
[In] Integrate[Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (12*a^2*c*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 4*b^2*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 - 6*a*b*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] + 12*a^2*Sqrt[d]*Sqrt[f]*Sqrt[1 + c
^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]]
+ 3*b^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] - 6*b*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(b*Cosh[2*ArcSinh[c*x]] - 2*a*
Sinh[2*ArcSinh[c*x]]) + 6*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x
]^2*(2*a + b*Sinh[2*ArcSinh[c*x]]))/(24*c*Sqrt[1 + c^2*x^2])
```

Maple [F]

$$
\int (a+b\operatorname{arcsinh}(cx))^2\sqrt{icdx+d}\,\sqrt{-icfx+f}dx
$$

```
[In] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x)
```

Fricas [F]

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\int \sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}(b\,\text{arsinh}\,(cx)+a)^2\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 +
1))^2 + 2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x^2 +
 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2, x)
```

Sympy [F]

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\int \sqrt{id\,(cx-i)}\sqrt{-if\,(cx+i)}(a+b\,\text{asinh}\,(cx))^2\;dx$$

```
[In] integrate((a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2)*(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral(sqrt(I*d*(c*x - I))*sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad Ar
gument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx$$
$$= \int (a+b\,\text{asinh}(c\,x))^2\,\sqrt{d+c\,d\,x\,1\text{i}}\,\sqrt{f-c\,f\,x\,1\text{i}}\,dx$$

```
[In] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2), x)
```

3.573 $\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 259

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \frac{2iabfx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ib^2f(1+c^2x^2)}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ib^2fx\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{if(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{f\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[Out] -2*I*b^2*f*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-I*f*(c^2*x^2+1
)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+2*I*a*b*f*x*(c
^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+2*I*b^2*f*x*arcsinh(c*x
)*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/3*f*(a+b*arcsinh(
c*x))^3*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used

= {5796, 5838, 5783, 5798, 5772, 267}

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \frac{f\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{if(c^2x^2+1)\,(a+b\text{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2iabfx\sqrt{c^2x^2+1}}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ib^2fx\sqrt{c^2x^2+1}\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ib^2f(c^2x^2+1)}{c\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[In] Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((2*I)*a*b*f*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (
(2*I)*b^2*f*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + ((2*I)
*b^2*f*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]) - (I*f*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]) + (f*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
```

```
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1+c^2x^2}\int \frac{(f-icfx)(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}} \\
&= \frac{\sqrt{1+c^2x^2}\int \left(\frac{f(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}-\frac{icfx(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}} \\
&= \frac{\left(f\sqrt{1+c^2x^2}\right)\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{\left(icf\sqrt{1+c^2x^2}\right)\int \frac{x(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}} \\
&= -\frac{if(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{f\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad+\frac{\left(2ibf\sqrt{1+c^2x^2}\right)\int (a+b\operatorname{arcsinh}(cx))\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}} \\
&= \frac{2iabfx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{if(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad+\frac{f\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{\left(2ib^2f\sqrt{1+c^2x^2}\right)\int \operatorname{arcsinh}(cx)\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}
\end{aligned}
$$

$$= \frac{2iabfx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ib^2fx\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{if(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{f\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(2ib^2cf\sqrt{1+c^2x^2}\right)\int \frac{x}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{2iabfx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ib^2f(1+c^2x^2)}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ib^2fx\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{if(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{f\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

Mathematica [A] (verified)

Time = 2.71 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \frac{-3i\sqrt{d+icdx}\sqrt{f-icfx}\left(-2abcx+a^2\sqrt{1+c^2x^2}+2b^2\sqrt{1+c^2x^2}\right)+6ib\sqrt{d+icdx}\sqrt{f-icfx}\left(bcx-a\sqrt{\right.}}{}$$

```
[In] Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((-3*I)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-2*a*b*c*x + a^2*Sqrt[1 + c^2*
x^2] + 2*b^2*Sqrt[1 + c^2*x^2]) + (6*I)*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]*(b*c*x - a*Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 3*b*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]*(a - I*b*Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2 + b^2*Sqrt[d + I*c*
d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 + 3*a^2*Sqrt[d]*Sqrt[f]*Sqrt[1 + c^2*
x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/(3
*c*d*Sqrt[1 + c^2*x^2])
```

Maple [F]

$$\int \frac{(a+b\,\text{arcsinh}\,(cx))^2\,\sqrt{-icfx+f}}{\sqrt{icdx+d}}dx$$

```
[In] int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{f - icfx}(a + b\text{arcsinh}(cx))^2}{\sqrt{d + icdx}}\,dx = \int \frac{\sqrt{-i\,cfx + f}(b\,\text{arsinh}\,(cx) + a)^2}{\sqrt{i\,cdx + d}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((-I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^
2 + 1))^2 - 2*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2
*x^2 + 1)) - I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c*d*x - I*d), x)
```

Sympy [F]

$$\int \frac{\sqrt{f - icfx}(a + b\text{arcsinh}(cx))^2}{\sqrt{d + icdx}}\,dx = \int \frac{\sqrt{-if\,(cx + i)}(a + b\,\text{asinh}\,(cx))^2}{\sqrt{id\,(cx - i)}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Integral(sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))**2/sqrt(I*d*(c*x - I)), x)
```

Maxima [F]

$$\int \frac{\sqrt{f - icfx}(a + b\text{arcsinh}(cx))^2}{\sqrt{d + icdx}}\,dx = \int \frac{\sqrt{-i\,cfx + f}(b\,\text{arsinh}\,(cx) + a)^2}{\sqrt{i\,cdx + d}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] a^2*(f*arcsinh(c*x)/(c*d*sqrt(f/d)) - I*sqrt(c^2*d*f*x^2 + d*f)/(c*d)) + in
tegrate(sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/sqrt(I*c*d*x
+ d) + 2*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/sqrt(I*c*d*x +
 d), x)
```

Giac [F]

$$\int \frac{\sqrt{f - icfx}(a + b\text{arcsinh}(cx))^2}{\sqrt{d + icdx}}\, dx = \int \frac{\sqrt{-i\, cfx + f}(b\,\text{arsinh}\,(cx) + a)^2}{\sqrt{i\, cdx + d}}\, dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)^2/sqrt(I*c*d*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f - icfx}(a + b\text{arcsinh}(cx))^2}{\sqrt{d + icdx}}\, dx = \int \frac{(a + b\,\text{asinh}(c\,x))^2\,\sqrt{f - c\,f\,x\,1\text{i}}}{\sqrt{d + c\,d\,x\,1\text{i}}}\, dx$$

```
[In] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(1/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(1/2), x)
```

3.574 $\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 544

$$
\begin{aligned}
\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx &= \frac{2if^2(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{2f^2x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{f^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{8ibf^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{4bf^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{4b^2f^2(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{4b^2f^2(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{2b^2f^2(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}
$$

```
[Out] 2*I*f^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3
/2)+2*f^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^
(3/2)+2*f^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I
*c*f*x)^(3/2)-1/3*f^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x
)^(3/2)/(f-I*c*f*x)^(3/2)-8*I*b*f^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*ar
ctan(c*x+(c^2*x^2+1)^(1/2))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*b*f^2*(
```

```
c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*
c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*b^2*f^2*(c^2*x^2+1)^(3/2)*polylog(2,-I*(c*
x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*b^2*f^2*(c^2*
x^2+1)^(3/2)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*
c*f*x)^(3/2)-2*b^2*f^2*(c^2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))
^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {5796, 5844, 5838, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 5789, 4265, 5783}

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = -\frac{8ibf^2(c^2x^2+1)^{3/2}\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2if^2(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2x(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4bf^2(c^2x^2+1)^{3/2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4b^2f^2(c^2x^2+1)^{3/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4b^2f^2(c^2x^2+1)^{3/2}\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2b^2f^2(c^2x^2+1)^{3/2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[In] Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((2*I)*f^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f
- I*c*f*x)^(3/2)) + (2*f^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*
c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (2*f^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSi
nh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (f^2*(1 + c^2*x^2
)^(3/2)*(a + b*ArcSinh[c*x])^3)/(3*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3
/2)) - ((8*I)*b*f^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSi
nh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b*f^2*(1 + c^2*x
^2)^(3/2)*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b^2*f^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-
I)*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*b^2*f^
```

```
2*(1 + c^2*x^2)^(3/2)*PolyLog[2, I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*
(f - I*c*f*x)^(3/2)) - (2*b^2*f^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcS
inh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
```

```
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{(1+c^2x^2)^{3/2}\int \frac{(f-icfx)^2(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{(1+c^2x^2)^{3/2}\int \left(-\frac{2i(if^2+cf^2x)(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}-\frac{f^2(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(2i(1+c^2x^2)^{3/2}\right)\int \frac{(if^2+cf^2x)(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{\left(f^2(1+c^2x^2)^{3/2}\right)\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{f^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2i(1+c^2x^2)^{3/2}\right)\int \left(\frac{if^2(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}+\frac{cf^2x(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{f^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(2f^2(1+c^2x^2)^{3/2}\right)\int \frac{(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2icf^2(1+c^2x^2)^{3/2}\right)\int \frac{x(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{2if^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(4ibf^2(1+c^2x^2)^{3/2}\right)\int \frac{a+b\text{arcsinh}(cx)}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(4bcf^2(1+c^2x^2)^{3/2}\right)\int \frac{x(a+b\text{arcsinh}(cx))}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{2if^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(4ibf^2(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int (a+bx)\text{sech}(x)\,dx, x, \text{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(4bf^2(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \text{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{2if^2(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{2f^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{8ibf^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{\left(8bf^2(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \text{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{\left(4b^2f^2(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int \log\left(1-ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{\left(4b^2f^2(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int \log\left(1+ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{2if^2(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{2f^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{8ibf^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{4bf^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{\left(4b^2f^2(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{\left(4b^2f^2(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{\left(4b^2f^2(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{2if^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ibf^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4bf^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4b^2f^2(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4b^2f^2(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(2b^2f^2(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int\frac{\log(1+x)}{x}dx,x,e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{2if^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ibf^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4bf^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4b^2f^2(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4b^2f^2(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2b^2f^2(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Mathematica [A] (verified)

Time = 5.32 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = \frac{\frac{6a^2\sqrt{d+icdx}\sqrt{f-icfx}}{-i+cx} - 3a^2\sqrt{d}\sqrt{f}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-ic}\right.}{}$$

```
[In] Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((6*a^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(-I + c*x) - 3*a^2*Sqrt[d]*Sqr
t[f]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - (
3*a*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(ArcSinh[c*x]*((-4*I)*Cosh[ArcSin
h[c*x]/2] - 4*Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2]
+ I*Sinh[ArcSinh[c*x]/2]) + 2*((4*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + Log[1 +
 c^2*x^2])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 + c^2*
x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (b^2*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]*((-6 + 6*I)*ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - Si
nh[ArcSinh[c*x]/2]) - ArcSinh[c*x]^3*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh
[c*x]/2]) + (12*I)*Pi*(Log[1 - I/E^ArcSinh[c*x]] + 2*Log[1 + E^ArcSinh[c*x]
] - 2*Log[Cosh[ArcSinh[c*x]/2]] - Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]])*(C
osh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 24*PolyLog[2, I/E^ArcSinh[c
*x]]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 6*ArcSinh[c*x]*(Pi -
 (4*I)*Log[1 - I/E^ArcSinh[c*x]])*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh
[c*x]/2])))/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/
2])))/(3*c*d^2)
```

Maple [F]

$$\int \frac{(a+b\,\text{arcsinh}\,(cx))^2\sqrt{-icfx+f}}{(icdx+d)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = \int \frac{\sqrt{-i\,cfx+f}(b\,\text{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2
+ 1))^2 + 2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x^2
 + 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c^2*d^2*x^2 - 2*I*c*d^2
*x - d^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx=\int \frac{\sqrt{-if\,(cx+i)}(a+b\operatorname{asinh}{(cx)})^2}{(id\,(cx-i))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(3/2),x)
```

```
[Out] Integral(sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))**2/(I*d*(c*x - I))**(3/2),
 x)
```

Maxima [F]

$$\int \frac{\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx=\int \frac{\sqrt{-i\,cfx+f}(b\operatorname{arsinh}{(cx)}+a)^2}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] a^2*(2*I*sqrt(c^2*d*f*x^2 + d*f)/(I*c^2*d^2*x + c*d^2) - f*arcsinh(c*x)/(c*
d^2*sqrt(f/d))) + integrate(sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 +
 1))^2/(I*c*d*x + d)^(3/2) + 2*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x^
2 + 1))/(I*c*d*x + d)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx=\int \frac{\sqrt{-i\,cfx+f}(b\operatorname{arsinh}{(cx)}+a)^2}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)^2/(I*c*d*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx=\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,\sqrt{f-c\,f\,x\,1\mathrm{i}}}{(d+c\,d\,x\,1\mathrm{i})^{3/2}}\,dx$$

```
[In] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(3/2),x)
[Out] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(3/2), x)
```

3.575 $\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 518

$$
\begin{aligned}
\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx = & -\frac{f^3(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
& -\frac{4ib^2f^3(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
& -\frac{if^3(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
& +\frac{2bf^3(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
& +\frac{if^3(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
& +\frac{4bf^3(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))\log\left(1+ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
& +\frac{4b^2f^3(1+c^2x^2)^{5/2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

```
[Out] -1/3*f^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*
f*x)^(5/2)-4/3*I*b^2*f^3*(c^2*x^2+1)^(5/2)*cot(1/4*Pi+1/2*I*arcsinh(c*x))/c
/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*I*f^3*(c^2*x^2+1)^(5/2)*(a+b*arcsi
nh(c*x))^2*cot(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(
5/2)+2/3*b*f^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*csc(1/4*Pi+1/2*I*arcsin
h(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*I*f^3*(c^2*x^2+1)^(5/2)
*(a+b*arcsinh(c*x))^2*cot(1/4*Pi+1/2*I*arcsinh(c*x))*csc(1/4*Pi+1/2*I*arcsi
nh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+4/3*b*f^3*(c^2*x^2+1)^(5/2
)*(a+b*arcsinh(c*x))*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f
```

```
-I*c*f*x)^(5/2)+4/3*b^2*f^3*(c^2*x^2+1)^(5/2)*polylog(2,-I*(c*x+(c^2*x^2+1)
^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {5796, 5844, 5843, 3399, 4271, 3852, 8, 4269, 3797, 2221, 2317, 2438}

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx = -\frac{f^3(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{4bf^3(c^2x^2+1)^{5/2}\log\left(1+ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{if^3(c^2x^2+1)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{2bf^3(c^2x^2+1)^{5/2}\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{if^3(c^2x^2+1)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{4b^2f^3(c^2x^2+1)^{5/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{4ib^2f^3(c^2x^2+1)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[In] Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),x]
```

```
[Out] -1/3*(f^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(5/2
)*(f - I*c*f*x)^(5/2)) - (((4*I)/3)*b^2*f^3*(1 + c^2*x^2)^(5/2)*Cot[Pi/4 +
(I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((I/3)*f
^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]
])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*b*f^3*(1 + c^2*x^2)^(5/
2)*(a + b*ArcSinh[c*x])*Csc[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c*(d + I*c*d*x
)^(5/2)*(f - I*c*f*x)^(5/2)) + ((I/3)*f^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSin
h[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]]*Csc[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/
(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (4*b*f^3*(1 + c^2*x^2)^(5/2)*
(a + b*ArcSinh[c*x])*Log[1 + I*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f
 - I*c*f*x)^(5/2)) + (4*b^2*f^3*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSi
nh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(1+c^2x^2)^{5/2}\int\frac{(f-icfx)^3(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&= \frac{(1+c^2x^2)^{5/2}\int\left(-\frac{2f^3(a+b\text{arcsinh}(cx))^2}{(-i+cx)^2\sqrt{1+c^2x^2}}+\frac{if^3(a+b\text{arcsinh}(cx))^2}{(-i+cx)\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&= \frac{\left(if^3(1+c^2x^2)^{5/2}\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{(-i+cx)\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{\left(2f^3(1+c^2x^2)^{5/2}\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{(-i+cx)^2\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}$$

$$= \frac{\left(if^3(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int \frac{(a+bx)^2}{-ic+c\sinh(x)}\,dx,x,\text{arcsinh}(cx)\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2cf^3(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int \frac{(a+bx)^2}{(-ic+c\sinh(x))^2}\,dx,x,\text{arcsinh}(cx)\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{\left(f^3(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int (a+bx)^2\csc^2\left(\frac{\pi}{4}+\frac{ix}{2}\right)\,dx,x,\text{arcsinh}(cx)\right)}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(f^3(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int (a+bx)^2\csc^4\left(\frac{\pi}{4}+\frac{ix}{2}\right)\,dx,x,\text{arcsinh}(cx)\right)}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{if^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2bf^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{if^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(f^3(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int (a+bx)^2\csc^2\left(\frac{\pi}{4}+\frac{ix}{2}\right)\,dx,x,\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2ibf^3(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int (a+bx)\cot\left(\frac{\pi}{4}+\frac{ix}{2}\right)\,dx,x,\text{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2b^2f^3(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int \csc^2\left(\frac{\pi}{4}+\frac{ix}{2}\right)\,dx,x,\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$
\begin{aligned}
&= -\frac{f^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{if^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{2bf^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{if^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{\left(4ibf^3(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int(a+bx)\cot\left(\frac{\pi}{4}+\frac{ix}{2}\right)\,dx,x,\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{\left(4ibf^3(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int\frac{e^x(a+bx)}{1+ie^x}\,dx,x,\text{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{\left(4ib^2f^3(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int 1\,dx,x,\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{f^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{4ib^2f^3(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{if^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{2bf^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{if^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{4bf^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\log\left(1+ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{\left(8ibf^3(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int\frac{e^x(a+bx)}{1+ie^x}\,dx,x,\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{\left(4b^2f^3(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int\log\left(1+ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

$$=-\frac{f^3(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{4ib^2f^3(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{if^3(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{2bf^3(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{if^3(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{4bf^3(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{\left(8b^2f^3(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\log\left(1+ie^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{\left(4b^2f^3(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$=-\frac{f^3(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{4ib^2f^3(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{if^3(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{2bf^3(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{if^3(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{4bf^3(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{4b^2f^3(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{\left(8b^2f^3(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{f^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{4ib^2f^3(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{if^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{2bf^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{if^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{4bf^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\log\left(1+ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{4b^2f^3(1+c^2x^2)^{5/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Mathematica [A] (warning: unable to verify)

Time = 9.85 (sec) , antiderivative size = 783, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx = \frac{\sqrt{id(-i+cx)}\sqrt{-if(i+cx)}\left(-\frac{2ia^2}{3d^3(-i+cx)^2}-\frac{a^2}{3d^3(-i+cx)}\right)}{c}$$
$$+ \frac{iab\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\sqrt{-df(1+c^2x^2)}\left(\cosh\left(\frac{1}{2}\text{arcsinh}(cx)\right)-i\sinh\left(\frac{1}{2}\text{arcsinh}(cx)\right)\right)\left(-i\cosh\right.}{}$$
$$+ \frac{ib^2(i+cx)\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\sqrt{-df(1+c^2x^2)}\left((-1+i)\text{arcsinh}(cx)^2-\frac{2\text{arcsinh}(cx)(-2i+\text{arcsi}}{-i+cx}\right.}{}$$

```
[In] Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((((-2*I)/3)*a^2)/(d^3*(-I + c
*x)^2) - a^2/(3*d^3*(-I + c*x))))/c + ((I/3)*a*b*Sqrt[I*((-I)*d + c*d*x)]*S
qrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] -
I*Sinh[ArcSinh[c*x]/2])*((-I)*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*Ar
cTan[Coth[ArcSinh[c*x]/2]] - I*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/
2]*(4 + (3*I)*ArcSinh[c*x] - (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 3*Log[Sqr
t[1 + c^2*x^2]]) + 2*(Sqrt[1 + c^2*x^2]*(ArcSinh[c*x] + 2*ArcTan[Coth[ArcSi
nh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]]) + 2*(I + ArcSinh[c*x] + 2*ArcTan[Co
th[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c*
d^3*(I + c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2]
 + I*Sinh[ArcSinh[c*x]/2])^4) + ((I/3)*b^2*(I + c*x)*Sqrt[I*((-I)*d + c*d*x
)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*((-1 + I)*ArcSinh[c*
x]^2 - (2*ArcSinh[c*x]*(-2*I + ArcSinh[c*x]))/(-I + c*x) + (2*I)*(Pi + (2*I
```

```
)*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] - I*Pi*(ArcSinh[c*x] - 4*Log[1 +
E^ArcSinh[c*x]] + 4*Log[Cosh[ArcSinh[c*x]/2]] + 2*Log[Sin[(Pi + (2*I)*ArcSi
nh[c*x])/4]]) + 4*PolyLog[2, I/E^ArcSinh[c*x]] - (4*ArcSinh[c*x]^2*Sinh[Arc
Sinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^3 + (2*(4 + A
rcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh
[c*x]/2])))/(c*d^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2
]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^2)
```

Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2\,\sqrt{-icfx+f}}{(icdx+d)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{f-icfx}(a+b\mathrm{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx = \int \frac{\sqrt{-i\,cfx+f}(b\,\mathrm{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/3*((b^2*c*x + I*b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt
(c^2*x^2 + 1))^2 - 3*(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3)*integral(1/3*(3*
I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + 2*(sqrt(c^2*x^2 + 1)*sqrt(I*c*
d*x + d)*sqrt(-I*c*f*x + f)*b^2 + 3*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*
a*b)*log(c*x + sqrt(c^2*x^2 + 1)))/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c*d^3
*x + I*d^3), x))/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3)
```

Sympy [F]

$$\int \frac{\sqrt{f-icfx}(a+b\mathrm{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx = \int \frac{\sqrt{-if\,(cx+i)}(a+b\,\mathrm{asinh}\,(cx))^2}{(id\,(cx-i))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(5/2),x)
```

```
[Out] Integral(sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))**2/(I*d*(c*x - I))**(5/2),
 x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Giac [F]

$$\int \frac{\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx = \int \frac{\sqrt{-i\,cfx+f}\,(b\operatorname{arsinh}(cx)+a)^2}{(i\,cdx+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)^2/(I*c*d*x + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx = \int \frac{(a+b\operatorname{asinh}(c\,x))^2\,\sqrt{f-c\,f\,x\,1\mathrm{i}}}{(d+c\,d\,x\,1\mathrm{i})^{5/2}}\,dx$$

```
[In] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(5/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(5/2), x)
```

3.576 $\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 37, antiderivative size = 774

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\frac{8ib^2d(d+icdx)^{3/2}(f-icfx)^{3/2}}{225c}$$
$$+\frac{1}{32}b^2dx(d+icdx)^{3/2}(f-icfx)^{3/2}+\frac{16ib^2d(d+icdx)^{3/2}(f-icfx)^{3/2}}{75c\,(1+c^2x^2)}+\frac{15b^2dx(d+icdx)^{3/2}(f-icfx)^{3/2}}{64\,(1+c^2x^2)}+\frac{2i}{}$$

```
[Out] 8/225*I*b^2*d*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/c+1/32*b^2*d*x*(d+I*c*d*x
)^(3/2)*(f-I*c*f*x)^(3/2)+16/75*I*b^2*d*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)
/c/(c^2*x^2+1)+15/64*b^2*d*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1
)+2/125*I*b^2*d*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(c^2*x^2+1)/c-9/64*b^2*
d*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*arcsinh(c*x)/c/(c^2*x^2+1)^(3/2)-2/5*
I*b*d*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^
(3/2)-3/8*b*c*d*x^2*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/
(c^2*x^2+1)^(3/2)-4/15*I*b*c^2*d*x^3*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a
+b*arcsinh(c*x))/(c^2*x^2+1)^(3/2)-2/25*I*b*c^4*d*x^5*(d+I*c*d*x)^(3/2)*(f-
I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(3/2)+1/4*d*x*(d+I*c*d*x)^(3/
2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2+3/8*d*x*(d+I*c*d*x)^(3/2)*(f-I*c*
f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(c^2*x^2+1)+1/5*I*d*(d+I*c*d*x)^(3/2)*(f-I*
c*f*x)^(3/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c+1/8*d*(d+I*c*d*x)^(3/2)*(f-
I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(c^2*x^2+1)^(3/2)-1/8*b*d*(d+I*c*d*
x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 774, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.405$, Rules used = {5796, 5838, 5786, 5785, 5783, 5776, 327, 221, 5798, 201, 200, 5784, 12, 1261, 712}

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx =$$
$$-\frac{3bcdx^2(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{8\left(c^2x^2+1\right)^{3/2}}$$
$$+\frac{3dx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{8\left(c^2x^2+1\right)}$$
$$-\frac{2ibdx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{5\left(c^2x^2+1\right)^{3/2}}$$
$$+\frac{d(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^3}{8bc\left(c^2x^2+1\right)^{3/2}}$$
$$+\frac{id(c^2x^2+1)\,(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{5c}$$
$$-\frac{bd\sqrt{c^2x^2+1}(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{8c}$$
$$-\frac{4ibc^2dx^3(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{15\left(c^2x^2+1\right)^{3/2}}$$
$$-\frac{2ibc^4dx^5(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{25\left(c^2x^2+1\right)^{3/2}}$$
$$+\frac{1}{4}dx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{9b^2d\text{arcsinh}(cx)(d+icdx)^{3/2}(f-icfx)^{3/2}}{64c\left(c^2x^2+1\right)^{3/2}}+\frac{15b^2dx(d+}{6}$$

```
[In] Int[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (((8*I)/225)*b^2*d*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/c + (b^2*d*x*(d
 + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/32 + (((16*I)/75)*b^2*d*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2))/(c*(1 + c^2*x^2)) + (15*b^2*d*x*(d + I*c*d*x)^
(3/2)*(f - I*c*f*x)^(3/2))/(64*(1 + c^2*x^2)) + (((2*I)/125)*b^2*d*(d + I*c
*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(1 + c^2*x^2))/c - (9*b^2*d*(d + I*c*d*x)^(
3/2)*(f - I*c*f*x)^(3/2)*ArcSinh[c*x])/(64*c*(1 + c^2*x^2)^(3/2)) - (((2*I)
/5)*b*d*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(1
+ c^2*x^2)^(3/2) - (3*b*c*d*x^2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a
+ b*ArcSinh[c*x]))/(8*(1 + c^2*x^2)^(3/2)) - (((4*I)/15)*b*c^2*d*x^3*(d + I
*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(1 + c^2*x^2)^(3/2)
 - (((2*I)/25)*b*c^4*d*x^5*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*A
rcSinh[c*x]))/(1 + c^2*x^2)^(3/2) - (b*d*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^
(3/2)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(8*c) + (d*x*(d + I*c*d*x)^(3
```

```
/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + (3*d*x*(d + I*c*d*x)^(3
/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(8*(1 + c^2*x^2)) + ((I/5)*
d*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
)^2)/c + (d*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^3)
/(8*b*c*(1 + c^2*x^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
```

```
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\text{integral} = \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int (d+icdx)\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(d(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2+icdx(1+c^2x^2)^{3/2}(a+b\text{arcsin}\right.}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{\left(d(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx}{\left(1+c^2x^2\right)^{3/2}} + \frac{\left(icd(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{1}{4}dx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{id(d+icdx)^{3/2}(f-icfx)^{3/2}\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2}{5c} + \frac{\left(3d(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\,dx}{4\left(1+c^2x^2\right)^{3/2}} - \frac{\left(2ibd(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))\,dx}{5\left(1+c^2x^2\right)^{3/2}} - \frac{\left(bcd(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int x\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))\,dx}{2\left(1+c^2x^2\right)^{3/2}}$$

$$= -\frac{2ibdx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{5\left(1+c^2x^2\right)^{3/2}} - \frac{4ibc^2dx^3(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{15\left(1+c^2x^2\right)^{3/2}} - \frac{2ibc^4dx^5(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{25\left(1+c^2x^2\right)^{3/2}} - \frac{bd(d+icdx)^{3/2}(f-icfx)^{3/2}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{8c} + \frac{1}{4}dx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{3dx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(c}{8\left(1+c^2x^2\right)}$$

$$= \frac{1}{32}b^2dx(d+icdx)^{3/2}(f-icfx)^{3/2} - \frac{2ibdx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{5\left(1+c^2x^2\right)^{3/2}} - \frac{3bcdx^2(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{8\left(1+c^2x^2\right)^{3/2}} - \frac{4ibc^2dx^3(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{15\left(1+c^2x^2\right)^{3/2}} - \frac{2ibc^4dx^5(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{25\left(1+c^2x^2\right)^{3/2}} - \frac{bd(d+icdx)^{3/2}(f-icfx)^{3/2}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{8c} + \frac{1}{4}dx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{3dx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(}{8\left(1+c^2x^2\right)}$$

$$= \frac{1}{32} b^2 dx (d+icdx)^{3/2} (f-icfx)^{3/2} + \frac{15 b^2 dx (d+icdx)^{3/2} (f-icfx)^{3/2}}{64 (1+c^2x^2)} - \frac{2ibdx (d+icdx)^{3/2} (f-icfx)^{3/2} (a+b\text{arcsinh}(cx))}{5 (1+c^2x^2)^{3/2}} - \frac{3bcdx^2 (d+icdx)^{3/2} (f-icfx)^{3/2} (a+b\text{arcsinh}(cx))}{8 (1+c^2x^2)^{3/2}} - \frac{4ibc^2dx^3 (d+icdx)^{3/2} (f-icfx)^{3/2} (a+b\text{arcsinh}(cx))}{15 (1+c^2x^2)^{3/2}} - \frac{2ibc^4dx^5 (d+icdx)^{3/2} (f-icfx)^{3/2} (a+b\text{arcsinh}(cx))}{25 (1+c^2x^2)^{3/2}} - \frac{bd (d+icdx)^{3/2} (f-icfx)^{3/2} \sqrt{1+c^2x^2} (a+b\text{arcsinh}(cx))}{8c} + \frac{1}{4} dx (d+icdx)^{3/2} (f-icfx)^{3/2} (a+b\text{arcsinh}(cx))^2 + \frac{3dx (d+icdx)^{3/2} (f-icfx)^{3/2} (a+b\text{arcsinh}(c}{8 (1+c^2x^2)}$$

$$= \frac{1}{32} b^2 dx (d+icdx)^{3/2} (f-icfx)^{3/2} + \frac{15 b^2 dx (d+icdx)^{3/2} (f-icfx)^{3/2}}{64 (1+c^2x^2)} - \frac{9 b^2 d (d+icdx)^{3/2} (f-icfx)^{3/2} \text{arcsinh}(cx)}{64c (1+c^2x^2)^{3/2}} - \frac{2ibdx (d+icdx)^{3/2} (f-icfx)^{3/2} (a+b\text{arcsinh}(cx))}{5 (1+c^2x^2)^{3/2}} - \frac{3bcdx^2 (d+icdx)^{3/2} (f-icfx)^{3/2} (a+b\text{arcsinh}(cx))}{8 (1+c^2x^2)^{3/2}} - \frac{4ibc^2dx^3 (d+icdx)^{3/2} (f-icfx)^{3/2} (a+b\text{arcsinh}(cx))}{15 (1+c^2x^2)^{3/2}} - \frac{2ibc^4dx^5 (d+icdx)^{3/2} (f-icfx)^{3/2} (a+b\text{arcsinh}(cx))}{25 (1+c^2x^2)^{3/2}} - \frac{bd (d+icdx)^{3/2} (f-icfx)^{3/2} \sqrt{1+c^2x^2} (a+b\text{arcsinh}(cx))}{8c} + \frac{1}{4} dx (d+icdx)^{3/2} (f-icfx)^{3/2} (a+b\text{arcsinh}(cx))^2 + \frac{3dx (d+icdx)^{3/2} (f-icfx)^{3/2} (a+b\text{arcsinh}(c}{8 (1+c^2x^2)}$$

$$= \frac{8ib^2 d (d+icdx)^{3/2} (f-icfx)^{3/2}}{225c} + \frac{1}{32} b^2 dx (d+icdx)^{3/2} (f-icfx)^{3/2} + \frac{16ib^2 d (d+icdx)^{3/2} (f-icfx)^{3/2}}{75c (1+c^2x^2)} + \frac{15 b^2 dx (d+icdx)^{3/2} (f-ic}{64 (1+c^2x^2)}$$

Mathematica [A] (verified)

Time = 4.44 (sec) , antiderivative size = 1084, normalized size of antiderivative = 1.40

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{-72000iabcd^2fx\sqrt{d+icdx}\sqrt{f-icfx}+57600ia^2d^2f\sqrt{d+icdx}\sqrt{f-ic}}{}$$

```
[In] Integrate[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]

[Out] ((-72000*I)*a*b*c*d^2*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (57600*I)*a
^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (72000*I)*
b^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 180000*a^
2*c*d^2*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (115200
*I)*a^2*c^2*d^2*f*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]
 + 72000*a^2*c^3*d^2*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2
*x^2] + (57600*I)*a^2*c^4*d^2*f*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqr
t[1 + c^2*x^2] + 36000*b^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSin
h[c*x]^3 - 72000*a*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSi
nh[c*x]] + (4000*I)*b^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[3*Ar
cSinh[c*x]] - 4500*a*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*Arc
Sinh[c*x]] + (288*I)*b^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[5*A
rcSinh[c*x]] + 108000*a^2*d^(5/2)*f^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + S
qrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 36000*b^2*d^2*f*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] - (12000*I)*a*b*d^2*f*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 1125*b^2*d^2*f*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] + 1800*b*d^2*f*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a + (20*I)*b*Sqrt[1 + c^2
*x^2] + (10*I)*b*Cosh[3*ArcSinh[c*x]] + (2*I)*b*Cosh[5*ArcSinh[c*x]] + 40*b
*Sinh[2*ArcSinh[c*x]] + 5*b*Sinh[4*ArcSinh[c*x]]) - (1440*I)*a*b*d^2*f*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[5*ArcSinh[c*x]] + 60*b*d^2*f*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((-1200*I)*b*c*x + (1200*I)*a*Sqrt[
1 + c^2*x^2] - 1200*b*Cosh[2*ArcSinh[c*x]] + (600*I)*a*Cosh[3*ArcSinh[c*x]]
 - 75*b*Cosh[4*ArcSinh[c*x]] + (120*I)*a*Cosh[5*ArcSinh[c*x]] + 2400*a*Sinh
[2*ArcSinh[c*x]] - (200*I)*b*Sinh[3*ArcSinh[c*x]] + 300*a*Sinh[4*ArcSinh[c*
x]] - (24*I)*b*Sinh[5*ArcSinh[c*x]]))/(288000*c*Sqrt[1 + c^2*x^2])
```

Maple [F]

$$\int (icdx+d)^{\frac{5}{2}} (-icfx+f)^{\frac{3}{2}} (a+b \operatorname{arcsinh}(cx))^2 dx$$

```
[In] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

Fricas [F]

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2 dx = \int (i\,cdx+d)^{\frac{5}{2}}(-i\,cfx+f)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2 dx$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="fricas")
```

```
[Out] integral((I*b^2*c^3*d^2*f*x^3 + b^2*c^2*d^2*f*x^2 + I*b^2*c*d^2*f*x + b^2*d
^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 -
 2*(-I*a*b*c^3*d^2*f*x^3 - a*b*c^2*d^2*f*x^2 - I*a*b*c*d^2*f*x - a*b*d^2*f)
*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a^2
*c^3*d^2*f*x^3 + a^2*c^2*d^2*f*x^2 + I*a^2*c*d^2*f*x + a^2*d^2*f)*sqrt(I*c*
d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2 dx = \text{Timed out}$$

```
[In] integrate((d+I*c*d*x)**(5/2)*(f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad Ar
gument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,(d+c\,d\,x\,1\text{i})^{5/2}\,(f-c\,f\,x\,1\text{i})^{3/2}\,dx$$

```
[In] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2), x)
```

3.577 $\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\,\textbf{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 37, antiderivative size = 396

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \frac{1}{32}b^2x(d+icdx)^{3/2}(f-icfx)^{3/2}+\frac{15b^2x(d+icdx)^{3/2}(f-icfx)^{3/2}}{64\,(1+c^2x^2)}-\frac{9b^2(d+$$

```
[Out] 1/32*b^2*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)+15/64*b^2*x*(d+I*c*d*x)^(3/2
)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)-9/64*b^2*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2
)*arcsinh(c*x)/c/(c^2*x^2+1)^(3/2)-3/8*b*c*x^2*(d+I*c*d*x)^(3/2)*(f-I*c*f*x
)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(3/2)+1/4*x*(d+I*c*d*x)^(3/2)*(f-I*c
*f*x)^(3/2)*(a+b*arcsinh(c*x))^2+3/8*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*
(a+b*arcsinh(c*x))^2/(c^2*x^2+1)+1/8*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a
+b*arcsinh(c*x))^3/b/c/(c^2*x^2+1)^(3/2)-1/8*b*(d+I*c*d*x)^(3/2)*(f-I*c*f*x
)^(3/2)*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {5796, 5786, 5785, 5783, 5776, 327, 221, 5798, 201}

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \frac{(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^3}{8bc\,(c^2x^2+1)^{3/2}} + \frac{3x(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{8\,(c^2x^2+1)} - \frac{b\sqrt{c^2x^2+1}(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{8c} - \frac{3bcx^2(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{8\,(c^2x^2+1)^{3/2}} + \frac{1}{4}x(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2 - \frac{9b^2\operatorname{arcsinh}(cx)(d+icdx)^{3/2}(f-icfx)^{3/2}}{64c\,(c^2x^2+1)^{3/2}} + \frac{15b^2x(d+icd}{64\,(}$$

```
[In] Int[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (b^2*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/32 + (15*b^2*x*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2))/(64*(1 + c^2*x^2)) - (9*b^2*(d + I*c*d*x)^(3/2
)*(f - I*c*f*x)^(3/2)*ArcSinh[c*x])/(64*c*(1 + c^2*x^2)^(3/2)) - (3*b*c*x^2
*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(8*(1 + c^2*
x^2)^(3/2)) - (b*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*Sqrt[1 + c^2*x^2]*
(a + b*ArcSinh[c*x]))/(8*c) + (x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a
 + b*ArcSinh[c*x])^2)/4 + (3*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a +
 b*ArcSinh[c*x])^2)/(8*(1 + c^2*x^2)) + ((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^
(3/2)*(a + b*ArcSinh[c*x])^3)/(8*b*c*(1 + c^2*x^2)^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{1}{4}x(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{\left(3(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\,dx}{4\left(1+c^2x^2\right)^{3/2}} - \frac{\left(bc(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int x\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))\,dx}{2\left(1+c^2x^2\right)^{3/2}}$$

$$= -\frac{b(d+icdx)^{3/2}(f-icfx)^{3/2}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{8c} + \frac{1}{4}x(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{3x(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx)}{8\left(1+c^2x^2\right)}$$

$$= \frac{1}{32}b^2x(d+icdx)^{3/2}(f-icfx)^{3/2} - \frac{3bcx^2(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{8\left(1+c^2x^2\right)^{3/2}} - \frac{b(d+icdx)^{3/2}(f-icfx)^{3/2}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{8c} + \frac{1}{4}x(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{3x(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx}{8\left(1+c^2x^2\right)}$$

$$= \frac{1}{32}b^2x(d+icdx)^{3/2}(f-icfx)^{3/2} + \frac{15b^2x(d+icdx)^{3/2}(f-icfx)^{3/2}}{64\left(1+c^2x^2\right)} - \frac{3bcx^2(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{8\left(1+c^2x^2\right)^{3/2}} - \frac{b(d+icdx)^{3/2}(f-icfx)^{3/2}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{8c} + \frac{1}{4}x(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{3x(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx}{8\left(1+c^2x^2\right)}$$

$$= \frac{1}{32}b^2x(d+icdx)^{3/2}(f-icfx)^{3/2} + \frac{15b^2x(d+icdx)^{3/2}(f-icfx)^{3/2}}{64(1+c^2x^2)} - \frac{9b^2(d+icdx)^{3/2}(f-icfx)^{3/2}\text{arcsinh}(cx)}{64c(1+c^2x^2)^{3/2}} - \frac{3bcx^2(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{8(1+c^2x^2)^{3/2}} - \frac{b(d+icdx)^{3/2}(f-icfx)^{3/2}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{8c} + \frac{1}{4}x(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{3x(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{8(1+c^2x^2)}$$

Mathematica [A] (verified)

Time = 3.19 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.32

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{160a^2cdfx\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+64a^2c^3dfx^3\sqrt{d+icdx}\sqrt{f-i}}{}$$

```
[In] Integrate[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (160*a^2*c*d*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 64
*a^2*c^3*d*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 32
*b^2*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 - 64*a*b*d*f*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - 4*a*b*d*f*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 96*a^2*d^(3/2)*f^(3/2)*S
qrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x]] + 32*b^2*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*
x]] + b^2*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] + 8*
b*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(12*a + 8*b*Sinh[2
*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]]) - 4*b*d*f*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]*ArcSinh[c*x]*(16*b*Cosh[2*ArcSinh[c*x]] + b*Cosh[4*ArcSinh[c*x]
] - 4*a*(8*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]])))/(256*c*Sqrt[1 + c
^2*x^2])
```

Maple [F]

$$\int (icdx + d)^{\frac{3}{2}} (-icfx + f)^{\frac{3}{2}} (a + b \operatorname{arcsinh}(cx))^2 \, dx$$

```
[In] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x)
[Out] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

Fricas [F]

$$\int (d + icdx)^{3/2}(f - icfx)^{3/2}(a + b\text{arcsinh}(cx))^2 \, dx = \int (i\,cdx + d)^{\frac{3}{2}}(-i\,cfx + f)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx) + a)^2 \, dx$$

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="fricas")
[Out] integral((b^2*c^2*d*f*x^2 + b^2*d*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*l
og(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c^2*d*f*x^2 + a*b*d*f)*sqrt(I*c*d*x
+ d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c^2*d*f*x^2 + a
^2*d*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d + icdx)^{3/2}(f - icfx)^{3/2}(a + b\text{arcsinh}(cx))^2 \, dx = \text{Timed out}$$

```
[In] integrate((d+I*c*d*x)**(3/2)*(f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d + icdx)^{3/2}(f - icfx)^{3/2}(a + b\text{arcsinh}(cx))^2 \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,(d+c\,d\,x\,1\text{i})^{3/2}\,(f-c\,f\,x\,1\text{i})^{3/2}\,dx$$

```
[In] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2), x)
```

3.578 $\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 37, antiderivative size = 508

$$
\begin{aligned}
&\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \\
&-\frac{4ib^2f\sqrt{d+icdx}\sqrt{f-icfx}}{9c}+\frac{1}{4}b^2fx\sqrt{d+icdx}\sqrt{f-icfx} \\
&-\frac{2ib^2f\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)}{27c}-\frac{b^2f\sqrt{d+icdx}\sqrt{f-icfx}\text{arcsinh}(cx)}{4c\sqrt{1+c^2x^2}} \\
&+\frac{2ibfx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} \\
&-\frac{bcfx^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}} \\
&+\frac{2ibc^2fx^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} \\
&+\frac{1}{2}fx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 \\
&-\frac{if\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c} \\
&+\frac{f\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{1+c^2x^2}}
\end{aligned}
$$

```
[Out] -4/9*I*b^2*f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+1/4*b^2*f*x*(d+I*c*d*x)^
(1/2)*(f-I*c*f*x)^(1/2)-2/27*I*b^2*f*(c^2*x^2+1)*(d+I*c*d*x)^(1/2)*(f-I*c*f
*x)^(1/2)/c+1/2*f*x*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2
)-1/3*I*f*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1
/2)/c-1/4*b^2*f*arcsinh(c*x)*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c/(c^2*x^2
+1)^(1/2)+2/3*I*b*f*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2
)/(c^2*x^2+1)^(1/2)-1/2*b*c*f*x^2*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I
```

```
*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+2/9*I*b*c^2*f*x^3*(a+b*arcsinh(c*x))*(d+I*c
*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/6*f*(a+b*arcsinh(c*x))^3*
(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.297, Rules used = {5796, 5838, 5785, 5783, 5776, 327, 221, 5798, 5784, 455, 45}

$$\begin{aligned}\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx= & -\frac{bcfx^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{2\sqrt{c^2x^2+1}} \\ & +\frac{2ibfx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}} \\ & +\frac{f\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{c^2x^2+1}} \\ & -\frac{if(c^2x^2+1)\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{3c} \\ & +\frac{2ibc^2fx^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{9\sqrt{c^2x^2+1}} \\ & +\frac{1}{2}fx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 \\ & -\frac{b^2f\text{arcsinh}(cx)\sqrt{d+icdx}\sqrt{f-icfx}}{4c\sqrt{c^2x^2+1}}-\frac{2ib^2f(c^2x^2+1)\sqrt{d+icdx}\sqrt{f-icfx}}{27c} \\ & +\frac{1}{4}b^2fx\sqrt{d+icdx}\sqrt{f-icfx}-\frac{4ib^2f\sqrt{d+icdx}\sqrt{f-icfx}}{9c}\end{aligned}$$

```
[In] Int[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (((-4*I)/9)*b^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/c + (b^2*f*x*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x])/4 - (((2*I)/27)*b^2*f*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]*(1 + c^2*x^2))/c - (b^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*
ArcSinh[c*x])/(4*c*Sqrt[1 + c^2*x^2]) + (((2*I)/3)*b*f*x*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] - (b*c*f*x^2*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(2*Sqrt[1 + c^2*x^2])
 + (((2*I)/9)*b*c^2*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSin
h[c*x]))/Sqrt[1 + c^2*x^2] + (f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a +
b*ArcSinh[c*x])^2)/2 - ((I/3)*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^
2*x^2)*(a + b*ArcSinh[c*x])^2)/c + (f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(
a + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[1 + c^2*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int (f-icfx)\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \left(f\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2 - icfx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\right)\,d}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(f\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\,dx}{\sqrt{1+c^2x^2}} - \frac{\left(icf\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{2} f x \sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2 - \frac{if\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))^2}{3c} + \frac{\left(f\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}} + \frac{\left(2ibf\sqrt{d+icdx}\sqrt{f-icfx}\right)\int (1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))\,dx}{3\sqrt{1+c^2x^2}} - \frac{\left(bcf\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x(a+b\operatorname{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{2ibfx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} - \frac{bcfx^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{2\sqrt{1+c^2x^2}} + \frac{2ibc^2fx^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} + \frac{1}{2} f x \sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2 - \frac{if\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))^2}{3c} + \frac{f\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^3}{6bc\sqrt{1+c^2x^2}} - \frac{\left(2ib^2cf\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \frac{x\left(1+\frac{c^2x^2}{3}\right)}{\sqrt{1+c^2x^2}}\,dx}{3\sqrt{1+c^2x^2}} + \frac{\left(b^2c^2f\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \frac{x^2}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}}$$

$$= \frac{1}{4}b^2fx\sqrt{d+icdx}\sqrt{f-icfx} + \frac{2ibfx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} - \frac{bcfx^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{2\sqrt{1+c^2x^2}} + \frac{2ibc^2fx^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} + \frac{1}{2}fx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2 - \frac{if\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3c} + \frac{f\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^3}{6bc\sqrt{1+c^2x^2}} - \frac{\left(b^2f\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{4\sqrt{1+c^2x^2}} - \frac{\left(ib^2cf\sqrt{d+icdx}\sqrt{f-icfx}\right)\text{Subst}\left(\int\frac{1+\frac{c^2x}{3}}{\sqrt{1+c^2x}}\,dx,x,x^2\right)}{3\sqrt{1+c^2x^2}}$$

$$= \frac{1}{4}b^2fx\sqrt{d+icdx}\sqrt{f-icfx} - \frac{b^2f\sqrt{d+icdx}\sqrt{f-icfx}\operatorname{arcsinh}(cx)}{4c\sqrt{1+c^2x^2}} + \frac{2ibfx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} - \frac{bcfx^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{2\sqrt{1+c^2x^2}} + \frac{2ibc^2fx^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} + \frac{1}{2}fx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2 - \frac{if\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3c} + \frac{f\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^3}{6bc\sqrt{1+c^2x^2}} - \frac{\left(ib^2cf\sqrt{d+icdx}\sqrt{f-icfx}\right)\text{Subst}\left(\int\left(\frac{2}{3\sqrt{1+c^2x}}+\frac{1}{3}\sqrt{1+c^2x}\right)dx,x,x^2\right)}{3\sqrt{1+c^2x^2}}$$

$$= -\frac{4ib^2f\sqrt{d+icdx}\sqrt{f-icfx}}{9c} + \frac{1}{4}b^2fx\sqrt{d+icdx}\sqrt{f-icfx} - \frac{2ib^2f\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)}{27c} - \frac{b^2f\sqrt{d+icdx}\sqrt{f-icfx}\text{arcsinh}(cx)}{4c\sqrt{1+c^2x^2}} + \frac{2ibfx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} - \frac{bcfx^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}} + \frac{2ibc^2fx^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} + \frac{1}{2}fx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 - \frac{if\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c} + \frac{f\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 3.09 (sec) , antiderivative size = 705, normalized size of antiderivative = 1.39

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{108iabcfx\sqrt{d+icdx}\sqrt{f-icfx} - 72ia^2f\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2} - 108ib^2f\sqrt{}}{}$$

```
[In] Integrate[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((108*I)*a*b*c*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (72*I)*a^2*f*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (108*I)*b^2*f*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 108*a^2*c*f*x*Sqrt[d + I*c*d*
x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (72*I)*a^2*c^2*f*x^2*Sqrt[d + I*c*
d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 36*b^2*f*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]*ArcSinh[c*x]^3 - 54*a*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*
Cosh[2*ArcSinh[c*x]] - (4*I)*b^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh
[3*ArcSinh[c*x]] + 108*a^2*Sqrt[d]*f^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x +
Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 27*b^2*f*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 18*b*f*Sqrt[d + I*c*d*x]*Sq
rt[f - I*c*f*x]*ArcSinh[c*x]^2*(6*a - (3*I)*b*Sqrt[1 + c^2*x^2] - I*b*Cosh[
3*ArcSinh[c*x]] + 3*b*Sinh[2*ArcSinh[c*x]]) + (12*I)*a*b*f*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 6*b*f*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]*ArcSinh[c*x]*(-9*b*Cosh[2*ArcSinh[c*x]] + 2*((9*I)*b*c*x - (9*I)*
a*Sqrt[1 + c^2*x^2] - (3*I)*a*Cosh[3*ArcSinh[c*x]] + 9*a*Sinh[2*ArcSinh[c*x
]] + I*b*Sinh[3*ArcSinh[c*x]])))/(216*c*Sqrt[1 + c^2*x^2])
```

Maple [F]

$$\int (-icfx+f)^{\frac{3}{2}} (a+b\,\mathrm{arcsinh}\,(cx))^2 \sqrt{icdx+d}dx$$

```
[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x)
```

Fricas [F]

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\mathrm{arcsinh}(cx))^2\,dx = \int \sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^2\,dx$$

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((-I*b^2*c*f*x + b^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1))^2 - 2*(I*a*b*c*f*x - a*b*f)*sqrt(I*c*d*x + d)*sqrt(-
I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a^2*c*f*x + a^2*f)*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F]

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\mathrm{arcsinh}(cx))^2\,dx = \int \sqrt{id\,(cx-i)}(-if(cx+i))^{\frac{3}{2}}\,(a+b\,\mathrm{asinh}\,(cx))^2\,\,dx$$

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2),x)
```

```
[Out] Integral(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))**2,
 x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad Ar
gument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\mathrm{asinh}(c\,x))^2\,\sqrt{d+c\,d\,x\,1\mathrm{i}}\,(f-c\,f\,x\,1\mathrm{i})^{3/2}\,dx$$

```
[In] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2), x)
```

3.579 $\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 436

$$\begin{aligned}\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx =& -\frac{4ib^2f^2(1+c^2x^2)}{c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{b^2f^2x(1+c^2x^2)}{4\sqrt{d+icdx}\sqrt{f-icfx}}\\&+\frac{b^2f^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{4c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{4ibf^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}\sqrt{f-icfx}}\\&+\frac{bcf^2x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{2if^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}}\\&-\frac{f^2x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{f^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}\end{aligned}$$

```
[Out] -4*I*b^2*f^2*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/4*b^2*f^2*
x*(c^2*x^2+1)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-2*I*f^2*(c^2*x^2+1)*(a+b*
arcsinh(c*x))^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/2*f^2*x*(c^2*x^2+1)
*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/4*b^2*f^2*arcsi
nh(c*x)*(c^2*x^2+1)^(1/2)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+4*I*b*f^2*x
*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1
/2*b*c*f^2*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*
c*f*x)^(1/2)+1/2*f^2*(a+b*arcsinh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)
^(1/2)/(f-I*c*f*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {5796, 5843, 3398, 3377, 2718, 3392, 32, 2715, 8}

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx=\frac{f^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{2if^2(c^2x^2+1)\,(a+b\text{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{f^2x(c^2x^2+1)\,(a+b\text{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{bcf^2x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{4ibf^2x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{b^2f^2\sqrt{c^2x^2+1}\text{arcsinh}(cx)}{4c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{4ib^2f^2(c^2x^2+1)}{c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{b^2f^2x(c^2x^2+1)}{4\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[In] Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((-4*I)*b^2*f^2*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (b
^2*f^2*x*(1 + c^2*x^2))/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (b^2*f^2*
Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(4*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) +
 ((4*I)*b*f^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]) + (b*c*f^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - ((2*I)*f^2*(1 + c^2*x^2)*(a + b*Ar
cSinh[c*x])^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (f^2*x*(1 + c^2*x^
2)*(a + b*ArcSinh[c*x])^2)/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (f^2*S
qrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(2*b*c*Sqrt[d + I*c*d*x]*Sqrt[f -
I*c*f*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+c^2x^2}\int \frac{(f-icfx)^2(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\operatorname{Subst}\left(\int (a+bx)^2(cf-icf\sinh(x))^2\,dx,x,\operatorname{arcsinh}(cx)\right)}{c^3\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\operatorname{Subst}\left(\int \left(c^2f^2(a+bx)^2-2ic^2f^2(a+bx)^2\sinh(x)-c^2f^2(a+bx)^2\sinh^2(x)\right)\,dx,x,\operatorname{arcs}}{c^3\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{f^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(2if^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int (a+bx)^2\sinh(x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(f^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int (a+bx)^2\sinh^2(x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{bcf^2x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2if^2(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{f^2x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{f^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(f^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int (a+bx)^2\,dx,x,\operatorname{arcsinh}(cx)\right)}{2c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(4ibf^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int (a+bx)\cosh(x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(b^2f^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \sinh^2(x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{2c\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= -\frac{b^2f^2x(1+c^2x^2)}{4\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{4ibf^2x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{bcf^2x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2if^2(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{f^2x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{f^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^3}{2bc\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(4ib^2f^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \sinh(x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(b^2f^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int 1\,dx,x,\operatorname{arcsinh}(cx)\right)}{4c\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= -\frac{4ib^2f^2(1+c^2x^2)}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{b^2f^2x(1+c^2x^2)}{4\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{b^2f^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{4c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{4ibf^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{bcf^2x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2if^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{f^2x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{f^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

Mathematica [A] (verified)

Time = 8.52 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.22

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \frac{32iabcfx\sqrt{d+icdx}\sqrt{f-icfx} - 16ia^2f\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+}}{}$$

```
[In] Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((32*I)*a*b*c*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (16*I)*a^2*f*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (32*I)*b^2*f*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*a^2*c*f*x*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 4*b^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c
*f*x]*ArcSinh[c*x]^3 + 2*a*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*A
rcSinh[c*x]] + 2*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((4*I
)*(4*b*c*x + a*(-4 + I*c*x)*Sqrt[1 + c^2*x^2]) + b*Cosh[2*ArcSinh[c*x]]) +
12*a^2*Sqrt[d]*f^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - b^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]*Sinh[2*ArcSinh[c*x]] + 2*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[
c*x]^2*(6*a - (8*I)*b*Sqrt[1 + c^2*x^2] - b*Sinh[2*ArcSinh[c*x]]))/(8*c*d*S
qrt[1 + c^2*x^2])
```

Maple [F]

$$\int \frac{(-icfx+f)^{\frac{3}{2}}(a+b\,\text{arcsinh}\,(cx))^2}{\sqrt{icdx+d}}dx$$

```
[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{(f - icfx)^{3/2}(a + b\text{arcsinh}(cx))^2}{\sqrt{d + icdx}}\,dx = \int \frac{(-i\,cfx + f)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx) + a)^2}{\sqrt{i\,cdx + d}}\,dx$$

[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algor ithm="fricas")

[Out] integral(-((b^2*c*f*x + I*b^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c *x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c*f*x + I*a*b*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c*f*x + I*a^2*f)*sqrt(I*c *d*x + d)*sqrt(-I*c*f*x + f))/(c*d*x - I*d), x)

Sympy [F]

$$\int \frac{(f - icfx)^{3/2}(a + b\text{arcsinh}(cx))^2}{\sqrt{d + icdx}}\,dx = \int \frac{(-if(cx + i))^{\frac{3}{2}}\,(a + b\,\text{asinh}\,(cx))^2}{\sqrt{id\,(cx - i)}}\,dx$$

[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(1/2),x)

[Out] Integral((-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))**2/sqrt(I*d*(c*x - I)), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f - icfx)^{3/2}(a + b\text{arcsinh}(cx))^2}{\sqrt{d + icdx}}\,dx = \text{Exception raised: RuntimeError}$$

[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algor ithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ e exponent.

Giac [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{i\,cdx+d}}\,dx$$

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((-I*c*f*x + f)^(3/2)*(b*arcsinh(c*x) + a)^2/sqrt(I*c*d*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(f-c\,f\,x\,1\text{i})^{3/2}}{\sqrt{d+c\,d\,x\,1\text{i}}}\,dx$$

```
[In] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(1/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(1/2), x)
```

3.580 $\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 752

$$\begin{aligned}
&\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = \\
&-\frac{2iabf^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2ib^2f^3(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&-\frac{2ib^2f^3x(1+c^2x^2)^{3/2}\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{4if^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+\frac{4f^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{4f^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+\frac{if^3(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{f^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^3}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&-\frac{16ibf^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&-\frac{8bf^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&-\frac{8b^2f^3(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+\frac{8b^2f^3(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&-\frac{4b^2f^3(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}$$

```
[Out] -2*I*a*b*f^3*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+2*I*b^
2*f^3*(c^2*x^2+1)^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*I*b^2*f^3*x*(c^
```

```
2*x^2+1)^(3/2)*arcsinh(c*x)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*I*f^3*(c^
2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*f^3*x
*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*f^3
*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/
2)+I*f^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)
^(3/2)-f^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(3/2)/(f-
I*c*f*x)^(3/2)-16*I*b*f^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*arctan(c*x+(
c^2*x^2+1)^(1/2))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*b*f^3*(c^2*x^2+1)
^(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/
2)/(f-I*c*f*x)^(3/2)-8*b^2*f^3*(c^2*x^2+1)^(3/2)*polylog(2,-I*(c*x+(c^2*x^2
+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*b^2*f^3*(c^2*x^2+1)^(3/
2)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/
2)-4*b^2*f^3*(c^2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I
*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 752, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.405$, Rules used = {5796, 5844, 5838, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 5789, 4265, 5783, 5772,

267}

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx =$$

$$-\frac{16ibf^3(c^2x^2+1)^{3/2}\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$-\frac{f^3(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^3}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{if^3(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+\frac{4f^3(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+\frac{4f^3x(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{4if^3(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$-\frac{8bf^3(c^2x^2+1)^{3/2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$-\frac{2iabf^3x(c^2x^2+1)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{8b^2f^3(c^2x^2+1)^{3/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+\frac{8b^2f^3(c^2x^2+1)^{3/2}\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$-\frac{4b^2f^3(c^2x^2+1)^{3/2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$-\frac{2ib^2f^3x(c^2x^2+1)^{3/2}\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2ib^2f^3(c^2x^2+1)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[In] Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((-2*I)*a*b*f^3*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(
3/2)) + ((2*I)*b^2*f^3*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x
)^(3/2)) - ((2*I)*b^2*f^3*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x])/((d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) + ((4*I)*f^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
)^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*f^3*x*(1 + c^2*x^2)*(
a + b*ArcSinh[c*x])^2)/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*f^3*(
1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*
f*x)^(3/2)) + (I*f^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*
x)^(3/2)*(f - I*c*f*x)^(3/2)) - (f^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x
])^3)/(b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((16*I)*b*f^3*(1 + c^
2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^
(3/2)*(f - I*c*f*x)^(3/2)) - (8*b*f^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*
x])*Log[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)
) - (8*b^2*f^3*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d +
 I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (8*b^2*f^3*(1 + c^2*x^2)^(3/2)*PolyL
og[2, I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b
```

```
^2*f^3*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
```

```
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(1+c^2x^2)^{3/2}\int\frac{(f-icfx)^3(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= \frac{(1+c^2x^2)^{3/2}\int\left(-\frac{4i(if^3+cf^3x)(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}-\frac{3f^3(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}+\frac{icf^3x(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= -\frac{\left(4i(1+c^2x^2)^{3/2}\right)\int\frac{(if^3+cf^3x)(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&\quad-\frac{\left(3f^3(1+c^2x^2)^{3/2}\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&\quad+\frac{\left(icf^3(1+c^2x^2)^{3/2}\right)\int\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= \frac{if^3(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{f^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^3}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&\quad-\frac{\left(4i(1+c^2x^2)^{3/2}\right)\int\left(\frac{if^3(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}+\frac{cf^3x(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&\quad-\frac{\left(2ibf^3(1+c^2x^2)^{3/2}\right)\int(a+b\text{arcsinh}(cx))\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}$$

$$= -\frac{2iabf^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{if^3(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^3}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(4f^3(1+c^2x^2)^{3/2}\right)\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2ib^2f^3(1+c^2x^2)^{3/2}\right)\int \operatorname{arcsinh}(cx)\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(4icf^3(1+c^2x^2)^{3/2}\right)\int \frac{x(a+b\operatorname{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2iabf^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2f^3x(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4f^3x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{if^3(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^3}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(8ibf^3(1+c^2x^2)^{3/2}\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(8bcf^3(1+c^2x^2)^{3/2}\right)\int \frac{x(a+b\operatorname{arcsinh}(cx))}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(2ib^2cf^3(1+c^2x^2)^{3/2}\right)\int \frac{x}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2iabf^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2f^3(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2f^3x(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4f^3x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{if^3(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^3}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(8ibf^3(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int (a+bx)\operatorname{sech}(x)\,dx, x, \operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(8bf^3(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2iabf^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2f^3(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2f^3x(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4f^3x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4f^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{if^3(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^3}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{16ibf^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(16bf^3(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(8b^2f^3(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int \log\left(1-ie^x\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(8b^2f^3(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int \log\left(1+ie^x\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2iabf^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2f^3(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2f^3x(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4f^3x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4f^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{if^3(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^3}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{16ibf^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8bf^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(8b^2f^3(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(8b^2f^3(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(8b^2f^3(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2iabf^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2f^3(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2f^3x(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4f^3x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4f^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{if^3(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^3}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{16ibf^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8bf^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8b^2f^3(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8b^2f^3(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(4b^2f^3(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2iabf^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2f^3(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2f^3x(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4f^3x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4f^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{if^3(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^3}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{16ibf^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8bf^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8b^2f^3(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8b^2f^3(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4b^2f^3(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Mathematica [A] (verified)

Time = 14.85 (sec) , antiderivative size = 1174, normalized size of antiderivative = 1.56

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = \frac{if\Big(-3a^2(-5i+cx)\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}\big(\cosh\big(\tfrac{1}{2}\text{arcsinh}}{}$$

```
[In] Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),
x]
```

```
[Out] ((I/3)*f*(-3*a^2*(-5*I + c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 +
c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 9*a^2*Sqrt[d]*Sq
rt[f]*(-I + c*x)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2
]) + 6*a*b*(I - c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]
/2]*(-(c*x) + (2 + Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + I*ArcSinh[c*x]^2 + 4*A
rcTan[Coth[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2]) + I*(-(c*x) + (-2 + Sqrt[
1 + c^2*x^2])*ArcSinh[c*x] + I*ArcSinh[c*x]^2 + 4*ArcTan[Coth[ArcSinh[c*x]/
2]] + I*Log[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2]) + (3*I)*a*b*(I - c*x)*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]/2]*(ArcSinh[c*x]*(-4*I +
ArcSinh[c*x]) + (8*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 2*Log[1 + c^2*x^2]) +
I*(ArcSinh[c*x]*(4*I + ArcSinh[c*x]) + (8*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] +
 2*Log[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2]) + I*b^2*(I - c*x)*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]*((6 - 6*I)*ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - Sin
h[ArcSinh[c*x]/2]) + ArcSinh[c*x]^3*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[
c*x]/2]) + 6*ArcSinh[c*x]*(I*Pi + 4*Log[1 - I/E^ArcSinh[c*x]])*(Cosh[ArcSin
h[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) - 24*PolyLog[2, I/E^ArcSinh[c*x]]*(Cosh
[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 12*Pi*(Log[1 - I/E^ArcSinh[c*x
]] + 2*Log[1 + E^ArcSinh[c*x]] - 2*Log[Cosh[ArcSinh[c*x]/2]] - Log[Sin[(Pi
+ (2*I)*ArcSinh[c*x])/4]])*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2
])) + b^2*(I - c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((2*I)*ArcSinh[c*x]
^3*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) - 6*ArcSinh[c*x]*(Pi + c
*x - (4*I)*Log[1 - I/E^ArcSinh[c*x]])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSin
h[c*x]/2]) + 6*(Sqrt[1 + c^2*x^2] + 2*Pi*Log[1 - I/E^ArcSinh[c*x]] + 4*Pi*L
og[1 + E^ArcSinh[c*x]] - 4*Pi*Log[Cosh[ArcSinh[c*x]/2]] - 2*Pi*Log[Sin[(Pi
+ (2*I)*ArcSinh[c*x])/4]])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])
+ 24*PolyLog[2, I/E^ArcSinh[c*x]]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh
[c*x]/2]) + 3*ArcSinh[c*x]^2*(((2 + 2*I) + Sqrt[1 + c^2*x^2])*Cosh[ArcSinh[
c*x]/2] + I*((-2 + 2*I) + Sqrt[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2]))))/(c*d^
2*(I - c*x)*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2
]))
```

Maple [F]

$$\int \frac{(-icfx+f)^{\frac{3}{2}} (a+b \operatorname{arcsinh}(cx))^2}{(icdx+d)^{\frac{3}{2}}} dx$$

```
[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x)
[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x)
```

Fricas [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}} dx = \int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)^2}{(i\,cdx+d)^{\frac{3}{2}}} dx$$

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algor
ithm="fricas")
[Out] integral(((I*b^2*c*f*x - b^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1))^2 - 2*(-I*a*b*c*f*x + a*b*f)*sqrt(I*c*d*x + d)*sqrt(
-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a^2*c*f*x - a^2*f)*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f))/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

Sympy [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}} dx = \int \frac{(-if(cx+i))^{\frac{3}{2}} (a+b\operatorname{asinh}(cx))^2}{(id(cx-i))^{\frac{3}{2}}} dx$$

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2),x)
[Out] Integral((-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))**2/(I*d*(c*x - I))**(3/
2), x)
```

Maxima [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}} dx = \int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)^2}{(i\,cdx+d)^{\frac{3}{2}}} dx$$

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] a^2*(I*(c^2*d*f*x^2 + d*f)^(3/2)/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3) + 6*
I*sqrt(c^2*d*f*x^2 + d*f)*f/(I*c^2*d^2*x + c*d^2) - 3*f^2*arcsinh(c*x)/(c*d
^2*sqrt(f/d))) + integrate((-I*c*f*x + f)^(3/2)*b^2*log(c*x + sqrt(c^2*x^2
+ 1))^2/(I*c*d*x + d)^(3/2) + 2*(-I*c*f*x + f)^(3/2)*a*b*log(c*x + sqrt(c^2
*x^2 + 1))/(I*c*d*x + d)^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx=\int \frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)^2\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}}{\left(d+c\,d\,x\,1\mathrm{i}\right)^{3/2}}\,dx$$

```
[In] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(3/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(3/2), x)
```

3.581 $\int \frac{(f-icfx)^{3/2}(a+b\,\mathrm{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 580

$$\begin{aligned}
\int \frac{(f-icfx)^{3/2}(a+b\,\mathrm{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx &= -\frac{8f^4(1+c^2x^2)^{5/2}(a+b\,\mathrm{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+\frac{f^4(1+c^2x^2)^{5/2}(a+b\,\mathrm{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{8ib^2f^4(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\,\mathrm{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&-\frac{8if^4(1+c^2x^2)^{5/2}(a+b\,\mathrm{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\,\mathrm{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+\frac{4bf^4(1+c^2x^2)^{5/2}(a+b\,\mathrm{arcsinh}(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\,\mathrm{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+\frac{2if^4(1+c^2x^2)^{5/2}(a+b\,\mathrm{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\,\mathrm{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\,\mathrm{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+\frac{32bf^4(1+c^2x^2)^{5/2}(a+b\,\mathrm{arcsinh}(cx))\log\left(1+ie^{\mathrm{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+\frac{32b^2f^4(1+c^2x^2)^{5/2}\,\mathrm{PolyLog}\left(2,-ie^{\mathrm{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}$$

```
[Out] -8/3*f^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*
f*x)^(5/2)+1/3*f^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(
5/2)/(f-I*c*f*x)^(5/2)-8/3*I*b^2*f^4*(c^2*x^2+1)^(5/2)*cot(1/4*Pi+1/2*I*arc
sinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-8/3*I*f^4*(c^2*x^2+1)^(5/2
)*(a+b*arcsinh(c*x))^2*cot(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(
f-I*c*f*x)^(5/2)+4/3*b*f^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*csc(1/4*Pi+
1/2*I*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3*I*f^4*(c^2*
x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2*cot(1/4*Pi+1/2*I*arcsinh(c*x))*csc(1/4*Pi
+1/2*I*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+32/3*b*f^4*(c^
```

```
2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*
d*x)^(5/2)/(f-I*c*f*x)^(5/2)+32/3*b^2*f^4*(c^2*x^2+1)^(5/2)*polylog(2,-I*(c
*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {5796, 5844, 5783, 5843, 3399, 4271, 3852, 8, 4269, 3797, 2221, 2317, 2438}

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx = \frac{f^4(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{8f^4(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{32bf^4(c^2x^2+1)^{5/2}\log\left(1+ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{8if^4(c^2x^2+1)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{4bf^4(c^2x^2+1)^{5/2}\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{2if^4(c^2x^2+1)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{32b^2f^4(c^2x^2+1)^{5/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{8ib^2f^4(c^2x^2+1)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[In] Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),x]

[Out] (-8*f^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*c*(d + I*c*d*x)^(5/2
)*(f - I*c*f*x)^(5/2)) + (f^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^3)/(
3*b*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((8*I)/3)*b^2*f^4*(1 + c^
2*x^2)^(5/2)*Cot[Pi/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*
c*f*x)^(5/2)) - (((8*I)/3)*f^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*C
ot[Pi/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
+ (4*b*f^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Csc[Pi/4 + (I/2)*ArcSin
h[c*x]]^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*f^4*(
1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]]*Cs
c[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
 + (32*b*f^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log[1 + I*E^ArcSinh[c
*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (32*b^2*f^4*(1 + c^2*
```

```
x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I
*c*f*x)^(5/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(1+c^2x^2\right)^{5/2} \int \frac{(f-icfx)^4(a+b\text{arcsinh}(cx))^2}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{5/2} \int \left(\frac{f^4(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}} - \frac{4f^4(a+b\text{arcsinh}(cx))^2}{(-i+cx)^2\sqrt{1+c^2x^2}} + \frac{4if^4(a+b\text{arcsinh}(cx))^2}{(-i+cx)\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(4if^4\left(1+c^2x^2\right)^{5/2}\right) \int \frac{(a+b\text{arcsinh}(cx))^2}{(-i+cx)\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(f^4\left(1+c^2x^2\right)^{5/2}\right) \int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(4f^4\left(1+c^2x^2\right)^{5/2}\right) \int \frac{(a+b\text{arcsinh}(cx))^2}{(-i+cx)^2\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{f^4\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(4if^4\left(1+c^2x^2\right)^{5/2}\right)\text{Subst}\left(\int \frac{(a+bx)^2}{-ic+c\sinh(x)}\,dx, x, \text{arcsinh}(cx)\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(4cf^4\left(1+c^2x^2\right)^{5/2}\right)\text{Subst}\left(\int \frac{(a+bx)^2}{(-ic+c\sinh(x))^2}\,dx, x, \text{arcsinh}(cx)\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{f^4\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(f^4\left(1+c^2x^2\right)^{5/2}\right)\text{Subst}\left(\int (a+bx)^2\csc^4\left(\frac{\pi}{4}+\frac{ix}{2}\right)dx, x, \text{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2f^4\left(1+c^2x^2\right)^{5/2}\right)\text{Subst}\left(\int (a+bx)^2\csc^2\left(\frac{\pi}{4}+\frac{ix}{2}\right)dx, x, \text{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$=\frac{f^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{4if^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{4bf^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2if^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{\left(2f^4(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int(a+bx)^2\csc^2\left(\frac{\pi}{4}+\frac{ix}{2}\right)\,dx,x,\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{\left(8ibf^4(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int(a+bx)\cot\left(\frac{\pi}{4}+\frac{ix}{2}\right)\,dx,x,\text{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{\left(4b^2f^4(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int\csc^2\left(\frac{\pi}{4}+\frac{ix}{2}\right)\,dx,x,\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$=-\frac{4f^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{f^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{8if^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{4bf^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2if^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{\left(8ibf^4(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int(a+bx)\cot\left(\frac{\pi}{4}+\frac{ix}{2}\right)\,dx,x,\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{\left(16ibf^4(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int\frac{e^x(a+bx)}{1+ie^x}\,dx,x,\text{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{\left(8ib^2f^4(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int 1\,dx,x,\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{8f^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{8ib^2f^4(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{8if^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{4bf^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{2if^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{16bf^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\log\left(1+ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{\left(16ibf^4(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int\frac{e^x(a+bx)}{1+ie^x}\,dx, x, \text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{\left(16b^2f^4(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int\log\left(1+ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{8f^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{8ib^2f^4(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{8if^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{4bf^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{2if^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{32bf^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\log\left(1+ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{\left(16b^2f^4(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int\log\left(1+ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{\left(16b^2f^4(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$
\begin{aligned}
&= -\frac{8f^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{f^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad -\frac{8ib^2f^4(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad -\frac{8if^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad +\frac{4bf^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad +\frac{2if^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad +\frac{32bf^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\log\left(1+ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad +\frac{16b^2f^4(1+c^2x^2)^{5/2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad +\frac{\left(16b^2f^4(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{8f^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{f^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad -\frac{8ib^2f^4(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad -\frac{8if^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad +\frac{4bf^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad +\frac{2if^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad +\frac{32bf^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\log\left(1+ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad +\frac{32b^2f^4(1+c^2x^2)^{5/2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1609 vs. $2(580) = 1160$.

Time = 17.16 (sec) , antiderivative size = 1609, normalized size of antiderivative = 2.77

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx = \frac{\sqrt{id(-i+cx)}\sqrt{-if(i+cx)}\left(-\frac{4ia^2f}{3d^3(-i+cx)^2}-\frac{8a^2f}{3d^3(-i+cx)}\right)}{c}$$
$$+\frac{a^2f^{3/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{id(-i+cx)}\sqrt{-if(i+cx)}\right)}{cd^{5/2}}$$
$$+\frac{iabf\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\sqrt{-df\left(1+c^2x^2\right)}\left(\cosh\left(\frac{1}{2}\operatorname{arcsinh}(cx)\right)-i\sinh\left(\frac{1}{2}\operatorname{arcsinh}(cx)\right)\right)(-i\operatorname{co}}{}$$
$$-\frac{abf\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\sqrt{-df\left(1+c^2x^2\right)}\left(\cosh\left(\frac{1}{2}\operatorname{arcsinh}(cx)\right)-i\sinh\left(\frac{1}{2}\operatorname{arcsinh}(cx)\right)\right)(\cosh(}{}$$
$$+\frac{ib^2f(i+cx)\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\sqrt{-df\left(1+c^2x^2\right)}\left((-1+i)\operatorname{arcsinh}(cx)^2-\frac{2\operatorname{arcsinh}(cx)(-2i+\operatorname{arc}}{-i+cx}\right.}{}$$
$$+\frac{b^2f(i+cx)\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\sqrt{-df\left(1+c^2x^2\right)}\left(7\pi\operatorname{arcsinh}(cx)-(7+7i)\operatorname{arcsinh}(cx)^2-i\operatorname{arc}\right.}{}$$

```
[In] Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),
x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((((-4*I)/3)*a^2*f)/(d^3*(-I +
 c*x)^2) - (8*a^2*f)/(3*d^3*(-I + c*x))))/c + (a^2*f^(3/2)*Log[c*d*f*x + Sq
rt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(c*d^(5/2)) + (
(I/3)*a*b*f*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1
 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*((-I)*Cosh[(3
*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] - I*Log[Sq
rt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/2]*(4 + (3*I)*ArcSinh[c*x] - (6*I)*Ar
cTan[Coth[ArcSinh[c*x]/2]] + 3*Log[Sqrt[1 + c^2*x^2]]) + 2*(Sqrt[1 + c^2*x^
2]*(ArcSinh[c*x] + 2*ArcTan[Coth[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]
]) + 2*(I + ArcSinh[c*x] + 2*ArcTan[Coth[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 +
c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c*d^3*(I + c*x)*Sqrt[-(((-I)*d + c*d*x)
*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^4) - (a*b*
f*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2
))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*ArcSinh[c*x])/
2]*((-14 + (3*I)*ArcSinh[c*x])*ArcSinh[c*x] - 28*ArcTan[Tanh[ArcSinh[c*x]/2
]] + (14*I)*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/2]*(84*ArcTan[Tanh[
```

```
ArcSinh[c*x]/2]] - I*(8 - (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 + 42*Log[Sq
rt[1 + c^2*x^2]])) + 2*(4 - (4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 + (56*I)*
ArcTan[Tanh[ArcSinh[c*x]/2]] + 28*Log[Sqrt[1 + c^2*x^2]] + Sqrt[1 + c^2*x^2
]*(ArcSinh[c*x]*(-14*I + 3*ArcSinh[c*x]) + (28*I)*ArcTan[Tanh[ArcSinh[c*x]/
2]] + 14*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(6*c*d^3*(I + c*x)
*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[Arc
Sinh[c*x]/2])^4) + ((I/3)*b^2*f*(I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I
)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*((-1 + I)*ArcSinh[c*x]^2 - (2*A
rcSinh[c*x]*(-2*I + ArcSinh[c*x]))/(-I + c*x) + (2*I)*(Pi + (2*I)*ArcSinh[c
*x])*Log[1 - I/E^ArcSinh[c*x]] - I*Pi*(ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c
*x]] + 4*Log[Cosh[ArcSinh[c*x]/2]] + 2*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]
]) + 4*PolyLog[2, I/E^ArcSinh[c*x]] - (4*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2
])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^3 + (2*(4 + ArcSinh[c*x]
^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))
/(c*d^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[Arc
Sinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^2) + (b^2*f*(I + c*x)*Sqrt[I*((-I)*d
 + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(7*Pi*ArcSin
h[c*x] - (7 + 7*I)*ArcSinh[c*x]^2 - I*ArcSinh[c*x]^3 + (2*ArcSinh[c*x]*(-2*
I + ArcSinh[c*x]))/(1 + I*c*x) - 14*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^A
rcSinh[c*x]] - 28*Pi*Log[1 + E^ArcSinh[c*x]] + 28*Pi*Log[Cosh[ArcSinh[c*x]/
2]] + 14*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]] + (28*I)*PolyLog[2, I/E^A
rcSinh[c*x]] - ((4*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*
x]/2] + I*Sinh[ArcSinh[c*x]/2])^3 + (2*(4 + 7*ArcSinh[c*x]^2)*Sinh[ArcSinh[
c*x]/2])/((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])))/(3*c*d^3*Sqrt
[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2]
 - I*Sinh[ArcSinh[c*x]/2])^2)
```

Maple [F]

$$\int \frac{(-icfx+f)^{\frac{3}{2}}\,(a+b\,\mathrm{arcsinh}\,(cx))^2}{(icdx+d)^{\frac{5}{2}}}dx$$

[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x)

[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x)

Fricas [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)^2}{(i\,cdx+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral(((b^2*c*f*x + I*b^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c*f*x + I*a*b*f)*sqrt(I*c*d*x + d)*sqrt(-
I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c*f*x + I*a^2*f)*sqrt(I*c*
d*x + d)*sqrt(-I*c*f*x + f))/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c*d^3*x + I
*d^3), x)
```

Sympy [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx = \int \frac{(-if(cx+i))^{\frac{3}{2}}\,(a+b\operatorname{asinh}(cx))^2}{(id\,(cx-i))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2),x)
```

```
[Out] Integral((-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))**2/(I*d*(c*x - I))**(5/
2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(f-c\,f\,x\,1\text{i})^{3/2}}{(d+c\,d\,x\,1\text{i})^{5/2}}\,dx$$

```
[In] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(5/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(5/2), x)
```

3.582 $\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 37, antiderivative size = 548

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{1}{108}b^2x(d+icdx)^{5/2}(f-icfx)^{5/2}+\frac{245b^2x(d+icdx)^{5/2}(f-icfx)^{5/2}}{1152\,(1+c^2x^2)^2}+\frac{65b}{}$$

```
[Out] 1/108*b^2*x*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)+245/1152*b^2*x*(d+I*c*d*x)^
(5/2)*(f-I*c*f*x)^(5/2)/(c^2*x^2+1)^2+65/1728*b^2*x*(d+I*c*d*x)^(5/2)*(f-I*
c*f*x)^(5/2)/(c^2*x^2+1)-115/1152*b^2*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*a
rcsinh(c*x)/c/(c^2*x^2+1)^(5/2)-5/16*b*c*x^2*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^
(5/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(5/2)+1/6*x*(d+I*c*d*x)^(5/2)*(f-I*c*f
*x)^(5/2)*(a+b*arcsinh(c*x))^2+5/16*x*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(
a+b*arcsinh(c*x))^2/(c^2*x^2+1)^2+5/24*x*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2
)*(a+b*arcsinh(c*x))^2/(c^2*x^2+1)+5/48*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)
*(a+b*arcsinh(c*x))^3/b/c/(c^2*x^2+1)^(5/2)-5/48*b*(d+I*c*d*x)^(5/2)*(f-I*c
*f*x)^(5/2)*(a+b*arcsinh(c*x))/c/(c^2*x^2+1)^(1/2)-1/18*b*(d+I*c*d*x)^(5/2)
*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used

$= \{5796, 5786, 5785, 5783, 5776, 327, 221, 5798, 201\}$

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{5(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^3}{48bc\,(c^2x^2+1)^{5/2}} + \frac{5x(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{24\,(c^2x^2+1)} + \frac{5x(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{16\,(c^2x^2+1)^2} - \frac{b\sqrt{c^2x^2+1}(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{18c} - \frac{5b(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{48c\sqrt{c^2x^2+1}} - \frac{5bcx^2(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{16\,(c^2x^2+1)^{5/2}} + \frac{1}{6}x(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2 - \frac{115b^2\text{arcsinh}(cx)(d+icdx)^{5/2}(f-icfx)^{5/2}}{1152c\,(c^2x^2+1)^{5/2}} + \frac{65b^2x(d+ic}{1728}$$

```
[In] Int[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (b^2*x*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))/108 + (245*b^2*x*(d + I*c*d
*x)^(5/2)*(f - I*c*f*x)^(5/2))/(1152*(1 + c^2*x^2)^2) + (65*b^2*x*(d + I*c*
d*x)^(5/2)*(f - I*c*f*x)^(5/2))/(1728*(1 + c^2*x^2)) - (115*b^2*(d + I*c*d*
x)^(5/2)*(f - I*c*f*x)^(5/2)*ArcSinh[c*x])/(1152*c*(1 + c^2*x^2)^(5/2)) - (
5*b*c*x^2*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(16
*(1 + c^2*x^2)^(5/2)) - (5*b*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b
*ArcSinh[c*x]))/(48*c*Sqrt[1 + c^2*x^2]) - (b*(d + I*c*d*x)^(5/2)*(f - I*c*
f*x)^(5/2)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(18*c) + (x*(d + I*c*d*x
)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/6 + (5*x*(d + I*c*d*x)^
(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(16*(1 + c^2*x^2)^2) + (5
*x*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(24*(1 +
 c^2*x^2)) + (5*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]
)^3)/(48*b*c*(1 + c^2*x^2)^(5/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
```

```
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\left((d+icdx)^{5/2}(f-icfx)^{5/2}\right)\int \left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx}{\left(1+c^2x^2\right)^{5/2}}$$

$$= \frac{1}{6}x(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2 + \frac{\left(5(d+icdx)^{5/2}(f-icfx)^{5/2}\right)\int \left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx}{6\left(1+c^2x^2\right)^{5/2}} - \frac{\left(bc(d+icdx)^{5/2}(f-icfx)^{5/2}\right)\int x\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))\,dx}{3\left(1+c^2x^2\right)^{5/2}}$$

$$= -\frac{b(d+icdx)^{5/2}(f-icfx)^{5/2}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{18c} + \frac{1}{6}x(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2 + \frac{5x(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{24\left(1+c^2x^2\right)}$$

$$= \frac{1}{108}b^2x(d+icdx)^{5/2}(f-icfx)^{5/2} - \frac{5b(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{48c\sqrt{1+c^2x^2}} - \frac{b(d+icdx)^{5/2}(f-icfx)^{5/2}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{18c} + \frac{1}{6}x(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2 + \frac{5x(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{16\left(1+c^2x^2\right)^2}$$

$$= \frac{1}{108} b^2 x (d+icdx)^{5/2} (f-icfx)^{5/2} + \frac{65 b^2 x (d+icdx)^{5/2} (f-icfx)^{5/2}}{1728 (1+c^2x^2)} - \frac{5bcx^2 (d+icdx)^{5/2} (f-icfx)^{5/2} (a+b\text{arcsinh}(cx))}{16 (1+c^2x^2)^{5/2}} - \frac{5b (d+icdx)^{5/2} (f-icfx)^{5/2} (a+b\text{arcsinh}(cx))}{48c\sqrt{1+c^2x^2}} - \frac{b (d+icdx)^{5/2} (f-icfx)^{5/2} \sqrt{1+c^2x^2} (a+b\text{arcsinh}(cx))}{18c} + \frac{1}{6} x (d+icdx)^{5/2} (f-icfx)^{5/2} (a+b\text{arcsinh}(cx))^2 + \frac{5x (d+icdx)^{5/2} (f-icfx)^{5/2} (a+b\text{arcsinh}(cx}{16 (1+c^2x^2)^2}$$

$$= \frac{1}{108} b^2 x (d+icdx)^{5/2} (f-icfx)^{5/2} + \frac{245 b^2 x (d+icdx)^{5/2} (f-icfx)^{5/2}}{1152 (1+c^2x^2)^2} + \frac{65 b^2 x (d+icdx)^{5/2} (f-icfx)^{5/2}}{1728 (1+c^2x^2)} - \frac{5bcx^2 (d+icdx)^{5/2} (f-icfx)^{5/2} (a+b\text{arcsinh}(cx))}{16 (1+c^2x^2)^{5/2}} - \frac{5b (d+icdx)^{5/2} (f-icfx)^{5/2} (a+b\text{arcsinh}(cx))}{48c\sqrt{1+c^2x^2}} - \frac{b (d+icdx)^{5/2} (f-icfx)^{5/2} \sqrt{1+c^2x^2} (a+b\text{arcsinh}(cx))}{18c} + \frac{1}{6} x (d+icdx)^{5/2} (f-icfx)^{5/2} (a+b\text{arcsinh}(cx))^2 + \frac{5x (d+icdx)^{5/2} (f-icfx)^{5/2} (a+b\text{arcsinh}(cx}{16 (1+c^2x^2)^2}$$

$$= \frac{1}{108} b^2 x (d+icdx)^{5/2} (f-icfx)^{5/2} + \frac{245 b^2 x (d+icdx)^{5/2} (f-icfx)^{5/2}}{1152 (1+c^2x^2)^2} + \frac{65 b^2 x (d+icdx)^{5/2} (f-icfx)^{5/2}}{1728 (1+c^2x^2)} - \frac{115 b^2 (d+icdx)^{5/2} (f-icfx)^{5/2} \text{arcsinh}(cx)}{1152c (1+c^2x^2)^{5/2}} - \frac{5bcx^2 (d+icdx)^{5/2} (f-icfx)^{5/2} (a+b\text{arcsinh}(cx))}{16 (1+c^2x^2)^{5/2}} - \frac{5b (d+icdx)^{5/2} (f-icfx)^{5/2} (a+b\text{arcsinh}(cx))}{48c\sqrt{1+c^2x^2}} - \frac{b (d+icdx)^{5/2} (f-icfx)^{5/2} \sqrt{1+c^2x^2} (a+b\text{arcsinh}(cx))}{18c} + \frac{1}{6} x (d+icdx)^{5/2} (f-icfx)^{5/2} (a+b\text{arcsinh}(cx))^2 + \frac{5x (d+icdx)^{5/2} (f-icfx)^{5/2} (a+b\text{arcsinh}(cx}{16 (1+c^2x^2)^2}$$

Mathematica [A] (verified)

Time = 3.95 (sec) , antiderivative size = 735, normalized size of antiderivative = 1.34

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\frac{9504a^2cd^2f^2x\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+7488a^2c^3d^2f^2x^3\sqrt{d+icdx}}{}$$

```
[In] Integrate[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (9504*a^2*c*d^2*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]
 + 7488*a^2*c^3*d^2*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^
2*x^2] + 2304*a^2*c^5*d^2*f^2*x^5*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[
1 + c^2*x^2] + 1440*b^2*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh
[c*x]^3 - 3240*a*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSi
nh[c*x]] - 324*a*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSi
nh[c*x]] - 24*a*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[6*ArcSin
h[c*x]] + 4320*a^2*d^(5/2)*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*
Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 1620*b^2*d^2*f^2*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 81*b^2*d^2*f^2*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] + 4*b^2*d^2*f^2*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]*Sinh[6*ArcSinh[c*x]] - 12*b*d^2*f^2*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(270*b*Cosh[2*ArcSinh[c*x]] + 27*b*Cosh[4*A
rcSinh[c*x]] + 2*b*Cosh[6*ArcSinh[c*x]] - 540*a*Sinh[2*ArcSinh[c*x]] - 108*
a*Sinh[4*ArcSinh[c*x]] - 12*a*Sinh[6*ArcSinh[c*x]]) + 72*b*d^2*f^2*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a + 45*b*Sinh[2*ArcSinh[c*x]
] + 9*b*Sinh[4*ArcSinh[c*x]] + b*Sinh[6*ArcSinh[c*x]]))/(13824*c*Sqrt[1 + c
^2*x^2])
```

Maple [F]

$$\int (icdx+d)^{\frac{5}{2}}\,(-icfx+f)^{\frac{5}{2}}\,(a+b\,\text{arcsinh}\,(cx))^2\,dx$$

```
[In] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

Fricas [F]

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \int (i\,cdx+d)^{\frac{5}{2}}(-i\,cfx+f)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2\,dx$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="fricas")
```

```
[Out] integral((b^2*c^4*d^2*f^2*x^4 + 2*b^2*c^2*d^2*f^2*x^2 + b^2*d^2*f^2)*sqrt(I
*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c^4*
d^2*f^2*x^4 + 2*a*b*c^2*d^2*f^2*x^2 + a*b*d^2*f^2)*sqrt(I*c*d*x + d)*sqrt(-
I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c^4*d^2*f^2*x^4 + 2*a^2*c^
2*d^2*f^2*x^2 + a^2*d^2*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Timed out}$$

```
[In] integrate((d+I*c*d*x)**(5/2)*(f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,(d+c\,d\,x\,1\text{i})^{5/2}\,(f-c\,f\,x\,1\text{i})^{5/2}\,dx$$

```
[In] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2), x)
```

3.583 $\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\textbf{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 37, antiderivative size = 774

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = -\frac{8ib^2f(d+icdx)^{3/2}(f-icfx)^{3/2}}{225c}$$

$$+\frac{1}{32}b^2fx(d+icdx)^{3/2}(f-icfx)^{3/2}-\frac{16ib^2f(d+icdx)^{3/2}(f-icfx)^{3/2}}{75c\,(1+c^2x^2)}+\frac{15b^2fx(d+icdx)^{3/2}(f-icfx)^{3/2}}{64\,(1+c^2x^2)}-$$

```
[Out] -8/225*I*b^2*f*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/c+1/32*b^2*f*x*(d+I*c*d*
x)^(3/2)*(f-I*c*f*x)^(3/2)-16/75*I*b^2*f*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2
)/c/(c^2*x^2+1)+15/64*b^2*f*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+
1)-2/125*I*b^2*f*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(c^2*x^2+1)/c-9/64*b^2
*f*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*arcsinh(c*x)/c/(c^2*x^2+1)^(3/2)+2/5
*I*b*f*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)
^(3/2)-3/8*b*c*f*x^2*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))
/(c^2*x^2+1)^(3/2)+4/15*I*b*c^2*f*x^3*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(
a+b*arcsinh(c*x))/(c^2*x^2+1)^(3/2)+2/25*I*b*c^4*f*x^5*(d+I*c*d*x)^(3/2)*(f
-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(3/2)+1/4*f*x*(d+I*c*d*x)^(3
/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2+3/8*f*x*(d+I*c*d*x)^(3/2)*(f-I*c
*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(c^2*x^2+1)-1/5*I*f*(d+I*c*d*x)^(3/2)*(f-I
*c*f*x)^(3/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c+1/8*f*(d+I*c*d*x)^(3/2)*(f
-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(c^2*x^2+1)^(3/2)-1/8*b*f*(d+I*c*d
*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 774, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.405, Rules used = {5796, 5838, 5786, 5785, 5783, 5776, 327, 221, 5798, 201, 200, 5784, 12, 1261, 712}

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx =$$
$$-\frac{3bcfx^2(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{8(c^2x^2+1)^{3/2}}$$
$$+\frac{3fx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{8(c^2x^2+1)}$$
$$+\frac{2ibfx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{5(c^2x^2+1)^{3/2}}$$
$$+\frac{f(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^3}{8bc(c^2x^2+1)^{3/2}}$$
$$-\frac{if(c^2x^2+1)(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{5c}$$
$$-\frac{bf\sqrt{c^2x^2+1}(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{8c}$$
$$+\frac{4ibc^2fx^3(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{15(c^2x^2+1)^{3/2}}$$
$$+\frac{2ibc^4fx^5(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{25(c^2x^2+1)^{3/2}}$$
$$+\frac{1}{4}fx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{9b^2f\text{arcsinh}(cx)(d+icdx)^{3/2}(f-icfx)^{3/2}}{64c(c^2x^2+1)^{3/2}}+\frac{15b^2fx(d+}{64}$$

```
[In] Int[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (((-8*I)/225)*b^2*f*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/c + (b^2*f*x*(
d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/32 - (((16*I)/75)*b^2*f*(d + I*c*d*
x)^(3/2)*(f - I*c*f*x)^(3/2))/(c*(1 + c^2*x^2)) + (15*b^2*f*x*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2))/(64*(1 + c^2*x^2)) - (((2*I)/125)*b^2*f*(d + I*
c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(1 + c^2*x^2))/c - (9*b^2*f*(d + I*c*d*x)^
(3/2)*(f - I*c*f*x)^(3/2)*ArcSinh[c*x])/(64*c*(1 + c^2*x^2)^(3/2)) + (((2*I
)/5)*b*f*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(1
 + c^2*x^2)^(3/2) - (3*b*c*f*x^2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a
 + b*ArcSinh[c*x]))/(8*(1 + c^2*x^2)^(3/2)) + (((4*I)/15)*b*c^2*f*x^3*(d +
I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(1 + c^2*x^2)^(3/2
) + (((2*I)/25)*b*c^4*f*x^5*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*
ArcSinh[c*x]))/(1 + c^2*x^2)^(3/2) - (b*f*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(8*c) + (f*x*(d + I*c*d*x)^(
```

```
3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + (3*f*x*(d + I*c*d*x)^(
3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(8*(1 + c^2*x^2)) - ((I/5)
*f*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x
])^2)/c + (f*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^3
)/(8*b*c*(1 + c^2*x^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
```

```
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\text{integral} = \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int (f-icfx)\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx}{(1+c^2x^2)^{3/2}}$$

$$= \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(f(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2-icfx(1+c^2x^2)^{3/2}(a+b\text{arcs}\right.}{(1+c^2x^2)^{3/2}}$$

$$= \frac{\left(f(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx}{(1+c^2x^2)^{3/2}} - \frac{\left(icf(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx}{(1+c^2x^2)^{3/2}}$$

$$= \frac{1}{4}fx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2 - \frac{if(d+icdx)^{3/2}(f-icfx)^{3/2}\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2}{5c} + \frac{\left(3f(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\,dx}{4\left(1+c^2x^2\right)^{3/2}} + \frac{\left(2ibf(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))\,dx}{5\left(1+c^2x^2\right)^{3/2}} - \frac{\left(bcf(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int x(1+c^2x^2)(a+b\text{arcsinh}(cx))\,dx}{2\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{2ibfx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{5(1+c^2x^2)^{3/2}} + \frac{4ibc^2fx^3(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{15(1+c^2x^2)^{3/2}} + \frac{2ibc^4fx^5(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{25(1+c^2x^2)^{3/2}} - \frac{bf(d+icdx)^{3/2}(f-icfx)^{3/2}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{8c} + \frac{1}{4}fx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{3fx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx}{8(1+c^2x^2)}$$

$$= \frac{1}{32}b^2fx(d+icdx)^{3/2}(f-icfx)^{3/2} + \frac{2ibfx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{5(1+c^2x^2)^{3/2}} - \frac{3bcfx^2(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{8(1+c^2x^2)^{3/2}} + \frac{4ibc^2fx^3(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{15(1+c^2x^2)^{3/2}} + \frac{2ibc^4fx^5(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{25(1+c^2x^2)^{3/2}} - \frac{bf(d+icdx)^{3/2}(f-icfx)^{3/2}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{8c} + \frac{1}{4}fx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{3fx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(c}{8(1+c^2x^2)}$$

$$= \frac{1}{32}b^2fx(d+icdx)^{3/2}(f-icfx)^{3/2} + \frac{15b^2fx(d+icdx)^{3/2}(f-icfx)^{3/2}}{64(1+c^2x^2)} + \frac{2ibfx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{5(1+c^2x^2)^{3/2}} - \frac{3bcfx^2(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{8(1+c^2x^2)^{3/2}} + \frac{4ibc^2fx^3(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{15(1+c^2x^2)^{3/2}} + \frac{2ibc^4fx^5(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{25(1+c^2x^2)^{3/2}} - \frac{bf(d+icdx)^{3/2}(f-icfx)^{3/2}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{8c} + \frac{1}{4}fx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{3fx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(}{8(1+c^2x^2)}$$

$$= \frac{1}{32}b^2fx(d+icdx)^{3/2}(f-icfx)^{3/2} + \frac{15b^2fx(d+icdx)^{3/2}(f-icfx)^{3/2}}{64(1+c^2x^2)} - \frac{9b^2f(d+icdx)^{3/2}(f-icfx)^{3/2}\text{arcsinh}(cx)}{64c(1+c^2x^2)^{3/2}} + \frac{2ibfx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{5(1+c^2x^2)^{3/2}} - \frac{3bcfx^2(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{8(1+c^2x^2)^{3/2}} + \frac{4ibc^2fx^3(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{15(1+c^2x^2)^{3/2}} + \frac{2ibc^4fx^5(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{25(1+c^2x^2)^{3/2}} - \frac{bf(d+icdx)^{3/2}(f-icfx)^{3/2}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{8c} + \frac{1}{4}fx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{3fx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(}{8(1+c^2x^2)}$$

$$= -\frac{8ib^2f(d+icdx)^{3/2}(f-icfx)^{3/2}}{225c} + \frac{1}{32}b^2fx(d+icdx)^{3/2}(f-icfx)^{3/2} - \frac{16ib^2f(d+icdx)^{3/2}(f-icfx)^{3/2}}{75c(1+c^2x^2)} + \frac{15b^2fx(d+icdx)^{3/2}(f-}{64(1+c^2x^2)}$$

Mathematica [A] (verified)

Time = 4.63 (sec) , antiderivative size = 1084, normalized size of antiderivative = 1.40

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{72000iabcdf^2x\sqrt{d+icdx}\sqrt{f-icfx}-57600ia^2df^2\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{}}{}$$

```
[In] Integrate[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((72000*I)*a*b*c*d*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (57600*I)*a^
2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (72000*I)*b
^2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 180000*a^2
*c*d*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (115200*
I)*a^2*c^2*d*f^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]
+ 72000*a^2*c^3*d*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*
x^2] - (57600*I)*a^2*c^4*d*f^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt
[1 + c^2*x^2] + 36000*b^2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh
[c*x]^3 - 72000*a*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSin
h[c*x]] - (4000*I)*b^2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[3*Arc
Sinh[c*x]] - 4500*a*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcS
inh[c*x]] - (288*I)*b^2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[5*Ar
cSinh[c*x]] + 108000*a^2*d^(3/2)*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sq
rt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 36000*b^2*d*f^2*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + (12000*I)*a*b*d*f^2*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 1125*b^2*d*f^2*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] + 1800*b*d*f^2*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a - (20*I)*b*Sqrt[1 + c^2*
x^2] - (10*I)*b*Cosh[3*ArcSinh[c*x]] - (2*I)*b*Cosh[5*ArcSinh[c*x]] + 40*b*
Sinh[2*ArcSinh[c*x]] + 5*b*Sinh[4*ArcSinh[c*x]]) + (1440*I)*a*b*d*f^2*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[5*ArcSinh[c*x]] + 60*b*d*f^2*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((1200*I)*b*c*x - (1200*I)*a*Sqrt[1
+ c^2*x^2] - 1200*b*Cosh[2*ArcSinh[c*x]] - (600*I)*a*Cosh[3*ArcSinh[c*x]] -
 75*b*Cosh[4*ArcSinh[c*x]] - (120*I)*a*Cosh[5*ArcSinh[c*x]] + 2400*a*Sinh[2
*ArcSinh[c*x]] + (200*I)*b*Sinh[3*ArcSinh[c*x]] + 300*a*Sinh[4*ArcSinh[c*x]
] + (24*I)*b*Sinh[5*ArcSinh[c*x]]))/(288000*c*Sqrt[1 + c^2*x^2])
```

Maple [F]

$$\int (icdx+d)^{\frac{3}{2}} (-icfx+f)^{\frac{5}{2}} (a+b\operatorname{arcsinh}(cx))^2\,dx$$

```
[In] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

Fricas [F]

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \int (i\,cdx+d)^{\frac{3}{2}}(-i\,cfx+f)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)^2\,dx$$

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="fricas")
```

```
[Out] integral((-I*b^2*c^3*d*f^2*x^3 + b^2*c^2*d*f^2*x^2 - I*b^2*c*d*f^2*x + b^2*
d*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2
- 2*(I*a*b*c^3*d*f^2*x^3 - a*b*c^2*d*f^2*x^2 + I*a*b*c*d*f^2*x - a*b*d*f^2)
*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a^
2*c^3*d*f^2*x^3 + a^2*c^2*d*f^2*x^2 - I*a^2*c*d*f^2*x + a^2*d*f^2)*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \text{Timed out}$$

```
[In] integrate((d+I*c*d*x)**(3/2)*(f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d + icdx)^{3/2}(f - icfx)^{5/2}(a + b\text{arcsinh}(cx))^2\, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (d + icdx)^{3/2}(f - icfx)^{5/2}(a + b\text{arcsinh}(cx))^2\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad Ar
gument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int (d + icdx)^{3/2}(f - icfx)^{5/2}(a + b\text{arcsinh}(cx))^2\, dx = \int (a + b\,\text{asinh}(c\,x))^2\,(d + c\,d\,x\,1\text{i})^{3/2}\,(f - c\,f\,x\,1\text{i})^{5/2}\,dx$$

```
[In] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2), x)
```

3.584 $\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 37, antiderivative size = 680

$$
\begin{aligned}
&\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \\
&-\frac{8ib^2f^2\sqrt{d+icdx}\sqrt{f-icfx}}{9c}+\frac{15}{64}b^2f^2x\sqrt{d+icdx}\sqrt{f-icfx} \\
&-\frac{1}{32}b^2c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}-\frac{4ib^2f^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)}{27c} \\
&-\frac{15b^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\text{arcsinh}(cx)}{64c\sqrt{1+c^2x^2}} \\
&+\frac{4ibf^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} \\
&-\frac{3bcf^2x^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} \\
&+\frac{4ibc^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} \\
&+\frac{bc^3f^2x^4\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} \\
&+\frac{3}{8}f^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 \\
&-\frac{1}{4}c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 \\
&-\frac{2if^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3c} \\
&+\frac{5f^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{24bc\sqrt{1+c^2x^2}}
\end{aligned}
$$

```
[Out] -8/9*I*b^2*f^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+15/64*b^2*f^2*x*(d+I*c
*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/32*b^2*c^2*f^2*x^3*(d+I*c*d*x)^(1/2)*(f-I*c
*f*x)^(1/2)-4/27*I*b^2*f^2*(c^2*x^2+1)*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/
c+3/8*f^2*x*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/4*c^
2*f^2*x^3*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-2/3*I*f^
2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c-15
/64*b^2*f^2*arcsinh(c*x)*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c/(c^2*x^2+1)^
(1/2)+4/3*I*b*f^2*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/
(c^2*x^2+1)^(1/2)-3/8*b*c*f^2*x^2*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I
*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+4/9*I*b*c^2*f^2*x^3*(a+b*arcsinh(c*x))*(d+I
*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/8*b*c^3*f^2*x^4*(a+b*ar
csinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+5/24*f^2*
(a+b*arcsinh(c*x))^3*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1
/2)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules

used = {5796, 5838, 5785, 5783, 5776, 327, 221, 5798, 5784, 455, 45, 5806, 5812}

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = -\frac{1}{4}c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2 - \frac{3bcf^2x^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{8\sqrt{c^2x^2+1}} + \frac{4ibf^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{3\sqrt{c^2x^2+1}} + \frac{5f^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^3}{24bc\sqrt{c^2x^2+1}} - \frac{2if^2(c^2x^2+1)\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2}{3c} + \frac{4ibc^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{9\sqrt{c^2x^2+1}} + \frac{bc^3f^2x^4\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{8\sqrt{c^2x^2+1}} + \frac{3}{8}f^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2 - \frac{15b^2f^2\operatorname{arcsinh}(cx)\sqrt{d+icdx}\sqrt{f-icfx}}{64c\sqrt{c^2x^2+1}} - \frac{1}{32}b^2c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx} - \frac{4ib^2f^2(c^2x^2+1)\sqrt{d+icdx}\sqrt{f-icfx}}{27c} + \frac{15}{64}b^2f^2x\sqrt{d+icdx}\sqrt{f-icfx} - \frac{8ib^2f^2\sqrt{d+icdx}\sqrt{f-icfx}}{9c}$$

```
[In] Int[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (((-8*I)/9)*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/c + (15*b^2*f^2*x*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/64 - (b^2*c^2*f^2*x^3*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x])/32 - (((4*I)/27)*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x]*(1 + c^2*x^2))/c - (15*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*A
rcSinh[c*x])/(64*c*Sqrt[1 + c^2*x^2]) + (((4*I)/3)*b*f^2*x*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] - (3*b*c*f^2*x^
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c^2
*x^2]) + (((4*I)/9)*b*c^2*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a +
b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] + (b*c^3*f^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c^2*x^2]) + (3*f^2*x*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/8 - (c^2*f^2*x^3*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/4 - (((2*I)/3)*f^2*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/c
+ (5*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^3)/(24*b*
c*Sqrt[1 + c^2*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5784

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
```

```
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
```

```
1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int (f-icfx)^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \left(f^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2-2icf^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(f^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\,dx}{\sqrt{1+c^2x^2}} - \frac{\left(2icf^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\,dx}{\sqrt{1+c^2x^2}} - \frac{\left(c^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{2}f^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 - \frac{1}{4}c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 - \frac{2if^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3c} + \frac{\left(f^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}} + \frac{\left(4ibf^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int (1+c^2x^2)\,(a+b\text{arcsinh}(cx))\,dx}{3\sqrt{1+c^2x^2}} - \frac{\left(bcf^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x(a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}} - \frac{\left(c^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{4\sqrt{1+c^2x^2}} + \frac{\left(bc^3f^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x^3(a+b\text{arcsinh}(cx))\,dx}{2\sqrt{1+c^2x^2}}$$

$$
\begin{aligned}
&= \frac{4ibf^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} \\
&\quad - \frac{bcf^2x^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}} \\
&\quad + \frac{4ibc^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} \\
&\quad + \frac{bc^3f^2x^4\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} \\
&\quad + \frac{3}{8}f^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 \\
&\quad - \frac{1}{4}c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 \\
&\quad - \frac{2if^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3c} \\
&\quad + \frac{f^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(f^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{8\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(bcf^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int x(a+b\text{arcsinh}(cx))\,dx}{4\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(4ib^2cf^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\frac{x\left(1+\frac{c^2x^2}{3}\right)}{\sqrt{1+c^2x^2}}\,dx}{3\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(b^2c^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\frac{x^2}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(b^2c^4f^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\frac{x^4}{\sqrt{1+c^2x^2}}\,dx}{8\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{4}b^2f^2x\sqrt{d+icdx}\sqrt{f-icfx} - \frac{1}{32}b^2c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx} \\
&+ \frac{4ibf^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} \\
&- \frac{3bcf^2x^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} \\
&+ \frac{4ibc^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} \\
&+ \frac{bc^3f^2x^4\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} \\
&+ \frac{3}{8}f^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 \\
&- \frac{1}{4}c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 \\
&- \frac{2if^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3c} \\
&+ \frac{5f^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{24bc\sqrt{1+c^2x^2}} \\
&- \frac{\left(b^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \frac{1}{\sqrt{1+c^2x^2}}\,dx}{4\sqrt{1+c^2x^2}} \\
&- \frac{\left(2ib^2cf^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\text{Subst}\left(\int \frac{1+\frac{c^2x}{3}}{\sqrt{1+c^2x}}\,dx, x, x^2\right)}{3\sqrt{1+c^2x^2}} \\
&+ \frac{\left(3b^2c^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \frac{x^2}{\sqrt{1+c^2x^2}}\,dx}{32\sqrt{1+c^2x^2}} \\
&- \frac{\left(b^2c^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \frac{x^2}{\sqrt{1+c^2x^2}}\,dx}{8\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{15}{64}b^2f^2x\sqrt{d+icdx}\sqrt{f-icfx} - \frac{1}{32}b^2c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx} \\
&\quad - \frac{b^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\text{arcsinh}(cx)}{4c\sqrt{1+c^2x^2}} \\
&\quad + \frac{4ibf^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} \\
&\quad - \frac{3bcf^2x^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} \\
&\quad + \frac{4ibc^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} \\
&\quad + \frac{bc^3f^2x^4\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} \\
&\quad + \frac{3}{8}f^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 \\
&\quad - \frac{1}{4}c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 \\
&\quad - \frac{2if^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3c} \\
&\quad + \frac{5f^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{24bc\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(3b^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \frac{1}{\sqrt{1+c^2x^2}}\,dx}{64\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(b^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \frac{1}{\sqrt{1+c^2x^2}}\,dx}{16\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(2ib^2cf^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\text{Subst}\left(\int \left(\frac{2}{3\sqrt{1+c^2x}}+\frac{1}{3}\sqrt{1+c^2x}\right)\,dx,x,x^2\right)}{3\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$= -\frac{8ib^2f^2\sqrt{d+icdx}\sqrt{f-icfx}}{9c} + \frac{15}{64}b^2f^2x\sqrt{d+icdx}\sqrt{f-icfx} - \frac{1}{32}b^2c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx} - \frac{4ib^2f^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)}{27c} - \frac{15b^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\text{arcsinh}(cx)}{64c\sqrt{1+c^2x^2}} + \frac{4ibf^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} - \frac{3bcf^2x^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} + \frac{4ibc^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} + \frac{bc^3f^2x^4\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} + \frac{3}{8}f^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 - \frac{1}{4}c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 - \frac{2if^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c} + \frac{5f^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{24bc\sqrt{1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 3.73 (sec) , antiderivative size = 890, normalized size of antiderivative = 1.31

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{6912iabcf^2x\sqrt{d+icdx}\sqrt{f-icfx} - 4608ia^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2} - 6912i}{}$$

```
[In] Integrate[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((6912*I)*a*b*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (4608*I)*a^2*f^
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (6912*I)*b^2*f^2*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 2592*a^2*c*f^2*x*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (4608*I)*a^2*c^2*f^2*
x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 1728*a^2*c^3*f^
2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 1440*b^2*f^2*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 - 1728*a*b*f^2*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - (256*I)*b^2*f^2*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[3*ArcSinh[c*x]] + 108*a*b*f^2*Sqrt[d + I*c*
```

```
d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 4320*a^2*Sqrt[d]*f^(5/2)*Sqrt
[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*
f*x]] + 864*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]
] + (768*I)*a*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]
] - 27*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] + 1
2*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((576*I)*b*c*x - (
576*I)*a*Sqrt[1 + c^2*x^2] - 144*b*Cosh[2*ArcSinh[c*x]] - (192*I)*a*Cosh[3*
ArcSinh[c*x]] + 9*b*Cosh[4*ArcSinh[c*x]] + 288*a*Sinh[2*ArcSinh[c*x]] + (64
*I)*b*Sinh[3*ArcSinh[c*x]] - 36*a*Sinh[4*ArcSinh[c*x]]) + 72*b*f^2*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a - (48*I)*b*Sqrt[1 + c^2*x^
2] - (16*I)*b*Cosh[3*ArcSinh[c*x]] + 24*b*Sinh[2*ArcSinh[c*x]] - 3*b*Sinh[4
*ArcSinh[c*x]]))/(6912*c*Sqrt[1 + c^2*x^2])
```

Maple [F]

$$\int (-icfx+f)^{\frac{5}{2}} (a+b\,\mathrm{arcsinh}\,(cx))^2 \sqrt{icdx+d}dx$$

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x)
```

Fricas [F]

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\mathrm{arcsinh}(cx))^2\,dx = \int \sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^2\,dx$$

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(b^2*c^2*f^2*x^2 + 2*I*b^2*c*f^2*x - b^2*f^2)*sqrt(I*c*d*x + d)*s
qrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 2*(a*b*c^2*f^2*x^2 + 2*I
*a*b*c*f^2*x - a*b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt
(c^2*x^2 + 1)) - (a^2*c^2*f^2*x^2 + 2*I*a^2*c*f^2*x - a^2*f^2)*sqrt(I*c*d*x
 + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\text{Timed out}$$

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algor
ithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algor
ithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \int (a+b\,\mathrm{asinh}(c\,x))^2\,\sqrt{d+c\,d\,x\,1\mathrm{i}}\,(f-c\,f\,x\,1\mathrm{i})^{5/2}\,dx$$

```
[In] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2),x)
[Out] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2), x)
```

3.585 $\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 615

$$\begin{aligned}\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx &= -\frac{68ib^2f^3(1+c^2x^2)}{9c\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{3b^2f^3x(1+c^2x^2)}{4\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ib^2f^3(1+c^2x^2)^2}{27c\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{3b^2f^3\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{4c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{22ibf^3x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{3bcf^3x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ibc^2f^3x^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{11if^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3f^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{icf^3x^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{5f^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{d+icdx}\sqrt{f-icfx}}\end{aligned}$$

```
[Out] -68/9*I*b^2*f^3*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-3/4*b^2*f
^3*x*(c^2*x^2+1)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+2/27*I*b^2*f^3*(c^2*x^
2+1)^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-11/3*I*f^3*(c^2*x^2+1)*(a+b*ar
csinh(c*x))^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-3/2*f^3*x*(c^2*x^2+1)*(
a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/3*I*c*f^3*x^2*(c^
2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+3/4*b^2*f
^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+22/
3*I*b*f^3*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f
*x)^(1/2)+3/2*b*c*f^3*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^
(1/2)/(f-I*c*f*x)^(1/2)-2/9*I*b*c^2*f^3*x^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^
(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+5/6*f^3*(a+b*arcsinh(c*x))^3*(c^2
*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.270, Rules used = {5796, 5843, 3398, 3377, 2718, 3392, 32, 2715, 8, 2713}

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \frac{5f^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{icf^3x^2(c^2x^2+1)\,(a+b\text{arcsinh}(cx))^2}{3\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3f^3x(c^2x^2+1)\,(a+b\text{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{11if^3(c^2x^2+1)\,(a+b\text{arcsinh}(cx))^2}{3c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{3bcf^3x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{22ibf^3x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ibc^2f^3x^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{9\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{3b^2f^3\sqrt{c^2x^2+1}\text{arcsinh}(cx)}{4c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ib^2f^3(c^2x^2+1)^2}{27c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3b^2f^3x(c^2x^2+1)}{4\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{68ib^2f^3(c^2x^2+1)}{9c\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[In] Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

```
[Out] (((-68*I)/9)*b^2*f^3*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
 - (3*b^2*f^3*x*(1 + c^2*x^2))/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + ((
(2*I)/27)*b^2*f^3*(1 + c^2*x^2)^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
+ (3*b^2*f^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(4*c*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]) + (((22*I)/3)*b*f^3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (3*b*c*f^3*x^2*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x]))/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (((2*I)/9)*b*c
^2*f^3*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]) - (((11*I)/3)*f^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (3*f^3*x*(1 + c^2*x^2)*(a + b*ArcSinh[
c*x])^2)/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + ((I/3)*c*f^3*x^2*(1 + c^
2*x^2)*(a + b*ArcSinh[c*x])^2)/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (5*f
^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
```

```
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+c^2x^2}\int \frac{(f-icfx)^3(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\text{Subst}\left(\int (a+bx)^2(cf-icf\sinh(x))^3\,dx,x,\text{arcsinh}(cx)\right)}{c^4\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\text{Subst}\left(\int \left(c^3f^3(a+bx)^2-3ic^3f^3(a+bx)^2\sinh(x)-3c^3f^3(a+bx)^2\sinh^2(x)+ic^3f^3(a\right.\right.}{c^4\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{f^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(if^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)^2\sinh^3(x)\,dx,x,\text{arcsinh}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(3if^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)^2\sinh(x)\,dx,x,\text{arcsinh}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(3f^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)^2\sinh^2(x)\,dx,x,\text{arcsinh}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{3bcf^3x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ibc^2f^3x^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3if^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3f^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{icf^3x^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{f^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(2if^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)^2\sinh(x)\,dx,x,\text{arcsinh}(cx)\right)}{3c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(3f^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)^2\,dx,x,\text{arcsinh}(cx)\right)}{2c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(6ibf^3\sqrt{1+c^2x^2}\right)\text{Subst}(\int(a+bx)\cosh(x)\,dx,x,\text{arcsinh}(cx))}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(2ib^2f^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\sinh^3(x)\,dx,x,\text{arcsinh}(cx)\right)}{9c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(3b^2f^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\sinh^2(x)\,dx,x,\text{arcsinh}(cx)\right)}{2c\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= -\frac{3b^2f^3x(1+c^2x^2)}{4\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{6ibf^3x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{3bcf^3x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ibc^2f^3x^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{11if^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3f^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{icf^3x^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{5f^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(4ibf^3\sqrt{1+c^2x^2}\right)\text{Subst}(\int(a+bx)\cosh(x)\,dx,x,\text{arcsinh}(cx))}{3c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(2ib^2f^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(1-x^2)\,dx,x,\sqrt{1+c^2x^2}\right)}{9c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(6ib^2f^3\sqrt{1+c^2x^2}\right)\text{Subst}(\int\sinh(x)\,dx,x,\text{arcsinh}(cx))}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(3b^2f^3\sqrt{1+c^2x^2}\right)\text{Subst}(\int 1\,dx,x,\text{arcsinh}(cx))}{4c\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= -\frac{56ib^2f^3(1+c^2x^2)}{9c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3b^2f^3x(1+c^2x^2)}{4\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ib^2f^3(1+c^2x^2)^2}{27c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{3b^2f^3\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{4c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{22ibf^3x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{3bcf^3x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ibc^2f^3x^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{11if^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3f^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{icf^3x^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{5f^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(4ib^2f^3\sqrt{1+c^2x^2}\right)\text{Subst}(\int\sinh(x)\,dx,x,\text{arcsinh}(cx))}{3c\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= -\frac{68ib^2f^3(1+c^2x^2)}{9c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3b^2f^3x(1+c^2x^2)}{4\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ib^2f^3(1+c^2x^2)^2}{27c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{3b^2f^3\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{4c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{22ibf^3x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{3bcf^3x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ibc^2f^3x^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{11if^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3f^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{icf^3x^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{5f^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

Mathematica [A] (verified)

Time = 14.05 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.18

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \frac{1620iabcf^2x\sqrt{d+icdx}\sqrt{f-icfx} - 792ia^2f^2\sqrt{d+icdx}\sqrt{f-icf}}{}$$

```
[In] Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((1620*I)*a*b*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (792*I)*a^2*f^2
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (1620*I)*b^2*f^2*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 324*a^2*c*f^2*x*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (72*I)*a^2*c^2*f^2*x^2*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 180*b^2*f^2*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 + 162*a*b*f^2*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] + (4*I)*b^2*f^2*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]*Cosh[3*ArcSinh[c*x]] + 6*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f -
```

```
I*c*f*x]*ArcSinh[c*x]*(27*b*Cosh[2*ArcSinh[c*x]] + (2*I)*(-4*b*c*x*(-33 + c
^2*x^2) + 27*a*(-5 + (2*I)*c*x)*Sqrt[1 + c^2*x^2] + 3*a*Cosh[3*ArcSinh[c*x]
])) + 540*a^2*Sqrt[d]*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[
f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - 81*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 18*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x]*ArcSinh[c*x]^2*(30*a - (45*I)*b*Sqrt[1 + c^2*x^2] + I*b*Cosh[3*ArcSi
nh[c*x]] - 9*b*Sinh[2*ArcSinh[c*x]]) - (12*I)*a*b*f^2*Sqrt[d + I*c*d*x]*Sqr
t[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]])/(216*c*d*Sqrt[1 + c^2*x^2])
```

Maple [F]

$$\int \frac{(-icfx+f)^{\frac{5}{2}} (a+b\,\mathrm{arcsinh}\,(cx))^2}{\sqrt{icdx+d}}dx$$

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\mathrm{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^2}{\sqrt{i\,cdx+d}}\,dx$$

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(((I*b^2*c^2*f^2*x^2 - 2*b^2*c*f^2*x - I*b^2*f^2)*sqrt(I*c*d*x + d)
*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 2*(-I*a*b*c^2*f^2*x^2
+ 2*a*b*c*f^2*x + I*a*b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) + (I*a^2*c^2*f^2*x^2 - 2*a^2*c*f^2*x - I*a^2*f^2)*sqrt(
I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*d*x - I*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\mathrm{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \text{Timed out}$$

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \int \frac{(a+b\operatorname{asinh}(c\,x))^2\,(f-c\,f\,x\,1\mathrm{i})^{5/2}}{\sqrt{d+c\,d\,x\,1\mathrm{i}}}\,dx$$

```
[In] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(1/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(1/2), x)
```

3.586 $\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 972

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = -\frac{8iabf^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+\frac{8ib^2f^4(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{b^2f^4x(1+c^2x^2)^2}{4(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$-\frac{b^2f^4(1+c^2x^2)^{3/2}\text{arcsinh}(cx)}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{8ib^2f^4x(1+c^2x^2)^{3/2}\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$-\frac{bcf^4x^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+\frac{8if^4(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{8f^4x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+\frac{8f^4(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{4if^4(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+\frac{f^4x(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{5f^4(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^3}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$-\frac{32ibf^4(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$-\frac{16bf^4(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$-\frac{16b^2f^4(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+\frac{16b^2f^4(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$-\frac{8b^2f^4(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] 4*I*f^4*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^
(3/2)-8*I*a*b*f^4*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+1
/4*b^2*f^4*x*(c^2*x^2+1)^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-1/4*b^2*f^4*
(c^2*x^2+1)^(3/2)*arcsinh(c*x)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-32*I*b
*f^4*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/c/(
d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-1/2*b*c*f^4*x^2*(c^2*x^2+1)^(3/2)*(a+b*a
rcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*I*b^2*f^4*(c^2*x^2+1)^2/
c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*f^4*x*(c^2*x^2+1)*(a+b*arcsinh(c*x)
)^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*f^4*(c^2*x^2+1)^(3/2)*(a+b*arcsin
h(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*I*f^4*(c^2*x^2+1)*(a+b*ar
csinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+1/2*f^4*x*(c^2*x^2+1)^2
*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-5/2*f^4*(c^2*x^2+
```

```
1)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*I*b
^2*f^4*x*(c^2*x^2+1)^(3/2)*arcsinh(c*x)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
-16*b*f^4*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))
^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-16*b^2*f^4*(c^2*x^2+1)^(3/2)*poly
log(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+16*
b^2*f^4*(c^2*x^2+1)^(3/2)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x
)^(3/2)/(f-I*c*f*x)^(3/2)-8*b^2*f^4*(c^2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^2*
x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 972, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.514$, Rules used = {5796, 5844, 5838, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 5789, 4265, 5783, 5772, 267, 5812, 5776, 327, 221}

$$\begin{aligned}
&\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = \\
&-\frac{5(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^3f^4}{2bc(icxd+d)^{3/2}(f-icfx)^{3/2}}+\frac{b^2x(c^2x^2+1)^2f^4}{4(icxd+d)^{3/2}(f-icfx)^{3/2}} \\
&+\frac{8ib^2(c^2x^2+1)^2f^4}{c(icxd+d)^{3/2}(f-icfx)^{3/2}}+\frac{x(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2f^4}{2(icxd+d)^{3/2}(f-icfx)^{3/2}} \\
&+\frac{4i(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2f^4}{c(icxd+d)^{3/2}(f-icfx)^{3/2}}+\frac{8(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^2f^4}{c(icxd+d)^{3/2}(f-icfx)^{3/2}} \\
&+\frac{8x(c^2x^2+1)(a+b\text{arcsinh}(cx))^2f^4}{(icxd+d)^{3/2}(f-icfx)^{3/2}}+\frac{8i(c^2x^2+1)(a+b\text{arcsinh}(cx))^2f^4}{c(icxd+d)^{3/2}(f-icfx)^{3/2}} \\
&-\frac{8iabx(c^2x^2+1)^{3/2}f^4}{(icxd+d)^{3/2}(f-icfx)^{3/2}}-\frac{8ib^2x(c^2x^2+1)^{3/2}\text{arcsinh}(cx)f^4}{(icxd+d)^{3/2}(f-icfx)^{3/2}} \\
&-\frac{b^2(c^2x^2+1)^{3/2}\text{arcsinh}(cx)f^4}{4c(icxd+d)^{3/2}(f-icfx)^{3/2}}-\frac{bcx^2(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))f^4}{2(icxd+d)^{3/2}(f-icfx)^{3/2}} \\
&-\frac{32ib(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)f^4}{c(icxd+d)^{3/2}(f-icfx)^{3/2}} \\
&-\frac{16b(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)f^4}{c(icxd+d)^{3/2}(f-icfx)^{3/2}} \\
&-\frac{16b^2(c^2x^2+1)^{3/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)f^4}{c(icxd+d)^{3/2}(f-icfx)^{3/2}} \\
&+\frac{16b^2(c^2x^2+1)^{3/2}\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)f^4}{c(icxd+d)^{3/2}(f-icfx)^{3/2}} \\
&-\frac{8b^2(c^2x^2+1)^{3/2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)f^4}{c(icxd+d)^{3/2}(f-icfx)^{3/2}}
\end{aligned}$$

```
[In] Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((-8*I)*a*b*f^4*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(
3/2)) + ((8*I)*b^2*f^4*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x
)^(3/2)) + (b^2*f^4*x*(1 + c^2*x^2)^2)/(4*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)) - (b^2*f^4*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x])/(4*c*(d + I*c*d*x)^(3/
2)*(f - I*c*f*x)^(3/2)) - ((8*I)*b^2*f^4*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]
)/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (b*c*f^4*x^2*(1 + c^2*x^2)^(3
/2)*(a + b*ArcSinh[c*x]))/(2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((8
*I)*f^4*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I
*c*f*x)^(3/2)) + (8*f^4*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*c*d
*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (8*f^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[
c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((4*I)*f^4*(1 + c^2*
x^2)^2*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
+ (f^4*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(2*(d + I*c*d*x)^(3/2)*(f
- I*c*f*x)^(3/2)) - (5*f^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^3)/(2*b
*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((32*I)*b*f^4*(1 + c^2*x^2)^(
3/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f
 - I*c*f*x)^(3/2)) - (16*b*f^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log
[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (16
*b^2*f^4*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d + I*c*d
*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (16*b^2*f^4*(1 + c^2*x^2)^(3/2)*PolyLog[2,
 I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (8*b^2*f^
4*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/
2)*(f - I*c*f*x)^(3/2))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
```

```
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{(1+c^2x^2)^{3/2}\int \frac{(f-icfx)^4(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{(1+c^2x^2)^{3/2}\int \left(-\frac{8i(if^4+cf^4x)(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}-\frac{7f^4(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}+\frac{4icf^4x(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}+\frac{c^2f^4x^2}{\ldots}\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(8i(1+c^2x^2)^{3/2}\right)\int \frac{(if^4+cf^4x)(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(7f^4(1+c^2x^2)^{3/2}\right)\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(4icf^4(1+c^2x^2)^{3/2}\right)\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(c^2f^4(1+c^2x^2)^{3/2}\right)\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{4if^4(1+c^2x^2)^2 (a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{f^4x(1+c^2x^2)^2 (a+b\text{arcsinh}(cx))^2}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{7f^4(1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(8i(1+c^2x^2)^{3/2}\right) \int \left(\frac{if^4(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}} + \frac{cf^4x(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\right) dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(f^4(1+c^2x^2)^{3/2}\right) \int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}} dx}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(8ibf^4(1+c^2x^2)^{3/2}\right) \int (a+b\text{arcsinh}(cx))\, dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(bcf^4(1+c^2x^2)^{3/2}\right) \int x(a+b\text{arcsinh}(cx))\, dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{8iabf^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{bcf^4x^2(1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^4(1+c^2x^2)^2 (a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{f^4x(1+c^2x^2)^2 (a+b\text{arcsinh}(cx))^2}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{5f^4(1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))^3}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(8f^4(1+c^2x^2)^{3/2}\right) \int \frac{(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}} dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(8ib^2f^4(1+c^2x^2)^{3/2}\right) \int \text{arcsinh}(cx)\, dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(8icf^4(1+c^2x^2)^{3/2}\right) \int \frac{x(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}} dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(b^2c^2f^4(1+c^2x^2)^{3/2}\right) \int \frac{x^2}{\sqrt{1+c^2x^2}} dx}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{8iabf^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2f^4x(1+c^2x^2)^2}{4(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ib^2f^4x(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{bcf^4x^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8if^4(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8f^4x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^4(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{f^4x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{5f^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^3}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(16ibf^4(1+c^2x^2)^{3/2}\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(b^2f^4(1+c^2x^2)^{3/2}\right)\int \frac{1}{\sqrt{1+c^2x^2}}\,dx}{4(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(16bcf^4(1+c^2x^2)^{3/2}\right)\int \frac{x(a+b\operatorname{arcsinh}(cx))}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(8ib^2cf^4(1+c^2x^2)^{3/2}\right)\int \frac{x}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{8iabf^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8ib^2f^4(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2f^4x(1+c^2x^2)^2}{4(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{b^2f^4(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ib^2f^4x(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{bcf^4x^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8if^4(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8f^4x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^4(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{f^4x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{5f^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^3}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(16ibf^4(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int (a+bx)\operatorname{sech}(x)\,dx, x, \operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(16bf^4(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{8iabf^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8ib^2f^4(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{b^2f^4x(1+c^2x^2)^2}{4(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{b^2f^4(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{8ib^2f^4x(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{bcf^4x^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{8if^4(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8f^4x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{8f^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^4(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{f^4x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{5f^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^3}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{32ibf^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{\left(32bf^4(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int\frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx,x,\operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{\left(16b^2f^4(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int\log\left(1-ie^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{\left(16b^2f^4(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int\log\left(1+ie^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$
\begin{aligned}
&= -\frac{8iabf^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8ib^2f^4(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{b^2f^4x(1+c^2x^2)^2}{4(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{b^2f^4(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{8ib^2f^4x(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{bcf^4x^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{8if^4(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8f^4x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{8f^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^4(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{f^4x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{5f^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^3}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{32ibf^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{16bf^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{\left(16b^2f^4(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int\log\left(1+e^{2x}\right)dx, x, \operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{\left(16b^2f^4(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int\frac{\log(1-ix)}{x}dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{\left(16b^2f^4(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int\frac{\log(1+ix)}{x}dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{8iabf^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8ib^2f^4(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad + \frac{b^2f^4x(1+c^2x^2)^2}{4(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{b^2f^4(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad - \frac{8ib^2f^4x(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{bcf^4x^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad + \frac{8if^4(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8f^4x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad + \frac{8f^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^4(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad + \frac{f^4x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{5f^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^3}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad - \frac{32ibf^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad - \frac{16bf^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad - \frac{16b^2f^4(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad + \frac{16b^2f^4(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad + \frac{\left(8b^2f^4(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{8iabf^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8ib^2f^4(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{b^2f^4x(1+c^2x^2)^2}{4(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{b^2f^4(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{8ib^2f^4x(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{bcf^4x^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{8if^4(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8f^4x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{8f^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^4(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{f^4x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{5f^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^3}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{32ibf^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{16bf^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{16b^2f^4(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{16b^2f^4(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{8b^2f^4(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2492 vs. 2(972) = 1944.

Time = 23.48 (sec) , antiderivative size = 2492, normalized size of antiderivative = 2.56

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),
x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(((4*I)*a^2*f^2)/d^2 + (a^2*c*
f^2*x)/(2*d^2) + (8*a^2*f^2)/(d^2*(-I + c*x))))/c - (15*a^2*f^(5/2)*Log[c*d
*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(2*c*d
^(3/2)) + ((4*I)*a*b*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*
Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*(-(c*x) + 2*ArcSinh[c*x] +
```

```
Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + I*ArcSinh[c*x]^2 + 4*ArcTan[Coth[ArcSinh[
c*x]/2]] + (2*I)*Log[Sqrt[1 + c^2*x^2]]) + I*(-(c*x) - 2*ArcSinh[c*x] + Sqr
t[1 + c^2*x^2]*ArcSinh[c*x] + I*ArcSinh[c*x]^2 + 4*ArcTan[Coth[ArcSinh[c*x]
/2]] + (2*I)*Log[Sqrt[1 + c^2*x^2]])*Sinh[ArcSinh[c*x]/2]))/(c*d^2*Sqrt[-((
(-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I
*Sinh[ArcSinh[c*x]/2])) - (a*b*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f
+ c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*(ArcSinh[c*x]*(-
4*I + ArcSinh[c*x]) + (8*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 4*Log[Sqrt[1 + c
^2*x^2]]) + I*(ArcSinh[c*x]*(4*I + ArcSinh[c*x]) + (8*I)*ArcTan[Tanh[ArcSin
h[c*x]/2]] + 4*Log[Sqrt[1 + c^2*x^2]])*Sinh[ArcSinh[c*x]/2]))/(c*d^2*Sqrt[-
(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] +
 I*Sinh[ArcSinh[c*x]/2])) - (b^2*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*
f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*((6*I)*Pi*ArcS
inh[c*x] + (6 - 6*I)*ArcSinh[c*x]^2 + ArcSinh[c*x]^3 + 12*((-I)*Pi + 2*ArcS
inh[c*x])*Log[1 - I/E^ArcSinh[c*x]] - (24*I)*Pi*Log[1 + E^ArcSinh[c*x]] + (
24*I)*Pi*Log[Cosh[ArcSinh[c*x]/2]] + (12*I)*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[
c*x])/4]]) - 24*PolyLog[2, I/E^ArcSinh[c*x]]*(Cosh[ArcSinh[c*x]/2] + I*Sinh
[ArcSinh[c*x]/2]) + (-6*Pi*ArcSinh[c*x] - (6 - 6*I)*ArcSinh[c*x]^2 + I*ArcS
inh[c*x]^3 + 12*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] + 24*Pi
*Log[1 + E^ArcSinh[c*x]] - 24*Pi*Log[Cosh[ArcSinh[c*x]/2]] - 12*Pi*Log[Sin[
(Pi + (2*I)*ArcSinh[c*x])/4]])*Sinh[ArcSinh[c*x]/2]))/(3*c*d^2*Sqrt[-(((-I)
*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sin
h[ArcSinh[c*x]/2])) + (((2*I)/3)*b^2*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)
*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*(-6*Pi*Arc
Sinh[c*x] - 6*c*x*ArcSinh[c*x] + (6 + 6*I)*ArcSinh[c*x]^2 + (2*I)*ArcSinh[c
*x]^3 + 3*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c*x]^2) + 12*Pi*Log[1 - I/E^ArcSin
h[c*x]] + (24*I)*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] + 24*Pi*Log[1 + E^A
rcSinh[c*x]] - 24*Pi*Log[Cosh[ArcSinh[c*x]/2]] - 12*Pi*Log[Sin[(Pi + (2*I)*
ArcSinh[c*x])/4]]) + I*(-6*Pi*ArcSinh[c*x] - 6*c*x*ArcSinh[c*x] - (6 - 6*I)
*ArcSinh[c*x]^2 + (2*I)*ArcSinh[c*x]^3 + 3*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c
*x]^2) + 12*Pi*Log[1 - I/E^ArcSinh[c*x]] + (24*I)*ArcSinh[c*x]*Log[1 - I/E^
ArcSinh[c*x]] + 24*Pi*Log[1 + E^ArcSinh[c*x]] - 24*Pi*Log[Cosh[ArcSinh[c*x]
/2]] - 12*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]])*Sinh[ArcSinh[c*x]/2] +
24*PolyLog[2, I/E^ArcSinh[c*x]]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c
*x]/2])))/(c*d^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*
(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (b^2*f^2*Sqrt[I*((-I)*d
+ c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(96*PolyLog[2
, I/E^ArcSinh[c*x]]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + Sinh[
ArcSinh[c*x]/2]*(24*Pi*ArcSinh[c*x] + 48*c*x*ArcSinh[c*x] + (24 - 24*I)*Arc
Sinh[c*x]^2 - (10*I)*ArcSinh[c*x]^3 + (3*I)*Sqrt[1 + c^2*x^2]*(c*x + (8*I)*
(2 + ArcSinh[c*x]^2)) - (3*I)*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]] - 48*Pi*Log
[1 - I/E^ArcSinh[c*x]] - (96*I)*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] - 96
*Pi*Log[1 + E^ArcSinh[c*x]] + 96*Pi*Log[Cosh[ArcSinh[c*x]/2]] + 48*Pi*Log[S
in[(Pi + (2*I)*ArcSinh[c*x])/4]] + (3*I)*ArcSinh[c*x]^2*Sinh[2*ArcSinh[c*x]
]) + Cosh[ArcSinh[c*x]/2]*(3*Sqrt[1 + c^2*x^2]*(c*x + (8*I)*(2 + ArcSinh[c*
```

```
x]^2)) - 3*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]] - I*(24*Pi*ArcSinh[c*x] + 48*c
*x*ArcSinh[c*x] - (24 + 24*I)*ArcSinh[c*x]^2 - (10*I)*ArcSinh[c*x]^3 - 48*P
i*Log[1 - I/E^ArcSinh[c*x]] - (96*I)*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]]
 - 96*Pi*Log[1 + E^ArcSinh[c*x]] + 96*Pi*Log[Cosh[ArcSinh[c*x]/2]] + 48*Pi*
Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]] + (3*I)*ArcSinh[c*x]^2*Sinh[2*ArcSinh
[c*x]]))))/(12*c*d^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x
^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (a*b*f^2*Sqrt[I*((-I
)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(-(Sinh[A
rcSinh[c*x]/2]*((-16*I)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + Cosh[2*ArcSinh[c*x
]] + 2*((8*I)*c*x + (8*I)*ArcSinh[c*x] + 5*ArcSinh[c*x]^2 + (16*I)*ArcTan[T
anh[ArcSinh[c*x]/2]] + 8*Log[Sqrt[1 + c^2*x^2]] - ArcSinh[c*x]*Sinh[2*ArcSi
nh[c*x]]))) + Cosh[ArcSinh[c*x]/2]*(16*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + I*(
Cosh[2*ArcSinh[c*x]] + 2*((8*I)*c*x - (8*I)*ArcSinh[c*x] + 5*ArcSinh[c*x]^2
 + (16*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 8*Log[Sqrt[1 + c^2*x^2]] - ArcSinh
[c*x]*Sinh[2*ArcSinh[c*x]])))))/(4*c*d^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f
*x))]*Sqrt[1 + c^2*x^2]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2]))
```

Maple [F]

$$\int \frac{(-icfx+f)^{\frac{5}{2}} (a+b\,\mathrm{arcsinh}\,(cx))^2}{(icdx+d)^{\frac{3}{2}}}dx$$

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x)
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x)
```

Fricas [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\mathrm{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algor
ithm="fricas")
[Out] integral(((b^2*c^2*f^2*x^2 + 2*I*b^2*c*f^2*x - b^2*f^2)*sqrt(I*c*d*x + d)*s
qrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c^2*f^2*x^2 + 2*I
*a*b*c*f^2*x - a*b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt
(c^2*x^2 + 1)) + (a^2*c^2*f^2*x^2 + 2*I*a^2*c*f^2*x - a^2*f^2)*sqrt(I*c*d*x
 + d)*sqrt(-I*c*f*x + f))/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f - icfx)^{5/2}(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(f - icfx)^{5/2}(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{3/2}}\,dx = \int \frac{(-i\,cfx + f)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx) + a)^2}{(i\,cdx + d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*(c^2*f^3*x^3/(sqrt(c^2*d*f*x^2 + d*f)*d) + 8*I*c*f^3*x^2/(sqrt(c^2*d*f*
x^2 + d*f)*d) + 17*f^3*x/(sqrt(c^2*d*f*x^2 + d*f)*d) - 15*f^3*arcsinh(c*x)/
(sqrt(d*f)*c*d) + 24*I*f^3/(sqrt(c^2*d*f*x^2 + d*f)*c*d))*a^2 + integrate((
-I*c*f*x + f)^(5/2)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(I*c*d*x + d)^(3/2)
+ 2*(-I*c*f*x + f)^(5/2)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(I*c*d*x + d)^(3/
2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f - icfx)^{5/2}(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx=\int \frac{\left(a+b\,\text{asinh}(c\,x)\right)^2\,\left(f-c\,f\,x\,1\text{i}\right)^{5/2}}{\left(d+c\,d\,x\,1\text{i}\right)^{3/2}}\,dx$$

```
[In] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(3/2),x)
[Out] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(3/2), x)
```

3.587 $\int \frac{(f-icfx)^{5/2}(a+b\textbf{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 790

$$
\begin{aligned}
\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx &= \frac{2iabf^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2ib^2f^5(1+c^2x^2)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2f^5x(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{28f^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{if^5(1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{5f^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{16ib^2f^5(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{28if^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{8bf^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{4if^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{112bf^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{112b^2f^5(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

```
[Out] 2*I*a*b*f^5*x*(c^2*x^2+1)^(5/2)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2*I*b^2
*f^5*(c^2*x^2+1)^3/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2*I*b^2*f^5*x*(c^2
*x^2+1)^(5/2)*arcsinh(c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-28/3*f^5*(c^
2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-I
```

```
*f^5*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/
2)+5/3*f^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(5/2)/(f-
I*c*f*x)^(5/2)-16/3*I*b^2*f^5*(c^2*x^2+1)^(5/2)*cot(1/4*Pi+1/2*I*arcsinh(c*
x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-28/3*I*f^5*(c^2*x^2+1)^(5/2)*(a+b
*arcsinh(c*x))^2*cot(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*
f*x)^(5/2)+8/3*b*f^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*csc(1/4*Pi+1/2*I*
arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+4/3*I*f^5*(c^2*x^2+1)
^(5/2)*(a+b*arcsinh(c*x))^2*cot(1/4*Pi+1/2*I*arcsinh(c*x))*csc(1/4*Pi+1/2*I
*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+112/3*b*f^5*(c^2*x^2
+1)^(5/2)*(a+b*arcsinh(c*x))*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^
(5/2)/(f-I*c*f*x)^(5/2)+112/3*b^2*f^5*(c^2*x^2+1)^(5/2)*polylog(2,-I*(c*x+(
c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 790, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.432$, Rules used = {5796, 5844, 5783, 5798, 5772, 267, 5843, 3399, 4271, 3852, 8, 4269, 3797, 2221, 2317, 2438}

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx = \frac{5f^5(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{if^5(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{28f^5(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{112bf^5(c^2x^2+1)^{5/2}\log\left(1+ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{28if^5(c^2x^2+1)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{8bf^5(c^2x^2+1)^{5/2}\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{4if^5(c^2x^2+1)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{2iabf^5x(c^2x^2+1)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{112b^2f^5(c^2x^2+1)^{5/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{2ib^2f^5x(c^2x^2+1)^{5/2}\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{16ib^2f^5(c^2x^2+1)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2ib^2f^5(c^2x^2+1)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[In] Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),x]
```

```
[Out] ((2*I)*a*b*f^5*x*(1 + c^2*x^2)^(5/2))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) - ((2*I)*b^2*f^5*(1 + c^2*x^2)^3)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)
^(5/2)) + ((2*I)*b^2*f^5*x*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x])/((d + I*c*d*x)
^(5/2)*(f - I*c*f*x)^(5/2)) - (28*f^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*
x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (I*f^5*(1 + c^2*x^2)
^3*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (5
*f^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^3)/(3*b*c*(d + I*c*d*x)^(5/2)
*(f - I*c*f*x)^(5/2)) - (((16*I)/3)*b^2*f^5*(1 + c^2*x^2)^(5/2)*Cot[Pi/4 +
(I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((28*I)
/3)*f^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh
[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (8*b*f^5*(1 + c^2*x^2
)^(5/2)*(a + b*ArcSinh[c*x])*Csc[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c*(d + I*
c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((4*I)/3)*f^5*(1 + c^2*x^2)^(5/2)*(a +
 b*ArcSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]]*Csc[Pi/4 + (I/2)*ArcSinh[
c*x]]^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (112*b*f^5*(1 + c^2*
x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log[1 + I*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*
x)^(5/2)*(f - I*c*f*x)^(5/2)) + (112*b^2*f^5*(1 + c^2*x^2)^(5/2)*PolyLog[2,
 (-I)*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
```

```
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{(1+c^2x^2)^{5/2}\int \frac{(f-icfx)^5(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{(1+c^2x^2)^{5/2}\int \left(\frac{5f^5(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}} - \frac{icf^5x(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}} - \frac{8f^5(a+b\text{arcsinh}(cx))^2}{(-i+cx)^2\sqrt{1+c^2x^2}} + \frac{12if^5(a+b\text{arcsin}}{(-i+cx)\sqrt{1+c}}\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(12 i f^5 (1+c^2x^2)^{5/2}\right) \int \frac{(a+b\text{arcsinh}(cx))^2}{(-i+cx)\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(5 f^5 (1+c^2x^2)^{5/2}\right) \int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(8 f^5 (1+c^2x^2)^{5/2}\right) \int \frac{(a+b\text{arcsinh}(cx))^2}{(-i+cx)^2\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(icf^5 (1+c^2x^2)^{5/2}\right) \int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{if^5(1+c^2x^2)^3 (a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5f^5(1+c^2x^2)^{5/2} (a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(12if^5(1+c^2x^2)^{5/2}\right) \text{Subst}\left(\int \frac{(a+bx)^2}{-ic+c\sinh(x)}\,dx, x, \text{arcsinh}(cx)\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2ibf^5(1+c^2x^2)^{5/2}\right) \int (a+b\text{arcsinh}(cx))\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(8cf^5(1+c^2x^2)^{5/2}\right) \text{Subst}\left(\int \frac{(a+bx)^2}{(-ic+c\sinh(x))^2}\,dx, x, \text{arcsinh}(cx)\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2iabf^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{if^5(1+c^2x^2)^3 (a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5f^5(1+c^2x^2)^{5/2} (a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2ib^2f^5(1+c^2x^2)^{5/2}\right) \int \text{arcsinh}(cx)\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2f^5(1+c^2x^2)^{5/2}\right) \text{Subst}\left(\int (a+bx)^2 \csc^4\left(\frac{\pi}{4}+\frac{ix}{2}\right)\,dx, x, \text{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(6f^5(1+c^2x^2)^{5/2}\right) \text{Subst}\left(\int (a+bx)^2 \csc^2\left(\frac{\pi}{4}+\frac{ix}{2}\right)\,dx, x, \text{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2iabf^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2f^5x(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{if^5(1+c^2x^2)^3\,(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5f^5(1+c^2x^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{12if^5(1+c^2x^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{8bf^5(1+c^2x^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{4if^5(1+c^2x^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{\left(4f^5(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int(a+bx)^2\csc^2\left(\frac{\pi}{4}+\frac{ix}{2}\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{\left(24ibf^5(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int(a+bx)\cot\left(\frac{\pi}{4}+\frac{ix}{2}\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{\left(8b^2f^5(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\csc^2\left(\frac{\pi}{4}+\frac{ix}{2}\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{\left(2ib^2cf^5(1+c^2x^2)^{5/2}\right)\int\frac{x}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$
\begin{aligned}
&= \frac{2iabf^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2f^5(1+c^2x^2)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{2ib^2f^5x(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{12f^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{if^5(1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5f^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{28if^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{8bf^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{4if^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{\left(16ibf^5(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int(a+bx)\cot\left(\frac{\pi}{4}+\frac{ix}{2}\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{\left(48ibf^5(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\frac{e^x(a+bx)}{1+ie^x}\,dx,x,\operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{\left(16ib^2f^5(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int 1\,dx,x,\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2iabf^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2f^5(1+c^2x^2)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2ib^2f^5x(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{28f^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{if^5(1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5f^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{16ib^2f^5(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{28if^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{8bf^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{4if^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{48bf^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{\left(32ibf^5(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\frac{e^x(a+bx)}{1+ie^x}\,dx,x,\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{\left(48b^2f^5(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\log\left(1+ie^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2iabf^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2f^5(1+c^2x^2)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{2ib^2f^5x(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{28f^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{if^5(1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5f^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{16ib^2f^5(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{28if^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{8bf^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{4if^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{112bf^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{\left(32b^2f^5(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\log\left(1+ie^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{\left(48b^2f^5(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2iabf^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2f^5(1+c^2x^2)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2ib^2f^5x(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{28f^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{if^5(1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5f^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{16ib^2f^5(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{28if^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{8bf^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{4if^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{112bf^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{48b^2f^5(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{\left(32b^2f^5(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2iabf^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2f^5(1+c^2x^2)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2ib^2f^5x(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{28f^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{if^5(1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5f^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{16ib^2f^5(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{28if^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{8bf^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{4if^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{112bf^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{112b^2f^5(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2622 vs. 2(790) = 1580.

Time = 24.70 (sec) , antiderivative size = 2622, normalized size of antiderivative = 3.32

$$
\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),
x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(((-I)*a^2*f^2)/d^3 - (((8*I)/
3)*a^2*f^2)/(d^3*(-I + c*x)^2) - (28*a^2*f^2)/(3*d^3*(-I + c*x))))/c + (5*a
^2*f^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(
I + c*x)]])/(c*d^(5/2)) + ((I/3)*a*b*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)
*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[A
rcSinh[c*x]/2])*((-I)*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Cot
h[ArcSinh[c*x]/2]] - I*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/2]*(4 +
(3*I)*ArcSinh[c*x] - (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 3*Log[Sqrt[1 + c^
2*x^2]]) + 2*(Sqrt[1 + c^2*x^2]*(ArcSinh[c*x] + 2*ArcTan[Coth[ArcSinh[c*x]/
2]] + I*Log[Sqrt[1 + c^2*x^2]]) + 2*(I + ArcSinh[c*x] + 2*ArcTan[Coth[ArcSi
```

nh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c*d^3*(I +
 c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*Sin
h[ArcSinh[c*x]/2])^4) - (a*b*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f +
c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c
x]/2])(Cosh[(3*ArcSinh[c*x])/2]*((-14 + (3*I)*ArcSinh[c*x])*ArcSinh[c*x]
- 28*ArcTan[Tanh[ArcSinh[c*x]/2]] + (14*I)*Log[Sqrt[1 + c^2*x^2]]) + Cosh[A
rcSinh[c*x]/2]*(84*ArcTan[Tanh[ArcSinh[c*x]/2]] - I*(8 - (6*I)*ArcSinh[c*x]
 + 9*ArcSinh[c*x]^2 + 42*Log[Sqrt[1 + c^2*x^2]])) + 2*(4 - (4*I)*ArcSinh[c*
x] + 6*ArcSinh[c*x]^2 + (56*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 28*Log[Sqrt[1
 + c^2*x^2]] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(-14*I + 3*ArcSinh[c*x]) + (
28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 14*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSi
nh[c*x]/2]))/(3*c*d^3*(I + c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Co
sh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^4) + ((I/3)*b^2*f^2*(I + c*x)*
Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))
]*((-1 + I)*ArcSinh[c*x]^2 - (2*ArcSinh[c*x]*(-2*I + ArcSinh[c*x]))/(-I + c
*x) + (2*I)*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] - I*Pi*(Arc
Sinh[c*x] - 4*Log[1 + E^ArcSinh[c*x]] + 4*Log[Cosh[ArcSinh[c*x]/2]] + 2*Log
[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]]) + 4*PolyLog[2, I/E^ArcSinh[c*x]] - (4*A
rcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[
c*x]/2])^3 + (2*(4 + ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*
x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(c*d^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f
*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^2)
- ((I/3)*b^2*f^2*(I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)
]*Sqrt[-(d*f*(1 + c^2*x^2))]*(((6*I)*c*x*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] +
((13 - 13*I)*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] + (3*ArcSinh[c*x]^3)/Sqrt[1
+ c^2*x^2] + (2*ArcSinh[c*x]*(-2*I + ArcSinh[c*x]))/((-I + c*x)*Sqrt[1 + c^
2*x^2]) - (3*I)*(2 + ArcSinh[c*x]^2) + ((13*I)*(-2*(Pi + (2*I)*ArcSinh[c*x]
)*Log[1 - I/E^ArcSinh[c*x]] + Pi*(ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c*x]]
+ 4*Log[Cosh[ArcSinh[c*x]/2]] + 2*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]]) +
(4*I)*PolyLog[2, I/E^ArcSinh[c*x]]))/Sqrt[1 + c^2*x^2] + (4*ArcSinh[c*x]^2*
Sinh[ArcSinh[c*x]/2])/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[Arc
Sinh[c*x]/2])^3) - (2*(4 + 13*ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Sqrt[1
 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]))))/(c*d^3*Sqrt[
-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c
*x]/2])^2) + (2*b^2*f^2*(I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f +
 c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(7*Pi*ArcSinh[c*x] - (7 + 7*I)*ArcSinh[
c*x]^2 - I*ArcSinh[c*x]^3 + (2*ArcSinh[c*x]*(-2*I + ArcSinh[c*x]))/(1 + I*c
x) - 14(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] - 28*Pi*Log[1
+ E^ArcSinh[c*x]] + 28*Pi*Log[Cosh[ArcSinh[c*x]/2]] + 14*Pi*Log[Sin[(Pi + (
2*I)*ArcSinh[c*x])/4]] + (28*I)*PolyLog[2, I/E^ArcSinh[c*x]] - ((4*I)*ArcSi
nh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]
/2])^3 + (2*(4 + 7*ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/((-I)*Cosh[ArcSinh
[c*x]/2] + Sinh[ArcSinh[c*x]/2])))/(3*c*d^3*Sqrt[-(((-I)*d + c*d*x)*(I*f +
c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^
2) + ((I/6)*a*b*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[

```
-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*(-3*C
osh[(5*ArcSinh[c*x])/2] + (3*I)*ArcSinh[c*x]*Cosh[(5*ArcSinh[c*x])/2] - Cos
h[(3*ArcSinh[c*x])/2]*(9 + (35*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 - (52*I)*
ArcTan[Coth[ArcSinh[c*x]/2]] + 26*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*
x]/2]*(20 - (24*I)*ArcSinh[c*x] + 27*ArcSinh[c*x]^2 - (156*I)*ArcTan[Coth[A
rcSinh[c*x]/2]] + 78*Log[Sqrt[1 + c^2*x^2]]) + (20*I)*Sinh[ArcSinh[c*x]/2]
- 24*ArcSinh[c*x]*Sinh[ArcSinh[c*x]/2] + (27*I)*ArcSinh[c*x]^2*Sinh[ArcSinh
[c*x]/2] + 156*ArcTan[Coth[ArcSinh[c*x]/2]]*Sinh[ArcSinh[c*x]/2] + (78*I)*L
og[Sqrt[1 + c^2*x^2]]*Sinh[ArcSinh[c*x]/2] + (9*I)*Sinh[(3*ArcSinh[c*x])/2]
 + 35*ArcSinh[c*x]*Sinh[(3*ArcSinh[c*x])/2] + (9*I)*ArcSinh[c*x]^2*Sinh[(3*
ArcSinh[c*x])/2] + 52*ArcTan[Coth[ArcSinh[c*x]/2]]*Sinh[(3*ArcSinh[c*x])/2]
 + (26*I)*Log[Sqrt[1 + c^2*x^2]]*Sinh[(3*ArcSinh[c*x])/2] - (3*I)*Sinh[(5*A
rcSinh[c*x])/2] + 3*ArcSinh[c*x]*Sinh[(5*ArcSinh[c*x])/2]))/(c*d^3*(I + c*x
)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[Ar
cSinh[c*x]/2])^4)
```

Maple [F]

$$\int \frac{(-icfx+f)^{\frac{5}{2}} (a+b\,\mathrm{arcsinh}\,(cx))^2}{(icdx+d)^{\frac{5}{2}}}dx$$

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x)
```

```
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x)
```

Fricas [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\mathrm{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral(((-I*b^2*c^2*f^2*x^2 + 2*b^2*c*f^2*x + I*b^2*f^2)*sqrt(I*c*d*x + d
)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 2*(I*a*b*c^2*f^2*x^2
- 2*a*b*c*f^2*x - I*a*b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) + (-I*a^2*c^2*f^2*x^2 + 2*a^2*c*f^2*x + I*a^2*f^2)*sqrt
(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c*d^3*
x + I*d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algor
ithm="maxima")
[Out] Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algor
ithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2\,(f-c\,f\,x\,1\text{i})^{5/2}}{(d+c\,d\,x\,1\text{i})^{5/2}}\,dx$$

```
[In] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(5/2),x)
[Out] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(5/2), x)
```

3.588 $\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 615

$$\begin{aligned}\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx &= \frac{68ib^2d^3(1+c^2x^2)}{9c\sqrt{d+icdx}\sqrt{f-icfx}} \\ &- \frac{3b^2d^3x(1+c^2x^2)}{4\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ib^2d^3(1+c^2x^2)^2}{27c\sqrt{d+icdx}\sqrt{f-icfx}} \\ &+ \frac{3b^2d^3\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{4c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{22ibd^3x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3\sqrt{d+icdx}\sqrt{f-icfx}} \\ &+ \frac{3bcd^3x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ibc^2d^3x^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9\sqrt{d+icdx}\sqrt{f-icfx}} \\ &+ \frac{11id^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3d^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}} \\ &- \frac{icd^3x^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{5d^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{d+icdx}\sqrt{f-icfx}}\end{aligned}$$

```
[Out] 68/9*I*b^2*d^3*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-3/4*b^2*d^
3*x*(c^2*x^2+1)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-2/27*I*b^2*d^3*(c^2*x^2
+1)^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+11/3*I*d^3*(c^2*x^2+1)*(a+b*arc
sinh(c*x))^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-3/2*d^3*x*(c^2*x^2+1)*(a
+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/3*I*c*d^3*x^2*(c^2
*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+3/4*b^2*d^
3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-22/3
*I*b*d^3*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*
x)^(1/2)+3/2*b*c*d^3*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(
1/2)/(f-I*c*f*x)^(1/2)+2/9*I*b*c^2*d^3*x^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(
1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+5/6*d^3*(a+b*arcsinh(c*x))^3*(c^2*
x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.270, Rules used = {5796, 5843, 3398, 3377, 2718, 3392, 32, 2715, 8, 2713}

$$\begin{aligned}
\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx &= \frac{5d^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad -\frac{icd^3x^2(c^2x^2+1)\,(a+b\text{arcsinh}(cx))^2}{3\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{3d^3x(c^2x^2+1)\,(a+b\text{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad +\frac{11id^3(c^2x^2+1)\,(a+b\text{arcsinh}(cx))^2}{3c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{3bcd^3x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad -\frac{22ibd^3x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{2ibc^2d^3x^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{9\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad +\frac{3b^2d^3\sqrt{c^2x^2+1}\text{arcsinh}(cx)}{4c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{2ib^2d^3(c^2x^2+1)^2}{27c\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad -\frac{3b^2d^3x(c^2x^2+1)}{4\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{68ib^2d^3(c^2x^2+1)}{9c\sqrt{d+icdx}\sqrt{f-icfx}}
\end{aligned}$$

```
[In] Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((((68*I)/9)*b^2*d^3*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
- (3*b^2*d^3*x*(1 + c^2*x^2))/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (((
2*I)/27)*b^2*d^3*(1 + c^2*x^2)^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) +
 (3*b^2*d^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(4*c*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]) - (((22*I)/3)*b*d^3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (3*b*c*d^3*x^2*Sqrt[1 + c^2*x^2]*(a +
 b*ArcSinh[c*x]))/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (((2*I)/9)*b*c^
2*d^3*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]) + (((11*I)/3)*d^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (3*d^3*x*(1 + c^2*x^2)*(a + b*ArcSinh[c
*x])^2)/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - ((I/3)*c*d^3*x^2*(1 + c^2
*x^2)*(a + b*ArcSinh[c*x])^2)/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (5*d^
3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
```

```
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+c^2x^2}\int \frac{(d+icdx)^3(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\text{Subst}\left(\int (a+bx)^2(cd+icd\sinh(x))^3\,dx, x, \text{arcsinh}(cx)\right)}{c^4\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\text{Subst}\left(\int \left(c^3d^3(a+bx)^2 + 3ic^3d^3(a+bx)^2\sinh(x) - 3c^3d^3(a+bx)^2\sinh^2(x) - ic^3d^3(a\right.\right.}{c^4\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{d^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(id^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)^2\sinh^3(x)\,dx, x, \text{arcsinh}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(3id^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)^2\sinh(x)\,dx, x, \text{arcsinh}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(3d^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)^2\sinh^2(x)\,dx, x, \text{arcsinh}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{3bcd^3x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ibc^2d^3x^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{3id^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3d^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{icd^3x^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{d^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(2id^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)^2\sinh(x)\,dx,x,\text{arcsinh}(cx)\right)}{3c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(3d^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)^2\,dx,x,\text{arcsinh}(cx)\right)}{2c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(6ibd^3\sqrt{1+c^2x^2}\right)\text{Subst}(\int(a+bx)\cosh(x)\,dx,x,\text{arcsinh}(cx))}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(2ib^2d^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\sinh^3(x)\,dx,x,\text{arcsinh}(cx)\right)}{9c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(3b^2d^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\sinh^2(x)\,dx,x,\text{arcsinh}(cx)\right)}{2c\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= -\frac{3b^2d^3x(1+c^2x^2)}{4\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{6ibd^3x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{3bcd^3x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ibc^2d^3x^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{11id^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3d^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{icd^3x^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{5d^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(4ibd^3\sqrt{1+c^2x^2}\right)\text{Subst}(\int(a+bx)\cosh(x)\,dx,x,\text{arcsinh}(cx))}{3c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(2ib^2d^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(1-x^2)\,dx,x,\sqrt{1+c^2x^2}\right)}{9c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(6ib^2d^3\sqrt{1+c^2x^2}\right)\text{Subst}(\int\sinh(x)\,dx,x,\text{arcsinh}(cx))}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(3b^2d^3\sqrt{1+c^2x^2}\right)\text{Subst}(\int 1\,dx,x,\text{arcsinh}(cx))}{4c\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$
\begin{aligned}
&= \frac{56ib^2d^3(1+c^2x^2)}{9c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3b^2d^3x(1+c^2x^2)}{4\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ib^2d^3(1+c^2x^2)^2}{27c\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad + \frac{3b^2d^3\sqrt{1+c^2x^2}\operatorname{arcsinh}(cx)}{4c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{22ibd^3x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad + \frac{3bcd^3x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ibc^2d^3x^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{9\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad + \frac{11id^3(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))^2}{3c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3d^3x(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad - \frac{icd^3x^2(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))^2}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{5d^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^3}{6bc\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad + \frac{\left(4ib^2d^3\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\sinh(x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{3c\sqrt{d+icdx}\sqrt{f-icfx}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{68ib^2d^3(1+c^2x^2)}{9c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3b^2d^3x(1+c^2x^2)}{4\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ib^2d^3(1+c^2x^2)^2}{27c\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad + \frac{3b^2d^3\sqrt{1+c^2x^2}\operatorname{arcsinh}(cx)}{4c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{22ibd^3x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad + \frac{3bcd^3x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ibc^2d^3x^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{9\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad + \frac{11id^3(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))^2}{3c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3d^3x(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&\quad - \frac{icd^3x^2(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))^2}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{5d^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^3}{6bc\sqrt{d+icdx}\sqrt{f-icfx}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 13.88 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.18

$$
\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \frac{-1620iabcd^2x\sqrt{d+icdx}\sqrt{f-icfx}+792ia^2d^2\sqrt{d+icdx}\sqrt{f-ic}
$$

```
[In] Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-1620*I)*a*b*c*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (792*I)*a^2*d^
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (1620*I)*b^2*d^2*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 324*a^2*c*d^2*x*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (72*I)*a^2*c^2*d^2*x^2
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 180*b^2*d^2*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 + 162*a*b*d^2*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - (4*I)*b^2*d^2*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*Cosh[3*ArcSinh[c*x]] + 6*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f -
```

```
I*c*f*x]*ArcSinh[c*x]*(27*b*Cosh[2*ArcSinh[c*x]] + (2*I)*(4*b*c*x*(-33 + c
^2*x^2) + 27*a*(5 + (2*I)*c*x)*Sqrt[1 + c^2*x^2] - 3*a*Cosh[3*ArcSinh[c*x]]
)) + 540*a^2*d^(5/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f
]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - 81*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 18*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c
*f*x]*ArcSinh[c*x]^2*(30*a + (45*I)*b*Sqrt[1 + c^2*x^2] - I*b*Cosh[3*ArcSin
h[c*x]] - 9*b*Sinh[2*ArcSinh[c*x]]) + (12*I)*a*b*d^2*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]])/(216*c*f*Sqrt[1 + c^2*x^2])
```

Maple [F]

$$\int \frac{(icdx+d)^{\frac{5}{2}} (a+b\operatorname{arcsinh}(cx))^2}{\sqrt{-icfx+f}}dx$$

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx=\int \frac{(i\,cdx+d)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{-i\,cfx+f}}\,dx$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(((-I*b^2*c^2*d^2*x^2 - 2*b^2*c*d^2*x + I*b^2*d^2)*sqrt(I*c*d*x + d
)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 2*(I*a*b*c^2*d^2*x^2
+ 2*a*b*c*d^2*x - I*a*b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) + (-I*a^2*c^2*d^2*x^2 - 2*a^2*c*d^2*x + I*a^2*d^2)*sqrt
(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*f*x + I*f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx=\text{Timed out}$$

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^{5/2}}{\sqrt{f-c\,f\,x\,1\mathrm{i}}}\,dx$$

```
[In] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(1/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(1/2), x)
```

3.589 $\int \frac{(d+icdx)^{3/2}(a+b\,\mathrm{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 436

$$\begin{aligned}\int \frac{(d+icdx)^{3/2}(a+b\,\mathrm{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx &= \frac{4ib^2d^2(1+c^2x^2)}{c\sqrt{d+icdx}\sqrt{f-icfx}} \\ &- \frac{b^2d^2x(1+c^2x^2)}{4\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{b^2d^2\sqrt{1+c^2x^2}\,\mathrm{arcsinh}(cx)}{4c\sqrt{d+icdx}\sqrt{f-icfx}} \\ &- \frac{4ibd^2x\sqrt{1+c^2x^2}(a+b\,\mathrm{arcsinh}(cx))}{\sqrt{d+icdx}\sqrt{f-icfx}} \\ &+ \frac{bcd^2x^2\sqrt{1+c^2x^2}(a+b\,\mathrm{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2id^2(1+c^2x^2)(a+b\,\mathrm{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} \\ &- \frac{d^2x(1+c^2x^2)(a+b\,\mathrm{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{d^2\sqrt{1+c^2x^2}(a+b\,\mathrm{arcsinh}(cx))^3}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}\end{aligned}$$

```
[Out] 4*I*b^2*d^2*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/4*b^2*d^2*x
*(c^2*x^2+1)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+2*I*d^2*(c^2*x^2+1)*(a+b*a
rcsinh(c*x))^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/2*d^2*x*(c^2*x^2+1)*
(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/4*b^2*d^2*arcsin
h(c*x)*(c^2*x^2+1)^(1/2)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-4*I*b*d^2*x*
(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/
2*b*c*d^2*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c
*f*x)^(1/2)+1/2*d^2*(a+b*arcsinh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^
(1/2)/(f-I*c*f*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {5796, 5843, 3398, 3377, 2718, 3392, 32, 2715, 8}

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx=\frac{d^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{2id^2(c^2x^2+1)\,(a+b\text{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{d^2x(c^2x^2+1)\,(a+b\text{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{bcd^2x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{4ibd^2x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{b^2d^2\sqrt{c^2x^2+1}\text{arcsinh}(cx)}{4c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{4ib^2d^2(c^2x^2+1)}{c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{b^2d^2x(c^2x^2+1)}{4\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[In] Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((4*I)*b^2*d^2*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (b^
2*d^2*x*(1 + c^2*x^2))/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (b^2*d^2*S
qrt[1 + c^2*x^2]*ArcSinh[c*x])/(4*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) -
((4*I)*b*d^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]) + (b*c*d^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + ((2*I)*d^2*(1 + c^2*x^2)*(a + b*Arc
Sinh[c*x])^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (d^2*x*(1 + c^2*x^2
)*(a + b*ArcSinh[c*x])^2)/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (d^2*Sq
rt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(2*b*c*Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+c^2x^2}\int \frac{(d+icdx)^2(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\text{Subst}\left(\int (a+bx)^2(cd+icd\sinh(x))^2\,dx,x,\text{arcsinh}(cx)\right)}{c^3\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\text{Subst}\left(\int \left(c^2d^2(a+bx)^2+2ic^2d^2(a+bx)^2\sinh(x)-c^2d^2(a+bx)^2\sinh^2(x)\right)\,dx,x,\text{arcs}\right.}{c^3\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{d^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(2id^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)^2\sinh(x)\,dx,x,\text{arcsinh}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(d^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)^2\sinh^2(x)\,dx,x,\text{arcsinh}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{bcd^2x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2id^2(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{d^2x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{d^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(d^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)^2\,dx,x,\text{arcsinh}(cx)\right)}{2c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(4ibd^2\sqrt{1+c^2x^2}\right)\text{Subst}(\int (a+bx)\cosh(x)\,dx,x,\text{arcsinh}(cx))}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(b^2d^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \sinh^2(x)\,dx,x,\text{arcsinh}(cx)\right)}{2c\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= -\frac{b^2d^2x(1+c^2x^2)}{4\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{4ibd^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{bcd^2x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2id^2(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{d^2x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{d^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{2bc\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(4ib^2d^2\sqrt{1+c^2x^2}\right)\text{Subst}(\int \sinh(x)\,dx,x,\text{arcsinh}(cx))}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(b^2d^2\sqrt{1+c^2x^2}\right)\text{Subst}(\int 1\,dx,x,\text{arcsinh}(cx))}{4c\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{4ib^2d^2(1+c^2x^2)}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{b^2d^2x(1+c^2x^2)}{4\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{b^2d^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{4c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{4ibd^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{bcd^2x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2id^2(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{d^2x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{d^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

Mathematica [A] (verified)

Time = 8.55 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.21

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \frac{-32iabcdx\sqrt{d+icdx}\sqrt{f-icfx}+16ia^2d\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1}}{}$$

```
[In] Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-32*I)*a*b*c*d*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (16*I)*a^2*d*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (32*I)*b^2*d*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*a^2*c*d*x*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 4*b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x]*ArcSinh[c*x]^3 + 2*a*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*
ArcSinh[c*x]] + 2*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((-1
6*I)*b*c*x - 4*a*(-4*I + c*x)*Sqrt[1 + c^2*x^2] + b*Cosh[2*ArcSinh[c*x]]) +
 12*a^2*d^(3/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]*Sinh[2*ArcSinh[c*x]] + 2*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh
[c*x]^2*(6*a + (8*I)*b*Sqrt[1 + c^2*x^2] - b*Sinh[2*ArcSinh[c*x]]))/(8*c*f*
Sqrt[1 + c^2*x^2])
```

Maple [F]

$$\int \frac{(icdx+d)^{\frac{3}{2}}\,(a+b\,\text{arcsinh}\,(cx))^2}{\sqrt{-icfx+f}}dx$$

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{-i\,cfx+f}}\,dx$$

[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algorithm="fricas")

[Out] integral(-((b^2*c*d*x - I*b^2*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c*d*x - I*a*b*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c*d*x - I*a^2*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*f*x + I*f), x)

Sympy [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \int \frac{(id(cx-i))^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(cx))^2}{\sqrt{-if\,(cx+i)}}\,dx$$

[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(1/2),x)

[Out] Integral((I*d*(c*x - I))**(3/2)*(a + b*asinh(c*x))**2/sqrt(-I*f*(c*x + I)), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \text{Exception raised: RuntimeError}$$

[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.

Giac [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{-i\,cfx+f}}\,dx$$

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
[Out] integrate((I*c*d*x + d)^(3/2)*(b*arcsinh(c*x) + a)^2/sqrt(-I*c*f*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d+c\,d\,x\,1\text{i})^{3/2}}{\sqrt{f-c\,f\,x\,1\text{i}}}\,dx$$

```
[In] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(1/2),x)
[Out] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(1/2), x)
```

3.590 $\int \frac{\sqrt{d+icdx}(a+b\,\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 259

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = -\frac{2iabdx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{2ib^2d(1+c^2x^2)}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ib^2dx\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{id(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{d\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[Out] 2*I*b^2*d*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+I*d*(c^2*x^2+1)
*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-2*I*a*b*d*x*(c^
2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-2*I*b^2*d*x*arcsinh(c*x)
*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/3*d*(a+b*arcsinh(c
*x))^3*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used

$= \{5796, 5838, 5783, 5798, 5772, 267\}$

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \frac{d\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{id(c^2x^2+1)\,(a+b\operatorname{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2iabdx\sqrt{c^2x^2+1}}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ib^2dx\sqrt{c^2x^2+1}\operatorname{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ib^2d(c^2x^2+1)}{c\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[In] Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-2*I)*a*b*d*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) +
((2*I)*b^2*d*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - ((2*I
)*b^2*d*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f
*x]) + (I*d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]) + (d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
```

```
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1+c^2x^2}\int \frac{(d+icdx)(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}\\
&= \frac{\sqrt{1+c^2x^2}\int \left(\frac{d(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}+\frac{icdx(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}\\
&= \frac{\left(d\sqrt{1+c^2x^2}\right)\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{\left(icd\sqrt{1+c^2x^2}\right)\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}\\
&= \frac{id(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{d\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}\\
&\quad-\frac{\left(2ibd\sqrt{1+c^2x^2}\right)\int (a+b\text{arcsinh}(cx))\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}\\
&= -\frac{2iabdx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{id(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}}\\
&\quad+\frac{d\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{\left(2ib^2d\sqrt{1+c^2x^2}\right)\int \text{arcsinh}(cx)\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}
\end{aligned}$$

$$= -\frac{2iabdx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ib^2dx\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{id(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{d\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(2ib^2cd\sqrt{1+c^2x^2}\right)\int\frac{x}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= -\frac{2iabdx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ib^2d(1+c^2x^2)}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ib^2dx\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{id(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{d\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

Mathematica [A] (verified)

Time = 2.64 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.22

$$\int\frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \frac{3i\sqrt{d+icdx}\sqrt{f-icfx}\left(-2abcx+a^2\sqrt{1+c^2x^2}+2b^2\sqrt{1+c^2x^2}\right)-6ib\sqrt{d+icdx}\sqrt{f-icfx}\left(bcx-a\sqrt{1+\right.}{}$$

```
[In] Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((3*I)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-2*a*b*c*x + a^2*Sqrt[1 + c^2*x
^2] + 2*b^2*Sqrt[1 + c^2*x^2]) - (6*I)*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]*(b*c*x - a*Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 3*b*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]*(a + I*b*Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2 + b^2*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 + 3*a^2*Sqrt[d]*Sqrt[f]*Sqrt[1 + c^2*x
^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/(3*
c*f*Sqrt[1 + c^2*x^2])
```

Maple [F]

$$\int\frac{(a+b\,\text{arcsinh}\,(cx))^2\sqrt{icdx+d}}{\sqrt{-icfx+f}}dx$$

```
[In] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx=\int \frac{\sqrt{i\,cdx+d}(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{-i\,cfx+f}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2
 + 1))^2 + 2*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*
x^2 + 1)) + I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c*f*x + I*f), x)
```

Sympy [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx=\int \frac{\sqrt{id\,(cx-i)}(a+b\,\text{asinh}\,(cx))^2}{\sqrt{-if\,(cx+i)}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral(sqrt(I*d*(c*x - I))*(a + b*asinh(c*x))**2/sqrt(-I*f*(c*x + I)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx=\int \frac{\sqrt{i\,cdx+d}(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{-i\,cfx+f}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] a^2*(d*arcsinh(c*x)/(c*f*sqrt(d/f)) + I*sqrt(c^2*d*f*x^2 + d*f)/(c*f)) + in
tegrate(sqrt(I*c*d*x + d)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/sqrt(-I*c*f*x
+ f) + 2*sqrt(I*c*d*x + d)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/sqrt(-I*c*f*x +
 f), x)
```

Giac [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx=\int \frac{\sqrt{i\,cdx+d}(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{-i\,cfx+f}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)^2/sqrt(-I*c*f*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2\,\sqrt{d+c\,d\,x\,1\text{i}}}{\sqrt{f-c\,f\,x\,1\text{i}}}\,dx$$

```
[In] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(1/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(1/2), x)
```

3.591 $\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 59

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx=\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[Out] 1/3*(a+b*arcsinh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x
)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {5796, 5783}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx=\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+c^2x^2}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$
$$= \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 168 vs. 2(59) = 118.

Time = 2.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.85

$$\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx = \frac{ab\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)^3}{3c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{a^2\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{c\sqrt{d}\sqrt{f}}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (a*b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]) + (b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^3)/(3*c*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]) + (a^2*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]])/(c*Sqrt[d]*Sqrt[f])
```

Maple [F]

$$\int\frac{(a+b\,\text{arcsinh}\,(cx))^2}{\sqrt{icdx+d}\,\sqrt{-icfx+f}}dx$$

```
[In] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 +
 1))^2 + 2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x^2
+ 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c^2*d*f*x^2 + d*f), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))^2}{\sqrt{id\,(cx-i)}\sqrt{-if\,(cx+i)}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(sqrt(I*d*(c*x - I))*sqrt(-I*f*(c*x + I))),
x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx = \frac{b^2\,\text{arsinh}\,(cx)^3}{3\,\sqrt{df}c} + \frac{ab\,\text{arsinh}\,(cx)^2}{\sqrt{df}c} + \frac{a^2\,\text{arsinh}\,(cx)}{\sqrt{df}c}$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/3*b^2*arcsinh(c*x)^3/(sqrt(d*f)*c) + a*b*arcsinh(c*x)^2/(sqrt(d*f)*c) + a
^2*arcsinh(c*x)/(sqrt(d*f)*c)
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2}{\sqrt{d+c\,d\,x\,1\text{i}}\,\sqrt{f-c\,f\,x\,1\text{i}}}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2)),x)
[Out] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2)), x)
```

3.592 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 464

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx = \frac{if(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{fx(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{f(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4ibf(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2bf(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2b^2f(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2b^2f(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{b^2f(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] I*f*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+
f*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+f*
(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2
)-4*I*b*f*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2)
)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*b*f*(c^2*x^2+1)^(3/2)*(a+b*arcsin
h(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/
2)-2*b^2*f*(c^2*x^2+1)^(3/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c
*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+2*b^2*f*(c^2*x^2+1)^(3/2)*polylog(2,I*(c*x+(c
^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-b^2*f*(c^2*x^2+1)^(
```

```
3/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^
(3/2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.297, Rules used = {5796, 5838, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 5789, 4265}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx = -\frac{4ibf(c^2x^2+1)^{3/2}\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{f(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{if(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{fx(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2bf(c^2x^2+1)^{3/2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2b^2f(c^2x^2+1)^{3/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2b^2f(c^2x^2+1)^{3/2}\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{b^2f(c^2x^2+1)^{3/2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (I*f*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*
f*x)^(3/2)) + (f*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*c*d*x)^(3/
2)*(f - I*c*f*x)^(3/2)) + (f*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(c
*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((4*I)*b*f*(1 + c^2*x^2)^(3/2)*
(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*
c*f*x)^(3/2)) - (2*b*f*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log[1 + E^(
2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (2*b^2*f*(1
 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(
f - I*c*f*x)^(3/2)) + (2*b^2*f*(1 + c^2*x^2)^(3/2)*PolyLog[2, I*E^ArcSinh[c
*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (b^2*f*(1 + c^2*x^2)^(3
/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(
3/2))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(1+c^2x^2)^{3/2}\int \frac{(f-icfx)(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&= \frac{(1+c^2x^2)^{3/2}\int \left(\frac{f(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}-\frac{icfx(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&= \frac{\left(f(1+c^2x^2)^{3/2}\right)\int \frac{(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{\left(icf(1+c^2x^2)^{3/2}\right)\int \frac{x(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}$$

$$= \frac{if(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{fx(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2ibf(1+c^2x^2)^{3/2}\right)\int \frac{a+b\text{arcsinh}(cx)}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2bcf(1+c^2x^2)^{3/2}\right)\int \frac{x(a+b\text{arcsinh}(cx))}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{if(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{fx(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2ibf(1+c^2x^2)^{3/2}\right)\text{Subst}(\int (a+bx)\text{sech}(x)\,dx, x, \text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2bf(1+c^2x^2)^{3/2}\right)\text{Subst}(\int (a+bx)\tanh(x)\,dx, x, \text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{if(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{fx(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{f(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4ibf(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(4bf(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \text{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2b^2f(1+c^2x^2)^{3/2}\right)\text{Subst}(\int \log(1-ie^x)\,dx, x, \text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(2b^2f(1+c^2x^2)^{3/2}\right)\text{Subst}(\int \log(1+ie^x)\,dx, x, \text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$=\frac{if(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{fx(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{f(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4ibf(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2bf(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{\left(2b^2f(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int\log\left(1+e^{2x}\right)\,dx,x,\text{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{\left(2b^2f(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{\left(2b^2f(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$=\frac{if(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{fx(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{f(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4ibf(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2bf(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2b^2f(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2b^2f(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{\left(b^2f(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$
\begin{aligned}
&= \frac{if(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{fx(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{f(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{4ibf(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{2bf(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{2b^2f(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{2b^2f(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{b^2f(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.26 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.09

$$
\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx = \frac{\sqrt{d+icdx}\sqrt{f-icfx}\left((-1+i)b^2\sqrt{1+c^2x^2}\operatorname{arcsinh}(cx)^2\left(\cosh\left(\tfrac{1}{2}\operatorname{arcsinh}(c\right.\right.\right.}{}
$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((-1 + I)*b^2*Sqrt[1 + c^2*x^2]*ArcSin
h[c*x]^2*(Cosh[ArcSinh[c*x]/2] - Sinh[ArcSinh[c*x]/2]) + (I*a^2 + a^2*c*x -
 (4*I)*a*b*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] + (2*I)*b^2*Pi*Sq
rt[1 + c^2*x^2]*Log[1 - I/E^ArcSinh[c*x]] + (4*I)*b^2*Pi*Sqrt[1 + c^2*x^2]*
Log[1 + E^ArcSinh[c*x]] - a*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] - (4*I)*b^
2*Pi*Sqrt[1 + c^2*x^2]*Log[Cosh[ArcSinh[c*x]/2]] - (2*I)*b^2*Pi*Sqrt[1 + c^
2*x^2]*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]])*(Cosh[ArcSinh[c*x]/2] + I*Sin
h[ArcSinh[c*x]/2]) + 4*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]]*(
Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + b*Sqrt[1 + c^2*x^2]*ArcSin
h[c*x]*(I*Cosh[ArcSinh[c*x]/2]*(2*a - b*Pi + (4*I)*b*Log[1 - I/E^ArcSinh[c*
x]]) + (2*a + b*Pi - (4*I)*b*Log[1 - I/E^ArcSinh[c*x]])*Sinh[ArcSinh[c*x]/2
])))/(c*d^2*f*(-I + c*x)*(I + c*x)*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c
*x]/2]))
```

Maple [F]

$$\int \frac{(a + b \operatorname{arcsinh}(cx))^2}{(icdx + d)^{\frac{3}{2}} \sqrt{-icfx + f}} dx$$

```
[In] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{3/2}\sqrt{f - icfx}} dx = \int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(i\, cdx + d)^{\frac{3}{2}} \sqrt{-i\, cfx + f}} dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] (sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2 +
(c^2*d^2*f*x - I*c*d^2*f)*integral((-I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)
*a^2 - 2*(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2 + I*sq
rt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b)*log(c*x + sqrt(c^2*x^2 + 1)))/(c^3*
d^2*f*x^3 - I*c^2*d^2*f*x^2 + c*d^2*f*x - I*d^2*f), x))/(c^2*d^2*f*x - I*c*
d^2*f)
```

Sympy [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{3/2}\sqrt{f - icfx}} dx = \int \frac{(a + b \operatorname{asinh}(cx))^2}{(id(cx - i))^{\frac{3}{2}} \sqrt{-if(cx + i)}} dx$$

```
[In] integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/((I*d*(c*x - I))**(3/2)*sqrt(-I*f*(c*x + I))
), x)
```

Maxima [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{3/2}\sqrt{f - icfx}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(i\,cdx + d)^{\frac{3}{2}}\sqrt{-i\,cfx + f}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/((I*c*d*x + d)^(3/2)*sqrt(-I*c
*f*x + f)), x) + 2*I*sqrt(c^2*d*f*x^2 + d*f)*a*b*arcsinh(c*x)/(I*c^2*d^2*f*
x + c*d^2*f) + I*sqrt(c^2*d*f*x^2 + d*f)*a^2/(I*c^2*d^2*f*x + c*d^2*f) - 2*
a*b*log(I*c*x + 1)/(c*d^(3/2)*sqrt(f))
```

Giac [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{3/2}\sqrt{f - icfx}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(i\,cdx + d)^{\frac{3}{2}}\sqrt{-i\,cfx + f}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((I*c*d*x + d)^(3/2)*sqrt(-I*c*f*x + f)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{3/2}\sqrt{f - icfx}}\,dx = \int \frac{(a + b\,\text{asinh}(c\,x))^2}{(d + c\,d\,x\,1\text{i})^{3/2}\,\sqrt{f - c\,f\,x\,1\text{i}}}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2)), x)
```

3.593 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 942

$$
\begin{aligned}
\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx &= -\frac{2ib^2f^2(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2b^2f^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2f^2(1+c^2x^2)^{5/2}\,\text{arcsinh}(cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{bf^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ibf^2x(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{bcf^2x^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^2(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{f^2x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{c^2f^2x^3(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2f^2x(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{4ibf^2(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2bf^2(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2b^2f^2(1+c^2x^2)^{5/2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2b^2f^2(1+c^2x^2)^{5/2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{b^2f^2(1+c^2x^2)^{5/2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

```
[Out] -4/3*I*b*f^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1
/2))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*b^2*f^2*x*(c^2*x^2+1)^2/(d+I
*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*b^2*f^2*(c^2*x^2+1)^(5/2)*arcsinh(c*x)/
c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*b*f^2*(c^2*x^2+1)^(3/2)*(a+b*arcs
inh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*I*b*f^2*x*(c^2*x^2+1)^(
3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b*c*f^2*x^2
*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2
/3*I*b^2*f^2*(c^2*x^2+1)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*f^2*x*
(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*c^
2*f^2*x^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5
/2)+2/3*f^2*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f
*x)^(5/2)+1/3*f^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2
)/(f-I*c*f*x)^(5/2)+2/3*I*f^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x
)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*b*f^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln
(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*b^2
*f^2*(c^2*x^2+1)^(5/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^
(5/2)/(f-I*c*f*x)^(5/2)+2/3*b^2*f^2*(c^2*x^2+1)^(5/2)*polylog(2,I*(c*x+(c^2
*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b^2*f^2*(c^2*x^2+
1)^(5/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f
*x)^(5/2)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 942, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.486$, Rules used = {5796, 5838, 5788, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 197, 5789, 4265, 267,

5800, 5810, 294, 221}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = -\frac{c^2f^2(c^2x^2+1)\,(a+b\text{arcsinh}(cx))^2x^3}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{bcf^2(c^2x^2+1)^{3/2}\,(a+b\text{arcsinh}(cx))x^2}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}-\frac{2b^2f^2(c^2x^2+1)^2\,x}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{2f^2(c^2x^2+1)^2\,(a+b\text{arcsinh}(cx))^2x}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}+\frac{f^2(c^2x^2+1)\,(a+b\text{arcsinh}(cx))^2x}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{2ibf^2(c^2x^2+1)^{3/2}\,(a+b\text{arcsinh}(cx))x}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}-\frac{2ib^2f^2(c^2x^2+1)^2}{3c(icxd+d)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{f^2(c^2x^2+1)^{5/2}\,(a+b\text{arcsinh}(cx))^2}{3c(icxd+d)^{5/2}(f-icfx)^{5/2}}+\frac{2if^2(c^2x^2+1)\,(a+b\text{arcsinh}(cx))^2}{3c(icxd+d)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{b^2f^2(c^2x^2+1)^{5/2}\,\text{arcsinh}(cx)}{3c(icxd+d)^{5/2}(f-icfx)^{5/2}}+\frac{bf^2(c^2x^2+1)^{3/2}\,(a+b\text{arcsinh}(cx))}{3c(icxd+d)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{4ibf^2(c^2x^2+1)^{5/2}\,(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3c(icxd+d)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{2bf^2(c^2x^2+1)^{5/2}\,(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3c(icxd+d)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{2b^2f^2(c^2x^2+1)^{5/2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3c(icxd+d)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{2b^2f^2(c^2x^2+1)^{5/2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3c(icxd+d)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{b^2f^2(c^2x^2+1)^{5/2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3c(icxd+d)^{5/2}(f-icfx)^{5/2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (((-2*I)/3)*b^2*f^2*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(
5/2)) - (2*b^2*f^2*x*(1 + c^2*x^2)^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^
(5/2)) + (b^2*f^2*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x])/(3*c*(d + I*c*d*x)^(5/2
)*(f - I*c*f*x)^(5/2)) + (b*f^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(
3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((2*I)/3)*b*f^2*x*(1 + c^2*
x^2)^(3/2)*(a + b*ArcSinh[c*x]))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
- (b*c*f^2*x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(
5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*f^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x
])^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (f^2*x*(1 + c^2*x^2)*(a
 + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (c^2*f^
2*x^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c
*f*x)^(5/2)) + (2*f^2*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (f^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[
c*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((4*I)/3)*b*f^2*(
```

```
1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b*f^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcS
inh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*
x)^(5/2)) - (2*b^2*f^2*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])
/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*b^2*f^2*(1 + c^2*x^2)^(
5/2)*PolyLog[2, I*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(
5/2)) - (b^2*f^2*(1 + c^2*x^2)^(5/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c*
(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
```

```
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\text{integral} = \frac{(1+c^2x^2)^{5/2}\int\frac{(f-icfx)^2(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{(1+c^2x^2)^{5/2}\int\left(\frac{f^2(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}-\frac{2icf^2x(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}-\frac{c^2f^2x^2(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(f^2(1+c^2x^2)^{5/2}\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2icf^2(1+c^2x^2)^{5/2}\right)\int\frac{x(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(c^2f^2(1+c^2x^2)^{5/2}\right)\int\frac{x^2(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2if^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{c^2f^2x^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2f^2(1+c^2x^2)^{5/2}\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(4ibf^2(1+c^2x^2)^{5/2}\right)\int\frac{a+b\text{arcsinh}(cx)}{(1+c^2x^2)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2bcf^2(1+c^2x^2)^{5/2}\right)\int\frac{x(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2bc^3f^2(1+c^2x^2)^{5/2}\right)\int\frac{x^3(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{bf^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ibf^2x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bcf^2x^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{c^2f^2x^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2f^2x(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2ibf^2(1+c^2x^2)^{5/2}\right)\int\frac{a+b\text{arcsinh}(cx)}{1+c^2x^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(b^2f^2(1+c^2x^2)^{5/2}\right)\int\frac{1}{(1+c^2x^2)^{3/2}}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2bcf^2(1+c^2x^2)^{5/2}\right)\int\frac{x(a+b\text{arcsinh}(cx))}{1+c^2x^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(4bcf^2(1+c^2x^2)^{5/2}\right)\int\frac{x(a+b\text{arcsinh}(cx))}{1+c^2x^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2ib^2cf^2(1+c^2x^2)^{5/2}\right)\int\frac{x}{(1+c^2x^2)^{3/2}}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(b^2c^2f^2(1+c^2x^2)^{5/2}\right)\int\frac{x^2}{(1+c^2x^2)^{3/2}}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{2ib^2f^2(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2f^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ibf^2x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bcf^2x^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{c^2f^2x^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2f^2x(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(b^2f^2(1+c^2x^2)^{5/2}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2ibf^2(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int(a+bx)\text{sech}(x)\,dx,x,\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2bf^2(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int(a+bx)\tanh(x)\,dx,x,\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(4bf^2(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int(a+bx)\tanh(x)\,dx,x,\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$
\begin{aligned}
&= -\frac{2ib^2f^2(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2f^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{b^2f^2(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2ibf^2x(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{bcf^2x^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^2(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{f^2x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{c^2f^2x^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2f^2x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{4ibf^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{\left(4bf^2(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{\left(8bf^2(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{\left(2b^2f^2(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int \log(1-ie^x)\,dx, x, \operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{\left(2b^2f^2(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int \log(1+ie^x)\,dx, x, \operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2ib^2f^2(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2f^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{b^2f^2(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf^2(1+c^2x^2)^{3/2}\,(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{2ibf^2x(1+c^2x^2)^{3/2}\,(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{bcf^2x^2(1+c^2x^2)^{3/2}\,(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^2(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{f^2x(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{c^2f^2x^3(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{2f^2x(1+c^2x^2)^2\,(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2(1+c^2x^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{4ibf^2(1+c^2x^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{2bf^2(1+c^2x^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{\left(2b^2f^2(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\log\left(1+e^{2x}\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{\left(2b^2f^2(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{\left(2b^2f^2(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{\left(4b^2f^2(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\log\left(1+e^{2x}\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

$$= -\frac{2ib^2f^2(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2f^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2f^2(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ibf^2x(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bcf^2x^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^2(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{c^2f^2x^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2f^2x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{4ibf^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2bf^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2f^2(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2b^2f^2(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(b^2f^2(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2b^2f^2(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$
\begin{aligned}
&= -\frac{2ib^2f^2(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2f^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{b^2f^2(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2ibf^2x(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{bcf^2x^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^2(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{f^2x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{c^2f^2x^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2f^2x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{4ibf^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2bf^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2b^2f^2(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2b^2f^2(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{b^2f^2(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

Mathematica [A] (warning: unable to verify)

Time = 6.91 (sec) , antiderivative size = 524, normalized size of antiderivative = 0.56

$$
\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{a^2(-2i+cx)}{(-i+cx)^2} - \frac{ab\left(-i\cosh\left(\frac{3}{2}\operatorname{arcsinh}(cx)\right)\left(\operatorname{arcsinh}(cx)-2\operatorname{arct}\right.\right.}{}\right.}{}
$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((a^2*(-2*I + c*x))/(-I + c*x)^2 - (a*
b*((-I)*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]
/2]] - (I/2)*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(-2 - (3*I)*ArcSinh[c
*x] - (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*Log[1 + c^2*x^2])/2) + 2*((-1
 + Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 2*(2 + Sqrt[1 + c^2*x^2])*ArcTan[Coth[
```

```
ArcSinh[c*x]/2]] + (I/2)*(-2 + (2 + Sqrt[1 + c^2*x^2])*Log[1 + c^2*x^2]))*S
inh[ArcSinh[c*x]/2]))/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[Arc
Sinh[c*x]/2])^3) - (b^2*((1 - I)*ArcSinh[c*x]^2 - (ArcSinh[c*x]*(-2*I + Arc
Sinh[c*x]))/(-I + c*x) + 2*((-I)*Pi + 2*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c
*x]] + I*Pi*(ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c*x]] + 4*Log[Cosh[ArcSinh[
c*x]/2]] + 2*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]]) - 4*PolyLog[2, I/E^ArcS
inh[c*x]] - (2*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] +
 I*Sinh[ArcSinh[c*x]/2])^3 - (2*(-2 + ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])
/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/Sqrt[1 + c^2*x^2]))/(3*c
*d^3*f)
```

Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{(icdx+d)^{\frac{5}{2}}\sqrt{-icfx+f}}dx$$

```
[In] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{5}{2}}\sqrt{-i\,cfx+f}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*((b^2*c*x - 2*I*b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqr
t(c^2*x^2 + 1))^2 + 3*(c^3*d^3*f*x^2 - 2*I*c^2*d^3*f*x - c*d^3*f)*integral(
-1/3*(3*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + 2*(3*sqrt(I*c*d*x + d)*s
qrt(-I*c*f*x + f)*a*b + (b^2*c*x - 2*I*b^2)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x
+ d)*sqrt(-I*c*f*x + f))*log(c*x + sqrt(c^2*x^2 + 1)))/(c^4*d^3*f*x^4 - 2*I
*c^3*d^3*f*x^3 - 2*I*c*d^3*f*x - d^3*f), x))/(c^3*d^3*f*x^2 - 2*I*c^2*d^3*f
*x - c*d^3*f)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \int \frac{\left(a+b\,\text{asinh}\left(cx\right)\right)^2}{\left(id\left(cx-i\right)\right)^{\frac{5}{2}}\sqrt{-if\left(cx+i\right)}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(1/2),x)
[Out] Integral((a + b*asinh(c*x))**2/((I*d*(c*x - I))**(5/2)*sqrt(-I*f*(c*x + I))
), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \int \frac{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}{\left(i\,cdx+d\right)^{\frac{5}{2}}\sqrt{-i\,cfx+f}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/((I*c*d*x + d)^(5/2)*sqrt(-I*c*f*x + f)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \int \frac{\left(a+b\,\text{asinh}(c\,x)\right)^2}{\left(d+c\,d\,x\,1\text{i}\right)^{5/2}\sqrt{f-c\,f\,x\,1\text{i}}}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2)),x)
[Out] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2)), x)
```

3.594 $\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 972

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \frac{8iabd^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ib^2d^4(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2d^4x(1+c^2x^2)^2}{4(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{b^2d^4(1+c^2x^2)^{3/2}\text{arcsinh}(cx)}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8ib^2d^4x(1+c^2x^2)^{3/2}\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{bcd^4x^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8id^4(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8d^4x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8d^4(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4id^4(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{d^4x(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{5d^4(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^3}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{32ibd^4(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{16bd^4(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{16b^2d^4(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{16b^2d^4(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8b^2d^4(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] 8*I*b^2*d^4*x*(c^2*x^2+1)^(3/2)*arcsinh(c*x)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^
(3/2)+32*I*b*d^4*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1
)^(1/2))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+1/4*b^2*d^4*x*(c^2*x^2+1)^2/
(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-1/4*b^2*d^4*(c^2*x^2+1)^(3/2)*arcsinh(c
*x)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*I*d^4*(c^2*x^2+1)*(a+b*arcsinh(
c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-1/2*b*c*d^4*x^2*(c^2*x^2+1)^(
3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*I*b^2*d^4*(c^
2*x^2+1)^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*d^4*x*(c^2*x^2+1)*(a+b*a
rcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*d^4*(c^2*x^2+1)^(3/2)*
(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*I*d^4*(c^2*x^2
+1)^2*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+1/2*d^4*x*
(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-5/2*
```

```
d^4*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x
)^(3/2)+8*I*a*b*d^4*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
-16*b*d^4*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))
^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+16*b^2*d^4*(c^2*x^2+1)^(3/2)*poly
log(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-16*
b^2*d^4*(c^2*x^2+1)^(3/2)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x
)^(3/2)/(f-I*c*f*x)^(3/2)-8*b^2*d^4*(c^2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^2*
x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 972, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.514$, Rules used = {5796, 5844, 5838, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 5789, 4265, 5783, 5772, 267, 5812, 5776, 327, 221}

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx =$$
$$-\frac{5(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^3d^4}{2bc(icxd+d)^{3/2}(f-icfx)^{3/2}}+\frac{b^2x(c^2x^2+1)^2d^4}{4(icxd+d)^{3/2}(f-icfx)^{3/2}}$$
$$-\frac{8ib^2(c^2x^2+1)^2d^4}{c(icxd+d)^{3/2}(f-icfx)^{3/2}}+\frac{x(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2d^4}{2(icxd+d)^{3/2}(f-icfx)^{3/2}}$$
$$-\frac{4i(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2d^4}{c(icxd+d)^{3/2}(f-icfx)^{3/2}}+\frac{8(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^2d^4}{c(icxd+d)^{3/2}(f-icfx)^{3/2}}$$
$$+\frac{8x(c^2x^2+1)(a+b\text{arcsinh}(cx))^2d^4}{(icxd+d)^{3/2}(f-icfx)^{3/2}}-\frac{8i(c^2x^2+1)(a+b\text{arcsinh}(cx))^2d^4}{c(icxd+d)^{3/2}(f-icfx)^{3/2}}$$
$$+\frac{8iabx(c^2x^2+1)^{3/2}d^4}{(icxd+d)^{3/2}(f-icfx)^{3/2}}+\frac{8ib^2x(c^2x^2+1)^{3/2}\text{arcsinh}(cx)d^4}{(icxd+d)^{3/2}(f-icfx)^{3/2}}$$
$$-\frac{b^2(c^2x^2+1)^{3/2}\text{arcsinh}(cx)d^4}{4c(icxd+d)^{3/2}(f-icfx)^{3/2}}-\frac{bcx^2(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))d^4}{2(icxd+d)^{3/2}(f-icfx)^{3/2}}$$
$$+\frac{32ib(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)d^4}{c(icxd+d)^{3/2}(f-icfx)^{3/2}}$$
$$-\frac{16b(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)d^4}{c(icxd+d)^{3/2}(f-icfx)^{3/2}}$$
$$+\frac{16b^2(c^2x^2+1)^{3/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)d^4}{c(icxd+d)^{3/2}(f-icfx)^{3/2}}$$
$$-\frac{16b^2(c^2x^2+1)^{3/2}\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)d^4}{c(icxd+d)^{3/2}(f-icfx)^{3/2}}$$
$$-\frac{8b^2(c^2x^2+1)^{3/2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)d^4}{c(icxd+d)^{3/2}(f-icfx)^{3/2}}$$

```
[In] Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),x]
```

```
[Out] ((8*I)*a*b*d^4*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3
/2)) - ((8*I)*b^2*d^4*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)) + (b^2*d^4*x*(1 + c^2*x^2)^2)/(4*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^
(3/2)) - (b^2*d^4*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x])/(4*c*(d + I*c*d*x)^(3/2
)*(f - I*c*f*x)^(3/2)) + ((8*I)*b^2*d^4*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x])
/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (b*c*d^4*x^2*(1 + c^2*x^2)^(3/
2)*(a + b*ArcSinh[c*x]))/(2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((8*
I)*d^4*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*
c*f*x)^(3/2)) + (8*d^4*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*c*d*
x)^(3/2)*(f - I*c*f*x)^(3/2)) + (8*d^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c
*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((4*I)*d^4*(1 + c^2*x
^2)^2*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) +
 (d^4*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(2*(d + I*c*d*x)^(3/2)*(f -
 I*c*f*x)^(3/2)) - (5*d^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^3)/(2*b*
c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((32*I)*b*d^4*(1 + c^2*x^2)^(3
/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f
- I*c*f*x)^(3/2)) - (16*b*d^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log[
1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (16*
b^2*d^4*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d + I*c*d*
x)^(3/2)*(f - I*c*f*x)^(3/2)) - (16*b^2*d^4*(1 + c^2*x^2)^(3/2)*PolyLog[2,
I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (8*b^2*d^4
*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2
)*(f - I*c*f*x)^(3/2))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
```

```
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{(1+c^2x^2)^{3/2}\int \frac{(d+icdx)^4(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{(1+c^2x^2)^{3/2}\int \left(-\frac{8i(id^4-cd^4x)(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}} - \frac{7d^4(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}} - \frac{4icd^4x(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}} + \frac{c^2d^4x^2(}{}\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(8i(1+c^2x^2)^{3/2}\right)\int \frac{(id^4-cd^4x)(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(7d^4(1+c^2x^2)^{3/2}\right)\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(4icd^4(1+c^2x^2)^{3/2}\right)\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(c^2d^4(1+c^2x^2)^{3/2}\right)\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{4id^4(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{d^4x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{7d^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{\left(8i(1+c^2x^2)^{3/2}\right)\int\left(\frac{id^4(a+b\operatorname{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}} - \frac{cd^4x(a+b\operatorname{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{\left(d^4(1+c^2x^2)^{3/2}\right)\int\frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}dx}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{\left(8ibd^4(1+c^2x^2)^{3/2}\right)\int(a+b\operatorname{arcsinh}(cx))\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{\left(bcd^4(1+c^2x^2)^{3/2}\right)\int x(a+b\operatorname{arcsinh}(cx))\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{8iabd^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{bcd^4x^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{4id^4(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{d^4x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{5d^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^3}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(8d^4(1+c^2x^2)^{3/2}\right)\int\frac{(a+b\operatorname{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{\left(8ib^2d^4(1+c^2x^2)^{3/2}\right)\int\operatorname{arcsinh}(cx)\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{\left(8icd^4(1+c^2x^2)^{3/2}\right)\int\frac{x(a+b\operatorname{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{\left(b^2c^2d^4(1+c^2x^2)^{3/2}\right)\int\frac{x^2}{\sqrt{1+c^2x^2}}dx}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$=\frac{8iabd^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{b^2d^4x(1+c^2x^2)^2}{4(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{8ib^2d^4x(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{bcd^4x^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{8id^4(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{8d^4x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4id^4(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{d^4x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{5d^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^3}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{\left(16ibd^4(1+c^2x^2)^{3/2}\right)\int\frac{a+b\operatorname{arcsinh}(cx)}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{\left(b^2d^4(1+c^2x^2)^{3/2}\right)\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{4(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{\left(16bcd^4(1+c^2x^2)^{3/2}\right)\int\frac{x(a+b\operatorname{arcsinh}(cx))}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{\left(8ib^2cd^4(1+c^2x^2)^{3/2}\right)\int\frac{x}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$=\frac{8iabd^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{8ib^2d^4(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{b^2d^4x(1+c^2x^2)^2}{4(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{b^2d^4(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{8ib^2d^4x(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{bcd^4x^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{8id^4(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{8d^4x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4id^4(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{d^4x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{5d^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^3}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{\left(16ibd^4(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int(a+bx)\operatorname{sech}(x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{\left(16bd^4(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int(a+bx)\tanh(x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{8iabd^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ib^2d^4(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{b^2d^4x(1+c^2x^2)^2}{4(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{b^2d^4(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{8ib^2d^4x(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{bcd^4x^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{8id^4(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8d^4x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{8d^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4id^4(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{d^4x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{5d^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^3}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{32ibd^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{\left(32bd^4(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int\frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx,x,\operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{\left(16b^2d^4(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int\log\left(1-ie^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{\left(16b^2d^4(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int\log\left(1+ie^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$
\begin{aligned}
&= \frac{8iabd^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ib^2d^4(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad + \frac{b^2d^4x(1+c^2x^2)^2}{4(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{b^2d^4(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad + \frac{8ib^2d^4x(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{bcd^4x^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad - \frac{8id^4(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8d^4x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad + \frac{8d^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4id^4(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad + \frac{d^4x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{5d^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^3}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad + \frac{32ibd^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad - \frac{16bd^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad + \frac{\left(16b^2d^4(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad + \frac{\left(16b^2d^4(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad - \frac{\left(16b^2d^4(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}
$$

$$= \frac{8iabd^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ib^2d^4(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{b^2d^4x(1+c^2x^2)^2}{4(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{b^2d^4(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{8ib^2d^4x(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{bcd^4x^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{8id^4(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8d^4x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{8d^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4id^4(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{d^4x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{5d^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^3}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{32ibd^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{16bd^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{16b^2d^4(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{16b^2d^4(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{\left(8b^2d^4(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$
\begin{aligned}
&= \frac{8iabd^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ib^2d^4(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{b^2d^4x(1+c^2x^2)^2}{4(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{b^2d^4(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{8ib^2d^4x(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{bcd^4x^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{8id^4(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8d^4x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{8d^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4id^4(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{d^4x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{5d^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^3}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{32ibd^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{16bd^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{16b^2d^4(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{16b^2d^4(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{8b^2d^4(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}
$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2143 vs. 2(972) = 1944.

Time = 25.04 (sec) , antiderivative size = 2143, normalized size of antiderivative = 2.20

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),
x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(((-4*I)*a^2*d^2)/f^2 + (a^2*c
*d^2*x)/(2*f^2) + (8*a^2*d^2)/(f^2*(I + c*x))))/c - (15*a^2*d^(5/2)*Log[c*d
*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(2*c*f
^(3/2)) - ((4*I)*a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*
Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*(-(c*x) + 2*ArcSinh[c*x] +
```

```
 Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - I*ArcSinh[c*x]^2 + 4*ArcTan[Coth[ArcSinh[
c*x]/2]] - (2*I)*Log[Sqrt[1 + c^2*x^2]]) - ((-I)*c*x - (2*I)*ArcSinh[c*x] +
 I*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]^2 + (4*I)*ArcTan[Coth[ArcS
inh[c*x]/2]] + 2*Log[Sqrt[1 + c^2*x^2]])*Sinh[ArcSinh[c*x]/2]))/(c*f^2*Sqrt
[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2]
 - I*Sinh[ArcSinh[c*x]/2])) - (a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(
I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*(8*ArcTan[Ta
nh[ArcSinh[c*x]/2]] + I*(ArcSinh[c*x]*(4*I + ArcSinh[c*x]) + 4*Log[Sqrt[1 +
 c^2*x^2]])) + (ArcSinh[c*x]*(-4*I + ArcSinh[c*x]) - (8*I)*ArcTan[Tanh[ArcS
inh[c*x]/2]] + 4*Log[Sqrt[1 + c^2*x^2]])*Sinh[ArcSinh[c*x]/2]))/(c*f^2*Sqrt
[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(I*Cosh[ArcSinh[c*x]/
2] + Sinh[ArcSinh[c*x]/2])) - (b^2*d^2*(-I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*
Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(-18*Pi*ArcSinh[c*x] -
(6 - 6*I)*ArcSinh[c*x]^2 + I*ArcSinh[c*x]^3 - 12*(Pi - (2*I)*ArcSinh[c*x])*
Log[1 + I/E^ArcSinh[c*x]] + 24*Pi*Log[1 + E^ArcSinh[c*x]] + 12*Pi*Log[-Cos[
(Pi + (2*I)*ArcSinh[c*x])/4]] - 24*Pi*Log[Cosh[ArcSinh[c*x]/2]] - (24*I)*Po
lyLog[2, (-I)/E^ArcSinh[c*x]] - ((12*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2]
)/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(3*c*f^2*Sqrt[-(((-I)*d
 + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[
ArcSinh[c*x]/2])^2) - (((2*I)/3)*b^2*d^2*(-I + c*x)*Sqrt[I*((-I)*d + c*d*x)
]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(((-6*I)*c*x*ArcSinh[
c*x])/Sqrt[1 + c^2*x^2] + ((6 + 6*I)*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] + (2
*ArcSinh[c*x]^3)/Sqrt[1 + c^2*x^2] + (3*I)*(2 + ArcSinh[c*x]^2) + ((6*I)*(2
*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + Pi*(3*ArcSinh[c*x] -
 4*Log[1 + E^ArcSinh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 4*L
og[Cosh[ArcSinh[c*x]/2]]) + (4*I)*PolyLog[2, (-I)/E^ArcSinh[c*x]]))/Sqrt[1
+ c^2*x^2] - (12*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Sqrt[1 + c^2*x^2]*(C
osh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]))))/(c*f^2*Sqrt[-(((-I)*d + c*
d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2) + (
b^2*d^2*(-I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-
(d*f*(1 + c^2*x^2))]*((-96*c*x*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + ((48 - 48*
I)*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] - ((20*I)*ArcSinh[c*x]^3)/Sqrt[1 + c^2
*x^2] + 48*(2 + ArcSinh[c*x]^2) + (6*I)*c*x*(1 + 2*ArcSinh[c*x]^2) - ((6*I)
*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (48*(2*(Pi - (2*I)*
ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + Pi*(3*ArcSinh[c*x] - 4*Log[1 + E^
ArcSinh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 4*Log[Cosh[ArcSi
nh[c*x]/2]]) + (4*I)*PolyLog[2, (-I)/E^ArcSinh[c*x]]))/Sqrt[1 + c^2*x^2] +
((96*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSi
nh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]))))/(24*c*f^2*Sqrt[-(((-I)*d + c*d*x)*(
I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2) + (a*b*d^
2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2
))]*(Sinh[ArcSinh[c*x]/2]*(-16*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + I*Cosh[2*Ar
cSinh[c*x]] + 2*(8*c*x + 8*ArcSinh[c*x] + (5*I)*ArcSinh[c*x]^2 + 16*ArcTan[
Tanh[ArcSinh[c*x]/2]] + (8*I)*Log[Sqrt[1 + c^2*x^2]] - I*ArcSinh[c*x]*Sinh[
2*ArcSinh[c*x]])) - Cosh[ArcSinh[c*x]/2]*((16*I)*Sqrt[1 + c^2*x^2]*ArcSinh[
```

```
c*x] + Cosh[2*ArcSinh[c*x]] - 2*((8*I)*c*x - (8*I)*ArcSinh[c*x] - 5*ArcSinh
[c*x]^2 + (16*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] - 8*Log[Sqrt[1 + c^2*x^2]] +
ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]]))))/(4*c*f^2*Sqrt[-(((-I)*d + c*d*x)*(I*f
 + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2
]))
```

Maple [F]

$$\int \frac{(icdx+d)^{\frac{5}{2}} (a+b\,\mathrm{arcsinh}\,(cx))^2}{(-icfx+f)^{\frac{3}{2}}}dx$$

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x)
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x)
```

Fricas [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\mathrm{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \int \frac{(i\,cdx+d)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^2}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algor
ithm="fricas")
[Out] integral(((b^2*c^2*d^2*x^2 - 2*I*b^2*c*d^2*x - b^2*d^2)*sqrt(I*c*d*x + d)*s
qrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c^2*d^2*x^2 - 2*I
*a*b*c*d^2*x - a*b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt
(c^2*x^2 + 1)) + (a^2*c^2*d^2*x^2 - 2*I*a^2*c*d^2*x - a^2*d^2)*sqrt(I*c*d*x
 + d)*sqrt(-I*c*f*x + f))/(c^2*f^2*x^2 + 2*I*c*f^2*x - f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\mathrm{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \int \frac{(i\,cdx+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*(c^2*d^3*x^3/(sqrt(c^2*d*f*x^2 + d*f)*f) - 8*I*c*d^3*x^2/(sqrt(c^2*d*f*
x^2 + d*f)*f) + 17*d^3*x/(sqrt(c^2*d*f*x^2 + d*f)*f) - 15*d^3*arcsinh(c*x)/
(sqrt(d*f)*c*f) - 24*I*d^3/(sqrt(c^2*d*f*x^2 + d*f)*c*f))*a^2 + integrate((
I*c*d*x + d)^(5/2)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(-I*c*f*x + f)^(3/2)
+ 2*(I*c*d*x + d)^(5/2)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(-I*c*f*x + f)^(3/
2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d+c\,d\,x\,1\text{i})^{5/2}}{(f-c\,f\,x\,1\text{i})^{3/2}}\,dx$$

```
[In] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(3/2), x)
```

3.595 $\int \frac{(d+icdx)^{3/2}(a+b\mathbf{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 752

$$\begin{aligned}
\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx &= \frac{2iabd^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{2ib^2d^3(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2d^3x(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{4id^3(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4d^3x(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{4d^3(1+c^2x^2)^{3/2}\,(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{id^3(1+c^2x^2)^2\,(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^3(1+c^2x^2)^{3/2}\,(a+b\operatorname{arcsinh}(cx))^3}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{16ibd^3(1+c^2x^2)^{3/2}\,(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{8bd^3(1+c^2x^2)^{3/2}\,(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{8b^2d^3(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{8b^2d^3(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{4b^2d^3(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}$$

```
[Out] 2*I*a*b*d^3*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*I*b^2
*d^3*(c^2*x^2+1)^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+2*I*b^2*d^3*x*(c^2
```

```
*x^2+1)^(3/2)*arcsinh(c*x)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*I*d^3*(c^2
*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*d^3*x*
(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*d^3*
(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2
)-I*d^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^
(3/2)-d^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(3/2)/(f-I
*c*f*x)^(3/2)+16*I*b*d^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c
^2*x^2+1)^(1/2))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*b*d^3*(c^2*x^2+1)^
(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2
)/(f-I*c*f*x)^(3/2)+8*b^2*d^3*(c^2*x^2+1)^(3/2)*polylog(2,-I*(c*x+(c^2*x^2+
1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*b^2*d^3*(c^2*x^2+1)^(3/2
)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2
)-4*b^2*d^3*(c^2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*
c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 752, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.405$, Rules used = {5796, 5844, 5838, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 5789, 4265, 5783, 5772, 267}

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}dx = \frac{16ibd^3(c^2x^2+1)^{3/2}\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^3(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^3}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{id^3(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4d^3(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4d^3x(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4id^3(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8bd^3(c^2x^2+1)^{3/2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2iabd^3x(c^2x^2+1)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8b^2d^3(c^2x^2+1)^{3/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8b^2d^3(c^2x^2+1)^{3/2}\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4b^2d^3(c^2x^2+1)^{3/2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2d^3x(c^2x^2+1)^{3/2}\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2d^3(c^2x^2+1)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[In] Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),x]
```

```
[Out] ((2*I)*a*b*d^3*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3
/2)) - ((2*I)*b^2*d^3*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)) + ((2*I)*b^2*d^3*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x])/((d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2)) - ((4*I)*d^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])
^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*d^3*x*(1 + c^2*x^2)*(a
 + b*ArcSinh[c*x])^2)/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*d^3*(1
 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f
*x)^(3/2)) - (I*d^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) - (d^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]
)^3)/(b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((16*I)*b*d^3*(1 + c^2
*x^2)^(3/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(
3/2)*(f - I*c*f*x)^(3/2)) - (8*b*d^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x
])*Log[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
 + (8*b^2*d^3*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d +
I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (8*b^2*d^3*(1 + c^2*x^2)^(3/2)*PolyLo
g[2, I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b^
2*d^3*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
```

```
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{(1+c^2x^2)^{3/2}\int \frac{(d+icdx)^3(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{(1+c^2x^2)^{3/2}\int \left(-\frac{4i(id^3-cd^3x)(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}} - \frac{3d^3(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}} - \frac{icd^3x(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(4i(1+c^2x^2)^{3/2}\right)\int \frac{(id^3-cd^3x)(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(3d^3(1+c^2x^2)^{3/2}\right)\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(icd^3(1+c^2x^2)^{3/2}\right)\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{id^3(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^3}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(4i(1+c^2x^2)^{3/2}\right)\int \left(\frac{id^3(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}} - \frac{cd^3x(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(2ibd^3(1+c^2x^2)^{3/2}\right)\int (a+b\text{arcsinh}(cx))\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{2iabd^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{id^3(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^3}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(4d^3(1+c^2x^2)^{3/2}\right)\int \frac{(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(2ib^2d^3(1+c^2x^2)^{3/2}\right)\int \text{arcsinh}(cx)\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(4icd^3(1+c^2x^2)^{3/2}\right)\int \frac{x(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{2iabd^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2d^3x(1+c^2x^2)^{3/2}\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4id^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4d^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{id^3(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^3}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(8ibd^3(1+c^2x^2)^{3/2}\right)\int \frac{a+b\text{arcsinh}(cx)}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(8bcd^3(1+c^2x^2)^{3/2}\right)\int \frac{x(a+b\text{arcsinh}(cx))}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2ib^2cd^3(1+c^2x^2)^{3/2}\right)\int \frac{x}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{2iabd^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2d^3(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2d^3x(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4id^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4d^3x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{id^3(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^3}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(8ibd^3(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int(a+bx)\operatorname{sech}(x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(8bd^3(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int(a+bx)\tanh(x)\,dx,x,\operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{2iabd^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2d^3(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2d^3x(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4id^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4d^3x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4d^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{id^3(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^3}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{16ibd^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(16bd^3(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int\frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx,x,\operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(8b^2d^3(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int\log\left(1-ie^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(8b^2d^3(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int\log\left(1+ie^x\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$
\begin{aligned}
&= \frac{2iabd^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2d^3(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{2ib^2d^3x(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4id^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{4d^3x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4d^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{id^3(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^3}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{16ibd^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{8bd^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{\left(8b^2d^3(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int\log\left(1+e^{2x}\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{\left(8b^2d^3(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{\left(8b^2d^3(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2iabd^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2d^3(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{2ib^2d^3x(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4id^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{4d^3x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4d^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{id^3(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^3}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{16ibd^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{8bd^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{8b^2d^3(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{8b^2d^3(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{\left(4b^2d^3(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}
$$

$$= \frac{2iabd^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2d^3(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{2ib^2d^3x(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4id^3(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{4d^3x(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4d^3(1+c^2x^2)^{3/2}\,(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{id^3(1+c^2x^2)^2\,(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^3(1+c^2x^2)^{3/2}\,(a+b\operatorname{arcsinh}(cx))^3}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{16ibd^3(1+c^2x^2)^{3/2}\,(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{8bd^3(1+c^2x^2)^{3/2}\,(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{8b^2d^3(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{8b^2d^3(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{4b^2d^3(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Mathematica [A] (warning: unable to verify)

Time = 17.84 (sec) , antiderivative size = 1346, normalized size of antiderivative = 1.79

$$\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \frac{\sqrt{id(-i+cx)}\sqrt{-if(i+cx)}\left(-\frac{ia^2d}{f^2}+\frac{4a^2d}{f^2(i+cx)}\right)}{c}$$
$$- \frac{3a^2d^{3/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{id(-i+cx)}\sqrt{-if(i+cx)}\right)}{cf^{3/2}}$$
$$- \frac{2iabd\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\sqrt{-df\,(1+c^2x^2)}\left(\cosh\left(\frac{1}{2}\operatorname{arcsinh}(cx)\right)\left(-cx+2\operatorname{arcsinh}(cx)+\sqrt{1+c^2\cdots}\right.\right.}{\cdots}$$
$$- \frac{abd\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\sqrt{-df\,(1+c^2x^2)}\left(\cosh\left(\frac{1}{2}\operatorname{arcsinh}(cx)\right)\left(8\arctan\left(\tanh\left(\frac{1}{2}\operatorname{arcsinh}(cx)\right)\right)\cdots\right.\right.}{cf^2\sqrt{-((-id\cdots}}$$
$$- \frac{b^2d(-i+cx)\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\sqrt{-df\,(1+c^2x^2)}\left(-18\pi\operatorname{arcsinh}(cx)-(6-6i)\operatorname{arcsinh}(cx)^2+\cdots\right.}{\cdots}$$
$$- \frac{ib^2d(-i+cx)\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\sqrt{-df\,(1+c^2x^2)}\left(-\frac{6icx\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}+\frac{(6+6i)\operatorname{arcsinh}(cx)^2}{\sqrt{1+c^2x^2}}+\frac{2\operatorname{arc}\cdots}{\sqrt{\cdots}}\right.}{\cdots}$$

```
[In] Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),
x]

[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(((-I)*a^2*d)/f^2 + (4*a^2*d)/
(f^2*(I + c*x))))/c - (3*a^2*d^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d
*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(c*f^(3/2)) - ((2*I)*a*b*d*Sqrt[I*((-
I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[Ar
cSinh[c*x]/2]*(-(c*x) + 2*ArcSinh[c*x] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - I
*ArcSinh[c*x]^2 + 4*ArcTan[Coth[ArcSinh[c*x]/2]] - (2*I)*Log[Sqrt[1 + c^2*x
^2]]) - ((-I)*c*x - (2*I)*ArcSinh[c*x] + I*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] +
 ArcSinh[c*x]^2 + (4*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 2*Log[Sqrt[1 + c^2*x
^2]])*Sinh[ArcSinh[c*x]/2]))/(c*f^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]
*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) - (a*b*
d*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2
))]*(Cosh[ArcSinh[c*x]/2]*(8*ArcTan[Tanh[ArcSinh[c*x]/2]] + I*(ArcSinh[c*x]
*(4*I + ArcSinh[c*x]) + 4*Log[Sqrt[1 + c^2*x^2]])) + (ArcSinh[c*x]*(-4*I +
ArcSinh[c*x]) - (8*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 4*Log[Sqrt[1 + c^2*x^2
]])*Sinh[ArcSinh[c*x]/2]))/(c*f^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*S
qrt[1 + c^2*x^2]*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])) - (b^2*d*
(-I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1
+ c^2*x^2))]*(-18*Pi*ArcSinh[c*x] - (6 - 6*I)*ArcSinh[c*x]^2 + I*ArcSinh[c*
x]^3 - 12*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + 24*Pi*Log[1
 + E^ArcSinh[c*x]] + 12*Pi*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] - 24*Pi*L
og[Cosh[ArcSinh[c*x]/2]] - (24*I)*PolyLog[2, (-I)/E^ArcSinh[c*x]] - ((12*I)
*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSin
h[c*x]/2])))/(3*c*f^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*
x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2) - ((I/3)*b^2*d*(-I
+ c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^
2*x^2))]*(((-6*I)*c*x*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + ((6 + 6*I)*ArcSinh[
c*x]^2)/Sqrt[1 + c^2*x^2] + (2*ArcSinh[c*x]^3)/Sqrt[1 + c^2*x^2] + (3*I)*(2
 + ArcSinh[c*x]^2) + ((6*I)*(2*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSin
h[c*x]] + Pi*(3*ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c*x]] - 2*Log[-Cos[(Pi +
 (2*I)*ArcSinh[c*x])/4]] + 4*Log[Cosh[ArcSinh[c*x]/2]]) + (4*I)*PolyLog[2,
(-I)/E^ArcSinh[c*x]]))/Sqrt[1 + c^2*x^2] - (12*ArcSinh[c*x]^2*Sinh[ArcSinh[
c*x]/2])/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])
)))/(c*f^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] +
I*Sinh[ArcSinh[c*x]/2])^2)
```

Maple [F]

$$\int \frac{(icdx+d)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{2}}{\left(-icfx+f\right)^{\frac{3}{2}}}dx$$

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x)
```

Fricas [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\mathrm{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \int \frac{\left(i\,cdx+d\right)^{\frac{3}{2}}\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^{2}}{\left(-i\,cfx+f\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(((-I*b^2*c*d*x - b^2*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c
*x + sqrt(c^2*x^2 + 1))^2 - 2*(I*a*b*c*d*x + a*b*d)*sqrt(I*c*d*x + d)*sqrt(
-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a^2*c*d*x - a^2*d)*sqrt(I*
c*d*x + d)*sqrt(-I*c*f*x + f))/(c^2*f^2*x^2 + 2*I*c*f^2*x - f^2), x)
```

Sympy [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\mathrm{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \int \frac{\left(id\left(cx-i\right)\right)^{\frac{3}{2}}\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^{2}}{\left(-if\left(cx+i\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Integral((I*d*(c*x - I))**(3/2)*(a + b*asinh(c*x))**2/(-I*f*(c*x + I))**(3/
2), x)
```

Maxima [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\mathrm{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \int \frac{\left(i\,cdx+d\right)^{\frac{3}{2}}\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^{2}}{\left(-i\,cfx+f\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] a^2*(-I*(c^2*d*f*x^2 + d*f)^(3/2)/(c^3*f^3*x^2 + 2*I*c^2*f^3*x - c*f^3) - 6
*I*sqrt(c^2*d*f*x^2 + d*f)*d/(-I*c^2*f^2*x + c*f^2) - 3*d^2*arcsinh(c*x)/(c
*f^2*sqrt(d/f))) + integrate((I*c*d*x + d)^(3/2)*b^2*log(c*x + sqrt(c^2*x^2
 + 1))^2/(-I*c*f*x + f)^(3/2) + 2*(I*c*d*x + d)^(3/2)*a*b*log(c*x + sqrt(c^
2*x^2 + 1))/(-I*c*f*x + f)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to transpose Error: Bad Argume
nt ValueUnable to transpose Error: Bad Argument Valuesym2poly/r2sym(const g
en & e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d+c\,d\,x\,1\text{i})^{3/2}}{(f-c\,f\,x\,1\text{i})^{3/2}}\,dx$$

```
[In] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(3/2), x)
```

3.596 $\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 544

$$
\begin{aligned}
\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx =
&-\frac{2id^2(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2d^2x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&+\frac{2d^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{d^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&+\frac{8ibd^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&-\frac{4bd^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&+\frac{4b^2d^2(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&-\frac{4b^2d^2(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&-\frac{2b^2d^2(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}
$$

```
[Out] -2*I*d^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(
3/2)+2*d^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)
^(3/2)+2*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-
I*c*f*x)^(3/2)-1/3*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*
x)^(3/2)/(f-I*c*f*x)^(3/2)+8*I*b*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*a
rctan(c*x+(c^2*x^2+1)^(1/2))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*b*d^2*
(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I
```

```
*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*b^2*d^2*(c^2*x^2+1)^(3/2)*polylog(2,-I*(c
*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*b^2*d^2*(c^2
*x^2+1)^(3/2)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I
*c*f*x)^(3/2)-2*b^2*d^2*(c^2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2)
)^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {5796, 5844, 5838, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 5789, 4265, 5783}

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \frac{8ibd^2(c^2x^2+1)^{3/2}\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^2(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d^2(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2id^2(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d^2x(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4bd^2(c^2x^2+1)^{3/2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4b^2d^2(c^2x^2+1)^{3/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4b^2d^2(c^2x^2+1)^{3/2}\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2b^2d^2(c^2x^2+1)^{3/2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[In] Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),x]
```

```
[Out] ((-2*I)*d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f
 - I*c*f*x)^(3/2)) + (2*d^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I
*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (2*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcS
inh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (d^2*(1 + c^2*x^
2)^(3/2)*(a + b*ArcSinh[c*x])^3)/(3*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(
3/2)) + ((8*I)*b*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcS
inh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b*d^2*(1 + c^2*
x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*
x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*b^2*d^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, (
-I)*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b^2*d
^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)
*(f - I*c*f*x)^(3/2)) - (2*b^2*d^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*Arc
Sinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{(1+c^2x^2)^{3/2}\int \frac{(d+icdx)^2(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{(1+c^2x^2)^{3/2}\int \left(-\frac{2i(id^2-cd^2x)(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}-\frac{d^2(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(2i(1+c^2x^2)^{3/2}\right)\int \frac{(id^2-cd^2x)(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{\left(d^2(1+c^2x^2)^{3/2}\right)\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{d^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2i(1+c^2x^2)^{3/2}\right)\int \left(\frac{id^2(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}-\frac{cd^2x(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{d^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(2d^2(1+c^2x^2)^{3/2}\right)\int \frac{(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(2icd^2(1+c^2x^2)^{3/2}\right)\int \frac{x(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2id^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d^2x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(4ibd^2(1+c^2x^2)^{3/2}\right)\int \frac{a+b\text{arcsinh}(cx)}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(4bcd^2(1+c^2x^2)^{3/2}\right)\int \frac{x(a+b\text{arcsinh}(cx))}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2id^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d^2x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(4ibd^2(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int (a+bx)\text{sech}(x)\,dx, x, \text{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(4bd^2(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \text{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2id^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d^2x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{2d^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{8ibd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{\left(8bd^2(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \text{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{\left(4b^2d^2(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int \log(1-ie^x)\,dx, x, \text{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{\left(4b^2d^2(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int \log(1+ie^x)\,dx, x, \text{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2id^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d^2x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{2d^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{8ibd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{4bd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{\left(4b^2d^2(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int \log(1+e^{2x})\,dx, x, \text{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{\left(4b^2d^2(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{\left(4b^2d^2(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2id^2(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d^2x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8ibd^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4bd^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4b^2d^2(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4b^2d^2(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(2b^2d^2(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2id^2(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d^2x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8ibd^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4bd^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4b^2d^2(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4b^2d^2(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2b^2d^2(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Mathematica [A] (warning: unable to verify)

Time = 6.69 (sec) , antiderivative size = 530, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \frac{\frac{6a^2\sqrt{d+icdx}\sqrt{f-icfx}}{i+cx} - 3a^2\sqrt{d}\sqrt{f}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-ic}\right.}{}$$

```
[In] Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),x]
```

```
[Out] ((6*a^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(I + c*x) - 3*a^2*Sqrt[d]*Sqrt
[f]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - (b
^2*(-I + c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-18*Pi*ArcSinh[c*x] - (6
 - 6*I)*ArcSinh[c*x]^2 + I*ArcSinh[c*x]^3 - 12*(Pi - (2*I)*ArcSinh[c*x])*Lo
g[1 + I/E^ArcSinh[c*x]] + 24*Pi*Log[1 + E^ArcSinh[c*x]] + 12*Pi*Log[-Cos[(P
i + (2*I)*ArcSinh[c*x])/4]] - 24*Pi*Log[Cosh[ArcSinh[c*x]/2]] - (24*I)*Poly
Log[2, (-I)/E^ArcSinh[c*x]] - ((12*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/
(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 + c^2*x^2]*(Cosh[
ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2) + (3*a*b*Sqrt[d + I*c*d*x]*Sqr
t[f - I*c*f*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x
]/2])) + 4*ArcSinh[c*x]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])
+ 2*(4*ArcTan[Tanh[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c
*x]/2] + Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] -
 I*Sinh[ArcSinh[c*x]/2])))/(3*c*f^2)
```

Maple [F]

$$\int \frac{(a+b\,\text{arcsinh}\,(cx))^2\sqrt{icdx+d}}{(-icfx+f)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \int \frac{\sqrt{i\,cdx+d}(b\,\text{arsinh}\,(cx)+a)^2}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2
+ 1))^2 + 2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x^2
 + 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c^2*f^2*x^2 + 2*I*c*f^2
*x - f^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \int \frac{\sqrt{id\,(cx-i)}(a+b\,\text{asinh}\,(cx))^2}{(-if\,(cx+i))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Integral(sqrt(I*d*(c*x - I))*(a + b*asinh(c*x))**2/(-I*f*(c*x + I))**(3/2),
 x)
```

Maxima [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \int \frac{\sqrt{i\,cdx+d}(b\,\text{arsinh}\,(cx)+a)^2}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] a^2*(-2*I*sqrt(c^2*d*f*x^2 + d*f)/(-I*c^2*f^2*x + c*f^2) - d*arcsinh(c*x)/(
c*f^2*sqrt(d/f))) + integrate(sqrt(I*c*d*x + d)*b^2*log(c*x + sqrt(c^2*x^2
+ 1))^2/(-I*c*f*x + f)^(3/2) + 2*sqrt(I*c*d*x + d)*a*b*log(c*x + sqrt(c^2*x
^2 + 1))/(-I*c*f*x + f)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \int \frac{\sqrt{i\,cdx+d}(b\,\text{arsinh}\,(cx)+a)^2}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)^2/(-I*c*f*x + f)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,\sqrt{d+c\,d\,x\,1\text{i}}}{(f-c\,f\,x\,1\text{i})^{3/2}}\,dx$$

```
[In] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(3/2),x)
[Out] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(3/2), x)
```

3.597 $\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 464

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx = -\frac{id(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{dx(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{d(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4ibd(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2bd(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2b^2d(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2b^2d(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{b^2d(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] -I*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
+d*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+d
*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/
2)+4*I*b*d*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2
))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*b*d*(c^2*x^2+1)^(3/2)*(a+b*arcsi
nh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3
/2)+2*b^2*d*(c^2*x^2+1)^(3/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*
c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*b^2*d*(c^2*x^2+1)^(3/2)*polylog(2,I*(c*x+(
c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-b^2*d*(c^2*x^2+1)^
```

```
(3/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)
^(3/2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.297, Rules used = {5796, 5838, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 5789, 4265}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx = \frac{4ibd(c^2x^2+1)^{3/2}\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{d(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{id(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{dx(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2bd(c^2x^2+1)^{3/2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2b^2d(c^2x^2+1)^{3/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2b^2d(c^2x^2+1)^{3/2}\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{b^2d(c^2x^2+1)^{3/2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] ((-I)*d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I
*c*f*x)^(3/2)) + (d*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*c*d*x)^
(3/2)*(f - I*c*f*x)^(3/2)) + (d*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)
/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((4*I)*b*d*(1 + c^2*x^2)^(3/
2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f -
 I*c*f*x)^(3/2)) - (2*b*d*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log[1 +
E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (2*b^2*d
*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2
)*(f - I*c*f*x)^(3/2)) - (2*b^2*d*(1 + c^2*x^2)^(3/2)*PolyLog[2, I*E^ArcSin
h[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (b^2*d*(1 + c^2*x^2)
^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x
)^(3/2))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
```

```
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(1+c^2x^2)^{3/2}\int \frac{(d+icdx)(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&= \frac{(1+c^2x^2)^{3/2}\int \left(\frac{d(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}+\frac{icdx(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&= \frac{\left(d(1+c^2x^2)^{3/2}\right)\int \frac{(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{\left(icd(1+c^2x^2)^{3/2}\right)\int \frac{x(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&= -\frac{id(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{dx(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad+\frac{\left(2ibd(1+c^2x^2)^{3/2}\right)\int \frac{a+b\text{arcsinh}(cx)}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&\quad-\frac{\left(2bcd(1+c^2x^2)^{3/2}\right)\int \frac{x(a+b\text{arcsinh}(cx))}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}$$

$$= -\frac{id(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{dx(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(2ibd(1+c^2x^2)^{3/2}\right)\text{Subst}(\int(a+bx)\text{sech}(x)\,dx,x,\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2bd(1+c^2x^2)^{3/2}\right)\text{Subst}(\int(a+bx)\tanh(x)\,dx,x,\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{id(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{dx(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{d(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4ibd(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(4bd(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int\frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx,x,\text{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(2b^2d(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int\log\left(1-ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2b^2d(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int\log\left(1+ie^x\right)\,dx,x,\text{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{id(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{dx(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{d(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4ibd(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2bd(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(2b^2d(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int\log\left(1+e^{2x}\right)\,dx,x,\text{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(2b^2d(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2b^2d(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{id(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{dx(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{d(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{4ibd(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{2bd(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{2b^2d(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{2b^2d(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{\left(b^2d(1+c^2x^2)^{3/2}\right)\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{id(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{dx(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{d(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{4ibd(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{2bd(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+ \frac{2b^2d(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{2b^2d(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$- \frac{b^2d(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Mathematica [A] (verified)

Time = 3.16 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx = \frac{\sqrt{d+icdx}\sqrt{f-icfx}\left((-1-i)b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2\left(\cosh\left(\frac{1}{2}\text{arcsinh}(cx\right.\right.\right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] (Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((-1 - I)*b^2*Sqrt[1 + c^2*x^2]*ArcSin
h[c*x]^2*(Cosh[ArcSinh[c*x]/2] - Sinh[ArcSinh[c*x]/2]) + ((-I)*a^2 + a^2*c*
x + (4*I)*a*b*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] - (2*I)*b^2*Pi
*Sqrt[1 + c^2*x^2]*Log[1 + I/E^ArcSinh[c*x]] + (4*I)*b^2*Pi*Sqrt[1 + c^2*x^
2]*Log[1 + E^ArcSinh[c*x]] - a*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] + (2*I)
*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] - (4*I)*b^
2*Pi*Sqrt[1 + c^2*x^2]*Log[Cosh[ArcSinh[c*x]/2]])*(Cosh[ArcSinh[c*x]/2] - I
*Sinh[ArcSinh[c*x]/2]) + 4*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[
c*x]]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]) + b*Sqrt[1 + c^2*x^2]
*ArcSinh[c*x]*((-I)*Cosh[ArcSinh[c*x]/2]*(2*a + 3*b*Pi - (4*I)*b*Log[1 + I/
E^ArcSinh[c*x]]) + (2*a - 3*b*Pi + (4*I)*b*Log[1 + I/E^ArcSinh[c*x]])*Sinh[
ArcSinh[c*x]/2])))/(c*d*f^2*(-I + c*x)*(I + c*x)*(Cosh[ArcSinh[c*x]/2] - I*
Sinh[ArcSinh[c*x]/2]))
```

Maple [F]

$$\int \frac{(a+b\,\text{arcsinh}\,(cx))^2}{(-icfx+f)^{\frac{3}{2}}\sqrt{icdx+d}}dx$$

```
[In] int((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] (sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2 +
(c^2*d*f^2*x + I*c*d*f^2)*integral((I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*
a^2 - 2*(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2 - I*sqr
t(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b)*log(c*x + sqrt(c^2*x^2 + 1)))/(c^3*d
*f^2*x^3 + I*c^2*d*f^2*x^2 + c*d*f^2*x + I*d*f^2), x))/(c^2*d*f^2*x + I*c*d
*f^2)
```

Sympy [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx = \int \frac{(a+b\operatorname{asinh}(cx))^2}{\sqrt{id(cx-i)}\,(-if(cx+i))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2/(f-I*c*f*x)**(3/2)/(d+I*c*d*x)**(1/2),x)
[Out] Integral((a + b*asinh(c*x))**2/(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(3/2)
), x)
```

Maxima [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="maxima")
[Out] b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/(sqrt(I*c*d*x + d)*(-I*c*f*x +
 f)^(3/2)), x) - 2*I*sqrt(c^2*d*f*x^2 + d*f)*a*b*arcsinh(c*x)/(-I*c^2*d*f^2
*x + c*d*f^2) - I*sqrt(c^2*d*f*x^2 + d*f)*a^2/(-I*c^2*d*f^2*x + c*d*f^2) -
2*a*b*log(I*c*x - 1)/(c*sqrt(d)*f^(3/2))
```

Giac [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/(sqrt(I*c*d*x + d)*(-I*c*f*x + f)^(3/2)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx=\int \frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)^2}{\sqrt{d+c\,d\,x\,1\mathrm{i}}\,{\left(f-c\,f\,x\,1\mathrm{i}\right)}^{3/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2)),x)
[Out] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2)), x)
```

3.598 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 224

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx &= \frac{x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} \\ &+ \frac{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\ &- \frac{2b(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\ &- \frac{b^2(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}\end{aligned}$$

```
[Out] x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+(c^2
*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*
b*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d
+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-b^2*(c^2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^
2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used

$= \{5796, 5787, 5797, 3799, 2221, 2317, 2438\}$

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx = \frac{(c^2x^2+1)^{3/2}\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{x(c^2x^2+1)\,(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2b(c^2x^2+1)^{3/2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{b^2(c^2x^2+1)^{3/2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] (x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)) + ((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/
2)*(f - I*c*f*x)^(3/2)) - (2*b*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log
[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (b^
2*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/
2)*(f - I*c*f*x)^(3/2))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{(1+c^2x^2)^{3/2}\int \frac{(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2bc(1+c^2x^2)^{3/2}\right)\int \frac{x(a+b\text{arcsinh}(cx))}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2b(1+c^2x^2)^{3/2}\right)\text{Subst}(\int (a+bx)\tanh(x)\,dx, x, \text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(4b(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \text{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{x(1+c^2x^2)\left(a+b\text{arcsinh}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{(1+c^2x^2)^{3/2}\left(a+b\text{arcsinh}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2b(1+c^2x^2)^{3/2}\left(a+b\text{arcsinh}(cx)\right)\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(2b^2(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int\log\left(1+e^{2x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{x(1+c^2x^2)\left(a+b\text{arcsinh}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{(1+c^2x^2)^{3/2}\left(a+b\text{arcsinh}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2b(1+c^2x^2)^{3/2}\left(a+b\text{arcsinh}(cx)\right)\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(b^2(1+c^2x^2)^{3/2}\right)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx, x, e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{x(1+c^2x^2)\left(a+b\text{arcsinh}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{(1+c^2x^2)^{3/2}\left(a+b\text{arcsinh}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2b(1+c^2x^2)^{3/2}\left(a+b\text{arcsinh}(cx)\right)\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{b^2(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 488 vs. $2(224) = 448$.

Time = 3.05 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.18

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx = \frac{a^2cx + 2abcx\text{arcsinh}(cx) - 2ib^2\pi\sqrt{1+c^2x^2}\text{arcsinh}(cx) + b^2cx\text{arcsinh}(cx)}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)),
x]
```

```
[Out] (a^2*c*x + 2*a*b*c*x*ArcSinh[c*x] - (2*I)*b^2*Pi*Sqrt[1 + c^2*x^2]*ArcSinh[
c*x] + b^2*c*x*ArcSinh[c*x]^2 - b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 + I*b^
2*Pi*Sqrt[1 + c^2*x^2]*Log[1 - I/E^ArcSinh[c*x]] - 2*b^2*Sqrt[1 + c^2*x^2]*
ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] - I*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[1 +
 I/E^ArcSinh[c*x]] - 2*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + I/E^ArcSi
nh[c*x]] + (4*I)*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[1 + E^ArcSinh[c*x]] - a*b*Sqr
t[1 + c^2*x^2]*Log[1 + c^2*x^2] + I*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[-Cos[(Pi +
 (2*I)*ArcSinh[c*x])/4]] - (4*I)*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[Cosh[ArcSinh[
```

```
c*x]/2]] - I*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]]
 + 2*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[c*x]] + 2*b^2*Sqrt[1 +
 c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]])/(c*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x])
```

Maple [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(icdx+d)^{\frac{3}{2}}(-icfx+f)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx=\int \frac{(b\,\operatorname{arsinh}(cx)+a)^2}{(i\,cdx+d)^{\frac{3}{2}}(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] (sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*x*log(c*x + sqrt(c^2*x^2 + 1))^2
+ (c^2*d^2*f^2*x^2 + d^2*f^2)*integral((sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f
)*a^2 - 2*(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*c*x -
 sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b)*log(c*x + sqrt(c^2*x^2 + 1)))/(c
^4*d^2*f^2*x^4 + 2*c^2*d^2*f^2*x^2 + d^2*f^2), x))/(c^2*d^2*f^2*x^2 + d^2*f
^2)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx=\int \frac{(a+b\,\operatorname{asinh}(cx))^2}{(id\,(cx-i))^{\frac{3}{2}}\,(-if\,(cx+i))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/((I*d*(c*x - I))**(3/2)*(-I*f*(c*x + I))**(3
/2)), x)
```

Maxima [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{3/2}(f - icfx)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(i\,cdx + d)^{\frac{3}{2}}(-i\,cfx + f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/((I*c*d*x + d)^(3/2)*(-I*c*f*x
 + f)^(3/2)), x) + 2*a*b*x*arcsinh(c*x)/(sqrt(c^2*d*f*x^2 + d*f)*d*f) + a^2
*x/(sqrt(c^2*d*f*x^2 + d*f)*d*f) - a*b*sqrt(1/(d*f))*log(x^2 + 1/c^2)/(c*d*
f)
```

Giac [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{3/2}(f - icfx)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(i\,cdx + d)^{\frac{3}{2}}(-i\,cfx + f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((I*c*d*x + d)^(3/2)*(-I*c*f*x + f)^(3/2))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{3/2}(f - icfx)^{3/2}}\,dx = \int \frac{(a + b\,\text{asinh}(c\,x))^2}{(d + c\,d\,x\,1\text{i})^{3/2}\,(f - c\,f\,x\,1\text{i})^{3/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2)), x)
```

3.599 $\int \frac{(a+b\,\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 743

$$
\begin{aligned}
\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx &= -\frac{ib^2f(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{b^2fx(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{ibfx(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{if(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{fx(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2fx(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2f(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2ibf(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{4bf(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{b^2f(1+c^2x^2)^{5/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{b^2f(1+c^2x^2)^{5/2}\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2b^2f(1+c^2x^2)^{5/2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

```
[Out] -1/3*I*b^2*f*(c^2*x^2+1)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b^2*f*
x*(c^2*x^2+1)^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*b*f*(c^2*x^2+1)^(3/
```

```
2)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*I*b*f*x*(c^
2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*I
*f*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1
/3*f*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
+2/3*f*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(
5/2)+2/3*f*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*
c*f*x)^(5/2)-2/3*I*b*f*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2
*x^2+1)^(1/2))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-4/3*b*f*(c^2*x^2+1)^(5
/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(5/2)/
(f-I*c*f*x)^(5/2)-1/3*b^2*f*(c^2*x^2+1)^(5/2)*polylog(2,-I*(c*x+(c^2*x^2+1)
^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*b^2*f*(c^2*x^2+1)^(5/2)*
polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-
2/3*b^2*f*(c^2*x^2+1)^(5/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*
d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 743, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.378$, Rules used = {5796, 5838, 5788, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 197, 5789, 4265, 267}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx =$$
$$-\frac{2ibf(c^2x^2+1)^{5/2}\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{2f(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2fx(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{bf(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{ibfx(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{if(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{fx(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{4bf(c^2x^2+1)^{5/2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{b^2f(c^2x^2+1)^{5/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{b^2f(c^2x^2+1)^{5/2}\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{2b^2f(c^2x^2+1)^{5/2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{ib^2f(c^2x^2+1)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{b^2fx(c^2x^2+1)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] ((-1/3*I)*b^2*f*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)
) - (b^2*f*x*(1 + c^2*x^2)^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) +
 (b*f*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c*(d + I*c*d*x)^(5/2)*(f
 - I*c*f*x)^(5/2)) - ((I/3)*b*f*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))
/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((I/3)*f*(1 + c^2*x^2)*(a + b*
ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (f*x*(1 + c^
2*x^2)*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
+ (2*f*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f
- I*c*f*x)^(5/2)) + (2*f*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*c*(
d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((2*I)/3)*b*f*(1 + c^2*x^2)^(5/2
)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f -
I*c*f*x)^(5/2)) - (4*b*f*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log[1 + E
^(2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (b^2*f*
(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/
2)*(f - I*c*f*x)^(5/2)) + (b^2*f*(1 + c^2*x^2)^(5/2)*PolyLog[2, I*E^ArcSinh
[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b^2*f*(1 + c^2*x
^2)^(5/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*
c*f*x)^(5/2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
```

```
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(1+c^2x^2)^{5/2}\int \frac{(f-icfx)(a+b\operatorname{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&= \frac{(1+c^2x^2)^{5/2}\int \left(\frac{f(a+b\operatorname{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}-\frac{icfx(a+b\operatorname{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&= \frac{\left(f(1+c^2x^2)^{5/2}\right)\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{\left(icf(1+c^2x^2)^{5/2}\right)\int \frac{x(a+b\operatorname{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

$$= \frac{if(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{fx(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2f(1+c^2x^2)^{5/2}\right)\int \frac{(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2ibf(1+c^2x^2)^{5/2}\right)\int \frac{a+b\text{arcsinh}(cx)}{(1+c^2x^2)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2bcf(1+c^2x^2)^{5/2}\right)\int \frac{x(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{bf(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{ibfx(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{if(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{fx(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2fx(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(ibf(1+c^2x^2)^{5/2}\right)\int \frac{a+b\text{arcsinh}(cx)}{1+c^2x^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(b^2f(1+c^2x^2)^{5/2}\right)\int \frac{1}{(1+c^2x^2)^{3/2}}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(4bcf(1+c^2x^2)^{5/2}\right)\int \frac{x(a+b\text{arcsinh}(cx))}{1+c^2x^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(ib^2cf(1+c^2x^2)^{5/2}\right)\int \frac{x}{(1+c^2x^2)^{3/2}}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{ib^2f(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2fx(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{ibfx(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{if(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{fx(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2fx(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(ibf(1+c^2x^2)^{5/2}\right)\text{Subst}(\int (a+bx)\text{sech}(x)\,dx, x, \text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(4bf(1+c^2x^2)^{5/2}\right)\text{Subst}(\int (a+bx)\tanh(x)\,dx, x, \text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{ib^2f(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2fx(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{ibfx(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{if(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{fx(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2fx(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2f(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ibf(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(8bf(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(b^2f(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int \log\left(1-ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(b^2f(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int \log\left(1+ie^x\right)\,dx, x, \text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{ib^2f(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2fx(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{ibfx(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{if(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{fx(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2fx(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2f(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ibf(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{4bf(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(b^2f(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(b^2f(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(4b^2f(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{ib^2f(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2fx(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{ibfx(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{if(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{fx(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2fx(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2f(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ibf(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{4bf(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2f(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2f(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2b^2f(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{ib^2f(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2fx(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{ibfx(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{if(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{fx(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2fx(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2f(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ibf(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{4bf(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2f(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2f(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2f(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Mathematica [A] (warning: unable to verify)

Time = 9.91 (sec) , antiderivative size = 754, normalized size of antiderivative = 1.01

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx = \frac{\sqrt{id(-i+cx)}\sqrt{-if(i+cx)}\left(-\frac{ia^2}{6d^3f^2(-i+cx)^2}+\frac{5a^2}{12d^3f^2(-i+cx)}+\frac{a^2}{4d^3f^2(i+cx)}\right.}{c}$$
$$+\frac{iab\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\left(4cx\text{arcsinh}(cx)+2i\text{arcsinh}(cx)\cosh(2\text{arcsinh}(cx))+\sqrt{1+c^2x^2}(1-2\right.}{3cd^2f(-i+cx)\sqrt{-((-id+}$$
$$+\frac{ib^2\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\sqrt{1+c^2x^2}\left(7\pi\text{arcsinh}(cx)+\frac{(2+i\text{arcsinh}(cx))\text{arcsinh}(cx)}{-i+cx}-(1+4i)\text{arcsin}\right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)),
x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(((-1/6*I)*a^2)/(d^3*f^2*(-I +
 c*x)^2) + (5*a^2)/(12*d^3*f^2*(-I + c*x)) + a^2/(4*d^3*f^2*(I + c*x))))/c
+ ((I/3)*a*b*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*(4*c*x*ArcSi
nh[c*x] + (2*I)*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]] + Sqrt[1 + c^2*x^2]*(1 -
(2*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 2*c*x*(ArcTan[Tanh[ArcSinh[c*x]/2]] -
(2*I)*Log[Sqrt[1 + c^2*x^2]]) - 4*Log[Sqrt[1 + c^2*x^2]])))/(c*d^2*f*(-I +
c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[-(d*f*(1 + c^2*x^2))]) +
((I/6)*b^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[1 + c^2*x
^2]*(7*Pi*ArcSinh[c*x] + ((2 + I*ArcSinh[c*x])*ArcSinh[c*x])/(-I + c*x) - (
1 + 4*I)*ArcSinh[c*x]^2 - 5*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c
*x]] + 3*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] - 16*Pi*Log[1
+ E^ArcSinh[c*x]] - 3*Pi*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 16*Pi*Log
[Cosh[ArcSinh[c*x]/2]] + 5*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]] + (6*I)
*PolyLog[2, (-I)/E^ArcSinh[c*x]] + (10*I)*PolyLog[2, I/E^ArcSinh[c*x]] + ((
3*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[Ar
cSinh[c*x]/2]) + ((2*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[
c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^3 + ((-4 + 5*ArcSinh[c*x]^2)*Sinh[ArcSinh
[c*x]/2])/((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])))/(c*d^2*f*Sqr
t[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[-(d*f*(1 + c^2*x^2))])
```

Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{(icdx+d)^{\frac{5}{2}}\,(-icfx+f)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{5}{2}}(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*((2*b^2*c^2*x^2 - 2*I*b^2*c*x + b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 3*(c^4*d^3*f^2*x^3 - I*c^3*d^3*f^2*x^2
+ c^2*d^3*f^2*x - I*c*d^3*f^2)*integral(1/3*(-3*I*sqrt(I*c*d*x + d)*sqrt(-I
*c*f*x + f)*a^2 - 2*(3*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b + (2*b^2*
c^2*x^2 - 2*I*b^2*c*x + b^2)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*
f*x + f))*log(c*x + sqrt(c^2*x^2 + 1)))/(c^5*d^3*f^2*x^5 - I*c^4*d^3*f^2*x^
4 + 2*c^3*d^3*f^2*x^3 - 2*I*c^2*d^3*f^2*x^2 + c*d^3*f^2*x - I*d^3*f^2), x))
/(c^4*d^3*f^2*x^3 - I*c^3*d^3*f^2*x^2 + c^2*d^3*f^2*x - I*c*d^3*f^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{5/2}(f - icfx)^{3/2}}\, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{5/2}(f - icfx)^{3/2}}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{5/2}(f - icfx)^{3/2}}\, dx = \int \frac{(a + b\,\text{asinh}(c\,x))^2}{(d + c\,d\,x\,1\text{i})^{5/2}\,(f - c\,f\,x\,1\text{i})^{3/2}}\, dx$$

```
[In] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2)), x)
```

3.600 $\int \frac{(d+icdx)^{5/2}(a+b\,\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 794

$$
\begin{aligned}
&\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \\
&-\frac{2iabd^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2ib^2d^5(1+c^2x^2)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&-\frac{2ib^2d^5x(1+c^2x^2)^{5/2}\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{28d^5(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+\frac{id^5(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{5d^5(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+\frac{112bd^5(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\log\left(1+ie^{-\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&-\frac{112b^2d^5(1+c^2x^2)^{5/2}\text{PolyLog}\left(2,-ie^{-\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+\frac{8bd^5(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+\frac{16ib^2d^5(1+c^2x^2)^{5/2}\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+\frac{28id^5(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&-\frac{4id^5(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

```
[Out] -2*I*a*b*d^5*x*(c^2*x^2+1)^(5/2)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2*I*b^
2*d^5*(c^2*x^2+1)^3/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2*I*b^2*d^5*x*(c^
```

```
2*x^2+1)^(5/2)*arcsinh(c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+28/3*d^5*(c
^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+
I*d^5*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5
/2)+5/3*d^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(5/2)/(f
-I*c*f*x)^(5/2)+112/3*b*d^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln(1+I/(c*
x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-112/3*b^2*d^5*(
c^2*x^2+1)^(5/2)*polylog(2,-I/(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/
(f-I*c*f*x)^(5/2)+8/3*b*d^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*sec(1/4*Pi
+1/2*I*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+16/3*I*b^2*d^5
*(c^2*x^2+1)^(5/2)*tan(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*
c*f*x)^(5/2)+28/3*I*d^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2*tan(1/4*Pi+1
/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-4/3*I*d^5*(c^2*x^2
+1)^(5/2)*(a+b*arcsinh(c*x))^2*sec(1/4*Pi+1/2*I*arcsinh(c*x))^2*tan(1/4*Pi+
1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 794, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.432, Rules used = {5796, 5844, 5783, 5798, 5772, 267, 5843, 3399, 4271, 3852, 8, 4269, 3797, 2221, 2317, 2438}

$$\begin{aligned}
\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx &= \frac{5d^5(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{id^5(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{28d^5(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{112bd^5(c^2x^2+1)^{5/2}\log\left(1+ie^{-\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{28id^5(c^2x^2+1)^{5/2}\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{8bd^5(c^2x^2+1)^{5/2}\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{4id^5(c^2x^2+1)^{5/2}\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2iabd^5x(c^2x^2+1)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{112b^2d^5(c^2x^2+1)^{5/2}\text{PolyLog}\left(2,-ie^{-\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2ib^2d^5x(c^2x^2+1)^{5/2}\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{16ib^2d^5(c^2x^2+1)^{5/2}\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2d^5(c^2x^2+1)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}$$

```
[In] Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),x]
```

```
[Out] ((-2*I)*a*b*d^5*x*(1 + c^2*x^2)^(5/2))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(
5/2)) + ((2*I)*b^2*d^5*(1 + c^2*x^2)^3)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x
)^(5/2)) - ((2*I)*b^2*d^5*x*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x])/((d + I*c*d*x
)^(5/2)*(f - I*c*f*x)^(5/2)) + (28*d^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c
*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (I*d^5*(1 + c^2*x^2
)^3*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (
5*d^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^3)/(3*b*c*(d + I*c*d*x)^(5/2
)*(f - I*c*f*x)^(5/2)) + (112*b*d^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]
)*Log[1 + I/E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
- (112*b^2*d^5*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)/E^ArcSinh[c*x]])/(3*c*(d
 + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (8*b*d^5*(1 + c^2*x^2)^(5/2)*(a +
b*ArcSinh[c*x])*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c*(d + I*c*d*x)^(5/2)*
(f - I*c*f*x)^(5/2)) + (((16*I)/3)*b^2*d^5*(1 + c^2*x^2)^(5/2)*Tan[Pi/4 + (
I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((28*I)/
3)*d^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Tan[Pi/4 + (I/2)*ArcSinh[
c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((4*I)/3)*d^5*(1 + c^
2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2*Tan[Pi
/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{(1+c^2x^2)^{5/2}\int\frac{(d+icdx)^5(a+b\operatorname{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{(1+c^2x^2)^{5/2}\int\left(\frac{5d^5(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}+\frac{icd^5x(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}-\frac{8d^5(a+b\operatorname{arcsinh}(cx))^2}{(i+cx)^2\sqrt{1+c^2x^2}}-\frac{12id^5(a+b\operatorname{arcsinh}(\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{\left(12id^5(1+c^2x^2)^{5/2}\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{(i+cx)\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(5d^5(1+c^2x^2)^{5/2}\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(8d^5(1+c^2x^2)^{5/2}\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{(i+cx)^2\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(icd^5(1+c^2x^2)^{5/2}\right)\int\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{id^5(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5d^5(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(12id^5(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int\frac{(a+bx)^2}{ic+c\sinh(x)}\,dx, x, \text{arcsinh}(cx)\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2ibd^5(1+c^2x^2)^{5/2}\right)\int(a+b\text{arcsinh}(cx))\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(8cd^5(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int\frac{(a+bx)^2}{(ic+c\sinh(x))^2}\,dx, x, \text{arcsinh}(cx)\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{2iabd^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{id^5(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5d^5(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2ib^2d^5(1+c^2x^2)^{5/2}\right)\int\text{arcsinh}(cx)\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2d^5(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int(a+bx)^2\csc^4\left(\frac{\pi}{4}-\frac{ix}{2}\right)\,dx, x, \text{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(6d^5(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int(a+bx)^2\csc^2\left(\frac{\pi}{4}-\frac{ix}{2}\right)\,dx, x, \text{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$
\begin{aligned}
&= -\frac{2iabd^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2d^5x(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{id^5(1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5d^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{8bd^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{12id^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{4id^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{\left(4d^5(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int(a+bx)^2\csc^2\left(\frac{\pi}{4}-\frac{ix}{2}\right)dx,x,\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{\left(24ibd^5(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int(a+bx)\cot\left(\frac{\pi}{4}-\frac{ix}{2}\right)dx,x,\operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{\left(8b^2d^5(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\csc^2\left(\frac{\pi}{4}-\frac{ix}{2}\right)dx,x,\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{\left(2ib^2cd^5(1+c^2x^2)^{5/2}\right)\int\frac{x}{\sqrt{1+c^2x^2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

$$= -\frac{2iabd^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2d^5(1+c^2x^2)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{2ib^2d^5x(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{12d^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{id^5(1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5d^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{8bd^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{28id^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{4id^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{\left(16ibd^5(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int(a+bx)\cot\left(\frac{\pi}{4}-\frac{ix}{2}\right)dx, x, \operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{\left(48ibd^5(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\frac{e^{-x}(a+bx)}{1+ie^{-x}}dx, x, \operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{\left(16ib^2d^5(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int 1\,dx, x, \cot\left(\frac{\pi}{4}-\frac{1}{2}i\operatorname{arcsinh}(cx)\right)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$
\begin{aligned}
&= -\frac{2iabd^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2d^5(1+c^2x^2)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{2ib^2d^5x(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{28d^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{id^5(1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5d^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{16ib^2d^5(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}-\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{48bd^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+ie^{-\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{8bd^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{28id^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{4id^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{\left(32ibd^5(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\frac{e^{-x}(a+bx)}{1+ie^{-x}}\,dx,x,\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{\left(48b^2d^5(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\log\left(1+ie^{-x}\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

$$= -\frac{2iabd^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2d^5(1+c^2x^2)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$- \frac{2ib^2d^5x(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{28d^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$+ \frac{id^5(1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5d^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$+ \frac{16ib^2d^5(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}-\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$+ \frac{112bd^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+ie^{-\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$+ \frac{8bd^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$+ \frac{28id^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$- \frac{4id^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$+ \frac{\left(32b^2d^5(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\log\left(1+ie^{-x}\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$+ \frac{\left(48b^2d^5(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{-\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{2iabd^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2d^5(1+c^2x^2)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{2ib^2d^5x(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{28d^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{id^5(1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5d^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{16ib^2d^5(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}-\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{112bd^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+ie^{-\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{48b^2d^5(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-ie^{-\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{8bd^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{28id^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{4id^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{\left(32b^2d^5(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{-\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$
\begin{aligned}
&= -\frac{2iabd^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2d^5(1+c^2x^2)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{2ib^2d^5x(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{28d^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{id^5(1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5d^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{16ib^2d^5(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}-\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{112bd^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+ie^{-\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{112b^2d^5(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-ie^{-\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{8bd^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{28id^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{4id^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2552 vs. 2(794) = 1588.

Time = 24.35 (sec) , antiderivative size = 2552, normalized size of antiderivative = 3.21

$$
\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),
x]

[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((I*a^2*d^2)/f^3 + (((8*I)/3)*
a^2*d^2)/(f^3*(I + c*x)^2) - (28*a^2*d^2)/(3*f^3*(I + c*x))))/c + (5*a^2*d^
(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c
*x)]])/(c*f^(5/2)) - ((I/3)*a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f
 + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSin
h[c*x]/2])*(-(Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSin
h[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]])) + Cosh[ArcSinh[c*x]/2]*(4*I + 3*Arc
Sinh[c*x] - 6*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*I)*Log[Sqrt[1 + c^2*x^2]])
+ 2*(Sqrt[1 + c^2*x^2]*(I*ArcSinh[c*x] + (2*I)*ArcTan[Coth[ArcSinh[c*x]/2]]
 + Log[Sqrt[1 + c^2*x^2]]) + 2*(1 + I*ArcSinh[c*x] + (2*I)*ArcTan[Coth[ArcS
```

inh[c*x]/2]] + Log[Sqrt[1 + c^2*x^2]]])*Sinh[ArcSinh[c*x]/2]))/(c*f^3*(1 + I*c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^4) + (a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*ArcSinh[c*x])/2]*((14*I - 3*ArcSinh[c*x])*ArcSinh[c*x] + (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] - 14*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/2]*(8 + (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 - (84*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 42*Log[Sqrt[1 + c^2*x^2]]) - (2*I)*(4 + (4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 - (56*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 28*Log[Sqrt[1 + c^2*x^2]] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(14*I + 3*ArcSinh[c*x]) - (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 14*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(3*c*f^3*(1 + I*c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^4) - ((I/3)*b^2*d^2*(-I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*((-1 - I)*ArcSinh[c*x]^2 - (2*ArcSinh[c*x]*(2*I + ArcSinh[c*x]))/(I + c*x) - (2*I)*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] - I*Pi*(3*ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 4*Log[Cosh[ArcSinh[c*x]/2]]) + 4*PolyLog[2, (-I)/E^ArcSinh[c*x]] - (4*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^3 + (2*(4 + ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(c*f^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2) + ((I/3)*b^2*d^2*(-I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(((-6*I)*c*x*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + ((13 + 13*I)*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] + (3*ArcSinh[c*x]^3)/Sqrt[1 + c^2*x^2] + (2*ArcSinh[c*x]*(2*I + ArcSinh[c*x]))/((I + c*x)*Sqrt[1 + c^2*x^2]) + (3*I)*(2 + ArcSinh[c*x]^2) + ((13*I)*(2*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + Pi*(3*ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 4*Log[Cosh[ArcSinh[c*x]/2]]) + (4*I)*PolyLog[2, (-I)/E^ArcSinh[c*x]]))/Sqrt[1 + c^2*x^2] + (4*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^3) - (2*(4 + 13*ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]))))/(c*f^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2) + (2*b^2*d^2*(-I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(-21*Pi*ArcSinh[c*x] - (7 - 7*I)*ArcSinh[c*x]^2 + I*ArcSinh[c*x]^3 + ((2*I)*ArcSinh[c*x]*(2*I + ArcSinh[c*x]))/(I + c*x) - 14*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + 28*Pi*Log[1 + E^ArcSinh[c*x]] + 14*Pi*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] - 28*Pi*Log[Cosh[ArcSinh[c*x]/2]] - (28*I)*PolyLog[2, (-I)/E^ArcSinh[c*x]] - ((2*I)*(4 + 7*ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]) + (4*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])^3))/(3*c*f^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2) - ((I/6)*a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I

```
*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcS
inh[c*x]/2])*(-(Cosh[(3*ArcSinh[c*x])/2]*(9 - (35*I)*ArcSinh[c*x] + 9*ArcSi
nh[c*x]^2 + (52*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 26*Log[Sqrt[1 + c^2*x^2]]
)) + Cosh[ArcSinh[c*x]/2]*(20 + (24*I)*ArcSinh[c*x] + 27*ArcSinh[c*x]^2 + (
156*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 78*Log[Sqrt[1 + c^2*x^2]]) - I*(3*(-I
 + ArcSinh[c*x])*Cosh[(5*ArcSinh[c*x])/2] + 2*(13 + (7*I)*ArcSinh[c*x] + 18
*ArcSinh[c*x]^2 + (104*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*I)*(I + ArcSinh
[c*x])*Cosh[2*ArcSinh[c*x]] + 52*Log[Sqrt[1 + c^2*x^2]] + Sqrt[1 + c^2*x^2]
*(6 + (38*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 + (52*I)*ArcTan[Coth[ArcSinh[c
*x]/2]] + 26*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2])))/(c*f^3*(-I +
c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh
[ArcSinh[c*x]/2])^4)
```

Maple [F]

$$\int \frac{(icdx+d)^{\frac{5}{2}} (a+b\,\text{arcsinh}\,(cx))^2}{(-icfx+f)^{\frac{5}{2}}}dx$$

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x)
```

Fricas [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \int \frac{(i\,cdx+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral(((I*b^2*c^2*d^2*x^2 + 2*b^2*c*d^2*x - I*b^2*d^2)*sqrt(I*c*d*x + d)
*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 2*(-I*a*b*c^2*d^2*x^2
- 2*a*b*c*d^2*x + I*a*b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) + (I*a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x - I*a^2*d^2)*sqrt(
I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*f^3*x^3 + 3*I*c^2*f^3*x^2 - 3*c*f^3*x
 - I*f^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(5/2),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algor
ithm="maxima")
[Out] Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algor
ithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d+c\,d\,x\,1\text{i})^{5/2}}{(f-c\,f\,x\,1\text{i})^{5/2}}\,dx$$

```
[In] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(5/2),x)
[Out] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(5/2), x)
```

3.601 $\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 584

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \frac{8d^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{d^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{32bd^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\log\left(1+ie^{-\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{32b^2d^4(1+c^2x^2)^{5/2}\text{PolyLog}\left(2,-ie^{-\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{4bd^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{8ib^2d^4(1+c^2x^2)^{5/2}\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{8id^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{2id^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[Out] 8/3*d^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f
*x)^(5/2)+1/3*d^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(5
/2)/(f-I*c*f*x)^(5/2)+32/3*b*d^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln(1+
I/(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-32/3*b^2*d
^4*(c^2*x^2+1)^(5/2)*polylog(2,-I/(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5
/2)/(f-I*c*f*x)^(5/2)+4/3*b*d^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*sec(1/
```

```
4*Pi+1/2*I*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+8/3*I*b^2*
d^4*(c^2*x^2+1)^(5/2)*tan(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f
-I*c*f*x)^(5/2)+8/3*I*d^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2*tan(1/4*Pi
+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*I*d^4*(c^2*x
^2+1)^(5/2)*(a+b*arcsinh(c*x))^2*sec(1/4*Pi+1/2*I*arcsinh(c*x))^2*tan(1/4*P
i+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {5796, 5844, 5783, 5843, 3399, 4271, 3852, 8, 4269, 3797, 2221, 2317, 2438}

$$\begin{aligned}
&\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \frac{d^4(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&+\frac{8d^4(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&+\frac{32bd^4(c^2x^2+1)^{5/2}\log\left(1+ie^{-\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&+\frac{8id^4(c^2x^2+1)^{5/2}\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&+\frac{4bd^4(c^2x^2+1)^{5/2}\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&-\frac{2id^4(c^2x^2+1)^{5/2}\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&-\frac{32b^2d^4(c^2x^2+1)^{5/2}\text{PolyLog}\left(2,-ie^{-\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}\\
&+\frac{8ib^2d^4(c^2x^2+1)^{5/2}\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}$$

```
[In] Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),x]
```

```
[Out] (8*d^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*c*(d + I*c*d*x)^(5/2)
*(f - I*c*f*x)^(5/2)) + (d^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^3)/(3
*b*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (32*b*d^4*(1 + c^2*x^2)^(5/
2)*(a + b*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)
*(f - I*c*f*x)^(5/2)) - (32*b^2*d^4*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)/E^A
rcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (4*b*d^4*(1 +
 c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c
*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((8*I)/3)*b^2*d^4*(1 + c^2*x^2
)^(5/2)*Tan[Pi/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x
)^(5/2)) + (((8*I)/3)*d^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Tan[Pi
```

```
/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((
2*I)/3)*d^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Sec[Pi/4 + (I/2)*Arc
Sinh[c*x]]^2*Tan[Pi/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*
c*f*x)^(5/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{(1+c^2x^2)^{5/2}\int\frac{(d+icdx)^4(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{(1+c^2x^2)^{5/2}\int\left(\frac{d^4(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}-\frac{4d^4(a+b\text{arcsinh}(cx))^2}{(i+cx)^2\sqrt{1+c^2x^2}}-\frac{4id^4(a+b\text{arcsinh}(cx))^2}{(i+cx)\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{\left(4id^4(1+c^2x^2)^{5/2}\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{(i+cx)\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(d^4(1+c^2x^2)^{5/2}\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(4d^4(1+c^2x^2)^{5/2}\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{(i+cx)^2\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{d^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(4id^4(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int\frac{(a+bx)^2}{ic+c\sinh(x)}\,dx,x,\text{arcsinh}(cx)\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(4cd^4(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int\frac{(a+bx)^2}{(ic+c\sinh(x))^2}\,dx,x,\text{arcsinh}(cx)\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{d^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(d^4(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int(a+bx)^2\csc^4\left(\frac{\pi}{4}-\frac{ix}{2}\right)dx,x,\text{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2d^4(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int(a+bx)^2\csc^2\left(\frac{\pi}{4}-\frac{ix}{2}\right)dx,x,\text{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{d^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{4bd^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{4id^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{2id^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{\left(2d^4(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int(a+bx)^2\csc^2\left(\frac{\pi}{4}-\frac{ix}{2}\right)dx,x,\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{\left(8ibd^4(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int(a+bx)\cot\left(\frac{\pi}{4}-\frac{ix}{2}\right)dx,x,\text{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{\left(4b^2d^4(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int\csc^2\left(\frac{\pi}{4}-\frac{ix}{2}\right)dx,x,\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{4d^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{4bd^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{8id^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{2id^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{\left(8ibd^4(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int(a+bx)\cot\left(\frac{\pi}{4}-\frac{ix}{2}\right)dx,x,\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{\left(16ibd^4(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int\frac{e^{-x}(a+bx)}{1+ie^{-x}}dx,x,\text{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{\left(8ib^2d^4(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int 1\,dx,x,\cot\left(\frac{\pi}{4}-\frac{1}{2}i\text{arcsinh}(cx)\right)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{8d^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{8ib^2d^4(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}-\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{16bd^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\log\left(1+ie^{-\text{arcsinh}(cx)}\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{4bd^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{8id^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{2id^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{\left(16ibd^4(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int \frac{e^{-x}(a+bx)}{1+ie^{-x}}\,dx, x, \text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{\left(16b^2d^4(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int \log\left(1+ie^{-x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{8d^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{8ib^2d^4(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}-\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{32bd^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\log\left(1+ie^{-\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{4bd^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{8id^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{2id^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{\left(16b^2d^4(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int \log\left(1+ie^{-x}\right)\,dx, x, \text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{\left(16b^2d^4(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{-\text{arcsinh}(cx)}\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$=\frac{8d^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{d^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{8ib^2d^4(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}-\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{32bd^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\log\left(1+ie^{-\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{16b^2d^4(1+c^2x^2)^{5/2}\text{PolyLog}\left(2,-ie^{-\text{arcsinh}(cx)}\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{4bd^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{8id^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{2id^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{\left(16b^2d^4(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{-\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$=\frac{8d^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{d^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{8ib^2d^4(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}-\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{32bd^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\log\left(1+ie^{-\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{32b^2d^4(1+c^2x^2)^{5/2}\text{PolyLog}\left(2,-ie^{-\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{4bd^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{8id^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{2id^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1617 vs. $2(584) = 1168$.

Time = 17.78 (sec) , antiderivative size = 1617, normalized size of antiderivative = 2.77

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \frac{\sqrt{id(-i+cx)}\sqrt{-if(i+cx)}\left(\frac{4ia^2d}{3f^3(i+cx)^2}-\frac{8a^2d}{3f^3(i+cx)}\right)}{c}$$
$$+\frac{a^2d^{3/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{id(-i+cx)}\sqrt{-if(i+cx)}\right)}{cf^{5/2}}$$
$$-\frac{iabd\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\sqrt{-df\left(1+c^2x^2\right)}\left(\cosh\left(\frac{1}{2}\text{arcsinh}(cx)\right)+i\sinh\left(\frac{1}{2}\text{arcsinh}(cx)\right)\right)\left(-\cos\right.}{}$$
$$+\frac{abd\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\sqrt{-df\left(1+c^2x^2\right)}\left(\cosh\left(\frac{1}{2}\text{arcsinh}(cx)\right)+i\sinh\left(\frac{1}{2}\text{arcsinh}(cx)\right)\right)\left(\cosh\left(\right.\right.}{}$$
$$-\frac{ib^2d(-i+cx)\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\sqrt{-df\left(1+c^2x^2\right)}\left((-1-i)\text{arcsinh}(cx)^2-\frac{2\text{arcsinh}(cx)(2i+\text{ar}}{i+cx}\right.}{}$$
$$+\frac{b^2d(-i+cx)\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\sqrt{-df\left(1+c^2x^2\right)}\left(-21\pi\text{arcsinh}(cx)-(7-7i)\text{arcsinh}(cx)^2+\right.}{}$$

```
[In] Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),
x]

[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((((4*I)/3)*a^2*d)/(f^3*(I + c
*x)^2) - (8*a^2*d)/(3*f^3*(I + c*x))))/c + (a^2*d^(3/2)*Log[c*d*f*x + Sqrt[
d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(c*f^(5/2)) - ((I/
3)*a*b*d*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 +
c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(-(Cosh[(3*ArcSi
nh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 +
 c^2*x^2]])) + Cosh[ArcSinh[c*x]/2]*(4*I + 3*ArcSinh[c*x] - 6*ArcTan[Coth[A
rcSinh[c*x]/2]] + (3*I)*Log[Sqrt[1 + c^2*x^2]]) + 2*(Sqrt[1 + c^2*x^2]*(I*A
rcSinh[c*x] + (2*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + Log[Sqrt[1 + c^2*x^2]])
+ 2*(1 + I*ArcSinh[c*x] + (2*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + Log[Sqrt[1 +
 c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c*f^3*(1 + I*c*x)*Sqrt[-(((-I)*d + c*d
*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^4) + (a
*b*d*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*
x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*ArcSinh[c*x
])/2]*((14*I - 3*ArcSinh[c*x])*ArcSinh[c*x] + (28*I)*ArcTan[Tanh[ArcSinh[c*
x]/2]] - 14*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/2]*(8 + (6*I)*ArcSi
```

```
nh[c*x] + 9*ArcSinh[c*x]^2 - (84*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 42*Log[S
qrt[1 + c^2*x^2]]) - (2*I)*(4 + (4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 - (56
*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 28*Log[Sqrt[1 + c^2*x^2]] + Sqrt[1 + c^2
*x^2]*(ArcSinh[c*x]*(14*I + 3*ArcSinh[c*x]) - (28*I)*ArcTan[Tanh[ArcSinh[c*
x]/2]] + 14*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(6*c*f^3*(1 + I
*c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] - I*Sin
h[ArcSinh[c*x]/2])^4) - ((I/3)*b^2*d*(-I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sq
rt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*((-1 - I)*ArcSinh[c*x]^2
- (2*ArcSinh[c*x]*(2*I + ArcSinh[c*x]))/(I + c*x) - (2*I)*(Pi - (2*I)*ArcSi
nh[c*x])*Log[1 + I/E^ArcSinh[c*x]] - I*Pi*(3*ArcSinh[c*x] - 4*Log[1 + E^Arc
Sinh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 4*Log[Cosh[ArcSinh[
c*x]/2]]) + 4*PolyLog[2, (-I)/E^ArcSinh[c*x]] - (4*ArcSinh[c*x]^2*Sinh[ArcS
inh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^3 + (2*(4 + Ar
cSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[
c*x]/2])))/(c*f^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]
*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2) + (b^2*d*(-I + c*x)*Sqr
t[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(
-21*Pi*ArcSinh[c*x] - (7 - 7*I)*ArcSinh[c*x]^2 + I*ArcSinh[c*x]^3 + ((2*I)*
ArcSinh[c*x]*(2*I + ArcSinh[c*x]))/(I + c*x) - 14*(Pi - (2*I)*ArcSinh[c*x])
*Log[1 + I/E^ArcSinh[c*x]] + 28*Pi*Log[1 + E^ArcSinh[c*x]] + 14*Pi*Log[-Cos
[(Pi + (2*I)*ArcSinh[c*x])/4]] - 28*Pi*Log[Cosh[ArcSinh[c*x]/2]] - (28*I)*P
olyLog[2, (-I)/E^ArcSinh[c*x]] - ((2*I)*(4 + 7*ArcSinh[c*x]^2)*Sinh[ArcSinh
[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]) + (4*ArcSinh[c*x]
^2*Sinh[ArcSinh[c*x]/2])/(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])^3)
)/(3*c*f^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[
ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2)
```

Maple [F]

$$\int \frac{(icdx + d)^{\frac{3}{2}} (a + b \operatorname{arcsinh}(cx))^2}{(-icfx + f)^{\frac{5}{2}}} dx$$

[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x)

[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x)

Fricas [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx=\int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral(((b^2*c*d*x - I*b^2*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c*d*x - I*a*b*d)*sqrt(I*c*d*x + d)*sqrt(-
I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c*d*x - I*a^2*d)*sqrt(I*c*
d*x + d)*sqrt(-I*c*f*x + f))/(c^3*f^3*x^3 + 3*I*c^2*f^3*x^2 - 3*c*f^3*x - I
*f^3), x)
```

Sympy [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx=\int \frac{(id(cx-i))^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(cx))^2}{(-if\,(cx+i))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Integral((I*d*(c*x - I))**(3/2)*(a + b*asinh(c*x))**2/(-I*f*(c*x + I))**(5/
2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d+c\,d\,x\,1\text{i})^{3/2}}{(f-c\,f\,x\,1\text{i})^{5/2}}\,dx$$

```
[In] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(5/2), x)
```

3.602 $\int \frac{\sqrt{d+icdx}(a+b\textbf{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 522

$$\begin{aligned}
\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx &= \frac{d^3(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{4bd^3(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))\log\left(1+ie^{-\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{4b^2d^3(1+c^2x^2)^{5/2}\,\text{PolyLog}\left(2,-ie^{-\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2bd^3(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{4ib^2d^3(1+c^2x^2)^{5/2}\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{id^3(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{id^3(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}$$

```
[Out] 1/3*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f
*x)^(5/2)+4/3*b*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln(1+I/(c*x+(c^2*x
^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-4/3*b^2*d^3*(c^2*x^2+1)
^(5/2)*polylog(2,-I/(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x
)^(5/2)+2/3*b*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*sec(1/4*Pi+1/2*I*arc
sinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+4/3*I*b^2*d^3*(c^2*x^2+1
)^(5/2)*tan(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2
)+1/3*I*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2*tan(1/4*Pi+1/2*I*arcsinh
(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*I*d^3*(c^2*x^2+1)^(5/2)*(a
```

```
+b*arcsinh(c*x))^2*sec(1/4*Pi+1/2*I*arcsinh(c*x))^2*tan(1/4*Pi+1/2*I*arcsin
h(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {5796, 5844, 5843, 3399, 4271, 3852, 8, 4269, 3797, 2221, 2317, 2438}

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \frac{d^3(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{4bd^3(c^2x^2+1)^{5/2}\log\left(1+ie^{-\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{id^3(c^2x^2+1)^{5/2}\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{2bd^3(c^2x^2+1)^{5/2}\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{id^3(c^2x^2+1)^{5/2}\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{4b^2d^3(c^2x^2+1)^{5/2}\text{PolyLog}\left(2,-ie^{-\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{4ib^2d^3(c^2x^2+1)^{5/2}\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[In] Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),x]
```

```
[Out] (d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(
f - I*c*f*x)^(5/2)) + (4*b*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log
[1 + I/E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (4*
b^2*d^3*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)/E^ArcSinh[c*x]])/(3*c*(d + I*c*
d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*b*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSi
nh[c*x])*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*
c*f*x)^(5/2)) + (((4*I)/3)*b^2*d^3*(1 + c^2*x^2)^(5/2)*Tan[Pi/4 + (I/2)*Arc
Sinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((I/3)*d^3*(1 + c
^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Tan[Pi/4 + (I/2)*ArcSinh[c*x]])/(c*(d
+ I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((I/3)*d^3*(1 + c^2*x^2)^(5/2)*(a +
 b*ArcSinh[c*x])^2*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2*Tan[Pi/4 + (I/2)*ArcSin
h[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5844

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{(1+c^2x^2)^{5/2}\int \frac{(d+icdx)^3(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{(1+c^2x^2)^{5/2}\int \left(-\frac{2d^3(a+b\text{arcsinh}(cx))^2}{(i+cx)^2\sqrt{1+c^2x^2}}-\frac{id^3(a+b\text{arcsinh}(cx))^2}{(i+cx)\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{\left(id^3(1+c^2x^2)^{5/2}\right)\int \frac{(a+b\text{arcsinh}(cx))^2}{(i+cx)\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{\left(2d^3(1+c^2x^2)^{5/2}\right)\int \frac{(a+b\text{arcsinh}(cx))^2}{(i+cx)^2\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{\left(id^3(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int \frac{(a+bx)^2}{ic+c\sinh(x)}\,dx, x, \text{arcsinh}(cx)\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2cd^3(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int \frac{(a+bx)^2}{(ic+c\sinh(x))^2}\,dx, x, \text{arcsinh}(cx)\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{\left(d^3(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int (a+bx)^2\csc^2\left(\frac{\pi}{4}-\frac{ix}{2}\right)\,dx, x, \text{arcsinh}(cx)\right)}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(d^3(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int (a+bx)^2\csc^4\left(\frac{\pi}{4}-\frac{ix}{2}\right)\,dx, x, \text{arcsinh}(cx)\right)}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{id^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{id^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(d^3(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int (a+bx)^2\csc^2\left(\frac{\pi}{4}-\frac{ix}{2}\right)\,dx, x, \text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2ibd^3(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int (a+bx)\cot\left(\frac{\pi}{4}-\frac{ix}{2}\right)\,dx, x, \text{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2b^2d^3(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int \csc^2\left(\frac{\pi}{4}-\frac{ix}{2}\right)\,dx, x, \text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{d^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{id^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{id^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(4ibd^3(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int(a+bx)\cot\left(\frac{\pi}{4}-\frac{ix}{2}\right)\,dx,x,\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(4ibd^3(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int\frac{e^{-x}(a+bx)}{1+ie^{-x}}\,dx,x,\text{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(4ib^2d^3(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int 1\,dx,x,\cot\left(\frac{\pi}{4}-\frac{1}{2}i\text{arcsinh}(cx)\right)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{d^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{4ib^2d^3(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}-\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{4bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\log\left(1+ie^{-\text{arcsinh}(cx)}\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{id^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{id^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(8ibd^3(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int\frac{e^{-x}(a+bx)}{1+ie^{-x}}\,dx,x,\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(4b^2d^3(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int\log\left(1+ie^{-x}\right)\,dx,x,\text{arcsinh}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$=\frac{d^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{4ib^2d^3(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}-\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{4bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\log\left(1+ie^{-\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{2bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{id^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{id^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{\left(8b^2d^3(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int\log\left(1+ie^{-x}\right)\,dx,x,\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{\left(4b^2d^3(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{-\text{arcsinh}(cx)}\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$=\frac{d^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{4ib^2d^3(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}-\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{4bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\log\left(1+ie^{-\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{4b^2d^3(1+c^2x^2)^{5/2}\text{PolyLog}\left(2,-ie^{-\text{arcsinh}(cx)}\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{2bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{id^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{id^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{\left(8b^2d^3(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{-\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{d^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{4ib^2d^3(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}-\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{4bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\log\left(1+ie^{-\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{4b^2d^3(1+c^2x^2)^{5/2}\text{PolyLog}\left(2,-ie^{-\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{2bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{id^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{id^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Mathematica [A] (warning: unable to verify)

Time = 10.07 (sec) , antiderivative size = 788, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \frac{\sqrt{id(-i+cx)}\sqrt{-if(i+cx)}\left(\frac{2ia^2}{3f^3(i+cx)^2}-\frac{a^2}{3f^3(i+cx)}\right)}{c}$$
$$- \frac{iab\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\sqrt{-df(1+c^2x^2)}\left(\cosh\left(\frac{1}{2}\text{arcsinh}(cx)\right)+i\sinh\left(\frac{1}{2}\text{arcsinh}(cx)\right)\right)\left(-\cosh\left(\right.\right.}{}$$
$$- \frac{ib^2(-i+cx)\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\sqrt{-df(1+c^2x^2)}\left((-1-i)\text{arcsinh}(cx)^2-\frac{2\text{arcsinh}(cx)(2i+\text{arcsi}}{i+cx}\right.}{}$$

```
[In] Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((((2*I)/3)*a^2)/(f^3*(I + c*x
)^2) - a^2/(3*f^3*(I + c*x))))/c - ((I/3)*a*b*Sqrt[I*((-I)*d + c*d*x)]*Sqrt
[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*S
inh[ArcSinh[c*x]/2])*(-(Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[C
oth[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]])) + Cosh[ArcSinh[c*x]/2]*(4
*I + 3*ArcSinh[c*x] - 6*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*I)*Log[Sqrt[1 + c
^2*x^2]]) + 2*(Sqrt[1 + c^2*x^2]*(I*ArcSinh[c*x] + (2*I)*ArcTan[Coth[ArcSin
h[c*x]/2]] + Log[Sqrt[1 + c^2*x^2]]) + 2*(1 + I*ArcSinh[c*x] + (2*I)*ArcTan
[Coth[ArcSinh[c*x]/2]] + Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c
*f^3*(1 + I*c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]
/2] - I*Sinh[ArcSinh[c*x]/2])^4) - ((I/3)*b^2*(-I + c*x)*Sqrt[I*((-I)*d + c
*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*((-1 - I)*ArcSin
h[c*x]^2 - (2*ArcSinh[c*x]*(2*I + ArcSinh[c*x]))/(I + c*x) - (2*I)*(Pi - (2
```

```
*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] - I*Pi*(3*ArcSinh[c*x] - 4*Log[
1 + E^ArcSinh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 4*Log[Cosh
[ArcSinh[c*x]/2]]) + 4*PolyLog[2, (-I)/E^ArcSinh[c*x]] - (4*ArcSinh[c*x]^2*
Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^3 + (
2*(4 + ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh
[ArcSinh[c*x]/2])))/(c*f^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 +
 c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2)
```

Maple [F]

$$\int \frac{(a+b\,\operatorname{arcsinh}(cx))^2\sqrt{icdx+d}}{(-icfx+f)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx=\int \frac{\sqrt{i\,cdx+d}(b\,\text{arsinh}\,(cx)+a)^2}{(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/3*((b^2*c*x - I*b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt
(c^2*x^2 + 1))^2 - 3*(c^3*f^3*x^2 + 2*I*c^2*f^3*x - c*f^3)*integral(1/3*(-3
*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + 2*(sqrt(c^2*x^2 + 1)*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f)*b^2 - 3*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)
*a*b)*log(c*x + sqrt(c^2*x^2 + 1)))/(c^3*f^3*x^3 + 3*I*c^2*f^3*x^2 - 3*c*f^
3*x - I*f^3), x))/(c^3*f^3*x^2 + 2*I*c^2*f^3*x - c*f^3)
```

Sympy [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx=\int \frac{\sqrt{id\,(cx-i)}(a+b\,\text{asinh}\,(cx))^2}{(-if\,(cx+i))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Integral(sqrt(I*d*(c*x - I))*(a + b*asinh(c*x))**2/(-I*f*(c*x + I))**(5/2),
 x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Giac [F]

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \int \frac{\sqrt{i\,cdx+d}(b\operatorname{arsinh}(cx)+a)^2}{(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)^2/(-I*c*f*x + f)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \int \frac{(a+b\,\operatorname{asinh}(c\,x))^2\,\sqrt{d+c\,d\,x\,1\mathrm{i}}}{(f-c\,f\,x\,1\mathrm{i})^{5/2}}\,dx$$

```
[In] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(5/2), x)
```

3.603 $\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 942

$$\begin{aligned}
\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx &= \frac{2ib^2d^2(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2b^2d^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2d^2(1+c^2x^2)^{5/2}\,\text{arcsinh}(cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{bd^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ibd^2x(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{bcd^2x^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^2(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{d^2x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{c^2d^2x^3(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2d^2x(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{4ibd^2(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2bd^2(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2b^2d^2(1+c^2x^2)^{5/2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2b^2d^2(1+c^2x^2)^{5/2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{b^2d^2(1+c^2x^2)^{5/2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}$$

```
[Out] 2/3*I*b*d^2*x*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c
*f*x)^(5/2)-2/3*b^2*d^2*x*(c^2*x^2+1)^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
+1/3*b^2*d^2*(c^2*x^2+1)^(5/2)*arcsinh(c*x)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)
^(5/2)+1/3*b*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(
f-I*c*f*x)^(5/2)+4/3*I*b*d^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*arctan(c*
x+(c^2*x^2+1)^(1/2))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b*c*d^2*x^2*
(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/
3*I*b^2*d^2*(c^2*x^2+1)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*d^2*x*(
c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*c^2
*d^2*x^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/
2)+2/3*d^2*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*
x)^(5/2)+1/3*d^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)
/(f-I*c*f*x)^(5/2)-2/3*I*d^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)
^(5/2)/(f-I*c*f*x)^(5/2)-2/3*b*d^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln(
1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3*b^2*
d^2*(c^2*x^2+1)^(5/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(
5/2)/(f-I*c*f*x)^(5/2)-2/3*b^2*d^2*(c^2*x^2+1)^(5/2)*polylog(2,I*(c*x+(c^2*
x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b^2*d^2*(c^2*x^2+1
)^(5/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*
x)^(5/2)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 942, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.486$, Rules used = {5796, 5838, 5788, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 197, 5789, 4265, 267,

5800, 5810, 294, 221}

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx = -\frac{c^2d^2(c^2x^2+1)(a+b\operatorname{arcsinh}(cx))^2x^3}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{bcd^2(c^2x^2+1)^{3/2}(a+b\operatorname{arcsinh}(cx))x^2}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}-\frac{2b^2d^2(c^2x^2+1)^2x}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{2d^2(c^2x^2+1)^2(a+b\operatorname{arcsinh}(cx))^2x}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}+\frac{d^2(c^2x^2+1)(a+b\operatorname{arcsinh}(cx))^2x}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{2ibd^2(c^2x^2+1)^{3/2}(a+b\operatorname{arcsinh}(cx))x}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}+\frac{2ib^2d^2(c^2x^2+1)^2}{3c(icxd+d)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{d^2(c^2x^2+1)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(icxd+d)^{5/2}(f-icfx)^{5/2}}-\frac{2id^2(c^2x^2+1)(a+b\operatorname{arcsinh}(cx))^2}{3c(icxd+d)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{b^2d^2(c^2x^2+1)^{5/2}\operatorname{arcsinh}(cx)}{3c(icxd+d)^{5/2}(f-icfx)^{5/2}}+\frac{bd^2(c^2x^2+1)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c(icxd+d)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{4ibd^2(c^2x^2+1)^{5/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{3c(icxd+d)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{2bd^2(c^2x^2+1)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{3c(icxd+d)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{2b^2d^2(c^2x^2+1)^{5/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{3c(icxd+d)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{2b^2d^2(c^2x^2+1)^{5/2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{3c(icxd+d)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{b^2d^2(c^2x^2+1)^{5/2}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{3c(icxd+d)^{5/2}(f-icfx)^{5/2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] (((2*I)/3)*b^2*d^2*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) - (2*b^2*d^2*x*(1 + c^2*x^2)^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(
5/2)) + (b^2*d^2*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x])/(3*c*(d + I*c*d*x)^(5/2)
*(f - I*c*f*x)^(5/2)) + (b*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3
*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*b*d^2*x*(1 + c^2*x
^2)^(3/2)*(a + b*ArcSinh[c*x]))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) -
 (b*c*d^2*x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(5
/2)*(f - I*c*f*x)^(5/2)) - (((2*I)/3)*d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
)^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (d^2*x*(1 + c^2*x^2)*(a
+ b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (c^2*d^2
*x^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*
f*x)^(5/2)) + (2*d^2*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*
d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (d^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c
*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((4*I)/3)*b*d^2*(1
```

```
 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*
d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b*d^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSi
nh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x
)^(5/2)) + (2*b^2*d^2*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/
(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b^2*d^2*(1 + c^2*x^2)^(5
/2)*PolyLog[2, I*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) - (b^2*d^2*(1 + c^2*x^2)^(5/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c*(
d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 5810

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m
- 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IGtQ
[m, 1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
```

```
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\text{integral} = \frac{(1+c^2x^2)^{5/2}\int \frac{(d+icdx)^2(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{(1+c^2x^2)^{5/2}\int \left(\frac{d^2(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}+\frac{2icd^2x(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}-\frac{c^2d^2x^2(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(d^2(1+c^2x^2)^{5/2}\right)\int \frac{(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2icd^2(1+c^2x^2)^{5/2}\right)\int \frac{x(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(c^2d^2(1+c^2x^2)^{5/2}\right)\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{2id^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{c^2d^2x^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2d^2(1+c^2x^2)^{5/2}\right)\int \frac{(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(4ibd^2(1+c^2x^2)^{5/2}\right)\int \frac{a+b\text{arcsinh}(cx)}{(1+c^2x^2)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2bcd^2(1+c^2x^2)^{5/2}\right)\int \frac{x(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2bc^3d^2(1+c^2x^2)^{5/2}\right)\int \frac{x^3(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{bd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ibd^2x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bcd^2x^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{c^2d^2x^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2d^2x(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2ibd^2(1+c^2x^2)^{5/2}\right)\int \frac{a+b\text{arcsinh}(cx)}{1+c^2x^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(b^2d^2(1+c^2x^2)^{5/2}\right)\int \frac{1}{(1+c^2x^2)^{3/2}}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2bcd^2(1+c^2x^2)^{5/2}\right)\int \frac{x(a+b\text{arcsinh}(cx))}{1+c^2x^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(4bcd^2(1+c^2x^2)^{5/2}\right)\int \frac{x(a+b\text{arcsinh}(cx))}{1+c^2x^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2ib^2cd^2(1+c^2x^2)^{5/2}\right)\int \frac{x}{(1+c^2x^2)^{3/2}}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(b^2c^2d^2(1+c^2x^2)^{5/2}\right)\int \frac{x^2}{(1+c^2x^2)^{3/2}}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2ib^2d^2(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2d^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ibd^2x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bcd^2x^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{c^2d^2x^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2d^2x(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(b^2d^2(1+c^2x^2)^{5/2}\right)\int \frac{1}{\sqrt{1+c^2x^2}}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2ibd^2(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int (a+bx)\text{sech}(x)\,dx, x, \text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2bd^2(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(4bd^2(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2ib^2d^2(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2d^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{b^2d^2(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{2ibd^2x(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{bcd^2x^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^2(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{d^2x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{c^2d^2x^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{2d^2x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{4ibd^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{\left(4bd^2(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{\left(8bd^2(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{\left(2b^2d^2(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int \log(1-ie^x)\,dx, x, \operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{\left(2b^2d^2(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int \log(1+ie^x)\,dx, x, \operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2ib^2d^2(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2d^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{b^2d^2(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{2ibd^2x(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{bcd^2x^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^2(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{d^2x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{c^2d^2x^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{2d^2x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{4ibd^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{2bd^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{\left(2b^2d^2(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\log\left(1+e^{2x}\right)dx,x,\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{\left(2b^2d^2(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\frac{\log(1-ix)}{x}dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{\left(2b^2d^2(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\frac{\log(1+ix)}{x}dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{\left(4b^2d^2(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\log\left(1+e^{2x}\right)dx,x,\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2ib^2d^2(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2d^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{b^2d^2(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{2ibd^2x(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{bcd^2x^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^2(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{d^2x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{c^2d^2x^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{2d^2x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{4ibd^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{2bd^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{2b^2d^2(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{2b^2d^2(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$- \frac{\left(b^2d^2(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+ \frac{\left(2b^2d^2(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$
\begin{aligned}
&= \frac{2ib^2d^2(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2d^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{b^2d^2(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{2ibd^2x(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{bcd^2x^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^2(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{d^2x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{c^2d^2x^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{2d^2x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{4ibd^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{2bd^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad + \frac{2b^2d^2(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{2b^2d^2(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&\quad - \frac{b^2d^2(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

Mathematica [A] (warning: unable to verify)

Time = 6.96 (sec) , antiderivative size = 528, normalized size of antiderivative = 0.56

$$
\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx = \frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{a^2(2i+cx)}{(i+cx)^2} - \frac{ab\left(i\cosh\left(\frac{3}{2}\operatorname{arcsinh}(cx)\right)\left(\operatorname{arcsinh}(cx)-2\arctan\left(c\right.\right.\right.}{}\right.}{}
$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] (Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((a^2*(2*I + c*x))/(I + c*x)^2 - (a*b*
(I*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]]
+ (I/2)*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(-2 + (3*I)*ArcSinh[c*x] +
 (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*Log[1 + c^2*x^2])/2) + 2*(I + (-1
+ Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 2*(2 + Sqrt[1 + c^2*x^2])*ArcTan[Coth[A
```

```
rcSinh[c*x]/2]] - (I/2)*(2 + Sqrt[1 + c^2*x^2])*Log[1 + c^2*x^2])*Sinh[ArcS
inh[c*x]/2]))/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x
]/2])^3) - (b^2*((1 + I)*ArcSinh[c*x]^2 - (ArcSinh[c*x]*(2*I + ArcSinh[c*x]
))/(I + c*x) + 2*(I*Pi + 2*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + I*Pi*(
3*ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh
[c*x])/4]] + 4*Log[Cosh[ArcSinh[c*x]/2]]) - 4*PolyLog[2, (-I)/E^ArcSinh[c*x
]] - (2*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh
[ArcSinh[c*x]/2])^3 - (2*(-2 + ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[
ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/Sqrt[1 + c^2*x^2]))/(3*c*d*f^3)
```

Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{(-icfx+f)^{\frac{5}{2}}\sqrt{icdx+d}}dx$$

[In] `int((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x)`

[Out] `int((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x)`

Fricas [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx=\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2}{\sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*((b^2*c*x + 2*I*b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqr
t(c^2*x^2 + 1))^2 + 3*(c^3*d*f^3*x^2 + 2*I*c^2*d*f^3*x - c*d*f^3)*integral(
-1/3*(3*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + 2*(3*sqrt(I*c*d*x + d)*s
qrt(-I*c*f*x + f)*a*b + (b^2*c*x + 2*I*b^2)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x
+ d)*sqrt(-I*c*f*x + f))*log(c*x + sqrt(c^2*x^2 + 1)))/(c^4*d*f^3*x^4 + 2*I
*c^3*d*f^3*x^3 + 2*I*c*d*f^3*x - d*f^3), x))/(c^3*d*f^3*x^2 + 2*I*c^2*d*f^3
*x - c*d*f^3)
```

Sympy [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx=\int \frac{(a+b\operatorname{asinh}(cx))^2}{\sqrt{id(cx-i)}\,(-if(cx+i))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2/(f-I*c*f*x)**(5/2)/(d+I*c*d*x)**(1/2),x)
[Out] Integral((a + b*asinh(c*x))**2/(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(5/2)
), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/(sqrt(I*c*d*x + d)*(-I*c*f*x + f)^(5/2)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx=\int \frac{(a+b\operatorname{asinh}(c\,x))^2}{\sqrt{d+c\,d\,x\,1\mathrm{i}}\,(f-c\,f\,x\,1\mathrm{i})^{5/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2)),x)
[Out] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2)), x)
```

3.604 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{5/2}} dx$

Optimal result

Integrand size = 37, antiderivative size = 743

$$\begin{aligned}
\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx &= \frac{ib^2d(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{b^2dx(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{ibdx(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{id(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{dx(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2dx(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2d(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2ibd(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{4bd(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{b^2d(1+c^2x^2)^{5/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{b^2d(1+c^2x^2)^{5/2}\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2b^2d(1+c^2x^2)^{5/2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}$$

```
[Out] 1/3*I*b^2*d*(c^2*x^2+1)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b^2*d*x
*(c^2*x^2+1)^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*b*d*(c^2*x^2+1)^(3/2
```

```
)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*I*b*d*x*(c^2
*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*I*
d*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/
3*d*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+
2/3*d*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5
/2)+2/3*d*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c
*f*x)^(5/2)+2/3*I*b*d*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*
x^2+1)^(1/2))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-4/3*b*d*(c^2*x^2+1)^(5/
2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(5/2)/(
f-I*c*f*x)^(5/2)+1/3*b^2*d*(c^2*x^2+1)^(5/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^
(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b^2*d*(c^2*x^2+1)^(5/2)*p
olylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2
/3*b^2*d*(c^2*x^2+1)^(5/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d
*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 743, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.378$, Rules used = {5796, 5838, 5788, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 197, 5789, 4265, 267}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx = \frac{2ibd(c^2x^2+1)^{5/2}\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{2d(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2dx(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{bd(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibdx(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{id(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{dx(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{4bd(c^2x^2+1)^{5/2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{b^2d(c^2x^2+1)^{5/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{b^2d(c^2x^2+1)^{5/2}\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$-\frac{2b^2d(c^2x^2+1)^{5/2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$+\frac{ib^2d(c^2x^2+1)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{b^2dx(c^2x^2+1)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] ((I/3)*b^2*d*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) -
 (b^2*d*x*(1 + c^2*x^2)^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (b
```

```
*d*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c*(d + I*c*d*x)^(5/2)*(f -
I*c*f*x)^(5/2)) + ((I/3)*b*d*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/((
d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((I/3)*d*(1 + c^2*x^2)*(a + b*Arc
Sinh[c*x])^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (d*x*(1 + c^2*x
^2)*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (
2*d*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I
*c*f*x)^(5/2)) + (2*d*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*c*(d +
 I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*b*d*(1 + c^2*x^2)^(5/2)*(
a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c
*f*x)^(5/2)) - (4*b*d*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log[1 + E^(2
*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (b^2*d*(1
+ c^2*x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*
(f - I*c*f*x)^(5/2)) - (b^2*d*(1 + c^2*x^2)^(5/2)*PolyLog[2, I*E^ArcSinh[c*
x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b^2*d*(1 + c^2*x^2)
^(5/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f
*x)^(5/2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
```

```
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(1+c^2x^2)^{5/2} \int \frac{(d+icdx)(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&= \frac{(1+c^2x^2)^{5/2} \int \left(\frac{d(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}} + \frac{icdx(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&= \frac{\left(d(1+c^2x^2)^{5/2}\right) \int \frac{(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(icd(1+c^2x^2)^{5/2}\right) \int \frac{x(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}$$

$$= -\frac{id(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{dx(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2d(1+c^2x^2)^{5/2}\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2ibd(1+c^2x^2)^{5/2}\right)\int\frac{a+b\text{arcsinh}(cx)}{(1+c^2x^2)^2}dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2bcd(1+c^2x^2)^{5/2}\right)\int\frac{x(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^2}dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{bd(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{ibdx(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{id(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{dx(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2dx(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(ibd(1+c^2x^2)^{5/2}\right)\int\frac{a+b\text{arcsinh}(cx)}{1+c^2x^2}dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(b^2d(1+c^2x^2)^{5/2}\right)\int\frac{1}{(1+c^2x^2)^{3/2}}dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(4bcd(1+c^2x^2)^{5/2}\right)\int\frac{x(a+b\text{arcsinh}(cx))}{1+c^2x^2}dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(ib^2cd(1+c^2x^2)^{5/2}\right)\int\frac{x}{(1+c^2x^2)^{3/2}}dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{ib^2d(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2dx(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{ibdx(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{id(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{dx(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2dx(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(ibd(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int(a+bx)\text{sech}(x)\,dx,x,\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(4bd(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int(a+bx)\tanh(x)\,dx,x,\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{ib^2d(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2dx(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{ibdx(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{id(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{dx(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2dx(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2d(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ibd(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(8bd(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(b^2d(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int \log\left(1-ie^x\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(b^2d(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int \log\left(1+ie^x\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{ib^2d(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2dx(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{ibdx(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{id(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{dx(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2dx(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2d(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ibd(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{4bd(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(b^2d(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(b^2d(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(4b^2d(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{ib^2d(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2dx(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{ibdx(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{id(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{dx(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2dx(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2d(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ibd(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{4bd(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2d(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2d(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2b^2d(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{ib^2d(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2dx(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{ibdx(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{id(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{dx(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2dx(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2d(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ibd(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{4bd(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2d(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2d(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2d(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Mathematica [A] (warning: unable to verify)

Time = 10.01 (sec) , antiderivative size = 757, normalized size of antiderivative = 1.02

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx = \frac{\sqrt{id(-i+cx)}\sqrt{-if(i+cx)}\left(\frac{a^2}{4d^2f^3(-i+cx)}+\frac{ia^2}{6d^2f^3(i+cx)^2}+\frac{5a^2}{12d^2f^3(i+cx)}\right)}{c}$$
$$-\frac{iab\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\left(4cx\text{arcsinh}(cx)-2i\text{arcsinh}(cx)\cosh(2\text{arcsinh}(cx))+\sqrt{1+c^2x^2}(1+\right.}{3cdf^2(i+cx)\sqrt{-((-id+}$$
$$-\frac{ib^2\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\sqrt{1+c^2x^2}\left(-9\pi\text{arcsinh}(cx)+\frac{(2-i\text{arcsinh}(cx))\text{arcsinh}(cx)}{i+cx}-(1-4i)\text{arcs}\right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)),
x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(a^2/(4*d^2*f^3*(-I + c*x)) +
((I/6)*a^2)/(d^2*f^3*(I + c*x)^2) + (5*a^2)/(12*d^2*f^3*(I + c*x))))/c - ((
I/3)*a*b*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*(4*c*x*ArcSinh[c
*x] - (2*I)*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]] + Sqrt[1 + c^2*x^2]*(1 + (2*I
)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 2*c*x*(ArcTan[Tanh[ArcSinh[c*x]/2]] + (2*I
)*Log[Sqrt[1 + c^2*x^2]]) - 4*Log[Sqrt[1 + c^2*x^2]])))/(c*d*f^2*(I + c*x)*
Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[-(d*f*(1 + c^2*x^2))]) - ((I/6
)*b^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[1 + c^2*x^2]*(
-9*Pi*ArcSinh[c*x] + ((2 - I*ArcSinh[c*x])*ArcSinh[c*x])/(I + c*x) - (1 - 4
*I)*ArcSinh[c*x]^2 + 3*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]]
- 5*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + 16*Pi*Log[1 + E^A
rcSinh[c*x]] + 5*Pi*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] - 16*Pi*Log[Cosh
[ArcSinh[c*x]/2]] - 3*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]] - (10*I)*Pol
yLog[2, (-I)/E^ArcSinh[c*x]] - (6*I)*PolyLog[2, I/E^ArcSinh[c*x]] - ((2*I)*
ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh
[c*x]/2])^3 + (I*(4 - 5*ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh
[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]) - ((3*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x
]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(c*d*f^2*Sqrt[-(((-
I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[-(d*f*(1 + c^2*x^2))])
```

Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{(icdx+d)^{\frac{3}{2}}\,(-icfx+f)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x)
```

Fricas [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{3}{2}}(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*((2*b^2*c^2*x^2 + 2*I*b^2*c*x + b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 3*(c^4*d^2*f^3*x^3 + I*c^3*d^2*f^3*x^2
+ c^2*d^2*f^3*x + I*c*d^2*f^3)*integral(1/3*(3*I*sqrt(I*c*d*x + d)*sqrt(-I*
c*f*x + f)*a^2 - 2*(-3*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b + (2*b^2*
c^2*x^2 + 2*I*b^2*c*x + b^2)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*
f*x + f))*log(c*x + sqrt(c^2*x^2 + 1)))/(c^5*d^2*f^3*x^5 + I*c^4*d^2*f^3*x^
4 + 2*c^3*d^2*f^3*x^3 + 2*I*c^2*d^2*f^3*x^2 + c*d^2*f^3*x + I*d^2*f^3), x))
/(c^4*d^2*f^3*x^3 + I*c^3*d^2*f^3*x^2 + c^2*d^2*f^3*x + I*c*d^2*f^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{3/2}(f - icfx)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{3/2}(f - icfx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{3/2}(f - icfx)^{5/2}}\,dx = \int \frac{(a + b\,\text{asinh}(c\,x))^2}{(d + c\,d\,x\,1\text{i})^{3/2}\,(f - c\,f\,x\,1\text{i})^{5/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2)), x)
```

3.605 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 386

$$\begin{aligned}
\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx &= -\frac{b^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{b(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2x(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{4b(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2b^2(1+c^2x^2)^{5/2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}$$

```
[Out] -1/3*b^2*x*(c^2*x^2+1)^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*b*(c^2*x^2
+1)^(3/2)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*x*(c
^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3*x*(c
^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3*(c
^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-
4/3*b*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/
c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*b^2*(c^2*x^2+1)^(5/2)*polylog(2,-
(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.270, Rules used = {5796, 5788, 5787, 5797, 3799, 2221, 2317, 2438, 5798, 197}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx = \frac{2(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2x(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{4b(c^2x^2+1)^{5/2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2(c^2x^2+1)^{5/2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2x(c^2x^2+1)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] -1/3*(b^2*x*(1 + c^2*x^2)^2)/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (b
*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c*(d + I*c*d*x)^(5/2)*(f - I*
c*f*x)^(5/2)) + (x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(
5/2)*(f - I*c*f*x)^(5/2)) + (2*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(3
*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*(1 + c^2*x^2)^(5/2)*(a + b*A
rcSinh[c*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (4*b*(1 + c
^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(3*c*(d + I
*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b^2*(1 + c^2*x^2)^(5/2)*PolyLog[2,
-E^(2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5788

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcSinh[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c
^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x],
 x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rule 5796

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q), Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{(1+c^2x^2)^{5/2}\int\frac{(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2(1+c^2x^2)^{5/2}\right)\int\frac{(a+b\text{arcsinh}(cx))^2}{(1+c^2x^2)^{3/2}}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2bc(1+c^2x^2)^{5/2}\right)\int\frac{x(a+b\text{arcsinh}(cx))}{(1+c^2x^2)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{b(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2x(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(b^2(1+c^2x^2)^{5/2}\right)\int\frac{1}{(1+c^2x^2)^{3/2}}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(4bc(1+c^2x^2)^{5/2}\right)\int\frac{x(a+b\text{arcsinh}(cx))}{1+c^2x^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{b^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2x(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(4b(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int(a+bx)\tanh(x)\,dx,x,\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{b^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2x(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(8b(1+c^2x^2)^{5/2}\right)\text{Subst}\left(\int\frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx,x,\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$
\begin{aligned}
&= -\frac{b^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{4b(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{\left(4b^2(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\log\left(1+e^{2x}\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{4b(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{\left(2b^2(1+c^2x^2)^{5/2}\right)\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{4b(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2b^2(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 8.66 (sec) , antiderivative size = 642, normalized size of antiderivative = 1.66

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx = \frac{4a^2cx(3+2c^2x^2)-b^2\Big(cx-6cx\text{arcsinh}(cx)^2+4i\pi\text{arcsinh}(cx)\cosh(3\text{ar}$$

[In] Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)), x]

[Out] (4*a^2*c*x*(3 + 2*c^2*x^2) - b^2*(c*x - 6*c*x*ArcSinh[c*x]^2 + (4*I)*Pi*ArcSinh[c*x]*Cosh[3*ArcSinh[c*x]] + 2*ArcSinh[c*x]^2*Cosh[3*ArcSinh[c*x]] - (2*I)*Pi*Cosh[3*ArcSinh[c*x]]*Log[1 - I/E^ArcSinh[c*x]] + 4*ArcSinh[c*x]*Cosh[3*ArcSinh[c*x]]*Log[1 - I/E^ArcSinh[c*x]] + (2*I)*Pi*Cosh[3*ArcSinh[c*x]]*Log[1 + I/E^ArcSinh[c*x]] + 4*ArcSinh[c*x]*Cosh[3*ArcSinh[c*x]]*Log[1 + I/E^ArcSinh[c*x]] - (8*I)*Pi*Cosh[3*ArcSinh[c*x]]*Log[1 + E^ArcSinh[c*x]] - (2*I)*Pi*Cosh[3*ArcSinh[c*x]]*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + (8*I)*Pi*Cosh[3*ArcSinh[c*x]]*Log[Cosh[ArcSinh[c*x]/2]] + (2*I)*Pi*Cosh[3*ArcSinh[c*x]]*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]] + 2*Sqrt[1 + c^2*x^2]*(((-3*I)*Pi + 6*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] + I*((2*I)*ArcSinh[c*x] + 6*Pi*ArcSinh[c*x] - (3*I)*ArcSinh[c*x]^2 + 3*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] - 12*Pi*Log[1 + E^ArcSinh[c*x]] - 3*Pi*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 12*Pi*Log[Cosh[ArcSinh[c*x]/2]] + 3*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]])) - 16*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)/E^ArcSinh[c*x]] - 16*(1 + c^2*x^2)^(3/2)*PolyLog[2, I/E^ArcSinh[c*x]] + Sinh[3*ArcSinh[c*x]] - 2*ArcSinh[c*x]^2*Sinh[3*ArcSinh[c*x]]) + 2*a*b*(Sqrt[1 + c^2*x^2]*(2 - 3*Log[1 + c^2*x^2]) - Cosh[3*ArcSinh[c*x]]*Log[1 + c^2*x^2] + 2*ArcSinh[c*x]*(3*c*x + Sinh[3*ArcSinh[c*x]])))/(12*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(c + c^3*x^2))

Maple [F]

$$\int \frac{(a+b\,\text{arcsinh}\,(cx))^2}{(icdx+d)^{\frac{5}{2}}\,(-icfx+f)^{\frac{5}{2}}}dx$$

[In] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x)

[Out] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x)

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx=\int \frac{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}{\left(i\,cdx+d\right)^{\frac{5}{2}}\left(-i\,cfx+f\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*((2*b^2*c^2*x^3 + 3*b^2*x)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x
 + sqrt(c^2*x^2 + 1))^2 + 3*(c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)
*integral(1/3*(3*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + 2*(3*sqrt(I*c*d
*x + d)*sqrt(-I*c*f*x + f)*a*b - (2*b^2*c^3*x^3 + 3*b^2*c*x)*sqrt(c^2*x^2 +
 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))*log(c*x + sqrt(c^2*x^2 + 1)))/(c^
6*d^3*f^3*x^6 + 3*c^4*d^3*f^3*x^4 + 3*c^2*d^3*f^3*x^2 + d^3*f^3), x))/(c^4*
d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx=\int \frac{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}{\left(i\,cdx+d\right)^{\frac{5}{2}}\left(-i\,cfx+f\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/3*a*b*c*(1/(c^4*d^(5/2)*f^(5/2)*x^2 + c^2*d^(5/2)*f^(5/2)) - 2*log(c^2*x^
2 + 1)/(c^2*d^(5/2)*f^(5/2))) + 2/3*a*b*(x/((c^2*d*f*x^2 + d*f)^(3/2)*d*f)
+ 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f^2))*arcsinh(c*x) + 1/3*a^2*(x/((c^2*d*
f*x^2 + d*f)^(3/2)*d*f) + 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f^2)) + b^2*inte
grate(log(c*x + sqrt(c^2*x^2 + 1))^2/((I*c*d*x + d)^(5/2)*(-I*c*f*x + f)^(5
/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2}{(d+c\,d\,x\,1\text{i})^{5/2}\,(f-c\,f\,x\,1\text{i})^{5/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2)), x)
```

3.606 $\int (d+ex^2)^4 (a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 312

$$\begin{aligned}
&\int (d+ex^2)^4 (a+b\text{arcsinh}(cx))\,dx \\
&= -\frac{b(315c^8d^4-420c^6d^3e+378c^4d^2e^2-180c^2de^3+35e^4)\sqrt{1+c^2x^2}}{315c^9} \\
&\quad -\frac{4be(105c^6d^3-189c^4d^2e+135c^2de^2-35e^3)(1+c^2x^2)^{3/2}}{945c^9} \\
&\quad -\frac{2be^2(63c^4d^2-90c^2de+35e^2)(1+c^2x^2)^{5/2}}{525c^9} \\
&\quad -\frac{4b(9c^2d-7e)e^3(1+c^2x^2)^{7/2}}{441c^9}-\frac{be^4(1+c^2x^2)^{9/2}}{81c^9} \\
&\quad +d^4x(a+b\text{arcsinh}(cx))+\frac{4}{3}d^3ex^3(a+b\text{arcsinh}(cx))+\frac{6}{5}d^2e^2x^5(a+b\text{arcsinh}(cx))+\frac{4}{7}de^3x^7(a+b\text{arcsinh}(cx))+
\end{aligned}$$

```
[Out] -4/945*b*e*(105*c^6*d^3-189*c^4*d^2*e+135*c^2*d*e^2-35*e^3)*(c^2*x^2+1)^(3/
2)/c^9-2/525*b*e^2*(63*c^4*d^2-90*c^2*d*e+35*e^2)*(c^2*x^2+1)^(5/2)/c^9-4/4
41*b*(9*c^2*d-7*e)*e^3*(c^2*x^2+1)^(7/2)/c^9-1/81*b*e^4*(c^2*x^2+1)^(9/2)/c
^9+d^4*x*(a+b*arcsinh(c*x))+4/3*d^3*e*x^3*(a+b*arcsinh(c*x))+6/5*d^2*e^2*x^
5*(a+b*arcsinh(c*x))+4/7*d*e^3*x^7*(a+b*arcsinh(c*x))+1/9*e^4*x^9*(a+b*arcs
inh(c*x))-1/315*b*(315*c^8*d^4-420*c^6*d^3*e+378*c^4*d^2*e^2-180*c^2*d*e^3+
35*e^4)*(c^2*x^2+1)^(1/2)/c^9
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {200, 5792, 12, 1813, 1864}

$$\begin{aligned}
&\int (d+ex^2)^4 (a+b\text{arcsinh}(cx))\,dx \\
&= d^4x(a+b\text{arcsinh}(cx)) + \frac{4}{3}d^3ex^3(a+b\text{arcsinh}(cx)) + \frac{6}{5}d^2e^2x^5(a+b\text{arcsinh}(cx)) \\
&\quad + \frac{4}{7}de^3x^7(a+b\text{arcsinh}(cx)) + \frac{1}{9}e^4x^9(a+b\text{arcsinh}(cx)) - \frac{4be^3(c^2x^2+1)^{7/2}(9c^2d-7e)}{441c^9} \\
&\quad - \frac{be^4(c^2x^2+1)^{9/2}}{81c^9} - \frac{2be^2(c^2x^2+1)^{5/2}(63c^4d^2-90c^2de+35e^2)}{525c^9} \\
&\quad - \frac{4be(c^2x^2+1)^{3/2}(105c^6d^3-189c^4d^2e+135c^2de^2-35e^3)}{945c^9} \\
&\quad - \frac{b\sqrt{c^2x^2+1}(315c^8d^4-420c^6d^3e+378c^4d^2e^2-180c^2de^3+35e^4)}{315c^9}
\end{aligned}$$

```
[In] Int[(d + e*x^2)^4*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/315*(b*(315*c^8*d^4 - 420*c^6*d^3*e + 378*c^4*d^2*e^2 - 180*c^2*d*e^3 +
35*e^4)*Sqrt[1 + c^2*x^2])/c^9 - (4*b*e*(105*c^6*d^3 - 189*c^4*d^2*e + 135*
c^2*d*e^2 - 35*e^3)*(1 + c^2*x^2)^(3/2))/(945*c^9) - (2*b*e^2*(63*c^4*d^2 -
 90*c^2*d*e + 35*e^2)*(1 + c^2*x^2)^(5/2))/(525*c^9) - (4*b*(9*c^2*d - 7*e)
*e^3*(1 + c^2*x^2)^(7/2))/(441*c^9) - (b*e^4*(1 + c^2*x^2)^(9/2))/(81*c^9)
+ d^4*x*(a + b*ArcSinh[c*x]) + (4*d^3*e*x^3*(a + b*ArcSinh[c*x]))/3 + (6*d^
2*e^2*x^5*(a + b*ArcSinh[c*x]))/5 + (4*d*e^3*x^7*(a + b*ArcSinh[c*x]))/7 +
(e^4*x^9*(a + b*ArcSinh[c*x]))/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 5792

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d^4x(a+b\operatorname{arcsinh}(cx))+\frac{4}{3}d^3ex^3(a+b\operatorname{arcsinh}(cx))+\frac{6}{5}d^2e^2x^5(a+b\operatorname{arcsinh}(cx)) \\
&\quad+\frac{4}{7}de^3x^7(a+b\operatorname{arcsinh}(cx))+\frac{1}{9}e^4x^9(a+b\operatorname{arcsinh}(cx)) \\
&\quad-(bc)\int\frac{x(315d^4+420d^3ex^2+378d^2e^2x^4+180de^3x^6+35e^4x^8)}{315\sqrt{1+c^2x^2}}\,dx \\
&= d^4x(a+b\operatorname{arcsinh}(cx))+\frac{4}{3}d^3ex^3(a+b\operatorname{arcsinh}(cx))+\frac{6}{5}d^2e^2x^5(a+b\operatorname{arcsinh}(cx)) \\
&\quad+\frac{4}{7}de^3x^7(a+b\operatorname{arcsinh}(cx))+\frac{1}{9}e^4x^9(a+b\operatorname{arcsinh}(cx)) \\
&\quad-\frac{1}{315}(bc)\int\frac{x(315d^4+420d^3ex^2+378d^2e^2x^4+180de^3x^6+35e^4x^8)}{\sqrt{1+c^2x^2}}\,dx \\
&= d^4x(a+b\operatorname{arcsinh}(cx))+\frac{4}{3}d^3ex^3(a+b\operatorname{arcsinh}(cx))+\frac{6}{5}d^2e^2x^5(a+b\operatorname{arcsinh}(cx)) \\
&\quad+\frac{4}{7}de^3x^7(a+b\operatorname{arcsinh}(cx))+\frac{1}{9}e^4x^9(a+b\operatorname{arcsinh}(cx)) \\
&\quad-\frac{1}{630}(bc)\text{Subst}\left(\int\frac{315d^4+420d^3ex+378d^2e^2x^2+180de^3x^3+35e^4x^4}{\sqrt{1+c^2x}}\,dx,x,x^2\right)
\end{aligned}$$

$$= d^4x(a+b\text{arcsinh}(cx)) + \frac{4}{3}d^3ex^3(a+b\text{arcsinh}(cx)) + \frac{6}{5}d^2e^2x^5(a+b\text{arcsinh}(cx)) + \frac{4}{7}de^3x^7(a+b\text{arcsinh}(cx)) + \frac{1}{9}e^4x^9(a+b\text{arcsinh}(cx)) - \frac{1}{630}(bc)\text{Subst}\left(\int\left(\frac{315c^8d^4-420c^6d^3e+378c^4d^2e^2-180c^2de^3+35e^4}{c^8\sqrt{1+c^2x}} + \frac{4e(105c^6d^3-189c^4d^2e+135c^2de^2-35e^3)\sqrt{1+c^2x}}{c^8} + \frac{6e^2(63c^4d^2-90c^2de+35e^2)(1+c^2x)^{3/2}}{c^8} + \frac{20(9c^2d-7e)e^3(1+c^2x)^{5/2}}{c^8} + \frac{35e^4(1+c^2x)^{7/2}}{c^8}\right)dx, x, x^2\right)$$

$$= -\frac{b(315c^8d^4-420c^6d^3e+378c^4d^2e^2-180c^2de^3+35e^4)\sqrt{1+c^2x^2}}{315c^9} - \frac{4be(105c^6d^3-189c^4d^2e+135c^2de^2-35e^3)(1+c^2x^2)^{3/2}}{945c^9} - \frac{2be^2(63c^4d^2-90c^2de+35e^2)(1+c^2x^2)^{5/2}}{525c^9} - \frac{4b(9c^2d-7e)e^3(1+c^2x^2)^{7/2}}{441c^9} - \frac{be^4(1+c^2x^2)^{9/2}}{81c^9} + d^4x(a+b\text{arcsinh}(cx)) + \frac{4}{3}d^3ex^3(a+b\text{arcsinh}(cx)) + \frac{6}{5}d^2e^2x^5(a+b\text{arcsinh}(cx)) + \frac{4}{7}de^3x^7(a+b\text{arcsi}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.83

$$\int (d+ex^2)^4 (a+b\text{arcsinh}(cx))\,dx = \frac{315ax(315d^4+420d^3ex^2+378d^2e^2x^4+180de^3x^6+35e^4x^8) - \frac{b\sqrt{1+c^2x^2}(4480e^4-320c^2e^3(81d+7ex^2)+48c^4e^2(1323d^2}{}}{}$$

```
[In] Integrate[(d + e*x^2)^4*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (315*a*x*(315*d^4 + 420*d^3*e*x^2 + 378*d^2*e^2*x^4 + 180*d*e^3*x^6 + 35*e^
4*x^8) - (b*Sqrt[1 + c^2*x^2]*(4480*e^4 - 320*c^2*e^3*(81*d + 7*e*x^2) + 48
*c^4*e^2*(1323*d^2 + 270*d*e*x^2 + 35*e^2*x^4) - 8*c^6*e*(11025*d^3 + 3969*
d^2*e*x^2 + 1215*d*e^2*x^4 + 175*e^3*x^6) + c^8*(99225*d^4 + 44100*d^3*e*x^
2 + 23814*d^2*e^2*x^4 + 8100*d*e^3*x^6 + 1225*e^4*x^8)))/c^9 + 315*b*x*(315
*d^4 + 420*d^3*e*x^2 + 378*d^2*e^2*x^4 + 180*d*e^3*x^6 + 35*e^4*x^8)*ArcSin
h[c*x])/99225
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.36

method	result
parts	$a\left(\frac{1}{9}e^4x^9+\frac{4}{7}d\,e^3x^7+\frac{6}{5}d^2e^2x^5+\frac{4}{3}d^3e\,x^3+d^4x\right)+\frac{b\left(\frac{c\ \mathrm{arcsinh}(cx)e^4x^9}{9}+\frac{4c\ \mathrm{arcsinh}(cx)d\,e^3x^7}{7}+\frac{6c\ \mathrm{arcsinh}(}{}\right.}{}$
derivativedivides	$\frac{\frac{a\left(d^4c^9x+\frac{4}{3}d^3c^9e\,x^3+\frac{6}{5}d^2c^9e^2x^5+\frac{4}{7}d\,c^9e^3x^7+\frac{1}{9}e^4c^9x^9\right)}{c^8}+\frac{b\left(\mathrm{arcsinh}(cx)d^4c^9x+\frac{4\ \mathrm{arcsinh}(cx)d^3c^9e\,x^3}{3}+\frac{6\ \mathrm{arcsinh}(cx)d^2c^9e^2x^5}{5}+\right.}{}}{}$
default	$\frac{\frac{a\left(d^4c^9x+\frac{4}{3}d^3c^9e\,x^3+\frac{6}{5}d^2c^9e^2x^5+\frac{4}{7}d\,c^9e^3x^7+\frac{1}{9}e^4c^9x^9\right)}{c^8}+\frac{b\left(\mathrm{arcsinh}(cx)d^4c^9x+\frac{4\ \mathrm{arcsinh}(cx)d^3c^9e\,x^3}{3}+\frac{6\ \mathrm{arcsinh}(cx)d^2c^9e^2x^5}{5}+\right.}{}}{}$

```
[In] int((e*x^2+d)^4*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/9*e^4*x^9+4/7*d*e^3*x^7+6/5*d^2*e^2*x^5+4/3*d^3*e*x^3+d^4*x)+b/c*(1/9*
c*arcsinh(c*x)*e^4*x^9+4/7*c*arcsinh(c*x)*d*e^3*x^7+6/5*c*arcsinh(c*x)*d^2*
e^2*x^5+4/3*c*arcsinh(c*x)*d^3*e*x^3+arcsinh(c*x)*d^4*c*x-1/315/c^8*(35*e^4
*(1/9*c^8*x^8*(c^2*x^2+1)^(1/2)-8/63*c^6*x^6*(c^2*x^2+1)^(1/2)+16/105*c^4*x
^4*(c^2*x^2+1)^(1/2)-64/315*c^2*x^2*(c^2*x^2+1)^(1/2)+128/315*(c^2*x^2+1)^(
1/2))+315*d^4*c^8*(c^2*x^2+1)^(1/2)+180*d*c^2*e^3*(1/7*c^6*x^6*(c^2*x^2+1)^
(1/2)-6/35*c^4*x^4*(c^2*x^2+1)^(1/2)+8/35*c^2*x^2*(c^2*x^2+1)^(1/2)-16/35*(
c^2*x^2+1)^(1/2))+378*d^2*c^4*e^2*(1/5*c^4*x^4*(c^2*x^2+1)^(1/2)-4/15*c^2*x
^2*(c^2*x^2+1)^(1/2)+8/15*(c^2*x^2+1)^(1/2))+420*d^3*c^6*e*(1/3*c^2*x^2*(c^
2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.07

$$\int \left(d+ex^2\right)^4 (a+b\mathrm{arcsinh}(cx))\,dx$$
$$=\frac{11025\,ac^9e^4x^9+56700\,ac^9de^3x^7+119070\,ac^9d^2e^2x^5+132300\,ac^9d^3ex^3+99225\,ac^9d^4x+315\,(35\,bc^9e^4x^9+}{}$$

```
[In] integrate((e*x^2+d)^4*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/99225*(11025*a*c^9*e^4*x^9 + 56700*a*c^9*d*e^3*x^7 + 119070*a*c^9*d^2*e^2
*x^5 + 132300*a*c^9*d^3*e*x^3 + 99225*a*c^9*d^4*x + 315*(35*b*c^9*e^4*x^9 +
 180*b*c^9*d*e^3*x^7 + 378*b*c^9*d^2*e^2*x^5 + 420*b*c^9*d^3*e*x^3 + 315*b*
c^9*d^4*x)*log(c*x + sqrt(c^2*x^2 + 1)) - (1225*b*c^8*e^4*x^8 + 99225*b*c^8
```

```
*d^4 - 88200*b*c^6*d^3*e + 63504*b*c^4*d^2*e^2 - 25920*b*c^2*d*e^3 + 100*(8
1*b*c^8*d*e^3 - 14*b*c^6*e^4)*x^6 + 4480*b*e^4 + 6*(3969*b*c^8*d^2*e^2 - 16
20*b*c^6*d*e^3 + 280*b*c^4*e^4)*x^4 + 4*(11025*b*c^8*d^3*e - 7938*b*c^6*d^2
*e^2 + 3240*b*c^4*d*e^3 - 560*b*c^2*e^4)*x^2)*sqrt(c^2*x^2 + 1))/c^9
```

Sympy [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 593, normalized size of antiderivative = 1.90

$$\int \left(d+ex^2\right)^4 (a+b\text{arcsinh}(cx))\,dx = \begin{cases} ad^4x+\frac{4ad^3ex^3}{3}+\frac{6ad^2e^2x^5}{5}+\frac{4ade^3x^7}{7}+\frac{ae^4x^9}{9}+bd^4x\,\text{asinh}\,(cx)+\frac{4bd^3ex^3\,\text{asinh}\,(cx)}{3}+\frac{6bd^2e^2x^5\,\text{asinh}\,(cx)}{5}+\frac{4bde^3x^7}{} \\ a\left(d^4x+\frac{4d^3ex^3}{3}+\frac{6d^2e^2x^5}{5}+\frac{4de^3x^7}{7}+\frac{e^4x^9}{9}\right) \end{cases}$$

```
[In] integrate((e*x**2+d)**4*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*d**4*x + 4*a*d**3*e*x**3/3 + 6*a*d**2*e**2*x**5/5 + 4*a*d*e**3
*x**7/7 + a*e**4*x**9/9 + b*d**4*x*asinh(c*x) + 4*b*d**3*e*x**3*asinh(c*x)/
3 + 6*b*d**2*e**2*x**5*asinh(c*x)/5 + 4*b*d*e**3*x**7*asinh(c*x)/7 + b*e**4
*x**9*asinh(c*x)/9 - b*d**4*sqrt(c**2*x**2 + 1)/c - 4*b*d**3*e*x**2*sqrt(c*
*2*x**2 + 1)/(9*c) - 6*b*d**2*e**2*x**4*sqrt(c**2*x**2 + 1)/(25*c) - 4*b*d*
e**3*x**6*sqrt(c**2*x**2 + 1)/(49*c) - b*e**4*x**8*sqrt(c**2*x**2 + 1)/(81*
c) + 8*b*d**3*e*sqrt(c**2*x**2 + 1)/(9*c**3) + 8*b*d**2*e**2*x**2*sqrt(c**2
*x**2 + 1)/(25*c**3) + 24*b*d*e**3*x**4*sqrt(c**2*x**2 + 1)/(245*c**3) + 8*
b*e**4*x**6*sqrt(c**2*x**2 + 1)/(567*c**3) - 16*b*d**2*e**2*sqrt(c**2*x**2
+ 1)/(25*c**5) - 32*b*d*e**3*x**2*sqrt(c**2*x**2 + 1)/(245*c**5) - 16*b*e**
4*x**4*sqrt(c**2*x**2 + 1)/(945*c**5) + 64*b*d*e**3*sqrt(c**2*x**2 + 1)/(24
5*c**7) + 64*b*e**4*x**2*sqrt(c**2*x**2 + 1)/(2835*c**7) - 128*b*e**4*sqrt(
c**2*x**2 + 1)/(2835*c**9), Ne(c, 0)), (a*(d**4*x + 4*d**3*e*x**3/3 + 6*d**
2*e**2*x**5/5 + 4*d*e**3*x**7/7 + e**4*x**9/9), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.33

$$\int \left(d+ex^2\right)^4 (a+b\text{arcsinh}(cx))\,dx = \frac{1}{9}\,ae^4x^9 + \frac{4}{7}\,ade^3x^7 + \frac{6}{5}\,ad^2e^2x^5 + \frac{4}{3}\,ad^3ex^3 + \frac{4}{9}\left(3\,x^3\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)bd^3e + \frac{2}{25}\left(15\,x^5\,\text{arsinh}\,(cx) - \left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\,\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\right)bd^2e^2 + \frac{4}{245}\left(35\,x^7\,\text{arsinh}\,(cx) - \left(\frac{5\,\sqrt{c^2x^2+1}x^6}{c^2} - \frac{6\,\sqrt{c^2x^2+1}x^4}{c^4} + \frac{8\,\sqrt{c^2x^2+1}x^2}{c^6} - \frac{16\,\sqrt{c^2x^2+1}}{c^8}\right)c\right)bde^3 + \frac{1}{2835}\left(315\,x^9\,\text{arsinh}\,(cx) - \left(\frac{35\,\sqrt{c^2x^2+1}x^8}{c^2} - \frac{40\,\sqrt{c^2x^2+1}x^6}{c^4} + \frac{48\,\sqrt{c^2x^2+1}x^4}{c^6} - \frac{64\,\sqrt{c^2x^2+1}x^2}{c^8} + \right.\right. + ad^4x + \frac{\left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)bd^4}{c}$$

```
[In] integrate((e*x^2+d)^4*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/9*a*e^4*x^9 + 4/7*a*d*e^3*x^7 + 6/5*a*d^2*e^2*x^5 + 4/3*a*d^3*e*x^3 + 4/9
*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c
^4))*b*d^3*e + 2/25*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4
*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*d^2*e^2 + 4/245*
(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x
^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*d*e^3
 + 1/2835*(315*x^9*arcsinh(c*x) - (35*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c
^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x
^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c)*b*e^4 + a*d^4*x + (c*x*arcsinh(c*x)
 - sqrt(c^2*x^2 + 1))*b*d^4/c
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^4 (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*x^2+d)^4*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^4 (a+b\text{arcsinh}(cx))\,dx=\int (a+b\,\text{asinh}(c\,x))\,\left(e\,x^2+d\right)^4\,dx$$

[In] int((a + b*asinh(c*x))*(d + e*x^2)^4,x)

[Out] int((a + b*asinh(c*x))*(d + e*x^2)^4, x)

3.607 $\int (d+ex^2)^3 (a+b\operatorname{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 221

$$\int (d+ex^2)^3 (a+b\operatorname{arcsinh}(cx))\,dx = -\frac{b(35c^6d^3-35c^4d^2e+21c^2de^2-5e^3)\sqrt{1+c^2x^2}}{35c^7} - \frac{be(35c^4d^2-42c^2de+15e^2)(1+c^2x^2)^{3/2}}{105c^7} - \frac{3b(7c^2d-5e)\,e^2(1+c^2x^2)^{5/2}}{175c^7} - \frac{be^3(1+c^2x^2)^{7/2}}{49c^7} + d^3x(a+b\operatorname{arcsinh}(cx)) + d^2ex^3(a+b\operatorname{arcsinh}(cx)) + \frac{3}{5}de^2x^5(a+b\operatorname{arcsinh}(cx)) + \frac{1}{7}e^3x^7(a+b\operatorname{arcsinh}(cx))$$

```
[Out] -1/105*b*e*(35*c^4*d^2-42*c^2*d*e+15*e^2)*(c^2*x^2+1)^(3/2)/c^7-3/175*b*(7*
c^2*d-5*e)*e^2*(c^2*x^2+1)^(5/2)/c^7-1/49*b*e^3*(c^2*x^2+1)^(7/2)/c^7+d^3*x
*(a+b*arcsinh(c*x))+d^2*e*x^3*(a+b*arcsinh(c*x))+3/5*d*e^2*x^5*(a+b*arcsinh
(c*x))+1/7*e^3*x^7*(a+b*arcsinh(c*x))-1/35*b*(35*c^6*d^3-35*c^4*d^2*e+21*c^
2*d*e^2-5*e^3)*(c^2*x^2+1)^(1/2)/c^7
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used

$= \{200, 5792, 12, 1813, 1864\}$

$$\int \left(d+ex^2\right)^3 (a+b\text{arcsinh}(cx))\,dx = d^3x(a+b\text{arcsinh}(cx)) + d^2ex^3(a+b\text{arcsinh}(cx)) + \frac{3}{5}de^2x^5(a+b\text{arcsinh}(cx)) + \frac{1}{7}e^3x^7(a+b\text{arcsinh}(cx)) - \frac{3be^2(c^2x^2+1)^{5/2}(7c^2d-5e)}{175c^7} - \frac{be^3(c^2x^2+1)^{7/2}}{49c^7} - \frac{be(c^2x^2+1)^{3/2}(35c^4d^2-42c^2de+15e^2)}{105c^7} - \frac{b\sqrt{c^2x^2+1}(35c^6d^3-35c^4d^2e+21c^2de^2-5e^3)}{35c^7}$$

```
[In] Int[(d + e*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/35*(b*(35*c^6*d^3 - 35*c^4*d^2*e + 21*c^2*d*e^2 - 5*e^3)*Sqrt[1 + c^2*x^
2])/c^7 - (b*e*(35*c^4*d^2 - 42*c^2*d*e + 15*e^2)*(1 + c^2*x^2)^(3/2))/(105
*c^7) - (3*b*(7*c^2*d - 5*e)*e^2*(1 + c^2*x^2)^(5/2))/(175*c^7) - (b*e^3*(1
 + c^2*x^2)^(7/2))/(49*c^7) + d^3*x*(a + b*ArcSinh[c*x]) + d^2*e*x^3*(a + b
*ArcSinh[c*x]) + (3*d*e^2*x^5*(a + b*ArcSinh[c*x]))/5 + (e^3*x^7*(a + b*Arc
Sinh[c*x]))/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 5792

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
```

```
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d^3x(a+b\text{arcsinh}(cx))+d^2ex^3(a+b\text{arcsinh}(cx))+\frac{3}{5}de^2x^5(a+b\text{arcsinh}(cx)) \\
&\quad +\frac{1}{7}e^3x^7(a+b\text{arcsinh}(cx))-(bc)\int\frac{x(35d^3+35d^2ex^2+21de^2x^4+5e^3x^6)}{35\sqrt{1+c^2x^2}}\,dx \\
&= d^3x(a+b\text{arcsinh}(cx))+d^2ex^3(a+b\text{arcsinh}(cx))+\frac{3}{5}de^2x^5(a+b\text{arcsinh}(cx)) \\
&\quad +\frac{1}{7}e^3x^7(a+b\text{arcsinh}(cx))-\frac{1}{35}(bc)\int\frac{x(35d^3+35d^2ex^2+21de^2x^4+5e^3x^6)}{\sqrt{1+c^2x^2}}\,dx \\
&= d^3x(a+b\text{arcsinh}(cx))+d^2ex^3(a+b\text{arcsinh}(cx))+\frac{3}{5}de^2x^5(a+b\text{arcsinh}(cx)) \\
&\quad +\frac{1}{7}e^3x^7(a+b\text{arcsinh}(cx))-\frac{1}{70}(bc)\text{Subst}\left(\int\frac{35d^3+35d^2ex+21de^2x^2+5e^3x^3}{\sqrt{1+c^2x}}\,dx,x,x^2\right) \\
&= d^3x(a+b\text{arcsinh}(cx))+d^2ex^3(a+b\text{arcsinh}(cx))+\frac{3}{5}de^2x^5(a+b\text{arcsinh}(cx)) \\
&\quad +\frac{1}{7}e^3x^7(a+b\text{arcsinh}(cx))-\frac{1}{70}(bc)\text{Subst}\left(\int\left(\frac{35c^6d^3-35c^4d^2e+21c^2de^2-5e^3}{c^6\sqrt{1+c^2x}}\right.\right. \\
&\quad +\frac{e(35c^4d^2-42c^2de+15e^2)\sqrt{1+c^2x}}{c^6}+\frac{3(7c^2d-5e)e^2(1+c^2x)^{3/2}}{c^6} \\
&\quad \left.\left.+\frac{5e^3(1+c^2x)^{5/2}}{c^6}\right)\,dx,x,x^2\right) \\
&= -\frac{b(35c^6d^3-35c^4d^2e+21c^2de^2-5e^3)\sqrt{1+c^2x^2}}{35c^7} \\
&\quad -\frac{be(35c^4d^2-42c^2de+15e^2)(1+c^2x^2)^{3/2}}{105c^7} \\
&\quad -\frac{3b(7c^2d-5e)e^2(1+c^2x^2)^{5/2}}{175c^7}-\frac{be^3(1+c^2x^2)^{7/2}}{49c^7} \\
&\quad +d^3x(a+b\text{arcsinh}(cx))+d^2ex^3(a+b\text{arcsinh}(cx))+\frac{3}{5}de^2x^5(a+b\text{arcsinh}(cx))+\frac{1}{7}e^3x^7(a+b\text{arcsinh}(c
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.85

$$\int (d+ex^2)^3 (a+b\text{arcsinh}(cx))\,dx = a\left(d^3x+d^2ex^3+\frac{3}{5}de^2x^5+\frac{e^3x^7}{7}\right)$$
$$-\frac{b\sqrt{1+c^2x^2}(-240e^3+24c^2e^2(49d+5ex^2)-2c^4e(1225d^2+294dex^2+45e^2x^4)+c^6(3675d^3+1225d^2e}{3675c^7}$$
$$+b\left(d^3x+d^2ex^3+\frac{3}{5}de^2x^5+\frac{e^3x^7}{7}\right)\text{arcsinh}(cx)$$

```
[In] Integrate[(d + e*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] a*(d^3*x + d^2*e*x^3 + (3*d*e^2*x^5)/5 + (e^3*x^7)/7) - (b*Sqrt[1 + c^2*x^2
]*(-240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) - 2*c^4*e*(1225*d^2 + 294*d*e*x^2
 + 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*d*e^2*x^4 + 75*e^3*x^
6)))/(3675*c^7) + b*(d^3*x + d^2*e*x^3 + (3*d*e^2*x^5)/5 + (e^3*x^7)/7)*Arc
Sinh[c*x]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.33

method	result
parts	$a\left(\frac{1}{7}e^3x^7+\frac{3}{5}d\,e^2x^5+d^2e\,x^3+d^3x\right)+\frac{b\left(\frac{c\,\text{arcsinh}(cx)e^3x^7}{7}+\frac{3c\,\text{arcsinh}(cx)d\,e^2x^5}{5}+c\,\text{arcsinh}(cx)d^2e\,x^3+\text{arc}\right.}{}$
derivativedivides	$\frac{a\left(d^3c^7x+d^2c^7e\,x^3+\frac{3}{5}d\,c^7e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6}+\frac{b\left(\text{arcsinh}(cx)d^3c^7x+\text{arcsinh}(cx)d^2c^7e\,x^3+\frac{3\,\text{arcsinh}(cx)d\,c^7e^2x^5}{5}+\frac{\text{arcsinh}(cx)e^3c^7x}{7}\right.}{}$
default	$\frac{a\left(d^3c^7x+d^2c^7e\,x^3+\frac{3}{5}d\,c^7e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6}+\frac{b\left(\text{arcsinh}(cx)d^3c^7x+\text{arcsinh}(cx)d^2c^7e\,x^3+\frac{3\,\text{arcsinh}(cx)d\,c^7e^2x^5}{5}+\frac{\text{arcsinh}(cx)e^3c^7x}{7}\right.}{}$

```
[In] int((e*x^2+d)^3*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/7*e^3*x^7+3/5*d*e^2*x^5+d^2*e*x^3+d^3*x)+b/c*(1/7*c*arcsinh(c*x)*e^3*x
^7+3/5*c*arcsinh(c*x)*d*e^2*x^5+c*arcsinh(c*x)*d^2*e*x^3+arcsinh(c*x)*c*x*d
^3-1/35/c^6*(5*e^3*(1/7*c^6*x^6*(c^2*x^2+1)^(1/2)-6/35*c^4*x^4*(c^2*x^2+1)^
(1/2)+8/35*c^2*x^2*(c^2*x^2+1)^(1/2)-16/35*(c^2*x^2+1)^(1/2))+35*d^3*c^6*(c
^2*x^2+1)^(1/2)+21*d*c^2*e^2*(1/5*c^4*x^4*(c^2*x^2+1)^(1/2)-4/15*c^2*x^2*(c
^2*x^2+1)^(1/2)+8/15*(c^2*x^2+1)^(1/2))+35*d^2*c^4*e*(1/3*c^2*x^2*(c^2*x^2+
1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.09

$$\int \left(d+ex^2\right)^3 (a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{525\,ac^7e^3x^7+2205\,ac^7de^2x^5+3675\,ac^7d^2ex^3+3675\,ac^7d^3x+105\,(5\,bc^7e^3x^7+21\,bc^7de^2x^5+35\,bc^7d^2ex^3+\cdots}{\cdots}$$

```
[In] integrate((e*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3675*(525*a*c^7*e^3*x^7 + 2205*a*c^7*d*e^2*x^5 + 3675*a*c^7*d^2*e*x^3 + 3
675*a*c^7*d^3*x + 105*(5*b*c^7*e^3*x^7 + 21*b*c^7*d*e^2*x^5 + 35*b*c^7*d^2*
e*x^3 + 35*b*c^7*d^3*x)*log(c*x + sqrt(c^2*x^2 + 1)) - (75*b*c^6*e^3*x^6 +
3675*b*c^6*d^3 - 2450*b*c^4*d^2*e + 1176*b*c^2*d*e^2 + 9*(49*b*c^6*d*e^2 -
10*b*c^4*e^3)*x^4 - 240*b*e^3 + (1225*b*c^6*d^2*e - 588*b*c^4*d*e^2 + 120*b
*c^2*e^3)*x^2)*sqrt(c^2*x^2 + 1))/c^7
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.76

$$\int \left(d+ex^2\right)^3 (a+b\text{arcsinh}(cx))\,dx$$
$$=\begin{cases} ad^3x+ad^2ex^3+\frac{3ade^2x^5}{5}+\frac{ae^3x^7}{7}+bd^3x\operatorname{asinh}(cx)+bd^2ex^3\operatorname{asinh}(cx)+\frac{3bde^2x^5\operatorname{asinh}(cx)}{5}+\frac{be^3x^7\operatorname{asinh}(cx)}{7}-\cdots \\ a\left(d^3x+d^2ex^3+\frac{3de^2x^5}{5}+\frac{e^3x^7}{7}\right) \end{cases}$$

```
[In] integrate((e*x**2+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x + a*d**2*e*x**3 + 3*a*d*e**2*x**5/5 + a*e**3*x**7/7 + b
*d**3*x*asinh(c*x) + b*d**2*e*x**3*asinh(c*x) + 3*b*d*e**2*x**5*asinh(c*x)/
5 + b*e**3*x**7*asinh(c*x)/7 - b*d**3*sqrt(c**2*x**2 + 1)/c - b*d**2*e*x**2
*sqrt(c**2*x**2 + 1)/(3*c) - 3*b*d*e**2*x**4*sqrt(c**2*x**2 + 1)/(25*c) - b
*e**3*x**6*sqrt(c**2*x**2 + 1)/(49*c) + 2*b*d**2*e*sqrt(c**2*x**2 + 1)/(3*c
**3) + 4*b*d*e**2*x**2*sqrt(c**2*x**2 + 1)/(25*c**3) + 6*b*e**3*x**4*sqrt(c
**2*x**2 + 1)/(245*c**3) - 8*b*d*e**2*sqrt(c**2*x**2 + 1)/(25*c**5) - 8*b*e
**3*x**2*sqrt(c**2*x**2 + 1)/(245*c**5) + 16*b*e**3*sqrt(c**2*x**2 + 1)/(24
5*c**7), Ne(c, 0)), (a*(d**3*x + d**2*e*x**3 + 3*d*e**2*x**5/5 + e**3*x**7/
7), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.30

$$\int \left(d+ex^2\right)^3 (a+b\text{arcsinh}(cx))\,dx = \frac{1}{7}ae^3x^7 + \frac{3}{5}ade^2x^5 + ad^2ex^3 + \frac{1}{3}\left(3x^3\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bd^2e + \frac{1}{25}\left(15x^5\,\text{arsinh}\,(cx) - \left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)bde^2 + \frac{1}{245}\left(35x^7\,\text{arsinh}\,(cx) - \left(\frac{5\sqrt{c^2x^2+1}x^6}{c^2} - \frac{6\sqrt{c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{c^2x^2+1}x^2}{c^6} - \frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\right)be^3 + ad^3x + \frac{\left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)bd^3}{c}$$

```
[In] integrate((e*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*e^3*x^7 + 3/5*a*d*e^2*x^5 + a*d^2*e*x^3 + 1/3*(3*x^3*arcsinh(c*x) - c
*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*d^2*e + 1/25*(15*
x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c
^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*d*e^2 + 1/245*(35*x^7*arcsinh(c*x) - (5*
sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 +
1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*e^3 + a*d^3*x + (c*x*arcsinh(c*
x) - sqrt(c^2*x^2 + 1))*b*d^3/c
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^3 (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^3 (a+b\text{arcsinh}(cx))\,dx=\int \left(a+b\,\text{asinh}(c\,x)\right)\,\left(e\,x^2+d\right)^3\,dx$$

```
[In] int((a + b*asinh(c*x))*(d + e*x^2)^3,x)
[Out] int((a + b*asinh(c*x))*(d + e*x^2)^3, x)
```

3.608 $\int (d+ex^2)^2 (a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 147

$$\int (d+ex^2)^2 (a+b\text{arcsinh}(cx))\,dx = -\frac{b(15c^4d^2-10c^2de+3e^2)\sqrt{1+c^2x^2}}{15c^5} - \frac{2b(5c^2d-3e)\,e(1+c^2x^2)^{3/2}}{45c^5} - \frac{be^2(1+c^2x^2)^{5/2}}{25c^5} + d^2x(a+b\text{arcsinh}(cx)) + \frac{2}{3}dex^3(a+b\text{arcsinh}(cx)) + \frac{1}{5}e^2x^5(a+b\text{arcsinh}(cx))$$

```
[Out] -2/45*b*(5*c^2*d-3*e)*e*(c^2*x^2+1)^(3/2)/c^5-1/25*b*e^2*(c^2*x^2+1)^(5/2)/
c^5+d^2*x*(a+b*arcsinh(c*x))+2/3*d*e*x^3*(a+b*arcsinh(c*x))+1/5*e^2*x^5*(a+
b*arcsinh(c*x))-1/15*b*(15*c^4*d^2-10*c^2*d*e+3*e^2)*(c^2*x^2+1)^(1/2)/c^5
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {200, 5792, 12, 1261, 712}

$$\int (d+ex^2)^2 (a+b\text{arcsinh}(cx))\,dx = d^2x(a+b\text{arcsinh}(cx)) + \frac{2}{3}dex^3(a+b\text{arcsinh}(cx)) + \frac{1}{5}e^2x^5(a+b\text{arcsinh}(cx)) - \frac{2be(c^2x^2+1)^{3/2}(5c^2d-3e)}{45c^5} - \frac{be^2(c^2x^2+1)^{5/2}}{25c^5} - \frac{b\sqrt{c^2x^2+1}(15c^4d^2-10c^2de+3e^2)}{15c^5}$$

```
[In] Int[(d + e*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/15*(b*(15*c^4*d^2 - 10*c^2*d*e + 3*e^2)*Sqrt[1 + c^2*x^2])/c^5 - (2*b*(5
*c^2*d - 3*e)*e*(1 + c^2*x^2)^(3/2))/(45*c^5) - (b*e^2*(1 + c^2*x^2)^(5/2))
/(25*c^5) + d^2*x*(a + b*ArcSinh[c*x]) + (2*d*e*x^3*(a + b*ArcSinh[c*x]))/3
 + (e^2*x^5*(a + b*ArcSinh[c*x]))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 5792

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= d^2x(a+b\text{arcsinh}(cx))+\frac{2}{3}dex^3(a+b\text{arcsinh}(cx)) \\
&\quad +\frac{1}{5}e^2x^5(a+b\text{arcsinh}(cx))-(bc)\int\frac{x(15d^2+10dex^2+3e^2x^4)}{15\sqrt{1+c^2x^2}}\,dx \\
&= d^2x(a+b\text{arcsinh}(cx))+\frac{2}{3}dex^3(a+b\text{arcsinh}(cx)) \\
&\quad +\frac{1}{5}e^2x^5(a+b\text{arcsinh}(cx))-\frac{1}{15}(bc)\int\frac{x(15d^2+10dex^2+3e^2x^4)}{\sqrt{1+c^2x^2}}\,dx
\end{aligned}
$$

$$= d^2x(a+b\operatorname{arcsinh}(cx)) + \frac{2}{3}dex^3(a+b\operatorname{arcsinh}(cx)) + \frac{1}{5}e^2x^5(a+b\operatorname{arcsinh}(cx)) - \frac{1}{30}(bc)\text{Subst}\left(\int \frac{15d^2+10dex+3e^2x^2}{\sqrt{1+c^2x}}\,dx, x, x^2\right)$$

$$= d^2x(a+b\operatorname{arcsinh}(cx)) + \frac{2}{3}dex^3(a+b\operatorname{arcsinh}(cx)) + \frac{1}{5}e^2x^5(a+b\operatorname{arcsinh}(cx)) - \frac{1}{30}(bc)\text{Subst}\left(\int \left(\frac{15c^4d^2-10c^2de+3e^2}{c^4\sqrt{1+c^2x}} + \frac{2(5c^2d-3e)\,e\sqrt{1+c^2x}}{c^4} + \frac{3e^2(1+c^2x)^{3/2}}{c^4}\right) dx, x, x^2\right)$$

$$= -\frac{b(15c^4d^2-10c^2de+3e^2)\sqrt{1+c^2x^2}}{15c^5} - \frac{2b(5c^2d-3e)\,e(1+c^2x^2)^{3/2}}{45c^5} - \frac{be^2(1+c^2x^2)^{5/2}}{25c^5} + d^2x(a+b\operatorname{arcsinh}(cx)) + \frac{2}{3}dex^3(a+b\operatorname{arcsinh}(cx)) + \frac{1}{5}e^2x^5(a+b\operatorname{arcsinh}(cx))$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.85

$$\int (d+ex^2)^2 (a+b\operatorname{arcsinh}(cx))\,dx = \frac{1}{225}\left(15ax(15d^2+10dex^2+3e^2x^4) - \frac{b\sqrt{1+c^2x^2}(24e^2-4c^2e(25d+3ex^2)+c^4(225d^2+50dex^2+9e^2x^4))}{c^5} + 15bx(15d^2+10dex^2+3e^2x^4)\operatorname{arcsinh}(cx)\right)$$

```
[In] Integrate[(d + e*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (15*a*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - (b*Sqrt[1 + c^2*x^2]*(24*e^2 -
4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2 + 50*d*e*x^2 + 9*e^2*x^4)))/c^5 + 1
5*b*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4)*ArcSinh[c*x])/225
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.28

method	result
parts	$a\left(\frac{1}{5}e^2x^5+\frac{2}{3}de\,x^3+d^2x\right)+\frac{b\left(\frac{c\,\mathrm{arcsinh}(cx)e^2x^5}{5}+\frac{2c\,\mathrm{arcsinh}(cx)de\,x^3}{3}+\mathrm{arcsinh}(cx)cx\,d^2-\frac{3e^2\left(\frac{c^4x^4\sqrt{c^2x^2+1}}{5}-\frac{4c \cdots}{}\right)}{}\right)}{c \cdots}$
derivativedivides	$\frac{\frac{a\left(d^2c^5x+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b\left(\mathrm{arcsinh}(cx)d^2c^5x+\frac{2\,\mathrm{arcsinh}(cx)d\,c^5e\,x^3}{3}+\frac{\mathrm{arcsinh}(cx)e^2c^5x^5}{5}-\frac{e^2\left(\frac{c^4x^4\sqrt{c^2x^2+1}}{5}-\frac{4c^2x^2\sqrt{c^2 \cdots}}{15}\right)}{5}\cdots\right)}{c^4}}{c}$
default	$\frac{\frac{a\left(d^2c^5x+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b\left(\mathrm{arcsinh}(cx)d^2c^5x+\frac{2\,\mathrm{arcsinh}(cx)d\,c^5e\,x^3}{3}+\frac{\mathrm{arcsinh}(cx)e^2c^5x^5}{5}-\frac{e^2\left(\frac{c^4x^4\sqrt{c^2x^2+1}}{5}-\frac{4c^2x^2\sqrt{c^2 \cdots}}{15}\right)}{5}\cdots\right)}{c^4}}{c}$

```
[In] int((e*x^2+d)^2*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/5*e^2*x^5+2/3*d*e*x^3+d^2*x)+b/c*(1/5*c*arcsinh(c*x)*e^2*x^5+2/3*c*arc
sinh(c*x)*d*e*x^3+arcsinh(c*x)*c*x*d^2-1/15/c^4*(3*e^2*(1/5*c^4*x^4*(c^2*x^
2+1)^(1/2)-4/15*c^2*x^2*(c^2*x^2+1)^(1/2)+8/15*(c^2*x^2+1)^(1/2))+15*d^2*c^
4*(c^2*x^2+1)^(1/2)+10*d*c^2*e*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+
1)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.11

$$\int \left(d+ex^2\right)^2\left(a+b\text{arcsinh}(cx)\right)dx$$
$$=\frac{45\,ac^5e^2x^5+150\,ac^5dex^3+225\,ac^5d^2x+15\left(3\,bc^5e^2x^5+10\,bc^5dex^3+15\,bc^5d^2x\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-(9 \cdots}{225\,c^5}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/225*(45*a*c^5*e^2*x^5 + 150*a*c^5*d*e*x^3 + 225*a*c^5*d^2*x + 15*(3*b*c^5
*e^2*x^5 + 10*b*c^5*d*e*x^3 + 15*b*c^5*d^2*x)*log(c*x + sqrt(c^2*x^2 + 1))
- (9*b*c^4*e^2*x^4 + 225*b*c^4*d^2 - 100*b*c^2*d*e + 24*b*e^2 + 2*(25*b*c^4
*d*e - 6*b*c^2*e^2)*x^2)*sqrt(c^2*x^2 + 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.63

$$\int \left(d+ex^2\right)^2 (a+b\text{arcsinh}(cx))\,dx$$
$$= \begin{cases} ad^2x + \frac{2adex^3}{3} + \frac{ae^2x^5}{5} + bd^2x\,\text{asinh}\,(cx) + \frac{2bdex^3\,\text{asinh}\,(cx)}{3} + \frac{be^2x^5\,\text{asinh}\,(cx)}{5} - \frac{bd^2\sqrt{c^2x^2+1}}{c} - \frac{2bdex^2\sqrt{c^2x^2+1}}{9c} - \frac{be}{} \\ a\left(d^2x + \frac{2dex^3}{3} + \frac{e^2x^5}{5}\right) \end{cases}$$

```
[In] integrate((e*x**2+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x + 2*a*d*e*x**3/3 + a*e**2*x**5/5 + b*d**2*x*asinh(c*x)
+ 2*b*d*e*x**3*asinh(c*x)/3 + b*e**2*x**5*asinh(c*x)/5 - b*d**2*sqrt(c**2*x
**2 + 1)/c - 2*b*d*e*x**2*sqrt(c**2*x**2 + 1)/(9*c) - b*e**2*x**4*sqrt(c**2
*x**2 + 1)/(25*c) + 4*b*d*e*sqrt(c**2*x**2 + 1)/(9*c**3) + 4*b*e**2*x**2*sq
rt(c**2*x**2 + 1)/(75*c**3) - 8*b*e**2*sqrt(c**2*x**2 + 1)/(75*c**5), Ne(c,
 0)), (a*(d**2*x + 2*d*e*x**3/3 + e**2*x**5/5), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.22

$$\int \left(d+ex^2\right)^2 (a+b\text{arcsinh}(cx))\,dx$$
$$= \frac{1}{5}\,ae^2x^5 + \frac{2}{3}\,adex^3 + \frac{2}{9}\left(3\,x^3\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)bde$$
$$+ \frac{1}{75}\left(15\,x^5\,\text{arsinh}\,(cx) - \left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\,\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\right)be^2$$
$$+ ad^2x + \frac{\left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)bd^2}{c}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 +
 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*d*e + 1/75*(15*x^5*arcsinh(c*x) -
 (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^
2 + 1)/c^6)*c)*b*e^2 + a*d^2*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*d
^2/c
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^2 (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^2 (a+b\text{arcsinh}(cx))\,dx = \int (a+b\,\text{asinh}(c\,x))\,\left(e\,x^2+d\right)^2\,dx$$

```
[In] int((a + b*asinh(c*x))*(d + e*x^2)^2,x)
```

```
[Out] int((a + b*asinh(c*x))*(d + e*x^2)^2, x)
```

3.609 $\int (d+ex^2) (a+b\text{arcsinh}(cx)) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 81

$$\int (d+ex^2) (a+b\text{arcsinh}(cx)) \, dx = -\frac{b(3c^2d-e)\sqrt{1+c^2x^2}}{3c^3}-\frac{be(1+c^2x^2)^{3/2}}{9c^3} + dx(a+b\text{arcsinh}(cx))+\frac{1}{3}ex^3(a+b\text{arcsinh}(cx))$$

```
[Out] -1/9*b*e*(c^2*x^2+1)^(3/2)/c^3+d*x*(a+b*arcsinh(c*x))+1/3*e*x^3*(a+b*arcsin
h(c*x))-1/3*b*(3*c^2*d-e)*(c^2*x^2+1)^(1/2)/c^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5792, 455, 45}

$$\int (d+ex^2) (a+b\text{arcsinh}(cx)) \, dx = dx(a+b\text{arcsinh}(cx))+\frac{1}{3}ex^3(a+b\text{arcsinh}(cx)) -\frac{b\sqrt{c^2x^2+1}(3c^2d-e)}{3c^3}-\frac{be(c^2x^2+1)^{3/2}}{9c^3}$$

```
[In] Int[(d + e*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/3*(b*(3*c^2*d - e)*Sqrt[1 + c^2*x^2])/c^3 - (b*e*(1 + c^2*x^2)^(3/2))/(9
*c^3) + d*x*(a + b*ArcSinh[c*x]) + (e*x^3*(a + b*ArcSinh[c*x]))/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5792

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= dx(a+b\text{arcsinh}(cx))+\frac{1}{3}ex^3(a+b\text{arcsinh}(cx))-(bc)\int\frac{x\left(d+\frac{ex^2}{3}\right)}{\sqrt{1+c^2x^2}}\,dx\\
&= dx(a+b\text{arcsinh}(cx))+\frac{1}{3}ex^3(a+b\text{arcsinh}(cx))-\frac{1}{2}(bc)\text{Subst}\left(\int\frac{d+\frac{ex}{3}}{\sqrt{1+c^2x}}\,dx,x,x^2\right)\\
&= dx(a+b\text{arcsinh}(cx))+\frac{1}{3}ex^3(a+b\text{arcsinh}(cx))\\
&\quad-\frac{1}{2}(bc)\text{Subst}\left(\int\left(\frac{3c^2d-e}{3c^2\sqrt{1+c^2x}}+\frac{e\sqrt{1+c^2x}}{3c^2}\right)\,dx,x,x^2\right)\\
&= -\frac{b(3c^2d-e)\sqrt{1+c^2x^2}}{3c^3}-\frac{be(1+c^2x^2)^{3/2}}{9c^3}+dx(a+b\text{arcsinh}(cx))+\frac{1}{3}ex^3(a+b\text{arcsinh}(cx))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int(d+ex^2)\,(a+b\text{arcsinh}(cx))\,dx=\frac{1}{9}\left(3ax(3d+ex^2)-\frac{b\sqrt{1+c^2x^2}(-2e+c^2(9d+ex^2))}{c^3}+3bx(3d+ex^2)\,\text{arcsinh}(cx)\right)$$

```
[In] Integrate[(d + e*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (3*a*x*(3*d + e*x^2) - (b*Sqrt[1 + c^2*x^2]*(-2*e + c^2*(9*d + e*x^2)))/c^3
 + 3*b*x*(3*d + e*x^2)*ArcSinh[c*x])/9
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.20

method	result	size
parts	$a\left(\frac{1}{3}x^{3}e+dx\right)+\frac{b\left(\frac{c\,\operatorname{arcsinh}(cx)x^{3}e}{3}+\operatorname{arcsinh}(cx)dcx-\frac{e\left(\frac{c^{2}x^{2}\sqrt{c^{2}x^{2}+1}}{3}-\frac{2\sqrt{c^{2}x^{2}+1}}{3}\right)+3d\,c^{2}\sqrt{c^{2}x^{2}+1}}{3c^{2}}\right)}{c}$	97
derivativedivides	$\frac{\frac{a\left(d\,c^{3}x+\frac{1}{3}e\,c^{3}x^{3}\right)}{c^{2}}+\frac{b\left(\operatorname{arcsinh}(cx)d\,c^{3}x+\frac{\operatorname{arcsinh}(cx)e\,c^{3}x^{3}}{3}-\frac{e\left(\frac{c^{2}x^{2}\sqrt{c^{2}x^{2}+1}}{3}-\frac{2\sqrt{c^{2}x^{2}+1}}{3}\right)}{3}-d\,c^{2}\sqrt{c^{2}x^{2}+1}\right)}{c^{2}}}{c}$	109
default	$\frac{\frac{a\left(d\,c^{3}x+\frac{1}{3}e\,c^{3}x^{3}\right)}{c^{2}}+\frac{b\left(\operatorname{arcsinh}(cx)d\,c^{3}x+\frac{\operatorname{arcsinh}(cx)e\,c^{3}x^{3}}{3}-\frac{e\left(\frac{c^{2}x^{2}\sqrt{c^{2}x^{2}+1}}{3}-\frac{2\sqrt{c^{2}x^{2}+1}}{3}\right)}{3}-d\,c^{2}\sqrt{c^{2}x^{2}+1}\right)}{c^{2}}}{c}$	109

```
[In] int((e*x^2+d)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
[Out] a*(1/3*x^3*e+d*x)+b/c*(1/3*c*arcsinh(c*x)*x^3*e+arcsinh(c*x)*d*c*x-1/3/c^2*
(e*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))+3*d*c^2*(c^2*x^2+1
)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int \left(d+ex^{2}\right)\left(a+b\operatorname{arcsinh}(cx)\right)dx$$
$$=\frac{3\,ac^{3}ex^{3}+9\,ac^{3}dx+3\,(bc^{3}ex^{3}+3\,bc^{3}dx)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)-(bc^{2}ex^{2}+9\,bc^{2}d-2\,be)\sqrt{c^{2}x^{2}+1}}{9\,c^{3}}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] 1/9*(3*a*c^3*e*x^3 + 9*a*c^3*d*x + 3*(b*c^3*e*x^3 + 3*b*c^3*d*x)*log(c*x +
sqrt(c^2*x^2 + 1)) - (b*c^2*e*x^2 + 9*b*c^2*d - 2*b*e)*sqrt(c^2*x^2 + 1))/c
^3
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.35

$$\int (d+ex^2)(a+b\text{arcsinh}(cx))\,dx$$
$$=\begin{cases} adx+\frac{aex^3}{3}+bdx\,\text{asinh}\,(cx)+\frac{bex^3\,\text{asinh}\,(cx)}{3}-\frac{bd\sqrt{c^2x^2+1}}{c}-\frac{bex^2\sqrt{c^2x^2+1}}{9c}+\frac{2be\sqrt{c^2x^2+1}}{9c^3} & \text{for } c\neq 0 \\ a\left(dx+\frac{ex^3}{3}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((e*x**2+d)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*d*x + a*e*x**3/3 + b*d*x*asinh(c*x) + b*e*x**3*asinh(c*x)/3 -
b*d*sqrt(c**2*x**2 + 1)/c - b*e*x**2*sqrt(c**2*x**2 + 1)/(9*c) + 2*b*e*sqrt
(c**2*x**2 + 1)/(9*c**3), Ne(c, 0)), (a*(d*x + e*x**3/3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12

$$\int (d+ex^2)(a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{1}{3}aex^3+\frac{1}{9}\left(3\,x^3\,\text{arsinh}\,(cx)-c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)be$$
$$+adx+\frac{\left(cx\,\text{arsinh}\,(cx)-\sqrt{c^2x^2+1}\right)bd}{c}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sq
rt(c^2*x^2 + 1)/c^4))*b*e + a*d*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*
b*d/c
```

Giac [F(-2)]

Exception generated.

$$\int (d+ex^2)(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d + ex^2) (a + b\text{arcsinh}(cx)) \, dx = \int (a + b \, \text{asinh}(c \, x)) \, (e \, x^2 + d) \, dx$$

```
[In] int((a + b*asinh(c*x))*(d + e*x^2),x)
[Out] int((a + b*asinh(c*x))*(d + e*x^2), x)
```

3.610 $\int (a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 8, antiderivative size = 30

$$\int (a+b\text{arcsinh}(cx))\,dx = ax - \frac{b\sqrt{1+c^2x^2}}{c} + bx\text{arcsinh}(cx)$$

[Out] a*x+b*x*arcsinh(c*x)-b*(c^2*x^2+1)^(1/2)/c

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5772, 267}

$$\int (a+b\text{arcsinh}(cx))\,dx = ax + bx\text{arcsinh}(cx) - \frac{b\sqrt{c^2x^2+1}}{c}$$

```
[In] Int[a + b*ArcSinh[c*x],x]
```

```
[Out] a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= ax + b\int \operatorname{arcsinh}(cx)\,dx \\
&= ax + bx\operatorname{arcsinh}(cx) - (bc)\int \frac{x}{\sqrt{1+c^2x^2}}\,dx \\
&= ax - \frac{b\sqrt{1+c^2x^2}}{c} + bx\operatorname{arcsinh}(cx)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (a + b\operatorname{arcsinh}(cx))\,dx = ax - \frac{b\sqrt{1+c^2x^2}}{c} + bx\operatorname{arcsinh}(cx)$$

```
[In] Integrate[a + b*ArcSinh[c*x],x]
[Out] a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

method	result	size
default	$ax+\frac{b\left(\operatorname{arcsinh}(cx)cx-\sqrt{c^2x^2+1}\right)}{c}$	31
parts	$ax+\frac{b\left(\operatorname{arcsinh}(cx)cx-\sqrt{c^2x^2+1}\right)}{c}$	31
derivativedivides	$\frac{cxa+b\left(\operatorname{arcsinh}(cx)cx-\sqrt{c^2x^2+1}\right)}{c}$	33

```
[In] int(a+b*arcsinh(c*x),x,method=_RETURNVERBOSE)
[Out] a*x+b/c*(arcsinh(c*x)*c*x-(c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int (a + b\text{arcsinh}(cx))\, dx = \frac{bcx \log \left(cx + \sqrt{c^2x^2 + 1}\right) + acx - \sqrt{c^2x^2 + 1}b}{c}$$

```
[In] integrate(a+b*arcsinh(c*x),x, algorithm="fricas")
[Out] (b*c*x*log(c*x + sqrt(c^2*x^2 + 1)) + a*c*x - sqrt(c^2*x^2 + 1)*b)/c
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int (a + b\text{arcsinh}(cx))\, dx = ax + b \left(\begin{cases} x \operatorname{asinh}(cx) - \frac{\sqrt{c^2x^2+1}}{c} & \text{for } c \neq 0 \\ 0 & \text{otherwise} \end{cases}\right)$$

```
[In] integrate(a+b*asinh(c*x),x)
[Out] a*x + b*Piecewise((x*asinh(c*x) - sqrt(c**2*x**2 + 1)/c, Ne(c, 0)), (0, Tru
e))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (a + b\text{arcsinh}(cx))\, dx = ax + \frac{\left(cx \operatorname{arsinh}(cx) - \sqrt{c^2x^2 + 1}\right)b}{c}$$

```
[In] integrate(a+b*arcsinh(c*x),x, algorithm="maxima")
[Out] a*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b/c
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int (a + b\text{arcsinh}(cx))\,dx = \left(x\log\left(cx + \sqrt{c^2x^2 + 1}\right) - \frac{\sqrt{c^2x^2 + 1}}{c}\right)b + ax$$

```
[In] integrate(a+b*arcsinh(c*x),x, algorithm="giac")
[Out] (x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*b + a*x
```

Mupad [B] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int (a + b\text{arcsinh}(cx))\,dx = a\,x - \frac{b\,\sqrt{c^2\,x^2 + 1}}{c} + b\,x\,\text{asinh}(c\,x)$$

```
[In] int(a + b*asinh(c*x),x)
[Out] a*x - (b*(c^2*x^2 + 1)^(1/2))/c + b*x*asinh(c*x)
```

3.611 $\int \frac{a+b\text{arcsinh}(cx)}{d+ex^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 485

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{d+ex^2}\,dx &= \frac{(a+b\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}\\
&-\frac{(a+b\text{arcsinh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}\\
&+\frac{(a+b\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}\\
&-\frac{(a+b\text{arcsinh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}\\
&-\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}\\
&-\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}\end{aligned}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))*ln(1-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(
-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*
x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1
/2*(a+b*arcsinh(c*x))*ln(1-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-
c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x
^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/
2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/
2)))/(-d)^(1/2)/e^(1/2)+1/2*b*polylog(2,(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*
(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*b*polylog(2,-(c*x+(c^2
```

```
*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+
1/2*b*polylog(2,(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1
/2)))/(-d)^(1/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5793, 5827, 5680, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{d+ex^2}\,dx=&\ \frac{(a+b\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}\\&-\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}+1\right)}{2\sqrt{-d}\sqrt{e}}\\&+\frac{(a+b\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}}\\&-\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}+1\right)}{2\sqrt{-d}\sqrt{e}}\\&-\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}\\&-\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{-d}c+\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{-d}c+\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}\end{aligned}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x^2),x]
```

```
[Out] ((a + b*ArcSinh[c*x])*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-
(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[c*x])*Log[1 + (Sqrt[
e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e])
 + ((a + b*ArcSinh[c*x])*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqr
t[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[c*x])*Log[1 + (Sq
rt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[
e]) - (b*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d)
+ e]))])/(2*Sqrt[-d]*Sqrt[e]) + (b*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*S
qrt[-d] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - (b*PolyLog[2, -((Sqr
t[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e]))])/(2*Sqrt[-d]*Sqrt[
e]) + (b*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) +
e])])/(2*Sqrt[-d]*Sqrt[e])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sqrt{-d}(a+b\text{arcsinh}(cx))}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}(a+b\text{arcsinh}(cx))}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx$$

$$= -\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{-d}-\sqrt{e}x}\,dx}{2\sqrt{-d}} - \frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{-d}+\sqrt{e}x}\,dx}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{c\sqrt{-d}-\sqrt{e}\sinh(x)}\,dx, x, \text{arcsinh}(cx)\right)}{2\sqrt{-d}} - \frac{\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{c\sqrt{-d}+\sqrt{e}\sinh(x)}\,dx, x, \text{arcsinh}(cx)\right)}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{c\sqrt{-d}-\sqrt{-c^2d+e}-\sqrt{e}e^x}\,dx, x, \text{arcsinh}(cx)\right)}{2\sqrt{-d}} - \frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{c\sqrt{-d}+\sqrt{-c^2d+e}-\sqrt{e}e^x}\,dx, x, \text{arcsinh}(cx)\right)}{2\sqrt{-d}} - \frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{c\sqrt{-d}-\sqrt{-c^2d+e}+\sqrt{e}e^x}\,dx, x, \text{arcsinh}(cx)\right)}{2\sqrt{-d}} - \frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{c\sqrt{-d}+\sqrt{-c^2d+e}+\sqrt{e}e^x}\,dx, x, \text{arcsinh}(cx)\right)}{2\sqrt{-d}}$$

$$= \frac{(a+b\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\text{arcsinh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+b\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\text{arcsinh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{b\text{Subst}\left(\int \log\left(1-\frac{\sqrt{e}e^x}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)dx, x, \text{arcsinh}(cx)\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b\text{Subst}\left(\int \log\left(1+\frac{\sqrt{e}e^x}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)dx, x, \text{arcsinh}(cx)\right)}{2\sqrt{-d}\sqrt{e}} - \frac{b\text{Subst}\left(\int \log\left(1-\frac{\sqrt{e}e^x}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)dx, x, \text{arcsinh}(cx)\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b\text{Subst}\left(\int \log\left(1+\frac{\sqrt{e}e^x}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)dx, x, \text{arcsinh}(cx)\right)}{2\sqrt{-d}\sqrt{e}}$$

$$= \frac{(a+b\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\text{arcsinh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+b\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\text{arcsinh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{b\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{e}x}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{x}dx,x,e^{\text{arcsinh}(cx)}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{e}x}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{x}dx,x,e^{\text{arcsinh}(cx)}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{b\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{e}x}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{x}dx,x,e^{\text{arcsinh}(cx)}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{e}x}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{x}dx,x,e^{\text{arcsinh}(cx)}\right)}{2\sqrt{-d}\sqrt{e}}$$

$$= \frac{(a+b\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\text{arcsinh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+b\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\text{arcsinh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 434, normalized size of antiderivative = 0.89

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{d+ex^2}\,dx$$
$$= \frac{2a\sqrt{-d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)-b\sqrt{d}\operatorname{arcsinh}(cx)\log\left(1+\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)+b\sqrt{d}\operatorname{arcsinh}(cx)\log\left(1+\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx}}{-c\sqrt{-d}+\sqrt{-c^2d}}\right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2),x]
```

```
[Out] (2*a*Sqrt[-d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - b*Sqrt[d]*ArcSinh[c*x]*Log[1 +
(Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])] + b*Sqrt[d]*Arc
Sinh[c*x]*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) +
 e])] + b*Sqrt[d]*ArcSinh[c*x]*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d]
 + Sqrt[-(c^2*d) + e])] - b*Sqrt[d]*ArcSinh[c*x]*Log[1 + (Sqrt[e]*E^ArcSinh
[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])] + b*Sqrt[d]*PolyLog[2, (Sqrt[e]*E
^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])] - b*Sqrt[d]*PolyLog[2, (S
qrt[e]*E^ArcSinh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) + e])] - b*Sqrt[d]*Po
lyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e]))] + b
*Sqrt[d]*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) +
e])])/(2*Sqrt[-d^2]*Sqrt[e])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.91 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.46

method	result
parts	$\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+\frac{bc\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d-2e\right)\_Z^2+e\right)}\frac{\mathrm{arcsinh}(cx)\ln\left(\frac{\_R1-cx-\sqrt{c^2x^2+1}}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-cx-}{\_R}\right.}{\_R1\left(\_R1^2e+2c^2d-e\right)}\right.}{2}$
derivativedivides	$\frac{\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+bc^2\left(\left(\frac{\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d-2e\right)\_Z^2+e\right)}\frac{\mathrm{arcsinh}(cx)\ln\left(\frac{\_R1-cx-\sqrt{c^2x^2+1}}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-cx-}{\_R}\right.}{\_R1\left(\_R1^2e+2c^2d-e\right)}}{2}\right.\right.}{}$
default	$\frac{\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+bc^2\left(\left(\frac{\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d-2e\right)\_Z^2+e\right)}\frac{\mathrm{arcsinh}(cx)\ln\left(\frac{\_R1-cx-\sqrt{c^2x^2+1}}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-cx-}{\_R}\right.}{\_R1\left(\_R1^2e+2c^2d-e\right)}}{2}\right.\right.}{}$

```
[In] int((a+b*arcsinh(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] a/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+1/2*b*c*sum(1/_R1/(_R1^2*e+2*c^2*d-e)
*(arcsinh(c*x)*ln((_R1-c*x-(c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x-(c^2*x^2+
1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d-2*e)*_Z^2+e))+1/2*b*c*sum(_R1/(_
R1^2*e+2*c^2*d-e)*(arcsinh(c*x)*ln((_R1-c*x-(c^2*x^2+1)^(1/2))/_R1)+dilog((
_R1-c*x-(c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d-2*e)*_Z^2+e))
```

Fricas [F]

$$\int \frac{a+b\mathrm{arcsinh}(cx)}{d+ex^2}\,dx=\int \frac{b\,\mathrm{arsinh}\,(cx)+a}{ex^2+d}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{d + ex^2} dx = \int \frac{a + b\,\text{asinh}\,(cx)}{d + ex^2} dx$$

```
[In] integrate((a+b*asinh(c*x))/(e*x**2+d),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{arcsinh}(cx)}{d + ex^2} dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{d + ex^2} dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{ex^2 + d} dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{d + ex^2} dx = \int \frac{a + b\,\text{asinh}(c\,x)}{e\,x^2 + d} dx$$

```
[In] int((a + b*asinh(c*x))/(d + e*x^2),x)
```

```
[Out] int((a + b*asinh(c*x))/(d + e*x^2), x)
```

3.612 $\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 707

$$\begin{aligned}
\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^2} dx = & -\frac{a+b\text{arcsinh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{a+b\text{arcsinh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)} \\
& - \frac{bc\arctan\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}} - \frac{bc\arctan\left(\frac{\sqrt{e}+c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}} \\
& - \frac{(a+b\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
& + \frac{(a+b\text{arcsinh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
& - \frac{(a+b\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
& + \frac{(a+b\text{arcsinh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
& + \frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
& + \frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}
\end{aligned}$$

```
[Out] -1/4*(a+b*arcsinh(c*x))*ln(1-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-
(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2
*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-
```

```
1/4*(a+b*arcsinh(c*x))*ln(1-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(
-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*
x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1
/4*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1
/2)))/(-d)^(3/2)/e^(1/2)-1/4*b*polylog(2,(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c
*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*b*polylog(2,-(c*x+(c^
2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)
-1/4*b*polylog(2,(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(
1/2)))/(-d)^(3/2)/e^(1/2)-1/4*b*c*arctan((-c^2*x*(-d)^(1/2)+e^(1/2))/(c^2*d
-e)^(1/2)/(c^2*x^2+1)^(1/2))/d/(c^2*d-e)^(1/2)/e^(1/2)-1/4*b*c*arctan((c^2*
x*(-d)^(1/2)+e^(1/2))/(c^2*d-e)^(1/2)/(c^2*x^2+1)^(1/2))/d/(c^2*d-e)^(1/2)/
e^(1/2)+1/4*(-a-b*arcsinh(c*x))/d/e^(1/2)/((-d)^(1/2)-x*e^(1/2))+1/4*(a+b*a
rcsinh(c*x))/d/e^(1/2)/((-d)^(1/2)+x*e^(1/2))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 707, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5793, 5828, 739, 210, 5827, 5680, 2221, 2317, 2438}

$$\begin{aligned}
\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^2}\,dx = &-\frac{(a+b\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
&+\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}+1\right)}{4(-d)^{3/2}\sqrt{e}} \\
&-\frac{(a+b\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
&+\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}+1\right)}{4(-d)^{3/2}\sqrt{e}} \\
&-\frac{a+b\text{arcsinh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\text{arcsinh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)} \\
&+\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
&+\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{-d}c+\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{-d}c+\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
&-\frac{bc\arctan\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{c^2x^2+1}\sqrt{c^2d-e}}\right)}{4d\sqrt{e}\sqrt{c^2d-e}}-\frac{bc\arctan\left(\frac{c^2\sqrt{-d}x+\sqrt{e}}{\sqrt{c^2x^2+1}\sqrt{c^2d-e}}\right)}{4d\sqrt{e}\sqrt{c^2d-e}}
\end{aligned}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x^2)^2,x]
```

```
[Out] -1/4*(a + b*ArcSinh[c*x])/(d*Sqrt[e]*(Sqrt[-d] - Sqrt[e]*x)) + (a + b*ArcSi
nh[c*x])/(4*d*Sqrt[e]*(Sqrt[-d] + Sqrt[e]*x)) - (b*c*ArcTan[(Sqrt[e] - c^2*
Sqrt[-d]*x)/(Sqrt[c^2*d - e]*Sqrt[1 + c^2*x^2])])/(4*d*Sqrt[c^2*d - e]*Sqrt
[e]) - (b*c*ArcTan[(Sqrt[e] + c^2*Sqrt[-d]*x)/(Sqrt[c^2*d - e]*Sqrt[1 + c^2
*x^2])])/(4*d*Sqrt[c^2*d - e]*Sqrt[e]) - ((a + b*ArcSinh[c*x])*Log[1 - (Sqr
t[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt
[e]) + ((a + b*ArcSinh[c*x])*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] -
 Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) - ((a + b*ArcSinh[c*x])*Log[1
 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/
2)*Sqrt[e]) + ((a + b*ArcSinh[c*x])*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqr
t[-d] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[2, -((Sqr
t[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e]))])/(4*(-d)^(3/2)*Sqr
t[e]) - (b*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d)
+ e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/(
c*Sqrt[-d] + Sqrt[-(c^2*d) + e]))])/(4*(-d)^(3/2)*Sqrt[e]) - (b*PolyLog[2,
(Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*
Sqrt[e])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{e(a+b\text{arcsinh}(cx))}{4d\left(\sqrt{-d}\sqrt{e}-ex\right)^2}-\frac{e(a+b\text{arcsinh}(cx))}{4d\left(\sqrt{-d}\sqrt{e}+ex\right)^2}-\frac{e(a+b\text{arcsinh}(cx))}{2d\left(-de-e^2x^2\right)}\right)dx$$

$$=-\frac{e\int\frac{a+b\text{arcsinh}(cx)}{\left(\sqrt{-d}\sqrt{e}-ex\right)^2}dx}{4d}-\frac{e\int\frac{a+b\text{arcsinh}(cx)}{\left(\sqrt{-d}\sqrt{e}+ex\right)^2}dx}{4d}-\frac{e\int\frac{a+b\text{arcsinh}(cx)}{-de-e^2x^2}dx}{2d}$$

$$=-\frac{a+b\text{arcsinh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\text{arcsinh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{(bc)\int\frac{1}{\left(\sqrt{-d}\sqrt{e}-ex\right)\sqrt{1+c^2x^2}}dx}{4d}$$

$$-\frac{(bc)\int\frac{1}{\left(\sqrt{-d}\sqrt{e}+ex\right)\sqrt{1+c^2x^2}}dx}{4d}-\frac{e\int\left(-\frac{\sqrt{-d}(a+b\text{arcsinh}(cx))}{2de\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{\sqrt{-d}(a+b\text{arcsinh}(cx))}{2de\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx}{2d}$$

$$= -\frac{a+b\text{arcsinh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{a+b\text{arcsinh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)} + \frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{-d}-\sqrt{e}x}\,dx}{4(-d)^{3/2}} + \frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{-d}+\sqrt{e}x}\,dx}{4(-d)^{3/2}} - \frac{(bc)\text{Subst}\left(\int \frac{1}{-c^2de+e^2-x^2}\,dx, x, \frac{-e-c^2\sqrt{-d}\sqrt{e}x}{\sqrt{1+c^2x^2}}\right)}{4d} + \frac{(bc)\text{Subst}\left(\int \frac{1}{-c^2de+e^2-x^2}\,dx, x, \frac{e-c^2\sqrt{-d}\sqrt{e}x}{\sqrt{1+c^2x^2}}\right)}{4d}$$

$$= -\frac{a+b\text{arcsinh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{a+b\text{arcsinh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{bc\arctan\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}} - \frac{bc\arctan\left(\frac{\sqrt{e}+c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}} + \frac{\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{c\sqrt{-d}-\sqrt{e}\sinh(x)}\,dx, x, \text{arcsinh}(cx)\right)}{4(-d)^{3/2}} + \frac{\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{c\sqrt{-d}+\sqrt{e}\sinh(x)}\,dx, x, \text{arcsinh}(cx)\right)}{4(-d)^{3/2}}$$

$$= -\frac{a+b\text{arcsinh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{a+b\text{arcsinh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{bc\arctan\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}} - \frac{bc\arctan\left(\frac{\sqrt{e}+c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}} + \frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{c\sqrt{-d}-\sqrt{-c^2d+e}-\sqrt{e}e^x}\,dx, x, \text{arcsinh}(cx)\right)}{4(-d)^{3/2}} + \frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{c\sqrt{-d}+\sqrt{-c^2d+e}-\sqrt{e}e^x}\,dx, x, \text{arcsinh}(cx)\right)}{4(-d)^{3/2}} + \frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{c\sqrt{-d}-\sqrt{-c^2d+e}+\sqrt{e}e^x}\,dx, x, \text{arcsinh}(cx)\right)}{4(-d)^{3/2}} + \frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{c\sqrt{-d}+\sqrt{-c^2d+e}+\sqrt{e}e^x}\,dx, x, \text{arcsinh}(cx)\right)}{4(-d)^{3/2}}$$

$$= -\frac{a+b\text{arcsinh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{a+b\text{arcsinh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{bc\arctan\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}}$$

$$- \frac{bc\arctan\left(\frac{\sqrt{e}+c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}} - \frac{(a+b\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{(a+b\text{arcsinh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$- \frac{(a+b\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{(a+b\text{arcsinh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1-\frac{\sqrt{e}e^x}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)\,dx, x, \text{arcsinh}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$- \frac{b\text{Subst}\left(\int\log\left(1+\frac{\sqrt{e}e^x}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)\,dx, x, \text{arcsinh}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1-\frac{\sqrt{e}e^x}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)\,dx, x, \text{arcsinh}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$- \frac{b\text{Subst}\left(\int\log\left(1+\frac{\sqrt{e}e^x}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)\,dx, x, \text{arcsinh}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$= -\frac{a+b\text{arcsinh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{a+b\text{arcsinh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{bc\arctan\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}}$$

$$- \frac{bc\arctan\left(\frac{\sqrt{e}+c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}} - \frac{(a+b\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{(a+b\text{arcsinh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$- \frac{(a+b\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{(a+b\text{arcsinh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{b\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{e}x}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{x}dx, x, e^{\text{arcsinh}(cx)}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$- \frac{b\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{e}x}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{x}dx, x, e^{\text{arcsinh}(cx)}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{b\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{e}x}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{x}dx, x, e^{\text{arcsinh}(cx)}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$- \frac{b\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{e}x}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{x}dx, x, e^{\text{arcsinh}(cx)}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$= -\frac{a+b\operatorname{arcsinh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{a+b\operatorname{arcsinh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{bc\arctan\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}}$$
$$- \frac{bc\arctan\left(\frac{\sqrt{e}+c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}} - \frac{(a+b\operatorname{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$+ \frac{(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$- \frac{(a+b\operatorname{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$+ \frac{(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$+ \frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$+ \frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.32 (sec) , antiderivative size = 622, normalized size of antiderivative = 0.88

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex^2)^2}\,dx = \frac{1}{2}\left(\frac{ax}{d^2+dex^2} + \frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\sqrt{e}}\right.$$
$$+ \frac{b\left(-2\sqrt{d}\left(-\frac{\operatorname{arcsinh}(cx)}{i\sqrt{d}+\sqrt{e}x} + \frac{c\arctan\left(\frac{\sqrt{e}-ic^2\sqrt{d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{\sqrt{c^2d-e}}\right) + 2i\sqrt{d}\left(\frac{\operatorname{arcsinh}(cx)}{\sqrt{d}+i\sqrt{e}x} + \frac{c\operatorname{arctanh}\left(\frac{i\sqrt{e}-c^2\sqrt{d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{\sqrt{c^2d-e}}\right) + \cdots\right.}{\cdots}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2)^2,x]
```

```
[Out] ((a*x)/(d^2 + d*e*x^2) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e])
+ (b*(-2*Sqrt[d]*(-(ArcSinh[c*x]/(I*Sqrt[d] + Sqrt[e]*x)) + (c*ArcTan[(Sqrt
[e] - I*c^2*Sqrt[d]*x)/(Sqrt[c^2*d - e]*Sqrt[1 + c^2*x^2])])/Sqrt[c^2*d - e
]) + (2*I)*Sqrt[d]*(ArcSinh[c*x]/(Sqrt[d] + I*Sqrt[e]*x) + (c*ArcTanh[(I*Sq
rt[e] - c^2*Sqrt[d]*x)/(Sqrt[c^2*d - e]*Sqrt[1 + c^2*x^2])])/Sqrt[c^2*d - e
```

```
]) + I*(ArcSinh[c*x]*(-ArcSinh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(
I*c*Sqrt[d] - Sqrt[-(c^2*d) + e])] + Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(I*c*
Sqrt[d] + Sqrt[-(c^2*d) + e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/((
-I)*c*Sqrt[d] + Sqrt[-(c^2*d) + e])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*
x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]))]) - I*(ArcSinh[c*x]*(-ArcSinh[c*x]
+ 2*(Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) + e])
] + Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e])]))
+ 2*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) +
 e]))] + 2*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d)
 + e])])))/(4*d^(3/2)*Sqrt[e]))/2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.96 (sec) , antiderivative size = 848, normalized size of antiderivative = 1.20

method	result
parts	$\frac{ax}{2d(e x^2+d)}+\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+\frac{b\left(\frac{c^3\operatorname{arcsinh}(cx)x}{2d\left(c^2e x^2+c^2d\right)}+\frac{c^2\left(\sum_{\_R1=\operatorname{RootOf}\left(e\_Z^4+\left(4c^2d-2e\right)\_Z^2+e\right)}\frac{\operatorname{arcsinh}(cx)\ln\left(\frac{\_R1\cdots}{\cdots}\right)\cdots}{\cdots}\cdots\right)}{4d}\cdots\right)}{\cdots}$
derivativedivides	$\frac{\frac{a c^3x}{2d\left(c^2e x^2+c^2d\right)}+\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+bc^4\left(\frac{\operatorname{arcsinh}(cx)x}{2cd\left(c^2e x^2+c^2d\right)}+\frac{\sum_{\_R1=\operatorname{RootOf}\left(e\_Z^4+\left(4c^2d-2e\right)\_Z^2+e\right)}\frac{\operatorname{arcsinh}(cx)\ln\left(\frac{\_R1\cdots}{\cdots}\right)\cdots}{\cdots}}{4c^2d}\cdots\right)}{\cdots}$
default	$\frac{\frac{a c^3x}{2d\left(c^2e x^2+c^2d\right)}+\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+bc^4\left(\frac{\operatorname{arcsinh}(cx)x}{2cd\left(c^2e x^2+c^2d\right)}+\frac{\sum_{\_R1=\operatorname{RootOf}\left(e\_Z^4+\left(4c^2d-2e\right)\_Z^2+e\right)}\frac{\operatorname{arcsinh}(cx)\ln\left(\frac{\_R1\cdots}{\cdots}\right)\cdots}{\cdots}}{4c^2d}\cdots\right)}{\cdots}$

```
[In] int((a+b*arcsinh(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x/d/(e*x^2+d)+1/2*a/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b/c*(1/2*c^
3*arcsinh(c*x)*x/d/(c^2*e*x^2+c^2*d)+1/4/d*c^2*sum(1/_R1/(_R1^2*e+2*c^2*d-e
)*(arcsinh(c*x)*ln((_R1-c*x-(c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x-(c^2*x^2
+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d-2*e)*_Z^2+e))+1/4/d*c^2*sum(_R1
```

```
/(_R1^2*e+2*c^2*d-e)*(arcsinh(c*x)*ln((_R1-c*x-(c^2*x^2+1)^(1/2))/_R1)+dilo
g((_R1-c*x-(c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d-2*e)*_Z^2+e)
)+1/2*(-(2*c^2*d-2*(c^2*d*(c^2*d-e))^(1/2)-e)*e)^(1/2)*(2*(c^2*d*(c^2*d-e))
^(1/2)*c^2*d+2*c^4*d^2-2*c^2*d*e-(c^2*d*(c^2*d-e))^(1/2)*e)*c^2*arctanh(e*(
c*x+(c^2*x^2+1)^(1/2))/((-2*c^2*d+2*(c^2*d*(c^2*d-e))^(1/2)+e)*e)^(1/2))/d/
(c^2*d-e)/e^3-1/2*(-(2*c^2*d-2*(c^2*d*(c^2*d-e))^(1/2)-e)*e)^(1/2)*(2*c^2*d
+2*(c^2*d*(c^2*d-e))^(1/2)-e)*arctanh(e*(c*x+(c^2*x^2+1)^(1/2))/((-2*c^2*d+
2*(c^2*d*(c^2*d-e))^(1/2)+e)*e)^(1/2))*c^2/d/e^3+1/2*((2*c^2*d+2*(c^2*d*(c^
2*d-e))^(1/2)-e)*e)^(1/2)*(-2*(c^2*d*(c^2*d-e))^(1/2)*c^2*d+2*c^4*d^2-2*c^2
*d*e+(c^2*d*(c^2*d-e))^(1/2)*e)*c^2*arctan(e*(c*x+(c^2*x^2+1)^(1/2))/((2*c^
2*d+2*(c^2*d*(c^2*d-e))^(1/2)-e)*e)^(1/2))/d/(c^2*d-e)/e^3-1/2*((2*c^2*d+2*
(c^2*d*(c^2*d-e))^(1/2)-e)*e)^(1/2)*(2*c^2*d-2*(c^2*d*(c^2*d-e))^(1/2)-e)*a
rctan(e*(c*x+(c^2*x^2+1)^(1/2))/((2*c^2*d+2*(c^2*d*(c^2*d-e))^(1/2)-e)*e)^(
1/2))*c^2/d/e^3)
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^2}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(ex^2+d)^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arcsinh(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^2}\,dx=\int \frac{a+b\,\text{asinh}\,(cx)}{(d+ex^2)^2}\,dx$$

```
[In] integrate((a+b*asinh(c*x))/(e*x**2+d)**2,x)
[Out] Integral((a + b*asinh(c*x))/(d + e*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex^2)^2}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{(ex^2 + d)^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex^2)^2}\,dx = \int \frac{a + b\,\text{asinh}(c\,x)}{(e\,x^2 + d)^2}\,dx$$

```
[In] int((a + b*asinh(c*x))/(d + e*x^2)^2,x)
[Out] int((a + b*asinh(c*x))/(d + e*x^2)^2, x)
```

3.613 $\int (d+ex^2)^3 (a+b\text{arcsinh}(cx))^2 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 559

$$\int \left(d+ex^2\right)^3 (a+b\text{arcsinh}(cx))^2\,dx = 2b^2d^3x-\frac{4b^2d^2ex}{3c^2}+\frac{16b^2de^2x}{25c^4}-\frac{32b^2e^3x}{245c^6}+\frac{2}{9}b^2d^2ex^3-\frac{8b^2de^2x^3}{75c^2}+\frac{16b^2e^3x^3}{735c^4}+\frac{6}{125}b^2de^2x^5-\frac{12b^2e^3x^5}{1225c^2}+\frac{2}{343}b^2e^3x^7-\frac{2bd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c}+\frac{4bd^2e\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3c^3}-\frac{16bde^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{25c^5}+\frac{32be^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{245c^7}-\frac{2bd^2ex^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3c}+\frac{8bde^2x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{25c^3}-\frac{16be^3x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{245c^5}-\frac{6bde^2x^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{25c}+\frac{12be^3x^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{245c^3}-\frac{2be^3x^6\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{49c}+d^3x(a+b\text{arcsinh}(cx))^2+d^2ex^3(a+b\text{arcsinh}(cx))^2+\frac{3}{5}de^2x^5(a+b\text{arcsinh}(cx))^2+\frac{1}{7}e^3x^7(a+b\text{arcsinh}(cx))^2$$

```
[Out] 2*b^2*d^3*x-4/3*b^2*d^2*e*x/c^2+16/25*b^2*d*e^2*x/c^4-32/245*b^2*e^3*x/c^6+
2/9*b^2*d^2*e*x^3-8/75*b^2*d*e^2*x^3/c^2+16/735*b^2*e^3*x^3/c^4+6/125*b^2*d
*e^2*x^5-12/1225*b^2*e^3*x^5/c^2+2/343*b^2*e^3*x^7+d^3*x*(a+b*arcsinh(c*x))
^2+d^2*e*x^3*(a+b*arcsinh(c*x))^2+3/5*d*e^2*x^5*(a+b*arcsinh(c*x))^2+1/7*e^
3*x^7*(a+b*arcsinh(c*x))^2-2*b*d^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c+4
/3*b*d^2*e*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-16/25*b*d*e^2*(a+b*arcs
inh(c*x))*(c^2*x^2+1)^(1/2)/c^5+32/245*b*e^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)
^(1/2)/c^7-2/3*b*d^2*e*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c+8/25*b*d*
e^2*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-16/245*b*e^3*x^2*(a+b*arcs
inh(c*x))*(c^2*x^2+1)^(1/2)/c^5-6/25*b*d*e^2*x^4*(a+b*arcsinh(c*x))*(c^2*x^
```

```
2+1)^(1/2)/c+12/245*b*e^3*x^4*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-2/49
*b*e^3*x^6*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {5793, 5772, 5798, 8, 5776, 5812, 30}

$$\int \left(d+ex^2\right)^3 (a+b\text{arcsinh}(cx))^2\,dx = -\frac{2bd^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c} - \frac{2bd^2ex^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c} - \frac{6bde^2x^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{25c} - \frac{2be^3x^6\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{49c} + \frac{32be^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{245c^7} - \frac{16bde^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{25c^5} - \frac{16be^3x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{245c^5} + \frac{4bd^2e\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^3} + \frac{8bde^2x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{25c^3} + \frac{12be^3x^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{245c^3} + d^3x(a+b\text{arcsinh}(cx))^2 + d^2ex^3(a+b\text{arcsinh}(cx))^2 + \frac{3}{5}de^2x^5(a+b\text{arcsinh}(cx))^2 + \frac{1}{7}e^3x^7(a+b\text{arcsinh}(cx))^2 - \frac{32b^2e^3x}{245c^6} + \frac{16b^2de^2x}{25c^4} + \frac{16b^2e^3x^3}{735c^4} - \frac{4b^2d^2ex}{3c^2} - \frac{8b^2de^2x^3}{75c^2} - \frac{12b^2e^3x^5}{1225c^2} + 2b^2d^3x + \frac{2}{9}b^2d^2ex^3 + \frac{6}{125}b^2de^2x^5 + \frac{2}{343}b^2e^3x^7$$

```
[In] Int[(d + e*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*d^3*x - (4*b^2*d^2*e*x)/(3*c^2) + (16*b^2*d*e^2*x)/(25*c^4) - (32*b^2
*e^3*x)/(245*c^6) + (2*b^2*d^2*e*x^3)/9 - (8*b^2*d*e^2*x^3)/(75*c^2) + (16*
b^2*e^3*x^3)/(735*c^4) + (6*b^2*d*e^2*x^5)/125 - (12*b^2*e^3*x^5)/(1225*c^2
```

```
) + (2*b^2*e^3*x^7)/343 - (2*b*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/
c + (4*b*d^2*e*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c^3) - (16*b*d*e^
2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(25*c^5) + (32*b*e^3*Sqrt[1 + c^2
*x^2]*(a + b*ArcSinh[c*x]))/(245*c^7) - (2*b*d^2*e*x^2*Sqrt[1 + c^2*x^2]*(a
 + b*ArcSinh[c*x]))/(3*c) + (8*b*d*e^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh
[c*x]))/(25*c^3) - (16*b*e^3*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2
45*c^5) - (6*b*d*e^2*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(25*c) + (
12*b*e^3*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(245*c^3) - (2*b*e^3*x
^6*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(49*c) + d^3*x*(a + b*ArcSinh[c*
x])^2 + d^2*e*x^3*(a + b*ArcSinh[c*x])^2 + (3*d*e^2*x^5*(a + b*ArcSinh[c*x]
)^2)/5 + (e^3*x^7*(a + b*ArcSinh[c*x])^2)/7
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
```

```
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(d^3(a+b\text{arcsinh}(cx))^2+3d^2ex^2(a+b\text{arcsinh}(cx))^2+3de^2x^4(a+b\text{arcsinh}(cx))^2 + e^3x^6(a+b\text{arcsinh}(cx))^2\right)\,dx \\
&= d^3\int (a+b\text{arcsinh}(cx))^2\,dx+\left(3d^2e\right)\int x^2(a+b\text{arcsinh}(cx))^2\,dx \\
&\quad +\left(3de^2\right)\int x^4(a+b\text{arcsinh}(cx))^2\,dx+e^3\int x^6(a+b\text{arcsinh}(cx))^2\,dx \\
&= d^3x(a+b\text{arcsinh}(cx))^2+d^2ex^3(a+b\text{arcsinh}(cx))^2 \\
&\quad +\frac{3}{5}de^2x^5(a+b\text{arcsinh}(cx))^2+\frac{1}{7}e^3x^7(a+b\text{arcsinh}(cx))^2 \\
&\quad -\left(2bcd^3\right)\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx-\left(2bcd^2e\right)\int\frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx \\
&\quad -\frac{1}{5}\left(6bcde^2\right)\int\frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx-\frac{1}{7}\left(2bce^3\right)\int\frac{x^7(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx \\
&= -\frac{2bd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c}-\frac{2bd^2ex^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3c} \\
&\quad -\frac{6bde^2x^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{25c}-\frac{2be^3x^6\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{49c} \\
&\quad +d^3x(a+b\text{arcsinh}(cx))^2+d^2ex^3(a+b\text{arcsinh}(cx))^2 \\
&\quad +\frac{3}{5}de^2x^5(a+b\text{arcsinh}(cx))^2+\frac{1}{7}e^3x^7(a+b\text{arcsinh}(cx))^2 \\
&\quad +\left(2b^2d^3\right)\int 1\,dx+\frac{1}{3}\left(2b^2d^2e\right)\int x^2\,dx+\frac{\left(4bd^2e\right)\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{3c} \\
&\quad +\frac{1}{25}\left(6b^2de^2\right)\int x^4\,dx+\frac{\left(24bde^2\right)\int\frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{25c} \\
&\quad +\frac{1}{49}\left(2b^2e^3\right)\int x^6\,dx+\frac{\left(12be^3\right)\int\frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{49c}
\end{aligned}$$

$$
\begin{aligned}
&= 2b^2d^3x + \frac{2}{9}b^2d^2ex^3 + \frac{6}{125}b^2de^2x^5 + \frac{2}{343}b^2e^3x^7 - \frac{2bd^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c} \\
&+ \frac{4bd^2e\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3c^3} - \frac{2bd^2ex^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3c} \\
&+ \frac{8bde^2x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{25c^3} - \frac{6bde^2x^4\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{25c} \\
&+ \frac{12be^3x^4\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{245c^3} - \frac{2be^3x^6\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{49c} \\
&+ d^3x(a+b\operatorname{arcsinh}(cx))^2 + d^2ex^3(a+b\operatorname{arcsinh}(cx))^2 + \frac{3}{5}de^2x^5(a+b\operatorname{arcsinh}(cx))^2 \\
&+ \frac{1}{7}e^3x^7(a+b\operatorname{arcsinh}(cx))^2 - \frac{(4b^2d^2e)\int 1\,dx}{3c^2} - \frac{(16bde^2)\int \frac{x(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{25c^3} \\
&- \frac{(8b^2de^2)\int x^2\,dx}{25c^2} - \frac{(48be^3)\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{245c^3} - \frac{(12b^2e^3)\int x^4\,dx}{245c^2} \\
&= 2b^2d^3x - \frac{4b^2d^2ex}{3c^2} + \frac{2}{9}b^2d^2ex^3 - \frac{8b^2de^2x^3}{75c^2} + \frac{6}{125}b^2de^2x^5 - \frac{12b^2e^3x^5}{1225c^2} + \frac{2}{343}b^2e^3x^7 \\
&- \frac{2bd^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c} + \frac{4bd^2e\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3c^3} \\
&- \frac{16bde^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{25c^5} - \frac{2bd^2ex^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3c} \\
&+ \frac{8bde^2x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{25c^3} - \frac{16be^3x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{245c^5} \\
&- \frac{6bde^2x^4\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{25c} + \frac{12be^3x^4\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{245c^3} \\
&- \frac{2be^3x^6\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{49c} + d^3x(a+b\operatorname{arcsinh}(cx))^2 \\
&+ d^2ex^3(a+b\operatorname{arcsinh}(cx))^2 + \frac{3}{5}de^2x^5(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{7}e^3x^7(a+b\operatorname{arcsinh}(cx))^2 \\
&+ \frac{(16b^2de^2)\int 1\,dx}{25c^4} + \frac{(32be^3)\int \frac{x(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{245c^5} + \frac{(16b^2e^3)\int x^2\,dx}{245c^4}
\end{aligned}
$$

$$= 2b^2d^3x - \frac{4b^2d^2ex}{3c^2} + \frac{16b^2de^2x}{25c^4} + \frac{2}{9}b^2d^2ex^3 - \frac{8b^2de^2x^3}{75c^2} + \frac{16b^2e^3x^3}{735c^4} + \frac{6}{125}b^2de^2x^5 - \frac{12b^2e^3x^5}{1225c^2} + \frac{2}{343}b^2e^3x^7 - \frac{2bd^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c} + \frac{4bd^2e\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3c^3} - \frac{16bde^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{25c^5} + \frac{32be^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{245c^7} - \frac{2bd^2ex^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3c} + \frac{8bde^2x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{25c^3} - \frac{16be^3x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{245c^5} - \frac{6bde^2x^4\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{25c} + \frac{12be^3x^4\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{245c^3} - \frac{2be^3x^6\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{49c} + d^3x(a+b\operatorname{arcsinh}(cx))^2 + d^2ex^3(a+b\operatorname{arcsinh}(cx))^2 + \frac{3}{5}de^2x^5(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{7}e^3x^7(a+b\operatorname{arcsinh}(cx))^2 - \frac{(32b^2e^3)\int 1\,dx}{245c^6}$$

$$= 2b^2d^3x - \frac{4b^2d^2ex}{3c^2} + \frac{16b^2de^2x}{25c^4} - \frac{32b^2e^3x}{245c^6} + \frac{2}{9}b^2d^2ex^3 - \frac{8b^2de^2x^3}{75c^2} + \frac{16b^2e^3x^3}{735c^4} + \frac{6}{125}b^2de^2x^5 - \frac{12b^2e^3x^5}{1225c^2} + \frac{2}{343}b^2e^3x^7 - \frac{2bd^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c} + \frac{4bd^2e\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3c^3} - \frac{16bde^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{25c^5} + \frac{32be^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{245c^7} - \frac{2bd^2ex^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3c} + \frac{8bde^2x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{25c^3} - \frac{16be^3x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{245c^5} - \frac{6bde^2x^4\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{25c} + \frac{12be^3x^4\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{245c^3} - \frac{2be^3x^6\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{49c} + d^3x(a+b\operatorname{arcsinh}(cx))^2 + d^2ex^3(a+b\operatorname{arcsinh}(cx))^2 + \frac{3}{5}de^2x^5(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{7}e^3x^7(a+b\operatorname{arcsinh}(cx))^2$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 443, normalized size of antiderivative = 0.79

$$\int \left(d+ex^2\right)^3 (a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{11025a^2c^7x(35d^3+35d^2ex^2+21de^2x^4+5e^3x^6)-210ab\sqrt{1+c^2x^2}(-240e^3+24c^2e^2(49d+5ex^2)-2c^4e(1}{}$$

```
[In] Integrate[(d + e*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (11025*a^2*c^7*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6) - 210*a
*b*Sqrt[1 + c^2*x^2]*(-240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) - 2*c^4*e*(122
5*d^2 + 294*d*e*x^2 + 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*d*
e^2*x^4 + 75*e^3*x^6)) + 2*b^2*c*x*(-25200*e^3 + 840*c^2*e^2*(147*d + 5*e*x
^2) - 210*c^4*e*(1225*d^2 + 98*d*e*x^2 + 9*e^2*x^4) + c^6*(385875*d^3 + 428
75*d^2*e*x^2 + 9261*d*e^2*x^4 + 1125*e^3*x^6)) - 210*b*(-105*a*c^7*x*(35*d^
3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6) + b*Sqrt[1 + c^2*x^2]*(-240*e^
3 + 24*c^2*e^2*(49*d + 5*e*x^2) - 2*c^4*e*(1225*d^2 + 294*d*e*x^2 + 45*e^2*
x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*d*e^2*x^4 + 75*e^3*x^6)))*ArcSi
nh[c*x] + 11025*b^2*c^7*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6
)*ArcSinh[c*x]^2)/(385875*c^7)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 683, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{\frac{a^2\left(d^3c^7x+d^2c^7e\,x^3+\frac{3}{5}d\,c^7e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6}+\frac{b^2\left(c^6d^3\left(\text{arcsinh}(cx)^2xc-2\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+\frac{c^4d^2e\left(9\,\text{arcsinh}(cx)^2x^3c^3-\right.}{}\right.}{}}{}$
default	$\frac{\frac{a^2\left(d^3c^7x+d^2c^7e\,x^3+\frac{3}{5}d\,c^7e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6}+\frac{b^2\left(c^6d^3\left(\text{arcsinh}(cx)^2xc-2\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+\frac{c^4d^2e\left(9\,\text{arcsinh}(cx)^2x^3c^3-\right.}{}\right.}{}}{}$
parts	$a^2\left(\frac{1}{7}e^3x^7+\frac{3}{5}d\,e^2x^5+d^2e\,x^3+d^3x\right)+\frac{b^2\left(55125\,\text{arcsinh}(cx)^2c^7x^7e^3+231525\,\text{arcsinh}(cx)^2c^7x^5d\,e^2+38587\right.}{}$

```
[In] int((e*x^2+d)^3*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a^2/c^6*(d^3*c^7*x+d^2*c^7*e*x^3+3/5*d*c^7*e^2*x^5+1/7*e^3*c^7*x^7)+b^
2/c^6*(c^6*d^3*(arcsinh(c*x)^2*x*c-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x)+
1/9*c^4*d^2*e*(9*arcsinh(c*x)^2*x^3*c^3-6*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^
2*c^2+2*c^3*x^3+12*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-12*c*x)+1/375*c^2*d*e^2*(
```

```
225*arcsinh(c*x)^2*c^5*x^5-90*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4*c^4+18*c^5
*x^5+120*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2-40*c^3*x^3-240*arcsinh(c*x)
*(c^2*x^2+1)^(1/2)+240*c*x)+1/25725*e^3*(3675*arcsinh(c*x)^2*c^7*x^7-1050*a
rcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^6*x^6+150*c^7*x^7+1260*arcsinh(c*x)*(c^2*x^
2+1)^(1/2)*x^4*c^4-252*c^5*x^5-1680*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2+
560*c^3*x^3+3360*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-3360*c*x))+2*a*b/c^6*(arcsi
nh(c*x)*d^3*c^7*x+arcsinh(c*x)*d^2*c^7*e*x^3+3/5*arcsinh(c*x)*d*c^7*e^2*x^5
+1/7*arcsinh(c*x)*e^3*c^7*x^7-1/7*e^3*(1/7*c^6*x^6*(c^2*x^2+1)^(1/2)-6/35*c
^4*x^4*(c^2*x^2+1)^(1/2)+8/35*c^2*x^2*(c^2*x^2+1)^(1/2)-16/35*(c^2*x^2+1)^(
1/2))-d^3*c^6*(c^2*x^2+1)^(1/2)-d^2*c^4*e*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/
3*(c^2*x^2+1)^(1/2))-3/5*d*c^2*e^2*(1/5*c^4*x^4*(c^2*x^2+1)^(1/2)-4/15*c^2*
x^2*(c^2*x^2+1)^(1/2)+8/15*(c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.05

$$\int (d+ex^2)^3 (a+b\operatorname{arcsinh}(cx))^2 \, dx$$

$$= \frac{1125\,(49\,a^2+2\,b^2)c^7e^3x^7+189\,(49\,(25\,a^2+2\,b^2)c^7de^2-20\,b^2c^5e^3)x^5+35\,(1225\,(9\,a^2+2\,b^2)c^7d^2e-1176}{}$$

```
[In] integrate((e*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/385875*(1125*(49*a^2 + 2*b^2)*c^7*e^3*x^7 + 189*(49*(25*a^2 + 2*b^2)*c^7*
d*e^2 - 20*b^2*c^5*e^3)*x^5 + 35*(1225*(9*a^2 + 2*b^2)*c^7*d^2*e - 1176*b^2
*c^5*d*e^2 + 240*b^2*c^3*e^3)*x^3 + 11025*(5*b^2*c^7*e^3*x^7 + 21*b^2*c^7*d
*e^2*x^5 + 35*b^2*c^7*d^2*e*x^3 + 35*b^2*c^7*d^3*x)*log(c*x + sqrt(c^2*x^2
+ 1))^2 + 105*(3675*(a^2 + 2*b^2)*c^7*d^3 - 4900*b^2*c^5*d^2*e + 2352*b^2*c
^3*d*e^2 - 480*b^2*c*e^3)*x + 210*(525*a*b*c^7*e^3*x^7 + 2205*a*b*c^7*d*e^2
*x^5 + 3675*a*b*c^7*d^2*e*x^3 + 3675*a*b*c^7*d^3*x - (75*b^2*c^6*e^3*x^6 +
3675*b^2*c^6*d^3 - 2450*b^2*c^4*d^2*e + 1176*b^2*c^2*d*e^2 - 240*b^2*e^3 +
9*(49*b^2*c^6*d*e^2 - 10*b^2*c^4*e^3)*x^4 + (1225*b^2*c^6*d^2*e - 588*b^2*c
^4*d*e^2 + 120*b^2*c^2*e^3)*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
+ 1)) - 210*(75*a*b*c^6*e^3*x^6 + 3675*a*b*c^6*d^3 - 2450*a*b*c^4*d^2*e + 1
176*a*b*c^2*d*e^2 - 240*a*b*e^3 + 9*(49*a*b*c^6*d*e^2 - 10*a*b*c^4*e^3)*x^4
 + (1225*a*b*c^6*d^2*e - 588*a*b*c^4*d*e^2 + 120*a*b*c^2*e^3)*x^2)*sqrt(c^2
*x^2 + 1))/c^7
```

Sympy [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 989, normalized size of antiderivative = 1.77

$$\int \left(d+ex^2\right)^3 (a+b\text{arcsinh}(cx))^2\,dx$$

$$=\begin{cases} a^2d^3x+a^2d^2ex^3+\frac{3a^2de^2x^5}{5}+\frac{a^2e^3x^7}{7}+2abd^3x\operatorname{asinh}(cx)+2abd^2ex^3\operatorname{asinh}(cx)+\frac{6abde^2x^5\operatorname{asinh}(cx)}{5}+\frac{2abe^3x^7}{} \\ a^2\left(d^3x+d^2ex^3+\frac{3de^2x^5}{5}+\frac{e^3x^7}{7}\right) \end{cases}$$

```
[In] integrate((e*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**3*x + a**2*d**2*e*x**3 + 3*a**2*d*e**2*x**5/5 + a**2*e**
3*x**7/7 + 2*a*b*d**3*x*asinh(c*x) + 2*a*b*d**2*e*x**3*asinh(c*x) + 6*a*b*d
*e**2*x**5*asinh(c*x)/5 + 2*a*b*e**3*x**7*asinh(c*x)/7 - 2*a*b*d**3*sqrt(c*
*2*x**2 + 1)/c - 2*a*b*d**2*e*x**2*sqrt(c**2*x**2 + 1)/(3*c) - 6*a*b*d*e**2
*x**4*sqrt(c**2*x**2 + 1)/(25*c) - 2*a*b*e**3*x**6*sqrt(c**2*x**2 + 1)/(49*
c) + 4*a*b*d**2*e*sqrt(c**2*x**2 + 1)/(3*c**3) + 8*a*b*d*e**2*x**2*sqrt(c**
2*x**2 + 1)/(25*c**3) + 12*a*b*e**3*x**4*sqrt(c**2*x**2 + 1)/(245*c**3) - 1
6*a*b*d*e**2*sqrt(c**2*x**2 + 1)/(25*c**5) - 16*a*b*e**3*x**2*sqrt(c**2*x**
2 + 1)/(245*c**5) + 32*a*b*e**3*sqrt(c**2*x**2 + 1)/(245*c**7) + b**2*d**3*
x*asinh(c*x)**2 + 2*b**2*d**3*x + b**2*d**2*e*x**3*asinh(c*x)**2 + 2*b**2*d
**2*e*x**3/9 + 3*b**2*d*e**2*x**5*asinh(c*x)**2/5 + 6*b**2*d*e**2*x**5/125
+ b**2*e**3*x**7*asinh(c*x)**2/7 + 2*b**2*e**3*x**7/343 - 2*b**2*d**3*sqrt(
c**2*x**2 + 1)*asinh(c*x)/c - 2*b**2*d**2*e*x**2*sqrt(c**2*x**2 + 1)*asinh(
c*x)/(3*c) - 6*b**2*d*e**2*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/(25*c) - 2*b
**2*e**3*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/(49*c) - 4*b**2*d**2*e*x/(3*c*
*2) - 8*b**2*d*e**2*x**3/(75*c**2) - 12*b**2*e**3*x**5/(1225*c**2) + 4*b**2
*d**2*e*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3*c**3) + 8*b**2*d*e**2*x**2*sqrt(c
**2*x**2 + 1)*asinh(c*x)/(25*c**3) + 12*b**2*e**3*x**4*sqrt(c**2*x**2 + 1)*
asinh(c*x)/(245*c**3) + 16*b**2*d*e**2*x/(25*c**4) + 16*b**2*e**3*x**3/(735
*c**4) - 16*b**2*d*e**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(25*c**5) - 16*b**2*
e**3*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(245*c**5) - 32*b**2*e**3*x/(245*c
**6) + 32*b**2*e**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/(245*c**7), Ne(c, 0)), (
a**2*(d**3*x + d**2*e*x**3 + 3*d*e**2*x**5/5 + e**3*x**7/7), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 684, normalized size of antiderivative = 1.22

$$\int \left(d+ex^2\right)^3 (a+b\text{arcsinh}(cx))^2\,dx = \frac{1}{7}\,b^2e^3x^7\,\text{arsinh}\,(cx)^2 + \frac{1}{7}\,a^2e^3x^7 + \frac{3}{5}\,b^2de^2x^5\,\text{arsinh}\,(cx)^2 + \frac{3}{5}\,a^2de^2x^5 + b^2d^2ex^3\,\text{arsinh}\,(cx)^2 + a^2d^2ex^3 + b^2d^3x\,\text{arsinh}\,(cx)^2 + \frac{2}{3}\left(3\,x^3\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)abd^2e - \frac{2}{9}\left(3\,c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\text{arsinh}\,(cx) - \frac{c^2x^3-6\,x}{c^2}\right)b^2d^2e + \frac{2}{25}\left(15\,x^5\,\text{arsinh}\,(cx) - \left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)abde^2 - \frac{2}{375}\left(15\left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\,\text{arsinh}\,(cx) - \frac{9\,c^4x^5-20\,c^2x^3+120\,x}{c^4}\right)b + \frac{2}{245}\left(35\,x^7\,\text{arsinh}\,(cx) - \left(\frac{5\sqrt{c^2x^2+1}x^6}{c^2} - \frac{6\sqrt{c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{c^2x^2+1}x^2}{c^6} - \frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\right)abe^3 - \frac{2}{25725}\left(105\left(\frac{5\sqrt{c^2x^2+1}x^6}{c^2} - \frac{6\sqrt{c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{c^2x^2+1}x^2}{c^6} - \frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\,\text{arsinh}\,(cx) - \frac{75\,c^6 \right. + 2\,b^2d^3\left(x - \frac{\sqrt{c^2x^2+1}\,\text{arsinh}\,(cx)}{c}\right) + a^2d^3x + \frac{2\left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)abd^3}{c}$$

```
[In] integrate((e*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*e^3*x^7*arcsinh(c*x)^2 + 1/7*a^2*e^3*x^7 + 3/5*b^2*d*e^2*x^5*arcsin
h(c*x)^2 + 3/5*a^2*d*e^2*x^5 + b^2*d^2*e*x^3*arcsinh(c*x)^2 + a^2*d^2*e*x^3
 + b^2*d^3*x*arcsinh(c*x)^2 + 2/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1
)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d^2*e - 2/9*(3*c*(sqrt(c^2*x^2 +
1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b
^2*d^2*e + 2/25*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqr
t(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*d*e^2 - 2/375*(15*
(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2
 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^2*d*e^2
 + 2/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x
^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c
)*a*b*e^3 - 2/25725*(105*(5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)
*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsin
h(c*x) - (75*c^6*x^7 - 126*c^4*x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*e^3 + 2
*b^2*d^3*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d^3*x + 2*(c*x*arcsin
h(c*x) - sqrt(c^2*x^2 + 1))*a*b*d^3/c
```

Giac [F(-2)]

Exception generated.

$$\int (d + ex^2)^3 (a + b\text{arcsinh}(cx))^2 \, dx = \text{Exception raised: RuntimeError}$$

[In] integrate((e*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")

[Out] Exception raised: RuntimeError >> an error occurred running a Giac command: INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int (d + ex^2)^3 (a + b\text{arcsinh}(cx))^2 \, dx = \int (a + b \,\text{asinh}(c\,x))^2 \left(e\,x^2 + d\right)^3 \, dx$$

[In] int((a + b*asinh(c*x))^2*(d + e*x^2)^3,x)

[Out] int((a + b*asinh(c*x))^2*(d + e*x^2)^3, x)

3.614 $\int (d+ex^2)^2 (a+b\text{arcsinh}(cx))^2 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 329

$$\begin{aligned}\int (d+ex^2)^2 (a+b\text{arcsinh}(cx))^2 \, dx &= 2b^2d^2x - \frac{8b^2dex}{9c^2} + \frac{16b^2e^2x}{75c^4} + \frac{4}{27}b^2dex^3 - \frac{8b^2e^2x^3}{225c^2} \\ &+ \frac{2}{125}b^2e^2x^5 - \frac{2bd^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c} \\ &+ \frac{8bde\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^3} \\ &- \frac{16be^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{75c^5} \\ &- \frac{4bdex^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c} \\ &+ \frac{8be^2x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{75c^3} \\ &- \frac{2be^2x^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{25c} \\ &+ d^2x(a+b\text{arcsinh}(cx))^2 + \frac{2}{3}dex^3(a+b\text{arcsinh}(cx))^2 \\ &+ \frac{1}{5}e^2x^5(a+b\text{arcsinh}(cx))^2\end{aligned}$$

```
[Out] 2*b^2*d^2*x-8/9*b^2*d*e*x/c^2+16/75*b^2*e^2*x/c^4+4/27*b^2*d*e*x^3-8/225*b^
2*e^2*x^3/c^2+2/125*b^2*e^2*x^5+d^2*x*(a+b*arcsinh(c*x))^2+2/3*d*e*x^3*(a+b
*arcsinh(c*x))^2+1/5*e^2*x^5*(a+b*arcsinh(c*x))^2-2*b*d^2*(a+b*arcsinh(c*x)
)*(c^2*x^2+1)^(1/2)/c+8/9*b*d*e*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-16
/75*b*e^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^5-4/9*b*d*e*x^2*(a+b*arcsi
nh(c*x))*(c^2*x^2+1)^(1/2)/c+8/75*b*e^2*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^
(1/2)/c^3-2/25*b*e^2*x^4*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {5793, 5772, 5798, 8, 5776, 5812, 30}

$$\int \left(d+ex^2\right)^2 (a+b\operatorname{arcsinh}(cx))^2\,dx = -\frac{2bd^2\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{c} - \frac{4bdex^2\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{9c} - \frac{2be^2x^4\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{25c} - \frac{16be^2\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{75c^5} + \frac{8bde\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{9c^3} + \frac{8be^2x^2\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{75c^3} + d^2x(a+b\operatorname{arcsinh}(cx))^2 + \frac{2}{3}dex^3(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{5}e^2x^5(a+b\operatorname{arcsinh}(cx))^2 + \frac{16b^2e^2x}{75c^4} - \frac{8b^2dex}{9c^2} - \frac{8b^2e^2x^3}{225c^2} + 2b^2d^2x + \frac{4}{27}b^2dex^3 + \frac{2}{125}b^2e^2x^5$$

```
[In] Int[(d + e*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*d^2*x - (8*b^2*d*e*x)/(9*c^2) + (16*b^2*e^2*x)/(75*c^4) + (4*b^2*d*e*
x^3)/27 - (8*b^2*e^2*x^3)/(225*c^2) + (2*b^2*e^2*x^5)/125 - (2*b*d^2*Sqrt[1
 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c + (8*b*d*e*Sqrt[1 + c^2*x^2]*(a + b*Arc
Sinh[c*x]))/(9*c^3) - (16*b*e^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(75
*c^5) - (4*b*d*e*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9*c) + (8*b*e
^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(75*c^3) - (2*b*e^2*x^4*Sqrt
[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(25*c) + d^2*x*(a + b*ArcSinh[c*x])^2 +
 (2*d*e*x^3*(a + b*ArcSinh[c*x])^2)/3 + (e^2*x^5*(a + b*ArcSinh[c*x])^2)/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = \int \left(d^2(a+b\operatorname{arcsinh}(cx))^2+2dex^2(a+b\operatorname{arcsinh}(cx))^2+e^2x^4(a+b\operatorname{arcsinh}(cx))^2\right)\,dx$$

$$= d^2\int (a+b\operatorname{arcsinh}(cx))^2\,dx+(2de)\int x^2(a+b\operatorname{arcsinh}(cx))^2\,dx + e^2\int x^4(a+b\operatorname{arcsinh}(cx))^2\,dx$$

$$= d^2x(a+b\text{arcsinh}(cx))^2 + \frac{2}{3}dex^3(a+b\text{arcsinh}(cx))^2 + \frac{1}{5}e^2x^5(a+b\text{arcsinh}(cx))^2 - (2bcd^2)\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx - \frac{1}{3}(4bcde)\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx - \frac{1}{5}(2bce^2)\int \frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx$$

$$= -\frac{2bd^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c} - \frac{4bdex^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c} - \frac{2be^2x^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{25c} + d^2x(a+b\text{arcsinh}(cx))^2 + \frac{2}{3}dex^3(a+b\text{arcsinh}(cx))^2 + \frac{1}{5}e^2x^5(a+b\text{arcsinh}(cx))^2 + (2b^2d^2)\int 1\,dx + \frac{1}{9}(4b^2de)\int x^2\,dx + \frac{(8bde)\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{9c} + \frac{1}{25}(2b^2e^2)\int x^4\,dx + \frac{(8be^2)\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{25c}$$

$$= 2b^2d^2x + \frac{4}{27}b^2dex^3 + \frac{2}{125}b^2e^2x^5 - \frac{2bd^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c} + \frac{8bde\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^3} - \frac{4bdex^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c} + \frac{8be^2x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{75c^3} - \frac{2be^2x^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{25c} + d^2x(a+b\text{arcsinh}(cx))^2 + \frac{2}{3}dex^3(a+b\text{arcsinh}(cx))^2 + \frac{1}{5}e^2x^5(a+b\text{arcsinh}(cx))^2 - \frac{(8b^2de)\int 1\,dx}{9c^2} - \frac{(16be^2)\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{75c^3} - \frac{(8b^2e^2)\int x^2\,dx}{75c^2}$$

$$= 2b^2d^2x - \frac{8b^2dex}{9c^2} + \frac{4}{27}b^2dex^3 - \frac{8b^2e^2x^3}{225c^2} + \frac{2}{125}b^2e^2x^5 - \frac{2bd^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c} + \frac{8bde\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^3} - \frac{16be^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{75c^5} - \frac{4bdex^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c} + \frac{8be^2x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{75c^3} - \frac{2be^2x^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{25c} + d^2x(a+b\text{arcsinh}(cx))^2 + \frac{2}{3}dex^3(a+b\text{arcsinh}(cx))^2 + \frac{1}{5}e^2x^5(a+b\text{arcsinh}(cx))^2 + \frac{(16b^2e^2)\int 1\,dx}{75c^4}$$

$$
\begin{aligned}
&= 2b^2d^2x - \frac{8b^2dex}{9c^2} + \frac{16b^2e^2x}{75c^4} + \frac{4}{27}b^2dex^3 - \frac{8b^2e^2x^3}{225c^2} + \frac{2}{125}b^2e^2x^5 \\
&\quad - \frac{2bd^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c} + \frac{8bde\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{9c^3} \\
&\quad - \frac{16be^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{75c^5} - \frac{4bdex^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{9c} \\
&\quad + \frac{8be^2x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{75c^3} - \frac{2be^2x^4\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{25c} \\
&\quad + d^2x(a+b\operatorname{arcsinh}(cx))^2 + \frac{2}{3}dex^3(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{5}e^2x^5(a+b\operatorname{arcsinh}(cx))^2
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.88

$$
\int \left(d+ex^2\right)^2 (a+b\operatorname{arcsinh}(cx))^2\,dx
$$

$$
= \frac{225a^2c^5x(15d^2+10dex^2+3e^2x^4) - 30ab\sqrt{1+c^2x^2}(24e^2-4c^2e(25d+3ex^2)+c^4(225d^2+50dex^2+9e^2
$$

```
[In] Integrate[(d + e*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (225*a^2*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - 30*a*b*Sqrt[1 + c^2*x^2]
*(24*e^2 - 4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2 + 50*d*e*x^2 + 9*e^2*x^4
)) + 2*b^2*c*x*(360*e^2 - 60*c^2*e*(25*d + e*x^2) + c^4*(3375*d^2 + 250*d*e
*x^2 + 27*e^2*x^4)) - 30*b*(-15*a*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) +
 b*Sqrt[1 + c^2*x^2]*(24*e^2 - 4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2 + 50
*d*e*x^2 + 9*e^2*x^4)))*ArcSinh[c*x] + 225*b^2*c^5*x*(15*d^2 + 10*d*e*x^2 +
 3*e^2*x^4)*ArcSinh[c*x]^2)/(3375*c^5)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{a^2\left(d^2c^5x+\frac{2}{3}dc^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b^2\left(c^4d^2\left(\operatorname{arcsinh}(cx)^2xc-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+\frac{2c^2de\left(9\operatorname{arcsinh}(cx)^2x^3c^3-6\operatorname{arcsinh}(cx\right.}{}\right.}{}$
default	$\frac{a^2\left(d^2c^5x+\frac{2}{3}dc^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b^2\left(c^4d^2\left(\operatorname{arcsinh}(cx)^2xc-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+\frac{2c^2de\left(9\operatorname{arcsinh}(cx)^2x^3c^3-6\operatorname{arcsinh}(cx\right.}{}\right.}{}$
parts	$a^2\left(\frac{1}{5}e^2x^5+\frac{2}{3}de\,x^3+d^2x\right)+\frac{b^2\left(675\operatorname{arcsinh}(cx)^2c^5x^5e^2+2250\operatorname{arcsinh}(cx)^2c^5x^3de+3375\operatorname{arcsinh}(cx)^2c^5x\,d^2-\right.}{}$

```
[In] int((e*x^2+d)^2*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a^2/c^4*(d^2*c^5*x+2/3*d*c^5*e*x^3+1/5*e^2*c^5*x^5)+b^2/c^4*(c^4*d^2*(
arcsinh(c*x)^2*x*c-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x)+2/27*c^2*d*e*(9*
arcsinh(c*x)^2*x^3*c^3-6*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+1
2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-12*c*x)+1/1125*e^2*(225*arcsinh(c*x)^2*c^5
*x^5-90*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4*c^4+18*c^5*x^5+120*arcsinh(c*x)*
(c^2*x^2+1)^(1/2)*x^2*c^2-40*c^3*x^3-240*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+240
*c*x))+2*a*b/c^4*(arcsinh(c*x)*d^2*c^5*x+2/3*arcsinh(c*x)*d*c^5*e*x^3+1/5*a
rcsinh(c*x)*e^2*c^5*x^5-1/5*e^2*(1/5*c^4*x^4*(c^2*x^2+1)^(1/2)-4/15*c^2*x^2
*(c^2*x^2+1)^(1/2)+8/15*(c^2*x^2+1)^(1/2))-d^2*c^4*(c^2*x^2+1)^(1/2)-2/3*d*
c^2*e*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.16

$$\int \left(d+ex^2\right)^2\left(a+b\operatorname{arcsinh}(cx)\right)^2dx$$
$$=\frac{27\left(25\,a^2+2\,b^2\right)c^5e^2x^5+10\left(25\left(9\,a^2+2\,b^2\right)c^5de-12\,b^2c^3e^2\right)x^3+225\left(3\,b^2c^5e^2x^5+10\,b^2c^5dex^3+15\,b^2c^5d^2\right.}{}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/3375*(27*(25*a^2 + 2*b^2)*c^5*e^2*x^5 + 10*(25*(9*a^2 + 2*b^2)*c^5*d*e -
12*b^2*c^3*e^2)*x^3 + 225*(3*b^2*c^5*e^2*x^5 + 10*b^2*c^5*d*e*x^3 + 15*b^2*
c^5*d^2*x)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 15*(225*(a^2 + 2*b^2)*c^5*d^2 -
 200*b^2*c^3*d*e + 48*b^2*c*e^2)*x + 30*(45*a*b*c^5*e^2*x^5 + 150*a*b*c^5*d
```

```
*e*x^3 + 225*a*b*c^5*d^2*x - (9*b^2*c^4*e^2*x^4 + 225*b^2*c^4*d^2 - 100*b^2
*c^2*d*e + 24*b^2*e^2 + 2*(25*b^2*c^4*d*e - 6*b^2*c^2*e^2)*x^2)*sqrt(c^2*x^
2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 30*(9*a*b*c^4*e^2*x^4 + 225*a*b*c^4*
d^2 - 100*a*b*c^2*d*e + 24*a*b*e^2 + 2*(25*a*b*c^4*d*e - 6*a*b*c^2*e^2)*x^2
)*sqrt(c^2*x^2 + 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.81

$$\int \left(d+ex^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx$$
$$=\begin{cases} a^2d^2x + \frac{2a^2dex^3}{3} + \frac{a^2e^2x^5}{5} + 2abd^2x\operatorname{asinh}(cx) + \frac{4abdex^3\operatorname{asinh}(cx)}{3} + \frac{2abe^2x^5\operatorname{asinh}(cx)}{5} - \frac{2abd^2\sqrt{c^2x^2+1}}{c} - \frac{4abdex^2\sqrt{}}{9c} \\ a^2\left(d^2x + \frac{2dex^3}{3} + \frac{e^2x^5}{5}\right) \end{cases}$$

```
[In] integrate((e*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**2*x + 2*a**2*d*e*x**3/3 + a**2*e**2*x**5/5 + 2*a*b*d**2*
x*asinh(c*x) + 4*a*b*d*e*x**3*asinh(c*x)/3 + 2*a*b*e**2*x**5*asinh(c*x)/5 -
 2*a*b*d**2*sqrt(c**2*x**2 + 1)/c - 4*a*b*d*e*x**2*sqrt(c**2*x**2 + 1)/(9*c
) - 2*a*b*e**2*x**4*sqrt(c**2*x**2 + 1)/(25*c) + 8*a*b*d*e*sqrt(c**2*x**2 +
 1)/(9*c**3) + 8*a*b*e**2*x**2*sqrt(c**2*x**2 + 1)/(75*c**3) - 16*a*b*e**2*
sqrt(c**2*x**2 + 1)/(75*c**5) + b**2*d**2*x*asinh(c*x)**2 + 2*b**2*d**2*x +
 2*b**2*d*e*x**3*asinh(c*x)**2/3 + 4*b**2*d*e*x**3/27 + b**2*e**2*x**5*asin
h(c*x)**2/5 + 2*b**2*e**2*x**5/125 - 2*b**2*d**2*sqrt(c**2*x**2 + 1)*asinh(
c*x)/c - 4*b**2*d*e*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(9*c) - 2*b**2*e**2
*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/(25*c) - 8*b**2*d*e*x/(9*c**2) - 8*b**
2*e**2*x**3/(225*c**2) + 8*b**2*d*e*sqrt(c**2*x**2 + 1)*asinh(c*x)/(9*c**3)
 + 8*b**2*e**2*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(75*c**3) + 16*b**2*e**2
*x/(75*c**4) - 16*b**2*e**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(75*c**5), Ne(c,
 0)), (a**2*(d**2*x + 2*d*e*x**3/3 + e**2*x**5/5), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.30

$$
\begin{aligned}
&\int \left(d+ex^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx \\
&= \frac{1}{5}\,b^2e^2x^5\,\text{arsinh}\,(cx)^2 + \frac{1}{5}\,a^2e^2x^5 + \frac{2}{3}\,b^2dex^3\,\text{arsinh}\,(cx)^2 + \frac{2}{3}\,a^2dex^3 \\
&\quad + b^2d^2x\,\text{arsinh}\,(cx)^2 + \frac{4}{9}\left(3\,x^3\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)abde \\
&\quad - \frac{4}{27}\left(3\,c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\text{arsinh}\,(cx) - \frac{c^2x^3-6\,x}{c^2}\right)b^2de \\
&\quad + \frac{2}{75}\left(15\,x^5\,\text{arsinh}\,(cx) - \left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\,\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\right)abe^2 \\
&\quad - \frac{2}{1125}\left(15\left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\,\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\,\text{arsinh}\,(cx) - \frac{9\,c^4x^5-20\,c^2x^3+120\,x}{c^4}\right)b \\
&\quad + 2\,b^2d^2\left(x - \frac{\sqrt{c^2x^2+1}\,\text{arsinh}\,(cx)}{c}\right) + a^2d^2x + \frac{2\left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)abd^2}{c}
\end{aligned}
$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*e^2*x^5*arcsinh(c*x)^2 + 1/5*a^2*e^2*x^5 + 2/3*b^2*d*e*x^3*arcsinh(
c*x)^2 + 2/3*a^2*d*e*x^3 + b^2*d^2*x*arcsinh(c*x)^2 + 4/9*(3*x^3*arcsinh(c*
x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d*e - 4/2
7*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) -
 (c^2*x^3 - 6*x)/c^2)*b^2*d*e + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2
 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a
*b*e^2 - 2/1125*(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/
c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 1
20*x)/c^4)*b^2*e^2 + 2*b^2*d^2*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2
*d^2*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*d^2/c
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx=\int \left(a+b\,\text{asinh}(c\,x)\right)^2\,\left(e\,x^2+d\right)^2\,dx$$

```
[In] int((a + b*asinh(c*x))^2*(d + e*x^2)^2,x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + e*x^2)^2, x)
```

3.615 $\int (d+ex^2)(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 153

$$\int (d+ex^2)(a+b\text{arcsinh}(cx))^2\,dx = 2b^2dx-\frac{4b^2ex}{9c^2}+\frac{2}{27}b^2ex^3-\frac{2bd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c}+\frac{4be\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^3}-\frac{2bex^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c}+dx(a+b\text{arcsinh}(cx))^2+\frac{1}{3}ex^3(a+b\text{arcsinh}(cx))^2$$

```
[Out] 2*b^2*d*x-4/9*b^2*e*x/c^2+2/27*b^2*e*x^3+d*x*(a+b*arcsinh(c*x))^2+1/3*e*x^3
*(a+b*arcsinh(c*x))^2-2*b*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c+4/9*b*e*
(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-2/9*b*e*x^2*(a+b*arcsinh(c*x))*(c^
2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used

$= \{5793, 5772, 5798, 8, 5776, 5812, 30\}$

$$\int (d+ex^2)(a+b\text{arcsinh}(cx))^2\,dx = -\frac{2bd\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c} - \frac{2bex^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{9c} + \frac{4be\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{9c^3} + dx(a+b\text{arcsinh}(cx))^2 + \frac{1}{3}ex^3(a+b\text{arcsinh}(cx))^2 - \frac{4b^2ex}{9c^2} + 2b^2dx + \frac{2}{27}b^2ex^3$$

```
[In] Int[(d + e*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*d*x - (4*b^2*e*x)/(9*c^2) + (2*b^2*e*x^3)/27 - (2*b*d*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x]))/c + (4*b*e*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))
/(9*c^3) - (2*b*e*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9*c) + d*x*(
a + b*ArcSinh[c*x])^2 + (e*x^3*(a + b*ArcSinh[c*x])^2)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
```

```
 0 || IGtQ[n, 0])
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(d(a+b\text{arcsinh}(cx))^2+ex^2(a+b\text{arcsinh}(cx))^2\right)\,dx \\
&= d\int (a+b\text{arcsinh}(cx))^2\,dx+e\int x^2(a+b\text{arcsinh}(cx))^2\,dx \\
&= dx(a+b\text{arcsinh}(cx))^2+\frac{1}{3}ex^3(a+b\text{arcsinh}(cx))^2 \\
&\quad -(2bcd)\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx-\frac{1}{3}(2bce)\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx \\
&= -\frac{2bd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c}-\frac{2bex^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c} \\
&\quad +dx(a+b\text{arcsinh}(cx))^2+\frac{1}{3}ex^3(a+b\text{arcsinh}(cx))^2 \\
&\quad +\left(2b^2d\right)\int 1\,dx+\frac{1}{9}\left(2b^2e\right)\int x^2\,dx+\frac{(4be)\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{9c} \\
&= 2b^2dx+\frac{2}{27}b^2ex^3-\frac{2bd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c} \\
&\quad +\frac{4be\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^3}-\frac{2bex^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c} \\
&\quad +dx(a+b\text{arcsinh}(cx))^2+\frac{1}{3}ex^3(a+b\text{arcsinh}(cx))^2-\frac{(4b^2e)\int 1\,dx}{9c^2}
\end{aligned}
$$

$$= 2b^2dx - \frac{4b^2ex}{9c^2} + \frac{2}{27}b^2ex^3 - \frac{2bd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c} + \frac{4be\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^3} - \frac{2bex^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c} + dx(a+b\text{arcsinh}(cx))^2 + \frac{1}{3}ex^3(a+b\text{arcsinh}(cx))^2$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.07

$$\int (d+ex^2)(a+b\text{arcsinh}(cx))^2 dx = \frac{9a^2c^3x(3d+ex^2) - 6ab\sqrt{1+c^2x^2}(-2e+c^2(9d+ex^2)) + 2b^2cx(-6e+c^2(27d+ex^2)) - 6b(-3ac^3x(3d+}{27c^3}$$

```
[In] Integrate[(d + e*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (9*a^2*c^3*x*(3*d + e*x^2) - 6*a*b*Sqrt[1 + c^2*x^2]*(-2*e + c^2*(9*d + e*x
^2)) + 2*b^2*c*x*(-6*e + c^2*(27*d + e*x^2)) - 6*b*(-3*a*c^3*x*(3*d + e*x^2
) + b*Sqrt[1 + c^2*x^2]*(-2*e + c^2*(9*d + e*x^2)))*ArcSinh[c*x] + 9*b^2*c^
3*x*(3*d + e*x^2)*ArcSinh[c*x]^2)/(27*c^3)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.41

method	result
parts	$a^2\left(\frac{1}{3}x^3e+dx\right)+\frac{b^2\left(\frac{e\left(9\,\text{arcsinh}(cx)^2x^3c^3-6\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^2c^2+2c^3x^3+12\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}-12cx\right)}{27c^2}+d\left(\text{arc}\right.}{c}$
derivativedivides	$\frac{\frac{a^2\left(d\,c^3x+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b^2\left(d\,c^2\left(\text{arcsinh}(cx)^2xc-2\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+\frac{e\left(9\,\text{arcsinh}(cx)^2x^3c^3-6\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^2c^2\right.}{27}\right.}{c^2}}{}$
default	$\frac{\frac{a^2\left(d\,c^3x+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b^2\left(d\,c^2\left(\text{arcsinh}(cx)^2xc-2\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+\frac{e\left(9\,\text{arcsinh}(cx)^2x^3c^3-6\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^2c^2\right.}{27}\right.}{c^2}}{}$

```
[In] int((e*x^2+d)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(1/3*x^3*e+d*x)+b^2/c*(1/27*e*(9*arcsinh(c*x)^2*x^3*c^3-6*arcsinh(c*x)*
(c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+12*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-12*c*
x)/c^2+d*(arcsinh(c*x)^2*x*c-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x))+2*a*b
```

```
/c*(1/3*c*arcsinh(c*x)*x^3*e+arcsinh(c*x)*d*c*x-1/3/c^2*(e*(1/3*c^2*x^2*(c^
2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))+3*d*c^2*(c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.37

$$\int (d+ex^2)(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{(9a^2+2b^2)c^3ex^3+9(b^2c^3ex^3+3b^2c^3dx)\log\left(cx+\sqrt{c^2x^2+1}\right)^2+3(9(a^2+2b^2)c^3d-4b^2ce)x+6\left(3abc\right.}{}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/27*((9*a^2 + 2*b^2)*c^3*e*x^3 + 9*(b^2*c^3*e*x^3 + 3*b^2*c^3*d*x)*log(c*x
 + sqrt(c^2*x^2 + 1))^2 + 3*(9*(a^2 + 2*b^2)*c^3*d - 4*b^2*c*e)*x + 6*(3*a*
b*c^3*e*x^3 + 9*a*b*c^3*d*x - (b^2*c^2*e*x^2 + 9*b^2*c^2*d - 2*b^2*e)*sqrt(
c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 6*(a*b*c^2*e*x^2 + 9*a*b*c^2*d
 - 2*a*b*e)*sqrt(c^2*x^2 + 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.82

$$\int (d+ex^2)(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\begin{cases} a^2dx+\frac{a^2ex^3}{3}+2abdx\operatorname{asinh}(cx)+\frac{2abex^3\operatorname{asinh}(cx)}{3}-\frac{2abd\sqrt{c^2x^2+1}}{c}-\frac{2abex^2\sqrt{c^2x^2+1}}{9c}+\frac{4abe\sqrt{c^2x^2+1}}{9c^3}+b^2dx\operatorname{asinh}^2 \\ a^2\left(dx+\frac{ex^3}{3}\right) \end{cases}$$

```
[In] integrate((e*x**2+d)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d*x + a**2*e*x**3/3 + 2*a*b*d*x*asinh(c*x) + 2*a*b*e*x**3*a
sinh(c*x)/3 - 2*a*b*d*sqrt(c**2*x**2 + 1)/c - 2*a*b*e*x**2*sqrt(c**2*x**2 +
 1)/(9*c) + 4*a*b*e*sqrt(c**2*x**2 + 1)/(9*c**3) + b**2*d*x*asinh(c*x)**2 +
 2*b**2*d*x + b**2*e*x**3*asinh(c*x)**2/3 + 2*b**2*e*x**3/27 - 2*b**2*d*sqr
t(c**2*x**2 + 1)*asinh(c*x)/c - 2*b**2*e*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x
)/(9*c) - 4*b**2*e*x/(9*c**2) + 4*b**2*e*sqrt(c**2*x**2 + 1)*asinh(c*x)/(9*
c**3), Ne(c, 0)), (a**2*(d*x + e*x**3/3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.42

$$\begin{aligned}
&\int (d+ex^2)\,(a+b\text{arcsinh}(cx))^2\,dx \\
&= \frac{1}{3}\,b^2ex^3\,\text{arsinh}\,(cx)^2 + \frac{1}{3}\,a^2ex^3 + b^2dx\,\text{arsinh}\,(cx)^2 \\
&\quad + \frac{2}{9}\left(3\,x^3\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)abe \\
&\quad - \frac{2}{27}\left(3\,c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\text{arsinh}\,(cx) - \frac{c^2x^3-6\,x}{c^2}\right)b^2e \\
&\quad + 2\,b^2d\left(x - \frac{\sqrt{c^2x^2+1}\,\text{arsinh}\,(cx)}{c}\right) + a^2dx + \frac{2\,\left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)abd}{c}
\end{aligned}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*e*x^3*arcsinh(c*x)^2 + 1/3*a^2*e*x^3 + b^2*d*x*arcsinh(c*x)^2 + 2/9
*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c
^4))*a*b*e - 2/27*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4
)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*e + 2*b^2*d*(x - sqrt(c^2*x^2 + 1
)*arcsinh(c*x)/c) + a^2*d*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*
d/c
```

Giac [F(-2)]

Exception generated.

$$\int (d+ex^2)\,(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d + ex^2) (a + b\text{arcsinh}(cx))^2 \, dx = \int (a + b \,\text{asinh}(c\,x))^2 \left(e\,x^2 + d\right) \, dx$$

```
[In] int((a + b*asinh(c*x))^2*(d + e*x^2),x)
[Out] int((a + b*asinh(c*x))^2*(d + e*x^2), x)
```

3.616 $\int (a + b\operatorname{arcsinh}(cx))^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 46

$$\int (a+b\operatorname{arcsinh}(cx))^2 \, dx = 2b^2x - \frac{2b\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c} + x(a+b\operatorname{arcsinh}(cx))^2$$

```
[Out] 2*b^2*x+x*(a+b*arcsinh(c*x))^2-2*b*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5772, 5798, 8}

$$\int (a+b\operatorname{arcsinh}(cx))^2 \, dx = -\frac{2b\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{c} + x(a+b\operatorname{arcsinh}(cx))^2 + 2b^2x$$

```
[In] Int[(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*x - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c + x*(a + b*ArcSinh
[c*x])^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x(a+b\text{arcsinh}(cx))^2-(2bc)\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx\\ &= -\frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c}+x(a+b\text{arcsinh}(cx))^2+\left(2b^2\right)\int 1\,dx\\ &= 2b^2x-\frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c}+x(a+b\text{arcsinh}(cx))^2\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.61

$$\int(a+b\text{arcsinh}(cx))^2\,dx=\left(a^2+2b^2\right)x-\frac{2ab\sqrt{1+c^2x^2}}{c}+\frac{2b\left(acx-b\sqrt{1+c^2x^2}\right)\text{arcsinh}(cx)}{c}+b^2x\text{arcsinh}(cx)^2$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2,x]
[Out] (a^2 + 2*b^2)*x - (2*a*b*Sqrt[1 + c^2*x^2])/c + (2*b*(a*c*x - b*Sqrt[1 + c^
2*x^2])*ArcSinh[c*x])/c + b^2*x*ArcSinh[c*x]^2
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.57

method	result	size
derivativedivides	$\frac{cx\,a^2+b^2\left(\text{arcsinh}(cx)^2xc-2\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+2ab\left(\text{arcsinh}(cx)cx-\sqrt{c^2x^2+1}\right)}{c}$	72
default	$\frac{cx\,a^2+b^2\left(\text{arcsinh}(cx)^2xc-2\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+2ab\left(\text{arcsinh}(cx)cx-\sqrt{c^2x^2+1}\right)}{c}$	72
parts	$a^2x+\frac{b^2\left(\text{arcsinh}(cx)^2xc-2\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)}{c}+\frac{2ab\left(\text{arcsinh}(cx)cx-\sqrt{c^2x^2+1}\right)}{c}$	73

```
[In] int((a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(c*x*a^2+b^2*(arcsinh(c*x)^2*x*c-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x
)+2*a*b*(arcsinh(c*x)*c*x-(c^2*x^2+1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(44) = 88$.

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.09

$$\int (a + b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{b^2cx\log\left(cx + \sqrt{c^2x^2 + 1}\right)^2 + (a^2 + 2\,b^2)cx - 2\,\sqrt{c^2x^2 + 1}ab + 2\left(abcx - \sqrt{c^2x^2 + 1}b^2\right)\log\left(cx + \sqrt{c^2x^2 +\right.}{c}$$

```
[In] integrate((a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] (b^2*c*x*log(c*x + sqrt(c^2*x^2 + 1))^2 + (a^2 + 2*b^2)*c*x - 2*sqrt(c^2*x^
2 + 1)*a*b + 2*(a*b*c*x - sqrt(c^2*x^2 + 1)*b^2)*log(c*x + sqrt(c^2*x^2 + 1
)))/c
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.78

$$\int (a + b\text{arcsinh}(cx))^2\,dx$$
$$= \begin{cases} a^2x + 2abx\operatorname{asinh}(cx) - \frac{2ab\sqrt{c^2x^2+1}}{c} + b^2x\operatorname{asinh}^2(cx) + 2b^2x - \frac{2b^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{c} & \text{for } c \neq 0 \\ a^2x & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*x*asinh(c*x) - 2*a*b*sqrt(c**2*x**2 + 1)/c + b**2
*x*asinh(c*x)**2 + 2*b**2*x - 2*b**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/c, Ne(c
, 0)), (a**2*x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.57

$$\int (a+b\mathrm{arcsinh}(cx))^2\,dx = b^2x\,\mathrm{arsinh}\,(cx)^2 + 2\,b^2\left(x - \frac{\sqrt{c^2x^2+1}\,\mathrm{arsinh}\,(cx)}{c}\right) + a^2x + \frac{2\,\left(cx\,\mathrm{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)ab}{c}$$

```
[In] integrate((a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] b^2*x*arcsinh(c*x)^2 + 2*b^2*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*x
 + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(44) = 88.

Time = 0.40 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.41

$$\int (a+b\mathrm{arcsinh}(cx))^2\,dx = 2\left(x\log\left(cx+\sqrt{c^2x^2+1}\right) - \frac{\sqrt{c^2x^2+1}}{c}\right)ab + \left(x\log\left(cx+\sqrt{c^2x^2+1}\right)^2 + 2\,c\left(\frac{x}{c} - \frac{\sqrt{c^2x^2+1}\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2}\right)\right)b^2 + a^2x$$

```
[In] integrate((a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] 2*(x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*a*b + (x*log(c*x +
 sqrt(c^2*x^2 + 1))^2 + 2*c*(x/c - sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2
 + 1))/c^2))*b^2 + a^2*x
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\mathrm{arcsinh}(cx))^2\,dx = \int (a+b\,\mathrm{asinh}(c\,x))^2\,dx$$

```
[In] int((a + b*asinh(c*x))^2,x)
[Out] int((a + b*asinh(c*x))^2, x)
```

3.617 $\int \frac{(a+b\mathbf{arcsinh}(cx))^2}{d+ex^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 739

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+ex^2}\,dx = \frac{(a+b\text{arcsinh}(cx))^2\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\text{arcsinh}(cx))^2\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+b\text{arcsinh}(cx))^2\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\text{arcsinh}(cx))^2\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{b^2\,\text{PolyLog}\left(3,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{b^2\,\text{PolyLog}\left(3,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{b^2\,\text{PolyLog}\left(3,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{b^2\,\text{PolyLog}\left(3,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))^2*ln(1-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)
-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsinh(c*x))^2*ln(1+(c*x+(
c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/
2)+1/2*(a+b*arcsinh(c*x))^2*ln(1-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1
/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsinh(c*x))^2*ln(1+(c*
x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^
```

```
(1/2)-b*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-
d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+b*(a+b*arcsinh(c*x))*polylog
(2,(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1
/2)/e^(1/2)-b*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)
/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+b*(a+b*arcsinh(c*x))*p
olylog(2,(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(
-d)^(1/2)/e^(1/2)+b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1
/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-b^2*polylog(3,(c*x+(c^2*x^2+1)^(1
/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+b^2*polylo
g(3,-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^
(1/2)/e^(1/2)-b^2*polylog(3,(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(
-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 739, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used

$= \{5793, 5827, 5680, 2221, 2611, 2320, 6724\}$

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+ex^2}\,dx = -\frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{-d}c+\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{-d}c+\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{(a+b\text{arcsinh}(cx))^2\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\text{arcsinh}(cx))^2\log\left(\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}+1\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+b\text{arcsinh}(cx))^2\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\text{arcsinh}(cx))^2\log\left(\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}+1\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b^2\,\text{PolyLog}\left(3,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{b^2\,\text{PolyLog}\left(3,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{b^2\,\text{PolyLog}\left(3,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{-d}c+\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{b^2\,\text{PolyLog}\left(3,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{-d}c+\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + e*x^2),x]
```

```
[Out] ((a + b*ArcSinh[c*x])^2*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt
[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[c*x])^2*Log[1 + (S
qrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt
[e]) + ((a + b*ArcSinh[c*x])^2*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d]
 + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[c*x])^2*Log
[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-
```

```
d]*Sqrt[e]) - (b*(a + b*ArcSinh[c*x])*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])
/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e]))])/(Sqrt[-d]*Sqrt[e]) + (b*(a + b*ArcSin
h[c*x])*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e
])])/(Sqrt[-d]*Sqrt[e]) - (b*(a + b*ArcSinh[c*x])*PolyLog[2, -((Sqrt[e]*E^A
rcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e]))])/(Sqrt[-d]*Sqrt[e]) + (b*(
a + b*ArcSinh[c*x])*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[
-(c^2*d) + e])])/(Sqrt[-d]*Sqrt[e]) + (b^2*PolyLog[3, -((Sqrt[e]*E^ArcSinh[
c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e]))])/(Sqrt[-d]*Sqrt[e]) - (b^2*PolyLo
g[3, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])])/(Sqrt[-d]
*Sqrt[e]) + (b^2*PolyLog[3, -((Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-
(c^2*d) + e]))])/(Sqrt[-d]*Sqrt[e]) - (b^2*PolyLog[3, (Sqrt[e]*E^ArcSinh[c*
x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(Sqrt[-d]*Sqrt[e])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sqrt{-d}(a+b\operatorname{arcsinh}(cx))^2}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}(a+b\operatorname{arcsinh}(cx))^2}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx$$

$$= -\frac{\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{-d}-\sqrt{e}x}\,dx}{2\sqrt{-d}} - \frac{\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{-d}+\sqrt{e}x}\,dx}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)^2\cosh(x)}{c\sqrt{-d}-\sqrt{e}\sinh(x)}\,dx, x, \operatorname{arcsinh}(cx)\right)}{2\sqrt{-d}} - \frac{\text{Subst}\left(\int \frac{(a+bx)^2\cosh(x)}{c\sqrt{-d}+\sqrt{e}\sinh(x)}\,dx, x, \operatorname{arcsinh}(cx)\right)}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^x(a+bx)^2}{c\sqrt{-d}-\sqrt{-c^2d+e}-\sqrt{e}e^x}\,dx, x, \operatorname{arcsinh}(cx)\right)}{2\sqrt{-d}}$$
$$- \frac{\text{Subst}\left(\int \frac{e^x(a+bx)^2}{c\sqrt{-d}+\sqrt{-c^2d+e}-\sqrt{e}e^x}\,dx, x, \operatorname{arcsinh}(cx)\right)}{2\sqrt{-d}}$$
$$- \frac{\text{Subst}\left(\int \frac{e^x(a+bx)^2}{c\sqrt{-d}-\sqrt{-c^2d+e}+\sqrt{e}e^x}\,dx, x, \operatorname{arcsinh}(cx)\right)}{2\sqrt{-d}}$$
$$- \frac{\text{Subst}\left(\int \frac{e^x(a+bx)^2}{c\sqrt{-d}+\sqrt{-c^2d+e}+\sqrt{e}e^x}\,dx, x, \operatorname{arcsinh}(cx)\right)}{2\sqrt{-d}}$$

$$= \frac{(a+b\text{arcsinh}(cx))^2\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\text{arcsinh}(cx))^2\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+b\text{arcsinh}(cx))^2\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\text{arcsinh}(cx))^2\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{b\text{Subst}\left(\int(a+bx)\log\left(1-\frac{\sqrt{e}e^x}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)dx,x,\text{arcsinh}(cx)\right)}{\sqrt{-d}\sqrt{e}} + \frac{b\text{Subst}\left(\int(a+bx)\log\left(1+\frac{\sqrt{e}e^x}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)dx,x,\text{arcsinh}(cx)\right)}{\sqrt{-d}\sqrt{e}} - \frac{b\text{Subst}\left(\int(a+bx)\log\left(1-\frac{\sqrt{e}e^x}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)dx,x,\text{arcsinh}(cx)\right)}{\sqrt{-d}\sqrt{e}} + \frac{b\text{Subst}\left(\int(a+bx)\log\left(1+\frac{\sqrt{e}e^x}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)dx,x,\text{arcsinh}(cx)\right)}{\sqrt{-d}\sqrt{e}}$$

$$= \frac{(a+b\text{arcsinh}(cx))^2 \log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}$$
$$- \frac{(a+b\text{arcsinh}(cx))^2 \log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}$$
$$+ \frac{(a+b\text{arcsinh}(cx))^2 \log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}$$
$$- \frac{(a+b\text{arcsinh}(cx))^2 \log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}$$
$$- \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}}$$
$$+ \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}}$$
$$- \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}}$$
$$+ \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}}$$
$$+ \frac{b^2\text{Subst}\left(\int \text{PolyLog}\left(2,-\frac{\sqrt{e}e^x}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)\,dx, x, \text{arcsinh}(cx)\right)}{\sqrt{-d}\sqrt{e}}$$
$$- \frac{b^2\text{Subst}\left(\int \text{PolyLog}\left(2,\frac{\sqrt{e}e^x}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)\,dx, x, \text{arcsinh}(cx)\right)}{\sqrt{-d}\sqrt{e}}$$
$$+ \frac{b^2\text{Subst}\left(\int \text{PolyLog}\left(2,-\frac{\sqrt{e}e^x}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)\,dx, x, \text{arcsinh}(cx)\right)}{\sqrt{-d}\sqrt{e}}$$
$$- \frac{b^2\text{Subst}\left(\int \text{PolyLog}\left(2,\frac{\sqrt{e}e^x}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)\,dx, x, \text{arcsinh}(cx)\right)}{\sqrt{-d}\sqrt{e}}$$

$$= \frac{(a+b\operatorname{arcsinh}(cx))^2\log\left(1-\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\operatorname{arcsinh}(cx))^2\log\left(1+\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+b\operatorname{arcsinh}(cx))^2\log\left(1-\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\operatorname{arcsinh}(cx))^2\log\left(1+\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{b(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{b(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{b(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{b(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{b^2\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt{e}x}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{x}dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{-d}\sqrt{e}} + \frac{b^2\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt{e}x}{-c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{x}dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{-d}\sqrt{e}} + \frac{b^2\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}x}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{x}dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{-d}\sqrt{e}} - \frac{b^2\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt{e}x}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{x}dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{-d}\sqrt{e}}$$

$$= \frac{(a+b\text{arcsinh}(cx))^2\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\text{arcsinh}(cx))^2\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+b\text{arcsinh}(cx))^2\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\text{arcsinh}(cx))^2\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{b(a+b\text{arcsinh}(cx))\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{b(a+b\text{arcsinh}(cx))\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{b(a+b\text{arcsinh}(cx))\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{b(a+b\text{arcsinh}(cx))\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{b^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{b^2\operatorname{PolyLog}\left(3,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{b^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{b^2\operatorname{PolyLog}\left(3,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 985, normalized size of antiderivative = 1.33

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+ex^2}\,dx$$
$$= \frac{2a^2\sqrt{-d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) - 2ab\sqrt{d}\text{arcsinh}(cx)\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right) - b^2\sqrt{d}\text{arcsinh}(cx)^2\log\left(1+\frac{\sqrt{e}e^{\text{arcsin}}}{c\sqrt{-d}-\sqrt{-}}\right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2),x]
```

```
[Out] (2*a^2*Sqrt[-d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - 2*a*b*Sqrt[d]*ArcSinh[c*x]*Lo
g[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])] - b^2*Sqr
t[d]*ArcSinh[c*x]^2*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c
^2*d) + e])] + 2*a*b*Sqrt[d]*ArcSinh[c*x]*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/
```

```
(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) + e])] + b^2*Sqrt[d]*ArcSinh[c*x]^2*Log[1 +
(Sqrt[e]*E^ArcSinh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) + e])] + 2*a*b*Sqrt
[d]*ArcSinh[c*x]*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*
d) + e])] + b^2*Sqrt[d]*ArcSinh[c*x]^2*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*
Sqrt[-d] + Sqrt[-(c^2*d) + e])] - 2*a*b*Sqrt[d]*ArcSinh[c*x]*Log[1 + (Sqrt[
e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])] - b^2*Sqrt[d]*ArcSinh
[c*x]^2*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])]
 + 2*b*Sqrt[d]*(a + b*ArcSinh[c*x])*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*
Sqrt[-d] - Sqrt[-(c^2*d) + e])] - 2*b*Sqrt[d]*(a + b*ArcSinh[c*x])*PolyLog[
2, (Sqrt[e]*E^ArcSinh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) + e])] - 2*a*b*S
qrt[d]*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) +
e]))] - 2*b^2*Sqrt[d]*ArcSinh[c*x]*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/(c
*Sqrt[-d] + Sqrt[-(c^2*d) + e]))] + 2*a*b*Sqrt[d]*PolyLog[2, (Sqrt[e]*E^Arc
Sinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])] + 2*b^2*Sqrt[d]*ArcSinh[c*x]*
PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])] - 2*
b^2*Sqrt[d]*PolyLog[3, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d)
 + e])] + 2*b^2*Sqrt[d]*PolyLog[3, (Sqrt[e]*E^ArcSinh[c*x])/(-(c*Sqrt[-d])
+ Sqrt[-(c^2*d) + e])] + 2*b^2*Sqrt[d]*PolyLog[3, -((Sqrt[e]*E^ArcSinh[c*x]
)/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e]))] - 2*b^2*Sqrt[d]*PolyLog[3, (Sqrt[e]*E
^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d^2]*Sqrt[e])
```

Maple [F]

$$\int \frac{(a+b\ \mathrm{arcsinh}\,(cx))^2}{e\,x^2+d}dx$$

[In] `int((a+b*arcsinh(c*x))^2/(e*x^2+d),x)`

[Out] `int((a+b*arcsinh(c*x))^2/(e*x^2+d),x)`

Fricas [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2}{d+ex^2}\,dx=\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2}{ex^2+d}\,dx$$

[In] `integrate((a+b*arcsinh(c*x))^2/(e*x^2+d),x, algorithm="fricas")`

[Out] `integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(e*x^2 + d), x)`

Sympy [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{d+ex^2}\,dx=\int \frac{(a+b\operatorname{asinh}(cx))^2}{d+ex^2}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2/(e*x**2+d),x)
[Out] Integral((a + b*asinh(c*x))**2/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{d+ex^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{d+ex^2}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{ex^2+d}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{d+ex^2}\,dx=\int \frac{(a+b\operatorname{asinh}(c\,x))^2}{e\,x^2+d}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(d + e*x^2),x)
[Out] int((a + b*asinh(c*x))^2/(d + e*x^2), x)
```

3.618 $\int \frac{(d+ex^2)^3}{a+b\,\mathbf{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 670

$$\int \frac{(d+ex^2)^3}{a+b\operatorname{arcsinh}(cx)}\,dx = \frac{d^3\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{bc} - \frac{3d^2e\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{4bc^3} + \frac{3de^2\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{8bc^5} - \frac{5e^3\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{64bc^7} + \frac{3d^2e\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{4bc^3} - \frac{9de^2\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5} + \frac{9e^3\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{64bc^7} + \frac{3de^2\cosh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5} - \frac{5e^3\cosh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{64bc^7} + \frac{e^3\cosh\left(\frac{7a}{b}\right)\operatorname{Chi}\left(\frac{7(a+b\operatorname{arcsinh}(cx))}{b}\right)}{64bc^7} - \frac{d^3\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{bc} + \frac{3d^2e\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{4bc^3} - \frac{3de^2\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{8bc^5} + \frac{5e^3\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{64bc^7} - \frac{3d^2e\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{4bc^3} + \frac{9de^2\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5} - \frac{9e^3\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{64bc^7} - \frac{3de^2\sinh\left(\frac{5a}{b}\right)\operatorname{Shi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5}$$

```
[Out] d^3*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c-3/4*d^2*e*Chi((a+b*arcsinh(c*x)
)/b)*cosh(a/b)/b/c^3+3/8*d*e^2*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c^5-5/
64*e^3*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c^7+3/4*d^2*e*Chi(3*(a+b*arcsi
nh(c*x))/b)*cosh(3*a/b)/b/c^3-9/16*d*e^2*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3
*a/b)/b/c^5+9/64*e^3*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b/c^7+3/16*d*e
^2*Chi(5*(a+b*arcsinh(c*x))/b)*cosh(5*a/b)/b/c^5-5/64*e^3*Chi(5*(a+b*arcsin
h(c*x))/b)*cosh(5*a/b)/b/c^7+1/64*e^3*Chi(7*(a+b*arcsinh(c*x))/b)*cosh(7*a/
b)/b/c^7-d^3*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c+3/4*d^2*e*Shi((a+b*arc
sinh(c*x))/b)*sinh(a/b)/b/c^3-3/8*d*e^2*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)
/b/c^5+5/64*e^3*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^7-3/4*d^2*e*Shi(3*(
a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^3+9/16*d*e^2*Shi(3*(a+b*arcsinh(c*x))/
b)*sinh(3*a/b)/b/c^5-9/64*e^3*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^7
-3/16*d*e^2*Shi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b/c^5+5/64*e^3*Shi(5*(a
+b*arcsinh(c*x))/b)*sinh(5*a/b)/b/c^7-1/64*e^3*Shi(7*(a+b*arcsinh(c*x))/b)*
sinh(7*a/b)/b/c^7
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 670, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used

$= \{5793, 5774, 3384, 3379, 3382, 5780, 5556\}$

$$\begin{aligned}
\int \frac{(d+ex^2)^3}{a+b\operatorname{arcsinh}(cx)}\,dx = & -\frac{5e^3\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{64bc^7} \\
& + \frac{9e^3\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{64bc^7} \\
& - \frac{5e^3\cosh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{64bc^7} \\
& + \frac{e^3\cosh\left(\frac{7a}{b}\right)\operatorname{Chi}\left(\frac{7(a+b\operatorname{arcsinh}(cx))}{b}\right)}{64bc^7} \\
& + \frac{5e^3\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{64bc^7} \\
& - \frac{9e^3\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{64bc^7} \\
& + \frac{5e^3\sinh\left(\frac{5a}{b}\right)\operatorname{Shi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{64bc^7} \\
& - \frac{e^3\sinh\left(\frac{7a}{b}\right)\operatorname{Shi}\left(\frac{7(a+b\operatorname{arcsinh}(cx))}{b}\right)}{64bc^7} \\
& + \frac{3de^2\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{8bc^5} \\
& - \frac{9de^2\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5} \\
& + \frac{3de^2\cosh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5} \\
& - \frac{3de^2\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{8bc^5} \\
& + \frac{9de^2\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5} \\
& - \frac{3de^2\sinh\left(\frac{5a}{b}\right)\operatorname{Shi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5} \\
& - \frac{3d^2e\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{4bc^3} \\
& + \frac{3d^2e\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{4bc^3} \\
& + \frac{3d^2e\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{4bc^3} \\
& - \frac{3d^2e\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{4bc^3} \\
& d^3\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)
\end{aligned}$$

```
[In] Int[(d + e*x^2)^3/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^3*Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(b*c) - (3*d^2*e*Cosh[
a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(4*b*c^3) + (3*d*e^2*Cosh[a/b]*C
oshIntegral[(a + b*ArcSinh[c*x])/b])/(8*b*c^5) - (5*e^3*Cosh[a/b]*CoshInteg
ral[(a + b*ArcSinh[c*x])/b])/(64*b*c^7) + (3*d^2*e*Cosh[(3*a)/b]*CoshIntegr
al[(3*(a + b*ArcSinh[c*x]))/b])/(4*b*c^3) - (9*d*e^2*Cosh[(3*a)/b]*CoshInte
gral[(3*(a + b*ArcSinh[c*x]))/b])/(16*b*c^5) + (9*e^3*Cosh[(3*a)/b]*CoshInt
egral[(3*(a + b*ArcSinh[c*x]))/b])/(64*b*c^7) + (3*d*e^2*Cosh[(5*a)/b]*Cosh
Integral[(5*(a + b*ArcSinh[c*x]))/b])/(16*b*c^5) - (5*e^3*Cosh[(5*a)/b]*Cos
hIntegral[(5*(a + b*ArcSinh[c*x]))/b])/(64*b*c^7) + (e^3*Cosh[(7*a)/b]*Cosh
Integral[(7*(a + b*ArcSinh[c*x]))/b])/(64*b*c^7) - (d^3*Sinh[a/b]*SinhInteg
ral[(a + b*ArcSinh[c*x])/b])/(b*c) + (3*d^2*e*Sinh[a/b]*SinhIntegral[(a + b
*ArcSinh[c*x])/b])/(4*b*c^3) - (3*d*e^2*Sinh[a/b]*SinhIntegral[(a + b*ArcSi
nh[c*x])/b])/(8*b*c^5) + (5*e^3*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])
/b])/(64*b*c^7) - (3*d^2*e*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x
]))/b])/(4*b*c^3) + (9*d*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c
*x]))/b])/(16*b*c^5) - (9*e^3*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[
c*x]))/b])/(64*b*c^7) - (3*d*e^2*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSi
nh[c*x]))/b])/(16*b*c^5) + (5*e^3*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcS
inh[c*x]))/b])/(64*b*c^7) - (e^3*Sinh[(7*a)/b]*SinhIntegral[(7*(a + b*ArcSi
nh[c*x]))/b])/(64*b*c^7)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{d^3}{a+b\text{arcsinh}(cx)}+\frac{3d^2ex^2}{a+b\text{arcsinh}(cx)}+\frac{3de^2x^4}{a+b\text{arcsinh}(cx)}+\frac{e^3x^6}{a+b\text{arcsinh}(cx)}\right)dx$$

$$= d^3\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx+\left(3d^2e\right)\int \frac{x^2}{a+b\text{arcsinh}(cx)}\,dx+\left(3de^2\right)\int \frac{x^4}{a+b\text{arcsinh}(cx)}\,dx+e^3\int \frac{x^6}{a+b\text{arcsinh}(cx)}\,dx$$

$$= \frac{d^3\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc}+\frac{\left(3d^2e\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc^3}+\frac{\left(3de^2\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc^5}+\frac{e^3\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^6\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc^7}$$

$$= \frac{(3d^2e)\,\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4x}-\frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{4x}\right)dx, x, a+b\text{arcsinh}(cx)\right)}{bc^3} + \frac{(3de^2)\,\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16x}-\frac{3\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16x}+\frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{8x}\right)dx, x, a+b\text{arcsinh}(cx)\right)}{bc^5} + \frac{e^3\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{7a}{b}-\frac{7x}{b}\right)}{64x}-\frac{5\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{64x}+\frac{9\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{64x}-\frac{5\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{64x}\right)dx, x, a+b\text{arcsinh}(cx)\right)}{bc^7} + \frac{\left(d^3\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}dx, x, a+b\text{arcsinh}(cx)\right)}{bc} - \frac{\left(d^3\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}dx, x, a+b\text{arcsinh}(cx)\right)}{bc}$$

$$= \frac{d^3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc} - \frac{d^3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc} + \frac{(3d^2e)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}dx, x, a+b\text{arcsinh}(cx)\right)}{4bc^3} - \frac{(3d^2e)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}dx, x, a+b\text{arcsinh}(cx)\right)}{4bc^3} + \frac{(3de^2)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{x}dx, x, a+b\text{arcsinh}(cx)\right)}{16bc^5} + \frac{(3de^2)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}dx, x, a+b\text{arcsinh}(cx)\right)}{8bc^5} - \frac{(9de^2)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}dx, x, a+b\text{arcsinh}(cx)\right)}{16bc^5} + \frac{e^3\text{Subst}\left(\int \frac{\cosh\left(\frac{7a}{b}-\frac{7x}{b}\right)}{x}dx, x, a+b\text{arcsinh}(cx)\right)}{64bc^7} - \frac{(5e^3)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{x}dx, x, a+b\text{arcsinh}(cx)\right)}{64bc^7} - \frac{(5e^3)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}dx, x, a+b\text{arcsinh}(cx)\right)}{64bc^7} + \frac{(9e^3)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}dx, x, a+b\text{arcsinh}(cx)\right)}{64bc^7}$$

$= \text{Too large to display}$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 444, normalized size of antiderivative = 0.66

$$\int \frac{(d+ex^2)^3}{a+b\text{arcsinh}(cx)}\,dx$$
$$= \frac{(64c^6d^3-48c^4d^2e+24c^2de^2-5e^3)\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)+3e(16c^4d^2-12c^2de+3e^2)\cosh\left(\frac{3a}{b}\right)\text{C}}{}$$

```
[In] Integrate[(d + e*x^2)^3/(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((64*c^6*d^3 - 48*c^4*d^2*e + 24*c^2*d*e^2 - 5*e^3)*Cosh[a/b]*CoshIntegral[
a/b + ArcSinh[c*x]] + 3*e*(16*c^4*d^2 - 12*c^2*d*e + 3*e^2)*Cosh[(3*a)/b]*C
oshIntegral[3*(a/b + ArcSinh[c*x])] + 12*c^2*d*e^2*Cosh[(5*a)/b]*CoshIntegr
al[5*(a/b + ArcSinh[c*x])] - 5*e^3*Cosh[(5*a)/b]*CoshIntegral[5*(a/b + ArcS
inh[c*x])] + e^3*Cosh[(7*a)/b]*CoshIntegral[7*(a/b + ArcSinh[c*x])] - 64*c^
6*d^3*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 48*c^4*d^2*e*Sinh[a/b]*S
inhIntegral[a/b + ArcSinh[c*x]] - 24*c^2*d*e^2*Sinh[a/b]*SinhIntegral[a/b +
 ArcSinh[c*x]] + 5*e^3*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 48*c^4*
d^2*e*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] + 36*c^2*d*e^2*Sin
h[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] - 9*e^3*Sinh[(3*a)/b]*SinhI
ntegral[3*(a/b + ArcSinh[c*x])] - 12*c^2*d*e^2*Sinh[(5*a)/b]*SinhIntegral[5
*(a/b + ArcSinh[c*x])] + 5*e^3*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[
c*x])] - e^3*Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])])/(64*b*c^7)
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 654, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{e^3 \mathrm{e}^{\frac{7a}{b}} \mathrm{Ei}_1\left(7\,\text{arcsinh}(cx)+\frac{7a}{b}\right)}{128c^6b}-\frac{e^3 \mathrm{e}^{-\frac{7a}{b}} \mathrm{Ei}_1\left(-7\,\text{arcsinh}(cx)-\frac{7a}{b}\right)}{128c^6b}-\frac{\mathrm{e}^{\frac{a}{b}} \mathrm{Ei}_1\left(\text{arcsinh}(cx)+\frac{a}{b}\right)d^3}{2b}+\frac{3\,\mathrm{e}^{\frac{a}{b}} \mathrm{Ei}_1\left(\text{arcsinh}(cx)+\frac{a}{b}\right)d^2e}{8c^2b}-}{}$
default	$\frac{-\frac{e^3 \mathrm{e}^{\frac{7a}{b}} \mathrm{Ei}_1\left(7\,\text{arcsinh}(cx)+\frac{7a}{b}\right)}{128c^6b}-\frac{e^3 \mathrm{e}^{-\frac{7a}{b}} \mathrm{Ei}_1\left(-7\,\text{arcsinh}(cx)-\frac{7a}{b}\right)}{128c^6b}-\frac{\mathrm{e}^{\frac{a}{b}} \mathrm{Ei}_1\left(\text{arcsinh}(cx)+\frac{a}{b}\right)d^3}{2b}+\frac{3\,\mathrm{e}^{\frac{a}{b}} \mathrm{Ei}_1\left(\text{arcsinh}(cx)+\frac{a}{b}\right)d^2e}{8c^2b}-}{}$

```
[In] int((e*x^2+d)^3/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/128/c^6*e^3/b*exp(7*a/b)*Ei(1,7*arcsinh(c*x)+7*a/b)-1/128/c^6*e^3/b
*exp(-7*a/b)*Ei(1,-7*arcsinh(c*x)-7*a/b)-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a
/b)*d^3+3/8/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*d^2*e-3/16/c^4/b*exp(a/b)
*Ei(1,arcsinh(c*x)+a/b)*d*e^2+5/128/c^6/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*e
^3-1/2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d^3+3/8/c^2/b*exp(-a/b)*Ei(1,-ar
csinh(c*x)-a/b)*d^2*e-3/16/c^4/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d*e^2+5/
128/c^6/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*e^3-3/8/c^2*e/b*exp(3*a/b)*Ei(1
```

```
,3*arcsinh(c*x)+3*a/b)*d^2+9/32/c^4*e^2/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*
a/b)*d-9/128/c^6*e^3/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-3/8/c^2*e/b*ex
p(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*d^2+9/32/c^4*e^2/b*exp(-3*a/b)*Ei(1,-
3*arcsinh(c*x)-3*a/b)*d-9/128/c^6*e^3/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*
a/b)-3/32/c^4*e^2/b*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)*d+5/128/c^6*e^3/b
*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-3/32/c^4*e^2/b*exp(-5*a/b)*Ei(1,-5*a
rcsinh(c*x)-5*a/b)*d+5/128/c^6*e^3/b*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b
))
```

Fricas [F]

$$\int \frac{(d+ex^2)^3}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(ex^2+d)^3}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate((e*x^2+d)^3/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)/(b*arcsinh(c*x) + a),
x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^3}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(d+ex^2)^3}{a+b\,\text{asinh}\,(cx)}\,dx$$

```
[In] integrate((e*x**2+d)**3/(a+b*asinh(c*x)),x)
[Out] Integral((d + e*x**2)**3/(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^3}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(ex^2+d)^3}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate((e*x^2+d)^3/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^3/(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^3}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(ex^2+d)^3}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate((e*x^2+d)^3/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^3/(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(e\,x^2+d)^3}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

```
[In] int((d + e*x^2)^3/(a + b*asinh(c*x)),x)
[Out] int((d + e*x^2)^3/(a + b*asinh(c*x)), x)
```

3.619 $\int \frac{(d+ex^2)^2}{a+b\,\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 388

$$\int \frac{(d+ex^2)^2}{a+b\operatorname{arcsinh}(cx)}\,dx = \frac{d^2\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{bc}-\frac{de\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{2bc^3}+\frac{e^2\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{8bc^5}+\frac{de\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{2bc^3}-\frac{3e^2\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5}+\frac{e^2\cosh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5}-\frac{d^2\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{bc}+\frac{de\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{2bc^3}-\frac{e^2\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{8bc^5}-\frac{de\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{2bc^3}+\frac{3e^2\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5}-\frac{e^2\sinh\left(\frac{5a}{b}\right)\operatorname{Shi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5}$$

```
[Out] d^2*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c-1/2*d*e*Chi((a+b*arcsinh(c*x))/
b)*cosh(a/b)/b/c^3+1/8*e^2*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c^5+1/2*d*
e*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b/c^3-3/16*e^2*Chi(3*(a+b*arcsinh
(c*x))/b)*cosh(3*a/b)/b/c^5+1/16*e^2*Chi(5*(a+b*arcsinh(c*x))/b)*cosh(5*a/b
)/b/c^5-d^2*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c+1/2*d*e*Shi((a+b*arcsin
h(c*x))/b)*sinh(a/b)/b/c^3-1/8*e^2*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^
5-1/2*d*e*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^3+3/16*e^2*Shi(3*(a+b
*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^5-1/16*e^2*Shi(5*(a+b*arcsinh(c*x))/b)*si
nh(5*a/b)/b/c^5
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {5793, 5774, 3384, 3379, 3382, 5780, 5556}

$$\int \frac{(d+ex^2)^2}{a+b\text{arcsinh}(cx)}\,dx = \frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8bc^5} - \frac{3e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^5} + \frac{e^2\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^5} - \frac{e^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8bc^5} + \frac{3e^2\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^5} - \frac{e^2\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^5} - \frac{de\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{2bc^3} + \frac{de\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{2bc^3} + \frac{de\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{2bc^3} - \frac{de\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{2bc^3} + \frac{d^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc} - \frac{d^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc}$$

```
[In] Int[(d + e*x^2)^2/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(b*c) - (d*e*Cosh[a/b]
*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(2*b*c^3) + (e^2*Cosh[a/b]*CoshInteg
ral[(a + b*ArcSinh[c*x])/b])/(8*b*c^5) + (d*e*Cosh[(3*a)/b]*CoshIntegral[(3
*(a + b*ArcSinh[c*x]))/b])/(2*b*c^3) - (3*e^2*Cosh[(3*a)/b]*CoshIntegral[(3
*(a + b*ArcSinh[c*x]))/b])/(16*b*c^5) + (e^2*Cosh[(5*a)/b]*CoshIntegral[(5*
```

```
(a + b*ArcSinh[c*x]))/b])/(16*b*c^5) - (d^2*Sinh[a/b]*SinhIntegral[(a + b*A
rcSinh[c*x])/b])/(b*c) + (d*e*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b
])/(2*b*c^3) - (e^2*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(8*b*c^
5) - (d*e*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(2*b*c^3)
 + (3*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(16*b*c^5
) - (e^2*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*x]))/b])/(16*b*c^5)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d^2}{a+b\text{arcsinh}(cx)}+\frac{2dex^2}{a+b\text{arcsinh}(cx)}+\frac{e^2x^4}{a+b\text{arcsinh}(cx)}\right)\,dx\\
&= d^2\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx+(2de)\int \frac{x^2}{a+b\text{arcsinh}(cx)}\,dx+e^2\int \frac{x^4}{a+b\text{arcsinh}(cx)}\,dx\\
&= \frac{d^2\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc}\\
&\quad+\frac{(2de)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc^3}\\
&\quad+\frac{e^2\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc^5}\\
&= \frac{(2de)\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4x}-\frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{4x}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc^3}\\
&\quad+\frac{e^2\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16x}-\frac{3\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16x}+\frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{8x}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc^5}\\
&\quad+\frac{\left(d^2\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc}\\
&\quad-\frac{\left(d^2\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc}
\end{aligned}$$

$$= \frac{d^2 \cosh\left(\frac{a}{b}\right) \text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc} - \frac{d^2 \sinh\left(\frac{a}{b}\right) \text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc} + \frac{(de)\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x} dx, x, a+b\text{arcsinh}(cx)\right)}{2bc^3} - \frac{(de)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x} dx, x, a+b\text{arcsinh}(cx)\right)}{2bc^3} + \frac{e^2\text{Subst}\left(\int \frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{x} dx, x, a+b\text{arcsinh}(cx)\right)}{16bc^5} + \frac{e^2\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x} dx, x, a+b\text{arcsinh}(cx)\right)}{8bc^5} - \frac{(3e^2)\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x} dx, x, a+b\text{arcsinh}(cx)\right)}{16bc^5}$$

$$= \frac{d^2 \cosh\left(\frac{a}{b}\right) \text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc} - \frac{d^2 \sinh\left(\frac{a}{b}\right) \text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc} - \frac{\left(de\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x} dx, x, a+b\text{arcsinh}(cx)\right)}{2bc^3} + \frac{\left(e^2\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x} dx, x, a+b\text{arcsinh}(cx)\right)}{8bc^5} + \frac{\left(de\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x} dx, x, a+b\text{arcsinh}(cx)\right)}{2bc^3} - \frac{\left(3e^2\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x} dx, x, a+b\text{arcsinh}(cx)\right)}{16bc^5} + \frac{\left(e^2\cosh\left(\frac{5a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{5x}{b}\right)}{x} dx, x, a+b\text{arcsinh}(cx)\right)}{16bc^5} + \frac{\left(de\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x} dx, x, a+b\text{arcsinh}(cx)\right)}{2bc^3} - \frac{\left(e^2\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x} dx, x, a+b\text{arcsinh}(cx)\right)}{8bc^5} - \frac{\left(de\sinh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x} dx, x, a+b\text{arcsinh}(cx)\right)}{2bc^3} + \frac{\left(3e^2\sinh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x} dx, x, a+b\text{arcsinh}(cx)\right)}{16bc^5} - \frac{\left(e^2\sinh\left(\frac{5a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{5x}{b}\right)}{x} dx, x, a+b\text{arcsinh}(cx)\right)}{16bc^5}$$

$$
\begin{aligned}
&= \frac{d^2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{bc} - \frac{de\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{2bc^3} \\
&+ \frac{e^2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{8bc^5} + \frac{de\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{2bc^3} \\
&- \frac{3e^2\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5} + \frac{e^2\cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5} \\
&- \frac{d^2\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{bc} + \frac{de\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{2bc^3} \\
&- \frac{e^2\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{8bc^5} - \frac{de\sinh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{2bc^3} \\
&+ \frac{3e^2\sinh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5} - \frac{e^2\sinh\left(\frac{5a}{b}\right)\mathrm{Shi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.65

$$
\int \frac{(d+ex^2)^2}{a+b\operatorname{arcsinh}(cx)}\,dx
= \frac{2(8c^4d^2-4c^2de+e^2)\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)+(8c^2d-3e)\,e\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(3\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\right)+
$$

```
[In] Integrate[(d + e*x^2)^2/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*(8*c^4*d^2 - 4*c^2*d*e + e^2)*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]]
 + (8*c^2*d - 3*e)*e*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] + e
^2*Cosh[(5*a)/b]*CoshIntegral[5*(a/b + ArcSinh[c*x])] - 16*c^4*d^2*Sinh[a/b
]*SinhIntegral[a/b + ArcSinh[c*x]] + 8*c^2*d*e*Sinh[a/b]*SinhIntegral[a/b +
 ArcSinh[c*x]] - 2*e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 8*c^2*d
*e*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] + 3*e^2*Sinh[(3*a)/b]
*SinhIntegral[3*(a/b + ArcSinh[c*x])] - e^2*Sinh[(5*a)/b]*SinhIntegral[5*(a
/b + ArcSinh[c*x])])/(16*b*c^5)
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.98

method	result
derivativedivides	$-\frac{e^2 \mathrm{e}^{\frac{5a}{b}} \mathrm{Ei}_1\left(5\,\mathrm{arcsinh}(cx)+\frac{5a}{b}\right)}{32c^4b}-\frac{e^2 \mathrm{e}^{-\frac{5a}{b}} \mathrm{Ei}_1\left(-5\,\mathrm{arcsinh}(cx)-\frac{5a}{b}\right)}{32c^4b}-\frac{\mathrm{e}^{\frac{a}{b}} \mathrm{Ei}_1\left(\mathrm{arcsinh}(cx)+\frac{a}{b}\right)d^2}{2b}+\frac{\mathrm{e}^{\frac{a}{b}} \mathrm{Ei}_1\left(\mathrm{arcsinh}(cx)+\frac{a}{b}\right)de}{4c^2b}-$
default	$-\frac{e^2 \mathrm{e}^{\frac{5a}{b}} \mathrm{Ei}_1\left(5\,\mathrm{arcsinh}(cx)+\frac{5a}{b}\right)}{32c^4b}-\frac{e^2 \mathrm{e}^{-\frac{5a}{b}} \mathrm{Ei}_1\left(-5\,\mathrm{arcsinh}(cx)-\frac{5a}{b}\right)}{32c^4b}-\frac{\mathrm{e}^{\frac{a}{b}} \mathrm{Ei}_1\left(\mathrm{arcsinh}(cx)+\frac{a}{b}\right)d^2}{2b}+\frac{\mathrm{e}^{\frac{a}{b}} \mathrm{Ei}_1\left(\mathrm{arcsinh}(cx)+\frac{a}{b}\right)de}{4c^2b}-$

```
[In] int((e*x^2+d)^2/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/32/c^4*e^2/b*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-1/32/c^4*e^2/b*e
xp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b
)*d^2+1/4/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*d*e-1/16/c^4/b*exp(a/b)*Ei(
1,arcsinh(c*x)+a/b)*e^2-1/2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d^2+1/4/c^2
/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d*e-1/16/c^4/b*exp(-a/b)*Ei(1,-arcsinh
(c*x)-a/b)*e^2-1/4/c^2*e/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)*d+3/32/c^4
*e^2/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/4/c^2*e/b*exp(-3*a/b)*Ei(1,-
3*arcsinh(c*x)-3*a/b)*d+3/32/c^4*e^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a
/b))
```

Fricas [F]

$$\int \frac{(d+ex^2)^2}{a+b\mathrm{arcsinh}(cx)}\,dx=\int \frac{(ex^2+d)^2}{b\,\mathrm{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*d*e*x^2 + d^2)/(b*arcsinh(c*x) + a), x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2}{a+b\mathrm{arcsinh}(cx)}\,dx=\int \frac{(d+ex^2)^2}{a+b\,\mathrm{asinh}\,(cx)}\,dx$$

```
[In] integrate((e*x**2+d)**2/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d + e*x**2)**2/(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(ex^2+d)^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^2/(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^2}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(ex^2+d)^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2/(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(e\,x^2+d)^2}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

```
[In] int((d + e*x^2)^2/(a + b*asinh(c*x)),x)
[Out] int((d + e*x^2)^2/(a + b*asinh(c*x)), x)
```

3.620 $\int \frac{d+ex^2}{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 180

$$\int \frac{d+ex^2}{a+b\text{arcsinh}(cx)}\,dx = \frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc} - \frac{e\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4bc^3} + \frac{e\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4bc^3} - \frac{d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc} + \frac{e\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4bc^3} - \frac{e\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4bc^3}$$

```
[Out] d*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c-1/4*e*Chi((a+b*arcsinh(c*x))/b)*c
osh(a/b)/b/c^3+1/4*e*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b/c^3-d*Shi((a
+b*arcsinh(c*x))/b)*sinh(a/b)/b/c+1/4*e*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)
/b/c^3-1/4*e*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used

$= \{5793, 5774, 3384, 3379, 3382, 5780, 5556\}$

$$\int \frac{d+ex^2}{a+b\operatorname{arcsinh}(cx)}\,dx = -\frac{e\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{4bc^3} + \frac{e\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{4bc^3} + \frac{e\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{4bc^3} - \frac{e\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{4bc^3} + \frac{d\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{bc} - \frac{d\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{bc}$$

```
[In] Int[(d + e*x^2)/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(b*c) - (e*Cosh[a/b]*Cos
hIntegral[(a + b*ArcSinh[c*x])/b])/(4*b*c^3) + (e*Cosh[(3*a)/b]*CoshIntegra
l[(3*(a + b*ArcSinh[c*x]))/b])/(4*b*c^3) - (d*Sinh[a/b]*SinhIntegral[(a + b
*ArcSinh[c*x])/b])/(b*c) + (e*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b
])/(4*b*c^3) - (e*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(
4*b*c^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d}{a+b\text{arcsinh}(cx)}+\frac{ex^2}{a+b\text{arcsinh}(cx)}\right)\,dx \\
&= d\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx+e\int \frac{x^2}{a+b\text{arcsinh}(cx)}\,dx \\
&= \frac{d\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc} \\
&\quad+\frac{e\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc^3} \\
&= \frac{e\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4x}-\frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{4x}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc^3} \\
&\quad+\frac{\left(d\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc} \\
&\quad-\frac{\left(d\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc}
\end{aligned}$$

$$= \frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc} - \frac{d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc} + \frac{e\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4bc^3} - \frac{e\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4bc^3}$$

$$= \frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc} - \frac{d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc} - \frac{\left(e\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4bc^3} + \frac{\left(e\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4bc^3} + \frac{\left(e\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4bc^3} - \frac{\left(e\sinh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4bc^3}$$

$$= \frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc} - \frac{e\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4bc^3} + \frac{e\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4bc^3} - \frac{d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc} + \frac{e\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4bc^3} - \frac{e\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4bc^3}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.70

$$\int \frac{d+ex^2}{a+b\text{arcsinh}(cx)}\,dx = \frac{(4c^2d-e)\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)+e\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)-4c^2d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\right.}{4bc^3}$$

```
[In] Integrate[(d + e*x^2)/(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((4*c^2*d - e)*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] + e*Cosh[(3*a)/b]
*CoshIntegral[3*(a/b + ArcSinh[c*x])] - 4*c^2*d*Sinh[a/b]*SinhIntegral[a/b
+ ArcSinh[c*x]] + e*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - e*Sinh[(3*
a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])])/(4*b*c^3)
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{-\frac{e\,e^{-\frac{3a}{b}}\operatorname{Ei}_1\left(-3\operatorname{arcsinh}(cx)-\frac{3a}{b}\right)}{8c^2b}-\frac{e\,e^{\frac{3a}{b}}\operatorname{Ei}_1\left(3\operatorname{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b}-\frac{e^{\frac{a}{b}}\operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)d}{2b}+\frac{e^{\frac{a}{b}}\operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)e}{8c^2b}-e^{-\frac{a}{b}}\ \mathrm{E}\cdots}{c}$
default	$\frac{-\frac{e\,e^{-\frac{3a}{b}}\operatorname{Ei}_1\left(-3\operatorname{arcsinh}(cx)-\frac{3a}{b}\right)}{8c^2b}-\frac{e\,e^{\frac{3a}{b}}\operatorname{Ei}_1\left(3\operatorname{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b}-\frac{e^{\frac{a}{b}}\operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)d}{2b}+\frac{e^{\frac{a}{b}}\operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)e}{8c^2b}-e^{-\frac{a}{b}}\ \mathrm{E}\cdots}{c}$

```
[In] int((e*x^2+d)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/8*e/c^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-1/8*e/c^2/b*exp(3
*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*d+1/
8/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*e-1/2/b*exp(-a/b)*Ei(1,-arcsinh(c*x
)-a/b)*d+1/8/c^2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*e)
```

Fricas [F]

$$\int \frac{d+ex^2}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{ex^2+d}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

Sympy [F]

$$\int \frac{d+ex^2}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{d+ex^2}{a+b\,\text{asinh}\,(cx)}\,dx$$

```
[In] integrate((e*x**2+d)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d + e*x**2)/(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{d+ex^2}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{ex^2+d}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{d+ex^2}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{ex^2+d}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{e\,x^2+d}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

```
[In] int((d + e*x^2)/(a + b*asinh(c*x)),x)
[Out] int((d + e*x^2)/(a + b*asinh(c*x)), x)
```

3.621 $\int \frac{1}{a+b\text{arcsinh}(cx)} dx$

Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{1}{a+b\text{arcsinh}(cx)} dx = \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc}$$

```
[Out] Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c-Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)
/b/c
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5774, 3384, 3379, 3382}

$$\int \frac{1}{a+b\text{arcsinh}(cx)} dx = \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc}$$

```
[In] Int[(a + b*ArcSinh[c*x])^(-1),x]
```

```
[Out] (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(b*c) - (Sinh[a/b]*SinhInt
egral[(a + b*ArcSinh[c*x])/b])/(b*c)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc}\\
&= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc}\\
&\quad - \frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc}\\
&= \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx = \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{bc}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-1),x]
[Out] (Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - Sinh[a/b]*SinhIntegral[a/b +
ArcSinh[c*x]])/(b*c)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{-\frac{e^{\frac{a}{b}} \operatorname{Ei}_{1}\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}} \operatorname{Ei}_{1}\left(-\operatorname{arcsinh}(cx)-\frac{a}{b}\right)}{2b}}{c}$	56
default	$\frac{-\frac{e^{\frac{a}{b}} \operatorname{Ei}_{1}\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}} \operatorname{Ei}_{1}\left(-\operatorname{arcsinh}(cx)-\frac{a}{b}\right)}{2b}}{c}$	56

```
[In] int(1/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
[Out] 1/c*(-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/2/b*exp(-a/b)*Ei(1,-arcsinh(c
*x)-a/b))
```

Fricas [F]

$$\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{1}{b\,\text{arsinh}\left(cx\right)+a}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*arcsinh(c*x) + a), x)
```

Sympy [F]

$$\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{1}{a+b\,\text{asinh}\left(cx\right)}\,dx$$

```
[In] integrate(1/(a+b*asinh(c*x)),x)
[Out] Integral(1/(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{1}{b\,\text{arsinh}\left(cx\right)+a}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(1/(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{1}{a + b\text{arcsinh}(cx)}\,dx = \int \frac{1}{b\,\text{arsinh}\,(cx) + a}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(1/(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a + b\text{arcsinh}(cx)}\,dx = \int \frac{1}{a + b\,\text{asinh}\,(c\,x)}\,dx$$

```
[In] int(1/(a + b*asinh(c*x)),x)
[Out] int(1/(a + b*asinh(c*x)), x)
```

3.622 $\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))}\,dx=\text{Int}\left(\frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Int[1/((d + e*x^2)*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))} dx = \int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))} dx$$

```
[In] Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(e\,x^2+d)\,(a+b\,\operatorname{arcsinh}\,(cx))}dx$$

```
[In] int(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x)
[Out] int(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))} dx = \int \frac{1}{(ex^2+d)(b\,\text{arsinh}\,(cx)+a)} dx$$

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(1/(a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 3.58 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))} dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))\,(d+ex^2)} dx$$

```
[In] integrate(1/(e*x**2+d)/(a+b*asinh(c*x)),x)
[Out] Integral(1/((a + b*asinh(c*x))*(d + e*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)\,(a+b\mathrm{arcsinh}(cx))}\,dx = \int \frac{1}{(ex^2+d)(b\,\mathrm{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(1/((e*x^2 + d)*(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)\,(a+b\mathrm{arcsinh}(cx))}\,dx = \int \frac{1}{(ex^2+d)(b\,\mathrm{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)*(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)\,(a+b\mathrm{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\mathrm{asinh}\,(c\,x))\,\left(e\,x^2+d\right)}\,dx$$

```
[In] int(1/((a + b*asinh(c*x))*(d + e*x^2)),x)
[Out] int(1/((a + b*asinh(c*x))*(d + e*x^2)), x)
```

3.623 $\int \frac{1}{(d+ex^2)^2(a+b\text{arcsinh}(cx))} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))} \, dx = \text{Int}\left(\frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))} \, dx = \int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))} \, dx$$

```
[In] Int[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))} \, dx$$

Mathematica [N/A]

Not integrable

Time = 2.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))} dx = \int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))} dx$$

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(e\,x^2+d)^2 (a+b\ \text{arcsinh}\,(cx))}dx$$

```
[In] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x)
[Out] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.65

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))} dx = \int \frac{1}{(ex^2+d)^2(b\,\text{arsinh}\,(cx)+a)} dx$$

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(1/(a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*
d^2)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 65.61 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))} dx = \int \frac{1}{(a+b\operatorname{asinh}(cx))(d+ex^2)^2} dx$$

```
[In] integrate(1/(e*x**2+d)**2/(a+b*asinh(c*x)),x)
[Out] Integral(1/((a + b*asinh(c*x))*(d + e*x**2)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))} dx = \int \frac{1}{(ex^2+d)^2(b\operatorname{arsinh}(cx)+a)} dx$$

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(1/((e*x^2 + d)^2*(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))} dx = \int \frac{1}{(ex^2+d)^2(b\operatorname{arsinh}(cx)+a)} dx$$

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)^2*(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\mathrm{asinh}\,(c\,x))\,\,(e\,x^2+d)^2}\,dx$$

```
[In] int(1/((a + b*asinh(c*x))*(d + e*x^2)^2),x)
[Out] int(1/((a + b*asinh(c*x))*(d + e*x^2)^2), x)
```

3.624 $\int \frac{(d+ex^2)^2}{(a+b\,\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 495

$$\int \frac{(d+ex^2)^2}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = -\frac{d^2\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{2dex^2\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{e^2x^4\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{d^2\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c} + \frac{de\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{2b^2c^3} - \frac{e^2\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8b^2c^5} - \frac{3de\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{2b^2c^3} + \frac{9e^2\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16b^2c^5} - \frac{5e^2\operatorname{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{16b^2c^5} + \frac{d^2\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{b^2c} - \frac{de\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{2b^2c^3} + \frac{e^2\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{8b^2c^5} + \frac{3de\cosh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{2b^2c^3} - \frac{9e^2\cosh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16b^2c^5} + \frac{5e^2\cosh\left(\frac{5a}{b}\right)\operatorname{Shi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16b^2c^5}$$

```
[Out] d^2*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c-1/2*d*e*cosh(a/b)*Shi((a+b*ar
csinh(c*x))/b)/b^2/c^3+1/8*e^2*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c^5+
3/2*d*e*cosh(3*a/b)*Shi(3*(a+b*arcsinh(c*x))/b)/b^2/c^3-9/16*e^2*cosh(3*a/b
)*Shi(3*(a+b*arcsinh(c*x))/b)/b^2/c^5+5/16*e^2*cosh(5*a/b)*Shi(5*(a+b*arcsi
nh(c*x))/b)/b^2/c^5-d^2*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c+1/2*d*e*C
hi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c^3-1/8*e^2*Chi((a+b*arcsinh(c*x))/b
)*sinh(a/b)/b^2/c^5-3/2*d*e*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^3
```

```
+9/16*e^2*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^5-5/16*e^2*Chi(5*(a
+b*arcsinh(c*x))/b)*sinh(5*a/b)/b^2/c^5-d^2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcs
inh(c*x))-2*d*e*x^2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))-e^2*x^4*(c^2*x
^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {5793, 5773, 5819, 3384, 3379, 3382, 5778}

$$\begin{aligned}\int \frac{(d+ex^2)^2}{(a+b\text{arcsinh}(cx))^2}\,dx = &-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^5}\\
&+\frac{9e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^5}\\
&-\frac{5e^2\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^5}\\
&+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^5}\\
&-\frac{9e^2\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^5}\\
&+\frac{5e^2\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^5}\\
&+\frac{de\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{2b^2c^3}\\
&-\frac{3de\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{2b^2c^3}\\
&-\frac{de\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{2b^2c^3}\\
&+\frac{3de\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{2b^2c^3}\\
&-\frac{d^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c}\\
&+\frac{d^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c}-\frac{d^2\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}\\
&-\frac{2dex^2\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}-\frac{e^2x^4\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}\end{aligned}$$

```
[In] Int[(d + e*x^2)^2/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((d^2*Sqrt[1 + c^2*x^2])/(b*c*(a + b*ArcSinh[c*x]))) - (2*d*e*x^2*Sqrt[1 +
 c^2*x^2])/(b*c*(a + b*ArcSinh[c*x])) - (e^2*x^4*Sqrt[1 + c^2*x^2])/(b*c*(a
 + b*ArcSinh[c*x])) - (d^2*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/
(b^2*c) + (d*e*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(2*b^2*c^3)
- (e^2*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(8*b^2*c^5) - (3*d*e
*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(2*b^2*c^3) + (9*e
^2*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(16*b^2*c^5) - (
5*e^2*CoshIntegral[(5*(a + b*ArcSinh[c*x]))/b]*Sinh[(5*a)/b])/(16*b^2*c^5)
+ (d^2*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b^2*c) - (d*e*Cosh[
a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(2*b^2*c^3) + (e^2*Cosh[a/b]*Sin
hIntegral[(a + b*ArcSinh[c*x])/b])/(8*b^2*c^5) + (3*d*e*Cosh[(3*a)/b]*SinhI
ntegral[(3*(a + b*ArcSinh[c*x]))/b])/(2*b^2*c^3) - (9*e^2*Cosh[(3*a)/b]*Sin
hIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(16*b^2*c^5) + (5*e^2*Cosh[(5*a)/b]*
SinhIntegral[(5*(a + b*ArcSinh[c*x]))/b])/(16*b^2*c^5)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
```

```
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{d^2}{(a+b\text{arcsinh}(cx))^2}+\frac{2dex^2}{(a+b\text{arcsinh}(cx))^2}+\frac{e^2x^4}{(a+b\text{arcsinh}(cx))^2}\right)dx\\
&= d^2\int \frac{1}{(a+b\text{arcsinh}(cx))^2}\,dx+(2de)\int \frac{x^2}{(a+b\text{arcsinh}(cx))^2}\,dx+e^2\int \frac{x^4}{(a+b\text{arcsinh}(cx))^2}\,dx\\
&= -\frac{d^2\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))}-\frac{2dex^2\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))}\\
&\quad -\frac{e^2x^4\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))}+\frac{(cd^2)\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx}{b}\\
&\quad +\frac{(2de)\text{Subst}\left(\int \left(-\frac{3\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4x}+\frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{4x}\right)dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^3}\\
&\quad +\frac{e^2\text{Subst}\left(\int \left(-\frac{5\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16x}+\frac{9\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16x}-\frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{8x}\right)dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^5}
\end{aligned}
$$

$$= -\frac{d^2\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{2dex^2\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{e^2x^4\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))}$$

$$- \frac{d^2\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b^2c}$$

$$+ \frac{(de)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{2b^2c^3}$$

$$- \frac{(3de)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{2b^2c^3}$$

$$- \frac{e^2\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{8b^2c^5}$$

$$- \frac{(5e^2)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{16b^2c^5}$$

$$+ \frac{(9e^2)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{16b^2c^5}$$

$$= -\frac{d^2\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{2dex^2\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{e^2x^4\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))}$$
$$+ \frac{\left(d^2\cosh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b^2c}$$
$$- \frac{\left(de\cosh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{2b^2c^3}$$
$$+ \frac{\left(e^2\cosh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{8b^2c^5}$$
$$+ \frac{\left(3de\cosh\left(\frac{3a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{2b^2c^3}$$
$$- \frac{\left(9e^2\cosh\left(\frac{3a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{16b^2c^5}$$
$$+ \frac{\left(5e^2\cosh\left(\frac{5a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{5x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{16b^2c^5}$$
$$- \frac{\left(d^2\sinh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b^2c}$$
$$+ \frac{\left(de\sinh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{2b^2c^3}$$
$$- \frac{\left(e^2\sinh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{8b^2c^5}$$
$$- \frac{\left(3de\sinh\left(\frac{3a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{2b^2c^3}$$
$$+ \frac{\left(9e^2\sinh\left(\frac{3a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{16b^2c^5}$$
$$- \frac{\left(5e^2\sinh\left(\frac{5a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{5x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{16b^2c^5}$$

$$
\begin{aligned}
&= -\frac{d^2\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))} - \frac{2dex^2\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))} - \frac{e^2x^4\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))} \\
&\quad - \frac{d^2\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c} + \frac{de\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{2b^2c^3} \\
&\quad - \frac{e^2\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8b^2c^5} - \frac{3de\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{2b^2c^3} \\
&\quad + \frac{9e^2\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16b^2c^5} - \frac{5e^2\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{16b^2c^5} \\
&\quad + \frac{d^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c} - \frac{de\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{2b^2c^3} \\
&\quad + \frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^5} + \frac{3de\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{2b^2c^3} \\
&\quad - \frac{9e^2\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^5} + \frac{5e^2\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.18 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.72

$$
\int \frac{(d+ex^2)^2}{(a+b\text{arcsinh}(cx))^2}\,dx =
$$

$$
-\frac{\frac{16bc^4d^2\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)} + \frac{32bc^4dex^2\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)} + \frac{16bc^4e^2x^4\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)} + 2(8c^4d^2-4c^2de+e^2)\,\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right.}{}
$$

```
[In] Integrate[(d + e*x^2)^2/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/16*((16*b*c^4*d^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (32*b*c^4*d*
e*x^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (16*b*c^4*e^2*x^4*Sqrt[1 +
c^2*x^2])/(a + b*ArcSinh[c*x]) + 2*(8*c^4*d^2 - 4*c^2*d*e + e^2)*CoshIntegr
al[a/b + ArcSinh[c*x]]*Sinh[a/b] + 3*(8*c^2*d - 3*e)*e*CoshIntegral[3*(a/b
+ ArcSinh[c*x])]*Sinh[(3*a)/b] + 5*e^2*CoshIntegral[5*(a/b + ArcSinh[c*x])]
*Sinh[(5*a)/b] - 16*c^4*d^2*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 8*
c^2*d*e*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 2*e^2*Cosh[a/b]*SinhIn
tegral[a/b + ArcSinh[c*x]] - 24*c^2*d*e*Cosh[(3*a)/b]*SinhIntegral[3*(a/b +
 ArcSinh[c*x])] + 9*e^2*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])]
- 5*e^2*Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])])/(b^2*c^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1035 vs. 2(469) = 938.

Time = 0.97 (sec) , antiderivative size = 1036, normalized size of antiderivative = 2.09

method	result	size
derivativedivides	Expression too large to display	1036
default	Expression too large to display	1036

```
[In] int((e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/32*(16*c^5*x^5-16*c^4*x^4*(c^2*x^2+1)^(1/2)+20*c^3*x^3-12*c^2*x^2*(c
^2*x^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))*e^2/c^4/b/(a+b*arcsinh(c*x))+5/32*
e^2/c^4/b^2*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-1/32/b*e^2/c^4*(16*c^5*x^
5+20*c^3*x^3+16*c^4*x^4*(c^2*x^2+1)^(1/2)+5*c*x+12*c^2*x^2*(c^2*x^2+1)^(1/2
)+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))-5/32/b^2*e^2/c^4*exp(-5*a/b)*Ei(1,-
5*arcsinh(c*x)-5*a/b)+1/2*(-(c^2*x^2+1)^(1/2)+c*x)*d^2/b/(a+b*arcsinh(c*x))
+1/2*d^2/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/4*(-(c^2*x^2+1)^(1/2)+c*x)*d
*e/c^2/b/(a+b*arcsinh(c*x))-1/4/c^2*d*e/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)
+1/16*(-(c^2*x^2+1)^(1/2)+c*x)*e^2/c^4/b/(a+b*arcsinh(c*x))+1/16/c^4*e^2/b^
2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/2/b*d^2*(c*x+(c^2*x^2+1)^(1/2))/(a+b*ar
csinh(c*x))-1/2/b^2*d^2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+1/4/c^2/b*d*e*(c*
x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))+1/4/c^2/b^2*d*e*exp(-a/b)*Ei(1,-arc
sinh(c*x)-a/b)-1/16/c^4/b*e^2*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))-1/
16/c^4/b^2*e^2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+1/4*(4*c^3*x^3-4*c^2*x^2*(
c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))*d*e/c^2/b/(a+b*arcsinh(c*x))-3/32
*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))*e^2/c^4/b/
(a+b*arcsinh(c*x))+3/4*e/c^2/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)*d-9/
32*e^2/c^4/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/4/c^2*e/b*(4*c^3*x^3
+3*c*x+4*c^2*x^2*(c^2*x^2+1)^(1/2)+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))*d+
3/32/c^4*e^2/b*(4*c^3*x^3+3*c*x+4*c^2*x^2*(c^2*x^2+1)^(1/2)+(c^2*x^2+1)^(1/
2))/(a+b*arcsinh(c*x))-3/4/c^2*e/b^2*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b
)*d+9/32/c^4*e^2/b^2*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b))
```

Fricas [F]

$$\int \frac{(d+ex^2)^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(ex^2+d)^2}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*d*e*x^2 + d^2)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*
x) + a^2), x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(d+ex^2)^2}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

```
[In] integrate((e*x**2+d)**2/(a+b*asinh(c*x))**2,x)
[Out] Integral((d + e*x**2)**2/(a + b*asinh(c*x))**2, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(ex^2+d)^2}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -(c^3*e^2*x^7 + (2*c^3*d*e + c*e^2)*x^5 + c*d^2*x + (c^3*d^2 + 2*c*d*e)*x^3
 + (c^2*e^2*x^6 + (2*c^2*d*e + e^2)*x^4 + (c^2*d^2 + 2*d*e)*x^2 + d^2)*sqrt
(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^
3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1)))
+ integrate((5*c^5*e^2*x^8 + 2*(3*c^5*d*e + 5*c^3*e^2)*x^6 + (c^5*d^2 + 12*
c^3*d*e + 5*c*e^2)*x^4 + c*d^2 + 2*(c^3*d^2 + 3*c*d*e)*x^2 + (5*c^3*e^2*x^6
 + 3*(2*c^3*d*e + c*e^2)*x^4 - c*d^2 + (c^3*d^2 + 2*c*d*e)*x^2)*(c^2*x^2 +
1) + (10*c^4*e^2*x^7 + (12*c^4*d*e + 13*c^2*e^2)*x^5 + 2*(c^4*d^2 + 7*c^2*d
*e + 2*e^2)*x^3 + (c^2*d^2 + 4*d*e)*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c
^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 +
 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(
c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sq
rt(c^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(ex^2+d)^2}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{{\left(e\,x^2+d\right)}^2}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

```
[In] int((d + e*x^2)^2/(a + b*asinh(c*x))^2,x)
[Out] int((d + e*x^2)^2/(a + b*asinh(c*x))^2, x)
```

3.625 $\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 247

$$\begin{aligned}\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^2}\,dx = & -\frac{d\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))}-\frac{ex^2\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))} \\ & -\frac{d\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c} \\ & +\frac{e\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4b^2c^3} \\ & -\frac{3e\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4b^2c^3} \\ & +\frac{d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c} \\ & -\frac{e\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^3} \\ & +\frac{3e\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^3}\end{aligned}$$

```
[Out] d*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c-1/4*e*cosh(a/b)*Shi((a+b*arcsin
h(c*x))/b)/b^2/c^3+3/4*e*cosh(3*a/b)*Shi(3*(a+b*arcsinh(c*x))/b)/b^2/c^3-d*
Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c+1/4*e*Chi((a+b*arcsinh(c*x))/b)*s
inh(a/b)/b^2/c^3-3/4*e*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^3-d*(c
^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))-e*x^2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arc
sinh(c*x))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5793, 5773, 5819, 3384, 3379, 3382, 5778}

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \frac{e\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^3} - \frac{3e\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^3} - \frac{e\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^3} + \frac{3e\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^3} - \frac{d\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c} + \frac{d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c} - \frac{d\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))} - \frac{ex^2\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}$$

```
[In] Int[(d + e*x^2)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((d*Sqrt[1 + c^2*x^2])/(b*c*(a + b*ArcSinh[c*x]))) - (e*x^2*Sqrt[1 + c^2*x
^2])/(b*c*(a + b*ArcSinh[c*x])) - (d*CoshIntegral[(a + b*ArcSinh[c*x])/b]*S
inh[a/b])/(b^2*c) + (e*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(4*b
^2*c^3) - (3*e*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(4*b
^2*c^3) + (d*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b^2*c) - (e*C
osh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(4*b^2*c^3) + (3*e*Cosh[(3*a
)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(4*b^2*c^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{d}{(a+b\operatorname{arcsinh}(cx))^2}+\frac{ex^2}{(a+b\operatorname{arcsinh}(cx))^2}\right)dx\\ &= d\int \frac{1}{(a+b\operatorname{arcsinh}(cx))^2}\,dx+e\int \frac{x^2}{(a+b\operatorname{arcsinh}(cx))^2}\,dx\end{aligned}$$

$$= -\frac{d\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))} - \frac{ex^2\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))} + \frac{(cd)\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx}{b} + \frac{e\text{Subst}\left(\int \left(-\frac{3\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4x}+\frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{4x}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^3}$$

$$= -\frac{d\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))} - \frac{ex^2\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))} - \frac{d\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c} + \frac{e\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^3} - \frac{(3e)\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^3}$$

$$= -\frac{d\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))} - \frac{ex^2\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))} + \frac{\left(d\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c} - \frac{\left(e\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^3} + \frac{\left(3e\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^3} - \frac{\left(d\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c} + \frac{\left(e\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^3} - \frac{\left(3e\sinh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^3}$$

$$= -\frac{d\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))} - \frac{ex^2\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))} - \frac{d\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c} + \frac{e\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4b^2c^3} - \frac{3e\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4b^2c^3} + \frac{d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c} - \frac{e\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^3} + \frac{3e\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^3}$$

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.77

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{\frac{4bc^2d\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}+\frac{4bc^2ex^2\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}+(4c^2d-e)\,\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right)+3e\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)$$

[In] Integrate[(d + e*x^2)/(a + b*ArcSinh[c*x])^2,x]

[Out] -1/4*((4*b*c^2*d*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (4*b*c^2*e*x^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (4*c^2*d - e)*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] + 3*e*CoshIntegral[3*(a/b + ArcSinh[c*x])]*Sinh[(3*a)/b] - 4*c^2*d*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + e*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 3*e*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])])/(b^2*c^3)

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.77

method	result
derivativedivides	$\frac{\left(4c^3x^3-4c^2x^2\sqrt{c^2x^2+1}+3cx-\sqrt{c^2x^2+1}\right)e}{8c^2b(a+b\,\text{arcsinh}(cx))}+\frac{3e\,e^{\frac{3a}{b}}\,\text{Ei}_1\left(3\,\text{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b^2}-\frac{e\left(4c^3x^3+3cx+4c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}\right)}{8c^2b(a+b\,\text{arcsinh}(cx))}-\frac{3e\,e^{-\frac{3}{}}}{}$
default	$\frac{\left(4c^3x^3-4c^2x^2\sqrt{c^2x^2+1}+3cx-\sqrt{c^2x^2+1}\right)e}{8c^2b(a+b\,\text{arcsinh}(cx))}+\frac{3e\,e^{\frac{3a}{b}}\,\text{Ei}_1\left(3\,\text{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b^2}-\frac{e\left(4c^3x^3+3cx+4c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}\right)}{8c^2b(a+b\,\text{arcsinh}(cx))}-\frac{3e\,e^{-\frac{3}{}}}{}$

[In] int((e*x^2+d)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)

[Out] 1/c*(1/8*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))*e/c^2/b/(a+b*arcsinh(c*x))+3/8*e/c^2/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/8*e/c^2/b*(4*c^3*x^3+3*c*x+4*c^2*x^2*(c^2*x^2+1)^(1/2)+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))-3/8*e/c^2/b^2*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)+1/2*(-(c^2*x^2+1)^(1/2)+c*x)*d/b/(a+b*arcsinh(c*x))-1/8*(-(c^2*x^2+1)^(1/2)+c*x)*e/c^2/b/(a+b*arcsinh(c*x))+1/2/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*d-1/8/c^2/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*e-1/2/b*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))*d+1/8/c^2/b*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))*e-1/2/b^2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d+1/8/c^2/b^2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*e)

Fricas [F]

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{ex^2+d}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")

[Out] integral((e*x^2 + d)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)

Sympy [F]

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{d+ex^2}{(a+b\,\text{asinh}\,(cx))^2}\,dx$$

[In] integrate((e*x**2+d)/(a+b*asinh(c*x))**2,x)

[Out] Integral((d + e*x**2)/(a + b*asinh(c*x))**2, x)

Maxima [F]

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{ex^2+d}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")

[Out] -(c^3*e*x^5 + (c^3*d + c*e)*x^3 + c*d*x + (c^2*e*x^4 + (c^2*d + e)*x^2 + d)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate((3*c^5*e*x^6 + (c^5*d + 6*c^3*e)*x^4 + (2*c^3*d + 3*c*e)*x^2 + (3*c^3*e*x^4 + (c^3*d + c*e)*x^2 - c*d)*(c^2*x^2 + 1) + c*d + (6*c^4*e*x^5 + (2*c^4*d + 7*c^2*e)*x^3 + (c^2*d + 2*e)*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 + 1)), x)

Giac [F]

$$\int \frac{d+ex^2}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{ex^2+d}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate((e*x^2 + d)/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{e\,x^2+d}{(a+b\operatorname{asinh}(c\,x))^2}\,dx$$

```
[In] int((d + e*x^2)/(a + b*asinh(c*x))^2,x)
[Out] int((d + e*x^2)/(a + b*asinh(c*x))^2, x)
```

3.626 $\int \frac{1}{(a+b\text{arcsinh}(cx))^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 85

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2} dx = -\frac{\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))} - \frac{\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c} + \frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c}$$

```
[Out] cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c-Chi((a+b*arcsinh(c*x))/b)*sinh(a/
b)/b^2/c-(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5773, 5819, 3384, 3379, 3382}

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2} dx = -\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c} + \frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c} - \frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}$$

```
[In] Int[(a + b*ArcSinh[c*x])^(-2),x]
```

```
[Out] -(Sqrt[1 + c^2*x^2]/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[(a + b*ArcS
inh[c*x])/b]*Sinh[a/b])/(b^2*c) + (Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*
x])/b])/(b^2*c)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))}+\frac{c\int\frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx}{b}\\
&= -\frac{\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))}-\frac{\text{Subst}\left(\int\frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c}\\
&= -\frac{\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))}+\frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c}\\
&\quad-\frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c}
\end{aligned}
$$

$$= -\frac{\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))} - \frac{\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c} + \frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2}\,dx = \frac{-\frac{b\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)} - \text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right) + \cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{b^2c}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-2),x]
```

```
[Out] (-((b*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])) - CoshIntegral[a/b + ArcSinh
[c*x]]*Sinh[a/b] + Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(b^2*c)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.39

method	result	size
derivativedivides	$\frac{\frac{-\sqrt{c^2x^2+1}+cx}{2b(a+b\,\text{arcsinh}(cx))}+\frac{e^{\frac{a}{b}}\,\text{Ei}_1\left(\text{arcsinh}(cx)+\frac{a}{b}\right)}{2b^2}-\frac{cx+\sqrt{c^2x^2+1}}{2b(a+b\,\text{arcsinh}(cx))}-\frac{e^{-\frac{a}{b}}\,\text{Ei}_1\left(-\text{arcsinh}(cx)-\frac{a}{b}\right)}{2b^2}}{c}$	118
default	$\frac{\frac{-\sqrt{c^2x^2+1}+cx}{2b(a+b\,\text{arcsinh}(cx))}+\frac{e^{\frac{a}{b}}\,\text{Ei}_1\left(\text{arcsinh}(cx)+\frac{a}{b}\right)}{2b^2}-\frac{cx+\sqrt{c^2x^2+1}}{2b(a+b\,\text{arcsinh}(cx))}-\frac{e^{-\frac{a}{b}}\,\text{Ei}_1\left(-\text{arcsinh}(cx)-\frac{a}{b}\right)}{2b^2}}{c}$	118

```
[In] int(1/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/2*(-(c^2*x^2+1)^(1/2)+c*x)/b/(a+b*arcsinh(c*x))+1/2/b^2*exp(a/b)*Ei(
1,arcsinh(c*x)+a/b)-1/2/b*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))-1/2/b^
2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b))
```

Fricas [F]

$$\int \frac{1}{(a + b\text{arcsinh}(cx))^2} \, dx = \int \frac{1}{(b \operatorname{arsinh}(cx) + a)^2} \, dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{1}{(a + b\text{arcsinh}(cx))^2} \, dx = \int \frac{1}{(a + b \operatorname{asinh}(cx))^2} \, dx$$

```
[In] integrate(1/(a+b*asinh(c*x))**2,x)
[Out] Integral((a + b*asinh(c*x))**(-2), x)
```

Maxima [F]

$$\int \frac{1}{(a + b\text{arcsinh}(cx))^2} \, dx = \int \frac{1}{(b \operatorname{arsinh}(cx) + a)^2} \, dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b
*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*
x + sqrt(c^2*x^2 + 1))) + integrate((c^4*x^4 + 2*c^2*x^2 + (c^2*x^2 + 1)*(c
^2*x^2 - 1) + (2*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1) + 1)/(a*b*c^4*x^4 + (c^2*
x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^2*x^2 + a*b + (b^2*c^4*x^4 + (c^2*x^2 + 1)*b
^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^2 + 2*(b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 +
 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2
+ 1)), x)
```

Giac [F]

$$\int \frac{1}{(a + b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\left(b\,\text{arsinh}\,(cx) + a\right)^2}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^(-2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\left(a + b\,\text{asinh}\,(c\,x)\right)^2}\,dx$$

```
[In] int(1/(a + b*asinh(c*x))^2,x)
[Out] int(1/(a + b*asinh(c*x))^2, x)
```

3.627 $\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^2} dx = \text{Int}\left(\frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^2}, x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^2} dx = \int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^2} dx$$

```
[In] Int[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][1/((d + e*x^2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^2} dx$$

Mathematica [N/A]

Not integrable

Time = 12.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(e\,x^2+d)\,(a+b\,\text{arcsinh}\,(cx))^2}dx$$

```
[In] int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x)
[Out] int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.85

$$\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex^2+d)(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(1/(a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arcsinh(c*x)^2 + 2*(a*b
*e*x^2 + a*b*d)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 33.78 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))^2\,(d+ex^2)}\,dx$$

```
[In] integrate(1/(e*x**2+d)/(a+b*asinh(c*x))**2,x)
[Out] Integral(1/((a + b*asinh(c*x))**2*(d + e*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 766, normalized size of antiderivative = 38.30

$$\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex^2+d)(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*e*x^4 + (c^3*d + c*e)*a*b*x
^2 + a*b*c*d + (b^2*c^3*e*x^4 + (c^3*d + c*e)*b^2*x^2 + b^2*c*d + (b^2*c^2*
e*x^3 + b^2*c^2*d*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b
*c^2*e*x^3 + a*b*c^2*d*x)*sqrt(c^2*x^2 + 1)) - integrate((c^5*e*x^6 - (c^5*
d - 2*c^3*e)*x^4 - (2*c^3*d - c*e)*x^2 + (c^3*e*x^4 - (c^3*d - 3*c*e)*x^2 +
 c*d)*(c^2*x^2 + 1) - c*d + (2*c^4*e*x^5 - (2*c^4*d - 5*c^2*e)*x^3 - (c^2*d
 - 2*e)*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*e^2*x^8 + 2*(c^5*d*e + c^3*e^2)*a*b*
x^6 + (c^5*d^2 + 4*c^3*d*e + c*e^2)*a*b*x^4 + a*b*c*d^2 + 2*(c^3*d^2 + c*d*
e)*a*b*x^2 + (a*b*c^3*e^2*x^6 + 2*a*b*c^3*d*e*x^4 + a*b*c^3*d^2*x^2)*(c^2*x
^2 + 1) + (b^2*c^5*e^2*x^8 + 2*(c^5*d*e + c^3*e^2)*b^2*x^6 + (c^5*d^2 + 4*c
^3*d*e + c*e^2)*b^2*x^4 + b^2*c*d^2 + 2*(c^3*d^2 + c*d*e)*b^2*x^2 + (b^2*c^
3*e^2*x^6 + 2*b^2*c^3*d*e*x^4 + b^2*c^3*d^2*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4
*e^2*x^7 + (2*c^4*d*e + c^2*e^2)*b^2*x^5 + b^2*c^2*d^2*x + (c^4*d^2 + 2*c^2
*d*e)*b^2*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4
*e^2*x^7 + (2*c^4*d*e + c^2*e^2)*a*b*x^5 + a*b*c^2*d^2*x + (c^4*d^2 + 2*c^2
*d*e)*a*b*x^3)*sqrt(c^2*x^2 + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)\left(a+b\operatorname{arcsinh}(cx)\right)^2}\,dx=\int \frac{1}{(ex^2+d)(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)\left(a+b\operatorname{arcsinh}(cx)\right)^2}\,dx=\int \frac{1}{\left(a+b\operatorname{asinh}(c\,x)\right)^2\,\left(e\,x^2+d\right)}\,dx$$

```
[In] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)),x)
[Out] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)), x)
```

3.628 $\int \frac{1}{(d+ex^2)^2(a+b\,\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(d+ex^2)^2\,(a+b\,\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)^2\,(a+b\,\text{arcsinh}(cx))^2},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex^2)^2\,(a+b\,\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(d+ex^2)^2\,(a+b\,\text{arcsinh}(cx))^2}\,dx$$

```
[In] Int[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(d+ex^2)^2\,(a+b\,\text{arcsinh}(cx))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 25.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^2} \, dx = \int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^2} \, dx$$

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.75 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(e\,x^2+d\right)^2 \left(a+b\,\operatorname{arcsinh}\left(c x\right)\right)^2}dx$$

```
[In] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x)
[Out] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 4.90

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^2} \, dx = \int \frac{1}{(ex^2+d)^2 (b\,\operatorname{arsinh}\left(cx\right)+a)^2} \, dx$$

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(1/(a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2*x^4 + 2*b^2*d*
e*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e*x^2 + a*b*d^2)
*arcsinh(c*x)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^2} dx = \text{Timed out}$$

```
[In] integrate(1/(e*x**2+d)**2/(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 1027, normalized size of antiderivative = 51.35

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^2} dx = \int \frac{1}{(ex^2+d)^2 (b \operatorname{arsinh}(cx)+a)^2} dx$$

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*e^2*x^6 + (2*c^3*d*e + c*e^
2)*a*b*x^4 + a*b*c*d^2 + (c^3*d^2 + 2*c*d*e)*a*b*x^2 + (b^2*c^3*e^2*x^6 + (
2*c^3*d*e + c*e^2)*b^2*x^4 + b^2*c*d^2 + (c^3*d^2 + 2*c*d*e)*b^2*x^2 + (b^2
*c^2*e^2*x^5 + 2*b^2*c^2*d*e*x^3 + b^2*c^2*d^2*x)*sqrt(c^2*x^2 + 1))*log(c*
x + sqrt(c^2*x^2 + 1)) + (a*b*c^2*e^2*x^5 + 2*a*b*c^2*d*e*x^3 + a*b*c^2*d^2
*x)*sqrt(c^2*x^2 + 1)) - integrate((3*c^5*e*x^6 - (c^5*d - 6*c^3*e)*x^4 - (
2*c^3*d - 3*c*e)*x^2 + (3*c^3*e*x^4 - (c^3*d - 5*c*e)*x^2 + c*d)*(c^2*x^2 +
 1) - c*d + (6*c^4*e*x^5 - (2*c^4*d - 11*c^2*e)*x^3 - (c^2*d - 4*e)*x)*sqrt
(c^2*x^2 + 1))/(a*b*c^5*e^3*x^10 + (3*c^5*d*e^2 + 2*c^3*e^3)*a*b*x^8 + (3*c
^5*d^2*e + 6*c^3*d*e^2 + c*e^3)*a*b*x^6 + (c^5*d^3 + 6*c^3*d^2*e + 3*c*d*e^
2)*a*b*x^4 + a*b*c*d^3 + (2*c^3*d^3 + 3*c*d^2*e)*a*b*x^2 + (a*b*c^3*e^3*x^8
 + 3*a*b*c^3*d*e^2*x^6 + 3*a*b*c^3*d^2*e*x^4 + a*b*c^3*d^3*x^2)*(c^2*x^2 +
1) + (b^2*c^5*e^3*x^10 + (3*c^5*d*e^2 + 2*c^3*e^3)*b^2*x^8 + (3*c^5*d^2*e +
 6*c^3*d*e^2 + c*e^3)*b^2*x^6 + (c^5*d^3 + 6*c^3*d^2*e + 3*c*d*e^2)*b^2*x^4
 + b^2*c*d^3 + (2*c^3*d^3 + 3*c*d^2*e)*b^2*x^2 + (b^2*c^3*e^3*x^8 + 3*b^2*c
^3*d*e^2*x^6 + 3*b^2*c^3*d^2*e*x^4 + b^2*c^3*d^3*x^2)*(c^2*x^2 + 1) + 2*(b^
2*c^4*e^3*x^9 + (3*c^4*d*e^2 + c^2*e^3)*b^2*x^7 + b^2*c^2*d^3*x + 3*(c^4*d^
2*e + c^2*d*e^2)*b^2*x^5 + (c^4*d^3 + 3*c^2*d^2*e)*b^2*x^3)*sqrt(c^2*x^2 +
1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*e^3*x^9 + (3*c^4*d*e^2 + c^2*
e^3)*a*b*x^7 + a*b*c^2*d^3*x + 3*(c^4*d^2*e + c^2*d*e^2)*a*b*x^5 + (c^4*d^3
 + 3*c^2*d^2*e)*a*b*x^3)*sqrt(c^2*x^2 + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^2} \, dx = \int \frac{1}{(ex^2+d)^2(b\,\text{arsinh}\,(cx)+a)^2} \, dx$$

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)^2*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^2} \, dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))^2\,(e\,x^2+d)^2} \, dx$$

```
[In] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^2),x)
[Out] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^2), x)
```

3.629 $\int (d + ex^2)^2 \sqrt{a + b\operatorname{arcsinh}(cx)}\, dx$

Optimal result

Integrand size = 22, antiderivative size = 672

$$\int (d+ex^2)^2 \sqrt{a+b\operatorname{arcsinh}(cx)}\,dx = d^2x\sqrt{a+b\operatorname{arcsinh}(cx)}+\frac{2}{3}dex^3\sqrt{a+b\operatorname{arcsinh}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+b\operatorname{arcsinh}(cx)} + \frac{\sqrt{b}d^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c} - \frac{\sqrt{b}dee^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c^3} + \frac{\sqrt{b}e^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^5} + \frac{\sqrt{b}dee^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{24c^3} - \frac{\sqrt{b}e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c^5} + \frac{\sqrt{b}e^2e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{320c^5} - \frac{\sqrt{b}d^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c} + \frac{\sqrt{b}dee^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c^3} - \frac{\sqrt{b}e^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^5} - \frac{\sqrt{b}dee^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{24c^3} + \frac{\sqrt{b}e^2e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c^5} - \frac{\sqrt{b}e^2e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{320c^5}$$

```
[Out] 1/1600*e^2*exp(5*a/b)*erf(5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)
*5^(1/2)*Pi^(1/2)/c^5-1/1600*e^2*erfi(5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1
/2))*b^(1/2)*5^(1/2)*Pi^(1/2)/c^5/exp(5*a/b)+1/72*d*e*exp(3*a/b)*erf(3^(1/2
)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/c^3-1/192*e^2*
exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*3^(1/2)*Pi
^(1/2)/c^5-1/72*d*e*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*
3^(1/2)*Pi^(1/2)/c^3/exp(3*a/b)+1/192*e^2*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(
1/2)/b^(1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/c^5/exp(3*a/b)+1/4*d^2*exp(a/b)*erf(
(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c-1/8*d*e*exp(a/b)*erf((
a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^3+1/32*e^2*exp(a/b)*erf
((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^5-1/4*d^2*erfi((a+b*a
rcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c/exp(a/b)+1/8*d*e*erfi((a+b*a
rcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^3/exp(a/b)-1/32*e^2*erfi((a+
b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^5/exp(a/b)+d^2*x*(a+b*arc
sinh(c*x))^(1/2)+2/3*d*e*x^3*(a+b*arcsinh(c*x))^(1/2)+1/5*e^2*x^5*(a+b*arcs
inh(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used

$= \{5793, 5772, 5819, 3389, 2211, 2236, 2235, 5777, 3393\}$

$$\int \left(d+ex^2\right)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx = \frac{\sqrt{\pi}\sqrt{b}e^2e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^5} - \frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^2e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c^5} + \frac{\sqrt{\frac{\pi}{5}}\sqrt{b}e^2e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{320c^5} - \frac{\sqrt{\pi}\sqrt{b}e^2e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^5} + \frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^2e^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c^5} - \frac{\sqrt{\frac{\pi}{5}}\sqrt{b}e^2e^{-\frac{5a}{b}}\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{320c^5} - \frac{\sqrt{\pi}\sqrt{b}dee^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c^3} + \frac{\sqrt{\frac{\pi}{3}}\sqrt{b}dee^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{24c^3} + \frac{\sqrt{\pi}\sqrt{b}dee^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c^3} - \frac{\sqrt{\frac{\pi}{3}}\sqrt{b}dee^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{24c^3} + \frac{\sqrt{\pi}\sqrt{b}d^2e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c} - \frac{\sqrt{\pi}\sqrt{b}d^2e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c} + d^2x\sqrt{a+b\text{arcsinh}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\text{arcsinh}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+b\text{arcsinh}(cx)}$$

```
[In] Int[(d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] d^2*x*Sqrt[a + b*ArcSinh[c*x]] + (2*d*e*x^3*Sqrt[a + b*ArcSinh[c*x]])/3 + (
e^2*x^5*Sqrt[a + b*ArcSinh[c*x]])/5 + (Sqrt[b]*d^2*E^(a/b)*Sqrt[Pi]*Erf[Sqr
t[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b]*d*e*E^(a/b)*Sqrt[Pi]*Erf[S
qrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*c^3) + (Sqrt[b]*e^2*E^(a/b)*Sqrt[Pi]*E
rf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(32*c^5) + (Sqrt[b]*d*e*E^((3*a)/b)*S
qrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(24*c^3) - (Sqrt
[b]*e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[
b]])/(64*c^5) + (Sqrt[b]*e^2*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b
*ArcSinh[c*x]])/Sqrt[b]])/(320*c^5) - (Sqrt[b]*d^2*Sqrt[Pi]*Erfi[Sqrt[a + b
*ArcSinh[c*x]]/Sqrt[b]])/(4*c*E^(a/b)) + (Sqrt[b]*d*e*Sqrt[Pi]*Erfi[Sqrt[a
+ b*ArcSinh[c*x]]/Sqrt[b]])/(8*c^3*E^(a/b)) - (Sqrt[b]*e^2*Sqrt[Pi]*Erfi[Sq
rt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(32*c^5*E^(a/b)) - (Sqrt[b]*d*e*Sqrt[Pi/3]
*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(24*c^3*E^((3*a)/b)) + (
Sqrt[b]*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(6
4*c^5*E^((3*a)/b)) - (Sqrt[b]*e^2*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSi
nh[c*x]])/Sqrt[b]])/(320*c^5*E^((5*a)/b))
```

<u>Rule 2211</u>

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

<u>Rule 2235</u>

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

<u>Rule 2236</u>

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

<u>Rule 3389</u>

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

<u>Rule 3393</u>

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(d^2\sqrt{a+b\text{arcsinh}(cx)}+2dex^2\sqrt{a+b\text{arcsinh}(cx)}+e^2x^4\sqrt{a+b\text{arcsinh}(cx)}\right)\,dx\\
&= d^2\int \sqrt{a+b\text{arcsinh}(cx)}\,dx+(2de)\int x^2\sqrt{a+b\text{arcsinh}(cx)}\,dx\\
&\quad +e^2\int x^4\sqrt{a+b\text{arcsinh}(cx)}\,dx\\
&= d^2x\sqrt{a+b\text{arcsinh}(cx)}+\frac{2}{3}dex^3\sqrt{a+b\text{arcsinh}(cx)}\\
&\quad +\frac{1}{5}e^2x^5\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{2}\left(bcd^2\right)\int \frac{x}{\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx\\
&\quad -\frac{1}{3}(bcde)\int \frac{x^3}{\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx\\
&\quad -\frac{1}{10}\left(bce^2\right)\int \frac{x^5}{\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx
\end{aligned}
$$

$$= d^2x\sqrt{a+b\operatorname{arcsinh}(cx)}+\frac{2}{3}dex^3\sqrt{a+b\operatorname{arcsinh}(cx)}+\frac{1}{5}e^2x^5\sqrt{a+b\operatorname{arcsinh}(cx)}+\frac{d^2\text{Subst}\left(\int\frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{2c}+\frac{(de)\text{Subst}\left(\int\frac{\sinh^3\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{3c^3}+\frac{e^2\text{Subst}\left(\int\frac{\sinh^5\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{10c^5}$$

$$= d^2x\sqrt{a+b\operatorname{arcsinh}(cx)}+\frac{2}{3}dex^3\sqrt{a+b\operatorname{arcsinh}(cx)}+\frac{1}{5}e^2x^5\sqrt{a+b\operatorname{arcsinh}(cx)}+\frac{d^2\text{Subst}\left(\int\frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{4c}-\frac{d^2\text{Subst}\left(\int\frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{4c}+\frac{(ide)\text{Subst}\left(\int\left(-\frac{i\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}}+\frac{3i\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)dx,x,a+b\operatorname{arcsinh}(cx)\right)}{3c^3}-\frac{(ie^2)\,\text{Subst}\left(\int\left(\frac{i\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16\sqrt{x}}-\frac{5i\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16\sqrt{x}}+\frac{5i\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{8\sqrt{x}}\right)dx,x,a+b\operatorname{arcsinh}(cx)\right)}{10c^5}$$

$$= d^2x\sqrt{a+b\operatorname{arcsinh}(cx)}+\frac{2}{3}dex^3\sqrt{a+b\operatorname{arcsinh}(cx)}+\frac{1}{5}e^2x^5\sqrt{a+b\operatorname{arcsinh}(cx)}+\frac{d^2\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{2c}-\frac{d^2\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{2c}+\frac{(de)\text{Subst}\left(\int\frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{12c^3}-\frac{(de)\text{Subst}\left(\int\frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{4c^3}+\frac{e^2\text{Subst}\left(\int\frac{\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{160c^5}-\frac{e^2\text{Subst}\left(\int\frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{32c^5}+\frac{e^2\text{Subst}\left(\int\frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{16c^5}$$

$$= d^2x\sqrt{a+b\text{arcsinh}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\text{arcsinh}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+b\text{arcsinh}(cx)}$$
$$+ \frac{\sqrt{b}d^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c} - \frac{\sqrt{b}d^2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}$$
$$+ \frac{(de)\text{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{24c^3}$$
$$- \frac{(de)\text{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{24c^3}$$
$$- \frac{(de)\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8c^3}$$
$$+ \frac{(de)\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8c^3}$$
$$+ \frac{e^2\text{Subst}\left(\int \frac{e^{-i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{320c^5}$$
$$- \frac{e^2\text{Subst}\left(\int \frac{e^{i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{320c^5}$$
$$- \frac{e^2\text{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64c^5}$$
$$+ \frac{e^2\text{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64c^5}$$
$$+ \frac{e^2\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{32c^5}$$
$$- \frac{e^2\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{32c^5}$$

$$
\begin{aligned}
&= d^2x\sqrt{a+b\text{arcsinh}(cx)}+\frac{2}{3}dex^3\sqrt{a+b\text{arcsinh}(cx)}+\frac{1}{5}e^2x^5\sqrt{a+b\text{arcsinh}(cx)} \\
&+\frac{\sqrt{b}d^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{b}d^2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c} \\
&+\frac{(de)\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx,x,\sqrt{a+b\text{arcsinh}(cx)}\right)}{12c^3} \\
&-\frac{(de)\text{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx,x,\sqrt{a+b\text{arcsinh}(cx)}\right)}{12c^3} \\
&-\frac{(de)\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\text{arcsinh}(cx)}\right)}{4c^3} \\
&+\frac{(de)\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\text{arcsinh}(cx)}\right)}{4c^3} \\
&+\frac{e^2\text{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx,x,\sqrt{a+b\text{arcsinh}(cx)}\right)}{160c^5} \\
&-\frac{e^2\text{Subst}\left(\int e^{-\frac{5a}{b}+\frac{5x^2}{b}}\,dx,x,\sqrt{a+b\text{arcsinh}(cx)}\right)}{160c^5} \\
&-\frac{e^2\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx,x,\sqrt{a+b\text{arcsinh}(cx)}\right)}{32c^5} \\
&+\frac{e^2\text{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx,x,\sqrt{a+b\text{arcsinh}(cx)}\right)}{32c^5} \\
&+\frac{e^2\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\text{arcsinh}(cx)}\right)}{16c^5} \\
&-\frac{e^2\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\text{arcsinh}(cx)}\right)}{16c^5}
\end{aligned}
$$

$$= d^2x\sqrt{a+b\text{arcsinh}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\text{arcsinh}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+b\text{arcsinh}(cx)} + \frac{\sqrt{b}d^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c} - \frac{\sqrt{b}dee^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c^3} + \frac{\sqrt{b}e^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^5} + \frac{\sqrt{b}dee^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{24c^3} - \frac{\sqrt{b}e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c^5} + \frac{\sqrt{b}e^2e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{320c^5} - \frac{\sqrt{b}d^2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c} + \frac{\sqrt{b}dee^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c^3} - \frac{\sqrt{b}e^2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^5} - \frac{\sqrt{b}dee^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{24c^3} + \frac{\sqrt{b}e^2e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c^5} - \frac{\sqrt{b}e^2e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{320c^5}$$

Mathematica [A] (verified)

Time = 4.82 (sec) , antiderivative size = 535, normalized size of antiderivative = 0.80

$$\int \left(d+ex^2\right)^2\sqrt{a+b\text{arcsinh}(cx)}\,dx = -\frac{be^{-\frac{5a}{b}}\left(450e^{\frac{6a}{b}}\left(8ac^4d^2\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}+8bc^4d^2\text{arcsinh}(cx)\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}+b(4c^2d-e)\,e\sqrt{-\frac{a+b}{}}\right.\right.}{}$$

```
[In] Integrate[(d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] -1/7200*(b*(450*E^((6*a)/b)*(8*a*c^4*d^2*Sqrt[a/b + ArcSinh[c*x]] + 8*b*c^4
*d^2*ArcSinh[c*x]*Sqrt[a/b + ArcSinh[c*x]] + b*(4*c^2*d - e)*e*Sqrt[-((a +
b*ArcSinh[c*x])/b)]*Sqrt[-((a + b*ArcSinh[c*x])^2/b^2)])*Gamma[3/2, a/b + A
rcSinh[c*x]] + 9*Sqrt[5]*b*e^2*Sqrt[a/b + ArcSinh[c*x]]*Sqrt[-((a + b*ArcSi
nh[c*x])^2/b^2)]*Gamma[3/2, (-5*(a + b*ArcSinh[c*x]))/b] + 25*Sqrt[3]*b*(8*
c^2*d - 3*e)*e*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Sqrt[-((a + b*ArcSinh[c
*x])^2/b^2)]*Gamma[3/2, (-3*(a + b*ArcSinh[c*x]))/b] + 450*E^((4*a)/b)*(8*a
*c^4*d^2*Sqrt[-((a + b*ArcSinh[c*x])/b)] + 8*b*c^4*d^2*ArcSinh[c*x]*Sqrt[-(
(a + b*ArcSinh[c*x])/b)] + b*e*(-4*c^2*d + e)*Sqrt[a/b + ArcSinh[c*x]]*Sqrt
[-((a + b*ArcSinh[c*x])^2/b^2)])*Gamma[3/2, -((a + b*ArcSinh[c*x])/b)] - b*
e*E^((8*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Sqrt[-((a + b*ArcSinh[c*x])^2
```

```
/b^2)]*(25*Sqrt[3]*(8*c^2*d - 3*e)*Gamma[3/2, (3*(a + b*ArcSinh[c*x]))/b] +
 9*Sqrt[5]*e*E^((2*a)/b)*Gamma[3/2, (5*(a + b*ArcSinh[c*x]))/b]))/(c^5*E^(
(5*a)/b)*(a + b*ArcSinh[c*x])^(3/2))
```

Maple [F]

$$\int \left(e\,x^{2}+d\right)^{2} \sqrt{a +b \,\operatorname{arcsinh}\left(c x \right)}d x$$

```
[In] int((e*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x)
[Out] int((e*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (d+ex^2)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (d+ex^2)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx = \int \sqrt{a+b\operatorname{asinh}(cx)}\left(d+ex^2\right)^2\,dx$$

```
[In] integrate((e*x**2+d)**2*(a+b*asinh(c*x))**(1/2),x)
[Out] Integral(sqrt(a + b*asinh(c*x))*(d + e*x**2)**2, x)
```

Maxima [F]

$$\int (d+ex^2)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx = \int \left(ex^2+d\right)^2\sqrt{b\operatorname{arsinh}(cx)+a}\,dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^2*sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \left(d+ex^2\right)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx = \int \left(ex^2+d\right)^2 \sqrt{b\,\text{arsinh}\left(cx\right)+a}\,dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2*sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx = \int \sqrt{a+b\,\text{asinh}\left(c\,x\right)}\,\left(e\,x^2+d\right)^2\,dx$$

```
[In] int((a + b*asinh(c*x))^(1/2)*(d + e*x^2)^2,x)
[Out] int((a + b*asinh(c*x))^(1/2)*(d + e*x^2)^2, x)
```

3.630 $\int (d+ex^2) \sqrt{a+b\text{arcsinh}(cx)} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 322

$$\begin{aligned}\int (d+ex^2) \sqrt{a+b\text{arcsinh}(cx)} \, dx &= dx\sqrt{a+b\text{arcsinh}(cx)} + \frac{1}{3}ex^3\sqrt{a+b\text{arcsinh}(cx)} \\ &+ \frac{\sqrt{b}de^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c} \\ &- \frac{\sqrt{b}ee^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^3} \\ &+ \frac{\sqrt{b}ee^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{48c^3} \\ &- \frac{\sqrt{b}de^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c} \\ &+ \frac{\sqrt{b}ee^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^3} \\ &- \frac{\sqrt{b}ee^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{48c^3}\end{aligned}$$

```
[Out] 1/144*e*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*3^
(1/2)*Pi^(1/2)/c^3-1/144*e*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b
^(1/2)*3^(1/2)*Pi^(1/2)/c^3/exp(3*a/b)+1/4*d*exp(a/b)*erf((a+b*arcsinh(c*x)
)^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c-1/16*e*exp(a/b)*erf((a+b*arcsinh(c*x))^
(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^3-1/4*d*erfi((a+b*arcsinh(c*x))^(1/2)/b^(
```

```
1/2))*b^(1/2)*Pi^(1/2)/c/exp(a/b)+1/16*e*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1
/2))*b^(1/2)*Pi^(1/2)/c^3/exp(a/b)+d*x*(a+b*arcsinh(c*x))^(1/2)+1/3*e*x^3*(
a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {5793, 5772, 5819, 3389, 2211, 2236, 2235, 5777, 3393}

$$\begin{aligned}\int \left(d+ex^2\right)\sqrt{a+b\text{arcsinh}(cx)}\,dx = &-\frac{\sqrt{\pi}\sqrt{b}ee^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^3}\\&+\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}ee^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{48c^3}\\&+\frac{\sqrt{\pi}\sqrt{b}ee^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^3}\\&-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}ee^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{48c^3}\\&+\frac{\sqrt{\pi}\sqrt{b}de^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}\\&-\frac{\sqrt{\pi}\sqrt{b}de^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}\\&+dx\sqrt{a+b\text{arcsinh}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\text{arcsinh}(cx)}\end{aligned}$$

```
[In] Int[(d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] d*x*Sqrt[a + b*ArcSinh[c*x]] + (e*x^3*Sqrt[a + b*ArcSinh[c*x]])/3 + (Sqrt[b
]*d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b
]*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*c^3) + (Sqr
t[b]*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b
]])/(48*c^3) - (Sqrt[b]*d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/
(4*c*E^(a/b)) + (Sqrt[b]*e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])
/(16*c^3*E^(a/b)) - (Sqrt[b]*e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[
c*x]])/Sqrt[b]])/(48*c^3*E^((3*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(d\sqrt{a+b\operatorname{arcsinh}(cx)}+ex^2\sqrt{a+b\operatorname{arcsinh}(cx)}\right)\,dx \\
&= d\int \sqrt{a+b\operatorname{arcsinh}(cx)}\,dx+e\int x^2\sqrt{a+b\operatorname{arcsinh}(cx)}\,dx \\
&= dx\sqrt{a+b\operatorname{arcsinh}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\operatorname{arcsinh}(cx)} \\
&\quad -\frac{1}{2}(bcd)\int \frac{x}{\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx \\
&\quad -\frac{1}{6}(bce)\int \frac{x^3}{\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx \\
&= dx\sqrt{a+b\operatorname{arcsinh}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\operatorname{arcsinh}(cx)} \\
&\quad +\frac{d\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{2c} \\
&\quad +\frac{e\text{Subst}\left(\int \frac{\sinh^3\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{6c^3} \\
&= dx\sqrt{a+b\operatorname{arcsinh}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\operatorname{arcsinh}(cx)} \\
&\quad +\frac{d\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{4c} \\
&\quad -\frac{d\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{4c} \\
&\quad +\frac{(ie)\text{Subst}\left(\int \left(-\frac{i\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}}+\frac{3i\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{6c^3}
\end{aligned}
$$

$$= dx\sqrt{a+b\text{arcsinh}(cx)} + \frac{1}{3}ex^3\sqrt{a+b\text{arcsinh}(cx)} + \frac{d\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{2c} - \frac{d\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{2c} + \frac{e\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{24c^3} - \frac{e\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8c^3}$$

$$= dx\sqrt{a+b\text{arcsinh}(cx)} + \frac{1}{3}ex^3\sqrt{a+b\text{arcsinh}(cx)} + \frac{\sqrt{b}de^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c} - \frac{\sqrt{b}de^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c} + \frac{e\text{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{48c^3} - \frac{e\text{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{48c^3} - \frac{e\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16c^3} + \frac{e\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16c^3}$$

$$
\begin{aligned}
&= dx\sqrt{a+b\operatorname{arcsinh}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\operatorname{arcsinh}(cx)} \\
&+\frac{\sqrt{b}de^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{b}de^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c} \\
&+\frac{e\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx,x,\sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{24c^3} \\
&-\frac{e\text{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx,x,\sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{24c^3} \\
&-\frac{e\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{8c^3} \\
&+\frac{e\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{8c^3} \\
&= dx\sqrt{a+b\operatorname{arcsinh}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\operatorname{arcsinh}(cx)} \\
&+\frac{\sqrt{b}de^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{b}ee^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^3} \\
&+\frac{\sqrt{b}ee^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{48c^3}-\frac{\sqrt{b}de^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c} \\
&+\frac{\sqrt{b}ee^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^3}-\frac{\sqrt{b}ee^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{48c^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.10 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.99

$$
\int (d+ex^2)\sqrt{a+b\operatorname{arcsinh}(cx)}\,dx
$$

$$
=\frac{de^{-\frac{a}{b}}\sqrt{a+b\operatorname{arcsinh}(cx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{\sqrt{\frac{a}{b}+\operatorname{arcsinh}(cx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{\sqrt{-\frac{a+b\operatorname{arcsinh}(cx)}{b}}}\right)}{2c}
$$
$$
+\frac{ee^{-\frac{3a}{b}}\sqrt{a+b\operatorname{arcsinh}(cx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\operatorname{arcsinh}(cx)}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\operatorname{arcsinh}(cx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(cx)}\Gamma\left(\frac{3}{2},-\right.\right.}{72c^3}
$$

```
[In] Integrate[(d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (d*Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]])
/Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-((
a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b)) + (e*Sqrt[a + b*ArcSinh[c*x]]*(9*E^
((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, a/b + ArcSinh[c*x]] +
Sqrt[3]*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, (-3*(a + b*ArcSinh[c*x]))/b] -
9*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, -((a + b*ArcSinh[c*x])/b)
] - Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, (3*(a +
b*ArcSinh[c*x]))/b]))/(72*c^3*E^((3*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])^2/b^2
)])
```

Maple [F]

$$\int \left(e\,x^{2}+d\right)\sqrt{a+b\,\operatorname{arcsinh}\left(c x\right)}dx$$

```
[In] int((e*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int((e*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}\,dx=\int \sqrt{a+b\operatorname{asinh}\left(cx\right)}\left(d+ex^{2}\right)\,dx$$

```
[In] integrate((e*x**2+d)*(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*asinh(c*x))*(d + e*x**2), x)
```

Maxima [F]

$$\int (d+ex^2) \sqrt{a+b\text{arcsinh}(cx)}\, dx = \int (ex^2+d)\sqrt{b\,\text{arsinh}\,(cx)+a}\, dx$$

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)*sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int (d+ex^2) \sqrt{a+b\text{arcsinh}(cx)}\, dx = \int (ex^2+d)\sqrt{b\,\text{arsinh}\,(cx)+a}\, dx$$

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)*sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2) \sqrt{a+b\text{arcsinh}(cx)}\, dx = \int \sqrt{a+b\,\text{asinh}\,(c\,x)}\,\left(e\,x^2+d\right)\, dx$$

```
[In] int((a + b*asinh(c*x))^(1/2)*(d + e*x^2),x)
[Out] int((a + b*asinh(c*x))^(1/2)*(d + e*x^2), x)
```

3.631 $\int \sqrt{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 102

$$\int \sqrt{a+b\text{arcsinh}(cx)}\,dx = x\sqrt{a+b\text{arcsinh}(cx)}+\frac{\sqrt{b}e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{b}e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}$$

```
[Out] 1/4*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c-1/4*e
rfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c/exp(a/b)+x*(a+b*ar
csinh(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5772, 5819, 3389, 2211, 2236, 2235}

$$\int \sqrt{a+b\text{arcsinh}(cx)}\,dx = \frac{\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}+x\sqrt{a+b\text{arcsinh}(cx)}$$

```
[In] Int[Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] x*Sqrt[a + b*ArcSinh[c*x]] + (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSi
nh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/
Sqrt[b]])/(4*c*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = x\sqrt{a+b\text{arcsinh}(cx)} - \frac{1}{2}(bc)\int \frac{x}{\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx$$

$$= x\sqrt{a+b\operatorname{arcsinh}(cx)} + \frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{2c}$$

$$= x\sqrt{a+b\operatorname{arcsinh}(cx)} + \frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{4c} - \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{4c}$$

$$= x\sqrt{a+b\operatorname{arcsinh}(cx)} + \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{2c} - \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{2c}$$

$$= x\sqrt{a+b\operatorname{arcsinh}(cx)} + \frac{\sqrt{b}e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c} - \frac{\sqrt{b}e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.99

$$\int \sqrt{a+b\operatorname{arcsinh}(cx)}\,dx = \frac{e^{-\frac{a}{b}}\sqrt{a+b\operatorname{arcsinh}(cx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{\sqrt{\frac{a}{b}+\operatorname{arcsinh}(cx)}} + \frac{\Gamma\left(\frac{3}{2},-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{\sqrt{-\frac{a+b\operatorname{arcsinh}(cx)}{b}}}\right)}{2c}$$

```
[In] Integrate[Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]])/S
qrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-((a
+ b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b))
```

Maple [F]

$$\int \sqrt{a + b \operatorname{arcsinh}(cx)}dx$$

```
[In] int((a+b*arcsinh(c*x))^(1/2),x)
[Out] int((a+b*arcsinh(c*x))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a + b\text{arcsinh}(cx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{a + b\text{arcsinh}(cx)}\,dx = \int \sqrt{a + b \operatorname{asinh}(cx)}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**(1/2),x)
[Out] Integral(sqrt(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \sqrt{a + b\text{arcsinh}(cx)}\,dx = \int \sqrt{b \operatorname{arsinh}(cx) + a}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \sqrt{a + b\text{arcsinh}(cx)}\, dx = \int \sqrt{b \operatorname{arsinh}(cx) + a}\, dx$$

```
[In] integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + b\text{arcsinh}(cx)}\, dx = \int \sqrt{a + b \operatorname{asinh}(c\, x)}\, dx$$

```
[In] int((a + b*asinh(c*x))^(1/2),x)
[Out] int((a + b*asinh(c*x))^(1/2), x)
```

3.632 $\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2}\,dx = \text{Int}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2},x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2}\,dx = \int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2}\,dx$$

```
[In] Int[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2),x]
[Out] Defer[Int][Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2}\,dx$$

Mathematica [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2}\,dx=\text{\$Aborted}$$

```
[In] Integrate[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2),x]
```

```
[Out] $Aborted
```

Maple [N/A] (verified)

Not integrable

Time = 0.63 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+b\,\text{arcsinh}\,(cx)}}{e\,x^2+d}dx$$

```
[In] int((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x)
```

```
[Out] int((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2}\,dx=\int \frac{\sqrt{a+b\,\text{asinh}\,(cx)}}{d+ex^2}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**(1/2)/(e*x**2+d),x)
```

```
[Out] Integral(sqrt(a + b*asinh(c*x))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a + b\operatorname{arcsinh}(cx)}}{d + ex^2} dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a + b\operatorname{arcsinh}(cx)}}{d + ex^2} dx = \int \frac{\sqrt{b \operatorname{arsinh}(cx) + a}}{ex^2 + d} dx$$

```
[In] integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arcsinh(c*x) + a)/(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a + b\operatorname{arcsinh}(cx)}}{d + ex^2} dx = \int \frac{\sqrt{a + b \operatorname{asinh}(c x)}}{e x^2 + d} dx$$

```
[In] int((a + b*asinh(c*x))^(1/2)/(d + e*x^2),x)
```

```
[Out] int((a + b*asinh(c*x))^(1/2)/(d + e*x^2), x)
```

3.633 $\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+ex^2)^2}\,dx = \text{Int}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+ex^2)^2},x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+ex^2)^2}\,dx = \int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+ex^2)^2}\,dx$$

```
[In] Int[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2)^2,x]
```

```
[Out] Defer[Int][Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+ex^2)^2}\,dx$$

Mathematica [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+ex^2)^2}\,dx=\text{\$Aborted}$$

```
[In] Integrate[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2)^2,x]
```

```
[Out] $Aborted
```

Maple [N/A] (verified)

Not integrable

Time = 1.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+b\,\text{arcsinh}\,(cx)}}{(e\,x^2+d)^2}dx$$

```
[In] int((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x)
```

```
[Out] int((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+ex^2)^2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 14.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+ex^2)^2}\,dx=\int \frac{\sqrt{a+b\,\text{asinh}\,(cx)}}{(d+ex^2)^2}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**(1/2)/(e*x**2+d)**2,x)
```

```
[Out] Integral(sqrt(a + b*asinh(c*x))/(d + e*x**2)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a + b\text{arcsinh}(cx)}}{(d + ex^2)^2}\, dx = \int \frac{\sqrt{b\,\text{arsinh}\,(cx) + a}}{(ex^2 + d)^2}\, dx$$

```
[In] integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="maxima")
[Out] integrate(sqrt(b*arcsinh(c*x) + a)/(e*x^2 + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a + b\text{arcsinh}(cx)}}{(d + ex^2)^2}\, dx = \int \frac{\sqrt{b\,\text{arsinh}\,(cx) + a}}{(ex^2 + d)^2}\, dx$$

```
[In] integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate(sqrt(b*arcsinh(c*x) + a)/(e*x^2 + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a + b\text{arcsinh}(cx)}}{(d + ex^2)^2}\, dx = \int \frac{\sqrt{a + b\,\text{asinh}\,(c\,x)}}{(e\,x^2 + d)^2}\, dx$$

```
[In] int((a + b*asinh(c*x))^(1/2)/(d + e*x^2)^2,x)
[Out] int((a + b*asinh(c*x))^(1/2)/(d + e*x^2)^2, x)
```

3.634 $\int (d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 427

$$\begin{aligned}\int (d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}\,dx &= -\frac{3bd\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{2c}\\
&+\frac{be\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^3}-\frac{bex^2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{6c}\\
&+dx(a+b\text{arcsinh}(cx))^{3/2}+\frac{1}{3}ex^3(a+b\text{arcsinh}(cx))^{3/2}\\
&+\frac{3b^{3/2}de^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c}-\frac{3b^{3/2}ee^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^3}\\
&+\frac{b^{3/2}ee^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{96c^3}+\frac{3b^{3/2}de^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c}\\
&-\frac{3b^{3/2}ee^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^3}+\frac{b^{3/2}ee^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{96c^3}\end{aligned}$$

```
[Out] d*x*(a+b*arcsinh(c*x))^(3/2)+1/3*e*x^3*(a+b*arcsinh(c*x))^(3/2)+1/288*b^(3/
2)*e*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1
/2)/c^3+1/288*b^(3/2)*e*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1
/2)*Pi^(1/2)/c^3/exp(3*a/b)+3/8*b^(3/2)*d*exp(a/b)*erf((a+b*arcsinh(c*x))^(
1/2)/b^(1/2))*Pi^(1/2)/c-3/32*b^(3/2)*e*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/
2)/b^(1/2))*Pi^(1/2)/c^3+3/8*b^(3/2)*d*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2
))*Pi^(1/2)/c/exp(a/b)-3/32*b^(3/2)*e*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2)
)*Pi^(1/2)/c^3/exp(a/b)-3/2*b*d*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/
c+1/3*b*e*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/c^3-1/6*b*e*x^2*(c^2*x
^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/c
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5793, 5772, 5798, 5774, 3388, 2211, 2236, 2235, 5777, 5812, 5780, 5556}

$$\begin{aligned}\int \left(d+ex^2\right)(a+b\text{arcsinh}(cx))^{3/2}\,dx=&-\frac{3\sqrt{\pi}b^{3/2}ee^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^3}\\&+\frac{\sqrt{\frac{\pi}{3}}b^{3/2}ee^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{96c^3}-\frac{3\sqrt{\pi}b^{3/2}ee^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^3}\\&+\frac{\sqrt{\frac{\pi}{3}}b^{3/2}ee^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{96c^3}+\frac{3\sqrt{\pi}b^{3/2}de^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c}\\&+\frac{3\sqrt{\pi}b^{3/2}de^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c}-\frac{3bd\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{2c}\\&-\frac{bex^2\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{6c}+\frac{be\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{3c^3}\\&+dx(a+b\text{arcsinh}(cx))^{3/2}+\frac{1}{3}ex^3(a+b\text{arcsinh}(cx))^{3/2}\end{aligned}$$

```
[In] Int[(d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-3*b*d*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(2*c) + (b*e*Sqrt[1 + c
^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(3*c^3) - (b*e*x^2*Sqrt[1 + c^2*x^2]*Sqrt
[a + b*ArcSinh[c*x]])/(6*c) + d*x*(a + b*ArcSinh[c*x])^(3/2) + (e*x^3*(a +
b*ArcSinh[c*x])^(3/2))/3 + (3*b^(3/2)*d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*Arc
Sinh[c*x]]/Sqrt[b]])/(8*c) - (3*b^(3/2)*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*A
rcSinh[c*x]]/Sqrt[b]])/(32*c^3) + (b^(3/2)*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sq
rt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(96*c^3) + (3*b^(3/2)*d*Sqrt[Pi]*
Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*c*E^(a/b)) - (3*b^(3/2)*e*Sqrt[P
i]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(32*c^3*E^(a/b)) + (b^(3/2)*e*Sq
rt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(96*c^3*E^((3*a)
/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(d(a+b\operatorname{arcsinh}(cx))^{3/2}+ex^2(a+b\operatorname{arcsinh}(cx))^{3/2}\right)\,dx\\
&= d\int (a+b\operatorname{arcsinh}(cx))^{3/2}\,dx+e\int x^2(a+b\operatorname{arcsinh}(cx))^{3/2}\,dx\\
&= dx(a+b\operatorname{arcsinh}(cx))^{3/2}+\frac{1}{3}ex^3(a+b\operatorname{arcsinh}(cx))^{3/2}\\
&\quad -\frac{1}{2}(3bcd)\int \frac{x\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{1+c^2x^2}}\,dx-\frac{1}{2}(bce)\int \frac{x^3\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{1+c^2x^2}}\,dx\\
&= -\frac{3bd\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{2c}\\
&\quad -\frac{bex^2\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{6c}+dx(a+b\operatorname{arcsinh}(cx))^{3/2}\\
&\quad +\frac{1}{3}ex^3(a+b\operatorname{arcsinh}(cx))^{3/2}+\frac{1}{4}(3b^2d)\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx\\
&\quad +\frac{1}{12}(b^2e)\int \frac{x^2}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx+\frac{(be)\int \frac{x\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{1+c^2x^2}}\,dx}{3c}
\end{aligned}
$$

$$= -\frac{3bd\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{2c} + \frac{be\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^3} - \frac{bex^2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{6c} + dx(a+b\text{arcsinh}(cx))^{3/2} + \frac{1}{3}ex^3(a+b\text{arcsinh}(cx))^{3/2} + \frac{(3bd)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4c} + \frac{(be)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{12c^3} - \frac{(b^2e)\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{6c^2}$$

$$= -\frac{3bd\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{2c} + \frac{be\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^3} - \frac{bex^2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{6c} + dx(a+b\text{arcsinh}(cx))^{3/2} + \frac{1}{3}ex^3(a+b\text{arcsinh}(cx))^{3/2} + \frac{(3bd)\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8c} + \frac{(3bd)\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8c} + \frac{(be)\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}} - \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right) dx, x, a+b\text{arcsinh}(cx)\right)}{12c^3} - \frac{(be)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{6c^3}$$

$$= -\frac{3bd\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{2c} + \frac{be\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{3c^3} - \frac{bex^2\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{6c} + dx(a+b\operatorname{arcsinh}(cx))^{3/2} + \frac{1}{3}ex^3(a+b\operatorname{arcsinh}(cx))^{3/2} + \frac{(3bd)\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{4c} + \frac{(3bd)\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{4c} + \frac{(be)\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{48c^3} - \frac{(be)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{48c^3} - \frac{(be)\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{12c^3} - \frac{(be)\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{12c^3}$$

$$= -\frac{3bd\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{2c} + \frac{be\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^3} - \frac{bex^2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{6c} + dx(a+b\text{arcsinh}(cx))^{3/2} + \frac{1}{3}ex^3(a+b\text{arcsinh}(cx))^{3/2} + \frac{3b^{3/2}de^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c} + \frac{3b^{3/2}de^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c} - \frac{(be)\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{96c^3} - \frac{(be)\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{96c^3} + \frac{(be)\text{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{96c^3} + \frac{(be)\text{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{96c^3} - \frac{(be)\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{6c^3} - \frac{(be)\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{6c^3}$$

$$= -\frac{3bd\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{2c} + \frac{be\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^3} - \frac{bex^2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{6c} + dx(a+b\text{arcsinh}(cx))^{3/2} + \frac{1}{3}ex^3(a+b\text{arcsinh}(cx))^{3/2} + \frac{3b^{3/2}de^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c} - \frac{b^{3/2}ee^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{12c^3} + \frac{3b^{3/2}de^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c} - \frac{b^{3/2}ee^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{12c^3} + \frac{(be)\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{48c^3} - \frac{(be)\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{48c^3} - \frac{(be)\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{48c^3} + \frac{(be)\text{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{48c^3}$$

$$= -\frac{3bd\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{2c} + \frac{be\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^3} - \frac{bex^2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{6c} + dx(a+b\text{arcsinh}(cx))^{3/2} + \frac{1}{3}ex^3(a+b\text{arcsinh}(cx))^{3/2} + \frac{3b^{3/2}de^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c} - \frac{3b^{3/2}ee^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^3} + \frac{b^{3/2}ee^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{96c^3} + \frac{3b^{3/2}de^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c} - \frac{3b^{3/2}ee^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^3} + \frac{b^{3/2}ee^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{96c^3}$$

Mathematica [A] (verified)

Time = 2.29 (sec) , antiderivative size = 770, normalized size of antiderivative = 1.80

$$\int (d+ex^2)(a+b\operatorname{arcsinh}(cx))^{3/2}\,dx = \frac{ade^{-\frac{a}{b}}\sqrt{a+b\operatorname{arcsinh}(cx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{\sqrt{\frac{a}{b}+\operatorname{arcsinh}(cx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{\sqrt{-\frac{a+b\operatorname{arcsinh}(cx)}{b}}}\right)}{2c}$$
$$+\frac{aee^{-\frac{3a}{b}}\sqrt{a+b\operatorname{arcsinh}(cx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\operatorname{arcsinh}(cx)}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\operatorname{arcsinh}(cx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(cx)}\Gamma\left(\frac{3}{2},-\frac{3(\cdots}{}\right.\right.}{72c^3\sqrt{-\cdots}}$$
$$+\frac{\sqrt{b}d\left(4\sqrt{b}\sqrt{a+b\operatorname{arcsinh}(cx)}\left(-3\sqrt{1+c^2x^2}+2cx\operatorname{arcsinh}(cx)\right)+(2a+3b)\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)(\cosh\cdots\right.}{8c}$$
$$+\frac{\sqrt{b}e\left(-9\left(4\sqrt{b}\sqrt{a+b\operatorname{arcsinh}(cx)}\left(-3\sqrt{1+c^2x^2}+2cx\operatorname{arcsinh}(cx)\right)+(2a+3b)\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)\cdots\right.\right.}{}$$

```
[In] Integrate[(d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (a*d*Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]
])/Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-
((a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b)) + (a*e*Sqrt[a + b*ArcSinh[c*x]]*(
9*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, a/b + ArcSinh[c*x]
] + Sqrt[3]*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, (-3*(a + b*ArcSinh[c*x]))/b
] - 9*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, -((a + b*ArcSinh[c*x]
)/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, (3*(
a + b*ArcSinh[c*x]))/b]))/(72*c^3*E^((3*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])^2
/b^2)]) + (Sqrt[b]*d*(4*Sqrt[b]*Sqrt[a + b*ArcSinh[c*x]]*(-3*Sqrt[1 + c^2*x
^2] + 2*c*x*ArcSinh[c*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*
x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b
*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(8*c) + (Sqrt[b]*e*(-9*(4
*Sqrt[b]*Sqrt[a + b*ArcSinh[c*x]]*(-3*Sqrt[1 + c^2*x^2] + 2*c*x*ArcSinh[c*x
]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b]
 - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]
*(Cosh[a/b] + Sinh[a/b])) + (2*a + b)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*A
rcSinh[c*x]])/Sqrt[b]]*(Cosh[(3*a)/b] - Sinh[(3*a)/b]) + (-2*a + b)*Sqrt[3*
Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]]*(Cosh[(3*a)/b] + Sinh[(
3*a)/b]) + 12*Sqrt[b]*Sqrt[a + b*ArcSinh[c*x]]*(-Cosh[3*ArcSinh[c*x]] + 2*A
rcSinh[c*x]*Sinh[3*ArcSinh[c*x]])))/(288*c^3)
```

Maple [F]

$$\int \left(e\,x^{2}+d\right) \left(a+b\,\operatorname{arcsinh}\left(c x\right)\right)^{\frac{3}{2}}d x$$

```
[In] int((e*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x)
[Out] int((e*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (d+ex^2) (a+b\text{arcsinh}(cx))^{3/2} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (d+ex^2) (a+b\text{arcsinh}(cx))^{3/2} dx = \int \left(a+b\operatorname{asinh}\left(cx\right)\right)^{\frac{3}{2}}\left(d+ex^{2}\right)\, dx$$

```
[In] integrate((e*x**2+d)*(a+b*asinh(c*x))**(3/2),x)
[Out] Integral((a + b*asinh(c*x))**(3/2)*(d + e*x**2), x)
```

Maxima [F]

$$\int (d+ex^2) (a+b\text{arcsinh}(cx))^{3/2} dx = \int (ex^{2}+d)(b\operatorname{arsinh}\left(cx\right)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)*(b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int (d+ex^2)(a+b\operatorname{arcsinh}(cx))^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)(a+b\operatorname{arcsinh}(cx))^{3/2}\,dx = \int (a+b\operatorname{asinh}(c\,x))^{3/2}\,\left(e\,x^2+d\right)\,dx$$

```
[In] int((a + b*asinh(c*x))^(3/2)*(d + e*x^2),x)
```

```
[Out] int((a + b*asinh(c*x))^(3/2)*(d + e*x^2), x)
```

3.635 $\int (a+b\text{arcsinh}(cx))^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 135

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx = -\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{2c}+x(a+b\text{arcsinh}(cx))^{3/2} + \frac{3b^{3/2}e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c}+\frac{3b^{3/2}e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c}$$

```
[Out] x*(a+b*arcsinh(c*x))^(3/2)+3/8*b^(3/2)*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2
)/b^(1/2))*Pi^(1/2)/c+3/8*b^(3/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi
^(1/2)/c/exp(a/b)-3/2*b*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/c
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5772, 5798, 5774, 3388, 2211, 2236, 2235}

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx = \frac{3\sqrt{\pi}b^{3/2}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c} + \frac{3\sqrt{\pi}b^{3/2}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c} - \frac{3b\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{2c}+x(a+b\text{arcsinh}(cx))^{3/2}$$

```
[In] Int[(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(2*c) + x*(a + b*ArcSinh[
c*x])^(3/2) + (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt
[b]])/(8*c) + (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(
8*c*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
```

```
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = x(a+b\text{arcsinh}(cx))^{3/2} - \frac{1}{2}(3bc)\int \frac{x\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{1+c^2x^2}}\,dx$$

$$= -\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{2c} + x(a+b\text{arcsinh}(cx))^{3/2} + \frac{1}{4}(3b^2)\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx$$

$$= -\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{2c} + x(a+b\text{arcsinh}(cx))^{3/2} + \frac{(3b)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4c}$$

$$= -\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{2c} + x(a+b\text{arcsinh}(cx))^{3/2} + \frac{(3b)\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8c} + \frac{(3b)\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8c}$$

$$= -\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{2c} + x(a+b\text{arcsinh}(cx))^{3/2} + \frac{(3b)\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{4c} + \frac{(3b)\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{4c}$$

$$= -\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{2c} + x(a+b\text{arcsinh}(cx))^{3/2} + \frac{3b^{3/2}e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c} + \frac{3b^{3/2}e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.86

$$\int (a+b\operatorname{arcsinh}(cx))^{3/2}\,dx=\frac{ae^{-\frac{a}{b}}\sqrt{a+b\operatorname{arcsinh}(cx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{\sqrt{\frac{a}{b}+\operatorname{arcsinh}(cx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{\sqrt{-\frac{a+b\operatorname{arcsinh}(cx)}{b}}}\right)}{2c}+\frac{\sqrt{b}\left(4\sqrt{b}\sqrt{a+b\operatorname{arcsinh}(cx)}\left(-3\sqrt{1+c^2x^2}+2cx\operatorname{arcsinh}(cx)\right)+(2a+3b)\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)(\cosh}{8c}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]])
/Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-((
a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b)) + (Sqrt[b]*(4*Sqrt[b]*Sqrt[a + b*Ar
cSinh[c*x]]*(-3*Sqrt[1 + c^2*x^2] + 2*c*x*ArcSinh[c*x]) + (2*a + 3*b)*Sqrt[
Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a
+ 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b
])))/(8*c)
```

Maple [F]

$$\int (a+b\operatorname{arcsinh}(cx))^{\frac{3}{2}}\,dx$$

```
[In] int((a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (a+b\operatorname{arcsinh}(cx))^{3/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx = \int \left(a+b\,\text{asinh}\,(cx)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**(3/2),x)
[Out] Integral((a + b*asinh(c*x))**(3/2), x)
```

Maxima [F]

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx = \int (b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx = \int (b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx = \int (a+b\,\text{asinh}(c\,x))^{3/2}\,dx$$

```
[In] int((a + b*asinh(c*x))^(3/2),x)
[Out] int((a + b*asinh(c*x))^(3/2), x)
```

3.636 $\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2}\,dx=\text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2},x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2}\,dx=\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2}\,dx$$

```
[In] Int[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2),x]
```

```
[Out] Defer[Int][(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2}\,dx$$

Mathematica [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2}\,dx = \$\text{Aborted}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2),x]
[Out] $Aborted
```

Maple [N/A] (verified)

Not integrable

Time = 0.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\,\operatorname{arcsinh}(cx))^{\frac{3}{2}}}{e\,x^2+d}dx$$

```
[In] int((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x)
[Out] int((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 9.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2}\,dx = \int \frac{(a+b\operatorname{asinh}(cx))^{\frac{3}{2}}}{d+ex^2}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**(3/2)/(e*x**2+d),x)
[Out] Integral((a + b*asinh(c*x))**(3/2)/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b\text{arcsinh}(cx))^{3/2}}{d + ex^2} dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a + b\text{arcsinh}(cx))^{3/2}}{d + ex^2} dx = \int \frac{(b \operatorname{arsinh}(cx) + a)^{\frac{3}{2}}}{ex^2 + d} dx$$

```
[In] integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(3/2)/(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a + b\text{arcsinh}(cx))^{3/2}}{d + ex^2} dx = \int \frac{(a + b \operatorname{asinh}(c x))^{3/2}}{e x^2 + d} dx$$

```
[In] int((a + b*asinh(c*x))^(3/2)/(d + e*x^2),x)
```

```
[Out] int((a + b*asinh(c*x))^(3/2)/(d + e*x^2), x)
```

3.637 $\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+ex^2)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+ex^2)^2} dx = \text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+ex^2)^2}, x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+ex^2)^2} dx = \int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+ex^2)^2} dx$$

```
[In] Int[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2)^2,x]
```

```
[Out] Defer[Int][(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+ex^2)^2} dx$$

Mathematica [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arcsinh}(cx))^{3/2}}{(d + ex^2)^2} \, dx = \$\text{Aborted}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2)^2,x]
[Out] $Aborted
```

Maple [N/A] (verified)

Not integrable

Time = 0.84 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a + b \operatorname{arcsinh}(cx))^{\frac{3}{2}}}{(e\,x^2 + d)^2} dx$$

```
[In] int((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x)
[Out] int((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a + b\text{arcsinh}(cx))^{3/2}}{(d + ex^2)^2} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 95.97 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a + b\text{arcsinh}(cx))^{3/2}}{(d + ex^2)^2} \, dx = \int \frac{(a + b \operatorname{asinh}(cx))^{\frac{3}{2}}}{(d + ex^2)^2} \, dx$$

```
[In] integrate((a+b*asinh(c*x))**(3/2)/(e*x**2+d)**2,x)
[Out] Integral((a + b*asinh(c*x))**(3/2)/(d + e*x**2)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+ex^2)^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}{(ex^2+d)^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="maxima")
[Out] integrate((b*arcsinh(c*x) + a)^(3/2)/(e*x^2 + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+ex^2)^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}{(ex^2+d)^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^(3/2)/(e*x^2 + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+ex^2)^2}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^{3/2}}{(e\,x^2+d)^2}\,dx$$

```
[In] int((a + b*asinh(c*x))^(3/2)/(d + e*x^2)^2,x)
[Out] int((a + b*asinh(c*x))^(3/2)/(d + e*x^2)^2, x)
```

3.638 $\int \frac{(d+ex^2)^2}{\sqrt{a+b\,\mathrm{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 608

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx = \frac{d^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} - \frac{dee^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3} + \frac{e^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{16\sqrt{b}c^5} + \frac{dee^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3} - \frac{e^2e^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5} + \frac{e^2e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5} + \frac{d^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} - \frac{dee^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3} + \frac{e^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{16\sqrt{b}c^5} + \frac{dee^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3} - \frac{e^2e^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5} + \frac{e^2e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5}$$

```
[Out] 1/160*e^2*exp(5*a/b)*erf(5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*5^(1/2)*
Pi^(1/2)/c^5/b^(1/2)+1/160*e^2*erfi(5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2
))*5^(1/2)*Pi^(1/2)/c^5/exp(5*a/b)/b^(1/2)+1/12*d*e*exp(3*a/b)*erf(3^(1/2)*
(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/c^3/b^(1/2)+1/12*d*e*erf
i(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/c^3/exp(3*a/b)
/b^(1/2)+1/2*d^2*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c/
b^(1/2)-1/4*d*e*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c^3
/b^(1/2)+1/16*e^2*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c
^5/b^(1/2)+1/2*d^2*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c/exp(a/
b)/b^(1/2)-1/4*d*e*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c^3/exp(
a/b)/b^(1/2)+1/16*e^2*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c^5/e
xp(a/b)/b^(1/2)-1/32*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^
(1/2))*3^(1/2)*Pi^(1/2)/c^5/b^(1/2)-1/32*e^2*erfi(3^(1/2)*(a+b*arcsinh(c*x)
)^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/c^5/exp(3*a/b)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 608, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used

$= \{5793, 5774, 3388, 2211, 2236, 2235, 5780, 5556\}$

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx = \frac{\sqrt{\pi}e^2e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{16\sqrt{b}c^5} - \frac{\sqrt{3\pi}e^2e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5} + \frac{\sqrt{\frac{\pi}{5}}e^2e^{\frac{5a}{b}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5} + \frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{16\sqrt{b}c^5} - \frac{\sqrt{3\pi}e^2e^{-\frac{3a}{b}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5} + \frac{\sqrt{\frac{\pi}{5}}e^2e^{-\frac{5a}{b}}\operatorname{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5} - \frac{\sqrt{\pi}dee^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3} + \frac{\sqrt{\frac{\pi}{3}}dee^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3} - \frac{\sqrt{\pi}dee^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3} + \frac{\sqrt{\frac{\pi}{3}}dee^{-\frac{3a}{b}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3} + \frac{\sqrt{\pi}d^2e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} + \frac{\sqrt{\pi}d^2e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}$$

```
[In] Int[(d + e*x^2)^2/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (d^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c)
- (d*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*Sqrt[b]*c
^3) + (e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*Sqrt
[b]*c^5) + (d*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]
])/Sqrt[b]])/(4*Sqrt[b]*c^3) - (e^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqr
t[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5) + (e^2*E^((5*a)/b)*Sqrt[P
i/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5) + (d
^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c*E^(a/b)) -
 (d*e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*Sqrt[b]*c^3*E^(a/
b)) + (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*Sqrt[b]*c^5
*E^(a/b)) + (d*e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]
])/(4*Sqrt[b]*c^3*E^((3*a)/b)) - (e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*A
rcSinh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5*E^((3*a)/b)) + (e^2*Sqrt[Pi/5]*Erfi
[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5*E^((5*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{d^2}{\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2dex^2}{\sqrt{a+b\text{arcsinh}(cx)}}+\frac{e^2x^4}{\sqrt{a+b\text{arcsinh}(cx)}}\right)dx$$

$$=d^2\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx+(2de)\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx+e^2\int \frac{x^4}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx$$

$$=\frac{d^2\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc}+\frac{(2de)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc^3}+\frac{e^2\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{bc^5}$$

$$=\frac{d^2\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{2bc}+\frac{d^2\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{2bc}+\frac{(2de)\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}}-\frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)dx,x,a+b\text{arcsinh}(cx)\right)}{bc^3}+\frac{e^2\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16\sqrt{x}}-\frac{3\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16\sqrt{x}}+\frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{8\sqrt{x}}\right)dx,x,a+b\text{arcsinh}(cx)\right)}{bc^5}$$

$$= \frac{d^2 \text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{bc} + \frac{d^2 \text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{bc} + \frac{(de)\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2bc^3} - \frac{(de)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2bc^3} + \frac{e^2 \text{Subst}\left(\int \frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16bc^5} + \frac{e^2 \text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8bc^5} - \frac{(3e^2)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16bc^5}$$

$$
= \frac{d^2 e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\,\mathrm{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} + \frac{d^2 e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\,\mathrm{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}
- \frac{(de)\mathrm{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\,\mathrm{arcsinh}(cx)\right)}{4bc^3}
- \frac{(de)\mathrm{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\,\mathrm{arcsinh}(cx)\right)}{4bc^3}
+ \frac{(de)\mathrm{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\,\mathrm{arcsinh}(cx)\right)}{4bc^3}
+ \frac{(de)\mathrm{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\,\mathrm{arcsinh}(cx)\right)}{4bc^3}
+ \frac{e^2\mathrm{Subst}\left(\int \frac{e^{-i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\,\mathrm{arcsinh}(cx)\right)}{32bc^5}
+ \frac{e^2\mathrm{Subst}\left(\int \frac{e^{i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\,\mathrm{arcsinh}(cx)\right)}{32bc^5}
+ \frac{e^2\mathrm{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\,\mathrm{arcsinh}(cx)\right)}{16bc^5}
+ \frac{e^2\mathrm{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\,\mathrm{arcsinh}(cx)\right)}{16bc^5}
- \frac{(3e^2)\,\mathrm{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\,\mathrm{arcsinh}(cx)\right)}{32bc^5}
- \frac{(3e^2)\,\mathrm{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\,\mathrm{arcsinh}(cx)\right)}{32bc^5}
$$

$$= \frac{d^2 e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} + \frac{d^2 e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}$$

$$+ \frac{(de)\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{2bc^3}$$

$$- \frac{(de)\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{2bc^3}$$

$$- \frac{(de)\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{2bc^3}$$

$$+ \frac{(de)\text{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{2bc^3}$$

$$+ \frac{e^2\text{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{16bc^5}$$

$$+ \frac{e^2\text{Subst}\left(\int e^{-\frac{5a}{b}+\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{16bc^5}$$

$$+ \frac{e^2\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{8bc^5}$$

$$+ \frac{e^2\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{8bc^5}$$

$$- \frac{(3e^2)\,\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{16bc^5}$$

$$- \frac{(3e^2)\,\text{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{16bc^5}$$

$$
\begin{aligned}
&= \frac{d^2 e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} - \frac{dee^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3} \\
&+ \frac{e^2 e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{16\sqrt{b}c^5} + \frac{dee^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3} \\
&- \frac{e^2 e^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5} + \frac{e^2 e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5} \\
&+ \frac{d^2 e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} - \frac{dee^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3} \\
&+ \frac{e^2 e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{16\sqrt{b}c^5} + \frac{dee^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3} \\
&- \frac{e^2 e^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5} + \frac{e^2 e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 530, normalized size of antiderivative = 0.87

$$
\int \frac{(d+ex^2)^2}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx
$$

$$
= \frac{e^{-\frac{5a}{b}}\left(-30(8c^4d^2-4c^2de+e^2)\,e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\operatorname{arcsinh}(cx)\right)+3\sqrt{5}e^2\sqrt{-\frac{a+b\operatorname{arcsinh}(cx)}{b}}\,\Gamma\left(\frac{1}{2}\right.\right.}{}
$$

```
[In] Integrate[(d + e*x^2)^2/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (-30*(8*c^4*d^2 - 4*c^2*d*e + e^2)*E^((6*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gam
ma[1/2, a/b + ArcSinh[c*x]] + 3*Sqrt[5]*e^2*Sqrt[-((a + b*ArcSinh[c*x])/b)]
*Gamma[1/2, (-5*(a + b*ArcSinh[c*x]))/b] + 40*Sqrt[3]*c^2*d*e*E^((2*a)/b)*S
qrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] - 15
*Sqrt[3]*e^2*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a
+ b*ArcSinh[c*x]))/b] + 240*c^4*d^2*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])
/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)] - 120*c^2*d*e*E^((4*a)/b)*Sqrt[-
((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)] + 30*e^2*E^
((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])
/b)] - 40*Sqrt[3]*c^2*d*e*E^((8*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (
3*(a + b*ArcSinh[c*x]))/b] + 15*Sqrt[3]*e^2*E^((8*a)/b)*Sqrt[a/b + ArcSinh[
```

```
c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b] - 3*Sqrt[5]*e^2*E^((10*a)/b)*S
qrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (5*(a + b*ArcSinh[c*x]))/b])/(480*c^5*E^
((5*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F]

$$\int \frac{(e\,x^2+d)^2}{\sqrt{a+b\,\operatorname{arcsinh}\left(cx\right)}}dx$$

```
[In] int((e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x)
[Out] int((e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{(d+ex^2)^2}{\sqrt{a+b\operatorname{asinh}\left(cx\right)}}\,dx$$

```
[In] integrate((e*x**2+d)**2/(a+b*asinh(c*x))**(1/2),x)
[Out] Integral((d + e*x**2)**2/sqrt(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{(ex^2+d)^2}{\sqrt{b\operatorname{arsinh}\left(cx\right)+a}}\,dx$$

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^2/sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{(ex^2+d)^2}{\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2/sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{(e\,x^2+d)^2}{\sqrt{a+b\,\text{asinh}\,(c\,x)}}\,dx$$

```
[In] int((d + e*x^2)^2/(a + b*asinh(c*x))^(1/2),x)
[Out] int((d + e*x^2)^2/(a + b*asinh(c*x))^(1/2), x)
```

3.639 $\int \frac{d+ex^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 287

$$\int \frac{d+ex^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \frac{de^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} - \frac{ee^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} + \frac{ee^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} + \frac{de^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} - \frac{ee^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} + \frac{ee^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}$$

```
[Out] 1/24*e*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^
(1/2)/c^3/b^(1/2)+1/24*e*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(
1/2)*Pi^(1/2)/c^3/exp(3*a/b)/b^(1/2)+1/2*d*exp(a/b)*erf((a+b*arcsinh(c*x))^
(1/2)/b^(1/2))*Pi^(1/2)/c/b^(1/2)-1/8*e*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/
2)/b^(1/2))*Pi^(1/2)/c^3/b^(1/2)+1/2*d*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2
))*Pi^(1/2)/c/exp(a/b)/b^(1/2)-1/8*e*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))
*Pi^(1/2)/c^3/exp(a/b)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5793, 5774, 3388, 2211, 2236, 2235, 5780, 5556}

$$\int \frac{d+ex^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = -\frac{\sqrt{\pi}ee^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} + \frac{\sqrt{\frac{\pi}{3}}ee^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} - \frac{\sqrt{\pi}ee^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} + \frac{\sqrt{\frac{\pi}{3}}ee^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} + \frac{\sqrt{\pi}de^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} + \frac{\sqrt{\pi}de^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}$$

```
[In] Int[(d + e*x^2)/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c) -
(e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*Sqrt[b]*c^3)
+ (e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]]
)/(8*Sqrt[b]*c^3) + (d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*
Sqrt[b]*c*E^(a/b)) - (e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8
*Sqrt[b]*c^3*E^(a/b)) + (e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]
])/Sqrt[b]])/(8*Sqrt[b]*c^3*E^((3*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{d}{\sqrt{a + b\text{arcsinh}(cx)}} + \frac{ex^2}{\sqrt{a + b\text{arcsinh}(cx)}} \right) dx$$

$$=d\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}dx+e\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx$$

$$=\frac{d\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}dx,x,a+b\text{arcsinh}(cx)\right)}{bc}+\frac{e\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}dx,x,a+b\text{arcsinh}(cx)\right)}{bc^3}$$

$$=\frac{d\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}dx,x,a+b\text{arcsinh}(cx)\right)}{2bc}+\frac{d\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}dx,x,a+b\text{arcsinh}(cx)\right)}{2bc}+\frac{e\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}}-\frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)dx,x,a+b\text{arcsinh}(cx)\right)}{bc^3}$$

$$=\frac{d\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}dx,x,\sqrt{a+b\text{arcsinh}(cx)}\right)}{bc}+\frac{d\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}dx,x,\sqrt{a+b\text{arcsinh}(cx)}\right)}{bc}+\frac{e\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}dx,x,a+b\text{arcsinh}(cx)\right)}{4bc^3}-\frac{e\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}dx,x,a+b\text{arcsinh}(cx)\right)}{4bc^3}$$

$$=\frac{de^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{de^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}-\frac{e\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}dx,x,a+b\text{arcsinh}(cx)\right)}{8bc^3}-\frac{e\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}dx,x,a+b\text{arcsinh}(cx)\right)}{8bc^3}+\frac{e\text{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}dx,x,a+b\text{arcsinh}(cx)\right)}{8bc^3}+\frac{e\text{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}dx,x,a+b\text{arcsinh}(cx)\right)}{8bc^3}$$

$$= \frac{de^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} + \frac{de^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} + \frac{e\operatorname{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{4bc^3} - \frac{e\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{4bc^3} - \frac{e\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{4bc^3} + \frac{e\operatorname{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{4bc^3}$$

$$= \frac{de^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} - \frac{ee^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} + \frac{ee^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} + \frac{de^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} - \frac{ee^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} + \frac{ee^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.76

$$\int \frac{d+ex^2}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx = \frac{e^{-\frac{3a}{b}}\left(-3(4c^2d-e)\,e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\operatorname{arcsinh}(cx)\right)+\sqrt{3}e\sqrt{-\frac{a+b\operatorname{arcsinh}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3(a+b\operatorname{arcsi}}{b}\right.\right.}{24c^3}$$

```
[In] Integrate[(d + e*x^2)/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (-3*(4*c^2*d - e)*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + Arc
Sinh[c*x]] + Sqrt[3]*e*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a +
b*ArcSinh[c*x]))/b] + 3*(4*c^2*d - e)*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x
])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)] - Sqrt[3]*e*E^((6*a)/b)*Sqrt[a
/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b])/(24*c^3*E^((3*a)
/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F]

$$\int \frac{e\,x^{2}+d}{\sqrt{a+b\,\operatorname{arcsinh}\left(c x\right)}}dx$$

```
[In] int((e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int((e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{d+ex^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{d+ex^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{d+ex^{2}}{\sqrt{a+b\operatorname{asinh}\left(cx\right)}}\,dx$$

```
[In] integrate((e*x**2+d)/(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/sqrt(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{ex^{2}+d}{\sqrt{b\operatorname{arsinh}\left(cx\right)+a}}\,dx$$

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{d+ex^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{ex^2+d}{\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)/sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{e\,x^2+d}{\sqrt{a+b\,\text{asinh}\,(c\,x)}}\,dx$$

```
[In] int((d + e*x^2)/(a + b*asinh(c*x))^(1/2),x)
[Out] int((d + e*x^2)/(a + b*asinh(c*x))^(1/2), x)
```

3.640 $\int \frac{1}{\sqrt{a+b\mathbf{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 88

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\frac{e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}$$

```
[Out] 1/2*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c/b^(1/2)+1/2*e
rfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c/exp(a/b)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5774, 3388, 2211, 2236, 2235}

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\frac{\sqrt{\pi}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}$$

```
[In] Int[1/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c) + (S
qrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc}\\
&= \frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2bc}+\frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2bc}\\
&= \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{bc}+\frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{bc}\\
&= \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.15

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \frac{e^{-\frac{a}{b}}\left(-e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)+\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)}{2c\sqrt{a+b\text{arcsinh}(cx)}}$$

```
[In] Integrate[1/Sqrt[a + b*ArcSinh[c*x]],x]
[Out] (-(E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]]) + S
qrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)])/(2*c*
E^(a/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F]

$$\int \frac{1}{\sqrt{a+b\ \text{arcsinh}\,(cx)}}dx$$

```
[In] int(1/(a+b*arcsinh(c*x))^(1/2),x)
[Out] int(1/(a+b*arcsinh(c*x))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a + b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{a + b\,\text{asinh}\,(cx)}}\,dx$$

```
[In] integrate(1/(a+b*asinh(c*x))**(1/2),x)
[Out] Integral(1/sqrt(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a + b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{b\,\text{arsinh}\,(cx) + a}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a + b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{b\,\text{arsinh}\,(cx) + a}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a + b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{a + b\,\text{asinh}\,(c\,x)}}\,dx$$

```
[In] int(1/(a + b*asinh(c*x))^(1/2),x)
[Out] int(1/(a + b*asinh(c*x))^(1/2), x)
```

3.641 $\int \frac{1}{(d+ex^2)\sqrt{a+b\mathbf{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx = \operatorname{Int}\left(\frac{1}{(d+ex^2)\sqrt{a+b\operatorname{arcsinh}(cx)}},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx = \int \frac{1}{(d+ex^2)\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx$$

```
[In] Int[1/((d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]]),x]
[Out] Defer[Int][1/((d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(d+ex^2)\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{(d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx$$

```
[In] Integrate[1/((d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]]),x]
[Out] Integrate[1/((d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)\sqrt{a+b\,\text{arcsinh}\,(cx)}}dx$$

```
[In] int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x)
[Out] int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 2.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{a+b\,\text{asinh}\,(cx)}\,(d+ex^2)}\,dx$$

```
[In] integrate(1/(e*x**2+d)/(a+b*asinh(c*x))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*asinh(c*x))*(d + e*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{(ex^2+d)\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(1/((e*x^2 + d)*sqrt(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{(ex^2+d)\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)*sqrt(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{a+b\,\text{asinh}\,(c\,x)}\,\,(e\,x^2+d)}\,dx$$

```
[In] int(1/((a + b*asinh(c*x))^(1/2)*(d + e*x^2)),x)
[Out] int(1/((a + b*asinh(c*x))^(1/2)*(d + e*x^2)), x)
```

3.642 $\int \frac{1}{(d+ex^2)^2\sqrt{a+b\textbf{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\text{Int}\left(\frac{1}{(d+ex^2)^2\sqrt{a+b\text{arcsinh}(cx)}},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{1}{(d+ex^2)^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx$$

```
[In] Int[1/((d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]]),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]]), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(d+ex^2)^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx$$

```
[In] Integrate[1/((d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]]),x]
[Out] Integrate[1/((d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.90 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(e\,x^{2}+d\right)^{2}\sqrt{a+b\,\text{arcsinh}\left(cx\right)}}dx$$

```
[In] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x)
[Out] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 41.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{1}{\sqrt{a+b\,\text{asinh}\,(cx)}\,(d+ex^2)^2}\,dx$$

```
[In] integrate(1/(e*x**2+d)**2/(a+b*asinh(c*x))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*asinh(c*x))*(d + e*x**2)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{(ex^2+d)^2\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(1/((e*x^2 + d)^2*sqrt(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{(ex^2+d)^2\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)^2*sqrt(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{a+b\,\text{asinh}\,(c\,x)}\,(e\,x^2+d)^2}\,dx$$

```
[In] int(1/((a + b*asinh(c*x))^(1/2)*(d + e*x^2)^2),x)
[Out] int(1/((a + b*asinh(c*x))^(1/2)*(d + e*x^2)^2), x)
```

3.643 $\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 349

$$\begin{aligned}\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = &-\frac{2d\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}-\frac{2ex^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}\\
&-\frac{de^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{ee^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}\\
&-\frac{ee^{\frac{3a}{b}}\sqrt{3\pi}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{de^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}\\
&-\frac{ee^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{ee^{-\frac{3a}{b}}\sqrt{3\pi}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}\end{aligned}$$

```
[Out] -d*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c+1/4*e*
exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3+d*erfi(
(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c/exp(a/b)-1/4*e*erfi((a
+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3/exp(a/b)-1/4*e*exp(3*a
/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(3/2)/
c^3+1/4*e*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b
^(3/2)/c^3/exp(3*a/b)-2*d*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))^(1/2)-2*
e*x^2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5793, 5773, 5819, 3389, 2211, 2236, 2235, 5778}

$$\int \frac{d+ex^2}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \frac{\sqrt{\pi}ee^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} - \frac{\sqrt{3\pi}ee^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} - \frac{\sqrt{\pi}ee^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} + \frac{\sqrt{3\pi}ee^{-\frac{3a}{b}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} - \frac{\sqrt{\pi}de^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} + \frac{\sqrt{\pi}de^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} - \frac{2d\sqrt{c^2x^2+1}}{bc\sqrt{a+b\operatorname{arcsinh}(cx)}} - \frac{2ex^2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\operatorname{arcsinh}(cx)}}$$

```
[In] Int[(d + e*x^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (2*e*x^2*Sqrt[1 +
 c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a
+ b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c) + (e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a +
b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3) - (e*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(
Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^3) + (d*Sqrt[Pi]*E
rfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c*E^(a/b)) - (e*Sqrt[Pi]*Er
fi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3*E^(a/b)) + (e*Sqrt[3*P
i]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^3*E^((3*a
)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{d}{(a+b\text{arcsinh}(cx))^{3/2}}+\frac{ex^2}{(a+b\text{arcsinh}(cx))^{3/2}}\right)dx$$

$$= d\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx+e\int \frac{x^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$$

$$= -\frac{2d\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{2ex^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{(2cd)\int \frac{x}{\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{b} + \frac{(2e)\text{Subst}\left(\int \left(-\frac{3\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}} + \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^3}$$

$$= -\frac{2d\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{2ex^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{(2d)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c} + \frac{e\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}dx, x, a+b\text{arcsinh}(cx)\right)}{2b^2c^3} - \frac{(3e)\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}dx, x, a+b\text{arcsinh}(cx)\right)}{2b^2c^3}$$

$$= -\frac{2d\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{2ex^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{d\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c} + \frac{d\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c} + \frac{e\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^3} - \frac{e\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^3} - \frac{(3e)\text{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^3} + \frac{(3e)\text{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}dx, x, a+b\text{arcsinh}(cx)\right)}{4b^2c^3}$$

$$= -\frac{2d\sqrt{1+c^2x^2}}{bc\sqrt{a+b\operatorname{arcsinh}(cx)}} - \frac{2ex^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\operatorname{arcsinh}(cx)}} - \frac{(2d)\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{b^2c} + \frac{(2d)\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{b^2c} + \frac{e\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{2b^2c^3} - \frac{e\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{2b^2c^3} - \frac{(3e)\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{2b^2c^3} + \frac{(3e)\text{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{2b^2c^3}$$

$$= -\frac{2d\sqrt{1+c^2x^2}}{bc\sqrt{a+b\operatorname{arcsinh}(cx)}} - \frac{2ex^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\operatorname{arcsinh}(cx)}} - \frac{de^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} + \frac{ee^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} - \frac{ee^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} + \frac{de^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} - \frac{ee^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} + \frac{ee^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}$$

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.87

$$\int \frac{d+ex^2}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \frac{e^{-3\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}\left(\left(4c^2d-e\right)e^{\frac{4a}{b}+3\operatorname{arcsinh}(cx)}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\operatorname{arcsinh}(c\right.\right.}{}$$

```
[In] Integrate[(d + e*x^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] ((4*c^2*d - e)*E^((4*a)/b + 3*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]*Gamma[
1/2, a/b + ArcSinh[c*x]] + Sqrt[3]*e*E^(3*ArcSinh[c*x])*Sqrt[-((a + b*ArcSi
nh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] + (4*c^2*d - e)*E^((2*
```

```
a)/b + 3*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*
ArcSinh[c*x])/b)] + E^((3*a)/b)*(-((1 + E^(2*ArcSinh[c*x]))*(4*c^2*d*E^(2*A
rcSinh[c*x]) + e*(-1 + E^(2*ArcSinh[c*x]))^2)) + Sqrt[3]*e*E^(3*(a/b + ArcS
inh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b])
)/(4*b*c^3*E^(3*(a/b + ArcSinh[c*x]))*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F]

$$\int \frac{e\,x^{2}+d}{\left(a +b \,\operatorname{arcsinh}\left(c x \right)\right)^{\frac{3}{2}}}d x$$

```
[In] int((e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int((e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{d + e x^{2}}{\left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{\frac{3}{2}}}\, dx$$

```
[In] integrate((e*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/(a + b*asinh(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{ex^2+d}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)/(b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{ex^2+d}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)/(b*arcsinh(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{e\,x^2+d}{(a+b\,\text{asinh}\,(c\,x))^{3/2}}\,dx$$

```
[In] int((d + e*x^2)/(a + b*asinh(c*x))^(3/2),x)
[Out] int((d + e*x^2)/(a + b*asinh(c*x))^(3/2), x)
```

3.644 $\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 116

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}$$

```
[Out] -exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c+erfi((a+
b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c/exp(a/b)-2*(c^2*x^2+1)^(1
/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5773, 5819, 3389, 2211, 2236, 2235}

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = -\frac{\sqrt{\pi}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} + \frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} - \frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

```
[In] Int[(a + b*ArcSinh[c*x])^(-3/2),x]
```

```
[Out] (-2*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (E^(a/b)*Sqrt[Pi]*E
rf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c) + (Sqrt[Pi]*Erfi[Sqrt[a +
 b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{(2c)\int \frac{x}{\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{b}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{2\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c} + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{2\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c} + \frac{2\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.18

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \frac{e^{-\frac{a+b\text{arcsinh}(cx)}{b}}\left(-e^{a/b}\left(1+e^{2\text{arcsinh}(cx)}\right)+e^{\frac{2a}{b}+\text{arcsinh}(cx)}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\,\Gamma\left(\frac{1}{2},\right.\right.}{bc\sqrt{a+b\text{arcs}}}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-3/2),x]
```

```
[Out] (-(E^(a/b)*(1 + E^(2*ArcSinh[c*x]))) + E^((2*a)/b + ArcSinh[c*x])*Sqrt[a/b
+ ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + E^ArcSinh[c*x]*Sqrt[-((a +
 b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)])/(b*c*E^((a + b*
ArcSinh[c*x])/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F]

$$\int \frac{1}{(a+b\,\text{arcsinh}\,(cx))^{\frac{3}{2}}}dx$$

```
[In] int(1/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(1/(a+b*arcsinh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*asinh(c*x))**(3/2),x)
[Out] Integral((a + b*asinh(c*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(c*x) + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

```
[In] int(1/(a + b*asinh(c*x))^(3/2),x)
[Out] int(1/(a + b*asinh(c*x))^(3/2), x)
```

3.645 $\int \frac{1}{(d+ex^2)(a+b\mathbf{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^{3/2}},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^{3/2}}\,dx$$

```
[In] Int[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2)),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}}\,dx$$

```
[In] Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2)),x]
[Out] Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)\,(a+b\,\text{arcsinh}\,(cx))^{\frac{3}{2}}}dx$$

```
[In] int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
[Out] int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 7.80 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{1}{(a+b\,\text{asinh}\,(cx))^{\frac{3}{2}}\,(d+ex^2)}\,dx$$

```
[In] integrate(1/(e*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
[Out] Integral(1/((a + b*asinh(c*x))**(3/2)*(d + e*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(ex^2+d)(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((e*x^2 + d)*(b*arcsinh(c*x) + a)^(3/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(ex^2+d)(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)*(b*arcsinh(c*x) + a)^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 2.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))^{3/2}\,\,(e\,x^2+d)}\,dx$$

```
[In] int(1/((a + b*asinh(c*x))^(3/2)*(d + e*x^2)),x)
[Out] int(1/((a + b*asinh(c*x))^(3/2)*(d + e*x^2)), x)
```

3.646 $\int \frac{1}{(d+ex^2)^2(a+b\,\mathrm{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^2 (a+b\,\mathrm{arcsinh}(cx))^{3/2}}\,dx = \mathrm{Int}\left(\frac{1}{(d+ex^2)^2 (a+b\,\mathrm{arcsinh}(cx))^{3/2}},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex^2)^2 (a+b\,\mathrm{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(d+ex^2)^2 (a+b\,\mathrm{arcsinh}(cx))^{3/2}}\,dx$$

```
[In] Int[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^(3/2)),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(d+ex^2)^2 (a+b\,\mathrm{arcsinh}(cx))^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^2 (a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(d+ex^2)^2 (a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx$$

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^(3/2)),x]
[Out] Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.80 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)^2 (a+b\operatorname{arcsinh}(cx))^{\frac{3}{2}}}dx$$

```
[In] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
[Out] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)^2 (a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 139.90 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2 (a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(a+b\operatorname{asinh}(cx))^{\frac{3}{2}} (d+ex^2)^2}\,dx$$

```
[In] integrate(1/(e*x**2+d)**2/(a+b*asinh(c*x))**(3/2),x)
[Out] Integral(1/((a + b*asinh(c*x))**(3/2)*(d + e*x**2)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(ex^2+d)^2(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((e*x^2 + d)^2*(b*arcsinh(c*x) + a)^(3/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(ex^2+d)^2(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)^2*(b*arcsinh(c*x) + a)^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 2.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))^{3/2}\,(e\,x^2+d)^2}\,dx$$

```
[In] int(1/((a + b*asinh(c*x))^(3/2)*(d + e*x^2)^2),x)
[Out] int(1/((a + b*asinh(c*x))^(3/2)*(d + e*x^2)^2), x)
```

3.647 $\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx=\text{Int}\left(\sqrt{d+ex^2}(a+b\text{arcsinh}(cx)),x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx=\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx$$

```
[In] Int[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] Defer[Int][Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x]), x]
```

Rubi steps

$$\text{integral}=\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx=\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx$$

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x]),x]
[Out] Integrate[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.55 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int (a+b\,\text{arcsinh}\,(cx))\,\sqrt{e\,x^2+d}dx$$

```
[In] int((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x)
[Out] int((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx=\int \sqrt{ex^2+d}(b\,\text{arsinh}\,(cx)+a)\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx=\int (a+b\,\text{asinh}\,(cx))\,\sqrt{d+ex^2}\,dx$$

```
[In] integrate((a+b*asinh(c*x))*(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*asinh(c*x))*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx = \int \sqrt{ex^2+d}(b\,\text{arsinh}\,(cx)+a)\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 2.97 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx = \int (a+b\,\text{asinh}(c\,x))\,\sqrt{e\,x^2+d}\,dx$$

```
[In] int((a + b*asinh(c*x))*(d + e*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + e*x^2)^(1/2), x)
```

3.648 $\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx=\text{Int}\left(\frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx$$

```
[In] Int[(a + b*ArcSinh[c*x])/Sqrt[d + e*x^2],x]
```

```
[Out] Defer[Int][(a + b*ArcSinh[c*x])/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\text{integral}=\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/Sqrt[d + e*x^2],x]
[Out] Integrate[(a + b*ArcSinh[c*x])/Sqrt[d + e*x^2], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a+b\,\operatorname{arcsinh}\left(cx\right)}{\sqrt{e\,x^{2}+d}}dx$$

```
[In] int((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
[Out] int((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{b\,\operatorname{arsinh}\left(cx\right)+a}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*arcsinh(c*x) + a)/sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\operatorname{asinh}\left(cx\right)}{\sqrt{d+ex^2}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*asinh(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{d + ex^2}} dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{d + ex^2}} dx = \int \frac{b \operatorname{arsinh}(cx) + a}{\sqrt{ex^2 + d}} dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/sqrt(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 2.86 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{d + ex^2}} dx = \int \frac{a + b \operatorname{asinh}(c x)}{\sqrt{e x^2 + d}} dx$$

```
[In] int((a + b*asinh(c*x))/(d + e*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))/(d + e*x^2)^(1/2), x)
```

3.649 $\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 70

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{3/2}}\,dx = \frac{x(a+b\text{arcsinh}(cx))}{d\sqrt{d+ex^2}} - \frac{b\text{arctanh}\left(\frac{\sqrt{e}\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}}$$

```
[Out] -b*arctanh(e^(1/2)*(c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/2))/d/e^(1/2)+x*(a+b*ar
csinh(c*x))/d/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {197, 5792, 12, 455, 65, 223, 212}

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{3/2}}\,dx = \frac{x(a+b\text{arcsinh}(cx))}{d\sqrt{d+ex^2}} - \frac{b\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2+1}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcSinh[c*x]))/(d*Sqrt[d + e*x^2]) - (b*ArcTanh[(Sqrt[e]*Sqrt[1 +
 c^2*x^2])/(c*Sqrt[d + e*x^2])])/(d*Sqrt[e])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5792

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x(a+b\operatorname{arcsinh}(cx))}{d\sqrt{d+ex^2}} - (bc)\int \frac{x}{d\sqrt{1+c^2x^2}\sqrt{d+ex^2}}\,dx \\ &= \frac{x(a+b\operatorname{arcsinh}(cx))}{d\sqrt{d+ex^2}} - \frac{(bc)\int \frac{x}{\sqrt{1+c^2x^2}\sqrt{d+ex^2}}\,dx}{d} \\ &= \frac{x(a+b\operatorname{arcsinh}(cx))}{d\sqrt{d+ex^2}} - \frac{(bc)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{2d}\end{aligned}$$

$$= \frac{x(a+b\text{arcsinh}(cx))}{d\sqrt{d+ex^2}} - \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{d-\frac{e}{c^2}+\frac{ex^2}{c^2}}}dx, x, \sqrt{1+c^2x^2}\right)}{cd}$$

$$= \frac{x(a+b\text{arcsinh}(cx))}{d\sqrt{d+ex^2}} - \frac{b\text{Subst}\left(\int \frac{1}{1-\frac{ex^2}{c^2}}dx, x, \frac{\sqrt{1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{cd}$$

$$= \frac{x(a+b\text{arcsinh}(cx))}{d\sqrt{d+ex^2}} - \frac{b\text{arctanh}\left(\frac{\sqrt{e}\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{3/2}}dx = \frac{x\left(-bcx\sqrt{1+\frac{ex^2}{d}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,-c^2x^2,-\frac{ex^2}{d}\right)+2(a+b\text{arcsinh}(cx))\right)}{2d\sqrt{d+ex^2}}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2)^(3/2),x]
[Out] (x*(-(b*c*x*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x^2), -((e*x
^2)/d)]) + 2*(a + b*ArcSinh[c*x])))/(2*d*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{a+b\,\text{arcsinh}\,(cx)}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arcsinh(c*x))/(e*x^2+d)^(3/2),x)
[Out] int((a+b*arcsinh(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(60) = 120.

Time = 0.28 (sec) , antiderivative size = 326, normalized size of antiderivative = 4.66

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{3/2}}dx = \left[\frac{4\sqrt{ex^2+d}bex\log\left(cx+\sqrt{c^2x^2+1}\right)+4\sqrt{ex^2+d}aex+(bex^2+bd)\sqrt{e}\log\left(8c^4e^2\right.}{4(}\right.$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(e*x^2 + d)*b*e*x*log(c*x + sqrt(c^2*x^2 + 1)) + 4*sqrt(e*x^2 +
 d)*a*e*x + (b*e*x^2 + b*d)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e
 + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d + c*e)*sqrt(c^2*x^2 +
 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2))/(d*e^2*x^2 + d^2*e), 1/2*(2*sqrt(e*x^2
+ d)*b*e*x*log(c*x + sqrt(c^2*x^2 + 1)) + 2*sqrt(e*x^2 + d)*a*e*x + (b*e*x^
2 + b*d)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d + e)*sqrt(c^2*x^2 + 1)*sq
rt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 + c*d*e + (c^3*d*e + c*e^2)*x^2)))/(d*e
^2*x^2 + d^2*e)]
```

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{3/2}}\,dx=\int \frac{a+b\,\text{asinh}\,(cx)}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e-c^2*d>0)', see `assume?` for more
 detail
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{3/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex^2)^{3/2}}\,dx = \int \frac{a + b\,\mathrm{asinh}(c\,x)}{(e\,x^2 + d)^{3/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))/(d + e*x^2)^(3/2),x)
[Out] int((a + b*asinh(c*x))/(d + e*x^2)^(3/2), x)
```

3.650 $\int \frac{a+b\textbf{arcsinh}(cx)}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 146

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{5/2}}\,dx = -\frac{bc\sqrt{1+c^2x^2}}{3d\left(c^2d-e\right)\sqrt{d+ex^2}}+\frac{x(a+b\text{arcsinh}(cx))}{3d\left(d+ex^2\right)^{3/2}} +\frac{2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+ex^2}}-\frac{2b\text{arctanh}\left(\frac{\sqrt{e}\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{3d^2\sqrt{e}}$$

```
[Out] 1/3*x*(a+b*arcsinh(c*x))/d/(e*x^2+d)^(3/2)-2/3*b*arctanh(e^(1/2)*(c^2*x^2+1
)^(1/2)/c/(e*x^2+d)^(1/2))/d^2/e^(1/2)+2/3*x*(a+b*arcsinh(c*x))/d^2/(e*x^2+
d)^(1/2)-1/3*b*c*(c^2*x^2+1)^(1/2)/d/(c^2*d-e)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {198, 197, 5792, 12, 585, 79, 65, 223, 212}

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\text{arcsinh}(cx))}{3d\left(d+ex^2\right)^{3/2}} -\frac{2b\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2+1}}{c\sqrt{d+ex^2}}\right)}{3d^2\sqrt{e}}-\frac{bc\sqrt{c^2x^2+1}}{3d\left(c^2d-e\right)\sqrt{d+ex^2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] -1/3*(b*c*Sqrt[1 + c^2*x^2])/(d*(c^2*d - e)*Sqrt[d + e*x^2]) + (x*(a + b*Ar
cSinh[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x]))/(3*d^2*Sq
```

```
rt[d + e*x^2]) - (2*b*ArcTanh[(Sqrt[e]*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + e*x^2
])])/(3*d^2*Sqrt[e])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 5792

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(a+b\text{arcsinh}(cx))}{3d\,(d+ex^2)^{3/2}}+\frac{2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+ex^2}}-(bc)\int\frac{x(3d+2ex^2)}{3d^2\sqrt{1+c^2x^2}\,(d+ex^2)^{3/2}}\,dx\\
&= \frac{x(a+b\text{arcsinh}(cx))}{3d\,(d+ex^2)^{3/2}}+\frac{2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+ex^2}}-\frac{(bc)\int\frac{x(3d+2ex^2)}{\sqrt{1+c^2x^2}(d+ex^2)^{3/2}}\,dx}{3d^2}\\
&= \frac{x(a+b\text{arcsinh}(cx))}{3d\,(d+ex^2)^{3/2}}+\frac{2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+ex^2}}-\frac{(bc)\text{Subst}\left(\int\frac{3d+2ex}{\sqrt{1+c^2x}(d+ex)^{3/2}}\,dx,x,x^2\right)}{6d^2}\\
&= -\frac{bc\sqrt{1+c^2x^2}}{3d\,(c^2d-e)\,\sqrt{d+ex^2}}+\frac{x(a+b\text{arcsinh}(cx))}{3d\,(d+ex^2)^{3/2}}\\
&\quad+\frac{2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+ex^2}}-\frac{(bc)\text{Subst}\left(\int\frac{1}{\sqrt{1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{3d^2}\\
&= -\frac{bc\sqrt{1+c^2x^2}}{3d\,(c^2d-e)\,\sqrt{d+ex^2}}+\frac{x(a+b\text{arcsinh}(cx))}{3d\,(d+ex^2)^{3/2}}\\
&\quad+\frac{2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+ex^2}}-\frac{(2b)\text{Subst}\left(\int\frac{1}{\sqrt{d-\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx,x,\sqrt{1+c^2x^2}\right)}{3cd^2}\\
&= -\frac{bc\sqrt{1+c^2x^2}}{3d\,(c^2d-e)\,\sqrt{d+ex^2}}+\frac{x(a+b\text{arcsinh}(cx))}{3d\,(d+ex^2)^{3/2}}\\
&\quad+\frac{2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+ex^2}}-\frac{(2b)\text{Subst}\left(\int\frac{1}{1-\frac{ex^2}{c^2}}\,dx,x,\frac{\sqrt{1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{3cd^2}
\end{aligned}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{3d\left(c^2d-e\right)\sqrt{d+ex^2}} + \frac{x(a+b\text{arcsinh}(cx))}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+ex^2}} - \frac{2b\text{arctanh}\left(\frac{\sqrt{e}\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{3d^2\sqrt{e}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.95

$$\int \frac{a+b\text{arcsinh}(cx)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{-\frac{bcd\sqrt{1+c^2x^2}(d+ex^2)}{c^2d-e} + ax(3d+2ex^2) - bcx^2(d+ex^2)\sqrt{1+\frac{ex^2}{d}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\right.}{3d^2\left(d+ex^2\right)^{3/2}}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] (-((b*c*d*Sqrt[1 + c^2*x^2]*(d + e*x^2))/(c^2*d - e)) + a*x*(3*d + 2*e*x^2)
 - b*c*x^2*(d + e*x^2)*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x
^2), -((e*x^2)/d)] + b*x*(3*d + 2*e*x^2)*ArcSinh[c*x])/(3*d^2*(d + e*x^2)^(
3/2))
```

Maple [F]

$$\int \frac{a+b\,\text{arcsinh}\,(cx)}{\left(e\,x^2+d\right)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*arcsinh(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. 2(122) = 244.

Time = 0.32 (sec) , antiderivative size = 738, normalized size of antiderivative = 5.05

$$\int \frac{a+b\text{arcsinh}(cx)}{\left(d+ex^2\right)^{5/2}}\,dx = \left[\frac{\left(bc^2d^3+(bc^2de^2-be^3)x^4-bd^2e+2\left(bc^2d^2e-bde^2\right)x^2\right)\sqrt{e}\log\left(8\,c^4e^2x^4+c^4d^2+\right.}{}\right.$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*((b*c^2*d^3 + (b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e - b
*d*e^2)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e +
 c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d + c*e)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2
```

```
+ d)*sqrt(e) + e^2) + 2*(2*(b*c^2*d*e^2 - b*e^3)*x^3 + 3*(b*c^2*d^2*e - b*d
*e^2)*x)*sqrt(e*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(2*(a*c^2*d*e^2 -
 a*e^3)*x^3 + 3*(a*c^2*d^2*e - a*d*e^2)*x - (b*c*d*e^2*x^2 + b*c*d^2*e)*sqr
t(c^2*x^2 + 1))*sqrt(e*x^2 + d))/(c^2*d^5*e - d^4*e^2 + (c^2*d^3*e^3 - d^2*
e^4)*x^4 + 2*(c^2*d^4*e^2 - d^3*e^3)*x^2), 1/3*((b*c^2*d^3 + (b*c^2*d*e^2 -
 b*e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(-e)*arctan(1/2*
(2*c^2*e*x^2 + c^2*d + e)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e
^2*x^4 + c*d*e + (c^3*d*e + c*e^2)*x^2)) + (2*(b*c^2*d*e^2 - b*e^3)*x^3 + 3
*(b*c^2*d^2*e - b*d*e^2)*x)*sqrt(e*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) +
(2*(a*c^2*d*e^2 - a*e^3)*x^3 + 3*(a*c^2*d^2*e - a*d*e^2)*x - (b*c*d*e^2*x^2
 + b*c*d^2*e)*sqrt(c^2*x^2 + 1))*sqrt(e*x^2 + d))/(c^2*d^5*e - d^4*e^2 + (c
^2*d^3*e^3 - d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 - d^3*e^3)*x^2)]
```

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{5/2}}\,dx=\int \frac{a+b\operatorname{asinh}(cx)}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))/(e*x**2+d)**(5/2),x)
[Out] Integral((a + b*asinh(c*x))/(d + e*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{5/2}}\,dx=\int \frac{b\operatorname{arsinh}(cx)+a}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] 1/3*a*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + b*integrate(l
og(c*x + sqrt(c^2*x^2 + 1))/(e*x^2 + d)^(5/2), x)
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{5/2}}\,dx=\int \frac{b\operatorname{arsinh}(cx)+a}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex^2)^{5/2}}\,dx = \int \frac{a + b\,\text{asinh}(c\,x)}{(e\,x^2 + d)^{5/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))/(d + e*x^2)^(5/2),x)
[Out] int((a + b*asinh(c*x))/(d + e*x^2)^(5/2), x)
```

3.651 $\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{7/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 227

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{7/2}}\,dx = -\frac{bc\sqrt{1+c^2x^2}}{15d\,(c^2d-e)\,(d+ex^2)^{3/2}} - \frac{2bc(3c^2d-2e)\sqrt{1+c^2x^2}}{15d^2\,(c^2d-e)^2\sqrt{d+ex^2}} + \frac{x(a+b\text{arcsinh}(cx))}{5d\,(d+ex^2)^{5/2}} + \frac{4x(a+b\text{arcsinh}(cx))}{15d^2\,(d+ex^2)^{3/2}} + \frac{8x(a+b\text{arcsinh}(cx))}{15d^3\sqrt{d+ex^2}} - \frac{8b\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{15d^3\sqrt{e}}$$

```
[Out] 1/5*x*(a+b*arcsinh(c*x))/d/(e*x^2+d)^(5/2)+4/15*x*(a+b*arcsinh(c*x))/d^2/(e
*x^2+d)^(3/2)-8/15*b*arctanh(e^(1/2)*(c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/2))/d
^3/e^(1/2)-1/15*b*c*(c^2*x^2+1)^(1/2)/d/(c^2*d-e)/(e*x^2+d)^(3/2)+8/15*x*(a
+b*arcsinh(c*x))/d^3/(e*x^2+d)^(1/2)-2/15*b*c*(3*c^2*d-2*e)*(c^2*x^2+1)^(1/
2)/d^2/(c^2*d-e)^2/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used

$= \{198, 197, 5792, 12, 6847, 963, 79, 65, 223, 212\}$

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{7/2}}\,dx = \frac{8x(a+b\text{arcsinh}(cx))}{15d^3\sqrt{d+ex^2}} + \frac{4x(a+b\text{arcsinh}(cx))}{15d^2\,(d+ex^2)^{3/2}} + \frac{x(a+b\text{arcsinh}(cx))}{5d\,(d+ex^2)^{5/2}} - \frac{8b\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2+1}}{c\sqrt{d+ex^2}}\right)}{15d^3\sqrt{e}} - \frac{2bc\sqrt{c^2x^2+1}(3c^2d-2e)}{15d^2\,(c^2d-e)^2\sqrt{d+ex^2}} - \frac{bc\sqrt{c^2x^2+1}}{15d\,(c^2d-e)\,(d+ex^2)^{3/2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x^2)^(7/2),x]
```

```
[Out] -1/15*(b*c*Sqrt[1 + c^2*x^2])/(d*(c^2*d - e)*(d + e*x^2)^(3/2)) - (2*b*c*(3
*c^2*d - 2*e)*Sqrt[1 + c^2*x^2])/(15*d^2*(c^2*d - e)^2*Sqrt[d + e*x^2]) + (
x*(a + b*ArcSinh[c*x]))/(5*d*(d + e*x^2)^(5/2)) + (4*x*(a + b*ArcSinh[c*x])
)/(15*d^2*(d + e*x^2)^(3/2)) + (8*x*(a + b*ArcSinh[c*x]))/(15*d^3*Sqrt[d +
e*x^2]) - (8*b*ArcTanh[(Sqrt[e]*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(1
5*d^3*Sqrt[e])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 5792

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\text{integral} = \frac{x(a+b\text{arcsinh}(cx))}{5d\,(d+ex^2)^{5/2}} + \frac{4x(a+b\text{arcsinh}(cx))}{15d^2\,(d+ex^2)^{3/2}} + \frac{8x(a+b\text{arcsinh}(cx))}{15d^3\sqrt{d+ex^2}} - (bc)\int \frac{x(15d^2+20dex^2+8e^2x^4)}{15d^3\sqrt{1+c^2x^2}\,(d+ex^2)^{5/2}}\,dx$$

$$= \frac{x(a+b\text{arcsinh}(cx))}{5d\,(d+ex^2)^{5/2}} + \frac{4x(a+b\text{arcsinh}(cx))}{15d^2\,(d+ex^2)^{3/2}} + \frac{8x(a+b\text{arcsinh}(cx))}{15d^3\sqrt{d+ex^2}} - \frac{(bc)\int \frac{x(15d^2+20dex^2+8e^2x^4)}{\sqrt{1+c^2x^2}(d+ex^2)^{5/2}}\,dx}{15d^3}$$

$$= \frac{x(a+b\text{arcsinh}(cx))}{5d\,(d+ex^2)^{5/2}} + \frac{4x(a+b\text{arcsinh}(cx))}{15d^2\,(d+ex^2)^{3/2}} + \frac{8x(a+b\text{arcsinh}(cx))}{15d^3\sqrt{d+ex^2}} - \frac{(bc)\text{Subst}\left(\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{1+c^2x}(d+ex)^{5/2}}\,dx, x, x^2\right)}{30d^3}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{15d\,(c^2d-e)\,(d+ex^2)^{3/2}} + \frac{x(a+b\text{arcsinh}(cx))}{5d\,(d+ex^2)^{5/2}} + \frac{4x(a+b\text{arcsinh}(cx))}{15d^2\,(d+ex^2)^{3/2}} + \frac{8x(a+b\text{arcsinh}(cx))}{15d^3\sqrt{d+ex^2}} - \frac{(bc)\text{Subst}\left(\int \frac{3d(7c^2d-6e)+12(c^2d-e)ex}{\sqrt{1+c^2x}(d+ex)^{3/2}}\,dx, x, x^2\right)}{45d^3\,(c^2d-e)}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{15d\,(c^2d-e)\,(d+ex^2)^{3/2}} - \frac{2bc(3c^2d-2e)\sqrt{1+c^2x^2}}{15d^2\,(c^2d-e)^2\sqrt{d+ex^2}} + \frac{x(a+b\text{arcsinh}(cx))}{5d\,(d+ex^2)^{5/2}} + \frac{4x(a+b\text{arcsinh}(cx))}{15d^2\,(d+ex^2)^{3/2}} + \frac{8x(a+b\text{arcsinh}(cx))}{15d^3\sqrt{d+ex^2}} - \frac{(4bc)\text{Subst}\left(\int \frac{1}{\sqrt{1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{15d^3}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{15d\,(c^2d-e)\,(d+ex^2)^{3/2}} - \frac{2bc(3c^2d-2e)\sqrt{1+c^2x^2}}{15d^2\,(c^2d-e)^2\sqrt{d+ex^2}} + \frac{x(a+b\text{arcsinh}(cx))}{5d\,(d+ex^2)^{5/2}} + \frac{4x(a+b\text{arcsinh}(cx))}{15d^2\,(d+ex^2)^{3/2}} + \frac{8x(a+b\text{arcsinh}(cx))}{15d^3\sqrt{d+ex^2}} - \frac{(8b)\text{Subst}\left(\int \frac{1}{\sqrt{d-\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx, x, \sqrt{1+c^2x^2}\right)}{15cd^3}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{15d\,(c^2d-e)\,(d+ex^2)^{3/2}} - \frac{2bc(3c^2d-2e)\sqrt{1+c^2x^2}}{15d^2\,(c^2d-e)^2\sqrt{d+ex^2}} + \frac{x(a+b\text{arcsinh}(cx))}{5d\,(d+ex^2)^{5/2}} + \frac{4x(a+b\text{arcsinh}(cx))}{15d^2\,(d+ex^2)^{3/2}} + \frac{8x(a+b\text{arcsinh}(cx))}{15d^3\sqrt{d+ex^2}} - \frac{(8b)\text{Subst}\left(\int \frac{1}{1-\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{15cd^3}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{15d\,(c^2d-e)\,(d+ex^2)^{3/2}} - \frac{2bc(3c^2d-2e)\,\sqrt{1+c^2x^2}}{15d^2\,(c^2d-e)^2\,\sqrt{d+ex^2}} + \frac{x(a+b\text{arcsinh}(cx))}{5d\,(d+ex^2)^{5/2}} + \frac{4x(a+b\text{arcsinh}(cx))}{15d^2\,(d+ex^2)^{3/2}} + \frac{8x(a+b\text{arcsinh}(cx))}{15d^3\sqrt{d+ex^2}} - \frac{8b\text{arctanh}\left(\frac{\sqrt{e}\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{15d^3\sqrt{e}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.29 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.84

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{7/2}}\,dx = \frac{ax(15d^2+20dex^2+8e^2x^4) - \frac{bcd\sqrt{1+c^2x^2}(d+ex^2)(-e(5d+4ex^2)+c^2d(7d+6ex^2))}{(-c^2d+e)^2} - 4bcx^2(d+ \ldots}{15 \ldots}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2)^(7/2),x]
```

```
[Out] (a*x*(15*d^2 + 20*d*e*x^2 + 8*e^2*x^4) - (b*c*d*Sqrt[1 + c^2*x^2]*(d + e*x^
2)*(-(e*(5*d + 4*e*x^2)) + c^2*d*(7*d + 6*e*x^2)))/(-(c^2*d) + e)^2 - 4*b*c
*x^2*(d + e*x^2)^2*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x^2),
 -((e*x^2)/d)] + b*x*(15*d^2 + 20*d*e*x^2 + 8*e^2*x^4)*ArcSinh[c*x])/(15*d^
3*(d + e*x^2)^(5/2))
```

Maple [F]

$$\int \frac{a+b\,\text{arcsinh}\,(cx)}{(e\,x^2+d)^{\frac{7}{2}}}dx$$

```
[In] int((a+b*arcsinh(c*x))/(e*x^2+d)^(7/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(e*x^2+d)^(7/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 670 vs. 2(193) = 386.

Time = 0.39 (sec) , antiderivative size = 1354, normalized size of antiderivative = 5.96

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/15*(2*(b*c^4*d^5 - 2*b*c^2*d^4*e + (b*c^4*d^2*e^3 - 2*b*c^2*d*e^4 + b*e^
5)*x^6 + b*d^3*e^2 + 3*(b*c^4*d^3*e^2 - 2*b*c^2*d^2*e^3 + b*d*e^4)*x^4 + 3*
```

```
(b*c^4*d^4*e - 2*b*c^2*d^3*e^2 + b*d^2*e^3)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4
+ c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d
+ c*e)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + (8*(b*c^4*d^2*e^3
 - 2*b*c^2*d*e^4 + b*e^5)*x^5 + 20*(b*c^4*d^3*e^2 - 2*b*c^2*d^2*e^3 + b*d*e
^4)*x^3 + 15*(b*c^4*d^4*e - 2*b*c^2*d^3*e^2 + b*d^2*e^3)*x)*sqrt(e*x^2 + d)
*log(c*x + sqrt(c^2*x^2 + 1)) + (8*(a*c^4*d^2*e^3 - 2*a*c^2*d*e^4 + a*e^5)*
x^5 + 20*(a*c^4*d^3*e^2 - 2*a*c^2*d^2*e^3 + a*d*e^4)*x^3 + 15*(a*c^4*d^4*e
- 2*a*c^2*d^3*e^2 + a*d^2*e^3)*x - (7*b*c^3*d^4*e - 5*b*c*d^3*e^2 + 2*(3*b*
c^3*d^2*e^3 - 2*b*c*d*e^4)*x^4 + (13*b*c^3*d^3*e^2 - 9*b*c*d^2*e^3)*x^2)*sq
rt(c^2*x^2 + 1))*sqrt(e*x^2 + d))/(c^4*d^8*e - 2*c^2*d^7*e^2 + d^6*e^3 + (c
^4*d^5*e^4 - 2*c^2*d^4*e^5 + d^3*e^6)*x^6 + 3*(c^4*d^6*e^3 - 2*c^2*d^5*e^4
+ d^4*e^5)*x^4 + 3*(c^4*d^7*e^2 - 2*c^2*d^6*e^3 + d^5*e^4)*x^2), 1/15*(4*(b
*c^4*d^5 - 2*b*c^2*d^4*e + (b*c^4*d^2*e^3 - 2*b*c^2*d*e^4 + b*e^5)*x^6 + b*
d^3*e^2 + 3*(b*c^4*d^3*e^2 - 2*b*c^2*d^2*e^3 + b*d*e^4)*x^4 + 3*(b*c^4*d^4*
e - 2*b*c^2*d^3*e^2 + b*d^2*e^3)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^
2*d + e)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 + c*d*e +
(c^3*d*e + c*e^2)*x^2)) + (8*(b*c^4*d^2*e^3 - 2*b*c^2*d*e^4 + b*e^5)*x^5 +
20*(b*c^4*d^3*e^2 - 2*b*c^2*d^2*e^3 + b*d*e^4)*x^3 + 15*(b*c^4*d^4*e - 2*b*
c^2*d^3*e^2 + b*d^2*e^3)*x)*sqrt(e*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) +
(8*(a*c^4*d^2*e^3 - 2*a*c^2*d*e^4 + a*e^5)*x^5 + 20*(a*c^4*d^3*e^2 - 2*a*c^
2*d^2*e^3 + a*d*e^4)*x^3 + 15*(a*c^4*d^4*e - 2*a*c^2*d^3*e^2 + a*d^2*e^3)*x
 - (7*b*c^3*d^4*e - 5*b*c*d^3*e^2 + 2*(3*b*c^3*d^2*e^3 - 2*b*c*d*e^4)*x^4 +
 (13*b*c^3*d^3*e^2 - 9*b*c*d^2*e^3)*x^2)*sqrt(c^2*x^2 + 1))*sqrt(e*x^2 + d)
)/(c^4*d^8*e - 2*c^2*d^7*e^2 + d^6*e^3 + (c^4*d^5*e^4 - 2*c^2*d^4*e^5 + d^3
*e^6)*x^6 + 3*(c^4*d^6*e^3 - 2*c^2*d^5*e^4 + d^4*e^5)*x^4 + 3*(c^4*d^7*e^2
- 2*c^2*d^6*e^3 + d^5*e^4)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{(d + ex^2)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*asinh(c*x))/(e*x**2+d)**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex^2)^{7/2}}\,dx = \int \frac{b\operatorname{arsinh}(cx)+a}{(ex^2+d)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(7/2),x, algorithm="maxima")
[Out] 1/15*a*(8*x/(sqrt(e*x^2 + d)*d^3) + 4*x/((e*x^2 + d)^(3/2)*d^2) + 3*x/((e*x
^2 + d)^(5/2)*d)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(e*x^2 + d)^(7
/2), x)
```

Giac [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex^2)^{7/2}}\,dx = \int \frac{b\operatorname{arsinh}(cx)+a}{(ex^2+d)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(7/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/(e*x^2 + d)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex^2)^{7/2}}\,dx = \int \frac{a+b\,\operatorname{asinh}(c\,x)}{(e\,x^2+d)^{7/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))/(d + e*x^2)^(7/2),x)
[Out] int((a + b*asinh(c*x))/(d + e*x^2)^(7/2), x)
```

3.652 $\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx=\text{Int}\left(\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2,x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx=\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx$$

```
[In] Int[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2,x]
[Out] Defer[Int][Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\text{integral}=\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx$$

Mathematica [N/A]

Not integrable

Time = 13.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx=\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx$$

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2,x]
[Out] Integrate[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int (a+b\,\text{arcsinh}\,(cx))^2\,\sqrt{e\,x^2+d}dx$$

```
[In] int((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x)
[Out] int((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx=\int \sqrt{ex^2+d}(b\,\text{arsinh}\,(cx)+a)^2\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*sqrt(e*x^2 + d), x
)
```

Sympy [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx=\int (a+b\,\text{asinh}\,(cx))^2\,\sqrt{d+ex^2}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2*(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*asinh(c*x))**2*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx = \int \sqrt{ex^2+d}(b\,\text{arsinh}\,(cx)+a)^2\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,\sqrt{e\,x^2+d}\,dx$$

```
[In] int((a + b*asinh(c*x))^2*(d + e*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + e*x^2)^(1/2), x)
```

3.653 $\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+ex^2}}\,dx=\text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+ex^2}}\,dx=\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+ex^2}}\,dx$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/Sqrt[d + e*x^2],x]
```

```
[Out] Defer[Int][(a + b*ArcSinh[c*x])^2/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\text{integral}=\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+ex^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 8.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{\sqrt{d + ex^2}}\,dx = \int \frac{(a + b\text{arcsinh}(cx))^2}{\sqrt{d + ex^2}}\,dx$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/Sqrt[d + e*x^2],x]
[Out] Integrate[(a + b*ArcSinh[c*x])^2/Sqrt[d + e*x^2], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a + b \,\text{arcsinh}\left(c x\right)\right)^{2}}{\sqrt{e \,x^{2} + d}}dx$$

```
[In] int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
[Out] int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{\sqrt{d + ex^2}}\,dx = \int \frac{(b\,\text{arsinh}\left(cx\right) + a)^2}{\sqrt{ex^2 + d}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/sqrt(e*x^2 + d), x
)
```

Sympy [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+ex^2}}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))^2}{\sqrt{d+ex^2}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2/(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*asinh(c*x))**2/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/sqrt(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 2.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+ex^2}}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{\sqrt{e\,x^2+d}}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(d + e*x^2)^(1/2),x)
[Out] int((a + b*asinh(c*x))^2/(d + e*x^2)^(1/2), x)
```

3.654 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}\,dx = \text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}\,dx = \int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}\,dx$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}\,dx=\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}\,dx$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(3/2),x]
[Out] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.68 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\,\operatorname{arcsinh}(cx))^2}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x)
[Out] int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.45

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}\,dx=\int \frac{(b\,\operatorname{arsinh}(cx)+a)^2}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*sqrt(e*x^2 + d)/(e
^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [N/A]

Not integrable

Time = 4.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}\,dx = \int \frac{(a+b\operatorname{asinh}(cx))^2}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2/(e*x**2+d)**(3/2),x)
[Out] Integral((a + b*asinh(c*x))**2/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e-c^2*d>0)', see `assume?` for more
 detail
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/(e*x^2 + d)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}\,dx=\int \frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)^2}{\left(e\,x^2+d\right)^{3/2}}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(d + e*x^2)^(3/2),x)
[Out] int((a + b*asinh(c*x))^2/(d + e*x^2)^(3/2), x)
```

3.655 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}\,dx=\text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}\,dx=\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}\,dx$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(5/2),x]
```

```
[Out] Defer[Int][(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(5/2), x]
```

Rubi steps

$$\text{integral}=\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 5.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}\,dx=\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}\,dx$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(5/2),x]
[Out] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(5/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.83 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\ \text{arcsinh}\left(cx\right)\right)^{2}}{\left(e\,x^{2}+d\right)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x)
[Out] int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.95

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}\,dx=\int \frac{\left(b\,\text{arsinh}\left(cx\right)+a\right)^{2}}{\left(ex^{2}+d\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*sqrt(e*x^2 + d)/(e
^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

Sympy [N/A]

Not integrable

Time = 59.51 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}\,dx = \int \frac{(a+b\operatorname{asinh}(cx))^2}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2/(e*x**2+d)**(5/2),x)
[Out] Integral((a + b*asinh(c*x))**2/(d + e*x**2)**(5/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 99, normalized size of antiderivative = 4.50

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] 1/3*a^2*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + integrate(b
^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(e*x^2 + d)^(5/2) + 2*a*b*log(c*x + sqrt(
c^2*x^2 + 1))/(e*x^2 + d)^(5/2), x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/(e*x^2 + d)^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

```
[In] int((a + b*asinh(c*x))^2/(d + e*x^2)^(5/2),x)
[Out] int((a + b*asinh(c*x))^2/(d + e*x^2)^(5/2), x)
```

3.656 $\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}\,dx = \text{Int}\left(\frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}\,dx$$

```
[In] Int[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x]),x]
```

```
[Out] Defer[Int][Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x]), x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}\,dx$$

```
[In] Integrate[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x]),x]
[Out] Integrate[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.60 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{e\,x^{2}+d}}{a+b\,\operatorname{arcsinh}\left(c x\right)}dx$$

```
[In] int((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x)
[Out] int((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{ex^2+d}}{b\,\text{arsinh}\left(cx\right)+a}\,dx$$

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{d+ex^2}}{a+b\,\text{asinh}\left(cx\right)}\,dx$$

```
[In] integrate((e*x**2+d)**(1/2)/(a+b*asinh(c*x)),x)
[Out] Integral(sqrt(d + e*x**2)/(a + b*asinh(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{\sqrt{ex^2+d}}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{\sqrt{ex^2+d}}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 2.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{\sqrt{e\,x^2+d}}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

```
[In] int((d + e*x^2)^(1/2)/(a + b*asinh(c*x)),x)
[Out] int((d + e*x^2)^(1/2)/(a + b*asinh(c*x)), x)
```

3.657 $\int \frac{1}{\sqrt{d+ex^2}(a+b\,\mathbf{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\,\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{1}{\sqrt{d+ex^2}(a+b\,\text{arcsinh}(cx))},x\right)$$

```
[Out] Unintegrable(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\,\text{arcsinh}(cx))}\,dx = \int \frac{1}{\sqrt{d+ex^2}(a+b\,\text{arcsinh}(cx))}\,dx$$

```
[In] Int[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\sqrt{d+ex^2}(a+b\,\text{arcsinh}(cx))}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.64 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a+b\,\text{arcsinh}\,(cx))\,\sqrt{e\,x^2+d}}dx$$

```
[In] int(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
[Out] int(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{\sqrt{ex^2+d}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)/(a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(a+b\,\text{asinh}\,(cx))\,\sqrt{d+ex^2}}\,dx$$

```
[In] integrate(1/(a+b*asinh(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral(1/((a + b*asinh(c*x))*sqrt(d + e*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{\sqrt{ex^2+d}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{\sqrt{ex^2+d}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))\,\sqrt{e\,x^2+d}}\,dx$$

```
[In] int(1/((a + b*asinh(c*x))*(d + e*x^2)^(1/2)),x)
[Out] int(1/((a + b*asinh(c*x))*(d + e*x^2)^(1/2)), x)
```

3.658 $\int \frac{1}{(d+ex^2)^{3/2}(a+b\textbf{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)^{3/2}\,(a+b\text{arcsinh}(cx))},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(d+ex^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Int[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(d+ex^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(d+ex^2\right)^{3/2} (a+b\text{arcsinh}(cx))} \, dx=\int \frac{1}{\left(d+ex^2\right)^{3/2} (a+b\text{arcsinh}(cx))} \, dx$$

```
[In] Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\ \operatorname{arcsinh}\left(c x\right)\right)}d x$$

```
[In] int(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x)
[Out] int(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.86

$$\int \frac{1}{\left(d+ex^2\right)^{3/2} (a+b\text{arcsinh}(cx))} \, dx=\int \frac{1}{{\left(e x^{2} + d\right)}^{\frac{3}{2}} {\left(b \operatorname{arsinh}\left(c x\right) + a\right)}} \,d x$$

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)/(a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*
b*d*e*x^2 + b*d^2)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(a+b\,\text{asinh}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*x**2+d)**(3/2)/(a+b*asinh(c*x)),x)
[Out] Integral(1/((a + b*asinh(c*x))*(d + e*x**2)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(ex^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(1/((e*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(ex^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

```
[In] int(1/((a + b*asinh(c*x))*(d + e*x^2)^(3/2)),x)
[Out] int(1/((a + b*asinh(c*x))*(d + e*x^2)^(3/2)), x)
```

3.659 $\int \frac{1}{(d+ex^2)^{5/2}(a+b\,\mathbf{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)^{5/2}\,(a+b\text{arcsinh}(cx))},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(d+ex^2)^{5/2}\,(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Int[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(d+ex^2)^{5/2}\,(a+b\text{arcsinh}(cx))}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.78 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)^{\frac{5}{2}}\,(a+b\,\text{arcsinh}\,(cx))}dx$$

```
[In] int(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x)
[Out] int(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.95

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)/(a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3
 + (b*e^3*x^6 + 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 7.80 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))\,(d+ex^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(e*x**2+d)**(5/2)/(a+b*asinh(c*x)),x)
[Out] Integral(1/((a + b*asinh(c*x))*(d + e*x**2)**(5/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(1/((e*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(e\,x^2+d\right)}^{5/2}} \,dx$$

```
[In] int(1/((a + b*asinh(c*x))*(d + e*x^2)^(5/2)),x)
[Out] int(1/((a + b*asinh(c*x))*(d + e*x^2)^(5/2)), x)
```

3.660 $\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2} dx = \text{Int}\left(\frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2}, x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2} dx = \int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2} dx$$

```
[In] Int[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Defer[Int][Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2} dx$$

Mathematica [N/A]

Not integrable

Time = 3.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x])^2,x]
[Out] Integrate[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{e\,x^2+d}}{\left(a+b\ \text{arcsinh}\left(cx\right)\right)^2}dx$$

```
[In] int((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x)
[Out] int((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{ex^2+d}}{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}\,dx$$

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x
)
```

Sympy [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{d+ex^2}}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}\,dx$$

```
[In] integrate((e*x**2+d)**(1/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(sqrt(d + e*x**2)/(a + b*asinh(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 575, normalized size of antiderivative = 26.14

$$\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{ex^2+d}}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}\,dx$$

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -((c^2*x^2 + 1)^(3/2)*sqrt(e*x^2 + d) + (c^3*x^3 + c*x)*sqrt(e*x^2 + d))/(a
*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*
x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((2*c
^3*e*x^4 + c^3*d*x^2 - c*d)*(c^2*x^2 + 1)*sqrt(e*x^2 + d) + (4*c^4*e*x^5 +
2*(c^4*d + 2*c^2*e)*x^3 + (c^2*d + e)*x)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)
+ (2*c^5*e*x^6 + (c^5*d + 4*c^3*e)*x^4 + 2*(c^3*d + c*e)*x^2 + c*d)*sqrt(e*
x^2 + d))/(a*b*c^5*e*x^6 + (c^5*d + 2*c^3*e)*a*b*x^4 + (2*c^3*d + c*e)*a*b*
x^2 + a*b*c*d + (a*b*c^3*e*x^4 + a*b*c^3*d*x^2)*(c^2*x^2 + 1) + (b^2*c^5*e*
x^6 + (c^5*d + 2*c^3*e)*b^2*x^4 + (2*c^3*d + c*e)*b^2*x^2 + b^2*c*d + (b^2*
c^3*e*x^4 + b^2*c^3*d*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4*e*x^5 + b^2*c^2*d*x +
 (c^4*d + c^2*e)*b^2*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) +
 2*(a*b*c^4*e*x^5 + a*b*c^2*d*x + (c^4*d + c^2*e)*a*b*x^3)*sqrt(c^2*x^2 + 1
)), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{ex^2+d}}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.65 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{e\,x^2+d}}{(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

```
[In] int((d + e*x^2)^(1/2)/(a + b*asinh(c*x))^2,x)
[Out] int((d + e*x^2)^(1/2)/(a + b*asinh(c*x))^2, x)
```

3.661 $\int \frac{1}{\sqrt{d+ex^2}(a+b\,\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\operatorname{arcsinh}(cx))^2}\,dx = \operatorname{Int}\left(\frac{1}{\sqrt{d+ex^2}(a+b\operatorname{arcsinh}(cx))^2},x\right)$$

```
[Out] Unintegrable(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{1}{\sqrt{d+ex^2}(a+b\operatorname{arcsinh}(cx))^2}\,dx$$

```
[In] Int[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\sqrt{d+ex^2}(a+b\operatorname{arcsinh}(cx))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 6.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\,\text{arcsinh}\left(cx\right)\right)^2\sqrt{e\,x^2+d}}dx$$

```
[In] int(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
[Out] int(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.05

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{\sqrt{ex^2+d}(b\,\text{arsinh}\left(cx\right)+a)^2}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)/(a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arcsinh(c
*x)^2 + 2*(a*b*e*x^2 + a*b*d)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\operatorname{asinh}(cx))^2\sqrt{d+ex^2}}\,dx$$

```
[In] integrate(1/(a+b*asinh(c*x))**2/(e*x**2+d)**(1/2),x)
[Out] Integral(1/((a + b*asinh(c*x))**2*sqrt(d + e*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 591, normalized size of antiderivative = 26.86

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\sqrt{ex^2+d}(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)*a
*b*c^2*x + (sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)*b^2*c^2*x + (b^2*c^3*x^2 + b^
2*c)*sqrt(e*x^2 + d))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*
sqrt(e*x^2 + d)) + integrate((c^5*d*x^4 + 2*c^3*d*x^2 + (c^2*x^2 + 1)*((c^3
*d - 2*c*e)*x^2 - c*d) + c*d + sqrt(c^2*x^2 + 1)*(2*(c^4*d - c^2*e)*x^3 + (
c^2*d - e)*x))/((a*b*c^3*e*x^4 + a*b*c^3*d*x^2)*(c^2*x^2 + 1)*sqrt(e*x^2 +
d) + 2*(a*b*c^4*e*x^5 + a*b*c^2*d*x + (c^4*d + c^2*e)*a*b*x^3)*sqrt(c^2*x^2
 + 1)*sqrt(e*x^2 + d) + ((b^2*c^3*e*x^4 + b^2*c^3*d*x^2)*(c^2*x^2 + 1)*sqrt
(e*x^2 + d) + 2*(b^2*c^4*e*x^5 + b^2*c^2*d*x + (c^4*d + c^2*e)*b^2*x^3)*sqr
t(c^2*x^2 + 1)*sqrt(e*x^2 + d) + (b^2*c^5*e*x^6 + (c^5*d + 2*c^3*e)*b^2*x^4
 + (2*c^3*d + c*e)*b^2*x^2 + b^2*c*d)*sqrt(e*x^2 + d))*log(c*x + sqrt(c^2*x
^2 + 1)) + (a*b*c^5*e*x^6 + (c^5*d + 2*c^3*e)*a*b*x^4 + (2*c^3*d + c*e)*a*b
*x^2 + a*b*c*d)*sqrt(e*x^2 + d)), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\sqrt{ex^2+d}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))^2\,\sqrt{e\,x^2+d}}\,dx$$

```
[In] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(1/2)),x)
[Out] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(1/2)), x)
```

3.662 $\int \frac{1}{(d+ex^2)^{3/2}(a+b\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}\,dx = \operatorname{Int}\left(\frac{1}{(d+ex^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{1}{(d+ex^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}\,dx$$

```
[In] Int[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(d+ex^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 13.99 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^{3/2} (a+b\text{arcsinh}(cx))^2} dx = \int \frac{1}{(d+ex^2)^{3/2} (a+b\text{arcsinh}(cx))^2} dx$$

```
[In] Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)^{\frac{3}{2}} (a+b\,\text{arcsinh}\,(cx))^2} dx$$

```
[In] int(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x)
[Out] int(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 4.91

$$\int \frac{1}{(d+ex^2)^{3/2} (a+b\text{arcsinh}(cx))^2} dx = \int \frac{1}{(ex^2+d)^{\frac{3}{2}} (b\,\text{arsinh}\,(cx)+a)^2} dx$$

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)/(a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2*
x^4 + 2*b^2*d*e*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e*
x^2 + a*b*d^2)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 4.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))^2\,(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*x**2+d)**(3/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(1/((a + b*asinh(c*x))**2*(d + e*x**2)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 864, normalized size of antiderivative = 39.27

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/((a*b*c^2*e*x^3 + a*b*c^2*d*x)*sqrt(
c^2*x^2 + 1)*sqrt(e*x^2 + d) + ((b^2*c^2*e*x^3 + b^2*c^2*d*x)*sqrt(c^2*x^2
+ 1)*sqrt(e*x^2 + d) + (b^2*c^3*e*x^4 + (c^3*d + c*e)*b^2*x^2 + b^2*c*d)*sq
rt(e*x^2 + d))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*e*x^4 + (c^3*d + c*e
)*a*b*x^2 + a*b*c*d)*sqrt(e*x^2 + d)) - integrate((2*c^5*e*x^6 - (c^5*d - 4
*c^3*e)*x^4 - 2*(c^3*d - c*e)*x^2 + (2*c^3*e*x^4 - (c^3*d - 4*c*e)*x^2 + c*
d)*(c^2*x^2 + 1) - c*d + (4*c^4*e*x^5 - 2*(c^4*d - 4*c^2*e)*x^3 - (c^2*d -
3*e)*x)*sqrt(c^2*x^2 + 1))/((a*b*c^3*e^2*x^6 + 2*a*b*c^3*d*e*x^4 + a*b*c^3*
d^2*x^2)*(c^2*x^2 + 1)*sqrt(e*x^2 + d) + 2*(a*b*c^4*e^2*x^7 + (2*c^4*d*e +
c^2*e^2)*a*b*x^5 + a*b*c^2*d^2*x + (c^4*d^2 + 2*c^2*d*e)*a*b*x^3)*sqrt(c^2*
x^2 + 1)*sqrt(e*x^2 + d) + ((b^2*c^3*e^2*x^6 + 2*b^2*c^3*d*e*x^4 + b^2*c^3*
d^2*x^2)*(c^2*x^2 + 1)*sqrt(e*x^2 + d) + 2*(b^2*c^4*e^2*x^7 + (2*c^4*d*e +
c^2*e^2)*b^2*x^5 + b^2*c^2*d^2*x + (c^4*d^2 + 2*c^2*d*e)*b^2*x^3)*sqrt(c^2*
x^2 + 1)*sqrt(e*x^2 + d) + (b^2*c^5*e^2*x^8 + 2*(c^5*d*e + c^3*e^2)*b^2*x^6
 + (c^5*d^2 + 4*c^3*d*e + c*e^2)*b^2*x^4 + b^2*c*d^2 + 2*(c^3*d^2 + c*d*e)*
b^2*x^2)*sqrt(e*x^2 + d))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*e^2*x^8 +
 2*(c^5*d*e + c^3*e^2)*a*b*x^6 + (c^5*d^2 + 4*c^3*d*e + c*e^2)*a*b*x^4 + a*
b*c*d^2 + 2*(c^3*d^2 + c*d*e)*a*b*x^2)*sqrt(e*x^2 + d)), x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.79 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))^2\,(e\,x^2+d)^{3/2}}\,dx$$

```
[In] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(3/2)),x)
[Out] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(3/2)), x)
```

3.663 $\int \frac{1}{(d+ex^2)^{5/2}(a+b\,\mathrm{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\,\mathrm{arcsinh}(cx))^2}\,dx=\mathrm{Int}\left(\frac{1}{(d+ex^2)^{5/2}\,(a+b\,\mathrm{arcsinh}(cx))^2},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\,\mathrm{arcsinh}(cx))^2}\,dx=\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\,\mathrm{arcsinh}(cx))^2}\,dx$$

```
[In] Int[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\,\mathrm{arcsinh}(cx))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 22.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)^{\frac{5}{2}}\,(a+b\,\text{arcsinh}\,(cx))^2}dx$$

```
[In] int(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x)
[Out] int(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 6.77

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)/(a^2*e^3*x^6 + 3*a^2*d*e^2*x^4 + 3*a^2*d^2*e*x^2 +
 a^2*d^3 + (b^2*e^3*x^6 + 3*b^2*d*e^2*x^4 + 3*b^2*d^2*e*x^2 + b^2*d^3)*arcs
inh(c*x)^2 + 2*(a*b*e^3*x^6 + 3*a*b*d*e^2*x^4 + 3*a*b*d^2*e*x^2 + a*b*d^3)*
arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 27.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{(a+b\,\text{asinh}\,(cx))^2\,(d+ex^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(e*x**2+d)**(5/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(1/((a + b*asinh(c*x))**2*(d + e*x**2)**(5/2)), x)
```

Maxima [N/A]

Not integrable

Time = 1.91 (sec) , antiderivative size = 1123, normalized size of antiderivative = 51.05

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/((a*b*c^2*e^2*x^5 + 2*a*b*c^2*d*e*x^
3 + a*b*c^2*d^2*x)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + ((b^2*c^2*e^2*x^5 +
2*b^2*c^2*d*e*x^3 + b^2*c^2*d^2*x)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + (b^2
*c^3*e^2*x^6 + (2*c^3*d*e + c*e^2)*b^2*x^4 + b^2*c*d^2 + (c^3*d^2 + 2*c*d*e
)*b^2*x^2)*sqrt(e*x^2 + d))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*e^2*x^6
 + (2*c^3*d*e + c*e^2)*a*b*x^4 + a*b*c*d^2 + (c^3*d^2 + 2*c*d*e)*a*b*x^2)*s
qrt(e*x^2 + d)) - integrate((4*c^5*e*x^6 - (c^5*d - 8*c^3*e)*x^4 - 2*(c^3*d
 - 2*c*e)*x^2 + (4*c^3*e*x^4 - (c^3*d - 6*c*e)*x^2 + c*d)*(c^2*x^2 + 1) - c
*d + (8*c^4*e*x^5 - 2*(c^4*d - 7*c^2*e)*x^3 - (c^2*d - 5*e)*x)*sqrt(c^2*x^2
 + 1))/((a*b*c^3*e^3*x^8 + 3*a*b*c^3*d*e^2*x^6 + 3*a*b*c^3*d^2*e*x^4 + a*b*
c^3*d^3*x^2)*(c^2*x^2 + 1)*sqrt(e*x^2 + d) + 2*(a*b*c^4*e^3*x^9 + (3*c^4*d*
e^2 + c^2*e^3)*a*b*x^7 + a*b*c^2*d^3*x + 3*(c^4*d^2*e + c^2*d*e^2)*a*b*x^5
+ (c^4*d^3 + 3*c^2*d^2*e)*a*b*x^3)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + ((b^
2*c^3*e^3*x^8 + 3*b^2*c^3*d*e^2*x^6 + 3*b^2*c^3*d^2*e*x^4 + b^2*c^3*d^3*x^2
)*(c^2*x^2 + 1)*sqrt(e*x^2 + d) + 2*(b^2*c^4*e^3*x^9 + (3*c^4*d*e^2 + c^2*e
^3)*b^2*x^7 + b^2*c^2*d^3*x + 3*(c^4*d^2*e + c^2*d*e^2)*b^2*x^5 + (c^4*d^3
+ 3*c^2*d^2*e)*b^2*x^3)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + (b^2*c^5*e^3*x^
10 + (3*c^5*d*e^2 + 2*c^3*e^3)*b^2*x^8 + (3*c^5*d^2*e + 6*c^3*d*e^2 + c*e^3
)*b^2*x^6 + (c^5*d^3 + 6*c^3*d^2*e + 3*c*d*e^2)*b^2*x^4 + b^2*c*d^3 + (2*c^
3*d^3 + 3*c*d^2*e)*b^2*x^2)*sqrt(e*x^2 + d))*log(c*x + sqrt(c^2*x^2 + 1)) +
 (a*b*c^5*e^3*x^10 + (3*c^5*d*e^2 + 2*c^3*e^3)*a*b*x^8 + (3*c^5*d^2*e + 6*c
^3*d*e^2 + c*e^3)*a*b*x^6 + (c^5*d^3 + 6*c^3*d^2*e + 3*c*d*e^2)*a*b*x^4 + a
*b*c*d^3 + (2*c^3*d^3 + 3*c*d^2*e)*a*b*x^2)*sqrt(e*x^2 + d)), x)
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{(a+b\,\text{asinh}\,(c\,x))^2\,(e\,x^2+d)^{5/2}}\,dx$$

```
[In] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(5/2)),x)
[Out] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(5/2)), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                  end if
              else #result contains complex but optimal is not
                  if debug then
                          print("result contains complex but optimal is not");
                  fi;
                  return "C","Result contains complex when optimal does not.";
              fi;
          else # result do not contain complex
               # this assumes optimal do not as well. No check is needed here.
              if debug then
                     print("result do not contain complex, this assumes optimal do not as well")
              fi;
              if leaf_count_result<=2*leaf_count_optimal then
                  if debug then
                      print("leaf_count_result<=2*leaf_count_optimal");
                  fi;
                  return "A"," ";
              else
                  if debug then
                      print("leaf_count_result>2*leaf_count_optimal");
                  fi;
                  return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
              fi;
          fi;
      else #ExpnType(result) > ExpnType(optimal)
          if debug then
              print("ExpnType(result) > ExpnType(optimal)");
          fi;
          return "C",cat("Result contains higher order function than in optimal. Order ",
                         convert(ExpnType_result,string)," vs. order ",
                         convert(ExpnType_optimal,string),".");
      fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```